LEARNING TRAJECTORIES OF PATTERN AND STRUCTURE RECOGNITION REGARDING REPEATING PATTERNS

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The recognition of patterns and structures is a central component of early mathematical learning and development, yet empirically grounded insights into children's learning trajectories in patterning remain limited. This study explores how first-grade students approach repeating patterning tasks and how their learning pathways relate to a theoretically grounded intended learning trajectory (LT). Using a qualitative study design, eye-tracking interviews were conducted before and after video-recorded support sessions. The analysis combines gaze data and verbal explanations to capture students' patterning strategies. Results indicate a shift from unsystematic and pattern-based approaches towards a structural focus on the repeating pattern, evident in both eye movements and justifications, and consistent with the posited intended LT.

INTRODUCTION

The recognition of patterns and structures is highly connected to mathematical development and achievement (Mulligan & Mitchelmore, 2009; Wijns et al., 2020). The first year of schooling is particularly important, as promoting pattern and structure recognition at this stage can positively impact arithmetic skills and provide valuable support for children who struggle with mathematics (Mulligan et al., 2018; Papic et al., 2011). Patterns can be categorized in repeating patterns and growing patterns. Repeating patterns are a sequence, that can be reduced to a smallest subset that is called *unit of repeat* (Threlfall, 1999). The difficulty of a repeating pattern depends especially on the complexity of the repeating unit (Clements & Sarama, 2021; Reuter & Reinhold, 2025).

To analyse and foster children's abilities with pattern sequences, various patterning tasks are used. Common tasks are: *explain*, where patterns are described; *copy* (*pattern hidden*), where the pattern is reproduced either in view or hidden; *repair*, where a missing element is filled in or a wrong element is corrected; *extend*, where the next element of a pattern needs to be identified; *name the last* element, where children identified the final element in an extended pattern, with the extension represented by uncoloured elements; *translate*, where the pattern is recreated with different materials and *identify the unit of repeat*, where the children should divide the pattern into its units of repeat (Lüken & Sauzet, 2021; Papic et al., 2011). These patterning tasks vary in difficulty and can be classified into three levels. *Repair* and *copy pattern in view* are considered the easiest. *Extend* and *copy pattern hidden* are more difficult and the most difficult are *name the last element*, *translate* and *identify the unit of repeat* (Lüken & Sauzet, 2021).

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THEORETICAL BACKGROUND

Theoretical considerations and empirical findings regarding the development of pattern and structure recognition of young learners

Previous literature has described the development of pattern and structure recognition of young learners in empirically grounded models. For example, building on their studies on pattern and structure recognition Mulligan & Mitchelmore (2018) describe four successive levels of Awareness of Mathematical Pattern and Structure (AMPS) that characterise children's developmental progression in patterning and structural thinking. At the lowest level, children show limited pattern awareness, relying on one-to-one matching and struggling with recognising very simple patterns. At the second level, they begin to recognise simple patterns, identify units of repeat and notice basic relationships. The third level reflects an emerging awareness of fundamental structures, such as the underlying structure of the unit of repeat. At the highest level, children demonstrate an understanding of the generality of these structures and can flexibly extend and apply them, for example by systematically partition a pattern.

Clements and Sarama (2021) propose a developmental progression of children's understanding of patterns. It progresses from an intuitive and implicit sensitivity to repetition (*Intuitive Patterner*), through recognition and construction of simple repeating patterns (*Pattern Recogniser*), to the identification and use of increasingly complex units of repeat (*Patterner*). More advanced stages involve translating patterns across representations and describing them structurally (*Pattern Translator and Unit Recogniser*) and numerically (*Numeric Patterner*). Overall, the progression highlights a developmental shift from perceptual and action-based patterning towards explicit, structural, and representational understanding. Following the idea of a learning trajectory (LT), Clements and Sarama (2021) also suggest instructional activities aligned with each stage to support this development.

While the existing LT guides researchers and educators on the development of children's general understanding of patterns, it remains unspecific regarding repeating patterns. Hence recent studies focus more closely on children's strategies regarding repeating patterns (e. g., Baumanns et al., 2024; Lüken & Sauzet, 2021; Papic et al., 2011).

Lüken and Sauzet (2021) describe five strategies that kindergarten children use when solving patterning tasks: (1) *No reference to pattern* describes responses in which children show no regularity and reproduce only the gestalt of the pattern or select elements randomly or due to subjective reasons. (2) *Attention to singular characteristics* refers to a focus on isolated features of the pattern (e.g., colour, shape) without recognising its structure, often resulting in arbitrary ordering or inappropriate pattern types. (3) *Comparison and classification* involves comparing, matching, or classifying individual elements within or between patterns, for example through one-to-one correspondence or reference to the pattern's beginning. (4) *Focus on sequence* characterises the

succession of elements without identifying the underlying structure, often through cycling or rhythmic repetition and (5) *View of unit of repeat* represents a structural understanding of the pattern, in which children identify, name, and intentionally use the unit of repeat. Papic et al. (2011) also distinguish five patterning strategies that three- to five-year-olds use. (1) *Random placement* refers to the use of arbitrary elements when solving patterning tasks. (2) *Direct comparison* involves copying a pattern by matching elements one by one. (3) *Alternation* describes switching between elements (e.g., colours) without identifying the underlying unit of repeat. (4) *Basic unit of repeat* indicates the ability to identify the unit of repeat and use it to extend a pattern. (5) *Advanced unit of repeat*, the most elaborated strategy, involves identifying the unit of repeat and flexibly applying it, for example by translating the pattern into different materials. Baumanns et al. (2024) found similar strategies to Lüken & Sauzet (2021) and Papic et al. (2011) for children in Year 1 and differentiated them using eye-tracking instead of verbalisations. (1) *Identifying one unit of repeat of the pattern* refers to gaze patterns that focus on the unit of repeat, either linearly or with recursive backtracking to elements preceding the unit. (2) *Identifying one unit of repeat and validating/applying it* involves locating a unit of repeat and confirming it by comparison with other units of repeat in the pattern before extending the pattern accordingly. (3*) Looking at each element* describes a sequential, element-by-element gaze of the pattern. (4) *Unsystematic jumping over the pattern* is characterised by rapid, irregular gaze movements without a recognisable systematic approach.

Taken together, the different studies describe a comparable progression of patterning strategies ranging from unsystematic and element-focused approaches to a more and more structurally use of the unit of repeat. These results build the basis for a LT specific to repeating patterns.

Developing a learning trajectory for pattern and structure recognition processes in repeating patterns

Based on the reviewed literature, the LT developed by Clements and Sarama (2021) can be refined into an intended LT that supports the recognition of patterns and structures in repeating patterns and allows for differentiating children's patterning competencies. The developmental progression is posited by the authors as follows:

(1) *Pre-formal pattern recognition*: At this level, children's responses show no awareness of the underlying structure and their approaches are unsystematic, such as guessing, choosing elements based on personal preferences, or randomly. The gaze movements are irregular without a recognisable systematic approach.

(2) *Informal pattern recognition*: Children focus on single characteristics of patterns (e.g., colour or shape), and begin to notice similarities in repeating patterns. Their strategies are intuitive and implicit. Children can *copy*, *extend*, and *repair* simple AB patterns, but their strategies remain limited to surface features and do not involve systematic identification of the unit of repeat.

(3) *Formal pattern recognition*: At this level, children analyse patterns more systematically by comparing and classifying elements within or across patterns. Although most relevant pattern elements are considered, the underlying structure is not yet fully or explicitly represented hence lacking a systematic identification of the unit of repeat.

(4) *Pre-structural pattern recognition*: This level is characterised by a systematic, sequential analysis of the pattern, in which children attend to each element one after another. This is evident in behaviours such as rhythmically verbalising the elements of the sequence and in gaze patterns that successively focus on individual elements. However, children at this level still experience difficulties in explicitly identifying the structure of the pattern, still lacking a systematic identification of the unit of repeat.

(5) *Structural pattern recognition*: At the most advanced level, children identify and intentionally use the unit of repeat. They can *describe* patterns structurally, generalise the underlying rule, *translate* patterns across representations and materials, and correctly determine distant elements. Their strategies reflect a clear and flexible understanding of the repeating unit as the core structural component of the pattern.

The intended learning trajectory as presented is still incomplete as it describes only the developmental progression. But before adding the instructional activities, the first step will be to examine the theoretically grounded developmental progression empirically.

AIM OF THE STUDY

The aim of the following study is to examine if the refined intended learning trajectory helps to describe and understand pathways of first-grade students in recognising patterns and structures in repeating patterns by following the research question: Which learning pathways do first-grade students follow in recognising patterns and structures in repeating patterns and how do these relate to the intended learning trajectory?

METHODS

In order to investigate the research question, this study uses a qualitative case study design. The data material consists of interviews using eye-tracking and video-recorded support sessions. The interviews were conducted at three time points: before the support sessions (pre-interview), immediately after completing the support sessions (post-interview) and seven months after the support sessions (follow-up interview).

Participants and data material

The sample consists of 27 first-grade students from two primary schools in Germany, selected from six different classrooms. At the start of the study, all students took a standardised mathematics test to assess their mathematical development.

The eye-tracking interviews consisted of 19 patterning tasks covering eight activities: *explain*, *copy* (*pattern hidden)*, *repair with a missing element*, *repair with a wrong element*, *extend*, *name the last element*, *translate* and *identify the unit of repeat*. The tasks included three different repeating patterns (AB, ABC, ABCC). The patterns were presented in the form of colour patterns, geometric shapes and numerical patterns,

either pictorially or through concrete materials, such as wooden cubes. The pre- and post-interviews were identical in structure and content. The follow-up interview consisted of nine patterning tasks covering all eight activities. All eye-tracking interviews were video-recorded to capture the students' verbalisations. In addition, their eye movements were recorded using Tobii Pro Glasses 3. For the analysis of eye movements, raw gaze-overlaid videos were generated.

To promote development in pattern and structure recognition, five support sessions were conducted in small groups over a period of three weeks between the first two interviews. Each lesson lasted approximately 30 minutes and focused exclusively on repeating patterns using a variety of activities, such as *creating*, *extending*, *copying*, *translating*, *repairing* repeating patterns and *identifying the unit of repeat*. The support sessions consisted of whole-group plenary phases, followed by individual and pair work phases. The follow-up interviews were conducted seven months after the post-interviews, during which time participants received two additional instructional interventions focusing on *the field of twenty* and *patterns in symbolic representation.*

Data analysis

To examine students' pattern and structure recognition processes deductive qualitative content analysis was used (Mayring, 2000). The coding categories reflect the intended learning trajectory for recognising patterns and structures in repeating patterns: (1) *Pre-formal pattern recognition*, (2) *Informal pattern recognition*, (3) *Formal pattern recognition*, (4) *Pre-structural pattern recognition* and (5) *Structural pattern recognition*. Eye movements and verbalisations were coded separately. Eye movements were coded from task presentation until task completion. Verbalisations were coded subsequently, taking gestures into account.

In order to investigate the extent to which the refined intended learning trajectory helps to describe and understand the pathways of first-grade students in recognising patterns and structures in repeating patterns, we selected a case with average mathematical performance but starting at the lowest level of pattern recognition (*pre-formal*).

PRELIMANARY RESULTS: THE CASE OF GIOVANNI

At the beginning of the study Giovanni solved 11 out of 22 tasks in the eye-tracking interviews correctly. After the support sessions he solved 15 out of 22 tasks correctly.

In the pre-interview, when asked to *extend* a repeating pattern of geometric shapes using materials, Giovanni's gaze moves unsystematically across the pattern and fails to extend it correctly. He justifies his solution by stating that he looked at the pattern. His pattern and structure recognition is therefore classified as *pre-formal*. When asked to *name the last element* in a sequence of items, his gaze sequentially fixates on each element of the sequence, justifying his answer by rhythmically naming each colour in the repeating pattern. Additionally, Giovanni makes comparisons between elements, for example when *translating* repeating patterns based on perceived similarities between colours, alternating his visual focus between elements. This behaviour is

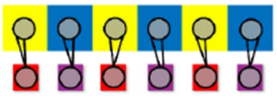
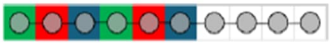
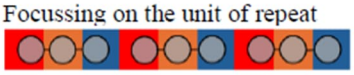
assigned to *formal pattern recognition*. An explicit focus on the repeating unit is not evident in the pre-interview. For example, he cuts a coloured cube pattern into individual cubes rather than the repeating units during a task to *identify the unit of repeat*. In the pre-interview, Giovanni predominantly shows lower levels of pattern recognition.

In the post-interview, Giovanni's approach has shifted towards more structural aspects of repeating patterns. Across several tasks (e.g., *copying a hidden pattern*, *extending* it and *identifying the unit of repeat*), his eye movements consistently focus on the units of repeat in the repeating patterns (*structural pattern recognition*), which he then uses to successfully extend the pattern or divide a coloured cube repeating pattern into its units of repeat. For example, when *extending* repeating patterns, his gaze focuses on the initial unit of repeat and continues the pattern accurately using the materials provided. He explains that he looked at the beginning of the pattern, pointing to the unit of repeat before extending the sequence. At the same time, eye movements still show fixations made element by element, for example when a pattern is being *repaired*. In these cases, Giovanni rhythmically recites the colours of the pattern sequence, which can be attributed to *pre-structural pattern recognition.* When *translating* repeating patterns, Giovanni’s gaze alternates between elements and he justifies his solutions based on comparisons between the elements (*formal pattern recognition*).

In the follow-up interviews (conducted seven months after the post-interviews), Giovanni mainly shows a structural approach. His eye movements primarily focus on the unit of repeat (*structural pattern recognition*), and in some cases, he even begins to copy the pattern before the pattern is removed (e.g., *copy pattern hidden*) and verbalises that he counted the colours and memorised the colours red, purple, green, green. When *repairing* repeating patterns, Giovanni’s gaze still shows fixations made element by element and he rhythmically recites the colours of the repeating pattern (*pre-structural*).

In summary, Giovanni’s case shows a developmental progression throughout the three interviews. In the pre-interview, he mainly relies on unsystematic and element-based approaches, for example when *extending* a repeating pattern using materials, his eye movements jump unsystematically over the pattern (*pre-formal*). In activities such as *naming the last element* of a sequence, his gaze focuses on each element of the repeating pattern and he justifies his solution by rhythmic recitation of the repeating pattern (*pre-structural*). In contrast, in the post-interview, Giovanni increasingly attends to more structural aspects in activities. For example, in *extending* repeating patterns, he focuses and uses the unit of repeat (*structural*), while still using element-based strategies in tasks like *repairing* a repeating pattern (*pre-structural*). In the follow-up interviews, he mostly focuses on the unit of repeat across activities (e.g., *extending*, *copy pattern hidden, name the last element*), indicating stable structural pattern recognition. When *repairing* repeating patterns, he still focuses on element-based approaches (*pre-structural*).

Figure 1: Giovanni's idealized gaze patterns and categorization to the pattern recognition level

Data material	Examples	Categorization to pattern recognition level
Pre-interview	Unsystematic and irregular gaze	*Pre-formal pattern recognition*
	Gaze alternates between elements 	*Formal pattern recognition*
	Focussing on each element 	*Pre-structural pattern recognition*
Post-interview	Gaze alternates between elements	*Formal pattern recognition*
	Focussing on each element	*Pre-structural pattern recognition*
	Focussing on the unit of repeat	*Structural pattern recognition*
Follow-up interview	Focussing on each element	*Pre-structural pattern recognition*
	Focussing on the unit of repeat 	*Structural pattern recognition*

Across the three interviews, Giovanni exhibits characteristics of four pattern recognition levels. However, no evidence is found for the *informal pattern recognition* level, in which children focus on single pattern features such as colour or shape.

DISCUSSION AND CONCLUSION

The study aimed to investigate whether the intended LT developed specific for recognising patterns and structures in repeating patterns fits to an empirically reconstructed learning pathway of a first-grader. The findings regarding the case of Giovanni are consistent with patterning strategies identified in previous research (Baumanns et al., 2024; Lüken & Sauzet, 2021; Papic et al., 2011) and align with documented developmental progressions in pattern and structure recognition processes (Clements & Sarama, 2021; Mulligan & Mitchelmore, 2018). Across the interview, a developmental shift in Giovanni's approaches becomes visible. In the pre-interviews, he mainly relied on unsystematic and element-based strategies, whereas in the post-interview and the

follow-up interview, he increasingly focused more on structural approaches and on the unit of repeat. This shift is reflected in both eye movement patterns and verbal justifications. Overall, Giovanni's case supports the intended learning trajectory in its general tendency, as his approaches to repeating patterns align with the proposed progression from less structured to more structural forms of pattern recognition. At the same time, Giovanni does not exhibit evidence of *informal pattern recognition* processes in the interview data. To further investigate whether and how *informal pattern recognition* occurs, ongoing analyses focus on the support sessions, where activity-specific interactions may provide additional evidence for this level. In this sense, Giovanni's case both confirms the learning trajectory in principle and highlights the need to examine learning pathways across a broader range of instructional contexts. In addition, further cases, especially with low mathematical performance, need to be examined.

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