THE UNKNOWN THAT DOES NOT HAVE TO BE KNOWN

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In response to the misconceptions students are experiencing in the algebraic domain, there has been a call to begin algebraic thinking early. Kaput (1999) believed that algebraic understanding evolves from viewing algebra as the study of structures abstracted from computation and relations, and as a study of functions (a static and dynamic dimension). The arithmetic knowledge base that is needed for algebra comprises an understanding of (i) arithmetic *operations,* (ii) the *equal* sign as equivalence, (iii) the operational *laws*, and (iv) the concept of *variable* (Ohlsson, 1993). Usiskin (1988) argued that the notion of variable could be introduced through three approaches: solving equations with *unknowns*; generalisations of *patterns*; and *relationships* between quantities. He contended that, in the long run, these notions had to be combined and abstracted to develop the concept that variable was a *member of an abstract system*. While some researchers (e.g., Chalouh & Herscovics, 1988) argued that unknown was not an appropriate algebraic conception for variable as it does not represent multiple meanings, Graham and Thomas (1997) contended that an appreciation of unknown could allow students to better assimilate later concepts. For this to happen, they argued that activities with unknown should cover a wide variety of situations including recognising unknown situations, substituting for unknowns, considering solutions as values that make the equation true, and finding solutions through arithmetic and algebraic methods.

Carraher, Schliemann, and Brizuela's paper explores young students' ability to operate on and represent unknowns in a relational situation as a precursor to developing understanding of the variable. They describe an example of classroom activity (two children starting with the same unknown amount of money) to provide evidence that "children as young as eight and nine years can learn to comfortably use letters to represent unknown values, and can operate on representations involving letters and numbers without having to instantiate them" (p. 7).

Power, formal letters and the limits of number lines

The power of the activity. We applaud the power of Carraher, Schliemann, and Brizuela's classroom activity. First, it presents arithmetic as *change* rather than solely as relationship; that is, it is a *dynamic* form of arithmetic in that it represents +3 as a movement on the number line from 2 to 5. Second, because of this focus on change, it allows the notion of *backtracking* (undoing the changes) to be introduced. Although limited (Stacey & McGregor, 1999), backtracking is a useful procedure for solving equations with one instance of unknown. Third, it involves sequences of operations which, initially, are not capable of *closure*. Fourth, it encourages children to interrelate a rich array of representations when articulating their understanding (e.g., verbal and written language, diagrams, number lines, and symbols), thus developing rich representational scheme. All these signify a significant move away from the traditional approach most children of this age experience in their everyday classroom.

The use of N. However, the use of the symbol N in the activity is a concern and raises questions. Even though the activity reflects Herscovics and Linchevski (1994) suggestion that the transition to formal algebra involves considering numerical relations of a situation, discussing them explicitly in simple everyday language, and eventually learning to represent them with letters, we wonder whether the use of N in the activity reflects a limited view of algebra (as "arithmetic with letters" rather than as the "mathematics of generalization"). Do you need letters to do algebra? To us, the answer is no; we see algebraic thinking as predominantly the ability to operate in generalized abstraction and prefer that the students use normal language (e.g., "the beginning money") or their own invented "symbolisation/notation" (e.g., "apb" - *amount in piggy bank*). We believe this use of language or invented terms are as algebraic as "N", and maybe less dangerous. We are not convinced that the children in the activity are not seeing the "N" as "the piggy bank" or as a specific number, understandings of N that are inappropriate for algebra (Küchemann, 1978).

The number line representation. The use of the number line in the activity also raises questions and concerns. While the number line contextualisation of the activity is very powerful, the change represented is linear movement (back and forward on the number line for addition and subtraction). How do we deal with situations where the change involves multiplication and division (e.g., he doubles the amount of money in the piggy bank or shares it among three friends)? How do we prevent prototypic thinking (Schwarz and Hershowitz, 1999). The number line also seems to restrict the type of problem that children can explore. It is difficult to see how the number line will model a problem where the unknown is not the starting point or it appears more than once. For example, Mary had \$15 in her piggy bank on *Sunday*, was given some money on *Monday*, spent \$6 on *Tuesday*, and opened her piggy bank to find \$30 on *Thursday*? And with the unknown as the starting point, closure is available for all other computations. In fact, it is possible to simply ignore the unknown.

Unknowns, young students' understanding and equals

The approach to variable used in the Carraher, Schliemann, and Brizuela's classroom activity falls into the category of *unknown* (Usiskin, 1988). While the activity uses a dynamic broader understanding of arithmetic that offers opportunities for developing algebraic ideas not available in traditional classrooms, we have concerns with young students' capacity to understand unknown. We have recently investigated this with a sample of 87 children of average age 8 years and 6 months attending four schools across metropolitan Brisbane. In an interview, the children were asked to explain how they could find the unknown in the following two situations:

16 + $\square$ = 49	54 = $\square$ - 12

The script was as follows: What is the card asking you to do? How can you find the missing number? What is the missing number?

Initial analysis of the scripts indicated that all the children understood that the task was to *find the missing number*, and most could do this for the first example. For this example, the common strategy was counting on and the common response was: *You go 16 and then you count from 16 to 49. Can I do it in my head - can I just count in my head 17, 18, 19, 20.* Most difficulties involved keeping track of how many had been counted on, *I just keep losing track of it*. Some simply counted from 6 to 9 and from 1 to 4 giving the solution of 33, *put a 3 on the 6 equals 9, put a 3 on the 1 equals 4*. Only four students found the unknown by using subtraction, *you can find the something by taking 16 from 49.* Very few children could find the unknown in the second task, for a variety of reasons that all seemed to relate to everyday classroom experiences. A common obstacle was the *non-standard* formatting of the question: *12 minus can't equal this. This is a wrong one because 12 minus can't equal 54. It's backwards*. Many children could not go beyond this point. This seemed to occur for two reasons. First the position of the = sign caused difficulties. When directed to explain what the problem was asking them to do, many said: *Fifty four take something equals 12 - you have to find the something.* Second, the position of the unknown also seemed to cause problems: *It is all mixed around. You can't have 12 minus something.* Most believed that the unknown should occur on its own after the equal sign: *12-54 = something. It would be a little number – it would be 0 because if you get 12 and take away a big number you would only get 1 and then an extra number would be 0 it would be 1 or 0.* Some simply dealt with this problem by: *flipping it over, 54-12=42. Or you could just do 54-42=12.* Only two students found the missing number by converting the problem into the correct addition situation: *You find the missing number by adding 12 onto 54.* Understandings abstracted from classroom experiences seemed to be acting as cognitive obstacles to solving equations with *unknowns.*

Conclusions

Commonly, classroom activities present arithmetic equations in the form 3+4=7, computation on the left and solution on the right. Unknowns presented in the same format (e.g., $\square$+6=11) are simpler for young children. Different formats, sequences of operations that cannot be closed (e.g. 4+$\square$+9) or two or more instances of unknowns (e.g., $\square$+7+$\square$=15) are much more difficult. “I am a number” activities can work at quite young ages because the “unknown” is first and can be ignored while the numbers are computed (e.g., *I am a number. I have been multiplied by 3 and 5 has been added. I am now 23, what was I?).* Does replacing $\square$ by “N” make the activity more algebraic? Does drawing a number line with N in the middle mean that the students are handling unknowns and understand the meaning of N+3?

Carraher, Schliemann, and Brizuela's classroom activity prepares students for algebra in its dynamic presentation of sequences operations, its potential to prevent closure (at least in the first operation), and its integration of problems, language and activity. However, the activity's use of N is not compelling and the number line places limitations on the position of the unknown that means it does not have to be known during the remainder of the operations. The activity should be extended to include all the components suggested by Graham and Thomas (1997) and combined with activity on operations that prepares students for non-standard formats and a variety of positions of unknowns. The challenge is to develop a number and operations sense that leads to algebra (and algebraic sense).

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