COMBINING LINGUISTIC AND COVARIATIONAL REASONING ANALYSES OF STUDENTS' DESCRIPTIONS OF CHANGE

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Quantitative and covariational reasoning are crucial for understanding graphs and relationships between varying quantities in secondary and undergraduate mathematics. Research on covariational reasoning has led to a fine-grained classification of quantitative and covariational reasoning. While much of this research draws on analysis of students' spoken and written responses to tasks, the language in these responses has rarely been analysed from a linguistic perspective. This study combines analysis of quantitative and covariational reasoning with systemic functional linguistics (SFL). We identify linguistic features of descriptions of change that foreground simultaneous change of variables and how these changes are unfolding as students account for graph shape and unpack relationships between quantities.

STUDY BACKGROUND

Research on covariational reasoning has highlighted the variety and value of ways students reason about quantities changing together (e.g., Karagöz Akar et al., 2022). Oftentimes, analysis of students' covariational reasoning stems from analysis of data collected from clinical interviews or teaching experiments. This data-rich method of data collection allows for the careful construction of models of students' mathematics. It also affords the potential for linguistic analysis, which enables considering language as a meaning-making resource. However, there are limited examples of linguistic analysis of transcripts of data involving students' covariational reasoning. In this exploratory study, we illustrate how systemic functional linguistics (SFL) can complement and contribute to research on students' covariational reasoning.

We report on a collaboration between USA and UK researchers, who bring together different analytic approaches to identifying covariational reasoning forms and grounds on which these may be attributed to students. We ask, *What linguistic features of students' descriptions of change are indicators of covariational reasoning?* The analysis presented in this report focuses on pre-service teachers' spoken and written work on tasks designed to elicit covariational reasoning in teaching experiment settings. Using transcripts previously analyzed and published in covariational reasoning research (Liang & Moore, 2021; Moore et al., 2013), we combine extant analysis with insights from analysis of the transcripts using tools from SFL. Through this research, we extend the tools available to covariational reasoning researchers and add to topic-specific mathematical discourse research, which is of interest to researchers in mathematical language.

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THEORETICAL BACKGROUND

Covariational Reasoning

Research on students' covariational reasoning has proliferated in the past few decades due to its importance for constructing productive understandings of STEM topics (Karagöz Akar et al., 2022; Rodriguez et al., 2019; Steffe et al., 2014). This proliferation has resulted in a variety of frameworks (e.g., Carlson et al., 2002) and framework refinements (e.g., Johnson, 2015; Thompson & Carlson, 2017; Yu, 2024) that provide researchers with tools to characterize and differentiate various forms of student thinking. The focus of our work is not to provide novel characterizations of students' covariational reasoning, but instead consider the linguistic structure of students' verbal descriptions as they engage in covariational reasoning. We thus draw on the well-established and widely-used covariation framework provided by Carlson et al. (2002). More specifically, we draw data from several peer-reviewed research journal articles (Liang & Moore, 2021; Moore et al., 2013) that have used their framework to characterize students' covariational reasoning.

To support this exploratory study, we narrowed our focus to data excerpts that indicated students reasoning about directional change and amounts of change, which their framework terms Mental Action 2 (MA2) and Mental Action 3 (MA3), respectively. MA2 involves a student reasoning about a quantity increasing, decreasing, or remaining constant as another quantity increases, decreases, or remains constant. MA3 involves a student reasoning about how the increase or decrease of a quantity increases, decreases, or remains constant as another quantity increases or decreases. For instance, a student could reason about Quantity A increasing as Quantity B increases (MA2), and additionally reason that the increase in Quantity A increases as Quantity B increases by equal, successive amounts (MA3). In introducing amounts of change reasoning and relevant to our work, Carlson et al. (2002) emphasized that rate of change language such as "Quantity A increases at an increasing rate" is often a pseudo-indicator of amounts of change reasoning. They identified numerous participants that had learned to use rate of change language but could not unpack such language in terms of quantities' covariation and amounts of change.

Language and Covariational Reasoning

Researchers in Europe working on mathematical language and language-responsive teaching have drawn attention to the need for more studies of topic-specific mathematics discourse. In the case of functions, language-focused analyses have built on covariational reasoning literature to identify features of language about graphs and change relating to lexical, syntactical and discursive dimensions of language and the epistemic role of these features. Findings include links between conceptual unfolding and language becoming more or less condensed (e.g., Prediger & Zindel, 2017; Zentgraf & Prediger, 2024), disentangling descriptions of change and rates of change, including covariational and correspondence perspectives (Şahin-Gür & Prediger,

2018), and the covariational meanings afforded by different verb choices in descriptions of graphs and change (Kimber, 2024; Kimber & Smith, 2023).

Thompson and Carlson (2017) called for researchers to articulate why and how they attribute different levels of covariational reasoning to participants, and articulations in the studies above vary in how they draw on linguistic analysis. We consider there to be benefits of using SFL to develop a more detailed understanding of how linguistic choices in descriptions of change may affect the level of covariational reasoning attributed to these descriptions. Carlson et al.'s (2002) finding regarding rate of change language reflects this consideration, as such a finding provides motivation for further exploring the nuanced and currently ambiguous relationships between students' language and researchers' characterizations of their covariational reasoning.

Systemic Functional Linguistics

Systemic Functional Linguistics (SFL) frames language as a resource for making meaning, and texts as being produced through ongoing choices of word and word order. These choices affect the three interrelated metafunctions of language: to portray experience and activity (ideational function), to build relationships between the author and audience of a text and its content (interpersonal function), and to organise the message and cohesion (textual function, which facilitates the other two functions). The metafunctions operate at the level of individual clauses and sequences of clauses that form a text (Halliday & Matthiessen, 2004). SFL analysis has identified features of mathematical discourse (Morgan, 1996; Morgan, 2006; Schleppegrell, 2007), including topic-specific classroom discourse on graphs (Kimber & Smith, 2023).

Our focus is the ideational function, namely how change in one or more variables is expressed through activity of participants and the circumstances of that activity. The ideational function depends on choices of participants (nouns), how their activity is portrayed through choices of processes (verbs, including tense), and the circumstances of this activity (adverbs or prepositional phrases). We illustrate this in the context of our study, drawing on foundational work by Halliday and Martin (1993) and Halliday and Matthiessen (2004).

(1): *The gradient is positive between 2 and 6.* (2): $f(x)$ *is increasing between 2 and 6.*

Example 1 uses a relational process *is*, which portrays a static relationship between the participant *the gradient* and its property *positive*, and the circumstance is *between 2 and 6.* Example 2 uses a material process *increasing*, which portrays material activity of change of $f(x)$, so $f(x)$ is a participant in a world of action and ongoing change. The tense in Example 2 is the 'present-in-present' *is increasing*, the usual everyday form for a material process, and narrows down to activity in the here and now. The 'simple present' *increases*, would be less usual, but portray more general occurrences and habits (Halliday & Matthiessen, 2004). In descriptions of change, Kimber (2024) suggested that these contrasting tenses may have affordances for emphasising change

as ongoing at every point in an interval (i.e., continuous change) or only between end points of intervals, but that these affordances may differ for experts and novices.

In an exploratory analysis of students' descriptions of graphs for bottle-filling tasks (e.g., Carlson et al., 2002), we identified a variety of ways in which students described change using single clauses and sequences of clauses. Although the basic unit of analysis in SFL is the clause, analysis also depends on the surrounding text. In particular, the meanings afforded (and even foregrounded) by a text depend on how clauses are combined into sequences, including use of grammatical tools such as conjunctions (e.g., *and*, *but*, *as*, *because*) that integrate the meaning of these clauses more tightly (Halliday & Matthiessen, 2004). For example:

(3): *y increases* // and *x increases* // (Extension: the second clause adjoins *x*'s activity to the activity of *y*, but does not indicate causal or temporal connections.)

(4): *y increases* // *as x increases* // (Enhancement: The second clause expands on the first by qualifying the activity of *y* using the activity of *x*.)

As our research is concerned with ideas of related change of two variables, there are thus theoretical as well as empirical motivations for analysing how students use single clauses and combine clauses to portray and relate change of variables in covariational reasoning tasks.

METHOD

In this exploratory study, we analyse transcripts from a covariational reasoning (CR) perspective and an SFL perspective to identify overlapping results in data analysis and to identify how SFL might be able to support claims of covariational reasoning made by researchers. Because this work is exploratory, we use transcripts that (i) include evidence of CR and (ii) can easily be broken into clauses suitable for SFL. Specifically, we use transcript data from existing literature because they have already been analyzed and peer-reviewed to be sufficient evidence from covariation researchers to indicate students' covariational reasoning while still being a manageable size for SFL analysis. We note that these were transcripts of video data, and included descriptions of gesture and were supplemented by images of students' written annotations on task sheets. After compiling published transcripts of students' CR, we collectively narrowed our focus for this paper to three transcripts that afforded considering students' in-the-moment or anticipated directional change covariation and amounts of change covariation in different contexts. Doing so would afford opportunities for different tense use (e.g., anticipated CR vs. in-the-moment CR), different participants (e.g., given points vs. constructed points), processes (e.g., directional CR vs. amounts of change CR), and settings (different tasks).

In choosing our excerpts, we also narrowed our focus on instances in which students were constructing a graph or considering a given graph. Our reason for this was two-fold. First, researchers have identified that graphs are a rich and productive context to investigate students' covariational reasoning (Moore et al., 2013). Second, graphs

provide a context in which students can enact operations on the one-dimensional quantity of length, which mitigates the quantitative complexity of the context when compared to other quantities like area, volume, temperature, time, etc. These affordances made the graphical context amenable to an exploratory analysis.

RESULTS

In this section, we detail three specific findings to highlight how SFL complements existing CR analysis: the role of describing (i) simultaneity and (ii) processes in covariational reasoning and (iii) recognizing that students' smooth and chunky imagery can be on hold as students operate on the quantities they have constructed. Due to space constraints, we cannot share full transcripts of what was analysed, but we provide the following sample transcript with Kate and Jack, a pair of secondary preservice teachers (PSTs) working on graphing a quadratic function in a polar coordinate system after having constructed one in a Cartesian coordinate system (Moore et al., 2013, p. 467).

Excerpt 1

1 Jack: Yeah, we can see better, with both of them, both graphs, that the change in radius (*referring to quadratic*) for every radian further that the angle is increasing (*rotating his hand in successive rotations while spreading his index and middle finger apart*)… Um, the radius, every time is increasing at an increasing rate (*referring to quadratic*).

[*A few lines of interaction later]*

2 Jack: We could do equal changes in theta and then…

3 Kate: Like, if we looked at first these two then these two points (*indicating the points (9, 3) to (16, 4), and then (16, 4) to (25, 5)*), the change of theta here would be this, that length (*drawing an arc from (9, 3) to (9, 4)*). But then the change is radius would be up that line (*drawing a segment from the point (9, 4) to (16, 4)*).

4 Jack: Which is seven.

5 Kate: And then we have the same thing (*draws an arc from (16, 4) to (16, 5) and a segment from (16, 5) to (25, 5)*)…so you can see these black lines, the [change in radius] is increasing.

Simultaneity of Quantities Varying

It is difficult to break (1) into clauses, but we consider for every radian further that *the angle is increasing Um, the radius, every time is increasing at an increasing rate* to be a single clause with participant *the radius*, process *increasing* (present-in-present), and circumstances *for every radian further that the angle is increasing*, *every time* and *at an increasing rate*. In this long and complicated clause, Jack packs in the activity of both variables. Radius is the participant, and angle (along with its activity as a defining relative clause) is a circumstance. A further circumstance *at an increasing rate* adds information about the activity of the radius. However, while *for every* and *every time* indicate a corresponding change, they do not portray these changes as occurring simultaneously in the way that two clauses linked with as would.

In contrast, a secondary PST Lydia reasoning about circular motion in a study on students' covariational reasoning in contexts and graphs (Liang & Moore, 2021, p. 310), states the following, using *as* which indicates simultaneity:

> Lydia: So say we were to partition the blue into equal increments [*motioning her hand as if she was drawing the light blue segments in Figure 13b*] we would be able to clearly see the change in green will become larger and larger as we increase [*tracing along the longest dark blue segment*].

Lydia moves from material, if hypothetical, action *partition* to a phenomenon to be *seen*. This phenomenon is the change of two variables, described using two further clauses *the change in green will become larger and larger // as we increase*. The second enhances the first, using *as*, to portray the activity of the participant *change in green* as occurring simultaneously with, and in the context of, the activity *we increase.* (Linguistically this meaning is quite unclear, but from Lydia's tracing gesture, *we increase* could be inferred as the activity of the blue segments, or these could be being increased through the activity).

In summary, covariational reasoning emphasizes the importance of quantities changing together. SFL provides a grammatical lens to identify that simultaneity and see how it is emphasized in determining students' covariational reasoning. That is, the word *as* in the excerpt to emphasize the simultaneous variation of both angle measure and distance from the origin.

Processes of Quantities Varying

Returning to Excerpt 1, Kate and Jack (2-5) unfold the activity described in (1) using relational clauses with processes *be*, *is* and participants *change of theta* and *change in radius*, which have turned change from an activity into a noun. These choices background the process of change, though Kate returns to a material description (5) *these black lines, the [change in radius] is increasing*, presenting this as a phenomenon to be seen.

In summary, covariational reasoning emphasizes the important of quantities changing together. SFL provides a grammatical lens to support identifying and distinguishing the participant and process in a students' language. Stating that *the angle is increasing* (1) has a quantity (i.e., theta) as participant and *increasing* as the process describes covariational reasoning, whereas *graph increasing* or *black lines increasing* would require unpacking what quantities, if any, students are referencing.

Reasoning with States to Reason Covariationally

Returning to Lydia's transcript, her description of amounts of change covariational reasoning includes both simultaneity, using *as*, and processes *change will become larger*, though her language is anticipatory *would be*, indicating that she is not determining the covariational relationship in that moment.

In contrast, examining Jack and Kate's lines 2-5, we note that the students are describing radius characteristics at particular points -- the *change in theta here would be this, that length* and *then* the *change in radius would be up that line* (3). Here, the students are intentionally making a gross comparison of quantities (changes in radius lengths) to make a conclusion about what would happen if they *do equal changes in theta* (2). This language does not point towards simultaneity or processes as we previously emphasized in our analysis, but we infer from students' intentional choice of equally spaced points that the simultaneous equal change in angle measure is critical to the students' reasoning. In summary, when quantifying covariation in the form of comparing states for amounts of change, we move out of an arena where the smooth and chunky distinction is important because we are operating on something we've created. Thus, the change has to stop so we can reflect on it and further quantify it.

DISCUSSION

The goal of our study was to understand how SFL analysis could be used to complement research on covariational reasoning. In our results, we detailed two ways in which SFL could do so via looking at how language can point to simultaneity and processes. We also identified indications of associations between student actions suggesting anticipatory reasoning (e.g., the ability to mentally generate amounts of change without having to physically carry out activity) and the linguistic structure of their verbal descriptions. These associations could be important because they indicate a student who has constructed a relationship as an object of thought that they can then use to assimilate other situations. Overall, we believe that there is more research to be done to identify how to build on our understanding of students' covariational reasoning via linguistic analysis, including implications for classroom language use that supports covariational reasoning. Our transcript analysis focused on students that researchers have identified as reasoning covariationally, using limited SFL constructs, but future research could focus on less quantitative ways of reasoning students engage in when reasoning about dynamic situations or incorporate different SFL constructs.

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