AFFECT AND MATHEMATICS LEARNING

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Beliefs, attitudes, emotions and values are the categories of affect used in mathematics education research. All these categories are founded on mental background systems which control thinking, learning and acting. To gain a deeper insight in the mechanisms of the mental systems and their forecasting character, results of neuroscience are used. By distinguishing between an explicit conscious memory system, which stores experiences about emotions, and an implicit emotional memory system, which operates unconsciously, it is possible to understand some emotional reactions in a better way.

1. Research in Mathematics Education on Affect

For a long time mathematics learning has been mainly seen as a problem of cognition. Researchers investigated the specificity of mathematical terms and concepts, steps of concept building, the influence of representations on the learning process, and so on. Consequences of research were theoretical models describing the learning process. The consideration of the social conditions in classroom learning led to an important extension. Mathematics learning was seen as a social process which controls the cognitive processes of the individual learners.

During the last decades there has been an increasing interest in affect and its influence on mathematics learning. The first step was to use the psychological "attitude concept" to explain differences in the students' mathematics performance. "Attitudes toward mathematics" (seen as a (learned) predisposition to respond in a consistent way with respect to a special object) was identified as a factor influencing the learning process and, as a consequence, the mathematics outcome. The most influential measure to investigate attitudes toward mathematics was the Fennema-Sherman Mathematics Attitude Scale (Hart, 1989; McLeod, 1994).

One consequence of this concept was the development of a "generic model for relating affect and outcome" (Fennema, 1989). In this model affect is a consequence of external influences like gender, social class etc. which influence learning activities and the mathematical test results. (For a further development of this model, see Evans, 2000).

A second very important concept in research on affect is the "belief concept" which stresses students' knowledge about mathematics in a very broad sense. The belief concept includes beliefs about mathematics, about the self, about mathematics teaching and about the social context (McLeod, 1992, 1994).

Furinghetti and Pehkonen (2000) describe the function of beliefs in the following way:

> (a) beliefs form a background system regulating our perception, thinking and actions; and therefore, (b) beliefs act as indicators for teaching and learning. Moreover, (c) beliefs can be seen as an inertial force that may work against change, and as a consequence, (d) beliefs have a forecasting character. (p. 8 - 9)

Research on attitudes and beliefs is based on questionnaires as instruments and statistical methods for evaluation. Qualitative methods especially in connection with research on nonroutine problem solving processes and higher-order thinking skills led to the interest in more intense affective reactions called "emotions". Goldin (2000) describes a model of affective states on different steps of a nonroutine problem solving process. These affective states can evoke special heuristic problem solving strategies.

The categories *beliefs - attitudes - emotions* are used by McLeod (1992) to reconstruct the research on the affective domain in mathematics eduction. To better distinguish these categories, McLeod (1989) uses the dimensions *intensity - direction - stability*. Longitudinal studies on mathematics learning processes and affect motivate DeBellis and Goldin (1997) to introduce a fourth affective category "values/morals/ethics". This new category emphasizes the appraisal aspect of affect.

A summary of the discussion of affect in research of mathematics education is provided by a quotation from Goldin (2001) which describes the categories of affective representations:

> (1) emotions (rapidly changing states of feeling, mild to very intense, that are usually local or embedded in context),
>
> (2) attitudes (moderately stable predispositions toward ways of feeling in classes of situations, involving a balance of affect and cognition),
>
> (3) beliefs (internal representations to which the holder attributes truth, validity, or applicability, usually stable and higly cognitive, may be highly structured).
>
> (4) values, ethics, and morals (deeply-held preferences), possibly characterized as "personal truth," stable, highly affective as well as cognitive, may also be highly structured). (p. 3)

The categories of affect in research on mathematics are on the one hand a consequence of research methods and on the other hand motivated by observed phenomena. The concepts don't describe the interaction between affect and cognition in a sufficient way to understand some affective reactions; especially, the relationship between conscious and unconscious is not really discussed.

2 Affect and Cognition -- Some Results of Neuroscience

First some remarks:

1) The use of terms "affect" and "emotion" is not uniform in neurosciences (Ciompi, 1999; Roth, 2001). Often they are used synonymously as generic terms for affective-emotional basic conditions.

2) The brain is the organ in which is represented all the knowledge about the world outside but also about inside of the body.

3) All systems of the brain are the result of evolution with the aim of helping the individual to survive. Emotion and cognition are also part of the brain system and therefore a result of evolution (Ciompi/Wimmer, 1996; Damasio, 1999; LeDoux, 1998; Roth, 2001).

4) A special brain system of an individual is a consequence of the evolution as well as of the ontogenetic development of the individual (Ciompi, 1999).

5) First of all we have to note that all processes on the neuronal level are unconscious. But some of these processes lead to conscious results. We are aware only of these conscious parts of the processes.

Emotion and cognition are both subsystems of the brain system. They are located in different parts of the brain (Damasio 1999; LeDoux 1998; Roth 2001) but there are connections between both systems which allow interactions. A very important consequence of these two different systems of concepts is the fact that we have to distinguish between "feeling" and "knowing that we have a "feeling" (Damasio, 1999; 26) or "emotional reactions" and "conscious emotional experience" (LeDoux, 1998; 296).

The emotion system has also connections to the arousal systems, therefore emotional reactions are often combined with "body reactions" (LeDoux, 1998). For this paper the interaction between affect and cognition are of particular interest:

* The emotion system influences memory processes. On the one hand, the emotion system is involved in the storage process. The system works like an appraisal filter. On the other hand, the results of the retrieval process are emotionally related (Roth, 2001).

* The emotions and consciousness interact: "While conscious control over emotions is weak, emotions can flood consciousness" (LeDoux, 1998; 19). That means that emotions have more influence on the cognition than the cognition has on emotions.

* LeDoux (1998) uses the concept of working memory to represent "the here and the now in the brain." In the working memory inputs from a sensory system meet inputs from the emotional and arousal system and information from the long-term memory.

3. Reconstructing the Theme by Results of Neuroscience

The most important research concepts of affect are beliefs and attitudes. Both are seen as the stable mental background which regulates perceptions, thinking and actions of a person (Furinghetti & Pekhonen, 2000). This stable mental background system is a consequence of all the processes of an individual to a special object, person or idea. Törner and Grigutsch (1994) speak therefore of a "mathematical world view" of a person; Ciompi (1999) of "one´s own truth".

Research methods for investigating beliefs and attitudes are in the most cases questionnaires; in some cases interviews. Both methods rely on the memory of the interviewees.

If we look at the results of neuroscience in relation to the memory, we have to distinguish between two memory systems with repect to emotions: the implicit emotional memory and the explicit memory about emotions (LeDoux, 1998). The implicit emotional memory operates unconsciously, is strongly connected to arousal systems and may often lead to bodily reactions.

The explicit memory about emotional situations contains all the conscious knowledge about emotional situations, emotional reactions to objects, persons and ideas etc.. The most important consequence is the fact that this memory system is part of the cognitive memory and there is no distinction between a remembrance of an emotion and a remembrance of a cognitive content (LeDoux, 1998).

The fact that the memory about emotions is a cognitive memory has some important consequences:

1) We have knowledge about our feelings and their origin. This knowledge is stored in memory systems as cognitive knowledge. Therefore we have the possibility of investigating knowledge about affect using cognition. Research methods used in cognition research (questionnaires and interviews) can help us to explore affect.
2) The memory about emotions is open for "rational" manipulations. That means we are able to think about our emotional remembrances. But we have to note all verbal statements are controlled by the cognition.
3) A very important control authority to all verbal statements is the "feeling of one´s own worth". Therefore we try to give our emotional remembrances a meaning which doesn´t destroy the feeling of one´s own worth. In this sense persons have in interviews a tendency to trivialize their own weaknesses. This trivialization can, for instance, take place by the expression of opinions that particular mathematical contents are unimportant if the person is unable to cope with these kinds of problems (Wagneder, 1998).
4) Research on emotion and memory suggests that humans "construct" their memories in a way that they are able to live with this memory. A part of this

process is that we forget unpleasant facts more easily than pleasant ones. And our memory has suppression mechanisms to handle unpleasant remembrances (Roth, 2001).

5) Group processes have an important influence on verbal statements concerning contents which are emotionally coloured. Humans are able to "learn" emotions in group processes (Ciompi, 1999; Damasio, 1999). These processes can lead to common shared emotions concerning special contents. Especially, shared value systems can influence the contents of the memory. In this sense we have to see the habitus concept (Gates, 2001) or the socially constructed "feeling rules" (Ulich & Kapfhammer, 1991) (Feeling rules mean that the group says what a member of the group has to feel in a particular situation) .

These reflections on the two memory systems give us a better insight into the meaning of the belief and attitude concepts. Both concepts are based in the outcomes of the cognitive memory system. This system contains not only our knowledge of cognitive contents but also the conscious knowledge of emotions. Therefore we are able by questionnaires and interviews to get information on cognitive as well as on emotional facts. The difference between the both concepts lies more in the kind of questions than in the quality of the memory. The restrictions of the remembrance of emotional situations and the handling of emotions by the memory also restricts the meaningfulness of the concepts. Belief and attitudes as a mental background system do not completely regulate our perception, thinking and actions, because the conscious controls on our emotions are only weak (LeDoux, 1998). Therefore we have learning situations, which are not ascertainable by the belief – attitude concept. In this sense it is not surprising that qualitative research methods led to a new affective category.

Studies on the problem solving process, especially with nonroutine problems, suggest that there exist emotional outbreaks during the problem solving process. These emotional reactions could be from mild to very intense, and are local and embedded in a special contextual situation. Goldin (2000) developed a model which describes states of the problem solving processes and the possible affective-cognitive reactions to the demands of this state. All these reactions are situational, caused by the demands of the problem solving process. The model describes two possible results, dependent on whether the problem solving process leads to success or not.

Interpreting these results by the concepts of neuroscience we have to note that there is a second memory in connection with emotions. This memory (LeDoux calls it emotional memory) operates unconsciously. Its evolutionary function is to help an individual to survive. Therefore it often works in dangerous situations, is strongly connected with arousal and bodily systems, and leads to reactions which are not controlled by cognition (LeDoux, 1998). This

emotional system is always activated if the appraisal of a situation leads to the result an activation is necessary. This appraisal of a situation is not conscious.

If we analyze the model of Goldin (2000) there are states in the problem solving process which can activate the emotional memory system. This activation can be observed by the researcher. The most well-known signs of an activation of the system are bodily reactions. To interpret bodily reactions is a qualification which is evolutionarily rooted in humans and therefore a valuable research method to evaluate emotional situations. As discussed above, these emotional reactions underlie only a weak control by the cognition (LeDoux, 1998) and are therefore especially valuable. Goldin describes the feelings by words which we can connect with the concerning expressions of the face, shaking of the hand, quavering of the voice, and so on. These signs of the body also allow us to recognize all changes of the emotional state.

It is important to note that emotional situations are not only stored in the memory which contains the knowlege about feelings they are also stored in the emotional memory and influence therefore further reactions to a special situation.

Activation of the emotional memory can lead in some cases to very intense emotional reactions. Reports on learning situations in adult education in mathematics tell from "learning blockades" (Lindenskov, 1996; Schlöglmann, 1999; Wedege, 1998). To elucidate these problems I will give an example which was reported on the internet discussion group "adult numeracy" by Bonnie Fortini:

> Has anyone run in to a case like the student I have had who seems unable to do any math that has unknowns or variables in it? She is mid 40s, very bright, English major going on to a Masters program. She can do all sorts of computations including fractions, percentages, ratios, and word problems are some of her favorite things to do. But as soon as you give her something like 4 + 2x - 6 + 5x= 95, she is totally frozen. She can't get past go when trying to combine like terms, and reacts physically (anxiety, tears, etc.).

Such an emotional reaction is not understandable within the conceptual frame which is used by research in mathematics education. This reaction is a traumatic reaction. The distinction between emotions used in mathematics education research is the stability. This is not a short sign of frustation described in papers to emotional reactions in problem solving processes. The mental system which caused this intense emotional reaction is stable, operating always if the relevant content is recognized.

LeDoux (1998) writes about such reactions:

> In traumatic situations, implicit and explicit systems function in parallel. Later, if you are exposed to stimuli, that where present during the trauma, both systems will most likely be reactivated. (p. 202)

The specificity of such emotional reactions is that there is no chance to help on the cognitive level. Such problems are not solvable in classroom situations. The emotion floods the cognition and the whole cognitive system is blocked and not able to be reached by "rational" arguments.

During the last year there has been an increasing interest in teachers' values and beliefs and the consequences for the classroom practice. Some researchers see a strong connection between teachers' values and beliefs and the classroom situation. But classroom observations and case studies suggest that there are differences between beliefs and values expressed in interviews and the real classroom practice (Skott, 2000). Bishop, FitzSimons, Seah, and Clarkson (2001) describe the situation in the following way:

> It is also recognized that there are differences between the values that are officially planned and those espoused by teachers, as well as between teachers` espoused beliefs and their actual classroom practices – due in part to differential positionings as interview subjects and as teachers. (p. 170)

We note that beliefs as well as values expressed in interviews are products of cognition. Both use the explicit memory as background. In the classroom reality implicit unconscious systems are also activated and these systems influence all actions in complex situations. We have in the classroom situation on the one side the cognitively controlled values and beliefs and on the other side the unconscious from the implicit emotional memory system influenced actions. It depends on the local situation which system dominates the concrete action. Especially in critical situations, the implicit system is more active because the implicit system leads to actions which allow survival independently of which values and beliefs are formulated in interviews.

Summarizing the results of this paper: Neuroscientifical results helps to prove the adequacy of the theoretical concepts used in research of mathematics education. Furthermore they can help us to understand these concepts in a better way. However, it is not the idea of this paper to say that research questions of mathematics education can only be solved by neuroscience.

4. References

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