CHAPTER 2: UNDERSTANDING –THE UNDERLYING GOAL OF TEACHER EDUCATION

Christoph Selter, Pedagogical University Heidelberg, Germany

Teachers' background knowledge is surely a precondition for their professionalism. However, they actually become professionals *while* they are teaching (Bromme 1994; Thompson 1992; Cooney & Krainer 1996). Consequently, the present paper sketches how (prospective) teachers should be encouraged to build up *background knowledge* as well as to develop *awareness* regarding (1) their own mathematical activity (*mathematical* component), (2) children's thinking and learning (*psychological* component), (3) subject-matter-specific didactics (*didactical* component) and (4) teaching practice (*practical* component). Activities from the context of one substantial learning environment are used as representative examples to illustrate this conception.

2.1 Understanding mathematics

An important goal of mathematical components in elementary teacher education is to contribute to breaking a vicious circle. Many (prospective) teachers do not feel confident with mathematics due to their own prior negative learning experiences. Thus, they are likely to perpetuate their limited understanding to their own students. In this context, (prospective) teachers' encounters with mathematics courses play a crucial role, as they offer opportunities to encourage them to develop a lively relation to the activity of *doing mathematics*.

Number chains - challenging mathematical acitivities

The context 'number chains' shall serve as a representative example (Price, Hoskin & May 1991, 12; Selter & Scherer 1996): Choose two start numbers and write them down beside each other; then note their sum as the third number. Now add the second and third number and write down this sum. Finally, put down the sum of the third and fourth numbers (the so-called target number), e.g., 1, 4, 5, 9, 14 or 66, 23, 89, 112, 201. One activity for third graders consists of finding two (non-negative integer) start numbers that lead to the target number 50. Besides this problem, there are many different ones up to university level, such as: (1) What happens, if you take a different target number? (2) If you work on number chains of a different length (4, 6, 7, …, n numbers)? Use examples first! (3) Is there a relationship between different target numbers and the number of solution pairs? (4) Is there a solution pair for each target number? (5) If not, is there a largest one that cannot be reached? (6) Is there a relation between the length of the chain, the target number and the number of solutions?

Doing mathematics (sensu Freudenthal)

Number chains are a typical example within the conception developed in the 'mathe 2000' project. In this context, one of its arch fathers, Hans Freudenthal, should be quoted who criticized what he called anti-didactic inversion: taking the ready-made system as the starting point of the teaching-learning process. In his terms, learning is not to be understood as duplicating, but as guided reinvention (Freudenthal 1991, 48). Children should learn *mathematizing* instead of consuming the finished product 'mathematics'. Thus, he developed his didactical phenomenology (*Freudenthal* 1983):

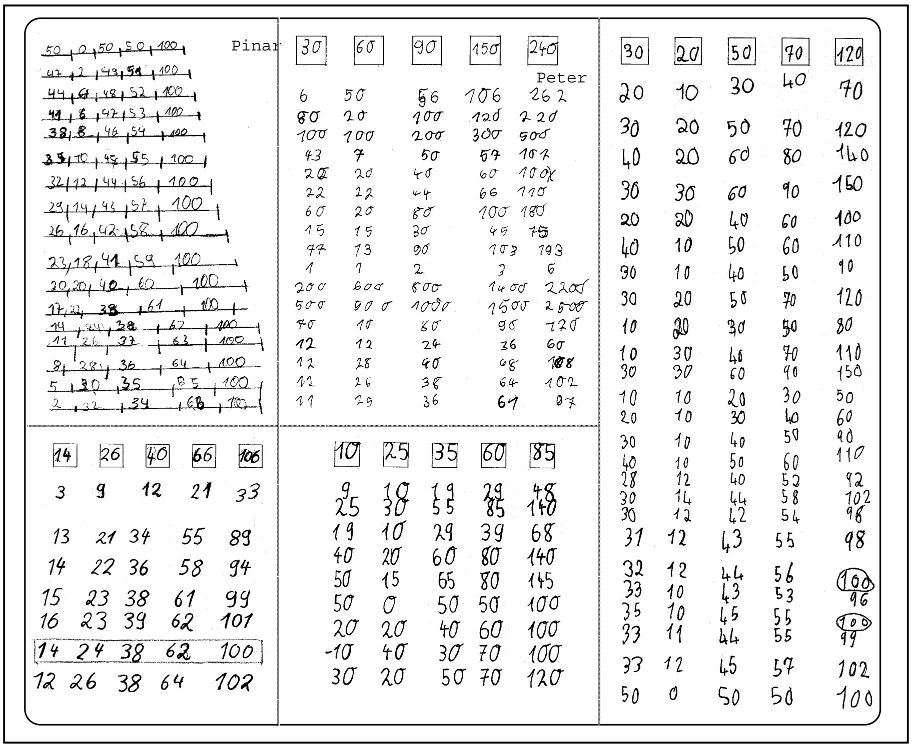
starting with phenomena that are meaningful for learners and thus provoke learning processes. Consequently, the mathematical components of teacher education courses should deal with an informal, process- and problem-oriented mathematics (Müller 1997; Wittmann, in press; Müller et al. 2001). In addition, the courses need to be organized in a way that provides (prospective) teachers with learning experiences that they will want *their students* to experience. The culture of teacher education should be similar to the favorable culture of teaching practice (Wittmann 1989).

2.2 Understanding children

With respect to the *psychological* component one is confronted with another vicious circle. Quite a couple of (prospective) teachers have learned as students, that it was of crucial importance to understand the ways their own teacher thought, whereas their own thinking often was regarded as not really important. They are likely to adopt this point of view for their own teaching as well. Thus, (prospective) teachers should learn about children's ways of thinking and learning.

Solution strategies for number chains

To illustrate this, the 'number chains' are revisited (Selter & Spiegel 1997, 68). Third graders were asked to find all number chains with target 100 (see besides). (1) How do/did you solve the problem yourself? (2) In how far did they work systematically? (3) Which similarities and differences do you notice between their ways of working and your own approach?



Looking through children's eyes (sensu Piaget)

Activities of this kind were designed for (prospective) teachers in order to learn to look through children's eyes, a crucial idea inseparably connected with Jean Piaget's work (1972). An extensive number of his writings has convincingly shown that even very small children interact with their environment and actively construct their own knowledge by modifying already existing schemes. Piaget was among the first who scientifically proved that children have their own ways of thinking, that make sense from their perspective, although it does not seem to be so at first sight.

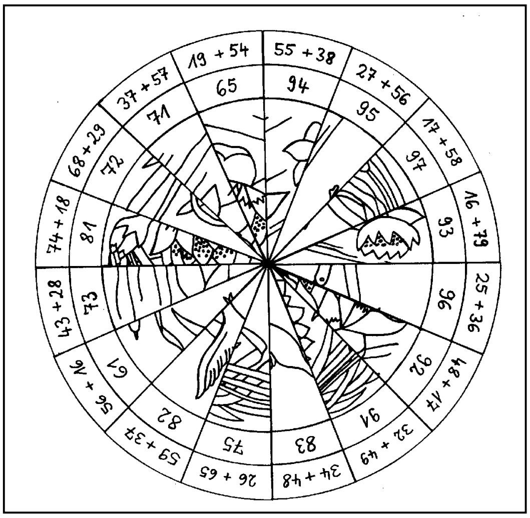
Reflecting on children's documents by analyzing documents like videos, transcripts or own productions can sensitize to their perspectives. (Prospective) Teachers can learn to understand that children often think differently from (1) how we think, (2) what we assume how they think, (3) how other children think and (4) how they think in different situations dealing with basically the same (Selter & Spiegel 1997).

2.3 Understanding didactics

A third vicious circle comes to the fore with respect to the didactical component of teacher education. The (prospective) teachers' own learning experiences shape their beliefs regarding what it means to teach and to learn. Formerly the teacher gave them the relevant information that had to be learned in order to pass the next test. Consequently, nowadays the mathematics educator is expected to tell them the subject matter needed to get good marks in the exams. Although (prospective) teachers often have a critical scepticism with respect to didactical ideas, they often do not make their doubts public. As they understand the mathematics educators' remarks as academic truth, they often accept it as part of their teacher education knowledge to be learned, without sufficiently making it part of their own thinking. Thus, a fruitful discussion should be encouraged instead in which arguments are exchanged and teachers as well as teacher educators can learn.

Number chains and arithmetic domino – two types of practicing

An example is given in the following. Work on the two tasks and analyze them according to following questions: (1) What are the goals of each of them? (2) Which are the advantages of each of both? (3) Which difficulties do you expect in teaching practice? (4) Which variations are possible in order to cope with different abilities of students? (5) Which task would you prefer, if addition in the domain up to 100 has to be practiced in your class? Give arguments for your decision!



Arithmetic domino: Cut out the domino cards. Take any card and work out the respective problem. Lay down the card containing the correct result next to it, work out its problem, and so forth. If you do not find a card that fits, you have to work out the problem again.

Number chains: Write down two start numbers one beside the other, add them and put down their sum as third number. Finally, write down the sum of the second and the third number, e.g., 36, 23, 59, 82. Add 1, 2, 3, … to the *first* (the *second, both)* start number(s). What happens to the other numbers? Explain!

Taking didactical theories as a basis for own decisions (sensu Kühnel)

Activities of this kind were designed to stimulate (prospective) teachers to critically relate knowledge input from different sources to their own theories. In this context, it is worthwhile to quote the German pedagogue and mathematics educator Johannes Kühnel (1925, 137) who postulated "Not *guidance* and *receptivity*, but *organization* and *activity!"* as central principles for teaching as well as for teacher education. His goal was to educate active and reflective personalities who do not just blindly copy didactical decisions others have made, but who are able to develop their own or modify existing conceptions based on different didactical theories (Kühnel 1923, 88).

2.4 Understanding teaching

The fourth and last vicious circle with respect to teacher education to be dealt with in this section is the following one: As German (prospective) teachers normally do not have any opportunities to experience something different, they continue to more or less teach as they were taught. This often happened according to a traditional approach counter-productive to modern conceptions of teaching and learning. One example shall be given to illustrate, how a higher degree of awareness can be developed with respect to a so-called progressive conception of teaching.

Number chains - analysis of a teaching episode

A group of (prospective) teachers commonly observes a lesson on number chains that one of them is giving. In order to focus their perceptions as well as their interpretations they have developed several criteria in collaboration with the teacher educator, such as (1) Did the teacher lucidly explain the problem? (2) Are the children aware of what (s)he expects? (3) Were the numbers chosen in the explanation appropriate (or too small, too big, too similar)? (4) Did (s)he give enough time for the children's own work? (5) Did (s)he maintain an atmosphere that, in principle, enabled all students to think for themselves and to contribute to whole-class or small-group discussions? (6) Did the teacher have advice available for children who experienced difficulties? (7) Did (s)he provide challenging activities for children who solved the problem faster than others?

Learning to teach according to the laboratory conception (sensu Dewey)

The goal of activities like these ones is to focus the (prospective) teachers' observations. This conception relates to a paper by John Dewey (1904), in which he distinguishes two different approaches: On the one hand, practical components can provide teachers with necessary tools of their profession, like the technique of whole-class instruction. With this aim in view, practical work is of the nature of an *apprenticeship training*, the aim is to form the actual teacher. On the other hand, practical work can make (prospective) teachers reflective and attentive, by relating theoretical aspects to what they can observe within the classroom – the *laboratory conception*. Here the goal is to equip the teacher with the intellectual methods and with materials of good workmanship instead of creating the good workman on the spot. Dewey does not reduce these two points of view to an 'either-or'. According to him, the apprenticeship and the laboratory conception give the limiting terms within all practical work falls.

2.5 Coherence of components

The present paper describes important principles of teacher education developed within the project 'mathe 2000' (see Selter 1995; Wittmann, in press). As teaching and learning are complex phenomena, it is obvious that a coherent view integrating all four components is needed. In this context, Wittmann (1984) has elaborated how substantial learning environments – like number chains – can fulfill this multiple function and form the integrating core of teacher education.

2.6 References

Bromme, R. (1994): Beyond Subject Matter: A Psychological Topology of Teachers' Professional Knowledge. In R. Biehler et al. (eds.): *Didactics of Mathematics as a Scientific Discipline*: Dordrecht, 73-88.

Cooney, T. & K. Krainer (1996): Inservice Mathematics Teacher Education: The Importance of Listening. In A. Bishop et al. (eds.): *International Handbook of Mathematics Education*. Dordrecht, 1155-1185.

Dewey, J. (1904): The Relation of Theory to Practice in Education (Part I). In *National Society for the Scientific Study of Education, Third Yearbook*. Bloomington, IL: Public School Publishing Company, 9-30.

Freudenthal, H. (1983): *Didactical Phenomenology of Mathematical Structures*. Dordrecht.

Freudenthal, H. (1991): *Revisiting Mathematics Education*. Dordrecht: Kluwer.

Kühnel, J. (1923): *Die Lehrerbildung auf der Hochschule* (Teacher Education at Universities). Dresden.

Kühnel, J. (1925): *Neubau des Rechenunterrichts, Bd. 1* (New Construction of Arithm. Teaching). Leipzig.

Müller, G. N. (1997): Mathematiklernen als konstruktiver, entdeckender Prozeß nicht nur in der Grundschule, sondern auch in der Lehrerbildung (Learning Mathematics as a Constructive Process). In P. Bardy (ed.): *Mathematische und mathematikdidaktische Ausbildung von Grundschullehrern*. Weinheim, 95-113.

Müller, G. N., H. Steinbring & E. Ch. Wittmann (in press; eds.): *Arithmetik als Prozess*. Leipzig.

Piaget, J. (1972): *Theorien und Methoden der modernen Erziehung* (Theories and Methods of Modern Education). Wien, München, Zürich.

Selter, C. (1995): Entwicklung von Bewußtheit – eine zentrale Aufgabe der Grundschullehrerbildung (Development of Awareness – A Central Goal of Elementary Teacher Education). *Journal für Mathematik-Didaktik* 16 (1/2), 115-144.

Selter, Ch. & P. Scherer (1996): Zahlenketten – ein Unterrichtsbeispiel für Schüler und für Lehrerstudenten (Number Chains – A Unit for Students and Prospective Teachers). *mathematica didactica* (1), 54-66.

Selter, Ch. & H. Spiegel (1997): *Wie Kinder rechnen* (How Children Do Arithmetic). Leipzig.

Thompson, A. (1992): Teachers' Beliefs and Conceptions: A Synthesis of the Research. In D. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York, 127-146.

Wittmann, E. Ch. (1984): Teaching Units as the Integrating Core of Mathematics Education. *Educational Studies in Mathematics* 15, 25-36.

Wittmann, E. Ch. (1989): The Mathematical Training of Teachers from the Point of View of Education. *Journal für Mathematik-Didaktik* 10 (4), 291-308.

Wittmann, E. Ch. (in press): The Alpha and Omega of Teacher Education: Organizing Mathematical Activities. In D. Holton (ed.): *Teaching Mathematics at the University Level. An ICMI Study*.