MATHEMATICS TEACHING PRACTICES IN TRANSITION: SOME MEANING CONSTRUCTION ISSUES

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Abstract. *There are a number of questions still to be addressed in coming to understand teacher change in the context of mathematics education reform. In this study, we examine the impact of a change in the organisation of the mathematical content and in the pupils' engagement in activities on the teachers' instructional practices. The results show that such changes have limited effect on both teachers' management and pupils' construction of the mathematical meaning.*

Theoretical issues

Until recently, mathematics classrooms were dominated by instructional practices which view knowledge as a static body of facts and techniques that can be broken down and "passed along" by the teacher through direct teaching (such as lecturing and demonstrating) to the pupils. However, in the last two decades, the findings of research in mathematics education has raised concerns about this approach to teaching mathematics and indicated a need for a reform in instruction in this area. Towards this, a number of approaches to mathematics teaching have been suggested, such as those promoted by the Realistic Mathematics Education model (e.g. Romberg, 1997) and the NCTM's document 'Principles and Standards for School Mathematics' (2000).

These alternative approaches, despite their differences, accept the premise that mathematical knowledge cannot be handed over by the teacher: it must be constructed by the pupil. Here, the teacher is viewed as an informed and reflective decision maker who must provide contexts in which children's own attempts to make sense of new ideas are valued and supported and their current understandings acknowledged. From this perspective, the teacher can no longer be seen as the ultimate source of knowledge and truth; neither can s/he be expected to be in absolute control of the class agenda.

These new forms of instruction often take teachers far beyond their traditional and familiar roles and practices, and they raise some difficult questions: How much of their professional persona can they dare to risk? How can they be sure that what is to be learned is indeed learned? What are the conditions that determine change in teachers and how can these be nurtured? How long are changes sustained? Are changes developed in one mathematical domain carried over into new topics?

For teachers, the shift from familiar instructional practice to a reformed approach is not easily accomplished (Fennema & Neslon, 1997). While they may invoke notions of "good practice", they do so without actually carrying out the practices which are entailed (Desforges & Cockburn, 1987). Research shows that providing teachers with experiences where their own practices are challenged and opportunities to reflect on and rethink about them, has the potential to facilitate new insights and understandings of the teaching process (Aichele & Caste, 1994). Much in the same vein, Yackel (1994) argues that making aspects of their current practice problematic for teachers constitutes a first priority for changing their teaching practice.

Teaching processes cannot be easily divested of particularity without distortion. Furthermore, a thorough analysis of processes that are not stripped of particulars can provide powerful interpretations of classroom events and explanations for common dilemmas. Groves et al. (2000) quote Stigler (1998) who highlights the importance of looking at examples and "say[ing] exactly what it is …that you'd like to see changed". Moreover, they note that the lack of "exemplars of conceptually focused problematic situations" is an important constraint on a more "coherent and conceptually based teaching practice".

Based on the above considerations, we consider the examination of what happens within the mathematics classroom, by focusing on particular instances of the teaching processes, as very important in the study of changes in teaching practices. This is because we can identify specific elements of teachers' practice that change or are resistant to change in various instructional approaches. Of course, it is evident that this study should be carried out from different perspectives - epistemological, sociological, psychological and – in order to provide a rich set of insights into the issues raised when teachers transfer from one approach to another, for example, from a traditional to a reformed one.

In previous studies (e.g. Kaldrimidou et al., 2000) we examined teaching practices in current (traditional) mathematics classrooms from an epistemological point of view. That is, we looked at the way teachers use the epistemological features of the subject matter: definitions, theorems (properties) and solving, proving and validation procedures. The results showed that in the Greek primary and secondary mathematics classrooms, independently of the mathematical topic discussed (algebraic or geometric), the teaching approach used tended to treat the epistemological features of mathematics in a unified manner. This homogeneity was seen as of crucial importance in the pupils' attempt to construct mathematical meaning for themselves. We further noted a dialectic relation between the communicative pattern and the management of the mathematical content within the classroom, arguing that the observed teachers' quick shifts to different pupils (communicative aspect) does not allow the control of the flow of the meaning construction by individual pupils.

Thus, communicative aspects, as they are related to the management of mathematical knowledge, seem to play an important role in pupils' construction of mathematical meaning. In the present study, we look at these two aspects (communication and the management of mathematical knowledge) as they are realised through the teaching practices in a constructive activity-based mathematics classroom, that is, in an alternative to traditional teaching and learning setting.

The study

The data reported here come from a large project[1] which focused on the teaching of mathematics in the nine years of the Greek compulsory educational system (6–15 year olds). The study consisted of two parts: the first was concerned with the current status of mathematics teaching in Greece, while the second focused on experimentation with alternative, pupil-centred mathematics teaching approaches. In particular, with respect to the latter, the research team, in co-operation with a group of well–experienced teachers of both primary and secondary schools, designed and prepared the teaching of eight mathematical topics for five different grades (11-15 years old) within an activity-driven framework which included: (a) the construction of a set of well-structured activities for each topic (unit) aiming at the targeted mathematical ideas, (b) the preparation of the teachers so that they could respond to the demands of the activities and (c) the observation and videotaping of the lessons as well as the provision of feedback to the teachers.

In each case, the activities were sequenced and set up in a such a way as to allow: (a) a gradual but methodical and global approach to the targeted idea in accordance with the mathematical framework, (b) the constructive engagement of the pupils, according to their stage of development and (c) the formulation of a final mathematical result, in accordance with the teaching objectives of the lesson. Teachers were advised to present all mathematical activities as problems to solve, and to challenge and expect the children to: solve them in their own ways; discuss, compare and reflect on different strategies; make sense of other pupils' solutions and strategies, and formulate generalisations.

The research problem addressed here examines the impact of change in the organisation of the content of a lesson and the children's activation (through appropriately designed and mathematically focused activities) on the interplay between the communicative patterns and the management of the mathematical knowledge within the classroom. This serves a double purpose: first, it allows for an investigation into whether teachers' practices using the traditional approach derive from the organisation of the mathematical content and second, it helps in the location of the "deeper" characteristics of their practice (making the assumption that in a new teaching environment, there is an activation of the strongest features of practice considered by the teachers as "good" and effective).

More specifically, in order to locate those elements of the teaching practice which resist and which may be held responsible for the "distortion" of pupils' mathematical meanings in both traditional and reformed approaches, we focus on the way in which the management of the mathematical knowledge and the communicative aspects interact to generate the mathematical meaning, addressing the following research questions: (a) what kind of ideas and meanings regarding mathematical knowledge are encouraged by the teacher? (b) how does the course of classroom interaction hinder or

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favour the mathematical thinking of the pupils? and (c) how does the communicative pattern elevate or weaken the mathematical meaning constructed by the children?

The data collected consisted of 48 mathematics lessons (from 23 teachers) observed in various primary and secondary classes (11–15 years old) from 13 different schools in three geographical areas of the country. For each teacher, at least two 45 minutes sessions on different topics were observed; these were then videotaped and transcribed. For the present paper, the transcripts of the lessons were analysed by looking at the interplay between the communicative patterns and the management of the mathematical knowledge by the teacher in two phases of the pupils' engagement with the activities and with respect to the epistemological differentiation that is achieved. The two phases concern the management of : (a) the pupils' outcomes at the completion of the unit's activities and (b) the pupils' mathematical thinking at the completion of a unit and the generalisation of the results.

Presentation of the data and discussion

In the following, exemplary episodes from various lessons are used to illustrate the findings. The focus of the analysis is on the interplay between the communicative patterns and the management of the mathematical knowledge as realised by the teacher, with reference to the epistemological elements of the mathematics generated.

1. The pupils' outcomes at the completion of activities: All the episodes below demonstrate the way in which the teachers dealt with the pupils' ideas and solutions after the latter had completed a unit's activity. The focus of the analysis is on the communicative aspects of this manipulation in relation to the mathematical meaning that may be generated by the approach employed.

Episode 1.1. [*The size of the angles of a triangle* (11 years old)]: The activity requires the calculation of the size of the angles of a right-angled and isosceles triangle (only the right angle is marked) and it constitutes part of similar activities. The objective is to help pupils identify those characteristics of a triangle that would allow them to determine the size of its angles.

T(eacher). *Pay attention. This triangle has two characteristics. First, what type of triangle is this Nik, with respect to its angles?*

P(upil). *Right-angled*

T. *Right-angled. With respect to its sides, what type of triangle is this? Tania?*

P. *Isosceles*

T. *Isosceles. Well done Tania. That is, this triangle is right-angled and isosceles. And we know one of the angles, the right angle,. Michael?*

P. *Angles b and c…*

T. *Yes….*

P. *They are each* 45^o

T. *But why?*

P. *Ehhh.. Because ….*

T. *The triangle is …*

P. *The triangle is right-angled and isosceles.*

The communicative features of the teacher's practice emerging from this episode are: he (a) tends to pose successive questions, often from one pupil to another, 'hunting for the correct' result, (b) breaks down the activities and the pupils' responses with questions, (c) allows very little space for the children to formulate ideas and complete their reasoning and (d) repeats or completes what he thinks important to be phrased overall. As for the mathematical meaning, in dealing with this activity, the teacher accepts a 'leap' in the reasoning concerning the justification of the 45^o for the equal angles (i.e. that $a+b=90^o$, $180^o-90^o=90^o$ and since $a=b$, each will be $90^o\div2=45^o$). This prevents the emergence of the general mathematical idea being targeted, that is, the justification of the result on the basis of the properties of a triangle. Only the final result appears to matter, which is related to a simple recognition process of the data (the right-angled and isosceles triangle has its acute angles equal to 45^o).

Episode 1.2. [*Percentages* (12 years old)]: Pupils are engaged in an activity asking for the equivalent of 36/45 as a percentage. This is part of an activity in which children learn how to turn fractions and decimals into percentages. One pupil calculates as follows: 100÷=2.22… and multiplies the result by 36 (=79.999…). But the teacher is looking for an accurate answer.

P. *We have found it, sir. We divided 36 by 45 and multiplied by 100.*

T. *How much did you find?*

P. *0.8 times 100 equals 80.*

T. (tries to generalise) *Thus, when we know a ratio of two quantities …*

P. *Yes…*

T. *What operation should we perform in order to find the percentage?*

P. *Sir, division!*

T. *Division. What do we divide?*

P. *The numerators by the denominator*

T. *Very nice*

Then the teacher generalises further with x: x/100=36/45 and standardises the rule. In this extract, the teacher asks for an accurate solution, rejecting an approximation. In doing so, he limits the development of an interesting process of reasoning and links the result with a rule and then with a formal procedure: we divide the numerator by the denominator. As far as the communicative characteristics are concerned, the same patterns as in episode 1.1 are observed.

2. The pupils' mathematical thinking at the completion of a unit: In this part, the episodes focus on communicative issues in the teachers' treatment of the pupils' mathematical thinking in relation to the pupils' justification which is encouraged.

Episode 2.1. [The sum of the angles of a triangle (11 years old)]: Through a series of activities, pupils are invited to conclude that the sum of the angles of a triangle is 180^o. After some construction and measurements, they initially estimate the sum of the three angles of special cases of triangles and then take measurements to confirm their estimations. The activity aims at leading children to a first level generalisation through a phenomenological contradiction (for either long or short sides, the sum remains 180^o). The activity should be treated as a whole in order for this contradiction to emerge.

T. *George, what did you find for triangle B. Tell us.*

P. *90^o*

T. *90^o. Why do you say 90^o? All together, eh?*

P. *All together?*

T. *Doesn't the exercise ask you for the sum of the angles?*

P. *Because they are small.*

T. *The angles are small. Right. Tell us Chris.*

P. *130^o madam.*

T. *About. Why my boy?*

P. *Madam, estimated by the eye.*

T. *By the eye, right. Charoula?*

P. *180^o, madam*

T. *Why 180^o Charoula?*

P. *Because, for the right angle, I say 90^o, for the other one which is acute, because it is very small, I say 10^o and for the other one towards the right angle .. but it is acute, I say 80^o.*

T. *About this, I don't know, it might be correct too.* .... Other pupils go on like this suggesting 60^o, 120^o, 185^o, 150^o, but the teacher doesn't ask for any more explanations, she simply says:

P. *Did you estimate it by the eye?*

With respect to the communicative patterns that are registered, the teacher: (a) tends to make frequently address successive questions from pupil to pupil, (b) breaks down the activities or the pupils' answers with questions, (c) provides little space for the pupils to complete their reasoning, (d) reduces the exchanges among the children, often repeats statements or completes anything she considers must be said. As for the mathematical meaning of this activity, the property targeted here (theorem: the sum of the angles) was never pushed forward, and the activity slipped into a process (estimation) of finding the result. Thus, the contradiction never emerged and the pupils missed the chance to see the necessity of resolving it by other means (checking by measuring), which would guide them to discover the common property. This also becomes apparent from the pupils' demand to be told the answer. It is clear that the children carry on considering the possibility that there are many different solutions and that it is the teacher who will provide the correct one.

Episode 2.2 [Quadrilaterals (12 years old)]: Following a series of activities, the pupils are invited to determine the characteristics of a rectangle. This last activity aims at encouraging pupils to formulate some definitions.

T. *Which are the characteristics of a parallelogram? Then what does it say? Which are the features of a rectangle?…..* A little later on

P. *The parallelogram has its opposite sides parallel and has 4 sides*

T. *Opposite sides parallel and 4 sides. But this is true for the rectangle too. Stefania?*

P. *In the parallelogram, all the opposite sides are parallel and equal*

T. (Repeats). *Do we have to add anything else? About the angles?*

P. *They are right angles.*

T. *Right angles?*

P. *Acute, obtuse maybe?*

T. *Two acute, two obtuse. Tell us Anna.*

P. *In the rectangle, all the angles are right angles and all the opposite sides …*

T. *Do we agree?*

P. *Yes* (all together)!

T. *Let's go for the last one, Katerina.*

P. *The square has 4 sides and 4 angles, that is, it is a quadrilateral and its sides are equal and parallel; its opposite sides and its angles are right angles.*

With regard to the communicative patterns, we find the same features recorded in the previous episodes. In addition, the teacher very often repeats and immediately corrects their answers, sometimes explains himself why it was wrong; but he never reasons or asks for reasoning about a correct answer. As for the mathematical meaning, the fact that each shape is treated separately does not allow pupils to appreciate the purpose of creating definitions, that is, to identify and differentiate. Other basic deriving properties, i.e. those which result from the application of a proving reasoning are also missed.

Concluding remarks

In the current era of educational reform, teachers all over the world are being asked to transform their mathematics teaching. This transformation entails more than changing the types of problems and questions posed; it requires changes in teachers' epistemological perspectives, their knowledge of how people learn mathematics and their classroom practices.

In the study described above, examining traditional and reformed mathematics classrooms through the interaction between the communicative patterns and the management of the mathematical knowledge that shapes the generated mathematical meaning, we found that teachers tended to augment reformed instruction with traditional practices and then to modify and change activities so that they resemble past lessons. In particular, we identified some patterns of communication that were repeated and persisted despite a change in the organisation of the lesson's content and the children’s activation via constructively designed and mathematically focused activities.

In the exemplary episodes analysed, and in both phases of the pupils’ engagement with the activities, three distinctive features were identified: (a) succession of questions and breaking down of the targeted mathematical idea with a simultaneous quick shifting from pupil to pupil in hunting for the “right” answer, (b) reduced opportunities for exchanges among children and insufficient encouragement for reasoning and justification and (c) elevation of the correct answers, repetition of pupils’ formulations, acceptance or cancellation of answers and correction of their mistakes by the teacher. All these features have also been identified in the traditional approach to mathematics teaching (see our previous studies) with the exception of the breaking down of an activity and the prevention of pupils from completing their reasoning. These latter two practices emerging in the reformed instructional settings result in a distortion of the characteristics of the content’s organisation with consequences on the mathematical meaning.

Thus, we can claim that the dialectic relation between the communicative patterns and the management of the mathematical content within the classroom is confirmed and argue that (a) the communicative aspects recorded are related to the mathematical meaning that emerges in the classroom and (b) the change in the mathematical context and of the classroom’s functioning do not affect significantly the way the management of the mathematical meaning is managed. This has important consequences for the development of pupils’ mathematical meaning in both traditional and reformed approaches.

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