UNKNOWNS OR PLACE HOLDERS?

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This reaction paper examines the cluster of algebraic concepts associated with variables used either as unknowns or as place holders. *Unknowns* are variables that represent particular numbers. As such, these variables appear in equations, whose solutions are a particular number, and the equations, themselves are about that number. In contrast, *place holders* are variables that represent any number. Such variables appear in identities and functions. These algebraic sentences are generalizations about particular operations and the order in which these are applied.

In the paper by Carraher, Schliemann, and Brizuela, a distinction is not clearly made between the particular algebraic meanings associated with each of these two variables. The term “unknown” is sometimes loosely used by the authors, and the children in the study make little distinction between “N” as a representative of an unknown quantity or as a generalization for “any number.”

Unknowns and Equations

The solution of the piggy bank problem utilizes unknowns. Even though the initial amount of money in each child’s bank is not given, the context of the problem makes it clear that this amount, while not specified, can only be a certain value. Thus, the reasoning that takes place as the children work through the activity is that of determining the exact value of this amount. The words that several of the children use as they talk about the problem do not align with this notion of variable.

For example, as the children discuss the quantity of money in the banks on Sunday, several students state that “N” could stand for “anything” or “any number.” Conceptually, this is not correct. N stands for a particular amount of money that is actually in the bank, an amount that the children can find if they reason through the given situation.

The classroom emphasis on N as a number that is not known is important. It is clear that the children are able to perform operations on such an entity and can comfortably explain the quantitative relationships that exits on the different days described in the piggy bank problem. However, it is also important to help the children understand that, while unknown, the number in the given context cannot be "anything."

The solution to the piggy bank problem embodies the algebraic concept of *equation*. The implicit understanding of the activity is that it is possible to perform operations on an unknown number, within the constraints of a given situation, in order to determine the value of this number. In the classroom, the use of expressions involving N and the N-number line provide an alternative representation for the conventional symbol string that comprises an algebraic equation in one unknown.

It would be useful to have held a discussion, before the children began their number line representations, about whether it could be possible to find the exact amount of money contained in each bank on Sunday. Children should have been able, given the result that Mary had $9 in her bank on Thursday, to have decided that the original amount would have had certain limits placed on its possible value. For instance, Mary would have needed to have spent a great deal more money if she had started with $100 in her bank on Sunday and ended up, on Thursday, with only $9 left.

Then, the activity could proceed as it did, with comments on the fact that the amount of money is not known at that time. After the specific value had been found at the end of the activity, it would also be appropriate for the teacher to remind the children that, by working through all the changes to the unknown that were given in the problem, they were able to find the exact amount originally in each piggy bank. It would have been a valuable extension to the activity if some reflective discussion could have been included to lay the foundation for the notion of an equation as a useful algebraic tool for finding the value of an unknown.

Dummy Variables and Functions

A second example of children dealing with an algebraic situation is briefly mentioned at the end of the paper. Here, children are discussing a comparison situation in which one child has three more candies than another. This example utilizes *place holders* rather than unknowns. However, the authors' use of the term "unknown" in the comment "that children can treat the unknowns … as having multiple solutions," has the potential for mixing conceptual entities. The authors do continue their discussion by commenting that "when children make statements of such a general nature they are essentially talking about relations among variables and not simply unknowns restricted to single variables."

In the comparison example, the children, as the authors correctly point out, are dealing with different conceptual entities than those used in the piggy bank problem. Here, the undetermined amount can take on any value, and the expression "N+3" represents a function. This expression provides information about a particular order of operations ("add three"), and the focus of the activity is not on finding a number, but on stating a relationship.

Discussion

It is important to use separate names to distinguish the two different uses for algebraic variables, since each use represents a different type of algebraic entity. Unknowns occur in equations that are essentially numeric in focus. The goal of using such symbols is to model a quantitative situation in which certain constraints make it possible to reach a numerical result. On the other hand, dummy variables are used as place holders to make general statements about mathematical relations, expressed as a particular sequence of operations.

The authors' descriptions (in the section "Some Reflections") of the ways in which the students used the number line illustrate how easy it is to blur the distinctions between the two clusters of algebraic entities. In a single paragraph, examples are given of representations for both functions and equations. In one case, students treat "minus four" as a displacement of four spaces, and recognize that the result of this operation can be represented as N-4 and as the movement from N+3 to N-1. In conventional algebraic symbolization, this is equivalent to, "if $f(x) = x - 4$, then $f(x+3) = x - 1$." In a second instance, students "infer" the value of N+43 from observing that N+7 is above the number 4. This is equivalent to solving the two-variable system of equations "$x + 7 = 4$ and $y = x + 43$."

In their concluding remarks, the authors state that "little is known about children's ability to make mathematical generalizations and to use algebraic notation." This statement is ambiguous in light of the previous remarks. It is not clear how the authors link "making generalizations" to particular uses of algebraic notation. I would prefer to make a clear distinction between using notation to make generalizations (place holders and functions) and using notation to find numerical results (unknowns and equations).

The authors' examples of young children using algebraic notation and number-line representations raise some interesting questions.

- Should the mathematical distinction between unknowns and place holders be a matter of concern in introductory algebraic experiences? (This distinction is one that many college-level students do not necessarily understand.)
- At what point in their educational experience should students first implicitly, and then explicitly deal with these conceptual issues?
- How deeply should teachers understand the algebraic structure behind particular introductory activities?

I agree with the authors that research is needed to investigate the extent to which difficulties in the development of algebraic understanding are either developmental or the result of educational practice. I would like to extend the focus of such investigations further, however, to include a careful understanding of algebraic entities themselves. Unless we as mathematics educators can fully unpack, for ourselves, this complex field of reasoning, we can only partially address the concerns of educating others.