THE ROLE OF THE FACILITATOR IN MATHEMATICS TEACHER LEARNING: RECOGNITION, DEBT, GRIEVANCE AND REFUSAL IN SOCIO-ECOLOGICAL RELATIONS

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In this conceptual report, we develop ideas relating to a socio-ecological approach to mathematics teacher professional learning, asking, what is the role of the facilitator? We focus on the related concepts of recognition, debt, grievance, and refusal, in socio-ecological relations. We conceptualise by thinking from place, a process of reading diffractively through scholarship and narrative interviews with teaching facilitators from two geopolitical South contexts. We theorise the facilitator role as creating and holding relational, communal spaces involving multiple ways of knowing, being and teaching. These spaces function as grievances: for recognising inequities and our needing one another as qualitative debt; and for confronting inequities in refusal. We offer our theorising to others for diffractive reading in their thinking from place.

INTRODUCTION

Socio-ecology is emerging as a meeting point for work in mathematics education that addresses questions of environmental sustainability, socio-ecological justice, and futures. While scholars enter and contribute to this work in multiple ways, a guiding, overarching question is: What might be the role of mathematics (education) in relation to socio-ecological challenges, and what might these challenges mean for mathematics (education)? Our primary aim in this conceptual report is to offer ideas on the role of the *teaching facilitator* (commonly 'mathematics teacher educator') in mathematics teacher professional learning, preservice and practising. Prediger et al. (2019) identify this as a growing area of interest within mathematics education broadly. Yet, as discussed below, we have found few studies developing ideas on this role when facilitating socio-ecologically framed concerns. Specifically, we pursue the potential of four related concepts: recognition, debt, grievance, and refusal.

Our secondary aim is to offer our process of theorising, the verb signalling our view of theory as 'ongoingness', rather than fixed closings (Haraway, 2016). In our collaboration, we have come to appreciate the inter-action of place and conceptual ideas, that is, the need to learn *from* and create *from* situated stories and histories (Haraway, 2015). The preposition 'from' emphasises our seeing the need to also engage complex, planetary-scale relations constituting our world (Coles et al., 2024), while not making claims to generalisability and universality. In this paper we think from place by reading diffractively (Barad, 2007) through stories of two forms: scholarship, primarily decolonial, ecofeminist, new materialist, and land-based

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education thought; and narrative interviews (Petitmengen, 2006) with teaching facilitators working with teachers in two geopolitical South contexts.

LITERATURE REVIEW AND THEORETICAL IDEAS

A socio-ecological approach to mathematics (teacher) education

We begin by summarising our current thinking around the use of nouns and connectives in the idea 'socio-ecology *and* mathematics education' (e.g. Coles et al., 2024; Makramalla et al., 2025). Ontologically, we consider the world and subjects therein – such as a teacher, mathematics, a learning activity, a river – as entangled social *and* ecological relations, and hence needing one another (Haraway, 2016; Velasco, 2017). They are constituted in relational processes that are biophysical, historical, cultural, discursive, symbolic, material, temporal, spatial, economic, political, and spiritual. Epistemologically, then, we view as agents of knowing *all* humans (living and ancestral, individuals and collectives) *and* the more-than-human (living and inanimate). A plurality of knowers, in relation, entails multiple ways of knowing, and knowing *in* relation, a notion we develop below. This view contrasts with thinking that gives mathematics (education) pride of place in knowledge hierarchies, on account of mathematics' 'completeness' and internal consistency, and ability to abstract from relations with the world. Yet, it is these inequitable social-ecological relations that have resulted in our precarious presents and futures (Escobar, 2006).

Within our socio-ecological thinking, our concern in this report is theorising the role of teaching facilitators acting in a world constituted in socio-ecological relations between multiple knowers and knowledges. We have identified empirical mathematics education work that describes this role as: raising teachers' awareness of the socio-ecological dimensions of mathematics education; supporting teachers' design of classroom tasks; and bring multiple knowers into relation. Methodologically, this work may involve facilitators analysing their own practice, or research on others' practice using as lenses: ideas from Critical Mathematics Education (e.g. Coles & Helliwell, 2023; Helliwell & Ng, 2022); principles of Education for Sustainable Development and reflective learning (e.g. Alsina & Mulà, 2019); or notions of community and power from socio-cultural and socio-political thought (e.g. Steffensen, 2025).

Recognition, debt, grievance, and refusal in mathematics (teacher) education

The words *recognition*, *debt*, *grievance* and *refusal* are not common in mathematics education, and if used, take on specific meanings within the unequal socio-ecological relations we seek to challenge. An example of a meaning we challenge is the idea that we 'recognise' a subject by using mathematics as a (neutral) tool to 'gaze' upon or model a phenomenon, bringing it into existence. This includes using mathematics to quantify 'debt' and 'wealth' in terms of things and people (Mbembe, 2018). Examples are calculating carbon offsetting and climate adaptation finance as debt. Here, mathematics' role is recognising "you are here" (Mbembe, 2021, p. 228) in neocolonial arenas founded in separation, boundaries, extraction, and inequitable socio-ecological

relations. The role of the teaching facilitator would be supporting teachers to design activities around such calculations.

In our socio-ecological thinking, we follow Mbembe (2018, 2021, 2023) in viewing our becoming-with in relation, as a form of *debt*. This debt is founded in qualitative wealth as relations with people and with the more-than-human, in our common past, present and future, and in our needing one another. *Recognition*, involves knowing in relation: knowing that recognises the debt of our constituting relations. This is a new form of "planetary consciousness" as recognising in unequal socio-ecological relations "that something has been broken and striving to put it back together" (Mbembe, 2023, p. 8). Thus, viewing our becoming-with in relation as debt includes "inheriting hard histories, for everybody, but not equally and in the same ways" (Haraway, 2016, p. 89), and "grieving with" to recognise what has changed (p. 38). The recognition comes with particular obligations, including for mathematics education, towards "justice and care" (Mbembe, 2023, p. 8) and the responsibility to develop new relations.

The word *grieving* echoes with Velasco's (2017) notion of *grievance*. Based on her studies in the Atoyac River Basin in Mexico, a place we think from in this report, she sees a grievance as a collective process – involving multiple knowers and knowledges – of recognising *and* confronting the relations that constitute a progressive loss of the now highly-polluted river, and the biophysical, economic, material, social, and symbolic relations that communities have with it. It is a complex process. While science may recognise socio-ecological devastations, the responses of individuals living the inequitable relations may seem contradictory – sometimes actively organising to defend their territory, but at other times denying the problem as a means of economic survival. This process may involve *refusal*, not simply as an act of disengagement, which in mathematics education would likely be seen as individual deficit and deviance (Valero, 2007). But it is a motivated, strategic and generative act, with possibilities of new relations (McGranahan, 2016).

METHODOLOGY FOR THEORISING FROM PLACE

Our thinking from place involves a diffractive reading (Barad, 2007) of our concepts and place-based narratives *through* one another. We do not fix or elevate either scholarship or practice, for example, using concepts as a fixed 'lens' on narratives that 'reflect' practice. Rather, both ideas and practice matter in the sense that they 'interfere' as they entangle in the reading. They give meaning to one another in ongoing in-betweenness, in potentially illuminating ways. We generate stories of practice using narrative interviewing (Petitmengen, 2006). This method is particularly appropriate for exploring rich, nuanced situations in depth, towards a new awarenesses for both interviewee and interviewer. For this report, Alf took the interviewing role, drawing on extensive experience with this methodology. He asked Kate and Armando, as interviewees, to prepare by choosing a context of their facilitation with mathematics teachers that had potential for thinking with the concepts, and to remind themselves of

the details. Each individual interview involved exploring the detail of the context, without explicitly asking about or focusing on the concepts.

The interview protocols were taken from Brown and Coles (2019, pp. 177-178), with the interviewer: (1) "Stabilising attention" by reformulating what the interviewee says, thus checking accuracy; (2) Guiding the attention from 'what' to 'how', and not 'why'; and (3) Moving from the general to the specific, and staying with detail of an experience "that allows access to the implicit." A comment by Armando during the interview illustrates the potential of the method to prompt new awareness: "That's now that you are asking me these questions, I am making these connections." In another example, Alf reflected back to Kate what he had heard, which she refuted ("I don't think it's been sort of inevitable"), offering a new articulation.

THINKING FROM AN EDUCATION PROJECT IN THE ATOYAC REGION

The context of this narrative is 'The Memorial Museum of the Atoyac River', an education project in a rural community of the Atoyac region of Mexico, the Atoyac river being a highly-polluted river (see Solares-Rojas et al., 2022). The project, led by Armando, was co-constructive by design, a recognition of the need for multiple, varied relations. It involved community members, scientists, local primary school teachers, (mathematics) education scholars, all with specific interests; education and community activities; the arts; mathematics; time; and space. A collective vision of a Memorial Museum school project (MM) represented an opportunity to, in relation, recognise (using multiple knowledges) and confront (in a school setting) the socio-ecological problem of a highly polluted river lived by the community. The Museum would have three halls, focusing on river-community relations in the past, present and future.

Armando said that when the team began designing activities for the halls, the teachers recognised particular relations between members, saying, "you are the experts, you are the scientists, and we are the teachers. So you have to tell us, if you have a clear idea of the activity, you have to tell us what to do." Inclined to refuse this approach, on account of the project aims, Armando consulted, including with the primary school principal. This led him to create an opportunity for the team to see and discuss activities designed by the researchers. For example, Armando's 'present' hall activity involved reading environmental risk maps of the Atoyac River region, the industrial zones located there, and the current levels of water pollution. Armando remembers, "teachers did not like that." They refused the activities as being too far from, unrelated to, the official school curriculum. However, by critically discussing the details of the MM proposal the teachers recognised knowledges – from their teaching experience *and* their experience as inhabitants of the Atoyac region – that related to the activities discussed. They brought into relation tasks they had previously used in their classrooms and those confronting socio-ecological inequities, as envisaged for the Museum halls. Armando recalled teachers' recognising the relations within the team as needed for this integration, but also recognising new relations beyond the team, at their schools.

Armando highlighted significant recognition by the scientists and education scholars of the debt, the relations needed, in the communal grievance process, and their responsibility in the arena of inequitable relations. The researchers came to recognise the teachers' needs, knowledges, and lives, despite some of these initially being in tension with their image of how a co-designed project might develop, such as teachers initially seeking a predefined educational design. The researchers came to recognise teachers as living inequitable socio-ecological relations in two roles:

> We knew that some teachers lived in the area. They are all not just teachers, but they are members of the communities. But we hadn't connected that they […] have the two roles as a person. I [Armando] hadn't connected that as one of the team members.

Armando's talk points to the MM functioning as a communal grievance process:

> …on one hand, through recovering the historical memory of the river and the community by sharing personal stories and discussing these matters with their grandparents and other children from their communities. On the other hand, it involves studying the current causes and effects of pollution on both people's health and the environment. Ultimately, it provides children with opportunities to envision potential futures for their relationship with the river.

The MM activities were a space to recognise, in relation, the progressive loss of the river and the complex, varied forms of connections that communities have with it.

THINKING FROM A SOUTH AFRICAN UNIVERSITY

Kate's work with lecturers and tutors (elsewhere 'teaching assistants') takes place in an ongoing undergraduate mathematics curriculum reform project at a South African university. This functions in the context of poor and inequitable student results, a legacy of the country's colonial and apartheid past. The reform involves creating opportunities for science-degree students to learn and use 'mathematics practices' seen as important for their roles as future scientists. Examples are curiosity, sitting with uncertainty, exploring new directions, and collaborating (Haraway, 2016). Kate's talk identified multiple, varied relations in the project, between: peoples; ways of learning, knowing, and teaching; project activities; time; and learning spaces.

Answering Alf's question on what the teachers "see themselves as trying to work on", Kate suggested recognition of "inequities", that they "know" from the statistics, but which are "very felt" and "lived" in the past and present. Lectures are "under pressure" about results, and new lecturers "have been through the process". Tutors (undergraduate and postgraduate students), have "lived that experience of particular struggle […] with others," and "are in communities of students where the struggles with maths and physics are constantly there." These inequities are not deemed "inevitable": Kate highlighted the "incredibly rich experience" of lecturers and tutors who had "been trying to do things in their space[s]." But these alone are "not going to solve the bigger problem". The project is a space of communal recognising *and* confronting inequities, as grievance: "I asked them to write why they were interested

in being in this community, and a lot was about their wanting to make a difference in maths performance and that this was a space where they could explore it."

Here the teachers recognise their needing one another in relation, their debt. Kate's responsibility, then, has become developing new relations for the collective grievance process, for example: designing activities involving tutors *and* lecturers together; and drawing on strengths she recognises, such as tutors redesigning tutorials, and lecturers crafting innovative worksheets. Trust is key to building relations:

> Tutors who have got good relationships with the lecturers based on their own studies and who trusted by the lecturers […] So there was enough trust that we could work with the students in that way. Yeah, that's about building relationships.

Project activities offer opportunities to recognise *and* confront inequities. She recalled discussing a book on equity in the reading group and a co-presenting a workshop on teacher noticing, and then hearing "talk around noticing where some students are participating more than others and who is participating and who isn't participating," and reflection "on their own experiences of what it felt like in that space." Kate's role is asking, "Does it have to be this way?", and prompting "thinking out of the box" towards refusal of the status quo. For example, recognising teachers' task design strengths, she prompted, "if you have that worksheet, our traditional method of a tutorial is not going to work. So how might you run that tutorial differently?" The teachers do experience challenges constructing a grievance: they feel "constrained" by the university timetable, and have varied styles for teaching mathematics practices. Kate's role is to prompt, "bring the voices all together", while allowing for difference.

DISCUSSION AND CONCLUSION

We conclude by discussing the two aims of this conceptual report: (1) theorising the role of the facilitator in teacher learning, offering the concepts of recognition, debt, grievance, and refusal, in socio-ecological relations; and (2) sharing our theorising process of reading diffractively with ideas not common in mathematics education, and narrative interviews around facilitator practice. Our key concepts emerged in our prior collaborations (e.g. Coles et al., 2024; Makramalla et al., 2025). Yet, rather than using these as a fixed, 'power-ful' lens on empirical practice, as commonly done, we seek in our thinking from place to be open to how both concepts and practice matter for illuminating insights when they entangle (Barad, 2007). Indeed, we began by arguing that thinking about socio-ecology *and* mathematics education requires new relations. Reading with the interviews from the two contexts, we have come to appreciate the potential of education projects as communal grievance processes, involving multiple knowers and knowledges. These are grievances in the sense of recognising inequities and the need for relations with one another, that is our debt, *and* confronting, possibly in refusal, (mathematics) education practices towards more just futures. The role of the teacher facilitator, then, involves creating and holding these relational communal spaces, while harnessing the necessary, multiple ways of knowing, being and teaching.

While each of the projects was originally designed around community of some form, the nuances and complexity of the relations in community are given meaning in our thinking in place. In the MM project, the environment (the river) is a presence in past, present and future relations, whereas, in the university curriculum project, the environment figures implicitly in its relation to mathematics practices in the present and in students' futures as scientists. Reading through Armando's interview, what emerges as important is recognising the needs of the community of teachers, and recognising teachers as members of the local community of their schools. In Kate's interview, it is the deliberate crafting of a community of teachers with lived experiences of education inequities and energy to challenge these. In both cases there is an element of contingency on the part of the facilitator. For Armando, this involves being open to re-thinking the very basis of his research plan. For Kate, designing and adapting activities to harness strengths and prompt the new, while learning herself, has emerged through an iterative process and could not have been known in advance. In the MM project, refusal involves the teachers negotiating their relations with the researchers into a space where they recognise their potential contribution. This requires recognition and humility on the part of the facilitator: accepting the teachers' request for activities, and then accepting their refusal of these. In reading Kate's interview, a refusal to be bound by past teaching practices emerges in the project activities, but also in other teaching spaces via relations of trust between tutors and lecturers.

We emphasise our theorising as an ongoingness and thinking from place, rather than offering fixed ideas that can be generalized. Thus offer this report, our theorising, to others for diffractive reading in their thinking from place.

Additional information

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References

Alsina, Á., & Mulà, I. (2019). Advancing towards a transformational professional competence model through reflective learning and sustainability: The case of mathematics teacher education. *Sustainability*, *11*(15), 4039. https://doi.org/10.3390/su11154039

Barad, K. (2007). *Meeting the universe halfway: Quantum physics and the entanglement of matter and meaning*. Duke University Press.

Brown, L., & Coles, A. (2019). Reciprocal narrative interviewing. In C. Comanducci & A. Wilkinson (Eds.), *Matters of telling: The impulse of story* (pp. 177-184). Brill.

Coles, A., & Helliwell, T. (2023). The role of mathematics teacher educators in preparing teachers of mathematics to respond to global challenges within their classrooms. *London Review of Education*, *21*(1), 2. https://doi.org/10.14324/LRE.21.1.02

Coles, A., Solares-Rojas, A., & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, *116*(2), 165-183.

Escobar, A. (2006). Difference and conflict in the struggle over natural resources: A political ecology framework. *Development*, *49*(3): 6-13.

Haraway, D. (2015). A curious practice. *Angelaki*, *20*(2), 5-14.

Haraway, D. J. (2016). *Staying with the trouble: Making kin in the Chthulucene*. Duke University Press.

Helliwell, T., & Ng, O. L. (2022). Imagining possibilities: Innovating mathematics (teacher) education for sustainable futures. *Research in Mathematics Education*, *24*(2), 128-149.

Makramalla, M., Coles, A., le Roux, K., & Wagner, D. (2025). Mathematics education for sustainable futures: a strengths-based survey of the field to invite further research action. *Educational Studies in Mathematics*, *119*(3), 535-556.

Mbembe, A. (2018). The idea of a borderless world. *Africa is a Country*, 11 November. https://africasacountry.com/2018/11/the-idea-of-a-borderless-world#

Mbembe, A. (2021). *Out of the dark night: Essays on decolonialization*. Wits University Press.

Mbembe, A. (2023). *Pathways of tomorrow: Contribution to thinking commensurate with the planet*. Education, research and foresight: Working paper 32. https://unesdoc.unesco.org/ark:/48223/pf0000387026_en

McGranahan, C. (2016). Theorizing refusal: An introduction. *Cultural Anthropology*, *31*(3), 319–325. https://doi.org/10.14506/ca31.3.01

Petitmengin, C. (2006). Describing one's subjective experience in the second person: An interview method for the science of consciousness. *Phenomenology and the Cognitive Sciences*, 5.3-4: 229– 269. https://doi.org/10.1007/s11097-006-9022-2

Prediger, S., Roesken-Winter, B., & Leuders, T. (2019). Which research can support PD facilitators? *Journal for Mathematics Teacher Education*, 22(4), 407–425.

Solares-Rojas, A., Arellano-Aguilar, O., García González, M. M., López-Vargas, M. R., Coles, A. & Méndez Serrano, A. (2022). Mathematics education and social-environmental crises: an interdisciplinary proposal for didactic innovation with rural communities in Mexico. *Research in Mathematics Education*, *24*(2), 202-223.

Steffenson, L. (2025). Network of collaborations and social practices to engage students in socio-ecological issues of peatlands. In K. le Roux, A. Coles, A. Solares-Rojas, A. Bose, C. Vistro-Yu et al. (Eds.), *Proceedings of the 27th ICMI Study Conference (Mathematics Education and the Socio-Ecological)* (pp. 259-266). MATHTED and ICMI.

Valero, P. (2007). A socio-political look at equity in the school organization of mathematics education. *ZDM*, *39*(3), 225-233. https://doi.org/10.1007/s11858-007-0027-2

Velasco, P. (2017). *Ríos de contradicción* [*Rivers of contradiction*]. Instituto de Investigaciones Antropológicas, UNAM. http://www.iia.unam.mx/publicaciones/detalles.php?clave=498