PROBLEM SOLVING IN THE MATHEMATICS CLASSROOM: A SOCIO-CONSTRUCTIVIST ACCOUNT OF THE ROLE OF STUDENTS' EMOTIONS

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Abstract. *The present study aims at documenting from a socio-constructivist perspective the relations between students' mathematics-related beliefs, their emotions, and their problem-solving behavior in the mathematics classroom. To investigate these relations we did a multiple-case study of 6 students (age 14). The use of questionnaires as well as more qualitative methods like thinking aloud and interviews, allowed us to trace students' problem-solving behavior in its affective, motivational, and cognitive dimension. Results indicate that students' beliefs as well as their task-specific perceptions determine the interpretation and appraisal processes underlying emotional experiences. These emotional experiences tend to be closely related to cognitive and metacognitive experiences and determine the problem-solving process in specific ways.*

Theoretical framework

Recent theories on cognition and learning stress the situatedness of every learning activity and point to the close interaction between cognitive, conative and affective factors in students' learning and problem solving. A socio-constructivist account of learning and problem solving (e.g., Cobb & Bowers, 1999) takes emotions and feelings to be as much a constitutive and integral part of problem solving as cognitive and metacognitive strategies. Moreover, within such a perspective emotions are not primarily characterized as typical neurological processes which, together with their corresponding expressions and feelings, can be studied independent of the individual and context. On the contrary, emotions are perceived as being fundamentally grounded in and defined by the broader social-historical context that constitutes the individual as well as by the immediate social-context wherein the problem solving activity is situated. For students this context is in the first place the instructional context. One can claim then that every emotion is situated in its instructional context by virtue of four characteristics. First, emotions are based on students' cognitive interpretations and appraisals of specific situations. Second, students construct interpretations and appraisals based on the knowledge they have and the beliefs they hold, and thus they vary by factors such as age, personal history and home culture. Third, emotions are contextualized because individuals create unique appraisals of "similar" events in different situations. Fourth, emotions are unstable because situations and also the person-in-the-situation continuously develop. One can conclude then that emotions clearly have a rationale with respect to the local social order (Cobb, Yackel, & Wood, 1989).

In the field of mathematics education, McLeod (1992) - although not specifically situating his research within a socio-constructivist perspective - already acknowledged the relevance of studying the interactions between students' beliefs, their emotions, and the specific context of the mathematics classroom when he argues

"Since beliefs provide an important part of the context within which emotional responses to mathematics develop, we need to establish stronger connections between research in beliefs and research on emotions in the context of mathematics classrooms." (p. 581)

Rarely scholars have addressed in their research this relation between students' mathematics-related beliefs and emotions experienced *in* the classroom. More generally, the emotional reactions of students have never been major factors in research on affect in mathematics education (for the exception see e.g., DeBellis & Goldin, 1993). This lack of attention to emotion is probably due to the fact that research on affective issues has generally looked for factors that are stable and can be measured by questionnaire. Nevertheless, in the last decade several researchers in the field of mathematics education advocating a situated perspective have begun to study students' learning and problem solving including analyses of motivational and affective processes, next to cognitive processes (e.g., Isoda & Nakagoshi, 2000; Lester, Garofalo, & Kroll, 1989; Prawatt & Anderson, 1994). They take a more socio-constructivist approach when investigating students' emotions and focus on the dynamic interplay between student and context, using a variety of research methods (e.g., interviews, video-stimulated recall interviews, on-line heart rate measures) that should enable them to represent the student's perspective on problem solving, rather than the researcher's perspective. After all, research from a socio-constructivist perspective that focuses on the individual has to document how students engage in classroom practices and dynamically reorganize their ways of participating in them (Op 't Eynde, De Corte, & Verschaffel, in press). This approach stresses intentionality and emotionality, next to intellectuality, and takes activity and meaning as its basic currency. It implies a shift for researchers from an observer's perspective to an actor's perspective (Cobb & Bowers, 1999). What matters is not so much the classroom environment, practices, and (emotional) experiences as observed by the researcher, but the meaning students (and teachers) give to it and upon which they act.

The results from the studies mentioned above point to several important relations between the classroom context, students' beliefs, their emotions and their problem-solving behavior. To further investigate these relationships and to advance the search for appropriate research instruments, we developed in our center a research project to study the role of emotions in students' mathematical problem solving, taking a socio-constructivist perspective and focusing on individual students as the unit of analysis. The pilot study discussed here was set up to find out if the research methods and instruments used would enable us to grasp some aspects of the dynamic interplay between the student and the class context, and in the mean while would learn us something more about the relations between students' mathematics-related beliefs, their emotions and their problem-solving behavior.

Research design

This pilot study took place in the second year of junior high school (age 14) in three classes from different schools. The classes had basically the same curriculum for mathematics but differed in the general level of secondary education they followed.

All students of these classes were administered a self-developed questionnaire on mathematics-related beliefs [MRBQ] (Op ‘t Eynde, De Corte, & Verschaffel, in preparation). Two months later (after presenting the questionnaire to the classes) we made a selection of six students, one high and one low achiever out of each class as evaluated by the teacher. They were asked to solve a complex realistic mathematical problem during a regular mathematics lesson and had to fill in the first part of the “On-line Motivation Questionnaire [OMQ]” (Boekaerts, 1987) after they had skimmed the task and before they actually started to work on it. The problem consisted of a one page long story about the Balkan war between the Serbs and the people from Kosovo. A group of Kosovarian refugees tried to go to Albania through the mountains. In the mountains a women gives birth to a baby that appears to be ill and urgently needs specialized medical care. There are two possibilities, one with a delta plain of the Red Cross, another on food and by car. The students had to calculate the fastest way, given the different speeds of the respective means of transport and the distances they have to travel. Basically the problem consisted of four sub-problems that had to be solved successfully to find the correct solution, i.e. the fastest way.

Every student was asked to think aloud during the whole problem-solving process that was also videotaped. Immediately after finishing, the student accompanied the researcher to a room adjacent to the classroom where a “Video Based Stimulated Recall Interview” [VBSRI] took place (Prawatt & Anderson, 1994). The interview procedure consisted of three phases. In the first phase the student and the researcher watched the videotape and the student was asked to recall what he did, thought and felt while he was solving the problem, especially during those episodes that he was not thinking aloud. In the second phase the interviewer asked questions for clarification, more specifically ‘what and how questions’ relating to what he saw on the screen, what the student told him, what the student wrote down on the OMQ, and what he wrote on the answer form. Finally, in the third phase, the researcher tried to unravel the subjective rationale for the student’s problem-solving behavior. He looked for the interpretations the student gave to certain situations. The “why questions” that were asked, made underlying beliefs more visible and as such clarified the relation between beliefs, emotions and problem-solving behavior.

A qualitative vertical analysis of the different data resulted in six rich narratives of the way students handled and experienced the problems. These narratives were then content analyzed. After these vertical analyses of each student’s problem-solving process, a more horizontal approach was taken to look for recurrent patterns and/or fundamental differences that might deepen our understanding of what happens during problem solving and more specifically the role of emotions in this process.

Results

The results indicate that, in general, there is an individually changing flow of emotional experiences that follows from students' interpretations and appraisals of the different events that occur during problem solving in class. We found that solving a problem in class, even the same problem, usually consists of an individually different chain of events for each student. For instance, whereas some students were confronted with a lot of obstacles when solving the problem, others encountered less difficulties they had to deal with; but all of them experience a number of different emotions in the course of solving the problem.

Frank, for example, after already having experienced some difficulties solving subtask 1 and becoming frustrated in the process of doing it, ended up relieved because he was finally able to solve subtask 2[1]:

Dakovica is another 14 km.

at 20 km/h

that is,…wait,…

"Wait,…. that 20 km/h…. I seem to have forgotten how I had to do it, then I took a quick look, and then…"

Frank takes his calculator

"Actually, I did not really need the calculator there. I wasn't thinking properly, and then I panic, and then I immediately want to go to my calculator, and if then I stop and think for a moment, I probably know again what I have to do"

INT: "If you are searching for the solution, you have tried something, you end up at the 20 km/h, you grasp your calculator, and you don't know what to do exactly, how do you feel then?"

Frank: "A little bit, I don't know how to put it, you don't feel well because you need to go to the calculator. I always want to do as much as possible without it."

"I did not know immediately how I had to go from 20 km to 14 km."

Franks moves his hand back to the calculator but he redraws it

INT: "At such a point, yes - no, yes - no, …, how do you feel then?"

Frank: "Then I get something like… come on what is this all about!!…. Usually I focus then on some point where there is nothing, and then I go through everything again, 20 km in one hour, how do I get to the 14 km."

INT: "How did you manage to find it then?"

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[1] *Thinking aloud data* are written in bold;
Students' comments in the VBSRI are placed between " ";
Questions of the interviewer are preceded by INT;
Data from observation by the researcher are written in italics.

Frank: "I just kept searching how I could get from the 20 km to 14 km. First, I thought to take some bigger numbers and that is why I was thinking of using the calculator. I don't know how I came to it, but at a certain point it worked. I was not really thinking, and then I thought, hey this is not possible, and then I start to think again and then I managed."

20 km in 1 hour

no, no, no; yes

20 km divided by 10

1 hour divided by 10

2 km times 7 is 14

6 minutes times 7 is 42 minutes

Clearly, Frank experiences different emotions when solving subtask 2 as, for example, panic, frustration, and angry. Not only do students encounter individually different obstacles when solving the same problem, but even when they are confronted with the same or comparable events these are in some cases interpreted and appraised differently according to the person (his knowledge and beliefs) and the class context.

For example, our data show that negative emotions (e.g., frustration) usually were experienced at moments that students were not able to solve the problem as fluently as they anticipated. Experiencing the inadequacy of a cognitive strategy used, is apparently as much an emotional as a (meta)cognitive process. However, the nature and the intensity of the emotion experienced, when confronted with a comparable cognitive block, can differ significantly between students. Confronted with a difficulty in an early stage of the problem-solving process, one of the students became hopeless, stating that “If I’m already not able to solve this, than I surely will not be able to solve the rest of the problem”. Another student also got stuck at the same point, became a bit annoyed, but experienced this as a challenge, and tried to find a way around it. Despite the fact that they both indicated to be highly motivated and confident to solve this problem (based on the results of the OMQ), they interpreted and appraised the first difficulty they encounter in an entirely different way. Differences in their more general mathematics-related beliefs (as measured by the MRBQ), more specifically their general competence beliefs, possibly grounded in the different social contexts they function in, can account for this (see also infra).

These examples also reveal another aspect of the role of emotions in mathematical problem solving. In most of our cases, an emotional experience always triggered students to redirect their behavior looking for alternative cognitive strategies or heuristics to find a way out of the problem. However, big differences were observed in the effectiveness and/or efficiency of the cognitive strategies used.

Frank, after panicking following a first attempt, keeps focusing on subtask 2. At a certain point he thinks about working with bigger numbers (a well-known heuristic strategy), but in fact he stays with the given numbers and looks for a way to bridge the gap between 20 km and 14 km, with good result.

Steve uses another strategy getting stuck with subtask 2, crying out "Come on pal". He decides to continue with the next subtask, hoping that this will help him to find out, how he has to solve subtask 2.

None of the students in the pilot study really used coping or emotional regulation strategies to control their behavior. Some thought of it, but did not use it. Ellen, for example, persists in going on although she has not made any progress for five minutes. Asked if she would act the same way at home, she answered.

Ellen: "No, then I would take a brake and relax for a moment, so that I can try it again afterwards."

Int: "You don't do it here, why not?"

Ellen: "Because I have to keep on working. You are not allowed. You don't do that in class. You are not just going to stop and leave, telling just leave me alone for a while guys, I will continue in a few minutes"

This example also illustrates how students' behavior is determined by the beliefs they have about the practices that are or are not allowed in the mathematics class context. More generally, students' descriptions and explanations of their emotional experiences in the video based stimulated recall interviews usually refer to underlying belief-systems. Combining these data with the results on the mathematics-related beliefs questionnaire and the on-line motivation questionnaire, we found that specifically students' general and task-specific competence and value beliefs appeared to determine the interpretation and appraisal processes underlying the emotional experiences. For example, the experience of a cognitive block as challenging rather than frightening or demotivating in many cases seemed to depend a lot on students' beliefs about their competence (see supra). Of course, one might assume that their specific beliefs about the nature of solving these kinds of problems and the class teacher's acceptance of getting stuck in the process also color their interpretations and appraisals. However, these very specific beliefs nor their social correlates, i.e. the classroom norms and practices, were the focus of attention in this study.

Conclusion and discussion

This pilot study already strongly suggests that emotions are very much part of problem solving in our mathematics classrooms. Especially negative emotions as, for example, frustration and anger were frequently experienced by the six participating students. These emotions almost seem to be an integral part of problem solving. Indeed the absence of an obvious method to immediately solve a presented task as a major defining characteristic of a problem (Mayer & Wittrock, 1996) implies that those who really want to reach the goal state, i.e. find the solution, will find themselves frustrated at some points in the process. Teaching students how to solve mathematical problems then necessarily implies that we have to learn them how to deal effectively with those feelings of frustration or sometimes anger.

The main aim of this pilot study was to test the general research approach and the quality of the methodology and instruments in function of the main study. It shows how the use of a variety of research methods and instruments in a complementary way can enable researchers to trace students' ongoing interpretations and appraisals of events constituting their problem-solving processes. As such the analysis of students' emotional experiences inevitably also includes an analysis of cognitive and conative processes, on the one hand, and of characteristics of the (subjective) task context (the events), on the other hand. It allows us to grasp the student's, actor's, perspective and the meanings underlying his activities when solving a problem, clarifying the dynamics that constitute his problem solving in class. In this way, this kind of research is a good example of how one can study the individual from a socio-constructivist perspective and what can be learned from it. Of course, a deeper and more complete understanding would have been obtained if the focus on the individual could have been complemented with an analysis of classroom interactions. This appeared to be very difficult to realize in one research project, given the available recourses and the constraints implied in doing research in the classroom. However, as argued above, the absence of an *explicit* analysis of the classroom context does not contradict the socio-constructivist nature of this study.

Although there is a lot to gain from these kind of in-depth studies of students' behavior, one has to stay aware of the restrictions implied in the methods and instruments used. For instance, bringing a researcher, video- and audio equipment into the classroom always interferes in some way with normal classroom life. There is sometimes a very clear influence on students' experiences in class, as can be illustrated by the following clarification given by a student during the video stimulated recall interview "I was nervous in the beginning, because there was a camera". However, from this study we have also learned that after a few minutes students' behavior in the presence of the camera becomes gradually more normal, i.e. similar to their behavior when the camera is not present. Teachers' observations in the classrooms confirm this. Nevertheless, researchers have to be aware that the research setting, even when it is situated in the classroom and stays as close as possible to authentic classroom activities, does influence students' behavior. By explicitly allowing students to deal with these aspects in the interviews researchers might be able to trace some of these unintended influences and take them into account when interpreting the results.

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