LEARNING NUMBER THEORY CONCEPTS VIA INTERACTION IN COMPUTERIZED ENVIRONMENT

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This paper presents a case study of two 7th-grade students during their explorations in the computer-based microworld Geoboard*. In their explorations of the connections between geometrical shapes and the corresponding computer instructions, the students needed to use elementary number theory concepts. The study uses inductive and discourse analyses to trace the development of the mathematical discourse between the students. It demonstrates how the computerized environment supports the construction of concepts which are tightly related both to the characteristics of the learning environment and to the relevant number theory content.*

Introduction

Understanding elementary number theory concepts is fundamental for many branches of mathematics, yet it has received only minor attention in the research literature. Existing studies fall into three main categories: one, studies in which elementary number theory concepts are used to investigate other mathematical issues (Martin & Harel, 1989; Leron, 1985; Lester & Mau, 1993; Movshovitz-Hadar & Hadass, 1990); two, studies that explore the multiplicative structure of numbers through problem solving (Ball, 1990; Graeber, Tirosh & Glover, 1989; Greer, 1992); and three, studies that explore understanding of elementary number theory concepts by pre-service teachers (Zazkis and Campbell, 1996a; Zazkis and Campbell, 1996b; Zazkis, 1998; Zazkis, 1999).

In this study we use the qualitative methods of inductive analysis and discourse analysis to trace the construction of elementary number theory concepts by seventh grade students, engaged in computerized explorations of number theory concepts such as prime number, divisor, and greatest common divisor (gcd). Under the present space limitations we can only sketch the theoretical framework and hint at some of the results. For full details, see Lavy (1999).

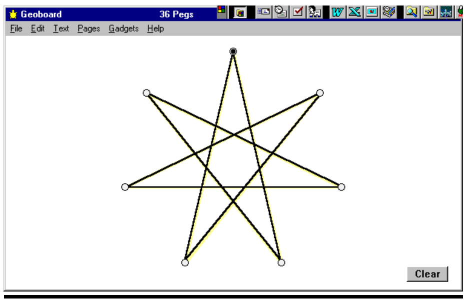
Methodology

The computerized environment under study consisted of MicroWorlds Project Builder (MWPB) – "a Logo-based construction environment which, in addition to retaining the expressive capacity of the Logo programming language, has a number of useful object-oriented features and facilities for direct manipulation which can be used to promote links between action- and conceptionally-based ideas." (Hoyles & Healy, 1997)

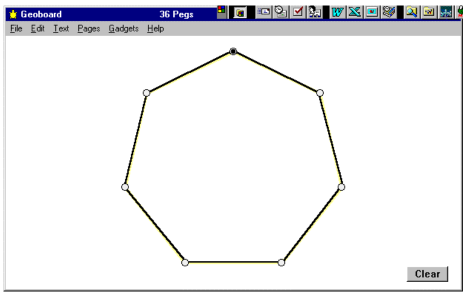
The *geoboard* is a mathematical package implemented on top of the basic MWPB environment. It consists of a simulated round board with pegs at constant intervals on its perimeter. The students can change the number of pegs by using the logo instruction: *newboard n*.

For example, *newboard 8* will create a 8-peg Geoboard.

A class of ten 7th-grade students met several times after school hours in the computer lab, and explored the effects of the instruction *repeat n [jump k]* on geoboards of varying number of pegs. The command *jump k* results in a line segment being drawn from the "current" peg to the k-th peg counting clockwise, which now becomes the new current peg. The statement *repeat n [command list]* results in the list being executed n times in succession. Each choice of specific values for n and k resulted in a screen display of a regular polygon or a star with varying number of vertices (Fig. 1). The students were encouraged to look for mathematical patterns connecting the input numbers and the shapes and the number of vertices of the resulting polygons or stars. These investigations led to the emergence in the students' discourse of concepts such as prime number, divisor and greatest common divisor (gcd).



A 7-peg star (jump size: 3 or 4)



A 7-peg polygon (jump size: 1 or 6)

Figure 1: Shapes obtained by the command *repeat n [jump k]*

We have chosen to concentrate on one couple, Noam and Jacob, whose actions and screen productions were captured by a Video camera. These students were selected because, more than all the other students, they tended to "think aloud" during their work.

The major part of the research data is the verbalized discourse, which took part between the students during the activities. The research data included in addition to the verbalized discourse, the screen pictures at every stage of the inquiry, the students' body language and every piece of written paper they produced.

The data were analyzed by two kind of qualitative methods: inductive analysis and discourse tools. Inductive analysis (Goetz & Lecompte, 1984) is a method which integrates between scanning the data looking for phenomenological categories, and successive refinement of them when confronted with the new events and interpretations. According to this approach, the investigator is the primary instrument of data collection and analysis. In keeping with this approach, there were no predetermined criteria or categories made.

To get a better understanding of the learning process, two discourse tools were applied: interaction analysis and focal analysis. These tools derive from the premise that in order to understand how students learn, we cannot separate cognition from socio-cultural influences: the way we express ourselves may throw light on how we understand. The research object is the discourse through which the pair under study interpret each other's ideas, thus gradually co-constructing their shared reality.

The first discourse tool is interaction analysis (Sfard & Kieran, 1997) through which we examined the interactions between discourse participants, with the help of a diagram constructed from the data according to a set of simple principles (ibid). The second discourse tool is focal analysis (Sfard, 1998), which is concerned with the discourse content and highlights additional aspects of the discourse between the students. Here the researcher interpreting students' utterances is trying to understand what students *actually see*[1] when they express their ideas.

Analyzing the mathematical discourse of the two students enabled us to get a better description of their personal learning profile, and to characterize more succinctly their individual contribution to the mathematical discourse. Triangulation of the three methods enabled us to obtain a comprehensive point of view of Noam and Jacob's learning process.

Results and discussion

Through a detailed analysis of the data scripts, we came up with four categories which were then refined in each iteration of the data review. We called the emerging categories *utterances types*, *number theory concepts*, *argumentation* and *rules*.

We used *utterance types* to categorize the different kinds of mathematical utterances Noam and Jacob used through the inquiry. These included general utterances like, "we need one [number] which has many divisors.

[1] a real or imaginative action.

You understand why" (Noam); specific utterances like, "fifteen. It has a common divisor with twenty" (Jacob); and general uttrences masked as specific: "twenty four less the jump". Noam phrased this utterance when they were investigating the equivalence of *repeat n [jump k]* and *repeat n [jump n-k]*. He used the number 24 even though they were working with a 15-peg board, hence our conclusion that 24 (the "canonical" number of pegs in these investigations) stood for a general number of pegs.

There were two types of *number theory concepts*: the first type involved explicit geoboard notions: "With such primes, there will be only one simple polygon and all the rest will be stars". In the second type the number theory concepts appeared in a purely mathematical context: "Is twenty seven prime?"

Argumentation included the instances in which one student phrased an argument and tried to convince his colleague of its validity.

Rules included all the instances in the discourse in which the students tried to formulate mathematical rules.

The process of categorization helped us to characterize the learning profile of Noam and Jacob, to describe the concept development during the inquiry, and to understand the contribution of each of the students to the shared learning process.

During two sessions of inquiry, the students phrased 46 tentative mathematical "rules", most of which being related to two main concepts. Adopting the students' own terminology we called the first concept *n-star* and the second concept *common denominators*. In what follows we will describe the concept of n- star.

n-star is a star polygon with the same number of vertices as the number of pegs on the geoboard. The actual name used by Noam during the investigation was in fact *24-star*, but since he used this name for all kind of geoboards, regardless of their number of pegs, we thought the name n-star was appropriate.

After Noam and Jacob had been exploring the Logo instruction *repeat n [jump k]* on a 24-peg geoboard, Noam noticed that certain values of k yielded a 24-star. He said to Jacob, "Wait, let me try again, I want to see something here". He could not quite formulate yet what he saw, and he needed to check a few more values of k, and then he said: "I want to check something, ok? I think I reached some pattern in 24-stars of all the vertices." Jacob tried to formulate his interpretation to Noam's discovery but he seemed unaware of what was special about this star: "look, if you put a prime number, it will give you a star". At this point Noam made a correction: "no, not just a star, a 24-star". Jacob then rephrased Noam's idea by saying "star of all [vertices]", and Noam repeated, "yes, star of all, this is the pattern I reached”. From this point on both students continue to use the new concept naturally in their discourse.

The development of the concept n-star reveals a process of *concept refinement*. At first the students use the utterance “a jump of prime number makes a star”; later they use the utterance “a jump of prime number creates an n-star”; then they refine their utterance further by saying, “a jump of a number that is disjoint to n [the number of pegs] makes an n-star”; finally, they use the utterance, “a jump of a prime number which is not a divisor of n, makes an n-star”.

The construction and the refinement of the concept n-star has been achieved via a mutual effort of Jacob and Noam, and we will now follow this construction by identifying the specific contribution of each student to the process (Figure 2).

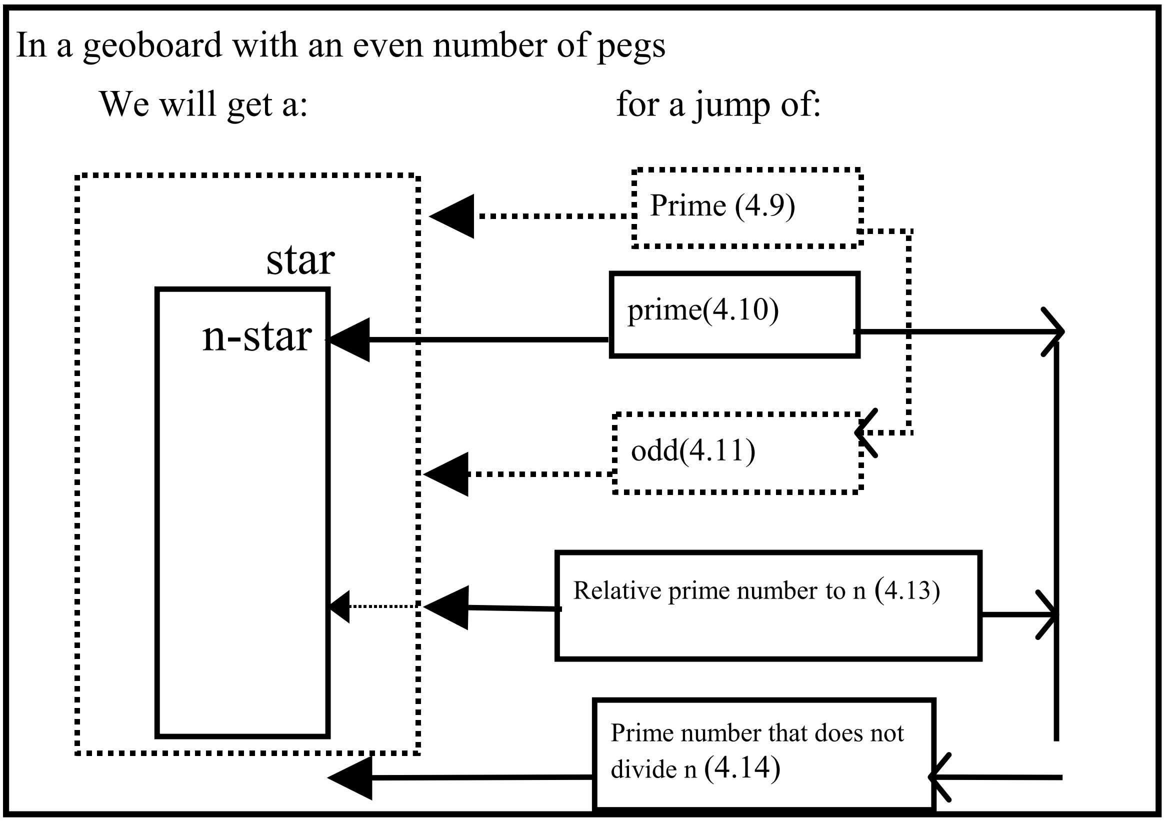


Figure 2: Schematic description of shared construction of the concept n-star

Fig. 2 describes the construction of the n-star concept as a joint endeavor of Noam and Jacob. The broken lines refer to Jacob's part and the solid lines refer to Noam's part. In the rectangles on the right appears the size of jump and the in the rectangles on the left appears the geometric outcome – a star or an n-star. (The numbers in parentheses refer to the utternace number in the original data).

The first rectangle in Figure 2 refers to Jacob's utterance (4.9), that in an *even geoboard* (this is our own shorthand for a geoboard with an even number of pegs) a jump by a prime number will create a star. Few minutes later Jacob phrases utterance 4.11, that it takes an odd number of jumps to get a star. (It looks as though Jacob has regressed in his understanding at this stage of the inquiry: analyzing the rest of the verbalized discourse reveals that Jacob confuses between odd and prime numbers.) The second rectangle refers to Noam's utterance (4.10) in which he characterizes the star as a special case of n-star. This is Noam's discovery and he is the one who first coined the concept n-star. This utterance is a refinement of Jacob's previous utterance (4.9), in which he was not yet aware that this was not just any star, but actually an n-star. The fourth rectangle refers to Noam's utterance (4.13) in which he emphasizes the fact that the size of jump is a number that does not divide n. Although Noam used the word "star" in this utterance, looking at the preceding and following transcript, it is clear that he meant an "n-star" (hence the broken arrow in Figure 2).

The fifth rectangle refers to Noam's utterance (4.14) in which he connects 4.13 with 4.9 and arrives at the conclusion that in an even geoboard, a jump by a prime number that does not divide n creates an n-star.

We have used interaction analysis to characterize the learning profile each student brought to the discourse, and how it helped them to construct a shared meaning of the learned concepts and how it contributed to each one’s individual learning. Applying interaction analysis to the discourse concerning the development of the concept n-star, reveals that Noam has a rich private channel which he uses in the inquiry process. Jacob, on the other hand, makes an effort to follow Noam’s actions and tries to communicate with him. Noam’s utterances are mainly concerned with mathematical interpretations that he makes for himself during the investigation, while Jacob tries to penetrate Noam’s private discourse in an effort to take part in Noam’s hidden processes. Significantly, most of the time Noam is the one who types the Logo instruction at the keyboard. When we pointed this out to Jacob he said, “he [Noam] is good at it”. Most of Noam’s utterance were completed or explained with certain screen images and came as reactions to Jacob’s questions. Noam was busy with his personal discourse while he was trying to figure out the connections between the polygons or stars appearing on the screen and the Logo instruction he was typing. Jacob, on the other hand, helped Noam to work systematically and to run a more systematic investigation. Noam uses an “examples language” even when he talks about patterns. Jacob, on the other hand, tries to generalize even when his generalizations are sometimes incorrect. Many times during the discourse Noam's utterances are corrections to those of Jacob, for example, when Jacob says to Noam, "look, if you take a prime number it will give you a star", Noam answers, "no, not just a star, a 24-star". Since Noam find it difficult to express himself verbally, he relies on Jacob's utterances, and by using them as a basis and rephrasing them, they finally arrive at the correct conclusion. In fact, each student functions as an "expert", promoting his partner further within his zone of proximal development (ZPD). Noam, who is adept at handling the computer, helps Jacob understand the mathematical regularity of the geoboard. Jacob, on the other hand, pushes Noam into a more systematic investigation, which in turn leads Noam towards new directions to explore. The way Jacob formulates his ideas helps Noam articulate his own more clearly.

The n-star concept emerged from a synthesis of the geoboard learning environment (stars and polygons, the number of pegs n and the jump k) and the related number theory notions (primes, divisors, gcd, etc.) The construction of the n-star concept was the result of a process in which the students learned how to generalize from examples, create and refine hypotheses, and see mathematical patterns through the screen images.

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