UNDERSTANDING HIGH SCHOOL MATHEMATICS TEACHER GROWTH

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This paper reports on a study of the professional growth of four inservice high school mathematics teachers who made fundamental changes to their teaching over their teaching career. The analysis focused on the nature of the teachers' belief structure for mathematics and the relationship between it and changes in their teaching. The findings indicated that belief structure played a key role in when and how changes occurred in the participants' teaching in terms of creating pedagogical tensions and as generative metaphor, which were important to facilitate the generation of new perceptions, explanations and behaviours in their teaching. The findings highlight the possible significance of consciously attending to these factors to assist mathematics teachers in achieving desired changes and choices in their teaching.

Background

Change seems to be a significant challenge for mathematics teachers. Even teachers who are interested in change do not necessarily succeed at making substantive or fundamental shifts in their teaching. In the last decade, the mathematics education literature has reflected a growing emphasis on beliefs as playing a key role on if, when or how change occurs because of their apparent relationship to behaviour. Ernest (1989), for e.g., argued that beliefs are a primary regulator for mathematics teachers' behaviours in the classroom. Although it is not clear that beliefs by themselves can account for mathematics teachers' classroom behaviours, studies do suggest that there is a strong and important influence (e.g., Cooney et al 1998; Chapman, 1997; Lloyd and Wilson, 1998; Pehkonen, 1994; Raymond, 1997; Thompson, 1992). The relationship to change, for the most part, has been deduced from such studies and others which have demonstrated or implied that shifts in beliefs accompanied shifts in teaching for inservice teachers participating in innovative approaches to professional development (e.g., Chapman, 1999; Cobbs et al, 1990; Simon & Schifter, 1991). The literature, however, seems to lack studies with an explicit focus on the relationship between belief structure and inservice mathematics teachers' growth. This relationship deserves attention given the ongoing importance of understanding the mathematics teacher and helping them to make fundamental changes in their teaching to reflect current reform recommendations in mathematics education. This paper reports on a study that investigated this relationship for inservice high school mathematics teachers who have changed their practice on their own from a teacher-centered to a student-centered perspective. Specifically, the focus is on the nature of the belief structure (how the beliefs are held) for mathematics and the relationship between it and changes in their teaching over their teaching career.

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Belief Structure

Green's (1971) metaphorical analysis of belief structures provides one way of interpreting the ways beliefs are held. Green described 3 dimensions of belief systems: (1) *primary or derivative* (i.e., the quasi-logical relation between beliefs), (2) *central or peripheral* (i.e., the relations between beliefs having to do with their spatial order or their psychological strength), and (3) *isolated clusters* (i.e., beliefs are held in clusters, more or less in isolation for other clusters and protected from any relationship with other sets of beliefs). Beliefs are also held evidentially or non-evidentially. Green (1971) explained that the importance of a belief to the believer is determined by whether it is psychologically central. Thus, the question of which beliefs are amenable to change may have to do not with their being primary or derivative, but with the strength with which they are held. For e.g., a teacher who holds a psychologically central belief that mathematics is a collection of facts may be less likely to change it regardless of whether it is primary or derivative. Change can also be restricted when, as Green (1971) explained, isolation of belief clusters occur to facilitate contradictory beliefs developed in contexts in which beliefs are not explicitly compared or when beliefs are held from a non-evidential perspective, a perspective immune from rational criticism.

Research Method

The participants of the study were 4 experienced (16 to 33 years) high school mathematics teachers (pseudonyms Linda, Elise, Mark, and Rose) who were known in the school system as excellent teachers. They were very articulate and open about their thinking and experiences in teaching mathematics. Data collection and analysis followed a humanistic approach (Chapman, 1999; Creswell, 1998). Data collection involved open-ended interviews, role-play, and classroom observations. The interviews focused on paradigmatic and narrative accounts (Bruner, 1986) of the teachers' past, present, and possible future teaching behaviours and their thinking in relation to mathematics pedagogy. Paradigmatic accounts, triggered in a variety of ways, highlighted the teachers' theories about a situation (e.g., problem solving). Narrative accounts highlighted the teachers' lived experiences and consisted of stories that described the experiences as they occurred, e.g., stories of lessons they taught that (i) were memorable, (ii) they liked, and (iii) they did not like. Classroom observations focused on recording what the teachers said and did and to identify scenarios for the teacher to talk about or role-play. All interviews and classroom talk were audio taped and transcribed.

The data were scrutinized for situations indicating change in teaching approach and for beliefs related to teaching mathematics. Beliefs were identified in terms of significant statements and actions that reflected, for e.g., personal judgements, intentions, expectations, and values of the participants in the context in which they were described. Belief about mathematics emerged as the most dominant belief in the teachers' story of change and was made the focus of the study. The nature of the belief structure and the relationship to the teachers' growth in teaching mathematics were determined by examining the beliefs in the contexts in which they occurred and the relationships among contexts. An abbreviated account of the findings follows.

Belief Structure for Mathematics

The dominant beliefs the participants held about mathematics were mathematics is play/game (Elise), mathematics is shared experience (Mark), and mathematics is language (Linda and Rose). The beliefs emerged from and were supported by the participants' personal experiences. For Elise and Rose, the beliefs emerged from their experience as students of mathematics. Elise explained,

> As a student in the classroom, I just thought it [math] was a blast… something that you absolutely love to do for no other reason. … I mean, I just loved the thrill of the chase. I loved proof. You know, it's the, it's a game, like it's just play, and it has a set of rules but it doesn't really have a set of rules.

Rose's experience as a student involved a lot of small-group discussions about mathematics that framed her view of mathematics as language, i.e., "something you speak, do and use to number the world."

Mark and Linda's beliefs emerged from their experiences as teachers. Mark's experience using manipulatives and small-group work when he started teaching elementary school mathematics along with his high school teaching framed his view of mathematics. He pointed out,

> Math to me is an experience. That's the way, that's how I started to see it with the elementary [school] kids.

Linda's experience working through mathematics problems in her own way in planning a mathematics course she was teaching for the first time framed her view of mathematics as language – "a tool used to understand our world."

There were two significant ways in which these beliefs were held – in terms of Green's (1971) belief structure and as metaphors. The way the participants held their beliefs about mathematics seemed to be primary, evidential and central (Green, 1971). The beliefs were primary because they were not explicitly based on other beliefs and evidential in that the teachers' lived experiences provided the primary evidence for them. Thus, they were amenable to change if these experiences changed. The beliefs seemed to be central, i.e., psychologically strong, because of the passion and conviction the participants displayed for them and their resistance to change. For example, each participant was very critical of any thinking or actions of teachers that was not consistent with her/his belief. He/she was very judgemental of her/his own teaching when in conflict with the belief and not vice versa. The belief also remained stable in that while there were extensions in its interpretation, it did not change conceptually/philosophically since it was constructed. The psychological strength of the beliefs was further validated and reinforced by current reform recommendations about mathematics adopted in the revised mathematics curriculum of the teachers' province. The humanistic perspective of mathematics embodied in the beliefs resonated positively with this orientation of the reform recommendations.

In addition to the preceding belief structure, another way in which the participants' beliefs about mathematics were held was as metaphors. Mathematics is play, experience and language can be viewed as descriptive metaphors from the teachers' perspective in that mathematics was being described and understood in terms of characteristics directly appropriate for some other domain. In the context of their stories, however, there were times when the beliefs also seemed to become generative metaphors (Schön, 1979) and helped to facilitate change as discussed later.

Relationship Between Belief Structure and Change in Teaching

The preceding structure of the participants' beliefs of mathematics played an important role in when and how changes occurred in their teaching. These roles are discussed in terms of *pedagogical tensions* and *generative metaphor*.

Pedagogical Tensions

Substantive changes in the participants' teaching were preceded by pedagogical tensions and a desire to resolve them. Both of these conditions seemed to be influenced by the psychological strength of their beliefs about mathematics. Fundamental shifts in the teachers' teaching were generally directed, often unconsciously, to eliminate or reduce tensions between their teaching and their beliefs about mathematics that created a state of disequilibrium for them. This tension was triggered by situations in their classroom experiences that made their teaching not feel right or students' learning seem to not meet their expectations. The tensions were resolved by extending the interpretation of doing mathematics without changing the primary belief of mathematics and modifying the teaching approach. Thus, belief about mathematics seemed to be held with more dominance, and had a stronger influence, than belief about teaching, in that, regardless of how the latter was held, e.g., central or peripheral, the change was to make actions reflect belief about mathematics. Elise's case will be used to illustrate these tensions and corresponding changes in teaching.

As a beginning teacher, Elise's expectation was that she would be able to teach to reflect mathematics as play even though she did not have a clear conception of what teaching looked like to realize this. This expectation was quickly smothered when Elise started her practice. She was told by her experienced colleagues that their approach (which Elise described as "stand and deliver") was the only realistic way to teach high school mathematics. There was nothing play-like about high school mathematics and she should abandon any thoughts of wanting to make it that. Not sure of what else to do, Elise reluctantly adopted her colleagues teaching approach and started to experience her first significant pedagogical tension. In order to deal with this conflict, instead of changing her belief about mathematics, she developed a position to protect it. She explained:

> You know, that that's the time when I separated mathematics from teaching mathematics. That's when … it became internalized to me that there must be a difference between teaching mathematics and doing mathematics. But those aren't the same thing and they can never be the same thing. And that to this day frustrates me because I don't want that to be the way it is.

With this ongoing desire to resolve the tension, Elise continued to think of how to make high school mathematics be play for her students. When she no longer felt under the influence of her experienced colleagues, since what she was doing lacked fun, she decided that if she added some fun activities, students might start to experience mathematics as play. These special activities, however, did not resolve the tension between what Elise believed about mathematics and how she was teaching it. They were "fun" in an isolated way and did not give her a feeling of play or a sense of the students engaging in play in terms of the mathematics being taught/learnt or mathematics in general.

After a few years, Elise realized that those "fun" activities did very little to foster her beliefs about mathematics in her classroom. They were too detached from the core content being taught and served more of a recreational purpose in the transition from one unit to the next. Elise also felt that, for the most part, the students were simply mimicking her instead of engaging in play or being problem solvers. As she focused on how to help them to become problem solvers, she eventually made a connection between game and problem solving, in particular, viewing problems as games and emphasizing the importance of strategies. She explained,

> I thought, if I'm going to be a good problem solver, I have …to think about what strategies to try. And I really firmly believe as a learner, what I need to do is look at them [problems] as a game. When I play monopoly, I know the rules but it's a dynamic, it changes. When I solve a problem, I have my strategies that colours the rules, but it's a dynamic situation, and so sometimes I use this strategy, sometimes I use that strategy, but I'm more relaxed because it's a game

Elise's interpretation of strategy included a way of thinking, seeing patterns, making connections, and reasoning and was seen as relevant to all areas of mathematics. For her, strategies were not just techniques to solve problems, but a way of viewing and learning mathematics. They were also "something you must see for yourself". This perspective of strategies provided a way for Elise to think of high school mathematics and her teaching of it differently. For her, in addition to fun, high school mathematics and doing mathematics became being and focusing on strategies, respectively, and consistent with her belief of mathematics as play/game. With this, Elise's teaching shifted from being "stand and deliver" to being more student-centered, but teacher guided. She tried to guide students to seeing strategies, e.g., looking for patterns in developing a procedure for themselves. She started to use more questioning and less telling. She followed the textbook less and selected or developed activities in which students could discover strategies through discussion. Students worked in small-groups to figure out strategies and shared them in whole-class discussions. However, Elise often intervened in the groups with questions to guide them to a strategy or led interactive, whole-class discussion to do so. This strategy-based approach created for Elise an acceptable level of harmony between her belief about mathematics as play and her teaching. The primary beliefs about mathematics remained unchanged while teaching behaviour was modified to an acceptable level of harmony.

The next significant pedagogical tension arose when Elise's view, that her approach of focusing on mathematics in terms of strategy was helping students to think for themselves, was challenged by her students' performance on the grade 12 provincial diploma examination (required for graduation in mathematics). In the mid-90's, this exam was revised to include genuine problem-solving items and Elise was surprised when even her best students did not perform at the level she expected. In trying to resolve this new tension, Elise eventually concluded that guiding students to see strategies was not enough for them to be good problem solvers. They must be able to think for themselves to be successful. Elise decided to facilitate this by getting students to think about their thinking through writing and self-questioning. She noted,

> I've focused much more the past 5 years on reflective thought for each person, each individual, and trying to not only encourage but in many ways force kids to do it. … Reflecting on what it is that you know and what does it mean to understand the remainder theorem [for e.g.] is very important. …

The reflective process Elise added to her teaching was, for her, both a strategy and a way of making sense of strategies. It also increased the level of harmony she was developing between her belief about mathematics as play/game and her teaching.

At the time of the study, Elise's teaching had evolved from a "straight stand and deliver" at the beginning of her practice to a combination of teacher-guided and teacher-facilitated situations. Her pedagogical tensions were resolved by modifying her teaching approach within the primary belief of mathematics as play/game. This belief did not change, but her understanding of it broadened (i.e., as fun, strategies, and reflection) in response to resolving the tensions and resulted in significant shifts in her teaching.

Elise's pedagogical tensions were unique to her but the underlying process of dealing with them was representative of that of the other participants. Mark, Rose and Linda had their own tensions that corresponded to significant turning points in their teaching. For example, in Mark's case, his teaching shifted from lecturing, which was in conflict with his belief of mathematics as experience, to teacher-guided/facilitated situations to correspond to his broadened interpretation of experience as communication, connection, and problem solving, each of which resulted from trying to resolve a pedagogical tension.

<u>Generative Metaphor</u>

While the preceding section highlighted the relationship between pedagogical tensions associated with the psychological strength of the participants' beliefs about mathematics and change, this section highlights the relationship between the metaphorical structure of these beliefs and change. One way of understanding the process of change in the participants' thinking and teaching is in terms of viewing the way the belief about mathematics was held, i.e., as play, experience and language, as a generative metaphor. Generative metaphor (Schön, 1979) or structural metaphor (Lakoff and Johnson, 1980) facilitates learning through a process that involves generating or structuring one concept in terms of another. As Schön (1979) noted,

> Metaphor refers to a certain kind of product … and to a certain kind of process, a process by which new perspectives on the world come into existence [p 254].

The latter situation refers to the generative quality of metaphor in terms of helping an individual to generate new perceptions, explanations and inventions to understand and deal with his/her world. This generative quality seemed to underlie the way in which the participants broadened their perspective of mathematics within the primary beliefs of play, experience and language and made changes to their teaching. The generative process was triggered by the pedagogical tensions as a way of resolving them. From the perspective of metaphor, these tensions occurred when the two domains of the metaphor seemed incompatible in relation to teaching high school mathematics, e.g., high school mathematics (first domain) as play (second domain) for Elise, or experience for Mark, was difficult for them to relate to their teaching. The generative process allowed for the elaboration of the interpretation of the second domain and corresponding changes in the first domain and their teaching. For continuity, Elise's case will be used as representative of the other participants' situation to illustrate the following five stages that seemed to constitute this process.

(i) *Hold belief as a metaphor:* This stage involved holding and articulating the belief about mathematics as a metaphor. Elise was able to do this at the beginning of her practice. Concrete, personal experiences were important to the creation and articulation of the metaphor.

(ii) *Experience pedagogical tension:* The generative process was triggered by these tensions described earlier.

(iii) *Elaborate assumptions/characteristics:* This stage involved becoming aware of and articulating assumptions/characteristics flowing out of the phenomenon providing the context of the metaphor (i.e., play for Elise) based on personal experience of the phenomenon. The first characteristic elaborated by Elise was fun.

(iv) *Connect two domains of metaphor:* The two domains of the metaphor were connected through the characteristics identified in stage (iii). For Elise, fun was mapped to high school mathematics.

(v) *Revise perception:* The mapping led to a revision of perception of both domains of the metaphors. Elise extended her understandings of high school mathematics, interpreting it as fun activities. Mathematics as play could now be seen in the context of high school mathematics and consequently the teaching of it. Change in Elise's teaching was then accomplished to reflect these ways of viewing mathematics.

Stages (ii) to (v) were repeated when the conflict in stage (ii) was not completely resolved or a new conflict arose. In repeating these stages, Elise generated strategies and reflection as further articulation of characteristics of play. These were accompanied by corresponding changes to her teaching. Thus, in the context of the generative process, only after further articulation of the nature of play was Elise able to expand her interpretation of high school mathematics and make substantive changes to her teaching.

Conclusion

The study suggests that the belief structure about mathematics and experiential/ concrete contexts for generating and interpreting pedagogical tensions related to the belief about mathematics are important factors that determine when and how changes in teaching occur. The belief structure in the form of metaphor could help to facilitate teachers' generation of new perceptions, explanations, and inventions in their teaching of mathematics. Thus, the study brings to light the possible importance of generative metaphors that may underlie mathematics teachers' personal story of growth and the possible significance of consciously attending to such metaphors to assist teachers in achieving desired changes and choices in their teaching.

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