GRAPH THEORY IN TEACHER EDUCATION: INVESTIGATING STUDENT TEACHERS' MOTIVATION AND INTENTION TO USE GRAPH THEORY IN LESSON PLANNING

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Graph theory is considered to have great educational and motivational potential for school teaching. However, so far, it has only been used peripherally in this context. This study investigates the impact of a graph theory lecture on student teachers' motivation and their intention to integrate this topic in future teaching. We examined 42 student teachers, divided into a group that attended a graph theory lecture and a control group. While no significant differences were found in motivation, attendees of the graph theory lecture showed a higher tendency to use graph-theoretical ideas in future teaching. Regression analyses show that students' intentions to use graph-theoretical ideas in future teaching are associated with graph theory lecture attendance, intrinsic motivation, and secondary school specialisation.

INTRODUCTION AND THEORETICAL FRAMEWORK

Graph theory is a mathematical discipline that has attracted growing interest in mathematics education in recent years, particularly due to its connections with computer science and computational thinking (Medová et al., 2022). Moreover, as part of discrete mathematics, there have been calls for greater emphasis to be placed on this in school teaching (e.g., Hart and Martin, 2018). At university level, occasional proposals have been made to integrate graph theory into mathematics education programmes (e.g., Hibi, 2025). These developments highlight graph theory as an area with great potential in mathematics education. Research areas dealing with graph theory at university level are often related to learning graph theory and focus on different aspects, such as reducing abstraction when learning graph theory (Hazzan & Hadar, 2005; Medová et al., 2022). Other studies evaluate the success of different approaches or applications in graph theory courses. Some of them also consider students' affective characteristics, such as joy or a positive rating (Coufal et al., 2021; Radmehr et al., 2023), or whether the tasks support students' problem solving (Wahyuningsih et al., 2020). Moreover, Milková (2011) reports that student engagement is high when they solve puzzles related to graph theory and combinatorial optimisation. At university level, therefore, some evidence exists for positive effects of teaching graph theory on both students' learning and their affective and motivational aspects. Furthermore, there is educational research in this area at pre-university level. For instance, in a summer course on network science, including graph theory, students' feedback was positive (Klimm & Maier, 2020). Likewise, Greefrath et al. (2022) showed that students were motivated to work on a graph-theoretical modelling task.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 179-186. PME.

Further research has produced numerous implications for introduction of graph theory content in schools, emphasizing that graph theory concepts and structures should be integrated into lessons more frequently (Prayitno et al., 2022; Uyangör, 2019).

The current state of research considers both students at school and at university as potential learners of graph theory. Mostly, the focus is on cognitive aspects of learning of graph theory. If considered at all, affective and motivational characteristics related to learning graph theory appear to be more incidental and qualitative observations rather than as the focus of systematic investigation. Thus, motivational factors of graph theory have not yet been investigated comprehensively. However, initial research has focussed on the influence of graph-theoretical concepts in primary school mathematics lessons on students' psychological characteristics. Windler (2018) showed that there were no significant differences in students' motivational orientations (including learning goals and work avoidance) between an experimental with a graph theory based intervention and a control group.

However, motivation is considered as important factor in learning processes. Looking at university level, Anthony (2009) concludes that "motivation was seen by both students and lecturers as the most influential factor related to levels of success" (p. 9). Accordingly, it is justified to assume this meaning when learning graph theory.

Although the potential of graph theory on both cognitive and affective characteristics is repeatedly emphasized, evidence for student motivation in graph theory is low. The goal of this article is hence to investigate effects of a graph theory lecture on students' motivational characteristics and their intention to use concepts for future teaching. We present the results of an initial study with university students. Hence, this study addresses the following research questions:

- RQ1: Does attending a graph theory lecture affect student teachers' motivation and/or intention to integrate graph theory in future teaching?
- RQ2: What factors might affect student teachers' intention to integrate graph theory content in future teaching?

METHOD

To answer the research questions, we conducted a quantitative questionnaire study with 42 undergraduate student teachers. At the University of Hildesheim, student teachers choose a specialisation lecture at the end of their mathematics bachelor's degree programme. Graph theory is one of these options. In the further analysis of the results, a distinction is made between two groups. We distinguished students who attended a graph theory course (n=19), and a control group, who attended a different specialisation lecture (n=23). The survey was conducted during the last week of the 2024 summer term, and comprised data on self-concept, motivation, and lesson design (Tab. 1). Further covariates are gender, school level (primary or secondary school) and average grade in mathematics studies.

Table 1: Scales and examples items statements (response options ranging from *strongly disagree* (0) to *completely agree* (4) on a Likert-scale)

Scale	Sample items of constructed statements	Cronbach's alpha
self-concept	I can rely on my maths skills.	0.81
intrinsic motivation	I am interested in the mathematical connections in the specialisation course.	0.92
instrumental motivation	The content of the specialisation course gives me a new perspective on mathematical content.	0.82
design of lessons	The contents of the specialisation course will support me in designing my lessons.	0.96

The items on the self-concept scale originate from Girnat (2019) and were adapted for this study. The mathematical self-concept scale is surveyed to see whether it affects intention to integrate graph theory in future teaching (RQ2). The study also aims to survey students' motivation. Two motivation scales based on Ryan and Deci (2000) were used. These distinguish between "intrinsic motivation, which refers to doing something because it is inherently interesting or enjoyable, and extrinsic motivation, which refers to doing something because it leads to a separable outcome" (Ryan & Deci, 2000, p. 55). We focus on instrumental motivation as part of extrinsic motivation, since current actions are seen as important for achieving future goals (Lens et al., 2009). In our study, items measuring instrumental motivation capture students' perception of the course content as being useful for their broader mathematical development. This includes broadening their mathematical horizons and fostering their ability to view mathematical content from different perspectives. In contrast, intrinsic motivation items refer to students' interest in the content of the specialisation course itself, regardless of any perceived instrumental value. The *design of lessons* scale measures students' perceived usefulness of the specialisation lecture for lesson planning by assessing whether they can imagine applying core ideas from the lecture and gaining ideas and impulses for lesson design.

After checking the internal consistency of the scales using Cronbach's alpha, all scales showed good reliability (α = .81–.96, see Tab. 1). Mean scale scores were then used for further analyses using t-tests and linear models to answer the research questions.

RESULTS

Table 2 shows the means and standard deviations of the scales used. While *self-concept*, *intrinsic motivation* and *instrumental motivation* are average, the values for the *design* scale are rather low.

Table 2: Mean, standard deviation and Cronbach's alpha of the different scales

Scale	M	SD
self-concept	2.35	0.54
intrinsic motivation	2.19	0.9
instrumental motivation	2.5	0.75
design of lessons	1.06	0.99

The following section analyses how these initial results are reflected in the research questions.

RQ1: In order to answer the first research question, we ran t-tests to compare the control and intervention groups. There are no significant deviations in the mean values of the *self-concept* (*graph theory*: *M*=2.46, *SD*=0.59; other: *M*=2.25, *SD* =0.48), *intrinsic motivation* (*graph theory*: *M*=2.48, *SD*=0.98; other: *M*=1.95, *SD*=0.76) and *instrumental motivation* (*graph theory*: *M*=2.57, *SD*=0.74; other: *M*=2.45, *SD*=0.75) scales. However, although mean *intrinsic motivation* is higher in the graph theory group than in the control group, the effect is not significant. Nevertheless, a significant effect can be observed for *design*, whereby Welch's t-test was carried out for unequal variances. There is a significant difference (t=3.5477, df=25.247, p<0.01) in *design* between students who attended the graph theory lecture (*M*=1.6, *SD*=1.12) and students who attended another lecture (*M*=0.6, *SD*=0.56). In order to determine the effect sizes, Cohen's d is used, showing a very large effect size (Cohen's d=1.2) (Sawilowsky, 2009).

RQ2: Linear models are used to analyse the data in order to answer the second research question (Rencher & Schaalje, 2007). For this purpose, the various factors are analysed in relation to the *design* variable as the dependent variable. The individual linear models were then refined through the inclusion of additional factors as predictors. As *design* shows a high positive Pearson correlation with the *intrinsic motivation* (r=0.63) and *instrumental motivation* (r=0.49) factors, these two factors are analysed. In addition, the t-tests related to RQ1 have shown that the group factor should also be analysed. If each the three factors mentioned is considered in isolation in relation to the variable *design*, remarkable R-squares are obtained, respectively. The factor *graph theory* (R^2 = 0.243***), *instrumental motivation* (R^2 = 0.225***) and *intrinsic motivation* (R^2 = 0.381***) all showed to be significant (p<0.001: ***, p<0.01: ** and p<0.05: *). *Intrinsic motivation* has the greatest explanatory power as an isolated factor. Next, these factors were combined in order to increase the explanatory power of the linear model. Combining *graph theory* and *instrumental motivation* leads to an increase of R^2 to 0.44 (*graph theory****, *instrumental motivation****). If *graph theory* and *intrinsic motivation* are considered in one linear model, R^2 increases to 0.48 (*graph theory***, *intrinsic motivation****). This model showed the highest coefficient of

determination among the models considered. When all three predictors were included simultaneously into the model, the explained variance slightly decreased ($R^2 = 0.47$). In this model, graph theory attendance remained a significant predictor, whereas neither instrumental nor intrinsic motivation showed statistical significance.

The two factors *instrumental motivation* and *intrinsic motivation* should now be considered together in relation to *design*. Integrated in one model results in R^2 of 0.36 (*instrumental motivation*, *intrinsic motivation***). The *instrumental motivation* variable is here no longer significant, R-square is lower than if only *intrinsic motivation* is considered (R^2=0.381). This could therefore be a suppression effect (Krus & Wilkinson, 1986), whereby *intrinsic motivation* is a variable that explains part of the variance that also explains *instrumental motivation*, but not vice versa. The *intrinsic motivation* variable might therefore suppress the influence of *instrumental motivation*. In summary, *intrinsic motivation* explains a large amount of the variance in *design*. For this reason, we consider the model of *graph theory* and *intrinsic motivation* for further analysis, as it has the highest coefficient of determination (R^2) among the models considered and does not include any additional interaction or suppression effects.

Furthermore, we tested for interactions. Looking at the interaction of *graph theory* and *intrinsic motivation* on the variable *design*, we obtain an R^2 of 0.55 (***), indicating that the factors complement and reinforce each other positively. Based on this model, other factors will be considered last. The variables *grade* and *gender* show no significant relations. However, including the school level *secondary school* (in contrast to *primary school*) is significant and increases the model to R^2=0.6 (*graph theory · intrinsic motivation + secondary school****). To better understand why this model accounts for the largest proportion of variance, we examine the contribution of each variable and their interaction in more detail (see Tab. 3).

Table 3: final regression model coefficients

Attending *graph theory*	school level	*design* = a · *graph theory* · *intrinsic motivation* + *b* · *secondary school* + *e*
attended	Secondary school=1	0.85 · *intrinsic motivation* + 0.31+ e
	Secondary school=0	0.85 · *intrinsic motivation* – 0.27+ e
not attended	Secondary school=1	0.36 · *intrinsic motivation* + 0.58 + e
	Secondary school=0	0.36 · *intrinsic motivation* + e

Table 3 shows the estimated coefficients of the final model, illustrating how each variable contributes to the dependent variable *design*. Here, *a* denotes the regression coefficient of the interaction between *graph theory* lecture attendance and *intrinsic motivation*, *b* represents the coefficient of *school level* (secondary vs. primary), and *e*

is the error term representing unexplained variance. The coefficient of the interaction is larger for students who attended the *graph theory* lecture (experimental group, *graph theory* = 1) than for those who did not (control group, *graph theory* = 0), indicating a stronger association between *intrinsic motivation* and *design* in the experimental group. The *school level* also contributes positively, with students specialising in secondary education showing higher *design* scores than those in primary education. Overall, the combination of *intrinsic motivation*, *lecture attendance*, and *school level* explains the largest portion of variance in the dependent variable (R^2= 0.6).

DISCUSSION

Our study addresses the research gap on motivation in graph theory. Like Windler (2018), who investigated the motivation of primary school children for graph theory based problems, we did not find significant differences between the two groups of university students on (*intrinsic* and *instrumental*) motivation. However, group differences were found for the variable *design*. Although attending a graph theory lecture has a statistically significant positive effect on students' intention to design lessons using its core ideas, the generally low mean scores suggest that this effect should be interpreted as students being less disinclined rather than strongly inclined to apply these ideas in future lessons.

The second focus was on the factors that influence student teachers' intentions to use the core ideas of graph theory in future teaching. Linear models were refined in several steps so that in the end a model was specified that can explain the variance of the *design* variable best. If students have a high intrinsic motivation for the content of the lecture attended, the graph theory students who also studying for secondary school have a high probability to use core ideas from their lecture for future teaching. The interaction of graph theory and intrinsic motivation on *design* plays a major role here. It should be mentioned that the influence of intrinsic motivation increases when attending the graph theory lecture than attending other lectures (see Table . 3). Using the core ideas learned to prepare for future teaching therefore depends not only on one variable, but on the combination of different variables. Students specialised on secondary school may benefit differently from the graph theory intervention, for example due to a different focus in teaching content. This could explain why the school level factor contributes to the model, which should be examined in further studies.

Overall, the results can be interpreted positively, as previous research also supports the integration of graph theory in schools (Prayitno et al., 2022; Uyangör, 2019). Students who attended a graph theory course were more likely to apply core ideas when designing their lessons compared to those who attended alternative courses.

However, this study also has some limitations. The research could be conducted again with a larger sample of students, extending beyond a single course, to obtain further research results. In the present study, the sample was small and all participants came from a single university. Conducting the study with a more diverse and larger sample

could help confirm that the results truly depend on the graph theory course and not on other factors.

CONCLUSION

Although there is a relatively large amount of research in the field of graph theory at university level, there is still a gap in research on motivation and the intention to use the content or core ideas later in school in relation to graph theory. This study attempts to gain initial insights in this area. Our analyses indicate a significant but modest effect of attending a graph theory lecture on students' intention to apply its core ideas in future teaching.

All in all, these results could serve as a plea to include graph theory content for student teachers in their degree programmes. The results highlight the potential of graph theory to support student teachers in applying core mathematical ideas in lesson design. In this way, the demand to introduce this content into mathematics lessons could be met by offering this content during the degree programme.

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