DESIGNING CURRICULUM-INTEGRATED MATHEMATICS INSTRUCTION IN VOCATIONAL CONTEXTS: A CASE OF ARITHMETIC SEQUENCES AND DEPRECIATION

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This study develops design principles and an instructional model for vocational context-based mathematics instruction to enhance foundational vocational competencies in vocational high schools. Utilizing Design-Based Research, we derived 12 design principles and a six-phase model grounded in curriculum integration and situated learning theories. Validation through expert review and field enactment confirmed significant improvements in students' perception of mathematical utility and practical application skills. This study contributes by demonstrating the function of mathematics as a practical tool within vocational contexts and providing a concrete instructional design framework.

INTRODUCTION

Modern industrial society, characterized by rapid technological shifts and complex job structures, demands practically competent individuals equipped with the ability to understand mathematical information, solve problems, and make quantitative judgments and decisions (Geiger et al., 2015; OECD, 2013). These demands emphasize numeracy not merely as computational proficiency but as the capacity to use mathematics meaningfully across vocational and real-life contexts (O'Sullivan et al., 2019). Accordingly, traditional concept-delivery-centered mathematics education is increasingly considered insufficient for developing the practical competencies required for workplace performance, calling for practice-centered approaches grounded in vocational contexts (Wedege, 2010).

This issue is particularly salient in vocational education. In South Korea, vocational high schools(officially referred to as 'Specialized High Schools' in the South Korean education system) aim to foster students' vocational competencies and job skills (Ministry of Education, 2022). However, mathematics instruction is still largely based on the same national curriculum used in general high schools (Kang & Tak, 2018). As an alternative, Kim and Oh (2012) highlighted the need for curriculum integration between mathematics and vocational subjects (i.e., specialized subjects) to enhance the practicality and career relevance of mathematics learning. Indeed, various studies have addressed curriculum integration with vocational subjects to ensure effective mathematics education in vocational high schools (Dalby & Noyes, 2016; Watters & Christensen, 2014; Xiao, 2023).

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Nevertheless, mathematics teachers continue to face difficulties in reconstructing vocationally meaningful problems and designing instruction that foregrounds the practical applicability of mathematics (Anderson, 2025; Roehrig et al., 2021). Moreover, theoretical foundations and actionable design principles and models to support such practices remain insufficient. Drawing on curriculum integration theory and situated learning theory, this study proposes a set of design principles and an instructional model for vocational context-based curriculum-integrated mathematics instruction and examines its enactment in a vocational high school case. This study addresses two research questions: (1) How are the design principles and an instructional model of vocational context-based curriculum-integrated mathematics instruction developed and structured? and (2) How do teachers and students respond to the proposed curriculum-integrated mathematics instruction in vocational contexts?

LITERATURE REVIEW

Based on knowledge organization through authentic inquiry (Beane, 1997) and perspectives on curriculum integration (Adler et al., 2000), mathematics instruction must be designed to position mathematics not as a mere subsidiary tool but as an equal partner in problem-solving. To achieve this, mathematical concepts must maintain a central status within integrated themes (Collins, 2022) and preserve their unique logical hierarchy (Drake & Burns, 2004). Furthermore, mathematical competencies should be organically linked to cross-curricular tasks, functioning as practical tools for decision-making (Burke & Lehane, 2023). To translate this integrated structure into authentic learner experiences, the application of situated learning theory is essential. As emphasized by Boaler (2000), instruction must be designed to enable learners to actively construct mathematical concepts while participating in real-world problem-solving. This process should utilize tools as mediators of thought (Greeno, 1998) and encourage learners to co-construct knowledge and identity through collaborative communities of practice (Lave & Wenger, 1998).

Vocational context-based curriculum-integrated mathematics instruction aims to bridge academic and practical knowledge to simultaneously foster mathematical thinking and vocational competencies (Dalby & Noyes, 2016). Research by Parr et al. (2009) has identified authentic problems, tool utilization, and collaborative structures as key mechanisms for realizing these objectives. Accordingly, instruction should be designed to help learners recognize the necessity of mathematics within vocational situations and strategically apply and transfer concepts mediated by tools and collaboration (Watters & Christensen, 2014).

RESEARCH METHODOLOGY

This study employed Design-Based Research (DBR) methodology, aiming to solve practical educational problems while simultaneously generating context-sensitive design principles and theoretical insights. According to McKenney and Reeves(2012), DBR is implemented through an iterative process of analysis and exploration, design

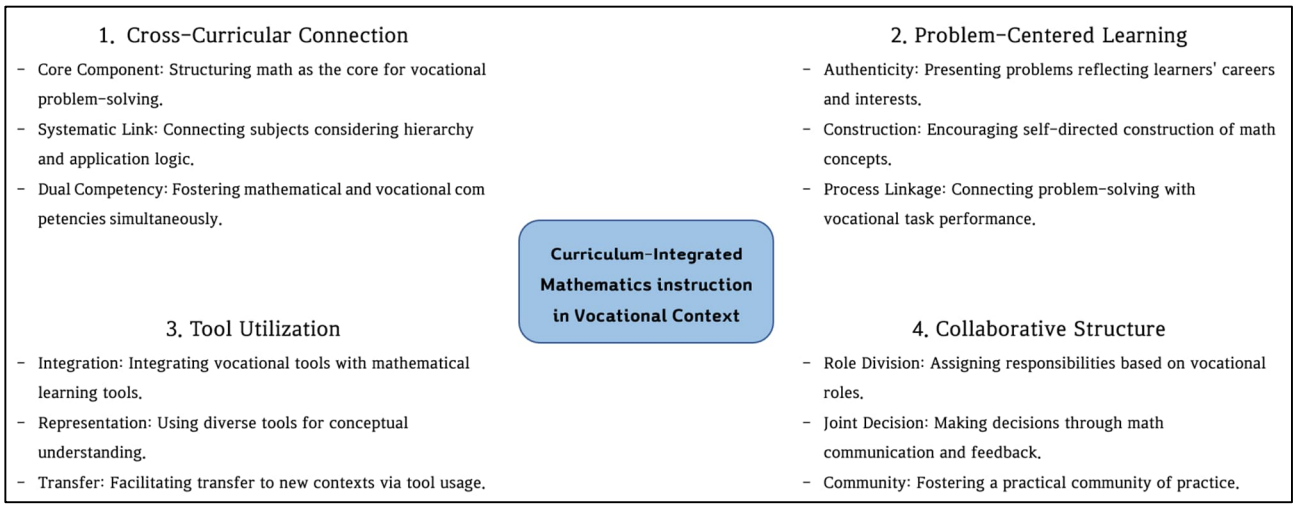
and construction, evaluation and reflection, and implementation and spread, which seeks to bridge the gap between practice and theory. The DBR process in this study consisted of three phases: First, initial design principles and an instructional model were developed through a literature review covering curriculum integration theory, situated learning theory, and existing cases of vocational context-based curriculum-integrated mathematics instruction. Second, internal validation was conducted using open-ended questionnaires with a panel of five experts in pedagogy, mathematics education, and vocational subjects, focusing on the theoretical soundness, clarity, contextual relevance, and feasibility of the proposed design to derive the first revision. Third, external validation was performed through the collaborative design and enactment of the instruction in a vocational high school involving one mathematics teacher and one vocational teacher. During this stage, the applicability and potential for improvement of the design principles and model were analyzed based on diverse data sources, including teacher open-ended questionnaires, interviews, instructional design meeting records, and classroom artifacts such as lesson plans, activity sheets, and student work. Additionally, semi-structured interviews were conducted with volunteering students among the six participating students to explore their learning experiences and perception shifts. All interviews and meetings were audio- and video-recorded and subsequently transcribed for analysis. The collected qualitative data were subjected to thematic analysis through semantic unit coding and categorization to identify recurring patterns and derive key themes supporting the finalization of the proposed design.

FINDINGS

Design Principles for Curriculum-Integrated Mathematics

Synthesizing theories of curriculum integration and situated learning with prior research on vocational context-based instruction, we derived 12 initial design principles categorized under four key elements to balance vocational authenticity with mathematical centrality. Specifically, the framework was grounded in Cross-Curricular Connection Structure, Problem-Centered Learning, Tool Utilization, and Collaborative Structure. Incorporating expert validation, the first revision focused on enhancing learner agency by addressing career interests and proficiency gaps, and ensuring terminological consistency. Furthermore, to realize legitimate peripheral participation, the collaborative structure was refined to emphasize role-based responsibility and mutual feedback beyond simple division of labor. External validation with field practitioners confirmed the ecological validity and scalability of the revised principles. However, responding to field demands for greater practical clarity, we further articulated abstract terminology to ensure ease of implementation. The final design principles, refined through this iterative process, are presented in Figure 1.

Figure 1: Final Design Principles for Curriculum-Integrated Mathematics instruction in Vocational Context.

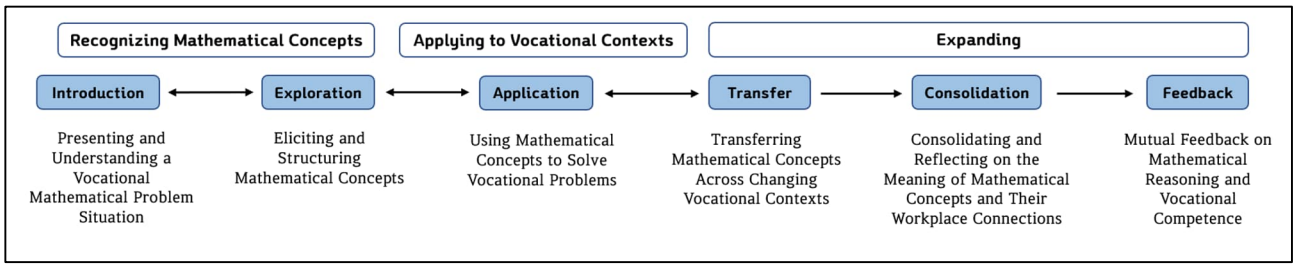


Instructional Model for Curriculum-Integrated Mathematics

Based on literature reviews including Parr et al. (2009) and Dalby and Noyes (2016), we developed an initial instructional model structured as 'Recognizing Mathematical Concepts (Introduction ↔ Exploration) ↔ Applying to Vocational Contexts (Application) ↔ Expanding (Transfer → Consolidation → Feedback).' This structure emphasizes recognizing and applying mathematical concepts within vocational tasks and extending them to new contexts. Furthermore, utilizing the BSCS 5E model (Boddy et al., 2003; Bybee et al., 2006) as a theoretical framework, the detailed procedures were organized to foster students' active exploration and reflection within vocational contexts. Through expert validation, the first revised model introduced a non-linear cyclic structure to overcome the limitations of linear steps and reflect the actual nature of problem-solving. Additionally, the revision strengthened learner-centered context design by emphasizing vocational contexts linked to students' careers and interests in the Introduction phase, and by clarifying multi-layered extensions based on task similarity and complexity in the Transfer phase. The Consolidation and Feedback phases were also supplemented to include students' subjective reflection and mutual feedback on vocational competencies, while terminology was refined for consistency and clarity to enhance teachers' ease of implementation.

Teachers participating in the field application positively evaluated the first revised model, noting that its structured phases facilitated lesson preparation and enactment while appropriately reflecting the characteristics of vocational high school students and vocational mathematics education. However, post-implementation surveys and in-depth interviews revealed the need for specific improvements to further enhance the model’s suitability for learner contexts and its feasibility. Incorporating this field feedback, particularly regarding the need to address gaps in students' prior knowledge, this study established the final instructional model as shown in Figure 2.

Figure 2: Final Instructional Model for Curriculum-Integrated Mathematics instruction in Vocational Context.



Design and Enactment: The Case of Arithmetic Sequences and Depreciation

At J High School, a commercial vocational high school, a Mathematics Teacher and Commercial Teacher collaboratively designed the vocational context-based curriculum-integrated instruction for second-year Marketing students through five offline consultation meetings. Through curriculum analysis, the teachers selected Arithmetic Sequences from Mathematics I and Depreciation(Straight-line Method) from Accounting Principles as the integrated topic and developed a lesson plan by connecting the achievement standards of both subjects(see Table 1). Throughout this process, the teachers exchanged feedback focusing on the authenticity of the vocational context and the connectivity of mathematical concepts, thereby refining the detailed design, including instructional tools and collaborative structures.

Table 1: Overview of Instructional case at J High School.

Topic	Management of Depreciation Using Arithmetic Sequences		
Subjects / Units	Mathematics I / Sequences Accounting Principles / Accounting for Assets	Grade	2nd year
Achievement Standard	[12MathI03-02] Understand the meaning of an arithmetic sequence and find the general term and the sum of the first n terms.		
	[AcctPrin02-02-03] Perform accounting treatments for transactions involving tangible assets.		
Lesson Flow	Understanding problems in accounting for assets related to arithmetic sequences → Understanding the concept of arithmetic sequences through accumulated depreciation and book value → Calculating depreciation using the general term of an arithmetic sequence → Solving various vocational problems using the general term of an arithmetic sequence		

The instruction was implemented following the six-phase model. In the Introduction phase, a problem situation was presented to explore changes in the asset value and accumulated depreciation of cargo trucks at a logistics company. In the Exploration phase, students investigated these changes using spreadsheets or statements of financial position on worksheets, deriving the concept of arithmetic sequences from the sequence of accumulated depreciation. In the Application phase, students efficiently solved accounting tasks utilizing the general term of arithmetic sequences. Subsequently, the Transfer phase addressed both analogous problems, such as calculating the book value of printing machines at a publishing company, and complex vocational tasks, including determining the useful life of assets. In the Consolidation phase, students discussed the relevance of their learning to vocational performance, including tangible asset value management and bookkeeping. Finally, in the Feedback phase, students calculated the negative terms of the sequence, interpreted their meaning within the vocational context, and made realistic judgments based on mathematical reasoning.

Teachers evaluated the model as highly practical, effectively reflecting the vocational education context. They noted that the structured phases facilitated lesson preparation and observed significant improvements in students' problem-solving skills and vocational basic competencies. While acknowledging the model's scalability to other subjects, they suggested supplementary activities in the introductory phase to address gaps in students' prior knowledge. Students also perceived a shift from rote calculation to practical utility. For instance, student M stated, “I realized mathematics is a useful tool for decision-making and analysis in accounting, …” expressing a strong intent to apply mathematical concepts to future vocational tasks. This indicates that the instruction successfully fostered students' recognition of the instrumental value of mathematics, leading to positive changes in both cognitive understanding and affective engagement.

CONCLUSION

Through the case of designing and applying vocational context-based curriculum-integrated mathematics instruction, this study explored design implications and practical possibilities for mathematics education to contribute to fostering vocational competencies. First, this study contributed to establishing a theoretical foundation for vocational context-based mathematics instruction by connecting curriculum integration theory and situated learning theory. Based on curriculum integration theory, this study sought to restructure the curriculum centering on tasks of real problem-solving within students' careers and vocational contexts, rather than treating mathematics and vocational subjects in isolation. Furthermore, by applying situated learning theory to mathematics education, this study emphasized the students’ learning experience through peripheral participation within a community of practice. Second, this study presented practical guidance for designing and implementing curriculum-integrated mathematics instruction linking mathematics and vocational subjects.

Specifically, this study holds practical significance by providing a case-based framework that supports teachers in designing vocational context-based instruction in a structured and consistent manner. However, given that this study reports on the results of instructional design and enactment based on a case, the refinement of design principles and models through repeated application across diverse vocational tracks and educational contexts is required in future research.

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