INTERACTIVE LEARNING AND MATHEMATICAL LEVEL RAISING: A MULTIPLE ANALYSIS OF LEARNING EVENTS

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Abstract

From different perspectives we have analysed an episode of two children working together on a mathematical task. Integration of our analyses brings to the fore authentic dilemmas and paradoxes, which are also experienced by students and teachers during collaborative work. We will present some examples.

Introduction

In the past, analyses of student learning have considered learning from cognitive, social or motivational aspects in isolation. Now in contrast to this earlier view, an interest exists in developing a "multidimensional framework for understanding mathematical learning" (Pintrich, 2000, p. 221). This is an approach that we have taken in collaborating in analyses of an episode of two children working together on a mathematical task.[i] We will first describe the different perspectives we use in research and then explain our recent work--a multiple analysis of learning events (Dekker, Elshout-Mohr, & Wood, in press). Although we differ in our perspectives, a common goal for research is to develop ways to describe events in students learning as they are occurring in classroom situations. In this case learning is defined as conceptual understanding and mathematical level raising.

THEORETICAL ORIENTATIONS

A Process Model for Mathematical Level Raising

Dekker and Elshout-Mohr (1996, 1998) developed a process model for interaction and mathematical level raising based on empirical analysis of students learning working in small groups while solving mathematical problems (Dekker, 1991). The problems are specifically developed to stimulate mathematical level raising among students (cf., Dekker & Elshout-Mohr, 1999). In the process model three types of activities are incorporated; they are *key activities* in the learning process; *regulating activities*; and *mental activities.* Each of these is further discussed below.

Key Activities

Dekker and Elshout-Mohr (1996, 1998) deem four key activities are primary conceptual level raising. Whenever these key activities occur, we speak of learning events. The key activities are:

- *to show one's work*
- *to explain one's work*
- *to justify one's work*
- *to reconstruct one's work*

The process model is presented in Table 1 with the key activities indicated in bold print. Dekker and Elshout-Mohr (1996, 1998) assume that the conjunction of these activities leads to the desired type of learning (i.e. conceptual level raising). The four key activities have the following characteristics:

a) They can be demonstrated by students who work individually, but more so by students who communicate with each other during their work;

b) They are easily observed;

c) They have a function in the learning process and contribute to mathematical understanding;

d) They can be influenced by didactic factors, such as the nature of problems and by teaching actions.

Regulating Activities

The process model further captures regulating activities, which elicit key activities, and mental activities, which concur with key activities. Table 1 shows the regulating activities identified in italics. Students asking for explanations, justifications, or transformations are described as regulating activities. For instance, when students are working together and show each other what they are doing and thinking, they may become aware that different participants have different knowledge about a central concept, and they may start thinking about these differences. They may also ask each other to show and explain their work, or to justify or transform it. In principle, a student who works alone can perform all the key activities, but it takes a great deal of self-regulation. However, by communicating with other students, the key activities will take place in a more natural way. Therefore, the process model also contains interactive and communicative activities that we call regulating activities.

Mental Activities

While key activities and regulating activities can be observed externally, mental activities occur which are not observable. Mental activities involve those activities which 'go along' with key activities. For instance, *to show one's work* includes the mental activity of becoming aware of one's own work. It has the effect that a focus on task-progress is temporarily replaced by taking a look at the work from the outside. *To explain one's work* means that one has to think about one's own work. It leads to elaboration of one's task-related conceptual knowledge. Attempts *to justify one's work* may include reinforcing prior knowledge or questioning it, and a prerequisite *to reconstruct one's work* is to criticise one's own work. Table 1 shows the process model of these mental activities in standard print.

Table 1

A Process Model for Interactive learning and Mathematical Level Raising

A and B are working on the same mathematical problem. Their work is different.		
A is working		B is working
A asks B to show his work	*what are you doing?* *what have you got?*	*B asks A to show her work*
A becomes aware of her own work		B becomes aware of his own work
A shows her own work	**I am doing this...** **I have got this...**	**B shows his own work**
A becomes aware of B's work		B becomes aware of A's work
A asks B to explain his work	*why are you doing that?* *how did you get that?*	*B asks A to explain her work*
A thinks about her own work		B thinks about his own work
A explains her own work	**I'm doing this, because...** **I have got this, because...**	**B explains his own work**
A thinks about B's work		B thinks about A's work
A criticises B's work	*but that's wrong, because...*	*B criticises A's work*
A thinks about B's criticism		B thinks about A's criticism
A justifies her own work	**I thought it was right, because...**	**B justifies his own work**
A thinks about her justification		B thinks about his justification
A criticises her own work	oh no, it isn't right, because...	B criticises his own work
A reconstructs her own work	**I'll better do it like this...**	**B reconstructs his own work**

Note: **Bold: key activities**
Standard: mental activities
Italic: regulating activities

Social Interaction and Learning Mathematics

The theoretical perspective taken by Wood (1996, 1999) in analysis of student learning is to consider the very social nature of children's learning and the fact that rich social interactions with others substantially contribute to children's opportunities for learning. Therefore, Wood claims that there is a need to consider an analysis of educational settings that attends to the social cognitive processes involved in learning as well as the cognitive processes (Wood & Turner-Vorbeck, in press).

This perspective is influenced by the view of Bruner (1990) and others that children need to adapt to a social existence and to develop a system of shared meanings in order to participate as members of their culture. Sociologists interested in the human need to adapt to social existence and develop a system of shared meanings provide insights into the importance of social structure in the lives of humans. Both Garfinkel (1967) and Goffman (1959) contend that the social structures in everyday life consist of normative patterns of interaction and discourse. Once established, these patterns become the reliable routines found in interactive situations. Individuals, when they participate, come to anticipate certain behaviours for themselves and for others so that much of what happens "goes without saying" (Garfinkel, 1967).

Several researchers argue that the social structures that are created in the classroom influence the 'mathematics' a child learns. They claim that the everyday patterns of interaction and the norms that are constituted contribute to children's beliefs about the nature of mathematical knowledge and the ways in which one learns and uses mathematics in everyday life (e.g., Carraher, Carraher, & Schliemann, 1985; Cobb, Wood, Yackel, and McNeal, 1992). In order to understand the children's mathematical learning we need to examine the social situations that teachers establish with their students. The norms (expectations for self and others behaviour) underlie the social interaction that reveals the 'practice' of mathematics in the classroom.

Moreover, it is thought that learning that is conceptual can benefit considerably from dialogue and collaboration with others (Yackel, Cobb, & Wood, 1991). Although social norms are initiated and established by the classroom teacher, the children's ability and commitment to adhere to the shared expectations is equally important. Kieran and Dreyfus (1998) provide evidence that the social cognitive facility of negotiation of meaning influences individual learning.

Using a qualitative research paradigm, drawing on microenthnographic procedures developed by Voigt (1990), analysis of learning situations is conducted using a line-by-line examination of the dialogue and interaction. This provides detailed description of the events that occur. Through this process interpretation can be made of the meanings held by students during collaborative problem solving classroom situations.

DATA SOURCE AND ANALYSIS

The multiple analysis is best explained in contrast to the approach that we followed to construct the above-mentioned process model. In the process model, we incorporate three types of activities: key activities in the learning process, regulating activities, and mental activities. The key activities are given the central place in the process model, whereas other activities are merely presented in so far as they are directly connected to the key activities. Thus, employment of the process model produced a coherent description of learning events in terms of the elements of the process model. In addition, a focus on social interaction provides information on the social conditions for learning including the influence of the teacher. The aim of the analysis is to reveal how the social norms established in the class affect the collaborative work of students and how these expectations actually provide the space for learning.

Towards Multiple Analyses and Integration

In the multiple analysis approach, however, we did not give priority to the elements of the process model or social interaction. Instead, we began by performing three separate analyses on one protocol of a student collaborative session wherein two primary-aged (8 year old) students worked together to solve a mathematical task.[ii] The mathematical task was developed to encourage students' conceptual understanding of multiplication beyond their intuitive notions of multiplication as repeated addition.

The first analysis of the episode was guided by theory about the role of key activities in achieving mathematical level raising. The second was guided by theory about social cognitive processes and the role of social interaction in learning, and the third was guided by theory about the role of time on task in learning outcomes. Once completed, the results of the three analyses were integrated in ways that allowed each perspective to be represented.

In the integrative stage, which we are currently in the process of conducting, preliminary findings reveal the complexity of multiple effects on students' learning activities. For instance, an activity that is evaluated positively from a social perspective on learning mathematics does not necessarily contribute to the occurrence of key activities, nor is it necessarily evaluated positively from a time on task perspective on learning. In the presentation, examples will be given that show how the methodology of multiple analyses brings to the fore authentic dilemmas and paradoxes that are also experienced by students and teachers during collaborative work in classrooms. One example is the fact that the social norm of collaboration leads to a lowering of level of one of the students, which conflicts with the aim of level raising. We will discuss this example in detail.

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[i] It was in 1988 at ICME 6 in Budapest where we first met. Dekker (1988) gave a presentation about her classroom observations. Inspired by the ideas of Freudenthal (1978) about learning of mathematics in small heterogeneous groups

and the Van Hiele's (1986) level theory, Dekker showed learning materials specifically developed for mathematical level raising and observations of small groups working with those materials. Wood was in the audience and expressed similar research interests. It was nine years later at the CIEAEM 49 in Setúbal where we met again, both giving plenary lectures on interactions in the mathematics classroom. Wood's focus was on the role of the teacher and the influence of the social norms on the learning opportunities for students (Wood, 1998). Together with Elshout-Mohr and Pijls, Dekker presented a process model for the analysis of interaction and mathematical level raising (Dekker, Elshout-Mohr, & Pijls, 1998). We again discussed our common interest albeit with different perspectives and decided to conduct a joint analysis of a classroom event.

[ii] The episode is from the project, Recreating Teaching Mathematics in the Elementary School, supported by the National Science Foundation under award RED 9254939. All opinions expressed are those of the authors.

References

Bruner, J. (1990). *Acts of meaning*. Cambridge, MA: Harvard University Press.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology, 3*, 21-29.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29*, 573-604.

Dekker, R. (1988). *Classroom observations, a connection between theory and practice.* Paper presented at the Sixth International Congress on Mathematical Education (ICME), Budapest, Hungary.

Dekker, R. (1991). *Wiskunde leren in kleine heterogene groepen* [Learning mathematics in small heterogeneous groups]. De Lier: Academisch Boeken Centrum.

Dekker, R., & Elshout-Mohr, M. (1996). Zelfstandig leren doe je niet alleen [Self-regulated learning is not a solitary thing]. Tijdschrift voor nascholing en onderzoek van het reken-wiskundeonderwijs 15, 20-27.

Dekker, R., & Elshout-Mohr, M. (1998). A process model for interaction and mathematical level raising. *Educational Studies in Mathematics 36*, 303-314.

Dekker, R., Elshout-Mohr, M., & Pijls, M. H. J. (1998). Interaction, self-regulated learning and the process of level raising. In P. Abrantes, J. Porfírio & M. Baía (Eds.), *The interactions in the mathematics classroom - Proceedings of CIEAEM 49* (pp. 27-34). Setúbal, Portugal: Escola Superior de Educação de Setúbal.

Dekker, R., Elshout-Mohr, M., & Wood, T. (in press). Working together on assignments: Multiple analysis of learning events. In J. Van der Linden & P. Renshaw (Eds.), *Dialogic learning.* Dordrecht: Kluwer Academic Publishers.

Elshout-Mohr, M, & Dekker, R. (1999). Opdrachten om samen van te leren [Assignments to learn from in collaboration]. In J. Van der Linden & E. Roelofs (Eds.), *Leren in dialoog* (pp. 37-67). Groningen: Wolters-Noordhoff.

Freudenthal, H. (1978). *Weeding and sowing*. Dordrecht: Reidel.

Garfinkel, H. (1967). *Studies in ethnomethodolgy*. Englewood Cliffs, NJ: Prentice-Hall.

Goffman, E. (1959). *The presentation of self in everyday life.* Garden City, NY: Anchor Books.

Kieran, C. & Dreyfus, T. (1998). Collaboration versus individual problem solving: Entering another's universe of thought. In A. Olivier & K. Newstead (Eds.), *Proceedings of the Twenty-second Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 112-119). Stellenbosch, South Africa: Psychology of Mathematics Education.

Pintrich, P. (2000). Educational psychology at the millennium: A look back and a look forward. *Educational Psychologist,35*, 221-226.

Van Hiele, P. M. (1986). *Structure and insight*. Orlando, FL: Academic Press, Inc.

Voigt, J. (1990). The microethnographical investigation of interactive constitution of mathematical meaning. In *Proceedings of the 2nd Bratislava International Symposium of Mathematics Education* (pp. 120-143). Bratislava: Bratislava University Press.

Wood, T. (1998). Creating classroom interactions for mathematical reasoning: Beyond “natural teaching”. In P. Abrantes, J. Porfírio & M. Baía (Eds.), *The interactions in the mathematics classroom - Proceedings of CIEAEM 49* (pp. 34-43). Setúbal, Portugal: Escola Superior de Educação de Setúbal.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education, 30,* 171-191.

Wood, T., & Turner-Vorbeck, T. (in press). Extending the conception of mathematics teaching. In T. Wood, B. S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.

Yackel, E., Cobb, P. & Wood, T. (1991). Small-group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education 22*, 390-408.