CHILDREN'S PROBLEM-SOLVING STRATEGIES AS A MEDIATOR BETWEEN SPATIAL VISUALIZATION AND ARITHMETIC WORD PROBLEM-SOLVING ACCURACY

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Despite a focus on mechanisms underlying the spatial–mathematical link, few studies have examined processes mediating spatial ability and young pupils' one-step additive word-problem solving. Adopting the Potential Outcomes Framework, this study investigates if 'Correct Number-Fact' strategy use—a mental strategy relying on the accurate representation of a problem's quantitative structure—mediates the relationship between spatial visualization and problem-solving accuracy. Using data from 137 pupils (6- to 7-year-olds) in England while controlling for key covariates (visuospatial working memory, fluid reasoning, language ability, numerical ability, age and gender), the analyses showed that spatial visualization exerted a significant indirect effect on problem-solving accuracy via strategy use.

INTRODUCTION

As the spatial–mathematical link is well established, focus has shifted to understanding the underlying mechanisms of this link (Hawes & Ansari, 2020). Although experimental methods provide the strongest basis for establishing causality (Bailey, 2017), examining potential mediators is an important first step in informing the design and target of subsequent experimental work. Arithmetic word problem solving remains an overlooked area within mechanistic accounts of the spatial–mathematical link. This study aims to contribute to addressing this gap in the literature by examining if young pupils' spatial visualization (SV) relates to one-step additive word problem-solving accuracy via their usage of problem-solving strategies. The study focuses on SV because this skill—which involves mentally manipulating and transforming visual-spatial images—has a particularly strong, and possibly causal, relationship with mathematical achievement (Hawes & Ansari, 2020).

One-step additive word problems

One-step additive word problems describe part-whole relations solvable by a single addition or subtraction (Nunes & Bryant, 1996). Adopting Vergnaud's (1982) framework, this study focuses on problems comprising two *quantities* (internal attribute of an object, e.g., set size or length) and one *quantitative relation* connecting them. While both quantity and quantitative relations can be represented with a numerical value, quantitative relations are best represented as "directed numbers" (Vergnaud, 1982, p. 46). For example: 'Ann has 8 books. Bob has 3 more books than Ann. How many books does Bob have?' This problem requires finding an unknown

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 1339-1346. PME.

quantity (Bob's books) given a known quantity (Ann's 8 books) and the quantitative relation linking the known and the unknown quantities (+3).

Problem-solving strategies

Young children solve additive word problems using informal methods of direct modeling (e.g., pretend play) and counting (Nunes & Bryant, 1996), well before formal instruction in the corresponding mathematical knowledge and skills (Verschaffel et al., 2020). As proficiency grows, children transition from laborious external *counting strategies* (e.g., Count-All, Separate-From), through more economical (e.g., Count-On, Count-Down) and non-perceptual (e.g., nodding) counting strategies, and eventually to efficient *number-fact strategies* (Carpenter & Moser, 1982). While *recalled* number-facts are retrieved from memory, *derived* number-facts involve decomposing a problem into known facts, e.g., solving 6 + 8 via (6 + 6) + 2. Longitudinal studies suggest these number-fact strategies, especially derived number-facts, play a role in the connection between young pupils' SV and arithmetic skills (Casey et al., 2017).

Beyond counting and number-fact strategies, a critical factor is whether pupils correctly represent the problem's quantitative structure (Nunes et al., 2015). Unlike arithmetic tasks presented in number sentences, word problems require understanding the problem's structure of quantities and the quantitative relation. Although research on young pupils is limited, studies with older pupils suggest that the correct representation of the problem's solution-relevant structure mediates the relationship between SV and problem-solving accuracy (Boonen et al., 2013). By extension, SV may facilitate the translation of verbal descriptions into correct quantitative structures in young pupils; this needs to be investigated. Based on the reviewed literature, this study develops a new synthesis of young pupils' problem-solving strategies (Table 1).

Table 1: Problem-solving strategies for one-step additive word problems

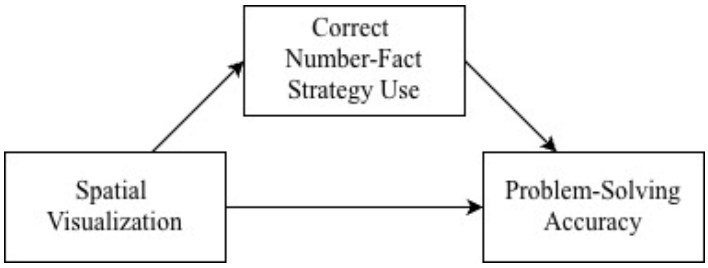
Strategy	Definition
Non-Quantitative	Not actively engage with the problem's quantitative structure (e.g., guessing without attempting)
Incorrect Counting	Incorrectly represent the quantitative structure using a counting strategy (e.g., counting-all for a subtraction problem)
Incorrect Number-Fact	Incorrectly represent the quantitative structure using a mental number-fact strategy (e.g., mentally subtracting for an addition problem)
Correct Counting	Correctly represent the quantitative structure using a counting strategy (e.g., separate-from for a subtraction problem)
Correct Number-Fact	Correctly represent the quantitative structure using a mental number-fact strategy (e.g., mentally adding for an addition problem)

THEORETICAL FRAMEWORK AND RESEARCH QUESTION

In the field of education, the most commonly used approach to mediation relies on a series of linear regressions to evaluate evidence for direct and indirect pathways (Baron & Kenny, 1986). However, this approach's key limitations are that it cannot accommodate nonlinear relationships or nonparametric models, and that it implicitly assumes sequential ignorability (i.e., the absence of unmeasured confounders), which is required for causal identification of the mediation pathway (Chi et al., 2022). To address these limitations, the present study adopts the Potential Outcomes Framework, which allows for the estimation of mediation effects in observational studies by simulating counterfactual scenarios based on observed data (Imai et al., 2010). While the non-experimental and cross-sectional design of the present study precludes causal conclusions, this approach provides more robust estimation than traditional linear methods through nonparametric bootstrap resampling and sensitivity checks used to evaluate assumption violations (Cuartas & McCoy, 2021).

This study investigates the following research question: to what extent does the frequency of children's (6- to 7-year-olds) 'Correct Number-Fact' strategy use mediate the relation between SV and problem-solving accuracy? Based on previous research on young pupils' arithmetic strategies (Casey et al., 2017; Möhring et al., 2017) and older pupils' mathematical word-problem solving strategies (Boonen et al., 2013), it is hypothesized that the frequency with which young pupils use 'Correct Number-Fact' strategies mediates the effect of SV on one-step additive word problem-solving accuracy (Figure 1). This study controls for factors known to affect SV and word problem-solving, namely, visuospatial working memory, fluid reasoning, language ability, numerical ability, age, and gender (Atit et al., 2022; Verschaffel et al., 2020).

Figure 1: Proposed mediation model



METHOD

Participants and procedure

Four schools in South East England were recruited via convenience sampling. From these, 149 Year 2 pupils were recruited using school-selected opt-in or opt-out parental consent. Twelve pupils were excluded due to task difficulty or withdrawal of assent, leaving a final sample of 137 pupils (no missing data). Pupils completed seven measures individually with the researcher in a quiet location at school, randomized

across two 30-minute sessions typically spaced an hour apart. The word-problem task followed Boonen et al.'s (2013) procedure. Problems were read aloud; pupils were provided 30 unifix cubes and a pencil and informed they could use them as necessary.

Measures

SV was assessed using the Wechsler Abbreviated Scale of Intelligence-Second Edition (WASI–II) Block Design subtest (Atit et al., 2022) and the Visual-spatial reasoning task (Hawes et al., 2019). These required pupils to recreate cube designs and mentally visualize parts-to-whole relationships, respectively. Word problem-solving accuracy and strategy use were measured using 24 one-step additive word problems adapted from Seo et al. (2025). Accuracy was scored binarily (0/1) per item and summed to create a total accuracy score (max = 24). Strategies were coded based on observable behavior and retrospective reports (Casey et al., 2017), using codes presented in Table 1. Visuospatial working memory (VSWM) was assessed using a digitized Corsi Block-Tapping task (Hawes et al., 2019), requiring pupils to replicate sequences of flashing dots. Fluid reasoning was measured using the WASI–II Matrix Reasoning subtest (pattern completion). Language ability was measured using the Wechsler Individual Achievement Test–Second UK Edition (WIAT–II^{UK}) Listening Comprehension subtest; this was prioritized over reading comprehension because the word problems were read aloud to the pupils (Spencer et al., 2022). Tasks included matching pictures to spoken prompts and generating words to match descriptions. Numerical ability was assessed using the WIAT–II^{UK} Numerical Operations subtest (arithmetic computation).

Statistical analysis

Mediation analysis was conducted with the 'mediation' package (Tingley et al., 2014) in R (version 4.4.0), using nonparametric 5,000 bootstrap simulations. The model included a composite SV score—derived via Principal Component Analysis (Seo et al., 2025)—as the predictor, problem-solving accuracy as the outcome, and the frequency of 'Correct Number-Fact' strategies as the mediator. Demographic (age and gender) and cognitive measures were included as covariates. All continuous predictors were standardized. Generalized Additive Models (GAMs) were used within the mediation framework to flexibly model the mediator and outcome equations using spline-based smooth terms (Harrell, 2015). Nonlinearity was assessed using the effective degrees of freedom (edf) of the smooth terms, with edf values substantially greater than 1 indicating departures from linearity. The average causal mediation effect (ACME) was estimated by comparing the mean of the composite SV score to a one-unit increase. Sensitivity analyses were conducted using a linear specification as a proxy model to evaluate the robustness of the mediation effect to potential unobserved confounding.

RESULTS

Preliminary correlation analysis (Table 2) revealed significant positive associations between SV and 'Correct Number-Fact' strategy use ($r = .55$, $p < .001$), and between

'Correct Number-Fact' strategy use and problem-solving accuracy ($r = .74$, $p < .001$), providing the empirical basis for the subsequent mediation analysis.

Diagnostic analyses indicated substantial departures from linearity in several predictors. In both the mediator and outcome models, SV (edf = 6.48 and 3.11, respectively), fluid reasoning (edf = 7.32 and 2.80), and numerical ability (edf = 3.18 and 4.76) exhibited nonlinear associations with the mediator and outcome. In contrast, VSWM did not show sufficient variability to support spline estimation, whereas language ability and age showed linear associations. Thus, penalized spline terms were retained for SV, numerical ability, and fluid reasoning in the final mediation models.

Table 2: Descriptive statistics and bivariate correlations (n = 137)

	1	2	3	4	5	6	7	8	9
1. SV	-	$.55^{***}$	$.44^{***}$	$.42^{***}$	$.26^{**}$	$.26^{**}$	$.55^{***}$	.02	.11
2. 'Correct Number-Fact' Strategy		-	$.74^{***}$	$.37^{***}$	$.32^{***}$	$.32^{***}$	$.64^{***}$	.15	$.20^{*}$
3. Problem-Solving Accuracy			-	$.35^{***}$	$.46^{***}$	$.46^{***}$	$.68^{***}$	$.24^{**}$	.14
4. VSWM				-	.16	.16	$.39^{***}$	.06	.07
5. Fluid Reasoning					-	$.42^{***}$	$.57^{***}$	.23	.02
6. Language Ability						-	$.36^{***}$	-.01	$-.17^{*}$
7. Numerical Ability							-	$.20^{*}$	.16
8. Age (years)								-	
9. Gender (0 = girls)									-
Mean (*SD*)	0.00 (1.00)	4.85 (5.98)	9.38 (6.96)	4.39 (1.21)	9.03 (4.22)	17.78 (3.89)	12.71 (3.26)	7.10 (0.30)	0.50 (0.50)

Note. $^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$.

A causal mediation analysis examined whether the frequency of 'Correct Number-Fact' strategy use mediated the relationship between SV and problem-solving accuracy. A significant indirect effect was observed (ACME = 0.07, 95% CI [0.01, 0.17], $p = .03$). The average direct effect (ADE) of SV on problem-solving accuracy was not significant (ADE = -0.03, 95% CI [-0.18, 0.07], $p = .37$). The total effect was also not significant (Total Effect = 0.04, 95% CI [-0.13, 0.14], $p = .66$), consistent with an indirect-only pattern of association.

A causal sensitivity analysis assessed the robustness of the mediation effect to potential unmeasured confounding. The ACME remained statistically significant under moderate levels of confounding ($\rho < .40$) and was robust to unobserved confounding

up to ρ = .50. Given that the model controlled for several key covariates, substantial unmeasured confounding would be required to fully account for the observed mediation effect.

DISCUSSION

To better understand the mechanisms underlying the spatial–mathematical link, this study adopted the Potential Outcomes Framework—an approach that remains underutilized in educational and developmental research (Cuartas & McCoy, 2021)—to examine whether 6- to 7-year-old's use of 'Correct Number-Fact' strategies serves as a potential mediator between SV and accuracy in solving one-step additive word problems. Results indicated a significant indirect effect but no direct effect, consistent with an indirect-only association. This pattern suggests that SV supports word-problem solving by enabling the use of mental strategies that rely on accurate representation of a problem's quantitative structure. Sensitivity analyses indicated that the mediation effect was robust to moderate unmeasured confounding, a particularly notable finding given that several key cognitive and demographic variables were already controlled for in the model. Importantly, additional exploratory analyses (not reported here due to space) suggested that other problem-solving strategies, including 'Correct Counting' strategies, did not mediate the relationship between SV and problem-solving accuracy.

This study's finding of an indirect effect is consistent with prior research showing that mental arithmetic strategies mediate the association between SV and arithmetic accuracy in young pupils (Möhring et al., 2024), as well as with studies demonstrating that accurate representations of a problem's solution-relevant structure mediate the relationship between SV and mathematical word problem-solving accuracy in older pupils (Boonen et al., 2013). Moreover, the present finding of an indirect-only association may help explain why some studies have reported weaker associations between SV and word problem-solving accuracy in young pupils (Nunes et al., 2009).

Together, these findings point to 'Correct Number-Fact' strategies as a promising target for leveraging the spatial–mathematical link in educational contexts and for future experimental investigations. The present findings highlight the potential value of embedding spatial supports within word-problem instruction. Rather than targeting SV directly or in isolation, young pupils may benefit from support in using a range of tools (e.g., drawings, part–whole models, number lines) that can scaffold the transition from counting-based strategies to number-fact strategies while also supporting the construction of representations of a problem's quantitative structure. Moreover, certain word problems may benefit more from the indirect association than others. Accordingly, it is important to expose young pupils to a wide range of problems, with explicit attention to how different problem types can be represented. Future intervention studies are needed to test whether such word-problem instructional approaches improve problem-solving outcomes.

Future research should examine whether the strength of this indirect association varies across different types of word problems. For example, Seo et al. (2025) propose a

framework for categorizing one-step additive word problems, which could be applied to develop a more nuanced understanding of the relationships observed in the present study. Furthermore, whereas this study aggregated derived and recalled number-fact strategies, prior research suggests that SV may relate differently to these two strategy types (Casey et al., 2017; Möhring et al., 2024), underscoring the need for more fine-grained classifications of problem-solving strategies in future work.

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