REACTION TO "ALGORITHMIC AND MEANINGFUL WAYS OF JOINING TOGETHER REPRESENTATIVES WITHIN THE SAME MATHEMATICAL ACTIVITY"

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Almost 20 years after I carried out my first mathematics education research on students' learning of algebraic ideas within a Logo environment I am again starting a new project on teaching and learning with new technologies. What have I learned in this intervening period? Firstly I know that although in the UK we are moving towards a ratio of 1 computer to 8 students in every school, mathematics teachers are amongst the most resistant of teachers to incorporate computer activities into their teaching. I know that although in our earlier research we theorised learning we did not adequately take into account the student's history of learning mathematics, the culture of the classroom, the culture of the school and the culture of the mathematics curriculum as influenced by both mathematics and national policy. I know that however beautiful the new technology there is nothing essential about this technology which implies it will be used in a particular way (although I do believe that it is possible to talk about the 'affordances' of a technology (Gibson, 1977)). Unfortunately in our consumer culture those who advocate new technologies tend to 'sell' them as if there is a cause and effect relationship between the use of the technology and student's learning. In particular I have sat through many presentations on the use of graphics calculators in which the presenter focuses on 'what the calculator can do' as opposed to 'what the student-plus-calculator might do when using a calculator'. Thus I welcomed an invitation to participate in the PME Research Forum on the potential and pitfalls of technological tools in learning mathematics.

In Hershkowitz and Kieran's paper they discuss the ways in which students in both Israel and Canada work with a graphics calculator on an investigative activity which involves predicting the growth patterns of the areas of three families of rectangles (which have been represented to them figuratively). The group of students in Israel chose from the outset to represent the areas of each family of rectangles as algebraic functions and then graphed these functions with their graphics calculators. This led to a graphical representation which they could interpret from the point of view of deciding on the relationship between the growth of each family of rectangles.

The group of students in Canada took a different approach. They used 2 x 10 tables to represent the growth for each family of rectangles. For each table they firstly entered a column 1,2,3…10. The task then became one of filling in the correct area for each family of rectangles for the first 10 years of growth. They managed to do this correctly. The rules to generate the 1st and 2nd family of rectangles were computed in a recursive manner 'in their heads' as opposed to using an algebraic rule in the calculator. However they did use a universal rule for the 2nd family of rectangles (which was increasing squares), although it is not clear in the paper whether they represented this rule in their calculators. By the end of this beginning activity they had produced a table of values for each family of squares. They knew that they needed to produce an algebraic model in order to plot graphs of growth and one student suggested “let’s do a linear regression”.

Hershkowitz and Kieran ask why the students so mechanistically used the calculator to produce an algebraic rule when the tables they had produced could be ‘read’ for the information which they had been asked to obtain. However the authors point out that these students had previously worked for several weeks on problems which involved finding models for real-world data and they had used linear regression in this respect. My interpretation of what is happening here is rather different. The students are firstly making sense of the problem presented to them by drawing on their previous experience of finding rules to represent data generated from real-world situations. In these previous situations fitting a straight line was a way of modelling a situation which may or may not have been underpinned by a linear relationship. Their approach to these messy real-world situations had been to start from data and then find a model which fitted the data. So for me it is not surprising that when presented with an investigative problem (which was underpinned by explicit algebraic models) they first decided to generate data from the situation and then tried to find rules to fit this data. They did not use the visual representations of the families of rectangles to find algebraic rules and they did not read the tables of data which they produced in order to find the relationship in the growth patterns. I would describe their behaviour as making-sense-for-them although it might not have made sense to an observer who had expected another approach. In this respect I disagree with Hershkowitz and Kieran’s analysis that “from the beginning of applying the regression option of the calculator tool and up to this moment, the group had appeared to be marching along a mechanistic path in a kind of automatic fashion, without doing very much in the way of reflection”.

When this group of students had produced graphs using linear regression they then began to question their approach, because the graphs produced did not tie up with how they were reading the table and the figurative representation of the problem. For me the story is one of students’ actively making sense, working with the calculator, the problem and multiple representations generated until they can make sense of the whole. The first phase of the problem seems to have been a getting-started phase which drew on previous experience of working with real-world messy data, in which it is not usually possible to read patterns from the data. They needed to generate algebraic rules from their data in order to produce graphs with their graphics calculators and the calculator’s regression function was how they had previously learned to do this. In the final section of the paper Hershkowitz and Kieran depart from their earlier analysis and start to discuss the students’ approach in a similar way: “Our second class had the experience of dealing with real world problems with nonidealized data, which usually do not fit perfectly an algebraic model. This encourages the use of regression techniques as a means of obtaining an algebraic model".

The starting point of the paper centred around the idea of two kinds of representatives "representatives that do not represent the properties of the mathematical objects involved at all, and representatives that do". This perspective, I suggest invests too much power in the representatives and not enough in the person plus representative (Perkins, 1999). It also suggests that 'if only students produced the 'prototypical' representative then they would be able to 'see' what the teacher wants them to see". The case discussed throughout the paper has raised very important issues about learning mathematics with new technologies. The comparison between the students in Israel and the students in Canada tells us that students with different backgrounds can approach the same problem and the same technology in very different ways (similar issues have been discussed in Sutherland & Balacheff, 1999).

The question for me is how can we understand student's sense making in these situations? The paper has provoked me into thinking whether it would be possible to develop a theoretical model of teaching and learning mathematics which would enable us to predict that the Canadian and Israeli students might have been likely to approach the problem in the ways in which they did. For me, such a model would place the student as a central sense-making agent who works with whatever resources and theories are available (technologies, representations, language) and brings a history of working with resources and theories in order to solve the problem presented to them. In this model there would be no 'best' representation and no 'best' technology although each representation and each technology would 'afford' a range of different potentialities.

Working with both the graphics calculator and real-world data problems as a way into algebra is likely to lead to an under-emphasis of the power of algebraic formulae for representing patterns. It could also be argued that this type of work is more like science than mathematics. The paper does not ask why the students in Israel took such a different approach and there is no discussion of the previous experiences of these students. My final question to the authors is where was the teacher in all this work? A teacher on-hand could discuss with the Canadian students the difference between data which derives from real-world situations and data which has already been generated by an algebraic formula. There is much work to be done here and I anticipate a very fruitful discussion at the PME Research Forum.

Gibson J J (1977) The theory of affordances, in Shaw R & Bransford J, *Perceiving, Acting and Knowing: Toward an Ecological Perspective*, LEA, NJ.

Sutherland R & Balacheff N (1999) Didactical complexity of computational environments for the learning of mathematics, *International Journal of Computers for Mathematical Learning*, Vol 4, pp1-26.

Perkins D N (1993) Person-plus: A distributed view of thinking and learning. In Salamon G (ed) *Distributed Cognitions: Psychological and Educational Considerations*, CUP, Cambridge.