The use of Real World Knowledge in solving Mathematical Problems

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The focus of this paper is the issue of reality in relation to mathematical non-routine problems. Findings pointed to three gaps: the gap between the world of in-school mathematics and the out-of school world; the gap among "realities", different realities between teachers and students but also different realities among the teachers and among the students; the gap between teachers' theoretical knowledge of the kind of problems they should teach and between what they actually teach in class.

Introduction

Mathematics has been a large part of the school curriculum. Students study mathematics from kindergarten through elementary and secondary school. Mathematics is also an integral part of our everyday life and much of the mathematics knowledge is acquired outside of school. Thus, it seems reasonable for students to use everyday life considerations when solving mathematical problems in school and use mathematics learned in school when dealing with everyday life situations. Moreover, it seems reasonable that the two mathematical worlds - the in-school world and the out-of -school world, will complement one another. However, Resnick (1987) describes the gap between learning mathematics in school and the out-of school world, saying that children frequently do not bring to school knowledge that was not studied formally, while knowledge acquired in school is not used adequately outside school. She concludes that the gap between "school intelligence" and "practical intelligence" should be minimized. Schoenfeld (1992) lists students' beliefs about the nature of mathematics. Students believe that mathematics learned in school has little or nothing to do with the real world. Nunes, Schliemann and Caharrer (1993) explored the use of "school mathematics" and "street mathematics". They found that unschooled street-vendors, fishermen and carpenters performed their street mathematics calculations competently. On the other hand, students did not know how to use mathematics studied in school and sometimes arrived at absurd solutions. Young street vendors who had some schooling performed their street mathematics calculations much better than when trying to solve the same problems using their school mathematics. These young street vendors solved problems they encountered in their jobs more easily than when the same problems were given to them in school. The detachment of school mathematics from the reality outside of school is documented in several research studies in which it was found that school children had difficulty in applying real world knowledge in problem solving (for example: Säljö, 1991; Greer, 1994). This difficulty is not unique to students. Verschaffel et al. (1997) found that pre-service teachers also had the same difficulties.

One of the reasons for this gap may be embedded in the mathematics itself. Resnick (1987) suggests that at school children learn rules and symbols but tend to lose the relationship to what these symbols represent. The detachment of the symbols from what they symbolize may cause difficulties in applying real world knowledge in mathematics lessons and vice versa. Another reason for this gap may derive from the repertoire of problems that appear in the textbooks and are studied in school. It is well known that teachers rely "heavily" on textbook problems. Researchers criticize the “stereotyped” problems studied at schools (for example, Nesher, 1980; Reusser, 1988; Gravenmeijer, 1997). Russell (1996) criticizes textbooks’ problems for not being “real life applications”. She gives an example of a problem found in a textbook in which the student is asked to find the average length of the world’s seven longest rivers. Russell claims that the problem is “silly”. “Why would we want to know the average length of these seven rivers?” Children should be given the opportunity to solve practical and realistic problems, otherwise they become bored, often causing them to dislike problem solving. On the other hand, when Nesher and Hershkovitz (1997) gave students non-routine problems taken from their lives, children did use real world considerations when solving those problems. (The problems dealt with dividing pizzas among children in a summer camp).

Our research adds one more piece to the puzzling relationship between school mathematics and everyday life. It is part of a larger research which investigated knowledge, beliefs and attitudes of elementary school teachers with different professional backgrounds toward non routine problems, as well as attitudes and beliefs of sixth graders to these problems (Asman, 2000). In four out of the 11 problems used in the research real world considerations had to be taken into account. In this paper we report some of the findings regarding these four problems.

Methodology

30 elementary school teachers were interviewed. Each interview lasted for about one hour and a half. After some personal information concerning the teacher’s background, each teacher was asked several questions regarding her beliefs and perceptions about problems in general. For example: why do we teach problem solving? Or, what is a "good" problem? Then each teacher was presented with 11 non-routine problems one at a time and asked to solve the problem. If the teacher was not able to solve it, or solved it incorrectly, she was given some hints by the interviewer who helped her to arrive at a reasonable solution. At this stage the teacher was asked about her beliefs regarding the non-routine problem she just solved: Would she give the problem to her class? In an examination? Has she encountered such problems in textbooks? In workshops for teacher development? After going through all 11 problems the teacher was asked for some feedback on the interview. Each interview was recorded and then transcribed.

In order to find out how do different professional backgrounds affect teacher knowledge and beliefs, we chose the 30 teachers as follows:

- Ten pre-service teachers specializing in mathematics, at the end of their third year of college. (PST)

- Ten in-service teachers, teaching in the higher grades of elementary school. (T)
- Ten in-service teachers teaching in the higher grades of elementary school, who had participated in long-term mathematics intervention programs. (TT)

265 sixth grade students participated in this research. They studied the 11 non-routine problems in class. Their teachers (T and TT teachers) who participated in this research study taught them. The students answered a short questionnaire regarding their beliefs and attitudes toward the problems.

In addition we observed two six grade classes studying these problems.

The Problems

The following are the four problems which in their solutions one needs to take into account everyday life considerations:

1. A bus can hold 40 students at the most. How many buses will be needed to transport 175 students? (Transportation Problem)

2. When looking at a 35^o angle through a magnifying glass that magnifies four times, what size angle will you see? (Angle Problem)

3. Four families live in Dan's building, altogether they have 10 children. What is the average number of children in each family? (Average Problem)

4. John's parents bought furniture for their dining room. The table cost $1.1 and each chair $0.4 . What is the total amount they should pay if they bought one table and four chairs? (Furniture Problem)

Results

The Transportation Problem

This problem is well known in research literature and was first used (with different data) by Carpenter et al. (1983). The exercise 175:40 is only the first step in solving this problem. The second step is adjusting the result to real life.

All 30 teachers solved the problem using everyday life considerations. For example, a (T) teacher said that five buses are needed. However, she would try to seat more students in each bus since she works in a neighborhood where the parents can not afford to pay for five buses. Another teacher (PST) commented that although each bus can hold 40 students there is no need for them to crowd the children and sit 40 in four buses and 15 in the fifth and they can divide the number of students more comfortably among the buses. Another teacher (T) said they could order four buses and one minibus in order to decrease the expenses. It seems that teachers used their experiences in ordering buses for field trips, which is something teachers frequently do. As for the students, many solved the problem just by performing the mathematical exercise, without using everyday life considerations, saying that 4.375 is the number of buses to be ordered. Some realized that 4.375 was an unrealistic number of buses, thus they wrote "4.375 – illogical", not knowing how to adjust the answer to everyday life or not knowing that the answer could be different from the result of the exercise they had just performed. This situation seems not to be a part of students' reality. One TT teacher commented that her students would probably know how to solve such a problem, since before going on a field trip they used to calculate the expenses per student. Thus, she said, for them this problem would be a “real” problem and a significant one.

The Angle Problem

Two numbers are involved in this problem (35 and 4), but in order to solve it one is not supposed to do any calculations with these two numbers. The use of real life considerations should be the clue to this problem. One needs to understand that when looking through a magnifying glass, the *shape* of the object being viewed does not change, thus the angle should remain the same. But even if the solver did not use everyday life considerations and obtained 140º as the answer her/his common sense should start working since they started with an acute angle, which turned into an obtuse angle. One should wonder what would happen to a right angle? Would it turn an angle of 360º when viewed through a magnifying glass? And what about a 150º? What would it look like?

23 teachers (4 T, 9 TT and 10 PST (all 10 PST were exposed to this problem in one of their courses)) said that the angle would remain the same. Some of them explicitly indicated that they relied on considerations from everyday life, as in the following explanation given by a pre-service student:

> *“I used my real world knowledge, I wear eyeglasses. When I put on my eyeglasses do I see a different angle? But I am not sure that children would see it the same way as I do”*

7 teachers (6T and 1 TT) gave 140º as their answer. During the interviews all of them were led to the correct solution. One of the teachers (T teacher) explained:

> *“It is a difficult problem, a magnifying glass is something that I have never worked with nor have the students.”*

Both teachers expressed their concerns about the way students will deal with this problem, since this is probably not from students' real life.

Many students did not look for reality when solving this problem. It seems that the numbers and the "key words" that appear in the problem confused them. Many of them explained that they were distracted by the words “magnifies four times”. They were surprised how simple this problem actually was.

One student who did take into account that the problem reflects out of school reality, explained her solution:

> *“The angle would not change in degrees, when we look at a bug through a magnifying glass, does it change its shape?”*

The Average Problem

Relying on everyday life considerations (without knowing the meaning of the term average) might be an obstacle in correctly solving this problem. This was the case

with 13 teachers (3 T, 2 TT and 8 PST). They used their "real world knowledge" but since *"We deal with children and not with tomatoes"* and since *"there is no half a child"* 2 ½ cannot be a reasonable answer. Some teachers rounded off the number either to 2 or to 3 so it would make sense. These teachers did not know that the average is a statistical datum and is not representative of reality. They were surprised to find out that the correct solution is that each family has an average of 2½ children. They complained that in their textbooks, when dealing with an average concerning people, the average is always a whole number, therefore they had never encountered such a problem. A (TT) teacher who solved the problem correctly said that since the material in textbooks is not authentic she brings her class authentic materials from the newspapers, like information about surveys. From this resource her students can learn about mathematics in real life.
In the classes we observed most students tended to round off the quotient. However there were students who left 2 ½ as the answer not paying attention at all to the possibility that it might be problematic. Only one student said that he saw in the newspapers that when dealing with averages regarding human beings it is o.k. to write fractions. This might take us back to the 13 teachers. We may wonder how could it be that they did not pay attention to the numbers in the newspapers, on the radio or television. Or is it that they did pay attention, but "their reality" in which we can not have 2 ½ children as an answer to a problem was the deciding factor in their reasoning?

The Furniture Problem
In this problem the prices are extremely unrealistic. The first purpose for including this problem in the research was to find out if teachers and students would notice the absurd prices. The second purpose was to find out more about teachers' beliefs regarding the question: Should the data that appears in math problems, stand the tests of reality? Only 5 teachers (4TT and 1PST) noted that the prices were unrealistic. The fact that most teachers solved the problem, expressed their beliefs toward it but were not bothered at all by the extremely unrealistic prices, illustrates the gap between in-school mathematics and out-of-school mathematics. When the interviewer raised the issue of the extremely unrealistic data, 25 teachers agreed that word problems should reflect reality and therefore if they had noticed that the data were unrealistic in the first place they would have changed the numbers. Five teachers (2T, 1TT and 2PST) thought that it does not matter if data reflect reality:

> *"I do not care about the numbers in the problem. It is important for me that they would be simple. I care about the strategy of solving, not the numbers since students calculate with calculators" (TT)*
> *"I would not change the prices since the problem is for an exercise, not a test of reasoning". (PST)*

Teachers who thought that prices should be changed to real ones expressed their opinions saying;

> *"I would change prices since we must bring reality into class". (TT)*
>
> *"We should adapt prices to reality since children constantly ask why should I study this or that? They are motivated to study only if they are convinced that what they learn will help them in everyday life or in the future. As a child I did not want to study stuff that I was not sure that it would be applicable in my everyday life. If the mathematical problem reflects a real problem from everyday life, the child will be interested". (PST)*
>
> *"A child should be prepared for real life. He should understand the value of things. If he vandalizes a desk in class, he should know its value, and this is surely not $1.1." (T)*

The last sentence is an authentic example from the real life experiences of a teacher who is teaching in a low socio-economical neighborhood. She explained why a child in her class should know the real price of furniture.

As for the students, most of them did not notice anything peculiar in this problem. In one of the classes we observed a student made a remark about the "funny" prices. This opened a discussion in class, whether prices should be real. Most students thought that the numbers in the problems are not important since teachers give problems in order to find out if the student knows how to solve and calculate. Only a few students thought that it would be better if the data would match reality.

One (TT) teacher reported that when she gave this problem to her students, a student (weak in mathematics) laughed and said that the prices are senseless. In the discussion held in class, as to weather the prices should be real, he was quite assertive saying that prices should be changed to real ones. The teacher was surprised since this student never showed any interest in mathematics lessons. The teacher explained that this student is helping his father in his shop after school and prices are part of his reality.

Discussion

It seems that the findings in this research may point to three gaps regarding the issue of reality and problems in mathematics. The first gap has to do with "theory and practice", between what teachers believe in theory and what they do in class. When asked about their beliefs of problem solving in general and "good problems" in particular, almost all teachers said that problems should be from students' lives, indicate authenticity, and promote reasoning. However, when asked to give examples of problems that they teach in class, most examples did not reflect these characteristics. Moreover, only very few paid attention to the unrealistic prices in the Furniture Problem. It might be that others did pay attention, but did not say anything since the numbers in the problem do not have to be realistic for them.

The second gap is the gap between in-school mathematics and the out-of-school world. This gap was prominent for example when teachers and students did not use their out-of-school knowledge when solving problems in school. Furthermore, unrealistic data did not bother them as long as the problem was solvable.

The third gap that emerged is the gap between different "realities". We could see that teachers' reality was different from students' reality, but we could also see different realities among teachers and different realities among students. It seems that when problems are from the solvers' reality it might reinforce their willingness to be involved and might increase their chances to solve the problem correctly. A (T) teacher aptly described this gap mentioning the student's world, her world and her husband's world:

> *"......"good problems" are surely not those in textbooks, which are irrelevant to a child's life or culture. There are some problems that do not belong to the student's cultural world. This is a disguise, it is not his world, therefore he does not understand it, and it frustrates him. ...sometimes I do not know myself what they mean, and I have to ask my husband for an explanation. If it is not even from my world, how could the child possibly understand it? For example, problems which involve filling a swimming pool with two pipes. Neither my students nor I have witnessed such a situation. The solution should be logical too, not with difficult fractions, and not with four or more digits after the decimal point. Such numbers may be relevant for my husband who is an engineer and works with small particles, but surely not for our students".*

There are probably many sources and many reasons for these gaps. The gap between what teachers say and what teachers do (a well-known gap among teachers in general) may be partly explained by the materials they use in class. During the interviews teachers complained about the textbooks, saying that textbooks are their main source for problems but criticized the textbook problems for being "stereotyped", not authentic, and boring. However, the TT teachers who participated in long-term mathematics intervention programs, reported that they also have other sources for word problems and that they do not rely on the textbooks only. The textbooks might also be the reason for the gap between in school mathematics and everyday life out of school. Another reason for this gap might be students' learning to play according to different rules. They have the "in school rules" to solve mathematical problems in class and "out of school rules" to deal with out of school situations which involve mathematics. They probably are familiar with considerations from everyday life but they do not think that they are supposed to use them in school (Bishop and de Abreu, 1991).

The existence of the gap of different "realities" is very clear. But does it mean that the teacher has to generate problems to fit the reality of all students in class,? Is it possible? Is it the right thing to do? Must all problems be realistic? Wouldn't academic mathematics be lost?

It seems that more research is needed in order to answer these questions and to more deeply explore the puzzling relationship between mathematics and everyday life.

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