INTEGRATION OF DOMAINS OF KNOWLEDGE IN MATHEMATICS TEACHERS' PRACTICE

Isabel Escudero and Victoria Sánchez
Departamento de Didáctica de las Matemáticas, Universidad de Sevilla, Spain

This study is a part of an ongoing research that attempts to characterise the relationship between mathematics teachers' knowledge and practice. Here we are focusing on the identification of the domains that are integrated in distinct moments of teachers' practice, in particular in the introduction of Thales theorem. Data were obtained through audiotapes of several semi-structured interviews, observations and videotapes. Although the teachers of our study had a similar background and experience, the analysis of data showed differences in the way of introducing the theorem, in the characteristics of the domains integrated in the previous decisions and in the changes related with the initial planning. The study contributes to highlight the complexity of the teachers' professional practice.

INTRODUCTION

Over the last decades, the relation between mathematics teachers' knowledge and practice has been studied from different perspectives. Within the cognitive perspective, Leinhardt and her colleagues have shown that teachers have a background that provides them resources for confronting the different teaching situations (Leinhardt et al., 1991). Other researchers have pointed out several ways in which teachers' knowledge and conceptions influence what the teachers are able to do in their classrooms (Lloyd and Wilson, 1998, Sherin, 2000). Furthermore, analytical models have been created to characterise 'the decisions and actions of teachers as they teach' (Schoenfeld, 2000, p.243), or to examine teachers' instructional practice with respect their underlying cognitions (Artzt and Armour-Thomas, 1999). The latter have tried to understand the role of the cognitions in the instructional practice by examining knowledge, beliefs and goals in different stages of teaching. For them, these components form a network of overarching cognition that controls the instructional behaviour of teachers in the classroom, assuming that differential instructional practice is related to differences in teacher cognitions.

FRAMEWORK AND AIMS

Our work draws upon the literature described just above and is situated in the line of researches that attempt to characterise the relationship between mathematics teachers' knowledge with different moments of their practice. In this study, we consider teachers' professional knowledge as 'a personal construction generated through the professional activity in which several domains of knowledge are integrated' (Garcia, 1997). This knowledge is developed over a long time and in different context, being a 'situated knowledge' (Brown et al, 1989). We assume that cognitive integration of the different domains of knowledge (pedagogical content knowledge, subject-matter knowledge) takes place in the different situations of professional practice.

In addition, we consider professional practice as 'the set of activities that the teacher generates when he carries out the tasks that define the teaching of mathematics, and the justifications that he provides about these activities' (Llinares, 1999, p. 110). Following this author, we consider that the professional practice is not limited to classroom events. It is conceptualised from a wide perspective that includes the relationships between peers, work in groups, etc., and the articulation, justification and communication of the decisions and actions. Therefore, although our study is built on cognitive perspectives, we try taking into account socio-cultural aspects.

Although we assume that teachers' practice is 'a conglomerate that cannot be understood by looking at parts split off from the whole' (Simon & Tzur, p.254), in our research project, we have focused on the teaching/learning process management of two secondary mathematics teachers' in a set of lessons to describe and characterise the professional practice. These lessons were related to the unit 'Thales theorem and similarity'. On other occasions within the above mentioned research project, we have studied what happens in mathematical classes when mathematical content appears different to that scheduled by the teacher (Escudero y Sánchez, 1999). The present study is focused on the identification of the domains that are integrated in distinct moments of the practice, (in particular in the introduction of Thales theorem). We are trying to characterise these domains, studying their role in the changes in relation to the initial planning.

Since we investigated the teachers' knowledge and practice in relation to Thales Theorem and similarity in our study, it is important to make explicit the framework that guide this inquiry. Our framework incorporates the consideration of the similarity in a 'intrafigural' way based on the properties of similar figures or as a geometrical transformation. We also consider the different modes of representation used (real world situations, manipulative, figurative, natural language, symbolic representations), and the type of tasks and the demanded activity in these tasks (calculation, construction or demonstration).

METHODOLOGY

Participants and Site

The research participants were two experienced Secondary teachers, Ismael and Juan (the names are pseudonymous). There was not a specific preparation as regards the design or implementation of the unit. Both prepared their units independently. The pupils belonged to two classes without any special characteristics (3rd and 4th years of Compulsory Secondary Education, 14-15 and 15-16 years of age, respectively). The schools were situated in the outskirts of the city.

The unit

The development of a new curriculum for Secondary level (12-16 years) in Spain commenced in the early 1990s. This reform has emphasised the teaching of Geometry. Objectives in this section include 'knowledge and use of basic geometrical properties: Pythagorean theorem, Thales configurations, similar figures and elementary notions about trigonometry (Junta de Andalucía, 1993, p.133). Supporting documents encourage teachers to include these objectives in their planning.

Instruments

Data were obtained through audiotapes of several semi-structured interviews, observations and videotapes. We videotaped each of the teachers teaching the whole unit of their own design (eight lessons and nine lessons respectively). During the initial interview, the teachers were asked to explain their lesson plans and to describe how they meant to develop the lessons. Previous and post lesson interviews were carried out. The field notes taken down during the observation provided information about classroom events that might not show on the videotape. Transcriptions were made of the audio part of the videotapes for analysis.

Data analysis

We analysed the previous interviews in terms of their goals, lesson plans, selected problems, etc. Thus, we obtained the teachers' agenda. The observation notes and the videotapes were used to categorise the teachers' instructional practice. Firstly, we identified 'segments' in the lessons (presentation segments, monitored practice, homework check, etc), considered as parts of a lesson with an objective, and perfectly differentiated by teacher and students (Leinhardt, 1989).

In the different segments, we characterised the actions developed by the teacher to reach the objectives, taking into account the grade of intervention of the teacher with respect the pupils and the type of data used. We identified different actions in the presentation segments: utilisation of an example/problem for reaching a definition, a property, a theorem, with the constant intervention of the teacher and pupils; explanation for reaching a definition or property, with minimum or nil intervention from the pupils, among others. In these cases, the data were whole numbers. The whole set of actions provided us with the structure of the segments. By integrating the data of the different instruments it was possible to know the reasons and justifications that made the teachers adopted a specific structure. This allowed us to identify and characterise the underlying domains of knowledge in the structure.

Through our analysis we were able to discern differences in teachers' instructional practice. We used the integration of domains as a way of better understanding these differences. In particular, within the segments identified in the unit, here we are going to focus on a presentation segment identified in the two teachers, whose objective was to introduce the Thales theorem. Firstly, we are going to present the structure we have identified. Afterwards, we will characterise the domains of knowledge that have influenced the adoption of this structure, and the aspects of these domains that were responsible for the changes.

RESULTS

The two teachers in our study had a similar background and experience. Nevertheless, we were able to observe differences in the structure of the presentation segments of Thales theorem and in the characteristics of the domains related with the previous decision of adopting these structures.

For Ismael, the unit of 'Thales theorem and similarity' was important because 'it helps to give sense and to understand the concept of numerical proportionality'. In particular, the theorem is considered as a context to visualise the numerical proportion in order to be able to connect 'the abstract numerical view with graphical images'. This unit was developed throughout 9 consecutive fifty-minute lessons. In these lessons, we identified 7 presentation segments; 3 monitored practice and 1 homework check, which were used alternately. In particular, the presentation segment of Thales theorem occupied the first place and was developed in 80 minutes (almost two lessons).

This segment belonged to a group of three segments, which structure was composed by the following actions: utilisation of an example or problem / explanation for reaching the definition or formula / definition / explanation for completing. This structure was identified in those cases in which the introduction of the definition or property was based in contents previously introduced by the teacher in other units. In the introduction of Thales theorem, former contents were numerical proportion and parallel lines cut off intersect lines. The teacher wanted the pupils to identify these contents in some proposed problems.

In the first action, utilisation of an example, Ismael proposed three problems previously selected in his planning. Through these problems, Ismael wanted 'to collect pupils' previous ideas that are related to the theorem' (initial interviews). The three problems were real world situations that included figural representations, pointing out the aspects of projection and homothety. They were proposed together to small groups of pupils. The teacher had justified the importance of this way of working by saying that 'the pupils work together sharing meanings and communicating their ideas' (initial interview).

In the following action, explanation for reaching the definition, the teacher pointed out the ideas that he had collected through the interaction with the groups of pupils, which he considered important for the introduction of the theorem. Ismael had mentioned that: 'after the work in groups, I am going to collect the important theoretical ideas, clarifying the difficulties' (initial interview). Ismael conducted a whole-class discussion, writing these ideas on the blackboard. He ended with the definition of the Thales theorem. Finally, through the explanation for completing, the teacher introduced other aspects that allowed him to connect with the monitored practice. He justified the introduction of this contents by saying: 'I would try to show several Thales configurations in relation to the position of data and unknown', adding 'the consequence of Thales theorem diminishes the difficulty that the pupils may have with a specific type of configurations' (initial interviews).

Globally considered the characteristics of the domains of knowledge that were decisive as regards the previous decisions of adopting this structure were related with his pedagogical content knowledge. In particular, the importance of establishing relationships between the earlier knowledge and the new concept by means of several problems previously selected, and working in small groups as a way of communicating and sharing ideas, followed by his intervention as 'organiser'. His knowledge of specific difficulties about the content played a more specific role, influencing his decision of including an explanation after the definition of the theorem.

Nevertheless, regarding the structure identified in the classroom, we appreciated variations in relation to the agenda. The teacher had indicated that his way of working 'was working in small groups/ setting out common grounds/ recollecting the important ideas and clarifying the difficulties to reach the formal definition' (initial interviews). Why did the setting out common grounds disappear? It was related to the difficulties that pupils had with the content. The teacher justified this omission by saying: 'I meant to start the explanation when the pupils had advanced more, but I had to intervene because of their difficulties' (Post-lesson interview). We want to point out two ideas. Firstly, the way in which Ismael understood the 'setting out common grounds', focused on his own intervention of collecting and clarifying the pupils' ideas that he considered important. This was coherent with the action 'explanation for reaching the definition' identified in our analysis.

Secondly, the important role of the difficulties in the change of action. Ismael related these difficulties to specific characteristics of the content. In particular, in the post-interview he pointed out two aspects, which were important for him. The first aspect refers to the difficulties related to the contents he wanted the pupils to identify in the proposed problems: 'with the first two problems, the pupils had difficulties with the proportion they had to identify' (post-lesson interview). The second aspect refers to the characteristics of the real world situation proposed in the problems. Initially, Ismael emphasised the use of real world situations and the importance of recognising the proportion in different figural representations, therefore, all three problems were real world situations with different geometric configurations. The problems showed both aspects projection and homothety. Two of the problems established a relationship between two data and the other was a missing value problem.

Nevertheless, the pupils' understanding of the enunciate of the problems was complicated by the real situations. Ismael concluded that this was a cause of difficulties: 'I realised that the enunciate of the problems was too long compared to what these pupils were habituated to' (post-lesson interview)'. This allowed him to think about changing the real world situations for other easier ones (even though these situations were no-real world situations) in the future. For Ismael, the role of real world situations was motivation. Nevertheless, he did not doubt of maintaining the use of several problems with variations in geometrical configurations. This aspect, and the importance that Ismael gave to the role of Thales theorem in the unit, was closely related to his subject matter knowledge.

All of this shows the role played by the perception in the classroom of the difficulties with the syntax of the real situations and the specific aspects of the content. Why did he have included these problems? As we have mentioned before, his decisions were related with his subject matter knowledge, but they were also influenced by his knowledge of other students. The teacher justified his election by saying: 'I had proposed these problems to older pupils. I think I overvaluated their capacity of abstraction' (post-lesson interview). The knowledge of students of Ismael was situated in other pupils.

For Juan, the inclusion of the Thales theorem in the unit was justified because it is used later on to establish the criteria of similar triangles, and also because of his previous experience 'I have done it like this for years' (initial interviews). This unit was developed in 8 fifty-minute lessons. We identified 11 presentation, 7 monitored practice and 4 homework check segments. The presentation segment of Thales theorem took about 20 minutes and was the second of the unit. It belonged to a group of 4 presentation segments with the following structure: utilisation of an example or problem/ definition/ explanation for completing. This structure was linked to the teacher's introduction of contents whenever he supposed the pupils had obtained some knowledge of the matter in previous courses ('the pupils have learned about numerical proportionality of previous courses' (initial interviews)). On this basis, Juan thought to introduce the concepts of ratio and proportion of segments, which he considered previous to approach the theorem, in the same unit.

In the first action, utilisation of an example, Juan developed the content in sequential steps by means of instructions to the whole group, intercalating questions and asking for the results. Juan was indicating to the pupils that made a verification of properties of parallel lines cut off intersect lines to equal and distinct distances by means of measurements carried out with drawing tools. The examples to be used, though previously selected by the teacher, were not known to the pupils at the beginning.

Juan justified this way of introducing the theorem because he considered that the pupils knew the geometrical elements that intervene in the examples to be used (parallel lines, intersect lines, segments) so that they will be able to follow his instructions. Furthermore, by assigning a numerical value to the segment pupils could connect with the arithmetic proportion ('already known'). During the initial interview Juan explained that the pupils 'more easily believed things' after having carried out the verification by them.

The enunciate of the theorem (by means of a transparency) collected all mathematical aspects that were sequentially emphasised in the performed verifications, showing a 'typical' configuration of the theorem, and emphasising the projection aspect. Finally, in the explanation for completing, the teacher stressed the generality of the lines in the drawing and placed emphasis on the general character of the theorem by specifically mentioned to the pupils that 'the things that we have done with these specific cases are going to be always true' (video transcriptions).

Globally considered, the domains of knowledge that intervened in the adoption of this structure were related to his knowledge of school mathematics as a teaching/learning object and the knowledge about learning processes, considered as a part of pedagogical content knowledge. Especially, as regards the use of a verification as a procedure of validation and the use of sequential steps that the pupils were carried out 'because they learn more' (post lesson interview). However, Juan specifically mentioned the low level and lack of motivation of these pupils in the initial interview. We think that this aspect was also integrated in the previous decision of using verification. Juan mentioned the characteristics of the general case only in the content of explanation.

This teacher only presented few variations as regards of his way of working compared to his agenda. In the case of Juan, difficulties played a different role in comparison to the case of Ismael. By dividing the content into sequential steps and giving a certain type of instructions to the pupils, few difficulties came up in the classroom that the teacher considered worth while to take into account with respect to his objective of introducing the theorem. Pupils' difficulties with the lack of precision at measuring the segments, which had an effect in the results of the verifications, were not relevant aspects from the perspective of his subject matter knowledge in relation with the introduction of the theorem. The algorithmic view, which was appreciated in the sequential steps, underlined the decision of including the theorem, in the sense that it was considered as a previous step towards the later introduction of the concept of similar triangles. Along with this, it was made evident the influence of the school culture, which traditionally assigns an important role to theorem.

CONCLUSIONS

This study is a part of an ongoing research into the relationships between teacher's knowledge and practice. The main objective of this paper was the identification and characterisation of domains of knowledge that intervened in this relationship. The objective of the presentation segments of both teachers was the introduction of the Thales theorem. Pedagogical content knowledge and subject matter knowledge were integrated in their decisions. Nevertheless, our study has shown that these teachers adopted different structures, being the initial decisions of adopting these structures linked to different characteristics of the domains of knowledge, which were integrated in a very different way.

Both teachers realised the importance of the pupils' difficulties in the adopted decisions. But, whereas Ismael was very sensitive to the difficulties, which came up during the interaction, Juan minimised the importance of these difficulties. He tried to avoid the difficulties globally in his previous decisions about his planning, admitting the verification as an easier procedure for the introduction of the content. The characteristics of the subject matter knowledge related to Thales theorem made that Ismael gave a great importance to the establishment of numerical/geometrical connections by means of the selected problems. Juan's algorithm approach of the contents was associated to a more general view of school mathematics.

There are many issues in our research that require further investigation. A better characterisation of domains of knowledge and its role in the different moments of professional practice can help to improve our knowledge about the complexities of mathematics teaching.

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