CHILDREN'S INTUITIVE KNOWLEDGE OF THE SHAPE AND STRUCTURE OF THREE DIMENSIONAL CONTAINERS

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Abstract.
Research about children's knowledge of the shape and structure of 3-dimensional containers or boxes was conducted with 137 boys and girls in grades 1,2,3 and 7. The children had to build cardboard boxes into which an empirical referent object had to fit. The problem solving processes were videotaped and analysed together with the children's spatial products. Grounded theory procedure was used to develop the following categories to describe the problem solving processes: spatial strategies, assignment of meaning, focus on influencing factors, measurement and planning and evaluation methods. The results showed age as well as gender differences and differences in the assignment of meaning which indicated that shape perception, as in contrast to object perception, is a minimum requirement for horizontal geometrisation of everyday 3-dimensional objects. Moreover, idiosyncratic representations of spatial aspects in childrens' drawings that were previously ascribed to lack of knowledge of conventions are suspected to have deeper conceptual roots.

Theoretical background

Liben (1981) proposed a model for the study of spatial development that distinguishes between three types of spatial representations, namely *spatial storage* (implicit spatial knowledge of which a person is largely unaware), *spatial thought* (spatial knowledge and processes such as imagery that one can reflect on and manipulate during problem solving) and *spatial products* (external products, 2-dimensional or 3-dimensional, that represent space in some way). These representations can be made of *specific spaces* (as in geographical or environmental cognition) or of *abstract spatial properties* (such as understanding of projective relationships). This study took children's *spatial storage* of boxes as the starting point and endeavoured to deduce aspects of spatial thought about *abstract spatial properties* by studying their *spatial products* such as their overt actions, verbalisations and the 3-dimensional boxes they built from cardboard.

Freudenthal suggested that geometry for young children should start in their living space and viewed the study of boxes as a sensible starting point for the geometrisation of 3-dimensional objects. Children should handle and study boxes through practical problem solving activities so that boxes can become mental objects for them, carrying properties of shape and structure that can be revisited on different levels (Freudenthal, 1983:228). However, Olson and Bialystok (1983:37) showed that children initially perceive objects without necessarily being aware of their spatial properties like shape and structure. In addition, Cassirer (1955) discussed the development of spatial thought and reasoned that the meaning given to spatial situations determines what is observed and represented. He suggested that spatial meaning and thought develop through three levels: from personal *mythical* meaning and thought, to *representational* meaning and thought, to *scientific* meaning and thought. Spatial representations in mythical space reflect the individual's imagination and personal interpretation (a box *is* a house in a play situation). In representational space greater objectivity and generality become possible through the use of language to negotiate common understanding (a box can be *used* as a house, but is a box) of representations in different media. Understanding in scientific space requires the development of concepts that often contradict experience, for example that a point has no dimension and a line has no thickness. In order to develop scientific spatial thought, or geometric thought, many primary intuitions must be challenged. Van Hiele's theory of geometric thought emphasizes development from a basic level ability to observe and identify geometric figures as a whole based on visual appearance (Van Hiele, 1960). His theory does not deal with the meaning (mythical/representational/scientific) children assign to the (geometric) objects they work with. Recently, a more basic level of geometric thought than Van Hiele's first level was proposed, based on research indicating that young children do not attend visually to a figure as a whole but rather to some salient aspects of a whole figure (Clements & Battista, 1992:429). However, revisiting Van Hiele's earlier description of levels (1959:8) confirms that he did view thought on the visual level or Level 0 as *pre-geometric* thought. Where geometric thought is concerned with qualitative and quantitative aspects of *shape and structure* of spatial objects and situations, the *pre-geometric* spatial thought of young children in particular has not yet been described adequately with regard to factors like intent and assigned meaning (see also Clements & Battista, 1992:430). This study was designed to research children's primary spatial/geometric intuitions of and meanings assigned to the concept of a box.

Research design

Elements of a revised clinical interview (Ginsburg et al, 1983) were combined with the observation of small groups of children who were free to interact while they solved the problem. A revised clinical interview allows the provision of an empirical referent to the subjects to help focus their thinking. Their overt actions on the referent object are recorded as data and analysed.

The task. The task had to be difficult enough to elicit a series of spatial thought processes. For this purpose, a box had to be built on the basis of an *image* of a box, in the absence of an example. The task was compared to Watanawaha's DIPT classification (Clements, 1983:16) and assigned a (3,2,2,1) level of difficulty, which is the most difficult level described. In addition, the task was designed to be independent of the need for knowledge of conventions for drawing 3-dimensional objects, and conducive to self-evaluation by the children during the process of construction.

Each child received an A2 sheet of cardboard, a 30 cm ruler, a pair of scissors and a roll of cellotape. The children could choose an object (empirical referent) from a selection of articles small enough to hold comfortably in their hands. The children were requested to build a box into which the object that they had chosen could fit. They discussed their understanding of the word *box* and used hand movements and language to describe the shapes and structural properties of boxes they imaged. During the task giving phase it was stressed that simply wrapping the empirical referent object was not allowed. After a pilot study involving 31 children in different grades, the task was given to altogether 106 children in grade 1 (average age: 7 years), grade 2, grade 3, and grade 7. These children were used to tackling problems for which standard answers and procedures are not available. The children were videotaped while they solved the problem. Video data made possible the use of grounded theory development as analytic process, because the researcher could return repeatedly to the original data as the research question was refined.

Categories for describing the problem-solving processes

The children constructed boxes from cardboard according to two predominant strategies that can be described as a *whole to parts* strategy and a *parts to whole* strategy. Both strategies showed developmental variance in the degree of analysis and synthesis of the structural properties of a box. The most primitive *whole to parts* strategy that was identified was that of cutting a single piece of cardboard and folding it in half to produce a flat envelope. An equivalent *parts to whole* strategy was the cutting of two similarly shaped pieces of cardboard and cellotaping them together on top of each other to produce a flat envelope. Midway along a possible continuum of progressive analysis of parts and synthesis into a structured 3-dimensional whole, were the following variations of the two strategies: A *whole to parts* strategy that comprised of folding a strip of cardboard into three parts, one of which formed the base of the intended box and the other two forming vertical faces on the left and right sides of the base. The equivalent *parts to whole* strategy entailed the cutting of three similar pieces (typically rectangles) and cellotaping them next to each other to be folded as in the *whole to parts* strategy. In both cases the resulting holes (vertical faces) were covered with custom made pieces of cardboard that fitted the direct shape of the missing vertical faces. The most sophisticated strategies incorporated pencil and paper design processes and can be described as follows: The *whole to parts* strategy entailed the folding of four vertical faces, creating a base in the process (at this stage the box looks like a flat tray). Then measurement and drawing were used to determine the exact position and size of a vertical face at the back of an envisaged closed box. Cuts or incisions were made at the correct places to allow the one half of the flat tray to fold over the other half. These cuts created a new vertical face separating top and bottom, so that a typical box used by confectioners or take-away food providers for cakes, pies, pizzas, etc., was formed. The equivalent *parts to whole strategy* entailed the design of a standard net for a rectangular prism, which in most cases approached the shape of a cube. The most typical primitive box that the children constructed was a flat tray (fig. 1).

Figure 1: Flat tray

Figure 2: Envelope

Figure 3: Wrapping

Factors that influenced the construction process. Three factors were identified.

1. The children's **focus** was either predominantly on the properties of the empirical referent *object* for which they had to build a box, or on the properties of the construction *materials*, or on a *mental image* of the box they wanted to make. The children who were able to integrate the demands of the referent object, the material and the image of the intended box, were successful and reached *integrated focus*. Some children's focus varied between object, material and image during the construction process. The children who focused on the object made boxes that were too small, and lacked structural synthesis of the parts into a 3-dimensional whole. Children who focused on the demands of the rigidity of the cardboard and difficulties with cutting and taping made boxes that were far too large and often diverged from their intended box. For example, triangular prisms were constructed non-intentionally, because gravity caused vertical faces to drop towards each other and the child would stick the top edges together in the position they came to rest. Children who focused predominantly on their image of a box made boxes that were far too large, but showed varied degrees of structural analysis and synthesis.
2. **Measurement** or the lack of measurement influenced the construction process. For the purpose of this research, measurement was defined as any intentional action to ensure fitting between parts of the box and between the empirical referent object and the box. Measurement strategies predominantly took the form of *manipulate and estimate* actions, where children handled the empirical object or the parts they constructed from cardboard to ensure fitting of various degrees of accuracy. Some children made no effort to fit parts intentionally while other children used their rulers to measure objectively with a standard unit.
3. **Planning and evaluation methods.** All children made use of kinesthetic imagery involving hand movements or manipulation of the parts of the box or the empirical referent object to plan and evaluate during construction. Excessive 3-dimensional modeling of parts of the box around the object was observed among children with object focus. They typically had to model the

position of each face in 3 dimensions to determine lengths of sides, orientation of faces and relative position of faces. Excessive 3-dimensional modeling usually vitiated the planning and evaluation process, since these children were unable to retain the structural relationships between the faces in 3 dimensions when they tried to tape the faces together in 2 dimensions. Excessive 3-dimensional modeling indicated ineffective imaging of the structural properties of the box.

Success. A successful box was defined as a container that provides a useful and appropriate (suitable) volume, has clearly visible faces that are integrated to a 3-dimensional whole and exists separately from the object for which it was made, retaining its 3-dimensional structure when the object is removed. Measured accuracy and closedness were not taken as criteria for success, since these aspects were not specified during task giving and lent themselves to estimation and interpretation. The boxes were classified according to shape and structure and the following variations were found: rectangular prisms (fig.1), triangular prisms, cylinders, envelopes (fig.2) and wrappings (fig.3). Clearly envelopes and wrappings were not successful boxes according to the criteria described above. In addition prisms and cylinders were judged unsuccessful when the faces were not differentiated clearly (*whole to part* strategies where fold lines were not intentionally constructed) or integrated properly (*part to whole* strategies where the edges of the faces were not joined intentionally).

Factors that caused failure. The following factors were identified.

1. Limiting effects of 2-dimensional thinking. Some children had difficulty in representing a third, vertical dimension. After 40 minutes of trying various methods, their boxes were flat envelopes. The following examples of acute limitations were found: Children who believed that a (perspective) drawing of a box or of the empirical referent object would produce a box when cut out (Daniel and Dale, gr 2); children who handled the cardboard as if it was a sheet of rubber and did not produce fold lines, resulting in the inability to form vertical faces (Elizabeth, gr 3); children who drew unfoldings (nets), but were unable to visualise the effect of folding these drawings into a 3-dimensional box (Adam and Lara, gr 1).
2. Limiting constructions in 3 dimensions. As already indicated in the discussion of criteria for successful containers, some children had difficulty in integrating the faces they constructed into 3-dimensional boxes. Three types of difficulties were identified. Children who lost the orientation of faces in relation to the whole when they had to tape the faces together in 2 dimensions (Yolande, gr 7); children who did not produce clear fold lines to separate the faces of the box (Hlubi, gr 7) and children who did not integrate the sides of the faces to form edges of a box. The lack of integration of sides was also evident while the children made the polygonal faces for their intended boxes from the cardboard. They would draw a rectangle with one side almost on the edge of the sheet of cardboard, yet would not use the edge of the cardboard as a side of the rectangle. In another case a child

(Shane, gr 3) used one rectangle as a template to cut two more rectangles side by side, the edge of the hole where the previous rectangle was cut out forming one side of the next rectangle. As he cut out the second rectangle, he taped the two rectangles together side by side and fitted them into the hole created by cutting them from the sheet of cardboard. He repeated the process with the third rectangle and ended with three rectangles taped together as if they had never been separate, fitting exactly into the hole they left in the sheet of cardboard.

3. Mythical meaning. Some children who had difficulty constructing their intended boxes regressed in terms of the meaning they had given to the task during discussion and made fancy objects instead of boxes. Examples of such objects, which the children themselves judged as not being boxes, were houses (complete with chimneys, doors and windows), mushrooms and bags. These terms were used by the children themselves when they talked about their products.

Discussion of the results

The results will be discussed according to age and gender variants.

Strategy. The choice of strategy shifted from *whole to parts* in grade 1 to *parts to whole* in grade 7, with the exception of grade 2 where *whole to parts* strategies (producing flat trays) dominated. In general boys used more *parts to whole* strategies and girls more *whole to parts* strategies. The girls in grade 7 that used *whole to parts* strategies used very primitive strategies, merely wrapping the empirical referent object or at most pressing in tentative fold lines after wrapping the object. The only boy in grade 7 that used a *whole to parts* strategy succeeded in fully explicating and representing the shape and structural properties of a box, producing a cake or pie box as described earlier. Most grade 7 boys used a *parts to whole* strategy that entailed the design of a net, while most grade 7 girls used a *parts to whole* strategy by which they assembled a net from cut out faces in a step by step fashion. This indicates that although the girls were able to fully analyse the shape and structural properties, they did not yet have a complete understanding of the synthesis of these aspects into a whole.

Focus. Children who followed *parts to whole* strategies showed predominantly image focus, while children who followed *whole to parts* strategies showed predominantly varying focus. Integrated focus occurred only among children who followed *parts to whole* strategies. Of grade 1 children, 50% showed varying focus, while the rest showed predominantly material focus, indicating a lack in skill with cutting and taping activities. Although 40% of the grade 2 children showed varying focus, image focus occurred most among the rest of the children. In grade 7, image focus and integrated focus occurred most and with the same frequency, indicating that skill in imaging has developed. The greatest difference between boys and girls was the degree to which they reached integrated focus. Only 9% of the girls compared to 28% of the boys reached integrated focus. On the other hand 19% of the girls compared to 9% of the boys showed object focus. As discussed earlier, integrated focus was a sure indicator of success, while object focus lead to too small and often unintegrated boxes.

Measurement. Most of the children who followed *whole to parts* strategies did no measuring. *Parts to whole* strategies were characterized by *manipulate and estimate* measurement methods. Objective measurement (with a standard unit) was only used by grade 3 and grade 7 children. More girls than boys in grade 7 made use of objective measurement, which may be an indication that accuracy becomes more important for girls than for boys in grade 7. On the other hand Piaget, Inhelder and Szeminska (1960:33) showed that visual estimates become more accurate once children are able to measure objectively, which may indicate that the boys trusted their estimated measurements and did not experience a need for more accuracy. Piaget et al mentioned the intuition that children have of vanishing volume between a 3-dimensional object and its 2-dimensional net, which results in nets that are too small to be folded into a copy of the referent object. A related intuition was noticed in this study, namely that children intuitively produced 3-dimensional boxes from 2-dimensional material that were far too large for their empirical referent objects, possibly in anticipation of extra volume needed in 3 dimensions.

Success. *Parts to whole* strategies were more successful than *whole to parts* strategies (84% compared to 45%). As can be expected, the success rate increased with age as the choice of strategies shifted towards *parts to whole* strategies. Across ages, with the exception of grade 2, the boys were more successful than the girls (see table 1).

Table 1. Success rate across ages (%)

	Grade1	Grade 2	Grade 3	Grade 7
Boys	64	18	80	100
Girls	30	78	53	84

In 57% of the cases failure was caused by the inability to produce vertical faces. This was also the reason for the dip in performance among grade 2 boys.

Meaning. As indicated before, some children (in grades 1 and 2) assigned personal mythical meaning to their boxes and constructed houses and other objects. It seems that the standard image of a box for most children was that of an open rectangular prism. From the strategies children used, two deeper representational meanings of the concept box can be deduced. On the one hand children assigned the meaning *a box covers an object.* This was evident from strategies that involved wrapping actions and favoured closure above structure. These children often constructed the vertical lateral faces of the box first and attached a base and a lid later. On the other hand children assigned the meaning *a box can receive an object* as was evident from the large number of flat trays and other open boxes that were constructed. These children constructed the base of the box first. The base served as a stable point of reference and therefore these children were better able to represent the structural properties of a box. Children who integrated the two meanings produced boxes with vertical faces high enough to cover the object and clearly visible structure in 3 dimensions.

Conclusion

This study showed that young children do not necessarily view boxes as geometric objects in the sense that they are aware of the shape and structural properties of such boxes. Moreover, they may be able to represent certain properties, noticeably properties of shape, in media like language, kinesthetic images and even plane drawings and yet they may not be able to construct a 3-dimensional model from 2-dimensional material. The development from object perception and the assignment of personal, mythical meaning to the perception and representation of abstract spatial/geometrical properties and relationships seems to require a transitional phase of shape perception and representation based on the assignment of representational meaning. This implies that the assignment of representational meaning and the accompanying perception of the shape of objects may be a pre-requisite for young children to start with geometry at an entry level. This study also showed that aspects of representation that are judged to be based on lack of knowledge of conventions in a medium such as drawing, may have deeper intuitive and conceptual roots. One notable example is the lack of integration of the edges of 3-dimensional objects and even the sides of 2-dimensional figures.

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