UNDERSTANDING TEACHERS' CHANGING APPROACHES TO SCHOOL ALGEBRA: CONTRIBUTIONS OF CONCEPT MAPS AS PART OF CLINICAL INTERVIEWS

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In this paper, we present a part of a larger study that examines changes in teachers' conceptualizations of school algebra. From a larger corpus of data, this paper examines data from two interviews with two teachers. Among other mathematical tasks the teachers were asked to draw a concept map for the concept "equation." Our hope was that these concept maps would help us identify teachers' conceptions of equation and how those conceptions fit inside a larger approach to school algebra.

Objectives/purpose

Within mathematics education, there is a strong assumption that teachers' "amount" of mathematical knowledge is a key component of their capacity to teach mathematics successfully. Quantitative attempts to support this assumption have used crude measures of prospective teachers' content knowledge and of the mathematical resources that they bring to teaching tasks and have not shown a positive correlation between the number of mathematics courses taken by teachers and their students' achievement (Begle, 1979). Thus, in order to examine this assumption more carefully in the future, it seems useful to develop better tools for describing and assessing teachers' mathematical understandings of the subjects they teach.

In science education, concept maps -- defined as a two-dimensional representation of relationships between selected concepts -- have been used extensively with a wide range of students, both as an instructional tool and as a mechanism for assessing the understandings of individuals (e.g., Novak, 1998). More recently, concept maps have been used in mathematics classrooms (e.g., Wilcox & Sahloff, 1998) and in research on student learning (e.g., Doerr & Bowers, 1999; Schmittau, 1991; Williams, 1998).

In this study, we will examine the strengths and weaknesses of concept maps as a tool for examining *teachers'* understandings of the mathematics that they teach. Are there reasons that concept maps might be especially useful in understanding how teachers conceptualize their approach to a topic? Or are concept maps limited in some principled ways when used to explore teachers' understandings?

In particular, we are interested in teachers' understandings of equations and the solving of equations. With the presence of new technologies, the algebra curriculum is in a state of flux; an extremely visible impact of the use of technology in school algebra involves using numerical and graphical representations of functions to identify solutions of equations. As a result, there is disequilibrium in the mathematics education community's views on relationships between algebra and functions (see, for example, Lee, 1996; Bednarz et al., 1996; or Lacampagne et al, 1995). Therefore, it seems especially useful to have tools to assess teachers' understandings in this area, as shared understandings of relationships between algebra and functions change.

Perspective or theoretical framework

Utilizing concepts from research on the psychology of student learning (Vinner, 1991), researchers interested in the understandings of high school teachers have studied their conception of function (see Cooney & Lloyd, 1993, for a review). As school algebra curricula have changed, some studies explore the conjecture that teachers' implementation of curricular topics related to functions are influences by the teachers' definitions of function. If that conjecture were true, teachers' definitions of function would shed light on their willingness and capacity to implement function-based curricula approaches to algebra. For example, Lloyd and Wilson (1998) focus on relationships between a teacher's implementation of the Core Plus curriculum and this teacher's conceptions of function. They suggest:

> "… because Mr. Allen was able to reconcile the Core-Plus approach to functions with the prominent features of his own conceptions of functions, the Core-Plus materials furnished a way for him to translate his understandings into new, but comfortable, pedagogical strategies" (p. 271).

We in addition suggest that it might be fruitful to create a construct "approach to school algebra" that indicates a teacher's overall orientation and to find tools to elicit and analyze such views.

Methods of inquiry

Sample: This research report describes a small piece of data from a larger study (Yerushalmy, Leikin, & Chazan, in press). We will focus on two in-service, high school mathematics teachers from a larger of sample of nine teachers. All of the interviewees in the larger study come from one U.S. school district. The district in which the interviews took place adopted a district-developed introductory algebra curriculum. This curriculum is built around the assumption of technological support, primarily in the form of graphing calculators. According to the criteria in Chazan and Yerushalmy (in press), this course takes a functions-based approach.

> "[It] initially emphasize[s] the interpretation of: letters as variables, rather than unknowns; expressions as the correspondence rules for functions; the Cartesian coordinate system as a space for displaying the results of calculation procedures, rather than the points in a solution set; and the equal sign as the assignment of a name to a particular computational process (f(x)=…) and as the indication of identity between two computational processes."

Interviews: Each interviewee was interviewed twice, once at the beginning and once at the end of the school year (1999-2000). Between the two interviews, the interviewees taught the district's curriculum and met five times during the year to discuss issues in algebra curriculum reform.

In the fall, interviewees were asked to compare and contrast different equations and to solve equations of different types. In the spring, the teachers were asked to discuss a set of statements about relationships between equations and functions and then relate these to their thoughts about teaching students what the solution to an equation is and how to recognize equivalent equations.

Besides these mathematical tasks, both in fall and in spring interviews, the teachers were given a table of concepts (e.g., variables, unknowns, statements about numbers, functions, …) and asked to draw a concept map of an equation. Of course, interviewees also added their own terms. In the fall, if an interviewee did not know what a concept map was, the interviewer provided an example of a concept map for the notion of quadrilateral. In spring interviews, the teachers were asked to compare new maps they had drawn in preparation for the interview with their fall maps.

Throughout the interviews, the teachers were asked to talk aloud. All the interviews were videotaped and any written work done during the interview was collected.

Data presentation and analysis: We will analyze the fall and spring interviews to identify the teachers' *conceptions of "equation"* and whether they hold a functions-based or an equations-based *approach to school algebra* (Kieran, 1997; Chazan & Yerushalmy, in press). The discussion section will reflect on contributions of the concept maps to our understanding of the teachers' views.

Results

Peter: In the fall, Peter was just starting to teach the introductory algebra curriculum. He consistently connected the concept of equation with a concept of function when performing tasks and discussing his performance. But, different tasks highlighted two different kinds of connections. For example, Peter viewed the equation $2^x=x^2$ as a comparison of two functions.

> Basically the way I would think about it first anyway is that I would think about what this problem means is where does the output of this function 2^x the same as the output of this function x^2.

In contrast, when solving the equation $x^2/_4+y^2/_9=1$ he said:

> … That one has the *y* in there and it doesn't look like we are comparing it to another function it seems like all those points in one relationship … So I would say I could think of this one as a function just *y* equals [by isolating *y*]…

More generally, whenever *y* was explicitly a second variable, in an equation in two variables, or in a system of two equations in two variables, he continued to think about functions, by turning equations of two variables into functions of a single variable.

Overall, Peter's fall concept map suggests that he is thinking about a function-based approach to school algebra. Almost all of the terms used for construction of this map and connections between the terms can be associated with function based approach (e.g., letters as variables, inputs as candidates for solution sets, …). Peter, for example, explained:

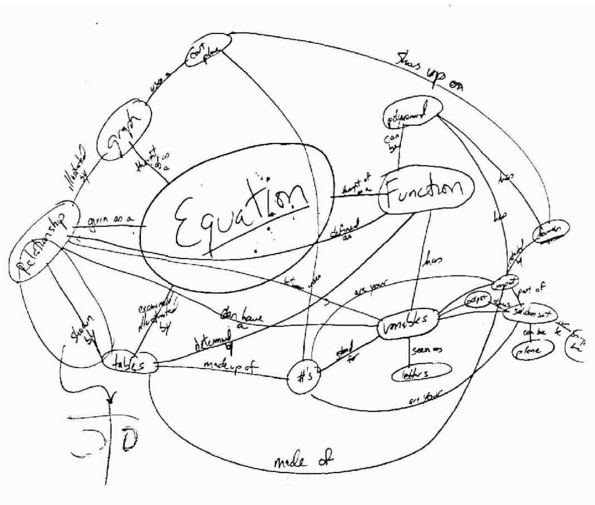
> …When I thought of an equation … the first thing that I saw was … a function. What I really thought of when I meant function was I kind of think of that is relationship between variables. … And form there spread out into the idea of variables, things that

are changing that include the input and output … The input was restricted by the domain, and … part of the inputs can be a solution set.

One link seems not to fit with this interpretation, viewing an equation as a function. However in his discussion of the map, he connected it back to his two views of the connections between equation and function:

> I wasn't sure … if equation and function were exactly the same. … You could think of an equation as a function. …Equation I [also see] kind of built of function obviously kind relate to that. … I thought also that graph went of an equation as you can think of equation as the function itself.

fall **spring**



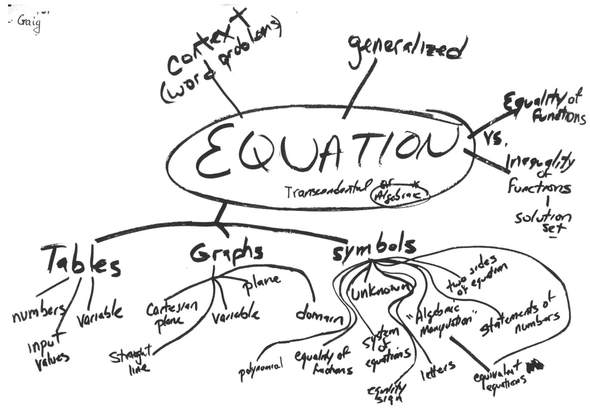


Figure 1: Peter's concept maps

While Peter's fall map was not organized hierarchically, his spring map is more tightly organized and has hierarchical components. In his spring map, equations were defined as an equality of functions (as opposed to inequality). He also connected equations directly with tables, graphs in the Cartesian plane, and symbols "because equation can be thought of as a graph, …, or can be illustrated by table."

In contrast to the fall interview, in the spring, Peter thought of equations in two variables as constructed out of two functions of two variables. For example he expressed his reasoning about the equation $x^2/9 + y^2/4=1$ as follows:

> We are trying to find where that output matches up with this different relationship between x and y. So here [in the right side] x and y are not written here because it is a constant function of 12 …To make more sense I will write I am just thinking of these as $f(x,y)=g(x,y)$, that's how I am, interpreting those.

Peter felt that his ideas had not changed much from fall to spring.

> My thinking hasn't changed much… I [am] actually pretty surprised how this [spring map, which] is done after a lot of thinking about teaching similar to this [fall map, which presents] just some random thoughts I had 6 month ago…

Interestingly, he suggested that his teaching effected his spring concept map and even seemed to say that it was organized according to "how it [solving of equations?] was taught in my class in the last … three months or so…" However, his spring map was more complicated than that. In his words, it represented both how he thinks, how the introductory course is organized, and "normal" approaches as well.

> This [spring map] is kind of a combination of the way I think about algebra and the way that school curriculum thinks about algebra and how that probably different from the traditional view about algebra or what you see in normal Algebra-1 book.

He continued and clarified why he placed "unknowns" in the branch of "Symbols," something that could be taken as in tension with the rest of his map. Variables were related to equations that are built of functions; unknowns were associated with either equations as they are presented in standard textbooks or points of intersection of the functions corresponding to equations' solutions.

> I put unknown here, which is a little bit different… I could have put variable here too [under symbols] but traditionally when you are talking about symbolic manipulation which is usually the algebraic manipulation, which is traditionally taught, variable can be thought as unknowns. Which is different than how I think about it…. The reason I think the unknown fits under equations is because we are looking for the point now….

Bob: Bob was a second year teacher in the fall of 1999. In 1998-99, he taught at junior high school. For the new year, he was transferred to the high school. At the junior high school, Bob taught the introductory algebra curriculum. This curriculum seemed different to him than the way he had thought about algebra in the past.

Throughout the fall interview, Bob consistently compared two approaches to school algebra. He discussed solving each one of the tasks in the interview according to two different approaches. He provided explanations from two perspectives, continually referring to what he had thought "before" and what he thought "now:"

> Before it [solving equations] was, … much more of trying to manipulate and do operations to each side… Now I think of that more like this is one function this is another function they got to be equal somewhere.

However, when he solved equations, he seemed more comfortable operating in the "before" mode. For Bob $x^2/9 + y^2/4=1$was an equation because it had an equal sign in it, an observation that fits his "before" mode. Then, shifting to his "now" mode, he discussed this equation in two variables as a comparison of two functions in two variables. At the same time, in discussing the graph of the solution set to this equation, he described the ellipse as "not necessarily a functional relationship." He concluded his reasoning about the similarities between this equation and an equation in one variable:

> Ah…it's still an equality um…this is in this has only one this [$2^x=x^2$] is really showing you only one variable this [$x^2/9+y^2/4=1$] is showing you two variables …It's [$x^2/9+y^2/4=1$] not necessarily a functional relationship except for again if we make that z it is in three dimensions.
>
> But, ah…similarly that their solutions would be values that if I'm solving this ($2^x=x^2$) I'm looking for the values and that makes this true. Solving this I'm looking for the values x and y that makes that true. So that will be similarity. So this [equation in two variables] is a curve and those ($2^x=x^2$) are curves.

Bob's fall concept map provides indications of two ways of thinking. First, an equation has unknowns that may be variables and it has an equal sign. But, it is not so easy to label his fall map as indicating an equations-based approach. For example, in discussing the map, he said:

> Both algebraic and transcendental [equations] can be solved by algebraic and symbolic manipulation, [by] looking at the Cartesian plane and [by] the graph and the intersection or tables of inputs and outputs… Solution set for either would be determine by domain and equivalent equations or [by] the intersection of the graph or where the tables … have the same output.

fall **Spring**

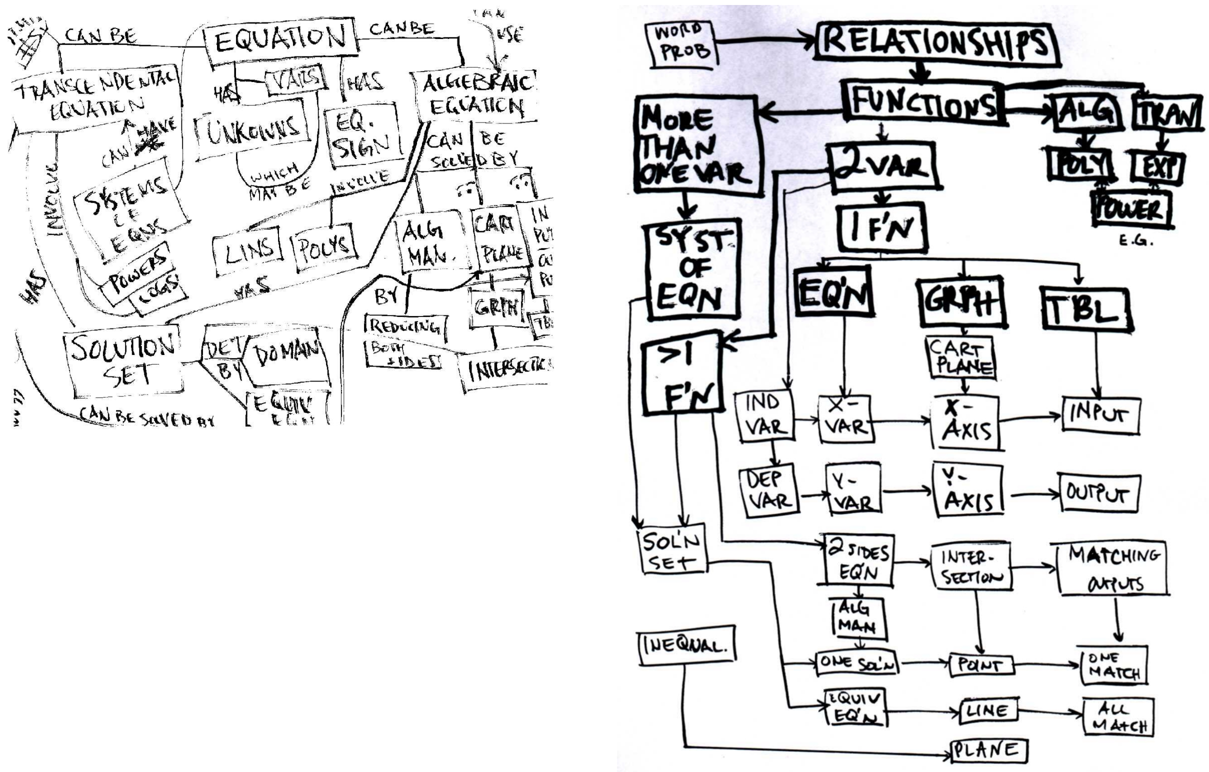


Figure 2: Bob's concept maps

Like his fall map, Bob's spring map includes both hierarchical and web-like components. In contrast, to the fall map, the definition that it provides for equations is less clear. Equation is under functions, along with graphs and tables. On the other hand, if one has "more than one function," it connects with "two sides of an equation." For him "An equation can frequently be made up of two functions then we can start talking about intersection points and solution set".

In the spring, Bob feels that both the organization and the content of his map have changed. Even with the ambiguity about the relationship between equations and functions, he spring map seems to reflect more explicitly Bob's "now" mode of thinking: "Here [in the spring], I started with relationships and functions, with equation as a part of function."

The rest of the spring interview supports these observations drawn from the concept map. When choosing a proposition that expresses a relationship between an equation and functions he, for example, said:

> I wouldn't agree with number 2: A function is a way to represent an equation. … Because I think of an equation as a subset of functions, as opposed to a function being a subset of equation… The function can be represented can be written out as an equation.

His interview comments revealed that he was referring to two views of equations. An equation can represent a function: "This [y=3x-5] is talking about a relationship: for any x value you are getting y values." This contrasts with his view of equations in one variable.

> I am trying to think, here [$x+2=3(x-1)$] we will be talking about two linear functions and we are looking at two linear functions and when does those two lines cross.

Equation in two variables bring forth a mixture of ideas:

> I can see that [5: $x^2+xy+y^2=12$] as … three variables three dimensions and then this is the value that you want I guess [if not in 3D] then I would have to refer back to what I said about 3 [$y+2=3(x-1)$] and 5 [$x^2+xy+y^2=12$] that this still representation a nonfunctional relationship. If x is supposed to be the independent and y the dependent or switch it around either way it is not a …function.

Discussion

Are there reasons that concept maps might be especially useful in understanding teachers' knowledge? One strength of concept maps is that they can indicate a conception of a concept as well as how this concept fits into a larger web, potentially revealing tensions in a teacher's thinking. For example, Bob's fall map suggests a definition of equation from his "before" mode and a web that has elements of his "now" mode.

Are concept maps limited in some principled ways when used to explore teachers' understandings? School algebra is a complicated domain to conceptualize. The teachers we interviewed knew different views (Bob's "before" and "now," Peter's own views and what appears in texts). There also is the question of how teaching appears in the maps. Does the way that teachers teach fit with how they think themselves? Which of these is being represented in the concept map?

As a result, we would certainly advocate the importance of discussion of maps with interviewees, as well as the combination of concept maps with other interview tasks. Without such supports, it seems very difficult to assess what one sees and to determine whether repeated use of concept maps over time reveals changes in perspective or representations of different aspects of a teacher's thinking.

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