MEASURES IN CABRI AS A BRIDGE BETWEEN PERCEPTION AND THEORY

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Abstract

In the context of teaching geometry at secondary school level, we study students' measurement activity in solving open problems in the Cabri environment. Different uses of measures are observed in the protocols' analysis, which points out that the students' approaches are double-sided, namely perceptual and/or theoretical. A tool like measures in Cabri (in the same way as dragging) can foster the passage from perception to theory and back again. The shifts back and forth are productive in the construction of a proof, after the conjecturing phase.

Introduction

At the level of Mathematics Education research, the issue of the use of new technologies has been addressed from different points of view. Studies on how the introduction of new technologies change learning in the classroom and how they can be integrated in the classroom practice have been carried out (e.g. Artigue, 1997; Arzarello et al., 1998b; Laborde, 1998; Healy, 2000; Hoyles & Healy, 1999; Mariotti & Bartolini Bussi, 1998; Sutherland & Balacheff, 1999).

The ongoing research project we are involved in, is concerned with students' cognitive behaviour when using a dynamic geometry environment, Cabri-Géomètre (Baulac et al,1988), in the context of the proving process[1] in geometry (Arzarello et al., 1998a). The classroom experiments involve secondary school students, who are asked to solve open problems (Arsac et al, 1988) in Cabri, with the aim of proving their conjectures.

The problem we study is the transition from conjectures to proofs (see e.g. Boero et al, 1996), as part of a long term activity: students' transitions from the perceptual level to the theoretical one and backwards. As perceptual level, we mean the activities in which the students use perception (e.g. to see if a quadrilateral is a square, only by eye). As theoretical level, we mean activities like producing a conjecture in a conditional form and validating it with a proof. The relationships between perception and theory prove a key element in mathematics, particularly in geometry (see for example Arsac, 1992).

The different tools provided by Cabri (e.g. dragging, constructions, measures) are useful mediators (Mariotti, to appear) in helping students in the transition mentioned

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[1]By proving process we mean the process of exploring, conjecturing and proving in an open problem.

above. In a previous study, we investigated in particular the role of dragging (Arzarello et al., 1998b; Olivero, 1999) within this framework[2].

In this paper we analyse the role of measures in Cabri in the above transition. Our hypothesis is that measures in Cabri constitute a bridge between the perceptual level and the theoretical one and may be a support for managing productively the interaction between the two levels.

The role of measures in Cabri: two transitions.

Our interest in the role of measuring in the proving process comes from the fact that students activity involves the use of measures in different phases of the process and with different purposes. Certainly, the need for measures comes from the perceptual level, when the students have the intuition that, for example, one side equals the double of another one. However, when they read the measures on the screen[3], they are no longer at a purely perceptual level: they are working at a higher level, because they are looking for an answer: *Is my intuition true or false?*

Measures work as a tool which can provide different kinds of answer, because they foster perception but at the same time lead towards the theory:

a. a dichotomic answer of qualitative kind (yes/no, e.g. knowing if two quantities are equal or not, or if one is bigger than another one);
b. a quantitative information (a number, which tells the measure of a quantity with respect to a measurement unit)[4];
c. a relational answer, that links the measures of two or more quantities (e.g. if one side equals the double of another one).

Therefore it is important that measurements are introduced correctly in the school curriculum, in order to avoid obstacles, misunderstandings or conflicts[5] in the learning process. The role of the teacher proves fundamental in relation to the different uses of measures described above, in order to teach the students:

a. to manage the qualitative answer;
b. to use the quantitative information as a number within an interval of uncertainty[6];

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[2]A variety of dragging modalities was identified, each one showing a particular aim to be achieved in the proving process. The different modalities used by students were considered as revealing different cognitive activities.

[3]Similar considerations may concern the use of measures on paper or using other tools.

[4]The quantitative side of the answer linked to the use of measures usually makes students feel safe and certain about results; in particular, weak students usually rely on measures.

[5]In another work (Olivero & Robutti, 2001) we point out some cognitive conflicts connected to the use of measures in Cabri.

[6]An example of good management of the interval of uncertainty is in Olivero & Robutti (submitted).

c. to understand the different status of the numbers obtained by measuring a quantity and the relationships between the quantity and other ones.

The students will then be conscious that measures are useful in the proving process in order to discover, to make conjectures and, finally, to prove, but that they are not sufficient to construct a proof. Measures (and dragging too) are useful when you want *to know* if something is true; however, they are no longer useful if you want *to prove* that something is true: at this point measures need to be substituted by relations of logical consequence. Measures in Cabri (in the same way as dragging) are mediation tools between the perceptual level of students' mathematical activity and the theoretical one.

Students' measurement activity reveals two typical cognitive shifts, which were also revealed by the use of dragging (see Figure 1):

1. the transition from the perceptual level toward the theoretical level;
2. the transition from the theoretical level toward the perceptual level.

If the pupils trust measures and 'believe' measures are absolutely exact, they stay at a perceptual level. While if they use the information provided by the feedback of measures to formulate a conjecture in a conditional form ('if … then') and to see the figure as a generic example (Balacheff, 1999), they pass to the theoretical level (Laborde, 1998).

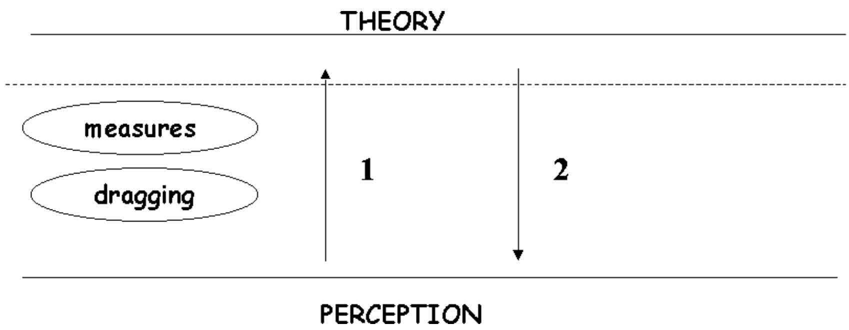


Figure 1

Classroom observations have shown that the modalities of using measurements in the first transition, are for example the followings:

- when the students do not have any precise ideas about the configuration, they explore the situation randomly: they take measurements of some elements of the configuration (“mesures exploratoire”, Vadcard, 1996), in the same way they as use wandering dragging (Arzarello et al., 1998b);
- when they do a guided exploration of the configuration, measurements are used to put in order a set of different cases, in order to explore them, in the same way as guided dragging (Olivero, 1999), or together with it.
- as a means of checking the validity of a perception: students see some features of the figure, but they are not sure of their perception, so they use measures on the

figure in order to validate the perception ("mesures probatoire", Vadcard, 1996), and they remain at a spatio-graphical level (Laborde, 1998).

As far as the second transition, the students use measures for example with these modalities:

- after formulating a conjecture, sometimes measurements are used to check the conjecture within Cabri, in order to refute or to accept it[7].
- after constructing a proof, the students go back to Cabri in order to understand the proof and to get a better explanation: new experiments are made in Cabri and measures are used, normally, on the static figure (see, for example, Olivero & Robutti, 2001).

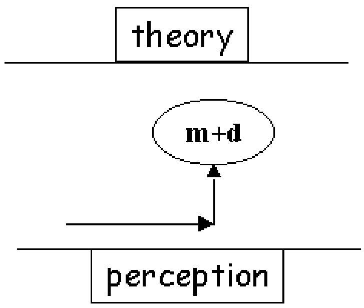
In the following we will present some exploratory examples which illustrate the previous ideas.

The two transitions: some exploratory examples.[8]

All these examples are taken from different Italian Secondary School classrooms (15-17 years old students) which took part in our research project. The classroom sessions consisted of both group work and classroom discussions (Bartolini Bussi, 1998). Two observers took fieldnotes and video-recorded one group for each session and the discussion.

EX1[9]. Students M & N, group work..

12.M: diametrically opposite points (D, E)
13.N: They are on the same line (D, B, E)
14.M (while N is dragging)*: They are on the line, right? OO' and DE are parallel. Put measures. I think it is 1/3, ...2/3, ...the double.*



The students are in the process of formulating a conjecture, so they are moving from perception to theory. At the beginning *(#12-13)* they formulate an observation coming from a perception. Then they use both dragging and measurements to go on with the exploration. The information M sees in the process is a relational answer: the relationship between two quantities is discovered *(#14)*.

[7]This use is very similar to the dragging test modality (Arzarello et al., 1998b), by which students check the exactness of a construction.

[8]We do not present the statements of each problem because they are not important for the analysis we carry out. However, here is an example of the kind of open problems students worked with:
"Let ABCD be a quadrilateral. Consider the bisectors of its internal angles and the intersection points H, K, L, M of pairs of consecutive bisectors.

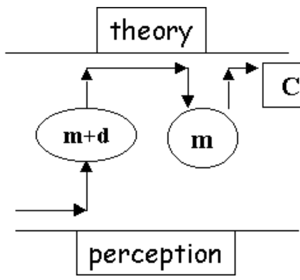
- Drag ABCD, considering different configurations. What happens to the quadrilateral HKLM? What kind of figure does it become?
- Can HKLM become a point? Which hypothesis on ABCD do you need in order to have a point?

Write down your conjectures and prove them."

[9] The figures represent the two transitions described above. m=measures, d=dragging, C=conjecture

EX2. Students R & M, teacher T; group work.

90.T: You need to find a different property from the one you used for the construction.
91.R: Let's try to put measures in a particular case.
92.M: Draw a square.
93.They draw a square and then drag one of the sides.
94.R: One side gets bigger and the other one gets smaller in the same proportion.
95.T: How would you say this in mathematics?
96.M: When one gets bigger and the other one gets smaller, they are inversely proportional.
97.T: Not really. They are inversely proportional when the product of two variables is constant. Is this the case?
98.They do some calculations on the measures of the sides of the quadrilateral in different configurations.
99.R: No, the product is not constant.
100. M: Let's have a look at the sum.
101. R: The sum is constant.
102. M: So the sum of the two opposite sides is equal. The conjecture is: if a quadrilateral is circumscribed to a circle, the sum of the opposite sides is equal.



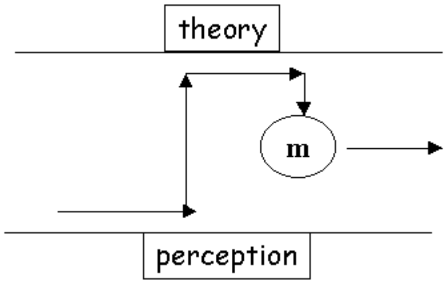
The students do not have any idea in mind. They know they have to find a property but they do not know what kind of property it can be. So they decide to start from a particular case. They put measures as a means to get ideas from. They start dragging *(#93)*. While dragging they look at how measures change on the sides of the quadrilateral and compare them. At first a relational information is seen (*#94)*. The teacher stimulates the students to move towards the theory *(#95)*. Some work at a theoretical level is done *(#96-97)*. In *#98* they go back again to the Cabri measurements and do some calculations. Relationships between the measurements of the sides are observed and then transformed in a conjecture, at a theoretical level.

EX3. Students D & G, group work.

25.*D: They do not coincide, because…look* (she points at the 4 axes)
26.G: And this is a square, I bet it!
27.D: Wait, what does the perpendicular bisector cut…It cuts the side in a half, doesn't it? …so this should be the midpoint…
28.*G: Ah…but I want to know if it is a square*
29.D: Use measurements!
30.G takes measurements of all the sides.
31.D: Maybe not…look, they are all different!

The starting point is the perceptual level: D observes a characteristic of the figure (*#25)*. G goes on with another perception (*#26)*. At this point D moves to a theoretical level, recollecting a property of the perpendicular bisector *(#27).* G goes back to the perceptive level and wants to check her perception of the figure being a square *(#28)*.

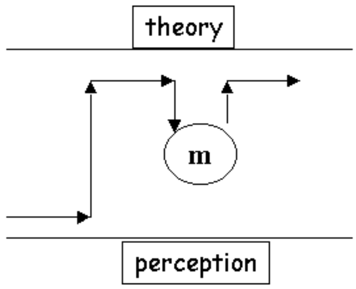
The means D chooses to validate the perception is the use of measurements. The conclusion D gathers from the measures is a dichotomic qualitative information: the sides are all different *(#31)*. At this point they are at an intermediate level between perception and theory.



EX4. Students L & S; group work.

110. L: UR is parallel to AB because you can see it by eye. But let's check it.
111. L marks a point on UR and a point on AB, draws the segment between these two points, marks the internal alternate angles, takes measurements.
112. S: they are equal!
113. L: yes, they are parallel.

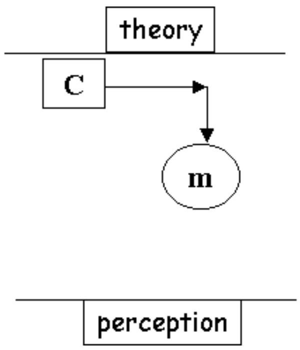
This episode starts with a perception *(#110)*; L explicitly says he is observing the figure *by eye*. However he wants to check this observation. There is a jump towards the theory when the students recollect a property of parallel lines *(#111)*. Then there is a move back to the use of measurements. At first a qualitative information is gathered *(#112)*. Then another move towards the theory is made *(#113)*.



EX5. Teacher T, student F, classroom discussion.

85.T: So your conjecture is…
86.F: if a quadrilateral can be circumscribed to a circle the bisectors all meet at a same point.
87.T: when can a quadrilateral be circumscribed to a circle?
88.F: When the opposite sides…when the sum of the opposite sides is equal.
89.F calculates the sum of the opposite sides using the measurements on the Cabri figure.

The students have a conjecture *(#85-86)*, so they are at a theoretical level. The teacher provokes them to explain the conjecture by characterising the circumscribed quadrilaterals *(#87)*. In order to do that, after mentioning the defining property for circumscribed quadrilaterals *(#88)*, they use measurements *(#89)* in order to show that this property is true. The function of measurements is to provide certainty.



Conclusions and issues for further research

From the cognitive point of view, the previous analysis shows that in the proving process there is a richness of back and forth shifts between the perceptual level and the theoretical one. This productive interaction is supported by the use of measures in

Cabri, both as a tool on its own and as a tool in addition to others, e.g. dragging. The cognitive analysis

A cognitive perspective on the use of technology seems to be particularly important if one wants to deeply understand how new technologies affect the teaching and learning process. Knowing in which way a cognitive behaviour changes when working with a software, for example, might be essential for developing suitable classroom activities in which the potential of technology is exploited together with the mathematics involved. The research results concerning dragging and measures we presented might be successfully exploited in the teaching practice, for example by showing students the different possibilities they have and 'teaching' them how to use these tools in a productive way.

And, last but not least, measures (not only in Cabri, but in general) should constitute a key topic in the school curricula as far as their epistemological role is concerned, as it is pointed out by many ongoing curricolar projects that are taking place in US (AAVV, 1998), Belgium, New Zealand, and, recently, Italy.

Some possible issues for further research may be: developing a deeper second order analysis (Arzarello & Bartolini Bussi, 1998) concerning the cognitive interpretation of measurements activities, in relation to the whole proving process; analysing in details the role of the teacher in managing the introduction of technologies in the classroom, with particular respect to the different tools they embed; studying this approach from the embodied cognition perspective (Arzarello, 2000; Nunez, 2000); analysing the role of other mediators, as for example language (Arzarello, 2000; Radford, 1999), in relation to the Cabri tools.

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