OPEN-ENDED TASKS: THE DILEMMA OF OPENNESS OR AMBIGUITY?

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Open-ended tasks have been shown to offer considerable potential for learning in mathematics. They have substantial benefits for learning and assessment. However, when using open-ended tasks, questions need to be posed as to how open can they be. In making tasks "open" there is some chance that the task can become ambiguous. This paper reports on a small study of students' responses to open-ended tasks where some tasks were clear in their goals whereas others were more open, and hence open to greater interpretation by the students.

Tirosh (1999) notes that questions and questioning have a significant role in reform in mathematics education since new movements in mathematics education compel teachers and teacher educators to move away from the traditional forms of teacher-directed and closed questioning techniques that have dominated mathematics classrooms in the past. One tool that has been recognized as having significant value is that of open-ended questions or tasks. The work of researchers and teachers across a range of nations has been particularly valuable in identifying the features of open-ended tasks and how these are of benefit to teaching and learning (Becker & Shimada, 1997; Boaler, 1998; Chappell & Thompson, 1999; Sullivan, Warren, & White, 1999). While such research has been particularly useful in demonstrating the value of open-ended approaches to teaching mathematics, it has been limited in its analysis of the questions being posed. This paper explores the use of open-ended tasks across three classrooms, noting the ways in which language influences the interpretations of the given tasks. It is proposed that the tasks are useful in facilitating greater responses from the students, and hence, making assessment more authentic. However, it is noted that there is some need for concern when tasks become so open as to become ambiguous.

Open-ended Tasks

Sullivan et al (1999) define open-ended tasks as having more than one possible response (goal) and where there are multiple pathways for resolution (activity). They propose that such openness in activity and goal "fosters some of the more important aspects of learning mathematics, specifically, investigating, creating, problematizing, mathematizing, communicating and thinking" (p. 250), which they see as being substantially different from the more restrictive processes associated with recall and rote learning. Closed tasks have typically and predominantly been used in mathematics classrooms and examinations and can be seen as having one answer, for example, *"What is the area of a rectangular piece of paper with the dimensions of 4 cm and 6 cm?"* Clearly, there is only one correct answer, 24 cm^2. While there may be a number of ways in which the answer can be calculated, such as repeated addition (6+6+6+6= 24), using a diagram marking the rectangle into small squares, or multiplying 4 by 6, in most cases there is a preferred activity for resolution. In contrast, an open-ended task is one that allows for a range of "correct" responses and a range of ways of achieving those responses. An open-ended task would be one such as: *"If the area of a rectangular piece of paper is 24 cm^2, what might be its dimensions?"* This type of question offers greater scope for teachers in assessing students' understanding of area, and also allows students' greater scope in demonstrating what they know about area.

Chappell and Thompson (1999) note that the advantages of using open-ended tasks in mathematics are that they encourage students to move beyond the skills-based approaches typical of mathematics classrooms. They demand that they students think more deeply about the concepts; and that they make connections between concepts. This point is reinforced by the work of Boaler (1998) who found that the students who used open-ended approaches in mathematics developed deeper and more connected forms of knowing than their peers in traditional classrooms. She argues that the learning in the open-ended classrooms was more like the learning that occurs in the world beyond school and hence there is greater transfer from school to beyond school.

Sullivan et al (1999) raise the issue of content specificity of open-ended tasks as this allows the mathematics to be made transparent to the students so enabling their learning to be more directed. An example these authors cite is "*A number is rounded off to 5.6. What might the number be?*" (Sullivan et al., 1999, p. 250). In these tasks, the content is specific to mathematics and students are able to answer it according to the current levels of understanding—some may use two decimal places whereas others use three or more decimal places. The rationale behind open-ended tasks is that they are to be seen as open enough for interpretation so that students answer them in ways that they understand.

The Project

This paper reports on the analysis of a number of tasks given to students in the upper primary school in Queensland, Australia. A set of five open-ended tasks was given to 115 students, but only two of the tasks will be discussed in this paper. The tasks were all open-ended and were designed to cover a range of areas of the mathematics curriculum. Students were given the tasks during a mathematics lesson, and conversation was allowed. Two multi-age classrooms participated in the study, with each classroom having approximately 60 students and two teachers. The students were in the final two years of primary schooling and aged 11–13. The students were given the tasks to complete and later a number of interviews were conducted with selected children in order to gain some appreciation of their thought processes when undertaking the tasks. These students were selected on the basis of the responses that they had offered in the tasks.

Task 1: Data Handling Task

The first task to be discussed involved the interpretation of statistics. The context was a real one to the students whose school was close to a major road that lead to an island where there was only a very small bridge to cover the main thoroughfare of traffic. There had been substantial reports on the need for a bridge or alternative thoroughfare to cater for the increasing traffic.

Task: At the Chevron Island Bridge, the average number of people per car is 2.5. Draw what this might look like if there are 16 cars on the bridge.

The task was open in its goal and method of resolution. It allowed the teachers to access students' understanding of what the statistics meant in the world beyond school mathematics. Frequently, the mean is something that is calculated: *"Forty people in 16 cars, what would the mean be?"* However, this task asked what this might look like when posed in the context beyond school. Similarly, it assessed whether or not students understood what a mean of 2.5 meant, and how this is manifested in the everyday. It also allowed for further information to be posed by students that would not otherwise be possible through a closed question. This type of task provided a rich source of information about students' understanding of measures of central tendency, interpretation of data, and application of data.

The responses to this task fell into a number of categories. Many of the students were able to work through the task so that they could provide a range of cars that would have different number of people in them and where the average number of people was 2.5 per car (see Fig 1a).

Figure 1a and 1b: *Responses offered by students*

Other responses indicated that students realised that there needed to be a total of 40 people in the 16 cars (see Fig 1b). In noting explicitly the need for a total of 40 people, students used a range of strategies, including those listed below, to arrive at 40 people.

Others took a more systematic approach and had alternate cars with two and three people in each (see Fig 2). While this produced a mean of 2.5, it did not show the depth of understanding evident in the responses above. Students appear to have calculated a simple method of means of 2.5 using a pairing strategy of 2 and 3 and then applied this to the 16 cars.

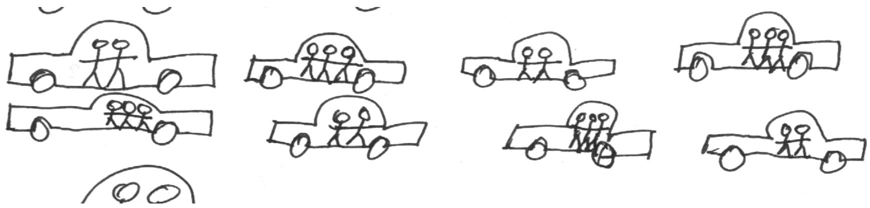

Figure 2: *[extract of] Systematic representation of 2:3 pairings*

Another strategy used involved the translation of "half". Some students (see Fig 3a) showed three people per car with one person being smaller than the others. This would suggest that these students interpreted the 0.5 to mean a "small" or "half" person, as was confirmed through the interviews. One student commented that a child was "half an adult so there are two adults and one child in each car." Others showed a similar representation, but with a code to show that there were two adults per car and one child whereby the one child represented "half a person" or "0.5 people".

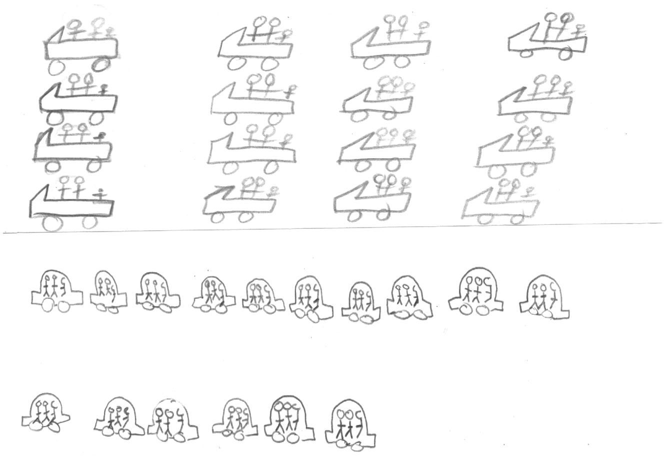

Figure 3a and 3b: *Cars showing representations of "half"*

Others (see Fig 3b) showed "half a person" by only drawing half a body so that there was literally a "half person" in each car. This representation indicates a literal translation of the data and hence would suggest that there is a need for further work to be undertaken with these students.

As can be seen from these responses, the open-ended task allowed for considerable diversity in responses and a range of representations. This offers potential for effective diagnosis of students' understandings. While the goal was restricted in some sense (in that there needed to be 16 cars on the bridge), the ways in which people were represented in these cars was open. The responses offered by the students varied in both the goal (by having different amounts of people in each car) and the activity through which they solved the task. Students had varying degrees of success with the task but all were able to produce some documentation of their understanding of the task and the concepts involved. This allowed for the teacher to make a range of judgements about students' levels of understanding.

Task 2: Estimation and Rounding Task

The role of language and openness need to be considered with open-ended tasks. The use of language in open-ended tasks may make them "open" to interpretation as well as open mathematically. Consider the following task:

Task: My dog weighs about 20 kilograms. How much could she weigh?

This task was designed to assess the estimation and rounding skills of students so that the words of "about" and "could" were central to the notion of estimation and rounding. However, the responses of the students indicated a number of possible interpretations of the task. Many of the responses were difficult to categorize since there would need to be some unsubstantiated interpretation of the results, so only those that clearly fell into a category are considered in the calculations.

Around half of the students interpreted the question to be one of estimation of weights. There was some degree of variability in responses with 38% of the students offering responses around the 20kg measure, as was the expected response. These responses centred on what would normally be considered those typical of a mathematical context. These students offered responses that were either a single weight such as *18.5k*g, or a range of weights such as *18-22kg;* or *between 22.5 and 19.8*, while others constructed a list of weights such as *17, 18 , 19, 20, 21, 22.* Others (16%) offered responses that were weights around 30–40kg that may be considered too high for estimation purposes but may indicate some conceptualization of estimation. Some of the students interviewed thought that this weight was close to 20kg while others thought that this is what their dog (at home) weighed. Hence, it is difficult to classify the answers as being correct or otherwise without further information as to the rationale behind the responses offered by these students.

What was interesting was the cluster of responses whereby students interpreted the task to be futures-orientated, where the task was translated as meaning "If my dog weighs 20 kg now, what might she weigh in the future?" Typical of this group of responses are the following answers:

"When she's bigger, she'll get to 25kg."
"A puppy could be a weight of 20kg but when it's older it could weigh 40 to 45 kg."
"If I had a Husky that weighed 20 kg as a puppy, it might weigh 60–70kg."
"My puppy is half grown. He weighs about 20kgs. When he is fully grown, he will way [sic] about 70kgs."

These responses indicate that the language of the task created a different interpretation from that intended. When asked about the response he made to this task, one boy offered the following comment. "In our group one of the girls who is good at maths said that we really needed to estimate that if a puppy would weigh 20 kgs, what might is be when it was fully grown." This comment indicates two important issues, the first being that of interpretation of the task, and second, that the role of group work also impacts on the translation of the task. While it is not possible to say how many students followed group dynamics in their responses, it is possible that in some groups dominant personalities were able to sway particular students in proposing answers of a particular type.

The use of “My dog” may cause students to personalize the task. Some students interpreted the question to be highly contextual and related to their own dog so that little or no mathematizing was undertaken. This group of consisted of 17% of the student responses. Typically the responses were written in a way that suggested that the students were interpreting the question as if it were about their own dogs, real or imagined. Responses in this category were typically:

“My dog is an obese German Shepherd who weighs 60kg.”

“My dog Wally weighs about 20kg. It is very old. The vet said it should weigh 40kgs at least.”

“I think my dog is a bit fat. She could weigh about 15-17kgs.”

Included in this group of responses was a cluster of responses that seemed to estimate what a dog might weigh and tried to think about which dogs might weigh particular weights around 20 kg such as:

“A cocker spaniel might weigh 30 kgs”

“A Labrador would weigh about 40-50kgs.”

There seems to be some transfer from the context of the question to the context beyond the mathematics classroom as if the question being posed asks the students to consider what types of dogs may weigh round 20kgs. While they could not accurately guess a dog that would be about 20 kgs, their reasoning is exemplified by the comment made by one girl—*“I tried to think about what dog might weigh about 20 kgs. A foxy [fox terrier] is only little so I think they would not even be 10kgs. Dogs like Rottweilers and Great Danes are really big and would weight lots. I could not think of a dog that would be about 20 kgs. A Labrador would be more than 20 kgs but it was the closest that I could guess that would be near to 20kgs.”*

With this question, there is some sense that the question could have been worded so that it would be less ambiguous. As Sullivan et al (1999) argued, the content specific nature of open-ended tasks can provide support for learners. This task may have been ambiguous and hence open to too much interpretation when the intention of the task was to assess/access students’ understanding of estimation and rounding of mass. The task might have been better worded if it were *“If a vet has rounded a dog’s weight to 20 kgs, what might the dog have weighed?”* However, such wording raises a dilemma as to the openness of the task. The use of signifiers such as “rounded” can serve as a key word and as such may provide the cue for what the students need to do to undertake the task. Where a key word approach has been used, it is difficult to ascertain whether or not students have understood what the task is asking or where there has been a reliance on key words. As a strategy, using key words to solve word problems has been found to be successful. Schoenfeld (1992), reporting on a text book series widely used in the USA, found that most word problems (approx. 90%) in the series could be solved using superficial key words rather than requiring any mathematical understanding or interpretation. Using tighter or more explicit wording may have resulted in more students being able to answer the question “correctly”, it raises the issue as to the purpose of questioning in mathematics classrooms.

One of the advantages of the open-ended tasks is that they allow students to see the application of tasks to contexts beyond the classroom. While tighter wording of the problem may have meant that students were then able to provide estimates of the pre-rounded weight, it raises issues as to whether or not students understand the transfer between contexts. As Boaler (1998) argues “It seemed that the act of using mathematical procedures within authentic activities allowed the students to view the procedures as tools that they could use and adapt. The understandings and perceptions that resulted from these experiences seemed to lead to increased competence in transfer situations” (p. 59). In this instance, the wording of this question was akin to what would be heard in many beyond-school contexts, such as a veterinarian’s surgery where the weight of the dog determines the dosage of a medicine. Following Boaler’s contention, it would suggest that such openness may be useful for its links, and hence transfer, to the non-school contexts.

Openness Versus Ambiguity

While the first example highlights the value of open-ended tasks for assessment, some questions regarding the value of the second task as an assessment item need to be raised. In contrast, the second task could be seen as being poorly worded due to the ambiguity of the task. As noted earlier, however, this ambiguity can also be of value insofar as creating an openness to the task. By tightening the wording of the task in order to reduce ambiguity, it is possible that the task becomes too prescriptive due to key words defining what is to be undertaken—in this case, the use of rounding would reduce ambiguity insofar as the goals of the task but would also provide a cue as to what needed to be done. Durkin and Shire (1991) have shown that ambiguity is a part of mathematics education. Words such as rational, odd, base and so on have particular meanings in mathematics that are very different from their in non-mathematical contexts. Similarly, homophones such as pi and pie; two and too; or whole and hole also produce ambiguities for students. Walkerdine (1982) has argued that students often identify a particular word as being key in a sentence or task and as a consequence select the wrong discourse in which to locate and respond to the task. For example, in the second task, the students have interpreted the word “could” to mean a futures perspective and have responded in this sense, rather than as a rounding context as intended by the teacher. They have identified a futures discourse and responded correctly in this context.

As has been recognised within the mathematics education community, there are particular social and cultural norms that work with mathematics classrooms that students must become conversant with. Part of such competency is recognizing the unspoken rules of interactions with mathematics and what are seen to be valid and legitimate forms of responding. Students need to become familiar with what are socially legitimate forms of knowledge within the classroom and what are not. Ambiguity in wording, for example, can confuse students in what are socially acceptable responses. In the case cited here, the potential for inclusion of futuresperspectives can be a legitimate part of the mathematics classrooms, so the responses can be seen as appropriate. However, many other tasks need to be considered carefully as the ambiguous wording may cause students to offer inappropriate responses as a consequence of misinterpretation of the words and their relevant contexts. For example, the use of “odd” numbers can result in students perceiving such numbers as being “strange” due to the ambiguity of the term between the mathematics and non-mathematics contexts. Students must make the transition from one context to another. Indeed, many of the errors made by students can be seen to be linguistically related rather than mathematical.

While the value of open-ended tasks has been shown in this and other studies, there is a need for support for students when beginning open-ended approaches to teaching. In the case cited here, these students had little or no experience with open-ended tasks. Their responses to the first task suggest that they are able to deal with the tasks, but as the second task indicates, some explicit teaching maybe of value in contexts where there is some ambiguity in the task. In this case, the ambiguity can be a feature of the openness of the task but also a hindrance, particularly if there is some assessment associated with the tasks. Coming to know mathematically and pedagogically, means coming to understand the expectations of the social and cultural norms that are embedded in such tasks. In the second task, where the ambiguity may be seen as a valuable characteristic, it may be of value to make this ambiguity an explicit teaching feature so that students come to know the unspoken rules or norms of the mathematics classroom.

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