INSIGHTS INTO CHILDREN'S RULER CONCEPTS – GRADE-2-STUDENTS` CONCEPTIONS AND KNOWLEDGE OF LENGTH MEASUREMENT AND PATHS OF DEVELOPMENT

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The paper is concerned with the qualitative investigation of pre- and postconceptual knowledge of grade-2-students with respect to their ruler concepts. The analysis of children`s drawings of rulers, their supporting explanations and their ruler techniques indicates that they have already constructed ruler concepts with different ways of understanding of the key measuring aspects prior to formal instruction. Their ruler concepts frequently exceed an instrumental understanding of linear measurement. However, some children interpret measuring in a familiar arithmetical framework simply as counting even after measurement has been formally introduced.

THEORETICAL BACKGROUND

We owe the understanding of the development of children`s length concepts primarily to the work of Piaget, who is widely recognized as the pioneer of research on measurement (PIAGET et al. 1974). He showed that the concept of length measurement depends on the comprehension of the construction and co-ordination of linear units. The process of linear measurement requires the ability of integrating space and number concepts in an idea of iterative, dividable and countable units. This idea is based on knowledge and conceptions of units and numbers and their integration into the process of linear measuring (BOULTON-LEWIS et al. 1996, HIEBERT 1984, NUNES et al. 1993).

Length concepts are an important part of elementary mathematics education. The introduction of length is the first formal measurement process to be taught and provides the basis for more sophisticated measurement concepts such as area or volume. Traditionally, the introduction of magnitudes such as length follows an instruction sequence (German: „didaktische Stufenfolge"). This instruction implies that children develop and use informal units, prior to the use of standard units and conventional measurement instruments. The underlying goal of this approach is that children on the one hand recognize the relationship between the one-dimensionality of length and the single countable objects and on the other hand realize the necessity of standard units (OSBORNE & WILSON 1992; RADATZ et al. 1998).

However, the iterative use of informal units requires above all the competence of counting and the accurate joining of objects. Students may not pay attention to the length of the objects nor consider them as units (HIEBERT 1984). The neglect of these aspects during formal instruction may explain why fourth and fifth graders frequently demonstrate insufficient knowledge and misconceptions of length units (CARPENTER et al. 1988; GRUND 1992). An increasing number of publications in the last decade questions the traditional sequential introduction of length (AINLEY 1991, BOULTON-LEWIS et al. 1996, BRAGG & OUTHRED 2000, NUNES et al. 1993). Recent studies indicate that students have already had experiences with culturally developed tools of length measurement in everyday life, e.g. with a ruler or a folding-rule. Even if classroom instruction is based on the use of informal units, students of all grades tend to compare visually or use their everyday measuring skills rather than using informal units (BOULTON-LEWIS et al. 1996, NUNES et al. 1993). A ruler has not only a concrete relation to the reference context of measuring but also a theoretical relation to the representation of the measurement process. It involves mathematical figures and presents an iconic illustration of structural connections between conventional units and numbers (STEINBRING 1993). BRAGG & OUTHRED (2000) as well as HIEBERT (1984) emphasize that many students are able to use a ruler for measuring and have mechanical knowledge about reading and using the ruler scale. But most of them obviously have only a poor understanding of the measurement process. They don't know more „than rules about rulers“ (BRAGG & OUTHRED 2000, 97).

According to the „moderate-constructivist“ view, children do not simply accept everyday practices or phenomenona in a passive way but rather construct subjectivly significant concepts in an active-ideosyncratic way in interaction with their environment (GERSTENMAIER & MANDL 1995). Hence, by using a ruler children construct individual understanding of length measuring and develop ideosyncratic concepts regarding the construction and co-ordination of marks, spaces and numbers. Therefore, students` ruler concepts imply on the one hand their measuring and drawing skills and on the other hand their understanding of the concepts underlying these procedures, i.e. their knowledge and conception of the relationship between the measurement of length and the numberline represented on a ruler scale.

In this paper the following questions with respect to the conceptual ruler knowledge of children will be investigated: In how far do grade-2-students focus on the represented key aspects of linear measuring (the structural connections between linear units and numbers) when they use a ruler? In how far do they interpret these symbols as a representation of measuring system?

METHODOLOGY

The qualitative longitudinal study underlying this paper is concerned with the investigation of the development of length measurement concepts of grade-2-students. 12 children from an urban school in Münster were selected as case study children according to their performance in a pretest about their knowledge of length and their teacher`s assessment of their mathematical abilities and performances. This process led to the selection of two girls and two boys in the following three categories: low, average and high achievers. These children were interviewed shortly before the formal sequential introduction of the magnitude „length” (pre-interview), a week as well as six months after this unit (1st and 2nd post-interview). The interview tasks were designed to include a variety of components of length measurement in different contexts. Instead of paper-pencil-tests practical tasks were chosen because of their correspondence to measuring in a real life context and to facilitate the use of conventional measurement tools and objects. The half-standardized interviews were evaluated following the „interpretative paradigm“ (BECK & MAIER 1993). The interview episodes referred to in this paper address iconic and verbal ruler representations and the techniques of ruler use.

Episodes	Questions	Measuring tools
Ruler pictures	*This is meant to be a ruler* (the children receive a sheet of paper with the drawing of a rectangle – 16,6 cm x 2,6 cm). *Please fill in, what is missing in order to make it look like a typical ruler.*	None
Verbal explanations	*You have drawn numbers and marks. Can you tell me why you used those numbers and marks?*	
Drawing	*a) Please draw a line with a ruler which is 10 cm (25 cm) long.* *b) Please draw a line with a ruler which is 2 mm (12 mm) long.*	They could choose between a plastic ruler (20 cm) or a wooden ruler (30 cm); on both rulers „0“ is marked 0,5 cm from the edge.
Measuring	*a) Which of these drawn lines is as long as this wooden path*? (The children are given 4 paper stripes with lines of 20, 23, 24, 25 cm and on the table in front of them is a wooden path made of two sticks, 20 cm and 4 cm long, like in this picture:) *b) How long is the distance between these two sticks?* (15 cm)	

Table 1: Questions to the ruler concepts

RESULTS AND DISCUSSION

The focus of analysis in this paper is on the ruler pictures, because they provide iconic evidences of the interpretations of the structural relations of measuring. When children draw an object or a phenomenon they consciously turn their attention to their rather vague mental images, organize their knowledge about the use and construction of the object and focus on the significant subjective structural characteristics (BIESTER 1991). Both, the ruler pictures, which are supported by verbal explanations, and the ruler techniques open a window in the students` world of ruler concept.

In this paper, the paths of development of ruler pictures are described and analyzed.

Ruler pictures: All the interviewed students were able to construct ruler pictures and to emphasize significant subjective key characteristics of the ruler scale. Their ruler pictures can be assigned to one of four types of drawings which result from the construction and co-ordination of numbers and marks (NÜHRENBÖRGER, in press):

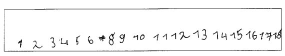
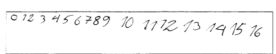
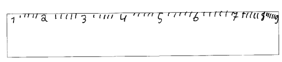
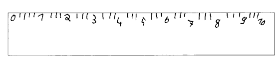
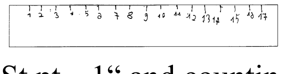
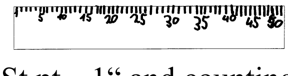
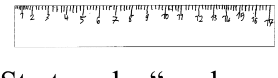
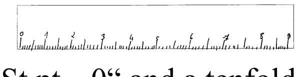
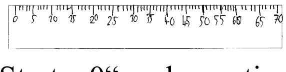
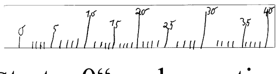
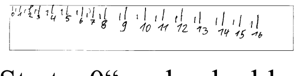
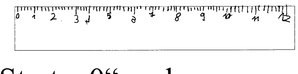
Number-Ruler		**Number-Intermarks-Ruler**	
 Starting point „1“	 Starting point „0“	 St.pt. „1“ and 5 intermarks	 St.pt. „0“ and at first 4, then 3 intermarks
Number-Mark-Ruler		**Unit-Ruler**	
 St.pt. „1“ and counting in steps of one	 St.pt. „1“ and counting in steps of five	 St.pt. „edge“ and a fivefold subdivision	 St.pt. „0“ and a tenfold subdivision
 St.pt. „0“ and counting in steps of five with a fivefold subdivision	 St.pt. „0“ and counting in steps of five with a tenfold subdivision	 St.pt. „0“ and a double subdivision with irregular intervals	 St.pt. „0“ and an irregular subdivision and irregular intervals

Table 2: Examples of types of ruler pictures with different chief characteristics

A **Number-Ruler** is characterized by numbers following the counting sequence from left to right and starting with the first counting number „1“ (if children started with „0“ they showed a deeper relation to the measurement scale). Children who drew these pictures consider numbers to be a subjective dominant and significant aspect of a ruler. They interpreted the measurement scale on the basis of their arithmetical and counting skills; that means that they connected the linearly ordered numbers of a ruler with their conceptions of numbers. This seems obvious because in general, numbers are mentally internalised in an intuitive way as a numberline from left to right. Children drew pictures of Number-Ruler merely in the pre-interview.

Most of the students drew marks and numbers which indicates that they have understood the marks to be a key aspect of the measurement scale. The **Number-Intermarks-Ruler** only has marks between the numbers without any visible subdivision into equal units. The marks rather seem to be an insignificant element which decorates the ruler. However, marks had a characteristic signification for the children so that some subjectively important marks were counted and noted in the familiar arithmetical context. The intervals between the numbers were constantly equal because the marks were counted and noted rhythmically. But it may be possible that the students have perceived a „conventional concept of equal intervals“ (PETITTO 1990, 71). They recognized that rulers usually have equal intervals, but could not explain their structural significance in connection with conventional units.

If children drew marks together with numbers as a visible iterated unit they drew a **Number-Mark-Ruler**. On the one hand this ruler picture shows again an arithmetical interpretation of the ruler as a numberline. But on the other hand the role of zero and especially the spatial distribution of numbers and marks indicate an already promising anchor of the interpretation of centimetre as a dividable unit. Some students counted a certain quantity of marks (or intervals) and consequently wrote numbers in an appropriate counting sequence. Few of the pictures show individual „sense constructions” regarding the unit subdivision presented on the scale.

Students` drawings of **Unit-Rulers** contained a connection between number and space concept which led to the idea of an iterated and subdivided unit. The students showed different facets of understanding of units, both for equal intervals and for rules of subdivisions. While one student drew only a double subdivision with irregular intervals, others drew a ruler picture containing a tenfold subdivision and equal intervals. The starting point of the ruler scale was particularly important. A ruler beginning with „0“ or starting right at the edge shows that the drawer recognizes that „1“ represents the first line segment. In contrast, a ruler with the starting point „1“ indicates an orientation on the sequence of number words. But they do not automatically exclude a deeper understanding of measurement, e.g. one student called his ruler picture with starting point „1“ a „one-minus-subtraction-ruler“.

Paths of development of the ruler pictures: The relative small sample of 12 students already demonstrates the diversity of children`s ruler concepts with respect to their drawings prior to formal instruction (see Table 3). It is remarkable, that four students drew an Unit-Ruler, one of them even with a tenfold subdivision.

Unit-Ruler with	- tenfold subdivision - fivefold subdivision - different subdivision - double subdivision
Number-Mark-Ruler with	- fivefold/tenfold subdivision - steps of two or five - steps of one
Number-Intermarks-Ruler with	- three/four/five intermarks - one intermark
Number-Ruler	

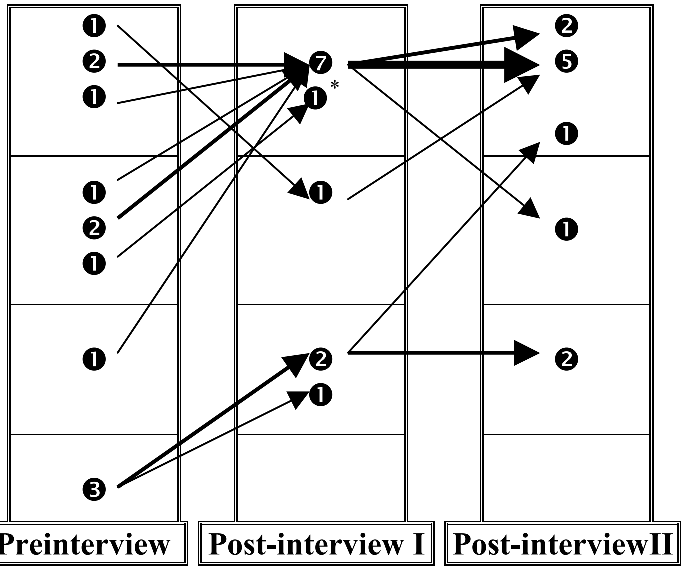


Table 3: Paths of development of the ruler pictures ❸: Number of children
*One child was absent in the 2nd post-interview

The development of the ruler pictures indicates a growing orientation towards the structural construction of the measurement scale. From the pre-interview to the first post-interview two different paths of development can be recognized:

- from the Number-Ruler to the Number-Intermarks-Ruler and
- from the other three ruler types to the Unit-Ruler with a fivefold subdivision.

It is remarkable that even directly after the measurement unit none of the students was able to draw a correct ruler and that three ruler pictures still referred to an arithmetical number concept. The analysis of the second post-interview shows that the majority of children developed further structural understanding of rulers in the absence of formal instruction. For example, two students show a development path from a fivefold subdivision Unit-Ruler to a tenfold one, while two other students still drew the less sophisticated Number-Intermarks-Ruler.

Verbal explanations: The explanations of the students about their ruler pictures contained key words according to the numbers and marks which generally describe a measurement or arithmetic context. Only a few students did not refer to any content.

Explanation of numbers		***Examples***
Reference to measurement	- The number of units	*„One can measure how many centimetres.“*
	- Information about length	*„One can measure how long something is.“*
	- Information about ruler length	*„In order to know how long the ruler is.“*
	- Visual help for measuring	*„There is no need to look at the marks.“*
Reference to arithmetic	- For counting	*„In order to count.“*
	- For calculating	*„In order to calculate.“*
No reference	- Conventional	
	- No idea	*„I don`t know.“*

Explanation of marks		Examples
Reference to measurement	- units: different	*„These little marks are millimetre, and the long ones are centimetres."*
	- units: only one known	*„These long marks are centimetres. The little ones are 1, 2, 3, ... 8 centimetres."*
	- „Distance-holder"	*„The four intermarks keep the distance."*
	- Visual help for measuring	*„In order to measure exactly"*
Reference to arithmetic	- For counting	*„In order to count."*
	- For calculating	*„In order to calculate."*
No reference	- Conventional	*„On every ruler there are little marks.*
	- No idea	*„I don`t know.*"

Table 4: Different explanations to the measurement scale with some examples

The interpretation of numbers and marks as units such as metre or centimetre indicated an anchor of measurement understanding. Few students already knew at the beginning of grade-2 that a ruler represents two different, but connected units. Other students saw an interval between two numbers (or „two long marks") as a basic iterable unit called centimetre without knowing that there is also another subdivided unit. Those children were merely able to use a unit for repeated counting. However, few of them were able to construct an explanation of the gaps between the marks that demonstrates „measurement sense" - they interpreted the interval as a „distance-holder" or as a unit which occasionally is also called centimetre. Some children used terms such as metre or centimetre without any unit concepts. They either interpreted marks as „number stations" or they were unable to explain them. In a sense these terms played the role of a „number companion" which refer to length.

Few students were not at all able to express insigths („measurement sense") with respect to the construction of the formal scale. They only explained the symbols on a ruler in a number and counting context or in a verifying-conventional way.

Measuring and drawing with the ruler: Nearly all students were able to use a ruler correctly for drawing centimetres. They aligned the lines with zero or back from the number to zero. Most the case study children were even able to construct a millimetre concept although millimetres are only taught in grade-3. While few students aligned millimetres correctly and utilized the references between one centimetre and ten millimetres, others interpreted the measurement scale in an idiosyncratic way – they explained e.g. a line of two millimetres as following: „*Up to ten, because one millimetre is up to five*" or „*up to the second longer mark*" or „*up to the longer mark just before the number two*" or „*up to two*". These creative „sense constructions" indicated subjective interpretations of the relations between numbers and marks on the formal scale. But they showed on the other hand that some students have a poor understanding of subdivision - they only knew the „number-unit" centimetre.

In contrast to the skills of drawing centimetres with a ruler, many students, even some who drew a ruler with starting point „0", had problems with measuring an object because they were not sure about ruler alignment: „*Should we align a ruler with the scale or with the other side - with the edge, with zero or with one?*"

The procedure of ruler alignment was not a stable one, many students varying their procedure with regard to the situation: Although they were able to identify the length of an object as the difference between its starting and end point on the ruler, they sometimes did not pay attention to how they aligned the object with „0". Dominating for them was obviously the result of the measurement solution, because they tried to place the ruler next to an object in a way that the end of the object was marked by a number and not by an intermark. It seems that they followed an inaccurate mechanical procedure without taking into consideration what the intervals and the numbers on the ruler scale represent.

CONCLUSION

The analysis of the ruler concepts suggests that children at the beginning of grade-2 already have a subjective understanding of the measurement scale represented on a ruler. Their ruler pictures in particular demonstrate that students are able to imagine a ruler and to perceive key aspects of measuring. Their everyday experiences with measurement not only promote an intuitive approach to the technical use of rulers, but also influence the development of individual ruler concepts. These concepts vary among children and change in different paths within a schoolyear.

However, the diverse representations of characteristic features of rulers, does not necessarily indicate an elaborate understanding of the relations between linear units and the measurement scale. Although students are able to use a ruler in measurement situations, they often do not possess structured insights into the construction and co-ordination of units. They connect instead their imagination of measuring with their number concept and interpret some of the measuring aspects in the context of their arithmetical understanding of numbers and counting. Their use of a ruler is based on rules, which merely refer to the application and reading of numbers. Therefore children do not have problems with drawing and (partly) measuring. This differentiated perspective on ruler concepts supports and elaborates on earlier findings of HIEBERT (1984) and BRAGG & OUTHRED (2000).

The current research results suggest that the traditional formal introduction of length with non-conventional units might not help students to develop an elaborated concept of linear measuring, because it does not allow them to make connections to their previous everyday measurement experiences with conventional tools. Instead, dealing with (mainly three-dimensional) informal units obscures the linear nature of the unit of measure and the children interpret measuring with informal units in a familiar arithmetical framework simply as counting. Hence, students cannot visualize the measuring process as a composition of linear units. „Neither zero nor the iteration of line segments can be made explicit when informal units themselves are counted, thus reducing the possibility that students are able to make the important link with the underlying linear unit concept" (BRAGG & OUTHRED 2000, 103). NUNES et al. (1993, 53) emphasized that „relying on conventional units, which have already been chosen and built into an instrument, does not make measurement more difficult." This study has shown that grade-2-students have idiosyncratic anchor imagination of centimetre and millimetre. Their everyday knowledge about the ruler scale and their skills of the ruler use should serve as a starting point for formal instruction and guide the development of an elaborated length concept. A sophisticated understanding of measurement requires both, „hands-on” measuring activities with rulers *and* especially discussions and reflections about the key aspects of measuring repeatedly stimulated and encouraged by the teacher. The comparison and analysis of individual ruler pictures of the children in the classroom for example can help the teacher to identify their „pre-conceptions” and also foster such a discussion and reflection.

REFERENCES:

Ainley, J. (1991). Is there any mathematics in measurement? In E. Love & D. Pimm (Eds.), *Teaching and Learning School Mathematics* (pp.69-76). London: Hodder&Stoughton

Beck, C. & Maier, H. (1993). Das Interview in der mathematikdidaktischen Forschung. *Journal für Mathematik-Didaktik, 14*(2), 147-179

Biester, W. (1991) (Ed.). *Denken über Natur und Technik*. Bad Heilbrunn: Klinkhardt

Boulton-Lewis, G., Wilss, L. & Mutch, S. (1996). An analysis of young children`s strategies and use of devices for measurement. *Journal of Mathematical Behaviour, 15*(3), 329-347

Bragg, P. & Outhred, L. (2000). Students` knowledge of length units: Do they know more than rules about rulers? In T. Nakahara & M. Koyama (Eds.), *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education* (Vol.2) (pp.97-104). Hiroshima: Hiroshima University

Carpenter, T., Lindquist, M.M., Brown, C.A., Kouba, V.L., Silver, F.A. & Swafford, J.O. (1988). Results of the fourth NAEP. Assessment of Mathematics: Trends and Conclusions. *Arithmetic Teacher, 36*(4), 38-41

Gerstenmaier, J. & Mandl, H. (1995). Wissenserwerb unter konstruktivistischer Perspektive. *Zeitschrift für Pädagogik, 41*(6), 867-887

Grund, K.H. (1992). Größenvorstellungen – eine wesentliche Voraussetzung beim Anwenden von Mathematik. *Die Grundschule, 24*(12), 42-44

Hiebert, J. (1984). Why do some children have trouble learning measurement concepts? *Arithmetic Teacher, 31*(7), 19-24

Nührenbörger, M. (2001, in press). Gezeichnete Linealbilder –Hinweise auf mentale Linealkonzepte. In Cusanuswerk, Bischöfliche Studienförderung (Ed.), *Das Bild als Medium der Erkenntnis* (Schriften Nr.14). Rheinbach: Union Betrieb

Nunes, T., Light, P & Mason, J. (1993). Tools for thought: The measurement of length and area. *Learning and Instruction, 3*(1), 39-54

Osborne, A. & Wilson, P. (1992). Foundational Ideas in Teaching about Measure. In T. Post (Ed.), *Teaching Mathematics in Grades K-8* (pp.89-121). Boston: Allyn and Bacon

Piaget, J., Inhelder, B. & Szeminska, A. (1974). *Die natürliche Geometrie des Kindes*. Stuttgart: Klett

Petitto, A.L. (1990). Development of Numberline and Measurement concepts. *Cognition and Instruction, 7*(1), 55-78

Radatz, H., Schipper, W., Dröge, R. & Ebeling, A. (1998). *Handbuch für den Mathematikunterricht. 2. Schuljahr*. Hannover: Schroedel

Steinbring, H. (1993). „Kann man auch ein Meter mit Komma schreiben?“ Konkrete Erfahrungen und theoretisches Denken im Mathematikunterricht der Grundschule. *mathematica didactica 16*(2), 30-55