Investigating of The Influence of Verbal Interaction And Real-World Settings In Children's Problem-Solving And Analogical Transfer

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Abstract

Eighty fourth-grade children in mixed-ability dyads were randomly assigned to four experimental conditions: with and without real-world settings and verbal interaction with peers. Dyads of children were asked to solve two daily mathematical problems in the first phase. Three weeks later children solved three daily mathematical problems as delayed transfer tasks after completed the first tasks. The results show that verbal interaction accompanied with real-world settings situation is the most effective way to improve children's performance in problem-solving. Either verbal interaction or real-world settings helps children's analogical transfer rather than solving problems individually without any real-world settings. Moreover, children tend to mix addition, multiplication and counting to solve problems through verbal interaction with peers and tasks.

Introduction and theoretical Framework

It's plausible that a stronger connection between school mathematics and everyday mathematics will enhance mathematical competencies and learning (Streefland,1991). Therefore, one of the focal points within problem-solving research is the impact of real-world settings and how children apply their skills and knowledge through verbal interaction with peers to find respective answers in solving everyday mathematical problems.

According to Piaget's theory, the young children are constantly experimenting with objects, language and situations to understand more about the world (Rogoff, 1990). A child is supposed to be an active experimenter who discovers facts and relationships when s/he is presented with materials and situations that encourage the design of his own experiments. This will in turn lead to deeper and more long-lasting knowledge than will a rote memorization of facts presented by teachers or in textbooks. Furthermore, everyday mathematics should be thought of as a process of knowing in which the same activity (arithmetic) takes different forms across situations and occasions (Lave, 1988). Thus the practices and the problem context in which people engage are supposed to be important for problem solvers, especially for the young learners.

The goal of making mathematics meaningful to students is to connect their everyday life understanding, in particular where arithmetic is concerned, to formal learning (Freudenthal, 1991; Streefland,1991). Strategies and patterns of problem-solving, initially are supported by situations and contexts, which in the long run are superseded by abstractions. Therefore, even though the practices and contexts of everyday mathematics differs from contexts of school mathematics, it is expected that students are able to transfer their mathematical competence in everyday context to do similar arithmetic in school or test contexts. Empirical studies have found that people can carry out new procedures or solve novel problems that are quite similar to those on which they had previously learned (Vousniado & Ortony, 1989). This study will explore the effects of real-world settings on children solving problems and applying their knowledge from a previous setting to novel and formal mathematical problems.

According to a number of models in cognitive science, cooperative verbal interaction with peers is a fundamental component in cognitive processes that operates at a deep level of conceptual understanding, for instance analogical reasoning in solving mathematical problems (e.g., Huang, 1997), as well as mathematical operations skills and application skills (e.g., Fuchs, Fuchs, Bentz, Phillips & Hamlett, 1994). From the social cognitive point of view, when students interact with their peers, they are exposed to new strategies, terminology and ways of thinking about problems, which may in turn affect their problem-solving behavior. For children as well as for their social partners, engagement in shared thinking yields the opportunities for development of greater skill and understanding (Rogoff, 1990).

Although some research found inconsistencies and contradict challenges in peer verbal interaction (e.g., Orsolini & Pontecorvo, 1992; Webb, 1989), the usual findings from empirical support for the premise is that one-to-one discussion environments might remove many of the barriers that prevent students from asking questions and learning actively (e.g., Fuchs et al., 1994; Fuchs, Fuchs, Hamlett, Phillips, Karns, & Dutka, 1997). Studies showing positive interactions and learning between high- and low-achieving classmates have incorporated thorough training in how to interact constructively (e.g., Fuchs et al., 1994; Fuchs et al., 1997; Graesser & Person, 1994). Therefore, casing this study within the context of same-age, good-and-poor dyads, the researcher expected to examine the effect on cooperative verbal interaction in dyads accompanied with/without real-world settings in solving daily mathematical problems and delayed transfer tasks.

The purpose of this study is to examine how verbal interaction and real-world settings affect children's skills in solving daily mathematical problems and analogical transfer. Children's strategies in solving multiplicative problems through verbal interaction with peers and tasks were also analyzed. In light of previous work on the importance of verbal interaction and real-world settings, the hypothesis supposes that either verbal interaction or real-world settings would significantly influence children problem-solving as opposed to solving problems individually without verbal interaction and real-world settings. Furthermore, the other hypothesis is that there is an interactive relationship between problems-solving conditions and solvers.

Method

Subjects. Eighty fourth grade children participated in this experiment. They were selected randomly from a public school in Taipei city, Taiwan. Subjects had been taught and had experience in verbal interaction and problem solving with small groups cooperatively in daily mathematics courses. All subjects worked in mixed-ability pairs. Therefore, good and poor solvers in this study were determined based on the mean of subjects' achievement in the previous semester's mathematics course. If a subject's score was higher than or equal to the mean score, s/he was defined as a good solver. If a subject's mathematics achievement score was lower than mean score, s/he was defined as a poor solver.

Design of the study. There were two problem-solving phases included in this study. The first phase was focused on dyads of children in solving two multiplication problems. The second phase was focused on children's transfer of learning in solving three novel multiplicative problems. The contexts of problems designed in this study were derived from children's daily life and Taiwanese society. The item difficulty index and item discrimination index of the five problems had been tested from 97 forth grade students. They were randomly selected from a public elementary school in Taipei, Taiwan. The values of item difficulty index of these five problems were from .39 to .50. The values of item discrimination index of these five problems were from .58 to .96. The idea of problem settings and arithmetical structures of these problems are referred to the research of Franke (1998) and Ruwisch (1998). The mathematical structures contents of the five problems are listed on Table 1. The design, materials and procedures in two phases are illustrated as follows.

A. *The first problem-solving phase.* 1.Task problems. Two multiplicative problem-solving settings were provided. In the first problem setting, called "Class Camping", the children were required to buy items for a camping trip with 30 classmates. In the second situation, called "Lantern Festival", children were required to buy colored envelopes for pasting on three different sizes of posters for Lantern Festival. Children had to determine the number of packages of items needed for solving these two tasks and write down their answers on shopping lists. Both "Class Camping" and "Lantern Festival" problems have similar arithmetical structures but differing in contexts. The underlying mathematical contents of "Class camping" and "Lantern Festival" are numbers and area, respectively. 2. Experimental conditions. The problem-solving situations were designed differently for each of the four experimental conditions. Subjects in dyads were assigned to the four conditions randomly. All subjects were provided with written and oral descriptions of the problems when they came in the experimental condition. The four experimental conditions are illustrated as follows. i. Items and Discussion (I&D): Children (n=20) were provided with real-world items and were encouraged to discuss with peers and work cooperatively. Children discussed cooperatively the purchase of the items in a fictitious store. ii. Items and No Discussion (I&ND): Children (n=20) were provided with real-world items but were not permitted to discuss with their peers. Children determined individually how to purchase the items in a fictitious store. iii. No Items and Discussion (NI&D): Children (n=20) were encouraged to discuss with their peers and work cooperatively without any real-world items. iv. No Items and No Discussion (NI&ND): Children (n=20) individually determined how to purchase the items according to the problems shown in the task without any real-world items and peer discussion. Videotapes and recorders were used to gather children's verbal interaction during solving problems. Analysis of variance indicated that the subjects' previous math achievement in the four conditions was not significantly different before the actual experiment began $F_{(3,76)}=.07$, $p>.05$.

Table1: The mathematical structures of problems in the first and transfer problem-solving phases.

	The first problem-solving phase		Delayed transfer problem-solving phase		
Problem Situation	Class Camping	Lantern Festival	Class Party	Tile Fitting	School Olympics
Items and objects in the problem	Each pack of items with different numbers of objects (*a*)	Three posters of different area (*a*) 2. Packs with a different number of envelopes(*b*).	Each pack of items with different numbers of elements (*a*)	1.Three rooms of different area *(a)* 2. Packs with a different number of tiles *(b)*	Packs of colored jungles with different number *(b)*
Multiplication model	Equal groups	1. Rectangular array. 2.Equal groups	Equal groups	1. Rectangular array. 2. Equal groups	Equal groups
Mathematical Content	Number	Area	Number	Area	Number
Arithmetical structure	$X \times b \geq 30$ Packs with b given as2,3,4,5, 6,7 or 8.	$X \times b \geq a$ a to be determined $b \in \{3,6,8\}$	$X \times b \geq 18$ Packs with b given as2,3,4,5, 6,7 or 8.	$X \times b \geq a$ a to be determined $b \in \{3,6,8\}$	$X \times b \geq a$ a given in the situations as 30,23,16,9,or 40 $b \in \{2,5,7\}$
Superfluous information	One item	Size of envelope	Two items	Size of tile	One item
Full score	35	15	35	15	25
Value of item Difficulty index	.49	.39	.50	.44	.43
Value of item discrimination index	.96	.78	.92	.58	.75

B. *The delayed transfer problem-solving phase.* Three word problems were presented on worksheets as delayed transfer tasks and were tested three weeks later after children completed the first phase of problem-solving tasks. As Table 1 shows, three problems in delayed transfer tasks have similar structural features but differing in superficial features.

C. *Evaluation.* Multiple evaluations (on a scale of 0-5) of written responses of each problem item were used. If a subject wrote the solutions on the shopping lists correctly, s/he was given a full score of 5 on each item.

Results

Table 2 shows the summaries of means and standard deviations of four experimental conditions in two problem-solving phases. A one between-subjects ANOVA indicated group comparison on performance. The results revealed that the difference among four conditions was significant, $F_{(3,76)}=7.02$, $p<.001$. The Tukey posteriori comparison analysis among the four experimental conditions suggested that there was a significant difference between I&D and I&ND, as well as I&D and NI&ND. There was no difference among I&ND, NI&D and NI&ND. The hypothesis is partially supported by the results. It suggests that cooperative verbal interaction with peers accompanied with real-world settings simultaneous would significantly improve children's performance in solving daily mathematical problems.

Table 2: Means and the standard deviations of four experimental conditions in two problem-solving phases.

	I & D		I & ND		NI & D		NI & ND	
	M	SD	M	SD	M	SD	M	SD
The first phase	45.70	4.28	31.70	14.20	36.10	12.68	29.25	14.79
Delayed transfer	59.45	14.83	53.65	21.45	57.95	9.36	39.50	23.31

Table3: Means and standards deviations of good and poor solvers in four experimental conditions in delayed transfer problem-solving phase.

	I & D		I & ND		NI & D		NI & ND	
	M	SD	M	SD	M	SD	M	SD
good solvers	65.70	7.42	67.40	3.75	62.70	6.41	54.60	13.43
poor solvers	53.20	17.95	39.90	23.17	53.20	9.68	24.40	21.44

In terms of analyzing good and poor solvers of the four experimental conditions in solving transfer task problems, the data was analyzed with a two (good Vs. poor solvers) and four (four experimental conditions) ANOVA. The results indicated that the significant interaction effect was not found with $F_{(3,72)}=2.56$, $p<.06$. The hypothesis is not supported by the results. However, a significant main effect of experimental conditions was found with $F_{(3,72)}=7.77$, $p<.001$. The results of follow up analysis and the Tukey posteriori comparison analysis for the four experimental conditions suggested children in I&D, I&ND, and NI&D conditions outperformed those in the NI&ND condition. Moreover, there was significant difference between good and poor solvers with $F_{(1,72)}=37.30$, $p<.001$. As shown in Table 3, good solvers demonstrated better analogical transfer performance than did poor ones.

Children's verbal interaction with peers in the discussion groups (I&D and NI&D) when they solved the "Class Camping" and "Lantern Festival" problems were collected and analyzed. Children's strategies used in solving problems were transcribed from videotapes and audio tapes. Two raters categorized children's problem solving strategies from the transcripts. The consistency between the two raters with the result of Kappa analysis was .57, p<.001. Table 4 shows the types of children's strategies in solving multiplicative problems.

Table 4: Children's strategies used in solving "Class Camping" and "Lantern Festival" problems through interaction with peers and tasks.

Strategies	Frequency	Percentage
1.Computation using multiplication and division directly.	27	34.6 %
2. Computation using addition and multiplication.	3	3.8 %
3. Computation using multiplication and subtraction.	9	11.5 %
4. Computation using addition, subtraction and multiplication.	2	2.6 %
5. Computation using addition, subtraction, multiplication and division.	2	2.6 %
6. Computation using counting, addition and multiplication.	20	25.6 %
7. Repeated addition.	11	14.1 %
8. Counting by ones or twos.	2	2.6 %
9. Drawing units and counting.	1	1.3 %
10. Calculation in head.	1	1.3 %
Total	78	100 %

Ten kinds of strategies were categorized from the children's peer interaction. Children's strategies used in solving daily mathematical problems are as follows: 1. Computation using multiplication and division directly. Around 34.6 % of the children determined the multiplier then applied multiplication algorithm as well as applied division directly to obtain the quotient. 2. Computation using addition and multiplication. About 3.8% of the children determined the needed amount and applied multiplicative facts and then added up the products. 3. Computation using multiplication and subtraction. About 11.5% of the children applied multiplicative facts to get the product then used the product to subtract the known number. 4. Computation using addition, subtraction and multiplication. Around 2.6% of the children used composite units in repeated addition and used multiplication partially, and then used the known quantity to subtract. 5. Computation using addition, subtraction, multiplication and division. About 2.6% of the children used the mixed computation. 6. Applying counting, addition and multiplication. About 25.6% of the children used skip counting by threes, twos or ones to determine the needed amount. 7. Repeated addition. About 14.1% of the children used a specific number as a unit to add up the numbers until the needed amount. 8.Counting by ones or twos. About 2.6% of the children accomplished computation by counting. 9.Children drew units or drew using a ruler on the given posters then counted the total squares to get the number of items needed. About 1.3% of the children used this strategy. 10. Calculation in head. About 1.3% of the children did not show any arithmetical strategies but stated the answer directly.

Discussion & Implication

Findings in this study clearly support the superiority of verbal interaction accompanied with real-world settings for children working on solving mathematics problems and analogical transfer. If either verbal interaction or real-world settings are provided separately, it is not sufficient to improve the children's performances particularly on their first time solving a specific problem. This may have been due to two reasons. First, a constructive discussion in the process of problem-solving was not found in all dyads. This was concluded from observation of peer discussion during problem solving. Some good solvers tended to solve problems on their own without verbal interaction with poor partners. On the other hand, several poor solvers tended to rely on their partners' explanation without questioning and analyzing. Second, children in the four conditions were able to deal with the items that represented figures shown on worksheets. Consequently, the effect of real-world settings did not significantly improve the children's performances. Although it was shown that the lack of either verbal interaction or real-world settings is sufficient to improve the children's performances on first time problem-solving, it did reveal quite a significant improvement on children's transfer performances.

With respect to the good/poor contrasts, the findings demonstrate the accuracy of transfer problem-solving was higher for good solvers rather than poor solvers. The good solvers seem to provide more procedural explanation, demonstration or checked work. Such instructional style is similar to tutoring and would in turn promote solvers' performances (Fuchs et al., 1994; Fuchs et al., 1997; Graesser & Person, 1994) as appeared in transfer superiority. It is possible that poor solvers tend to watch and listen to their partners with little opportunity to analyze and apply their partners' explanations. When comparing a good solver's in-depth learning with a poor solver's passive learning, it is not surprising to find that the poor solver's performance is inferior to the good one's.

In terms of the strategies applied in solving multiplicative problems, children seem to tend to mix addition, multiplication and counting to solve problems. A lot of fourth grade children might have been aware of the commutative principle of multiplication and the inverse relationship between multiplication and division. Some children counted each item one by one instead of count in multiples. Such perceptual counting is also found in lower graders(e.g., Franke, 1998; Ruwisch, 1998)

Results of this study suggest possibilities for improving children's problem solving. Effective training of peer tutoring and conceptual explanation during solving mathematical problems (e.g., Fuchs et al., 1994; Graesser & Person, 1994) are needed for increasing children's abilities to provide complete explanations and work in constructive, interactive fashion during instruction.

Acknowledgement

This research was supported by a grant from the National Science Council under grant NSC

89-2413-H-133-003. The opinions do not reflect the views of the foundation. I would like to thanks to assistants Yi-Wei Wang, Yi-An Chen, Hua-Chun Liu and Hsiao-Ting Tsi for their efforts. Moreover, I also thanks to Tung-men primary school in Taipei city for participating in this study.

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