ACTIVITY ANALYSES AT THE SERVICE OF TASK DESIGN

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The goal of this paper is to exemplify a model of design-oriented research, based on four different analyses of an inquiry activity, concerning proving in geometry. The designers' intention in this activity, was to create geometrical situations in which students will confront contradictions between their conjectures and findings, face uncertainties concerning the right conclusions, and thus search for explanations that will settle the uncertainties. In this way normative explanations in geometry might stem from students' need for justification. The first two analyses reflect the beliefs, hypotheses and intentions of the designer-researcher concerning the potential of appropriate situations to lead to a meaningful activity. The third and the forth analyses are experimental: students' conjectures and explanations serve as the database against which the task potential is validated.

INTRODUCTION

In this paper we exemplify a model of design oriented research, based on four different analyses of an inquiry activity, concerning proving in geometry. This model is based on the hypothesis, that it is possible to design geometrical inquiry situations, in which students are confronted with contradictions between their conjectures and the findings of their investigations in a Dynamic Geometry (DG) learning environment. These contradictions might lead students to uncertainties concerning the right conclusion, and thus push them to search for explanations, which may settle the contradictions. Since the only satisfactory explanations for this kind of situation are deductive (satisfactory in the sense that they can settle the mathematical contradictions), students might naturally appeal to such explanations.

For investigating this hypothesis, a few inquiry activities were designed in a DG environment in a process of *design-research-design,* as described in Hershkowitz et al. (in press). The design of these activities, intended to lead students towards contradictions in two ways, (see Figure 1).

Design of activities with potential for contradictions between conjectures and conclusions in two different ways.	(1) The findings contradict the conjecture. →	Explanation for the surprising conclusion.
	(2) The findings do not lead to a definite conclusion and thus, create uncertainty. →	An explanation that points to the right conclusion, and provides conviction.

Figure 1: Two ways to contradiction

Our approach to the design of meaningful ways for teaching/learning to prove in geometry, is in agreement with the wide consensus of mathematics educators and researchers that proving activity should involve different actions, like discovery and reinvention, conjecturing followed by confirmation or refutation, including confrontation with situations of uncertainty (Chazan & Yerushalmy, 1998; Hoyles, 1997; Goldenberg, Cuoco & Mark, 1998; de Villiers, 1998).

THE TASK AND THE RESEARCH POPULATION

In this paper we will relate to one activity and exemplify the four analyses mentioned above.

<u>*The task:*</u>

Divide the side AC of a dynamic triangle into 3 equal segments. Connect the division points to the vertex B (Figure 2).

Make conjectures about the 3 varying angles (<ABD, <DBE, and <EBC).

Use the software to decide when (if ever) the three angles are equal. You may change your conjecture and check.

Describe your investigation and explain your conclusion.

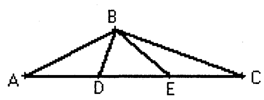

Figure 2: The three angles task

The above task is the product of a first stage of the *design-research-design process,* described in Hadas & Hershkowitz (1998), and has served as the "research tool" for the model mentioned above, and which will be described in detail below. The task was first used in semi-structured interviews with three pairs of students and then given as a written activity to all students in three classes, which were invited to work collaboratively in pairs. The students were all ninth graders, and had already had a year-long course in Euclidean geometry. Altogether 32 reports were collected (three from the interviews and 29 from the three classes in two different schools).

The interviews enabled us to qualitatively trace the changes in students' conjecturing and explanation processes. The transcriptions of the interviews and the written reports of students working in pairs in classrooms were collected and analysed. The results of these analyses enabled us to investigate quantitatively and qualitatively students' conjectures and explanations, and hence to clarify to what extent students faced contradictions as well as to make categories of students' explanations.

THE FOUR ANALYSES

Analysis no. 1: Epistemological analysis of the activity

The epistemological analysis demonstrates all possible investigation paths in the designed activity, without giving preference to any of them. This analysis is summarised in Figure 3.

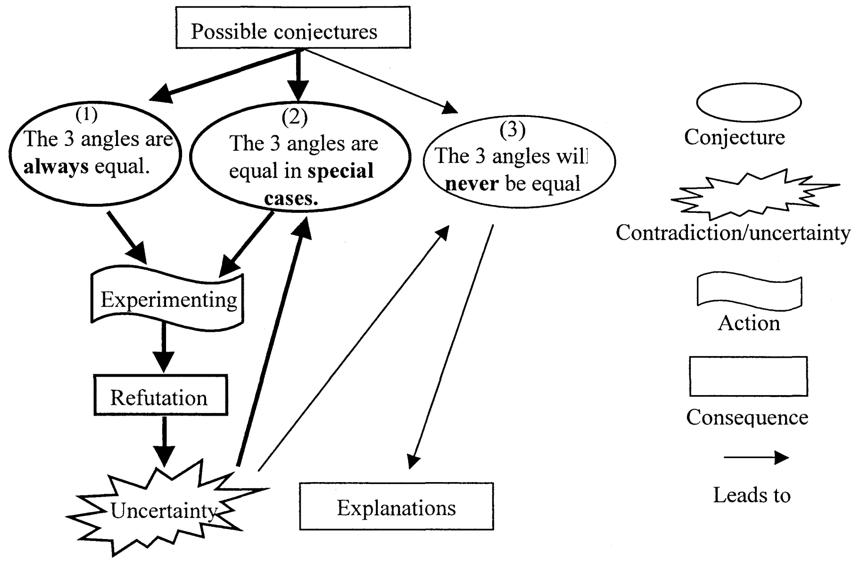

Figure 3: Diagram of the possible investigation paths

Starting with the conjecture in ellipse 1, 'the three angles are **always** equal', may lead students towards experimentation, which refutes this conjecture. This path exemplifies the first way of contradictions, (see Figure 1). After refuting this conjecture students may omit the **'always'**, and join those who conjecture 'equality in **special cases**' (ellipse 2). For checking this conjecture students may move from example to example, checking with the software and refuting every new conjecture concerning a special case in which equality seems possible (left and middle 'bold paths'). Thus, students are led to a situation of uncertainty where they don't know whether an example of equal angles exists. This may lead them either to raise a different conjecture about equality or to the conjecture that 'the three angles will **never** be equal' (ellipse 3) followed by an explanation (represented in the path on the right side of Figure 1).

Analysis no. 2: Didactical characteristics of the activity

This analysis is based on the previous one, and reflects our intention as designers, to create favourable conditions for contradictions and uncertainties. In this design we took into consideration students' common belief that opposite equal segments in a triangle, there are equal angles, and the power of DG to check if there are 'equal angles situations' ('bold paths' starting at ellipses 1 or 2 in Figure 3.) As the angles' equality situation is impossible, it was hypothesise that students will confront repeating refutations ('bold middle path in Figure 3). Thus the investigation involves uncertainty, namely not knowing whether one must look for an existence example or for an explanation why such examples do not exist (ellipse 3). The role of the DG software in this activity is not to provide an answer but to enable students to experiment (i.e. to manipulate geometrical entities involved, to refine their conjectures, then to refute them). The DG environment may also influence students' explanations.

The following third and fourth analyses describe the data concerning the students' real actions (their ways of investigating the task), and study their meaning.

Analysis no. 3: Conjectures raised by students

Here we describe students' initial conjectures and investigate to what extent they contradict the findings. We also describe how these conjectures are changed during the students' work. The analysis of the conjectures is based on the first analysis of the possible investigation paths (Analysis no. 1), and will be described here on the basis of Figure 3.

From the 32 reports that were collected, we had altogether 35 conjectures (the students in the three interviews conjectured individually).

Stage I: In 18 conjectures it was claimed that the three angles 'would **always** be equal'. The other 17 conjectures specified that the angles 'would be equal in **special cases**' like in isosceles or at least in equilateral triangles. At this stage, no one raised the conjecture, represented in ellipse 3, that 'the three angles would **never** be equal'. Constructing, measuring and dragging actions, using the software, immediately refuted the conjecture in ellipse 1, and convinced the 18 students that the angles 'are **not always** equal'. After this conjecture was refuted these students joined the 17 others and dragged the figure, trying to find cases of equality. This first stage of conjecturing is demonstrated in Figure 4 (Stage I).

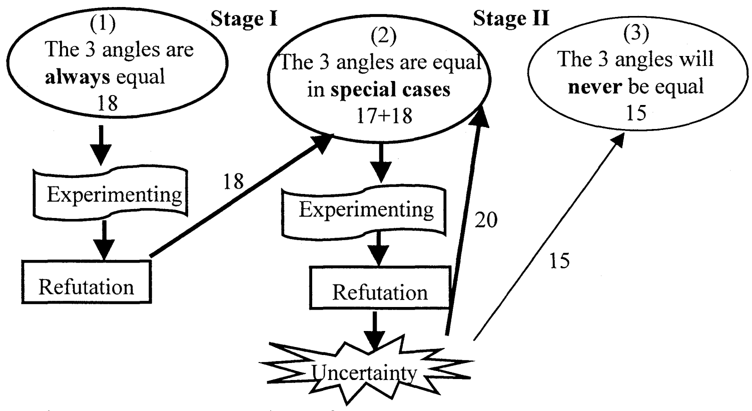

Figure 4: Stages I and II of conjecturing

Stage II: In 15 out of the 35 reports, after dragging the figure and measuring, students moved towards the conjecture that 'the angles will **never** be equal'. The remaining twenty moved in circles, from the search for examples to new conjecture. This second stage of conjecturing is also demonstrated in Figure 4 (Stage II).

Stage III: This process, of moving in circles, continued until 19 of the pairs joined the previous 15, in conjecturing that the three angles 'might **never** be equal' (ellipse 3). These 34 pairs of students began to search for a satisfactory explanation for the impossibility. The 35th pair removed vertex B so far from edge AC that the angles became small and numerically equal on the computer. They then concluded that there is a case for which the angles are equal. This Third stage of conjecturing is demonstrated in Figure 5.

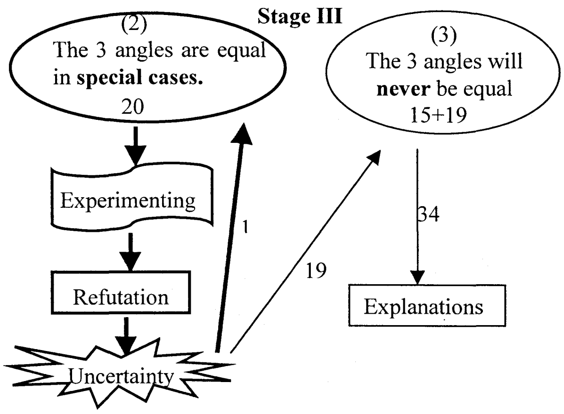

Figure 5: Third stage of conjecturing

In summary: All students began their work with conjectures, which contradict their final conclusion. Following the stages demonstrated in Figures 4 & 5, we conclude that all students choose investigation paths that lead to refutations and confrontation with uncertainty, and thus acted out the didactical intention described in Analysis no. 2.

Analysis no. 4: Explanations

In the previous analysis we concluded that all but one of the pairs followed at some stage the path starting in ellipse 3, which led them to search for an explanation why the three angles will never be equal (see Figure 5). We collected 34 explanations, which were categorised by the first author and two experienced teachers with agreement among all three in 88% of the cases. When full agreement was not attained, explanations were categorised according to agreement between two of them.

The goal of the categorisation is to find qualitatively and quantitatively (1) whether and how students use deductive considerations in their explanations; (2) whether and how students are influenced by the DG environment in their explanations (whether they use examples created on the screen to explain the findings inductively, or whether they make use of the dynamic variation of the figures on the screen, to explain their findings). Thus, we defined five categories emphasising these aspects.

In the following we will first describe each of the five categories with examples. (Examples from two other activities of the research are described in Hadas, Hershkowitz & Schwarz, 2000.) Then some quantitative information, concerning the number of explanations classified in each category will be given.

Categories of explanations

<u>* No explanation</u>

This category includes responses without any argument, and responses that were mainly tautological (in which students rephrased their conclusion or the results obtained on the screen).

Example 1: One pair explained why the three angles cannot be equal as follows:

> *Such a case cannot be obtained, as there is always something that messed it up.*

<u>* Inductive explanation</u>

This category includes responses in which students based their explanations on one or more examples. Sometimes students' examples were taken from what appeared on the screen and sometimes students drew their own examples on paper.

Example 2:

> *There is no possibility that the three angles will be equal because we checked with the computer several triangles (equilateral, isosceles etc.)*

<u>* Partial deductive explanation</u>

This category includes responses in which students constructed a chain of deductive arguments but at least one link is missing or wrong.

Example 3:

> *There is no situation in which, the three angles will be equal because if there was one, the four segments from B would be equal too (BA=BD=BE=BC), but this is impossible since only two segments from B to line AC can be equal.*

Students in this category constructed a short chain of deductive arguments. It is partial because some arguments were neglected; for instance, they didn't explain the connection between their conclusion concerning the equality of the four segments from B, and the assumption of the angles' equality.

<u>* Visual – variation explanation</u>

In this category we include explanations in which students made use of visual variation, as a result of the dragging action or of their imagery. As such this category is typical for explanations in a DG environment.

Example 4: After trying to find a case of equality for the three angles and concluding that such a case doesn't exist, one girl in an interview explained while dragging the triangle on the screen:

> *I thought that maybe if I drag B up and angle B will become small, the angles might be equal, but then it* [the figure] *doesn't look like a triangle anymore, it will be more like a line. So it is impossible to get the 3 angles to be equal.*

She accompanied her explanation with moving her hands up showing how the triangle becomes thinner.

<u>* Deductive explanation</u>

In this category we include explanations that consist of a complete chain of logical arguments.

Example 5:

> *Let us assume that the 3 angles are equal* (see Figure 2 above). *EC=ED and thus ΔBDC is an isosceles triangle as the median and the angle bisector coincide. ΔBEA is also isosceles as the median* [BD] *coincides with the angle bisector. Thus BE and BD must both be altitudes and we have two different perpendicular segments from point B to AC which is impossible.*

The quantitative data (number of explanations and percentages in each category) are given in Table 1.

Table 1: Classification of students' explanations

No explanation	Inductive	Partial deductive	Visual-variation	Deductive	Total
4 (12%)	8 (23%)	14 (41%)	4 (12%)	4 (12%)	34

It is important to note that: (1) The examples of partial deductive explanations (Category 3), described here and in Hadas, Hershkowitz & Schwarz (2000) demonstrate that in spite of the fact that these explanations do not comply with the criteria for normative proofs, students tend to rely on their geometrical knowledge and use deductive strategies to explain their conclusions. Thus, in eighteen out of the 34 explanations, students used deductive arguments (fourteen partial deductive explanations and four deductive). (2) We found that twelve explanations were based on experimenting with the DG software and resulted either in inductive generalisation (eight explanations), or in a visual explanation based on the variations of the figure caused by dragging (four explanations).

CONCLUDING REMARKS

This work exemplifies two issues: The first is the 'four analyses model' of design oriented research, in which the planning work and the intentions of the designer are the basis for experimental research, and the research findings concerning students' responses serve as the database against which this planning work and these intentions are validated. The second issue relates to the specific content, in which the model was demonstrated, namely proving in geometry. Through the four analyses of the above model we demonstrated our approach to teaching/learning this controversial topic.

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