SIXTH-GRADE STUDENTS' SENSE-MAKING OF RATIO THROUGH PROBLEM POSING

Dhea Anisah Putri[1], Zulkardi[1], Ratu Ilma Indra Putri[1], Michiel Doorman[2]

[1]Sriwijaya University, [2]Utrecht University

This study explores how sixth-grade students make sense of ratio situations, as reflected in the problems they pose in response to a given context. Adopting a sense-making perspective that views understanding as the process of structuring a situation, we analyze students' posed problems and interviews in a familiar mixture context. The analysis shows that different framings and interpretations of the same situation can lead to different forms of ratio reasoning and shape the ratio structure students construct. In particular, some framings afford multiplicative and unit-rate reasoning, while others remain oriented toward direct quantitative comparison grounded in the context. These findings contributed to ratio learning by showing how problem posing reveals both the possibilities and limits of students' emerging sense-making of ratio.

INTRODUCTION

Problem posing has received considerable attention, particularly in facilitating students' understanding of mathematical concepts and ideas (Liljedahl & Cai, 2021; Cai & Hwang, 2020) It refers to activities that engage students in formulating (or reformulating) and expressing problems or tasks based on a given context (Cai & Rott, 2024; Silver & Cai, 2005). It requires students to interpret situations meaningfully, identify relevant mathematical relationships, and translate their interpretations to formulate new mathematical problems (Ran et al., 2025; Possamai & Allevato, 2024). In this way, problem posing is closely connected to students' sense-making (Christou et al., 2024; Walkington & Bernacki, 2015)

This study adopts McCallum's (2023) sense-making stance, which emphasizes students' active efforts to interpret and structure mathematical situations. Sense-making involves students' cognitive strategies for giving or extracting mathematical meaning from situations, concepts, or representations (Pinto, 2018). In the domain of ratio learning, this perspective is particularly salient for sixth-grade students. It is because ratio involves more than simply performing multiplication calculations (Cengiz & Rathouz, 2018). Instead, understanding ratios requires students to reason about relationships between quantities, to coordinate covarying measures, and to distinguish multiplicative structures from additive ones (Lobato et al., 2010). Problem posing, therefore, offers a promising avenue for examining students' sense-making of ratios, as suggested by previous research (Cai, 2022). When students create their own problems from a given situation, they must decide which quantities are important, how they relate to each other, and what mathematical structure underlies the situation

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(Zhang et al., 2022; Bevan & Capraro, 2021). This process may reveal how students conceptualize ratios, whether they notice multiplicative relationships, and how they interpret comparison or proportionality embedded in a context. However, posing problems to specific contexts requires high cognitive abilities (Chen & Cai, 2020; Xu et al., 2020), especially for sixth-grade students who are still developing basic proportional reasoning (Sawatzki et al., 2019). Consequently, it remains unclear how students' interpretation of a given situation shapes the ratio structures they construct when posing problems.

There remains a limited understanding of how students' distinct ways of framing the same context shape the ratio structures that emerge in their posed problems. This study addresses these gaps by examining how sixth-grade students frame familiar ratio situations and how these framings are reflected in the problems they pose. In particular, this study aims to clarify how sense-making in ratio learning is manifested through different framings of the given mixture context.

METHOD

Research Design

This study uses a qualitative approach to examine sixth-grade students' sense-making of ratios through problem-posing activities. Following McCallum's sense-making stance (McCallum, 2023), we treat students' posed problems as situated expressions of how they understand and structure the ratio situation. Therefore, we focus on what students choose to ask, compare, and connect in the problems they pose, as it will reflect their understanding of ratios. Participants were 40 sixth-grade students from a primary school in Palembang, Indonesia. The class included in this study had previously studied ratio topics but had not previously engaged in a formal problem-posing experience. Data were collected during a 90-minute session and included students' posed problems, students' written responses, and semi-structured interviews with selected students conducted after the problem-posing activity.

Task and Context

Students were provided with a familiar context in line with the principle of Realistic Mathematics Education (RME), which is grounded in meaningful situations that actually occur and can be imagined by students (Van den Heuvel-Panhuizen & Drijvers, 2020; Zulkardi et al., 2020). We presented students with situations close to their everyday lives and asked them to pose a ratio problem based on the context, following Carmona-Medeiro et al. (2024) and Cai & Hwang (2023). The context was designed to potentially allow multiple interpretations of ratio relationships (e.g., unit rate, invariant ratio, part-to-whole), so that students' choices in problem formulation could reveal how they structured the situation. No explicit constraints were imposed on problem type, level of difficulty, or expected numerical operations to preserve students' autonomy in interpreting the situation. The following is a prompt given to students.

Figure 1: Prompts and Contexts of Problem Posing Given to Students

Making Juice
Mother is making juice at home. To make the juice, she mixes several ingredients, such as tomatoes, carrots, and oranges. The amount of each ingredient used will affect the amount of juice produced and also its taste.
Create at least one math problem about ratios based on the context.

Data Analysis

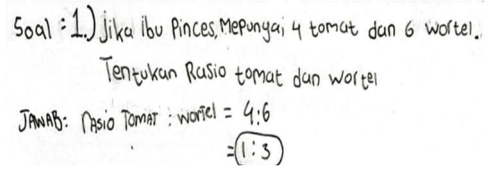
Data analysis focused on how students framed the given context and how these framing were reflected in the expressed ratio structure in their posed problems. We define 'framing' as students' way of structuring what they consider mathematically important to the situation as evidenced by the quantities chosen, the relationships highlighted, and the form of the questions posed. The analysis was conducted iteratively. First, we examined the problems to identify students' ways of framing the context. Second, within each framing, we analyzed ratio structure expressed (e.g., unit rate, multiplicative ratio, part-to-part or part-to-whole relationship) to examine how particular framings enabled or constrained the emergence of multiplicative reasoning. Third, interviews were used to refine the researchers' interpretation by clarifying the students' intended meanings.

RESULT

Students' Framing of the Context in Their Posed Problems

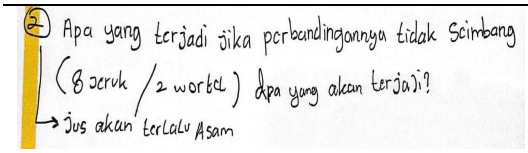
The analysis of students' posed problems shows that students made sense of the given mixture context in qualitatively different ways. The first framing (see Figure 2) treats the context as a direct comparison of quantities. S01's question focused on identifying two quantities (tomatoes and carrots) and asking for their ratio without referring to the juicing process or the resulting product. The juicing situation itself plays no structural role in establishing the mathematical relationship. The student simply asked about the ratio of tomatoes to carrots after determining their amount. Subsequent interviews also confirmed that S01 understood ratio as a comparison between two quantities. In this case, the student's sense-making of ratio focused on constructing ratio as a relationship between two quantities, but had not yet used the context to structure it. The following is S01's posed problem:

Figure 2: Student (S01) Posed Problem – Direct Comparison of Quantities

	Translation: If Mrs. Pinces has 4 tomatoes and 6 carrots, determine the ratio of tomatoes to carrots. Answer: Ratio of tomatoes to carrots= 4 : 6 =1 : 3.

The second framing interprets the context as a mixture situation affecting quality. Student (S02) framed question with the idea that ratio of the ingredients affect the taste or quality of juice. This indicates how the student made sense of ratios as a relationship whose significance lies in its influence on the whole. In explaining his reasoning, S02 noted that his primary concern was '*how the juice will taste*' if one ingredient is used significantly more than the other. As a result, S02's question focused on unbalanced ratios rather than calculating or directly comparing quantities. This frame of thinking explains an understanding in which the student coordinates quantitative relationships with qualitative consequences embedded in the context. The following Figure 3 shows the problem the student created.

Figure 3: Student (S02) Posed Problem - Mixture Situation



Translation: What happens if the ratio is not balanced (8 oranges/2 carrots)?

Answer: The juice will be too sour.

Ratio Structures and Relationships Expressed in Students' Posed Problems

An analysis of the questions posed by students shows that the meaning of ratio is constructed through several different quantitative structures. From the students' work, several dominant ratio structures emerged, including unit rate, multiplicative comparison, and part-to-whole.

One of the problems students successfully designed included the concept of unit rate and the invariant multiplicative structure of a ratio (see Figure 4). First, student (S03) normalized the ratio of ingredients into a per-unit relationship. Based on the student's initial information that 200 ml of juice requires 2 tomatoes and 1 carrot, the student explicitly reconstructed the situation in 100 ml units, stating that 100 ml of juice requires 1 tomato and 0.5 carrots. We asked students about why the quantities were stated per 100 ml in the interview results below.

Interviewer: Why do you state 100 ml for each juice in your problem?

S03: Because every 100 ml of juice always uses the same amount of tomatoes and carrots, it is 2 tomatoes and 1 carrot.

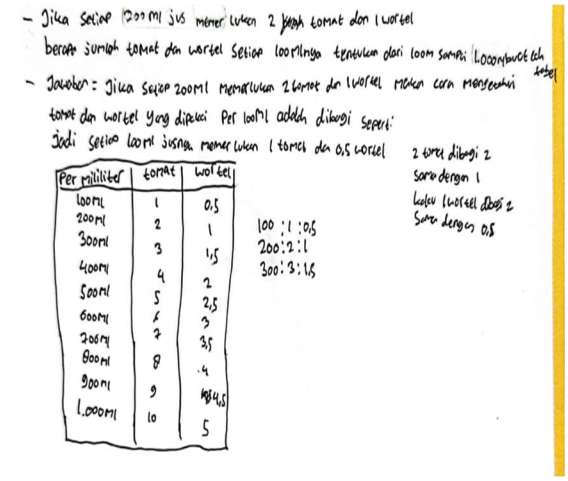
The interview results reveal how the student views ratios as a relationship between two quantities through the unit-rate relationship. In the following steps, S03 created a table that expanded the per-100 ml relationship from 100 ml to 1000 ml, showing the ratio as an invariant multiplicative structure. Each increase in volume was always followed by a proportional multiplicative scaling of the number of tomatoes and carrots.

Interviewer: Does the amount of tomatoes and carrots you use per 100 ml of juice affect the desired outcome of this problem?

S03: Yes. Since the ratio of tomatoes and carrots per 100 ml is 2 to 1, if I double the juice, the ingredients must also be doubled to maintain the same ratio.

The interview result confirms S03's interpretation of the ratio as an invariant multiplicative structure that governs how quantities scale simultaneously. Figure 4 shows the S03's posed problem.

Figure 4: Student's (S03) Posed Problem



Translation
The student's question: If every 200 ml of juice requires 2 tomatoes and 1 carrot, how many tomatoes and carrots are needed for every 100 ml of juice? Then, determine the number of tomatoes and carrots needed from 100 ml to 1,000 ml of juice.
Student answer: If every 200 ml of juice requires 2 tomatoes and 1 carrot, then to find out the tomatoes and carrots used per 100 ml, divide as follows: 2 tomatoes divided by 2 equals 1, if 1 carrot divided by 2 equals 0.5
100 : 1 : 0.5
200 : 2 : 1
300 : 3 : 1.5
So for every 100 ml of juice you need 1 tomato and 0.5 carrots

Volume of juice (per mililiter)	Number of Tomatoes	Number of Carrots
100ml	1	0.5
200ml	2	1
300ml	3	1.5
400ml	4	2
500ml	5	2.5
600ml	6	3
700ml	7	3.5
800ml	8	4
900ml	9	4.5
1000ml	10	5

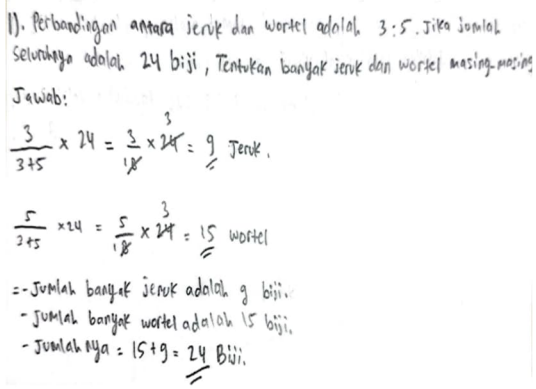
Another problem in Figure 5 shows that the student was able to construct involved part-to-whole problems. In this problem, the student (S04) compared two parts of an orange and a carrot, then framed the situation as the distribution of a predetermined total of 24 pieces. S04 was also able to determine the ratio 3:5 as a basis for dividing the two quantities into two proportional parts. S04 reconstructed the ratio as a division operator with respect to a whole. S04 then verified that the total remained 24 pieces by adding the results (9 oranges and 15 carrots) again.

Interviewer: Why did you need to consider the total before determining the number of oranges and carrots?

S04: It is because a ratio only applies if they all come from the exact total, and the ingredients must be adjusted together so the total remains unchanged

The interview shows that she viewed ratios as determining proportional relationships within a unit. S04 also explained that quantities must be adjusted together so that 'the total remains unchanged,' indicating that ratios were treated as fixed structures that govern the distribution of parts within the same whole. In this case, sense-making is evident in the students' process of interpreting the ratio in terms of part-to-whole relationships, revealing student's early proportional relationships. Figure 5 shows the S04's posed problem and answer.

Figure 5: Students (S04) Posed Problem



Translation: The ratio between oranges and carrots is 3:5. If the total number is 24, determine each number of oranges and carrots.
Answer:
$\frac{3}{3+5} \times 24 = \frac{3}{8} \times 24 = 9$ oranges
$\frac{5}{3+5} \times 24 = \frac{5}{8} \times 24 = 15$ carrots
The number of oranges is 9
The number of carrots is 15
Total $= 15 + 9 = 24$

DISCUSSION

This study demonstrates that problem-posing activities facilitate sixth-grade students' sense-making of ratios by structuring situations. The main finding is that differences in students' meanings of the same context prompt have direct implications for the structure of the ratios that emerge in the problems they pose. This suggests that students' sense-making processes in learning ratios are reflected in the problems they pose, the procedures of solution they use and how they determine what is considered mathematically relevant (Christou et al., 2024). The role of context in this sense-making process is selective and depends on how students interpret the situation. This finding supports previous research showing that problem-posing can reveal how students construct meaning through framing situations (Walkington & Bernacki, 2015) and confirms that such framing can differ substantially even in simple and familiar contexts. Students frame mixture contexts differently, for example, by making direct comparisons of quantities and by considering situations that affect quality. In the student's framing of 'mixture situation affecting quality,' context is invoked by linking ratios to qualitative consequences, but it has not been mathematized multiplicatively. This finding aligns with previous research showing that students' high cognitive abilities are required in problem-posing (Chen & Cai, 2020; Xu et al., 2020). Further findings show that the ratio structures expressed in students' problems were also varied, including unit rates, invariant multiplicative relationships, and part-whole relationships. This suggests that the same context prompt can produce different mathematical structures depending on how students frame it (Bevan & Capraro, 2021). These structures reflect how students coordinate quantities and the relationships between quantities in their self-constructed problems. These results align with the relational understanding of ratios that involves the coordination of quantities that change together (Cengiz & Rathouz, 2018; Lobato et al., 2010). Unlike studies that focus on students' ability to solve ratio problems, this research presents that these structures can be identified through the way students formulate their own questions.

CONCLUSION

Problem-posing activities provide space for sixth-grade students to actively construct the meaning of ratios through how they interpret and structure given situations. The problems posed by students demonstrate how their understanding and sense-making of ratios can emerge from the diversity of the context framing and structure of ratios. This suggests that problem-posing activities become a process by which students determine what is 'mathematically important' in a situation. In this case, the ratio concepts that emerge in students' posed problems indicate that proportional reasoning develops from procedural exercises and opportunities to structure ratio relationships themselves in meaningful contexts. Some of the ratio framings and structures that emerge lack explicit multiplicative relationships and remain limited to comparisons of quantities. This limitation is not seen as a failure but rather as a representation of how students currently structure ratio situations based on their experiences and understanding. Future research should design more responsive learning interventions to students' understanding by giving attention to how students frame contexts and the mathematical structures they express, in order to gain a more in-depth picture of students' sense-making processes.

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