STRUCTURING MATHEMATICAL KNOWLEDGE DURING A SPECIFIC EDUCATIONAL TASK

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The set of all triads $(a,b,a+b) \in N^3$, $(0 \notin N)$, $a \leq b$ with the mappings Lt: $(a,b,a+b) \rightarrow (a,a+b,2a+b)$, and Rt: $(a,b,a+b) \rightarrow (b,a+b,a+2b)$ creates the structure. This was used as a tool in the research aimed at investigating the building of an infinite arithmetic structure.

Research

The set of triads that is equipped with left and right mappings serves as a good tool for research, diagnosing pupils` abilities to build structure and is an educational field within recreational mathematics. The structure of triads as a research tool enables us to observe a whole process of creating an arithmetical structure because the graphical model of structure evidences many thinking structural processes of the pupils. The research has illustrated the following four facts:

1. Creating a global structure presumes necessarily previous insight into local structures (involving from two to five elements – the triads joined by the mappings).
2. The ability to create a concept of the structure of triads is profoundly individual. Pupils showing the same level of understanding of the structure of natural numbers get insights into the structure of triads at different rates.
3. A powerful tool is the ability to grasp the structure graphically and leads to the understanding of structure which does not depend whether a tree, representing the structure, is orientated up or down.
4. As the pupils solved the problems step by step they created other elements of the structure. During these activities the pupils had misconceptions. By realising this and by understanding the reasons for them, contributed to getting an insight into the structure. The following four cases of misconceptions were found:

A. The notation of triads can be reduced, e.g. triads coded as dyads or monads.
B. The triads on a line higher than line 1 can be written down automatically without thinking about mappings.
C. The generated pattern from the left 'branch' can be applied to the right 'branch'.
D. The number of line can be used as an operator for generating an appropriate triad on the line (the triad on line 10 was made by doubling the numbers in the triad on line 5).

I will give specific examples of pupils` solutions to illustrate the four cases of misconceptions mentioned above in my presentation.

References:

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