

A consumer's point of view

Fred Goffree

1 What are we aiming for exactly?

There have been twenty-five PME's since the forming of the study group in 1976 (Karlsruhe) and the first meeting in Utrecht. Were the initial expectations too high? Probably, although I can't produce any figures from the PME 1977 proceedings that would prove the validity of this assumption – they could not be found anywhere. What no one certainly would have expected was the tremendous growth of the PME community in those 25 years. The proceedings from Osnabrück 1978 (a very modest brochure) to Hiroshima 2000 (a four-part, wonderfully finished volume) all bear witness to that. As a comparative PME outsider, I had to make do with the proceedings. Arranged next to each other in the FI bookcase, they take up almost a metre of shelf space, so I needed seven-league boots to get through them. Fortunately, to provide an answer to the question 'what are we aiming for exactly?', I had already chosen a context, a domain, a position, a perspective and a viewing direction:

Context	Primary teacher education
Domain	Research in primary school mathematics
Position	Active consumer of research (processes and outcomes)
Perspective	Can the education of primary teachers gain more from PME research?
Viewing direction	Where theory and practice come together

To run ahead of my findings, I can say right away that there are indeed prospects, but only on condition that researchers opt more frequently for developmental research as defined by Hans Freudenthal and as conducted in the institute bearing his name. Besides that, other methods of reporting need to be learned, maybe using IT potential.

2 Background

It is obvious that the above did not appear just like that; as is the case with every researcher, my personal history did enter into it. Although researchers have the tendency to side-step personal issues in their work, I would like to say something on that score.

After ten years of designing maths lessons whilst working as a teacher educator with student teachers at a Teacher Training College in the Netherlands, round about 1968, I joined the CMLW, the predecessor of IOWO (Institute for the Development of Mathematics Education) that later became the Freudenthal Institute. The ten IOWO years were dominated by the Wiskobas (mathematics in primary schools) project, which modernised mathematics education in the Netherlands. The working methods

of developmental research flourished in the Wiskobas team: idea, design, debate, try-out, reflection, discussion, revision, etc. Practice and theory converged in this developmental research and, in addition to that, new theories were framed. As Freudenthal used to say, 'like sparks flying off real work', or, as Leen Streefland later put it: 'The development of a researcher is the source of theory formation.'

I was one of the two members of the Wiskobas team who were mainly interested in the effects the developmental research activities had on teacher education and I learned to see educational research work from the teacher education perspective. Not only were practical results and theoretical conclusions of interest, so was the cycle of designing and research in the classroom. Piaget's method of clinical interviews with children was adopted and modified to Freudenthal's method, which had become known from publications as 'Walks with Bastiaan'. In those days, instead of teaching the 'class as a whole', student teachers conducted conversations with small groups of children about mathematical tasks or phenomena from daily life which they attempted to link to mathematics with the children.

After the IOWO had closed, on 1 January 1981, I wrote a 3-volume course book for student teachers, 'Mathematics & Didactics', a reflection on my 10 years of research and development work. The book included admirable designs, discussions of theory in the team, practical experience with it in class, comments made by pupils, to name a few. That way, students were invited to experience for themselves the Wiskobas routine – linking theory and practice, the recurring bottleneck in this corner of the scientific community, as a matter of course.

Midway through the 1990s, the work done by Lampert and Bell in Michigan (1998) offered a new stimulus to research and development work on teacher education in the Netherlands, resulting in the Mile project. Wil Oonk will be telling us more about this project in his presentation.

3 The (pre-service) education of primary mathematics teachers.

The first conference of a new European research community (in formation) was held in 1999, in Osnabrück: European Research in Mathematics Education. Working Group No. 3 was engaged in research work on mathematics teacher education. (Krainer, Goffree and Berger, 1999). There we distinguished between:

1. Research in the perspective of teacher education (the researcher does not have teacher education in mind).
2. Research in the context of teacher education (the researcher is a teacher educator, a teacher or a student teacher).
3. Research in teacher education (teacher education is itself the object of research).

Keeping my five special points of interest in mind, I glance through the list of proceedings of 24 PMEs.

Most hits fall under category 1. My attention was drawn to the paper by Bednar and Janvier (Bernadette), which they presented in Warwick (1979): 'The understanding of place value'. In my view, the points that receive special attention in this paper are of relevance to student teachers and the way the research is presented and written certainly has student teachers in mind. I would mention the following:

- The mathematical beginning: clarify the notion of place value (content knowledge, according to Shulman 1986).
- Next, a didactical follow-up: what is meant by 'understanding place value' (pedagogical content knowledge).
- Apply this knowledge to design a series of problems:
 - for diagnostic testing;
 - for learning units.
- Single out the main difficulties and strategies of the children.

What is it about this research that makes it so appropriate from the teacher education perspective? It takes as its point of departure the student teacher's own level of proficiency (what is place value?). The topic (understanding) is then 'didacticised'. Following that, the 'theory' is applied to the design of teaching after which focused tests are done with children. The long and the short of it is that previously acquired (theoretical) knowledge is applied both for the benefit of practice and in the practical situation itself.

Whilst saying this, I could also cite Alan Bell, who also ventured into this field at the PME in Berkeley a year later: 'Designing teaching in the light of research on understanding'. So this is about theoretical knowledge that has been produced in research done by 'others', and can be regarded as a continuation of what Bednar and Janvier proposed.

In the 1997 proceedings my attention is also attracted by several papers related to the use made of 'clinical interviews' in the spirit of Piaget and Freudenthal ("Some reflections on the construction of the idea of number in 6-year-olds", "How does reflective thinking develop?", "A child's eye view of learning mathematics. Has Piaget found it?").

4 A philosophy of primary mathematics teacher education

The way in which I have picked out the best bits of PME research in the examples I have given above has a lot to do with my outlook on teacher education. As I mentioned before, I wrote a course (work)book for student teachers based on my experiences as member of the Wiskobas team (1971-1981). That was why in the early 1980s I was frequently on the look-out for useful material in research and development work. Material I then found suitable was material that illustrated, as it were, my vision of training, which itself was still in the making. Now, a good many years, development projects and investigations later, I am able to enumerate the building blocks that constitute my perception. Together, they form the underlying

criteria for picking out the best bits, far more so than the 5 limitations I referred to previously. In connection with this, I would like to bring them to the fore to use them as an impetus towards the wishes I want to give to PME.

- As a school subject, mathematics provides excellent working material for prospective primary school teachers. (PML 1998)
- Where possible, practical situations form a starting point in teacher education.
- Elementary school practice is not solely represented by what crops up during fieldwork. Digitised teaching episodes (real teaching and real TV) make the study of practice possible. (Goffree & Oonk, 2001^{a en b})
- Student teachers 'construct' practical knowledge through the investigation of real teaching practice. (Lampert & Ball 1998; Oonk 1997)
- 'Practical knowledge is what 'moves' the teacher. It is a way of narrative knowing'. (Gudmundsdottir 1995)
- Theory becomes integrated in practice (situations) by following Donald Schön's 'reflective conversations'. (Schön 1983)

The latter especially is of vital importance because teacher educators can demonstrate their expertise in the form of 'theory in action'.

I hope therefore that the work done by researchers will yield multiple practice situations complete with theoretical reflections by the researchers themselves. The researcher could, in his reflections, also include his own development and the onset of theory formulation.

The PME 1985 proceedings again caught my eye. It was one of the two meetings I attended in person. Richard Lesh presented his 'idea analysis', which had been worked out two years earlier in the book (Lesh and Landau 1983) that made a profound impression on me. How do mathematical concepts come about, from intuitive notions to formal mathematics, the influence of education and environment, and so on. Food for the reflective practitioners among the teacher educators, was my first thought.

5 Narrative knowing

Practical knowledge is therefore to a considerable extent made up of practical episodes. I had also collected a wide selection of episodes of my own when I wrote my Mathematics & Didactics. Those are written episodes about education in the classroom, with pupils and a teacher. (Among the favourites were John Holt, Herbert Ginsburg, Erlwänger). Later on, the filmed episodes from Mile were added, which the student teachers are actually required to use for making up their own episodes.

At the time, I opted for a very short episode that Alan Bishop showed me whilst we were working together in BACOMET on the chapter 'Classroom organisation and dynamics'. (Christiansen, Howson and Otte 1986). You can find it in the 1985 PME

proceedings. I cite the episode and a possible reflective conversation. The background to this is the question to what extent PME research can contribute to realising such episodes + theories, which contain relevant practical knowledge (prospective) teachers have. (Harvey et al. 1982, p.28).

David

D	15's odd and $q \frac{1}{2}$'s even.
RH	15's odd and $q \frac{1}{2}$'s even? Is it?
D	Yes.
RH	Why is a $\frac{1}{2}$ even?
D	Because, erm, $\frac{1}{4}$'s odd and $\frac{1}{2}$ must be even.
RH	Why is $\frac{1}{4}$ odd?
D	Because it is only 3.
RH	What is only 3?
	A $\frac{1}{4}$.
RH	A $\frac{1}{4}$ is only 3?
D	That's what I did in my division.

At this point another child joined in to explain to the teacher:

R	Yes, there's three parts in a quarter, like on a clock. It goes 5, 10, 15.
RH	Oh, I see.
R	There's only three parts in it.
RH	Oh, so you've got three lots of 5 minutes makes a quarter of an hour.
D	Yes. No. Yes, yes, yes.

A reflective conversation with the situation

Here, 'David' is the situation and in his reflective conversation, the educator can approach this from three different angles, thinking out aloud.

"Starting from the structure of pedagogical content knowledge:

It is possible to envisage 'models' between the intuitive notions of children and formal mathematics, which make connecting the two easier. These models are sometimes given in the book or provided by the teacher (just think of....), but sometimes the children think up models themselves. That is what David does, he links the dial of a clock to fractions. This is not really that extraordinary, several educationalists did the same thing with fraction circles, except that, in principle, they did not include the ready-made 12x5 minute structure. A lucky thing for David that

you can divide the number 12 (the dial) up in so many ways. Just think how many fractions are possible using it. (...)

But there is more than that. We can learn something from the teacher as well. He attempted to project himself into the role of pupil, on the basis of his own knowledge of fractions (educational content and teaching method). That didn't go very well. We can understand how and as a result of what he was set on the wrong track. But take care, he did not proceed on the assumption that David was talking nonsense. An important didactic principle: Do not presume that a pupil just says things off the top of his head. He has thought it through, although it might be completely wrong. (...)

Imagining the thought process pupils go through is vital in arithmetic and mathematics and indispensable in interactive lessons and discussing solutions afterwards."

6 As a conclusion

Here, I was adopting the teacher educator position and that of a consumer of educational research. I take an eclectic stance as I draw what I want from the rich source that can be used. My wish to create practice situations and to couple them to a (descriptive, explanatory or creative) theory so as to enable student teachers to acquire (construct) adequate knowledge of practice, calls for research to be done on the shop floor of mathematics teaching. The researcher, who, with regard to teacher education, is customer-oriented, allows none of the data to go to waste and he also keeps his own reflective notes and thoughts on his personal development during the research work. Video and audio recordings can be made (using information technology), anecdotes, remarks made by pupils, their written work, teachers' logbooks, tests and marks, and any other things that make mathematical classrooms a rich fund of learning for children and their future teachers.

My wish is that future PME research be conducted and reported in large part in the way outlined above, from the perspective of teacher education.

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