PROMOTING MATHEMATICAL THINKING: A PILOT STUDY FOR INNOVATIVE LEARNING ENVIRONMENTS

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***Abstract**: The aim of this pilot study was to experiment and test some novel innovative teaching methods in practical teaching situations with primary level pupils (10 year-olds). In the experiment on learning environments four different teaching methods was implemented with that aimed primarily at using of collaborative communication, supporting pupils' meta-cognition and improving problem solving processes. The development of teaching methods used in the study were based on the earlier research on word problems, the research on meta-cognition connected with them, and research done on the use of open problems. The results of the pilot study show that with the use of such a many-sided learning environment and the teaching methods used it is possible to support the development of pupils' meta-cognition and problem solving skills.*

Background

The theoretical background for the pilot study was a compound of theories on mathematical thinking and and learning with have been developed in the current constructivist tradition (e.g. Davis & al. 1990). Particularly our approach is based on theories that emphasise the role of communication, interaction, and meaningful tasks in supporting the development of pupils' meta-cognition and problem solving skills. A teacher's role in mathematics lessons is seen primarily as an activator of pupils' thinking skills, to help pupils to understand mathematical structures and ideas as well as to help pupils when developing their own mathematical knowledge (Schoenfeld 1985, Sfard 1998, Lehtinen & al.1999). The central element in developing the teaching methods of the pilot study is the activation of a pupil's own thinking.

Mathematical thinking should be distinguished from mathematical contents and techniques. Burton (1984, 35) states that "mathematical thinking is not thinking on mathematics, but a thinking style which is a function of special operations, processes and dynamics characteristic to mathematics". Of them especially the processes of mathematical thinking are interesting for problem solving. An individual's meta-cognitions are regulating elements of his thinking (cf. Schoenfeld 1987). In mathematics, meta-cognitions are often connected to problem solving skills (e.g. Schoenfeld 1985). An important component of meta-cognitions is self-regulation (cf. De Corte & al. 1998). To practice problem solving is seen to promote the development of pupils' higher order thinking and understanding (e.g. Verschaffel & al. 1999). It is stated in many sources that i.a. verbal elaboration of mathematics to be learned (e.g. Cockcroft 1982, NCTM 1996, 2000) and a pupil's own experiences in problem solving (e.g. Schoenfeld 1985) promote the development of his meta-cognitive skills. For example, self-regulation can be promoted with proper problems and their treatment (De Corte & al. 1998). Just with the aid of many-sided communication in the pilot study, it is searched for different verbal methods with which a teacher could support the development of his/her pupils' meta-cognitions and problem solving skills.

Research on mathematical thinking will often be operationalized through problem solving (e.g. Burton 1984). Since in the middle of the 1980's it was realized that teaching of heuristics was not a successful solution, alternative approaches were searched for (e.g. Schoenfeld 1985). Among other methods, it was developed for teaching problem solving an approach that emphasized creativity as an important part of problem solving (e.g. Mason 1982). During the last ten years, the so-called open problem solving has become popular which has near connections to creative problem solving (Pehkonen 1997a) and also to the Japanese "open approach" (e.g. Nohda 1991) and the use of investigations (e.g. Wiliam 1994).

Creativity and logic are central elements in mathematical problem solving (e.g. Mason 1982, Pehkonen 1997a). Recently one less known component of problem solving – problem posing – has been pushed up (e.g. Silver 1995). It belongs also to the open understanding of problem solving (cf. Pehkonen 1997b). A teacher's role is of paramount importance when selecting proper tasks and problems for mathematics lessons. The selection of problems and their application properly in lessons is a difficult task in mathematics teaching. The role of the teacher is very important when constructing a proper atmosphere in class where pupils can frankly investigate, make mistakes, share their success und failure, and exchange their ideas with each others without fear that they will be critically assessed (NCTM 2000). With the aid of such a learning environment it is possible to educate pupils who trust in their own abilities and who are willing to participate and investigate problems.

Hakkarainen & al. (2000) state that in order to be able to help intensive and pedagogically intentional learning with the aid of information technology, it would be very important to locate computers in classes and integrate working with computers with many different working methods and disciplines during the working phase in classroom. Computers should not be seen as separate tools for learning of an individual pupil, but rather as a method of learning in a co-operational learning environment (e.g. Reusser 1991, Lehtinen & Repo 1996). Computers should not be used as a substituting tool when striving for mathematical understanding and intuition, but their use should promote and strengthen processes of mathematical understanding (NCTM 2000).

The focus of the paper

The pilot study in question experiments and investigates the possibilities and limits of new and innovative teaching methods for promoting mathematical thinking and reasoning skills when used in the teaching practice of 10 year-old pupils. Furthermore, the focus lies in the possibilities of developing pupils' problem solving skills, pupils' communication skills, pupils' added awareness of their meta-cognitive skills and abilities as well as pupils' motivation. Based on the results of the pilot study, the usefulness of different teaching methods in the learning environment used are evaluated for the implementation of the main experiment. Teacher students who implemented a part of teaching form an additional factor in research. A side objective of the pilot study is to clarify the possibilities to integrate such research projects into the teacher education, in order students would get experiences from a new kind of mathematics teaching.

Implementation

The pilot study on promoting mathematical thinking and reasoning skills was implemented in the teacher training school in Turku in the spring 2000. A forth-grade class (10 year-olds, N=17) formed the experiment group; the class has a special emphasis on mathematics and computers. This class has four lessons mathematics pro week during the experiment period of ten weeks there were altogether 40 experimental mathematics lessons. During the intervention special emphases was given on promoting pupils' thinking and reasoning skills, communication, added awareness on their own knowledge and skills (i.e. meta-cognition) and motivation. The teacher of the class, Mr. Jari Sorvari (one of the authors) with five student teachers was responsible for the practical implementation in class. The teaching methods used in the pilot study were planned together by the authors, in the way that a teacher in a usual school could also implement them alone. The researcher wrote beforehand written plans for all experiment lessons. Each student teacher got in advance the written lesson plan for his future lessons with instructions. Before each lesson, there was still a brief discussion on the lesson plan with the student teacher, if needed. During the discussion, possible difficulties or lack of clarity in the lesson plan were dealt with. Each student teacher implemented during his week of practice four experimental mathematics lessons. The personal teaching style of each student teacher was not influenced in any other way.

The pilot study was built of four different teaching methods which were implemented always in the same weekday and which aimed to promote pupils' mathematical thinking. Additionally, computers were used in routine exercises. The method called *Scattered-Minded Moses* was based on the teacher's own loud-thinking, modelling of word problems, and conscious "thinking mistakes" during the solving process before the class. Pupils' task during the lesson was to perceive these mistakes that were then discussed together. With the learning game *Quest of the Golden Chalice* (Vauras, Rauhanummi, Kinnunen, & Lepola, 1999) we tried to develop pupils' solution skills in word problems and meta-cognitive skills as well as to improve their calculation skills and to promote mathematical communication in class. With the aid of the method *Concrete Problem Solving*, we tried to build creative, open and motivational learning situations with hand-on material. Problem solving gave an opportunity to develop pupils' thinking skills and logical reasoning. The method *Concept Lessons* concentrated in mathematical concepts using problem-oriented approach and instructional discussion. Beside these four methods *Computers* were used to intense routine exercising Computers offered for the teacher a possibility to work with a smaller group in class.

Data was gathered in the pilot study by using multiple observation methods Since we wanted with a small number of participants to find out the appropriateness of different teaching methods, observational methods were stressed in data gathering. There were teachers' field notes (both the researcher's and the student teachers') on implemented lessons, the student teachers' own notes and experiences of mathematics lessons, videos on the part of the lessons and pupils' interviews. Observations were unstructured, and thus, the field notes are based on each observer's own personal view on teaching situation.

Results

Based on the analyses of the video records of the lessons in which the method *Scattered-Minded Moses* was used, it seemed that the teacher-pupils interaction functioned well, especially in short word problems. Pupils were able to follow excellently the teacher's modelling and thinking process in these problems. Almost all pupils could perceive the teacher's "mistakes" in loud-thinking processes and amend them during the common discussion. The use of the method seemed really to motivate pupils to follow their teacher's loud-modelling process. Since the teacher did not make mistakes in each problem, pupils were compelled to follow very keen their teacher’s thinking process, in order to perceive his mistakes. The motivational level seemed to be really high when solving so-called short word problems with the help of the loud-modelling method. Whereas in the case of long problems, similar effect was no more so clear to observe. Difficulties in long problems seemed on one hand to be partly connected with the complexity of the teacher’s own thinking process, and on the other hand with the fact that pupils were simply not able to concentrate and to comprehend the structure of the long problems.

The learning game *The Quest of the Golden Chalice* is planned for small-group teaching or remedial teaching of third-graders (9 year-olds). In this study, the learning game was accommodated with small changes in rules to normal teaching in mathematics. The changes in rules done seemed, according to video analysis, to produce some especially interesting observations. Differently as in normal case, the pupils played the games during the experiment in pairs. This seemed to change the nature of the game. When playing in pairs, the communication between players (pupils) increased very much. The teacher’s role as a game leader was also changed. Since the pupils played in pairs, main instructional discussion was done within the pairs, and not between the teacher and the pupils as usual. The teacher’s role was mainly to act as a discussion leader. The communication within pupil pairs seemed to increase and develop, when the pupils gathered experiences with the game. Pupil pairs advised each other and could give those pupil pairs in trouble good hints for solutions of problems. Furthermore, pupil pairs’ loud-modelling of solutions seemed to get more clear with time. The additional changes in rules done in the middle of the experiment seemed to still add spontaneous use of drawing, in order to clarify one’s own solution process. Drawings seemed to clear and strengthen pupils’ thinking processes, when they explained and described their solutions to other players.

In the analyses of *Concrete Problem Solving*, observations made were based on the researcher's own perceptions as well as student teachers' observations and field notes. On the ground of these we can state that instructional discussion had a meaningful share in the problem solving lessons with hands-on materials. Concrete materials offered a tool to "externalize" different phases of the problem at hand, and their possible problematic points. This gave an opportunity to consider also difficult topics with the aid of common instructional discussion. The pupils were very motivated to solve problems using concrete hands-on materials. Such material used were match sticks, soma cubes, multilink cubes, fraction cards and normal playing cards.

Working with matches seemed to interest the pupils very much. In addition, the problems used supported well the objectives set for the use of concrete materials. An interesting observation was connected to the behaviour of two pupils, when giving a home problem. The home problem was a continuation to the following problem dealt with in class. A square sequence should be constructed with matches, where the starting point was a square of four matches. The home problem was formulated, as follows: "How many matches are needed to construct a sequence of 500 squares? Try to find a possible formula / solution not using the help of matches." The giving of the problem was just finished, when two boys in class wanted to give the solution: *"It is an easy piece, Mister. Those matches will be needed altogether 1501, since in the first square there are four matches, thus 3*500+1 is 1501."*

During the *Concept Lesson*, the teacher had an opportunity to work with a smaller pupil group. Half of the class worked at the same time with computers on routine tasks connected to the topics to be learned. Based on the observations done we could state that during the Concept Lessons the interaction between the teacher and pupils could be intensified. The time the teacher used for a pupil doubled. Furthermore, it seemed that materials used for concretization motivated pupils. Working with a smaller group made it also possible that the level of pupils' understanding was more effective secured. The big amount of instructional discussion should be also mentioned. A special emphasis seemed to be in discussions on different solution alternatives.

In this study, six *computers* were used in classroom. The programs used were two in Finnish, of which the first concentrated on routine tasks of mathematics to be learned, and the second contained mainly problems of proper level. Since there were only six computers at hand, and half of the group (N=17) at the same time, a part of pupils were compelled to work in pairs on one computer. Observations made during the computer working phase showed that on one hand to transfer such amount of mechanical exercises on computer seemed to be a functioning solution. On the other hand, the programs used did not offer enough purposeful and useful tasks, in order to implement instruction effectively, especially there was a lack of problems for individual differentiation. In addition, the program did not contain characteristics needed for following effectively an individual's learning process.

The communication and interaction within pupils pairs was very active almost all the time of the experiment period on computers. The instruction given and the awareness that the computer will remember mistakes seemed to activate the discussion within a pair. It seemed that the interaction between the pupil pairs could have acted as a feedback system, and the pairs did not need the computer's feedback system when solving the problems. Observations on those pupils working alone were slightly different. They seemed to base their work more on the computer's feedback system than their mates working in pairs.

Discussion

In Finland, a teacher is compelled to work, especially in growing counties, with bigger and bigger classes. This results that time allotted to help pupils individually, e.g. in mathematics lessons, will be decreased all the time. Therefore, one is compelled to search new ways and methods to organize learning environments. There is a need to develop different ways of group working which will support effectively the development of pupils' mathematical thinking. Research results and observations gained during the experiment showed that the methods used in this study are worthwhile developing further. With the aid of the teaching methods developed through research, we should be able to find functioning working methods for the class, and through using them to conduct pupils toward higher order mathematical knowledge and understanding.

The objective of developing such learning environments is to find out pedagogically purposeful teaching practices. Some research questions which can been seen as results of the pilot study are, as follows: Is it possible to reach deeper and better level learning with such a learning environment? Could it be shown that some individual teaching method would support some special area of learning? In the light of the pilot study, none of these questions could be answered exactly, but the study showed these questions to be sensible to ask in the main experiment.

The benefit of the pilot study lies just in its concrete context. The experiment in real teaching situation and the data gathered during it made it possible to implement the learning environment with different teaching methods at one time. On one hand, such an implementation gave much valuable information on the advantages and difficulties of combining individual teaching methods. On the other hand, we could experiment and plan the learning environment as a whole. Thus, we gained information on of what kind of changes in classroom, teaching organization, working order etc. should be taken account of when implementing such a learning experiment.

All the student teachers considered the participation to such an experiment very important. In the following, there is a comment in one student teacher' notes which comment reflects the feeling and thinking of all participative student teachers: *"I am* *glad of the possibility to participate the mathematics intervention. For me mathematics has always been a challenging topic, since my best method of learning has been doing, but mathematics has been traditionally very abstract and theoretical. It was absolutely wonderful to have an opportunity to experiment that mathematics, and especially challenging problem solving, could be taught so concretely. A pupil has a possibility himself to do, to experiment and to have insights."* (student teacher #2)

Such kind of developing projects on learning environments should be connected to teacher education also more generally. Thus we may construct important links, e.g. in mathematics, between research and practice. To see this connection is not necessary always self-evident. This view of the gap between research and practice has been described very hitting in a comment of a student teacher: *"The mathematics project which I was allowed to participate seemed to be very interesting. In the model used, a teacher was a guide for learning instead of a knowledge transmitter. This was at last the concretization, which was long heard in the speech of the theoreticians. The ultimate aim was to get pupils aware of their thinking processes. In the matter of fact, we are in the core of learning (a pupil understands his thinking), if I have condensed rightly. This starting point awakes me a desire to know more."* (student teacher #1) The combination of learning theories, didactics and teaching pedagogy would add student teachers understanding on the teaching / learning process.

What were the pupils experiences in the learning environment? One third of the pupils could see nothing special done during last weeks in class. Another third of the pupils described the classroom working very many-sided. All pupils thought that mathematics has been nice and interesting. On one hand, some did not like mathematics, when there were too difficult problems to solve. On the other hand, many pupils experienced mathematics interesting such when having difficult tasks to solve. This view is described very touching in one pupil's interview: *"difficult tasks are really nice to solve, ... such where is some idea and not only a mere calculation, ... such where you should yourself pick the numbers, ... such straight calculations are dull"* (pupil #38). Finally one student teacher's view on the behaviour of the class during the last weeks of the experiment: *"In this class, it was self-evident that questions like "Why?" and "What did you think?" were presented. I had a good feeling: pupils' pre-knowledge and beliefs were activated, and these were the starting point for further development."* (student teacher #5)

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