REACTION 2: DESIGN AND USE OF SUBSTANTIAL LEARNING ENVIRONMENTS IN MATHE 2000

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This short reaction is organized around the four principles at the heart of the MATHE 2000 project that are listed in Wittmann's introduction to the Research Forum. However, I want to begin by expressing my admiration for the project, which entails a unique, provocative and successful concept of how to organize and do research in the field of mathematics education. The decision to place substantial learning environments (SLE) at the core of the research and the developmental work and to use these SLE as the common starting point for the design of curricula and textbooks, empirical studies into children's thinking and communication, and teacher education, has proven to be a very valuable and effective strategy both from a theoretical and a practical point of view. Although the project has many similarities with other projects, such as the Realistic Mathematics Education (RME) project in The Netherlands, there seem to be some differences with these other approaches in terms of aims, content and research strategy, -- some of which will be pointed out below.

Fundamental ideas of mathematics as guidelines. As stressed and illustrated in all papers, SLE are at the very heart of the MATHE 2000 project. Throughout the papers it is shown how the design of SLE -- both for pupils and for student-teachers -- is based upon the progressive elaboration of the epistemological structure of elementary mathematics. Major characteristics of SLE are that they represent central objectives and contents at a certain level of instruction, and that they are related to significant mathematical objectives and contents beyond that level. Quite strikingly, almost all examples of SLE relate to pattern finding (see, e.g., Wittmann's "rechendreiecke", Selter's "number chains", Steinweg's "series of beautiful calculation tasks", and Steinbring's "number walls"). I consider this as one of the clearest manifestations of taking the (quintessens of the) subject of mathematics as a "science of patterns" -- already from the very first years of elementary mathematics education on -- very seriously. Equally strikingly, these pattern-finding tasks are given in a purely symbolic format and not embedded in a real-life context. Although it should be acknowledged that some of these pattern-finding tasks are briefly introduced through a realistic or fantasy context and that these symbolically presented pattern-findings activities are complemented with more "applied" kinds of problem-solving activities in the textbook series "Das Zahlenbuch", the early and frequent recurrence of these context-lean pattern-finding activities -- is remarkable. Interestingly, Steinweg's study of 6-10-year old children's understanding of patterns reveals that these children do not have any reservations against such unfamiliar, context-lean tasks, that they fully accept them, and do not need any extrinsic motivation.

Learning as a constructive and a social process. The view of learning that lies at the basis of the MATHE 2000 project is in line with current conceptions in instructional psychology in general, and in the psychology of mathematics education in particular, wherein learning is considered an active, constructive, cumulative, self-regulated, goal-oriented, situated and social process of knowledge building and meaning construction. Throughout the papers, it becomes clear that in the MATHE 2000 project not only the subject (i.e., mathematics) but also the learner -- with his or her own constructions, interpretations, beliefs and goals -- is taken very seriously. The documentation that this process of knowledge building and meaning construction is fundamentally social in nature and that it is shaped by the norms, rules and agreements that constitute the culture of the mathematics classroom (especially in Steinbring's paper) fits with recent analyses by others (e.g., Brousseau, Cobb).

Teaching as organizing learning processes. Complementary to the above view of learning, teaching is conceived in the MATHE 2000 project as inducing and supporting children's active process of knowledge building and meaning construction, rather than providing them with the necessary knowledge and skills to pass the next test. As in most other countries where similar new conceptions of teaching and learning mathematics are being developed and implemented, MATHE 2000 risks of being criticized for giving children too much freedom in developing their own strategies, for paying too less attention at the basic skills, for not progressing quickly enough towards abstraction and formalisation… For many reasons, these criticisms do not hold to the MATHE 2000 project. The carefully designed SLE, grounded in a thorough epistemological and historical analysis of the domain and supported by empirical research on pupils' reactions to the materials, provide ample opportunities for the well-balanced combination of the practice of skills with higher mathematical activities. Moser-Opitz's evaluation study of children with special needs provides some empirical support for this claim, -- even among those children for whom this new approach is deemed to be most detrimental.

Cooperation with teachers. Finally, in the MATHE 2000 project, not only the subject and the child, but also the teacher is taken very seriously. This is evidenced, among other things, by the great attention given to teacher pre-service and in-service education and to cooperation with teachers. Selter's approach to pre-service teacher education, with its multi-dimensional view of the components of teacher professionality, its emphasis on "de-constructing" student-teachers' erroneous beliefs about and negative attitudes towards mathematics (education), and its focus on the development of teachers' background knowledge and awareness rather than of their executive skills, convincingly demonstrates the value of SLE as the keystone, not only for elementary mathematics education, but also for teacher education.