Atomistic and holistic approaches to the early primary mathematics curriculum for addition

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Abstract

A study of teaching and learning in early primary British mathematics classrooms (children aged 4 -6 years) has identified contrasting ways in which the curriculum is approached. Focusing on the teaching of addition, one of the first formal arithmetical concepts taught in school, the researcher identified that children taught in through an atomistic approach to the curriculum, taught sequentially with reference to psychological research on children's conceptual development of addition, made less progress than those taught through a holistic approach, where the focus was on counting and the number system. The mathematical structure of number, the opportunities given to children to mathematize, and alternate views on cognition are used to consider the reasons for these findings.

Introduction

In the twentieth century the curriculum in British primary schools, and especially the mathematics curriculum, has been heavily influenced by Piagetian constructivism (Walkerdine 1984). Sociocultural (Lerman 1996) and Social Practice theories (Lave et al. 1991) while influencing views of learning in primary education (e.g. Wood 1998) have had little observable effect on the mathematics curriculum. Activities for young primary children emphasise practical activity, with a range of structure apparatus (Dienes 1964) in order to develop relational understanding (Skemp 1979). Lacking confidence in their ability to provide for the developmental needs of individual children, many teachers have tended to rely on structured published mathematics schemes (Walkerdine 1984; Desforges and Cockburn 1987). These schemes tend to be based on research evidence on the order in which concepts and strategies are developed, with the assumption that teaching should follow the same order; despite evidence to the contrary (Denvir and Brown 1986a, b).

The development of addition strategies has been summarised by Nunes and Bryant (1996) drawing on work by Carpenter, Romberg and Moser (1992) Gelman and Gallistel (1978) and others. The first strategy, and the most naive, involves counting out each set (e.g. 1,2,3, then 1,2,3,4,5) combining the sets and counting all of the new, larger set (1,2,3,4,5,6,7,8), and is referred to as counting-all (or the sum procedure). Subsequently children realise that they do not need to recount the first set of objects, but can 'count-on' from this set (3...4, 5, 6, 7, 8). Baroody and Ginsberg (1986) showed that this counting-on strategy might be 'invented' by children, while Fuson and Fuson (1992) developed ways to teach it. At the same time children begin to learn some of the number facts, especially the double facts (e.g. 5 + 5 are always 10)

(Gray 1997), and then to use these facts to derive others (if 5 + 5 are 10 then 5 + 6 are 11) (Thompson 1997). Further developments in addition strategies require an understanding of place value to solve multidigit addition (Nunes and Bryant 1996).

Context

The research is part of a wider study which examined the teaching and early learning of addition in early primary classrooms in Britain. The relationship between teaching and learning was examined at the level of classroom interaction, in the carrying out of mathematical tasks. The mathematics lessons of two classes in each of two schools were observed over a period of six months, involving four teachers and 112 children aged 4 to 6 years. The mathematical focus of the study was the learning of addition, one of the first formal mathematical concepts taught in school. This formed a basis for exploring the factors involved in the teaching of mathematics to young children, and their learning.

The methodology was qualitative, with participant observation the main method of data collection. Detailed fieldnotes were taken of all mathematics lessons observed; short unstructured interviews with teachers were carried out before and after the lessons. The children's understanding of number concepts and addition was assessed at both the beginning and the end of the observation period through a series of informal activities and games. The data was analysed using a grounded theory approach, which produced patterns of recurring variables. Analysis of these variables, grounded in the theoretical framework of the researcher, provided analytical pictures of teaching and learning, from which the findings emerged. One set of findings related to the way that the curriculum was structured and presented to the children, and it is this aspect that will be considered here.

Approaches to the mathematics curriculum

The study identified two contrasting approaches to the primary mathematics curriculum evident in the two schools studied.

Analysis of the addition curriculum at Ashburne School identified key characteristics of the way that the mathematics was broken down and taught to the children, which I have defined as '**atomistic**'. Atomism is defined as 'any doctrine or theory which propounds or implies the existence of irreducible constituent units' (SOED, Brown 1993). The most significant characteristic of the atomistic approach to the teaching of addition was that the curriculum was broken down into elements which were taught in a set order, which related to the findings on children's development of arithmetic strategies. There was an assumption that if research has found that children develop increasingly complex addition strategies, then the teaching of these strategies in a set order would result in learning. Analysis of the lessons observed showed that other significant characteristics were found to be related to this developmental approach. An Atomistic Approach showed:

- an initial emphasis on small numbers [small numbers],
- teaching addition in isolation from subtraction [isolation];
- emphasis on procedures rather than patterns and relationships [procedural];
- the use of physical and predominantly cardinal representations of number [cardinal];
- progression from one 'stage of development' to the next [developmental];
- the repetition of similar activities in order to reinforce the procedure [repetition].

These characteristics can be seen in the following transcript from one of the classes studied at Ashburne School. The lesson was designed to encourage the children (aged 6 years), who currently used 'count-all' to solve addition, to use a 'counting-on' strategy. It was one of a series of similar lessons in which the children used the number line to model addition [repetition; isolation]. The children had made a 1 to 20 number track [small numbers] which they used to play a game. The children took turns to throw a die and put that number of counters on the track [physical and predominantly cardinal]. On second and subsequent turns they were required to use the language of adding-on.

> Barry threw five and placed five counters, one each on the squares marked 1 to 5. The next time it was Barry's turn he threw another five.
>
> Teacher M. So, count out five to one side first of all, and then count-on from that number and put the cubes down one at a time.
>
> Barry 5, 6, 7, 8, 9, 10.
>
> (Teacher M:3)

This game formed a link between the use of discrete objects e.g. counters to model addition and the use of the number track, between cardinal and ordinal number. The children were encouraged to count-on (i.e. to say 6, 7, 8, 9, 10 rather than 1, 2, 3, 4, 5 as they added on five) and the teacher reinforced the language of counting-on: "So, 5 count-on 5 is 10" [procedural]. Over time the children learnt the procedure and could use the number track and counters to model addition, but in subsequent lessons and without these aids were seen to be no further on in their use of counting-on to solve addition.

The characteristics were not only seen in the lessons observed; they were also evident in the school's planning documents for mathematics and the mathematics scheme used in the school, showing that it was a curriculum, rather than individual teachers', approach.

Analysis of the addition curriculum at the second school, St David's, identified very different characteristics in the way that the mathematics was broken down and taught to the children, which I have defined as '**holistic**'. The most significant characteristic of the holistic approach to the addition curriculum was an emphasis on the number system, with calculation seen as one element of the relationship between numbers within the overall system. Further analysis showed other characteristics were related to this approach. A Holistic Approach showed:

- an emphasis on large numbers as well as small ones [large numbers],
- an emphasis on patterns in number and pattern spotting [pattern];
- discussion of the inverse relationship between addition and subtraction [inverse];
- emphasis on relationships rather than procedures [relationship];
- emphasis on ordinal representations of number and the development of mental imagery [ordinal];
- a eclectic curriculum in which the children are immersed in a wide range of activities, with little apparent sequence [immersion].

In both classes observed at St David's there was a heavy emphasis on counting, especially in the whole class introduction to the mathematics lessons. This was not the counting of objects, nor counting along the number track to encourage counting-on which I had seen in Ashburne School, but oral repetition of the number words in units and in larger steps, counting in twos, fives, tens etc., forwards and backwards. While I had initially not seen this as directly related to the teaching of addition, analysis showed that it was teaching essential skills towards the development of mental addition strategies where numbers are added by counting-on in units or larger steps. The children were developing mental and oral, ordinal representations of number which would help them count-on [ordinal]. The counting was not restricted to small numbers. During discussion of a hundred square [ordinal], Teacher D. highlighted the multiples of ten and she and the children (aged 5 and 6) counted in tens to one thousand.

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Debbie      ... count with me and we'll go all the way down[1] to one
            hundred and then see if we can keep going, 10, 20, 30, ...
            100, 110, 120, 130, ... 190, 200 ...
Some children are still counting with Debbie on two hundred, others
saying a hundred and twenty, all picked up again at 210.   They
continued  counting to 800.
Debbie      I am absolutely amazed.  Give yourselves a big clap.  Can
            anyone tell me what happens after 970, 980, 990 ?
Zeb         991, 992
Will        A thousand
Rita        it goes on and on                            (Teacher D:7)
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Many of the children were not confident in keeping track of the hundreds as they counted and they relied on their teacher to provide the next hundred number, but they were confident at the pattern of tens. They were praised and felt good about counting with such high numbers [large numbers]. The counting sparked further discussion which showed the children's wider understanding of the number system including negative numbers [relationships].

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[1] I believe that the teacher talked of counting 'down' rather than the more usual counting 'up' to 100, since the number square showed the tens increasing down the right hand column.

These relationships can also be seen in an earlier lesson, centred around the number seven. Before the lesson Teacher D. talked to me about what she was going to do.

> "Some days we just look at a number and see how many different ways we can make sums that have that answer. Some of the children will just be able to do simple addition or subtraction, some might get into patterns [patterns]. I hope that all the children will be able at least to try. I think it is important for them to look at sums this way ... there can be lots of different sums with the same answer, you can get there by doing different things" [relationships].
>
> (Teacher D:4 - pre-lesson discussion)

D. wrote a large number 7 in the centre of the board.

D. Who can think of a way to make seven?

Rob Three and four.

[At each suggestion D. wrote the calculation in symbols on left-hand side of the board (3 + 4). Subtractions were recorded on the right-hand side of the board.]

Nozumo Four and three.

Ruth Seven and zero.

Lucia Twelve take away three ... er ... five.

John Twenty take twelve.

D. Nearly

John Twenty take thirteen.

Nozumo Two add five.

Ellen Six and one.

Zeb Five and two.

John Twenty-one take fourteen; twenty-two take fifteen.

Ruth Twenty-three take away sixteen.

Ellen Twenty-four minus seventeen...

Beaty one add one add one add one add one add one add one, and 10 take away 3.

John Seven take seven add seven; eleven take four, twenty-seven take twenty... (Teacher D. 4)

I observed that the children were using pattern [patterns] to generate more examples, applying their knowledge of pattern to addition. Nozumo (one of the new reception children, aged 5) seemed aware of commutativity, providing 4 + 3 after Rob gave 3 + 4, and later 1 + 6 when 6 + 1 was already on the board [relationship]. John uses inverse (7+7-7) [inverse] and equal addition. The children were working mentally and at their own level. Nozumo, Zeb and Rob (aged 5) were able to concentrate on simple one step addition, while experiencing the subtraction and more complex multi-step calculations offered by the others [immersion].

The advantages of a holistic approach to the curriculum

I have identified differences between the two school's approaches to the curriculum which I have categorized as Atomistic and Holistic. While these differences were found between the two schools, by analysing the data for counter examples, I found that the approaches were also influenced by the individual understanding of the teachers and their beliefs about mathematics and learning, though this paper does not allow for further discussion of this. In this section I want to look at why a holistic approach may be preferable to an atomistic approach to teaching addition, drawing on three areas of theory.

The first of these relates to the structure of the number system itself and, in particular, the additive composition of number, which Nunes and Bryant (1996) define as " any number *n* can be decomposed into two others that come before it in the ordinal list of numbers, in such a way that these two add up exactly to *n*" (p. 46). A special, and important, case is that of place value, since larger numbers can be decomposed into their constituent multiples of ten and units. Nunes and Bryant found that an understanding of this characteristic of number is important not only for addition of large numbers, but is also essential for children to learn to count-on. It is the basis for understanding that a number can be added to another number directly, without them having to be reduced to their unitary elements and counted singly. Nunes and Bryant emphasize the importance not only of counting, but also of "understanding the relative value of counting units and their additive composition" (p. 51). It therefore follows that teaching children to count-on using a number track is likely to be ineffective if they do not understand this additive structure of number, and demonstrates why direct teaching of small areas of arithmetic do not necessarily result in the learning of those particular areas (Denvir & Brown 1986). The concept of additive composition of number provides insight into the advantages of a holistic curriculum. Such a curriculum allows an understanding of the relationships between numbers and patterns within numbers, which characterize the additive composition of number, and are essential for addition strategies more complex than counting-all. This is more than an argument that place value should be taught in advance of addition (an atomistic approach). It is to argue that addition must be seen as an integral part of the number structure rather than a mathematical topic in its own right.

A second reason why a holistic approach to the curriculum may be preferable to an atomistic one relates to the nature of mathematics and learning. A holistic curriculum gives children greater opportunities to mathematize, to act in a mathematical way. Patterns and relationships are an essential part of mathematics which enable the development of conceptual structures (Skemp 1971; Hiebert 1986). Procedural learning, identified as a characteristic of an atomistic curriculum, can result in limited understanding: children are able to carry out the procedure but not understand why. When the procedure has been forgotten, or the necessary materials are not to hand, the learner has no way to reconstruct it. I have shown how breaking the curriculum down into very small pieces which are learnt in isolation (atomism), can result in such procedural learning. Teaching children the 'bigger picture' first, an understanding of the way that the number system fits together (holism), offers them an overall structure in which constituent sub-topics within arithmetic, such as addition, can be defined.

For the third and final explanation of the advantages of a holistic curriculum I want to appeal to the wider human experience of learning. Chomsky (Chomsky 1980) proposed a language acquisition device (LAD) to explain how children learn to speak. He reasoned that, since language was extremely complex, young children must have some special way to make sense of it. They appear to learn not only by copying the language that they hear, but by constructing their own logical rules. For example, young children will often generalize regular forms of the past tense to include goed (went), seed (saw) or buyed (bought). Chomsky therefore argued for a specialized area of the brain, specific to language learning and complete with a LAD. Bruner argued that such a LAD was in fact culturally influenced, offering instead a language acquisition support system (LASS, Bruner 1986).

I believe that language acquisition is not a unique part of learning. All children's learning in the world outside school is as an experience of immersion, in language and in culture. It is in the nature of children to make sense of the world around them. It would seem logical to assume that to teach mathematics through immersion into the number system, is to take advantage of the way that children learn. To break it down into 'bite sized pieces' is analogous to teaching children to speak using only nouns first, or allowing them only to relate socially to one other human being because more than one may confuse them. While the curriculum in British primary school mathematics classrooms has been based on a constructivist perspective on learning, founded on an active, practical approach to learning and a developmental (and therefore atomistic) view of progression, to view learning as resulting from 'immersion' is to see learning from a sociocultural perspective. Such a perspective appears to accord more clearly with the findings of this research.

Summary and Implications

In this article I have identified differences in the way that the mathematics curriculum is presented to young children found in a study of two schools and which I have described as atomistic or holistic. An atomistic approach to the curriculum breaks the addition curriculum into its developmental stages and teachers target teaching to the next stage, according to the teacher's perception of the children's needs. Activities are designed to address these stages, to teach counting-all, counting-on, and place value, starting from cardinal representations of number. This was the predominant curriculum experience at Ashburne School. A holistic approach to the curriculum sees addition as part of the relationships within the number system. Teachers offer children a range of activities which develop facility with number including counting and locating, and patterns in number as well as more formal addition tasks. This was the predominant way of presenting the curriculum at St David's.

I have shown how these findings are based on the data collected in this study, and finally given some explanations for why a holistic approach may be preferable to an atomistic approach to teaching young children addition. I have considered three ways to explain why a holistic approach to the curriculum may be preferable to am atomistic approach. These have been based on an understanding of the additive composition of the number system, an understanding of mathematical reasoning, and a personal hypothesis about children's learning.

This implies that children will learn early addition more easily if they experience a holistic curriculum. More complex addition strategies will not be developed through direct teaching of these strategies but through an understanding of the complexities of the number system. Children, offered a broad range of activities which explore the number system in its complexity, are able to develop understanding and skills which are specific to addition. However, in order to teach in a holistic way teachers may have to change their views of mathematics, addition and how children learn. Offering teachers a holistic curriculum may not prevent them from reverting to an atomistic approach if this is all that they understand.

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