INVESTIGATING THE INFLUENCE OF THE INTUITIVE RULES IN ISRAEL AND IN TAIWAN

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Mathematics education researchers are always on the lookout for theories with explanatory and predictive powers – i.e., theories that enable the analysis of students' reactions and the prediction of their likely responses to given tasks. One such theory is the intuitive rules theory, which has been described in a series of articles. In this paper we describe a study which was carried out in Israel and in Taiwan to examine the influence of the intuitive rules More A – more B and Same A – same B on Israeli and Taiwanese students' responses to comparison tasks. Our findings confirm that in both countries the reactions to such tasks of students in various grades were indeed influenced by these intuitive rules.

In their work in science and mathematics education, Stavy and Tirosh have observed that students react in similar ways to a wide variety of conceptually non-related tasks (e.g., Stavy & Tirosh, 2000). Although these tasks differ with regard to their content area and/or to the type of reasoning required, they share some common, external features. So far four types of responses were identified, two of which, i.e., *More A - more B* and *Same A-same B*, are related to comparison tasks. For example, when students are told that Tom saves 15% of his salary, and Marry saves 20% of her salary, they tend to claim that Marry saves more money than Tom, because 20 is larger than 15, in line with the intuitive rule more A (percentage)-more B (money). Similarly, when students are told that John saves 10% of his salary, and Daffy also saves 10% of her salary, children tend to claim that John and Daffy save the same amount of money, in line with the intuitive rule same A (percentage)-same B (money).

Based on these observations, the intuitive rules theory has been proposed to explain and predict students' responses to mathematics and science tasks. Many responses that the literature describes as alternative conceptions could be interpreted as evolving from the intuitive rules *More A - more B, Same A-same B*, which are activated by specific external task features (for detailed

descriptions see, for instance, Tsamir, Tirosh &. Stavy, 1997; Tsamir, Tirosh & Stavy, 1998; Tsamir & Mandel, 2000).

The intuitive rules theory is based on data collected in the western world. It is, however, very interesting and important from both the theoretical as well as practical points of view to test the universality of this theory. For this purpose a study, parallel to the one conducted in Israel, was carried out in Taiwan, relating to a wide number of comparison tasks. Here we focus on Israeli and Taiwanese students' reactions to tasks related to three topics: vertical angles, temperatures, and the volume of cylinders, with regard to the intuitive rules *More A - more B* and *Same A - same B*.

Methodology

Background

In Israel, the topic of angles is introduced in Grade 4 and the proof that vertical angles are equal is dealt with in Grades 8-9. The topic of volume is introduced in grade 3-6 and formally discussed in Grades 10 to 12, when dealing with three dimensional geometry, trigonometry and analysis (e.g., extreme problems). The concept of temperature is presented in Grades 4 -5 and children are introduced to the terminator as a measuring tool. Further issues related to heat and temperature are and discussed in Grades 7-9.

In Taiwan, the topic of angles is introduced in grade 3-4 and the proof that vertical angles are equal is presented in Grade 9. The topic of volume is introduced in grade 3-6 and formally discussed in Grades 9 and up, when dealing with three dimensional geometry, trigonometry and analysis (e.g., extreme problems). The concept of temperature in presented in Grades 3-6 and children are introduced to the terminator as a measuring tool. Further issues related to heat and temperature are and discussed in Grades 8-9.

A question that naturally arises is – Are Taiwanese children similarly affected by intuitive rules as are Israeli children? More specifically, will Taiwanese students tend to claim, in line with the intuitive rule more A more B that "The larger the arms the larger the angle"? "The more water- the higher the temperature"?; and in line with the intuitive rule *Same A - same B*, that "Same area-same volume?"

Participants and Procedure

In Israel: Two hundred-and-forty-three students from Grades 2, 4, 6, and 9 – sixty nine, 65, 70 and 60 students, respectively, answered a written questionnaire that included a *comparison-of-vertical-angles* task.

One-hundred-and-twenty students, from Grades 2, 3, 5, 6, 7, 8, twenty from each grade, were individually interviewed regarding the *comparison of temperatures* task.

Three-hundred-and-seventy-five students from grades 1-6, and 10, 12, forty students each from grade levels 1-6, 110 10th graders and 29 12th graders. In Grades 1-6 each participant was individually interviewed, and the 10th and 12th graders were given a written questionnaire regarding the *comparison of volume of cylinders* task.

In Taiwan: Nine-hundred-and-sixty-six students from Grades 3, 4, 5, 6, 10 -- 206, 339, 345, 67, respectively, answered a written questionnaire that included a *comparison-of-vertical-angles* task.

Nine-hundred-and-twenty-one students from Grades 2, 3, 4, 5, 6, 10, -- 33, 197, 210, 205, 209, 67, respectively, answered a written questionnaire that included a *comparison of temperatures* task.

On-thousand-two-hundred-and-sixty-nine students from Grades 1-6, 10, 11, -- 28, 33, 34, 117, 443, 428, 65, 121, respectively. All participants answered a written questionnaire that included a *comparison of cylinder volumes* task.
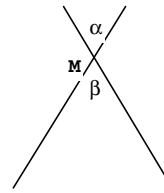
Materials

Comparison of Vertical Angles Task

Consider the following drawing:

Is angle β smaller than / equal to / larger than / angle α?
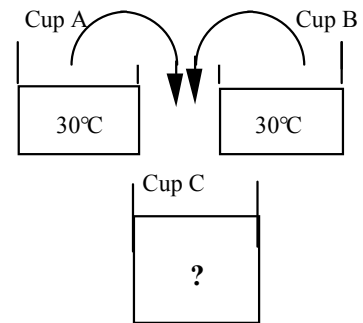
Explain your answer.

Comparison of Temperatures Task

Consider the following drawing:

The water from Cup A and the water from Cup B were

poured into Cup C. What is the temperature of the water in

Cup C?

Explain your answer.

Comparison of Cylinder Volumes Task

Take two identical rectangular (non-square) sheets of papers
(Sheet 1 and Sheet 2):

Rotate one sheet (sheet 2) by 90º
    Is the area of sheet 1 smaller than / equal to /
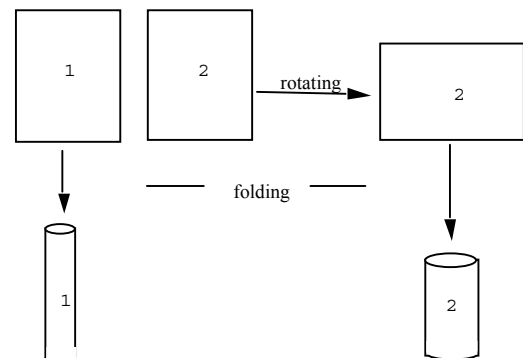   larger than / the area of Sheet 2?
Explain your answer.

Fold each sheet (as shown in the drawing).
    You get two cylinders: Cylinder 1 and
    Cylinder 2.
    Is the volume of Cylinder 1 smaller than /
    equal to / larger than / the volume of Cylinder
    2?
Explain your answer.

The tasks were originally formulated in Hebrew, the language in which they were administered in Israel (see for instance, Stavy & Berkovitz, 1980; Stavy, Tirosh & Tsamir, 1997; Tirosh, Stavy & Ronen, 1996). The mathematics and science educators who conducted these researches in Israel were familiar with the intuitive rules theory. They were responsible for writing, editing, administering the tasks as well as for the analysis of the data.

In order to use these tasks in Taiwan a workshop about the intuitive rule theory was carried out by one of the authors. Forty-five Taiwanese mathematics and science educators participated in ten four-hour weekly sessions, in which they studied the theory and had to carry out a relevant mini study in school classes.   The tasks were translated into English, and then one of the workshop participants re-translated it into Chinese. In order to validate the translation, the other participants commented on the translated version. The final agreed upon version was again translated into English and the meanings were verified. Then, the tasks were administered by the workshop participants, in the form of a written questionnaire, in several cities in Taiwan. Students responded in Chinese and the results were analyzed by each of the researchers.

Results

First we present the results related to the intuitive rule *More A - more B*, and then the results related to the intuitive rule *Same A - same B*.

Results related to the intuitive rule more A - more B

Vertical angles

Figures 1 and 2 show that in both Israel and Taiwan the tendency to correctly solve the comparison of vertical angles task increased with age. While among young children only about 30% of the 3rd graders in Taiwan, and about 10% of the 2nd graders in Israel provided correct responses, most Israeli 9th graders (over 80%) and all Taiwanese 10th graders correctly responded that the angles were equal.

Of the participants who did not respond correctly, almost all answered in line with the intuitive rule more A (longer arms, larger enclosed area) -- more B (larger angle).

Comparison of Temperatures Task

Figures 3 and 4 show that both Israel and Taiwan the tendency to correctly solve the comparison of temperatures task increased with age. While among 2nd graders of both countries there were no correct responses at all, most Israeli 8th graders (over 80%) and all Taiwanese 10th graders correctly responded that the temperature remained thirty degrees.

Of the participants who did not respond correctly, almost all answered in line with the intuitive rule more A (amount of water) -- more B (higher temperature).

Results related to the intuitive rule *Same A - same B*

Comparison of Cylinder Volumes Task

Figures 5 and 6 show that in both Israel and Taiwan the tendency to correctly solve the comparison of surface area of cylinders task increased with age, accompanied by an incorrect response to the comparison of volume task. In all cases students claimed in line with the intuitive rule *Same A - same B* that the volumes of both cylinders were equal as they were made from identically sized sheets of paper.

Final Comments

The findings of this study clearly indicate that Taiwanese students, much like the Israeli peers are strongly affected by the intuitive rules *More A - more B* and *Same   A - same B*, when presented with relevant comparison tasks. When relating to tasks whose correct answer was not in line with the intuitive rules, two main findings should be highlighted. First, to all three tasks two major types of responses were evident – the correct, and an incorrect response in line with one of the two intuitive rules. Also, the developmental pattern with age was rather similar.

Consequently, we suggest that the intuitive rules affect students' responses in both countries regardless of culture. As mentioned before, additional data were collected with regard to other relevant tasks, and similar findings pointing to the influence of the intuitive rules *More A – more B* and *Same A – same B*, were obtained.

These conclusions should be taken with some caution, due to practical constrains: the methodologies applied were not completely identical. Still we find the picture presented by these preliminary results a very relevant one. Clearly, further research is needed to bolster our knowledge about the universality of the role of the intuitive rules in students' mathematical and scientific thinking

References

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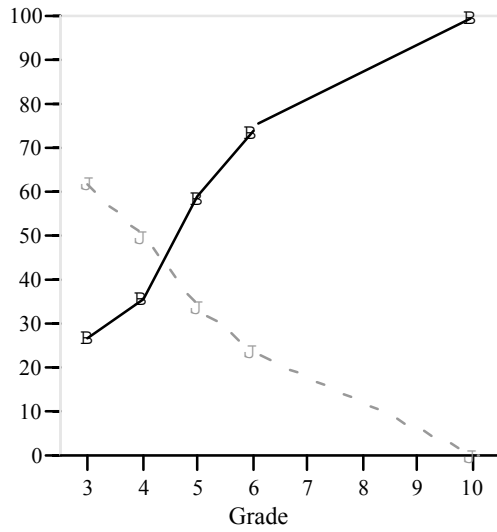Fig. 1: Taiwanese students' responses to the vertical angles task

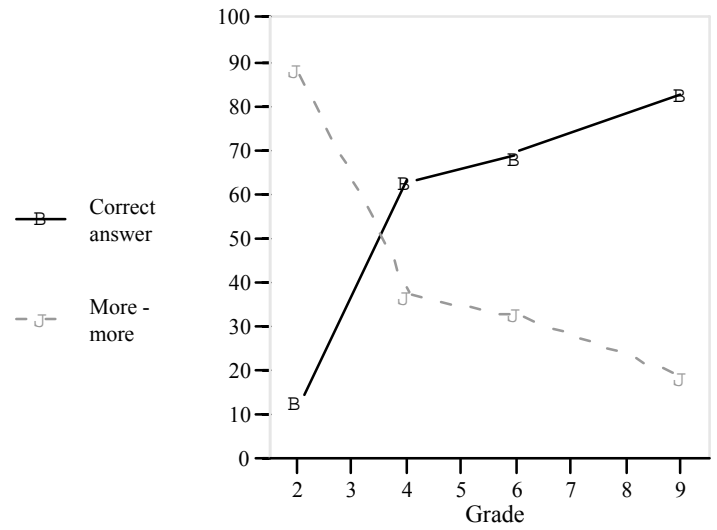Fig. 2: Israeli students' responses to the vertical angles task

Correct answer

More - more

Fig. 3: Taiwanese students' responses to the temperature task

Fig. 4: Israeli students' responses to the temperature task

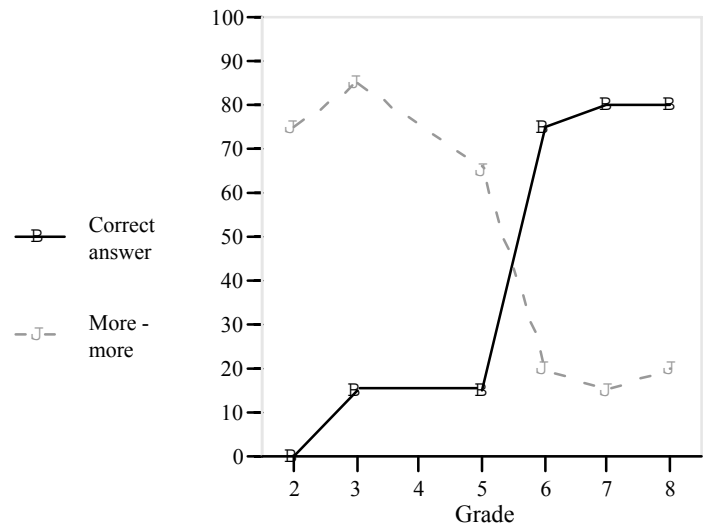Correct answer

More - more

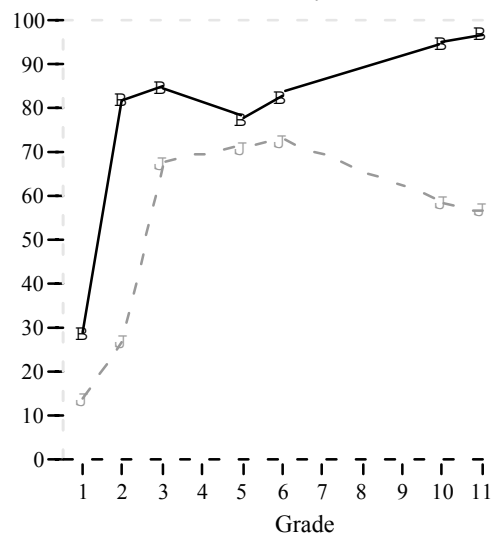
Fig. 5: Taiwanese students' responses to the volume of cylinder task

Fig. 6: Israeli students' responses to the volume of cylinder task

Same - same area (correct)

Same - same volume