CHALLENGING A PURELY MATHEMATICAL PERSPECTIVE ON TEACHERS' COMPETENCE

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This paper has two main parts. First it describes some recent trends in research on mathematics teachers (sections 2 and 3). One trend concerns the development of relatively concrete pieces of methodological advice; another is to re-conceptualise the mathematical knowledge that teachers need. Both these trends are based on analyses of school *mathematics and of classroom interactions. Second, I present a study of three novice teachers (section 4). One conclusion of this study is that the teachers' activity is often directed at broader educational motives than facilitating the students' learning. Therefore the efforts at redefining relevant mathematical qualifications for teachers should not be pursued at the expense of general pedagogical emphases in teacher education. Rather there is a need to ground both general pedagogical perspectives, pedagogical subject matter knowledge and mathematical qualifications in analyses of classroom practice.*

1: AN EXTENDED VIEW OF TEACHING PRACTICE

Current initiatives in mathematics education present the teacher with very different challenges in comparison with 20 years ago. I have previously argued that the role of the teacher in reform classrooms - inspired by fallibilism, constructivism, and socio-cultural theory - may be summarised as one of *forced autonomy* (Skott, 2000): The teacher is required to manoeuvre autonomously and independently in order to support individual students and to orchestrate small-group and whole class discourse so as to facilitate individual and collective conceptual development by balancing communal involvement in processes of for instance experimenting, conjecturing, reasoning, generalising, formalising, and refuting with more traditional teaching-learning processes. In this sense the teacher has been recognised to move to centre stage of curriculum enactment and is expected to become involved in instantaneous decision-making on the basis of his or her reflective activity. For instance (s)he is required to

- develop and flexibly use a wide range of experientially and mathematically rich tasks and contexts, some of which have open beginnings and/or open ends;
- to interpret the students' current understanding and potentials for learning and to decide on the types of support needed by individuals and groups of students;
- consider when and how to introduce small-group interaction, either to provoke individual cognitive disequilibria through social interaction or to pave the way for the creation of small communities of mathematical practice;
- capitalise on students' contributions to the discourse by instantaneously evaluating their pedagogical and mathematical potential and if appropriate to involve the rest of the class to become involved in developing taken-as-shared concepts, procedures, and meta-mathematical understandings.

The majority of the teacher decisions corresponding to these requirements cannot be made de-contextually in more than very general terms. Consequently the domain of on-the-spot teacher decision making has expanded and the notion of teaching practice is extended beyond the teaching methods in the narrow sense of the term, i.e. beyond the set of observable teacher actions.

The recognition that many of the most important educational decisions have to made by the teacher in the classroom seems to be one reason why - until recently - the literature has remained remarkably silent with regard to methodological advice: If the specific interactions between teacher and student(s) are essential to the types social or cognitive of support required, any attempt to provide a general set of suggestions for classroom teaching may be in vain. As a consequence the recommendations for teachers have in practice often degenerated into a caricature of what they are expected *not* to do (use whole class approaches, stand at the board, use routine tasks, etc.). Instead the situation of forced autonomy and the inherent emphasis on teachers' reflective activity has fuelled a large amount of belief research, that has focussed on their meta-mathematical conceptions and views of the teaching-learning process[i]. One rationale behind this latter approach seems to be that the teachers' views of mathematics and of the teaching-learning process plays a fundamental role for the ways in which they cope with the extended conception of teaching practice (e.g. Schoenfeld (1992); Ernest (1989)).

2: THE DEMANDS OF FA: A METHODOLOGICAL PERSPECTIVE

Over the last few years the lack of recommendations on how to cope with classroom interactions has to some extent been remedied, and a number attempts have been made to indicate how teachers may proactively support student learning in ways that comply with the views of mathematics and learning that dominate the reform.

Cobb, Boufi, McClain, and Whitenack (1997) described how the teacher's introduction of symbolic records of the students' suggestions provides him or her with opportunities to provoke a collective meta-cognitive shift taking the students' previous activities as objects of the continued discourse. In doing so the teacher supported individual students' learning, facilitated the development of taken-as-shared concepts and at the same time contributed to the emergence of a meta-mathematical conceptions and norms for action compatible with the reform.

Chazan and Ball (1999) challenged the understanding that teachers' non-interference is the most dominant feature of reformist teaching. They suggested that teacher reflections on the mathematical value of the topic in question in relation to the students' future learning, the direction and momentum of the discourse, and the social and emotional tone of the classroom form the basis for decisions of for instance inserting disagreements in order to create a productive learning environment.

Stephan (2000), using Toulmin's model of argumentation, argued that the teacher should elicit warrants and backings from the students in order that they substantiate their initial claims. Stephan suggests that profound taken-as-shared mathematical understandings may emerge as a consequence, while reform oriented visions of mathematics and of its teaching and learning is maintained.

Skott (2000) suggests to explore the potential of intentional methodological discontinuities (IMDs) as a way of coping with situations in which the intended mathematical focal point of a classroom interaction is discarded, for instance as a

funnelling type of interaction emerges. IMDs require the teacher to play an active part in the classroom discourse by breaking with the dominant methodological and organisational framework of the situation in question, and by doing so to ensure a continued emphasis on its potential for student learning.

It is a common feature of these suggestions that they highlight and specify the teacher's role in mathematically qualifying the classroom discourse. In other words, they are all based on the assumption that the teacher's activity in terms of support of the students' individual and collective mathematical learning is essential, and that even though teaching requires competent, on-the-spot decision-making some methodological recommendations are needed that facilitate it. In this sense the suggestions mentioned may be seen as attempts to present relatively concrete pieces of methodological advice that are in line with the meta-mathematical priorities and the approaches to learning that dominate the reform. The suggestions do obviously not relieve the teacher from the obligations of forced autonomy, i.e. from becoming involved in the types reflective activities described above. It cannot be decided de-contextually when and how to introduce a meta-cognitive shift, insert a disagreement, call for a backing to an argument, or propose a methodological discontinuity. The teacher has to make these decisions based on the specific set of interactions in the classroom in question. But the recommendation to consider doing so is in itself a relative concretisation of the reform's demands, as it introduces possible focal points for the teacher's reflective activity.

These suggestions for practice, then, share two characteristics. First, they recommend teacher activities that are methodologically oriented, but based on the set of meta-mathematical understandings and conceptions of learning that frame the reform. Second, they do not do away with the teacher's obligations in a situation of forced autonomy, but they may assist him or her in handling these obligations more competently and with greater ease or deeper comprehension.

3: A MATHEMATICAL PERSPECTIVE ON TEACHERS' COMPETENCE

Recently a few publications have emerged that address the issue of the mathematical qualifications required, if the teacher is to play role of flexibly supporting student learning. These studies, then, view teaching competence from the perspective of the teachers' subject matter knowledge.

Ma's (1999) study, at first sight a comparison between the qualifications of American and Chinese teachers, convincingly argues that teachers at the elementary levels need *profound understanding of fundamental mathematics* (PUFM). PUFM is more than conceptual understanding of the topic in question. The teacher must also be aware and make use of connections between different concepts and procedures in different domains of mathematics both at a given educational level and in a longitudinal sense; (s)he should be able to use multiple perspectives on a mathematical domain and various approaches to the solution of a problem; and (s)he should guide students' mathematical activity by use of basic but powerful mathematical concepts and

principles. The essence of the argument is that one of the most significant obstacles to substantial mathematical learning on the part of the students is poor understanding of *school* mathematics on the part of the teacher. The teacher, then, needs profound understanding of fundamental mathematics rather than superficial acquaintance with more advanced mathematical topics, in order to facilitate significant student learning.

In a somewhat similar sense Ball and Bass (2000) focus on the mathematical qualifications of the teachers and challenge the idea that the main teacher related obstacle to reformist classroom practices is pedagogical or meta-mathematical. Using a terminology that acknowledges the challenges of forced autonomy, they claim that:

> "... teachers need mathematical knowledge that equip them to navigate these complex mathematical transactions flexibly and sensitively with diverse students in different lessons" (p. 94)

Their argument is that the divides between teaching methods, mathematical qualifications and teaching practice, institutionalised in most teacher education programmes, need to be bridged, and that this may be achieved by "grounding the problem of teachers' content preparation in problems and sites of practice" (p. 101). When doing so they also build on the research on what Shulman (1986, 1987) has called *pedagogical content knowledge*. Pedagogical content knowledge is "that special amalgam of content and pedagogy that is uniquely the province of teachers" (Shulman (1987), p. 8), "... the ways of representing and formulating the subject that make it comprehensible to others" (Shulman (1986), p. 9). However, Ball and Bass argue that this type of knowledge does not suffice as a basis for the teacher's instantaneous decision making. What the teacher needs, they claim, is

> "a kind of mathematical understanding that is pedagogically useful and ready, not bundled in advance with other considerations of students or learning or pedagogy" (p. 88)

Both Ma and Ball & Bass, then, point to the teacher's mathematical qualifications as the main obstacle to the enactment of reformist intentions in mathematics classrooms, and attempt to redefine the required qualifications. They do so on the basis of investigations of teacher competence related to *school* mathematics and of an analysis of what it may take in terms of teacher qualifications for the conceived potentials for student learning to materialise. In other words, the argument is that significant student learning requires teacher qualifications in mathematics, that must be defined not with reference to mathematics in its own right, but to the contents and envisaged interactions of mathematics classrooms at school level. This may be interpreted as compatible with the methodological recommendations described in the previous paragraph as it specifies the types of mathematical qualifications required for the teacher to successfully employ the types of teaching methods suggested: the meta-cognitive shifts, mathematical disagreements, the warrants and backings and the methodological discontinuities. In short, these studies present a long-needed reversal of a dominant approach to teaching competence. This tradition asks what it takes to transform a set of mathematical ideas (concepts, procedures, conceptions) based in traditional college or university courses to classroom practice. The two studies referred here start with a perspective on *school* mathematics and on classroom

practice. They then use this perspective as a basis to redefine the mathematical qualifications for teachers that are deemed necessary for the realisation of the reform, for instance as conceived in the methodological recommendations described in the previous paragraph.

However in doing so the two studies disregard the need to integrate a general pedagogical perspective in the mathematical qualifications of the teachers. In the remaining part of this paper I shall argue that by doing so they in effect endanger the enactment of the very reform initiatives they seem to pursue.

4: Including a pedagogical perspective on teaching competence

The study described in this section aimed to understand how novice teachers deal with the situation of forced autonomy. The study followed three teachers for 2 to 3 weeks each. The teachers were selected because they presented strongly reformist priorities of school mathematics both in a questionnaire before and in research interviews after their graduation from college. They all described the students' activity in terms of investigations and experimentation; they conceived mathematics as a way of approaching and posing problems; and they presented their visions of teaching in terms that reflected intentions of being unobtrusively supportive in relation to student learning. In short, the school mathematical priorities of these teachers were strongly inspired by the reform, and they all seemed confident to take on the responsibilities inherent in the extended notion of teaching practice (section 1).

The teachers' classrooms were videotaped and if possible I had an informal discussion with them after each lesson, asking a few questions based on my field notes. Also a final interview was conducted with each teacher, and he was asked to comment on a number of clips from the video recordings. These clips were selected in order to exemplify situations in which the teacher's role appeared to be crucial for the further development of the classroom interactions and for the learning potential.

In the case of all three teachers the classroom interactions often developed in ways that resembled their school mathematical priorities. For instance, they frequently invited the students to explore open problems - often using manipulatives - and to present their own hypotheses for further investigation. In some cases the continued investigations would be the responsibility of a one or a few students, while in others the teacher would attempt to involve the whole class in it. Also the teachers all tried to support student learning by asking them to explain and reword their current understandings and by taking these understandings as the starting point for their own contributions to the interaction. When doing so the teachers generally used everyday language when discussing new concepts and procedures and only later introduced standard mathematical terminology that in turn was used to focus the students' attention on particularly important aspects of the concepts in question.

However, there were also episodes in which the classroom interactions were at odds with the teachers' professed school mathematical priorities. In some of these the teachers' themselves referred to mathematical insecurity on their part as the main

reason why the interaction developed the way it did. For example one of the teachers, Larry, - after having been shown an episode from his grade 5 classroom - explained why he in effect avoided pursuing an idea developed by one of his students:

> **"**For one thing I hadn't considered the question beforehand, and if I am to pursue something like this at the instant, I at least have to have some idea about where I want to go, and if I'm too much in doubt about the direction, if I haven't understood it properly myself, it becomes difficult to convey it to the children." (From the final interview with Larry).

There were, though, also a number of episodes in which the learning opportunities were strongly inhibited by the Larry's contributions to the interactions, but in which mathematical insecurity did not appear to be the main reason. For instance Larry, who teaches at a very conservative private school, often struggled to reconcile his own educational priorities and those of the school. In particular he often referred to school's emphasis on covering the syllabus and preparing for the next test as incompatible with his own intentions. At other times the classroom interactions and Larry's comments on them indicated that his contributions to the discourse were directed primarily towards manifesting his own professional authority rather than to facilitating the students' mathematical learning. In these situations he became very direct in his explanations and at times took over the students' suggestions.

Christopher, a 28-year-old teacher working at a municipal primary/lower secondary school, was also asked to comment on episodes in which his contributions to the interaction seemed particularly significant, especially when they appeared in dissonance with his explicit images of school mathematics. Also in his case, mathematical insecurity appeared as *one* reason why he did not always exploit the mathematical potential of students' questions and comments. However, there seemed to be several other and more frequent such reasons, reasons that - in a sense similar to the situation for Larry - were related to a shift in the motives of his activity: He was sometimes more concerned with building students' self-confidence by ensuring that they provided an acceptable solution to a textbook task than with supporting their mathematical learning, and in consequence he got involved in funnelling types of interaction that in effect depleted the task in question of its mathematical contents. Also his activity was sometimes primarily directed towards managing the rather noisy classroom in which many different (groups of) students simultaneously asked for his assistance. And - like Larry - he sometimes tried to manifest professional and mathematical authority in ways that seemed counterproductive to student learning.

John, the last teacher in the study, works at small village school with only 120-130 students. Like the others he was asked to comment on episodes in which his school mathematical priorities were challenged. In some of these episodes the interactions with the students developed very much along lines compatible with his expressed views of school mathematics, while in others this was clearly not the case. Describing his reaction in some of the latter ones he says:

> "There are some children in here, some of the weak ones, with whom I've had to choose [...] especially with Louise, I've had to say to myself 'If only she acquires a system [of how to

> solve the tasks], then it doesn't matter, if I've provided her with it, because at least she can follow what goes on'. I've chosen that. And she does [follow]. So up to now she is part of the team. She largely makes the same tasks as the rest, although she finds mathematics very difficult." (From the final interview with John).

John was more concerned that Louise remained part of the classroom community than with providing the types of support he found best suited to facilitate her mathematical learning. In other situations he tried to integrate his support to particular students' learning with concerns of for instance avoiding to challenge what he conceived as their weak and vulnerable self-perceptions and of taking their family background into consideration. Asked if he ever experienced a conflict between his intentions teaching mathematics and of taking broader educational considerations into account he said:

> "No I don't think so. Because I can't imagine being a teacher without taking all the other things into account. You know. But that's my attitude to being a teacher. [...] [these other things] are always part of being a teacher, and I don't consider it a conflict. It is just another type of professional challenge." (From the final interview with John).

An important characteristic of the episodes described above is the simultaneous existence of multiple motives of the teacher's activity. In each of them the teacher's intention of facilitating mathematical learning is challenged, as the energising element of his activity is changed to for instance complying with the dominant school culture, supporting students' self-confidence, managing the classroom, or manifesting professional authority[ii]. The teacher, then, is often playing a very different game than one of teaching mathematics, and consequently the most dominant obstacle to the enactment of their reformist school mathematical priorities is not insufficient mathematical qualifications on their part.

5. SUMMARY AND CONCLUSIONS

In this paper I have outlined some of the responses on the part of the research community to the pressures on teachers as a result of forced autonomy. One response is to develop of a set of relatively concrete pieces of advice on how to proactively support student learning. Such advice suggests possible focal points for the teacher's reflective activity in the classroom without relieving him or her from the obligations of forced autonomy. Another response is to investigate teachers' mathematical qualifications, and recently it has been suggested that the most significant teacher related obstacle to the enactment of the reform is the teacher's (insufficient) mathematical competence.

These two sets of responses are similar in that they are both based on analyses of interactions in mathematics classroom and of *school* mathematics rather than on general pedagogical theory or on mathematics per se. They may, then, be seen as attempts to develop conceptions of teachers' pedagogical content knowledge and subject matter knowledge from the practices of mathematics classrooms.

The study described in section 4 also investigated teacher-related potentials for and obstacles to student learning on the basis of classroom observations. The results of

that study point not only to the teachers' mathematical and pedagogical content knowledge as relevant factors, but to the ways in which the teachers sometimes shift the objects and motives of their activity to much broader educational issues than those related to mathematics. This change of the teacher's activity turned out to be one of the most important challenges to his intention of unobtrusively supporting the student's learning. It follows that teachers' pedagogical knowledge is not irrelevant for student learning, and it should - just like the pedagogical content knowledge and the subject matter knowledge - be re-defined on the basis of analyses of classroom interactions and integrated with the other two types of knowledge. The second quotation from Ball and Bass, then should be turned upside down (cf. section 3). Maybe what is needed, is exactly 'a kind of mathematical understanding that is [...] bundled in advance with other considerations of students and learning and pedagogy'.

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[i] In Zeitschrift für Didaktik der Mathematik DM 96/4 Törner and Pehkonen list 764 papers in belief research in mathematics education. This indicates an interest in the field that does not seem to have diminished since then.

[ii] I have previously called such episodes *Critical Incidents of Practice* (CIPs). For a discussion of CIPs based on the classroom observations of and interviews with Christopher see (Skott, in press).

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