On "How to Make Our Ideas Clear":
A Pragmaticist Critique of Explication in the Mathematics Classroom

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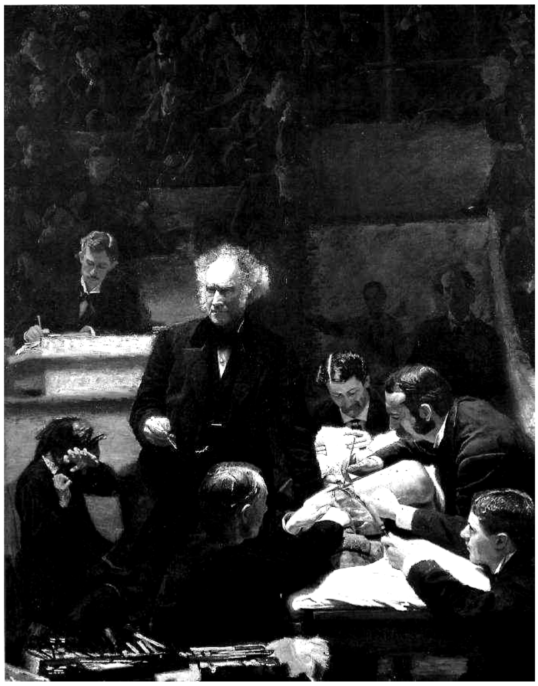
Instruction in the mathematics classroom often involves a paradigmatic explication of material. Children are asked to follow along the reasoning of an "experienced" adult who actually acts on the part of the children. Knowledge and understanding of the material by the children are measured by the ability of the children to catch up with the reasoning of the teacher. In this paper I intend to argue that paradigmatic instruction is based in false interpretation of a pragmatic logic as the translation of ideas into action. I will support my argument with discursive analysis of a short interactional sequence between teacher and children in a sixth-grade class in a primary school in Greece.

At Easter last year I was busy organizing a conference for primary school teachers. A local school—the only one with an amphitheatre large enough to accommodate all the teachers—offered to host the event. While the principal was giving me a tour, he happened to describe another in-service session on mathematics education that had taken place recently in that same amphitheatre. Participating teachers, he told me, observed a tutor teaching a class, which, for the purposes of the session, had moved to the amphitheatre.[1]

Later on, I stood alone in the amphitheatre, trying to imagine the scene: the tutor teaching the class, while the teachers looked on. My musings brought to mind two of Thomas Eakins' most famous paintings, *The Gross Clinic* and *The Agnew Clinic*. In the painting reproduced here the master surgeon and his assistants operate on a patient. While several students observe the operation, the patient's mother sits in the corner, writhing in agony. The master surgeon is depicted in a state of apparent withdrawal from the act of operating, still holding his surgical blade, his facial expression revealing intense thinking.



Thomas Eakins, *The Gross Clinic*, 1875-76

Ostensibly, paradigmatic instruction follows a pragmatic logic since the master-teacher must translate ideas into action. Even if the "experts" might be said, in this way, to bridge

[1]Readers should not perceive this description as prototypical of current in-service training programs in Greece, even though it was chacteristic in the past. Nevertheless, the mentality it reflects still prevails in beliefs on learning and teaching, albeit in more subtle ways.

the gap between theory and practice, the fact remains that “novices” are taught to observe the center of activity from a distance. In the case of the in-service session, paradigmatic teaching contributed not only to the formation of methods for teaching the content of mathematics, but also, even if it was not explicitly stated, to the formation of the essence of belief concerning how the learning and teaching of mathematics should be approached in the classroom.

In this paper I will analyze an episode from a mathematics lesson in a sixth-grade class in Volos, Greece. The purpose of my analysis is to bring attention to certain linguistic phenomena that characterize social encounters and, therefore, are partly responsible for framing the interactional space of the encounter. Elaborating on Peirce’s *pragmatic maxim*, which locates the utmost clearness of ideas in “all the conceivable practical bearings we conceive the object of our conception to have” (Peirce, 1991: 169), I differentiate a ‘pragmaticist’ framework for mathematics education from an instrumental one, which presents itself as ‘hands-on’.

From paradigms to the production of a “Practionary”

> “Well, that’s what we do first. We open the little trunk where we keep the materials for the circle, we empty it, and what do we have? We have a center, a diameter, a curve … Everything I might need I take out. Same thing as if we wanted to install a faucet. I go and take the toolbox…”
>
> [6th grade teacher addressing his class] (Volos, Nov 2000)

The phrasing of Peirce’s *pragmatic maxim* can easily mislead. In his famous essay on “How to Make Our Ideas Clear” (1878), Peirce identifies the first stage in the clarity of ideas as familiarity with the common uses or nature of an idea. Standard dictionaries provide us with linguistic definitions, which for Peirce comprise the second degree of clarity. It is tempting to consider, then, the possibility of composing a kind of super-dictionary of praxis—a *practionary*—that would furnish the reader with descriptions of all the conceivable practical effects a thing could have in experience, which is Peirce’s third stage in the clarity of ideas. The possessor of such a “practionary” could acquire a clearer understanding of an idea simply by emptying its “little trunk”—to use the metaphor of the teacher, whose pedagogical style I will describe below. For Peirce, though, knowledge of the practical effects of an idea can only be achieved through personal hands-on experience with an idea or a thing (Parker, 1998; Peirce, 1991).

Peirce’s discussion of the three grades of clearness of ideas and the notion of *practionary* are all too relevant for mathematics education. A number of researchers have stressed, for instance, the fallibility of the assumption that concepts are acquired mainly through their definitions (Sfard, 2000; Vinner, 1991). A definition is a configuration of signs that assists in establishing the meanings of other signs. The definition of an idea is an exercise of power over the idea: its utterance conveys an appropriation of complete knowledge of the idea. Even within the deductive theory of mathematics education, signs are depleted of meaning, if considered without reference to their anthropological milieu (Cobb, 1990). Characteristic of the irreducibity of mathematical concepts to strings of words are the notions of “concept image” (Tall and Vinner, 1981) and “figural concept” (Fischbein, 1993).

The processes of symbol manipulation present in traditional school mathematics is often linked with a mechanistic instruction of “basic skills” and ideas (Romberg and Kaput, 1999). Even though research suggests otherwise (Hewitt, 1997; Voigt, 1995), teaching is still considered, or at least approached, as an act of explication. In his critique of the “explicative order,” Jacques Rancière describes explication as a series of reasonings used in order to explain a series of reasonings that already exist within the material being taught. If a student cannot understand the first series of reasonings, why should we assume that he or she will understand the teacher’s reasonings. And, if the teacher’s reasonings themselves need to be explained, we can see how, in Rancière’s words, “the logic of explication calls for the principle of a regression ad infinitum” (1991: 4).

Over-reliance on the act of explication inevitably places emphasis on what Jackobson defined as the poetic function of language, a focus “on the message for its own sake” (1960: 356). Indeed, what speakers say is always evaluated according to aesthetic rules, in other words, for its efficacy, in “moving” an audience. The master-teacher of the in-service session described above had to employ a series of reasonings in order to explicate to the students the material that he himself had chosen. The outcome of the series of series of reasonings, including those of the attending teachers, judged by the standards of the explicator, created the illusion of an instance of successful mathematics teaching. The ability of the master-teacher to “move” the audience of teachers made this session a successful instance of paradigmatic teaching—judging from the story that was told to me by the school’s principal.

I suggest that teaching, conceived as an act of explication, represents an attempt to produce a *practionary*, a false substitute for Peirce’s third step in the clarity of ideas. Reaching or, to be more precise, approaching this third stage is, thus, trivialized and reduced to cramming practices or a series of reasoning, stored in a “little trunk.” The appropriation of knowledge by the students is measured by their ability to follow and/or reproduce these practices, even if this requires, in many cases, an act of collusion by all the engaged parties (McDermott and Tylbor, 1995).

Explication as action? Analyzing instances of explication

I now turn to the description and analysis of an episode from a mathematics session from a sixth-grade class in Volos, Greece. The educational system in Greece is nationally standardized. Mathematics syllabi are prescriptive in the sense that they allocate a specific number of hours to the teaching of each area. Instruction in the classroom is restricted to the textbook provided by the state, with teachers usually teaching “from the front.”

With its 140,000 inhabitants, Volos is the fifth largest city in Greece and the third largest port. The primary school that I visited is in a lower middle-class area far from the center of the city. My almost daily visits to the school began in October, 2000, though I was familiar with the school from a project I had carried out there two years before. It was then that I first met with the children of this year's sixth-grade class. Their teacher had changed, though. I was not surprised to find a male teacher; reflecting gender hierarchies and stereotypes, the few men among primary school teachers tend to be overwhelmingly represented in the higher grades of primary school. My presence in the classroom was limited to observing and audio-taping the mathematics sessions. I was conscious of the fact, though, that even as a bystander I was co-constructing the course of events (Goffman, 1981).

Linguistic anthropologists view language as a set of practices "which play an essential role in mediating the ideational and material aspects of human existence, and, hence, in bringing about particular ways of being-in-the-world" (Duranti, 1997: 4-5). Delineating types of organization that characterize the interactional space of a mathematics classroom will highlight how particular ways of "being-in-the-world" are formed, sustained and altered.

The extract that follows comes from a session on geometry. The class is discussing the concept of perimeter, having already described the characteristic features of a square. Perimeter is a concept formally introduced in the fourth-grade, so the current lesson served as an opportunity for the teacher to freshen up children's memory of the concept. In the transcript that follows, I interpolate brief explanatory comments and theoretical elaborations. (A description of the transcription symbols are explained below.)

(The class has already discussed equal angles and equal, parallel, and perpendicular sides in a square. The teacher had drawn a square EZHK on the board.)

1 Teacher: **NOW**. (3.0) We are looking for (2.0) the perimeter of the square EZHK . WHAT INFORMATION do we have for this unknown quantity you see here?
2 Child 1: Sir!
3 Teacher: Yes Elena.
4 Child 1: We know the length of all sides.
5 Teacher: Well **LENGTH** (0.5) TOTAL LENGTH (0.5) of all sides. Splendid!
[
6 Child 2: Sir it also has equal sides.

The teacher takes the turn to speak as soon as he receives the first response (line 4) from the class. He restates what Child 1 has just said, adding the phrase "total length," an expression that frames the concept of perimeter (line 5). At the same time, he speaks louder in an assertive tone, underlining the significance of what he had just mentioned for the wanted answer. As soon as Child 2 chips in another piece of information (line 6), the teacher interrupts the process once again to restate the task and suggest how to proceed (line 7). Child 2's comment concerning the equality of sides is ignored.

7 Teacher: = What we have to do is take this information, what we know (0.5) and apply it here in this certain (0.5) square.
[ ]
8 Child 3: Sir! Sir!

9 Teacher: = TOTAL length (0.5) of **ALL** (0.5) sides. What element do you think I get from this information that I can match up here?

The teacher refuses to give to Child 3 permission to speak (line 8). He repeats and emphasizes the words "total" and "all" using them as cues that would evoke further information about the concept of perimeter. Child 3's hesitant and faintly-uttered responses in line 10 and 12 below indicate the bafflement of the class as to what the aim of the process is.

10 Child 3: °°Perimeter? Perimeter? ° That the::
11 Teacher: It says (0.5) **length** (0.5) **of** (0.5) **all** (0.5) **sides**.
12 Child 3: We know how many: °are all the sides:: *((almost like asking))*
13 Teacher: BRAVO! How many are all the sides. = Where are they? Here. (1.0) We write them down again (12.0) *((teacher writes on the board))*
I made the BEST OUT (0.5) of the element that **says** (0.5) SIDES. Did~I~know? = I~did~know. = I~took~them~and (1.0) I~placed~them~there. = **WHAT~ELSE~DOES~THIS~INFORMATION~TELL~US?** = It~says~sides~only? = It~says~perimeter (1.0) **TOTAL** (1.0) **length**. **= A::::** that's right! Addition.
[ ] [ ]
14 Child 4: Total (1.0). Total (0.5) with addition.

Sensing the hesitance of the class, the teacher interjects another explication in line 11. This time he pauses briefly between words, implying that "this is it!" Child 3 strives to follow the teacher's line of argument. Even though her response is expressed in almost a questioning mode (line 12), the teacher congratulates her and proceeds with yet another explication of the argument so far (line 13). Next he asks the children to think of other characteristics of the square that would be relevant in finding its perimeter. Child 4 (line 14) interprets correctly the cues given by the teacher and takes the process a step further. Notice that in line 13 there are no "transition-relevant points," i.e. moments when a change of speaker may take place (Sacks, Schegloff, and Jefferson, 1974). Besides speaking too fast, the teacher leaves no interval between the end of a prior unit of speech and the next piece of talk.

15 Child 5: Sir we have done these before.
16 Teacher: Everything's fine?
17 Child: Yes.
16 Teacher: **What else** did we discover today or better did we mention again? = WE KNEW IT BUT WE SEE IT AGAIN (0.5) FOR THE SIDES:
17 Child 6: That these *((pointing towards the board))* are:: opposite to one another.
18 Child 4: That these:: are:: parallel and do not meet.
[
19 Teacher: They are parallel. Yes. **This element** (3.0) can it be used? (6.0) At~the~point~where~we~are~now? = In order to see let's say =
20 Child 4: The parallels?
21 Teacher: **YES**. That they are parallel (2.0) does it interest us in finding the total length?
22 Child 6: Eh sir::
23 Child 4: Ho-? How much are all the parallels? *((she wants to say how long))*
24 Child 7: **SIR the parallels are equal!** *((the child does not express himself adequately))*
25 Teacher: **A::::!** *We have another element*. = Look here. = That sides:=

26 Child 6: Are equal.

In line 16, the teacher seeks reassurance that the class comprehends. More information is needed in order to calculate the perimeter, though. Children 4 and 6 suggest that the missing link is the fact that the sides of a square are parallel to one another (lines 17 and 18). The teacher interrupts (line 19) making an implicit statement that the concept of "parallels" is not the information missing. Child 4's question in line 20, though, indicates that the class is not following the process consciously. Children appear to be choosing information at random from the "little trunk." The teacher attempts again to lead the argument away from the information "parallel sides" (line 21), and Child 6 with the help of Child 7 seem to be making the required connection (line 23 and 24).

27 Teacher: **They are** (2.0) equal. Therefore
[
28 Child 4: We can multiply.
29 Teacher: **BRAVO!** We move to the property that says that multiplication~is~a~**SHORT:?** (1.5)
30 Class: °Addition.
[
31 Teacher: Addition. When~the~terms~**ARE:?** (3.5)
32 Class: ° Same.
33 Teacher: **THE SAME.** That's righ-. = Instead of saying 4~and~4~and~4~and~4~I~say 4 times 4. = Or 3~and~3~and~3~and~3 (0.5) I~say 4 times (1.0) 3. Is that so? If here then (1.0) instead of saying side EZHK I put a general name (2.0) *α ((he writes a on all sides of the square))* for the sides (0.5) of the square α ZH α and KH α (0.5) and this (1.0) and the other (1.0) = *((pointing to the figure on the board))*
34 Child 4: **4~TIMES α** (4.0)
35 Teacher: With a 4 times α then I define:
36 Child 4: The::: the:: perimeter!
37 Teacher: **BRAVO**! I define the magnitude that is called perimeter (1.5) of a square.

Concluding remarks

Participation in any social scene requires a minimum consensus on what is getting done (Gumperz, 1992; McDermott and Tylbor, 1995). On a micro-perspective level, teacher and children appear to be discussing already familiar concepts in an attempt to deepen or widen their understanding of the concepts (see lines 15 and 16). Despite this observation though, the fact that children's responses, like those in lines 14 and 18, are overlooked, indicates that the teacher ranks the correct use of information and the pious following of a process higher than understanding the concepts. On a macro-perspective level, children are disciplined into following the series of reasonings used by the teacher in explicating the process of finding the perimeter of a square with side α.

As McDermott and Tylbor suggest, "it is possible to live lies without having to tell them" (1995: 281). Judging by Child 5's response in line 36, we remain puzzled as to whether the teacher actually moved through this series of reasoning alone or not. A fair amount of collusion was involved in this process of explication, both by the teacher and the children, in the sense that a particular state of affairs was sought that could be accepted as an indication of understanding and, therefore, of closure and successful or satisfactory teaching (see lines 5, 12, 13, 16-17, 29-30, 31-32, 37).

The confusion of the class concerning the task at hand, along with their hesitant decision to follow the teacher's explication of the task, may suggest a form of resistance to the way of "being-in-the-mathematics-classroom" proposed by the teacher. One cannot force reasoning to follow a specific order of steps. What the teacher does is lay out his own interpretation of what "being-in-the-mathematics-classroom" might mean. Knowledge and understanding are then measured by the degree to which the students collude in the teacher's mode of explication. Analyzing modes of explication can show us the subtle ways that states of "being-in-the-mathematics-classroom" are being sustained and reproduced. A discursive analysis of conversational exchanges in the classroom may be one tool for understanding this process. Further research would entail connecting these to a broader fabric of social structures and relationships.

Transcription features

Boldface indicates emphasis signaled by changes in pitch and/or amplitude.
A *left bracket*, connecting talk on separate lines, marks points at which one speaker's talk overlap the talk of another.
A *right bracket* marks the place where the overlap ends.
Colons indicate that the sound just before the colon has been noticeably lengthened.
A *dash* marks a sudden breaking-off of a particular sound.
A *degree sign* (^{o}) indicates that the talk following it is spoken with noticeably lower volume.
An *equals sign* is used to indicate that there is no interval between the end of a prior unit of speech and the next piece of talk.
Capitals indicate increased volume.
Numbers in parentheses mark silence in seconds.
Text in *italics* between *double parentheses* mark the author's comments.

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