SCAFFOLDING STUDENTS' COLLABORATIVE PROBLEM-SOLVING ACTIVITIES IN AN ADVANCED UPPER SECONDARY MATHEMATICS CLASSROOM

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This paper reports from a study investigating collaborative problem solving in upper secondary mathematics classrooms following the principles of the Thinking classrooms approach, focusing on teacher scaffolding strategies. In the observed lessons, students worked in groups of three attempting to solve problems involving solids of revolution and trigonometry. Overall, conceptual scaffolding was the most often observed scaffold. In addition, procedural, metacognitive, participatory, and epistemic scaffolding was observed. The most common scaffolding practice was questioning. Throughout the lesson the teacher managed to maintain the complexity of the task by withdrawing scaffolding at an early stage, thereby transferring responsibility to the students and allowing them to struggle with the problem.

INTRODUCTION

In recent years, the teaching approach *Thinking classrooms*, as explicated by Liljedahl (2021), has received increased attention and seen an uptake in classroom teaching internationally (Muir, 2025), as a way of introducing student-centred, collaborative problem-solving activities. This approach comprises students working collaboratively in groups of 2-3 to solve mathematical problems on vertical, non-permanent, surfaces (Liljedahl, 2021). The approach also delineates a set of practices that are meant to get students to start *thinking*, comprising guidelines on how to for example set up the student groups, how to consolidate a lesson, and how the teacher should act during the activity. Despite the approach being implemented in increasingly more classrooms, empirical studies investigating teaching in these classrooms remain few (Muir, 2025), particularly at the upper secondary level.

Problem solving and student-centred activities are not a novelty in mathematics education, however, despite much knowledge acquired from research over the past decades, practicing them can still be challenging to teachers. In classrooms focusing on collaborative problem solving, the role of the teacher is to guide and facilitate students learning processes. This comes with a risk of reducing the complexity and opportunities provided in the problem-solving activities. However, it is vital that students are offered opportunities to discuss amongst each other and "struggle" with the mathematics.

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The aim of this paper is to discuss scaffolding practices observed in one advanced upper secondary mathematics classroom implementing the Thinking classrooms approach to mathematics teaching. The research question reads:

> *What characterizes the scaffolding offered during two collaborative problem-solving lessons in a classroom adhering to the Thinking classroom approach?*

THEORETICAL BACKGROUND

The teaching approach Thinking classrooms includes collaborative problem solving; students should be divided into random groups of three and work on vertical surfaces to explore and solve unfamiliar tasks (Liljedahl, 2021; Muir, 2025). Liljedahl (2021) argues that students should struggle with the problem-solving tasks, to enhance their mathematical thinking, and according to Schoenfeld (1987) they should also engage in reflection, evaluation, and justification as part of the problem-solving process. However, the teacher might interact and offer scaffolding, for instance by offering reflective questions to students who reach an impasse.

The concept of scaffolding was introduced by Wood et al. (1976) as a "process that enables a child or novice to solve a problem, carry out a task, or achieve a goal which would be beyond his unassisted efforts" (p. 90). While initially intended as a practice applicable in one-to-one settings, scaffolding has been described in both group and whole-class settings later (cf. Calder, 2015). While defined in different ways, scaffolding typically has three common elements: 1) contingency, meaning that the support is adapted to the group or student, 2) fading, which is the gradual withdrawing of the scaffolds, and 3) transfer of responsibility, wherein the student or group are gradually granted control over own learning (van de Pol, 2010).

During problem-solving activities, scaffolding can play a crucial role for instance by supporting students when clarifying and explaining their thinking, correcting misconceptions, and highlighting important ideas (Anghileri, 2006; Webb et al., 2019). Scaffolding purposes such as explicitly promoting clear explanations and justification (Anghileri, 2006), remain relevant in group settings. However, in group-based problem-solving settings, scaffolding can also be used to foster productive discussions and participation amongst the students, and guide the group towards a shared understanding of key ideas (Webb et al., 2019). A main challenge is to facilitate student participation in high quality within-group discourse, maintaining a sufficiently high cognitive demand, and helping students move beyond emerging difficulties, but without lowering the cognitive demand or creating confusion (Goos et al., 2002).

In Thinking classroom activities such as the ones investigated in the current study, scaffolding collaborative problem solving could entail asking students to elaborate their thinking to their group members, asking specific students in a group to provide alternative explanations and justifications, or helping students' resolve conflicts.

In terms of collaborative problem-solving contexts, student collaboration can be seen as a scaffolding element in itself (Anghileri, 2006). Moreover, using scaffolding to

ensure equitable participation can be seen as a part of the teachers' responsibility in collaborative problem-solving activities. Overall, scaffolding can have different intentions and functions, ranging from supporting students in solving complex mathematical problems, to helping students collaborate and build on each other's ideas, to being able to explicate their own thinking processes. Therefore, we can understand scaffolding to have different purposes, such as conceptual, procedural, and metacognitive (cf. Gou et al., 2025).

METHODS

In this study we employ video observation suitable to our aim to gain in-depth understanding of teacher scaffolding during collaborative problem-solving activities. Video data was suitable for this study, as it allowed us to analyze complex interactions in-depth and re-analyze the interactions (Klette, 2023), which was critical due to our use of abductive thematic analysis. The study can be said to align with a socio-constructivist perspective, assuming the teacher aims to foster active engagement in the classrooms. Our data comprised multiple video recordings from two Norwegian upper secondary advanced mathematics lessons, where a Thinking classroom approach was utilized. The data was naturally occurring, in that the observed lessons were not designed for this study, but part of the class's regular lesson plan.

Participants

The participating teacher can be considered an experienced mathematics teacher with ten years of teaching experience and a Master's degree in mathematics education. The students were 18–19 years old, enrolled in the most advanced mathematics course available in Norwegian upper secondary school (Mathematics R2). This course can be understood as a mix of pre-calculus and calculus.

Data collection

The video data was collected using one camera per observed student group, placed to capture the whiteboard utilized by the group. One student per group was allocated a lapel microphone, wirelessly connected to the camera, ensuring audio was captured from each participant in the group, as well as the teacher when addressing the group. In total, four randomized student groups per lesson were observed, providing a total of 8 video recordings. The first lesson aimed at helping students understand how to calculate the volume of a solid of revolution. The students were provided with a series of tasks, all revolving around visualizing and calculating volumes of solids, with heights depending on some function $f(x)$. In the first part of the series, the students were asked to find an expression for the volume of a wall comprised of square prisms, with heights following some function $f(x)$. The students were prompted to first use summation to find an expression, then reflect on what happens and what the volume is if the number of discs increases, but the thickness decreases, so that the original wall is equally long as the new one. The students were then asked to consider what happens if the prisms are exchanged for cylinders, by creating a sketch and an expression for

the volume. In the latter part of the series of tasks, the student drew the solids of revolution and calculated their volume, based on provided functions ($f(x) = x^2 + 1$ and $g(x) = \frac{x}{2}$), their limits, and the axis of revolution.

In the second lesson, the students worked with trigonometrical equations. They were provided with two tasks. First they were to evaluate and explain the thinking behind a provided solution to an equation $\cos = \frac{\sqrt{2}}{2}$, for $x \in \mathbb{R}$, and then solve it themselves for $x \in (0, 20)$. In the second task, containing three sub-tasks, the students were again provided with a solution to an equation, $\sin x - \cos x = 0$, in addition to a "discussion" between the person solving the task and another, arguing the validity of the solution. The students were asked to evaluate the solution and comment on the discussion. Next, the students were asked to solve the equation $\sin 2x - \cos 2x = 0$, without using tools. Finally, the students were asked to answer a claim regarding a strategy applied to solve a third equation, $2 \sin x \cdot \cos x - \cos x = 0$, before solving it themselves without tools.

Data Analysis

The video data was analyzed at a sentence level, focusing on episodes within each video where the teacher interacted with a student group being recorded. As mentioned, we conducted an abductive thematic analysis (cf. Thompson, 2022). In the first phase, the episodes relevant for this analysis were identified and transcribed, allowing us to also be familiarized with the data. In the next phase, we coded the data in two rounds, with a first all-encompassing round, leading to a second, more selective one, per recommendations (Thompson, 2022). A rudimentary codebook was also created following this, providing definitions, application rubrics, and examples, followed by a check of the second round of codes to identify any overlaps or errors.

Themes were developed from the codes, supported by our theoretical perspectives. For example, different kinds of questions, such as asking about how to execute an operation or eliciting elaborations from students, led to the overarching theme questioning, supported by literature underscoring this as a central scaffolding practice (e.g. Anghileri, 2006). In the last phase, we worked on theorizing our themes, aiming to understand how and if our themes could be explained by, or aligned with, our theoretical perspectives (e.g. Anghileri, 2006; Gou et al., 2025; van de Pol et al., 2010). Here, we found that two emerging concepts from our data, given the terms epistemic and participatory scaffolding, extended the previously described theory on scaffolding purposes.

The themes could be seen as divided into two sets. One set describes foci of scaffolding, comprising procedural, conceptual, metacognitive, participatory and epistemic scaffolding. The other set of themes comprised scaffolding practices, including questioning, hints, feedback, explaining, parallel modelling, negotiating meaning, clarifying aim, and prompting peer interaction.

RESULTS AND DISCUSSION

In the observed lessons, the students quickly found their places at the board and entered into problem solving when the teacher provided the problem. The teacher circled the classroom, regularly approaching a group. Sometimes the student groups called on the teacher or addressed a neighboring group. Our observation reveals that all student groups received scaffolding in both lessons. Scaffolding was initiated both by the teacher and the student groups themselves or by individual students.

Our observations yield two lessons in one classroom, and as such, our aim is not to generalize, rather to deepen our understanding of how scaffolding might take place during collaborative problem solving in a Thinking classroom context, and to offer illustrative examples of what characterizes the observed scaffolding. Overall, our observations of the two observed lessons reveal that the teacher was able to maintain a high cognitive demand during both lessons, by seemingly adapting the scaffolding to the needs of the different groups and not providing too much guidance or information. After a scaffolding instance, the teacher gradually removed support, until the student groups were allowed to take the responsibility for moving their problem-solving processes forward themselves, suggested by the group discussions following episodes of scaffolding.

The scaffolding provided by the teacher could mostly be characterized as conceptual. For example, by scaffolding students' understanding of mathematical concepts, such as what certain variables represent. We also found evidence of procedural and metacognitive scaffolding, thus showing that the teacher was able to utilize scaffolding foci aligned with those identified in previous research (e.g., Gou et al., 2025) in the thinking classroom context. However, the thematic analysis also revealed two additional categories of scaffolding, which we termed epistemic and participatory scaffolding. All five categories are expanded upon and exemplified with findings from our data subsequently.

Procedural scaffolding can be described as supporting students in carrying out calculations, strategies, methods, or operations, with a focus on execution and progression within tasks. An example of procedural scaffolding that was observed involved the teacher walking up to a group, after having observed their whiteboard, asking:

> "I was also wondering, could you try to draw the figure in task a) again? Perhaps in a way that if you imagine that you were filming, right? And it looks like you moved the camera a bit to the right [gestures to the whiteboard]. If you try to move the camera a bit more to the right? How would it look then?"

This shows the teacher specifying how to proceed with the task, by prompting a specific strategy. Another, typical example of procedural scaffolding that was observed included questioning, such as *"So you have the general solution to the equation here, but then you would like to only have x-values that are within this interval, how could*

you manage to do that then?". Showcasing teacher asking students how to execute a strategy.

Conceptual scaffolding is about supporting students understanding of concepts (Gou et al., 2025), including relationships, constraints, limitations, and possibilities of mathematical entities and concepts. Typical examples of the observed conceptual scaffolding include questions relating to mathematical entities and concepts, such as: *"What does this k represents?"* and *"What kind of numbers could that k be?"*, showing two examples where the teacher asked the students to reflect on what a variable represents and the limitations of the variable. Similarly, the teacher had several instances of explanations that acted as conceptual scaffolds, for example: *"So you shouldn't let yourselves be tricked by it being multiplied by pi, that is only a factor, and you can set it outside if you would like to"*, in reference to simplifying an expression before finding an integral.

Metacognitive scaffolding refers to the teachers' support of students' awareness, monitoring, and regulation of thinking and strategies (Gou et al., 2025). This form of scaffolding draws attention to students' decisions and strategy choices, to support metacognitive thinking. This form of scaffolding can be exemplified by instances where the teacher drew attention to and challenged the students' strategy choices or thinking, *"And why do you want to use partial integration?"*. It could also entail episodes where the teacher asks the students to think about their own thinking, such as *"How did you think on the tasks you solved on the whiteboards?"*.

The fourth observed category, *epistemic scaffolding*, can be described as scaffolding that supports students in engaging in justification, evaluation, comparisons, and validation of mathematical knowledge, solutions, and strategies. This scaffolding focuses on guiding students in how to understand why and when something works or is valid. For example, the teacher asked students to explain why certain strategies or solutions were possible to use or valid: *"So it is obviously possible, you have done it. But why is it unproblematic?"*. In other situations, the teacher asked students to compare strategies or solutions, such as *"What is it that happens here and not there?"*, asking students to understand the validity of different strategies in comparison to each other.

Participatory scaffolding, identified in the observed classrooms, revolves around the teacher scaffolding students' access to participating in activities and discourse, through actively inviting interactions between students. This form of scaffolding focuses on ensuring that all students are granted opportunities to partake in the problem-solving activity and contribute both within their groups and in whole class activities. We for instance observed the teacher asking the students to take turns practicing drawing representations of the solids of revolution, as it is an important skill to practice. Similarly, observations of the teacher prompting the groups to discuss within-group how they could proceed with a strategy, was interpreted and categorized as participatory scaffolding.

In terms of what type of scaffolding practices were utilized by the teacher, our observations showed that the most common practice by far was questioning, followed by hints and feedback. This finding aligns with previous research, where such practices have been viewed as core scaffolding practices (see e.g. Anghileri, 2006; Webb et al., 2019). Even so, other practices were also identified, such as clarifying the aim of the task at hand, which is an action forwarded in early scaffolding literature (cf. Wood et al., 1976) and prompting students to interact with their peers. It should be noted that questioning is a broad category, with many kinds of questions, that can function as, for example, both procedural and epistemic scaffolds depending on their formulation. The prevalence of questioning also aligns with the practices outlined by Liljedahl (2021) when describing Thinking classrooms, which could be one reason for our findings, where this practice was observed as the main scaffolding practice.

Our observations of the teacher's scaffolding of student problem solving showed that the different scaffolding practices were used dynamically throughout the teacher-student group interactions, thereby offering different foci for the scaffolds. This meant that the teacher switched between and used several different forms of scaffolding within one episode, as well as throughout the lesson. For example, for a group that was struggling with the problem, the teacher began with explaining a concept, followed by conceptual questioning. After the students had responded, by discussing together and negotiating an answer, the teacher challenged their answer with a counter example and reminded them of the aim of the problem. As the students still struggled, the teacher provided a hint, connecting to the students' prior knowledge, finally leading to them being able to proceed their own discussion. In addition to showing a variation in terms of scaffolding, the teacher was able to adapt the scaffolding to the group. He also withdrew gradually and left the students to carry the discussion themselves. This aligns with the elements of scaffolding that have been highlighted in previous research (van de Pol, 2010; Wood et al., 1976).

CONCLUDING REMARK

These findings demonstrate how established key elements in scaffolding described in previous research (e.g. Anghileri, 2006; van de Pol et al., 2010; Webb et al., 2019), could be identified in the observed collaborative problem-solving setting. Building on theory describing scaffolding purposes (cf. Gou et al., 2025), we were able to identify five scaffolding *foci* through our abductive thematic analysis: procedural, conceptual, and metacognitive scaffolding all inspired by the terminology used in previous research (e.g. Gou et al., 2025) but operationalized for mathematical problem solving in this study. However, we also identified two additional foci, epistemic and participatory scaffolding. Epistemic scaffolding was evident in the teachers' focus on guiding students to engage in justification, evaluation, and validating mathematical strategies, solutions, and claims. This kind of mathematical processes have been highlighted as key processes in problem solving (Schoenfeld, 1987) and are in line with the main purpose of the Thinking classroom approach, namely getting students to start thinking

about mathematics (Liljedahl, 2021). Aligned with this, participatory scaffolding also addresses a crucial factor in the collaborative problem-solving approach of the Thinking classrooms, namely collaboration. Our observations indicated that specific types of scaffolding were utilized to prompt within-group interactions and discussions, as well as to reaffirm norms for participation, indicating that this was a focus in itself.

Overall, our findings showed that questioning was the most used practice, and conceptual the most common focus. This could partly be due to the teacher and students having worked with Thinking classrooms for the last 2.5 years, and thus, it is reasonable to assume that they felt familiar with this approach and could afford focusing on conceptual aspects of mathematics. The extent of the conceptual focus is somewhat surprising, given that the course in question (Mathematics R2), is often seen as focusing on learning and using methods and procedures.

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