

Three Ways of Using the Arithmetic Mean

José Luis Cortina, Vanderbilt University

A problematic issue in research on understanding the arithmetic mean has been to develop a satisfactory conceptual and instructional definition. My research in the area of statistics education has brought me to distinguish between three different uses of the arithmetic mean in statistics, each of which requires different sophisticated understandings of statistical analysis. Hence, I believe each of these uses should be introduced at a different time in the curriculum.

1) Measure of the aggregate. The mean is used to measure the aggregate of a group when the total becomes inadequate because of differences in group size. The salient property of the mean in this usage is that of addressing the multiplicative relation between total accumulation in a group, and the number of units that generated that accumulation. This way of using the mean is common in science and in social sciences to construct normalized units of measure (e.g. kilometers per liter). My research shows that 12 and 13-year-old students with limited exposure to statistics, can make sense of this way of using the mean (cf. Cortina et al., 2001).

2) Measure of the center. The mean is used to characterize a distribution. Its salient property becomes the indication of the point of symmetry (or balance) of the deviations of values in a data set. The mean in this case is used in combination with other measures to characterize a distribution. To use the mean in this way, students need to at least understand "center" as a stable characteristic of a data set. Recent research suggests that even after 6 months of instruction, many 13 and 14-year-old students do not easily develop this understanding of center (cf. Cobb et al, in press). Hence, relatively long exposure to statistical instruction might be necessary in order for students to develop an understanding of the mean as a center.

3) Inferential tool. In this case the mean is used to infer, from a sample, characteristics of the aggregate and/or distribution of a population (i.e. central tendency). This use is based on the Law of Large Numbers, and requires relatively sophisticated understandings of sampling and probability. Hence, this use should be addressed in a rather advanced stage of statistical instruction.

Cobb, P., McClain, K. & Gravemeijer, K. (In press). Learning About Statistical Covariation. Curriculum and Instruction

Cortina, J. L., Cobb, P., & McClain K., (2001, April) Understanding Means and Ratios as Measures. Paper presented at the annual meeting of the AERA, Seattle, USA.