CHILDREN'S STRATEGIES FOR NUMEROSITY JUDGEMENT IN SQUARE GRIDS: THE RELATIONSHIP BETWEEN PROCESS AND PRODUCT DATA

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Abstract

This study investigated the relationship between children's strategies, their accuracy and speed measures in a numerosity judgement task. Based upon a rational analysis of the task at hand, several strategies were specified in terms of their course of response times and accuracy. Several significant relations were found between pupils' accuracy data and the distinct strategies that were derived from their response-time patterns. These results support our rational task analysis and validate previous findings concerning numerosity judgement strategies.

1. Theoretical and empirical background

Many studies (for an overview see Siegler, 1996) have shown that children of a particular age use several strategies to solve the same cognitive task. This multiple strategy use allows them to adapt their strategies to inherent task characteristics such as problem difficulty as well as to situational demands such as the need to answer quickly or accurately. The larger the repertoire of strategies, the better one can adapt one's strategy choice in function of the task requirements.

In a series of studies (Luwel, Verschaffel, Onghena, & De Corte, 2000; Luwel, Verschaffel, Onghena, & De Corte, in press; Verschaffel, De Corte, Lamote & Dhert, 1998), we investigated the development of children's strategy use in a numerosity judgement task from the perspective of "strategic change" (Lemaire and Siegler, 1995). This theoretical framework distinguishes between four dimensions of strategic competence. Changes in any of those dimensions can yield overall improvements in speed and accuracy of performance: (a) acquisition of new strategies and abandonment of old ones, (b) shift towards greater use of more efficient available strategies, (c) improvement in the fluency and efficiency with with strategies are executed, and (d) increase of the adaptive nature of the choice among available strategies.

In all of our studies participants were asked to determine numerosities of blocks that were presented in a square grid structure (see Figure 1).

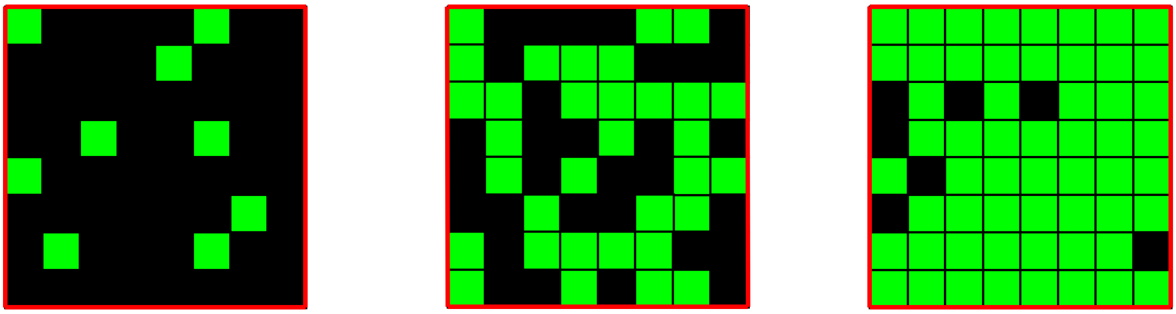

Figure 1. Three examples of stimuli from the experimental task.

Based on a rational task analysis and on the results of Verschaffel et al. (1998), three major strategies for solving this task were hypothesized. Application of each of these strategies depended on the ratio of blocks to empty squares in the grid. When there were few blocks but many empty squares in the grid the use of an *addition strategy* by means of which (groups of) blocks are counted (and added) was hypothesized. When there were many blocks but few empty squares, we hypothesized that participants would use a *subtraction strategy* in which the number of empty squares is subtracted from the (estimated or computed) total number of blocks in the grid (i.e., the anchor). When the number of blocks and empty squares is too large to be counted within the given time constraint participants will fall back on a rough *estimation strategy*, whereby the number of blocks is determined in a quick but imprecise way.

It was assumed that the addition and subtraction strategy were relative accurate strategies leading to, respectively, linearly increasing and decreasing response times with an increasing number of blocks. Due to its nature, the estimation strategy will elicit less accurate responses with relatively quick response times that lie within the same range. The assumed connection between the use of a particular strategy on a particular numerosity judgement item and the time needed to solve that item, led to four hypothetical response-time patterns (see Figure 2).

2. Method

One hundred and nine children participated in the study: 59 second graders (aged 7-8 years) and 50 sixth graders (aged 11-12 years). All participants were asked to judge different numerosities of blocks that were presented in square grids of three different sizes (7 x 7, 8 x 8, and 9 x 9) as accurately as possible within the given time constraint of 20 s. All pupils ran three sessions and in each session all possible numerosities of blocks in one grid size (i.e., 49, 64, and 81) were presented randomly. Children's responses and response times were recorded by the computer.

Half of the pupils was given information about the total amount of squares in that particular grid at the beginning of each trial. For instance, in the case of the 7 x 7 grid, children in the information condition were shown the number 49 at the beginning of each trial. We included this manipulation to investigate the effect of information on children's strategy use[1].

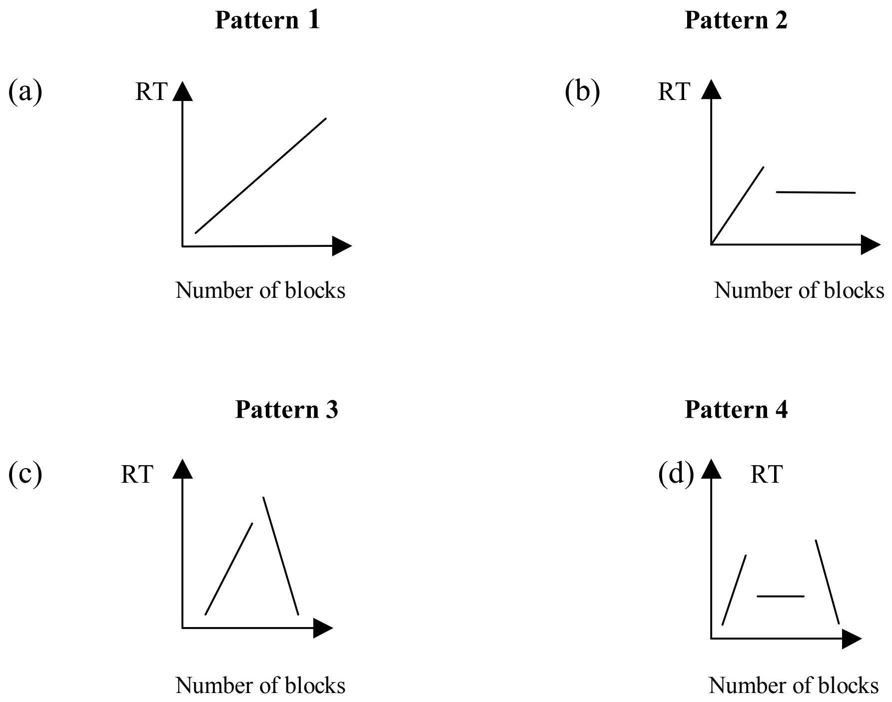

Figure 2. Hypothetical response-time patterns with (a) application of the addition strategy, (b) use of the addition and estimation strategy, (c) execution of the addition and subtraction strategy, and (d) application of the addition, estimation, and subtraction strategy.

3. Research questions and hypotheses

The results with respect to the effect of different task and subject variables are already reported elsewhere (see Luwel, Verschaffel, Onghena, & De Corte, in press). In the present paper we focus on several hypotheses about the relationship between the process data (i.e., the applied strategies) and the product data (i.e., response times and error rates).

Hypothesis 1: The accuracy of the addition and subtraction strategy will be considerably greater than the accuracy of the estimation strategy. Furthermore, there will be no significant difference in accuracy between the addition and the subtraction strategy.

Hypothesis 2: There will be a *__negative__* *relationship between the number of trials on which the estimation strategy is used, on the one hand, and the accuracy of the pupil on the task as a whole, on the other hand.* The more the less accurate estimation strategy is used, the lower the overall accuracy will be.

Hypothesis 3: The overall accuracy of children fitting Pattern 3 will be higher than the accuracy of pupils who fit Pattern 4, and both will be higher than the accuracy of participants with a type 1 or 2 response-time pattern. Using only the more accurate addition and subtraction strategies (Pattern 3) will result in more accurate responses than when the less accurate estimation strategy is used in addition to the other two strategies (Pattern 4). Moreover, because of the use of the subtraction strategy for the largest numerosities, both strategy repertoires will lead to a higher overall accuracy than the combined use of the addition and estimation strategy (Pattern 2) or merely applying the addition strategy (Pattern 1).

4. Results

Before we discuss the results with respect to each of the four hypotheses, we will shortly describe how we identified children's strategies. For this purpose we compared the individual response-time patterns with the four hypothetical response-time patterns in Figure 2 by means of two- and three-phase segmented linear regression models (Beem, 1993, 1999). These models look, respectively, for one or two change points in the data pattern and describe the relationship between the independent and dependent variable by means of, respectively, two or three regression equations. After having determined the number of segments in a data pattern by statistically testing for the presence of one or two change points, we looked for a possible fit with one of these hypothetical patterns by comparing the b-parameters in the different equations under two hypotheses. For a detailed description of this procedure we refer to Luwel, Beem, Onghena, and Verschaffel (2000). This method resulted in 77% of the second graders and 83% of the sixth graders fitting one of the four hypothetical data patterns. For reasons of clarity the data in all analyses were aggregated over grid size and type of information condition.

Hypothesis 1 was tested for each type of pattern separately. Due to an unequal division of the pupils over the different patterns (see Luwel et al., in press), we were only able to test this hypothesis for the second graders fitting Pattern 2, for the sixth graders fitting Pattern 3, and in both age groups for Pattern 4. Accuracy was measured in terms of the absolute deviation from the given response to the actual numerosity (i.e., the error rate). The error-rate patterns of the subjects included in the analysis were divided into two or three segments by means of the change points computed in the individual response-time patterns and for each segment the mean error rate was computed.

For Pattern 2, the mean error rates of the different segments were compared by means of a t-test for dependent samples. This test showed that the mean error rates of Segment 1 (addition strategy) ($M = 1.38$) were significantly smaller than the mean error rates of Segment 2 (estimation strategy) ($M = 11.78$), $t(24) = 2.67$, $p = .01$.

For Pattern 3, the same test did not show a significant difference in mean error rates between Segment 1 (addition strategy) and Segment 2 (subtraction strategy).

For Pattern 4, we conducted an analysis of variance with age as independent between-subjects variable and segment as independent within-subjects variable. This analysis revealed a significant main effect of age, $F(1, 94) = 7.06$, $p = .009$, and of segment, $F(2, 188) = 177.00$, $p < .0001$. Both variables were involved in a significant interaction effect, $F(2, 188) = 5.12$, $p = .007$. A posteriori Tukey tests revealed that in both age groups, Segment 2 showed significantly larger mean error rates than Segment 1 and 3 (*M*s: 6.05, 0.80 and 1.58 for the second graders and *M*s: 4.42, 0.72 and 1.21 for the sixth graders), all *p*'s < .0001. The mean error rates in Segment 1 and 3 did not differ significantly in both age groups. Furthermore, the mean error rates of the second graders in Segment 2 ($M = 6.05$) were significantly larger compared to the mean error rates of the sixth graders in that segment ($M = 4.42$), $p = .0001$, whereas we did not observe an age difference for Segment 1 and 3.

These findings confirm our first hypothesis: for Pattern 3 and 4 there was no significant difference in mean error rates produced by the addition strategy, on the one hand, and the subtraction strategy, on the other hand. Moreover, for Pattern 2 and 4 the mean error rates resulting from the use of the estimation strategy were significantly larger than the mean error rates produced by the addition strategy (and the subtraction strategy in the case of Pattern 4).

To test *Hypothesis 2*, we computed the correlation between the number of trials on which the estimation strategy was applied and the mean error rate of the pupil on the task as a whole. For both age groups this correlation was computed for pupils fitting Pattern 4.

This analysis revealed a significant positive correlation for the second graders, $r(41) = .60$, as well as for the sixth graders, $r(53) = .41$, all *p*s < .05, suggesting a negative effect of the use of the estimation strategy on the global accuracy.

For *Hypothesis 3*, we compared the mean error rate on the whole task for the different types of patterns by means of a *t*-test for dependent samples. For the second graders we could only compare Pattern 2 with Pattern 4. The *t*-test revealed that the mean error rates for Pattern 2 ($M = 4.64$) were significantly larger than the mean error rates in Pattern 4 ($M = 2.49$), $t(24) = 3.45$, $p = .002$. For the sixth graders, the only comparison was between Pattern 3 and Pattern 4. Results showed that the mean error rates in Pattern 4 ($M = 2.09$) were significantly larger than the mean error rates of Pattern 3 ($M = 1.01$), $t(44) = 4.42$, $p < .0001$.

5. Discussion

The present results confirmed our hypotheses in which we predicted a number of relationships between the strategies that were identified on the basis of the individual response-time patterns, on the one hand, and their accuracy data, on the other hand. These findings support our rational task analysis in which we specified the different types of strategies in terms of their expected course of response times and their degree of accuracy. It was shown that the addition and subtraction strategy were relatively accurate strategies, whereas the estimation strategy was a less accurate strategy. Moreover, this study also validated previous findings concerning numerosity judgement strategies.

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[1] A pilot study revealed that some children made large numerosity judgement errors due to an incorrect determination of the anchor. We wanted to investigate whether the provision of information about the anchor could prevent this kind of errors.