WHY AND HOW TO PROVE:
THE PYTHAGORAS' THEOREM IN TWO CLASSROOMS

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We have long recognised the pedagogical significance of proofs, even informal proofs, in the mathematics classroom. The teaching of proof concerns itself more with meaning and understanding than validity. Wittmann (1996), in his case with the teaching of the Pythagoras' Theorem, has suggested that those diagrammatic proofs are conducive to students' construction of conceptual relationships.

Despite such theoretical concerns, however, we have been increasingly aware that for mathematics learning in the classroom context, a wide range of factors, intertwining with each other, are impinging on students' understanding of what purposes a 'proof' serves and how it constitutes a mathematical proof (e.g. Sekiguchi (1992)). Taking on board the significant shaping effect of everyday classroom practice, analyses of teacher-student interaction (mainly found in recent German mathematics-education research) from the sociological interactionist position look closely into the communication and negotiation process occurring in the mathematics classroom.

Against this general background and following the method of analysis proposed by Steinbring (1998), teaching episodes are taken from two classrooms (at Secondary 2 level in Hong Kong) during the teaching and learning of the Pythagoras' Theorem. Attempts in using Steinbring's conceptual tool, the epistemological triangle, open a window on our understanding of how students conceptualise the Pythagorean formula (i.e. the symbolic representation $a^2 + b^2 = c^2$) in relation to certain reference contexts. Although the teachers took more or less the same approach (i.e. started from a few examples and then went through the same 'dissection proof' to reach the general theorem), results of analysis reveal that the development of the mathematical arguments and explanations in the classrooms unfolded in their own self-referential ways, qualitatively different from each other. Among others, the difference lies in why the proof was perceived as needed, and also in how the proof was done. These two aspects are particularly considered as regards the possible meaning of proof conjointly constructed and emerged amidst the teacher-student interaction.

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