GEOMETRY AT WORK
(or Pytagoras will never fail you!)

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Abstract: *This research report presents an empirical study of the geometrical knowledge 'hidden' in the working practices of Argentinian bricklayers. The aim of the research was to find clues to improve the mathematics education of adults within the vocational and professional training system. The results of the study concerning the construction of right angles are partially presented to illustrate some of the differences between the geometric knowledge situated in the school context and in the work place context. The paper finishes by raising some questions about the need and the possibility of trying to establish links between the different contexts for everyday mathematical practices and the school practices.*

Some of the reasons for the study

In the Argentinian province of Rio Negro, where the study was developed, the illiterate population reaches 34.04% (illiterate and functionally illiterate). Certainly, there are economic and social reasons that explain why a significant number of young people drop out school before reaching the end of compulsory school age. These reasons are reinforced by the academic failure of some students. Mathematics, as a school subject, has a big share of the responsibility for this failure, since it still operates as a selection instrument. This selection, moreover, takes place within a system where the meaning of the mathematical knowledge decreases for the student as she/he progresses.

On the other hand, only 3.1% of the illiterate adult population attend vocational schools, adult schools or literacy and numeracy programs. Moreover, the programs offered have little connection with the real needs of their potential students, or with their possibilities to gain knowledge, or with their abilities, or with procedures that would facilitate their effectiveness in the work place and their participation in the social life as citizens.

Within this context, the main goal of this study was to analyze the characteristics of the mathematical knowledge as used and valued by those that could benefit from attending vocational and adult programs[1] compared to the mathematical knowledge as it is promoted for them. In particular, we studied the practical problems that building workers face, as members of a community of practice, in their work-places to identify the geometrical knowledge, either hidden or explicit, that they use to solve them. We also analyzed how the workers relate to the knowledge learnt in school and to the knowledge in-practice, and how they understand and value them as knowledge. We chose to work with the community of bricklayers and building workers because of our interest in the teaching and learning of geometry and of spatial abilities and concepts.

Framework and research questions.

Our study is framed within the everyday mathematics cognition trend that sees everyday mathematics as a body of mathematical knowledge and practices accumulated through the development of everyday activities (Nunes, Schliemann and Carraher, 1993). Gerdes (1988, p.140) referring to the traditional forms and shapes given to objects affirms that this *'It constitutes not only biological and physical knowledge about the materials that are used, but also mathematical knowledge, knowledge about the properties and relations of circles, angles, rectangles, squares....'*. Our starting point is that the traditional procedures used by building workers also reflect a lot of 'hidden' or 'frozen' mathematics (Gerdes, op.cit.).

The authors of this paper would not make a strong argument in favour of the 'utilitarian mathematics culture' but are convinced that knowing more about this hidden mathematics in the adult context could contribute to the development of a realistic curriculum and would help to give meaning to mathematical tasks, to motivate adult students, and to them valuing their own situated knowledge. Nunes and Bryant (1996, p.108) referring to the mathematical knowledge that can be found embedded in many everyday activities, state that *'These points are not mere curiosities about mathematics: they have an impact on children's future chances because they can actually affect how well they do in school'*.

Our purpose is not to discuss whether or not the geometrical knowledge used at the work place to solve practical problems should be considered 'true' geometrical knowledge, but to find clues that contribute to create learning situations that establish links between the knowledge at work and the school knowledge.

With this purpose the research aimed at identifying and analyzing:

[1] Due to length reasons, we do not report here the analysis of the formal educational system that refers to vocational, professional or adult schools.

a) work tasks, as developed in practice, that involved the use of geometrical knowledge and the planning and the verification of those tasks by the workers,
b) the use of work tools that materialize mathematical knowledge, and
c) the acknowledgment and valuing by the workers of their using of mathematical knowledge and its usefulness.

The problem that motivated our analysis was essentially to establish some of the differential traits between the two forms of geometrical knowledge, that embedded in work and that promoted at school.

Nunes (1992) points out that there are two distinct approaches to the study of cultural influences on mathematical knowledge, one based on the content of the knowledge and another based on the use of the knowledge. We wanted to know if the knowledge, in the sense of content of knowledge, present in both contexts of practice was the same and to identify and describe work tasks in which geometrical knowledge was embedded. However, our main focus of interest was on how this knowledge was used, what was its function in the thinking processes, in the way of approaching and solving the problems, not only from the point of view of mathematics, but also from the perspective of the relationship that the person establishes with the knowledge.

Research procedures

When deciding how to approach the world of the building workers it was clear to us that our research methods would depend not only on the research aims but also on the research context. We really felt outsiders in that particular world: our knowledge of the work tasks was sparse or zero, our gender made us 'uncommon' among them, our social norms came from the academic world, our register of language was completely different from theirs… and we had *'plenty of time to waste, while they were there to work'* (as Juan confessed when we reached a friendly relationship).

To overcome this obstacle the talking of the first author with the engineers, architects and responsible persons in the building company was a great help to us. Through those conversations, we were not only granted permision to develop the study, but more importantly, we were 'introduced' to the field. The languages and the contexts being closer, the building company's personnel understood the aims of the research, and they provided us with descriptions of the roles and the tasks performed at the work place, and they also helped with the selection of workers to be interviewed.

Observing the actual working practices, and reading some previous interviews, were useful to identify certain activities where geometrical knowledge was potentially in use or embedded. However, observation was not the main procedure used. We could have gained information about how knowledge is transmitted from experts to learners in the community of practice, but we discarded this method: it was too expensive on time, too dependent on our interpretation, and it would give us little information on how decisions were taken.

Interviewing the workers was the research method chosen. The first author interviewed 12 workers selected according different criteria: the tasks developed and level of responsibility in the work place, their experience and their abilities as workers, and their previous school experience. The suggestions from the company people were very useful, since they helped to ensure that we would interview those bricklayers who would be ready to help and open enough to make the interviewing smooth.

Only bricklayers, building carpenters and foremen were interviewed but not building labourers, since they, being unskilled workers, rarely took decisions on their own regarding the planning or supervising of the work. All of them had more than four years practice, and regarding their level of formal education there was a range varying from illiterate workers to some who had passed succesfully through part of secondary school education.

The content of the interview was directly related to their actual work or required to evoke it. This means, for instance, there was no question asking *how do you draw a right angle?*, but *how do you proceed to make the frame of a door?*. The questions asked the reasons for a technique observed or the explanation of a particular procedure. Some examples of the topics that guided the interviews are:

a) *how to ensure the perpendicularity of the walls and the floor, how to ensure that the floor is flat and horizontal, how to know where the walls have to be built, how to build a doorframe, a windowframe, an arch…*

b) *how to use some of the working tools like the carpenter's square or the angle iron, the plumb-line, the spirit level …*

c) *how conscious they were of having learnt at school (if they attended) any knowledge that was useful to their job, of using any mathematical or geometrical knowledge and how they valued that knowledge …*

The interviews took place in the work place, some of them in the open air, some of them in the storeroom. The interviews were tape-recorded. The difficulties of convincing them to allow us to use the tape-recorder discouraged the use of a video camera, despite the fact that the gestural language was of great importance in our study. During the interviews, drawing tools like those normally used in the work place were at hand and spontaneously used. The interviewer took notes of the gestural language and drawn schemes to help with the later transcription. Keeping a record of the gestural language was of great importance, because many times it was the clue to understand the non-formal language. Data was interpreted through succesive and recursive analysis.

Some findings

Following we present some of the findings to illustrate the tasks identified as involving geometrical knowledge and to describe how this knowledge is used. The findings correspond to the procedures used in the construction of right angles and its verification. Six of the workers interviewed referred to and explained the construction of right angles in different tasks. The procedures reported were repeated in different interviews and involved:

1. <u>The Pytagorean relationship</u>

One procedure uses a particular Pytagorean triplet to construct and control the right angle when building window or door frames. Once having fixed the two sides for the right angle - 0.60m and 0.80m - then the length of the hypotenuse being 1m will ensure that the constructed angle is a right one.

> **Juan:** *I tell you a trick that will never fail you. You make it here 80 and there 60 and then you open or close the frame until you get one meter, then the sides are squared.*

Some of the workers show a functional use of this knowledge that goes beyond just a routine use

> **Carlos:** *You measure 80 and then 60 and you have to have one meter, and you have there the squared corner and then you increase when the length of your square increases.*
>
> **Interviewer:** *And how do you increase?*
>
> **Carlos:** *Instead of 60, 120 and you increase that way, and the same there, 80, 160 and to obtain the square, you have one meter, two meters…*

We want to note that while in school the 'direction of the implication' of the Pytagorean relationship used is the 'if' (if you get a right angle, then the relationship verify itself), the workers use the other direction of the implication, the 'only if', to construct and to control the right angles:

> **José:** *for instance, to square, we make on one side 60, on the other side we make 80, and we see, if we get one meter, then it is because it is squared ... but if we get a little more, this is out of square, it is out of shape.*

2. <u>Properties of the diagonals of a rectangle</u>

Another geometrical fact hidden in one of the procedures used to construct and control right angles concerns the properties of the diagonals of a rectangle or of those of a square. The first one is very similar to that found among Mozambican peasants by Gerdes (1988).

Santiago gave the following explanation, accompanied by a drawing, of how he proceeded when he needed to draw a right angle while 'copying' the basic plan of the house onto the actual ground.

Santiago: (he wants a perpendicular to AO at O)…*like being squared here*[2] (pointing at O)*, the axis* (the vertex O), ... *we start here* (O) *to all the sides, this is the main axis* (pointing at the segment AO)*, we tighten the thread from here* (from A to B by O)*, and another from here* (OC, a thread not yet fixed to the ground)*, and then we have 5 meters* (as required by the plan) *here* (OA)*, 5 here* (OB*), and 5 here* (OC)*, then to have this squared* (right angle at O) *you need* (to get) *the same* (length) *from here to here* (AC) *and from here* to here (BC).

Again the direction of the implication used by Santiago ('only if') is the contrary to the one more commonly used at school ('if').

3. Concretizing the idea of perpendicularity

Another procedure used to construct right angles when building door or window frames is based on the idea of perpendicularity concretized in the intersection of vertical and horizontal lines, concepts that correspond to the physical world and that are materialized in the working tools: the spirit level and the plumb-line.

Daniel: *You hang the plumb-line at the two corners, then the frame is 'plumbed', and then you have to get it at 'level', at the corners, with the spirit level.*

4. Using a working tool that concretizes the geometrical concept

Finally, another procedure used was based on the use of the iron angle, a metal square, which materializes the geometrical concept of right angle.

Daniel: *This is an iron angle, you put it at the corner of the door frame, and you square it, if your square is false, then you get it more open, and then with the iron angle you should adjust the frame, the frame is not yet fixed.*

Some results and further questions

The range and richness of content of knowledge we could identify throughout the different interviews is impressive, to the point that we found a procedure to construct angles of any measure smaller than 90º based on the concept of tangent. The content is related to the ideas of parallelism, angle, plane, straight line, proportionality, circle, triangle, square and rectangle. We also observed abilities related to measurement, visualization, interpretation of plans, and locating. Concerning how this knowledge is used there are some constant features that repeatedly appeared. We summarize them below staying with the example of right angles.

[2] The letters in the drawing are ours to help the reader following the explanation.

In the interviews, most of the workers referred to the use of more than one procedure and related its use to the requirement of the task or the particular situation. We may talk about two different kind of procedures: those based on geometrical facts, and those based on the use of physical tools that materialize geometrical concepts. For instance, only one of the workers referred to using the iron angle to construct right angles; the other two that referred to the instrument explained how to use it to verify the squareness.

One of the workers, Eduardo, referred to using three different procedures (1, 2 and 4) and made explicit the fact that he is able to adapt the procedure to the particular requirement of the task. Eduardo is the one who has both a bigger responsibility in the work place and a 'better' academic history. The richness of the procedures used could depend on both the task to be solved and the worker's knowledge of relevant geometric facts, with this knowledge having been acquired 'in situ' or at school.

Referring to different uses of the knowledge we want to note that the different use of 'implications' we have illustrated in the previous section appears repeatedly and relates our findings to the idea of inversion in the works of Lave (1988), Carraher (1986) and Schliemann and Carraher (1990).

The interviews also showed us that the relationship established with the knowledge in use was different from that promoted in school. In particular, the workers made explicit that they could use more than one method in every situation, one to control the construction and the other to verify it. For instance, two of the workers use the Pytagorean relationship and the properties of the rectangle to control the construction process, and to verify the result one of them used the iron angle. It is important to note that this idea of further verification is one of the characteristics that makes a distinction with what is taught at school and is linked to the fact that in work places the idea of 'error' is linked to 'loss of effectiveness'.

We observed a wide range of possible procedures to solve particular tasks, and that the 'frozen' knowledge can be made operative, adapting it to the requirements of the context, for instance adapting the procedure to the size of the object by using proportionality facts. Also different personal experiences may engender different degrees of flexibility for the procedures used by the participants.

Some of the procedures used can be thought as being general, as being applicable to several circumstances, some of them being based on geometrical properties or facts, flexible in their application, and some of them being concretized in tools. Among the first we could include the use of the Pytagorean triplets, and among the second the use of the iron angle. Some of the procedures are specific, being like algorithms, as set of rules for getting a specific output from a specific input, like the use of the plumb-line and the spirit level to make a door frame. When geometrical knowledge is flexibly used we can think of the procedure as being a problem solving strategy, not a simple routine technique.

As a conclusion, we want to share with the reader further issues that we have raised ourselves. If one wants to obtain new insights for the teaching of mathematics, more detailed descriptions are needed about the informal and formal procedures for solving geometrical problems and, in particular, how the particular situation presents conditions that elicit different types of strategies. However, at which point can the informal content of the lessons become an effective foundation for more formal mathematical reasoning, and what are the limits of engaging students in contextualized reasoning and learning?

The situation in which the problems are embedded may have a strong impact on how they are solved, the impact seeming to result from the meaning that problems have for the ones engaged in problem solving. School mathematics promote classroom exercices that elicit algorithmic solutions, making the students lose track of the meaning of the problem within the situation and within the structure of the knowledge.

The divorce between understanding and the use of algorithms is a by-product of an educational system that leads children to focus not on meaning but on routines. The algorithmic approach promoted in our classes makes students use knowledge as a tool, not as an object in itself to be thought of. Therefore we have two more questions: Can everyday mathematics help with the transition from tools of knowledge to objects of knowledge? Can actual mathematics teaching benefit from teachers and text-book writers, among others, being acquainted with non-formal mathematical knowledge and everyday procedures?

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