A CASE STUDY OF FOUR GRADE 7 GEOMETRY CLASSES

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Traditionally, geometry at school starts on a formal level, largely ignoring prerequisite skills needed for formal spatial reasoning. This tradition may lead to ineffective teaching and learning. The Van Hiele theory postulates learner progression through levels of geometry thinking, from a Gestalt-like visual level through increasing sophisticated levels of description, analysis, abstraction, and proof. Progression does not depend on biological maturation or development only, but also on appropriate teaching/learning experiences. A Van Hiele-based geometry learning and teaching program was designed and implemented in four Grade 7 classes (133 learners) at two South African schools. The study investigated some factors and conditions influencing the effective learning and teaching of spatial concepts, processes and skills in different contexts. Results suggest that the implementation of a Van Hiele-based learning and teaching program had a positive effect on the learners' levels of geometric thought. Learners who completed the program reasoned on a higher level, gave more complete answers, demonstrated less confusion, and generally exhibited higher order thinking skills than their counterparts who did not take part in the program. For efficacy of this program, a prerequisite is that the teacher should consistently teach from a learner-centered approach.

INTRODUCTION

South Africa has a history of inadequate geometry instruction in primary schools (Van Niekerk, 1997:270). Taylor and Vinjevold (1999:143) state that lessons are generally characterized by a lack of structure, and the absence of activities which promote higher order skills such as investigation and understanding relationships. At present in most schools geometry teaching and learning in the school curriculum starts on a formal level, ignoring prerequisite skills needed for formal reasoning. The effect of ignoring the sequential and hierarchical nature of the learning of geometry causes ineffective teaching and learning because learners are expected to perform without the necessary prior knowledge and/or prerequisite skills (Clements & Battista, 1992:421). It is necessary that the foundation of formal geometry learning (that starts in grade 7) be laid according to a widely respected and acceptable theory, for example the Van Hiele theory.

THE VAN HIELE THEORY

The Van Hiele theory postulates that learners progress through progressive levels of geometrical thought from a Gestalt-like visual level through increasing sophisticated levels of description, analysis, abstraction, and proof (Van Hiele, 1986:39). At the *first level* (Recognition) learners identify and operate on shapes and other geometric configurations according to their appearance alone (Mason, 1997:39). On the *second level* (Descriptive / Analytic) learners are able to recognize and explicitly characterize shapes by their properties (Van Hiele, 1986:40), but can not recognize relationships between classes of figures (Battista, 1994:89) or even redundancies (repetitions) (Spear, 1993:393). Learners at *level three* (Abstract / Relational) can form abstract meaningful definitions (Mason, 1997:39), distinguish between necessary and sufficient sets of conditions for a concept, classify figures hierarchically (by ordering their properties), give informal arguments to justify their classification (Battista, 1994:89), and understand and sometimes even provide logical arguments in the geometric domain (Clements & Battista, 1992:427). At l*evel four* (Formal Deduction) learners are able to establish theorems within an axiomatic system. They recognize the differences among undefined terms, definitions, axioms, and theorems and are capable of constructing original proofs (Clements & Battista, 1992:428). At the *fifth level* (Rigor / Metamathematical) learners reason formally about mathematical systems, understand the formal aspects of deduction (Presmeg, 1991:9), establish and compare mathematical systems (Mason, 1997:40), and reason by formally manipulating geometric statements such as axioms, definitions, and theorems.

Gutiérrez, Jaime and Fortuny (1991:237-238) theorized that the Van Hiele levels are not discrete and presented an alternative method to evaluate and identify those learners who are in transition between levels. Gutiérrez *et al*. (1991:238-241) quantify the acquisition of a level by representing it with a segment from 0 to 100 thus creating a scale of degrees of acquisition. A division is also made to divide this continuous process into five stages (no-, low, intermediate, high and complete) acquisition characterized by the qualitatively different ways in which the learners reason.

Progression from one level to the next does not depend only on biological maturation or development (Piaget,1970), but also on appropriate teaching/learning experiences (Koehler & Grouws, 1992:123). The effect of appropriate teaching/learning experiences (in accordance with the Van Hiele theory) on South African grade 7 learners in varied context will be investigated. Attention will be given to the (possible) change in geometric thought levels as well as specific details emanating from the implementation of the program. The impact of second language instruction and it's inevitable influence was noted but will not be discussed in this paper but in a later paper.

METHOD

SUBJECTS

The study population consisted of Grade 7 learners (n=221) from a large town in the North West Province of South Africa. A non-randomized experimental-control group design was used with 133 learners in the experimental group and 88 learners in the control group. The experimental group was further divided into two naturally occurring classes, C1 and C3, and similarly the control group consisted of C2 and C4 (naturally occurring classes). Class 1 (C1) and Class 2 (C2) were both instructed in their second language, while class 3 (C3) and class 4 (C4) received instruction in their first language.

PROCEDURE

Before the beginning of the program the two teachers in classes C1 and C3 were trained for one month in the Van Hiele theory as well as the activities developed by the researchers. The activities were implemented after completion of the training and the progression through these activities was continuously videotaped. Despite the initial training and re-training C1's teacher continued with a teacher-centered approach, as she believed that a program such as this would take up too much time, which could result in her not completing the syllabus. The learners in C1 and C3 were randomly arranged into sub-groups. A sub-group was randomly selected to be the group on which the videotaping would focus. The other groups in each class were still included in the videotaping for comparison when analyzing the progress. The learners remained in the same groups for the duration of the study. The mathematics teacher taught the classes while they were being taped for analyses and transcription afterwards. The same sub-groups (school C1: 12; school C3: 13) that were focussed on in the video taping were also used to complete the Van Hiele post-test.

At the same time that the C1 and C3 groups were progressing through the designed program (as developed by the researcher), the C2 and C4 groups continued with their spiral syllabus system where the teacher and the textbook formed the main sources of information with little or no learner involvement in the classroom activities. From the C2 and C4 group, a randomized sample was taken to complete the Van Hiele post-test (from C2: 11 and C4: 13 learners were selected).

INSTRUMENTATION

Post-testing took place after conclusion of (both) programs. The post-test was compiled using selected items of The Mayberry Test (Lewin and Pegg Version) and items from a test developed by the unit for Research in Mathematics Education of the University of Stellenbosch (RUMEUS -1984). The final product (post-test) included 21 items (with some sub-items), on the concept of parallel lines and shapes such as the square, right angle and isosceles triangle. The answers to the items were quantified according to the acquisition scales of Gutiérrez *et al.* (1991:237-241). No pre-testing could be done as the C2 and C4 groups started Geometry teaching already at the beginning of the year (January) while the (experiment and the) C1 and C3 groups started only in May. No interviews were conducted with the learners thus the analysis can not attain the fine grain of qualitative depth that interviews would have provided.

RESULTS AND DISCUSSION

Degrees of acquisition

Investigation through the post-test into the differences of degrees of acquisition in geometric thought provided encouraging results (see figure 1).

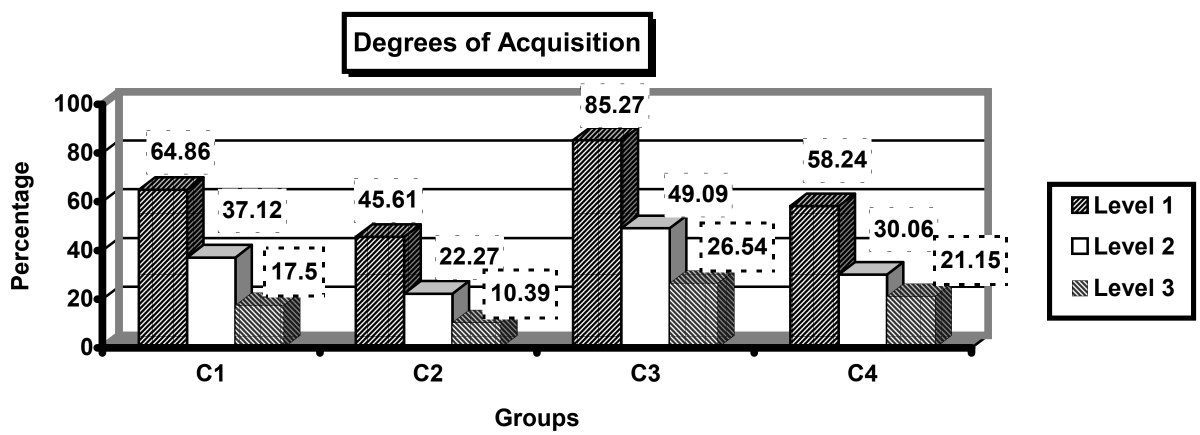


FIGURE 1: DEGREES OF ACQUISITION

The experimental groups consistently achieved higher degrees of acquisition than the control groups, leading to the conclusion that the program did have a positive effect on the acquisition of high(er) degrees of geometric thought.

Class 3 achieved the highest degrees of acquisition in all 3 levels considered. Between C1 and C3 clear differences exist. Possible reasons can include the complex influence of the difference of mother tongue and medium of instruction. A second and maybe more important reason for the difference is the teaching approach followed, with C1's teacher following a content-based teacher-centered approach and C3's teacher following a problem orientated learner-centered teaching approach. C1's teacher persisted in the teacher-centered approach in spite of the training (and re-training) by for example denying learners the opportunity to sort (for example triangles) for themselves and so discover properties. This teacher sorted the triangles herself on the board (with little to no learner participation) and later even provided the properties in the way she wanted them "back" in the test/exam.

- ***Right angles and triangles***

In testing the general acquisition of concepts of triangles, learners were asked to decide if certain figures were triangles and to explain their answers (see table 1). While doing analysis of relevant data concerning triangles, confusion was noticed between right angles and a right-angled triangle in some groups (see table 1). It is not clear whether the incorrect answers concerning a right angle and/or a right-angled triangle were given because of confusion between the concepts (due to second language instruction) or just because of carelessness on the part of the learners. To shed light on this uncertainty more in depth investigation would be necessary.

TABLE 1: IDENTIFICATION AND DRAWINGS OF TRIANGLES AND RIGHT ANGLES

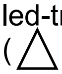
		Groups (with number of learners in each group)			
		C 1 (12)	C 2 (11)	C 3 (11)	C 4 (13)
General identification of triangles					
Answers	Correct	6	3 1[a]	10	8 4[a]
	Incorrect	6	7	1	1
(Reasons:)	3 angles	4	1	7	7
	3 sides	3	1	9	2
	More specific answer	3	1	3	
Identification of right angle and drawings of a right-angled triangle					
Correct identification of right ∠ and correct drawing of right-angled ∇		3	3	9	8
Correct identification of right ∠ but incorrect drawing of right-angled ∇		4	4	1	3
Incorrect identification of a right ∠ and correct drawing of right-angled ∇		1	1	1	1
Incorrect identification of a right ∠ and incorrect drawing of right-angled ∇		4	3	0	1

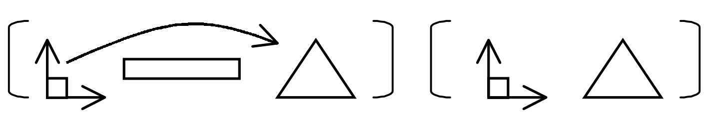
[a] These answers were classified as being correct, but unconvincing or incorrect motivation was given.

Class 1

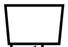
Learners in C1 who decided that the shapes were not triangles rejected them because the sides were not equal (as for an equilateral triangle). Learners pertinently stated "no, because triangle all side are equal" and "no, triangles it has two equal size at the sides and the bottom one is shorter than two equal size in the sides[2]". Most of the learners who decided that the figures were triangles gave more than one answer that usually consisted of a reference to the angles and sides. In total four learners correctly drew a right-angled triangle. The eight learners who drew the right angled-triangle incorrectly all drew it as a right angle without a third side (). Seven learners in total could correctly identify the right-angle but only three could also correctly draw a right-angled triangle.

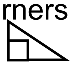
Class 2

Learners who decided that the figures were not triangles all motivated their answers by referring to the shapes' visual gestalt as the following answers[3] demonstrate: "no it is not triangls because is not the same" and " no Because they don't look like the triangles". Responses such as these demonstrate classical level 1 responses. Only one learner tried to give a more specific answer, but even this learner's thoughts seem scattered: "Yes Because they are all the same but the shape is very different but is all the same way but the shape are not the same[3]". Of the four learners who incorrectly drew a right angled-triangle, one learner tried to draw the triangle but only managed to draw an acute-angled triangle (). Two learners' confusion surrounding these concepts was clear when looking at their drawings:



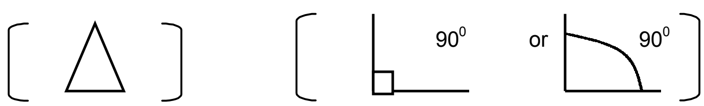
[2] These answers are reported in verbatim from the (written) Van Hiele post-test.

It is noticeable that the learners who incorrectly identified right angles even drew more “strange” figures () that resembled a rectangle or a trapezium, instead of a right-angled triangle.
It is disappointing to notice that C1 faired very poorly in correctly drawing a right-angled triangle. Their statistics are as low as C2’s and even worse in correctly identifying a right angle and drawing a right-angled triangle. A possible reason for C2’s and C1’s poor performance could be the influence of second language instruction – an issue for further investigation.

Class 3
Most learners gave a combination of answers[3] that included references to the number of sides and angles needed to be classified as a triangle, for example: “Yes, it has three sides, three angles” and “Yes, Because they all have three sides and they are have three angles they just have different names and sizes.” Three learners gave some additional information even when not required to for example: “Yes, There are 3 corners, They don’t have to be equal as long Their are 3 sides and 2 acute angles”[3]. An overwhelming number (82%) of learners correctly identified a right angle and correctly drew a right-angled triangle. Most of these learners’ drawings also appeared to be “classical text-book” right-angled triangles () while one learner drew the triangle in an “unusual” position ().
Some of the learners also exhibited more detailed drawings by placing a right angle sign in the relevant angle ().

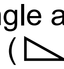
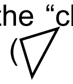
Class 4
Learners judged shapes by appearance as this learner’s answer[3] demonstrates: “ … all 4 looks like triangles…”- typical level 1 reasoning as defined by various authors (Mason, 1997:39; Flores, 1993:152; Spear, 1993:393; Presmeg, 1991:9). Motivation for defining a triangle was varied, ranging from answers[3] referring to angle shapes (“it has 3 sharp points”) to answers referring to sides (“their sides al have angles….they are sharp”). A learner that indicated that the shapes are not triangles motivated his answer by writing: ”the sides are not the same length”. It seems to be a common phenomenon that learners perceive a triangle to have equal sides, as some learners in all the groups rejected isosceles and scalene triangles on the grounds that their sides were not all equal.

All four learners who drew the right-angled triangle incorrectly, drew it as an acute angled triangle (). It stands to reason that these learners may have focussed on the “triangle” part of the question without giving attention to the specification (right-angled). Confusion surrounding right angles in this group seems to be deeper than first discovered as the following drawing shows when learners were asked to draw a rectangle:



In summary it can be concluded that C3’s answers proved to be more correct and complete with a total of 19 reasons (from 11 learners) explaining why the figures were triangles (see table 1), compared with C4’s 9 reasons (from 13 learners), C1’s 10 answers (from 12 learners) and C2’s 3 answers (from 11 learners). A synopsis of the data reveals that C3 emerged as to be the top achiever once again with 81,8% of the learners being able to correctly identify a right angle and drawing a right-angled triangle (see table 1).

- ***Spatial Orientation***

Some questions in the Van Hiele post-test require learners to identify specific concepts in a variety of orientations. These concepts (such as a right-angled triangle and isosceles triangle) appear with other concepts in the “classical or conventional textbook form” () as well as in an “unconventional or turned form” (). When analyzing the answers to these two questions (see table 2) it was believed

[3] These answers are reported in verbatim as transcribed from the videos recorded during the class activities.

and confirmed that recognizing one kind of triangle (right-angled triangle) in a “rotated form” does not guarantee recognition of a different kind of triangle (isosceles triangle) in a “rotated form” (as illustrated in the data shown in table 2).

TABLE 2 IDENTIFICATION OF A VARIETY OF SHAPES IN DIFFERENT SPATIAL ORIENTATIONS

	C1 (12)	C2 (11)	C3 (11)	C4 (13)
Identify right-angled ∇ only in “traditional form”	7 / 1[a]	1 / 5[a]	0	3
Do not identify right-angled ∇ in “traditional form”	1	3	1	3
Identify right-angled ∇ only in “rotated form”	0	0 / 2[a]	1	1 / 1[a]
Identify right-angled ∇ in both forms	**3**	**0**	**9**	**1** / **3[a]**
Identify isosceles ∇ only in “traditional form”	1 / 2[b]	0 / 4[a]	0 / 1[a]	0 / 1[b]
Do not identify isosceles ∇ in “traditional form”	2	1	2	5
Identify isosceles ∇ only in “rotated form”	0	0	0	0
Identify isosceles ∇ in both forms	**7**	**3** / **2[a]** / **1[b]**	**6** / **1[a]** / **1[b]**	**3** / **3[a]** / **1[b]**

[a] Learners identified figure in specified manner but also identified incorrect shapes with the correct answer.
[b] Learners correctly identified figure in specified manner, but the answer is incomplete.

Of the four categories for each figure, the last category that entailed identification of figures in both the “traditional” and “rotated” form, is considered the highest category of identification. C3 consistently achieved the highest number of correct answers (when comparing the number of learners in this category with the number of learners in the group) in both of these categories. In the light of the results of C3’s learners’ recognition of the two kind of triangles (81,8% and 72,7% respectively) a conjecture was formed that if a learner could recognize one kind of triangle (right-angled) in both forms, he/she could most probably recognize any other kind of triangle (e.g. isosceles in both forms. This conjecture was not supported in the data from C4. Only 30,8% of the learners could recognize a right-angled triangle in the “traditional” and “turned” form, but this percentage reached 53,8% in recognizing an isosceles triangle in both forms. It is noticeable that C1, C2 and C4 found recognizing the right-angled triangle problematic, as it is here that they scored the lowest marks of all the figures. It becomes evident that C1 fared as well or as poorly as the classes who did not partake in the study in many aspects, leading to the conclusion that the program is not efficient in raising understanding if it is not implemented using a learner-centered teaching approach.

IMPLICATIONS AND CONCLUSIONS

It can be theorized that reaching higher order thinking in geometry relies not merely on a suitable choice of activities, but also on the active participation of both teacher and learners. The program in a teacher-centered environment (as in class 1) can produce results that are higher than the comparable class, but it is probable that the results could have been more dramatic if the program had been taught in a learner-centered problem solving environment (as in class 3). The fact that class 1 received instruction in their second language could also have had a great impact on their results. The results that were achieved in class 1 (second language education) are comparable with class 4 (mother tongue education) which may indicate that class1 has shown some progress, and to such an extent that their results are equal or higher than the learners who received first language instruction such as class 4 (and whose school is considered to be an advantaged school).

Results furthermore suggest that the implementation of a Van Hiele based learning and teaching program had a positive effect on the degree of geometric thought. Learners who completed the program reasoned on a higher level, gave more complete answers (see table 1), demonstrated less confusion (see table 2), and generally exhibited higher order thinking skills than their counterparts who did not take part in the program. The possible influence of receiving instruction in a second language (class 1) on learning was not investigated, but could have had extensive influence on these learners performances. A prerequisite for achieving success is that the teacher should consistently teach from a learner-centered approach as the program will deliver little or no advantages if the program is presented in a teacher-centered content-based context.

Teachers should take note of the profound part they play in the learning that takes place in their classrooms. The learning profit of their students could (partly) be determined by the teacher's interaction with the learning material and it is therefore necessary for teachers to make sure that their teaching approach and style complement rather than hinder the function of learning material.

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