ARITHMETIC OR ALGEBRA?
PRE-SERVICE TEACHERS' PREFERENTIAL STRATEGIES FOR SOLVING ARITHMETIC AND ALGEBRA WORD PROBLEMS

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Abstract

This study investigated the arithmetic and algebra word-problem-solving skills and strategies of pre-service primary and secondary school teachers both at the beginning and at the end of their teacher training, and the way in which these groups of pre-service teachers evaluated different kinds of algebraic and arithmetical solutions of pupils. The results showed that future secondary school mathematics teachers clearly preferred algebra, even for solving very easy problems for which arithmetic is more appropriate. About half of the future primary school teachers adaptively switched between arithmetic and algebra, while the other half experienced serious difficulties with algebra. Finally, it was found that the problem-solving behavior of the future teachers is strongly related to their evaluations of pupils' solutions.

1. Theoretical and empirical background

Acquiring an algebraic way of reasoning and problem solving is one of the major learning tasks for pupils in the transitional stage from primary to secondary school. However, a vast amount of research has shown that learning algebra creates serious difficulties for a lot of pupils (Kieran, 1992; Filloy & Sutherland, 1996). The focus of the current study is somewhat different; its attention goes to the *mathematics teacher* who has to stimulate and support the transition from arithmetic to algebra.

The starting point of our study was the work of Schmidt (1994; Schmidt & Bednarz, 1997). She states that the complexity of the algebra learning process makes appeal to both primary and secondary school teachers. Primary school teachers should develop in pupils a rich base of mathematical concepts and skills which are the psychological foundations and precursors for algebraic thinking. Secondary school teachers should have a very good understanding of the 'arithmetical histories' of pupils entering secondary education, and be able to show pupils the validity and necessity of the new algebraic way of thinking. At the same time, they should develop in the pupils a disposition to apply arithmetical and algebraic strategies flexibly, taking into account the characteristics of the problem to be solved. In sum, Schmidt claims that primary and secondary school teachers must understand, master and appreciate both arithmetical and algebraic problem-solving strategies themselves, and be able to use them whenever necessary.

These considerations led Schmidt to conduct a study with three different groups of Canadian students at the start of their teacher training. By means of a paper-and-pencil test with typical arithmetic and algebra word problems (see the Method section), and by means of semi-structured interviews, she studied their spontaneous problem-solving behavior, their arithmetical and algebraic skills as well as their related domain-specific beliefs. The *first group* were students who just subscribed to a training to become a remedial teacher (in primary or secondary school). She found that the majority of them had to rely exclusively on arithmetical strategies because they had no good understanding and/or mastery of the algebraic approach. This need to rely always on arithmetic had a negative influence on their performance on the (complex) algebra problems. The *second group* were students wanting to become primary school teachers. Schmidt found that about half of these students were adaptive problem solvers, using arithmetic strategies for easy arithmetic problems, with good success, and algebra for more complex algebra problems, with moderate success. The other half of this second group had a similar profile as future remedial teachers. The *third group* consisted of beginning pre-service secondary school teachers. Here, the vast majority of participants exclusively used algebraic strategies (with good success), even for those problems that could easily be solved arithmetically. This preference was accompanied by a perception of arithmetic as being a primitive, mathematically worthless approach.

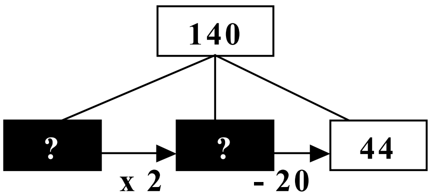
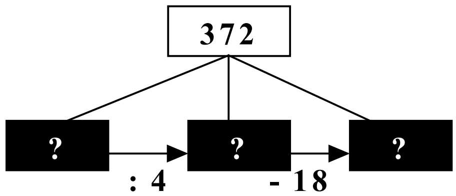
The study presented in this paper replicates Schmidt's study in the Flemish teacher training context, but at the same time elaborates it in two important aspects First we expanded the research group with students arrived *at the end* of their teacher training. Second, we wanted to shed light on the relationship between the student-teachers' pedagogical content knowledge and beliefs, on the one hand, and the quality of their future teaching, on the other hand. Therefore, we also collected data about the way in which the pre-service teachers evaluated pupils' arithmetical and algebraic solution strategies.

2. Method

Participants were 97 pre-service teachers of one typical training institute in Flanders. These future teachers belonged to four different groups according to the specific training they were subscribed to (primary versus secondary school) and the moment in their training (the beginning of their first year versus the end of their third and last year of teacher training). In the group of future primary school teachers, there were 26 participants in the first and 36 in the third year. In the group of future secondary school teachers, these groups contained 19 and 16 participants, respectively.

Two research instruments were administered: a paper-and-pencil test containing 12 word problems, and a questionnaire wherein the student-teachers had to score pupils' solution strategies for six problems from the test.

The paper-and-pencil test contained – in randomized order – six word problems that could easily be solved with a few arithmetic calculations and six more difficult word problems for which an algebraic strategy was more efficient. The problems were generated by means of an analysis method designed by Bednarz and Janvier (1993, in Schmidt & Bednarz, 1997). This method schematizes data in such a way that problems can be characterized as "connected" (two known values can easily be used to calculate a third, and so forth, so that arithmetic solutions are easy), or "disconnected" (a calculation with two known values to generate a third cannot be made, so that an algebraic strategy, which represents all known and unknown data in one equation, becomes more appropriate). In the table below we give an example of an arithmetic and a (semantically equivalent but structurally different) algebra problem.

Arithmetic problem	**Algebra problem**
In our farm we have 140 animals: cows, pigs and horses. The number of cows is the double of the number of pigs, and there are 20 horses less than cows. We have 44 horses in the farm. How many pigs and cows do we have?	In a large company, there work 372 people. There are 4 times as many workmen as clerks, and 18 clerks more than managers. How many people of each group are there then in the company?
	

Student-teachers' solutions were analyzed according to the correctness of the answer, and according to the solution strategy used. For this last scoring we applied a classification schema that was influenced by the findings of several researchers (e.g. Filloy & Sutherland, 1996; Hall, Kibler, Wenger & Truxaw, 1989). The table below presents a global characterization of the strategies for solving algebra and arithmetic word problems distinguished in this classification. For more details and examples we refer to Van Dooren, Verschaffel and Onghena (in press).

Strategies for algebraic word problems		
Algebra: An equation is written and transformed to calculate the unknown.	**Manipulating the structure**: The problem is restructured in a clever way so that it becomes solvable arithmetically.	**Guess-and-check**: The value of one unknown is "guessed", the correctness of the guess is checked. Repeats this until the correct value is found.
Strategies for arithmetical word problems		
Algebra: Same as for algebra problems.	**Manipulating the structure**: Same as for algebra problems.	**Generating numbers**: The missing values are directly calculated by performing the correct arithmetic operations on the known values.

Immediately after finishing the paper-and-pencil test, the student-teachers received a questionnaire with three arithmetic word problems and three algebra problems from this test. Each problem was accompanied by three handwritten correct solutions (one of each category from the table mentioned above). Student-teachers had to give a score on 10 points to express their appreciation of the quality of each of these three strategies. By presenting only strategies that led to *correct* answers, we could interpret differences in scores as evidence for differences in appreciation of the underlying solution strategy. We also asked the student-teachers to motivate their scores in a short written comment. As in the paper-and-pencil test, the order of the word problems (as well as the answers accompanying them) was randomized.

3. Research questions and hypotheses

Question 1: How (well) do pre-service teachers solve arithmetic and algebra word problems?

Our main hypothesis here was that the Flemish student-teachers would have a similar pattern of solution strategies as in Schmidt's (1994) study. More particularly, we predicted that future secondary school teachers would use mainly algebra to solve the word problems (including the arithmetic problems), and that they would do it successfully (**Hypothesis 1a**). Furthermore, we expected to find two groups of future primary school teachers: an "adaptive" group using alternately arithmetic and algebra with moderate success, and another group with a strong preference for arithmetic strategies (and, therefore, a weak performance on the algebra problems) (**Hypothesis 1b**). With respect to the factor "years of teacher training", the following two effects were expected. First, we predicted that the initial differences in strategy use between future primary and secondary-school teachers would increase during teacher training. In the training of secondary school teachers, pivotal attention is given to algebra; therefore, we expected that they will use more algebra at the end of their teacher training. In contrast, in the primary school teachers' training program, large attention is paid to the arithmetical method "manipulating the structure"; therefore, the use of this strategy among pre-service primary school teachers should increase with years of training (**Hypothesis 2a**) Second, we predicted that the test performance of the student-teachers would improve, since the Flemish teacher training does not only aim at the development of the students' *pedagogical* content knowledge, but also of their mathematical knowledge and skills as such (Van de Plas, 1995) (**Hypothesis 2b**).

Question 2: How do pre-service teachers evaluate pupils' solution strategies for arithmetic and algebra word problems?

Our main hypothesis with respect to this second research question was that the way in which the student-teachers solved the word problems themselves would be reflected in their evaluations of pupils' solutions (**Hypothesis 3**). This hypothesis was based on the general claim – which is supported by a lot of research (e.g. Fennema & Loef, 1992; Verschaffel, De Corte & Borghart, 1997) – that teachers content-specific knowledge and skills shape to a large extent their teaching behavior. Therefore, we predicted strong and positive correlations between the number of times a student-teacher used a certain strategy and the average score he gave to a solution that works with that strategy. Furthermore, we predicted that all (solution-strategy related) differences we hypothesized with respect to the first research question would also be present in the student-teachers' evaluations (i.e. differences between primary and secondary school teachers, and between first and third year student-teachers).

4. Results for research question 1

To statistically test the hypothesis concerning the *solution strategies* used by the student-teachers, we performed a 2×2×2 ANOVA on the number of algebraic strategies: type of word problem × group (primary versus secondary school teacher) × study year (1^{st} versus 3^{rd} year). This ANOVA first of all showed that the algebra problems elicited more algebraic solutions (on average 3.65 for 6 problems) than the arithmetic problems (1.75 on average), *Wilks'* $\lambda(1, 93) = 0.47$, $p < .00015$. Moreover, there was a significant difference in the use of algebra between the future primary and secondary school teachers, $F(1, 93) = 11.33$, $p = .0011$. The table below gives an overview of their solution strategies for the arithmetic and algebra word problems, together with the percentage of unanswered problems.

		Type of solution strategy			
Group	**Type of problem**	**Algebra**	**Manipulating the structure**	**Generating numbers / Guess and check**	**No answer**
Primary school	Arithmetic	11.0%	4.7%	78.8%	5.4%
	Algebra	42.5%	20.2%	19.9%	17.5%
Secondary school	Arithmetic	61.4%	1.9%	34.8%	1.9%
	Algebra	93.3%	2.9%	1.4%	2.4%

As expected, the solution patterns of the future primary and secondary school teachers were very different. Among future secondary school teachers, using algebra was the most common method for solving both types of word problems (which is in accordance with **Hypothesis 1a**). The large majority of the future primary school teachers solved the arithmetic problems arithmetically, and, more particularly, by the most straightforward strategy called generating numbers. For the algebra problems, many of them switched to algebra, while several others tried to solve these problems using more cumbersome methods like manipulating the structure or guess-and-check, and left problems unanswered. Again, this is in line with our expectancies (**Hypothesis 1b**). In additional analyses, it was found that there were two subgroups among the future primary school teachers: about half of them exclusively used algebra, the others almost exclusively applied arithmetic. These additional analyses also revealed that this last group was responsible for the unanswered problems.

The ANOVA showed no significant differences between the solution strategies of first and third year student-teachers, $F(1, 93) = 1.84, p = .2564$, which means that **Hypothesis 2a** was rejected: contrary to our expectations, future primary school teachers did *not* use more arithmetic at the end of their training, and future secondary school teachers did *not* use more algebra.

To test the hypotheses about the student-teachers' performances on the word problems, another 2×2×2 ANOVA was performed with 'correctness of answer' as the dependent variable. This ANOVA firstly revealed a main effect of the type of problem, $Wilks'\ \lambda(1, 93) = 0.58$, $p < .00015$, indicating that the average score on the arithmetic problems (5.19 on a total of 6) was much higher than on the algebra problems (3.69). Second, the future secondary school teachers (with an average score of 9.89 on a maximum of 12) scored significantly higher on the word problems test than the future primary school teachers (8.31), but this effect was, as expected, only caused by a different performance for the algebra problems (with respective average scores of 3.19 and 4.57), $F(1, 93) = 15.97, p < .00015$.

The ANOVA revealed no significant main effect of grade, $F(1, 93) = 3.62$, $p = .0601$, but – again – there were differences on the algebra problems (a significant type of problem × study year effect was found, $Wilks'\ \lambda(1, 93) = 0.91$, $p = .00038$): third year students (with a mean score of 4.04) performed considerably better than first year students (3.29), and this difference was observed for future primary as well as secondary school teachers. This finding confirms **Hypothesis 2b**. A further analysis of our data showed that the student-teachers became particularly more skillful in the strategy that is envisaged in the educational level of their future pupils: future secondary school teachers became considerably more skilled in using algebra from the first to the third year, while the future primary school teachers showed an increased mastery of manipulating the structure from first to third year.

5. Results for research question 2

According to **Hypothesis 3**, student-teachers' evaluations of pupils' solutions would reflect their own problem solving pattern. The correlations between the future teachers' use of a certain strategy and their appreciation of the strategy, already confirm this hypothesis. All correlation coefficients are positive and significant at the .05-level, varying from 0.30 to 0.46 Further evidence for this hypothesis was provided by a 2×2×2×3 ANOVA on the scores given by the student-teachers, with the same independent variables as for the previous ANOVA's, but with one extra variable: the type of solution (consisting of 3 categories). The table below gives the average scores of the two different groups of student-teachers for the three distinct kinds of solution strategies for the arithmetic and the algebra problems. Since there were no differences between the strategies of first and third year student-teachers on the word problems test, there are no reasons to expect differences here either. Therefore, we will not differentiate in the evaluations of the first and third year student-teachers here.

Group	**Type of problem**	**Type of solution strategy**		
		Algebra	**Manipulating the structure**	**Generating numbers / Guess and check**
Primary school	Arithmetic	7.84	6.52	9.28
	Algebra	8.42	7.67	5.91
Secondary school	Arithmetic	9.20	6.60	8.16
	Algebra	9.33	7.18	5.50

The ANOVA showed that – as for the effect on the solution strategies on the word problems test – the overall score of the future secondary school teachers for the algebraic strategy was higher than the primary school teachers' score, *Wilks'* $\lambda(2, 93) = 125.16$, $p < .00015$. Here too, future secondary school teachers had a strong and overall preference for algebra, independent from the problem to be solved. Although the future primary school teachers adapted their appreciations more to the nature of the problem, their score for algebra remained lower than the future secondary school teachers' score, $F(1, 93) = 13.51$, $p = .0004$. In sum, as expected in **Hypothesis 3**, the differences we found in the solution pattern of future primary and secondary school teachers were also present in their evaluations.

6. Discussion

The findings of the present study confirm Schmidt's (1994) concerns about the ability and the inclination of (future) teachers to support their pupils in the difficult transition from arithmetic to algebra. Our findings force us to be even more seriously concerned, because they show that at the end of their teacher training future teachers demonstrated still problem-solving behavior with the same problematic characteristics as the student-teachers who were just starting their teacher training. Moreover, we documented that these problematic characteristics of future teachers' problem-solving behavior have an impact on at least one crucial aspect of their teaching behavior, namely the way they appreciate and score pupils' solution strategies. We doubt whether the subgroup of future primary school teachers experiencing great problems with algebra will have the proper disposition to prepare their pupils for the transition to algebra, but also whether the future secondary school teachers will be empathic towards pupils coming straight from primary school and bringing with them a strong arithmetic background.

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