SCIENCE TEACHERS' LEARNING ABOUT RATIO TABLES

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Abstract

In Dutch lower secondary mathematics curricula, the ratio table is used as a tool for developing ratio and proportion calculation skills. Some recent physics and chemistry textbooks use ratio tables when dealing with proportional quantities. This makes the ratio table a suitable issue for co-operation between mathematics and science departments in schools. This article reports a developmental research study to starting science teachers attending the ratio table part of the IVLOS teacher education course. Factors that promote or impede teachers using ratio tables in their classrooms were identified. Main promoting factors appeared to be teaching lower secondary science and consulting mathematics teachers and textbooks. A main impeding factor was senior chemistry textbooks linking the ratio table to cross multiplication, affirming teachers' idea of the ratio table as a trick instead of a tool for promoting understanding.

Context of the study

Ratio and proportion is a core topic in primary as well as in secondary mathematics education. To many pupils it causes various conceptual problems (Streefland 1984, 1985; Tourniaire and Pulos 1985). In Dutch primary and secondary mathematics ratio tables are used as tools for structuring and doing calculations.

In science, many quantities have a proportional character, e.g. speed, density, concentration, concepts that usually are explained taking the mathematics of ratio and proportion for granted. In fact, students face many problems when using proportional reasoning in the sciences, even in the upper secondary (Akatugba and Wallace 1999). Many science teachers complain that students "cannot calculate any more". These problems can only be solved when science teachers and textbooks account for the ratio and proportion tools pupils have learned in mathematics. Recently, some physics and chemistry textbooks are published that do use ratio tables.

Nowadays, a curriculum reform being introduced in Dutch secondary education urges science and mathematics department to co-operate. The reform aims at presenting students a more coherent curriculum and opportunities for autonomous learning. In some schools, participating in the BPS-project of Utrecht University, departments are supported in finding ways to co-operate (Van der Valk *et al.* 1998). As it was noted in the BPS-school that some science teachers in the schools were apt to bring ratio tables in their classes but others objected, it was suggested to focus on ratio tables as a topic for co-operation. That resulted in an inventory of knowledge about the ratio table teacher have to master (Broekman *et al.* 2000) and identification of issues related to applying it into the sciences (Van der Valk *et al*. 2000). These results were used to introduce the ratio table in initial science teacher education.

The ratio table in mathematics education

A ratio table is a tool for structuring and calculating ratio and proportion problems. It can be described by the following characteristics:

1. the table consists of two rows and a variable number of columns
2. the rows have a label, indicating the meaning of the numbers and specifying, if needed, the units used
3. the upper or lower row can be changed: there is no preference
4. the ratio between the numbers in the columns is the same for all columns; this can be used to calculate an empty place in a column
5. so to get the numbers of a column, the numbers of another column can be multiplied or divided by a certain number; proportionately adding or subtracting are possible as well.

x7 : 2

Spring roll	1	2	3	4	28	14
Guilders	1,25	2,50	3,75	5,00	35,00	17,50

x 7 : 2

Fig. 1: Example of a ratio table from a Dutch 7th grade textbook.

These characteristics indicate that the ratio table attaches to intuitions young children already have (Lo and Watanaba 1997, Singer *et al*. 1997).

Ratio tables provide students with a structure they can use to find an answer to ratio and proportion problems in several ways. They:

- are allowed freedom which one of the steps that are admitted to choose for filling in numbers into a next column;
- are allowed to use as many steps as (s)he needs or finds useful
- however, have to be careful that the ratio between the numbers in the upper and lower row stays the same.

This is an example of what Dolk (1997) called ‘construction space’. It can be seen as a sixth characteristic of the ratio table in mathematics education. Growing insight results in increasing efficiency. An efficient strategy is normalising to one. Thus, every proportion problem can be solved in two steps. For the actual calculating, the pocket calculator can be used. This way of working provides an alternative for 'mechanistic' algorithms like cross multiplication. Middleton and Van den Heuvel-Panhuizen (1995) described how the ratio table can be used in US middle school (grade 5-8).

The ratio table in Dutch science textbooks

In recently published secondary science textbooks (junior and senior level), several versions of ratio tables can be found. They are used to introduce physics and chemistry quantities having a proportional characteristic.

***First way of working*: [use of formula, not transcribed here]**

***Second way of working:* use the ratio table.**

1,000 mol $FeCl_3$	**162,2 g**
***n* ($FeCl_3$)**	**$16,4 . 10^3$ g**

***n* is the number of mol $FeCl_3$**

$$n . 162{,}2\ g = 16{,}4 . 10^3\ g . 1{,}000\ mol$$

$$n = \frac{16{,}4 . 10^3\ g . 1{,}000\ mol}{162{,}2\ g} = 101\ mol$$

Figure 2. Text taken from: Antwerpen *et al* (2000) *Curie 1*, p. 129

To show that some ratio tables in science textbook are quite different from the one used in mathematics, one textbook, *Curie*, chemistry for grade 10, is analysed. In *Curie*:

- rows and columns are changed, compared to a mathematics-like table
- there are no more than 4 cells
- labels are missing; the cells contain information, like units and '$FeCl_3$' that should be included in the labels
- no choice of strategy within the ratio table is allowed as only cross multiplication is suggested (however, some choice of algorithm: formula or ratio table).

The ratio table as a topic in IVLOS teacher education

IVLOS teacher education is post-graduate and takes a full year (Wubbels 1992). The participants are pre-service as well as in-service teachers. They do their teaching practice in secondary schools and come to the university for the institute-based part of their teacher education. Main characteristics are its focus on reflection (Korthagen

1993), the integration of theory and practice and its attention to teacher learning (Korthagen and Kessels 1999). A course 'didactics of science and mathematics' is incorporated, focusing on areas of common interest to science and mathematics teachers, the ratio table being one of them.

In the 1999 'didactics of mathematics and science' course, the ratio table class was taught for the first time. It was noted that part of the started science teachers embraced the ratio table as an important solution for the problems they met in their classes, but others refused it. A phenomenon that was also observed in BPS-project schools. Therefore, it was decided to investigate during the 2000 course *what factors impede or promote starting science teachers' willingness to use ratio tables in their teaching.*

Research design and data collection

The research design was based on 'developmental research method' (Lijnse 1995; Gravemeijer 1994). This method is useful to get insight into problems of teaching and learning and to develop ideas how to solve them. Emphasis is mainly on the use of interpretative qualitative methods, which includes classroom observation, content analysis, interviews and analysis of student notes.

We regarded the following as candidate factors:

- teachers' own secondary school experiences with ratio and proportion
- experiencing the 'construction space' possibilities of the ratio table
- discussions with mathematics and science colleagues in the school
- (not) successful experiences with ratio tables or other tools in the classroom.

With these factors in mind, the ratio table part of the 'didactics' course was structured consisting of a 2½-hour class, a pre-class and a post-class assignment that aimed at promoting reflection and integration of theory and practice. The pre-class assignment had three tasks:

- study the way ratio and proportion are dealt with in your student textbooks
- look at the ways your students deal with ratio and proportion
- have a discussion with school colleagues about the ratio table.

At the start of the class, the participants reported about the assignment. They discussed the question 'what is ratio and proportion?' Then a short introduction was given into the way the ratio table is introduced in grade 7 mathematics, along with activities and discussions aiming at teachers experiencing 'construction space' in ratio tables in mathematics as well as in the sciences.

The post-class assignment asked the teachers to read a paper on the ratio table and to write a report about one of three tasks:

- collect students’ notes showing ratio tables and give your comments
- teach a lesson on ratio tables in your subject and write down reflections
- prepare a discussion between the mathematics and the sciences departments about fine-tuning on the topic.

The study focused on nine science teachers’ (2 female, 7 male) that handed in their pre- as well as post-class reports, 3 physicists and 6 chemists. All of the physicists and 3 chemists taught combined science (physics/chemistry) to junior students. Their reports were analysed by two researchers in discussion with each other. Comparing pre- and post-class arguments resulted in identifying factors that promoted or impeded the use of ratio tables in the classroom.

The implicit premise of the post-class assignment was that ratio tables can help students. One teacher did not agree: he refused to use the ratio table in the classroom because[1] *it will not be a success as I don’t believe in it*, and excused himself to the teacher educator, writing: *it seems too much a trick that the students have to follow (I hope I don’t offend you, it is just my opinion).* No indications were found that this premise was a problem to others doing the assignments.

Results

Only one teacher referred to her experiences as a student.

In their pre-class reports, four teachers already were positive about the ratio table. Three of them had some experience with using ratio tables in junior secondary science, the fourth had studied ratio tables in a mathematics textbook. Their arguments referred to:

- developmental psychology[2] (3): *[in the ratio table] the relation of the three quantities (m, V and density) is illustrated; this could make somebody with few feeling for formulae, with few sense of abstraction, more sensible to it*
- structuring (2): *To me, it is important that proportional calculations are taught in a structured way*
- students like it (2).

In their post-class reports, all four gave new arguments in favour of using the ratio table in science teaching.

The other 5 teachers, all chemists, related the ratio table with the cross product algorithm: *it is a kind of crosswise multiplication, but because there is no × sign it suddenly is proportional calculating*, three of them referring to senior level chemistry textbooks. In their pre-class reports they argued that students can use the ratio table/ cross product as a calculation trick, e.g.:

[1] Statements from teachers’ reports translated from Dutch, are reproduced in *italics.*
[2] The number indicates to how many teachers it applies.

- *some students just try the trick and hope the answer will be 'a bit normal'*
- *students will fill in figures on the wrong place in the ratio.*

However, four of them knew that it can be a support to students as well:

- *I have heard from a colleague that the number of mistakes made using ratio tables is much less than before. Calculating with formulae apparently is more difficult than filling in a ratio table*
- *Students remarked that using the cross product felt like a trick. They did not understand better how the calculations worked, but made less mistakes.*

Two of these five teachers left the 'trick' argument in their post-class report. The other three, however, left the help-argument. They did not use the ratio table in their teaching, but invited studied senior chemistry students to use it. Studying the results, they were reinforced in their 'trick' argument as all students applied cross multiplication. In their analysis they only used some of the features of ratio tables that were taught in the 'didactics' class. They:

- did not refer to tables having more than two columns (3)
- showed no (2) or few (1) knowledge of strategies alternative to cross multiplication (e.g. of 'reduction-to-one', multiplication factors)
- did not consult mathematics colleagues and books (3)
- reported about disagreement on the use of ratio tables in the science departments (2). One of them *met a difficulty: my mentor teacher did not like the students to use ratio tables.*

The six teachers that got more positive about the use of ratio tables in science:

- consulted maths and science colleagues (5) science textbooks (5) and studied a mathematics textbook (2)
- applied it in the classroom teaching in junior level combined science (4)
- using labels (3): "*now I pay attention to writing the quantities and units in it*"
- using multiplication factors (3)
- applying the reduction-to-one method and so tables with more than two columns (3)
- analysed their students' ratio tables (6)
- experienced success (3): *the students now looked less glassy, asked less questions and showed understanding.*

Conclusions

All starting science teachers initially had the feeling that the ratio table can help students in calculating proportional science problems. However, some teachers feared that it would only support students in doing calculations and would not help them in understanding the proportional character of the science concepts. With three teachers, this fear was not taken away by the ratio table class nor by the assignments because their idea of the ratio table being a ‘calculation trick’ because of its link to cross multiplication was affirmed:

- by studying upper secondary chemistry textbooks
- by analysing upper secondary students’ use of the ratio table.

Indications were found that they were affirmed by colleague teachers in the schools as well, as they met departments that did not agree on using the ratio table in teaching science.

However, with six teachers the idea that the ratio table can support the understanding of proportional science concepts was affirmed by:

- consulting mathematics colleagues and textbooks
- having the opportunity to use ratio tables in lower secondary science classes.

An important factor appeared to be to have some experiences with ratio tables in lower secondary classes before teacher education on ratio tables. Teachers who had, were able to improve their lessons using ‘construction space’ alternatives to cross multiplication strategy of the ratio table, like indicating multiplication factors with the table, normalising to one. The ones that did not teach the ratio table in the classroom, did hardly apply the construction space aspects of the ratio table doing the post-class assignment, however involved they had been in applying ratio tables during the ratio table class.

To teacher education it is recommended to stimulate science teachers to discuss ratio tables with their mathematics colleagues and to let them reconstruct ratio tables from science textbooks that link to cross multiplication, applying the ‘construction space’ aspects of the ratio table.

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