"POLDER MATHEMATICS"
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MATHEMATICS EDUCATION IN THE NETHERLANDS

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Sheltered behind its dikes, the Netherlands more or less escaped the New-Math wave that swept the world in the 1960's. Inspired by Freudenthal, the Netherlands developed its own brand of mathematics education, currently known as realistic mathematics education (RME). His adagio of "mathematics as a human activity" was worked out by members of the Freudenthal Institute and its predecessors, IOWO, and OW&OC. New ideas have taken shape in prototypes of instructional sequences that are construed in developmental research (or design research). Teacher enhancement materials, tests, and background publications accompany these prototypical sequences. These materials form a source for textbook authors, teacher trainers, and school councillors and test developers. Mediated by this group, the new ideas have found their way to the instructional practice in schools.
In our presentation we will try to give an overview of what is going on in mathematics education in Dutch schools. We will use the innovation of mathematics education as pursued by the Freudenthal Institute as a background. In doing so, we will start by elaborating Freudenthal's idea of mathematics as a human activity. Subsequently we will describe the Dutch school system and the curricular innovations that lead to the current curriculum. Next we will look at the use of context problems, followed by examples of the reinvention approach in Dutch textbooks. Next, will look at the role of technology, and we will close with a discussion of the Dutch approach to geometry.

Mathematics as a human activity
Freudenthal was an outspoken opponent of the 'new mathematics' of the 1960s that took its starting point in a one-side structuralistic interpretation of the attainments of modern mathematics, especially set theory. Since the applicability of mathematics was also often problematic, he concluded that mathematics had to be taught in order to be useful. He observed that this could not be accomplished by simply teaching a 'useful mathematics'; that would inevitably result in a kind of mathematics that was useful only in a limited set of contexts. However, he also rejected the alternative: 'If this means teaching pure mathematics and afterwards showing how to apply it, I'm afraid we shall be no better off. I think this is just the wrong order' (Freudenthal 1968: 5). Instead, mathematics should be taught as mathematizing, and this view of the task of school mathematics was not only motivated by its importance for usefulness. For Freudenthal mathematics was first and foremost an activity. As a research mathematician, doing mathematics

was more important to Freudenthal than mathematics as a ready-made product. In his view, the same should hold true for mathematics education: mathematics education was a process of doing mathematics that led to a result, mathematics-as-a-product. In traditional mathematics education the result of the mathematical activities of others was taken as a starting point for instruction. Freudenthal (1973) characterized this as an anti-didactical inversion. Things were upside down if one started by teaching the result of an activity rather than by teaching the activity itself.

History & school system

We will discuss some of the history of the many innovations in mathematics education in the Netherlands to offer some background for a sketch the mathematics curriculum in the Dutch schools. We will start out with the first projects of the IOWO, the Wiskobas project that aimed at primary school and Wiskivon that aimed at lower secondary. While changes in the primary-school curriculum were the result of a cumulative effect of a long-term process of gradual changes, the government usually mandated the changes in secondary education. In the former case, there was an indirect influence by the Wiskobas project and its successors. In the latter case, the government systematically gave our institute the responsibility of developing (prototypes of) new curricula—as was the case with the Hewet project and the Hawex project (both upper secondary), the project W12-16 (for the age group 12-16; together with the National Institute for Curriculum Development, SLO), and the recent Profi project (new curriculum for the 'exact' stream in upper secondary education). Current projects involve goals and learning route for primary school, special education, lower secondary and vocational training.

What is "realistic"?

In RME context problems play a role from the start onwards. Here they are defined as problems of which the problem situation is experientially real to the student. Under this definition, a pure mathematical problem can be a context problem too. Provided that the mathematics involved offers a context, that is to say, is experientially real for the student. In RME, the point of departure is that context problems can function as anchoring points for the reinvention of mathematics by the students themselves.

Reinvention

Freudenthal proposed 'guided reinvention' as an alternative for the 'anti-didactical inversion'. Moreover, guided reinvention offers a way out of the generally perceived dilemma of how to bridge the gap between informal knowledge and formal mathematics. This principle is elaborated in many instructional sequences. As an example of the reinvention of an algorithm in primary school, we will discuss the reinvention of the long division. Further we will describe how reinvention plays out in some topics in algebra.

Text books
Most curricular changes in secondary school are based on government decisions. Usually the Freudenthal Institute would be asked to develop new curricula. The government then would mandate these new curricula, and textbook authors would use the prototypical materials that were developed to ground the new curriculum as the basis for the new textbook series. In primary school the influence was more indirect, here inspiring results of developmental research would find their way to textbooks via (journal) publications, conferences and personal contacts. In both cases, differences are to be expected between the original intent of the researchers and the actual textbooks, and their use.

Technology
The role of technology in mathematics education is growing that also holds for the Netherlands. We will show some examples of software that is used in primary and secondary education. This will include the use of web-based applets. In addition to this we will special attention to the use of graphic calculators. These are integrated in the Dutch curriculum; moreover, the use of graphic calculators is an integral part of the final exams.

Geometry
Geometry in the Dutch curriculum can be type-casted as “vision geometry”. The informal experiential knowledge that students have is taken as a starting point for geometry instruction. We will present some examples of vision geometry from primary- and secondary-school textbooks. Further we will discuss how geometry is used in the new Profi-curriculum to foster the students’ experience with proving.