Difficulties Confronting Young Children Undertaking Investigations

Carmel M Diezmann, James J Watters, Lyn D English

Queensland University of Technology, Australia

Abstract

One approach to providing a mathematically rich curriculum is to involve young children in mathematical investigations in which they engage in the exploration of meaningful problems, and problem posing. However, there is limited research on how teachers can facilitate young children's learning through investigations. This study explored the difficulties seven-to-eight year old students experienced when they began an investigatory program. We present examples of specific difficulties students confronted in conceptualising and conducting investigations, as well as general difficulties that they experienced which hindered their investigations, such as limited observation skills. Our contention is that mathematical investigations can enhance young children's learning provided that their difficulties are addressed.

Background

The importance of providing students with opportunities to work as mathematicians has held credence for at least the past three decades (e.g., Papert, 1972; Wells, 1985) and has recently been strongly advocated (National Council of Teachers of Mathematics, 1998, 1999). As mathematical investigations are central to the work of mathematicians (e.g., Hoffman, 1998), they are fundamental to children's work as young mathematicians (Baroody & Coslick, 1998; Wells, 1985). Investigations are defined as "something more than solving the problem" in which "there will be questions to ask as well as questions to answer" and that require "speculation and conjecture" coupled with opportunities "to test out ideas and to convince others of their validity" (Jaworski, 1986, p. 3). The cognitive activities fundamental to investigations are consistent with those advocated in reform classrooms. In these mathematics classrooms, students should raise questions, pose and solve problems, participate in constructive dialogue and debate, and explain, clarify, and revise their mathematical ideas and problem constructions (Baroody & Coslick, 1998; Bowers, Cobb, & McClain, 1999; English, 1998). Thus, mathematical investigations are ideal for implementing these practices and supporting the child-centred approach that underpins reform initiatives (Borasi, 1992; Jaworski, 1994; Shifter, 1996).

However, there is an urgent need for classroom-based research that addresses the teaching and learning issues associated with young children undertaking investigations. While classroom-based research on the teaching and learning of children conducting mathematical investigations exists for the upper primary years (e.g., Oliveira, Segurado, da Ponte, & Cunha, 1997), there is limited research in the early primary years. The existing research on young children implementing investigations tends to focus on the implementation of an investigation with an individual child (e.g., Juraschek & Evans, 1997), and hence, provides scant guidance for implementing investigations with a class. As engaging young children in investigations requires teachers to teach mathematics in new and different ways (Baroody & Coslick, 1998; Skinner, 1999; Taber, 1998), such research is fundamental to the reform vision for improving student achievement (Hiebert, 1999).

Ideally, classroom investigations should parallel real life problems and provide children with opportunities to apply their basic knowledge (Holding, 1991). Such problems might involve making decisions that are influenced by aesthetics, economics, pragmatism or safety. Associated tasks that may involve observation, collecting data, seeking patterns and relationships, characterise original thinking in mathematics and provide authentic circumstances for conjecture, logical thinking and proof, all of which are cornerstones of authentic mathematics (e.g., Greenes, 1996).

Although investigatory tasks for young children need to be commensurate with their interests, experiences, and mathematical capacity, the tasks needs to be relatively challenging to have cognitive benefit (Lappan & Briars, 1995; Stein, Grover, & Henningsen, 1996). Due to the cognitive demand that occurs when students are engaged in challenging tasks, teachers may scaffold students' problem solving by simplifying tasks or providing hints (e.g., Rosenshine & Meister, 1992). However, even with young children, scaffolding should be used judiciously because when a teacher takes over the challenging aspects of the task, it becomes routinized (Stein et al., 1996) and the cognitive value of the task is reduced (Henningsen & Stein, 1997). Routine investigations in which the "investigation degenerates into an algorithm" have limited cognitive value (Roper, 1999). Thus, while teachers may initially pose and guide children's investigations (Baroody & Coslick, 1998), children should ultimately develop and implement their own solution plans (Brahier, Kelly, & Swihart, 1999), and pose investigations (Rowan & Bourne, 1994).

A key consideration for facilitating learning is teachers' pedagogical content knowledge, which includes an understanding of students' difficulties (Carpenter, Fennema, & Franke, 1996). In this paper, we report on some of the difficulties that confronted young children when they began a program of investigations. This is part of a larger project exploring how young children in the early years of primary school engage in mathematical investigations.

Design and Methods

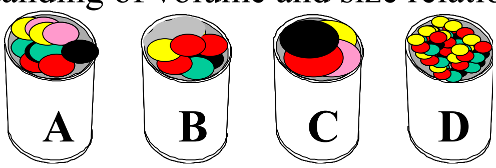
The research adopts an exploratory case study design (Yin, 1994) in which a teaching experiment was conducted with the goal of supporting the development of investigatory abilities in young children. This study was implemented in class in which one of the researchers (CMD) assumed the role of the teacher while the other researchers provided feedback as a non-participant observer (JJW) and "critical friend" (LDE). Twenty-seven seven to eight-year-old students were selected for the investigations program on the basis of their interest and strength in mathematics from four class groups within the same school. Students worked as a "class group" and received 90 minutes weekly of investigatory activities over a 14-week period.

This paper reports on the initial five-week phase of the program, which was implemented in the early part of the school year. In this phase, students worked on a series of mathematical investigations involving Smarties[1] (Table 1). The first three investigations were teacher-initiated, although questions posed by students during these investigations were followed up. The fourth investigation was a student-initiated task, which the students undertook with a partner. These investigations are described in detail elsewhere (Diezmann, Watters, & English, 2001).

Table 1. Overview of the Smartie Investigations

Investigation 1 (I-1): How many Smarties in the can?
Students were asked to investigate the numerical contents of small, white, translucent, sealed (film) canisters that had been filled with Smarties. Pairs of students were provided with a few Smarties, an empty can and a filled, sealed can. Students had access to a range of common tools, such as kitchen scales, balance scales, rulers, calculators, and magnifying glasses.

Investigation 2 (I-2): Smartie Cans
Students were asked to explore and predict the numerical contents of a series of Smartie Cans that varied in fullness and contained different sizes of Smarties. This task was designed to develop students' skills of observing, predicting, collecting and analysing data, and reasoning. Additionally, this task provided a rich environment for developing an understanding of volume and size relationships.



Investigation 3 (I-3): Distribution of Smartie Colours
The students were each given a small packet of Smarties to explore the distribution of colours. This involved representing the number of each colour Smartie on a table and a graph, answering questions about these representations, and comparing their results with other students.

Investigation 4 (I-4): Independent Smartie Investigation
The students were given support to identify investigable questions about Smarties. (E.g., What is the most popular coloured Smartie?). Their findings were presented as pages for a class book about Smartie facts. Students had access to various common-place resource materials.

The case study database comprised video and audio records, classroom artefacts, the teacher's lesson plans and reflections, and notes by the research team. Four video cameras recorded events during each lesson supplemented by audio taping of selected

[1] "Smarties" are sweets similar to "M & M's" and "Beanies".

individual group interactions. A research assistant made focussed observations that involved an ongoing record of the interactions of particular children. The data also included children's written work. After each lesson, the video-tapes were reviewed and salient events discussed among the researchers. These discussions permitted the team to analyse behaviours, develop conjectures, and plan strategies. Summaries of discussions were compiled as a diary containing descriptions of events, hypotheses, and reflections about teaching and learning. For this paper, the data were analysed to identify the range of difficulties encountered by students when they were introduced to investigations. Only examples of these difficulties are presented due to space limitations.

Selected Results

In this initial phase of five weeks, the students encountered two types of difficulties. These were Investigation-Specific Difficulties and General Difficulties that impact on other school work in which students engage apart from investigations.

Investigation-Specific Difficulties

Students experienced a range of difficulties when engaged in teacher-initiated and self-initiated investigations. Four examples of these difficulties follow.

1. A lack of understanding of the problem under investigation. For example, in I-4 (Investigation 4) Melissa was asked to explain how to investigate the most popular colour Smartie. Her response suggests that she interpreted the term "popular" to mean the most frequently occurring item instead of a consumer preference. Other students interpreted the term similarly.

> I think you would open all the Smartie jars you had and then, and then put the colours into groups say, purple, yellow pink and different colours and when you are finished putting them into groups well you count them up and (find) … the colour that has the highest number.

2. Failure to link the findings of an investigation to the answer to the problem. For example, in I-1 students used a variety of tools, including rulers, scales, calculators, and magnifying glasses to investigate "How many Smarties in the Smartie can?" Though they generally used these tools proficiently, most students did not use their measurements in producing their answers. Catherine's response is typical of students' responses: "We used the ruler to measure the Smartie Can to see how many Smarties there were." A couple of students, for example, Caroline, provided further information: "We used the ruler for the height. The height was four and a half centimetres". Robert was the only student to explicitly link his measurement to his answer. He and his partner weighed a can containing four Smarties and the full can. They then determined how many partially-filled cans were equivalent in mass to the full can. Finally, they multiplied the result of their calculation by four, as there were four Smarties in the partially-filled can. However, they failed to realise that the mass of the partially-filled can should have only been included once in their calculations. They never attempted to weigh an empty can.

We used the scales to measure the can with four Smarties to see how many it weighed to help us find out the answer to the problem…Well, if we weighed the "four can" then we could multiply the four can on the calculator.

3. Difficulty posing a problem to investigate. For example, in I-4 Jason was unable to identify a problem to investigate. He wrote, "Different Smarties go down the slide" (Figure 1). The Smartie slide was a cardboard construction that was used for measuring the speed of Smarties as they travelled down the slide. However, while some students were unable to spontaneously pose their own problems in Phase 1 of the program, other students, such as Tim, clearly articulated a problem: "How long does it take different types of Smarties to go down the Smartie slide?

4. A lack of prerequisite mathematical knowledge to complete an investigation of interest. For example, in I-4 Robert's initial problem was "What is the chance of the first Smartie out of the box being your favourite colour?" Although Robert recorded the outcomes of his trials and identified the colours that were drawn the most and least times, he was unable to proceed further with his exploration of "chance". He chose not to present this relatively sophisticated investigation to the class, but presented a simpler investigation designed by his partner. Robert's sense of self-efficacy might have been diminished because he was unable to complete the initial investigation to his satisfaction. This is an instance of where an investigation requires more advanced mathematics and intervention by the teacher.

It is not surprising that students experienced difficulties with teacher-initiated or self-initiated investigations, as they were novices. The teacher provided students with support to overcome investigation-specific difficulties and addressed these difficulties in subsequent lessons. None of these difficulties was considered sufficiently insurmountable to obviate the benefits of an investigations program.

Figure 1. Slide.

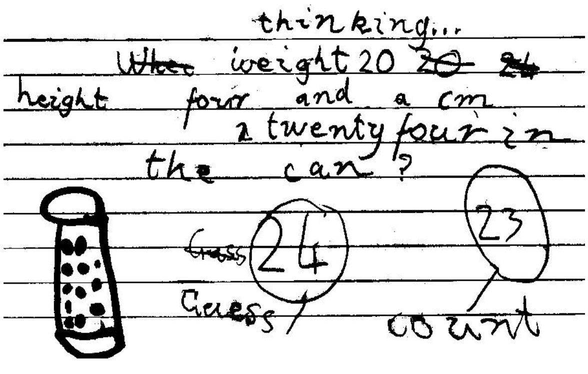


Figure 2. Inappropriate calculation.

Figure 3. Using scales

General Difficulties

In addition to investigation-specific difficulties, students experienced various general difficulties that hindered their investigations. Six of these difficulties are outlined here. The first three difficulties relate to mathematics and the latter three difficulties relate to communication and representation.

1. A failure to detect critical differences. For example, in I-2 students were unable to explain the discrepancy between their prediction and the actual count for the partially filled Can B. After prompting to compare the full Can A with Can B, Robert picked up the cans and explained, "Well this one here (Can B) it's not as full as this one (Can A)".

2. A lack of an understanding of what can be added. For example, in I-1 Leanne calculated the number of Smarties in her can to be 24 by combining the can's mass of "20" with the can's height of " four" (Figure 2).

3. Difficulty in identifying how to use a tool for a particular purpose. For example, in I-4, Leanne and Libby encountered difficulty in trying to weigh a single giant Smartie on kitchen scales and balance scales. This difficulty was overcome by prompting the students to weigh more than one giant Smartie (Figure 3).

4. Difficulty conveying ideas clearly orally or in writing or in a drawing. Throughout Phase 1, students were frequently asked to clarify and elaborate on their oral and written responses. Additionally, drawing did not appear to be a regular feature of their mathematical thinking or communication. Even when students were instructed to include drawings in their reports, some students failed to complete a drawing or their drawing lacked adequate detail.

5. Difficulty using common mathematical representations. For example, in I-3 many students needed considerable support to produce a simple table and bar graph.

6. A lack of understanding of the correspondence between objects and their symbolic and pictorial representations. For example, in I-3 some students had difficulty understanding that their count of a particular colour Smartie could be written on the table beside the corresponding colour and could also be represented on the bar graph.

These general difficulties highlight the range of knowledge or skills that students utilise in undertaking investigations. Hence, teachers may need to provide individualized and differential support to address particular difficulties that hinder students' investigatory work. While problematic, these difficulties provide invaluable opportunities for learning within a meaningful context.

Conclusion

The results indicate that young students are capable of planning and implementing investigations but they encounter a range of difficulties in the process. Knowledge of specific difficulties experienced by students enables the teacher to structure an investigations program to pre-empt and address likely difficulties, and provide students with opportunities for success on challenging tasks. Knowledge of the general difficulties that impact on students' capacity to engage effectively in investigations assists the teacher to determine the preparedness of particular students for investigatory work, and the type of support they may require to successfully engage in investigations. General difficulties experienced by students also provide teachers with an insight into the students' capacity to apply previously learnt knowledge or skills within a new and challenging context.

Engaging young students in investigations requires that teachers reconsider their understanding of the nature of mathematics and how mathematics is learnt. Mathematical investigations are one of the few classroom mathematics activities in the early years that require high-level thinking and task commitment. However, investigations provide students with the satisfaction of successfully completing a challenging task and being able to identify and investigate their own problems. Hence, the time and effort invested by teachers in planning and supporting children's investigatory work can yield worthwhile cognitive and motivational dividends.

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