THE DECA-BASED APPROACH: A MATHEMATICAL PROGRAM TO LEARN AND APPLY TWO-DIGIT NUMBERS

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This study examines the effects of an instructional program designed to promote students' understanding of two-digit numbers. The program enables students to identify the tens and ones in two-digit numbers and to add two-digit numbers of different levels of difficulty. One student was taught, using a multiple baseline experimental design to track the learning associated with teaching procedures during the process of schema acquisition. Results show marked gains in both the understanding and addition of two-digit numbers, generalization and expansion of these concepts in the classroom and at home, and a more positive attitude.

Introduction

This study, which is part of a larger investigation, concerns one child, Jim, who at age 10 had still not learned to identify tens and ones or to add two-digit numbers. It uses a methodology rarely seen in mathematics education, although common in applied behavior analysis. This methodology tracks a student's learning with precision, so that the effects of teaching are clear.

The study is based on the theory, proposed by Fuson (1990), that children use particular schemas in developing whole number concepts, especially for multidigit numbers. She suggested that children begin with a unitary conceptual structure that links cardinal meanings of spoken and written numerals (five, 5) as to perceived objects (5 dots). They extend this unitary conception to numbers over 10, in which the separate numerals (e.g. 15) have no quantitative referents in themselves. They then move on to develop further conceptual structures to accommodate the meaning of a decade with extra ones, a decade-ones conception. This is described by two further conceptions, a sequence-tens and a separate-tens-and-ones conception. In a sequence-tens conception, single units of ten are formed within the decade part of the quantity. This is an extension of the unitary counting except that the sequence is now counted in units of tens (e.g. 10, 20, 30). The separate-tens-and-ones conception is built through experiences in which a child comes to think of a two-digit quantity as comprising two separate kinds of units -

units of ten and units of ones. Both kinds of units are counted by ones (e.g. fifty three is 53 - 1,2,3,4,5 tens and 1,2,3 ones) but the units of ten means that each digit in that position comprises a "ten-unitness". With this conception, the child understands that each ten is made up of 10 ones and can then switch to the unitary thinking of ten ones if that approach is required. This last conceptual level is required for additive decomposition as used by Gravemeijer, McClain and Stephan (1998).

A child's construction of the sequence-tens and separate-tens conception depends heavily on their learning environments, including that of linguistic structures (e.g. Fuson & Kwon, 1992; Miura & Stigler, 1987). Some languages are seen as having direct and systematic structures for labeling multi-digit numbers. For example, the Chinese number word for 51 is "five ten one". The direct correspondence of the spoken number word to the written numeral makes it easier to match the two forms of the number. This linguistic structure moves the learner directly into the separate-tens-and-ones conception. Combined with other cultural factors and experiences such as being taught with a strong emphasis on rote counting, this schema is constantly being reinforced for the learner. English has a much less regular structure, particularly from 11 to 19.

Fuson and colleagues trialed a teaching program to facilitate the separate-tens-and-ones conception with two groups of six-year-olds in a lower economic district (Fuson, Smith & Cicero, 1997). Results suggested that the program helped raise the performance of the participants to be substantially above that reported in other studies of children of higher SES and for older children. Their responses were reported to look more like those of East Asian children than of U.S children in other studies. The theoretical basis of this study was expanded for use with two digits (Fuson, Wearne, et al., 1997).

This case study builds on the theoretical framework of Fuson and colleagues. It follows the learning of one boy who started with a unitary understanding of two-digit numbers and follows his progress through to the addition of two-digit numbers using a separate-tens-and-ones conception. The question behind this research was whether or not an intervention program based on this theoretical framework could be shown to help a slow learner develop these conceptual structures for use in two-digit addition.

Methodology

Jim was a ten-year-old boy enrolled in a suburban primary school. He was referred by the school's special needs coordinator because of difficulties with his learning. The class teacher reported that he was unable to identify the tens and ones in a two-digit number even after an intensive year of remedial mathematics with a teacher aide. In a pre-intervention assessment., he gained a score on the Basic Skills portion of the KeyMath Diagnostic Test (Revised) of 5 years 10 months. An assessment with the WISC-III placed him on the 5th percentile with an overall score of 70-82.

The initial focus of the instruction was identifying the tens and ones in a two-digit-number (Fuson's decade-ones structure) as the boy had serious difficulties in this area. To accomplish this, teaching emphasized the use of the "deca-language" as well as the tens and ones value in a given two-digit number. In the created deca-language, numbers are named as in Chinese, so that 11 is called "ten one", and 71 is "seven ten one". Instructional charts associated with the deca-language program were prepared to provide him with an understanding of the separation-tens-and-ones concept. These are similar to standard 100s (first numbers on the left are multiples of ten) charts except that the single digits are at the bottom, so that it read, in part:

20	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
0**0**	0**1**	0**2**	0**3**	0**4**	0**5**	0**6**	0**7**	0**8**	0**9**

A multiple baseline across behaviors design (Barlow & Hersen, 1984) was used to examine the effects of the intervention on different areas of mathematical performance. This design clarifies the relationship between instruction and performance on measures that are not directly part of the instruction: in this case on ten dependent variables used to assess this learning. The first set of three measured Jim's ability to identify two-digit numbers, the second set of six measured competence in horizontal addition, and the third set of four measured generalization in a different setting as well as to vertical addition.

The researcher conducted all sessions during class time, three times a week for 8 weeks, in a quiet room within the school compound. Each session lasted about 30 minutes including the testing which followed the individual teaching.

Intervention

There were six phases in the intervention. These were a baseline phase, when initial measures of his performance were found through tests were administered during the session, four instructional or treatment programs and a maintenance phase when his performance was assessed but he received no special teaching. Treatment 1 (T1) used the deca-language to reinforce the separation-tens-and-ones conception. The theory was taught using different sets of charts as the basis of discussion. As Jim had demonstrated the ability to read and write two-digit numbers in English during baseline, the instruction focused on the ability to differentiate the tens and ones in a given two-digit number, using deca language. Treatment 2 (T2) focused on the addition component of the program. Again, the treatment was streamlined to meet the Jim's needs as identified from baseline data. Therefore, there was particular emphasis on the addition of numbers 6-9 and of multiples of ten as the second addend. Treatment 3 (T3) focused on the addition of two-digit numbers as the second addend, both with and without regrouping. Treatment 4 (T4) focused on adding by visualizing a chart without it being present. Each treatment included a practice phase with a number of different questions that involved the use of the theory that had been discussed.

Figure 1 shows Jim's test scores for two set of measures across each of these phases. The introduction of T1 was associated with an increased performance in the first set of measures: the acquisition of a number system, especially for two-digit numbers . Results for the ability to read a given number in the deca-language (Deca Read) show that he moved from a mean of 20% correct during baseline to 80%

with the introduction of treatment. Similarly, Jim moved to 93.3% for writing a given number said in the deca-language (Deca Write). He moved from a mean of 4% to 86.7% for the ability to state the tens and ones of a given two-digit number. These scores improved further to 100% for Deca Read and Deca Write, and 96% for the Tens and Ones during maintenance. Four weeks later, all scores were 100%.

For the next set of 6 measures, baseline data on addition suggest that Jim was able to add numbers 1-5 with reasonable accuracy. This performance was maintained throughout the treatments and maintenance. There was an improvement in performance for adding numbers 6-9 with the introduction of T2, from a mean baseline of 60% to 100%. This level of performance (100%) also occurred during maintenance. Although addition was not targeted in T1 the test scores suggest that T1 has had an impact on the adding skills of single digit numbers. This was also true for addition of multiples of ten, where addition moved from a mean of 25% (session 1-5) to 66.7% (Session 7-9).

Treatment 2 also brought about a further improvement of test performance for the addition of single digit numbers with regrouping. Scores improved from a baseline of 42.8% to 100% during treatment, with a score of 93.3% during maintenance. For the addition of multiples of ten, the introduction of T2 was associated with an increase in performance from a mean of 25.7% during baseline to 53.3% during treatment and 100% during maintenance.

Treatment 3 focused on two-digit addition. This brought about a change from a baseline mean of 14% to 100% during intervention. This score remained high (90% mean) during the next phase without the use of the 0-100 charts. A similar improvement was evident at the onset of Treatment 3 with two-digit addition with regrouping. Test scores improved from a mean of 14% during baseline to a mean of 86.7% during intervention. However, test scores dropped to 0% on this during the next phase of return to baseline without the use of the charts.

During Treatment 4, charts were no longer available and Jim was asked to visualize them. This was done through an emphasis on mental mathematics. He was encouraged to visualize the previously learnt process of movement across the 0-100 chart. The visualization process was reinforced through a return to the tens-and-ones structure learnt during T1. Test scores in two-digit addition with regrouping improved from a mean baseline (B2) of 0% to a mean of 60% with the introduction of Treatment 4. This further improved to a score of 100% during maintenance.

Generalization was the focus of the third set of four measures and was assessed through the application of these skills in the classroom. This is seen by his ability to state the tens and ones in a two-digit number, to demonstrate them with place value blocks and to add in horizontal and vertical formats. Initially he was correct only for single digit adding (31%). At the end of the 8th week he was correct on 87.5% of the items. Least progress was made in problems in a vertical framework. Therefore the last session (session 21) was used to apply his adding skills to the vertical format. This involved pointing out to the participant that the addends given in a vertical format consisted of two-digit numbers had tens and ones that could be added in the same way as for addends in the horizontal format. A score for the test items in the vertical format after that session suggests that this knowledge was sufficient for the participant to then apply the adding skills to the questions.

Evaluation

In a meeting held 7 weeks after the conclusion of the program, the class teacher reported that Jim had further improved on the skills acquired during intervention. He was able to solve two-digit addition questions given in a vertical form as class work. She also reported that he was able to identify and write three-digit numbers. He was said to be much more confident in mathematics, more involved in the class discussion, more willing to take risks and attempt questions that were previously difficult. His parents reported a change in attitude towards numbers, so that he was more willing to attempt homework questions in mathematics and demonstrated confidence that his answers would be correct. Jim himself reported that he now felt that he could do more mathematics and did not struggle as much as in the past. He recognized that he was learning more and had made much progress since the beginning of the program.

These results support the hypothesis that the deca-based number system as introduced in Treatment 1 provided a scaffolding structure in enabling the student to separate the tens and ones in a given two-digit number. Using the skills that he had already acquired in adding single digit numbers (1-5), this number structure was then applied to areas in addition where he had previously struggled. With this knowledge, and further instruction (T2, T3 and T4) using the same number system in the addition of two-digit numbers, results suggests a greater mastery of skills in addition, generalization into the classroom and an increase in confidence towards mathematics. These results support the hypothesis that the mastery of the separation tens-and-ones conception in the two-digit number will facilitate the learning of operational mathematics.

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