# CHARTING ARGUMENTATION SPACE IN CONCEPTUAL LOCALES

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*We describe a procedure for developing pedagogical knowledge about the potential argumentation space of groups of children in a conceptual locale. The method used involved the collection of small groups of children who had made significantly different responses to diagnostic test items, and the recording and analysis of their subsequent researcher-managed arguments in discussion. An example of the method is presented which shows how groups of 11-year old children developed arguments about the ordering of decimals, in response to a classic diagnostic item involving the ordering of 185, 73.5, 73.32, 57, 73.64. The analyses of these discussions led to a chart of the key elements of argument that arose, as well as general strategies for managing such discussions that were productive. These are considered as devices for helping teachers to plan argumentation in their classrooms.*

## Introduction

The identification and characterisation of the way children understand the mathematics presented to them in the curriculum has long been the focus of research in the psychology of mathematics education and continues to be a source of empirical and theoretical investigation. While much of this work was, and is still, conducted within a 'misconceptions' or 'alternative frameworks' paradigm, there is continuing development of work on children's mathematical thinking which elaborates on contextual, social and socio-cultural factors, and on the significance of inquiry discourse. (See for instance, Kirschner & Whitson, 1998, Forman & van Oers, 1998, Cobb & Bauersfeld, 1995 and Cobb et al, 2000.) In this study we are interested in how children may reveal and develop their understanding through collaborative argument in group discussion. We are particularly interested in discovering and describing productive lines of argument in relation to particular content conceptual locales, which may help teachers to develop productive discussions in their regular classrooms.

It has long been 'known' that children's errors and misconceptions can be the starting point for effective *diagnostically-designed mathematics teaching*. The key mathematical work on this in the UK was done in the 1980s by the ESRC Diagnostic Teaching Project (Bell et al, 1983), in which cognitive conflict was seen as the route to developing understanding. Argument in discussion between conflicting positions is seen as one important source of such conflict. The TIMSS video study reported that Japanese mathematics teaching typically makes use of a diagnostic approach: teachers are prepared with notes on a variety of *likely responses* to a key lead question, with guidance as to the thinking these responses indicate, and constructive teaching suggested related to each (Schmidt et al, 1996). This was related to the success of Japanese children's mathematical learning and particularly their problem solving capabilities. Dialogic methods involve the characteristics of conversation and the rigours of reason and persuasion: *sustained* talk and listening, statements of understanding or thinking-in-progress, the use and consideration of evidence, cognitive conflict and the making of new connections (Andrews et al, 1993; Costello et al, 1995; Inagaki, Hatano & Morita, 1998; Ryan and Williams, 2000).

## Charting argumentation space: a methodology

There can be no genuine discussion or argument without a 'problematic', ie. an unresolved or not trivially-resolvable problem. This induces some purpose and some tension that sustains a discussion. The problematic for a particular group of children was established through prior testing which provided a range of student responses and methods of solution. The children were set the task of persuading each other by clear explanation and reasonable argument of the answer. The giving of clarifications, reasons, justifications and informal 'proof' was the rationale for the discussion. We use the term 'argumentation space' to describe the collection of relevant arguments likely to be used productively in children's arguments about a particular problematic. In this paper we show how we are beginning to chart argumentation spaces in ways which may help teachers to plan classroom discussions to develop productive arguments. In addition we outline the main strategies which we found supported productive argument in group discussions.

In this study a primary school cohort of 74 'year 6' (i.e. 11-year old) children was *screened* with a test that was designed to reveal common errors that had already been identified as relevant to their mathematics curriculum and level (Ryan & Williams, 2000). Essentially this involved identifying the most important common errors on tests for which we had collected a National sample of data (N = 1759) covering the entire mathematics curriculum for Key Stage 2 (end of UK primary school). From these errors, which had been coded and entered into a Rasch analysis (Ryan, Doig & Williams, 1998), we identified the most interesting errors based on the criteria that they should be: (a) common enough to reward a teacher's attention, (b) relevant to a significant locale of the curriculum being taught at the given age level in focus and (c) significant in terms of the literature on the psychology of learning.

The result was a diagnostic pencil and paper test of some 30 items lasting about 30 to 40 minutes and (later) a 20 minute mental test. The test items were drawn from the whole primary school curriculum. By way of an example, we will cite the case of an item called 'Ordering', which asked children to sort the numbers 185, 73.5, 73.32, 57, 73.64 from smallest to largest. In the National sample we found two common errors as expected: 57, 73.5, 73.32, 73.64, 185 ('decimal point ignored') and 57, 73.32, 73.64, 73.5, 185 ('longest is smallest'). These errors had been identified in the APU study of the early 1980s (Assessment of Performance Unit, 1982). The 'decimal point ignored' error is believed to have an important bearing on the development of children's number concept, and is typical of children's over-generalisation of whole number conceptions to the wider field of rational numbers. From each of the three year 6 classes, we selected 4 children for each discussion group on the basis that they had provided a range of responses on the test items. There were 9 groups (36 children). The children from each group were from the same class and knew each other well, though were not necessarily from the same friendship group. Their teachers advised us on the likely successful dynamics for each group. They were mixed groups of boys and girls. Each group was withdrawn for discussion and videotaped in sessions lasting from between 30 to 50 minutes. Most children were involved in two sessions of taping. They recalled their test item response (an interval of a few days only) and were invited to present an argument for their response to the group. We, as researchers, adopted the teacher's role in discussion: generally four students at a time. All discussions were transcribed and analysed.

The analysis of argument follows Toulmin's scheme in general (developed by Cobb & Bauersfeld, 1995 and Cobb et al, 2000; Krummheuer, 1997, and others). Propositions relevant to the issue are 'backed' by arguments that are then subject to testing. In general children find it unnecessary to argue propositions which are believed to be shared, (i.e. taken-as-shared) so any particular discourse reflects the presumed shared points of departure, including the rules of argument in such situations. In this, the Researcher as a quasi-teacher, assumes the authority and seeks to ensure reasonableness, the need for the inquiry to persuade by good thinking and argument, and so on (Costello et al, 1995). An important role in productive argument may be played by tools in practice, which may provoke the formulation of connections between components of mathematical knowledge, new constructions and hence productive backing. The number line has been shown to play a significant role in many such contexts, and does so in the following example.

## Results

Here we present part of one transcript for an argument about the ordering of decimals. Some commentary and analytical categories used are shown in bold to the right:

Kim: OK. I put 57 there – …. Then I put 73.5, … Then I put 73 point thirty-two, then I put 73.64 point sixty-four, then I put 185. **Everyday language**

Natalie: Well, I got 57 at the beginning too. And then I got 73.5. Then I got 73 point three- two Then I got 73 point six- four. Then I got 185. **Mathematical language**

RES: Could you explain why you put 73.5 before 73.32 (three, two)? **Focus**

Natalie: Because 73.32 (three, two) has got two digits after the decimal point and 73.5 has only got one. **Backing: separating decimal as wholes**

Elise: I'm not so sure, because 73.5 is basically 73 and a half. 73.64 (six, four) is, I'm not sure if it would be over a half or under…Actually I think the same as

Kim… because, like Natalie said, there are two digits there, and two digits there, and only one digit there.

**Intro fraction referent: conflict, backing**

RES: What do you think Richard?

Richard: Same as Elise. **Check alternatives**

RES: The same… If I had a number line… Are you used to seeing a number line? (*children nod*). And I had 72. 72 would be back there. 73 would be there. 74 would be there. Where would you put 73.5? Do you want to do that Richard? **Intro tool: number line**

Richard: (*puts 73.5 half way between 73 and 74*) **Number line product**

RES: Can anybody put any other numbers in between 73 and 74?

**Check alternatives. Press**

Kim: Yeah (*puts 73.64 above 73.5*) **Number line product**

RES: Why have you put in bigger than 73.5? **Check backing**

Kim: Because it's over a half. **Backing:fraction equivalent**

RES: Any other numbers you could put on that number line?
Do you want to have a go Natalie? **Press**

Natalie: 73 point two-five **Number line produces new argument!**

RES: 73.25 (?), where would that go?…
Could you tell us why you put 73.25 *just* there? **Focus on 0.25**

Natalie: It's a quarter of the number. **backing: fraction equivalent**

RES: Do you agree with that? (*children nod.*) So, it's gone… why has it gone exactly there? Is that because it is halfway towards a half? **Check backing**

Natalie: Yeah.

RES: Could you put a number on that number line Richard? **Develop number line**

Richard: Erm, 73.45…(*places it between 73.25 and 73.5… places 73.75 between 73.5 and 74*).

RES: 73.75, right? That's …? **More 2-place decs**

Richard: Three-quarters. **Backing: fraction equivalence**

RES: So you put that halfway between 73 and a half, and 74…Where do you think 73.32 should go

Kim: Before 73.5

RES: Why?

Kim: Because 73.5 is a half and 73.32 (?) is just after a quarter. **Resolution of referents**

RES: Could you say *why* it's just after a quarter? **Check backing**

Kim: Because a quarter is 73.25 (?) and 73.32 is bigger than 73.25 (*All agree*)
I now think 73.32 is there, and 73.5 is there. **Kim sees change of mind**

RES: You all want to change your minds now?
Now why did we go wrong in the first place? **Seeks reflection**

Kim: Because we saw them as two-digit numbers, and we thought that the two-digit numbers were more than a one-digit number **Making new explicit**

Elise: I would say that 73.25 is a quarter, and it's less than 73.5 because that's a half, and 73.32 is just over a quarter, so it would be just under 73.5

**Fraction-decimal explicit**

Extracting the most productive and essential elements of this and other arguments about 'decimal point ignored' allow us to make a summary chart (Fig 1 below). This summarises the lines of argument we found that we think teachers will find useful in preparing for a particular discussion about 'ordering'.

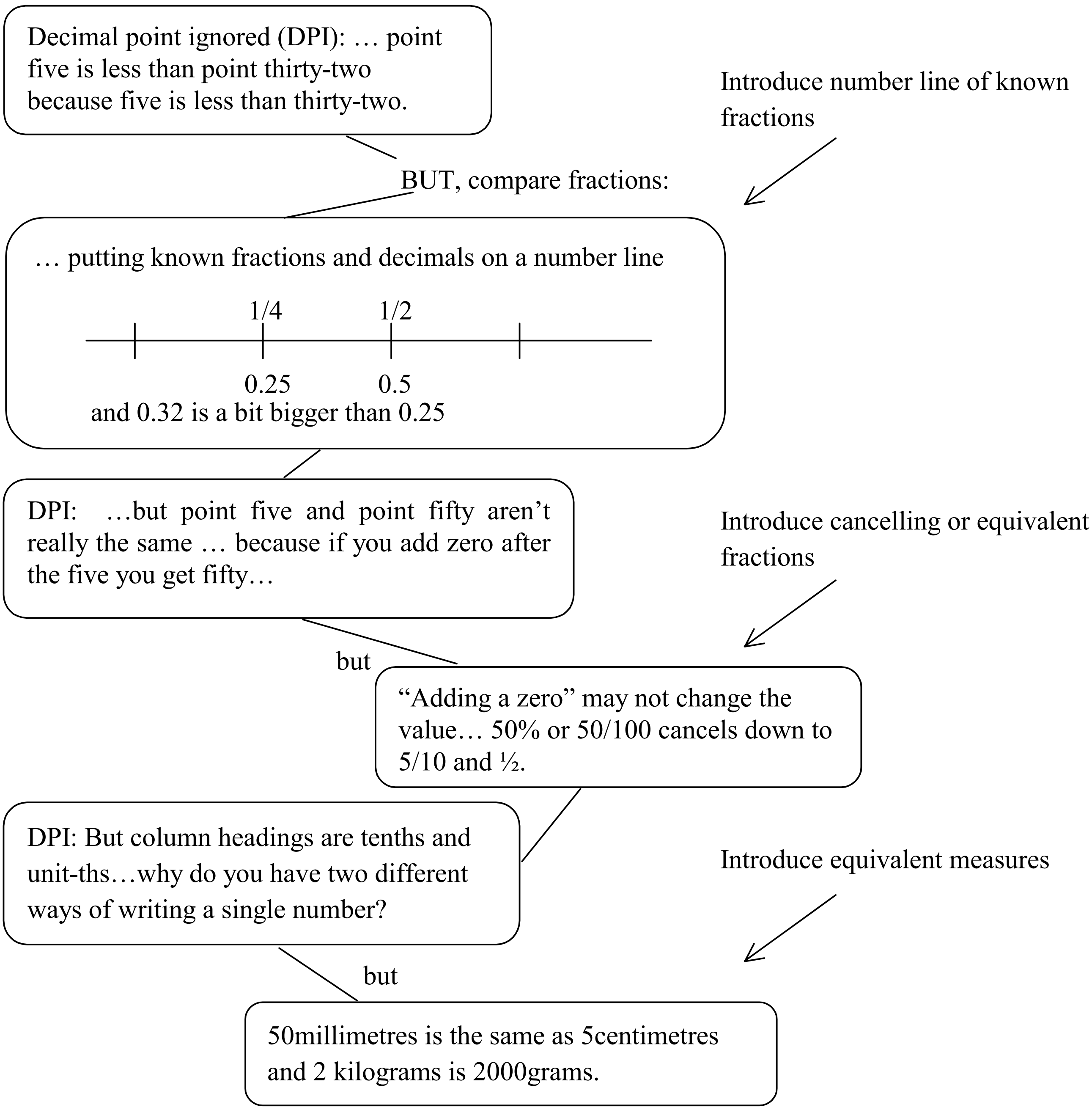


Fig 1: A chart of argumentation space for 'Decimal Point Ignored'

Note the arguments advanced are 'backed' in Toulmin's sense (Toulmin, 1969) by the introduction of three key tools and references, without the introduction of these by the teacher/researcher or by a child, the arguments may critically follow different lines. These were:

- the placing of known decimals and fractions on a number line (e.g. 0.5 at ½, 0.25 at ¼).
- the equivalence or cancelling of fractions (e.g. 50% = 50/100 = 5/10 = ½)
- the equivalence of metric measures (e.g. 500mm = 50cm = 0.5m).

The difference between numbers with one decimal place and two decimal places is particularly critical here, as was evident in this episode when 0.25 was placed at ¼ on the number line. In another discussion the critical difference was manifest in argument about the value of 0.5 and 0.50, which were considered by some children to represent different numbers, even with different places on the number line. One child actually suggested that 'nought point five' should perhaps be written as 0.05 when ordered with 'other' two decimal place numbers. This discourse manifests a conceptual world of whole numbers extended to decimal numbers which consists of pairs of numbers separated by a point, ( i.e. *x* point *y*) and that the number after the point is, or should be, a fixed length of string digits. This is the root of DPI errors and their backing arguments.

In the next stage of the analysis we sought to categorise the researchers' inputs to the discourse and influence on the discussions in general. Being aware of the children who made the common errors in advance, the researcher manifestly sought to ensure that the arguments for the error were clearly voiced, as well as to ensure that potentially productive tools and referents were introduced at some point: hence the significance of the particular conceptual locale to pedagogical content knowledge.

However, we examined general teaching strategies that seemed generally productive across problem contexts and conceptual locales. In eliciting and sustaining argument, we include the eliciting of variety of 'answers' and arguments, asking children to listen and sometimes paraphrase others' views, seeking further clarification of arguments, (sometimes helping to formulate and encourage a minority point of view), seeking alternatives and dissent, and seeking reasons and 'backing'. These are all strategies that forestall closure and encourage productive conflict. In the final post-resolution stage of discussion, strategies that encouraged reflection included asking children whether and why they had changed their mind, what the argument or misconception had been, and how they would summarise what they had learnt.

## Conclusions and discussion

We have shown that it is possible to use dialogue generated in research to chart an argumentation space which describes the children's arguments in response to provocative diagnostic items in a conceptual locale. The concept of an argumentation space located around a diagnostic item is designed to be helpful in supporting teachers' pedagogical content knowledge. We interpret these spaces as providing potential classroom discourses structuring potential zones of proximal development of individuals within a class. The dialogues teachers might generate in replication of the research setting might thereby provide opportunities for individuals to learn by testing their responses against those of their peers, and being given an opportunity to evaluate and shift their position accordingly.

We are currently investigating and evaluating how helpful these 'charts' can be to teachers in practice, and whether the resulting dialogues will be successful in helping children learn. The next step in the project involves a study with teachers delivering, marking and interpreting the diagnostic test and observation of their subsequent teaching through discussions based on these argumentation spaces. We will present some evaluation of its effectiveness in helping teachers to develop their practice at the PME presentation.

We believe that the approach adopted here has certain conceptual strengths and weaknesses. Clearly, desirable pedagogical content knowledge with respect to a conceptual field or even locale cannot be altogether encapsulated in one chart. In fact, even the literature in diagnostic teaching – a conflict-based method – is far from solely based on conflicting students in discussion: the use of particular tools and representations in particular have a most significant role. Furthermore there is a danger in peer discussion, often cited by teachers and in the literature, that students will be persuaded by the weakest of arguments.

However, the strength of developing this methodology for teachers as a tool in their practice is that it has certain general features as well as the particularities in the locally structured chart for a locale. Thus, we would hope that this approach as a teaching method will help teachers to improve their practice very generally. We are optimistic that the method will encourage at least some teachers to see themselves as teacher-researchers, and that they will wish to extend these or begin constructing their own charts for locales we have not yet explored.

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