DESIGNING SOFTWARE TO ALLOCATE APPROPRIATE DEMANDS ON MEMORY AND AWARENESS

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I begin by considering the division into necessary and contingent which exists within philosophy and discuss ways in which this can be applied to the mathematics curriculum (the divide having been re–named arbitrary and necessary). By looking at implications of this divide, I develop a pedagogic approach which clarifies when a teacher's role is to assist memory and when it is to educate awareness. I consider how this approach has informed the development of one of the dynamic geometry computer files in the package ACTIVE GEOMETRY.

Arbitrary and necessary

Within philosophy there is a divide made between those things which are logically true and can be derived in some way from other truths, and those which cannot be so derived and where some sort of choice is possible. The former are known as *necessary* and the latter as *contingent*. Kripke (1996, p36) expressed the divide as follows: *If* [something] *is true, might it have been otherwise?… If the answer is 'no', then this fact about the world is a necessary one. If the answer is 'yes', then this fact about the world is a contingent one.* In my application of this divide to mathematics education (Hewitt, 1999), the issue of whether the question *why?* can be suitably answered can be an alternative tool for deciding whether a 'fact' might be considered necessary or not. Within Nozick's (1984, pp140-141) discussion of statements concerning the principle of sufficient reason, he wrote:

> Let us state the principle of sufficient reason as: every truth has an explanation. For every truth *p* there is some truth *q* which stands in the explanatory relation E to *p*… When any other truth holds without an explanation it is an arbitrary brute fact.

Using a combination of Nozick and Kripke, if the question *why?* can be answered by providing an explanation which shows that this 'fact' could not have been otherwise, then this 'fact' is one which is necessary, otherwise it is arbitrary. I have chosen to use the term *arbitrary* rather than *contingent* since within education a student is rarely party to the moment when some people were aware of choice and had reasons for making the choice they did. So rather than being party to this choice, students are presented with the 'fact' in a text book, or by a teacher, some decades or centuries after the choice was actually made, which makes it appear arbitrary. For example:

> Student: What is the name of a 2D shape which has four sides the same length?
>
> Teacher: A rhombus.

Student: Why?

Teacher:…

Consider how you might answer the student's question before reading on.

Personally, I do not know why such a shape is named a *rhombus* I just accept the use of the name as a social convention within the English speaking mathematics community. I suspect there were reasons why this name was chosen at the time, but for me, working on mathematics today, I merely have to accept the term in order to communicate with colleagues successfully within the field of mathematics.

Walkerdine (1990, p2) wrote that *Saussure is credited with recognizing the importance of the fact that the relationship between the signifier and the signified is arbitrary; that is to say, conventional rather than necessary*. Names are signifiers and all names are arbitrary in my usage of the word. Names are the result of choices, as are conventions. Thus all names and conventions within the mathematics curriculum are arbitrary and as such are to be *accepted* rather than *understood* by a student. This is because there are no reasons for why any of these *have* to be how they are. Different choices could have been made which would have resulted in different names and conventions without affecting the mathematics of the situation at all. Only the language of description would have been changed. For example, there is no reason for why the x co–ordinate *must* be written before the y co–ordinate. It could have been the other way round and none of the mathematical properties concerning co–ordinates would change, only the way in which they are described.

I will now consider how a student may come to know the arbitrary. I am usually known as Dave Hewitt but I also have other names between Dave and Hewitt in my full name. What are they? *Consider this before reading on*. These other names are arbitrary and the only way in which it is possible for you to know them is to be informed by someone who already knows them, such as me, or perhaps an official document which has them written down. Students cannot know for sure how a co–ordinate is written unless they consult someone who knows. They can invent plenty of ways of writing co–ordinates, but if we expect them to learn what is a convention within the mathematics community, then those students will have to be told of the convention. This gives a clear role for both students and teachers: students need to be told, accept and memorise the arbitrary; teachers will need to inform and offer ways to practise the use of the arbitrary.

The name of the shape in Figure 1 may be arbitrary but there are other things about the shape which are not. The shape has certain properties, such as sides which are the same length, which I can work out and not have to ask someone else who knows. This is an example of something which is not arbitrary but necessary. There is an explanation which I can provide which convinces me that the sides *must* be the same length. There is no choice about the matter, it is a property which has to be so.

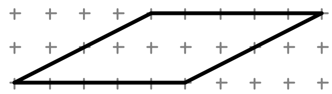

Figure 1. What properties has this shape?

Not everyone may be able to become aware of the property of equal length sides. A student who does not know about Pythagoras' Theorem may struggle to know for sure that the shape has this property. So the necessary is not available for *all* students to know as necessary. Whether students can come to know a property will depend upon the awareness they already have of the mathematics involved. If they do have sufficient awareness, then that awareness can become educated in the process of finding out that this property of the shape must be true. It is the role of a teacher to consider the awareness of students and make choices about what are appropriate challenges for certain students. Whatever the situation though, properties of shapes are not arbitrary and so I propose as a guiding principle that therefore they are not to be told. Instead, either particular properties are considered not to be within the awareness of the students and so will not form the focus of a lesson, or they are accessible to the awareness of students and so an activity needs to be provided to educate students' awareness so that they can come to know that these properties must be true.

	Student	**Teacher**	**Mode of teaching**
Arbitrary	*All* students *need* to be informed of the arbitrary by someone else	A teacher *needs* to inform students of the arbitrary	*Assisting Memory*
Necessary	*Some* students *can* become aware of what is necessary without being informed of it by someone else	A teacher does *not* need to inform students of what is necessary	*Educating Awareness*

Figure 2. When to inform and when not to inform.

Figure 2 summarises the differences between arbitrary and necessary. In particular, the arbitrary is in the realm of memory and a teacher's role is to assist students in memorising. The necessary, however, can be known through awareness and so a teacher's role is not to inform but to devise activities which help students to educate their awareness as I have just mentioned above.

The necessity of the sides being the same length in Figure 1 is based upon some information which is provided or assumed, such as the drawing itself, the assumption that the corners lie exactly on grid points, it is a square grid, that we are in Euclidean geometry, etc. These I describe as *givens* and provided certain givens are accepted, there are situations where there is underlying necessity which comes as a consequence of these givens. It is working on what is necessary where mathematics really lies. The arbitrary is for ease of communication and is not mathematics itself. Sadly, too much attention is often paid to the arbitrary when mathematics is to be found in the necessary. Richard Feynman (1988, pp13-14), a famous physicist of the 20th century, gave the following story:

> One kid says to me, "See that bird? What kind of bird is that?"
>
> I said, "I haven't the slightest idea what kind of bird it is."
>
> He says, "It's a brown–throated thrush. Your father doesn't teach you anything!"
>
> But it was the opposite. He had already taught me: "See that bird?" he says. "It's a Spencer's warbler." (I knew he didn't know the real name.) "Well, in Italian, it's a *Chutto Lapittida*. In Chinese, it's a *Chung–long–tah*, and in Japanese, it's a *Katano Tekeda*. You can know the name of that bird in all the languages of the world, but when you're finished, you'll know absolutely nothing whatever about the bird. You'll only know about humans in different places, and what they call the bird. So let's look at the bird and see what it's *doing* – that's what counts." (I learned very early the difference between knowing the name of something and knowing something.).

The linking together of memory and awareness is often exhibited by skilled practitioners. Bird watchers attend to properties of birds and with the noticing of certain properties (size, markings, birdsong, etc.) comes the name of the bird, and likewise mathematicians attend to properties of the shape in Figure 1 and the word *rhombus* or *parallelogram* appears to come as if from thin air. Walkerdine (1990, p3) posed the question *how do children come to read the myriad of arbitrary signifiers – the words, gestures, objects, etc. – with which they are surrounded, such that their arbitrariness is banished and they appear to have that meaning which is conventional?* The arbitrary signifier *rhombus* only gains an (apparent) meaning when the relevant properties are associated with the word within a student's mind. As Saussure (1974, p75) said in relation to the signifier and signified coming together to form a 'sign': *Both terms involved in the linguistic sign are psychological and are united in the brain by an associative bond.* It is only when the name and related properties are brought together through association that the name stops having the sense of arbitrariness (in the usual English use of the word) and takes on an apparent meaning. This issue is developed within the next section where I consider some pedagogic decisions involved in the development of a software file.

An example of bringing together the roles of memory and awareness: the design of a dynamic geometry file

Dynamic geometry software, such as *Geometer's Sketchpad* and *Cabri–Géomètre*, has offered new ways for students to engage with geometry. The possibility of movement and the focus on properties, especially concerning geometric relationships through construction, has brought new pedagogic possibilities (for example, see Laborde, 1995, and Jones, 1998). Some files for *Geometer's Sketchpad* have been commercially available (for example, *Exploring Geometry with the Geometer's Sketchpad* from Key Curriculum Press) which generally take the form of demonstrating dynamically a mathematical property or setting up a mathematical construction where the locus of a point, for example, is to be explored. The emphasis on these files is on the mathematical content. The *Active Geometry* files available from the Association of Teachers of Mathematics (ATM) for either *Geometer's Sketchpad* or *Cabri–Géomètre II* consider ways in which parts of the UK National Curriculum can be directly addressed through dynamic geometry software without the need for students or teachers to know about construction techniques. I will consider one of these files, *Quadinc*, and discuss some pedagogic issues which underpin the design of the file.

Quadinc has a quadrilateral whose corners can be moved to any point on a square grid. As a corner is moved, so the properties of the quadrilateral change and names associated with the quadrilateral may also change as a consequence. For example, Figure 3 shows the quadrilateral ABCD. Such a shape is usually assigned the name *square*, however, it is also a *trapezium* (accepting an 'inclusive' definition for a trapezium), *parallelogram*, *kite*, *cyclic*, *rhombus* and a *rectangle*. All these names appear on the screen. If corner B is moved one grid space to the right then all the names disappear except for *trapezium*. Thus all the names associated with the particular quadrilateral appear on the screen and change dynamically as the quadrilateral is changed dynamically. At any given positioning of the quadrilateral, the associated names are displayed alongside. Thus the file provides the arbitrary – the names – by simply having the appropriate names displayed. However, the file does not attempt to explain or define any of the names. This is similar to situations that all young children face when they learn their first language. Words appear (are said), and appear within a context. The fact that certain words appear frequently (such as *no* or *yes*) means that a child has an opportunity to recognise them, and improve the way in which that child says the word through checking against the way adults say the word. The fact that words are said within context gives a child the opportunity of developing meaning for those words. Within *Quadinc*, mathematical meaning will come from a student considering the context within which the word *rhombus*, for example, appears. The power to abstract rules and meanings from examples is one which all children have used when learning their first language and which can be utilised in learning mathematics as an older child. Ginsburg (1977), Bruner (1960) and Gattegno (1971) have all made references to the powers that very young children use in learning their first language and yet these are rarely called upon by many traditional teaching approaches.

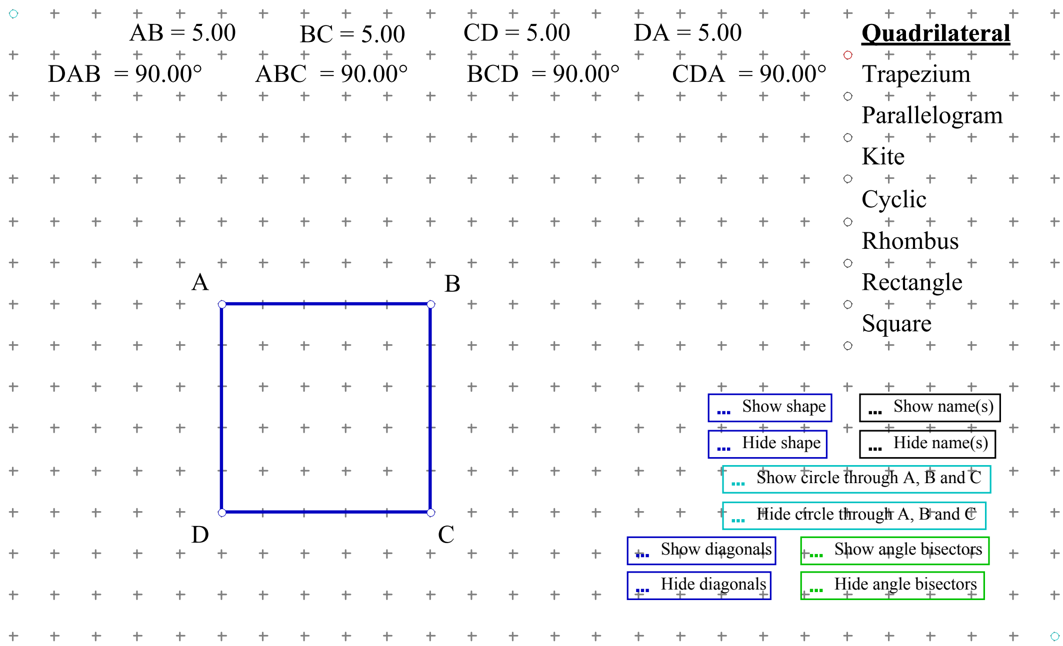

Figure 3. A screen from *Quadinc*.

Quadinc informs students of what is arbitrary – the names. It leaves to the students' awareness those things which are necessary – properties and relationships. In fact some properties are given, namely the length of sides and interior angles at the top of the screen. This has been done in order that attention can remain at the level of properties such as *this angle is the same as that* rather than attention having frequently to be taken away to the level of calculation such as the use of Pythagoras and trigonometry. There would be many students for whom the requirement to use Pythagoras, for example, to calculate lengths of sides would exclude them from knowing the lengths of sides since they may not be aware of Pythagoras or such calculations may be difficult to carry out. Choosing to provide the lengths and angles means that noticing properties such as equal length sides now becomes accessible for such students. Thus this is a pedagogic decision to make the task of finding properties associated with names more accessible when it may not otherwise have been or have required attention being taken onto a sub-task for a considerable period of time. For example, Ginsburg (1977, p164) wrote about a similar issue regarding the task of noticing commutativity when the sub-tasks of actually carrying out calculations take up so much time and attention for some students:

> Over the years, they work 6+7, 7+6, 7x6, 6x7, 7÷6, 6÷7. At first, the very act of calculation may be so difficult that they focus attention only on individual problems. They concentrate on getting the correct sum for say 6+7 and so they cannot notice that 7+6 yields the same result. Or it is so hard for them to do 7÷6 that they cannot see that 6÷7 gives a different answer.

If it is desired that students' attention is with the *results* of such calculations so that the issue of commutativity might be noticed, then asking some students to also carry out the calculations may result in attention being taken away from where it is required since the task of calculating may be sufficiently challenging in itself. For similar reasons, there is a button which can be double–clicked which will show on the screen a circle going through the corners A, B and C. It is necessary that such a circle exists (allowing for certain 'degenerate' cases such the circle having infinite radius) but providing it makes the property of *cyclic* become highly accessible since a student need only attend to whether the fourth point, D, lies on the circle as well.

There are many possible activities which a teacher may provide which help to focus a student's attention on to properties of the quadrilateral. Several are included within the activity tasks in *Active Geometry* of which a section brief section regarding the exploration of a rhombus below is given as an example:

> See if you can state a rule which makes a quadrilateral a rhombus, where the rule is *only* about:
>
> a) the lengths of the sides;
>
> b) the direction of the sides;
>
> c) the angles inside the quadrilateral;
>
> d) the diagonals of the quadrilateral (use the *Show diagonals* button);
>
> e) the angle bisectors of the quadrilateral (use the *Show angle bisectors* button).

(Note: not all of these may be possible)

Check each rule *carefully* to see whether *any* quadrilateral which obeys the rule *must* be a rhombus. One idea is to pass on your rules to someone else and see whether they can find a quadrilateral which obeys the rule but is not a rhombus. If they find one, then you need to think again about your rule.

Through such activities the names are practised (in order to assist memory) whilst attention is placed with properties and relationships (in order to educate awareness). The simultaneity of names and properties appearing and disappearing together helps develop an associative bond between the names and the properties of the quadrilateral. The appearance of a name indicates that associated properties are present within the quadrilateral. The student’s role is to abstract from examples which properties remain the same when a certain name appears on the screen. Meanwhile the use of the name itself is being met and practised again and again.

Summary

Viewing the mathematics curriculum in terms of arbitrary and necessary can assist in clarifying the role a teacher has with respect to whether or not to inform students of certain parts of the curriculum. It also helps the way in which software is developed so that what is arbitrary is provided and is practised whilst working on the necessary. This way memory is assisted whilst awareness is educated. This contrasts with the use of tests as the main method of practice (where awareness is rarely educated) or treating properties as things which have to be memorised as well (thus placing a burden on memory as well as not educating awareness). Asking students to use memory for the arbitrary, and awareness for the necessary, means that students in mathematics classrooms can begin to use the powers they used so effectively as young children, such as the power of abstraction.

References

Bruner, J. S. (1960) *The Process of Education*. Cambridge, Massachusetts: Harvard University Press.

Feynman, R. (1988) *'What do <u>you</u> care what other people think?'*. London: Unwin Hyman.

Gattegno, C. (1971) *What we owe children. The Subordination of Teaching to Learning.* London: Routledge and Kegan Paul Ltd.

Ginsburg, H. (1977) *Children's Arithmetic. The Learning Process*. New York: D. Van Nostrand Company.

Hewitt, D. (1999). Arbitrary and Necessary: Part 1 a Way of Viewing the Mathematics Curriculum. *For the Learning of Mathematics*, 19(3), 2–9.

Jones, K. (1998). The Mediation of Learning within Dynamic Geometry Environment. In A. Olivier, K Newstead, K (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, (pp. 96–103), Stellenbosch, South Africa: University of Stellenbosch.

Kripke, S. (1996) *Naming and Necessity*. Oxford: Blackwell.

Laborde, C. (1995). Designing Tasks for Learning Geometry in a Computer–based Environment. In L. Burton and B. Jaworski (Eds.), *Technology in Mathematics Teaching – a bridge between teaching and learning* (pp. 35–67). Bromley, UK: Chartwell–Bratt.

Nozick, R. (1984) *Philosophical Explanations*. Oxford: Clarendon Press.

Saussure, F. de (1974) *Course in General Linguistics*. London: Fontana.

Walkerdine, Valerie (1990) *The Mastery of Reason: cognitive development and the production of rationality*, London: Routledge.