THE UNREASONABLE EFFECTIVENESS OF DIVERGENT PERSPECTIVES IN COOPERATIVE PROBLEM SOLVING: A MICROGENETIC STUDY OF A GEOMETRY 3D CONSTRUCTION ACTIVITY

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'Heteroperception' is a socio-cognitive phenomenon where participants in a joint activity perceive shared resources differently. Rather than viewing such divergence as disruptive, we consider advantages of heteroperception for problem solving by investigating how 3 prospective math teachers co-construct a body-scale icosahedron. Data include multi-person mobile eye-tracking (MET) and synchronized MET-overlaid point-of-view videos. Across 3 microgenetic episodes, we trace a recurring triadic progression of heteroperception: implicit, explicit, resolved. We show how divergent perceptual foci coalesce discursively, tacitly negotiated through situated operations, to formulate actionable construction solutions. Heteroperception is not a bug but a feature, which teachers should anticipate, foster, valorise, and mediate.

INTRODUCTION

Wigner (1990) noted the "unreasonable effectiveness" of mathematical concepts in providing uncannily accurate descriptions of the natural world. We observe an equally unreasonable feature of collaborative work: participants may act jointly on shared resources while perceiving them differently). These differences, nevertheless, have been theorized as not necessarily compromising—and perhaps even facilitating—the solution of shared problems (Benally et al., 2022; Schwarz & Baker, 2017). However, little is known about the detailed mechanics of such "strength in diversity," in part due to methodological limitations of prior seminal efforts. As such, we still lack an empirically validated account of how this process unfolds. Perceptual sense making is important to study, because it offers researchers of collaborative learning an overarching axis for modelling the unfolding dynamics of individual experience during the emergence of mathematical discourse, in particular for curating the tacit meanings that students each bring to bear as they explore, execute, and evaluate together

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 1571-1578. PME.

candidate solutions to assigned problems (Goodwin, 2013). In this paper, we use multimodal learning data to trace the "life cycle" of divergent perception in collaborative 3D construction, addressing the following research question:

RQ. When participants in collaborative construction activities implicitly bear competing perceptual orientations on emerging structures, how do these perspectives interface to mutually inform each other? Specifically, how might we account theoretically, with evidentiary detail, for the apparent ontological shift from multiple idiosyncratic imaginary perceptions to a single collaborative material construction?

THEORETICAL FRAMEWORK

Heteroperception, a socio-cognitive construct, articulates apparent tensions between individual imagination and joint action (Abrahamson et al., 2025). Biological psychology suggests the evolutionary advantage of individual group members sustaining diverse orientations in executing efficacious solutions to collective problems, such as foraging (Ohlinger & Sasaki, 2026). For humans, cognitive linguistics, too, models a vital function for unshared knowledge in collective practice: meaning in situated communication is distributed across interaction, both common (sameness) and privileged (diversity) information must be pooled task-dependently across participants (Rączaszek-Leonardi et al., 2014, p. 1).

Such cross-species empirical evidence of functional heteroperception invites a view of language as facilitating co-operative *action* rather than signifying would-be shared *meanings* (Wittgenstein, 1953). Thus, in collaborative construction, apparent talk/gesture alignment may not mask misaligned perceptual orientations. Drawing on Palatnik and Abrahamson (in press), we model heteroperception by networking ecological dynamics (Chow et al., 2007) and co-operative action (Goodwin, 2013).

Attentional anchors: An ecological-dynamics view on individual perception

At the individual level, participants engage in perceptually guided action (Varela, 1999) as they imagine, build, and transform the environment. So doing, they spontaneously generate *attentional anchors* (AAs), imaginary perceptual structures that serve as handles for coordinating ecologically coupled action (Hutto & Sánchez–García, 2015). In co-construction settings, diverse AAs might come forth to avail group members in engaging the same material features. These AAs upgrade the materials' visibility for group members performing concrete operations (placing, holding, pointing).

The semiotic substrate: A co-operative-action view on group problem-solving

For collaboration to yield a new practice, participants' private AAs must become actionable within a public field of coordination—Goodwin's (2013) semiotic *substrate* of talk, gesture, and material moves. Palatnik and Abrahamson (in press) used Goodwin's discursive construct of professional vision to model this perception-to-practice transition as a discursive process, where participants align their perceptions through highlighting, coding, and elaborating on features in the shared domain of

scrutiny. Mobile eye-tracking point-of-view data enhances this line of research by providing access to individual participants' allocation of visual attention while their coordination coalesce, illuminating the tacit sociogenesis of perceptual consensus.

The secret triadic life of heteroperception

Collaborative efficacy can be supported by functionally sufficient overlap rather than by shared perception in a strong sense. Collaborators may coordinate under a thin layer of mutual intelligibility even as their perceptual structures for joint action differ in consequential ways. We organize analysis using a triadic model of heteroperception's rise and fall (Abrahamson et al., 2025): (1) *Implicit heteroperception*: An illusion of mutual intelligibility enables individuals to operate effectively in their common ground; (2) *Explicit heteroperception*: As the activity progresses, resource paucity intensifies while structural constraints tighten, precipitating communication breakdowns that surface the heteroperception, revealing individuals' imaginary construction goals as divergent action plans warranting repair, debate, and negotiation; and (3) *Resolved heteroperception*: By refining linguistic indexicality, an operational substrate emerges that, in turn, constitutes an ad hoc geometric discourse bearing disciplinary normativity.

Implicit to the heteroperception model of emergent professional vision is an appreciation of breakdown as productive moments when tacit variation becomes inspectable and negotiable—when the group is compelled to hone their semiotic means of coordinating their collaboration.

A 2D–3D MAPPING PROBLEM IN BODY-SCALE CONSTRUCTION

The empirical case is a small group of three prospective math teachers collaboratively constructing a body-scale icosahedron from a 2D image using dowels and plastic connectors (Rosenbaum et al., 2024). The task poses a 2D–3D mapping problem: participants must translate an opaque diagram into a 3D structure while coordinating local decisions (which triangle/edge/vertex) with global form. Although naturally collaborative, 2D source is perceptually polysemous (i.e. the struggle of imaginatively volumizing a tangle of flat lines according to its author's intention), affording diverse idiosyncratic Gestalts that each participant then "casts" multimodally into the shared working space as publicly inspectable virtual objects (cf. Murphy, 2004).

An emergent technical feature of building together a large voluminous object from a small sheet of paper is the consequent spatial distance between the instructions and the construction: in our case study, participants repeatedly examine the 2D image and then instantiate their perceptual constructions thereof through talk, gesture, and material action. This source-to-target spatial interval, where individual intentions may be lost in translation, compounds their heteroperception, rendering the coordination task into collectively making the invisible visible.

METHOD

Participants and setting

Participants were three bachelor-level students (prospective math teachers) enrolled in a West European university. The task, embedded in a larger pilot program on embodied mathematics, invited them to collaboratively assemble a body-scale icosahedron based on a 2D image provided on an instruction sheet.

Multimodal data sources

We collected mobile eye-tracking (MET) and point-of-view (POV) videos from all three participants as well as general audio recordings and multi-angle videography. Using MET technology, we generated three synchronized MET-overlaid POV videos, each showing a participant's gaze superimposed on their first-person view of the shared workspace. This configuration enables comparison of participants' contemporaneous MET-overlaid POV gaze streams on the same scene. Thus, one can examine how heteroperception brews under the semiotic surface until it boils over in material conflict, where MET provides the methodological link that visualizes the ontological alchemy of this instruction-perception-construction process.

Analysis: Interpretive microgenesis

We conducted a microgenetic qualitative analysis of selected episodes, moments when the group initiates a plan, attempts to realize it, encounters breakdown, and negotiates repair. We triangulated the MET-POV streams with talk, gesture and material actions to trace the purported triadic life of heteroperception: (1) Implicit heteroperception: verbal alignment (shared labels, agreement tokens) appearing with divergent gaze foci/trajectories and/or divergent embodied preparations; (2) Explicit heteroperception: breakdowns in action (stalling, incompatible placements) and escalations in meta-talk (clarification requests, challenges, re-descriptions); and (3) Resolved heteroperception: tighter indexicality (more precise pointing and naming), stabilization of a shared AA, and coordinated construction moves that "make sense" to all three participants.

FINDINGS: THE TRIADIC LIFE OF HETEROPERCEPTION

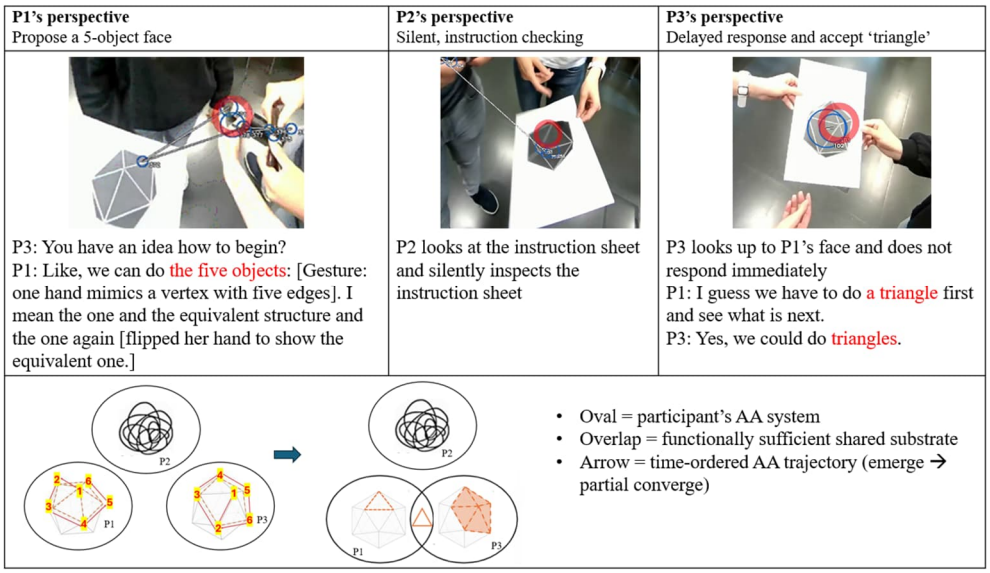
In this section, we present three episodes that together constitute one triadic life cycle of heteroperception, showing how participants' AAs are enacted in micro-operations and how divergence surfaces and is repaired.

Episode A: Implicit heteroperception

Early in the session, the group converges verbally on an initial shared AA: a *triangle(s)*. This term enters the semiotic substrate as a tenuous-yet-workable coordinating handle to begin construction. MET-POVs reveal a covert divergence beneath this verbal alignment: while the *triangle* is shared, each participant's gaze selects a different face of the 2D diagram as their first triangle. In other words, the group aligns on a label while diverging in what that label indexes. See Figure 1 for different AAs of P1 and P3, who see the "five objects" through different paths (see the numbered gaze-order paths in Figure 1). Despite their covert divergence, the activity proceeds, because the

material configuration's degrees of freedom still accommodate different imagined triangles as compatible with the ongoing construction—these idiosyncratic AAs can coexist without immediate repair.

Figure 1: Implicit heteroperception: simultaneous MET views and Venn diagram



Episode B: Explicit heteroperception

As the activity progresses, resource paucity intensifies while structural constraints tighten, so that the material configuration becomes less tolerant: connectors commit edges; partial assemblies constrain future possibilities; and the emerging structure demands agreement on local AAs. The group experiences breakdown when they have to connect an edge to the existing first triangle. The earlier polysemy can no longer be carried, since the AAs of P1 and P3 are structurally incompatible. Participants become cognizant of competing perceptions, because the configuration makes next moves impossible to reconcile across orientations.

Figure 2: Explicit heteroperception: simultaneous MET views and Venn diagram

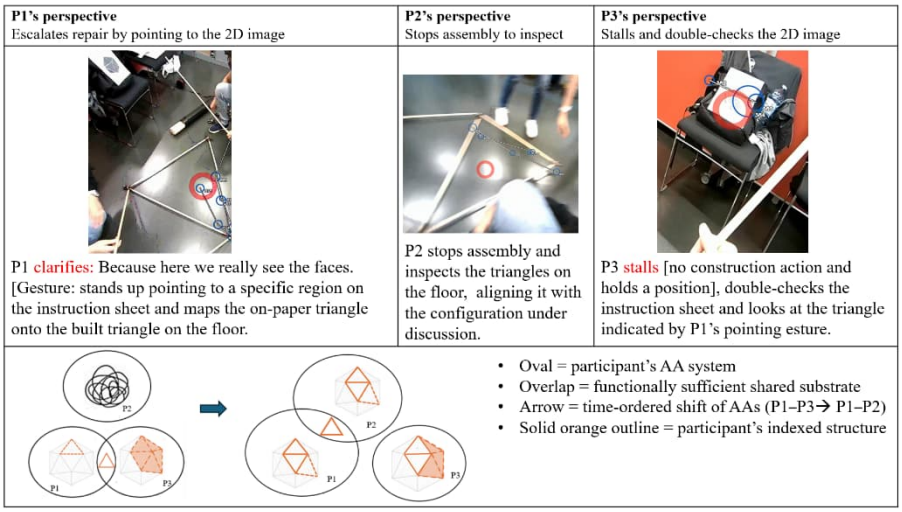
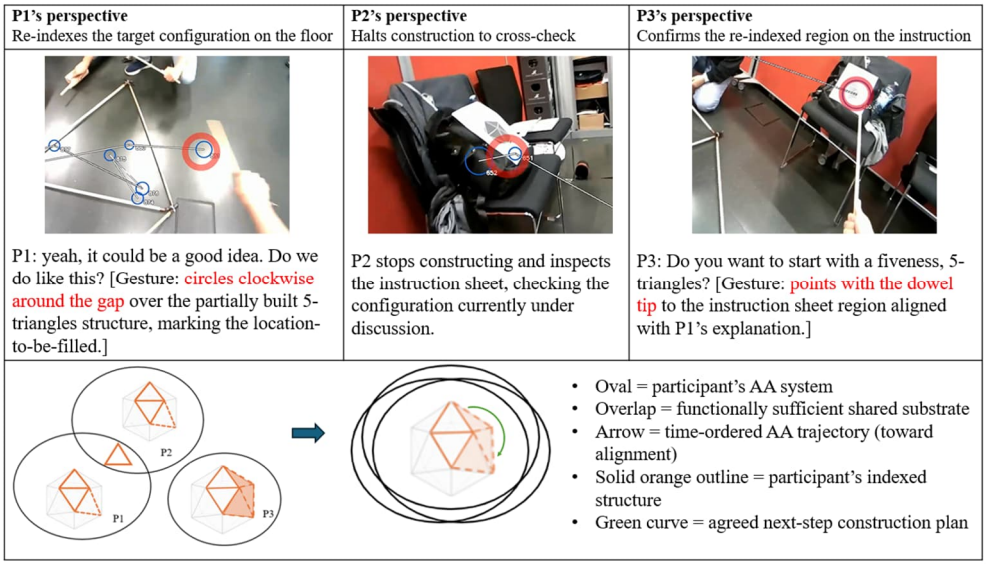


Figure 2 captures micro socio-material phenomena (e.g., stalling, attempts to force alignment via gesture and re-description) that surface the breakdown. The impasse emerges not due to any explicit verbal disagreement, but because the shared material field refuses to accommodate mismatched imaginations. The *Tower of Babel* quality emerges as participants continue speaking in shared tongues (triangle, five-objects, five-triangles) while their gaze data confirms they are visually gripping different structural possibilities. This is the transition to explicit heteroperception. The group is "forced" to engage in repair: to inspect what the terms are indexing for each participant and to renegotiate the construction plan.

Episode C: resolved heteroperception

The group repairs heteroperception by bootstrapping professional vision: more precise pointing, more specific reference to diagram features, tighter re-descriptions, and material demonstrations that calibrate meaning. P3 highlights and codes to specify the part being built (pointing to the 2D image while specifying it as "five-triangles"). P1 gestures a complex rotation, lifting the "five-triangles" structure off the page to demonstrate how it must be deployed in the 3D assembly.

Figure 3: Resolved heteroperception: simultaneous MET views and Venn diagram



This moment shows how the team makes the indexical grounds of five-triangles publicly inspectable and thus repairable. MET-POVs corroborate that the repair is not verbal alone (Figure 3): gaze and AAs increasingly converge. This convergence is sufficient to build a jointly agreed next-step construction plan and resume coordinated building.

A later locus of tension

Later in the session, similar tensions recur around what counts as the to-be-built pyramid: 5 objects vs. 5 triangles. The persistent recurrence of heteroperception

suggests its normative persistence as an iteratively reemerging feature of ongoing co-operative action. A term may be stabilized at one level (e.g., a workable meaning of "triangle") only to be destabilized at another (e.g., how triangles compose a pentagonal pyramid; how five faces meet; how local closure relates to global symmetry). Therefore, heteroperception appears to be not a single or rare event but an ever-recurring feature driving spirals of divergence and convergence: implicit compatibility, explicit breakdown, and partial resolution—repeated across substructures and across time.

DISCUSSION

Our findings support a paradoxical hypothesis: individuals collaborate productively not despite their different perceptions, but because of them. A group's pooled non-shared orientations toward a shared domain of scrutiny can be productive in solving collective problems (Surowiecki, 2004). We account for the ontological shift from covert perception to overt material structure by showing how idiosyncratic AAs are progressively externalized into the semiotic substrate, and then stabilized, through material operations and indexical repair. This shift dialogues curiously with Activity Theory (Leont'ev, 1981): whereas participants agree at the "top" level of shared activity (building an icosahedron) and "bottom" level of moment-to-moment operations (placing, holding, pointing), the activity's novelty prohibits tacit consensus at the "middle" level of actions, implicating the discursive root of heteroperception.

Implicit heteroperception enables tacit division of labour, parallel exploration, and varied attempts at sense-making, where each member brings to bear their idiosyncratic domain knowledge and epistemic practices. Divergent AAs can function as exploratory probes, enabling early progress under polysemy. Explicit heteroperception, however, may break the flow of diverse roles, exposing non-shared perceptions to critique, debate, and negotiation: once an individual perception is realized concretely as a material commitment, its installation reduces prospective degrees of freedom, rendering incompatible the materialization of competing perceptions. Still, in the microgenetic "slow motion" of materializing imagination, preliminary installations—one dowel at a time—might overlap with team-members' vying perceptions, seducing a temporary illusion of consensus, only to be disillusioned once the next dowel is placed. In resolved heteroperception, the group may transform divergence into a resource for learning by establishing technical indexicality supporting a more efficacious operational coordination of prospective actions. As such, heteroperception constitutes both a challenge and an opportunity of collaborative problem solving, at least in the case of collectively building a geometric structure in the absence of assembly instructions. Heteroperception appears inevitable whenever individuals collaborate on a shared task, yet this tension drives the refinement of how the object is grasped, talked into being, and materially realized.

Methodologically, we demonstrate the utility of MET-POVs for tracking the life cycle of heteroperception, where implicit attentional anchors are reified through multimodal displays, eventually manifested concretely. Such ontological alchemy transforms the

unreasonable effectiveness of collaborative problem solving into a traceable, socio-material process driven by, and then unravelling heteroperception.

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