EARLY MATHEMATICAL DISCOURSE

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This study analysed the language of three boys, aged 5, who were engaged in play that was judged to have a high degree of mathematical content. The boys were selected because of the diversity of their language. One was relatively taciturn, one used a great deal of language but of limited complexity, and one used complex language that appeared to describe and lead his thought and that of others. Analysis suggests that the last of these three has the skills to succeed at school while the other two will need help in expanding their language to describe and advance their concepts.

Introduction

Students' ability to discuss mathematical activities and express mathematical thinking is crucial in current methods for teaching and learning mathematics. Teachers rely on children's language to decipher their thought so that mathematics instruction can be built upon the children's existing. Teachers must pay attention to children's conversations (e.g., Gooding & Stacey, 1993; Solomon & Nimerovsky, 1999), their answers to tasks set by teachers (e.g., Brodie, 1999), and their self-talk. Yet there is considerable variation in the specificity with which different students discuss their mathematical activity. Gooding and Stacey explored the nature of discussion in groups of 10 and 11 year-old students who did or did not gain in understanding in cooperative problem solving. The unsuccessful groups talked less than did the successful groups and used a smaller proportion of explanations. Unsuccessful groups used hardly any explicit mathematical discussion while successful groups named and discussed key mathematical aspects of their problems.

While teachers at all levels of school need to understand children's thinking, teachers in children's first year of school may have the most difficulty in coming to an understanding of what their students know. It is hard for these teachers to make appropriate judgements because many students do not use the same terms as their teachers do, may use the same terms in different ways, or may be unable or unwilling to talk about what they know.

One guide to young children's concepts can be found in observing their play. A study by Ginsburg and colleagues (Ginsburg, Inoue & Seo, 1999) has demonstrated that mathematical thinking can be identified in a large portion of the play of 4- and 5-year-old children. Although this is important information, teachers have little time for an extensive analysis of individual children's play, and are generally dependent on children's language to understand their concepts.

The study by Ginsburg and colleagues involved an intensive analysis of the play of 80 children who were from lower, middle and high-income families in New York City and who attended pre-school facilities. The investigators videotaped individual children for 15 minutes during times designated as free play. Then each 1-minute period of these videotapes was coded for the presence of five mathematical concepts: magnitude comparison, enumeration, pattern and shape, spatial relations, and dynamics. Overall, these mathematical concepts appeared in about 45% of the 1-minute episodes. This was true for children from lower income, middle income and upper-income homes. Play with blocks, Lego™, or jigsaw puzzles was found to be particularly rich in mathematical concepts.

When these children were interviewed about addition and subtraction, differences in performance and underlying concepts were seen among children from different economic backgrounds (Ginsburg, Pappas & Seo, in press). In general, the upper income children showed a somewhat higher level of mathematical thinking and metacognition. This may have been related to their use of language.

The case studies reported here are an early step in a wider analysis of the ways in which children from this study use language in the course of their mathematical play. Understanding the nature and uses of language in free play may help teachers gain insight into children's mathematical thinking, foster their language development, and encourage language use in mathematics instruction.

In examining children's mathematical language, we considered the views of both Piaget and the Vygotsky. On the one hand, there is evidence (e.g., Jordan, Huttenlocher, & Levine, 1994) that children often have mathematical competencies they cannot express in words. As Piaget maintained, there is a sense in which thought precedes language. At the same time, Vygotsky (1978) discusses two ways in which language may facilitate intellectual growth: intrapersonal speech in which children use language to plan a solution or to control behavior, and interpersonal language to share ideas and to stimulate development, as when a more competent peer provides assistance in the zone of proximal development. In this study, we considered both Piaget and Vygotsky's perspectives. One purpose of this analysis was to see if patterns of language use could be seen that would form the basis for analysis of the rest of the sample and thus provide recommendations for teaching.

Case studies

Three boys, aged 5, were selected for exploration of mathematical language employed in free play. They were selected because they all demonstrated a high level of mathematical concepts in their play, but differed in their language. They were: (1) Nigel, an African American who was building a "roller coaster", (2) Franco, an Hispanic American who was building a robot and then a tower with Lego, and (3) Isaac, a European American who was helping to make a large jigsaw puzzle of a train. All spoke English. Franco's family spoke Spanish at home.

Table 1 shows the mathematical codes assigned to each minute of play observed for these three boys. Mathematics was coded in five categories: Magnitude Comparison (MC), Enumeration (EN), Pattern and Shape (PS), Spatial Relations (SR) and Dynamic or exploring the process of change (DY). Where more than one code is given, the first code is the one that was judged to dominate that episode. These codes are based on observations of behaviour and do not necessarily imply that any language was employed.

Nigel and Isaac were coded as engaging in play with a mathematical content in 14 of the 15 one-minute periods (93%), and Franco was coded in engaging in mathematical play in all of the 15 minutes coded (100%).

Table 1. Mathematical behaviour evident in the children's play

	Nigel	Franco	Isaac
Minute 1	MC	DY	MC, PS, SR
Minute 2		DY	PS, MC
Minute 3	PS	MC, DY, PS, SR	PS
Minute 4	PS	MC, DY, PS, SR	PS, MC, SR
Minute 5	MC	MC, DY, PS, SR	MC
Minute 6	MC, PS	MC, PS	MC
Minute 7	MC	MC, PS SR	MC, EN
Minute 8	MC, PS	MC, PS, SR	MC
Minute 9	PS, MC, EN	PS, DY	PS
Minute 10	MC, PS	PS, MC, DY	
Minute 11	MC EN, PS	PS MC DY	PS
Minute 12	MC, PS	MC DY PS	PS
Minute 13	PS, MC	MC DY PS	PS
Minute 14	MC, PS	PS MC DY	PS
Minute 15		MC, PS	PS, EN

Nigel was relatively taciturn in this episode. He spoke in only 7 of the 15 minute intervals, making a total of 14 intelligible utterances. Six (42%) of these utterances were judged to have a mathematical content. Each utterance identified as mathematical related to the type of block he needed or had found. Examples included "*There it is, two*" (judged to be self-talk, as no one else was in the vicinity, and enumeration) and "*Gimme one of these blocks*" (instruction to another boy, enumeration). His longest utterance involved a complaint to the teacher that someone had upset his structure: "*Ms M., look what he's done. You've broke my roller coaster, I don't like it*."

Franco was talkative, and spoke in all of the minute episodes, making a total of 47 utterances, 18 (38%) of which were judged to have mathematical content (one containing two categories). A feature of his utterances was that he used many non-specific terms to accompany his play. For example while talking about adding blocks to parts of his structure he said, "*R, Lookit, I did it over there and I put over there and I put over there*." He was employing concepts of position, but could not be understood by someone who was not present. Most of his utterances related to his building but he also exchanged comments about his father's bicycle and sharing. Most of his comments drew attention to his activity or structure. The following utterances came from minutes 14 and 15. Codes are given beside each utterance considered mathematical, to give a picture of the analysis.

"*Long bigger, long bigger, long bigger*"(while building his tower) [Description, MC]

"*I still more bigger, I still more bigger*" (comparing his tower with R's [Description, MC]

"*I put mine bigger. You see, I told you*" [Description, MC]

"*Wanna see mine is bigger?*" (to R) [Question MC]

"*No Matthew had it first.*" (claiming a block for M)

"*No, no Matthew, we are making robots. I'ma show you. Put yours next to mine*" (persuading M to join his Lego with his) [Instruction, SR]

"*Look Matthew, it's long bigger*" (showing the taller tower to M) [Description, MC]

Isaac was also talkative. He spoke in all of the 15 minute sessions, making a total of 61 utterances, 22 (36%) of which had mathematical content. When not commenting on the task, he sang, discussed a television cartoon, or made noises into the microphone. While Nigel's utterances tended to be the minimum that would get what he wanted, Isaac's were more complex, expanding ideas from one sentence to the following one. His self-talk included both repetition of a previous statement, "We have to take it apart and build it all over again", and reading aloud from the box "Over five feet long, thirty-eight [pieces], age four and up. Wow!" The following sample is from minute 1 of his transcript.

"*Guess we need a little help. Excuse me. Are we making pieces? I don't know. Are we making any pieces?*" (watching 3 children working on a puzzle)

"*Okay, that goes in the caboose*" (has joined in working on the puzzle) [Instruction SR]

"*And the caboose is in the back.*" [Description SR]

"*We're making a real big train, just like a real train actually.*" (taking more pieces from the box) [Description MC]

"*Looks like a real olden-day train. This looks like a real olden day train, for good*" (singing) "*just like I love you…this goes away*"

Judgements were made on what would be considered emergent mathematical language. For example, "one" could be classified as a pronoun or enumeration depending on the context. Time sequences were not classified, nor were iterations or the concept of 'broken' which could be a precursor to part-whole, although an argument could be made for including all of these as early stages of mathematical language. Mathematical language was classified by its function, as self-talk, instruction, questions, or comments. Table 2 gives the number of utterances with a mathematical content by function and mathematical code.

Table 2. Number of utterances with a mathematical content, given by function of language and mathematical categories covered

	Nigel	Franco	Isaac
Self talk	1 EN		1 EN & MC
Description	1 EN	8 MC	5 SR
		2 SR	1 MC
			3 PS
			1 EN
Questions		2 MC	1 PS
		1 SR	
Instructions	2 MC	1 MC	5 PS
	2 EN	4 SR	1 MC
			1 SR
Word play			1 SR
			2 EN
Total	6	18	22

Interpretation

The number of episodes of play considered mathematical was similar for these boys, as was the proportion of their utterances judged to have a mathematical content. The differences noted were qualitative. There was a marked difference in the variety of mathematical contexts that the boys spoke about and in the complexity of the structure of their language.

Nigel's limited use of language is interesting in itself, although it makes it difficult to judge his competence. His play covered three spheres of early mathematics, and he used language for two of these. His limited language did include evidence of specific nouns, modifiers, and logical connection between sentences in an utterances not classified as mathematical, that in which he complained to his teacher about someone breaking his roller coaster. However, his limited use of language, if typical, could make him like the students in Gooding and Stacey's groups who did not learn in cooperative groups.

Franco was not fully fluent in English. His play was very rich in early mathematical concepts, with a greater number of mathematical episodes being identified than for the other boys. While his play in this episode covered four categories of mathematical ideas, he used language to describe only two of these. The fact that he talked a lot is likely to be an advantage, as he was practicing putting the process of comparison into words. In this episode he showed a limited vocabulary for mathematical concepts, using pronouns for both processes and objects. While all three boys used pronouns when the object of their play was self evident, the other two also used nouns and modifiers. Franco only occasionally used a noun and used no modifiers. He used no words to describe either dynamic changes or pattern and shape.

Isaac's play covered four mathematical categories, and he used language for all of them. This sample gives many examples of his analytic thought. His sentences showed continuation of ideas that became more specific in succeeding utterances. The nature of his self-talk suggests that he will continue to generate ideas, so that his concepts grow with minimal external influence.

The language of three boys gives us initial insights that will be explored further with the full data. It appeared that descriptive language supported thinking for all three boys and, in Isaac's case, questions and instructions led both his activity and possibly thinking for him and for others. Nigel and Franco demonstrated more concepts than they talked about and there was less evidence that it led their activity. Although two boys used some self talk, it is difficult to judge the extent to which this may have led thinking.

The presence or absence of attributes that Gooding and Stacey found in the conversations of successful groups indicates the relevance of these factors in the language of young children. Studies of functional grammar suggest that the analytic nature and logical connectedness of language is the aspect that enables students to be successful in classroom discourse. The size of vocabulary is known to be a good predictor of success in school, and this is likely to be true in mathematics as well. All of these are factors that can be fostered in good teaching. We know that all of these boys exhibit mathematical concepts in their everyday behavior and it is important for teachers to be aware of what they know. Two of them are likely to need more help in using words to describe mathematical objects and activities and ask relevant questions.

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