MATHEMATICAL GIFTEDNESS IN THE CLASSROOM: A CASE STUDY EXPLORING THE INTERACTIONAL DYNAMICS

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This study explores how mathematical giftedness is actualized through interactional dynamics in the mathematics classroom. Adopting a single-case study design, we analyzed a critical episode in which a mathematically gifted student's new insight was introduced and adapted into the classroom. The analysis identified three distinctive phases, each involving diverse interactions among the gifted student, teacher, classmates, and shared mathematical context. As an initial attempt to elucidate the socio-dynamic nature of mathematical giftedness, this study provides insights into how each classroom element contributes to the actualization of mathematical giftedness.

INTRODUCTION

In recent years, giftedness and gifted education have become a global topic of interest across several disciplines. In the field of mathematics education, growing interest in this area since the early 2000s (Singer et al., 2017) has led previous studies to elucidate the cognitive characteristics of mathematically gifted students (e.g., Sriraman, 2003; Mora et al., 2022), primarily focusing on giftedness as an internal individual trait. Although it is recognized that giftedness is a socially constructed concept emerging through interactions with the environment (e.g., Brandl, 2011), the social dimension of mathematical giftedness remains unexplored. Furthermore, it is sometimes argued that students have difficulty with the manifestation of their creativity within the constraints of a classroom setting (Singer et al., 2017), suggesting a gap between individual potential and the classroom reality.

The complexity of classroom setting has led researchers to restrict variables and study decontextualized creativity (Beghetto, 2019). However, Beghetto suggests that it is necessary to approach the socio-dynamic nature of creativity in the classroom by addressing such complexity. By the same token, this study aims to shed light on a socio-dynamic aspect of mathematical giftedness in a complex classroom interaction.

THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

Socio-dynamic perspective on mathematical giftedness

Early psychological studies on giftedness in the 20th century primarily focused on general intelligence as an internal trait of individual (Lo & Porath, 2017). Later, this perspective expanded to recognize the domain-specific and developmental aspects of giftedness. Consequently, it came to be understood as a multidimensional concept that manifests through diverse trajectories (e.g., Renzulli, 1978). More recently, researchers

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have acknowledged giftedness not only as an internal trait but also as a socio-dynamic construct that emerges through continuous interactions between individuals and their environments, corresponding to the view of systemism (Lo & Porath, 2017).

Beghetto (2019) argues that balancing individualistic and social perspective is crucial in the research of creativity. He points out that those focusing solely on individual traits tend to disregard the socio-dynamic dimension as "statistical noise" (p. 474), whereas others emphasizing general strategies or processes that lead to creative outcomes beyond the individual often overlook "the unique contributions of particular individuals or the dynamic features of a particular situation" (p. 474). Addressing these gaps, Beghetto highlights the dynamic and developmental nature of creativity. From this perspective, an individual's creative insight initially emerges in unpredictable form–since uncertainty is central to creativity (Beghetto, 2019; Sriraman, 2022)–and its value is gradually elaborated through social interaction.

Such a shift in how giftedness and creativity are understood has had a slight but discernible impact on research in mathematics education (e.g., Brandl, 2011; Sriraman, 2009). For instance, Sriraman (2009) illuminates the nature of mathematical creativity through the interview with mathematicians, by adding the role of social interaction into the process of their creative works. Moreover, it is noted that innovation occurs only when one's creative work gains recognition within a community.

Building on these perspectives, this study focuses on the socio-dynamic nature of mathematical giftedness as it emerges through interaction within the surrounding environment, which is further elaborated in the next section.

Iteraction between mathematical giftedness and surrounding environment

Drawing on Csikszentmihalyi's systemic view of creativity, Sriraman (2009) describes mathematicians' creative achievement as a sociocultural process involving dynamic interactions among the individual, domain, and field. The domain is "a cultural organism that preserves and transmits creative products to other individuals in the field" (p. 17). This corresponds to Brandl's (2011) "environment 2" (p. 37) namely, a particular mathematical context. Creativity occurs when individuals introduce new insights into a given domain. These variations are then evaluated to determine their eligibility for acceptance into the domain by the field, which "consists of people who have influence over a domain" (Sriraman, 2009, p. 17). Such a group of people are called "gatekeepers" (Csikszentmihalyi, 2014, p.52), who have a privileged role in selecting which variations are worth being incorporated into the domain.

Classroom environment provides a distinctive site to examine these dynamics, as it constitutes a sociocultural setting where participants' roles and instructional flows are largely predetermined, yet underpinned by inherent uncertainty (Beghetto, 2019).

However, how variations produced by mathematically gifted students interact with the classroom environment including a teacher, classmates, and shared mathematical context remains unexplored. While previous research has illustrated the individualistic

aspects of mathematical giftedness or creative process beyond the individual, there is still a gap in understanding how such potential is actualized through social interaction.

Research questions

As an initial attempt to elucidate the socio-dynamic nature of mathematical giftedness, the current study empirically tracks the interactional trajectory of a mathematically gifted student's idea, specifically aims to address the following research questions.

- Through what process is the variation produced by a mathematically gifted student adapted into the sociocultural setting of the mathematics classroom?
- How do mathematically gifted students and introduced variations interact with the classroom environment during the process of adaptation?

METHODOLOGY

Research design

This study adopted a qualitative single-case study design to elucidate an unexplored nature of mathematical giftedness. As Yin (2018) notes, a single-case study is appropriate when the case is unique and revelatory. This study especially focused on a critical event during 2 months of observing mathematics lessons covering the whole instructional unit of "the function $y = ax^2$."

Participants

The survey was conducted at a junior high school affiliated with a national university in Japan. It is recognized as one of the most academically competitive schools in Japan and offers a unique environment in which gifted students' abilities and creativity are supported rather than suppressed, as a pilot site for a project by Japan's Ministry of Education to explore an effective educational environment for gifted students.

This study focused on a ninth-grade class. Sora who demonstrated exceptional intention and mathematical ability not only at school but also outside school was identified as a mathematically gifted student there. At the time of the survey, he had competed in the Japan Junior Mathematical Olympiad (JJMO) since his first year of junior high school and had advanced to the final round for two consecutive years, placing him in the top 3% of participants nationwide who qualified for the JJMO finals.

Data collection and analysis

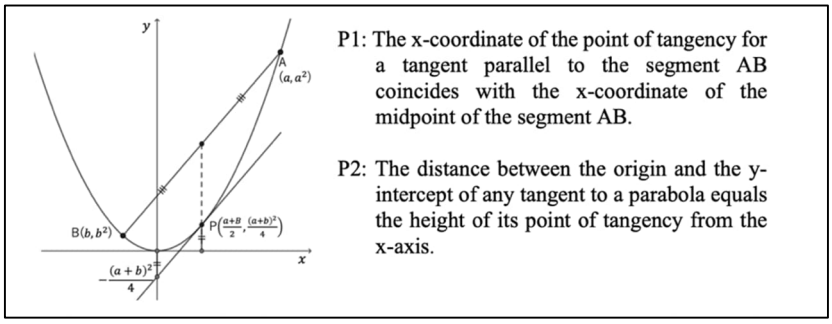
Data were collected through participant observation over 2 months in the above-mentioned class. Throughout the survey, taking the role of an "observer as participant" (Merriam & Tisdell, 2016, p. 144), the author observed a total of 13 mathematics lessons (50 minutes each) covering the whole instructional unit of "the function $y = ax^2$," including informal mathematical discourse during recesses. Classroom discourse was recorded using a fixed-point camera and two audio recorders. Individual activities were documented using a handheld camera and field notes, with a specific focus on Sora. To triangulate these observations, semi-structured interviews were conducted

with Sora, three other students, and their mathematics teacher. The collected data were fully transcribed, including non-verbal actions for classroom observation.

This study analyzed a critical episode from the third lesson, when Sora introduced a new insight, which became adapted through interactions in the classroom.

At the beginning of the lesson, students were given a worksheet with five mathematical problems and started to solve them by themselves. Nearly 20 minutes later, the teacher shared the solutions to the first problem and presented two properties identified from the problem (Figure1; hereafter P1 and P2). He remarked on their aesthetic value and asked, “Can you provide a satisfying interpretation of why this is the case?” While P1 was discussed over the next 15 minutes, P2 remained without a satisfactory explanation. The focal episode consisted of the remaining 15 minutes of the lesson, during which Sora shifted the classroom discourse toward a collective exploration of P2.

Figure 1: P1 and P2 presented by the teacher



By analyzing the transcribed data using the methodology of constant comparisons (Corbin & Strauss, 2014), three phases emerged based on the nature of interactions:

- Phase 1: Manifestation of a new insight introduced by Sora
- Phase 2: Adaptation of Sora’s idea by the teacher
- Phase 3: Progressive acceptance and contextualization of Sora’s idea

Each phases involved diverse interactions between Sora, the teacher, classmates, and a shared mathematical context of the classroom. The following section provides detailed descriptions of these interactional dynamics.

RESULTS

Phase 1: Manifestation of a new insight introduced by Sora

This phase began with Sora raising his hand as the teacher was about to move on to the latter part of the lesson. At this point, 15 min were left until the end of the lesson, and four of the five mathematical problems had not yet been addressed.

1 T: How about this one (P2)? Should we hold this for now? It’s fine. We’ve spent too much time (on P1).

2 Sora: (Sora raises his hand.)

3	T:	(The teacher, unaware of Sora, stares at the blackboard.) Well, let's just leave this part a bit vague for now.
4	Sora:	(Sora stands up, trying to step forward.) Teacher.
5	T:	(The teacher is unaware of Sora) Well then, uh
6	Sora:	(Sora stops and steps back toward his seat.)
7	Yuki:	(Yuki gestures to push Sora forward while smiling.)
8	Sora:	(Sora glances at Yuki and trots toward the blackboard.) Uh, teacher, this
9	T:	Oh. Yeah. That's fine. He's coming out. Yes. What do you have for us?
10	Sora:	Well, this point comes here, and this part, uh, […] since this parabola shares same coefficient of $x^2$with that one, this should be the same as c, and

This transcript illustrates the interaction between Sora, who attempted to introduce a new insight, and the teacher, whose focus on advancing the lesson led to an inadvertent marginalization of Sora's contribution (lines 1–6). Triggered by Yuki, Sora decided to introduce it, interrupting the lesson flow (lines 7–8). Despite no prior contact with Sora, noticing Sora's actions, Yuki became a catalyst for Sora's insight to be introduced into the classroom. Finally, the teacher welcomed Sora's interruption, providing an affordance for his idea to emerge in the classroom (lines 9–10).

Phase 2: Adaptation of Sora's idea by the teacher

Sora's initial attempt to introduce his idea (line 10) was met with laughter and confusion among classmates and the teacher, indicating that his idea failed to be understood. In response, the teacher acted as a gatekeeper as illustrated below.

15	Sora:	First, I will do the same operation as this. (Sora points out the figure on the blackboard.)
16	T:	Yes. The same operation as this. OK.
17	Sora:	Then, from this point, this one, isn't it. (Sora explains the phenomenon observed by performing his operation.)
18	T:	Yes. Then?

The teacher confirmed Sora's explanations step-by-step, while Sora monitored the teacher's understanding (lines 15–18). Sora also interacted with the shared mathematical context by intentionally linking his idea to existing resources on the blackboard. Finally, the teacher, acting as a gatekeeper, recognized the value of Sora's idea and incorporated into the mathematical context of the classroom (line 26).

25	C2:	He's saying something brilliant, probably.
26	T:	Brilliant! Now I understood! […] I've got it, but now, how can I break this down and explain it to you? Just give me a moment.

Throughout this phase, most classmates remained bystanders, frequently expressing their confusion (e.g., "I don't get it."). However, some students acknowledged Sora's ideas regardless of their understanding (line 25), which illustrates their expectations for Sora's unpredictable idea. This served as a prerequisite for the subsequent phase.

Phase 3: Progressive acceptance and contextualization of Sora's idea

In this phase, the teacher, who had played a privileged role as a gatekeeper, began to delegate his role to the students. This shift facilitated the socialization of Sora's insight.

47	T:	Hold on, can anyone else explain this? […]
50	Kei:	(Kei points the graph on the blackboard.) Well, I'm not sure, but it should be more comprehensible if we overlay it on that, that diagram.
51	T:	It should be more comprehensible if we overlay it on that diagram. Simply put, it means that this length and that length are the same, right?
52	Sora:	(Sora nods.)

Responding to the teacher's inquiry, Kei facilitated the contextualization of Sora's idea by expressing his understanding of the idea as a classmate and linking it to existing resources. This indicates that Sora's idea, which had been exclusively validated by the teacher in the previous phase, became accessible to the classmates. In turn, Sora provided feedback to confirm the shared interpretations (line 52). Consequently, the idea moved beyond individual ownership to become a subject of collective discourse.

54	C1:	(Shaking head) I don't get it.
55	T	Wait, so, to make it comprehensible; uh, this one, and that way. These figures of red and yellow are the results of "*Yoisho,*" right?

During this phase, the students' expressions of confusion served as a catalyst for the teacher and for some students who understood the idea of rephrasing it in a more accessible language, which lead Sora's idea to be elaborated in the shared mathematical context of the classroom (lines 54–55).

94	T:	I feel like the discomfort I had with solving them like geometry saying, "this length and that length are the same," has largely faded.

Finally, the teacher mentioned about the remaining four problems (line 94), which illustrates the adaptation of Sora's idea into the mathematical context of the classroom.

DISCUSSION AND IMPLICATIONS

This study explored how a mathematically gifted student's idea was adapted through the classroom interaction. As an initial attempt to elucidate the socio-dynamic nature of mathematical giftedness, the findings offer implications for future research.

Mathematical context as a catalyst for actualizing mathematical giftedness

In this study, shared mathematical context of the classroom functioned as a crucial catalyst for actualizing mathematical giftedness.

First, it stimulated gifted students' intrinsic curiosity; when reflecting on the lesson in the interview, Sora expressed his interest in P2 itself rather than highlighting his idea that had involved the whole class into the discussion. This finding is in line with previous findings that mathematically gifted students are driven to make sense of underlying principles of mathematics (e.g., Sriraman, 2003).

Second, the thickness of the shared mathematical context in the surveyed class provided a common language that enabled Sora to introduce his idea to the classroom and facilitated collective understanding of the teacher and other students. As such, this study further suggests that individual drive alone is insufficient for the social construction of mathematical giftedness. Without the fertile ground for mathematical ideas cultivated by the classroom community, Sora's insight might have remained isolated or dismissed as an outlier. Thus, to explore the socio-dynamic nature of mathematical giftedness, it is essential to focus on not only a unique interest of individuals but also a mathematical context that is rich enough to adapt a new variation.

The dual role of a teacher in the classroom

Throughout the survey, the teacher seemed to avoid being an "intellectual authority" (Lampert, 1990, p.32) while playing a crucial role as a gatekeeper. Such a teacher's behavior created a classroom culture in which not only did Sora feel comfortable sharing his idea, but also other students were not forced to accept it just because the teacher did so. In the world of professional mathematicians, "the opinion of a very small number of leading researchers was enough" (Sriraman, 2009, p.17) to adapt a variation. This study, however, suggest that the adaptation of a new variation is much more complicated in a heterogeneous classroom under the culture of egalitarianism like in Japan. The teacher seemed to navigate the dilemma between two identities: an educator responsible for leading all classmates, and a member of the field who reviewed the new idea. Indeed, he reflected on his practice in the interview as follows.

> T: Perhaps the idea was so talented, as you see today, that not all 40 students could share the same sense of it. Considering that my practice is to lead all 40 students, I could have just acknowledged the property and said, "Alright, let's solve No.4", which would have kept all 40 students together.

Classmates' expectations for unpredictable ideas

Finally, the role of classmates was fundamental in the adaptation of mathematically gifted student's idea. Although classmates were not forced to accept Sora's idea, they neither abandoned their attempts to understand it nor became indifferent. Rather, they maintained the expectation that his idea held inherent value. This is notable given that mathematically gifted students often curtail their process of reasoning (Krutetskii, 1976), leading to a lack of proper recognition from others. Sora mentioned in his interview that he rarely worked on mathematics with peers because "it takes effort to explain my solution." However, the classmates' attitudes of expectation and willingness to comprehend unpredictable idea appeared to encourage Sora to voluntarily share his thinking in the classroom. This suggests that the realization of mathematical giftedness depends not only on gatekeepers but also on peers.

This study specifically analyzed a critical episode. Future research should examine more diverse cases in which the mathematical context and participants' roles vary and illuminate the socio-dynamic nature of mathematical giftedness.

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