TEACHING ANGLE CONCEPTS BY ABSTRACTION AND GENERALISATION: AN EXPLORATORY STUDY

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Mitchelmore & White (2000) have recently advanced a developmental theory, based on the theory of abstraction, which attempts to explain the formation of a single concept for angle which results from recognising the underlying similarity between a variety of angle contexts which all involve two lines meeting at a point with some meaning (often "turning") attached to the opening between them. An angle diagram represents the common features of all the different angle contexts.

The authors have shown conclusively that children rarely recognise the dynamic similarity (turning) between two angle situations. and more easily recognise the static similarity (two lines meeting at a point), the facility depending on the salience of the two lines which form the angle (over 90% of Grade 2 children where the two lines are obvious in situations such as tile corners and scissors; but only about 40% of Grade 6 children where one or both lines has to be imagined in situations such as hills, doors, and wheels). Based on their findings the authors tested a method for teaching angles which focused on the lines (in particular), vertex and opening in paired situations where one situation had both arms of the angle visible and one situation did not.

The data showed that focusing students' attention to the individual attributes of an angle helped them to make appropriate matches, to identify the critical features of an angle, to recognise irrelevant matches, and to draw abstract angle diagrams. However, it was not as effective as hoped because it sometimes seems to have led to an angle being viewed just as a line, a vertex, or an opening, and not as a single mental object integrating all three. The data also showed that focusing on these features in a 2-line situation often did little to help students recognise angles in 1-line situations or 0-line situations. The data on drawing abstract angle diagrams at first seems paradoxical because even though students could not match the angles in a pairs of situation, they easily learnt to make an abstract drawing of an angle. This finding can be regarded as a *non-transitivity* effect: Being able to match a drawing to two angle situations does not imply being able to match the two situations.

In hindsight, we realise that we failed to correctly interpret our own research findings and in fact used an approach which is the exact opposite to that suggested by the theory of abstraction. Hence, taken overall, the teaching sequence suggested is supported, but with a specific focus on the three critical features of an angle occurring *after* the appropriate similarity has been recognised, not before. Further teaching experiments are currently being undertaken to test the revised approach.

Reference

Mitchelmore, M. C. & White, P. (2000) Development of angle concepts by progressive abstraction and generalisation. *Educational Studies in Mathematics*, *41*, 209 - 238.