USING PROBLEM SOLVING TO IDENTIFY MATHEMATICALLY GIFTED STUDENTS

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Despite growing awareness of the characteristic behaviours of mathematically gifted students, the identification method of choice in most New Zealand schools continues to be results on standardised tests. This study looked at a range of measures for identifying mathematically gifted students in elementary school. Of the six measures identified by the literature as being useful, student's responses to a series of difficult mathematical problems proved most useful in identifying giftedness. Two cases are discussed in which standardised test results differed from ability to solve problems.

Introduction

As the general description of a gifted individual has evolved over the years, so too has the description of the mathematically gifted. In the past, the mathematically gifted were usually identified by means of their results on achievement tests. While this practise continues to a large extent today, a number of researchers have shown that high scores on achievement tests do not necessarily imply mathematical giftedness (e.g., Span and Overtoom-Corsmit, 1986). Researchers and educators alike are looking more and more at the behaviours of students that might indicate mathematical giftedness (e.g., House, 1987). Many of these characteristic behaviours are evident when the mathematically gifted student is engaged in problem solving.

Over the last thirty years there has been an increasing awareness of the importance of problem solving in the teaching and learning of mathematics and an increasing understanding of the particular characteristics of mathematically gifted students that make them adept problem solvers. Krutetskii (1976) studied 192 students aged between six and sixteen, 34 of whom were judged as mathematically gifted. The students were asked to work on a variety of non-standard mathematical problems. He found that mathematically gifted students differed from less able students in their problem solving behaviour in that: they were able to view the content of a problem analytically and synthetically, generalise problem content and solution method, exhibit curtailment when solving similar problems, be flexible, readjust solution techniques where necessary, look for simple, direct, elegant solutions, investigate aspects of problems before trying to solve them, remember and generalise curtailed structures of problems and their solutions, and tire less when doing mathematics than when doing other subjects.

Recent research on problem solving and problem posing has highlighted the potential within these related areas for assessment of mathematical ability. Lowrie and Whitland (2000) studied problem posing in Grade Three students and found valuable information on the students' mathematical understanding and ability was gained by studying the problems they posed. Cifarelli (2000) describes in depth the problem solving of a college student, focussing on her methods to gain a solution as much as on the solution itself. The study revealed much about the mathematical understanding and ability of the student.

The measures that are most commonly recommended for the identification of gifted students in New Zealand are standardised tests, teacher nomination, parent nomination, self nomination and peer nomination (Dale, 1993). This study looked at these five measures and their usefulness in identifying mathematically gifted students. It also added a sixth measure, the student's problem solving ability, because of the strong links that have been identified between problem solving ability and mathematical giftedness.

This study

Sixty-six students participated in this study. Fifty-six of the students were from two mixed ability year six classes (age 10) and ten of the students, of a similar age, were in a class for gifted students. The gifted students attended normal state schools for four days each week and the private gifted school for one day a week.

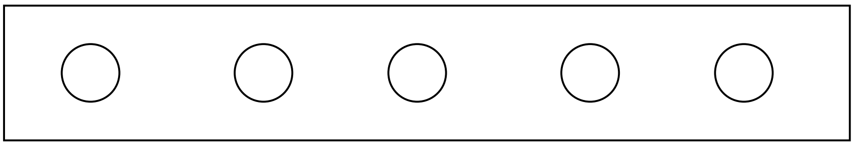
Seven of these students were already judged to be mathematically gifted and provided a benchmark for the study.

Information on each of the six measures used for identification of mathematical giftedness in this study was collected in the following manner.

1. Standardised test: Progressive Achievement Test in Mathematics (P.A.T.) (NZCER, 1998) age percentiles were obtained for each student. Students routinely sit this test in the first month of the new school year.
2. Teacher nomination of which of their students were mathematically talented.
3. Self-appraisal: students were asked to rank their own ability in mathematics.
4. Peer nomination: peers were asked whom they thought were the best mathematicians in their class.
5. Parent nomination: parents were asked whether or not they thought their child was mathematically talented.
6. Problem solving: each student sat two problem solving tests involving a total of six problems. The problems were based on those used in a study on the problem solving attributes of the mathematically gifted by Span and Overtoom-Corsmit (1986). The six problems used are shown below.

1. *On a summer day a grandmother walked along the beach with her granddaughter. They saw bike tracks in the sand. They tried to work out the order in which the bike tracks were made. In which order do you think they might have been made? (A diagram followed showing a series of four intersecting tracks with some of the intersections covered by footprints)*
2. *If I add a father's age to that of his son's the total is 50 years. The father is 28 years older than the son is. How old is the father and how old is the son?*
3. *Sarah went to the shops and bought 4 magazines; Metro, the Listener, More and the New Zealand Woman's Weekly. In how many different sequences can she read her magazines?*
4. *I have two barrels of water. One barrel contains twice as much water as the other barrel. I pour 20 litres of water out of each barrel. Now one barrel contains three times as much water as the other. How much water was there in each of the two barrels to start with?*
5. *Tim's neighbours have just moved to another town. New neighbours will arrive next week. Tim has discovered that two of the new neighbours are children. He wonders what the chances are that at least one of the children will be a boy. What do you think?*
6. *In an office in town people are called by blinking lights. Each employee has a personal combination of one or more lights. There are exactly as many*

combinations as there are workers. There are 5 different coloured lights in a straight line as shown below. How many workers are there?



Each problem was scored using a 7-point rubric. The problems chosen were expected to be difficult for the more able mathematicians in the classes to solve. This proved to be the case, with only 17 of the 66 students who attempted the problems scoring 50% or higher and only two students scoring 90% or more on the problems.

Despite the care taken in choosing the six problems, inevitably some of them proved to be more useful than others in differentiating between the more and less able problem solvers. Problems that a majority of students solved correctly, or came close to solving, were seen as too easy. Problems that very few of the students could solve, or could come close to solving, differentiated between the very able and the less able problem solvers. Problems three, five and six proved to be the most difficult problems. Two of these are permutation problems and the other is a probability problem. Of the three problems which were less successful in differentiating between the more and less able problem solvers, two could be solved using a guess and check method. Problem one proved relatively easy for students to solve for no very obvious reason.

Of the five measures tested, self-appraisal, peer nomination and parental nomination proved to be unreliable as a means for assessing mathematical giftedness and are not discussed further here.

There were strong links between teacher nomination and high standardised test results. Of the 23 students who were nominated as mathematically gifted by their teachers, 19 had test results of 90% or higher. This was largely to be expected as these test results are used in many schools to identify the mathematically gifted.

The problem solving results proved to be most interesting. Looking at the students' answers and their solution methods told us much about their mathematical ability. To illustrate this point I have chosen two students who have similar test results but very dissimilar problem solving results.

Tof and Dan scored very highly on their standardised tests, gaining 99% and 98% respectively. From their similar high test results we might expect them both to achieve well in the problem solving tests. This was not the case. While Tof gained the highest problem solving score of 95%, Dan only achieved 26%. A close look at the methods these two students used to obtain solutions to the problems yields still more interesting information.

For the second problem Dan tried to find an appropriate algorithm to help him solve this problem. He wrote:

$$\begin{array}{rl} 50 & \\ \underline{-\ 28} & \qquad \text{Son } 22 \\ 22 & \qquad \text{Father } 38 \\ +\underline{38} & \\ 50 & \end{array}$$

His addition was incorrect for the second algorithm and he reached an incorrect solution. He was not sure how to solve this problem and seems to be 'grabbing at numbers'.

In contrast, Tof wrote:

$$\begin{array}{rl} 50 & \\ -\underline{28} & \qquad 22 \div 2 = 11 = \text{Son} \\ 22 & \qquad \text{Dad} = 11+28=39 \end{array}$$

This is a very clear and appropriate way of solving the problem. Tof understood the nature of the problem and used appropriate strategies to reach a solution. His mathematical understanding is evident in his response.

A similar difference between these two boys' mathematical ability was evident in their solutions to problem six. Dan used a listing strategy but unfortunately ignored the fact that the coloured lights could not change their position. He labelled the lights r, y, o, b, g. His first five solutions were:

r y o b g

y o b g r

o b g r y

b g r y o

g r y o b

He then went on to provide combinations using four lights, two lights and one light. His answer was ‘25’.

In comparison, Tof recognised the mathematical nature of this problem and set about solving it appropriately. He wrote:

2 x 2 x 2 x 2 x 2

$2^3 = 8$ x $2^2 = 4$

8 x 4 = 32 – 1 = 31 workers

He was the only student to get a correct answer on this particularly difficult problem. He recognised the nature of the problem and used appropriate mathematical tools to reach a solution. His solutions have much in common with Krutetskii’s (1976) descriptions of the work of mathematically gifted children.

The wide difference in Tof and Dan’s mathematical abilities was very apparent in their responses to the problems, yet their test results were almost identical. Teachers relying on these test results to identify giftedness would rate these two children as similarly capable and similarly entitled to any extension programme. Their different abilities, as evidenced by their problem solving solutions, would suggest otherwise.

If standardised test results were used as the primary assessment of giftedness, the top 10 students from the 66 students studied would be as shown on Table 1. Note the range of problem results, from a high of 95% percent to a low of 26%. All of the mixed class students with high test results were nominated as mathematically gifted by their teachers. In contrast, the gifted class teacher (myself) did not nominate Dan, even though he had a test result of 99%. Having worked with gifted children for some years now I relied more on my own judgement of Dan than on his test result when deciding whether or not to nominate him as mathematically gifted. His low problem solving results supported this judgement.

Table 1. Problem solving, standardised test percentile and teacher nomination results for children scoring in top ten (age percentile) of the standardised test

Student and class (Mixed or gifted)	Tof (G)	Rav (G)	Dac (G)	Has (G)	Bob (G)	Dar (G)	Gem (M)	Jup (G)	Dan (M)	Stn (M)
Problem total as %	95	93	76	67	50	48	69	67	26	52
P.A.T. age percentile	99	99	99	99	99	99	98	98	98	96
Teacher thinks talented (Yes)	Y	Y	Y	Y	Y		Y	Y	Y	Y

Analysis of the problem solving scores for students with relatively low standardised test result yielded interesting information also. Thp and Rog, who both scored 60% on the standardised test, scored very differently on the problem solving. Thp scored a very low 14% and Rog a much higher 52%. Their solutions showed a marked degree of difference in their mathematical understanding.

Discussion

While information accrues on the characteristics of mathematically gifted students there remains no easy method for their assessment. Teachers faced with a barrage of behaviours to identify and quantify tend to defer to what is straightforward and seemingly defensible in the form of standardised tests. This study sounds a strong note of caution for this practise. Using a score of 90% or more as an identifier of giftedness failed to identify six excellent problem solvers and identified eleven students with low problem solving results. Standardised tests may not be the most appropriate means for assessing giftedness for a variety of reasons. Firstly, the tests may require students to do little more then complete algorithms. This may test a student's memory and ability to repeat a learned procedure, rather than a student's mathematical ability. Secondly, the more able students may experience a ceiling effect on sitting a grade level test. They may be capable of high results in tests one or more years ahead of their grade. Grade level tests may mask this. Thirdly, the test used for this study was a multi-choice test.

Whether students guessed the correct answer or whether they understood the questions and used appropriate means to solve them is unclear in such a test. The problem-solving test gave a real insight into the mathematical ability of the students who sat it.

Of the problems used in this assessment, three stood out in discrimination of the top children. Two of these questions involved permutations, including the magazine question above, and one involved probability, both of which are known to be challenging concepts.

It is apparent that standardised tests alone are a poor test of giftedness. A short additional assessment is needed to identify those children in need of extension. Problems similar to those used in this study may provide the appropriate assessment.

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