SPONTANEOUS TENDENCY TO FOCUS ON NUMEROSITIES IN THE DEVELOPMENT OF CARDINALITY

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In this paper we report on a longitudinal study where we investigated whether there are differences in 3 – 4 year-old children's tendency to focus on numerosities, and whether these differences are related to the children's development of cardinality. It was found that the two groups of children at the age of three formed on the basis of their spontaneous tendency to focus on amounts of objects differed in their development of recognising and producing small amounts. Those children who spontaneously regarded the numerosities as relevant factors in the test situations developed faster in cardinality related skills than those children that focused on other features in the tasks.

Introduction

Recent research in cognitive, comparative and developmental psychology supports the position that infants have biologically primary quantitative abilities, which encompass their implicit understanding of numerosity, ordinality, counting and simple arithmetic (for a review, see Geary, 2000). However, in these studies on infant abilities there seems to regularly appear a small group of children who do not present these quantitative abilities at all.

In the research on the development of exceptional skills it has been established that the amount of deliberate practice as well as the age at which it is started is related to the level of performance (Ericsson & Lehmann, 1996). The child's social environment influences the ways in which he/she engages with the mathematical world around him/her. Also the differences in opportunities the child finds in his surroundings to practice pre-mathematical skills might explain the differences in the development of early mathematical skills of children. The world could appear to be full of numerosities and opportunities for practising early mathematical skills to some children, whilst others focus on other features in the environment and involve themselves much less with pre-mathematical ideas. According to Ginsburg et al.'s (1999) observational studies, 5 year-old children deal with mathematical activities as much as 45 percent of their time of free play in day care, of which 11% is enumeration.

In the development of cardinality a child moves from innate, preverbal counting system called subitizing to fluent usage of traditional counting in determining numbers of objects. Counting skills enable the child to recognise and produce bigger amounts than innate capability of subitizing does. "The cardinal number word refers to the whole set of entities and tells how many entities there are, that is, describes the manyness of the set" (Fuson, 1988, 5). The concept of cardinality grows out of

children's experience with counting in a manner similar to that in which a symbolic concept of print grows out of children's experiences with the alphabet. Cardinality requires both explicit knowledge of the relation between numbers and quantity, and the attentional procedures for focusing on that which is counted (Bialystok & Codd, 1997.) According to Schaeffer, Eggleston, and Scott (1974), cardinality results from the integration of two prior processes: subitizing and counting. Among other primitive processes, subitizing provides the basis for acquisition of the number concept and orients a child to the numerosities of sets (s)he experiences (English & Halford, 1995, 60). Subitizing, or enumeration of small sets up to about four elements, is a rapid and accurate pre-attentive process, which requires no explicit teaching, and possibly little or no experience (Gallistel & Gelman, 1978; Sathian et al., 1999).

A major step in early mathematical development is learning to count. Counting enables the child to make quantitative determinations of amounts, rather than relying on perceptual or quantitative judgements. Cardinal situations and counting are, at first, separate and different situations for children, but are gradually connected when children understand that counting is not an isolated activity, but has a result (Fuson, 1988, 206.) The remarkably slow development of cardinality and counting skills (e.g. Fuson, 1988; Wynn, 1990) could be explained by major differences in preverbal and verbal counting systems (Wynn, 1992a, Wynn 1992b). One potential explanation for the differences in pre-schoolers mathematical skills (see, e.g Reusser, 2000) might be children's different amount of spontaneous dealing with mathematical activities.

In the present study, we followed up children from the age of 3 to the age of 4 years in terms of spontaneous focusing on numerosities and cardinality. This study investigates whether there are differences in children's focusing on numerosities, and whether the groups formed on the basis of differences in focusing on amounts differ in their development of cardinality. Our hypothesis is that the differences in spontaneous focusing on amounts might indicate the children's overall tendency to focus on numerosities in his/her surroundings, and this could cause considerable differences on the amounts of practice in enumeration and counting skills. So, those children who do not regard numerosities immediately as relevant factors in the tasks would not develop as quickly in recognising and producing small amounts.

Method

Participants

39 children (18 girls and 21 boys) from the city of Turku participated in this one year follow up study. The children's mean age at the start of the follow up was 2 years and 11 months (s=1,5 months). The children were tested for the second time when they were on average 3 years and 5 months (s=1,5 months) old, and for the third time when they were on average 3 years and 11 months (s=1,5 months) old. The children attended seven day-care centres located in the middle-class areas of Turku at the beginning of the follow-up period.

Procedure and tasks

The video-recorded tasks were presented individually in a familiar room of the child's day-care centre during the morning. The carrot task took approximately 5 minutes and the caterpillar/pig task about 10-15 minutes, depending on the child's skills.

Spontaneous focusing on numerosities was assessed at the beginning of the follow up by the carrot task and the spontaneous focusing section of the caterpillar task. *Cardinal tests* were presented at the age of 3, 3½ and 4 years. At the beginning and in the middle of the follow up period the cardinal section of the caterpillar task was presented, and in the end of the follow up a modified but parallel version of the caterpillar task, called the pig task, was performed.

Carrot task

The experimenter placed two similar cuddly toys in the form of "bunnies" and then a plate of 5cm long "carrots" on the table in front of the child. The bunnies and the carrots were identified together with the child. The experimenter asked the child, "Look at what I do. I do this, this. Now, you do what I did." While saying "this", the experimenter lifted two carrots, one at a time, into a row in front of the experimenter's bunny. The child imitated the experimenter as well as (s)he could and placed carrots in front of his/her bunny.

The amounts of carrots in the items were 2, 1, 3, 4, 5, ...10. If the child did not lift as many carrots to his/her bunny as the experimenter did, the failed quantity was repeated. After two failures with the same quantity, the experimenter returned once again to the previously successful quantities. In every case the first items with two, one and three carrots were presented to the child.

Caterpillar task

The materials in the caterpillar task were a black sack, a small (17cm x 14cm) box, ten sewed green fabric "caterpillars" (length 60cm), who had either 1,2,3... or 10 legs (length 6cm) every 2.5cm. Feet were 3.5cm in length. The one-legged caterpillar had its leg in the middle, the two-legged caterpillar had a leg in the middle and a leg 2.5cm from the other leg. This was the way in which all the caterpillars' legs were situated. There were also 24 red socks in the small box, which were elastic and suitable for the caterpillars. In the task, a child sat at a table. The experimenter sat to their left, and the sack that contained the caterpillars was on the experimenter's left. The box with the socks was placed on a tall stool on the opposite side of the table at the beginning of the task.

The caterpillar tasks did not to assume that the child would use any verbal number words. The child had to determine the number of objects (legs on the caterpillar), keep it in mind for few seconds and pick up the same amount of objects (socks) and bring them to the original objects (legs). The tasks enabled the child to spontaneously use either subitizing, estimating or counting to solve the items. The child could check

the original number of legs on the way whilst picking up the socks, if s/he wanted to. The location of the caterpillar and the box of socks prevented the use of one-to-one correspondence. The child could not easily see the legs of caterpillars when standing next to the box of socks.

In the spontaneous focusing section of the caterpillar task the experimenter asked the child what clothes (s)he put on when (s)he goes out (e.g. hat, gloves, shoes and socks). Then the experimenter told the child about the sack, in which there lived a family of friendly caterpillars. The caterpillars were about to go out, but first they needed to put on socks. After introducing the first 6-legged caterpillar, which already had socks on, the experimenter showed, by pointing to the legs one at a time, that there was a sock on every foot of the caterpillar. The child was told that was the way that the other caterpillars should be dressed too. The 6-legged caterpillar "waited" in the sack and the 2-legged caterpillar was lifted from the sack. The child was asked to bring the caterpillar as many socks as the caterpillar needed from the box on opposite side of the table. All the socks that the caterpillar needed were to be brought in one go. After bringing the socks, the child was asked if (s)he had brought the correct number of socks. Then the experimenter put the socks on the caterpillar and asked if the caterpillar was ready to go. If the child did not bring exactly two socks for the caterpillar, the item was repeated to make sure that the child considered the amount of legs as a relevant factor in the task. The spontaneous focusing on amounts was assessed from these two trials in the first section of the caterpillar task.

The cardinal section of the caterpillar task was conducted immediately after the spontaneous focusing section by advising the child to bring exactly as many socks as the caterpillar needed if the child had not brought the right amount of socks for the caterpillar. This was to make sure that all the children understood that it was necessary to focus on the amount of legs and socks in this task.

The caterpillars were presented in the order 2, 1, 3, 4, 2, 5, 6, 1, 7, 8, 2, 9, and 10, where the numbers represent the number of legs of the caterpillars. The one- and two legged caterpillars were presented between the sequence of test caterpillars for two reasons: to motivate and relax the child with easier tasks, and to break the growing order of test caterpillars. If the child did not bring the right number of socks for the caterpillar, the failed amount was repeated, and if the child failed twice with the same amount, the previous successful amount was repeated. The amount at which the child got both attempts right determined the level of the child's performance.

Pig task

The materials in the pig task were a flat box, where the sows stayed, a small (17 x 14cm) box for pigs, 12 plastic sows (length 60cm), which had either 1,2,3…or 12 teats (length 3.5cm) every 2cms. The teats were located similarly to the legs of the caterpillars. There were also 24 piglets in a box on the opposite side of the table. The child and the experimenter sat as in the caterpillar tasks at a table.

In the pig task, the experimenter told the child about the sows that stayed in a box, and needed help. The child was asked, if s/he knew how sows feed little pigs, and the 6-teated sow, who nursed six pigs was introduced. The experimenter showed that this was the way in which sows feed their pigs: a piglet suckling at every teat. The children were instructed to help the other sows feed their piglets. The 6-teated sow went back to feeding her piglets in the box, and the 2-teated sow was lifted from the box. The child was asked to bring as many piglets as the sow could feed from the box on the opposite side of the table. The pig task continued in a manner similar to the caterpillar task. The maximum amount of teats the sow had was 12.

Analyses

It was carefully observed if the child immediately focused his/her attention on the numerosities of objects instead of other features in the first items of the tasks. The child was considered as spontaneously focusing on numerosities, if (s)he produced the same number of carrots as the experimenter had produced, or if (s)he immediately brought the same number of socks as there were legs on the caterpillar in the first items of the caterpillar task. The child was considered as not spontaneously focusing on amounts if (s)he did not focus on the amount of carrots, and either tried to imitate the way in which the experimenter lifted the carrots, or concentrated on feeding the bunny. Consequently in the caterpillar task the lack of spontaneous focusing on amounts appeared if the child did not pay attention to the number of legs and socks and brought either a handful of socks or just one sock for the first 2-legged caterpillars.

Cardinality was analysed from the cardinal section of the caterpillar task and the pig task. The carrot task could not be used because those children who did not focus on the number of carrots in the task obviously considered the whole task as an imitation of movements or feeding task instead of producing the same number of carrots for the bunny. After the child's first attempts in the caterpillar task, it was made clear to the child that the number of socks was a relevant factor in the task.

The level of a child's skills of recognising and producing numerosities was determined through successful trials in bringing socks for the caterpillar. The child had to succeed twice with the same number of legs to be considered as capable of producing the given number of socks. The task was finished after two failures at the same amount.

Results

Spontaneous focusing on numerosities

The children of this study had differences in spontaneous focusing on numerosities. There were 14 children who did not spontaneously focus on numerosities in either one (11 children) or both tasks (3 children), called the *non-spontaneous group*, and 25 children who focused immediately on numerosities in both tasks in the beginning of the follow up, called the *spontaneous group*. Four children did not focus on the amount of carrots in the carrot task, and 11 children did not focus on the numbers of

legs and socks in the caterpillar task. Those children who did not focus on the amounts either imitated the movements of experimenter, concentrated on feeding the bunny, or brought only one sock or a handful of socks to the 2-legged caterpillar. In the caterpillar task, 10 children out of those 11 who did not focus on the amount of legs and socks in the first item started focusing on the number of legs when advised, and proving that they were capable of recognising and producing the amounts of one and two.

Cardinality

To examine the question of whether the non-spontaneous group of children differed from the spontaneous group in their cardinal skills during the follow up period we investigated their achievements in cardinal tests (see Figure 1.).

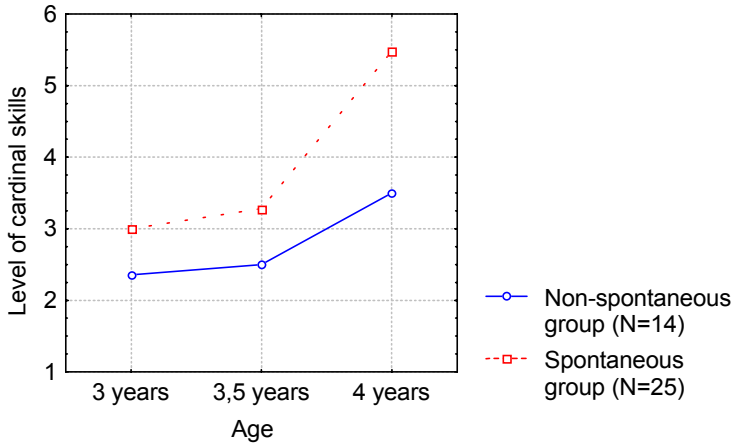

Figure 1. The results of the non-spontaneous and spontaneous groups in cardinal tests in the age of 3, 3½ and 4 years.

Separate 2 x 3 (group x age) ANOVAs with repeated measures were performed for their performance, together with Tukey's HSD tests, from 3 to 4 years of age. The main effects of group ($F(1,37)=7,18$; $p=0.011$) and age ($F(2,74)=21,20$; $p<0.001$) were statistically significant. The non-spontaneous group displayed weaker skills in cardinal tests and the children's skills developed during the follow up period. Symptomatically significant ($F(2,74)=2,92$; $p=0.060$) interaction of Group x Age in performance suggest that there were developmental differences in cardinal skills between the groups. The spontaneous group developed faster than non-spontaneous group in recognising and producing small amounts.

In spite of the slight trend of differences in the groups' performances in favour of the spontaneous group, Tukey HSD tests revealed that there were no statistically significant differences in the groups' results at the age of 3 and 3½ years. However, after 12 months, group differences began to appear. The cardinal skills of the non-spontaneous group were weaker ($p<0.001$) when the children were 4 years old. The results of group comparisons were confirmed by Kruskall-Wallis's non-parametric test.

Discussion

The children of this study had differences in their tendency to focus spontaneously on numerosities. The spontaneous group of children (25 out of 39 children) interpreted the carrot and the caterpillar task instantly as numerical and focused their attention to the amounts in the tasks. Those children (14 out of 39) who did not spontaneously focus on the numerosities in the tasks when they were three years old, developed more slowly in cardinality than those who immediately perceived the task situations in terms of various numbers of objects or events. The spontaneous group outperformed the non-spontaneous group at the age of four years unlike than at the age of 3 and 3½ years in their skills to recognise and produce small amounts.

According to the results, it seems that there are significant differences in the ways in which children pay attention to numerosities in situations and these differences in the amount and the quality of spontaneous activity might be of great importance for the children's later development of number concept. The fact that all the children except for one in the non-spontaneous group were able to bring the 2-legged caterpillar two socks when advised to focus on the amount of legs in the caterpillar task, supports the idea that spontaneous focusing on numerosities is a separate process from enumeration. It is possible that every child manages to deal with numbers within their subitizable range once they realise that number of objects in the situation is a relevant feature for his/her action. What causes these differences in the ways that children interpret their perception is a subject for a later study, where the follow-up of children will begin earlier than in this study. This should be investigated because this study raises the question of whether the differences in the spontaneous tendency to focus on numerosities produce differences in the development of cardinality, or vice versa. There is also a question about the nature of spontaneous tendency to focus on numerosities: is it inherently different in children, or as a result of social support? Could there be such differences in the linguistic skills or subitizing range of the children which would explain the different development of cardinal skills?

In Hannula's (2000) study it was reported that the spontaneous group of this study tried to use more counting to solve the caterpillar task at the age of three years (64% of the children) than the non-spontaneous group of children (21% of the children). Considering the long time (e.g. Fuson, 1988; Wynn, 1990) that counting skills take to develop accurately, it is plausible that the spontaneous group of children was further in their development of counting skills at the beginning the follow up, though the levels of children's performances did not differ. It is a common phenomenon in the development of skills that the level of performance does not rise immediately, but after a training phase, when a more advanced and demanding strategy is mastered. In this case the spontaneous group of children would have focused on numerosities more often, because the quantities of objects had become more meaningful to them. Even so, the differences in the skills of spontaneous and non-spontaneous groups in the beginning of follow up do not eliminate the possibility of spontaneous numerical tendency to have a significant role in the development of cardinality. The effects of

differences in the amount of training quantitative skills produced by spontaneous activity could still derive to different developmental profiles in early mathematical skills. Therefore, our preliminary hypothesis for future studies is that spontaneous tendency to focus on numerosities serves as one of the building blocks for the child's early numerical development, possibly besides innate quantitative abilities.

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