MATHEMATICS GOALS AND ACHIEVEMENT IN THE UNITED STATES

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Abstract: In this paper, the goals of both traditional and contemporary reform school mathematics are first situated in the complex public school system of the United States. This is followed by a discussion of the methods of gathering achievement in schools and states. The conclusion is that there is no way one can aggregate data across districts or states to summarize achievement for the three different categories of students at the three levels of achievement for the nation.

Describing the mathematics goals and achievement in U.S. schools is not easy for two reasons: (a) the complexity of the public school systems and (b) the current efforts to reform school mathematics. One of the most striking features of U.S. schools to foreign visitors is the diversity of schooling practices, particularly with respect to governance and to the ways policy decisions are made, a result of the fact that educational policy is not national. The writers of the Constitution of the United States, in omitting any reference to education, left decisions about education to the states. With the exception of Hawaii, the states have, in varying degrees, turned over the control of schools to local communities with locally elected school boards. Today there are over 15,000 school districts that hire administrators and teachers, approve programs, select texts, and so on. As a consequence of shared state and local control, and shared state and local taxes to support schools, there are vast differences in the quality of programs, facilities, staff, and teachers both across and within states. There is no national curriculum, no national set of standards for the licensing or retention of teachers, no common policies for student assessment of progress or admission to higher education, and so forth.

Traditional Mathematics Goals

Until the past decade, in spite of the diversity in school governance, there was considerable similarity in practice and expectations for school mathematics. The curriculum reflected the 19th century compromise, described by Jahnke (1986), to formalize school mathematics as a closed unified system rather than as a sequence of methods for analyzing and understanding our world. Scientific management of this system resulted in a fragmented, hierarchical classification of mathematical concepts and skills. Scope-and-sequence charts, which specify behavioral objectives to be mastered by students at each grade level, were commonly produced. The goal for all students was that they sequentially master one concept or skill after another, and their primary task was to get correct answers to well-defined problems or exercises. This method of segmenting and sequencing school mathematics led to the assumption that there was a strict partial ordering to the discipline.

Another way of describing what mathematics has been taught to students in schools is to describe the topics covered in various grades. The emphasis in the elementary and middle schools was on computational proficiency in arithmetic. The standard topics included addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals; some experience-based geometry; and a few word problems. At the secondary school level, there was a 4-year "layer-cake" sequence with a year of algebra at Grade 9, followed by a year of Euclidean geometry at Grade 10, another year of algebra at Grade 11, and a year of pre-calculus mathematics at Grade 12. The goals for different groups of students varied only in how far along the sequence of topics students were expected to study. The college-bound mathematics and science students were to complete the entire 12-year sequence. Students planning to go to college but not planning to study mathematics or the sciences were expected to complete 10 years of the same sequence of courses. For students not college bound, eight or nine years were usually required. Finally, in some schools a few "accelerated" students started algebra in Grade 8 and took a calculus course in Grade 12. This portrayal of school mathematics was universal in the United States until the past decade and is still the dominant picture in the majority of schools today.

Reform Mathematics Goals

With the publication of *A Nation at Risk* (National Commission on Excellence in Education, 1983) and *Educating Americans for the 21st Century* (National Science Board Commission, 1983), it was apparent that our schools were not adequately preparing most of our students to participate meaningfully in the real world of work, personal life, and higher education; or in the country's social and political institutions. The initial response to these concerns in most states and school districts was to continue the same sequence of mathematics courses but to shift the expectations for the college-bound mathematics and science students to algebra in Grade 8 and a year of calculus in Grade 12 and to add an additional year of mathematics to the education of both college-bound liberal arts students and non-college-bound students.

However, on reflection it was clear that the traditional course sequence in our schools was designed to meet the demands of an earlier industrial age. The transition to an information society has created new demands on U.S. citizens. The mathematical sciences education community argued that learning must be generative: that students learn mathematics in ways that provide a basis for lifelong learning and for solving problems that cannot be anticipated today. The "standards-based reform movement" now under way in many states and schools is based on NCTM's three standards documents (NCTM, 1989, 1991, 1995) and its recently published *Principles and Standards for School Mathematics* (2000). The vision espoused in these documents involves a shift in the epistemology of learning mathematics, systemic notions about schooling that follow from that shift, the need for appropriate evidence related to the notions of schooling practices, and new assessments. The central tenet underlying the shift in epistemology is that students should become mathematically "literate." For a person to be literate in a language implies that the person knows many of the design resources of the language and is able to use those resources for several different social functions (Gee, 1998). Analogously becoming mathematically literate implies that students must not only learn the concepts and procedures of mathematics (its design features), but must also learn to use such ideas to solve nonroutine problems and to be able to mathematize a variety of situations (its social functions).

A set of assumptions about instruction and schooling practices has been associated with this vision of mathematical literacy. First, all students can and must learn more and somewhat different mathematics than has been expected in the past in order to be productive citizens in tomorrow's world. In particular, all students need to have the opportunity to learn important mathematics regardless of socioeconomic class, gender, and ethnicity. Second, some of the important notions we expect students to learn have changed due to changes in technology and new applications. Thus, at every stage in the design of instructional settings one must continually ask, Are these ideas in mathematics important for students to understand? Third, technological tools increasingly make it possible to create new, different, and engaging instructional environments. Finally, the critical learning of mathematics by students occurs as a consequence of building on prior knowledge via purposeful engagement in activities and by discourse with other students and teachers in classrooms. The point is that, with appropriate guidance from teachers, a student's informal notions can evolve into models for increasingly abstract mathematical and scientific reasoning. The development of ways of symbolizing problem situations and the transition from informal to formal semiotics are important aspects of these instructional assumptions.

What is envisioned in the reform documents is that all students will study an integrated mathematics program for at least 11 years. Integration includes activities from such strands as number, algebra, geometry, and statistics, and so forth from the early grades through Grade 11. This provides students with an opportunity to explore, in an informal manner, topics traditionally taught in high school and to proceed from such informal notions to more formal mathematics in the later grades. It also provides teachers with considerable flexibility to organize instruction to meet the specific needs of their students. In this vision, there is no distinction in goals between the non-college-bound and the college-bound liberal arts students. For the college-bound mathematics and science students, an additional year of more formal mathematics is required.

The problem with the reform vision of school mathematics is that it is based on ideas put forward by educational leaders, policymakers, and professors about what mathematical content and pedagogy *should* be. Implementation of such ideals can be undermined by a number of factors. For example, not everyone agrees with the goal of mathematical literacy for all; some influential people believe that the current course of study works reasonably well (particularly for their children), and so forth. In fact, as Labaree pointed out, during the past century, calls for reform have had "remarkably little effect on the character of teaching and learning in American classrooms" (1999, p. 42). Instead of changing conventional practices, the common response to calls for reform has been "nominal" adoption of the reform ideas. Schools adopted the reform *labels* but not most of the *practices* advocated, and it is often a political necessity for schools and teachers to claim they are using a standards-based, reform program even if classroom practices have not changed. Thus, to document the impact of any reform efforts in U.S. classrooms, one needs to examine the degree to which the reform vision has actually been implemented.

In summary, there are no consensual mathematics goals in the United States for the three groups of students However, what traditionally was and is proposed for the college-bound mathematics and science students seems reasonable. Traditionally, no special provisions were considered for college-bound liberal arts students or non-college-bound students. The reform recommendations, on the other hand, are tailored to meet the assumed needs of these groups.

Student Achievement

Traditionally, U.S. schools judge students' knowledge of mathematics either from quizzes and tests made and administered by teachers in order to prepare a formal report (usually to give a grade) or from externally developed (and often mandated) tests. Although grades are commonly used for a variety of purposes, including admission to higher education, there are no common criteria for assigning grades and no way to summarize achievement for the three categories of students based on the grades.

Most school districts periodically administer an external norm-referenced standardized test, and all but one state administers some form of state test at one or more grades. The typical test used by school districts in the United States measures the number of correct answers to questions about knowledge of facts, representing, recognizing equivalents, recalling mathematical objects and properties, performing routine procedures, applying standard algorithms, manipulating expressions containing symbols and formulae in standard form, and doing calculations. Such tests reflect the fragmentation of content and the corresponding emphasis on low-level objectives of the curriculum. Multiple-choice questions on concepts and skills emphasize the independence rather than the interdependence of ideas and reward right answers rather than the use of reasonable procedures. Unfortunately, none of the existing instruments commonly used to judge student performances in mathematics were designed to assess mathematical literacy. As such, at best they measure a student's knowledge of some of the "design features" associated with mathematical literacy. Some items on these tests may measure understanding of such features, but none make any serious attempt to assess student ability to mathematize. Thus, because of these characteristics and the variety of different tests used, there is no way to aggregate data across districts or states to summarize achievement for the three different categories of students at the three levels of achievement for the nation.

To be consistent with the standards-based vision, the quality of student performance should be judged in terms of whether students are mathematically literate. Information needs to be gathered about what concepts and procedures students know with understanding and the ways students use such knowledge to mathematize a variety of nonroutine problem situations. Only then can one judge whether student performance meets the reform vision and, in turn, whether the curriculum and teaching changes meet society's needs. To assess the intended impact of standards-based reforms in mathematics education, new assessment systems are now being developed. For example, the new international assessment framework emphasizing literacy (reading, mathematical, and scientific) prepared for the Programme for International Student Assessment (PISA) by the Organisation for Economic Cooperation and Development (OECD, 1999) was designed to monitor on a regular basis the mathematical literacy of students as they approach the end of secondary school.

The only summary data for the nation comes from the periodically administered National Assessment of Educational Progress (NAEP; OERI, 1997) and from international studies such as the Third International Mathematics and Science Study (TIMSS; OERI, 1996). These external tests are administered to a national sample of students via matrix sampling. They provide a general profile of achievement and can be summarized for different groups of students (e.g., by gender, ethnicity), but not for districts, schools, or classrooms. In 1995, on the TIMSS tests, U.S. students tested slightly above the international average in mathematics at Grade 4 and below at Grades 8 and 12. On the NAEP tests in 1996, students in 44 (of 50) states were tested and showed improvement in scores at Grades 4, 8, and 12 when compared to scores in 1990 and 1992. It should also be noted that although these assessments included a few open-response items designed to assess understanding, they do not test mathematizing and, thus, do not provide information about the three levels of achievement. Neither do they provide information about the three groups of students. However, in TIMSS, the results of a sample of the top 10–20% of students who had taken or were taking precalculus or calculus (generally college-bound mathematics and science students) were compared to those of advanced mathematics students in other countries. The U.S. students scored considerably lower than the international average.

Overall, no summary data is available to judge how well the three groups of students were achieving the three levels of performance for either the traditional or reform goals for school mathematics.

Conclusion

For the United States, characterizing the goals (both traditional and reform) for three groups of mathematics students and their achievements at three performance levels is not possible, at least with any confidence. This fact, however, does not stop policymakers, administrators, and politicians from making inferences about U.S. students.

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