# SELF-ESTEEM AND PERFORMANCE IN SCHOOL MATHEMATICS: A CONTRIBUTION TO THE DEBATE ABOUT THE RELATIONSHIP BETWEEN COGNITION AND AFFECT

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*This research work was aimed to analyze the relationship between cognitive and affective aspects in the particular context of mathematical education. Self-esteem was chosen to represent affective pole involved in this process, while mathematical performance at school was focused in terms of performance in a specific mathematical evaluation test. Data collected suggest empirical evidence for a connection between the level of self-esteem, interaction patterns and mathematical performance inside the pairs, high-level self-esteem pairs with cooperative patterns of work performing clearly better than low self-esteem level subjects. These data seem to reinforce the idea that it is not possible to split children difficulties at school in a dichotomy of cognitive and affective aspects.*

## 1. Introduction

The theoretical question about the place of affectivity in mathematical sense-making activities emerged strongly in previous research efforts of our research group (see Da Rocha Falcão and cols., 2000; Brito Lima & Da Rocha Falcão, 1997; Da Rocha Falcão, 1996), and is the main motivation of this research. These two poles of human psychological functioning cannot, of course, be scientifically addressed in such generic terms; because of this, the first step in operationally building this research enterprise was the translation of "affectivity" and "mathematical activities" is better-delimited and narrower representative variables. On the other hand, we share the perspective of V.A. De Bellis and G.A. Goldin about the affective system as including *changing states of feeling* (local affect) as well as *more stable, longer-term constructs* (global affect) (De Bellis and Goldin, 1999, pp.250; italics added). We are here particularly interested in the sphere of the so-called global aspect, concerning "(...) emotions about and within emotional states, emotions about and within cognitive states, and the *monitoring* and *regulation* of emotion (De Bellis and Goldin, op. cit., pp. 250; italics added).
*Self-esteem* was then chosen as representative of the global-affective sphere, since this complex process can be seen as a component of the general representation that someone has of himself/herself (Palacios & Hidalgo, 1995; Aberastury & Knobel, 1992). This representation has a more conceptual pole, the *self-concept*, and an affect-evaluative pole, the *self-esteem*. The data shown here try to offer empirical evidence of a connection between self-esteem and performance in school mathematics; this attempt, in fact, intend to go further, offering evidence for the debate about connections between affect and cognition in psychology. This debate is a hard theoretical task; since René Descartes, there is a strong tradition of splitting human nature in rational/spiritual aspects (*res cogitans*) and somatic/emotional/animal aspects (*res extensa*) (Descartes, 1973). This philosophical and epistemological background nourished theoretical

systems in psychology stressing one of these poles (in detriment of the other one), without an integrative approach showing the functional interconnection between affectivity and cognition. Two important theoretical systems in psychology are good examples of this Cartesian heritage: Piagetian genetic epistemology and Freudian psychoanalysis. Cognition, from a Piagetian point of view, is related to a biological need of equilibration, where affective aspects are seen as “combustible” for logical structures (the “engine”): “(...) *affectivity is considered as the energetic pole of behavior*” (Piaget, 1980, pg. 135). Freud, for his turn, will stress unconscious pulsional (libidinal) aspects as central in the theoretical explanation of human behavior, viewing cognition (or epistemophilic motivation) as a derivative of libidinal impulse by sublimation or neurosis (Freud, 1973). There are certainly other important theoretical contributions addressing this specific aspect of an integrative view of human behavior (see, for example, Henry Wallon, Erick Erikson and Donald W. Winnicot’s works on child development); none of them, nevertheless, seem to propose empirical data concerning specific aspects of cognition (i.e., specific knowledge domains or conceptual fields) taking into account affective aspects as constitutive, not merely adjuvant. The present research work tries to contribute in this direction, showing empirical evidence of the interaction between self-esteem and school achievement in mathematical problem-solving tasks.

## 2. Procedure

The sample of this study was initially constituted by 81 students of the 5$^{th}$ grade (elementary level) of a public school from Recife (Brazil), with ages varying from 12 to 14. This specific school level was chosen because it represents the first important moment of clear school difficulties in mathematics for Brazilian children (Ministério da Educação e do Desporto-Brazil, 1995). This initial sample was submitted to the HTP (House, Tree, Person) Test, used here to access levels of self-esteem among the students, according to administration criteria proposed by E. Hammer (Hammer, 1991). We could then arrive to a research sample of 20 students showing low or high levels of self-esteem (see Figure 1 below for examples of HTP data). These students were put together in pairs, controlling gender and self-esteem, as illustrated by Table 1 below. These ten pairs of students were submitted to a set of 20 evaluation questions in school mathematics, created specifically for northeastern Brazilian children (NAPE Evaluation Test – for more detailed information, see Neves e Souza, 1997). These questions addressed aspects considered important by Brazilian educational authorities for this school level, and are briefly described in Table 2. Both the achievement as well as the problem-solving strategies and difficulties in the NAPE Evaluation Test were analyzed, together with descriptive data like constitution of pairs concerning gender and self-esteem level (according to the procedure described in Table 1 below) and patterns of interaction and leadership showed by the subjects during problem-solving sessions. This set of data was analyzed with the help of multidimensional descriptive tools (cluster analysis of nominal data; for more detailed information about these statistical descriptive multidensional procedures, see Rouanet, Bernard & Le Roux, 1990); results are described in the next session.

**Subject N:** drawing occupying more than 1/3 of the paper, presence of details in the body, presence of the soil and additional elements (sun, clouds, flowers) (Obs: this drawing was originally produced with color pencils).

**Subject R:** drawing occupying less than 1/3 of the paper, few details in human body, absence of the soil (Obs: this drawing was originally produced with black pencil only).

**Figure 1:** Drawings of a person produced by two subjects considered, respectively, as being examples of high-level (subject N.) and low-level (subject R.) self-esteem, according to Hammer (op. cit.) classificatory propositions for HTP analysis.

| | Boys | Boy & Girl | Girls | |
|---|---|---|---|---|
| **Elevated self-esteem** | ■■ A1<br>■ C1 | ■● A2<br>C2 ■    C3 ● | ●● A3<br>● C4 | **Elevated self-esteem** |
| **Low self-esteem** | □<br>□□ B1 | □    ○<br>□○ B2 | ○<br>○○ B3 | **Low self-esteem** |

**Table 1:** Design for the constitution of 10 pairs of students, taking into account gender and self-esteem level (Boys= ■ and □; girls= ● and ○).

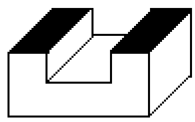
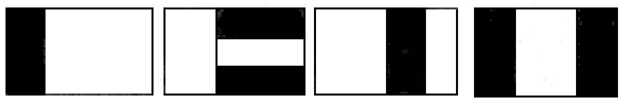
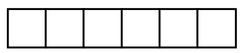
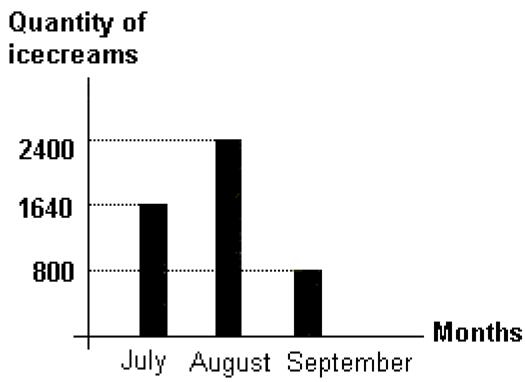
| **Mathematical aspect addressed [$G_n$ =Group] and number of questions** | **Example of a test item** |
|---|---|
| $G_1$= Problems of addictive structure<br>**- 3 questions** | An airplane must fly over a distance of 962 Km in two stages: the first stage will cover a distance of 642 Km. How many kilometers this airplane will have to cover in the second stage? |
| $G_2$= Problems of multiplicative structure<br>**- 3 questions** | A ship can carry a maximum of 200 kilograms per trip. What would be the minimum number of trips that this ship has to do in order to carry 8 people, each person weighting 60 kilograms? |
| $G_3$= Questions involving the use of operational algorithms<br>**- 2 questions** | Make these operations: $3529 \div 15$<br>$3847 + 5 + 98$ |
| $G_4$= Questions involving the comprehension of the numerical decimal system of notation<br>**- 2 questions** | The number ***eight thousand and two*** written in hindu-arabic algorisms is: __________ |
| $G_5$= Questions of geometry<br>**- 3 questions** |  Look carefully to this piece of a fitting game: how would be this piece of game seen from up to down? [*Options given are shown below*:]<br> |
| $G_6$= Questions involving fractions<br>**- 3 questions** | The figure below represents a chocolate bar. Draw in black the part that corresponds, in this figure, to the following addition: 2/6 + 1/6 of the chocolate bar.<br> |
| $G_7$= Questions concerning the comprehension of statistical descriptive graphics and mesures.<br>- In the graphic on the right, how many icecreams would be sold if this amount was half the icecreams sold in September?<br>**- 4 questions** |  |

**Table 2:** Profile of questions in the NAPE Evaluation Test.

## 3. Results

Two descriptive multidimensional clusters of pairs of subjects were obtained from data analyzed (see Figure 2 below). These clusters represent a hierarchical classification of the pairs of subjects according to two sets of information: classification ① was obtained from descriptive data concerning gender and self-esteem level in the composition of the pair, interaction and leadership patterns verified in the pair interactions during problem-solving activities, and need of help from the teacher during problem-solving. Interpretation of this first classification showed that two descriptive variables combined, *level of self-esteem* (LSE) and *pattern of interaction of the pair* (INT), could explain the partition obtained (the other variables having low contributions to the general variance of the data, i.e., contributions under the average value of explained contributions; this interpretation was confirmed by correspondence factor analysis). The first of these variables, LSE, admitted three analytical categories concerning the composition of the pairs in terms of self-esteem level of each subject: a) high-high; b) low-low; c) high-low.

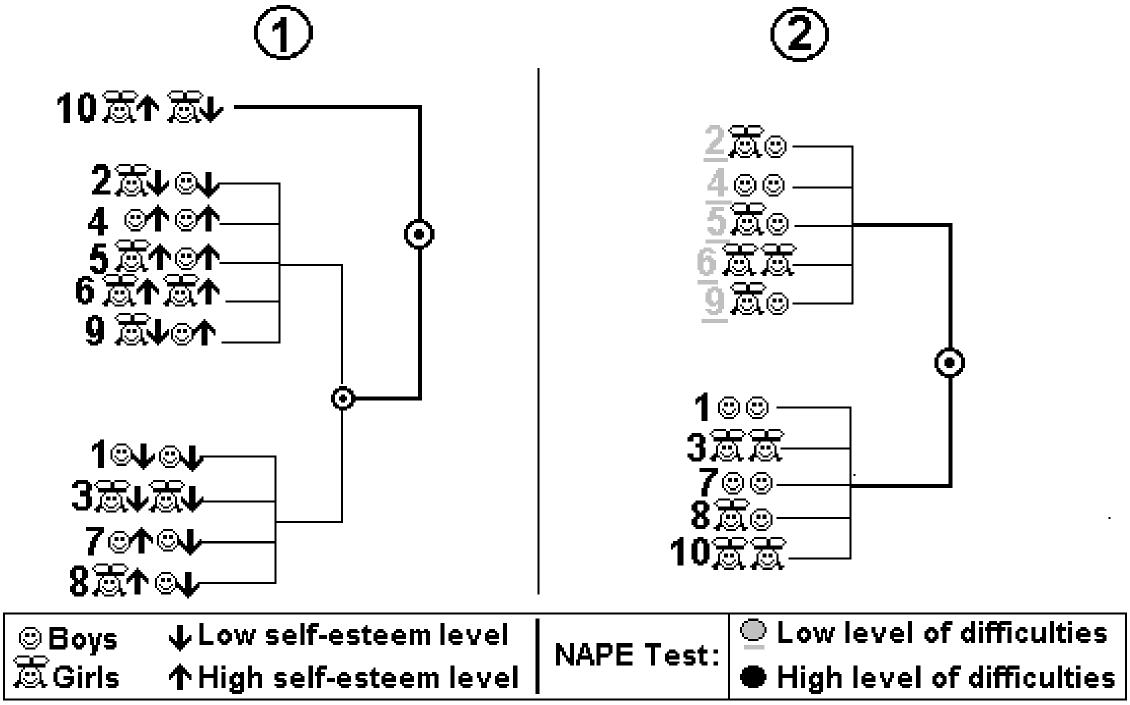



**Figure 2:** Hierarchical classificatory trees produced from data issued of pairs of subjects

The variable INT, for its turn, had equally three analytical categories concerning the patterns of interaction showed by the pair during problem-solving activities: a) Strong cooperation between subjects, with dialogues and step-by-step sense making in problem-solving procedures; b) One of the subjects is clearly the "guide", thinking aloud in order to share with the other subject his/her procedure; this other subject supports and encourages the first one; c) One of the subjects ignores the other, working alone and silently, the ignored subject accepting this scenario. Classificatory tree number ① shows two main clusters of pairs (and an isolated pair [number 10], which will be referred latter): the first one is formed by pairs 2, 4, 5, 6 and 9, and the second one being formed by pairs numbers 1, 3, 7 and 8. The first cluster is formed by three LSE high-high pairs, one low-low and one high-low; the second cluster is formed by two LSE low-low pairs and two high-low; the first pair was strongly characterized by INT pattern type (a), while the second cluster was characterized by INT pattern type (c). Briefly, we can interpret the partition obtained in terms of two groups characterized respectively by low self-esteem, combined with important difficulties in interaction between subjects of the pair (cluster 2: 1, 3, 7, 8), and high self-esteem, combined with good interaction pattern (cluster 1: 2, 4, 5, 6, 9). It is important to observe that this is not a *pure* partition (since there are pairs more and less representatives of the partition scheme), but the most suitable one, in terms of the contribution to the total explained variance of the analyzed data. In this context of analysis, the pair number 10 is explained as a member of cluster 2, since clinical analysis of protocols shows the same LSE and INT characteristics of this cluster. The exclusion of this pair from cluster 2 seems to be explained by very peculiar and strong characteristics detected in clinical analysis, specifically a clear INT pattern type (b) and a very strong dependence to the teacher during the problem-solving session.

Classification ②, for its turn, was obtained from data concerning the performance of pairs along the problems of NAPE Evaluation Test. The categorization of these data took into account the specific groups of mathematical aspects (in accordance with summary reproduced in table 2 above). The same multidimensional classificatory algorithm used for classification 1 analyzed these data, and the partition obtained showed a very interesting result: this partition is almost identical to the first one, *these two classifications being produced by different set of data.* The interpretation of classification 2 showed that the partition obtained is strongly explained by aspects that could be summarized in terms of difficulties in solving 6 of the 20 problems (the other 14 problems showed an irrelevant contribution to the partition obtained); cluster 1, formed by pairs numbers 2, 4, 5, 6 and 9, showed very few difficulties, while cluster 2, formed by pairs numbers 1, 3, 7, 8 and 10, showed important difficulties, summarized by table 3 below. A global interpretation of the main differences between clusters 1 and 2 of the second classification seems to indicate that the cluster 1 is clearly able to capitalize help from the teacher, and also from discussion inside the pairs; cluster 2 has important difficulties in dealing with formal representations like graphics, numeric systems and school algorithms. The partition obtained in classification 2 reproduces almost perfectly the previous partition obtained in classification 1, based upon self-esteem and patters of interaction inside the pairs; this result offers empirical support to a possible connection between the two sets of data.

| Relevant questions of NAPE Evaluation test | Pattern of problem-solving shown by cluster 1 | Pattern of problem-solving shown by cluster 2 |
|---|---|---|
| **Question 8**: question involving fraction [$G_6$] | Has some difficulty in considering part-whole relations, but arrives to the solution after help offered by the teacher. | Absence of one specific pattern for this question. |
| **Questions 9 and 10**: Questions involving the comprehension of statistical graphics [$G_7$] | Can easily discuss and solve these questions. | Important difficulties in interpreting graphic representation even after teacher 's help. |
| **Question 14**: Question involving the comprehension of the numerical decimal system of notation [$G_4$] | Shows comprehension of the numeric-decimal system. | Has difficulties in dealing with intermediary zeros in transposing four-digit numbers from current language to algorism representation |
| **Question 17**: Problems of multiplicative structure [$G_2$] | Has some difficulty in making explicit the multiplicative structure of the problem, and in explaining how to solve it, but arrives to the solution. | Absence of one specific pattern for this question. |
| **Question 19**: Question involving the use of operational algorithms [$G_3$] | Shows comprehension of the numeric-decimal system, and of the algorithmic procedures related to this system. | Difficulties in intermediary operational steps of the use of algorithms (especially when dealing with division). |

**Table 3:** Relevant questions of NAPE Evaluation Test (with respective patterns of problem-solving performance) which contributed for the partition obtained in classification ②.

## 4. Concluding remarks

This study tried to offer empirical evidences to the need of taking into account both affective and cognitive aspects in the research about mathematical learning. Efforts were made, first of all, in proposing clearly defined variables in order to arrive to a replicable study. As a result of this effort, the affective pole was operationally defined in terms of self-esteem, which cannot be considered as covering affectivity in its complexity, but seems to be an important aspect of *global affect* (De Bellis & Goldin, op. cit). This variable was analyzed together with other descriptive variables, and an important relation between self-esteem and interaction patterns was detected. On the other hand, mathematical activity was operationally defined in terms of performance in a specific evaluation test, what is certainly a narrow sample of mathematical performance, but is a perfectly public, explicit tool in the context of research.

The two very similar partition of subjects obtained as a function of descriptive variables like self-esteem and interaction patterns and performance in the NAPE Evaluation Test is a first empirical evidence of a possible connection between performance in school mathematics and affective aspects. This connection is still a very tentative interpretation, since we cannot assume a causal relation in any direction, neither we can discard the role of other intervening variables not addressed by this study. Interaction patterns, an aspect already explored in previous research (see, for example, Leikin and Zaslavsky, 1997), appears here in contact with self-esteem, and this empirical connection certainly deserves additional research.

Affect is undoubtedly a difficult aspect to take into account in the context of research and theorizing in psychology of mathematics education. Nevertheless, data collected here reinforce the need of taking mathematical learning into account as a human activity, impregnated of fears, self-evaluation, social roles and interaction possibilities.

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