VERBALIZATION AS A MEDIATOR BETWEEN FIGURAL AND THEORETICAL ASPECTS

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ABSTRACT: The study reported in this paper concerns the dialectic relationship between the figural register and the natural language register when students try to solve plane geometry problems. I will present a theoretical framework and some preliminary results concerning the following problem: how and to what extent does natural language act as a mediator and a control tool between the operational handling *(Duval) of the drawing and the theoretical reference (in our case, Euclidian geometry)?*

1. The research problem and the theoretical framework

In geometry, since we deal with theoretical objects and their representations, we need to state what we mean by *drawing*, *figure* and *geometric object*. I considered the definitions given by Parzysz (1988) and by Laborde and Capponi (1994).

Parzysz suggests that "*the FIGURE is the geometrical object which is described by the text defining it* " and "*The figure is most often REPRESENTED*" (Parzysz pg. 80). Parzysz calls *drawing* the illustration of a figure.

Referring to Parzysz' elaboration, Laborde and Capponi propose the following definition: *"Drawing can be considered as a signifier of a theoretical reference (an object of a geometric theory, like Euclidean Geometry or Projective Geometry). A geometric figure involves the joining of a given reference to all of its drawings: it can be defined as the set of all couples which have the reference as the first term, while the second term belongs to the universe of all possible drawings of the reference".*

Referring to the abovementioned elaborations, from now on I will consider *geometric object* the *object of a geometric theory related to a definition*. The *description* will be the *verbal presentation of the geometric object* (i.e. the text of the definition). By *drawing* I then mean *one of the different graphical expressions of the definition itself*.

I can now define the *figure* (F) as the set of couples made up by the geometrical object (O) and one among the drawings (d_i) that are material representations of that geometrical object (O):

$F = \{(O, d_1), (O, d_2), (O, d_3), \ldots (O, d_i)\}$.

In this way the theoretical aspect is linked to the graphic one and a kind of bridge is established between them.

The differences (and relations) between drawing, figure and geometrical object play a very important role in handling the drawing when trying to solve a plane geometry problem. Therefore, we adopted the *operational handling of a drawing* considered in Duval's theory (Duval, 1994): operational handling of drawing (*appréhension opératoire* [1]) involves an immediate perception of the drawing and its different variations (*mereologique,* optical or of podition[2]). Our research concentrates mainly on analysing the influence of natural language on the relationship between the operational handling of drawings and the theoretical reference to which it is related. We define *theoretical reference* in a given geometric theory as theorems and definitions of that theory which are related to the figure by the student who is solving the problem. Since solving a plane geometry problem involves reciprocal relationships between drawing and theory, we note a two-way relationship between the handling of the drawing and the choosing of a particular theoretical reference: choosing a particular theoretical reference leads to the operational handling of the drawing and vice versa the drawing operational handling can suggest how to choose a particular theoretical reference.

Three hypotheses can be formulated:

a) The above-described relationship is guided and controlled by natural language.

b) Recognising some useful sub-configurations or some useful geometrical properties may be due not only to the perception of the drawing, but also to the *description* of the considered *geometrical objects*. This might even have a certain influence on the way to consider and analyse the actual drawing.

c) A verbal discourse about the project of resolution (meta-discourse) is necessary in order to give a *status* of hypothesis or of conclusion to the information which we get by handling the drawing procedures and by the description of geometrical objects.

In this paper I will elaborate a research methodology suitable for testing the above hypotheses and present some preliminary experimental results (intended to support further investigations).

2. Research methodology

Hypotheses a), b) and c) in particular bring to the following questions:

1. Is natural language a tool of mediation and of control between operational handling of drawing and theoretical reference in the procedure for solving a plane geometry problem?
2. What role does language play in the meta-discourses elaborated while solving the problem?

These questions immediately pose a problem concerning research methodology: How can we access the students' solving process? Basically, I tried to elaborate a methodology that should be suitable for tackling this problem. Such a problem depends on the fact that language plays two different roles: it is a tool for the researcher (as a revealer of students' processes) and, at the same time, it is a tool for students, because they use it to solve the problem (and our inquiry concerns its role in the solving process). We elaborated a model for analysing protocols, based on the use of language as a revealer, which allowed us to point out the role of language as a

[1] "L'appréhension opératoire est l'appréhension d'une figure en ses différents modifications" .

[2] A variation is called "*mereologique*" when it divides the drawing into parts; it is designated as *optical* if it is an enlargement or a reduction of the drawing ; it is called *positional* when the figure background changes position.

problem-solving tool for students. The model is based on the assumption that solving processes are mainly expressed through two registers: the linguistic and the figural. Then, the model distinguishes between two strategies: one developing from the figure drawn after reading the text of the problem, and the other developing from the question posed in the text or from the sub-questions obtained by transforming that question.

We named these strategies *drawing strategy* and *discourse strategy.*

The *drawing strategy* involves handling the drawing (in Duval's sense of operational handling of drawing), or its perceptive apprehension (cf. Duval) in order to construct a *work environment* by means of a list of information

The *discourse strategy* consists of a structured sequence of questions, starting from the question of the text; or from some key words taken from the text, by talking with a schoolmate, or with a teacher; or from a key configuration isolated in the drawing.

So, the *discourse strategy* is closely linked to the text of the problem but, on the contrary, the *drawing strategy* is not strictly related to the text.

Both strategies may intervene in the same student's solution.

• *Aim of strategies*

If the *discourse strategy* consists of a structured sequence of questions, its aim is a structured sequence of answers. On the contrary, the aim of the *drawing strategy* is collecting information starting from the drawing or by acting on the drawing itself. So, the change of aim in the procedure is the key element that reveals the intention to go on to another solving strategy. The *discourse strategy* usually is a part of a deductive strategy, in which the aim is to prove something. On the contrary, the aim of the *drawing strategy* isn't proving (indeed this strategy is used to create a set of information that constitutes the working environment).

• *Criteria for distinguishing between the two strategies*

We now try to provide some criteria that are useful for recognising a *drawing strategy*. In detail, language makes it possible to recognise this strategy when we can detect:

- words that refer to perception, such as *" you can see that..";*
- words and adverbs indicating space, such *"here, there,.."*; besides demonstrative adjective or pronouns, such *"this (one), that (one), ...",* accompanied by gestures;
- the simple present tense recurring frequently
- a descriptive rather than deductive discourse, without any connection linking the information in the list.
- unjustified inferences: they carry the formal shape of ordinary inferences such as *"since we know that..., then it follows necessarily that...".* But the term "*necessarily*" introduces perceptive evidence and takes the place of *"then, since...".* The following is an example containing some of the above-mentioned inferences, made at the drawing level.

123. Taina: "because, since we have OD diagonal, I go on tracing the OD line, then we have the parallel, no, the perpendicular, which is AE, since it is a circle, since we know than OA, OD and OE are circle radii and that AO is equal to AD and that OE is also equal to AD, then necessarily DE is equal too”.

We now try to provide some rules that are useful for recognising a *discourse strategy :*

- the variation in the use of verb forms and tenses. For instance a sentence like *"we should be able to demonstrate that"* points out an attempt to get out of the solving procedure, in order to provide a plan of it.
- the complexity of sentences: coordination between several complete propositions.
- the *final* [3] structure of a sentence as *“to have…it is necessary that…”*. This structure allows us to determine the theoretical reference that guided the answers to the questions.
- the presence of *key words* such as *perpendicular* or *isosceles triangle, height, medians.* These words play a key role for the subject, which refers back to a concept belonging to his/her knowledge system. Therefore, these words are a kind of bridge between the subject's knowledge and the text of the problem or the discussion with some schoolmates. Let’s take a look at the word *parallelogram*, for example: it reminds the subject of the quadrilateral figure, then the student will relate it to all the theorems and properties defining it which are part of his/her knowledge system, thus becoming capable of handling the drawing.
- the presence of *Key configurations*, which is recognised and isolated by the subject in the drawing.

3. Experimental situation and early research results

As pointed out above, the object of our research is the role of language in the link between the operational handling of a drawing and the theoretical reference to which it is related. So, analysing the students' oral and written texts using the model of *drawing strategy* and *discourse strategy*, we try to distinguish among various behaviours, which we call *action models*, in the students’ solving processes. Such models should enable us to define how these working moments are structured and what the switchovers from one to another are. Such models should be made operational by finding ways of relating them to students' behaviour. A short description of the first experiment performed is presented, followed by the early results.

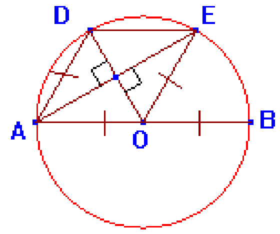
3.1 The experimental situation

We performed a preliminary experiment involving Italian and French Grade X students. They worked in pairs, trying to solve a plane geometry problem involving

[3] This term indicates that the subject begins to search for the “cause” starting from the “effect” (consequence). The action focuses on a search of the theoretical reference to reach the “effect” that, in the specific case of our experimentation, is the rhombus.

geometrical objects already studied by the students in the middle school. Audio and video-recordings as well as students' written texts were collected.

Task

Given a circle C; its centre O; its diameter AB; *D is a point on this circle, so that AD = AO.* *The perpendicular to DO through A meets the circle C again in point E.* *Prove that OADE is a rhombus.*	

3.2 Early research results

We identified several *verbal action models* implemented by pupils: some of them will be described (along with the criteria for detecting them).

It is within the context of these models that we are trying to determine the role of language as a problem-solving tool.

Action models in the Drawing strategy

Among the results obtained by analysing the students' *drawing strategy*, there is one action model involving the creation of a list of information and the handling of such data. Let's use a French student's work as an example (Taina worked with Sophie - see later for additional excerpts).

40. Taina: diagonals AE and OD cut each other in their middle point, making a right angle, and AO is equal to EO, EO is equal to AD.

The information in the list is, obviously:

1) Diagonals AE and OD intersect each other in their middle point.
2) They both make a right angle
3) AO is equal to EO
4) EO is equal to AD.

As we can see, the information in the list is not related to each other.

How and where do students get the information (theoretical references, geometrical relations, properties, etc.) for making their own list? We already said that the information in a list can be collected from the drawing, through operational handling or through the perception of it, but it can also be collected through implicit or explicit inferences. Here are two examples: Taina - Sophie and Gaelle - Camille:

- explicit inference:

59. Sophie: Look! AO is a radius of the circle and EO is a radius of the circle too.
60. Taina: then, AO is equal to EO too
61. Sophie: and AO is equal to AD too
62. Taina: so, and AO is equal to AD, so AD is equal to OE

- implicit inference:

36. Gaelle: maybe, look! this one is symmetrical to this one *(OA and DE)* then it is the same.

The above inference is implicit, since it comes out from the drawing interpretation field[4] and not from a transition to a deductive procedure. We can see some revealing signs: the verbs *"to look at",* related to a perception of the figure, and indicative words *("this one").*

Experience shows that students try to handle the list whenever it becomes too long to be managed. Such handling involves some operations geared to modify the list. Some of these operations include: putting the information in the correct order, picking up useful information and leaving useless information off the list through inference, adding some information to the list. For instance, the following dialogue is an example of how information is deleted from the list:

26 O: But …wait, …look: this one is equal to that one *(Ad = AO)* 27 D: NO, but… look: OD is equal to OE, which is equal to OA, because they are radii of the circle, three radii…then… since this one is equal to that one *(OD = OE)* and OE is equal to AO, then DE is equal to OE 31 D: Since in the text they say that AD is equal to OA, and that OE is a radius, then it (OE) is equal to AO, because AO is a radius. Then OE is equal to OA, which is equal to AD.	Intervention 26/27 List : C_1 : AD = AO C_2 : OE radius C_3 : AO radius By inference we can get to the information C_4: OE=AO Intervention 31 Handled list (useless information C_2, C_3 kept out of the list through inference) : C_4 : OE=AO C_1: AO=OD

In order to point out the role played by language in the drawing and its handling in the relationship between the operational handling and the theoretical reference, let's see the result of analysing the student's oral and written work:

1) The ordering function of language (language as an *organiser*): there is no order in the information carried by drawing, because it is global and two-dimensional (the operational handling of the drawing doesn't give any ordered information). On the contrary, language is straight and sequential and because of these qualities the information must come out in order.

2) The selective memory function of language (language as a selective memory tool), which makes it possible to select only the useful part of the information given in the drawing. The drawing has everything, but there's even too much! Therefore we need to select the information and thus build a system to keep such data in mind. (For instance, deleting an useless information: see the interventions 26-31)

[4] The drawing interpretation field was defined by Laborde as the set of spatial drawing's properties which are related to the geometrical properties of the object.
(Laborde and Capponi, 1994, pp. 171 – 172).

3) The role of control in handling the drawing : the presence of language in handling the *drawing* is a tool for controlling the entire operation. (For an example, see the interventions 36-46)

Action models in the Discourse strategy

Based on the results obtained by analysing the students' output related to the *discourse strategy*, two *action models* were identified: one related to the discourse procedure starting from the question of the problem, and the other related to the procedure started by key words. The latter will be described in detail in this report while the former will not be discussed at this time.

The *key words* act like a kind of label and carry out two functions: they let students recall a particular theoretical reference and they can refer to a linking concept, by which it is possible to switch to another theoretical field, leaving the given one (such a situation is not described in this report).

The action model that refers to *key words* can be used to recall the theoretical reference needed for the solution. In this action the word is associated to the concept (theoretical reference belonging to the subject's knowledge system). Usually, the associative operations are started by pronouncing a word or by reading it. The concept allows us to consider particular geometrical objects. In this sense, based on the definition of Parallelogram, we can consider two equal segments, which are also parallel segments. This defines a set of information that must be found again in the drawing by handling it: we need to identify two opposite and parallel segments.

Here is an example of how *key words* work:

36. Gaelle: maybe we can prove… well, look at it! This one is symmetrical to the other one (AO and DE), so it is the same.

37. Camille: and then?

38. Gaelle: <u>and then we should be able to prove that</u> (*this is a meta discourse*) it is parallel to that one there (*AO parallel to DE*).

39. Camille: yes, but what we have to say is that this one is the middle point (*the diagonals intersection*).

40. Gaelle: yes.

41.Camille: it is the middle point of this one and of that one (*DO and AE*)…wait! AO is equal to AD… and *what if we could prove that (meta discourse)* <u>triangle DAO is isosceles</u>? Because, you know, it is important with reference to this one (*DO*).

42: Gaelle: yes, <u>because it is the height.</u>

43. Camille: yes, <u>it is the height.</u>

44. Gaelle: Yes, it is also <u>the median ….</u>Yeees!!! it is <u>the median!!</u>!

45. Camille: and this means that it is an <u>isosceles triangle</u> because the <u>height</u> is equal to the <u>median</u>… AE is perpendicular to OD and AH is the <u>height</u> in the triangle ADO (*H intersection of the two diagonals*).

46. Gaelle: then AE cuts OD in the middle.

The sequence of these pieces of information (isosceles triangle, height and medians, as underlined in the text) is the standard sequence of the properties by which an isosceles triangle is described in France. After naming the medians, Gaelle realizes that it is connected to *middle point*; then she relates this word to the theoretical reference, the isosceles triangle, and then she goes on to the discourse procedure.

We notice that the meta-discourse usually reveals the transition to a new strategy (see the above-mentioned interventions no. 38 and no. 41) and it plays a role of control on the solving procedure.

The dialogue reported above is an example showing quite clearly how a change of aim is decisive in transforming the descriptive structure of the discourse into a deductive one, to go on to the *discourse strategy*.

4. Conclusion

In the previous sections I presented a description and a means of interpreting students' works in the specific field of plane geometry problem solving, in particular by concentrating on the role of language in the problem solving process. Up to now, results show that natural language plays a truly important role in plane geometry problem solving because it acts not only as a bridge but also as a guide and a mediator, in the two-way relationships between the handling of the drawing and the theoretical reference which is useful for handling the figure. By using a set of criteria to identify students' main strategies, I was able to partly describe this mediation process and give a list of same functions of the language.

I presented the first steps a long-term research project about language as a didactic tool, aimed at obtaining results to be used in classroom didactic engineering. The first experiment I carried out suggests that further steps in the research project should be made at a micro-analysis level concerning the identification of students' *action models* related to their macro-strategies. *Action models* (once identified) should make it possible to find the appropriate area where the teacher can intervene in students' problem-solving activities.

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