STUDENTS' UNDERSTANDING OF GRAPHICAL ASPECTS OF DERIVATIVE AND OF SOME OF ITS UNDERLYING CONCEPTS

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The purpose of the study, on which this paper is based, was to examine university students' understanding of fundamental calculus concepts, after the concepts concerned had been dealt with in first-year calculus courses. 630 students were subjected to the diagnostic tests (a pre-test and a post-test) pertaining to this study. Fifteen of these students also participated in individual interviews. The analysis of students' written and verbal responses to test items revealed a great deal of detail about the nature and characteristics of students' understanding of key calculus concepts and their symbolic representations (Bezuidenhout, 1998; Bezuidenhout and Olivier, 2000; Bezuidenhout, in press). This paper focuses on students' understanding of graphical aspects of the derivative and of some of its underlying concepts, including the derivative as a rate of change.

The following three kinds of function representations were most commonly used in the diagnostic tests: tabular and graphical representations and the function defined in algebraic symbols. The main reason for the utilization of different representations of functions, was to examine students' understanding of concepts within different modes of representation. Concerning this paper the graph of a non-linear function provides the graphical context for examining students' "graphical" understanding of aspects relating to derivative or rate of change. These aspects include the following: increase/decrease of a function at a point; rate of increase/decrease; increase/decrease at the greatest rate; the average rate of change over a small interval of the independent variable as an approximate value of the derivative at a point; slope of a secant line; slope of a tangent line; increase/decrease of the rate of change.

The interview format served to reveal the nature of some misconceptions of students. One such misconception suggests that "the average rate of change over an interval corresponds to the average of the rates of change at the two endpoints of the interval". Knowledge associated with the arithmetic mean was identified as a key source of this misconception.

References

Bezuidenhout, J. (1998). First-year university students' understanding of rate of change. *International Journal of Mathematical Education in Science and Technology*, 29(3), pp. 389 – 399.

Bezuidenhout, J. & Olivier, A. (2000). Students' conceptions of the integral. *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, 2, pp. 73 – 80.

Bezuidenhout, J. (in press). Limits and continuity : Some conceptions of first-year students. *International Journal of Mathematical Education in Science and Technology.*