INFLUENCES ON AND FACTORS CHANGING TECHNOLOGY PRIVILEGING

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Based on classroom observations and interviews over two years, this study reports on the different ways two teachers incorporated use of a computer algebra system into their teaching of introductory differential calculus. Factors that contribute to successful CAS teaching were monitored: teaching approach, preference for representations, and use of technology. In the first trial the teachers adopted distinctly different ways of using CAS. In the second trial, these differences persisted, but they made some changes. In response to new knowledge about the testing, one teacher extended his focus on rules by teaching new CAS procedures. In response to new students, the second teacher reduced CAS use for routine symbolic procedures while continuing to use it to support conceptual understanding.

Background

It is generally acknowledged that teachers' classroom teaching practices are influenced by their underlying beliefs and knowledge about mathematics and mathematics teaching. Reporting on a professional development project during which teachers explored learning and teaching with computer activities, Noss and Hoyles (1996) monitored changes in the ways the teachers used Microworlds technology. These changes served as a 'window' on the teachers' mathematical beliefs and pedagogy. Noss and Hoyles observed that ″there is a mutually constructive relationship between what teachers believe and what they do″(p.201). They maintained that while changing their pedagogy, ″Teachers are not pushed arbitrarily by 'constraints'. Neither are they free agents″(p.201). The present study provides a detailed description of how such changes in teaching with technology come about and how they are linked to changes in teacher knowledge and beliefs.

Attempts have been made to classify teaching practices and to relate them to teachers' knowledge and beliefs. Shulman (1986) distinguishes between teachers' content knowledge (understanding and organization of knowledge of specific topics) and content pedagogical knowledge (knowing ways of presenting knowledge to students, including representations, and understanding what makes the learning easy or hard). These types of knowledge are interconnected (Even & Tirosh, 1995). Fennema and Franke (1992) describe other types of teacher knowledge; pedagogical knowledge (knowledge and planning of teaching procedures, behaviour management and motivational techniques), knowledge of students and knowledge of institutional constraints.

Teacher content knowledge impacts on teaching practice which in turn impacts on student learning. Jablonka and Keitel (2000) believe that teachers with highly developed content knowledge are ″more flexible in structuring content for teaching and in discussing students' ideas″(p.120). Gutstein and Mack (1998), in a study of teaching fractions, show that a teacher's teaching decisions and practices were derived from the depth of her interrelated knowledge of content, pedagogical content, and of students.

Kuhn and Ball's (1986) model of teaching practice was commended by Thompson (1992), in a comprehensive survey of teachers' beliefs and conceptions, ″as constituting a consensual knowledge base regarding models of teaching″(p.136). The model identifies four different teaching practices linked to particular beliefs about mathematics and goals for teaching, and characterized by related teaching styles.

> 1. Learner focused: mathematics teaching that focuses on the learner's personal construction of mathematical knowledge;
> 2. Content focused with an emphasis on conceptual understanding: mathematics teaching that is driven by the content itself but emphasizes conceptual understanding;
> 3. Content-focused with an emphasis on performance: mathematics teaching that emphasizes student performance and mastery of mathematical rules and procedures;
> 4. Classroom focused: mathematics teaching based on knowledge about effective classrooms. (Kuhn & Ball, 1986, p.2)

The relationship between teaching styles and student achievement for teaching with technology is a topic of current interest. For example, student success (i.e., improved conceptual understanding) on calculus courses taught with technology is attributed to the adoption of student-centred teaching practices within a constructivist perspective (Keller, Russell & Thompson, 1999; and others). Kendal & Stacey (1999 & in press) use the word *privileging* (originally used by Wertsch, 1990) to describe a teacher's individual way of teaching. It includes decisions about what is taught and how it is taught including: what is emphasised in the content (what is stressed and what is not stressed), what representations are preferred and ignored, the attention paid to procedures and concepts, rules and meaning, and how much is explained or left to students to work out for themselves. Privileging reflects the teacher's underlying beliefs about the nature of mathematics and how it should be taught. It is derived from an interplay of teachers' beliefs and interrelated knowledge sources (content, content pedagogical, pedagogical), is moderated by institutional knowledge about students and school constraints, is manifested through teachers' practice and attitudes, and is highly influential in student learning. Privileging has several components and in this study of the teaching of calculus with technology, we focus on teacher choices about:
(1) *Teaching approach* (evidenced by teaching method and style using Kuhn and Ball's classification above).
(2) *Calculus content* (evidenced by representations of differentiation taught).
(3) *Use of technology* (evidenced by the nature of use of the CAS calculator).

This study reports on the three aspects of each teacher's privileging during the teaching of an introductory calculus unit. It monitors the changes that occurred over the two years and explores the impact of new knowledge and a new situation on the changes in technology privileging, linking them to each teacher's beliefs and pedagogy.

Method

In two successive years, Teachers A and B volunteered to teach the same 25 lessons on introductory differential calculus course to their Year 11 classes (16-17 year olds). They were both experienced teachers of mathematics and had used graphics calculators in their classrooms for several years. They participated in the development of the teaching program that focused on numerical, graphical and symbolic representations of derivative and links between them (see Kendal & Stacey, 1999) and integrated the use of a CAS calculator (TI-92).

Observation of lessons during teaching trials 1 and 2. Half the lessons in the first trial, and every lesson in the second trial were observed and audiotaped. Teacher behaviour was closely monitored (e.g., time spent on each type of differentiation activity, the nature of every teacher-student interaction, and attitudes displayed towards calculus, technology and students) and a comprehensive checklist of 52 observations was completed immediately after each lesson. Finally, a privileging profile (with three components) for each teacher was developed. It consisted of teaching approach (style & manner), calculus content (the representations of differentiation they chose to teach), and ways the CAS calculator was incorporated into their lessons (frequency & nature of use). The nature of calculator use was classified as functional (primarily to get answers), pedagogical (primarily for learning) or neutral.

First interview after the first teaching trial. Nine months after the first trial and ten weeks prior to the commencement of the second trial, each teacher was interviewed separately, to identify their personal knowledge of differentiation. This also provided a basis for comparison with the privileging that occurred during the second trial. During the interview the teachers were asked to discuss their proposed solutions (and to predict their students' responses) to a set of problems most of which would be included on the students' tests six months later. A wide spectrum of teacher characteristics was monitored including: personal knowledge of multiple representations of differentiation; awareness of alternative ways to solve differentiation problems; preference for representation; attitude towards the CAS calculator; personal CAS calculator use; knowledge of students; awareness of subtle school pressures and explicit constraints; and evidence of teaching methods and teaching styles.

Second interview after second teaching trial. Ten weeks after the teaching second trial, a second teacher interview was conducted to substantiate the privileging identified by observation of lessons during the second trial. Teachers were asked to reflect about their teaching practices, particularly the way they had used the CAS calculator to support conceptual understanding of the concept of derivative.

The following results summarise observations from all of these sources.

Results

Teacher knowledge

Personal teacher knowledge of calculus (differentiation). During the first interview, Teacher A knew how to differentiate symbolically (using algebraic rules) and graphically (finding the gradient of the tangent at the point), and how to translate between the two representations. He nominated up to two different differentiation strategies to answer particular test items, reasoned successfully about numerical derivative (limiting value of a difference quotient), reasoned with difficulty about graphical derivative and was unable to reason about a symbolic derivative. Teacher B performed symbolic, graphical and numerical differentiation and translated between the three representations of derivative. He nominated up to four different differentiation strategies to answer particular test items, and reasoned successfully about numerical, graphical, and symbolic derivatives.

Overall, Teacher A displayed less depth and less integrated knowledge about the concept of derivative than Teacher B who displayed deep, holistic, and integrated knowledge. This was reflected in both trials by Teacher A's focus on teaching rules and Teacher B's focus on developing students' understanding.

Institutional knowledge. During both trials, the teachers were cognizant of the fact that although their students could use the CAS calculator for the trial tests, they would not be permitted on school examinations to be held in three months time and on official state school examinations in fifteen months time. They were also aware that the style of assessment on trial test 1 was similar to the official school examination (based mostly on the symbolic derivative) and whereas for trial test 2, the assessment involved numerical, graphical and symbolic derivatives.

Three components of teacher privileging

1. Privileging related to teaching approach during both trials

Teaching method. Teacher A mostly taught rules for procedures and during both interviews talked about routines to solve problems. In contrast, Teacher B emphasized understanding of the concept of derivative. He employed enactive representations and encouraged students to use visualization techniques. During the first interview he solved each problem several ways, explained his use of different representations and made sense of each answer. During the second interview he talked about conceptual understanding: ″Getting the tangent idea through to them, what the gradient actually represents, what the derivative represents and the relationship between them - I think we've done very nicely with the calculator.″

Teaching style. Teacher A lectured his students who were expected to copy down his lesson notes. In contrast, Teacher B orchestrated discussions between 'student and teacher' and 'student and student' and his student-centred teaching style fostered student construction of meaning. In both trials, Teacher A's teaching approach (which emphasised student performance and mastery of mathematical rules and procedures) is classified as *Content-focused with an emphasis on performance (using*

Kuhn and Ball's (1986) model). Teacher B's approach which emphasised conceptual understanding of content and student construction of meaning is classified as *Content-focused with an emphasis on conceptual understanding.* This involves the ″dual influence of content and learner. On one hand, content is focal, but on the other, understanding is viewed as constructed by the individual″ (p.15).

2. Privileging of calculus content

During the first trial and in the first interview, Teacher A focused almost exclusively on the symbolic derivative. However, during the second teaching trial, he expanded his use of representations to include graphical and numerical derivatives. This came about after the first interview during which he realized that the students' assessment would involve numerical and graphical differentiation, unlike the tests in first trial that were essentially symbolic. In consequence, during the second trial he decided to include the numerical and graphical representations of derivative.

During both teaching trials, Teacher B consistently stressed the symbolic derivative and involved the graphical derivative in explanations. Although during the interview he personally demonstrated ability to differentiate numerically (i.e. use a rate of change or difference quotient), he actively rejected teaching about the numerical representation in the second trial. He explained that this was because he believed that his students were a low attaining group and would not be able to cope with three representations of differentiation (i.e., he made changes because of knowledge of new students).

3. Privileging of technology

In the first trial, Teacher A linked the CAS calculator to an overhead projector and frequently demonstrated symbolic procedures to the students and allowed them to use CAS freely. He avoided using graphs. In the second trial, he taught his students the additional CAS numerical and graphical differentiation routines (described as *functional* use by Etlinger, 1974) and provided them with a step-by-step flowchart of corresponding CAS calculator procedures. In addition, he used the calculator to explain the links between the numerical and graphical derivatives (described as pedagogical use by Etlinger, 1974)

> I'd say, when you see these words it means between two points, and when you see this word that means at a point. . . [I am] giving them strategies. . . and we did it [used dynamic graphing program] to understand the straight line against the curve.

In the first trial, Teacher B used graphs freely but noticeably controlled student use of the CAS calculator for symbolic procedures. In the second trial, he actually reduced his functional use of the CAS calculator while maintaining his pedagogical emphasis on the symbolic and graphical links.

> It's [the CAS] good for discovery because it takes a lot of the hack work out of teaching for understanding but you still need to teach pen and paper skill. I think there are certain skills that the kids have to have, even if you can use the technology

> to do it. I think the kids have to have the [algebraic] skills as well, without the technology. I think that's essential for their understanding. It's not sufficient to just use the calculator; they have to have the understanding of what's behind it.

Table 1 below summarizes Teacher A and B's privileging demonstrated during the first trial lesson observations (reported by Kendal & Stacey, 1999) and the changes in their technology privileging that occurred during the second trial.

Table 1. *Teacher A and Teacher B's Privileging in Trial 1 and Changes in Technology Privileging that Occurred During Trial 2*

	Teacher A	Teacher B
	Privileging in First Trial	
	1. Teaching approach	
Teaching style	• Lectured students	• Orchestrated student centred discussion between 'teacher-student' and 'student-student'
Teaching method	• Used rules for routine procedures	• Promoted understanding of routine procedures and problem solving, used enactive representations and visualization
	2. Calculus content	
Preference for representation	• Preferred symbolic derivative	• Preferred symbolic & graphical derivatives
	3. Technology	
Functional use of CAS	• Strongly promoted use of CAS for symbolic procedures • Disliked graphical procedures	• Restricted use for symbolic procedures • Permitted use of CAS for graphical procedures
Neutral use of CAS	• Checked by-hand solutions with CAS	• Checked by-hand solutions with CAS
Pedagogical use of CAS		• Used CAS for algebraic and graphical procedures to save time on activities that linked symbolic and graphical derivatives • Incorporated activities that stressed the links between the symbolic and graphical derivatives
	Changes in Technological Privileging in Second Trial	
Functional use of CAS	• Adopted additional CAS numerical and graphical differentiation procedures	• Reduced CAS use for symbolic procedures • Rejected CAS use for numerical procedures
Pedagogical use of CAS	• Incorporated activities that stressed the links between the numerical and graphical derivatives	

Conclusions and discussion

Teacher A's personal knowledge of differentiation was limited and the interviews revealed he believed his main responsibility was to help students pass examinations. During both trials, his teaching approach involved teaching rules and procedures using a lecture style of delivery. He focused on symbolic differentiation because he believed this was exact (stated in first interview). He appreciated the symbolic power of CAS, enjoyed using it himself, and encouraged his students to use it for symbolic procedures. During the first interview, which occurred between the two teaching trials, Teacher A realized for the first time, that the second trial assessment would involve multiple representations of differentiation. He responded to this “new” institutional knowledge by expanding his repertoire of calculator procedures to include numerical and graphical differentiation. In addition, his initial preference for the symbolic representation had an unexpected consequence - he gave a stronger emphasis to numerical differentiation. He showed his students how to use CAS procedures to substitute into functions to find ordered pairs and create a difference quotient calculation. This usually gives an ‘excellent’ approximation to the gradient of the tangent (and curve) and Teacher A led his students to believe it was exact. He also believed that with CAS, graphical differentiation was “exact”.

Teacher B’s knowledge of differentiation was deep and holistic and he believed that it was his responsibility to foster student understanding. During both trials, his privileging included teaching approaches that supported conceptual understanding using a student-centred style of delivery. He believed that the symbolic representation was the most powerful and useful for his students but he limited their use of CAS in order to prepare them for future examinations without it. However, to support student understanding, he adopted the graphical representation of derivative (gradient) using the CAS calculator. He also showed gradients enactively, and encouraged his students to visualize graphs of symbolic functions and derivatives. From the first to the second trial, Teacher B’s privileging was essentially stable but he reacted to “new” knowledge about his students: that the second group was algebraically weaker. In the second trial, he totally rejected the numerical representation, believing they could not cope with three representations. He also reduced their opportunity to individually differentiate symbolically with CAS (but allowed pedagogical class activities) strongly believing they needed practice with by-hand symbolic differentiation to cope with future examinations.

Both teachers, in response to new knowledge, made changes to their technological privileging in ways that were consistent with their own beliefs and knowledge. These results are consistent with other research. For example, Tharp, Fitzsimmons, and Brown Ayers (1997) showed that teaching style tended to be unchanged by the addition of technology. In their study, after an initial attempt at more enquiry based learning using a graphics calculator, teachers with a rule-based (procedural) view tended to revert to their procedural style of teaching, whilst teachers who were not rule-based remained more likely to focus on student conceptual understanding and thinking.

The institutional constraints and teachers’ knowledge or assessment of the needs of their students were important determinants of the change in privileging for both of our teachers from the first to the second trial. The constraints of the assessment system were important for both, although in different ways. Teacher A was more accepting of aims the research project, allowing students to use CAS and adding new techniques in response to new knowledge of expectations of students. Teacher B was always more concerned that students developed by hand skills and rejected some of the aims of the research trial for a group he assessed as weak.

As hand-held CAS calculators are now becoming more affordable and easily incorporated into the teaching of mathematics in secondary school classrooms, the issue of teachers changing their pedagogy for more effective teaching with technology is becoming increasingly urgent.

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