



# DESIGNING, RESEARCHING AND IMPLEMENTING MATHEMATICAL LEARNING ENVIRONMENTS



## THE RESEARCH GROUP “MATHE 2000”

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### Origin

In 1985 the State of Nordrhein-Westfalen adopted a new syllabus for mathematics at the primary level (grades 1 to 4). This syllabus is essentially due to Heinrich Winter, one of the leading German mathematics educators, who chaired the commission preparing the document. For three reasons this syllabus marked an important turning point in the history of mathematical education in Germany:

- The list of objectives also contains the so-called general objectives ”mathematizing”, ”exploring”, ”reasoning” and ”communicating” which reflect basic components of doing mathematics *at all levels*.
- The complementarity of the structural and the applied aspect of mathematics is stated explicitly and its consequences for teaching are described in some detail.
- The principle of learning by discovery is explicitly prescribed as the basic principle of teaching and learning.

In order to support teachers in putting this syllabus into practice the project ”mathe 2000” was founded at the University of Dortmund in 1987 as a joint venture of the chairs ”Didactics of Mathematics at the Primary Level” (Gerhard N. Müller) and ”Foundations of Didactics of Mathematics” (Erich Ch. Wittmann). In 1993 Heinz Steinbring joined the project and brought in the missing empirical component. Since its inception ”mathe 2000” has been based mainly on the brains of its members and so has been independent of funds although some of its research was funded.

## **Basic philosophy**

According to a conception of mathematics education as a "design science" three areas of research and development are closely linked and pursued simultaneously:

- the design of substantial learning environments and curricula,
- practical work (pre-service and in-service teacher education in both mathematics and didactics, school development, counselling),
- empirical studies into children's thinking and into communication in the classroom.

The five papers presented at the research forum try in an exemplary way to cover the whole range of the project work and to display a special feature of the project, namely the systematic reference to learning environments: Erich Ch. Wittmann gives an example of design ("Arithmogons") and points to its implications for teacher education. By referring to "Number chains" Christoph Selter explains how teacher education can be systematically related to developmental research. The other three papers are concerned with empirical research: Anna S. Steinweg's paper indicates how number patterns from the "mathe 2000" textbook can be used for qualitative research into children's thinking. Elisabeth Moser Opitz reports on a quantitative study which revealed that the "mathe 2000" approach is feasible also for children with special needs. Heinz Steinbring gives an example of his qualitative studies into classroom interaction in which he used "number pyramids" as research instruments. A special mark of his studies is that they relate psychological processes and communication in the classroom to the epistemological structure of subject matter.

The five papers also shed light on four principles which are at the heart of the project:

- Fundamental ideas of mathematics as guidelines (epistemological orientation)
- Learning as a constructive and social process (socio-psychological orientation)
- Teaching as organising learning processes (practical orientation)
- Co-operation with teachers (systemic orientation).

## **The arch fathers**

The roots of the present reform of mathematical education date back to at least the late 19<sup>th</sup> century. "mathe 2000" is understood as part of this process and draws heavily on ideas developed in the past, in particular on four "arch fathers":

- John Dewey (1859-1953), for his clear decision for education in a democratic society and for conceiving of theory as a guide to an enlightened societal practice at all levels (as developed, for example, in "Democracy and Education", "The Child and the Curriculum", "The Relation of Theory and Practice in Education", "The Sources of a Science of Education")
- Johannes Kühnel (1869-1928), for his precise description of the shift from "guidance and receptivity" towards "organisation and activity" (cf., his classical books "Neubau des Rechenunterrichts" [Reconstructing the Teaching of Arithmetic] and "Die alte Schule" [The Old School])
- Jean Piaget (1896-1980), for displaying the overwhelming importance of the learner's "constructive activity" in his pioneering work in genetic epistemology and psychology ("Psychology of Intelligence", "Mathematical Epistemology and Psychology", "Biology and Knowledge", "Theories and Models of Modern Education")
- Hans Freudenthal (1905-1990), for his fundamental contributions to understanding mathematics as a human activity and as an educational task as well as for his insistence on developmental research as the core of mathematics education ("Mathematics as an Educational Task", "Weeding and Sowing", "Didactical Phenomenology of Mathematical Structures", "Revisiting Mathematics Education").

### **Publications**

The research conducted by project members is documented in numerous articles and some books. As mentioned before a special mark of almost all papers is that they are systematically related to learning environments. A booklet with selected papers in English will be available at the conference.

### **Materials**

It was a strategic decision at the very beginning to present the basic message of "mathe 2000" in a "Handbuch produktiver Rechenübungen" [Handbook of practicing skills in a productive way] (2 vols. published in 1990 and 1992). The "Handbuch" contains a systematic epistemological analysis of arithmetic from grades 1 to 4 in the form of substantial learning environments which combine the practice of skills with higher mathematical activities.

The "Handbuch" inspired many teachers to conduct teaching experiments. These experiments were very successful. So it was teachers who demanded a new textbook consistent with the "Handbuch". In collaboration with a group of teachers the four volumes of "Das Zahlenbuch" were developed and tested from 1993 to 1997.

Since the middle of the nineties the book has spread over Germany and crossed the borders to some neighbouring countries (Switzerland, Belgium, and the Netherlands).

In addition to the "Handbuch" and the "Zahlenbuch" the "Programm mathe 2000" published by Ernst Klett Grundschulverlag contains other materials for teaching primary mathematics (including the CD ROM "Blitzrechnen"/ "Calculightning", with an option in English, and the booklet "Double Mirror Magic" in English). The complete "Programm mathe 2000" will be exhibited at the conference.

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