# PEER INTERACTIONS AND STATISTICS LEARNING

Carvalho, Carolina & César, Margarida (1)
Centro de Investigação em Educação da Faculdade de Ciências da Universidade de Lisboa

**Abstract:** This research is part of the project Interaction and Knowledge, whose main aim is to study and promote peer interactions as one of the possible forms of developing pupils' socialisation and positive attitudes towards mathematics, as well as to promote their socio-cognitive development and enhance their school achievement. In this paper we analyse a case that illustrates the role of peer interactions in knowledge appropriation in a statistical task.

## Introduction

The curricular reforms implemented in the last few decades have stressed the need to stop considering only aims related to contents and start taking into account the development of attitudes and values, as well as abilities and skills (Abrantes, Serrazina and Oliveira, 1999). These authors claim that "these three aspects (knowledge, abilities and attitudes) are inseparable, not just in the new tasks pupils are presented with but also in the learning process itself" (p. 22).

[1]One of the suggestions most often made in official documents regards turning to group work and giving importance to interactive processes that are present in the didactic relation (NCTM, 1991; van der Linden et al., in press). Abrantes, Serrazina and Oliveira (1999) state that "since pupils are different from one another and build different images and conceptions about the topics under study, the teacher has to value the interactions between pupils and between these and the teacher" (p. 29). This is precisely the main goal of the project *Interaction and Knowledge*: to study in detail and promote peer interactions as a way of fostering pupils' full development and their school achievement in Mathematics. At the same time, the intrinsic advantages of a greater link to reality and pupils' experiences have been highlighted, and Ponte, Matos and Abrantes (1998) have declared that "Statistics and Probabilities are essential themes that allow for a link between school mathematical knowledge and the Maths used in everyday life" (p. 170). This way, when we chose Statistics as a curricular unit for detailed studying in our project, we had in mind that it adapted particularly well to the development of studies of an interactive nature and would encourage exercising a critical and participative citizenship.

The main aim of this report is to analyse the performances of the dyad in tasks related to the concept of mean, stressing the role of peer interactions in the co-construction of solving strategies, in statistical tasks.

## Theoretical background

According to Shaughnessy (1992), twenty years ago there was hardly any research in the field of statistical education. In Portugal, despite the topics of

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Statistics and Probability being a part of the curricula since the 80s, only in the 90s did they begin to be taught by most elementary-level teachers (Ponte, Matos and Abrantes, 1998). The aims of Statistics contents include aspects such as: developing communicational abilities, autonomy and solidarity (including showing a critical and rigorous spirit; trust in one's reasonings; approaching new situations with interest and initiative; assessing situations and making decisions) or the capacity to use quantitative methods to analyse real-life situations. Reaching this kind of aim is not compatible with teacher centred teaching, neither with solving routine activities, where pupils only have to turn to instrumental knowledge, such as using formulas and algorithms, without needing to interpret the proposed situation (Batanero, 1998, 2000; Carvalho and César, 2000a; César and Silva de Sousa, 2000; Ng and Wong, 1999).

Putting into practice the suggestions of the current curricula and educational policy documents means creating novel tasks, promoting horizontal interactions (pupil/pupil) and not just vertical interactions (teacher/pupil), being able to explore pupils' reasonings and response strategies, posing stimulating questions that increase their involvement in the tasks. Thus, the didactic contract or the experimental contract established with the pupils or subjects of a certain research is of the utmost importance. These are the (generally implicit) rules that legitimise many of the mutual expectations of the various social partners involved in an interactive process, with which the appropriation of knowledge and the mobilisation of relational competencies are supposed to exist. Therefore, implementing novel classroom practices or experimental activities also involves defining contracts that are novel themselves and that encourage peer interactions (César, 2000a, 2000b, 2000c; Grossen and Py, 1997; Schubauer-Leoni and Perret-Clermont, 1997).

Several investigations have discussed the role of social interactions in cognitive development and in the promotion of pupils' school achievement, namely in Mathematics (César, 2000b; Perret-Clermont and Nicolet, 1988; Schubauer-Leoni and Perret-Clermont, 1997). In Portugal, the first studies by César (1994) were an attempt to find answers to the challenges brought forth by teachers who wanted to know how to form efficient groups in their daily practices in order to improve pupils' mathematical knowledge. These studies were contextualised and they allowed us to understand the mechanisms related to peer interactions and their role in the promotion of pupils' socio-cognitive development and knowledge appropriation. In recent studies (Carvalho and César, 2000c) we showed that working in peers was an effective way of promoting pupils' cognitive development and better statistical performances. But in order to understand how interactive processes contribute to these progresses we need to undertake an in-depth analysis of the interaction itself.

The contributions of the Vygoskian theory (1962, 1978) were essential in order to understand the difference between actual development and a potential one. This difference leads to the notion of Zone of Proximal Development (ZPD) which is one of the most fruitful constructs arising from this theory and one of the most explored in educational settings (Allal and Ducrey, 2000; Moll, 1990). But in order to work collaboratively pupils need to construct an intersubjectivity (Werstch, 1991) that allows for exchanging ideas and knowledge, i.e., to be able to follow each other's solving strategies as well as to manage every aspect related to the relational context.

**Method**

The project *Interaction and Knowledge* is divided into two different levels: 1) - A micro-analysis level, in which we studied different types of peers, their interactions, the tasks we propose, the mistakes they make and the progress that peer interactions are able to generate in statistical contents (Carvalho and César, 2000a, 2000b, 2000c, in press; César, 2000a); 2) - An action research level, in which some mathematics teachers implemented peer interactions as a daily practice during a school year (César, 1998a, 1998b, 2000a, 2000c; César and Torres, 1998; César and Silva de Sousa, 2000). The data we are going to present are from level 1.

*Subjects*

The sample was formed by 136 dyads. Subjects attended the 7th grade in two public schools near Lisbon. Their ages were between 11 and 15 years (Average=12,5 and Sd=0,8). The case we are going to analyse is representative of most dyads.

*Instruments*

The statistical tasks used in peer work sessions were "unusual" ones according to teachers' statements and to our previous observations of their classes (Carvalho and César, 2000c; César, 2000a). They corresponded to more open and innovative tasks.

*Procedure*

In this research pupils had three sessions in which we promoted collaborative work. Pupils were grouped in dyads and the interactions were audio taped only in the second session of peer work. The episode we are going to analyse is related to one of the tasks presented in the second session. The working instructions that were given asked them to discuss before writing their answers and to explain everything they thought. Each dyad only had an answering sheet.

**Results**

The task that pupils were solving in these episodes is the following one: *"The mean of four numbers is 25. Three of those numbers are 15, 25 and 50. Which is the missing number?"*

*Case 1 - Co-constructing statistical knowledge in an asymmetric dyad*

***14 A - First we must sum up 15 + 25 + 50.***

***15 C - 15 + 25 + 50 is 90 [She uses the calculator]. If this was the mean of 90, 90 dividing by three would be 30.***

***16 A – Let's try with 94 divided by four. It must be an integer number. So, adding four doesn't make it right.***

***17 C - 90 plus what? 6? Wait a moment, I'll do it. [She uses the calculator] 25 isn't right either***

***18 A - So, how do you calculate the mean? When you calculate the mean you sum up all this [she points to the values] and you divide by a number.***

***19 C - How many they are?***

***20 A – 4. So, what you should do is this, 15 + 25 + 50 + X and divide by 4 which equals 25.***

***21 C - How much is the X?***

***22 A – It's the result of 15 + 25 + 50 + X divided by four which was 25.***

***23 C – It's 6.***

***24 A - It can't be. In order to solve this [equation] you have to do 25 times 4.***

***25 C - Wait a moment!***

***26 A - Let me do it. 25 times 4 is 100, eh! Delete it [C is doing the computations with the calculator]. Now try to do 100 dividing by four to see whether... [C finishes her computations and she agrees moving her head] it's 25. Then, it has to be 10. It's 10.***

***27 C - 15 + 25 + 50 + 10 is 100.***

***28 A - Now, if you do 100 dividing by four, which is the total of the data. Do it: 100 dividing by 4 is...***

***29 C - 100 dividing by four is 25 [She writes down]***

***30 A - Equals four, how dumb! Equals 10, because 25 times 4 is 100 and then... If we had 100 to 90 there's 10 left. Let's move on to the next one.***

In this episode A.'s leadership is clear although, according to the rules of the experimental contract we established, she always explains everything she is thinking to C. and takes care to check if C. is following her solving procedures for the proposed task. So, C.'s first phrase is to show A. she knows how to calculate the mean. However, this phrase does not show a solving strategy for the problem, for C. just says what the mean of the 3 numbers on the sheet would be. The fact that classroom practices usually stick to applying algorithms and procedures (Batanero, 2000; Carvalho and César, 2000a; Shaughnessy, 1992; Skemp, 1979) may explain why this pupil associates solving this problem with merely applying an algorithm, regardless of this leading or not to the solution of the problem. This way, C. would dominate what Skemp (1979) calls instrumental knowledge but, if she worked individually, she wouldn't manage to reach a relational knowledge.

As the mean is 30 instead of 25, A. realises that the missing number mustn't be very high. So she decides to start applying a trial-and-error strategy, which César (1994) considers to adapt well to problems of a medium complexity, but in this case her attempt fails from the start. However, we can see that despite C. having the calculator, it is A. who controls the credibility of the results*: "It must be an integer number"*. So she is the one who understands, through the result on the calculator, that her hypothesis isn't the result they want, for the number is not an integer.

C., who is having more calculation difficulties, doesn't want to leave the solution of the problem to A. alone. Therefore repeatedly she tells her *"Wait a moment"*, which not only shows that she acknowledges that she works at a slower rhythm than her peer but also that work should be carried out in dyads and she intends to contribute to it, however slow she is. Curiously, the several temporal expressions present in this dialogue have a well defined role: for C., they mean to tell A. that she needs more time to think, so A. doesn't answer just yet. So they serve to put the brake on the speed of the solution. For A., who starts several times by saying *"Now..."*, they serve to accelerate the rhythm, make suggestions or even give orders, so C. can collaborate but not put the task solution at risk. This is also visible in the last phrase of the episode, when A. wraps up the task solution, adding, *"Let's move on to the next one"*.

On the other hand, since C. has less knowledge that is necessary for the task solution, the pauses she imposes are also a way of negotiating – in the relational context – the appropriation of the knowledge in question in the task solution. This probably explains how C. progressed, between pre- and post-test (see Carvalho and César, 2000c for more details) in cognitive terms and in terms of her level of performance in statistical tasks. Besides, this kind of attitude also shows she understood and stood by the proposed experimental contract, since it suggested they only write down an answer when both agreed with it and when both had collaborated in the task solution.

These rules of the didactic contract give rise to a few words of a pedagogic nature, on behalf of the more competent peers*: "So, how do you calculate the mean? When you calculate the mean you sum up all this* [she pointed out to the values] *and you divide by a number"*. This way, we may state that the rules of the experimental contract play a very important role in the type of interaction that pupils establish, as several authors stress (César, 1994; Schubauer-Leoni and Perret-Clermont, 1997).

As soon as A. realises that through trial and error she is not reaching the result she wants, she drops this tactic and changes to an algebraic strategy, typical of pupils who already have access to formal reasoning (César, 1994; Perret-Clermont and Nicolet, 1988). She easily verbalises the translation of the problem on the sheet in an equation: *"15 + 25 + 50 + X dividing by 4 equals 25"*, which means A. does not only master Statistics contents. In this part of the interaction the difference between the operatory level of each of the subjects is clear: for A., the value X really represents an unknown quantity and does not need to be immediately replaced with the value it represents; C., who is on a less advanced operatory level, needs to ask *"How much is the X?"*, for she needs to materialise this value. Actually, it is funny that C. seems to adopt A.'s initial strategy for a moment, for she still tries to see whether X can be replaced with 6, before calculating its value by solving the equation A. formulated.

Another very interesting point is the understanding A. gains through C.'s difficulties. When she realises C. has difficulty in reasoning in formal terms, she abandons the equation solution that had been her second suggestion and chooses an arithmetic strategy: 25 times 4 is 100; the previous numbers had summed 90. So, the missing number is 10. This way A. does not drop the leadership of the problem solution but adapts her speed and solving strategies to the difficulties revealed by C. They are able to construct something that Werstch (1991) calls an intersubjectivity, whereby both contribute to find an adequate solving strategy. This capacity to simultaneously deal with the cognitive as well as the relational aspects of the task is what enables them to work in their ZPD, which, according to Vygotsky (1962, 1978), promotes cognitive development. This is what happened to the less competent peer, who progressed to performances that are typical of the formal stage.

Regarding the relational aspect for a moment more, it seems important to us that there is an opposition between A.'s patience and the type of vocabulary she uses when C. has difficulties or makes a mistake, and the way she reacts when she makes a mistake herself. Only one expression with a negative connotation is used in this last case: *"How dumb!"*. This means that A. has a true concern in supporting C., in contributing so that she gains confidence in her competencies and mobilises them more easily. When it comes to judging herself, A. becomes a harsher evaluator, probably because she doesn't feel her competency threatened by the fact.

It seems clear to us that for either one of the elements of this pair, it is the fact that they work collaboratively and with certain rules of the experimental contract that allows them to develop a relational knowledge. The fact that they have to explain everything they do, discuss several solution hypotheses, experiment with several possible strategies, makes the level of appropriate statistical knowledge more complex, for they discuss the inherent context of the proposed task between each other, enriching the meaning they are capable of giving it.

As for statistical performances, in both cases they had better results in the post-test than in the pre-test, which is in accordance with results reached by several pupils, namely in Portuguese investigations (Carvalho and César, 2000c, in press; César 1998a, 1998b, 2000a, 2000c; César and Torres, 1998). Contrary to what Vygotsky (1962, 1978) believed, it is not just the less competent peer who benefits from the interactive process. Peer interactions have shown to be a powerful mediator in knowledge appropriation and in the mobilisation of competencies on behalf of the pupils. Therefore, when we promote peer interactions in the Maths class we are facilitating pupils' full development and their school achievement in this subject, for on a post-test level, which is already solved individually, subjects are often capable of using solving and reasoning strategies that we saw them developing for the first time during the collaborative working sessions.

Statistical tasks adapt particularly well to collaborative work, as long as teachers use open tasks that stimulate the use of varied strategies and the development of a critical spirit (Carvalho and César, 2000a). In this case, more positive attitudes towards Maths are also promoted, besides contributing to the development of an academic self-esteem on behalf of the pupils who usually have difficulty in reaching good performances. Thus, as we see in the example we presented, pupils show their intent in solving tasks instead of abandoning or rejecting them. Hearing and talking to a colleague forces them to express their doubts and solving processes clearly, and this contributes to them being able to appropriate more knowledge and do so in a relational way instead of a merely instrumental way (Skemp, 1979). This is in conformity with the aims suggested in the educational policy documents.

## Final Remarks

Attributing a meaning is a fundamental step in solving a mathematical task and for knowledge appropriation and the mobilisation of competencies. As Vygotsky (1962, 1978) stressed, pupils need to de-contextualise and re-contextualise knowledge so that it goes from being external and social to being internal and personal. Facilitating the attribution of meaning is essential in order to promote school achievement and numeracy, so the nature of the tasks we propose to pupils must be taken into account. Besides this, social interactions, namely peer interactions, play a truly relevant role in the facilitation of meaning attribution, so the implementation of a novel didactic or experimental contract is also a very important element to take into account if we intend to reach the aims of current curricula and educational policy documents.

In order to promote school achievement in Maths, namely in Statistics contents, we need to facilitate the possibility of pupils going from an instrumental knowledge to a relational knowledge. But for this to become a reality and not just an intention, we need to learn to observe pupils' solving strategies and reasonings in detail. Only this way can we, as researchers and teachers, promote practices that allow pupils to work in their zone of proximal development, as several authors suggest.

The recommendations of several official documents point to the need to value social interactions in the Maths classroom, particularly if we consider, as Abrantes, Serrazina and Oliveira (1999) state, that "Maths education may contribute, in a significant and irreplaceable way, to help pupils become individuals who are not dependent but, on the contrary, competent, critical and confident in the essential aspects in which their lives relate to Maths" (p. 18). However, we feel this aim will only be fully reached if we know how to implement rich and fruitful social interactions in the classroom.

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