Pathway between Text and Solution of Word Problems

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Solution of a word problem begins with the given text. The students' task is to discover the mathematical model that will lead them to the correct solution. We endeavored in our study to detect the text representation by analyzing the students' repetition of the text on the one hand and their mathematical solution on the other hand. While revealing the representation mechanism, we analyzed the performance of students who were successful, as well as those who failed in the mathematical solution of the problem. The findings demonstrate that the solvers have their own representation of the given text that affects both their retelling and the mathematical solution.

Theoretical Background

A solution of a mathematical word problem begins with a given text. The processes involved in reading the text, are linked on the one hand to text comprehension and discovering the formal mathematical model on the other hand. Comprehension in this context means building a representation of the textual information. According to Gick (1986) there are three major stages in solving a problem: constructing a representation of the problem, searching for a solution and implementing the solution. Our main interest in this study is the first stage, constructing the representation.

One method of revealing the construction of representation is to ask the students to retell the text they have read. The retelling can provide evidence of how the student has interpreted the original text. Kintsch (1986, 1994) and Verschaffel (1994), employing this methodology in their studies, found the source for subsequent erroneous solutions in the manner the solver represented the text. Thus, the subsequent erroneous solutions are reflected in the retold text. The solver, instead of retelling the original text, which may be too complicated for him, relates a simplified version of the story. For example, a relative quantity might become an absolute quantity.

Reading a given word problem, the student relates to the textbase in order to build a situation model (Kintsch 1986, 1994). To elaborate it to a mathematical model requires understanding of the situation described in the text and using mathematics to complete the missing data in the given text.

Similarly, Bilsky (1986) demonstrated that the context and the goal of reading a text affect the way the subject constructs the representation of the text. She found that the manner in which a subject views the text, whether a "math problem" or a "story", was decisive regarding the text representation. Similarly, Anderson and Pichert (1978) found that shifting perspectives on the same text (second recall) add additional information and recalls less irrelevant information, thus adopting a new perspective led subjects to invoke a schema that provided implicit cues for the different categories of story information.

Numerous evidence exists for schemas directing comprehension and representation Reed (1999), Marshall (1995). A schema provides a framework for integrating new information into old knowledge in order to construct a general structure for a variety of specific instances.

The purpose of our study was the verbal text and its representation. The method used was retelling. We endeavored in our study to detect the relationship between the text representation and the mathematical solution by analyzing the retelling. While revealing the representation mechanism we analyzed students who were successful as well as those who failed in the mathematical solution of the problem.

In analyzing the retelling of successful solvers, we also noted what Raney et al. (2000) wrote about the model of text repetition effects in which wording is represented in an abstract, context-independent manner, whereas the situation described by the text is represented in an episodic, context-dependent manner.

Method

The texts used in our study were three word problems, typically given in math classes, all of which lead to two-step solutions.
Forty-nine fifth and sixth grade Israeli students, who had already studied such problems, were individually interviewed in a single 45-minute session.

Task description:

Three word problems were presented to the students:

> Problem No. 1: *I have a book with 320 pages. I have already read 80 pages of the book. How many days are needed to finish reading the book if every day I read 60 pages?*
> Problem No. 2: *In the morning the seller distributed the roses equally into 6 vases. How many roses did he place in each vase if*

during the day he sold 120 roses and at the end of the day he saw that 60 roses were left?

Problem No. 3: *For the journey lunch-boxes were prepared for all participants. Each lunch box had 5 pieces of fruit of which 2 were apples and the rest were plums. In preparing the lunch-boxes 240 plums were used. How many participants received lunch-boxes?*

For each of the three problems each student was asked to read aloud the text (original text) of the word problem, to retell it (first retelling), and then to solve it. After the solution he was asked again to tell the story (second retelling). The complete record of the session was used in our analysis.

Method of analyzing the session records

Our initial analysis was performed on two levels: 1) Deviation of the repetitions from the original text, and 2) Correct or incorrect solution. Regarding deviation from the original text, we classified it into four major categories:

One) Changing the wording **without changing the schema** of the text. For example, in this category Yael retold problem No. 2 in this way: *There was a seller. He received roses and equally distributed them in vases. During the day a lot of people arrived and bought a lot of roses. Then he found out that 60 roses were left?*

Two) Changing the order or the flow of information of the original text. *For example, in this category Michal retold problem No. 2 in this way: A seller sold 120 flowers and 60 flowers were left. The flowers (those sold and those left) were in 6 vases. How many flowers were there in each vase?*

Three) Accurately retelling the original text.

Four) Changing the text so that it describes a different schema. For example, in this category Joseph retold problem No. 1 in this way: *There is a book with 320 pages. Up to now I have read 80 pages, one day I read 60 pages. How many more pages do I have to read?*

Findings

In Table No.1 we can see the distribution among the above four categories of deviations from the original text in the repetition, and the correctness of the solution for each word problem.

Table No. 1: Deviation from the Original Text
and Correctness of the Solution.

Problem	Correct Solutions			Incorrect Solutions	Not * Included
	(a) Changing the wording without changing the schema	(b) Changing the order	(c) Retelling precisely	(d) Changing the text into different schema.	
1	21 (43%)	14 (29%)	4 (8%)	8 (16%)	2 (4%)
2	17 (35%)	14 (18) (29%-38%)	2 (4%)	13 (26%)	3 (6%)
3	13 (26%)	14 (16) (29%-33%)	4 (8%)	16 (33%)	2 (4%)

* Due to any reason

We can see in Table 1 that very few students are included in category (c) in which the subjects retold the original text exactly. This means that most students engaged in some elaboration while retelling the original text, thus supporting the findings of Raney (2000) that the wording is abstract.

What is the difference between those who rephrased the text and correctly solved the problem and those who rephrased the text and erred?
Those who succeeded in solving the task, did not change the basic schema of the text. Their variations were of two kinds: (a) changing the wording without changing the schema, and (b) changing the order of the text. Those of category (a) usually added details to the description of the situation (episode) taken from their general world knowledge, which were not mentioned in the text.
For example there were students who told about the people who came to buy the flowers, or added some information about the flower-seller.
This also conforms to the findings of Raney et al. The situation described in the text is represented in an episodic context- dependent manner. Once the subjects grasped the situation they were ready to add their own details. On the other hand, frequently the wording was changed to express the original semantics.

Those included in category (b – of the correct answer) also preserved the text schema, but elaborated on the text in a way so as to help them detect the mathematical model,
Michal, who changed the flow of problem 2 (see explanation of category b above), immediately began to solve the problem and claimed: *" first I have to know how many flowers he had in the morning, the sold flowers and the left ones 120+60. Only then can I find out how many flowers he placed in each vase. I have to do 180:6=30"*

Six students belong to both categories (a) and (b). For example, Johnny in problem No. 3, rather than adding to the story some details from his world knowledge, also solved a part of the problem in the course of retelling it.
In this way he retained the problem's underlying structure while changing its surface structure.
Johnny's repetition was:

A member of the entertainment committee, or somebody else, I don't know exactly who, prepared the lunch-boxes for the journey the committee organized. In each lunch box they put 5 pieces of fruit of which there were 2 apples and 3 plums. 240 plums were needed to prepare all the lunch-boxes. How many children got lunch-boxes?

While solving the problem he wrote 2 math expressions as follows:

"3 + 2 = 5 and 240 : 3 = 80"
plums apples fruits

He summarized *"80 children will get lunch-boxes"*.
He continued and said: *"Now I can find out how many apples were needed as well (80X2=160)."*

All those who solved the problems incorrectly, also consistently changed the schema of the text in their repetition (category d). As found by Kintsch (1986), the changes they made in the text fitted their erroneous mathematical solution. Moreover, when asked to give the second repetition (after the solution) they were again consistent in their story, The second repetition fitted their erroneous solution.
Changes in the schema were also of two kinds:

1. Those who entirely changed the mathematical model (e.g. from multiplicative to additive),

   For example: Joseph, whose repetition was demonstrated for category (d), changed the problem from two different structures (additive and multiplicative) to two similar structures (two additive

structures). He continued to solve the problem by solving the expressions: 80 + 60 = 140, and 320 – 140 = 180.

2. Those who changed the relationships of the sets involved in the situation text (Bilsky, 1986). For example, Shay retold problem No.3 as follows:

   Lunch-boxes were prepared. There were 5 fruits in each lunch box, of which 3 were plums and 2 were apples. Shay continued to speak aloud while solving: 240:5=48 and said:
   "240 are all the fruit. Each child received 5 fruits".

   In this case the 240 plums were changed into 240 fruits, so that it would be reasonable to distribute 240 fruits - 5 fruits per each child, which was not the original question.

Discussion:

It is clear that the first step in the problem solving process is constructing a representation of the situation. This process does not take place in a vacuum, but is strongly affected by its context (Bilsky1986, Bransford and Johnson 1972).

Verschaffel et al. (1994) described the role of real world knowledge in the different phases of problem solving. Beginning with the initial phase of problem understanding, modeling that precedes the computation, and the final phase in which the result of the computational work is interpreted and evaluated. We also tried to trace the students' representation of the text read by asking them to retell the text. We examined their solution (thus, their modeling) and we noted their solution by asking them to retell the text again after their solution.

We found some students who retold the text precisely. According to Fletcher & Chrysler (1990): "*The most superficial level of representation, called surface memory, captures the exact wording of a text. This representation is viewed as a product of highly automated lexical and syntactic process."* All those students solved the problem correctly. They probably found no reason to change the wording and they immediately saw the mathematical model, which led them to the correct answer. According to Fletcher & Chrysler (1990), in studies on reading, accurate repetition of the text is viewed to be the lowest level. In mathematics, we have thus far been unable to relate the precise retelling to a low level of understanding. In our sample, those who retold the text exactly did not support the claim that they were on a lower level.

Most students changed the wording while retelling the given text. We believe that this occurs while the student is constructing the situation model of the given textbase (Kintsch 1986). Fletcher & Chrysler (1990) add*, "the final level of representation is referred to as the situation model. This representation results from processes that integrate the information conveyed by the text with a reader's or listener's prior knowledge and produce the most lasting trace in long memory. The situation model corresponds to the equivalence class of all experiences that convey the same situation."*

We found evidence that most students do not "translate" the textbase directly into a mathematical expression. In order to construct the situation model from a given text, the student has to invoke qualitative considerations, which are necessary for constructing the situation in his mind. He frequently adds details not originally mentioned in the text but taken from his world experience (Nesher 1980). Anderson et al. (1983) claim that *"the content schema embodies the reader's existing knowledge of real and imaginary worlds. What the reader already believes about the topic helps to structure the interpretation of new messages about the topic"*. We found that all students in our sample who built a richer text by adding detailed information, did so because it was useful for them in order to construct a complete understanding of the text, and thus they correctly solved the problems.

Another path from the original text to correct solutions was found when students changed the original order of the text. This happened in problems in which the numbers given in the text were not in the same order they appeared in the mathematical expression. In this case the students changed the order so that the numbers appeared in their retelling in the same order they were supposed to appear in the mathematical expression.

All students who <u>failed</u> to solve the problems changed the text into another situation, mostly to a simpler one. These changes related to changing the text so that it described different mathematical structures. Kintsch (1986) and Verschaffel (1994) found this type of behavior in simple one-step additive problems. In our study, in accordance with theirs, the students felt comfortable with the changes they made. This was consistent even in the second repetition made after they already solved the problem. When they had to interpret and evaluate their solution, they did not feel that they were referring to another story.
Once again, we would note that solving a word problem is not a matter of a direct solution. Zwaan and Radvansky (1998) found similar findings in

relation to other expertise domains. In the path that leads from a given text to its mathematical solution, the situation model plays a crucial role. This model is constructed by the solver after reading the given text and adding interpretations based on his world experience.

We believe that our findings might have pedagogical implications, as teachers could gain some access to the students' internal representations by asking them to retell the text. This may be particularly helpful in tracing the source of erroneous solutions.

References:

Anderson R.C., (1978): "Recall of Previously Unrecallable Information Following a Shift in perspective" Journal of Verbal Learning and Verbal Behavior 17, 1-12

Anderson R. C., Pichert J. W. and Shirey L.L., (1983): "Effects of Reader's Schema at Different Points in Time" Journal of Educational Psychology, Vol. 75(2), 271-179

Bilsky L.H., (1986): "Comprehension Strategies in Math Problem and Story Contexts"
Cognition and Instruction, 3(2), 109-126

Fletcher C.R., Chysler S.T. (1990): "Surface Forms, Textbases, and Situation Models: Recognition Memory for Three Types of Textual Information" Discourse Processes 13, 175-190

Gick M.L., (1986): "Problem –Solving Strategies" Educational Psychologist 21 (1&2), 99-120

Kintsch W., (1986): "Learning from Text" Cognition and Instruction, 3(2), 87-108

Kintsch W., (1994): "Text Comprehension, Memory, and Learning" American Psychologist 49, 294-303

Marshall S.P. (1995): "Schemas in Problem Solving" Cambridge, England: Cambridge University Press.

Nesher P., (1980): "The Stereotyped Nature of School Word Problems"
For the Learning of Mathematics 1(1), 41-48

Raney G.E., Therriault D.J., Minkoff S.R.B. (2000): "Repetition Effects from Paraphrased Text: Evidence for an Integrated Representation" Discourse-Processes. Vol 29(1), pp. 61-81

Reed S.K. (1999): "Word Problems: Research and Curriculum Reform" Lawrence Erlbaum
Associates, publishers

Verschaffel L., De Corte E. And Lasure S. (1994): "Realistic Considerations in Mathematical Modeling of School Arithmetic Word Problems" Learning and Instruction Vol. 4, 273-294

Verschaffel L., (1994): "Using Retelling Data to Study Elementary School Children's Representations and Solutions of Compare Problems". Journal for Research in Mathematics Education 2, pp. 141-165

Zwaan A.Z. and Radvansky G.A.,(1998): " Situation Models in Language Comprehension and Memory" Psychological Bulletin Vol. 123(2) 162-185