SO THAT'S WHAT A CENTIMETRE LOOKS LIKE: STUDENTS' UNDERSTANDINGS OF LINEAR UNITS

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In this paper the results of a set of tasks designed to investigate student understanding of linear measurement units and the process of measuring length are presented. By Grade 5 the majority of the students were able to use informal paper-clip units to measure length and to identify linear units. However, few students in Grades 1 to 4 showed an understanding of the linear nature of units when they were asked to show a centimetre unit length in a variety of contexts. The results indicate that teachers need to identify units explicitly when they are teaching measurement because many students do not seem to have abstracted this concept in grades where it was assumed they had done so.

INTRODUCTION

In 1976 Carpenter reviewed the research on students' learning of measurement, based largely on the work of Piaget. Carpenter questioned how students might benefit from the information that had been gained because, in his opinion, training on specific concepts of conservation and transitivity seemed to be less important than training in measurement itself. At that time he referred to the lack of direct research relating the results of research to the measurement curriculum. He pointed out that "although research has identified levels of development of measurement concepts and rough age approximations for the development of certain operations, it is not immediately clear what implications this has for the curriculum" (Carpenter, 1976, p.71). Other researchers have queried the idea that concepts such as transitivity and conservation must be learned prior to measurement. Nunes, Light, & Mason (1991) have argued that measuring activities themselves assist in the understanding of transitivity and conservation.

Since that time there have been a number of fundamental studies of how students learn measurement concepts, in particular, Battista, Clements, Arnoff, Battista, K., & Borrow (1998); Hart, Johnson, Brown, Dickson, & Clarkson (1989); Outhred & Mitchelmore (2000); Wilson & Osborne (1988); Wilson & Rowland (1992). The emphasis of these studies is an understanding of how students interpret measurement concepts linked to the mathematics curriculum. However, not only curriculum knowledge is important. Hiebert (1984) has commented that "many children do not connect the mathematical concepts and skills they possess with the symbols and rules they are taught in school" (p. 498). He points out that if learning is to be applicable then students need to connect classroom and real-life experiences with the formal mathematical abstractions.

The first topic primary-school students usually encounter is linear measurement, and their knowledge of length provides the basis for the later development of area and volume concepts, as well as understanding of measurement scales which are essential for mass, time and temperature. A useful way of examining the way children think about linear measurement is to use Hiebert's (1986) distinction between the formal symbols, skills and procedures (procedural knowledge) and the intuitions and ideas about how mathematics works (conceptual knowledge). He states that the critical connection between procedural and conceptual knowledge is required when students have to know 'how the system works' to solve tasks or problems.

An example of procedural and conceptual knowledge in measurement would be the scale on a ruler. A ruler involves symbols (marks representing the beginning and end of each unit linked with a numeric scale, as well as shorter marks representing subdivisions) and procedures for use (aligning the ruler with the object to be measured and reading off the scale). These involve procedural knowledge. By Grade 5 almost all students can measure the lengths of objects using a ruler, that is they have mastered the "form" of ruler use but they do not understand its construction (Bragg & Outhred, 2000). Making an accurate ruler to measure in informal units (say, paper clips) would seem to involve an "understanding" of how the measurement process works in Hiebert's use of the term.

Hiebert suggests that "Many of children's observed difficulties can be described as a failure to link the understandings they already have with the symbols and rules they are expected to learn. Even though teachers illustrate the symbols and operations with pictures and objects, many children still have trouble establishing important links" (1984, p. 501). An understanding of measurement units would seem to be fundamental to establishing links across different measurement topics. The aim of this paper is to investigate the growth of students' knowledge of linear units across the primary school years. The paper reports the results of five tasks from a larger study of the development of children's understanding of linear measurement.

METHODOLOGY

The study was cross-sectional; 120 students from Grades 1-5 (aged 6-10 years) were selected from three state primary schools in a medium to low socio-economic area of Sydney. Each class teacher selected six students: one girl and one boy considered 'above average', 'average', and 'below average' in terms of mathematical concepts. However, in one school twelve students were selected from each grade. The first researcher interviewed individual students towards the end of the school year (September-November). Thus, they had been exposed to a large part of the measurement program for their grades. The interview tasks were designed to elicit information about the students' understanding of length measurement. Five interview tasks, a subset of the larger study, were used to determine what the students understood about the linear nature of units of measure using both informal and formal units. An understanding of measurement units would seem to be a fundamental aspect of learning to measure. Work with informal units is used to help students become familiar with important properties of units and how lengths may be measured and compared (Campbell, 1990). The subset of tasks (see Table 1) was used to determine how students might represent linear units that are formalised on portable tools such as rulers and tape measures.

The first task (Task 1) required students to apply their knowledge of informal units to construct a ruler, using the length of the paper clips as the unit. The second task (Task 2) was used to establish if students could measure with informal units. Tasks 3, 4 and 5 required students to identify and represent units of linear measure (centimetres). In Tasks 3 and 4 students were asked to either mark the centimetre unit on a printed ruler or on a washable plastic 1cm cube. In Task 5 students were shown a picture of a gesture commonly used to indicate a centimetre (thumb and forefinger opposed to show a gap of approximately one centimetre) and to mark what one centimetre would look like if you could see it.

The tasks were presented in the same order to all students. The paper clip items (Tasks 1 and 2) were presented first followed in order by Tasks 3, 4 and 5. However, the five tasks were separated by other tasks not listed.

Table 1 The five tasks involving linear units.

Task	Description	Knowledge
1	Make a ruler using paper clips as the unit of measure (students were given a long rectangular strip of light cardboard).	A scale can be constructed by iterating a unit and marking each endpoint. These marks can be associated with numbers.
2	Use 2 paper clips to measure a line 28cm long (noting the fractional unit).	A length can be measured by iterating a constant-size unit with no gaps or overlaps. Fractional units may result.
3	Count 5 sea horses shown on a card and state what the ‘5’ represents. Then explain what ‘5’ on a ruler represents and identify a single unit.	Linear units are separated by marks. A numeric scale aligned with the marks gives the number of linear units from the origin.
4	Draw the linear unit on a picture depicting a familiar representation of a centimetre: thumb and forefinger placed 1cm apart.	Identification of the linear unit in a pictorial representation.
5	State which part of a 1cm cube (a ‘short’) is used when measuring a length.	The length of an object gives the measurement unit (its area and volume are not relevant).

RESULTS AND DISCUSSION

The results for Tasks 1 and 2 involving constructing a paper clip ruler and measuring with paper clips are presented in Figure 1 for each grade level. There were 24 students at each grade level.

The results for Task 1 (see Figure 1) show a gradual increase in the construction of an accurate paper clip ruler. No Grade 1 students and only a few Grade 2 (21%) students were successful but, by Grade 5, 75% of the students constructed an accurate ruler. Students were not successful for the following reasons: they used the paper clips as unit markers; they did not maintain a constant-size unit; or they used an arbitrary unit length. There was a very clear distinction between those students who successfully used a paper-clip length as the unit of measure and those who used them as markers. Successful students were observed to mark each unit length carefully and then add the correct numeral.

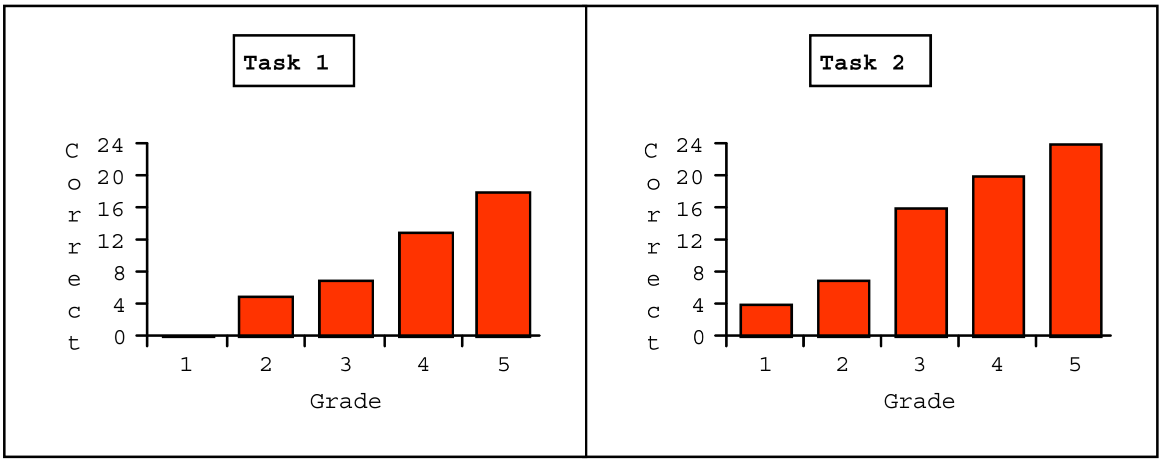


Figure 1 Number of correct responses for each grade for the tasks involving informal units (Task 1 and Task 2)

The graph for Task 2 (see Figure 1) shows an increase from Grade 1 to Grade 5 with the greatest change between Grade 2 and Grade 3, when there is an emphasis on teaching length measurement. Only a small number of Grade 1 students could measure a length by iterating paper clips and indicate that the result involved a fraction of a paper clip, but by Grade 5 all students could successfully complete this task.

However, when the results of Tasks 1 and 2 are compared with Tasks 3 to 5, it is evident that being able to measure with informal units and to construct an accurate paper clip ruler are not sufficient to show that students understand the linear nature of the units. The results for Tasks 3 to 5 (see Figure 2) indicate that until Grade 5, very few children could show what a centimetre would look like on a ruler (Task 3), between a picture of an opposed finger and thumb (Task 4) or on a cube (or "short") (Task 5). Their responses showed that, in the case of the centimetre on the ruler students represented the unit either as a space (e.g., by placing their finger on the space) or as a feature of the ruler (e.g., as marks).

Daniel's (Grade 4) remark was typical of many students who had a 'spatial' view: "It's these spaces here, you just count them". His finger fitted the 1cm space on the ruler. He, like many others, coloured in the space between the printed fingers (Task 4) and said that you counted the face of the 'short' (Task 5).

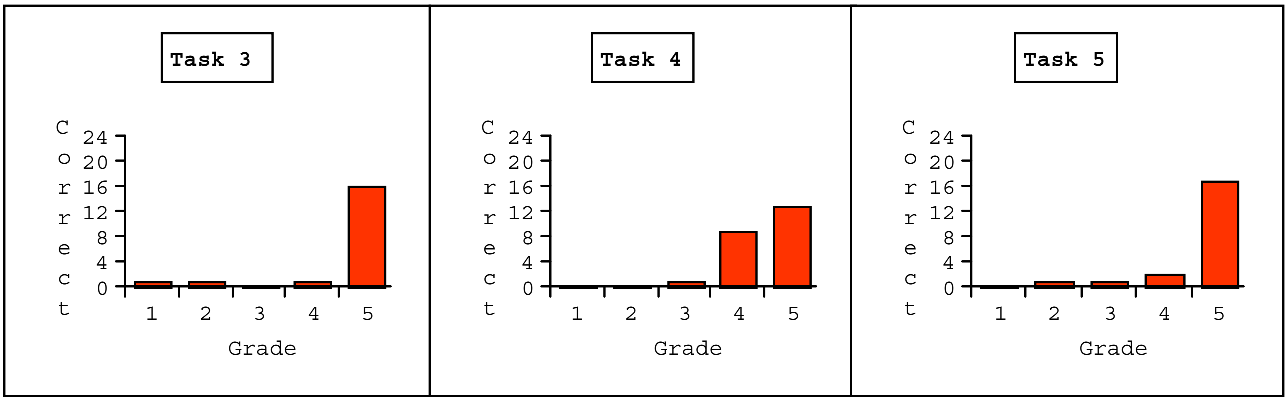


Figure 2 Number of correct responses for each grade for the tasks involving identification of the linear unit (Task 3, Task 4 and Task 5)

For Task 3, in which students were shown a picture of five seahorses and asked how many. If the answer given was five, the interviewer prompted "five what?" Then the student was asked what the "five" on a ruler represented and if the response was 5 cm, the student was asked to show one centimetre. A number of students represented centimetres as a mark or as marks perpendicular to the length. These students would either say that the measure (5cm) was "Where the line ended", or they would indicate that the *5* centimetres were the five marks: Comments similar to "The lines on the ruler point to the numbers you need." were very common. A large proportion of students (38%) from Grades 3 to 5 made observations such as "…there's nothing at the edge of a ruler anyway, that's where you rule lines." Similar misunderstandings were reflected in the responses to Task 5. In Grades 1 to 4 students usually counted the cubes to measure lengths and most said it was "the flat part" that was used to measure lines. These results suggest that many students have not abstracted the concept of a centimetre as a linear unit from their experiences of measuring length.

Students' different representations of a centimetre unit in Task 4 are shown in Figure 3. The percentage of students who gave each response is shown in Figure 3. There appeared to be four main forms of representations: unrelated to length, area representations; ruler-like representations; and linear representations. Eduardo's response cannot be interpreted as he has transformed the finger-thumb opposition into a "C' cue with a small m inside to remind him of "cm". There is not indication that he has a linear unit in mind when this representation is shown.

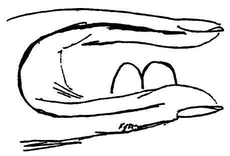
The area representations highlight why a response of pointing to a space on a ruler is not sufficient to assume that a student has abstracted the idea of a linear unit. Older students who made this error were more likely to draw a 1cm square whereas the younger ones were more likely to colour in the whole space. Such students may have a mental representation of a two-dimensional unit in the space between the marks.

The confusion with a centimetre cube is evident in the second and third examples (2(b) and 2(c)).

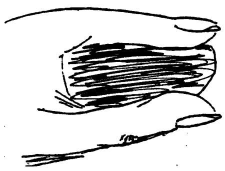
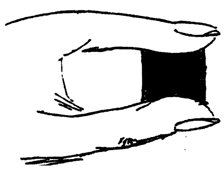
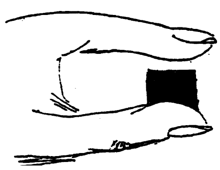
The first two ruler-like representations involve marks to indicate a scale (3(a)), and numbers to indicate a scale (3(b)) between finger and thumb rather than a single 1cm unit. In 3(c) students draw a ruler perhaps because they link rulers and centimetres but they cannot isolate one unit whereas in 3(d) they seem to be indicating the marks delineating a unit (similar to a ruler) but the unit itself could be two-dimensional.

The first and second linear representations (4(a) and 4(b)) suggest that students have no idea of the part of the diagram that is meaningful. Similar responses were found in Task 5 in which students did not know what part of the cube was relevant when measuring length. Only 19% of students in the sample, almost all from Grade 4 and Grade 5, were able to draw an accurate representation of the linear unit. This inability to represent a single linear unit was also found in the other two tasks, the sea horses and the cube.

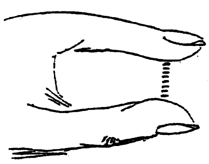
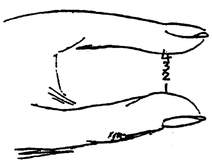
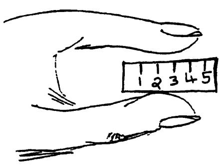
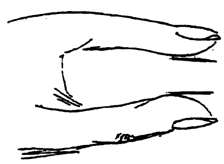
(1) Unrelated representation (3%):

"I learned how to remember a centimetre this way 'cause I got mixed up with a metre." Eduardo, Year 4.	

(2) Examples of area representations (31%):

 (a) 12%	 (b) 7%	 (c) 12%

(3) Examples of ruler-like representations (38%):

 (a) 13%	 (b) 5%	 (c) 8%	 (d) 12%

(4) Examples of linear representations (25%):

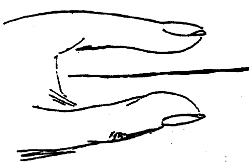
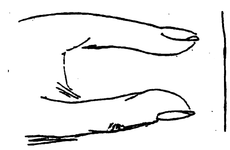
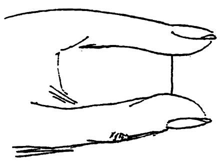
 (a) 4%	 (b) 3%	 (c) 19%

Figure 3 Categories of responses to Task 4 and the percentages for each response.

CONCLUSION

According to Kamii & Clark (1997) student performance on National Assessment of Education Performance (NAEP) items remains disappointing. The results of this study suggest possible reasons why performance on measurement items may be disappointing. Although these students were able to manipulate informal units to measure lengths, there is little evidence to show that they have constructed an understanding of the linear nature of the units of measure until Grade 5. While students in Grades 3 and 4 would have had many opportunities to measure using both informal and formal units, few of them were able to identify a cm unit on a ruler, on a cm cube or on a drawing showing a thumb and finger 1cm apart.

Learning about measuring and the identification of units of measure is very complex (Campbell, 1990) and the dimensionality of the units contributes to this complexity. The analysis of students' drawings of the centimetre unit on a drawing showing a thumb and finger indicated that the majority of the students in Grades 1 to 4 had constructed either a ‘spatial’ or a ‘ruler-feature’ concept of a centimetre. Teachers may be unaware of the multiplicity of representations that students construct for this "convention" of showing the size of a centimetre unit.

These results support the theories of Hiebert (1990, 1986, 1984) and Skemp (1979) that many students are learning the "procedures" of mathematics but not the understanding of, or relationships among, the fundamental concepts, linear measurement units in this particular case. Paper and pencil tests and exercises with informal units often assess only student knowledge of routines and procedures and do not reveal if students possess an understanding of units of measure, that length may be represented by a line and that the units of measure are also linear. For example, use of cubes to measure lines may contribute to students' confusion unless teachers make explicit the part of the cube being used as a unit.

The results from this study have shown that, in spite of their facility with informal units, the majority of students have not constructed a clear representation of linear units of measure. Since students rarely establish explicit and unambiguous connections (Hiebert, 1984) researchers and teachers need to work together to investigate how to teach young students to link experiences with informal units with formal measurement, especially the construction of rulers and scales. Students construct “mental images” (Shaw & Cliatt, 1989) and referents that make sense to them from classroom contexts. Incorrect or confused representations may remain undetected if assessment relies on paper and pencil questions and procedural tasks, such as ruling lines or measuring objects. While teachers are encouraged to help students ‘make sense’ of measurement and not rely on procedures (NCTM, 1989), there has been insufficient research to recommend to teachers how they can best assist students to understand key measurement concepts.

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