VALUE-LOADED ACTIVITIES IN MATHEMATICS CLASSROOM

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ABSTRACT

This paper describes a form of value-loaded activities emerged in teaching and learning of mathematical induction in which the value of pleasure is shared by an expert teacher and his students. Using case study method, values are explored with classroom observation, teacher and student interview, and questionnaire survey. Conceiving identity theory from the pedagogical aspect, values are identified in terms of the significant value statements attached to and reflected by teacher and student's pedagogical identities. In consequence, implications about the teaching of pedagogical values and the need for more research on pedagogical values are suggested.

INTRODUCTION

S_i (whole class replies): Oh! No, you are cheating ---.
S_3 (single student replies): My goodness! What if I do ---.

Why the students above would react like that? We will show in this paper that these reactions have to do with the value-loaded activity that the students engage. Goals of the school mathematics curriculum in many countries indicate explicitly desirable values, for example 'pleasure' and 'appreciation' in Australian curriculum statements (AEC, 1991). There were considerable discussions of the link between affective and cognitive elements, in particular the role of values in mathematics teaching, in the PME conferences. For example, at the 1993 conference in Japan the overall theme was "How to link affective and cognitive aspects in mathematics education", and there was also substantial exchange of implicit values in various sessions at the 1998 conference in South Africa. There is a strong concern both to question and to challenge the values currently being taught in mathematics classroom. And yet, the school curriculum following these goal-initiated values, for example the values of *happy* and *knowledge for learning* that the new Taiwanese curriculum statements "happy to learn" and "knowing how to learn" underline (ME, 2000, p.135), say little about their practice, particularly in the teaching of such values. Although there are several levels of values transmission (e.g., Bishop, 1988) but we are still lack of the relevant researches looking particularly at the form of teacher-student values interaction in mathematics classroom. To conceptualize values about mathematics and pedagogy, we have to consider carefully the question of 'What implicit values could classroom teaching convey and through what kind of teaching activities could such values be taught?'

Values have been conceived as personal experiences, objects of thought, or

psychological phenomena (Frondizi, 1970), as individuals' feelings (Meinong, 1894) or objects to be desired (Scheler, 1954), as individual principles of selection and judgment (Samuel, 1937), as ideas or concepts concerning the worth of something (Swadener & Soedjadi, 1988). In this case, they are personal *preferences* concerning individuals' *standards* for considering *the importance* or *worthwhile of* something for themselves to *think* and *act*. A domain of research relevant to values is beliefs. Mathematics teachers may hold various pedagogical beliefs, which differ in forms as mathematical or pedagogical, or in levels as enacted or espoused. Nevertheless, beliefs have to do with an individual's propositions about mathematics and pedagogy, and values are more about personal principles or standards of thinking and action across such propositions. Rokeach (1973) in studying the nature of human values suggests that values are prescriptive or proscriptive beliefs wherein some means or end of action is judged to be desirable or undesirable. Allport (1961) further contends that a value is a belief upon which a person acts by preference. These arguments conceive value as a preference, a desirable mode of conduct, or a desirable end-state of existence, concerning the conception of something that is importance and worthwhile of thinking and doing for the person (Rokeach, 1973). Therefore, beliefs are more about "the nature of *propositions* about phenomena" (Bishop, 2001), however, values are likely to be the key *substances* underlying such propositions for people to think and act.

The Value In Mathematics Teaching (VIMT) project funded by the Taiwanese National Science Council (NSC, 1997-2000) aimed to: explore mathematics teachers' values about mathematics and pedagogy; examine and increase the extent to which mathematics teachers' can clarify their own pedagogical values; and investigate students' values about mathematics and mathematics learning, and the values interaction activity in the classroom between teacher and student. Based on a portion of VIMT results (Chin & Lin, 2000a; Chin & Lin, 2000b), this paper aims to describe an activity observed in the teaching and learning of mathematical induction in which the value of *pleasure* was loaded, and through which a teacher and his students interacted overtly. Other values can be found in Chin and Lin (2001, submitted)

THEORETICAL BACKGROUNDS

The Value of Pleasure in Mathematics Teaching

The analog/unconscious mathematics (Davis & Hersh, 1981) is related to the idea of intuition and feeling, in which sense making and free thought are the two important elements. It is our belief that the more efforts that we make for students to learn mathematics through understanding the better they would appreciate the value of pleasure. The statement, "The thing that is important is that doing math is fun. That's what I try to put across to the kids --- What I try to do is to tell math to kids on the basis that it's fun" (ibid, 1981, p.272), may convey "Learning mathematics is interesting and doing it is a pleasant thing" undrelining the value of *pleasure* in pursuit of having fun in the teaching and learning of mathematics. The

analytic/conscious aspects conceive mathematics from the other way round as a combination of forms, rules, and proofs. This is to addresses the concept of mathematical induction as a formalized subject for learning through a step-by-step mechanical process of proofs. As a result, these effects may create the value of *un-pleasure*. In discussion of the emotional influences on learning school mathematics, Skemp (1989) proposed a model of conceptualizing affective responses such as *pleasure* and *un-pleasure* in terms of the orientations towards goal state or away from the goal state. Pleasure signal changes towards a goal state, and leaving from the goal state creates un-pleasure. The goal state here depends on the implicit learning orientations about mathematical induction that individual students stand.

Values Statement as Carrier of Values

We conceive a statement of values as the carrier that contexturalizes individual pedagogical values into a concise sentence, representing his/her core principle for thinking and action. A value carrier thus contains a set of values that the teacher endorses. A values statement, as Taylor (2000) pointed out, is a goal-directed description indicating the values by which the school intends its practices to be guided, and setting out the values the school intends to promote and which it intends to demonstrate through all aspects of its life. An example of a secondary teacher's values statement is "We want our school to be caring and Christian, disciplining, encouraging, happy" (ibid, p.157). This statement includes the values of care, discipline, encouragement and happiness that secondary school is expected to address across the curriculum. Values, in the light of this view, are situated in and entertained with the propositional statements of teacher and student conceptions about mathematics and pedagogy (e.g., Haydon, 2000).

The Pedagogical Aspect of Identity and Its Relationship to Value

Values are conceived here as a sort of individual identities concerning mathematics and pedagogy. Fereshteh's (1996) definition of teaching, as an intellectual activity requiring varied abilities in educating students certain knowledge, as guiding and evaluating students' learning processes, and as an artistic and scientific activity, seem to suggest that the underlying pedagogical values for such an activity are *intellectual* and *knowledge acquisition*, *guidance* and *evaluation of learning process*, and *artistically scientific discovery*. In pursuit of such values, a teacher is expected to play as a manager, creating and organizing the lessons, as a motivator, fulfilling the student needs, and as a professional, enjoying and developing their career. To accomplish these roles, a teacher identifies him/herself with the professional identities of a manager whose classroom teaching reveals some role specifics. This aspect of identity seems to shift its sociological nature into a pedagogical realm, reflecting how and in what ways a teacher should think and act '*as if*' he/she is a person accompanying with such identity. In an article of reporting the role of values in pedagogical content knowledge (PCK) from four experienced English and History teachers, Gudmundsdottir (1990) concluded that values, as implicit personal curricula, are integral part of teachers' excellence in teaching influencing several aspects of PCK, such as choice of pedagogical strategies and

perception of the students' needs. These elements, reflecting a teacher's excellence in teaching, play an integral part of his/her pedagogical identities. In discussion of a continual development of values from early youth to old age, Erikson (1963) indicated that the value systems properly reflected those features of the development of eight identity stages. In Rokeach's (1973) researches, values are integral to self-identity that people strive to be authentic, moral beings by acting on the basis of values tied to their desired self-conceptions. Therefore, pedagogical identities seem to tie to the values of individuals.

RESEARCH METHODS

The case study method, including questionnaire survey, interviews, and classroom observations, was used as the major approach of inquiry to explore an expert teacher (Ming) and his students' pedagogical values. Ming had a master degree in mathematics and taught mathematics in a public senior high school for 21 consecutive years. We used critical teaching events as probes for post-lesson interviews. Another teacher, Yuh who taught mathematics in the same school, participated in each interview. Four teaching topics were videotaped and transcribed during 1997-1999 including mathematical induction. A senior secondary mathematics teacher acted as an independent checker to examine the reliability of the observational data. A questionnaire was used for all students, designed to uncover students' preferences on selecting two of the six problems relating to the topic taught and to collect students' reasons of doing so. First part of this questionnaire consists of six open-ended problems designed by Ming based on his core pedagogical values, and the second part has several items for students to express their agreements on each statement, concerning the reasons of doing and not doing so. The second questionnaire consisted of 20 questions, using a five-point Likert format. It asked students to express their views on each statement according to two different contexts, test scores taken for grading or not to be taken. The 6 sample students were selected according to their mathematical performance in the first questionnaire and willingness to talk, and the representative of the student class. These questionnaires were piloted and revised with Yuh's students.

RESULTS

A Value-loaded Teaching Activity

According to the school curriculum, the topic should be taught in duration of five 50 minutes lessons. Usually teachers, for example Yuh, would take less than ten minutes to introduce the format of mathematical induction, and the rest for exercises. Ming spent 15 minutes in the activity of Hanoi tower to develop student ideas of 'potential infinity'. Although the activity is well known, however, Ming re-framed it to address student manipulation and teacher-student dialogue. Two critical questions used by him to guide the student thinking were: 'Can you do it?' and 'Do you believe

that if N=3 is possible then N=4 will also be possible?' Ming told the rules first followed by student manipulations and teacher-student dialogue. Finally, he showed the solution. A brief snapshot of the activity after three students manipulated was:

Ming: Let me show how I solve it. First of all, can you do it if the number is 3?
S_i (whole class replies): Yes, of course we can.
Ming: If it is 4, could I pack it up as a unit and move the package from A to B?
S_i: It is okay.
Ming: Then, I move the fourth one to C, is it okay?
S_i: Yes.
Ming: Then, if I move again this package from B to C, can I do it?
S_i: ***Oh! No, you are cheating.***
S_3(single student replies): ***My goodness! What if I do it the way that you just showed us by packing up the case of 4 to solve the case of 5, and then to solve the case of 6, and so on?***
Ming: Are you sure?
S_3: Why not?
Ming: Excellent (he smiles expressively), are you convinced that I didn't cheat you?
S_i: Yes! There should not be any problem.
(a recursive procedure of potential infinity through teacher-student dialogues)
S_i: It can also be done by the same way. There will not be any problem.
Ming: Therefore, we can do it all the way through in the same method?
S_i: Yes, why not.
Ming: Are you convinced that for any counting number we can always do it this way?
S_i: Yes, we can do it by counting up.

The expressions "Oh! No, you are cheating" and "My goodness! What if ---", referring to the freshness and power of interpreting mathematical knowledge, are related to the value of *pleasure*, which will be discussed later. The subsequent dialogues, in which a recursive procedure of step-by-step reasoning format is introduced, develops meanwhile the ideas of potential infinity that the values of *infinite* and *reasoning* underlay (see Chin & Lin, 2001, submitted).

The Underlying Teacher's Pedagogical Values

The activity of Hanoi Tower was used to inculcate the value of *pleasure*. As Ming said that "the idea of 'infinite reasoning' underlying the 'empirically counting truth' conviction, should play the significant role to encourage students to do *mathematical* investigations in which *enjoyment* of knowledge are of paramount importance". In particular, he concerns the affective aspect of learning and teaching school mathematics as "I really hope that all of my students will feel happiness, enjoyment, and pleasure in their own processes of investigating mathematical knowledge in these activities. They are the affective and humanistic concerns that I have been trying very hard to express in my teaching, such as mathematical induction". Referring to the pre-lesson planning of the activity, Ming professed that "I intended to develop activities in which my students would feel that mathematics could be very interesting and they might in this case be eager to attend the subsequent lessons. The Hanoi Tower activity was just designed to initiate such student motivation through learning the concept of mathematical induction". He explained further "Most students are *not happy* in the mathematics lessons --- *they don't feel that the knowledge is useful or practical in their life*. Therefore, most of them feel

panic and *anxiety* when learning mathematics. This is the reason I have been trying so hard to motivate them to learn mathematics through *enjoyment, pleasure, and anticipation* using investigative games or activities, and focusing on the nature of the knowledge---".

The acceptance of such a value of Ming for the students will be further examined in terms of the data collected from the student phase.

The Attainable Students' Pedagogical Values

One question in the first questionnaire, "Suppose that the concept of human is well defined, and the life in the earth has gone through about 4 billions year. Prove that there is a human being whose mother is not a human (called Genesis)", was used to examine student understanding and the ability of application from an unconscious/analog aspects and meanwhile loading specifically with the value of *pleasure*. The implicit pedagogical values that Ming intends to pass to the students are then conextualised into 6 tasks in assessment, including Genesis, for students to select and solve. It was supposed that in the process of selection that the values of students would become explicitly.

When we asked 'Why did you choose Genesis rather than other questions?' S_1 said "Because I felt that these two questions ask me to elaborate and the context of the problem make sense to me". The text and situation of Genesis was much easier for him to get access. They created motives of *pleasure* such as interesting and fun, and he was eager to solve the problem since "These are questions that I have not seen before. They are *new* for me. But, I am quite familiar with the other questions that I am not so interested. They are so *boring* because I have already known the answer of the questions and the procedure of solving them". This value encouraged curiosity and willingness for the student to solve individually, for "The questions raise my *curiosity* to solve them and I *like* to find out myself. I really like to know whether if I can solve a totally new problem like this on my own". Routine questions could not create any pleasure for him, as he said that "There is *no pleasure* for me at all to face a mathematical question which has no practical use or not realistic. I don't know what could be of interest if you have already known the answer or method of solving the question?" Another student S_2 also claimed for searching an unfamiliar question to solve, as "I would have got *bored* if a mathematical question were solved easily according to certain familiar steps. It is no fun at all for doing or answering a question like that". Therefore, it was *the pleasure of reasoning* that encouraged him to solve Genesis. Because, "I like to solve the question through my own efforts. There are lots of *pleasant* during the processes of solving such questions" and "I am now telling you that I hope all mathematical will like this, encouraging me to *think* and *reason freely* according to the ways whatever I like, fulfilling my *curiosity*, and being *full of pleasure* in the process of solving them". These statements indicate the crucial role that affect played in the mathematics learning concerning the value of *pleasure*.

Therefore, the mathematical ideas of *infinite reasoning*/*potential infinity* and its accompanied affective element of *pleasure* are interwoven within the process of teaching and learning of the Hanoi tower.

DISCUSSIONS

A foreseeable relationship between pedagogical values, value statements, and identities is become clear. Value statement as carrier of pedagogical values portrays

teacher and student principles of evaluation on teaching incidents and learning tasks. A text like this has its syntactical structure of wordings in which certain values are embedded implicitly. The key words included in a value sentence may represent different pedagogical values. Values are in this sense represented and embodied in words or a combination of words, underlying the connected value statements to which they apply. Peoples, who agree or identify oneself with a particular value statement, are conceived as carrying the underlying fundamental elements of the statement, that is, the pedagogical values that the statement portrays. This is to conceive and analyze pedagogical values from human discourse in terms of a syntactical and psychological analysis of talks and words. It is this process of pedagogical identification that reflects a person's preferences to aspects of teaching. These preferences are in connection to the teacher and student's pedagogical identities. In the light of this, values are integral to self-identity and that teacher and student strive to be authentic (Rokeach, 1973). Australian researchers also referred values to the pedagogical aspect of personal identity as in "The values taught, whether explicitly or more like implicitly, seems to depend heavily on *one's personal set of values as a person and as a teacher*" (Bishop, Clarkson, FitzSimon, & Seah, 2000, p.148). Halstead (1996, p.5) used the concept of identity to define values, as he put it "The term values --- which act as general guides to behaviors or as points of reference in decision-making or the evaluation of beliefs or action and which are closely connected to *personal integrity* and *personal identity*".

To inform classroom practices, the practitioners need more supports on learning and constructing value-loaded activities that might be useful, and also the framework for developing and elaborating on pedagogical values for different subject matters. This is an area that Tomlinson and Quinton (1986) called "De facto implemented curriculum" and Bishop (2001) claimed "the Meso classroom level" in which means of planning and enacting values are the foci. Researches on this line are urgent. It is also important for mathematics teacher educators to try to develop and examine plausible ways of inculcating specific pedagogical values at "the Macro curriculum level" (Bishop, 2001) of pedagogical value education for mathematics teachers. Therefore, we need to know more about the processes of valuing and value clarification from both teachers and students that may in turn contribute to the development of values education curriculum for them.

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