# WHAT ARE WE TRYING TO ACHIEVE IN TEACHING STANDARD CALCULATING PROCEDURES?

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*Abstract*

*In societies that depend on calculators and computers for all important calculations, questions must be asked about the purpose of written calculation. Reforms in arithmetic teaching have led to a shift from the repetition and rehearsal of algorithmic approaches to a focus on developing pupils' own methods, but mastery of standard procedures remains a fundamental goal. This presents a dilemma for pupils who try to replicate a taught procedure which may not be their most effective solution strategy. This paper reports a study involving English and Dutch pupils (n=535) and highlights the way their efforts to implement taught procedures can inhibit more appropriate strategies using number sense.*

## Background

Classroom activities today involve pupils in observing patterns and explaining relationships so that they develop understanding of connections among different numbers and operations gaining a 'feel' for numbers often referred to as 'number sense' (Anghileri 2000). This term 'number sense' is widely used across the world in reform documents (NCTM 1989; AEC 1991) and refers to 'flexibility' and 'inventiveness' in strategies for calculating. It is a reaction against overemphasis on computational procedures and reflects 'new numeracies' (Noss 1998) that are more relevant to the skills and understanding needed in our social and working lives. At the same time, computational procedures remain an important element of the curriculum, for example, the National Numeracy Strategy for England states that 'at least one standard written method of calculation should be taught in primary schools' as these 'offer reliable and efficient procedures which, once mastered, can be used in many different contexts' (DfEE, 1998: 52).

## English and Dutch teaching approaches

The 'standard' procedures to be taught will vary from one country to another and will reflect the teaching that has led up to the stage of written calculations. England and the Netherlands have different teaching priorities culminating in different standard methods. The role of place value is emphasised from an early age in England where 'understanding about place value is required as a sound basis for efficient and correct mental and written calculation' (SCAA 1997: 4). In the Netherlands, in contrast, holistic approaches to numbers include the development of counting skills as the basis for calculating (Beishuizen and Anghileri

1998). For the operation of division these contrasting approaches culminate in different written procedures. Standard written methods for division in England are based on the traditional algorithm while the Dutch approach involves repeated subtraction with appropriately chosen multiples (chunks) for developing efficiency (Anghileri 2001). Progression from informal methods to standard written procedures is more clearly evident in the Dutch approach as will be shown in the results reported in this paper.

## Purposes of written calculations

Ruthven (1998) identifies two distinct purposes for using pencil and paper for calculating: 'to augment working memory by *recording* key items of information' and 'to cue sequences of actions through *schematising* such information within a standard spatial configuration'. The traditional algorithm is structured to 'direct and organise' (Anghileri 1998?), providing a highly efficient written method for solving problems but is not easy to reconcile with 'the way people naturally think about numbers' (Plunkett 1979). The formal procedure is also prone to errors in some cases due to its incompatiblity with intuitive approaches (Anghileri and Beishuizen 1998; Anghileri 2000). The Dutch Realistic approach uses contextual problems as a starting point and a standard procedure is evolved from informal approaches based on repeated subtraction (Gravemeijer, 1994) with whole numbers retained at all stages.

## Comparing the effectiveness of different approaches

Effectiveness of the different teaching approaches is compared in a study in the two countries. In cities with similar cultural characteristics, whole classes of year 5 pupils in ten English schools (n=276) and in parallel grade 4 classes in ten Dutch schools (n=259) were asked to write solutions to ten division problems. Pupils completed written tests in January involving five word problems and five symbolic ('bare') problems with similar numbers (Table 1).

**Table 1 - Ten problems used in the first test**

| context | bare | type |
|---|---|---|
| 1. 98 flowers are bundled in bunches of 7. How many bunches can be made? | 6. 96÷6 | grouping: 2-digit divided by 1-digit - no remainder |
| 2. 64 pencils have to be packed in boxes of 16. How many boxes will be needed? | 7. 84÷14 | grouping: 2-digit divided by 2-digit - no remainder |
| 3. 432 children have to be transported by 15 seater buses. How many buses will be needed? | 8. 538÷15 | grouping: 3-digit divided by 2-digit - remainder |
| 4. 604 blocks are laid down in rows of 10. How many rows will there be? | 9. 804÷10 | grouping: 3-digit divided by 10 - remainder |
| 5.1256 apples are divided among 6 | 10. 1542÷5 | sharing: 4-digit divided by |

| | | |
|---|---|---|
| shopkeepers. How many apples will each shopkeeper get? How many apples will be left? | | 1-digit - remainder |

The numbers were selected to encourage mental strategies and to invite the use of known number facts. The tests were repeated in June to establish changes in pupils' strategies. In the second test, numbers in the context and non-context problems were interchanged to reduce the influence of memory.

**Results**

Solution strategies were classified into 8 categories. Low level strategies involved making tally marks or repeatedly adding or subtracting the divisor **1(S)**, partitioning the divisor or dividend (or both) **2(P)**, or use of small multiples of the divisor in low level chunking **3(L)**. Efficiency gains were evident with repeated subtraction of large chunks **4(H)**, or use of the traditional algorithm **5(AL)**. Where there was a solution but no working the strategy was classified as mental **6(ME)**. Some solutions involved the wrong operation **7(WR)** or the strategy was unclear **8(UN)**.

Overall success was greater for the Dutch pupils who successfully completed 47% of the items in test 1 and 68% in test 2. English pupils successfully completed 38% in test 1 and 44% in test 2 (Table 2).

**Table 2: Percentage of questions successfully completed**

| Dutch test1 | English test1 | | Dutch test2 | English test2 |
|---|---|---|---|---|
| 47% | 38% | | 68% | 44% |

English pupils persisted longer in using low level strategies 1(S), 2(P) and 3(L) with 28% of attempts in test 1 and 22% in test 2. Dutch pupils used these strategies for 33% of items in test 1 but this reduced to 13% in test 2. With the large numbers involved pupils struggled to reach a successful solution using these strategies.

The most popular Dutch strategy in both tests involved repeated subtraction of large chunks, 4(H), which was often structured in a standard written format and was used for 41% of the items in test 1 and 69% in test 2. English pupils used the traditional algorithm, 5(AL), most extensively with 38% of items in the test 1, and 49% in test 2 attempted using this approach. The Dutch standard method, 4(H), led to a correct solution in 74% of attempts while only 47% of the English attempts to use the traditional algorithm 5(AL) were successful.

Mental methods were used equally by the Dutch and English pupils (11% of all items) with almost equal success (6% Dutch/5% English).

**Progression**

In the solutions of Dutch pupils there was evidence of progression from repeated subtraction of the divisor, to subtraction of small multiples/chunks which often involved long calculations, to efficient use of large multiples/chunks in a standardised written procedure (Beishuizen and Anghileri, 1998). At all stages, whole numbers were used and the written structure developed was the same for 1-digit and 2-digit divisors. Pupils written solutions showed extensive use of the structured written procedure at different levels of efficiency (Figure 1).

**Figure 1: Dutch written procedure for division**

Progression was not evident in the English strategies where idiosyncratic written methods based on mental strategies did not appear to relate well to the traditional algorithm. Informal methods generally lacked any written structure and it was evident that difficulties arose for some pupils in following through their own working to give a correct solution (Figure 2).

**Figure 2: Unstructured recording of an English pupil**

For a 2-digit divisor, attempts were made to partition the divisor or to operate on separate digits. A typical example was the problem 64 ÷ 16 which was solved by first dividing by 10 and then by 6, or as 6 ÷ 1 = 6 and 4 ÷ 6 = 1 r 2 (wrong use of the commutative rule) (Figure 3). Some pupils appeared to get stuck trying to divide 60 by 16. Use of the formal procedure appeared to preclude any return to an informal approach and inappropriate results written in the answer space suggest that a written procedure had been followed which took no account of the approximate answer (Figure 3).

**Figure 3: Errors with the traditional algorithm**

## Single digit divisors

In addition to having a procedure that related well to informal thinking about division, better results for the Dutch pupils may be explained by the fact that they meet division by a 2-digit divisor in grade 4 (Y 5) while most English pupils will meet only 1-digit divisors. Results were compared for those items involving only a single digit divisor. Scores in test 1 were close for the English and Dutch pupils with averages of 45.5% and 47.25%. Both were more successful in dividing the 2-digit numbers that in dividing the 4-digit numbers. In test 2 the English results improved to average 55% over the four problems while the Dutch result was 71% successful.

Improvements were similar for the items, 96÷6 and 98÷7, with Dutch/English increase in correct answers 8%/5% and 22%/21% respectively. For the 4-digit numbers, 1256÷6 and 1542÷5, the Dutch improvements were higher than those of the English children, with increases of 29% and 36% compared with 2% and 10%.

English pupils used the algorithm with low success rate for the 4-digit numbers. The Dutch pupils used repeated subtraction with large chunks and although the success rate is not as high for the 4-digit numbers, differences were less marked (Table 3).

**Table 3:Percentage use and effectiveness of the most popular strategies for test 2**

| | Problem | 96÷6 | 1256÷6 | 98÷7 | 1542÷5 |
|---|---|---|---|---|---|
| English test 2 | traditional algorithm 5(AL) | 66 (51) | 67 (21) | 66 (52) | 70 (34) |
| Dutch test 2 | repeated subtraction of large chunks 4(H) | 78 (69) | 72 (50) | 76 (69) | 71 (52) |

The figures in brackets give the percentage of correct attempts.

Errors by the English pupils included missing digits in the answer, but also many confused attempts often leading to impossible (and sometimes bizarre) answers (see Figure 3).

## Overall improvements

When individuals' scores were compared for test 1 and test 2, changes in score varied from +9 (e.g. 1 correct in test 1 and 10 correct in test 2) to –6 (e.g. 8 correct in test 1 and 2 correct in test 2). Again, the scores of Dutch pupils showed better improvements with 69% improving their score while almost half of the English pupils (49%) showed no improvement or a deterioration (Figure 4).

**Figure 4: Changes in score from test 1 to test 2**

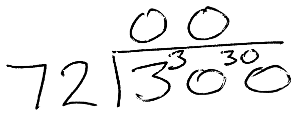
## Conclusions

The Dutch approach to written division calculations, involving repeated subtraction using increasingly large chunks, builds progressively on a mental strategy and retains whole numbers at all stages. The success of the Dutch pupils reflects their mastery of an increasingly efficient approach that has the flexibility for individuals to use the knowledge of multiplication facts that they have. On the other hand, the traditional algorithm extensively used by the English children, introduces a schematic approach that focuses on separate digits with their true value implicit, rather than explicit. Not only is the traditional algorithm more difficult to understand and prone to errors, but also progression to division by a 2-digit divisor requires substantial adaptation that is not intuitively clear to pupils. There is no flexibility in the choice of multiplication facts that can be used and links with mental methods are not clear.

When a standard procedure for calculating is taught in school it appears to take precedence over informal methods and implementing the procedure can be at the expense of making sense of a calculation. A problem such as 64÷16 caused great difficulty to the English pupils because it does not respond readily to the traditional algorithm which was used in preference to informal approaches. Instead of *recognising* the number relationships involved, pupils used a *procedure* cued by the operation.

Developing efficient procedures that relate to pupils' knowledge of numbers and to their intuitive understanding is crucial for developing the confidence that will encourage pupils to work on making sense of problems they meet. When presented with a meaningful problem the two approaches illustrated below show how the algorithm can lead back to the original calculation while a procedure that is better understood can encourage a solution that goes beyond the minimal requirements of a pure arithmetic calculation.

A farm shop sells about 72 eggs each day. How many days will 300 eggs take to sell?

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  300
- 144  (2 days)
  156
- 144  (2 days)
   12
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So it will take 4 days and a bit so probably about 11:00 on the 5th day.

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