FROM A STUDENT INSTITUTIONAL POSITION TO A TEACHER ONE : WHAT CHANGES IN THE RELATIONSHIP TO ALGEBRA ?

Agnès Lenfant, IUFM de Reims, Equipe DIDIREM

Abstract : *In this report, we present an on-going research about the professional development of pre-service secondary mathematics teachers. This professional development is analysed through their relationship with algebra. After presenting our theoretical frame and methodology, we describe the multidimensional grid of professional competencies in algebra that we have developed in order to analyse the professional relationships with algebra and their evolution. We face then our research hypothesis with the first results obtained through the analysis of questionnaires and interviews.*

I. INTRODUCTION

Various research studies have recently addressed the issue of teachers' professional development (see e.g. (Cooney, 2001) for a synthesis). Our on-going research belongs to this category. It focuses on the professional development of pre-service teachers (called PLC2) who, in France, have just passed the national competition called CAPES in order to become secondary mathematics teachers and are given one year of professional training in an IUFM (Institut Universitaire de Formation des Maîtres). Note that during this year of professional training, they also have one class in full responsibility, 6 hours per week. Research also focuses on one particular mathematical domain : algebra, as we hypothesise that reconceptualising mathematics in an important part of professional development, which includes changes in general epistemological views and changes in the perception of specific mathematical domains. The choice of algebra is motivated by the following reasons :

- algebra is a domain which poses severe learning difficulties and is the first step towards mathematics rejection for many students,
- algebra is a domain, all our PLC are concerned with as they have to teach algebra or pre-algebra in their full responsibility class,
- algebra is a domain which, a priori, they feel easy to teach. Algebraic thinking is for them a natural thinking mode and they have a great familiarity with the algebraic techniques they have to teach. Understanding pupils' difficulties requires thus from them a lot of efforts,
- algebra is a domain where naïve epistemological views are strongly inadequate,
- and, finally, this is a domain which has been extensively investigated by research. We have now a fairly good knowledge of the main difficulties pupils meet with elementary algebra, and research also provides engineering designs whose aim is

to tackle these difficulties more efficiently. These research results strongly inspire the short professional training they receive at the IUFM.

Through this research work, we want to better understand how the PLC's relationships with algebra change when they move from a student position to a teacher position, thanks both to the training they receive at the IUFM and their practical experience as teachers. We hypothesise that the different professional competencies which can be attached to algebra are not equally accessible, for reasons which have also to be understood and that a better knowledge of such differences could help to design more efficient pre-service training strategies. Conceiving efficient training strategies in the context of the IUFM is not an easy task, due to severe institutional constraints. For French secondary teachers, pre-service professional training is concentrated during the two years they spend at the IUFM. During the first year, they essentially prepare the very difficult national competition whose professional component reduces to the choice and presentation of a selection of exercises on a given mathematical theme, during the oral part of the competition. During the second year, the part of training officially devoted to the didactics of mathematics, different from one IUFM to another one, rarely exceeds 120h.

II. THEORETICAL FRAMES AND METHODOLOGY

Theoretical frames :

For this research, we rely on different theoretical frames. At a global level, we rely on the anthropological theory developed by Chevallard (1997, 1999) in order to analyse "mathematical and didactical organisations", the associated "mathematics praxeologies" (in terms of tasks, techniques, technologies and theories), and also the mathematical professional work of the teacher via its different "gestures". As regards the analysis of the teacher, we also refer to the frame developed by Robert (1999), relying on research in ergonomy in order to give account of the functioning of the teacher, seen as an actor in a "dynamical open environment". These theoretical frames lead us to hypothesise that professional development is a very complex process, that its understanding requires a multidimensional analysis taking into account and connecting the different professional gestures of the teacher, both in and out of the classroom. Of course, we also rely on educational research in algebra and especially on synthesis such as those produced by Bednarz, Kieran and Lee (1996) or Grugeon (1995). We hypothesise that professional expertise in algebra is a complex mixture of competencies specific to this domain and more transversal competencies which strongly intertwine in professional practices.

Methodology :

Taking into account our "problématique" and the nature of our hypothesis, we have chosen a qualitative methodology based on the triangulation on multiple sources of data and analysis which allow to take into account the multidimensionality of professional competencies and professional development as a process. It includes:

- questionnaires and interviews with students just before they take the CAPES competition, that is to say at the end of the first year at the IUFM,
- observation of the training process organised by the IUFM for the PLC2 as regards algebra,
- questionnaires taken by the complete cohort of PLC2 at the beginning and at the end of the academic year,
- and, most important, the following up of a group of PLC2 (6 in 1998-99 and 6 in 1999-2000), selected among volunteers, through regular interviews, videotaped classroom observations, collection of teaching material (personal teachers notes, assessment tasks, selected students' copy-books excerpts and productions).

Moreover, in order to analyse the whole data, we have built a methodological tool: a multidimensional grid of professional competence in elementary algebra (MGPCA), thanks to the results and categories coming from the research work used as a theoretical or experimental reference. We briefly introduce this grid in the next paragraph.

III The multidimensional grid of professional competence in elementary algebra

As stressed above, we consider that professional competence has to be analysed in multidimensional terms. It can be modelled as a multivariate function which shapes the decisions the teacher takes in his (her) different professional gestures, the way (s)he faces unforeseen situations, the discourse and analysis (s)he develops at a more reflective level. This professional competence, as regards algebra, relies both on transversal and specific competencies. The MGPCA focuses on the specific competencies and tries to describe potential underlying knowledge. We hypothesise that such knowledge influences the different professional gestures in a non-uniform way and that it cannot necessarily be made explicit. One ambition of the research is to explore the complexity of relationships between knowledge and competencies, to try to find the real influence professional algebraic knowledge plays with respect to other determinants of PLC2's behaviour, and to evidence some regularities which could help to better understand teacher's behaviour and improve training strategies. The grid is structured around three non-independent dimensions : the epistemological, the cognitive and the didactic ones. In the following, we synthesise the contents of knowledge which structure the grid according to each dimension.

The epistemological dimension :

Epistemological knowledge in the grid, is structured around, on the one hand, some important features of the historical development of algebra, on the other hand, the distinction between the "tool and object" dimensions of algebra (Douady, 1984).

As regards the first point, one essential epistemological characteristics is the complexity of the algebraic symbolic system and the difficulties of its historical development. Such knowledge can help to understand the difficulties met by present pupils. Another important point which arise from historical development is the

extension and diversity of the algebraic domain. Such knowledge can support the change of epistemological views about algebra, allow to better understand the rationale for the progression in algebraic knowledge organised by the curriculum and eventually discuss this. We conjecture that, at the beginning of the academic year, our PLC2 are unaware of this complexity and tend to reduce algebra to the algebraic structures and theories they have been taught at university.

As regards the second point, the distinction between the tool and object facets of algebra allows to structure algebraic knowledge around two perspectives: a tool perspective where algebra is considered as a set of tools allowing to solve different kind of problems internal or external to the mathematics field, an object perspective where algebra is considered as a structured set of objects with specific properties, semiotic representations, treatment modes, which are studied for themselves (Grugeon, 1995).

We conjecture that PLC2, at the beginning of the academic year, mainly see algebra as a tool for solving problems which can be modelled in terms of equations and that, as teachers, they will tend to over-emphasise the work on algebraic techniques.

The cognitive dimension :

This component deals with potential professional knowledge about learning processes in algebra. We have organised this part of the grid around three main points linked to resistant learning difficulties evidenced by didactic research: the relationships between arithmetic and algebra, the symbolic system of algebra and the relationships between different semiotic representations used in algebra, the role algebra can play in the development of mathematical rationality. For this part of the grid, we especially rely on the synthesis made by Grugeon (1995) in her attempt to define a multidimensional grid of analysis of pupils' competencies in elementary algebra.

Algebraic knowledge builds on arithmetic knowledge but also against it. Algebra, when introduced, shares with arithmetic many objects and symbols but these have to take new meanings : for instance, equality has to become an equivalence sign in order to allow algebraic manipulations, letters have to take several different new meanings. Beyond that, arithmetic and algebraic thinking modes are of a very different nature, the first one belonging to the synthetic mode and the second one to the analytic mode. Understanding algebraic thinking modes also imposes semantic changes. Algebraic work contrarily to arithmetic work cannot only rely on external semantics, semantics has to take also an internal dimension proper to algebraic expressions. Other kind of knowledge deals with the specificity of the symbolic system of algebra, already evoked in the epistemological part, and the resistant difficulties it induces (for instance those resulting from the lack of closure). Learning difficulties can also be linked to the necessary interaction in algebraic work, as in any mathematical work, between different semiotic registers, especially here: the symbolic register, the natural language register, the numerical register and the graphical register, and the

difficulties of conversion between non-congruent semiotic representations. Finally, we integrate in this cognitive part, knowledge about the role algebra can play in the construction of students' mathematics rationality.

As regards this cognitive dimension, we conjecture that, at the beginning of the academic year, PLC2 are not aware of the learning difficulties evoked above, but that, through their practice and discussions with pairs, they soon become sensitive to most of these, even if they are not able to interpret them in coherent and analytic ways as research allows to do, and thus react efficiently. As regards rationality, we conjecture that, due to the general educational French culture which over-emphasises the links between rationality and geometry at the expense of any other one, integrating this pole of knowledge will result more difficult.

The didactic dimension :

Knowledge relevant to these two first dimensions certainly influences the didactic and mathematical organisations, the teachers develop. But these are also shaped by what we will call here more specific didactic knowledge: knowledge of the curriculum, of the specific goals of algebraic teaching at a given grade, of possible progressions and activities for the teaching of algebra compatible with these and well adapted assessment tasks, knowledge of educational resources: textbooks but also publications from the IREM (Instituts de Recherche sur l'Enseignement des Mathématiques), websites (especially as regards the use of computer tools such as spreadsheets for the teaching of algebra), etc.

IV. SOME PRELIMINARY RESULTS

The first results we present here result from the analysis of questionnaires and interviews. In the near future, we will face them with the detailed analysis of the different elements of the PLC2's following up. This will allow us, we hope, to better understand the possible relationships between systems of knowledge and competencies. We summarise these results according to the three dimensions of knowledge structuring the MGPCA.

The epistemological dimension :

The first analysis tend to confirm the conjectures made about the initial PLC2s' state. Nevertheless, the training at the IUFM seems to easily destabilise the vision of algebra as a domain reduced to the field of algebraic structures and theories. For most students, boarders of algebra tend to blur or to become questionable, for some others a duality seems to install between two points of view: the mathematical and the educational, resulting in two distinct conceptions of algebra which coexist. As regards the diverse facets of algebra and the tool/object dimensions, the object dimension strongly predominates and this priority tends to resist to the training. Their initial vision of the different tool facets of algebra is limited and integrating a functionality of proof for numerical properties seems specially difficult. The strength of the cultural obstacle is especially visible here.

The cognitive dimension :

As conjectured, initially, the PLC2 are not aware of the difficulties of the transition between arithmetic and algebra. The short training they have (6h) seems to make them aware of the differences between the arithmetic and the algebraic processes for solving numerical problems. But they tend to see the algebraic method as the "right method" safer, simpler and more rigorous, which has to take the place of arithmetic methods, more intuitive and, at the same time, requiring more astuteness. As one of them expresses at the end of the year: "An arithmetic approach is more a reasoning out of standard ways. It is less framed. One cannot really say that there exists a method". PLC2 teaching grade 8, where the solving of problems which can be modelled by first grade equations is an important point in the syllabus, seem specially sensitive to this difference. Nevertheless, even at the end of the academic year, deeper interpretations of the arithmetic / algebraic cut are not spontaneously evoked by students, at the exception for some of them of situations where they are asked to interpret students' errors.

As regards the symbolic system of algebra, students are very soon aware that they have to face resistant learning difficulties. Some of these are helped by previous experience of giving private remedial courses. But we notice evident differences in the way the PLC2 interpret these difficulties and in the way they tackle these. Once more, the grade they teach in full responsibility seems to have an evident influence on their perception. Knowledge related to the internal semantics of algebraic expressions and the role this knowledge plays for piloting and controlling algebraic work seems the most difficult to internalise. What predominates in algebra for them is the existence of formal rules for transforming expressions and equations, and the algorithmic character of most elementary algebraic practices. This doesn't help them to pay the necessary attention to what becomes crucial in more complex algebraic computations. Once more, we see here the strength of a cultural obstacle equalling algebraic computation to computation without intelligence. As regards the connections between different semiotic registers, they identify difficulties in the mutual conversions between the symbolic register and the natural language but are not necessarily able to analyse these in more technical terms.

The didactic dimension :

As regards the building of a progression, textbooks and the local advisor the PLC2 have in the high school where they teach in full responsibility have a predominant influence. Generally, the PLC2 don't meet difficulties when deciding the different points they want to address, but they have more difficulties at identifying the real aims of the teaching of pre-algebra or algebra, at the grade level corresponding to their class or at foreseeing the time they will have to spend on such or such chapter. As expressed by one of them in the final interview: "At the beginning of the year, I tried to build a progression but… I don't succeed in knowing how many time I will stay on one notion. So, now I mainly try to see in what order and with what spirit I want to approach the notion".

The PLC2 also have some difficulties at taking into account the initial state of knowledge of their students when they come to teach algebra. Such a difficulty is stronger, as can be easily foreseen, at the beginning of the academic year. As expressed by one PLC2 in the first interview: “I have to say that I didn’t give a lot of exercises on this topic because I thought that it was well known. At the beginning of the year, I wanted to arrive very quickly to algebraic developments and factorisations. The remainder was only matter of recall. I realised too late that it didn’t work. I didn’t choose to go back, but it was perhaps a wrong decision. My problem is that for some of these students, I can explain one thousand times, that doesn’t change anything. I don’t know how to do”. This excerpt shows the PLC2 becoming aware of the problem, doubting about the adequacy of his initial strategy and, at the same time, we can understand up to what point he is deprived from means in order to tackle the problem. Visibly, spontaneously, he remains in a state of global approach, insufficient to analyse real students’ difficulties and needs, and conceive appropriate educational strategies, beyond the explanatory strategy which is the sole to be evoked.

As regards the mathematical and didactical organisations, at a more local level, textbooks once more play an essential role, but most PLC2 don’t only use the textbook selected by the high school for their class. Visibly, the work they did the year before, when preparing the CAPES, which obliged them to search and select exercises on specific mathematical themes by looking at different textbooks has created some “habitus”. Two great tendencies emerge: in the first one, the PLC2 tries to cover extensively the diversity of types of tasks proposed by the textbook, this result in very few examples for each type ; in the second one, the PLC2 focuses on some selected tasks and one can see, through the progression along the exercises of the same type, a progressive attempt to play on specific didactic variables, even if this expertise mainly remains implicit. The same diversity occurs as regards the relationships between techniques and technologies (this word being taken here with the Chevallard’s acception of discourse explaining or justifying a technique). Some PLC2 mainly stay at the technical level (including of course the description of the technique), other give real importance to the technological dimension. But, even at the end of the academic year, the elaboration of technological discourse seems to remain under their sole responsibility. The different data coming from the questionnaires and interviews show that there is a sensible evolution during the academic year: the lesson conception evolves, the relationship to assessment evolve, but each evolution seems to have its idiosyncrasy. As expressed by one PLC2: “Let us say that, every time, I have tried to ask myself more questions before introducing something new. A sort of a priori analysis. I ask myself what will be the students’ difficulties. But it is difficult to foresee. And also, how to introduce that new thing. At the beginning of the year, I delivered the course more directly, now I always try to do some activity which prepares the notion”. The interviews show the difficulties they have at analysing classroom situations, but we notice a great difference between the analysis they are able to produce personally and the analysis they are able to

produce when working in small groups on the same activity designed for students or on classroom videotapes.

V. DISCUSSION

In this research project, we try to better understand the change in the relationships with algebra of French pre-service teachers, moving from a student position to a teacher position, and the development of an initial professional expertise in that area. We consider this development as a muldimensional and complex process, involving epistemological, cognitive and didactic changes. We are perfectly aware that what can be reached through this first year of professional training is necessarily very limited. These evident limitations make all the more important to detect possible germs for priming professional development. The results we have obtained up to now are certainly very partial but they clearly tend to show that some interesting and subtle evolutions can take place. All of these don't directly affect the design and management of classroom situations. At a first level, they seem more able to express in a priori and a posteriori analysis of classroom sessions, and more in a collective way than in an individual way. Results also show important differences in the accessibility of the respective parts of what can be considered today as professional expertise in algebra. We would like to add that the influence of the short didactic training offered by the IUFM will remain anecdotal if it is not properly echoed by a reflective analysis of PLC2's practices in the class they have in full responsibility. This is, both from an experiential and affective point of views, the fundamental object in the transition they live. There is no doubt that for the PLC2 who have been selected for the following up, we are in some sense in the optimistic conditions, taking into account the strong institutional constraints of this pre-service training.

REFERENCES

Bednarz N., Kieran C. & Lee L. (eds.) (1996). Approaches to algebra: Perspectives for research and teaching. Dordrecht : Kluwer Academic Publishers.

Chevallard Y. (1997). Familière et problématique, la figure du professeur, Recherches en Didactique des Mathématiques, vol. 17/3, 17-54.

Chevallard Y. (1999). Analyse des pratiques enseignantes en théorie anthropologique du didactique. Recherches en Didactique des Mathématiques, vol. 19/2, 221-266.

Cooney T.J. (2001). Using research to inform preservice teacher education programs in, D. Holton & al. (eds.), The Teaching and Learning of Mathematics at University Level. Dordrecht : Kluwer Academic Publishers (to appear).

Douady R. (1984). Jeux de cadre et dialectique outil/objet dans l'enseignement des mathématiques. Thèse d'Etat. Université Paris 7.

Grugeon B. (1995). Etude des rapports institutionnels et des rapports personnels des élèves dans la transition entre deux cycles d'enseignement. Doctorate Thesis. Université Paris 7.

Robert A. (1999). Les pratiques des enseignants de mathématiques en classe de seconde. Cahier de DIDIREM n°33. Paris : IREM Paris 7.