PROOF, PROOFS, PROVING AND PROBING: RESEARCH RELATED TO PROOF

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Research on proof and proving in mathematics education makes use of several different meanings for the words "proof" and "proving". In some cases this can lead to seeming contradictions in research findings. This paper identifies four current usages.

The concept of proof: Most professional mathematicians would say a proof of a statement confers absolute certainty on it. Some research has concluded that many secondary students do not understand proof in this way (see, e.g., Healy & Hoyles 2000). On the other hand, unpublished data (Zack 2000) suggests that some younger students may understand this concept of proof. For example, one Grade 5 student stated: "I think proving means showing that your answer is correct and it can't be wrong."

Proofs: There are sections of writing in mathematics textbooks and journals, which are called "proofs". They are characterized by a particular form and style. Writing proofs is a goal of recent reform documents but further research in this area is needed.

Proving: "Proving" is usually connected with deductive reasoning. Quite young children have been observed to be reasoning deductively (see, e.g., Zack 1997) but their reasoning seems to depend strongly on context. Additional research on the contexts in which children find deductive reasoning useful is needed.

Probing: Lakatos (1976) describes a process of "proof-analysis" in which "proving" is probing, testing the truth of a statement. Lakatos claims that his version of the nature of proving is incompatible with the concept of proof described above, which implies that researchers working from either perspective need to be careful they are not misunderstood as working from the other.

Note: A longer version of this paper is available at
http://plato.acadiau.ca/courses/educ/reid/papers/Pme-paper-2001.htm

References

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