FRAMING TEACHER NOTICING THROUGH THE LENS OF HABERMAS' THEORY OF RATIONALITY IN MATHEMATICS EDUCATION

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This paper argues that students' mathematical thinking can be reframed as rational behaviour manifested in their participation in mathematical activity. Drawing on Habermas's theory of rationality, we conceptualise students' engagement through epistemic, teleological, and communicative components, which provide observable and theoretically grounded indicators of understanding. To address the long-standing challenge of specifying what teachers should notice, we integrate this perspective with the expanded teacher noticing framework and propose the Teacher Noticing of Rationality Framework. The paper contributes to the networking of theories in mathematics education by articulating connections, and offers implications for research on noticing and the design of teacher educations

WHY RETHINK WHAT TEACHERS NOTICE?

Teacher noticing enables teachers to attend to and make sense of complex classroom interactions (Blömeke et al., 2015; Sherin et al., 2011; van Es & Sherin, 2021). In mathematics education research, teacher noticing is conceptualised as a dynamic process involving selective attention to classroom events and the interpretation of students' thinking (Jazby, 2023; Mason, 2025). The first dimension of noticing involves attending to noteworthy events in the classroom (van Es & Sherin, 2002, 2021; Jacobs et al., 2010). Although teachers may attend to classroom management, climate, pedagogy, and students' mathematical thinking (Sherin & van Es, 2009), it is not possible to focus on all aspects equally and simultaneously. Therefore, what teachers choose to ignore is as important as what they attend to (Erickson, 2011; van Es & Sherin, 2021). The second dimension of noticing is interpreting, which refers to teachers' reasoning processes based on their knowledge and experience to make sense of students' thinking (van Es & Sherin, 2002, 2021). This requires an inquiry-oriented stance in which teachers go beyond describing or evaluating events and instead infer the underlying reasons for students' mathematical ideas. van Es and Sherin (2021) further propose shaping as the third dimension of teacher noticing, referring to teachers' instructional actions to access and develop students' thinking through questioning. After attending to and interpreting students' ideas, teachers intervene to support deeper and more sophisticated understanding. Thus, shaping involves structuring and extending students' thinking during instruction.

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Over the past two decades, a growing body of empirical research has documented the importance of noticing for instructional quality, leading to an expanding literature on how teachers learn to notice and how noticing can be supported in teacher education (König et al., 2022). Despite this progress, an enduring challenge in the teacher noticing literature concerns the question of what exactly teachers are expected to notice. While there is broad agreement that noticing in mathematics classrooms should primarily focus on students' mathematical thinking (e.g., Jacobs et al., 2010; Santagata & Yeh, 2014; van Es & Sherin, 2021), mathematical thinking itself remains a theoretically complex and empirically elusive construct. Mathematical understanding involves the integration of new concepts with students' prior knowledge, resulting in the construction of beliefs and mathematical connections within the learner's cognitive structure (Cai & Ding, 2017; Kastberg, 2002). These internal cognitive connections, however, are not directly observable. Instead, they are manifested through students' written and verbal arguments produced during mathematical activity, which provide evidence of how meanings are constructed and relationships are coordinated (García-García & Dolores-Flores, 2018). Students' actions and solution steps can therefore be regarded as observable reflections of their mathematical thinking. From this perspective, what is expected of students in mathematics classrooms is not merely the production of correct answers, but engagement in mathematical activity in a rational manner, including justifying claims, selecting appropriate strategies, and communicating reasoning in ways that are acceptable within the classroom community (Boero, 2006; Boero & Morselli, 2009; Morselli & Boero, 2009).

This conceptual ambiguity regarding the object of noticing may help explain why empirical studies often report considerable variation in what teachers attend to and how they interpret students' understanding. If the object of noticing remains underspecified, then both the assessment and the development of noticing skills become methodologically unstable. Teachers may attend primarily to surface features of performance, such as answer correctness or procedural fluency, while overlooking deeper aspects of reasoning, justification, or strategic decision-making. Similarly, interpretations of student understanding may rely on implicit or inconsistent criteria rather than shared analytical frameworks, making findings across studies difficult to compare. In response to this challenge, we argue that students' mathematical thinking can be productively reframed in terms of their rational behaviour during mathematical activity. Drawing on Habermas' theory of rationality (1999) and following Boero's (2006) proposal to use this theory as a didactical tool in mathematics education, we conceptualise students' observable actions as oriented toward three interrelated components: epistemic, teleological, and communicative rationality. Epistemic rationality concerns how students justify claims using accepted mathematical knowledge, such as definitions, properties, and representations. Teleological rationality refers to how students select and coordinate strategies and tools in relation to the goals of a task. Communicative rationality involves how students express and negotiate meanings in ways that are understandable and acceptable to others in the

classroom community. This perspective is consistent with prior studies that have applied Habermas's construct of rationality to the analysis of mathematical proving and problem solving (Urhan & Bülbül, 2023a; 2023b; Urhan & Zengin, 2025). These studies conceptualise learning mathematics not merely as acquiring correct procedures, but as developing forms of rational participation in mathematical practices. Reframing mathematical thinking as rational behaviour allows teacher noticing to be grounded in theoretically principled and empirically accessible indicators of student understanding. Rather than treating thinking as an internal cognitive state to be inferred indirectly, this perspective foregrounds participation in mathematical practices as the primary site where understanding becomes visible and actionable for instruction. However, while Habermas's framework provides powerful analytical categories for describing students' mathematical engagement, it does not address how teachers attend to and interpret these forms of rational behaviour in real time. Conversely, teacher noticing frameworks specify how teachers attend to, interpret, and shape classroom events but typically leave the object of noticing only broadly defined (van Es & Sherin, 2021, 2025). This theoretical gap motivates the integration of these perspectives. By combining them, we are able to specify both what teachers are expected to notice and how noticing unfolds in instructional practice, thereby laying the foundation for the Teacher Noticing of Rationality Framework (TNRF) introduced in the following section.

TEACHER NOTICING OF RATIONALITY FRAMEWORK (TNRF)

We propose reframing teacher noticing as noticing of rationality (Table 1), where the object of noticing is students' epistemic, teleological, and communicative rational behaviour during a mathematical task. This proposal does not replace existing conceptualisations of teacher noticing, but rather specifies what teachers attend to, interpret, and shape when they engage with students' mathematical thinking.

We adopt the expanded conceptualisation of teacher noticing proposed by van Es and Sherin (2021), which comprises three interrelated dimensions: attending, interpreting, and shaping. Within this structure, Habermas's (1999) components of rationality provide the analytical focus for each dimension. Teachers are expected to attend to manifestations of epistemic, teleological, and communicative rationality in students' actions; to interpret these manifestations using their professional knowledge; and to shape classroom interaction in ways that elicit and develop students' rational engagement with mathematical tasks.

Attending to Rationality

In the attending dimension, teachers selectively focus on features of students' activity that are indicative of rational behaviour. With respect to epistemic rationality, this involves noticing whether students justify claims, reference mathematical properties, or rely on intuitive or unsupported assertions. For teleological rationality, teachers attend to whether students' strategies and representations align with the goals of the

task and whether actions appear purposeful rather than routine or habitual. Regarding communicative rationality, teachers notice how students express reasoning, including the clarity of explanations, the use of mathematical language and symbols, and responsiveness to peers' contributions.

Table 1: Teacher noticing of rationality framework (TNRF)

Noticing of Rationality	Epistemic Rationality	Teleological Rationality	Communicative Rationality
Attending	Seeking any justification (definition, theorem, etc.) in the student's behaviours	Seeking mathematical tool (strategy, etc.) suitable for the purpose in the student's behaviour	Seeking for the student's use of numerical representation, formal notation, equation, expression
Interpreting	Comparing the teacher's own mathematical facts with the student's behaviour	Determining whether the tool chosen by the student is appropriate for the purpose	Determining whether the student uses representation and formal notation in accordance with the problem and solution strategy
Shaping	To require students to justify and defend their arguments, steps, and claims with a reliable mathematical warrant.	To require students to explain selection an appropriate strategy, theorem, definition, or representation and to engage in the process of performing the task.	To require students to present their performance in a manner that allows for comprehensive understanding by others, demonstrating clear and coherent verbal and written explanations, and using the language of mathematical symbols accurately and consistently.

This focus shifts attention away from isolated outcomes or procedural steps and towards patterns of engagement that reveal how students are reasoning. Attending to rationality also involves selective inattention. Teachers may choose to temporarily set aside issues such as classroom management or task completion in order to concentrate on students' reasoning processes, consistent with van Es and Sherin's (2021) emphasis on selective attention in noticing.

Interpreting Rationality

Interpreting involves making sense of what has been observed by drawing on professional knowledge of mathematics, learning, and instruction. When interpreting epistemic rationality, teachers assess whether students' justifications are mathematically valid, whether warrants are appropriate, and whether misconceptions or gaps in reasoning are present. In interpreting teleological rationality, teachers make

sense of whether students' strategies are efficient, coherent, and aligned with task goals, and whether alternative approaches might better support conceptual understanding. Interpreting communicative rationality entails judging whether students' explanations are sufficiently clear and mathematically meaningful for others to follow, and whether representational choices support or hinder shared understanding. It is important to emphasise that interpreting rationality requires a questioning stance. Rather than merely categorising student responses as correct or incorrect, teachers consider why students acted in ways and what these actions reveal about underlying reasoning processes. This aligns with van Es and Sherin's (2021) emphasis on inquiry-oriented interpretation, where teachers seek to understand the sense-making behind student behaviour rather than focusing solely on surface features of performance.

Shaping Rationality

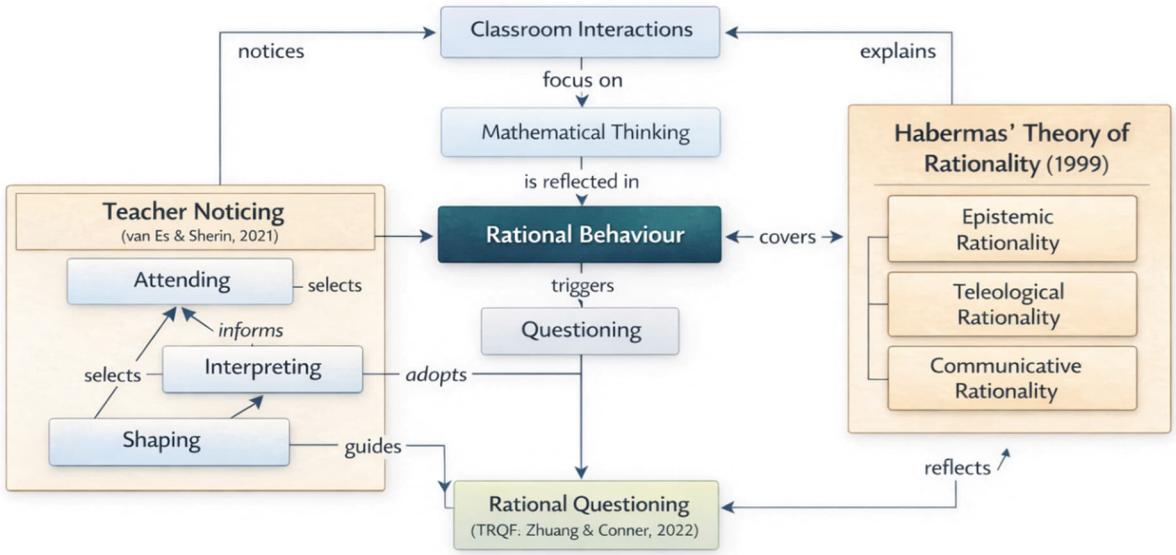
Shaping refers to teachers' active role in creating interactional conditions that support deeper insight into students' reasoning and promote further development of rational behaviour. In the context of noticing of rationality, shaping involves posing questions and designing interactions that prompt students to justify claims (epistemic), articulate goal-oriented strategies (teleological), and communicate reasoning clearly (communicative).

In the shaping dimension, teachers are expected to pose questions that create contexts and interactions enabling them to access additional information (van Es & Sherin, 2021; 2025) about students' rationality. While the focus of attending and interpreting is on students' rational behaviours, shaping requires teachers to take an active role in classroom interactions by engaging students through questions that elicit and stimulate rational inquiry. Although the approaches of Habermas (1999) and Boero (2006) emphasise analysing rationality in students' behaviours, the shaping dimension highlights the teacher's active involvement in fostering such rationality through an inquiry-oriented stance. However, existing frameworks remain limited in specifying how teachers' questions can trigger students' rational engagement in inquiry. Building on studies that stress the importance of students learning and habituating rational behaviour in mathematical activities, Zhuang and Conner (2022) argue that teachers play a central role in designing and enacting such learning environments and therefore proposed the Teacher Rational Questioning Framework (TRQF). Grounded in components of Habermas's theory of rationality, TRQF specifies the types of questions teachers can ask to elicit students' rational engagement across each component of rationality. In this sense, TRQF provides concrete support for the shaping dimension of teacher noticing.

THE STRUCTURE AND IMPLICATION OF TNRF IN TEACHER EDUCATION

This study proposes an innovative response to the problem of how to assess and develop teacher noticing by integrating teacher noticing (van Es & Sherin, 2021), Habermas's theory of rationality (1999), and TRQF (Zhuang & Conner, 2022) into the TNRF. We address the question of what to notice through Habermas's theory of rationality (1999), conceptualising mathematical thinking as reflected in students' rational behaviours rather than as a directly observable mental process. We aim to provide a clearer focus for both research and practice and to contribute to a more coherent understanding of noticing in mathematics education. Figure 1 illustrates how Habermas's theory of rationality (1999), and TRQF (Zhuang & Conner, 2022) are integrated with the attending, interpreting, and shaping dimensions of teacher noticing (van es & Sherin, 2021).

Figure 1: Connecting frameworks



Although noticing is considered teachable and learnable, how teachers should learn to notice remains an open question. Our framework addresses this by focusing on attending to components of rationality in students' behaviours, interpreting these behaviours through an inquiry-oriented stance, and shaping students' thinking through rational questioning. By grounding noticing in Habermas's theory of rationality, as suggested for mathematics education by Boero (2006), and integrating it with TRQF (Zhuang & Conner, 2022), our framework provides a theoretically coherent basis for both developing and assessing noticing. We strengthen our claims with evidence from a prior study in which a prototype of our framework was used to analyse mathematics teachers' noticing of rationality according to three components of rationality (Galiç et al., 2025). Although these findings are promising, further empirical evidence is needed.

Habermas's theory of rationality (1999), a general sociocultural theory of human action, was proposed by Boero (2006) as a useful didactical tool for mathematics education and as a theory that can be productively networked with other mathematics education frameworks. By adopting this perspective, we situate teacher noticing within a theory of rational action and tailor it to educational contexts. While TRQF (Zhuang & Conner, 2022) assumes that teachers can engage in rational questioning, such questioning can only be responsive to students' needs if teachers first attend to and interpret students' rational behaviours. We therefore argue that integrating teacher noticing with TRQF bridges in-the-moment teacher–student interactions with student-centred rational questioning. Integrating these three perspectives offers a richer and more practical approach to studying and supporting teacher noticing in mathematics education.

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