MEDIATING ARTEFACTS IN CLASSROOM INTERACTIONS

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This study uses an activity theory approach to explore artefact mediation within a middle high school lesson relating to the reduction of fractions. The first part of the paper discusses the main concepts, including the Bakhtinian concept of 'voice'. These are then applied to outline the activity system of a classroom - episodes of which are analysed in greater detail. Artefacts including the 'teacher voice', the expression 'simplest terms' and various strategies for calculation are identified. It is argued that the choices made by the students reflected the playing out of an hypothesised contradiction between use values and exchange values lying at the heart of the lesson.

The point of departure for this study is the belief that mathematical knowledge is dependent on both the mind and society (Gerdes, 1985; Bishop, 1988; Lave and Wenger, 1991; Nunes, Schlieman & Carraher, 1993; Bloor, 1983; Solomon, 1989; Cobb, 1994; von Glasersfeld, 1995; Walkerdine, 1988, 1994). A common finding/assumption associated with this view is that mathematical thinking is not valid by virtue of correspondence with what is factual (the so called Platonist view) - rather, its truth value depends on its coherence with what is already taken to be true and with what works. These concerns (coherence and utility) throw into focus the questions of how mathematical tools are generated and related to a problem situation and, if all knowledge is situated, how knowledge can be transferred from one site to the next. Related to these larger questions is the issue of how mediating artefacts facilitate the construction of mathematical knowledge within the local domain of a mathematics classroom. It is this topic which is the immediate focus for this paper.

In the next section a theoretical framework for the paper, drawing on activity theory (Scribner, 1997), is set out. Following this, the paper illustrates its main ideas with an examination of transcripts depicting interactions within a middle school classroom. Tensional qualities within situated artefacts are then discussed, and this leads to the conclusion.

'Mediating artefacts' in the processes of knowledge use

The idea of cognitive mediation develops from the work of Vygotsky. In *Mind and Society* (1978), for instance, he enriches the simple idea of stimulus-response processes by introducing a third element described by him as a second order stimulus or "mediated act". For Vygotsky, the mediating act "possesses the important characteristic of reverse action (that is, it operates on the individual, not the environment)" and in so doing "permits humans, by the aid of extrinsic stimuli, *to control their behavior from the outside*. The use of signs leads humans to a specific structure of behaviour that breaks away from biological development and creates new forms of a culturally-based psychological process" (italics in the original, p 40). In Vygotsky's work, signs are an instance of psychological tools, and in the work of later theorists (for instance, Leont'ev, 1981) these are subsumed into a larger category of cultural-historical artifacts associated with the physical, social and cultural context of knowledge production and use. What is especially interesting about Vygotsky's formulation is that mediating artifacts are not merely technical components of task performance - but they “transfer the psychological operation to higher and qualitatively new forms” (p 40). In other words, mediating artefacts transform knowledge in the course of their operation (Wertsch, 1985); thus, knowledge use and knowledge production are seen as interrelated on the Vygotskian view.

These insights have given rise to an ‘object oriented’ cultural-historical approach to human cognition (Leont’ev, 1981). In Engeström’s analysis (1987), for instance, 'subjects' collaborate within an activity driven by a common motive or ‘object’. Activities are mediated by tools, rules, and other social characteristics such as the community and division of labour specific to action. However, the object is characterised by internal or primary contradictions and these are played out across multiple domains of mediation leading to an expansive redevelopment of the object and subsequent re-mediation of the activity system.

In addition, because communicative events call up socio-historical processes and thereby express value positions as well as systems of belief and historical placement, an important artefact within an activity is the Bakhtinian concept of ‘voice’. When applied to the classroom, for instance, this means we can sensibly talk about the “voice of the teacher” - and mean by this an artefact within the dialogue which both shapes and is shaped by activity. For Bakhtin, when two voices are in dialogue a “hybrid construction” of utterances is produced. By this he means that “an utterance that belongs, by its grammatical (syntactic) and compositional markers, to a single speaker, but that actually contains mixed within it two utterances, two speech manners, two styles, two ‘languages’, two semantic and axiological belief systems” (Bakhtin, 1991, p. 304, cited Wertsch, 1985, p 227; Engeström, 1995; Wells, 1996). Hybrid constructions such as these facilitate the trajectory of the object on its developmental path.

Looking at the data

In order to illustrate the ideas introduced above, I have selected four excerpts from a corpus of observed mathematics middle high school classroom interactions (Epidsodes, A, B, C, and D). These involve the teacher, Mrs R and two students, M and D. Prior to the action depicted, Mrs R requires her class to do questions (a) - (e) of the following list taken from a work sheet entitled the 'Ratio pep test'.

Reduce to simplest terms

(a) 4/12 =	(d) 240/360 =	(g) $2^1/_2$:4 =
(b) 36/12 =	(e) 5:5 =	(h) 3x/2x =
(c) 16/40 =	(f) ab:ab =	(i) 6ab/3b =

As the class settles down to this task, she calls forward to the blackboard a number of students including M whom she says "like to work ahead" (also see A1 below). Here she illustrates certain algebraic techniques (see Episode A below) useful in undertaking items (f) - (i) which she then sets for this subgroup and sends them back to their desks to begin work. M takes up his position with D and the two commence to work collaboratively (Episodes B, C, D). In the course of this we see M and D checking their progress with Mrs R (Episode B) and making progress through the exercises (Episodes C and D).

Episode B starts with M concerned about his response to (h) which was '1.5'.

Episode B

B1 M: Oh, I've got to reduce to the simplest terms. [crosses out 1.5] Oh, OK, um. [pause, writes, works calculator] Alright. Shit. [pause, both work]
B2 D: [inaudible] here and speed.
B3 M: One point 5 - that's what we had [inaudible] [T talks with nearby student]
B4 M: Mrs R? Mrs R, do we simplify that as well? Those two numbers [not clear which referring].
B5 T: Yes, you do.
B6 M: OK.
B7 T: And you're right. [looking at M's work]
B8 M: OK.
B9 T: So you got one right and you got two right. [T then picks up a pencil of M's and ticks (i) and (h), respectively]
B10 M: And that one. [pointing to (f)]
B11 T: Yep. Ahhh, no because it's a ratio. Oh - I guess you could say yes. I'll accept that. Yep.
B12 M: Thank you. [T goes to A and L's desks]

Episode C occurs shortly after the preceding episode. In this M and D engage on the topic of what is the correct response to question (e) which asked for a reduction to simplest terms of 5:5.

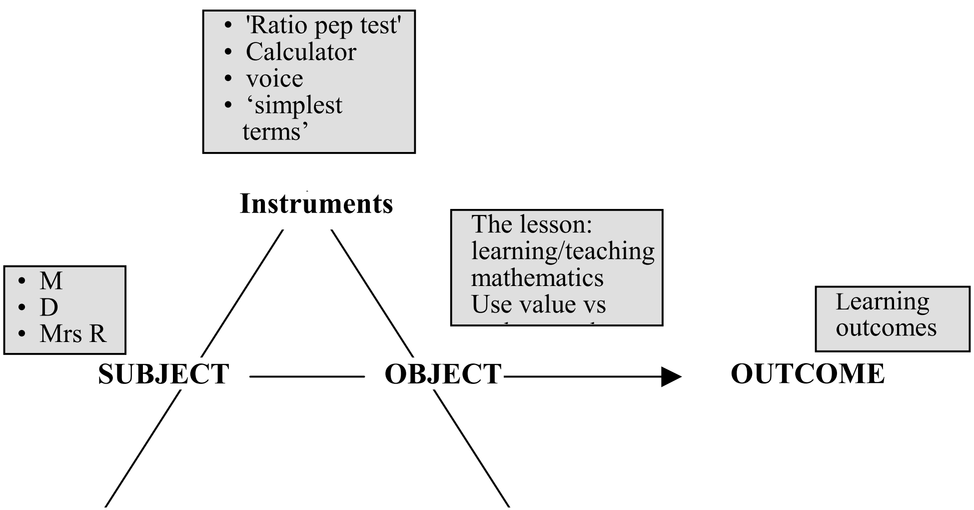
Episode C

C1 D: Five to the ratio of five, is one.
C2 M: This was [inaudible], I don't know.
C3 D: One over one.
C4 M: No, we've got to reduce to simplest terms.
C5 D: Yeah, it's just one. The five over five one.
C6 M: It's just one to one.
C7 D: Yeah.

In analysing these episodes, the first step is to examine M’s statement to D “no, we've got to reduce to simplest terms” (C4).

Already in B1 M’s exclamation “oh, I’ve got to reduce to the simplest terms” coincides with expressions of hesitation, frustration and uncertainty(B1 - B5). This suggests that the voice used in C4 derives from elsewhere - I argue a hybridisation of the ‘teacher’ voice and the imputed voice of the text which required students to “reduce to simplest terms”. In C4 the form of M’s retort resembles the teacher’s response given to M previously in B11 - “no, because it’s a ratio”. On this earlier occasion the teacher expressed a reservation concerning the legitimacy of the form of a response M gave to exercise question (f); in C4 M expresses the same kind of concern (albeit in relation to D’s work), and does so in a similar way. In particular, the function of the voice for each instance (B11 and C4) is to progress problem solving attempts by redefining the problem into an alternative version of the question under examination. In B11 this was achieved by drawing attention to specific features of the question asked - the importance of ratio form. In C4 this was achieved by emphasising the salience of the need for “simplest terms”. This shows that M’s utterance in C4 ventriloquises the teacher’s voice in B11. However, C4 also ventriloquises the imputed voice of the text for the text actually requires students to express in “simplest terms”. Further, it is noted that M when in C4 M says to D “no, we’ve got to reduce to simplest terms” words crucial to his meaning are left out. From the context it is possible to tell that he means “No, we’ve got to reduce *the ratio* to simplest terms”. Indeed, because he leaves the words *the ratio* out D is quite legitimately able to respond “Yeah, it’s just one”(C5). It is concluded that, for M, the term “simplest” does not function strictly as a mathematical term, but as a tool used to think and perform tasks in accordance with the expectations of the mathematics lesson.

This example illustrates how M and D adopt and synthesise the vocal and textual media in which they are immersed; and use the artifacts so synthesised to reconfigure the problems they encounter in their progress towards accepted outcomes. These relations are expressed in the top triangle of Figure 1. Further, it is clear that the character of this kind of intersubjective knowledge was reflexive in that Mrs R and the students shared knowledge relating to the lesson and shared the knowledge that this knowledge is shared. In the section following, I propose to consider more closely the development of this intersubjectivity and, in particular, explore the issues of tensions among mediated artefacts.



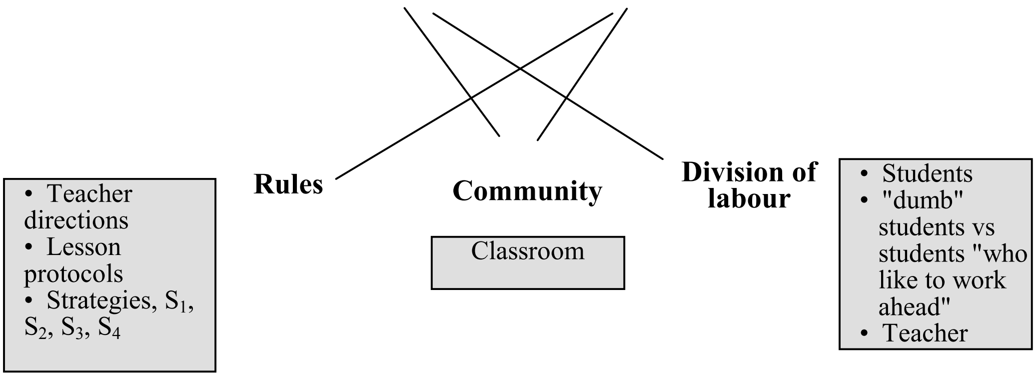


Figure 1: Model of the classroom activity system

Tensions among artefacts

Episode D occurs immediately after Episode C.

Episode D

[Question (a) reduce to simplest terms: 4/12]

D1 D: It's one third, the top one, then there's—that's one third as well. [pause] What's the forty one? [referring to question 1 (c) reduce to simplest terms: 16/40] Four over. [M writes 1/3]
D2 M: That's not right, the …
D3 D: Divide it by four, it's, yeah. [M writes 4/]
D4 M: No. It's got to be two over five. [writes 2/5]
D5 D: Yeah, two over five. I knew that one. I was just [inaudible]. [writes 2/5]
D6 M: Oh sure you did, that's what they all say.
[Question 1 (d) reduce to simplest terms: 240/300]
D7 D: Um, two-forty, yeah, three hundred divided by two-forty.
D8 M: Hang on, twelve, so we'd have twelve ...[pause - writes 12/]
D9 D: No, that ... it's divided by it, right? That's one ...
D10 M: ... over four [interrupting - writes 4]. Four divided by ...
D11 D: ... or one third.
D12 M: ... it's going to be three over one. [writes 3/1]
D13 D: Why?
D14 M: 'Cause, it's just going to be the opposite. Look.
D15 D: Yes.
D16 M: Times twelve.
D17 D: 'Cause that's bigger number. Yep.
D18 M: Um. Um. Two forty over three hundred. Um, two four 'o' divide—no.
D19 D: Three hundred.
D20 M: Three hundred, divide two four 'o' —no. No, um. [Presses 300/4 = 75 on calculator] Seventy-five.
D21 D: Did you get twelve over. [pause]
D22 M: [Presses 240/6 = 40 on calculator] ... six ... forty. And that was one. [writes 40/75]
D23 D: Is it—yeah, it's eight over.
D24 M: Seventy-five divide eight doesn't work, no, can't be eight.
D25 D: It's um—try ten.
D26 M: Five. No, seventy-five, it's going to be five there.
D27 D: Yeah, five over—no, try five.
D28 M: By five, is fifteen.
D29 D: It's eight over fifteen.
D30 M: I think, hang on.

In this episode M and D exhibit evidence of four strategies (S_1, S_2, S_3 and S_4) relating to the task of simplifying fractions, as follows. In S_1 a fraction a/b is reduced to the form 1/n where n is the natural number for which either b = na or a = nb. D uses this strategy in D7 and D9 - it gives him the correct answer to (a) and a wrong answer to (b). S_2 provides a necessary condition for any reduced form: if a/b is reduced to c/d and if a>b, then c>d. This test enables M to state in D2 by inspection that D's response to (b) must be wrong (see also D14). Strategy S_3 is to divide both the numerator and denominator by a common factor (D3 - D5, and D8, D10 and D12), thereby determining a reduced fractional form. Finally, S_4 deals with the case where the natural number n as in S_1, does not exist. In this case factors of a, say n_1 and n_2, are sought such that $a = n_1n_2$ and there exists a natural number n_3 such that $b = n_2n_3$. The reduced fraction is then written as n_2/n_3. We see S_4 applied in D18 - D30. In D18 and D20 M carefully checks that strategy S_1 does not apply. Then he uses S_4 twice: first in order to reduce 240/360 to 40/75 (D20 and D22); and second, with D's help, in order to reduce 40/75 to 8/15 (D23 - D30).

Of these strategies, of course, S_2 is correct only in the limited cases to which it applies, and S_3 is a correct general strategy; S_1 and S_4 are both false. My focus in the following is on S_4 . Where does come from? Preceding their work together, a number of other students including M who like "to work ahead" (Mrs R's words) is gathered at the board in the front of the classroom. Here Mrs R rehearses a series of algebraic manipulations working towards a cancellation algorithm involving algebraic expressions. Her method (a sample is illustrated below) involves an intricate construction of pronumerals and arithmetic operations involving factors.

Episode A

A1 T: Just working ahead a little bit? OK, guys I'm going to think of two numbers. OK? Oh, come on Alistair. Now stand over here because. See, Dale, we'll talk behind their back, you see. And so will L. Over this side guys, over this side. See if you can see. You stay, the small ones can stay here, you go over there. OK. All right, that's it, that's fine. Just squat down. OK. Now, I'm going to think of three numbers, right?, x is going to be 7, y is going to be 9, and uh, m is going to be 3. OK? Now I'm going to multiply x by 5. I would write it as 5x. OK? I'm going to multiply y by 5, how would I write it?
A2 Ss: 5y.
A3 T: OK. And I'm going to multiply m by 8.
A4 Ss: 8m.
A5 T: All right. Now, I'm now going to divide x by 5. Now what's going to happen if I do that?
A6 Ss: Be the same number.
A7 T: Ah. It's going to go back to the same number. All right. I'm now going to take this, um, I multiplied m by 8, I'm now going to divide it by m. What am I going to be left with?
A8 S51: Eight.
A9 T: Um. Is it?
A10 S52: Yes.
A11 T: Right, m is 3, 8 times 3 is 24, divided by 3, brings it back to 8. Do you notice that this one you told me brought it back to 7. Seven times—5 times 7 is 35, divided by 5 is 7. Good. And this one here you told me went to 8. Now can you see a pattern?

At the conclusion of this instructional episode, students are instructed to return to their seats and do the entire question set - that is, including those questions (f) - (i) she requested the general class to "cross out". Mrs R's figuring in this episode and M's construction of S_4 are comparable in their intricacy and in their emphasis on the multiplication and division of factors. Whether M takes more than these kinds of manipulations from Mrs R's work is unclear - nevertheless, in his choice of S_4, M exhibits a motivation to engage in a similar kind of exploration of factors relevant to the questions considered. It emerges that in his subsequent work with D, M expresses a stronger preference for the erroneous and more convoluted S_4 rather than the correct and apparently more straightforward, S_3. M seems to find the form of S_4 more appealing than the substance of S_3. What can explain this apparently anomalous situation?

My approach is to analyse the tensions among mediating artefacts within the activity system of the classroom, as set out in the application of Engeström's model of the activity system in Figure 1, above. At the heart of my argument is the assumption that there is a tension within the core of the lesson itself - namely, a primary contradiction between the ‘use values’ and the ‘exchange values’ of mathematics learning (Engeström, 1987, 1991, 2000). In the classroom, use value is associated with getting the answers correct; exchange value, with being seen by the teacher and other students as "liking to work ahead" (as Mrs R put it) or not being “a dumb person" (as the students put it). This primary contradiction is played out as a tension between the correct strategy S_3 on one hand, and the choice of the status conferring though erroneous strategy S_4 on the other. What this analysis shows is that ‘the lesson’ observed is more than merely a mathematics lesson - it is also a lesson about the social standing of mathematical knowledge, and about how critical value sometimes attaches to the forms of knowledge (such as are evident in S_4) rather than its substance (as in S_3). This paper therefore highlights the importance of recognising contextual and axiological variables when analysing cognitive events in the classroom.

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