MATHEMATIZING (WITH) THE BODY: SENSING "INSCRIBABILITY" WHILE MODELING AND 3D PRINTING

Charikleia Korompli, Chronis Kynigos

National and Kapodistrian University of Athens

This study examines the embodied mathematical exploration of students when programming digital models for 3D printing. The challenge of fitting geometrical entities within others, creates a new Conceptual Field, "Inscribability", within which our qualitative empirical research maps students' lived experiences, strategies and tech-mediated practices (coding, dynamic manipulation) when creating functional physical models (3D printing). Findings highlight how digital tools actively mediate expression, artefacts and perception, negotiated through the body.

INTRODUCTION

Abstract concepts in Geometry and Stereometry are often presented through symbolic formalism and static representations, detached from bodily experience. Formal instruction continues to rely mainly on predesigned visual tools (manipulative or digital) without paying sufficient attention to the dynamic creation of objects by the children themselves (Ng & Chan, 2019). Even though dynamic geometry software (e.g., GeoGebra) has transformed mathematics education, it often still prioritizes premade visualizations over student initiative and leave learners' bodies disconnected and passive. Contemporary digital technologies offer a profound opportunity to challenge this long-established narrative. By taking advantage of the interactive and materializing capacities of digital interfaces, we can now design experiences that extend the mathematical senses, transforming learners from passive observers into active agents who see, feel, and model their perception of mathematical reality. Although the educational benefits of mathematization and modeling activities have been well documented in the literature for many years (Blum & Niss, 1991) and the possibilities offered by 3D design and printing technologies have also been studied (Ng & Ferrara 2020; Ng & Ye, 2022), the relationship between modeling geometric concepts and tangible construction of their instances through 3D printing remains underexplored.

Addressing this, we attempt to investigate the nature of students' embodied mathematical sense-making while modeling geometrical entities, and we proceed to assemble "Inscribability" as a new Conceptual Field (Vergnaud, 2009), as described further below. This study aims to report on students' learning experience during such a technologically mediated exploration. The objective was to answer the following questions:

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 851-858. PME.

1. How students make sense of Inscribability during their constructions and how do they capture (verbally, gesturing, symbolically) their lived experiences?

2. How do digital tools (programming and 3D printing) mediate and transform students' embodied exploration as they navigate their path from 2D shape to 3D solid?

THEORETICAL FRAMEWORK

Embodied Mathematical Thinking

Embodied Mathematical Thinking posits that mathematical meaning emerges through bodily engagement with tools and environment (Alibali & Nathan, 2011). Research confirms physical interaction (gestures, object manipulation) dynamically shapes mathematical understanding (Nemirovsky et al., 2013) with movement and cognition constituting dynamically interrelated phenomena (Abrahamson & Bakker, 2016; Roth, 2016). Mathematical thinking is not limited to virtual representations but manifests itself through creative processes where body and mind work together seamlessly (de Freitas & Sinclair, 2014). “Mathematical instruments” thus become active systems that embody material, practical and multimodal interactions, where knowledge emerges through bodily engagement, implying transformations in children's lived experiences (Nemirovsky et al, 2020).

A postphenomenological prespective

A postphenomenological perspective focuses on the embodied and sensuous qualities of the material world as experienced by human actors. Within learning contexts, it provides a valuable lens for studying how digital tools actively mediate students' experiential worlds (Psaros, 2025). Technologies are in no way innocent, but simultaneously enhance and limit perceptions (Hasse, 2008). From this point of view, digital tools are framed as active mediators that co-constitute the very relationship between the learner and the mathematical world, as potential bridges between “micro-perception” (the immediate sensory-bodily interaction with a tool) and “macro-perception” (the social, cultural, and disciplinary frameworks that give meaning to that interaction) (Ihde, 1990). In our study, a postphenomenological perspective is adopted to explore and analyse how the specific digital technologies (programming and 3D printing) mediate mathematical sense-making and free exploration of an abstract geometrical property (“Inscribability”).

Conceptual Fields [CF]

A Conceptual Field [CF] (Vergnaud, 2009) is assembled as a triad consisting of: 1) Representations (linguistic/symbolic systems) that are currently available in the subject's mental arsenal for the concept, 2) Problem situations where the concept can be encountered or used and, 3) Other affiliated concepts/meanings interconnecting mathematical ideas. This triadic framework rejects the notion of concepts as isolated entities and emphasizes instead, their situational emergence in personally relevant

problem contexts, with progressive stabilization via social negotiation, strategy development and adaptive formation through concept-field interactions.

METHODOLOGY

Assembling the CF "Inscribability"

In this study the CF "Inscribability" has been structured through the following triplet:
1. i) Representations of the children through their prior lived experience (how they perceive it with their own bodies and their spatial awareness) and ii) Representations of the children being built during the intervention, mediated by the digital tools

2. Problem-activity: a design-thinking project consisting of: i) digital creation by textual programming and ii) the physical construction (3D printable models)

3. Other related/affiliated concepts (e.g. geometry and trigonometry concepts) and students' spatial reasoning

The MaLT2 Environment

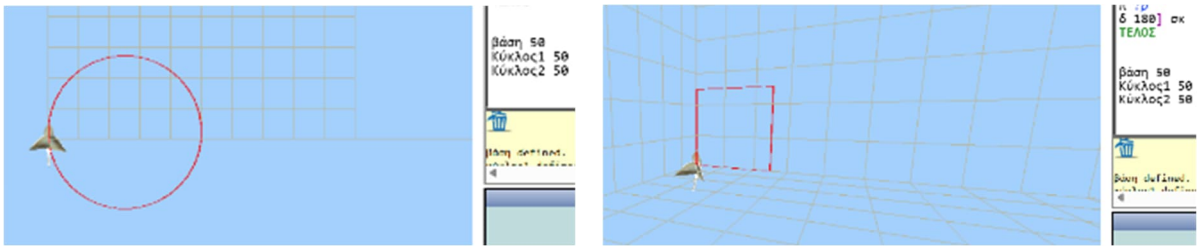
MaLT2 (http://etl.ppp.uoa.gr/malt2/#Gr) is a computational environment for creating and manipulating 3D dynamic digital models. It unifies multimodal representations, symbolic (Logo-like) and figural (graphical objects), and allows dynamic manipulation of the parameters of an executed process by interconnecting functionalities that interact dynamically. At the same time, it enables the changing of the viewing angle of the 3D space. It allows the user to define computational procedures (sets of commands), which, when executed, produce geometric shapes in a 3D "scene", bridging programming with mathematical concepts and mathematical formalism. The dynamic manipulation of models enables the user to introduce variables into the code and modify them through sliders, thus promoting direct monitoring of the effect of variables on graphical results, enhancing the understanding of the relationship between algebraic parameters and geometric properties.

Research Design

This pilot study was conducted as part of a larger Design Based Research project. The intervention was implemented as a Design Thinking activity at an upper secondary vocational school in Athens, Greece, consisting of five two-hour sessions. Participants were 20 students (aged 16-17) with no prior experience with the MaLT2 environment. They had basic knowledge of Python programming, Algebra, Geometry, and Trigonometry. Working in groups of 2-3 with one computer per group, they collaborated to create functional 3D printed objects for storage and organization (e.g., for cables, keys) or personalized everyday items (e.g., cup holders). Students first explored MaLT2 and were introduced to some predefined procedures (for creating a circle and a square; Figure1), which they could modify and/or use as building blocks. Then groups defined the purpose of their model, considered practical constraints (dimensions, functionality), and created their digital models in MaLT2. After 3D

printing, a presentation and feedback session were held in the classroom. Students examined each other's physical models, provided comments, and subsequently returned to MaLT2 to refine their digital designs based on the new ideas generated.

Figure1: The 2D-models initially given to students in MaLT2



Data collection and analysis

Data were collected from six complementary sources to allow for methodological triangulation: (i) screen recordings, (ii) audio recordings of group discussions, (iii) students' sketches, (iv) digital .mlt files, (v) physical 3D printed models, and (vi) field notes. The unit of analysis was the "critical incident" (Angelides, 2001). Incidents were considered critical to the research questions when the tools (programming and 3D printing) and the students' embodied experience played decisive role in collaborative exploring and overcoming challenges during the construction process. The analysis focused on human-technology relations alongside their multimodal interaction.

PRELIMINARY FINDINGS

Critical Incident 1: Inscribing a square in a circle

Group 7 (consisted of students M11 and M12) attempted to create a prototype for a cup holder with a hook to attach to the car dashboard. So, they faced the challenge of inscribing a square into a circle, to create the "skeleton" of the cupholder, as depicted in their very first draft sketch (Figure 2A). They decided not to use the given procedures for circle and square (Figure 1) and, instead, write their own codes.

M11: So, if we start from here (*points somewhere on the circle*).. and move.. let' s say.. (*he pauses to think the length of the side of the square*) hmmm..

M12: Let's try 30, like the size of the radius.. (*they execute the command*; Figure 2B).. oh no! It doesn't ***reach*** it.. we need to pull it a little further so that they ***meet***.. ***perfectly***..

M11: We try 5 more, and see? (*they try randomly different values, but trial and error method fails*)

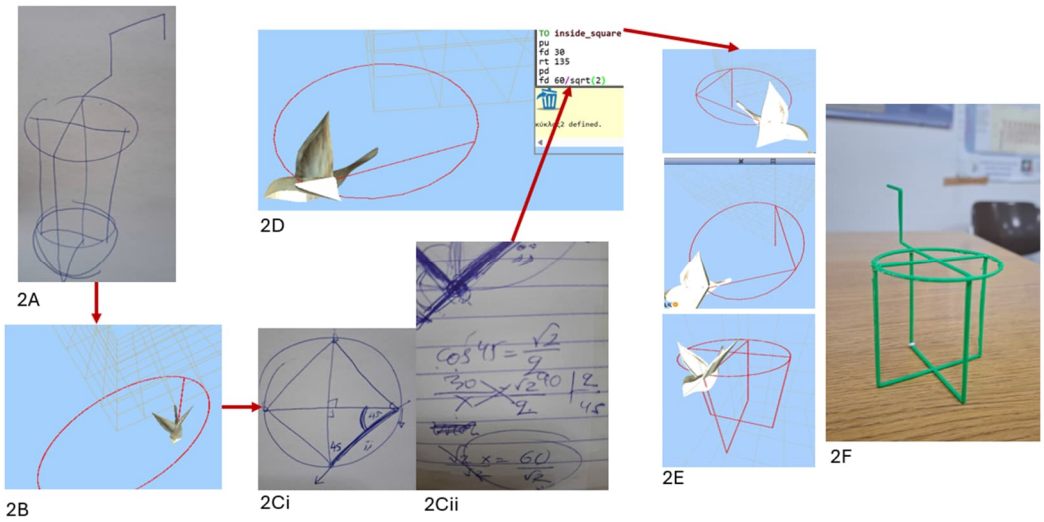
M11 suggests thinking about "what they would see ***as if they looked at it from above***". He draws the floor plan (Figure 2Ci) and M12 gestures to indicate the turn the entity will make as soon as it reaches the circle.

M12: Oh!! So, let's find the angles... (*They are writing down the angles*; Figure 2Ci)

M11: Look! These triangles are rectangular, aren't they? So, Trigonometry!!

They calculated the angles of each of the four right-angled triangles formed by the diagonals of the square (Figure 2Ci) and expressed some trigonometric relationships (Figure 2Cii). Soon, they found the appropriate relationship between the length of the side and the radius (Figure 2D), corrected the code and continued, completing the inscription of the square in the circle with repetition loops for the rotation-moving forward set (Figure 2E).

Figure 2: Sketch (2A), bugged digital model (2B), M11's floor plan (2Ci), Students' calculations (2Cii), Debugging (2D), Finalizing the digital model (2E) and 3D printed model (2F)



Another group of students, Group 1 (consisted of students M1 and M2) decided to construct a cup holder with a square base on which a cylindrical receptacle would be developed (Figure 4E). It is worth noting that they faced the exact opposite mathematical challenge to Group 7, namely inscribing a circle in a square that would form the basis of their model (Figure 4A). This brought up the relative position of the centres of the two shapes and the relationship between the radius of the circle and the side of the square. While findings from this case are congruent with the conference's aims, we have chosen, respecting page limitations, not to analyse them here. They would be, however, available for discussion during the presentation.

Critical Incident 2: A cylinder "emerging" from a circle

As shown in the following dialogue, Group 1 perceives the construction of the cylinder as the per-height-development through the repetition of equal circles.

M1: Look, its surface, let's say.. its bottom ***lies on*** the table, and then this ***sticks out***, like that... like a holder for the glass... I can't even describe it... (*His hands are joined together in a circle and raised parallel to the desk, like forming a cylinder*)

M2: And it raises... it ***comes up***, it ***rises***.. like it's ***getting taller and taller***.. so we need a lot of circles, but the same size all of them.

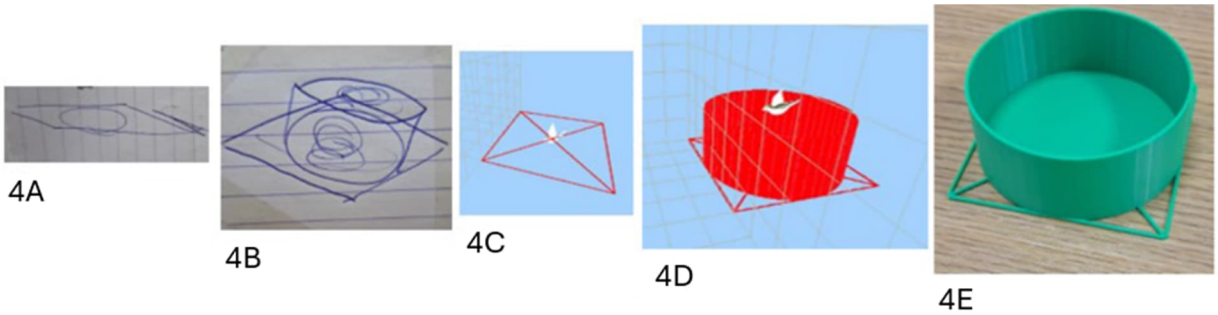
M1: Yes, and each new circle is just above the other (*From the level of the desk, M1's hand moves parallel and upward, but not in a continuous way, leaving tiny spaces as if "cutting" the air in many layers*), so that it ***rises***. (Figure 4B; *M2 sketches a rough draft. As can be seen, he thinks of it as a spiral that "rises" into the invisible interior of the cylinder*) ...

Throughout their attempt to create the cylinder, students gestured emphatically and used words such as "***comes up***," "***rising***," and "***sticks out***", reflecting in their speech the emergence of the three-dimensional object from its two-dimensional generator, the circle.

A 3D print failure (Figure 3) initiated an iterative cycle of refinement in the digital model. They specifically altered two parameters based on their physical observation: the spacing between circles (from 1 to 0.5) and adding the diagonals (Figure 4C) to the square-basis (for better support of the overall construction). This demonstrates a recursive loop where the tangible construction directly informed a new, more robust digital design.

Figure 3: Failed first attempt of 3D print

Figure 4: Sketches (4A, 4B), Iterated Digital (4C, 4D) and 3D printed model (4E)



DISCUSSION

This pilot study demonstrates the potential of enabling a form of exploratory mathematical sense-making, deeply rooted in bodily experience with multimodal, material, and digital interaction. First findings suggest that models acted as "Mathematical instruments" (Nemirovsky et al., 2020), synthesizing three dimensions:

1) materiality, 2) embodied practices: 2i) bodily actions (e.g., drawing a floor plan, hands' movement, gestures, etc), 2ii) verbal descriptions in spatial language and, 3) semantic expressions (mathematical formalism in coding, expressing geometric relationships). In response to our first research question, our results inform on how the synergy and interrelation of these three dimensions intertwine and shape students' perceptuomotor activity when attempting to make sense of Inscribability through this technologically mediated experience. The CF of "Inscribability" frames students' lived experiences, interpreting them as representations (embodied, linguistic, semiotic) expressed in a mathematical language (coding) and materialized to face the design problem/challenge (3D printing) to finally "give them a body". Regarding our second research question, the digital tools utilized in this study (programming and 3D printing) crucially mediated the transition from the 2D shape to the 3D solid object. They played a crucial role in bridging "micro-perception" (sensory, embodied interaction with the digital and physical model) and "macro-perception" (the collaborative production of a model/artefact). A continuous, bidirectional, and dynamic interaction was observed between the embodied elements (gestures, phrases, sketches), the coding activity and the feedback that children were receiving from the tools. For example, the case of the failed 3D printed model and group's revisions to the code, highlight a process of reciprocal transformation where students and tools mutually reshaped each other.

Limitations, Critical Reflections and Future Work

Our study has several limitations that inform the interpretation of its findings and outline directions for future work. The homogenic of the sample (all participants were male students at the same vocational school) and the short-term intervention within the context of a specific Design Thinking project limit, of course, its generalizability. Future work aims to extend this research to different learning environments, educational contexts and populations.

References

Abrahamson, D., & Bakker, A. (2016). Making sense of movement in embodied design for mathematics learning. *Cognitive Research: Principles and Implications, 1*, 33. https://doi.org/10.1186/s41235-016-0034-3

Alibali, M. W., & Nathan, M. J. (2011). Embodiment in mathematics teaching and learning: Evidence from learners' and teachers' gestures. *Journal of the Learning Sciences, 21*(2), 247–286. https://doi.org/10.1080/10508406.2011.611446

Angelides, P. (2001). The development of an efficient technique for collecting and analyzing qualitative data: The analysis of critical incidents. *International Journal of Qualitative Studies in Education, 14*(3), 429–442. https://doi.org/10.1080/09518390110029058

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects: State, trends and issues in mathematics instruction. *Educational Studies in Mathematics, 22*(1), 37–68. https://www.jstor.org/stable/3482238

de Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom.* Cambridge University Press.

Hasse, C. (2008). Postphenomenology: Learning cultural perception in science. *Human Studies, 31*(1), 43–61. https://doi.org/10.1007/s10746-007-9075-4

Ihde, D. (1990). *Technology and the lifeworld: From garden to earth.* Indiana University Press.

Nemirovsky, R., Kelton, M. L., & Rhodehamel, B. (2013). Playing mathematical instruments: Emerging perceptuomotor integration with an interactive mathematics exhibit. *Journal for Research in Mathematics Education, 44*(2), 372–415. https://www.jstor.org/stable/10.5951/jresematheduc.44.2.0372

Nemirovsky, R., Ferrara, F., Ferrari, G., Adamuz-Povedano, N., & Bracho López, R. (2020). Body motion, early algebra, and the colours of abstraction. *Educational Studies in Mathematics, 104*, 261–283. https://doi.org/10.1007/s10649-020-09955-2

Ng, O., & Chan, T. (2019). Learning as making: Using 3D computer-aided design to enhance the learning of shapes and space in STEM-integrated ways. *British Journal of Educational Technology, 50*(1), 294–308. https://doi.org/10.1111/bjet.12643

Ng, O., & Ferrara, F. (2020). Towards a materialist vision of 'learning as making': The case of 3D Printing Pens in school mathematics. *International Journal of Science and Mathematics Education, 18*, 925–944. https://doi.org/10.1007/s10763-019-10000-9

Ng, O., & Ye, H. (2022). Mathematics learning as embodied making: Primary students' investigation of 3D geometry with handheld 3D printing technology. *Asia Pacific Education Review, 23*, 311–323. https://doi.org/10.1007/s12564-022-09755-8

Psaros, H. (2025). Toward a metatheoretical dialogue on learning with digital technologies: The case of three postcognitivist approaches. *Journal of Theoretical and Philosophical Psychology.* Advance online publication. https://doi.org/10.1037/teo0000331

Roth, W.-M. (2016). Growing-making mathematics: A dynamic perspective on people, materials, and movement in classrooms. *Educational Studies in Mathematics, 93*, 87–103. https://doi.org/10.1007/s10649-016-9695-6

Vergnaud, G. (2009). The theory of conceptual fields. *Human Development, 52*(2), 83–94. https://doi.org/10.1159/000202727