ATTITUDE TOWARD MATHEMATICS: SOME THEORETICAL ISSUES

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Abstract

Research on affect has produced many meaningful results in the context of mathematics education. Nevertheless, the theoretical framework needs further development, in order to grant effective tools for observing, interpreting, and possibly modifying students' decisions in the context of mathematics activity. In particular the construct of attitude toward mathematics appears to be an ambiguous one. After a brief survey of some theoretical issues that are still open, we propose further questions involving the definition itself of attitude.

Attitude toward mathematics: an ambiguous construct

The attitude construct finds its origin in the context of social psychology (Allport, 1935), in connection with the question of foreseeing individual's choices in contexts like voting, buying goods, etc. The construct appears ambiguous from the beginning, but the research develops more toward the formulation of measuring instruments than toward the theoretical definition of the construct. Furthermore in these times the quantification is considered a warrant of the discipline's scientific nature. The instruments that have been produced have given theoretical and methodological contributes of great importance (such as those of Thurstone and Likert), but the attitude's "measurement" soon found itself facing the problem of identifying the possible variables.

In the field of mathematics education the construct gains renewed popularity with the reevaluation of affect in the learning of mathematics. This reevaluation, according to us, has two reasons, which are both very important:

-The needs to explain the failure in the problem solving context of individuals who possess the necessary cognitive resources. The studies in this field underline the role of metacognitive abilities (Schoenfeld, 1985; Zan, 2000) but also the influence of the emotional factors on the control processes (Borkosky, Carr, Rellinger & Pressley, 1990; De Bellis & Goldin, 1999).

-The mathematical activity itself, as described by important mathematicians, such as Hardy, Hadamard, Poincarè, is marked out by a strong interaction between cognitive and emotional aspects.

In the field of mathematics education there is a general agreement in seeing the affective domain as divided into emotions, attitudes and beliefs (McLeod, 1992). The

agreement is not as unanimous in the definition of these constructs (Hart, 1989), and the studies on attitude, in mathematics education also, privilege measurement problems more than definition problems. With the development of research in this field, and especially with the increase of awareness of the affect's role in mathematics learning, the necessity of a theoretical framework has grown too. Attitude research in mathematics education has been criticized in several respects:

1. The construct of attitude appears to be vague and ambiguous.

Moreover mathematics educators often do not clearly describe the definition used in their own research (Hart, 1989).

2. The first attempts to measure attitudes seem "exceptionally primitive" (Leder, 1987), and "the driving force in much of this research seems to be the statistical methodology rather than the theory" (McLeod, 1987, p. 134). Kulm (1980) points out that: "The measurement of mathematics attitudes in the future should make use of many approaches, and researchers should not believe that scales with proper names attached to them are the only acceptable way to measure attitudes" (p. 365).

According to Hart (1989): "Beliefs, attitudes and emotions have been examined via scores on paper-and-pencil instruments and occasionally via individual student interviews. This view of beliefs, attitudes, and emotions might be called a black-box approach (…) The time and effort required to collect and analyze the data obtained from the think-aloud interview are much greater than the time and effort required for the paper-and-pencil instrument, but the information gained is a richer reflection of the student." (pp. 43-44)

3. Despite the fact that research lately has shifted from quantitative methods toward a multidimensional approach, including qualitative methods, like interviews, classroom observations, and essays, several problems have not been solved yet. One of these problems is linked with the relationship between beliefs and attitudes. Silver (1985) points out that: "(…) we need to investigate the relationship between beliefs and attitudes. Are all attitudes also beliefs; if not, then how do we distinguish those that are from those that are not?" (p. 256)

Independently from the chosen definition of attitude (whether implicit or explicit), the attitude observation instruments always include beliefs observation instruments. Thus the attitude research has to deal with problems that the beliefs research has highlighted, especially the mismatch between exposed beliefs and beliefs-in-practice: the beliefs that students declare are in the end definitely different from those that guide their solving processes and their behavior in general. A way of interpreting this mismatch is to distinguish the beliefs toward school mathematics from those toward abstract mathematics (Schoenfeld, 1989). If the measurement of attitude includes the observation of beliefs, this beliefs differentiation implies a similar distinction of attitudes.

4. More in general, some researchers think that the term "attitude toward mathematics" should be divided in several components. Kulm (1980) delineates the

objects and situations on which attention is focused for mathematics attitudes: mathematics content, mathematics characteristics, teaching practices, mathematics classroom activities, and mathematics teacher. Tirosh (1993) suggests the opportunity to describe students' beliefs, attitudes, and emotions toward mathematics in terms of "specific affects", namely their reaction and feelings toward specific mathematical topics, specific tasks and specific activities.

In conclusion, the problem of 'measuring' attitudes is often faced without an appropriate theoretical framework (McLeod, 1992).

Further theoretical questions

A deep analysis of the construct 'attitude' requires first of all a definition of the term itself. In the variety of definitions of attitude toward mathematics used in the different studies, we can identify two important typologies:

1. A 'simple' definition of attitude describes it as the positive or negative degree of affect associated to a certain subject. According to this point of view the attitude toward mathematics is just a positive or negative emotional disposition toward mathematics (McLeod, 1992; Haladyna, Shaughnessy J. & Shaughnessy M., 1983).
2. A more 'articulated' one recognizes three components in the attitude: an emotional response, the beliefs regarding the subject, the behavior toward the subject. From this point of view an individual's attitude toward mathematics is defined in a more articulated way by the emotions that he/she associates to mathematics (which, however, have a positive or negative value), by the beliefs that the individual has regarding mathematics, and by how he/she behaves (Hart, 1989).

According to us the acceptance of either definition brings up several and distinct problems.

Remarks on the 'simple' definition

-Apparently the absence of a connection with the cognitive aspects could be criticized in this definition. As a matter of fact many studies, explicitly or implicitly based on this definition, follow models (Mandler, 1984, 1989; Ortony, Clore & Collins, 1988) which emphasize the cognitive source of emotions: "Mandler's view is that most affective factors arise out of the emotional responses to the interruptions of plans or planned behavior. In Mandler's term, plans arise from the activation of a schema. The schema produces an action sequence, and if the anticipated sequence of actions cannot be completed as planned, the blockage or discrepancy is followed by a physiological response (…) at the same time the arousal occurs, the individual attempts to evaluate the meaning of this unexpected or otherwise troublesome blockage. (…) The cognitive evaluation of the interruption provides the meaning to the arousal." (McLeod, 1992, page 578)

-Even those who accept this definition, when measuring attitudes use questionnaires based on beliefs. In this way it is implicitly assumed that certain beliefs imply a positive emotional disposition.

For example a widely used item is "mathematics is useful". This belief is considered positive, assuming implicitly that it gives place to a positive emotion. As a matter of fact this implication is not natural at all, as the opinions and reported feelings of important mathematicians demonstrate (see for example Hardy, 1940).

Furthermore we agree with Gal and Ginsburg's remark (1994), about the attitude toward statistics: "Thus, a student's responses to items assessing usefulness-of-statistics issues might have little to do with feelings or attitudes towards statistics as a subject; instead they may only reflect on the student's vocational development (…) or knowledge about requirements or content of certain jobs."

-Accepting this definition, it is quite clear that 'positive attitude' means 'positive' emotional disposition. It is thus important to question *why* a positive emotional attitude is meaningful in the context of mathematics education.

The goal of promoting a positive attitude may have two reasons:

i) The attitude's influences on an individual's choices about mathematics courses to take. As Hart says (1989): "It is relatively clear that decisions about how many and which mathematics courses to take in middle school, high school, and college can be influenced by affective characteristics of the student (…)." (p. 38)

However it seems to us that to promote a general positive emotional disposition toward mathematics is not very significant, if this disposition is not linked with an epistemologically correct view of the discipline. In other words an affective goal of mathematics education is to promote a "view of mathematics as vibrant, challenging, creative, interesting, and constructive" (Silver, 1987, p. 57).

ii) The idea that a positive attitude is connected to success. As a matter of fact this connection is far from being clear. McLeod (1992) refers data from the Second International Mathematics Study, that indicate that Japanese students had a greater dislike for mathematics than students in other countries, even though Japanese achievement was very high. Ma & Kishor (1997), after analyzing the correlation attitude / achievement in 113 studies, underline that this correlation is not statistically significant: they explain this as caused by the inappropriateness of the observing instruments that had been used. According to us, on the contrary, this limitation is a natural consequence of the 'simple' definition of attitude. In fact it is not enough that the mathematical experience is generally associated with positive feelings: it is also important for such an experience to be meaningful. For example a distinction is needed between a child that likes mathematics because of the calculation involved and another one that likes it because of problem solving. Furthermore also negative emotions play an important role in mathematical activity.

Mc Leod, Metzger and Craviotto (1989) found that experts and novices have the same emotional reactions in problem solving situations, but differ in that experts are better able to control their reactions than novices. In particular a minimum degree of anxiety seems to be necessary to invest adequate resources in the task.

Remarks on the 'articulated' definition

-In this case both the emotional and cognitive dimensions are explicitly underlined. But too often the *interaction* between these dimensions is not properly considered. In particular it is important to consider beliefs together with the emotions that they elicit. For example the belief "In mathematics there is always a reason for every rule" is to be considered differently whether it elicits a positive emotion ("and I like this") or a negative one ("and I don't like this").

Similarly it is essential to distinguish emotions according to their cognitive source (Ortony et al., 1988). This suggests the opportunity to differentiate 'simple' emotions (associated with an individual's tastes) and 'complex' ones (associated with an individual's beliefs).

-According to the 'articulated' definition the attitude construct is multidimensional, so it can not be quantified with a single score. The possible alternatives are:

(i) Give a score for each dimension (beliefs, emotions and behavior). This is close to the original idea that Thurstone & Chave (1929) suggested. They pointed out that even if attitude is a complex construct (that can not be measured with a single numerical index) it can be measured using several indices. They underline the fact that the same process is followed in measuring physical objects (like a table). We find interesting their remark that the choice of the characteristics to be measured depends on the context.

(ii) Give up 'measuring' attitude, and describe it qualitatively with the pattern beliefs / emotions / behavior and the connections between them (it may be interesting to notice that in an article written recently by Ruffell, Mason & Allen (1998) the term 'probing' is used instead of 'measuring').

Regarding the dimension "beliefs", in our opinion a questionable aspect is the observation of single beliefs rather than of belief systems. For example the belief "Only those who are intuitive are successful in mathematics" does not give significant information about the individual, unless we know the belief that the individual has about his own intuition. Cooney, Shealy & Arvold (1998) underline this aspect regarding research on teachers' beliefs: "It is important to understand not only what teachers believe but also how their beliefs are structured and held" (p. 306). Grigutsch and Törner (1998) tried to carry on a research similarly focused on this aspect, analyzing the beliefs of expert mathematicians.

-In this case what a 'positive' attitude should mean is not clear: but referring only to the emotional dimension seems lessening to us. We think that two other possibilities are preferable. The first is to find out whether a typical pattern beliefs / emotions /

behavior of students who are successful in mathematics exists, and to define this pattern as positive. The second one is to define as positive the attitude typical of experts, which obviously brings up the problem of the existence of such attitude (Grigutsch & Törner, 1998). This rises other questions: recognizing if these two possibilities lead to two different definitions of positive attitude, and, if that is the case, deciding which is more useful.

Conclusions

Research in mathematics education has already accepted the importance of affective factors to interpret behavior of individuals who are involved in problem solving in the mathematics context. The research in this field has produced a substantial amount of knowledge. But for affect to be a real powerful theoretical instrument, a further analysis of the affective constructs is necessary. In each discipline, after a first period when the introduction of new concepts plays an important role in building up the theory and in the production of results (without considering how clearly and rigorously these concepts are used), a second period follows when the researchers feel the necessity of clarifying the nature of the concepts that have been used. The analysis of such concepts may create 'monsters' (i.e. ideas that apparently undermine the knowledge produced up to that moment), but this crisis is extremely fruitful for the further development of the discipline. This is what happened with concepts like those of function or continuity in mathematics history: monsters created by Dirichlet, Cantor, Weierstrass (i.e. discontinuos functions at every point) have, as a matter of fact, opened up new horizons in the field of mathematics.There is still another reason for this analysis. Because of its nature, mathematics education deals with constructs, theories and methods that are taken from other disciplinary contexts, like psychology, sociology, anthropology, etc. (Sierpinska & Kilpatrick 1998). The problems that researchers have to face, nevertheless, are exactly what pushes them to use such theories and constructs, and these problems are specific of mathematics education. As regard to the construct ‘attitude’, the research in mathematics education deals with problems that are different from those typical of social psychology, where the construct originated. Mathematics educators are interested not only in foreseeing the choices of the students, but in observing, interpreting and possibly modifying their decisions in the context of mathematical activity. In our opinion this difference is important, and is another reason that forces us to a deep reflection, and in particular to re-think the meaning and the sense of the terms involved.

Our aim with this paper is to contribute to this type of reflection.

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