THE ‘PARTICULAR’, ‘GENERIC’ AND ‘GENERAL’ IN YOUNG CHILDREN’S MENTAL CALCULATIONS

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When asked to perform a mental calculation and then to say what was in their head when doing it, young pupils sometimes describe what they did with the particular numbers given. In some descriptions, however, pupils use the numbers as generic examples to explain the procedure that is used. In other descriptions they may simply state a general rule. Responses given in six interviews over a two year period by UK pupils aged 7 to 9 years have been analysed in terms of these categories of generality. Results suggest that the use of non-particular expressions of generality is a characteristic of pupils who are successful in mental calculation. There was evidence also that the use of these modes of expression did not necessarily bring success in difficult questions where flexibility was needed.

INTRODUCTION

Studies of the influence of classroom activities on young children’s mental representations have been described in Bills & Gray (1999, 2000). In a two-year longitudinal study pupils have been asked to perform a total of 45 mental calculations in the six interviews. Their subsequent descriptions of what was ‘in their head’ when they performed a calculation fall broadly into three categories. After calculating 48 add 23, for instance, some descriptions have been categorised as ‘particular’ because the pupil simply said what they had done with those numbers, e.g. “I just added 20 then I just added 8”. Other descriptions are ‘generic’ because they use the numbers as a vehicle to describe a procedure, e.g. “if it’s 40 with 20 is 60, 48 add 20 comes 68 and then you add 3 on”. The ‘general’ category of response makes little mention of the numbers, e.g. “I just added the tens and added the units and then added them both together”. When questions answered correctly were compared with incorrect answers it was found that ‘particular’ expressions most frequently accompanied wrong answers. Correct answers were more often accompanied by ‘generic’ or ‘general’ expressions. Moreover, high-achieving pupils were more likely to use non-‘particular’ expressions than low-achieving pupils.

Responses to questions about non-numeric procedures and non-mathematical concepts were also categorised into three similarly designated categories. The data shows that pupils of all achievement levels, in mental calculation, can use ‘general’ expressions in non-calculation contexts. This suggests that the use of non-‘particular’ expressions in descriptions of mental calculations is not simply a sign of a pupil’s linguistic sophistication. The analysis of responses to difficult questions suggests, however, that flexibility in mode of response is required. The implication for the teacher is that the style in which a pupil describes a calculation procedure may be a useful indicator of understanding.

ASPECTS OF GENERALITY

Procedural and Proceptual Thinking

The distinction has been made between 'instrumental' and 'relational' understanding (Skemp 1976) where relational understanding requires knowing both what to do and why, whilst instrumental understanding involves simply knowing how to do something. Gray and Tall (1993, 1994) argue that those who fail at mathematics have failed to progress satisfactorily from the procedures of counting to the processes of arithmetic and similarly fail to generalise from other learned procedures in other areas of mathematics. They distinguish between flexible, 'proceptual' thinkers, for whom a symbol is a mathematical object that can be manipulated in the mind, and instrumental, 'procedural' thinkers for whom the symbol simply signifies a procedure to be carried out.

Pupils may thus use a procedure without understanding. This view was endorsed by Harries (1997) in a study with low attaining children. He described one pupil's use of the written algorithm for two-digit addition where she failed to carry (her answer for 49 + 22 was 611). Her consideration of the correctness of the answer was based on her perceived accuracy of the process not on the basis of the objects she was working with.

Particular, Generic and General

Mason and Pimm (1984) in their paper "Generic Examples: Seeing the General in the Particular" use the term 'generic example' when a particular number is used to stand in for others and doesn't rely on any specific property of that number. More generally, a generic example has been described by Balacheff (1988,p 219) as "an object that is not there in its own right, but as a characteristic representative of the class". The use of the word 'generic' to imply a representation of something more general is common, Johnson (1987) for instance, gave a definition of 'schema' as a cluster of knowledge representing a particular 'generic' procedure, object, percept, event, sequence of events, or social situation. He suggests that this cluster provides a 'skeleton structure' for a concept that can be instantiated with the detailed properties of the particular instance being represented.

The notion that a particular instance is recognised as typical of a class and is used to represent that class is also the key to Rosch's theory of categorisation. In her view (Rosch, 1977) categories are not coded in the mind as lists of individual members of the category nor as lists of category inclusion criteria but as 'prototypes' of the most characteristic members of the categories. This theory suggests that learners construct concepts by comparing new experiences with prototypical or generic examples which represent their current knowledge. Thus our representation of the general is in terms of the particular. For the purposes of this paper 'generic' will be used as a label for both generic examples used in procedures and proto-typical exemplifications of concepts. The terms 'particular' 'generic' and 'general' will be used as categories both of modes of expression and of the mental representations that they might reveal.

Pavio (1971, p18) insisted that it is "simply asserting a truism" to say that modes of representation evolve within the individual from the more concrete to the more abstract. Luria (1982) for instance found that young children, asked for a definition such as "What is a dog?", gave proto-typical associations ("a dog barks") but older children responded with a more abstract verbal-logical category definitions ("is an animal"). In this paper 'particular' is used to signify a 'concrete' representation both in pupils responses to procedure questions, were descriptions involve what was done with particular numbers, and to concept questions where a particular object is given. 'General' is the most abstract representation for procedures, where a rule is expressed without reference to numbers, and for concepts, where a definition is given. Between these two levels of abstraction 'generic' uses a particular instance as an exemplar of the general. As previously noted, however, (Bills and Gray, 2000) literature on mental representation generally supports a view of variability within and between individuals rather than the existence of developmental levels.

METHOD

Lesson observations and pupil interviews were first conducted with two classes from Year 3 (pupils aged 7 and 8 years) in a school for children aged 5 to 11 years in a large middle-income village near Birmingham U.K., from September 1998 to July 1999. The same pupils were observed and interviewed in the following year. The 80 children in the year group had been placed in one of three groups for Mathematics based on their previous attainments. Lessons with the high attainment and the middle attainment groups were observed and a sample of 14 pupils from the first and 12 from the second was interviewed in December, March and July in each year. The samples were chosen to represent the spread of achievement levels in each group.

Examples have been given in the introduction for each of the categories of response for calculation questions. In the numerical procedure questions, such as "Tell me how to multiply by ten", pupils described what to do with a particular number (e.g. "ten times ten you add ten ten times") or chose a generic example (e.g. "like if it was 8, just add a nought on to it, so it'll equal 80") or gave a general rule ("just add a zero"). The non-numeric procedure questions evoked similar types of response. For instance "Tell me how to tell the time":

'particular'	if the big one was at the top and the small one was at the bottom it would be 6 o'clock
'generic'	I'd look at the big hand first so if it was like at 8 past I'd round it to the nearest 5, which would be 10 so I'd say like 10 past 6
'general'	The big hand points to the minutes, the little hand points to the hours.

Pupils were also asked first "What is the first thing that comes into your head when I say ...?" for mathematical concepts (centimetre, three, millions, fraction, polygon) and then to say more about each. Similarly for non-mathematical concepts (shadow, ball, adjective, Christmas, animal). The responses fell into three categories: pupils mentioned a particular object, gave a proto-typical property (also termed 'generic') or a general property that amounted to a definition. For example "three" and "ball" elicited the following:

category	"three"	"ball"
particular	A picture of the number 3	a football
generic	It's in the 3 times table	it's bouncy
general	It's a number.	It's a sphere shape and you can play with it

Over the six interviews 78 questions were used. They were classified into 10 calculation types and 6 non-calculation type. Each was presented verbally and followed by "What was in your head when you were thinking of that?"

Type	Description	Examples of questions
1	1-digit addend	17 + 8, 17 + 9 (repeated in each interview)
2	Missing addend	13 + * = 18, 30 + * = 80, 27+*=65
3	2-digit addition	48 + 23 (repeated in each interview)
4	Addition of multiple of 10	97 + 10, 597 + 10, 1097 + 10, 1197 + 10
5	Counting	What comes before 380, 2380, 12100; after 12386
6	Rounding	Round 2462 to the nearest ten, 239 to nearest hundred
7	Recent topic	What is difference between 27 and 65, 0.6+0.7
8	Recent topic	65 subtract 29, Read time (11:40), 0.1 times by 10
9	Division and fractions	quarter of 40, third of 48, 140 divided by 3
10	Multiplication	48 multiplied by 3, 47 multiplied by 5
11	Numerical procedure	Tell me how to add 23, find a third, times by ten,
12	Non-numerical procedure	Tell me how to cross road, tell the time, do subtraction
13	Maths concept, first	First thing in head when I say centimetre, three, million
14	Maths concept, more	What else can you tell me about centimetre, three, million
15	Non-Maths concept, first	First thing in head when I say shadow, ball, adjective
16	Non-Maths concept more	What else can you tell me about shadow, ball, adjective

RESULTS

Context differences

In the calculation questions there was a marked difference in the distribution of the categories of generality between questions answered correctly and those answered incorrectly (chi-square test significant, $p<0.005$). Those who gave a 'generic' or 'general' response were more likely to be correct and those who expressed themselves in 'particular' terms were more likely to be wrong:

	Number of responses (percentages of row totals in bold)							
	Particular		Generic		General		Totals	
Right	184	**30**	324	**53**	107	**17**	615	**100**
Wrong	147	**51**	110	**38**	30	**10**	287	**100**
Totals	331	**37**	434	**48**	137	**15**	902	**100**

Descriptions of the procedure in some way other than what was done with the particular numbers are thus associated with accuracy.

Categories of generality were also compared across all aspects of the interviews and there are distinct differences between distributions in the different contexts:

	Number of responses (percentages of row totals in bold)							
	Particular		Generic		General		Totals	
Maths Calculation	331	**37**	434	**48**	137	**15**	902	**100**
Maths Non-calc	103	**27**	206	**53**	77	**20**	386	**100**
Non-Mathematics	87	**31**	101	**36**	93	**33**	281	**100**
Totals	521	**33**	741	**47**	307	**20**	1569	**100**

Pupils demonstrate in non-mathematical contexts that they can give responses at all levels of generality but in calculation questions, mathematics-procedure and mathematics-image questions they are more likely to express themselves in ‘particular’ or ‘generic’ terms. This is emphasised when mathematics questions are grouped:

	Number of responses (percentages of row totals in bold)							
	Particular		Generic		General		Totals	
Mathematics	434	**34**	640	**50**	214	**17**	1288	**100**
Non-mathematics	87	**31**	101	**36**	93	**33**	281	**100**
Totals	521	**33**	741	**47**	307	**20**	1569	**100**

Pupil differences

When the pupils were grouped by their level of achievement in the three written mathematics tests (SAT) conducted at the end of each year it became clear that the higher achieving pupils use more non-‘particular’ expressions of generality than the lower scoring pupils in calculation questions ($p<0.005$):

	Number of responses (percentages of row totals in bold)							
	Particular		Generic		General		Totals	
Higher SAT scores	136	**28**	253	**52**	100	**20**	489	**100**
Lower SAT scores	195	**47**	181	**44**	37	**9**	413	**100**
Totals	331	**37**	434	**48**	137	**15**	902	**100**

The difference between the groups is not significant in non-mathematical questions:

	Number of responses (percentages of row totals in bold)							
	Particular		Generic		General		Totals	
Higher SAT scores	48	**32**	53	**36**	48	**32**	149	**100**
Lower SAT scores	39	**30**	48	**36**	45	**34**	132	**100**
Totals	87	**31**	101	**36**	93	**33**	281	**100**

The differences in pupils is more pronounced when grouped by their performances in the interview calculation questions. The three groups: High- (scores greater than 1 sd above mean), Middle- (scores within 1 sd of mean) and Low- (scores less than 1 sd below mean) accuracy pupils, express themselves quite differently ($p<0.005$):

	Number of responses (percentages of row totals in bold)							
	Particular		Generic		General		Totals	
High-accuracy	59	**32**	89	**48**	36	**20**	184	**100**
Middle-accuracy	191	**33**	305	**52**	87	**15**	583	**100**
Low-accuracy	81	**60**	40	**30**	14	**10**	135	**100**
Totals	331	**37**	434	**48**	137	**15**	902	**100**

Once again there is no statistically significant difference between the groups in non-mathematics contexts though here high-accuracy pupils use a higher proportion of ‘particular’ expressions than the other pupils:

	Number of responses (percentages of row totals in bold)							
	Particular		Generic		General		Totals	
High-accuracy	22	**39**	16	**28**	19	**33**	57	**100**
Middle-accuracy	50	**29**	63	**36**	61	**35**	174	**100**
Low-accuracy	15	**30**	22	**44**	13	**26**	50	**100**
Totals	87	**31**	101	**36**	93	**33**	281	**100**

Difficult and easy questions

The facility level of questions varied from 0% to 100%. Eleven questions were answered correctly by ten or fewer pupils. When these 'difficult' questions are compared with the others the change in styles of response are similar for groups of children with different levels of success. 'High-success' pupils were correct in more than 4 of these. 'Low-success' pupils did not answer any correctly. Each group is less likely to use non-'particular' expressions in difficult questions and the swing toward 'particular' is most marked in the response of the most successful:

	Number of responses (percentages of row totals in bold)							
	Particular		Generic		General		Totals	
High-success easy	47	**28**	91	**54**	32	**19**	170	**100**
High-success difficult	25	**42**	26	**43**	9	**15**	60	**100**
Mid-success easy	109	**31**	183	**52**	59	**17**	351	**100**
Mid-success difficult	48	**41**	58	**50**	10	**9**	116	**100**
Low-success easy	78	**48**	61	**37**	24	**15**	163	**100**
Low-success difficult	24	**57**	15	**36**	3	**7**	42	**100**
Overall easy	234	**34**	335	**49**	115	**17**	684	**100**
Overall difficult	97	**44**	99	**45**	22	**10**	218	**100**

The most successful pupils use a more even spread of 'particular' and 'generic' in difficult questions and their proportion of 'general' expressions is not as reduced as the other pupils. The sense that pupils carry on describing procedures even in questions they get wrong by failing to use the procedure correctly, is re-enforced when two similar questions are compared:

Year	Term		Correct	No response	Particular	Generic	General
3	1	597+10	17	2	6	13	5
4	1	1097+10	8	3	6	14	3

Many pupils described the procedure in similar terms in each instance but failed to deal with the thousand appropriately, 2007 being a common wrong answer.

Differences over time

There appears to be no evidence of progression from concrete to abstract when individuals' categories of generality are analysed. Some pupils were consistent in their mode of explanation but most showed variability over the six interviews. A few did move from 'particular' to 'general' in comparable questions with an accompanying improvement in accuracy. There was little change in the distribution of categories for groups of children over the two years. When the total number of responses in each category in Y3 is compared with totals for Y4 the main difference between the groups is in the number of questions that they could respond to:

	Number of responses (percentages of row totals in bold)									
	No response		Particular		Generic		General		Totals	
High-accuracy Y3	16	**15**	31	**30**	43	**41**	15	**14**	105	**100**
High-accuracy Y4	25	**21**	28	**23**	46	**38**	21	**18**	120	**100**
Middle-accuracy Y3	60	**18**	91	**27**	141	**42**	44	**13**	336	**100**
Middle-accuracy Y4	77	**20**	100	**26**	164	**43**	43	**11**	384	**100**
Low-accuracy Y3	36	**34**	40	**38**	23	**22**	6	**6**	105	**100**
Low-accuracy Y4	54	**45**	41	**34**	17	**14**	8	**7**	120	**100**
Total Y3	112	**21**	162	**30**	207	**38**	65	**12**	546	**100**
Total Y4	156	**25**	169	**27**	227	**36**	72	**12**	624	**100**

Against this background of global lack of change it is instructive to consider performance in one question, “48 add 23”, used in the first five interviews:

	Number of responses (percentages of row totals in bold)							
	Particular		Generic		General		Totals	
48 + 23 Correct	15	**19**	44	**56**	20	**25**	79	**100**
48 + 23 Incorrect	15	**50**	10	**10**	5	**17**	30	**100**
Totals	30	**28**	54	**50**	25	**23**	109	**100**

The proportion of non-‘particular’ is higher than the average for correct answers as might be expected for a relatively easy question but the breakdown for the number of responses in separate interviews in each term in each year is more revealing:

Year	Term	Correct	No response	Particular	Generic	General
3	1	13	6	3	14	3
3	2	16	5	4	14	3
3	3	14	3	3	15	5
4	1	17	3	7	7	9
4	2	23	2	8	11	5

This shows a high level of ‘recipe-following’ in Y3 when the ‘generic’ responses reflect the use of frequently practised written algorithms. In Y4 pupils were encouraged to use more of their own mental strategies and this meant initially both more ‘particular’ and more ‘general’ responses. In the final interview the pupils were most accurate yet used a high proportion of ‘particular’ expressions

DISCUSSION

The initial picture presented by the data suggests that expressions at a ‘generic’ or ‘general’ level are strongly associated with accuracy. Accurate answers are most often explained in one of these modes and accurate pupils use more of these expressions than the least accurate. They are good at ‘instantiating’ their ‘skeleton’ mental representations of procedures. This does not imply, however, that achievement in mental arithmetic is due simply to an ability to express oneself in this way. The analysis of non-mathematical items shows that all pupils are capable of non-‘particular’ expressions of generality and, if anything, the most accurate reserve this mode for calculation questions more than in non-mathematical contexts.

When the responses to difficult questions are considered the pupils use fewer non-‘particular’ expressions than in easy questions. This is predictable because these questions are associated with low accuracy. Those pupils who are most successful in the difficult questions, however, show a greater flexibility by switching more to 'particular' modes of expressions. The others continued to use similar proportions of 'generic' expressions to those they had used with easy questions. This same picture of advantage in flexibility of expression emerged when the responses to one question which occurred in five interviews were examined. Initially pupils spoke predominantly in procedural terms, typically "you add the ...", but when the standard procedure was practised less often in the classroom they showed greater variation in expressions of generality and there was a higher level of success.

At one level this paper seems to provide one more measure which discriminates the successful from the unsuccessful. The least successful are less likely to describe their calculations in 'generic' and 'general' terms than the more successful. The data also demonstrates, however, that pupils can describe procedures without being aware of the correctness of their answers. They use a similar proportion of 'generic' expressions in easy questions, that they get right, to the proportion in harder questions, that they get wrong, using the same procedures. There is evidence here of 'instrumental' understanding. Teachers may have an indication that pupils know a procedure by their use of expressions of generality but need to be aware that this does not imply that the pupils understand what they are doing.

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