REACTION 1: BETWEEN PSYCHOLOGISING AND MATHEMATISING

Kenneth Ruthven, University of Cambridge School of Education

This necessarily brief response will focus on the central idea informing the rich body of work developed within the Mathe 2000 project: that of 'substantial learning environments' as organising systems for mathematical and didactical transactions between students and teachers, and between teachers and teacher educators.

As Wittmann illustrates, while common epistemological, psychological and pedagogical principles underpin these environments, what characterises each of them is a distinctive flexible mathematical artefact around which classroom activity can be organised. Steinbring and Selter variously show how such environments are brought to life through classroom activity in which mathematical structure latent in an artefact is actively (re)constructed by learners with assistance from their tutor. Underpinning successful learning through this co-operative and communicative activity are the complementary processes by which codified mathematical knowledge and reasoning are -in Dewey's term- 'psychologised' to recast them in forms more accessible to, and usable by, students; and through which student experience is -in Freudenthal's term- 'mathematised' to recast it in disciplinary terms. Evident too, in the design principles and exemplary environments, is an orientation towards -in Bartlett's terms- 'simplification by integration' over 'simplification by isolation'. Indeed, Moser Opitz conceives her study in this way, and provides some empirical support for the integrative position through evidence of successful pre-school learning as well as of distinctive trends in the development of student competence and strategy over the first year of schooling.

Some of the complexities of these processes emerge from Steinweg's study which explores -and largely rejects- the conjecture of 'natural or genetic' development in understanding of number patterns 'with advancing age'. Intriguingly, an important socio-cultural dimension is suggested by the illustrative student responses to the unfamiliar type of 'beautiful package' task. When Jana 'suspects the calculations to be wrong', she seems to be interpreting the task simply as one of checking the solutions to a conventional school exercise. From the way in which Réné 'repairs' the exercise, he appears to be interpreting it as a puzzle devised so that the answers form some pattern (somewhat analogous to the picture pattern formed by the arithmetic domino illustrated by Selter). Finally, Kim forms the 'epistemologically evident' interpretation that not only the answers but the problems themselves are intended to conform to an overarching mathematical pattern. Arguably, then, Jana, Réné and Kim were not tackling the 'same' task, and only Kim was tackling the task as envisaged by the designer. 'Success' in the task seems to depend on students' anticipation of new didactical norms as much as their grasp of mathematical structures. While over half the spontaneous responses fitted the maximally mathematically formatted interpretation of the task, one suspects that further analysis would reveal some troubling relationships between social position and task interpretation. Considerations of equity in making these (new) rules of the classroom mathematics game explicit lend further support to Steinweg's suggestion that such ideas may need to be taught more actively.

Nevertheless, the idea that teacher and students are working on 'the same task' and seeing ‘the same structure’ is a useful strategic fiction where classroom participants are seeking to co-ordinate and articulate their interpretations of a task and their resulting constructions. Steinbring’s microgenetic analysis of the production of a public explanation shows such processes in operation. Through interaction between teacher and student, the focus of this episode of classroom communication shifts from the articulation of a generic mathematical structure which the student perceives as fundamental to an artefact, towards the concretisation of this idea in specific examples inscribed on the board. Far from the front of the classroom, Timo uses positional language to evoke the common structure he perceives in number walls. But as he moves closer to the board and starts physically pointing, ‘the outmost’ becomes ‘here’, and his account becomes more tied to a designated example. In response to teacher solicitations to concretize his ideas, his accounts become progressively more focused on the manifest content of specific number walls. As Steinbring notes, this layered explanation casts latent structure into a form of relief, making it more readily accessible to other participants.

These accounts lead one to appreciate the considerable demands of managing these forms of classroom activity so as to promote effective learning, and both Moser Opitz and Selter emphasise the change and challenge this presents for many teachers. In this respect, one admires the sense and subtlety with which substantial learning environments seem to extend and reshape the familiar didactical form of the exercise rather than wholly rejecting it. An important aspect of this is the way in which tasks seek to ‘combine the practice of skills with higher mathematical activities’. Correspondingly, Selter emphasises the potential of substantial learning environments as organising structures within teacher education around which concerns to develop the mathematical, psychological, didactical and practical expertise of primary teachers can be co-ordinated.

One crucial question is how to conceive mathematical expertise for primary teaching. For the honourable tradition of ‘elementary mathematics from an advanced standpoint’ may lead to an over-extension of a single-minded idea of progression. Take the treatment of a generalised form of arithmogon in terms of the theory of systems of linear equations. Whilst appreciating that the theory is developed ‘just as far as is necessary to frame a certain class of problems’, and noting student teachers interest ‘to see how abstract algebra applies to concrete numbers’, it is not clear how such ideas -estimable in their own right- might readily be brought to bear in shaping children’s mathematical enquiry and argumentation. There, the teacher is required to operate with great ingenuity, flexibility and fluency within the construction zone of the students. So, while a matrix representation neatly captures the relationship between ‘inner’ and ‘outer’ values of the Rechendreiecke, a more didactically pertinent representation might highlight the complementary arithmetic relationship between each ‘inner’ value and its spatially opposite ‘outer’ counterpart, and between ‘inner’ and ‘outer’ sums. In short, the question is one of finding a sound balance between ‘psychologising’ and ‘mathematising’ in professional education.