WHAT FACILITATORS EXPECT AND WHAT TEACHERS THINK: COMPARING CONCEPTIONS OF HIGH-QUALITY MATHEMATICS TEACHING

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This study examines the extent to which teachers' conceptions of good mathematics teaching correspond to facilitators' anticipations. Open-ended responses from 333 facilitators and 120 secondary school teachers were analysed using qualitative content analysis with inductive and deductive coding. Results show that both facilitators and teachers refer early on to deep-structures of instruction, indicating a shared research-based orientation toward instructional quality. However, facilitators systematically underestimated teachers' emphasis on these aspects. Overall, facilitators' and teachers' conceptions were more closely aligned than facilitators had anticipated, highlighting implications for participant-oriented PD and facilitator expertise.

THEORETICAL CONSIDERATIONS

The German QuaMath program (Developing Teaching and Professional Development Quality in Mathematics Education) aims to strengthen mathematics education sustainably from early childhood through upper secondary school (Prediger & Selter, 2024). Planned over ten years and implemented across 15 federal German states, the program builds on scientific developments conducted by twelve universities. In practice, three to five mathematics teachers from a single school participate in a professional development (PD) course. Several of these school teams form a school network that serves as the central unit for PD and is supported by a tandem of facilitators. These facilitators are professionalized through a multi-stage qualification process as part of the QuaMath program. With regard to scaling up, the program is intended to reach approximately 10.000 schools.

High-quality mathematics teaching is shaped by a variety of characteristics that have been the focus of both general and subject-specific research. A distinction between surface and deep structures of instruction has become increasingly important in describing and analysing teaching (Decristan et al., 2020, Klieme et a., 2009). Surface structures include easily observable features, such as instructional methods, organizational arrangements, and teacher-student behaviours. In contrast, deep structures refer to content-related engagement and cognitive interactions that are less directly observable but critically influence student learning (ebd.). In the context of mathematics education, the work of the QuaMath program is oriented towards five principles of high-quality mathematics teaching: Conceptual Focus, Cognitive Demand, Student Focus and Adaptivity, Longitudinal Coherence, and Enhanced

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Communication, which are closely linked to deep structures (Prediger et al., 2022). These *QuaMath principles* provide a coherent program-wide framework.

Facilitators play a crucial role in conveying the QuaMath principles in PD sessions. Research shows that facilitators' generic and content-related beliefs, together with pedagogical attitudes, shape their practices and the way they engage with teachers (Prediger et al., 2021). Understanding facilitators' conceptions of good mathematics teaching is therefore essential, particularly regarding alignment with the QuaMath principles. Furthermore, effective PD relies, among other things, on participant-orientation, meaning that facilitators must be able to anticipate teachers' prior knowledge, beliefs, and conceptions of teaching (Barzel & Selter, 2015). Such anticipation constitutes a form of diagnostic competence and reflects facilitators' professional vision. At the same time, facilitators often navigate multiple roles, serving as both classroom teachers and PD facilitators. Novice facilitators, in particular, must negotiate this transition, balancing their own professional beliefs with the perspectives of their participants (Perry & Boylan, 2018).

In a previous study (Zastrow et al., 2025), facilitators' conceptions of good mathematics teaching were largely aligned with the five QuaMath principles and emphasized deep-structure features, indicating an emerging professional identity as advocates of high-quality instruction. However, facilitators tended to assume that teachers would focus primarily on surface-structure features, suggesting a potential misalignment in perceived and actual participant understanding. This raises the question of whether facilitators' anticipations accurately reflect teachers' conceptions. Investigating this correspondence is theoretically and practically relevant: participant-oriented PD depends on facilitators' realistic understanding of teachers' conceptions, and underestimating teachers' attention to deep-structure features could influence the design and delivery of PD courses. Comparing facilitators' own conceptions with their anticipations and teachers' actual responses provides insights into facilitator expertise, the development of professional vision, and the dynamics of role transition within PD contexts.

RESEARCH QUESTION

Building on these findings, the present article addresses the following research questions:

1. How do teachers' conceptions of good mathematics teaching compare with facilitators' conception and their anticipations of teachers' conceptions?
2. To what extent do the characteristics identified by teachers align with the QuaMath principles, and how does this alignment compare with facilitators' own conceptions and their anticipations of teachers' responses?
3. What level of instructional understanding (surface versus deep structures) is reflected in teachers' responses, and how does this compare with facilitators' anticipations and facilitators' own conceptions?

METHOD

The study employed a qualitative content analysis with descriptive quantitative reporting. This approach was chosen to capture facilitators' and teachers' conceptions of good mathematics teaching based on open-ended responses and to allow for systematic, profile-based comparisons between facilitators' own views, their anticipations of teachers' conceptions, and teachers' actual responses. Data were collected at two different points within the QuaMath PD program. Facilitators completed the questionnaire at the beginning of their facilitator qualification during an initial workshop. Similarly, teachers were also surveyed at the beginning of the first PD module. Both groups responded to an open-ended question: "What do you think constitutes quality mathematics teaching? Write down three characteristics of quality.". In addition, facilitators were asked: "Consider potential teachers' responses to the question: What do you think constitutes quality mathematics teaching?".

N = 333 facilitators were questioned, 155 from primary schools (46.5%) and 178 from secondary schools (53.5%). Of these, 300 have studied a university teaching program in mathematics, 18 had not, and 15 did not specify. Teaching experience ranged from 1 to 40 years, with a median of 14 years. The median experience as a facilitator is two years and varies from 0 to a maximum of 34 years. A total of N = 991 teachers participated in the survey. The present study is based on an initial subsample of 120 responses of secondary teachers, which had been fully coded and analysed at the time of writing.

Qualitative data were analyzed using qualitative content analysis following the procedures outlined by Kuckartz and Rädiker (2023). In an initial step, an inductive coding scheme was developed to identify key characteristics of effective mathematics teaching as articulated by the facilitators. The development of categories was informed by a review of the literature on (mathematics) teaching quality. The final coding scheme comprised 30 categories. All responses were coded using this category system. This step aimed to capture the content and scope of facilitators' and teachers' conceptions without imposing predefined distinctions regarding instructional depth. For each group, percentages of coded segments were calculated relative to the total number of codings to account for differences in overall coding volume and to examine patterns of thematic emphasis across the three types of responses. Percentages reflect relative distributions of coded segments and do not represent person-level frequencies.

In a second step of analysis, the facilitators' and teachers' responses were classified with respect to the level of instructional quality: *Surface structures of instruction*, referring to observable, organizational, or methodological aspects of instruction (e.g., instructional methods), and *deep structures of instruction*, referring to aspects related to underlying learning processes and instructional quality (e.g., cognitive demand, conceptual understanding). Based on this classification, each response was aggregated and assigned a global rating indicating the relative emphasis on surface versus deep structures. Ratings were made on a four-point ordinal scale (1 = all responses refer to

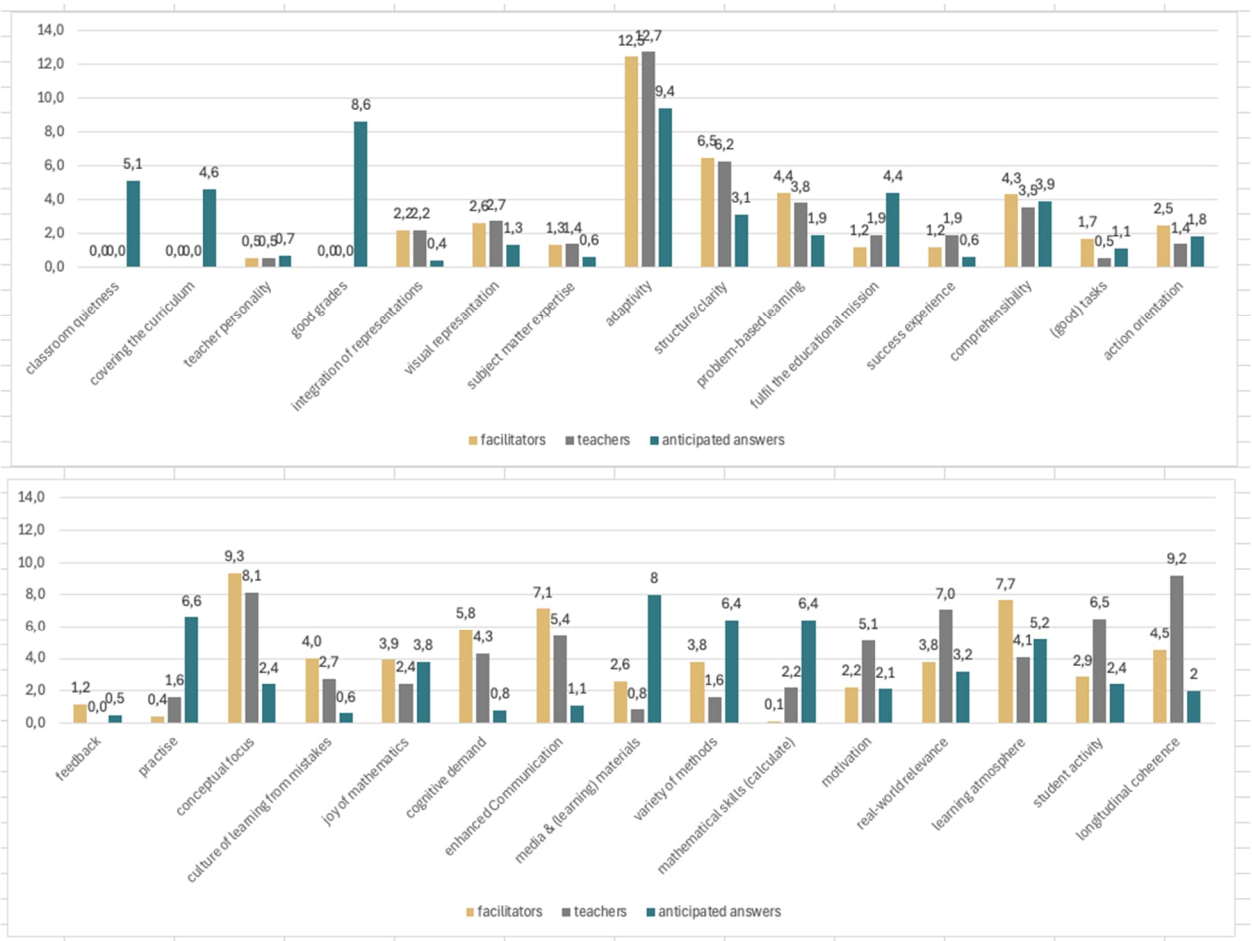
surface structures; 2 = the majority of responses refer to surface structures; 3 = most responses refer to deep structures; 4 = all responses refer to deep structures).

To assess interrater reliability, 25% of the data were independently coded by a second researcher. Cohen's kappa coefficients ranged from 0.77 for the facilitators' responses and 0.70 for the teachers' responses, indicating substantial agreement (Landis & Koch, 1977).

RESULTS

RQ1: *How do teachers' conceptions of good mathematics teaching compare with facilitators' conception and their anticipations of teachers' conceptions?* To address the research question, we examined the relative frequency of coded categories across the three types of responses. Figure 1 displays the percentage of coded segments assigned to each category for facilitators' own responses, facilitators' anticipated teacher responses, and teachers' actual responses. The categories in the figure are ordered from left to right according to the increasing difference between facilitators' responses and teachers' responses, with categories showing no difference (classroom quietness) on the left and large difference (longitudinal coherence) on the right. For improved readability, the figure has been split into two parts and arranged vertically.

Figure 1: Percentage of coded segments by category, ordered by increasing discrepancy between facilitators' and teachers' responses



This visualization highlights similarities and differences in thematic emphasis between the response types, providing a descriptive overview of how facilitators' conceptions and anticipations align with teachers' actual conceptions of high-quality mathematics teaching. Percentages reflect the relative distribution of coded segments within each group and do not represent person-level frequencies. Overall, facilitators and teachers' responses were similar across some categories, with both groups mentioning comparable aspects of good mathematics teaching similarly often (e.g., integration of representation, visual representation, subject matter expertise, integration of adaptivity, structure/clarity). The largest differences between facilitators and teachers emerged as follows: longitudinal coherence and learning atmosphere were emphasized more by facilitators, while student activity and real-world relevance were highlighted more by teachers. This indicates that while facilitators and teachers differ in their emphases on aspects of high-quality mathematics teaching, their responses are much more closely aligned than the facilitators had expected.

A comparison of the five most frequently mentioned categories (Table 1) shows that both facilitators and teachers cited adaptivity most often, and conceptual focus appeared among the top five for both groups. Interestingly, adaptivity was also the most frequently mentioned category in facilitators' anticipated responses; however, the remaining top categories in anticipated responses did not correspond to either facilitators' own responses or teachers' actual responses. This indicates that while facilitators accurately anticipate the prominence of adaptivity, they otherwise underestimate or mispredict the emphasis teachers place on other instructional aspects.

Table 1: The five most frequently mentioned categories

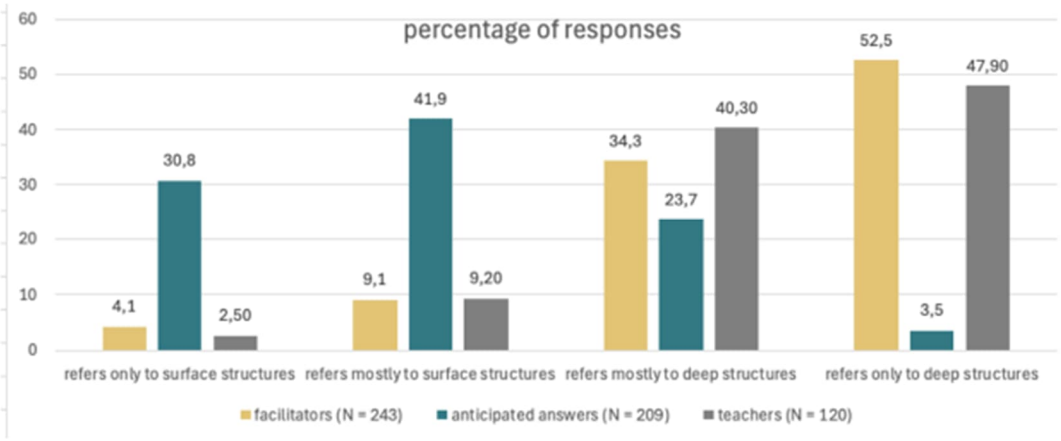
Facilitators	Teachers	Anticipated answers
Adaptivity	Adaptivity	Adaptivity
Conceptual focus	Longitudinal coherence	Good grades
Learning atmosphere	Conceptual focus	Practice
Enhanced Communication	Real World Relevance	Media & Learning Materials
Structure/Clarity	Student Activity	Variety of Methods

RQ2: *To what extent do the characteristics identified by teachers align with the QuaMath principles, and how does this alignment compare with facilitators' own conceptions and their anticipations of teachers' responses?* To further examine the alignment of responses with the QuaMath program, we analysed the extent to which coded segments could be assigned to the five central QuaMath principles. Results show that a substantial proportion of responses in both facilitators' own conceptions and teachers' actual responses referred to these principles: 39% of facilitators' coded segments and 33% of teachers' coded segments were aligned with the QuaMath principles. When examining the five most frequently mentioned categories (Tab 1), it

becomes apparent that both teachers and facilitators referenced three of the five QuaMath principles among their top five categories. In contrast, only 16% of the anticipated teacher responses were assigned to the QuaMath principles. These findings indicate that facilitators underestimated the degree to which teachers already reference the core QuaMath principles, while both facilitators and teachers themselves show a considerable focus on these program principles.

RQ3: *What level of instructional understanding (surface versus deep structures) is reflected in teachers' responses, and how does this compare with facilitators' anticipations and facilitators' own conceptions?* Following the findings from RQ1, which suggested that facilitators tended to attribute surface-level features to teachers' responses, we conducted a further analysis in which all respones, were classified as reflecting either surface or deep structures of instruction. Based on this classification, each response was aggregated and assigned a global rating indicating the relative emphasis on surface versus deep structures. The results, shown in Figure 2, indicate that there is no substantial difference between facilitators' own responses and teachers' actual responses: both groups predominantly emphasize criteria related to deep structures of instruction. In contrast, facilitators' anticipated responses differed notably, with a greater assumed emphasis on surface-structure features.

Figure 2: Distribution of Surface vs. Deep Structure Ratings



These findings indicate that facilitators' own responses and teachers' actual responses are similarly distributed with respect to surface and deep structures of instruction; however, facilitators misjudged teachers' thinking when anticipating their responses, expecting more surface-structure-oriented answers than teachers actually provided.

DISCUSSION

The present study examined the extent to which teachers' conceptions of good mathematics teaching correspond to facilitators' anticipations, extending previous findings on facilitators' own conceptions. Both facilitators and teachers reference to deep-structure aspects of instruction and several QuaMath principles early in the program, providing a shared foundation for engaging with PD content and fostering a

coherent, research-based understanding of high-quality mathematics teaching. From the QuaMath program's perspective, this constitutes a positive outcome. While these results provide insights into facilitators' self-perceptions, it remains an open question how these conceptions manifest in their actual teaching and facilitation practice, which would be a valuable direction for future research.

Furthermore the results indicate systematic biases in specific areas, where facilitators underestimate teachers' orientations. This is particularly relevant given that participant-orientation is widely recognized as a central aspect of facilitator expertise (Barzel & Selter, 2015). Anticipating teachers' conceptions represents a key form of diagnostic competence. Our results indicate that this skill is not yet fully developed in all facilitators, highlighting the need for targeted support and reflection to improve their understanding of participants' instructional thinking. From a theoretical perspective, the observed discrepancies between facilitators' anticipated and teachers' actual responses can be interpreted through social psychological theories of professional identity and group dynamics. In line with Optimal Distinctiveness Theory (Brewer, 1991) and related work on in-group differentiation (Marques et al., 1988), novice facilitators—many of whom are still active teachers—appear to navigate a transitional phase in which they renegotiate their professional self-understanding. As part of this identity work, facilitators may consciously or unconsciously anticipate teachers' responses as more surface-oriented than they actually are, thereby highlighting the distinction between their emerging role and that of classroom teachers. This differentiation can serve to justify their position as facilitators and their capacity to guide teachers toward deeper instructional practices, suggesting that these discrepancies reflect role development rather than simple misjudgement (Murray & Male, 2005; Tajfel & Turner, 1979).

Two key limitations should be considered: First, only a small subset of teacher responses was analysed, limiting generalizability. Second, it is unclear whether participants' responses fully reflect their true beliefs or whether they were influenced by social desirability, providing answers they believed the researchers expected. Future research could examine how facilitators' anticipations develop over time and whether experience in PD facilitation increases the accuracy of these perceptions, while also exploring methods to mitigate social desirability effects in data collection.

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