A DESCRIPTION FROM GERMANY

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Abstract: This paper concentrates on the goals of mathematics education in Germany after compulsory education. The school system is first explained briefly. In what then follows, it is demonstrated that, based on an analysis of the curricula and results from the TIMSS study, German general education and the various kinds of vocational education differ very much from each other concerning goals of mathematics teaching as well as in the students' achievements. Finally, it is stated that mathematics teaching only barely attains the goals formulated in the curricula and that in international comparisons the weaknesses of German mathematical teaching are becoming obvious.

The Structure of the German School System

This paper concentrates on mathematics education after compulsory education has been completed. In order to better understand what follows, the German school system is first briefly described.

In Germany, compulsory schooling commences at the age of 6 and finishes at 18. Nine or 10 of these years, depending on the school system of each federal state, must be spent in full-time schooling, and the remaining 2 or 3 years spent either in full-time schooling or in part-time vocational schools in conjunction with a trade or apprenticeship programme.

Lower secondary level for students aged 10 to 16 offers differentiated teaching in accordance with student ability, talent, and inclination. Students are placed into one column of a three-column-system, within which there is no streaming.

Upper secondary level, for students aged 16 to 19, takes place, as already mentioned, in two systems: a general education system and a vocational system.

Overview of the General Education System

In the general education system, a three-year course qualifying students to enter university is offered. Courses are offered at two different levels in terms of academic standards and teaching time: basic or advanced courses (*Grundkurse* and *Leistungskurse*, respectively). The basic courses are intended to ensure that all students acquire a broad general education of the subject in question whereas the advanced courses are meant to provide further specialist knowledge and serve as an in-depth introduction to academic study.

Many limitations are placed on students' choices to ensure that all students achieve a broad range of knowledge. Students must select at least two advanced courses, one of which must be either German, a foreign language, mathematics, or a science subject.

Mathematics as advanced course is chosen as a second course most often by 35% of the students, but with a great difference between boys (47%) and girls (26%); as well as between the western (32%) and the eastern (41%) federal states. There is a significant difference in teaching time (3 vs. 5 hours weekly) between basic and advanced courses. Currently, mathematics is a compulsory subject and must be taken

at least at a basic level until Grade 13. Until recently, students in some states were able to drop mathematics in Grade 13, which on average 10% of the students did.

Those students whose aim after school is to study mathematics, science, or technical sciences choose mathematics as an advanced course; those oriented more toward linguistics and social sciences tend to take basic courses in mathematics. Those attending basic courses of mathematics can be described as college-bound liberal arts students; the students participating in advanced courses are generally college-bound mathematics and science students.

The upper secondary level also encompasses full-time and part-time vocational education. The Western German dual system of vocational education involves cooperative apprenticeship at two learning sites: the school and the workplace. Enterprise-based vocational training has two sponsors: the governments of the federal states, which establish and finance vocational schools; and the enterprises themselves, which finance and provide apprenticeships. Full-time vocational education comprises many mixed forms of schooling, which shall not be differentiated here. These students may be considered non-college-bound.

Of students 17–19 years old in the upper secondary level, 31% are in the gymnasium and comprehensive schools, Grades 11 to 13; nearly 16% are in full-time vocational education; and about 53% are in part-time vocational education.

Goals of Mathematics Education for General Education

The curriculum for mathematics in Germany is laid down in curricula for each state and for each of the different types of schools.

In general, the curricula state that the general aims of mathematics education are to—

- Provide fundamental knowledge and skills in important areas of mathematics.
- Provide mastery of the techniques, algorithms, and concepts necessary for everyday life in society.
- Develop the ability to describe facts mathematically, to interpret the contents of mathematical formulae, and to enhance the solving and understanding of nonmathematical or environmental phenomenon through mathematics.
- Teach pupils to think critically and to question.
- Give examples of mathematics as a cultural creation in the historical development of civilisation.
- Provide terms, methods, and ways of thinking useful in other subjects.

During the last few years, the curricula have undergone changes, but the trends of changes are more apparent in approach than in content. Thus, the new orientation for mathematics teaching is to—

- Present mathematics both as a theoretical study and as a tool for solving problems in the natural and social sciences.

- Provide experience with the fundamental mathematical idea of generalisation, the need for proofs, structural aspects, algorithms, the idea of infinity, and deterministic versus stochastic thinking.
- Use inductive and deductive reasoning, methods for providing proof, axiomatics, normalisation, generalisation and specification, and heuristic work.
- Provide variation in argumentation and representation levels in all fields and aspects of mathematics teaching.
- Teach historical aspects of mathematics.

These goals, indicated as common learning objectives of mathematics teaching, can be regarded as consensus among mathematics educators and, among others, are therefore explicated in the expertise on mathematics teaching in the upper secondary level (Borneleit et al., 2000). The main difference between basic and advanced courses is that objectives formulated for advanced courses are oriented more toward mathematics as science, whereas those for basic courses stress algorithms and mathematics formulae. Both courses are based on the same three pillars of mathematics education: calculus, linear algebra/analytical geometry, and probability and statistics. Among these, calculus is the most important and gets the largest portion of teaching time. The first two strands were found in curricula already at the beginning of the 20th century, but probability and statistics have been added only in the last 10–15 years. These unshakable pillars of mathematics teaching are firmly laid down in the German baccalaureate (*Abitur*) standards for all states of the German Republic. Further specifications and differentiation of goals vary very much from state to state.

Goals of Mathematics Education for Vocational Education

In vocational education, the importance given to mathematics varies very much, and there is a wide spectrum of goals in mathematics teaching. The TIMSS study on the upper secondary level divides the fields of vocational education into three groups: (a) professions closely related to mathematics, including professions in the trades; (b) professions closely related to technology, including professions in the fields of metal, electrical/electronic, and construction industries; and (c) professions not closely related to mathematics or technology professions, including those found in the fields of agriculture, housekeeping, social services, and nursing (see Baumert, Bos, & Lehmann, 2000, Vol. 1). In each of these three categories, mathematics is of different importance. Often, in vocational education not closely related to mathematics, mathematics does not exist as a separate subject, but is integrated into content-related subjects. In nearly all fields, in fact, the learning objectives depend on content-related contexts. Therefore, mathematical literacy should be taught, as it has been tested by the TIMSS study, focusing on the following competencies:

- Everyday real-world-related conclusions.
- Application of basic routines.

- Elementary modeling and linking of mathematical operations.
- Mathematical reasoning, especially through graphs.

Interviews with experts show that, in technical education and education for manual work, mathematical modeling is given special emphasis and that, in the commercial fields beyond those dealing with graphs, that emphasis is also needed. Mathematics in social services and nursing mostly focuses on everyday real-world reasoning and conclusions as well as application of routines.

Achievement of German Students

The above mentioned TIMSS study provides an overview of the real achievements of German students. In the following, I refer to these results.

Achievement of Students in General Education Schools

In basic course lessons, only a small portion of the students (almost one fifth) reached a level that enabled them to apply safely and independently what they had learned to solve standard problems. If the context of items they were familiar with was changed, nearly all students had great problems in solving the problems. The results of more than four fifths of the students in the basic course did not exceed the level of application of simple mathematical concepts and rules. Students who dropped mathematics almost never stepped beyond this threshold. As one would assume, in advanced courses, a significantly higher level of student achievement was found. Nevertheless, fewer than one in eight students showed an ability to deal successfully with mathematical problems when the solutions were not directly evident (see Table 1). With the tested topic fields divided as for TIMSS, in geometry German students showed above-average results, whereas in algebra (number and equations) and calculus, they perform more or less below average. In international comparisons, women obtained clearly worse results (Mullis et al., 1998, pp. 145ff).

> Altogether, from interviews of the students it became clear that German mathematics lessons are structured strongly receptive and focusing on practising skills. Unlike the learning objectives formulated in the curricula, the support of understanding and applying of mathematics to everyday problems plays only a minor role. However, obviously the basic courses did not develop a didactic form of its own and give the impression as being advanced course on a reduced level of achievement demand. Furthermore, teaching in these courses is less application and understanding oriented, which leads students to take a more strongly receptive attitude (Baumert, Bos, Lehmann, 2000, Vol. 2, pp 275ff).

Table 1

Student Results, General Education Schools

Ability Components	Dropped Basic Course	Continuously Attended Basic Course	Attended Advanced Course	Total
Elementary reasoning and conclusions	57%	29%	8%	24%
Application of simple concepts and rules	37%	53%	38%	46%
Application of upper secondary level mathematics for standard problems	6%	17%	45%	25%
Individual problem solving on upper secondary level	9%	1%	12%	5%

Note. From Baumert, Bos, & Lehmann, 2000, Vol. 2, pp. 193ff)

Achievement of Students in Vocational Education

The TIMSS results make clear that in the broad tendency, German students face greater difficulties with items that demand more complex operations, the application of mathematical models, and individual mathematical argumentation. Generally, the strongest capabilities of German students were found in solving routine mathematical items and items close to their mathematical experiences. Specifically, 22% of the students achieved only a competency level of everyday reasoning/conclusions, 43% remained at the level of applying simple routines, 32% reached the level of simple modeling, and only 3% achieved the highest standard of mathematical argumentation. There was a great gap between classroom reality and the curricula claiming conceptual understanding, the ability to combine elementary operations, and the transfer of trained skills to new contexts (Baumert, Bos, & Lehmann, 2000, Vol. 1, pp. 199ff).

An analysis of achievement in the three groups of vocational education show substantially great leaps between the levels, and four achievement groups can be recognised: Bankers achieved the best test results, followed by garage mechanics and industrial mechanics. The third group consisted of retail merchants and trained office clerks; hairdresser trainees obtained the worst results. The differences between achievement levels strongly depended on the educational level in the lower secondary level. Students achieving better results generally had a higher education qualification than those who obtained worse results. Furthermore, great gender-dependent differences, to the disadvantage of women, can be recognised. Foreign students whose families were still strongly rooted in a non-German culture also produced clearly worse results (Baumert, Bos, & Lehmann, 2000, Vol. 1, pp. 227ff, 295ff).

Final Remarks

Summarising, it can be stated that German mathematics teaching in reality only barely attains the goals formulated in the curricula. This applies to students of all three education groups: students in vocational education/non-college-bound students and students in general education with its two levels of differentiation, college-bound liberal arts students and college-bound mathematics and science students. In international comparisons, in which German students ranked in the lower or average achievement level, the weaknesses of the German mathematical teaching are also becoming extremely clear.

References

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