UTILIZE CAS TO CLOSE THE GAP BETWEEN TWO VIEWPOINTS ON PARABOLAS

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Our aim was to develop a unit for Grade 10 that deals with parabolas. We started by identifying, with the help of teachers, two main limitations of the traditional presentation of parabolas in the high school curriculum:

1. The difficulties students have in expressing the relationship between the parameter 'a' of the quadratic function $y = ax^2$ and the shape of its graph. Most of the students can determine when the parabola has a cup/hat shape. Students also say something about 'stretching/shrinking', or 'opening/closing' of parabolas, while the teachers are not sure which description is the right one. A common misconception is that 'a' is the 'slope' of the parabola (analogous to a line).

2. The gap between the two viewpoints on the parabola in the traditional curriculum: the algebraic view of the graph of a quadratic equation, and the analytic-geometry view of loci. This gap causes students to think that there are two different kinds of parabolas.

Our plan was to use the symbolic power and rich graphics of CAS to make connections between analyzing the shape of the parabola and viewing it as a locus of points. Naturally, we concentrated on the *Focus point* of the parabola. But, we did not find a constructivist approach that could direct students at this age (15-16). Consequently, we came with another idea - to develop the notion of "a special chord in a parabola" whose mid-point is the focal point. The chord, which passes through the Focus $F(0, \frac{1}{4a})$, has special properties. The length of this chord is $\frac{1}{|a|}$, which agrees visually with the fact that as *'a' increases*, the parabola 'shrinks'. When we calculate the distance between F and any point on the parabola $P(m, am^2)$, using our 'talented mathematical assistant', we get the expression $|P - F| = m^2 |a| + \dfrac{1}{4|a|}$. In addition, the slopes at the end-points of the chord are 1 or −1.
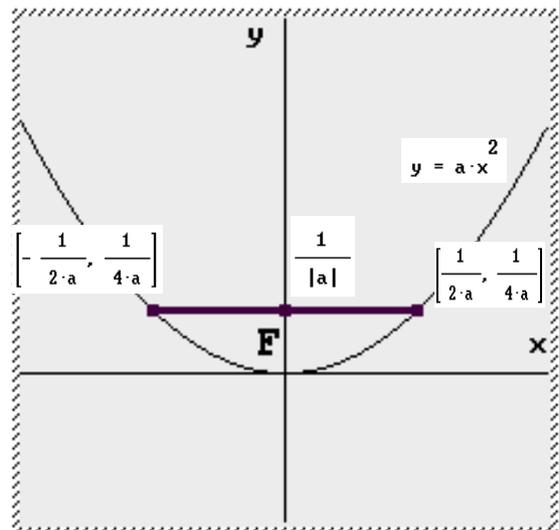
Our goal was then to develop a sequence of tasks that will help students to open a geometric window on the parabola. The poster will demonstrate critical stages in the formative development of the learning unit.