25 Years of PME – and the next 25

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To review adequately 25 years of work in PME and make suggestions for the future is a daunting task. In the time available for preparation, and for presentation now, I can only highlight some trends which I perceive; I hope to provoke every one here to contribute and to make this a review more representative of the perceptions of all of us of this very substantial and growing work of ours. I decided to look at the first published PME proceedings - PME2, Osnabrueck 1978, and to compare them with those of a recent meeting, PME23 in Haifa, 1999. focusing on questions such as

- What new *knowledge* do we now have on the themes addressed in that early meeting?
- In what ways have the *methodologies* changed?

At PME2 there were 70 participants from 11 countries, and 26 talks were given –so they were all plenary. In the one 400-page volume, some of them occupy over 30 pages; the mean is about 15. The following were among the main themes addressed; these are the ones I will try to trace in later work.

- ***Students' understanding of particular mathematical concepts*****,** common intuitive preconceptions & misconceptions & normal development
- ***Students' understanding and learning of general strategies*** **-** of Mathematics and of Problem-solving, including mathematical thinking and the place of these aspects in the curriculum
- ***Teaching/Learning Interactions*** in the classroom
- ***The Learning Process*** and the role of ***Reflection***
- ***Dissemination and Implementation in School Practice***

Students' understanding of concepts

This theme has attracted by far the greatest volume of work in PME. In 1978, Vergnaud spoke about practical arithmetic problems, requiring identification and use of one of the four basic operations. Such problems embody a much greater variety of mathematical structures than the four operations themselves, when one takes into account the actual context, the order of events, and whether quantities are involved or pure numbers, and are typically solved by students in many different ways, relating to different symbolic and diagrammatic representations..
Problems with the initial quantity unknown are markedly more difficult than the others, requiring a reversal of thought and resistance to the conventional associations of cue-words for subtraction or addition.

Studies such as this, which analyse the space of problems in a given conceptual field, and observe students' approaches to them, identifying key difficulties and misconceptions, have formed a rich vein of study throughout the 25 years of PME. For example, there have been many studies focusing on difficulties with decimals, and, more recently, on probability. At PME23, Amir & Linchevski identified a 'representativeness' misconception which fails to distinguish sufficiently between the probability of a particular *order* of TTHHTH compared with TTTTTT, and the probability of these (unordered) combinations of T and H; and Ayres & Way show how children's predictions in sampling are influenced by the success or otherwise of their immediately prior prediction.

Also at PME23, Silver reported a study of students' generation of all the different nets of a cube, identifying the modes of classification they used, This year (PME25) there is a study of the stages young children pass through in coming to make up composite geometric shapes from basic pieces. These last two were descriptive studies in classroom learning settings, in contrast to the quoted earlier ones, which used a more rigorous comparison of responses to carefully designed questions. This tension between realism and rigour is visible in many recent studies. There is also the question Are these stages obvious, or is there anything surprising about the results?

- **Do we now have enough knowledge of student concepts and misconceptions in the major mathematical fields, or is more still needed?**
- **Is such work best incorporated in teaching studies?**

Some of this earlier work has led to the development of general teaching methodologies. Our own Diagnostic Teaching Project (Bell, 1993) used a 'conflict-discussion' method, in which a lesson started with a few critical problems (chosen in the light of the previous research), some soluble in the 'obvious' way, others intended to elicit a particular misconception and thus creating a cognitive conflict which would be resolved by inter-pupil discussion. In an equal-time comparison with a 'positive-only' method, which taught the correct concepts before posing the critical problems, the conflict method showed great superiority in retention over several months, in spite of the smaller (because more intensive) coverage of the material. Carpenter's Cognitively Guided Instruction project had a similar philosophy.

The Learning of General Processes

My 1978 paper considered both general problem-solving strategies and the more mathematics-specific processes of generalisation and proof. On the former I was able to quote a study reported by Scott (1977) which showed superior results in 16 year-old maths exams for students who, in primary school, had taken part in weekly scientific problem-solving sessions, in which they were faced with puzzling science situations (eg some objects floating & others sinking) and had to find explanations by formulating questions to ask of the teacher, who would only respond Yes or No. In view of the significance of this transfer of general strategy training to concept learning it is surprising that it has not been followed up.

More recent studies of students' work in open problem-situations include one in PME23 by English on problem posing and model generation and a Discussion Group in PME22 supported by a substantial publication by Pehkonen on open-ended problems and their use in the classroom. But these were descriptive, not experimental studies of transfer. What we have (in the paper by English) is more detailed report of the students' work on one of the tasks.

On the learning of general *mathematical* strategies, there has been ongoing work focused on the learning of proof. In my 1978 talk, using a number of simple generalisations, to be found or verified for truth, I distinguished levels of proof-explanations in students' responses. Most pupils up to age 15 relied on incomplete checks of a few cases, few gave explanations in terms of more basic principles, and none reached a higher stage of using explicitly stated starting points.

Three examples can be given from PME23 of work in this field. Boero identifies four 'processes generating conditionality' which link with proving activities. This could be the beginning of a study which might develop in a similar way to that on arithmetic problems discussed earlier. In another paper, Douek explores the explicit and implicit problem-solving and proof strategies which undergraduates use; she argues that the imposition of formal proof structures is harmful and that effective proof activity depends on intellectual qualities fully developed during ordinary, demanding argumentative activities other than proving.
In this field there are broad underlying questions about **transfer.**

- **How far does the acquisition of general strategies for problem-solving, generalising and proving improve the subsequent learning of particular mathematical concepts, skills and applications – or of learning in general?**

- **What aspects of mathematics are the most important for students' education – the knowledge of particular concepts, OR the experience of exploring problems like a mathematician?**

How do we attack these questions?

The social environment for learning

These questions can be seen as relating to the **depth** of learning, since it is in the quest for deeper understanding of mathematics that we are led to think about these general strategies. Another step along the same path raises questions about the social and personal environment in which learning takes place, and how far it is attuned to the ways of thinking and feeling of the learners. A relevant study in PME23 was that of Boaler, who compared the very different mathematical learning environments in two English schools. One was traditional and hard working; in the other, the students engaged in a variety of individual and group projects in an informal social environment. This group out-performed the other in the standard exams. We cannot be sure, of course, that all other aspects of the two situations were similar, which suggests a need for a more extended study.

Teaching/learning interactions

Bauersfeld's 1978 talk identified a number of characteristic question-and-answer sequences common in mathematics classrooms; one he called 'funnelling', in which the questions led the pupils steadily closer to an answer the teacher had in mind. Subsequent work has included the study of transcripts of such exchanges, more recently using videotape as well. Such activity is clearly especially valuable for groups of teachers, who can discuss their interpretations of what they see or read, and become aware of possibly unsuspected aspects of their own practice. In PME23, this field was the subject of a plenary lecture by Steinbring and of meetings of an ongoing project group and discussion group. My question here is

Do we have here mainly a methodology for gaining insights, or are there also significant new concepts for teachers to use?

Learning Processes and The Role of Reflection

Though this is widely acknowledged as a most important theme, it has received little explicit attention. It can be argued that reflection is an essential component of the notion of *advance organisers*, of the *institutionalisation* phase of the French didacticians' teaching/learning problematique, and of the cognitive conflict which figures in the teaching experiments mentioned above. But if the psychologists are right, it is a powerful enhancer of learning which should appear in every teaching episode. Two recent projects with this focus are the Australian Project for Enhancing Effective Learning (PEEL) and our own Pupils' Awareness of their Learning Processes. (for both, see Bell et al, PME21). The former work arose from the perception that, in the typical secondary classroom, students participated rather passively in an activity designed and controlled by teacher and school, and that learning could be improved if they became more aware of the purposes of the various activities and took part in decisions about what and how they studied. This achieved some success, but was hard to establish and maintain, as it proved to require the involvement of the teachers taking the classes in question for most of their subjects. In our work, pupils devised their own tests, taught other pupils, interviewed each other on recent work and took part in their own assessment. They also discussed the purposes of different lessons, such as those for practising skills, learning concepts, developing strategies for investigation. Questionnaire results showed improvements in these perceptions, but the circumstances of the project (insufficient length, in particular) did not enable us to evaluate changes in mathematics test performance.

- **Do we need more focused studies of the ways in which reflection can enhance learning?**

Dissemination and Implementation

The general problem of assimilation of new insights into the school system is a long-term one. Jack Easley addressed it in a talk at PME2. He had conducted an intensive study of the lack of adoption of well-founded innovations, particularly in science education, reporting that teachers reject the recommendations of experts as unworkable with the students they have, and much inservice activity fails to achieve a meeting of minds. Not so many studies of dissemination have been reported at PME, in spite of its great importance. Such work requires more substantial time and resources than are available to most of our members. However, there was a welcome contribution to PME23 on this theme, in a plenary lecture by Ruthven on the development and attempted dissemination of a calculator-aware curriculum in the UK, initially led by Hilary Shuard. He concluded that a successful innovation required a very thorough analysis of the entire content, progression and teaching methods of the curriculum, and its adoption as part of a coherent and committed programme of school development and ultimately of systemic reform, rather than the isolated responsibility of individual teachers. Such a programme the National Numeracy Initiative, has in fact been developed and prescribed by the UK government, with detailed lesson programmes and considerable inservice support. However, this has coincided with a politically inspired ban on calculator use until the last two years of primary education! Such are the hazards securing useful implementation of our work.

Another substantial study was conducted by Brown and her London team, on Effective Teachers of Numeracy. A substantial sample of Primary schools was used, and each teacher's success in improving pupils' performance on the standard tests over a year was correlated with various teacher characteristics. The most successful were those who had a well-connected relational understanding of the mathematics curriculum, often gained from in-service courses with this aim.

- **How can we find out more about the processes by which new knowledge and insights eventually influence the education of our students?**

Methodological Trends

In general, there has been a great expansion in the number of works presented, and many have been on a smaller scale than in 1978. Much detail has been filled in, especially in the field of students' understanding, and more work has been done in realistic classroom settings. At the same time, there has been perhaps less work integrating all these particulars into coherent bodies of knowledge. The Research Fora give more opportunities for extended exposition and critique, and this is a significant development; but they tend to adopt their own theories, and aim to display the power of the theory to provide a framework for constructing a teaching material, and explaining the results of its use. Aspects of the theory itself are not generally tested.

- **How can we encourage more critical tests of major theories?**
- **Do we need more integrative review studies?**

The trend in ***methodology*** I see is towards more observational studies of students' responses, often in classroom settings, to some innovatory curriculum element. The purpose of these is usually to demonstrate the superior effectiveness of the innovation, but there is often no formal comparison; one draws conclusions by reference to a mental comparison with what one would normally expect in the topic in question.

This raises the question of how to read and interpret such reports. We have moved a long way from the assumption that we are establishing hard scientific generalisations beyond reasonable doubt. So the way in which knowledge accumulates for each of us is by reading many reports, each very specific in terms of topic, method, underlying theoretical assumptions, and integrating them in much the same way as we do our general knowledge of the world. I used to think of this as being like reading good novels, which generally have some general insights to convey about the human condition, but clothed in a specific story which adds interest and also convinces us by showing how such things might happen. The analogy is not perfect, because our reports are not meant to contain a fictional element! But the fact remains that what is taken from it depends on the perceptions and preconceptions of the reader.

- **Is this an appropriate way of conducting and reporting research in our field?**

References

For volume numbers for PME references, see text

Bell, A, 1993, Principles for the Design of Teaching, *Educational Studies in Mathematics* **24**(1), 5-34.

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