LINKS BETWEEN INTUITIVE THINKING AND CLASSROOM AREA TASKS

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This paper investigates the links between intuitive area integration rules and classroom area tasks in students 9 to 13 years of age. The study used rectangular and near rectangular regions of varying areas and perimeters, as well as common classroom area tasks. Information Integration Theory and functional measurement procedures were used to reveal the students' intuitive additive and multiplicative rules. It was found that intuitive judgement rules are strongly linked to a students' responses to and success on common classroom area tasks.

Area is the most commonly used domain of measurement and the basis for many models used by teachers and textbook authors to explain computational strategies (Hirstein, Lamb & Osborn, 1978; Woodward & Byrd, 1983; and Baturo & Nason, 1996). It is also a concept that textbooks commonly used in Australia either fail to define or, often (for example, Blane & Booth, 1989), discuss with the apparent assumption that students already understand it.

This apparent lack of definition, and an emphasis on formulae, seems to be contributing to the documented confusion between area and perimeter (Kidman, 1999; Kidman & Cooper, 1996a; Outhred & Mitchelmore, 1996; Clements & Ellerton, 1995; Bell, Costello & Kuchemann, 1983; Hirstein, 1981; Hirstein, et. al. 1978; Bell, Hughes, & Rogers, 1975). In particular, as Kidman and Cooper (1996b) found, students have difficulty with the process of obtaining shapes' measurements, determining which dimensions to consider, and how to integrate these dimensions when calculating either area or perimeter.

The Information Integration Theory (IIT) method Anderson and Cuneo (1978) offer an excellent opportunity to explain the process of area concept development. IIT been widely used to identify the intuitive rules applied by students to integrate dimension information. In particular, recent studies have employed IIT to investigate students' perceptual judgement of area (Kidman, 1999; Kidman and Cooper, 1996b; Lautrey, Mullet & Paques, 1989; Silverman & Paskewitz, 1988; Schlottman & Anderson, 1994; Wolf, 1995); while functional measurement, the methodological counterpart of IIT, has allowed the diagnosis, in simple algebraic terms, "… of the rules which govern integration of information about perceived stimuli." (Wolf, 1995, p. 49-50).

In these studies, students have been provided with different rectangular shapes and asked to rate their area on a linear scale. The general consensus of these studies has been that students' judgements of area obeyed two-dimensional rules. They have also shown a transition from additive to multiplicative judgement rules. The expectation is that children will make the transition from an additive integration rule to the normative multiplicative integration rule at some stage between the ages of 5 and 12.

This paper describes and reports on an investigation to determine the link between students' intuitive judgement rules and their progress on common classroom area tasks. The investigation was based on the body of literature and the functional measurement methodology stemming from the work of Anderson and Cuneo (1978).

METHOD

The investigation used a multi-method design. The quantitative methodology of functional measurement was combined with the qualitative methodology of semi-structured interview. (A comprehensive outline of the methodology of the study, including how IIT determines area judgement rules, is provided in Kidman, 1997, and Kidman & Cooper, 1996b). The sample consisted of 36 students aged 9 to 13 years with an equal number of boys and girls and a range of mathematical abilities, one third each of above average, average and below average.

The instruments used were three experiments and an interview. The first experiment contained 16 rectangular wooden pieces painted to represent chocolate and with dimensions corresponding to the factorial combinations of 3, 6, 9, and 12 cm. The pieces were presented to students who were asked to judge the area of the rectangular pieces in relation to two end anchors. To obtain measures of the students' area judgements, the students were provided with a 19 point scale with two end anchors, two 'special' pieces of dimensions 2.7 x 2.7 cm and 15.8 x 15.8 cm. The second experiment used 16 rectangular pieces identical in dimensions to the first experiment, but with a rectangular corner 'bitten' off producing a figure of equal perimeter, but less area. The dimensions of the 'bitten' off corner were all one third of the width and one third of the height of the rectangular pieces. The third experiment again used 16 rectangular pieces identical to the first experiment, with the exception that they had a semi-circular 'bite' out of one side producing a figure with less area but greater perimeter. The 'bite' was centred along one dimension with the radius of the 'bite' one third of the length of the shortest dimension. Throughout the three experiments, each student was quizzed as to the method they were using to rate each piece, they were asked as to whether they were aware of any changes they made to their method over the course of the three experiments, and diagrams were sought.

At the conclusion of the three experiments, the students were interviewed. Initially, the students' understanding of both area and perimeter were discussed. The students were asked to identify if they had employed either or both of these concepts to rate the chocolate pieces. To conclude the interview, the students were asked to complete the 5 classroom tasks shown in Figure 1.

The tasks were designed to cover a range of ability and instructional levels. All students were expected to be able to complete the task involving the congruent subregions as this should be among initial area activities presented to students 7 and 8 years of age. The task involving the diagram of a rectangle marked with 3 cm and 5 cm was designed to investigate the student's computational knowledge. The youngest students in the study were expected to be able to subdivide the region into a grid, while the older students were expected to be able to calculate the area using a formulae.

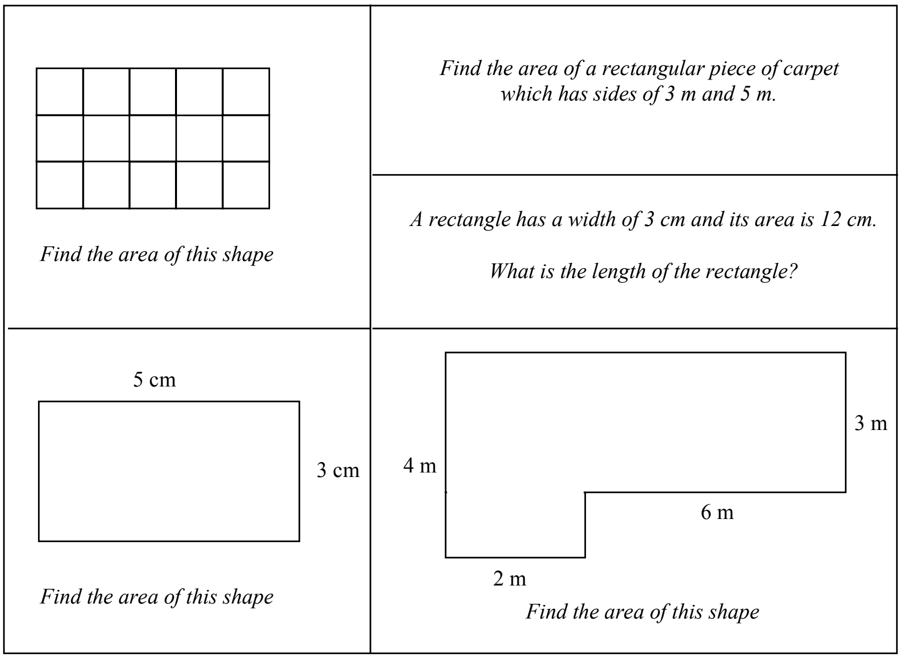

Figure 1. The 5 classroom tasks

The real-world word task was designed to determine if the "draw a sketch" technique (Department of Education, 1988, p. 196) would be utilised. All but the youngest of the students should have been familiar with this technique. Both the "draw a sketch" and knowledge of the area formulae were investigated in the other word problem. The remaining task of the L-shaped figure investigated the student's principled conceptual knowledge (Baturo and Nason, 1996) and problem solving approach.

RESULTS AND DISCUSSION

The functional measurement technique of the IIT method (Anderson & Cuneo, 1978) revealed both additive and multiplicative intuitive judgement rules were present in the sample of students (Kidman, 1997). It was found that a student could be categorised as being either predominantly additive (that is, the student intuitively had a perception of area where doubling the lengths of the sides of a rectangular region can be seen as doubling the area) or predominantly multiplicative (that is, the student intuitively had a perception of area where doubling the lengths of the sides of a rectangular region more than doubles the area) by noting the three judgement rules used by the student over the three experiments. Codes existed which were used to create the two categories. This is summarised in Table 1.

It can be seen that the predominantly additive category was composed of 16 students, from five different codes. It was not possible to determine a judgement rule for one student, in Experiment 1, due to intersecting locations on the factorial plot. The 20 students in the predominantly multiplicative category were less variant in their codes. 60% of the students experienced 'intra-individual' rule changes (for example, the removal of a corner in Experiment 2 caused the student to alter their judgement rule from additive to multiplicative.

It is interesting to note that for the students in this study, the proportions perceiving area as either predominantly additive or predominantly multiplicative is consistent across the age range (Kidman, 1997). Thus the differences between the ages was not as obvious as could be expected.

Table 1. Judgement rule codes and categories for Experiments 1, 2 and 3

Overall Judgement Rule	**Judgement Rules**			**Number of Students**
	Exp 1	**Exp 2**	**Exp 3**	
	A	A	A	5
Additive	A	A	M	7
	A	M	A	1
	M	A	A	2
	?	A	A	1
			TOTAL	16
	M	M	M	9
Multiplicative	M	M	A	5
	A	M	M	6
	M	A	M	0
			TOTAL	20

KEY A = additive judgement rule M = Multiplicative judgement rule
? = unknown judgement rule

The older students had had increased levels of instruction, but do not seem to have advanced much beyond the younger students. It is clear that many students, irrespective of age, are experiencing confusion between area and perimeter.

The two categories, predominantly additive and predominantly multiplicative, were linked with the strategies that emerged throughout the three experiments (a comprehensive description of the strategies is provided in Kidman, 1999), and from the completion of the five classroom tasks. These links are shown in Figure 2.

Predominantly additive thinkers. These thinkers perceive area to double when the lengths of the sides of rectangular regions double. Additive thinkers tend to think in terms of units of one (Steffe, 1992). 16 students displayed this thinking style. The actions and statements of these students throughout the three experiments revealed 4 strategies and 1 descriptor clearly linked to their additive thinking.

The *test piece rotation* strategy was particularly robust across the three experiments. In this strategy, the student would measure the outside edge of the rectangular region, by rotating the test piece so each edge was compared, around the end anchors.

The *index finger* strategy was also very robust across the three experiments. Initially the student placed an index finger adjacent and parallel to one side of the test piece. This is then repeated by moving the finger along to the next adjacent parallel position. The student repeated this procedure along the edge of the rectangular piece to the opposite side.

The *vertical alignment* strategy involved a rotation of the test piece prior to a rating being made. Each of the wooden pieces were presented to the students in a uniform manner, but some pieces were presented in a horizontal alignment (lying flat), while others had a vertical alignment (standing tall). During Experiments 1 and 2, many additive thinkers rotated the test pieces when presented with a horizontal alignment so that it became vertical. One student indicated the need for the pieces to be presented "like chocolate on shop shelves ... like the way the wrapper would go" (Rhea, 10 yrs).

The *request a ruler* strategy occurred among additive thinkers in Experiment 1. Students commented prior to starting the experiment: "I need a ruler to do this" (Jack, 9 Yrs); "This can't be done properly without a ruler" (Jodie, 11 Yrs). Jodie insisted that "we always use rulers to measure. You see, without one you can't measure something". Jack (9 Yrs) explained that he needed to "measure the chocolate pieces to see if one was bigger or not". He wanted to measure the longest sides of each piece.

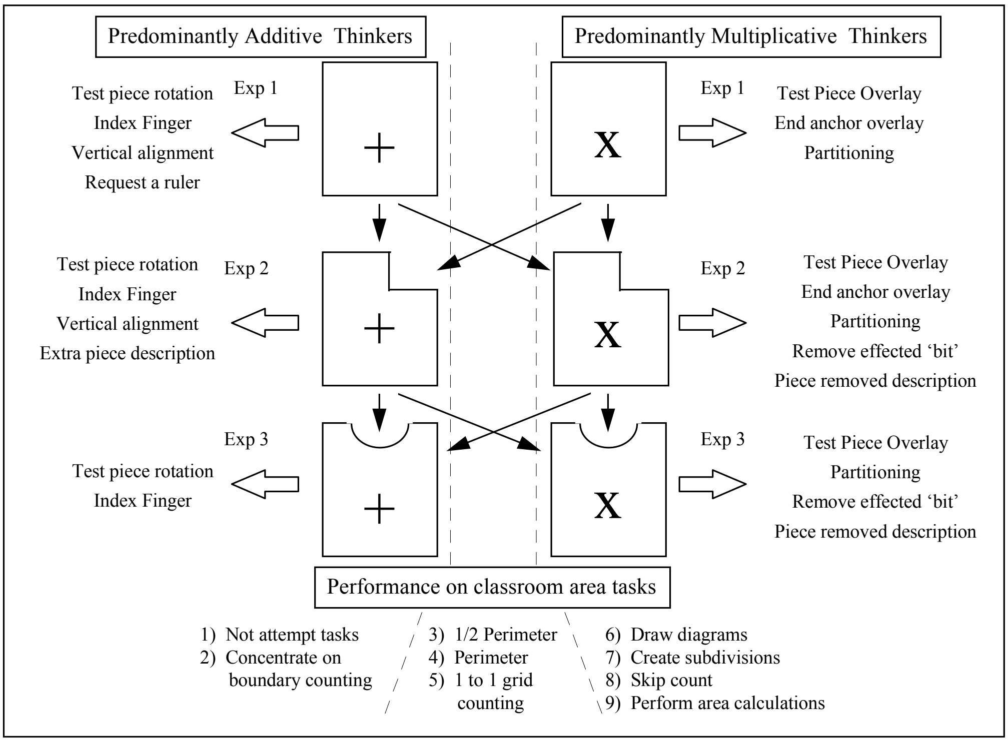

Figure 1. The links between intuitive thinking and classroom tasks

The *extra piece* descriptor of the Experiment 2 test pieces was quite prevalent among students employing predominantly additive thinking styles. Such students perceived the near rectangular region as being basically rectangular with an 'extra piece' added onto it, despite having been introduced to the second set of test pieces as being "identical to the first set of pieces except they have a rectangular corner 'bitten' off". It is possible that this view originates from the students' perception that the area has additive properties, and therefore the 'extra bit' is also added.

Predominantly multiplicative thinkers. These thinkers have the perception where doubling the lengths of the sides of a rectangular region more than doubles the area. 20 students displayed this thinking style. Such multiplicative thinkers have the ability to think of units of one, and about units of more than one, simultaneously (Steffe, 1992). The actions and statements of these students throughout the three experiments also revealed 4 strategies and 1 descriptor clearly linked to their multiplicative thinking.

The *test piece overlay* strategy was particularly robust across the three experiments. It involved overlaying the test pieces onto the large end anchor in a series of flip and slide transformations.

The *end anchor overlay* strategy involved the small end anchor being overlaid onto the test pieces using a series of randomised flip and slide transformations also. It was also particularly robust across the three experiments. Jenny (11 Yrs) claimed she "… counted how many squares, see these little squares [indicating the small end anchor], I want to see how many of them cover this bit [holding up the test piece]". Jay (9 Yrs) explained "it has to be the small square that you use because it doesn't change each time like the bigger ones do".

The *partitioning* strategy also occurred among multiplicative thinkers across the three experiments. These students were familiar with the method of partitioning both the length and width of a rectangular shape, and integrating these values using multiplicative reasoning. One of the students made imaginary marks along the two salient dimensions with her finger. She partitioned the width of the test piece into what appeared to be 1cm lengths, maintaining a mental count of the partitions. Upon completion of the width, her attention focused on the length of the test piece and she repeated the partitioning process, again with 1cm partitions, maintaining a mental count as she progressed. Other partitioning students explained that they imagined the chocolate pieces already divided into squares, counted the squares on two dimensions, and multiplied "… them together, like you do for area sums" (Phillip, 13 Yrs).

The *remove effected 'bit'* strategy emerged, through drawings, during Experiment 2 and maintained its presence through Experiment 3. Students using this strategy mentally removed the "effected bit". Phillip (13 Yrs) explained that when he worked with "the boards with nails in and rubber bands … and if my shape was wonky, the teacher told me to get rid of the crooked bit and work with the rest of it". As a result of his classroom geo board lessons, Phillip mentally removed the altered sections of the test pieces.

The *piece removed* descriptor existed for students who thought multiplicatively, where they perceived the basic shape as rectangular, but with a piece removed. Such students either remembered being told a piece had been removed, or they have a more wholistic view of the shape, than the additive thinkers.

Performance on classroom area tasks. Table 1 indicated that five students are truly additive thinkers as they only employed additive integration rules across all three experiments. For these students, their performance on the five classroom area tasks was characterised by 2 strategies - *failure to even attempt a task*, and if attempted, the student would concentrate on some form of *boundary counting*. The failure to attempt a task was partially characterised by students who simply said "pass" (Ben, 9 Yrs) presumably because they were tired or bored. Students who read a task a number of times but did not actually attempt the task were also included in this category. They were not able or prepared to offer a solution even when told a solution was possible; for example, "This doesn't have all the numbers, so I can't do it" and, after being told it was possible, "Next" (Anne, 12 years). Boundary countingincluded either grid line counting around the four sides of the figure, or a count of the spaces around the four sides of the figure.

Nine students were truly multiplicative thinkers as they only employed multiplicative integration rules across all three experiments. For these students, their performance on the five classroom area tasks was characterised by four strategies – *drawing diagrams, creating subdivisions, skip counting, and performing area calculations*. Two tasks were word problems. While both could have been solved without diagrams, six of the nine students successfully used diagrams. Two students (aged 11 Yrs) attempted to use diagrams for the carpet task but had difficulty deciding which measurements belonged on which dimension. Steven (11 Yrs) finally decided "it doesn't matter anyway, 3 times 5 is 5 times 3". The creating of subdivisions could be seen as an extension to the draw a diagram strategy. The majority of truly multiplicative thinkers automatically drew subdivisions on the L-shaped figure, however only a minority successfully found the solution. Computational errors caused two of the 9-year-old students to obtain an incorrect area. Skip counting in the task involving congruent subregions was also evident only among the truly multiplicative thinkers. The only students to obtain correct solutions for the classroom tasks, using multiplicative processes were the truly multiplicative thinkers.

The remaining 22 students (11 additive thinkers and 11 multiplicative thinkers) experienced 'intra-individual' rule changes. In the case of the additive thinkers, for one of the three experiments they thought multiplicatively. Similarly, for the multiplicative thinkers, for one of the three experiments they thought additively. This rule change may be the result of the students' having the ability to think of units of one, and of units of more than one, but not both simultaneously. The classroom strategies for the additive thinkers and the multiplicative thinkers in this group were the same. They attempted all tasks, and these attempts included calculations of *perimeter, ½ perimeter* as well as calculations of area through *1 to 1 counting* of the congruent subregions. Computational error also prevented these students from obtaining a correct area solution. For this group there was a lot of confusion between area and perimeter. The younger students particularly would calculate a ½ perimeter by simply adding the given dimensions.

CONCLUSION

This experiment extends the body of literature stemming from the work of Anderson and Cuneo (1978) by using non-rectangular regions. The findings confirm that two area judgement rules do exist, that an individual can alter their judgement rule, and that there is confusion between area and perimeter, possibly resulting from the student's inability to think multiplicatively .

The misconception of area of rectangles being dependent on the sum of the dimensions is fairly constant across the age range especially for students who experience inter-individual rule changes. Such students may have the ability to think of units of one, and of units of more than one, but not both simultaneously. Students making rule changes experience difficulties with classroom area tasks in that they confused area with perimeter. It may be beyond the student's ability to think multiplicatively for area tasks, irrespective of their academic backgound.

Students using additive thinking need to measure regions with a ruler or index finger and prefer a vertical alignment. Truly additive students tend not to attempt classroom area tasks. It is possible that such students do not have a workable method for some area problems, and as a result they choose not to attempt them.

Students using multiplicative thinking tended to use overlay strategies as well as partitioning, and some are capable of interpreting word problems and drawing diagrams to solve classroom area tasks.

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