Highschool students' conceptions of graphic representations associated to the construction of a straight line of positive abscissas.

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Abstract: *The construction of a straight line of positive abscissas is a task that require to know how make a graph, you can use the point-by-point strategy, but if you don't have any equation is necessary to know where the graph is. We ask to tree different samples of students (16 to 19 years old) where a straight line of positive abscissas have to build. We use a semiotic classification of variables visual and categorial for to find no relevant aspects in their answers, our analysis suggest that some of the students have an "one dimensional conception" about the Cartesian plane, they consider variations only in one coordinate and this conception seems coexist with the point-by-point graphics. They take the y-axis like an anchorage in their attention, it cut the x-axis leaving in one side negative numbers and the positive in the other side. And they use often one part of the graph y=x although they think a complete straight line*

Resumen: *La construcción de una recta de abscisas positivas es una tarea que requiere de conocer cómo se hace una gráfica, pero si además no se tiene la ecuación entonces es necesario saber donde debe estar la gráfica. Proposumos a tres muestras distintas de estudiantes, de 16 a 19 años, la construcción de una recta de abscisas positivas. Utilizamos una clasificación semiótica de variables visuales y categoriales para detectar los aspectos relevantes de la tarea así como los que no lo eran en las respuestas, nuestro análisis sugiere que algunos de nuestros estudiantes tienen una "concepción unidimensional" sobre le plano Euclídeo, es decir, consideran solamente variación en una de las dos coordenadas y esta concepción parece coexitir con la graficación punto a punto. Toman al eje y como un elemento que ancla su atención y corta al eje x dejando de un lado a los números negativos y por otro a los positivos. Utilizan frecuentemente una parte de la gráfica de la recta y= x pensando en la recta completa.*

Introduction

When studying Analytic Geometry, in which objects and conceptions may be expressed through different kinds of representations, Defining and Definiens must be clearly distinguished in accordance to current education, based on the construction of relevant concepts which cannot take place on the basis of only some of their representations.

For to build graphics is neccesary to pass from equation to the graphic and converse, this articulation is a *one to one* relation and always it is possible have one equation for each graphic although you don´t know it.

The idea that *there is a correspondence between the graph and the equation* was used by Pierre de Fermat and Rene Descartes in the creation of Analytic Geometry, according to C.Boyer (1956) which -for teaching purposes- should be understood, as we mentioned before, in the sense that there is *a one to one* correspondance between each algebraic representations of straight lines and their graphic representations and converse.

In fact, the possibility of establishing the relationship between the graphication of the straight line and its equation can see since a semiotic point of view. In this case graph and equation are linking in a univocal way through visual elements (visual variables) in the one case and algebraics (categorical variables) [1] in the other (Duval 1999).

In this paper we make a suggestion about the linking between categorical and visual variables in a construction task: the straight line of positive abscisas. We have been working with tree different samples to see if the typical graphic answers appear on them. We describe the relevant aspects against no relevant aspects about the answers of the students too.

Theoretical framework

The Raymond Duval's point of view is that the conceptualization process involved in mathematical concepts or objects (noesis) through conversion of different semiotic representations (semiosis). Such point of view enables us to observe the conceptualization of mathematical objects or concepts in which all the meaningful variables between the graph and the equation are displayed, thus showing meaningfulness and allowing for the learning of the concept from a sign/significance perspective.

R. Duval (1999) develops a semiotic viewpoint based on graphic and algebraic representations that belong to different semiotic registers and articulation between them, they must be carried out through *conversion*[2].

As to the conversion between registers -which is necessary to make a conversion the graphic and the algebraic representations- one would have to establish the relationships between the visual and the categorical variables on the basis of general variables defined by Duval (1988), as follows, the general variables are: 1.Task implantation, figure sticking out from ground: a line or region 2.Task form: the line, whether it limits a region or not, whatever it is straight or curve. lf it is curve, whether it is open or closed.

And three particular variables: 1.Direction of line's slope (goes up, goes down). 2.Angle and axes (symmetrical partition, major angle, minor angle). 3. Position on the y axis (cuts above the axis, below the axis or on origin).

We notice the following changes in its school treatment about straight line:

[1] Categorical variables herein mentioned are those which make a difference from a semiotic point of view between one equation and another, between one graph and another, in case of visual variables.

[2] Duval (1994) defines thus: "conversion is an operation which transforms a representation into a register change (p.222). lt is also: the transformation of such representation into one of different register where all or only part of the former content is preserved". (1988, p. 21).

a) Graphism associated to the Euclidean straight line where position has no referents. Only the figure-form exists. Position is contemplated as a visual variable acquiring value when it is associated to other elements as a tangent or a bicectrice. The kind of process is called here an *Euclidean straight line.*

b) The straight line is treated almost like the Euclidean straight line. With almost complete freedom as to position, restricted only by the figure-ground, this is, by the coordinase axes. The equations are not the most important subjet here. Relationship between figure-ground and figure-form is given on a figural manner. This kind of process is called here *post-Euclidean straight line.*

c) Straight lines` graphism is associated to linear equations. Equation use and its variables determine the straight line's position. The straight line's characteristics are manifested through points such as (0,b) and/or (-b/m, 0) or by the slope m of straight line $y = mx + b$. Visual variables are not considered, since the position is now expressed algebraically. We call this treatment *algebraic straight line*[3] .

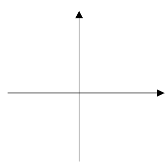
In the mathematics straight-line process, two principles can be noted: 1) The straight line is formed by an infinite number of points; 2) The straight line can be extended indefinitely to both sides.

These principles are strictly theoretical but they have a gestalt importance that is often missing. Every attempt to represent them figuraly or algebraically is but an attempt to build an infinite set with a finite number of elements through a finite process as well, however is not possible miss these gestalt characters when they build this straight line.

For the current paper we must consider some works have demonstrated that when handling definitions, are influenced by certain kinds of examples called prototype, which are more popular than the others are. There is also a tendency to increase secondary or irrelevant characteristics in the definitions (Hershkowits, 1989; Vinner, 1991).

The problem

The main problem we have used for our observation is the following; the task is about geometric description on graphic register:

6.Draw a line where all the abscissas of the points are positive, this is, where x is always x > 0.	

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[3] Front a point of view of the historical development of the mathematical knowledge the stages can be a) Euclidian line, b)Analytic line and c) Afin line, but here we want to consider the school treatment by programs.

In this task we only used one way in the double link of the conversion between algebraic and graphic registers and the algebraic register is support with the natural language.

In the sense of Duval (1988), we describe the general visual variables in our task: 1. Task implantation: a line; 2.Task form: Straight line. About particular variables we have: 1. Angle and axes: symmetrical position; 2. Lines slope: infinity; 3. Position on the y-axis: no cut.. The draw conditions are: $x = k, k > 0$

Categorical variables	Visual variables
x	Parallel straight line to y-axis
k	Cut x axis at k
$k>0$	The line put on the right side of y-axis.

Methodology

In early papers, we have been doing research with highschool students between 15 and 19 years old, in relation to the graphication of points, straight lines and semiplanes, with special attention to their ordinate spatial character and observing the relationship between figure-form (graph) and figure-ground (axes) under a coordinate-based treatment, on the one hand, and to the relationships established by its graph, on the other. In the present paper we want to ask about how the student think the build a straight line no typical, where they have to show theirs conceptions about the straight line in the Cartesian plane. Our samples were:

Sample	School	Age	Number	Questionnaire
A	CCH Sur	16	87	C2
B	CC Sur	17	77	C6
C	Sciences Department	19	42	C7

The sample C students are highschool graduates coming from different schools that answered the questionnaires on the first day of class at the Sciences Department, Math division, University of Mexico.

Students of the other samples were on 5o semester but the ones who are 17 years old were repeating the course, our intention with different samples was to see if the typical descriptions are present in different school information and age stages.

Localization of points on the plane may be considered a condition for graphicating point-to-point, although it is not enough for conversion between representations because the continuity gestalt condition is lacking. The following table shows performance when locating points on the plane.

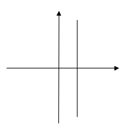
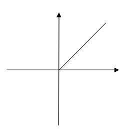
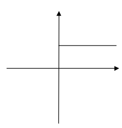
Sample	Point (4,7)	Point (-5,8)	Point (-4,-7)	Point (0,9)
A	95.4%	88.5%	94.2%	
B	89.6%	85.6%	84.45%	74%
C	95.2%	97.6%	97.6%	95.2%

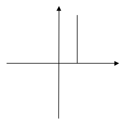
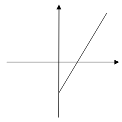
We have introduced in all of the questionnaires we have worked with a legend elucidating the difference between ordinate and abscissa to prevent confusion about axes from being the most relevant error. Table results indicate that once the difference is made, the task is reduced to applying the localization algorithm.

The problem dealt with in the present paper is proposed as a pencil and paper task on the Cartesian plane where we work with whole units around the origin. Students distinguish positivity on the plane as we have noted on their localization results, and they have built straight-line graphs point-to-point, supported by their equations as part of the highschool curriculum.

We consider their answers reflect their interpretation of the question in terms of their own conceptions since they have no background on questions like this; therefore, they are not influenced by their teacher's indications.

Below we have the answers to the problem:

Answers	Graphics	Sample A	Sample B	Sample C
I Right answer		0%	5.2%	40.5%
II		62.1%	26%	19%
III		1.1%	29%	16.7%

IV		3.4%	6.5%	7.1%
V		24.1%	9.1%	2.4%
Others		3.4%	10.4%	14.3%
Absent		5.7%	13%	0%

The nest table show the relevant and no relevant aspects in the answers of the students we classify them like follow:

Relevant aspects:

$x = k$
$k > 0$

0% in A, 6.5% in B and 40.5% in C

No relevant aspects:

In II the points are: $(x,y) = \{ y = x, x > 0 \}$ 62.1% A; 26%, B; 19% C
In III $(x,y) = \{ y = k, f(x) = y, x > 0 \}$ 1.1%A; 29% B; 16.7% C
In IV $(x,y) = \{ y = m x + b, 0 < m$ from infinity; $x > 0\}$ 24.1%A; 9.1% B;2.4% C
In V $(x,y) = \{ x = k, k > 0, y > 0 \}$ 3.4% A; 6.5% B; 7.1% C

In all wrong answers the students missed a hidden aspect, the continuity of the line, this aspect own to the *treatments*[4], it is not a visual variable but it is necessary for focus the visual variable that we need for the conversion. The number of no relevant aspects that students consider in their answers are: one in IV (y > 0), two in II and V , and three in III, besides the continuity condition (see the next before table).

Analysis and Conclusions

Previous results evidence that almost all of the students in sample A and B and almost 60% of sample C students make no difference between a straight and a cut straight line and, once the student incorporases this idea, he might think that, complementarily, the negative abscissas may be avoided if axis y is not exceeded as in answers II, III and V.

[4]Trearment of one representation is when it hava a converting inner the own semiotic register.

If we consider the item referring to the semi-straight line at 45º , we may say that this prototype example is rather relevant in the teaching of Analytic Geometry. In fact, it cannot be missing in school texts or in class. This straight line has a figural characteristic in the sense that its special slope is synonymous of its angle.

We found that the wrong answers start when they no consider the gestalt continuity condition about the continuity and that the straight line can be extended indefinitely to both sides.

Among the answers given to our question, answer II has anchorage on the origin of the coordinate axes, this point is privileged because it makes the difference between positive and negative abscissas.

The cut straight line of answer III seems to be stopped by the *x* axis; the same happens with answer V. Anchorage is produced on the same axis in both cases.

How the requested answer is interpretad in the exercise might be rooted in the school legend that says: *positive to the right, negative to the left,* which is used when establishing positivity on the real straight line.

An anchorage would explain answer IV on the horizontal axis where the legend could be *positive above, negative below.*

In the present case, as far as figural treatment is concerned, we observe that students alienate figure-ground from figure-form in their graphic description. The straight line is treated like the post-Euclidean straight line with almost complete freedom as to position, restricted only by the figure-ground, this is, by the coordinate axes, though not in its coordinate character but as limits to positivity, the same way zero is the cutting limit between positive and negative numbers.

We may draw the following conclusions:

1. The figure-ground (the axes), in the case of no correct answers, is reduced to a general reference of the plane's positivity in one dimensional terms. Reduction is very important in the figural treatment produced by our students, because it is the reference, that the axes support, brings meaningfulness to the straight line's position, the reference's frame make the difference with a straight line in the Euclidean Space, they use the straight line like a post Euclidian striaght line.

The straight line's position, for students that proposed some types of cut straight lines as solution, is linked to the legend *positive above, negative below;* on one side and other side *positive to the right, negative to the left.* When they use both conditions separate in fact they use it like a one dimensional element, defining like a single ordinate not like a couple.

2. Most of our students -mainly the youngest- use a cut straight line related to a prototype straight line where the general visual variable regarding the problem solution is not considered by them. Their conception were building with only one

part of the defintion, they miss that 1) The straight line is formed by an infinite number of points and 2) The straight line is extended indefinitely to both sides.The most important cause of mistakes in this task were the usefulness miss of this two conditions.

3. In the three samples they use the straight line and the cut straight line equally, according to convenience. This happen more in sample with youngest students but the oldest too made it. The straight line's figural properties as a geometrical object (using part of it or parting it) is used in solution without even noticing the difference.

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