The Students' Processes of Transforming the Use of Technology in Mathematics Problem Solving Tools

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The students' use of technology plays an important role in their learning of mathematics. Here we report the work shown by high school students who participated in problem solving activities that involves the use of dynamic software (Cabri-Geometry). A task proposed by the students themselves is used as a means to illustrate three different approaches that appeared during the students' work. Each approach shows diverse mathematical processes and resources that helped them explore and solve the task.

Recent curriculum proposals identify the use of technology as a powerful tool in the learning of mathematics (NCTM, 2000). There are different ways in which technology can be used by students, in particular, the idea that with the help of some software or calculators they can achieve easy representations, explore different cases, and find loci or trajectories of points (segments or figures) seems to be attractive in designing students' learning activities. What type of mathematical resources do students need in order to show an efficient or significant use of technology during their learning experiences? When does the use of technology become a powerful tool for students? These are questions that provide information to explain what students achieve in classroom that promote the use of technology. In this paper, we document features of students learning that show a process of students' adaptation in the use of technology. At this latest stage, students not only search for different approaches to represent and solve problems, but also they explicit redesign or formulate their own questions or problems.

Conceptual Framework

In a changing and demanding society the study of mathematics becomes an important need for all students; however, as Romberg and Kaput mentioned:

> the changes make it imperative that any answer to the question "What mathematics is worth teaching"? Be periodically considered". … regardless of the specific content, the aims of mathematics teaching can

> be described in terms, as teaching students to use mathematics to build and communicate ideas, to use it as a powerful analytic and problem-solving tool, and to be fascinated by the patterns it embodies and exposes (pp.15-16).

The use of technology can play an important role in helping students represent, identify, and explore behaviors of diverse mathematical relationships. An important goal during the process of learning mathematics is that students develop an appreciation and disposition to practice genuine mathematical inquiry during their school learning experiences. The idea that students should pose questions, search for diverse types of representations, and present different arguments during their interaction with mathematical tasks has become an important component in current curriculum proposals (NCTM, 2000). Here, the role of students goes further than viewing mathematics as a fixed, static body of knowledge; it includes that they need to conceptualize the study of mathematics as an activity in which they have to participate in order to identify, explore, and communicate ideas attached to mathematical situations.

> …Students themselves become reflective about the activities they engage in while learning or solving problems. They develop relationships that may give meaning to a new idea, and they critically examine their existing knowledge by looking for new and more productive relationships. They come to view learning as problem solving in which the goal is to extend their knowledge (Carpenter & Lehrer, 1999, p. 23).

It is also recognized that instructors should provide a class environment that promotes students' experiences in reflecting, conjecturing, and persisting. In this context, the design and implementation of tasks, which favor the use of these experiences, continue to be a great challenge in problem solving instruction (Santos, 1998). We document that the use of technology eventually becomes a powerful tool for students to make sense of information, to propose conjectures, and to examine different approaches to the problems. Students were encouraged to work as a community in which they valued not only personal contributions, but also the participation as a group. Students' engagement in processes of inquiring and explaining became the key ingredient while working with the tasks.

Methods and Procedures

Sixteen grade 12 students participated in a four weeks seminar that included two sessions per week (2.5 hours each session). The general idea of the seminar was to employ dynamic software to solve mathematical tasks initially provided by the instructor. Later, the same students were encouraged to propose their own tasks or problems. During the first two sessions the instructor gave a general introduction to the use of the software and illustrated the use of some commands to the whole class. In general, a student worked individually first, later in small groups of four members, and at the end of each session there was a general discussion with the whole group. Students could also exchange files and receive feedback from other participants. For the analysis of the students' work, we have chosen a task that was proposed by a small group. This task was solved during the last two sessions of the seminar. Throughout the analysis, we attached some comments or observations to describe particular students' behavior that appeared during this implementation; however, there is no attempt to show a detailed analysis of transcripts of their work. Instead, we identify a set of observations that illustrate mathematical relationships that emerged from students' interaction with the task. In some cases, the teacher' participation played an important role in orienting the students' discussion which eventually led them to propose and examine those relationships.

Origin of the task. An important activity that appeared during the sessions was to ask students to formulate their own questions or problems. So, during the students' interaction with tasks or situations, they were free to explore connections or change original statements to examine and document the behavior of other relationships. A member of a small group mentioned that in order to formulate questions, it was important to identify basic properties attached to different figures. For example, what do we know about rectangles? They have four sides (two pairs of parallel sides, perpendicular sides, four right angles, two diagonals, one center (diagonal intersection), and attributes such as areas perimeters and include pair of congruent right triangles (Pythagorean Theorem). Indeed, students agreed that in order to represent a task via the software, it was important to think of all figures in terms of properties and then select proper commands to achieve particular representation.

Can we construct a rectangle if we know only its perimeter and one of its diagonal? This was one question proposed by one student to the whole class. Three different students' approaches emerged from the students' work in this task. Although in all of them the use of technology appeared to be relevant, we focused on identifying two approaches in which the software functioned as powerful tool not only in achieving the solution but also in exploring other geometric properties of present figures.

Solution Process shown by three small groups

How can I represent the perimeter geometrically? What information does the perimeter provide about the sides of the rectangle? ¿How the perimeter information is related to the diagonal? These were some of the initial questions discussed within a small group that eventually led students to represent basic information and use the software dynamic to connect such information. The important stages are described next:

(i) Students represented the semi-perimeter as segment AB and chose point Q on it. That is, a + b is segment AB where a & b are sides of a rectangle. With this information they constructed the corresponding rectangle EHGF (figure 1). Here a = AQ = EH and b = QB = HG.

(ii) Students realized that by moving point Q along segment AB, a family of rectangles with a fixed perimeter was generated. Indeed, they decided to find the locus of point G when point Q is moved along AB (figure 2).

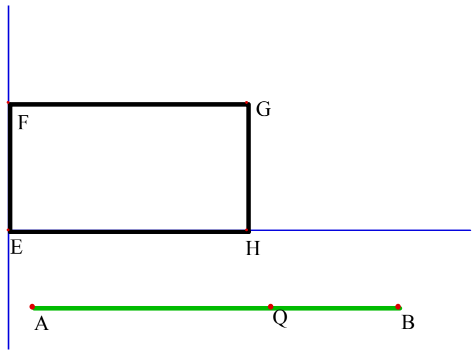


Figure 1

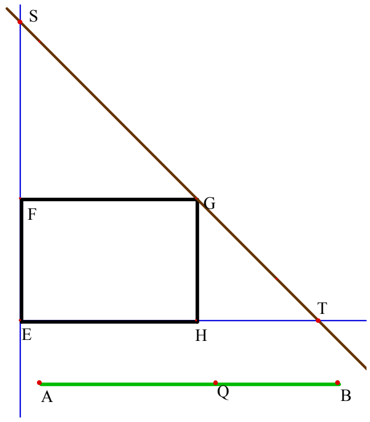


Figure 2

(iii) They found that the locus was the segment ST and explained that when point Q becomes point B, them ET will become segment AB. Similarly, when point Q coincides with Point A, and then segment ES becomes AB. That is, they noticed that the rectangle they wanted to find was one of those that can be inscribed in the right triangle EST. Indeed, they realized that the rectangle could be drawn in two different positions except when the rectangle became a square (figure 3).

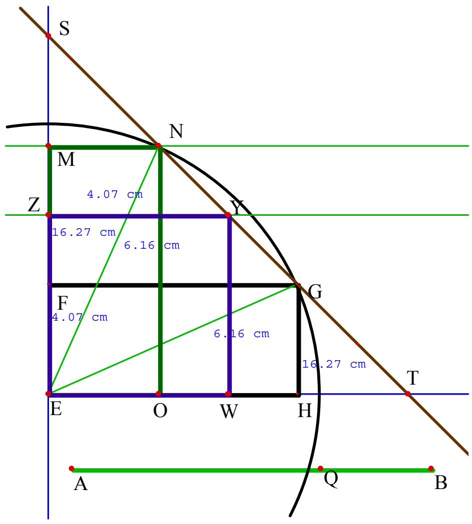
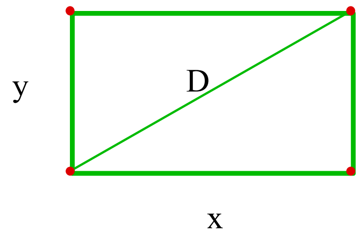


Figure 3

Another approach shown by two small groups was to focus on the algebraic representation of the situation. That is, they decided to use x and y for sides of the possible rectangle and wrote the following equations:



Here, a student suggested to graph both equations, he mentioned that since P and D were given numbers, then the first equation represented a line and the second a circle. They showed the following procedures and representation:

$$y = -x + \frac{p}{2}$$
$$x^2 + y^2 = D^2$$

Here, it is important to mention that students spent significant time analyzing the cases in which it was not possible to construct the rectangle. Eventually the graph became a referent to explain the existence of such rectangle (the circle might intersect the line in one point, the case of square, two point, the above figure, and no intersection points). The other small group which followed this approached gave an algebraic explanation regarding the solution of the system of equation.

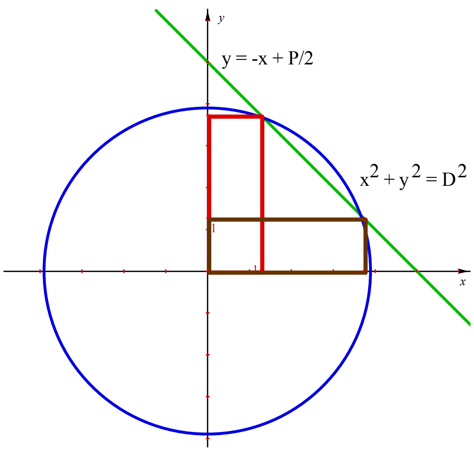


Figure 4

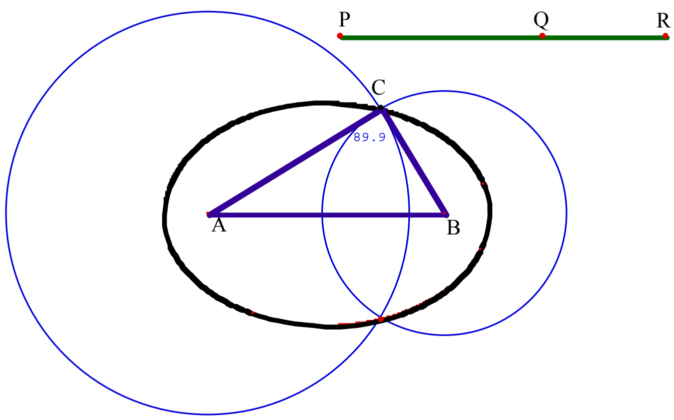


Figure 5

Yet another students' approach was to construct a family of triangles with perimeter equal to the sum of two sides of the rectangle plus the length of the diagonal. Here, they chose the given diagonal as a fixed side of the triangle and the other two sides of the triangle as the semi-perimeter of the rectangle. The software became a powerful tool to find the family of triangles with fixed perimeter (figure

5).

Segment AB represents the given diagonal and segment PR is the semi-perimeter. Students drew two circles, one with center on A and radius PQ and other with center on B and radius QR. These two circles get intercepted in C. The locus of point C when Q is moved along PR is an ellipse (foci A & B and constant PR). Here, students focused on finding the triangle with angle ACB a right angle. To find it, they drew a circle with center, middle point of the diagonal AB, and radius half of the diagonal. The intersection of the ellipse and the circle will determine the vertex of the right triangle. Here, they also noted that there were cases in which angle ACB never became a right angle. In this case, it is observed that there is no intersection between the circle and the ellipse.

When these solutions were discussed within the whole class, it was evident that students realized that the use of the software provided a means to explore the task from diverse angles and perspectives. In particular, they were surprised that a variety of mathematical resources and ideas were present in each approach. At the end of the session, a student asked: Can we construct a rectangle if we have its diagonal and its area (instead of its perimeter? Here, students again were ready to explore this question through the software.

Remarks

The work shown by students during their interaction with the task illustrated different mathematical qualities that allowed them explore strengths and limitations of their approaches to the task. For example, the dynamic approach in which students focused on finding the locus of the fourth vertex provided enough information for them to identify all rectangles with fixed perimeter. Here, they introduced the diagonal information to find the rectangle. The students' algebraic approach relied on a static representation in which they basically represented a particular case and discussed other possibilities of behavior of the two graphs in terms of the graphs intersection. The third approach in which students decided to construct a family of triangles with a fixed perimeter combines both a partial representation of the rectangle, that is a right triangle and the power of technology to find all of them with a fixed base (the diagonal). When students moved points, found diverse loci, assigned measurements, and formulated and supported conjectures, it was clear that the software became a powerful mathematical tool for the students.

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