EFFICACY IN PROBLEM POSING AND TEACHING PROBLEM POSING

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In this paper we examine the efficacy beliefs of prospective teachers' in posing problems and teaching problem posing in relation to their ability to construct problems. We used data from 115 questionnaires and 25 interviews to study the structure of perceived efficacy beliefs in problem posing, to examine the relationship between efficacy and actual performance in constructing problems, and efficacy to teach problem posing. The results indicated that students with high efficacy beliefs were more able to construct problems of more advanced complexity than low efficacy students. Significant differences were also found in the level of efficacy beliefs between subjects in terms of their mathematical background, prior involvement in related tasks, and gender.

AFFECT AND PROBLEM SOLVING

Despite the recently intensified interest on the affective domain, Schoenfeld (1992) argued that the arena of beliefs was under-conceptualised and stressed the need for new methodological and exploratory frames. He claimed specifically that "we are still a long way from a unified perspective that allows for a meaningful integration of cognition and affect or, if such unification is not possible, form understanding why it is not" (p. 364). In the following years research on the affective domain has resulted in notable theoretical advances, and there is now an expert consensus that affect is an essential factor in learning interacting with cognition during problem solving activities (De Bellis, 1997). Goldin (1998) proposed a five-component unified model for mathematical learning and problem solving. He considered the affective system as the most important among the five; the other four representations systems proposed were the verbal syntactic, the imagistic, the auditory, the formal notational, and the system of planning, monitoring and executive control.

Some of the questions already cleaned, to some extent, concern the structure and the development of the domain; the construct of affect, however, is still far from being well defined. The affective system includes components such as beliefs, conceptions, views, attitudes, emotions etc. related to mathematics and mathematical learning. If we define *learning*, as the development of one's general and specific "competencies", then affective competencies can be learned and consequently taught in the same sense as cognitive competencies can (Goldin, 1998). Regarding the teaching of affective competencies, the teacher's own belief system has a major role; it functions as a filter influencing knowledge and behaviour. Several components of the affective system have been so far investigated, including self-confidence, self-esteem, self-concept, and self-efficacy. In particular, research has shown that a teacher's sense of efficacy is a reliable indicator of his/her teaching behaviour and effectiveness in bringing about desired learning outcomes. Teacher education programs should, therefore, enhance both the cognitive and the affective domain.

Bandura (1997) defined self-efficacy as one´s conviction that he/she is able to achieve a certain task. By analogy, teaching efficacy can be defined as ones' belief in his/her capability to achieve learning outcomes. Self-efficacy is a context-specific construct in contrast to self-esteem, which is more global. That means that the study of teacher efficacy is more meaningful when carried out in terms of specific teaching tasks rather than in general. Several researchers found that the ability to construct problems and confidence in posing problems are among the most important competencies in mathematics learning, closely related to mathematics achievement. For instance, in models using path analysis the direct influences of efficacy beliefs on students' performance were estimated to range from .349 to .545 (Pajares, 1996). Furthermore, Pajares and Miller (1994) asserted that efficacy in problem solving had a causal effect on students' performance; they found that efficacy beliefs are better predictors of performance in problem solving than beliefs about the usefulness of mathematics, the involvement of students in mathematics, students' gender and experience with mathematics. In general, efficacy to perform a certain task was found to be the most reliable predictor of one´s behaviour in the course of achieving this task (Bandura, 1997; Tschannen-Moran, Woolfock-Hoy & Hoy, 1998).

Efforts to reform mathematics education had a direct impact on the philosophy of mathematics and consequently on instruction. *Knowing mathematics* has been widely identified as "doing mathematics" and *learning mathematics* as equivalent to constructing meaning for oneself and the ability to handle non-routine problems. In this context, problem posing comprises a primary factor that contributes to enhancing students' ability to solve mathematical problems (Leung, 1994). On the same line, a growing consensus that constructivism epistemology could provide the basis to prepare teachers in reform-oriented ways has led to an increased emphasis on developing teachers' ability to construct problems. One of the main responsibilities of primary teachers consists of constructing and/or selecting appropriate, pedagogically rich problem situations and orchestrates classroom activities, which facilitate students' effort to do mathematics on their own.

Problem-posing tasks may take various forms, and thus students can be involved in problem posing through a variety of situations. In this study, preservice teachers were asked to pose problems a) from a mathematical situation (Leung, 1994), b) from a given number sentence (English, 1997), and c) by modifying the structure, the data or the information given in certain problems (Gonzales, 1998). The main purpose of the study was to explore the relationship between the prospective teachers' efficacy beliefs on problem posing in each of the above-mentioned situations and their competence to complete the tasks. In this respect the present study sought for answers to the following questions: a) Is there a significant relation between students' efficacy beliefs in problem posing and their ability to construct problems? b) Is there a significant relation between students' efficacy beliefs in problem posing and their efficacy to teach problem posing? And c) are there significant differences in students' efficacy beliefs about problem posing in terms of gender, prior involvement in problem posing, and mathematical background?

METHODOLOGY

Data collection: The data were gathered through a questionnaire consisting of 31 statements aiming at the clarification of students' involvement in problem posing and problem solving activities, and their efficacy beliefs with respect to these tasks. The questionnaire was administered to 115 preservice teachers during the final stages of their teaching practice. After a first analysis of the responses, 25 students were selected and interviewed. Six of the subjects involved in the interviews were from the low efficacy (LE) group, ten from the average efficacy (AE), and nine were from the high efficacy group (HE). The same tasks were administered to the rest of the students who did not take part in interviews.

The interviews: The interviews were semi-structured and conducted by one of the researchers. During the interviews we used tasks similar to the problems involved in the questionnaire. Each student was asked to construct problems given a mathematical situation, a number sentence, and a problem to modify. The tasks and the initial directions were as follows:

Mathematical situation (Task 1): Construct three problems based on the following story: "Michael, Nicolas, and John drove in succession on their way back from a trip. Michael drove for 80 km more than John. John drove for double the distance Nicolas did. Nicolas drove for 50 km ".

Number sentence (Task 2): "Construct three problems all of which could be solved using the equation 56: 6 = n".

Problem modification (Task 3): Read the following problem and construct up to seven different problems modifying the problem: "The students in a certain school were talking about their favourite singers. One fourth of them voted for singer A, one sixth for singer B, one eighth for singer C, and one twelfth for singer D. What is the student population of the school, if 90 students were undecided?"

The students were at first given time to construct problems on their own. Later on, whenever a student got stuck, the interviewer provided progressively clearer hints about possible ways of performing the tasks. For instance, in the case of Task 1, a common hint was "find a problem in which the answer is not fractional", in the case of Task 3, the students were advised to "insert new information", or "change the unknown", "impose new constraints", etc. If students were proposing an impossible or non-sensible problem, they were considered as failing to achieve the specified task.

RESULTS

Analysis of the questionnaires

Students' responses were factor analysed using Principal Axis factoring with varimax rotation. A five-factor solution, explaining 75% of the variance, was identified as being the most appropriate in isolating distinct scales to identify efficacy beliefs (the loadings of all items were large and statistically significant). The first factor indicated efficacy in Task 1 and Task 2 and explained 21.4% of the variance. The second factor explained 19.71% of the variance and reflected confidence in Task 3, the third factor explained 12.58% of the variance and reflected the subjects' efficacy to *teach problem posing* strategies. The fourth factor explained 11.7% of the variance reflected prior *involvement in problem posing* activities and the fifth factor reflected students' *experience in problem solving* and explained 10.26% of the variance.

In exploring differences among students, we used extracted factors, which reflected efficacy beliefs, as dependent variables and the students' mathematical background (high-school strand[1]), prior involvement in related tasks, and gender as independent variables. Analysis of variance showed that students from the science strand have more desirable efficacy beliefs than students in any of the other two strands (classical and the economics) in all three tasks of problem construction. The same pattern was also found on the teaching efficacy factor. There were no significant differences among the efficacy beliefs of the students from the economics strand and the students from the classical strand.

The subjects expressed significantly higher involvement with problem solving than with problem posing activities ($\overline{X}_{ps}= 2.01$, $\overline{X}_{pp} =1.66$, $p < .01$). The students' prior involvement with problem posing and problem solving was found to be related to their expressed efficacy beliefs; students with extensive experience in such tasks had a higher level of beliefs in the their ability to construct problems and teach problem posing, than students with limited experience. Significant differences were also found between males and females on the factor efficacy to teach problem posing ($\overline{X}_m= 3.11$, $\overline{X}_f = 2.71$, $p < .05$). These differences can partially be attributed to the male students' superior efficacy beliefs in their ability to construct problems from a given number sentence over female students ($\overline{X}_m = 3.62$, $\overline{X}_f = 3.10$, $p < .05$).

Efficacy in Tasks 1, 2 and 3 was strongly correlated to the efficacy to teach problem posing ($r = .58$, $r = .55$, $r = .52$, respectively, $p< .01$). The analysis also showed that "efficacy to construct problems" and "efficacy to teach problem posing" were significantly correlated ($r = .62$, $p < .0$. Finally, the level of students' efficacy beliefs to construct problems was significantly higher than the level of efficacy beliefs to teach problem posing ($\overline{X}_{pp} = 3.27$, $\overline{X}_{tpp.} = 2.81$, $p < .001$).

Analysis of the interviews

The interviews showed that a) all participants realized the importance of developing problem posing competencies, b) irrespective of efficacy level, they considered problem posing as harder than problem solving, and c) students valued

[1] The students come from three high school strands: the science section (emphasis on mathematics and science), the economics section (emphasis and economics), and the classical strand (only core mathematics).

problem posing as the ultimate goal of mathematics learning. “A thorough understanding of problems and problem solving is evidenced when teachers and pupils reach the level of problem posing” (extract from interviews).

When facing a problem-posing task, 52% of the interviewed students felt uneasy (two of the LE students felt even anxious when assigned such a task). Five of the AE students said that even "hearing the term problem posing makes them feel insecure", and they consider problem posing to be "a very complicated process". The majority of the HE students felt quite comfortable with the task, though one of the students in this group mentioned that he "did not have any real experience and hence he did not like being assigned such a task". The differences among the three efficacy groups of students were more obvious, when the discussion was focused on the specific tasks of problem posing from a mathematical situation, a number sentence, or a given problem.

Concerning efficacy beliefs with respect to teaching problem posing, LE students were less confident than AE and HE students. Specifically, three LE students felt that they were "not well prepared to involve their students in problem posing activities". Explaining their position, they stated that they themselves "faced so many troubles in problem posing” and were "not confident in undertaking such a task". Two others expressed their "reservations…" and "felt more comfortable in teaching problem posing in the lower school grades". The AE subjects were more or less ready to pursue the task, though "they needed more experience with problem posing". On the contrary, five of the HE students stated that they were well prepared to integrate problem posing in their teaching, while two others held the same beliefs as the AE students, i.e., they said that they needed "additional experiences".

A student was rated as successful in a task when he or she was able to construct a good problem with little or no help. Given the initial efficacy statements of the students in each of the 13 assigned tasks, we tested the correlation coefficient between stated efficacy and performance in each of these cases. The correlations were in the range of .724 and .866 and they were significant at the .01 level.

Task 1: To rate the quality of the students’ problems we adopted the linguistic and structural criteria established by English (1997) and Silver and Cai (1996). The most important elements in a problem are (a) the type of the question, and (b) the number of important relations involved in the problem structure. The same authors classified problem questions as *conditional, relational, and assignment*; conditional and relational questions are more complicated than the direct assignment questions. Similarly, the number of relations involved in a problem is an indication of the complexity of the problem. Bandura (1997) asserted that efficacy beliefs could be a good predictor of the quality of peoples’ work. The average number of conditional and relational problems constructed by each efficacy group was 1.13, .92, and .64 by the high, the average and the low efficacy group, respectively (significant at the .05 level). Finally, HE students constructed eleven assignment questions, eight relational, and two conditional questions. We also observed an increasing trend of complexity from the first constructed problem onwards. The average number of relations per proposed problem was 2.40 for LE subjects, 2.59 relations for AE subjects, and 2.84 for the HE subjects (non-significant difference). Table 1 shows indicative problems constructed by the students in each efficacy group.

Table 1

Examples of problems constructed by the subjects of the three groups

	Low Efficacy	**Average Efficacy**	**High Efficacy**
Ass	How many km did John drive? How many km did John and Michael drive?	How many km did John drive? How many km did the three friends drive altogether?	How many km did the three friends drive altogether? How many km did each of them drive?
Rel	How many more km did Michael drive than John did? How many km did Michael drive more than Nicolas?	Did both Nicolas and John drive more km than Michael did alone? Who drove the more km?	How many more km did Michael drive than Nicolas? How many less km did Michael drive than Nicolas?
Con	none	If Michael drove for 3 hours, John for 4 hours and Nicolas for 2 hours, compare their driving speed. If the average speed of the car were 50 km/h, how long would the journey last?	If the distance of their journey was X km, how many km would each of them had driven in order to reach their destination? If they travelled 800 km and continued driving in the same way, how many more km would John had driven than Nicolas?

Task 2: The students had great difficulties in constructing problems given a number sentence. Most of the students explained their difficulties saying "there is no story to start with…one has to start from the beginning, to create everything in his/her own mind". Three LE students were unable to construct problems eliciting answers other than 9 2/6 (56: 6 = n) despite of being helped by the interviewer. The AE and the HE students could somehow construct a problem, but it was evident that this task was more difficult even for them.

Task 3: The second easier task was constructing a problem by modifying a given one for the majority of students (Task 1 was generally judged as the easiest). About 64% of the subjects initially thought to change the story of the problem, 56% to change the values of the variables or the unknown, 28% to introduce new information and 28% to delete some information. However, only two of the subjects thought on their own to impose new constraints or extend the problem using “what if” strategy.

Another dimension differentiating students in different efficacy groups concerns checking the constructed problems by solving them. The majority of the AE and the HE students checked the solution of the problems they constructed, in contrast to LE students of which only three attempted to check anyone of the posed problems.

DISCUSSION

The findings of this study underline the importance of students' background and involvement with problem solving and particularly with problem posing. The analysis of the questionnaire data provides support to Bandura's (1997) claim that the main source of efficacy beliefs comes from the individuals' experiences with similar and related tasks. As one LE students mentioned, "What I really lack is confidence in myself, that I will succeed in doing a problem … my prior experience was, so far, to solve a problem than to construct a good problem to assign to students". The high school strand was related to efficacy in problem posing. Since the science students are generally involved in more extensive and rich mathematical experiences than the rest of the students, this factor is not different from one's overall mathematical involvement. Males were in some cases found to hold higher efficacy beliefs than females. One possible explanation could be the masculine "aggressive" attitude to overestimate own capabilities against the feminine moderate attitude, influenced by the well-known role stereotypes.

The significant correlations between the prospective teachers' efficacy beliefs in problem posing and their ability to construct problems indicate that efficacy constitutes a reliable predictor of the subjects performance in problem posing from the types of sources examined in the present study. Furthermore, these beliefs provide a clue about the quality of the results in such a task. For instance, the AE and the HE students were able to construct more problems and of higher complexity, as indicated by the type of the questions raised and the number of relations involved. In addition, these subjects used to test their problems and felt more comfortable with this task of problem posing, in contrast to the LE students who felt anxious, when facing a problem-posing task and seldom did they test the problems they constructed. For instance, one of the LE subjects mentioned, "What I really lack is the confidence in myself, … that I will succeed in making a problem. My prior experience was so far in solving problems, … not to make up a good problem to assign the to others".

In conclusion, the above findings suggest that developing efficacy beliefs in problem posing should be an integral part in any preservice teacher education program. Efficacy beliefs constitute "an important component of motivation and behaviour" (Pajares, 1996, p. 341) and consequently are important for integrating problem posing and problem solving in classroom instruction. The correlation found among the efficacy in problem posing and the students' beliefs about teaching this activity suggests a possible focus for further research.

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