THE AESTHETIC IS RELEVANT

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Many would agree that we need to make more mathematics relevant and interesting to students, yet most recommendations for increased relevance have ignored the aesthetic dimension of student interest and cognition. In this paper, I argue that the aesthetic dimension plays a central role in determining what mathematics is personally or epistemologically relevant to children. I present an example of a learning environment that attempts to explore this dimension—both mathematically and pedagogically—and then briefly describe a small study that examined the responses of middle school students to this environment.

Recently, researchers have argued that all human abstract thinking is metaphorical, based on our sensory-motor experiences (Lakoff and Johnson, 1999) and that humans possess an innate aesthetic sensibility that acts as one of our primary meaning-making capacities (Dissanakye, 1992; Wilson, 1998). This conception of aesthetics is not limited to the formal, detached, and objective judgements of beauty and elegance. Rather, an aesthetic response is a cognisance of fit, of structure or order, perceived in part as being intuitive and recognised at an emotional level as being pleasurable. It is reflective in the sense of resulting from an awareness of the perceiver in relation to the environment. The role of the aesthetic in mathematics has been explored by many mathematicians (e.g. Penrose, 1974; Poincaré, 1956 Tymoczko, 1993). The emerging picture is that aesthetics is involved in: (a) motivating the choice of certain problems to solve; (b) guiding the mathematician to discovery; and (c) helping a mathematician decide on the significance of a certain result (Sinclair, 2000). In a challenge to traditional epistemologies, some researchers have argued that the aesthetic is in fact a mode of cognition used by scientists and mathematicians (Burton, 1999; Papert, 1978; Weschler, 1978). Based on these claims, and more generally on aesthetics’ perceived role in learning (Dewey, 1933; Eisner, 1985), a growing number of educators have argued that aesthetic considerations should be of primary importance in children’s learning of mathematics (e.g. Brown, 1993; Silver 1994; Whitcombe, 1988). However, adequate understandings of how aesthetic considerations play into mathematics learning have yet to be developed.

I propose that aesthetically rich learning environments enable children to wonder, to notice, to imagine alternatives, to appreciate contingencies, and to experience pleasure and pride. They are characterised by two facets; first, they legitimise students’ expressions of innate sensibilities and subjective impressions—they “work with” such perceptions rather than exclude or deny them. Second, they uphold Dewey’s (1933) sense of the fourfold interests of children: communicating, finding things out, making things, and expressing themselves artistically. These dual facets—of perception and of action—permit children to become absorbed in and identify themselves with some object or idea, to become interested.

I wanted to explore the possibility of creating an aesthetically rich learning environment that would make accessible to middle school students the pattern possibilities of the real numbers and to explore the potential of students' aesthetic engagement with the much abhorred topic of fractions and decimals. I therefore developed an internet-based calculator[1] designed to facilitate the visualisation and manipulation of real numbers.

Description of the Colour Calculator

The Colour Calculator (CC) is a regular calculator that produces numerical results, but that also outputs its results in a colour-coded table. Conventional operations are provided; the division operation allows rational numbers while the square root operator allows irrational numbers. Each digit of the result corresponds to one of ten distinctly coloured swatches in the table, as shown in Figure 1. The calculator operates at a precision of 100 decimals digits, and thus each result represented by a (long) decimal string and an array or matrix of colour swatches. It is also possible to change the dimension, or the width, of colour table.

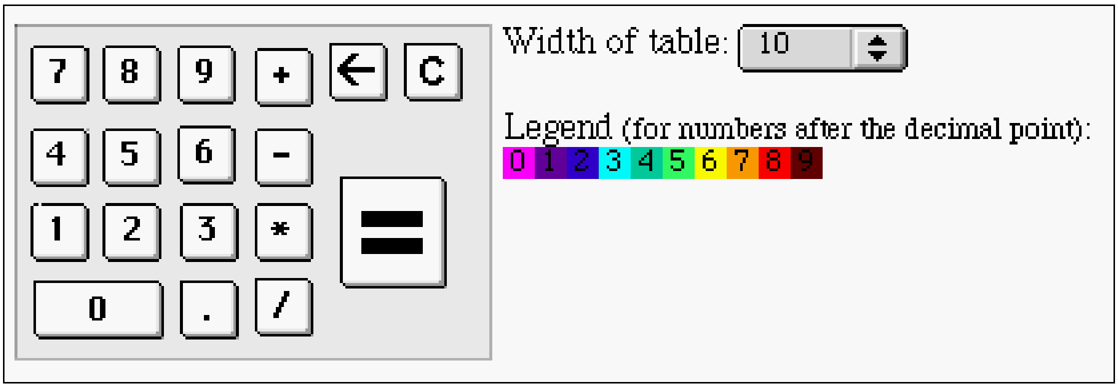


Figure 1. The Colour Calculator

In this following screenshot, 1/7 has been typed into the calculator with the table width set at 20. The associated table of colour has been generated:

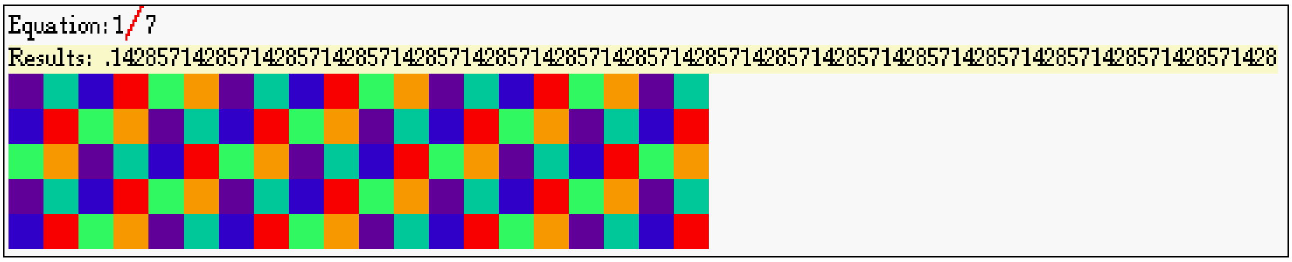
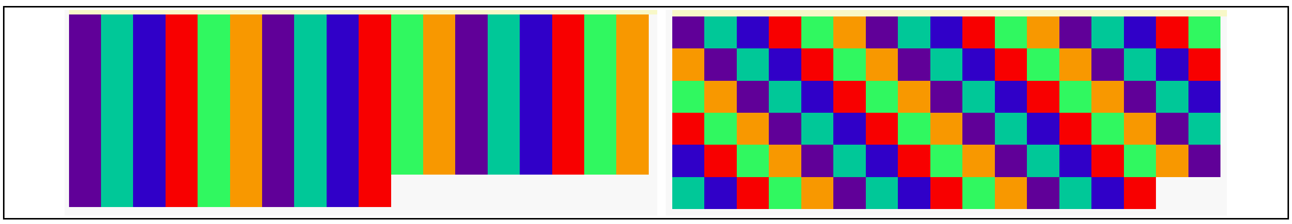


Figure 2. The Colour Calculator outputting 1/7

Using the button that controls the width of the table of colours, the student can create different colour patterns, some which highlight different aspects of the number's period.



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[1] The calculator is part of a larger project called Alive Maths (on-line at http://math.ai.iit.nrc.ca) which, as an internet –based environment is not only platform independent and freely accessible by students both at school and at home, but allows students to chronicle—in writing and through activity recording—their discoveries and questions on their own personalised web pages. This work was funded by SchoolNet through the National Research Council of Canada.

Figure 3. Different tabular representations of 1/7: widths 18 and 17

I designed the environment with three hypotheses in mind:

1. The pattern-rich table of colours patterns resulting from rational number calculations would surprise and engage the students.
2. The students' sensitivity to visual patterns would prompt and facilitate their sense-making of some characteristics and relationships of rational numbers
3. The CC would provide a setting in which students could develop more positive relationships with fractions and decimals.

The structure of the CC mathematical environment emphasises two aspects of learning. The first is to encourage students to make sense of mathematical ideas such as fractions, which they often find almost repelling, using some of their aesthetic sensitivities such as symmetry, repetition, rhythm, and pattern. This type of sense-making is part of the cognitive processes that students use to understand the form and meaning of objects and ideas. The second is to facilitate a process—one of exploration, research, and discovery—that potentially gives rise to a chain of sensory and emotive responses (Dewey, 1934).). This process is initiated by surprise (or novelty) and ambiguity. It culminates in the grasping of new knowledge that has been experientially developed.

Methodology

I conducted structured task-based interviews with 15 middle school students, 8 male and 7 female, of mixed ability (as rated by their regular classroom teacher). The students were all in grade 8, and came from lower to middle class, small town backgrounds. The interviews were task-based in that each student worked through a mathematical task using the CC as I observed and asked questions. They were structured in the sense of my facilitating the problem-posing and problem-solving process for each student. Each interview began with the student reading the instructions for the CC out loud. The student then started on the task while I asked a series of questions designed to elicit some of her thought processes as well as to guide her through the exploration. I occasionally intervened to provide guidance, following a set sequence of prompts that were only given when I judged that the student could no longer progress either in identifying a problem or solving it. The interview continued until the student had concluded at least one exploration; that is, until the student had resolved one problem. Following this, I asked each student to reflect on their experience, first asking them what they thought about what they had just done, then asking them to compare what they had just done with their other mathematics activities, and finally asking them how they felt about the open-ended nature of the activity. Each interview lasted between 20 and 30 minutes.

The interviews were all audio-taped and then transcribed. In addition, while interviewing, I kept notes of their facial and bodily reactions as they interacted with the environment, particularly at the beginning when they tried their first fraction and also when they were approaching the resolution of their problem.

Findings

I will discuss each of the three hypotheses presented above about how students would interact with the CC environment. At an obvious and almost trivial level, every student expressed that they had never seen fractions or decimals like this, together and with colours—many of them realised for the first time that a fractional and its corresponding decimal are the *same*[2], one student noting "you never see them together like this." Every student also expressed how different this type of mathematical activity was from their regular classroom work, one explaining that "you actually have to *do* things" while another observed that "you have to *notice* things."

Of the 15 students I interviewed, thirteen of them showed obvious physical signs of surprise, which they expressed either through one or more of the following actions: widening their eyes, sitting upright or moving forward, making a sound such as "ooh," or saying some form of "wow." One student, whom I will call Nadia, showed no physical surprise at all, and answered "I don't see anything" when I asked her what she saw in the table of colours. Nadia was either completely insensitive to the patterns in the table or, because of her timidity and lack of confidence, she may have been under too great of an affective barrier to even attempt to engage. The other student who showed no physical reaction was Cameron, a very ends-oriented student, who remarked flatly: "there are lots of colours and patterns there."

Of course, initial surprise is only desirable if its effect is to engage the student in sense-making; that is, if it prompts the student to try to understand something about what they are seeing. This was easiest to observe with the more articulate students who provided a running commentary of their thought processes, like Sean:

> Okay. Ah. It looks like an abstract painting. Not exactly like a math problem. I'm trying to figure out how it calculates that. Uh. Well, it says that the results are 0.142857 and it repeats. So this is a repeating pattern. I can see it because the red sticks out and the purple, and ooh the green. They kind of go in a diagonal which shows a standard repeating pattern but I'm trying to figure out how things are working. So the number corresponds to the colour…

There were a few other students who provided such spontaneous descriptions of their thought processes, but most of the students had to be prompted to share their thoughts and perceptions. All the students quickly made the connection between the table of colours and the colour legend (Figure 1), and between the decimal number and the table of colours. A few of the students failed to see the connection between the fraction and the table of colours, needing some further experimentation to be able to conceive of them as the same number. However, beyond suggesting these obvious relationships, I wanted to know whether the CC environment would produce a generative engagement: for example, would the students wonder *why* the

[2] It is interesting in fact to recall that a regular calculator replaces its input with its output so that a student calculating 1/7 on the calculator never actually sees both the fraction 1/7 and its decimal expansion simultaneously. Though I am sure that the students think that a fraction and its decimal are equal, they seemed struck by an ontological equivalence.

1/7 fraction produced the table of colour or *why* the table showed the patterns it did?

I judged a student to be generatively engaged if, after their initiation to the CC, they made observations or took actions that indicated an emerging question or conjecture. For example, Ann's observation that "every seventh box is a purple" indicated a conjecture that the period of 1/7 is 6, and was followed by her experimentation with the width of the table (which, perhaps not surprisingly, she first tried at 7 before realising she really wanted 6). Sean's immediate experimentation with 1/3, then 1/2 indicated an emerging question of how other fractions will contrast with 1/7. Julie took a slightly different approach by experimenting first with the width of the table of colours, describing a width of 7 as "it's like a staircase" and a width of 3 as "it's doubled up," indicating an emerging question about the types of possible patterns. She went on to characterise diagonal patterns as those that were one more or less than the width that makes the colours of the table line up. Four other students each embarked on explorations similar to the three described above.

The other students either paused, waiting for instructions or guidance, or asked me whether I wanted them to make the colours line up (as was suggested in the instructions to the activity). These students, either because of their ends-oriented approach, their lack of confidence, or a lack of interest in the activity, did not quickly become generatively engaged. Four students required some guidance and prompts, as if they needed to know what was interesting or significant enough to pursue. After they had formed a question or conjecture, they were able to experiment and all but one of them added a personal variation to their experimentation. For example, Robert started by following my prompt of figuring out what kinds of numbers are non-terminating, but then decided to investigate what kinds of fractions gave solid tables of colours, discovering that n/9 (for $0 < n < 9$) would always give a solid table in the colour corresponding to n.

I now turn to my second hypothesis about whether the students' sensitivity to visual patterns and engagement would prompt and facilitate their sense-making of some of the characteristics and relationships of rational numbers. There were two types of sense-making exhibited by the students. The first type was around the characteristics and relationships of rational numbers that these students had encountered or "already learned" in their regular mathematics classes. The second type was around the characteristics and relationships of rational numbers that were new to them, and mediated by the CC environment.

Of course, not all the students made the same inquiries and discoveries; In fact, the wide range of inquiries and discoveries made by the revealed much of the students' existing understanding of fractions and decimals. Within the first type of sense-making, the majority of the students realised, some to a greater extent than others, that fractions aren't just the canonical 1/2, 2/3, 3/4, 1/10 numbers they have often encountered during "fraction class," but that they can have a denominator greater than 10, and that they can even be *any* integer over *any* integer, as Steve's question shows: "You mean I can put any number on the bottom?" Several students also expressed surprise at seeing the fraction and the decimal at the same time—as

I mentioned above—and seemed to gain a new understanding of their equivalency, as Alice concluded: “they mean the same number.” Related to this understanding of equivalence, a few of the students became intrigued with trying several equivalent fractions to see what the table of colours would depict, allaying any small doubts they were having that 1/2, 5/10, 20/40 were really the same number. A few of the students were somewhat fluent at the outset with decimals (i.e., knowing that 1/2 is 0.5 and that 1/3 is 0.33…) but most of the students seemed to have very little sense of which decimal would result from a given fraction, even with fractions whose denominators were multiples of 10. This is perhaps due to the situatedness of their fraction-decimal knowledge in classroom worksheets but it would be interesting to see what impact their brief exposure to fraction-decimal pairs has on their future classroom work with fractions and decimals. These findings highlight some of the basic conceptions with respect to fractions and decimals that students rarely have a chance to develop, yet that are almost assumed to be part of their ability to operate on fractions, convert them, and estimate them.

I now turn to sense-making of the second type. Since many of the students experimented with changing the width of the table, they were able to see what the period of a fraction is, how long the period of 1/7 is, and how any multiple of the period of the fraction makes the colours in the table line up. These are not typically the kind of rational number characteristics and relationships taught in school curricula, but are ones that were both accessible and interesting for this group of students in the CC environment. Other than making these common realisations, the students embarked on quite individual investigations. The different investigations were entirely student generated in that I only proposed questions during the interviews that had already been posed by other students in this study. Here is an incomplete list of the topics explored by the students, to various degrees of generality: What values of the width of the table would create diagonal patterns? What values of the denominator yield non-terminating decimals? What values yield terminating decimals? How is the period of the fraction related to its denominator? When does the decimal only start repeating after a certain point? What kinds of numbers neither terminate nor repeat? How can you get a solid red (or blue or green) table of colour? What is the effect when you square a fraction that has a certain period?

This environment certainly prompted the students to make new understandings of fractions and decimals and in particular, to explore characteristics and relationships they are not usually encouraged to explore. The CC environment highlighted some of the incomplete fraction and decimal understanding that students have and allowed them to gain a new understanding of what a fraction *is*, as opposed to what you can do to fractions—add them, generate equivalent ones, etc. Additionally, the CC appears to be an environment in which students are interested and motivated to discover certain things about numbers, using fractions and decimals, that are different than what is emphasised in current school curricula. The ideas explored by these students are not easier than the ones we typically emphasise, but, in this CC environment, they are perhaps more relevant to students’ personal and epistemological interests.

This brings me to my third hypothesis, of whether the CC would provide a setting in which students could develop a more positive relationship with fractions and decimals. The only data I collected that is useful in verifying this hypothesis is the students' reflections at the end of their interviews. In these reflections, I asked them how they compared what they had just done with their usual mathematics activities. I found it difficult to determine the cause of their unanimous beliefs that this environment provided them with a more positive experience. Comments such as it's "fun because you can work with patterns," or "good because it helps you out more," or "creative because you can make patterns" or "fun because you don't just have to look at numbers" suggest that the colourful patterns were enjoyable but do not ascertain whether the students have a different relationship with fractions and decimals now than they did before. Some students may also have had positive experiences just because they like working on the computer or because they like having an adult's attention and help. And still, for others, the fact that they weren't set up for failure at the outset (as often is the case in mathematics class) may have made their experiences more enjoyable. That this third hypothesis remains unclearly substantiated is due both to the paucity of data in this particular study and partly to the methodological challenges of assessing students' emotional responses.

Conclusions

A majority of the 15 students called upon their aesthetic modes of cognition to explore and make sense of the visual patterns depicted by the CC. An even larger majority of the students initially became engaged either through surprise, novelty, or perceptual attraction, prompting them into a varying degree of sophisticated mathematical meaning making about fractions and decimals. These are promising findings given that each student had less than half an hour to interact with this aesthetically rich learning environment.

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