# Can interaction between inferior strategies lead to a superior one? The case of proportional thinking

Liora Linchevski and Baruch Schwarz, The Hebrew University of Jerusalem

Abstract

*The idea that interaction among peers leads to cognitive development is not new. However, this study focuses on an issue hardly been studied so far: this is the case when two children who do not succeed at solving a task by themselves, succeed to solve it when working in collaboration. A study by Schwarz, Neuman, and Biezuner )2000( showed that this phenomenon )we refer to as – "the two-wrongs-make-a-right phenomenon"( is possible. It depends on the kinds of tasks and of interaction between students, though. In this paper we report on a research-study that investigated "the two-wrongs-make-a-right phenomenon" in the domain of proportional reasoning. We focus on: a) the cognitive gains of couples in which both students are "wrongs" in comparison to the gains of couples in which one is an "expert" and the other is "wrong"; b( The effect of hypothesis testing on the cognitive gains of the individual.*

## Scientific background

The role of a child's interactions with her/his age peers in the development of his/her cognition, has long been discussed both from a Piagetian and a socio-historical perspective )e.g., Hartup, 1970; Vygotski, 1986). This study is in line with these efforts. However, it focuses on a phenomenon that has not been systematically investigated, the cognitive gains of two "wrongs" interacting to solve a task they are unable to solve individually.

A priori, it seems dubious that when two wrongs interact, at least one of them reaches cognitive gains. At any rate, it seems obvious that in two other kinds of interaction, the wrong should gain more: In modeling studies (in which one child observes a more competent child) and in active interaction studies in which the "wrong" interacts with a more competent child (e.g., Botvin and Murray, 1975; Murray, 1974; Kuhn, 1972(.

In this section we argue that a cognitive gain from interaction between two wrongs is theoretically possible. For this purpose, we analyze the factors suggested by researchers to be responsible for the cognitive development of peers in active interaction and in modeling studies. Two factors have been recognized, disagreement and being strategic (also designated as being able to give reasons/arguments for a specific solution or offering an operational solution e.g. Miller & Brownell, 1975(. It was suggested that when two interacting solvers disagree, their cognitive gains originate not only from a pragmatic component, the disagreement, but also from the contradicting solution itself. This suggestion was confirmed by a study conducted by Doise, Mugny and Perret-Clermont (1975) in which students gained from being presented a contradicting solution by an adult, whether the solution was correct or not. In another study, Doise and Mugny (1979) showed that interaction with a less capable child proposing a contradicting solution led even the more capable child to progress. In the same study, Doise and Mugny showed that when interacting students used different strategies, they progressed, whereas when they used the same strategies, they could not. A key result obtained by Doise and Mugny in their study of interactions between children with different levels of competence is that if the difference of level between the two students is too big, low-level students did not progress. As for modeling studies, Botvin and Murray (1975) showed that observing peers expressing and defending different (counter)-arguments is responsible for cognitive gains. Also Kuhn (1972) showed that in modeling studies, the observer who disagrees with the modeler is in a situation of mismatch, and he actually converses with the modeler in a kind of "tacit interaction".

In addition to disagreement and to being strategic Doise (1978) has showed that hypothesis testing, that is, the manipulation of materials available in the task to check whether the solution found is correct, also leads to cognitive gains. However, Doise (1978) showed that the more competent student is often too dominant when undertaking manipulations, preventing the less competent student from making gains on the task.

Glachan and Light (1982), on the basis of their review of modeling and active interaction studies, hypothesize that "interaction between inferior strategies can lead to superior strategies or, in other words, two wrongs can make a right" (p. 258). Glachan and Light grounded their hypothesis on the analysis of the very conditions that peer interaction studies have found as affording cognitive gains: disagreement, being strategic, and hypothesis testing. Such conditions do not depend on whether the peers are right or wrong. It ensues then that the active interaction among two wrongs can lead to cognitive gains when such conditions are fulfilled. The researchers suggest that such an interaction can be beneficial because the differing strategies being pursued by the two children lead to the making of moves inconsistent with those strategies. "The child is thus led to (jointly) make moves which he would never otherwise had made, so that established inefficient strategies are disrupted. As a consequence of this disruption one or both of the children may see possibilities for better strategies. Interaction is thus envisaged as a destabilizing influence" (ibid.). In sum, Glachan and Light hypothesized that two wrongs can make a right if they have different strategies and have opportunities to resolve their conflict.

The "destabilizing influence" alluded to by Glachan and Light has, without any doubt, Piagetian roots. It relates to stages of development. But what about the process that turns wrongs to rights? The analysis of the studies on adequate conditions for cognitive development through peer interaction shows that cognitive gains are intermingled with the utterance of arguments and counter-arguments. For example, Miller and Brownell (1985) showed that non-conservers are more likely to yield to conservers because conservers produce counter-arguments (see also, Doise et al., 1975). Botvin and Murray (1975) have shown that in modeling studies, the cognitive gains of observing peers originated from listening at solvers expressing and defending different (counter)-arguments leads to cognitive gains (see also Kuhn, 1972). And indeed in subsequent studies (Murray, Ames & Botvin, 1977), it was shown that cognitive dissonance taking place in argumentation or in observing conservers giving reasons for their points of view was the main source for significant gains.

The studies on peer interaction and cognitive development we just reviewed are to be related to studies directly focusing on argumentation. For example, Kuhn, Shaw and Felton (1997( and Schwarz, Neuman, Gil and Ilya (in press) showed that peer interaction fosters argumentative reasoning. In these studies, the researchers observed the arguments as outcomes of dyadic interaction and showed that their quality increased as a result of the dyadic interaction. The researchers did not analyze the conditions under which peer interaction occurs, but it seems that at least the two first conditions, disagreement and being strategists were fulfilled.

In a recent study Schwarz and colleagues (Schwarz, Neuman & Biezuner, 2000) have experimentally identified the "two wrongs make a write phenomenon" in the domain of decimal numbers. In this experiment, weak high-school students with conceptual bugs regarding comparison of decimal numbers were invited to collaboratively solve problems. They had a calculator at their disposal. It was shown that in pairs of "wrong" students having different bug, at least one of the "wrongs" turned to be "right". In contrast, this effect was not identified for wrongs in pairs in which one was wrong and one was right. Analysis of several case-studies in this experiment showed that the calculator was used to test hypotheses and was important in the argumentative process that led to change. However, the conditions that led to change were not systematically manipulated. The object of the current research is to deepen the study of the "two wrongs make a right phenomenon". This phenomenon needs to be examined in new domains and under new conditions in order to lead to general principles concerning gains in peer interaction. In the present study we aimed at empirically checking the "two wrongs make a right" phenomenon, in a new mathematical domain – proportional reasoning. In light of the research-review on peer interaction, we studied the "two-wrongs-make-a-right phenomenon", under three working assumptions: (a) the two wrongs disagree, (b) they have different strategies, and (c) active hypothesis testing is made possible. We aimed at evidencing this phenomenon by studying the effectiveness of the interaction between two-wrongs and comparing it with the effectiveness of the interaction between one wrong and one right (expert).

## Research Design

The choice of the mathematical domain and the specific tasks within this domain, has been made according to the following criteria: (1) there exist a diagnostic task measuring students' competence in the domain; (2) the number of strategies used to solve the task is limited; (3) The strategies are hierarchical; (4) The students are consistent in using the strategies. The topic of proportional reasoning and the "blocks task" )Harel, Behr, Post, & Lesh, 1992( fulfill these criteria. A detailed theoretical analysis of the blocks task appears in Harel et al. (1992). A short description of the task is given here.

*<u>Description of the blocks task</u>*: In Harel et al. the student was given a drawing representing two pairs of blocks (A, B) and (C, D(. A and C are "built" of big cubes while B and D are "built" of small cubes. The tasks appear in nine configurations (a configuration is a set of two pairs of blocks). In A there are always less cubes than in C. The variables are the number of cubes in B and D. There are three possibilities: the number of cubes in B (resp. in D) differs from the number of cubes in A (resp. in C) by 1, 0 or -1. The students were informed about the relative weight of A and B. The three alternatives are A>B (A heavier than B), A=B and A<B. The students are then asked to decide about the relative weight of C and D (4 possible answers are always displayed: C>D; C<D; C=D; "It is impossible to decide"(.

Harel et al. gave the 9 configurations of the blocks task to 18 Grade 7 students. Three types of strategies were uncovered. Harel et al. showed that the match representation-strategy was not random and only six such pairs (called solution process) could be identified. From these findings, it appears that the blocks task lead students to adopt solution processes that attest their level of proportional reasoning. However, it appears that for none of the nine configurations proposed by Harel and al. it is necessary to adopt a solution process attesting explicit considerations concerning the comparison of two ratios.

For our study we have made some modifications: )a( in Harel et al. the cubes in the structures A, B, C, and D had different magnitudes, and sometimes students needed to infer that a small cube weighted more than a big one, a fact creating a physical "disturbance" regarding proportional reasoning. In our case all cubes had the same size. The different weights were represented by different colors; (b( the concrete models of the blocks were presented in addition to the drawing of the blocks; (c( Harel's configurations led students to opt for additive solution processes (yielding correct answers); in our configurations the difference between the number of cubes in the different blocks were various; )d( the students were provided with a balance to check their hypotheses.

*Procedure*

32 lower-level Grade 10 and 11 students participated in the study. The study had three phases: individual pre-test, pair-interaction, and individual post-test.

*<u>Pre-test:</u>* The students were given the 9 configurations, one at a time, and were asked to justify their answers. Five levels of strategies were expected:

S0: Guessing [included in SP1 in Harel et al.[. The student totally ignores configurations A and B and does not provide any justification.

S1: Visual Shape level ]included in SP1 in Harel et al.[ The student ignores A and B, however he/she refers to the difference between C and D.

S2: Additive proportional level [corresponds to SP2, SP3 and SP4 in Harel et al.[ The student takes into consideration A and B. The explanations, however, are based only on the difference in the numbers of blocks among the 4 structures.

S3: Quasi-multiplicative proportional level ]includes SP5 and SP6 in Harel et al.[ The student's considerations are based on the proportional weight of a single block. However, he/she does not use it for a complete proportional reasoning.
S4: Multiplicative proportional level ]new solution process [A full multiplicative proportional reasoning. S4 was not found in the students' responses to the 9 tasks.

After the pre-test, tasks 1,2 and 6 were excluded. Tasks 1 and 2 were found to be trivial and task 6 was identical to task 8. Tasks 5, 7, and 9 were chosen for the interaction phase, and tasks 3, 4, and 8 were used as the post-test.

The students' answers were scored according to two scales: (1) Competence: right – R=1, wrong – W=0; (2) Strategies 0 to 3: S0=0, S1=1, S2=4, S3=7. Since in each phase of the research 3 configurations were involved, the jump of three in the scores between S1 and S2 ensured that a student that used S2 once will score higher than a student who used S1 three times. It was found that students might use a high strategy and give a wrong answer or vice versa. The decision was to score strategies higher than competence.

*Conflict Scale:* The "conflict" between two students was defined as the difference between them in competence and strategy on tasks 5, 7, and 9 in the pre-test. From competence perspective there were 3 possibilities: R&R (the two students are right on the task); W&R (one wrong one right); W&W (the two are wrong). In the framework of this research theoretical focus, the following scores were assigned: R&R = 0; W&R = 1; W&W = 2, the highest score (2) was assigned to two wrongs. For each task the possible range was 0-2 and for three tasks it was 0-6. From strategy perspective the range between the students' strategies (S0-S3) was 0-7 and for the three tasks it was 0-21. The final conflict-score between two students was calculated as the sum of these two components.

*Cognitive Gain*: Each student was given a *cognitive score* for each task. The score was defined as the sum of competence (0,1) and strategy (0,1,4,7). The cognitive gain was defined as the difference between the average of a student's cognitive-scores in the post-test (tasks 3, 4, 8) and the pre-test (on the same tasks). We calculated also the cognitive gain on the basis of the interaction tasks (tasks 5, 7, 9) and the post-test (tasks 3, 4, 8).

*The interaction phase:* The students were randomly arranged in couples. The conflict-score of each couple was calculated. A minimum conflict-score of 1 was required. The couples that did not meet this requirement were placed again in the list and reallocated. All together we had 16 pairs. In the interaction phase each couple solved together 3 tasks – tasks 5,7,9. The interaction phase was done under two conditions, with and without balance, that concretize the possibility or the impossibility to check hypotheses. The students were encouraged by the interviewers to verbalize and explain their considerations and decisions. In each task, immediately after the couple had reached an agreement or agreed not to agree they were given the opportunity to check their solution by the balance. After the weighting process they could change their solution/s and explanations. All the sessions were videotaped. Each student got for every task two cognitive-scores, for performance before and after the weighting process. The sum of the three singles cognitive-scores served as the student's cognitive-scores.

*Post-test*: The students solved individually tasks 3, 4 and 8. The cognitive-score for each task and a total score for the post-test were calculated.

## Results and Discussion

I. Students' performance in the 3 phases of the research are displayed in Table 1.

| Research Phase | Average | S.D. |
|---|---|---|
| Pre-test | 4.8542 | 1.702 |
| Interaction I (before weighting) | 5.0312 | 1.031 |
| Interaction II (after weighting) | 5.8854 | 0.721 |
| Post-test | 5.6458 | 1.54 |

The difference between the students' average cognitive-score in the post-test and the pre-test was statistically significant ($t(32)=3.37$, $p=.002$). The difference between the students' average cognitive-score in the interaction phase II and the post-test phase was significant ($t(32)=4.09$, $p=.000$). The difference between interaction phase II and interaction phase I was significant ($t(32)=6.27$, $p=.000$). These results show that interaction advances students' cognitive gains, it also shows that their performance during interaction phase II is higher than their performance in the post-test. However, analysis of the individual scores shows that in interaction phase II the jump in the cognitive-scores was due more to the competence component of the score than to strategy. Analysis of the individual scores shows that students who scored lower than their partners while working in pairs regressed more in the post-test than students who scored the same or higher than their partners.

II. The research main theoretical focus was "the two-wrongs-make-a-right phenomenon". Our assumption was that a couple of two "wrongs" would gain more than a couple of "wrong" and "right" (expert). The definition of a couple was done in the following way: An expert was defined as someone whose average cognitive-score in the pre-test was higher or equal to 6, and a "wrong" as someone whose score was lower than 6. The data showed that we had eight couples of two "wrongs", six couples of an expert and wrong and two couples of two experts. For the analysis the 2 couples of experts were excluded because of a possible roof effect. The difference between the pre- and post-test was statistically insignificant ($t(28)=0.72$, ns). However, the results indicated a tendency of a higher gain for couples of two "wrongs" (average of 1.313) in comparison to couples of expert and wrong (0.931). These results, albeit not statistically significant, support our theoretical assumption that when two wrongs interact the possibility for at least one of the wrongs to progress is real. Although the "wrong" student is exposed to incorrect arguments, the quality of the cognitive process is not inferior to the one that takes place when one of the interacting students is right. On the contrary, the results support our theoretical framework that indicates the possible benefits of such an interaction and the potential limitations of the other one. For example, expert students tend to dominant the interaction and thus to suppress counter-arguments to be raised.

III. Students' cognitive-gain as a factor of the conflict-score between the interacting couple is shown in Table 2.

| Level of Conflict-Score | Average Cognitive-gain | Number of students |
|---|---|---|
| 1-2 | 1.3 | 10 |
| 3-5 | 1.6 | 8 |
| 6 | 0.7 | 8 |
| 7-15 | 0.2 | 6 |

The results show that the maximal gain was obtained in the second level of conflict and the minimal one in the fourth level. These results support the theoretical assumption that a high level of conflict is not always beneficial. It may implies domination of one of the partners that prevents the interacting students from gaining while a moderate conflict, even between two "wrongs", may lead to significant cognitive-gains. However, these results should be re-checked in the light of a possible roof effect.

## Summary

The paper reported on a study aimed at examining the "two-wrongs-make-a-right phenomenon" - where the terms "wrong" and "right" imply hierarchy of competence in a well-defined mathematical domain - in an empirical research. The mathematical domain in which this study took place was proportional reasoning. We argued that when certain theoretical conditions are fulfilled – disagreement, being strategic, and hypothesis testing – the quality of an interaction between two "wrongs" is not inferior to the one that takes place when one of the interacting students is right. The findings of the study support our assumptions. The first finding of the study is not surprising - interaction between peers yields cognitive gains. The second finding support our assumption that interaction between two "wrongs" generates cognitive gains. Moreover, the results suggest that interaction between two wrongs is cognitively more productive than interaction between wrong and right. The results also show that hypothesis testing effects the cognitive gain of the interacting couple. It was also interesting to see that the students' cognitive-gains reached their highest level immediately after the process of hypothesis testing and dropped in the post-test. However, this study is only the first step in the investigation and leaves many research-questions open. For example: What are the interaction processes between the two "wrongs" that lead to the cognitive changes of the individual? How wrongs turn to be right? What is the optimum level of cognitive heterogeneity? Will a student working on his own, while having the opportunity the check his hypotheses gain the same? These questions are the focus of our current research.

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