DESIGNING TASKS TO EXPLORE DRAGGING WITHIN SOFT CONSTRUCTIONS USING CABRI-GEOMETRE.

Cathy Smith

Faculty of Education, Cambridge University, UK

Tasks were designed to study the relationship between students' actions and reasoning in Cabri geometry problems. In task-based interviews with two UK students of differing geometric experience, I recorded the simultaneous words, actions and visual results whose interrelation expressed their problem solving approach. This data was analysed using Arzarello's (1998) classification of dragging modes, and relationship with transitions between inductive and deductive reasoning. This indicated that these students spontaneously used dragging modes related to soft construction and inductive reasoning, sometimes also finding deductive explanations. Implications of these results for future task design include the importance of promoting construction as a means of gaining control of a figure for future enquiry.

AIMS

In this study I design and evaluate tasks that exploit and promote distinctive heuristics identified in the research concerning learning with Cabri-géomètre. Cabri is a dynamic geometry environment characterised by: direct manipulation of screen geometric "drawings", construction tools based on Euclidean geometry, and - its key design feature - the invariance of geometric dependencies under dynamic variation of their initial objects (*dragging*). Any software medium changes not only the accessibility of mathematical objects of study, but also their nature, the way that they are acted upon and how meaning is made of them. We can therefore distinguish between the geometry of the curriculum, and the meanings constructed through Cabri.

A review of case studies of children's work with Cabri illustrates the variation in children's geometric knowledge and in the foci of researchers' interest across different countries (Hölzl, 1996; Arzarello, 1998b; Jahn, 2000 Healy, 2000). The background to my study is the UK curriculum, which traditionally has little in the way of extended problem solving or explicit teaching in deductive geometric reasoning., Given appropriately designed tasks, could such students find that dragging heuristics supported deductive geometric learning?

DRAGGING AS MEDIATOR BETWEEN CONCRETE AND ABSTRACT

The functional uses made of Cabri – the actual sequences of drags and clicks – are of interest in pedagogic research because they are both indicative of the learner's understanding, and formative in making meaning. The computer provides a *window* that enables reconstruction of the learner's conceptions from her actions in a context that is both *exploratory* and *expressive* (Noss et al., 1997). Cabri users recursively exploit the dual exploratory/expressive dialectic by developing heuristics of dragging in which geometrical conceptions, already reorganised by the Cabri medium, are made accessible at a perceptual level to permit further reorganisation (Arzarello, 2000).

Noss et al. (1997) identify a *situated abstraction:* a reorganised mental concept which does not replace a concrete visual representation but simultaneously derives from it and gives meaning to it through actions. Hancock (1995) finds a similar two-way interaction in his notion of *transparency*: a computer modelling tool embodies mathematics in a way that makes it easier to "see" its subjects; but equally the subjects are a background against which the mathematics can be seen. He traces a phenomenological-epistemological progression from *naïve transparency*, as learners use Cabri to express what they already know, to *opacity* as they struggle with the tool and with their geometric understanding, to *co-ordinated transparency* as the two are reconciled. The progression from opacity is the hardest, and of most interest to teaching.

Different modes of dragging

Recent literature has distinguished modes of dragging according to how they are used and their mathematical nature. This emergent classification has been summarised and extended by Arzarello (1998b), who focuses on the interplay of perception, movement and thought, and the distinction between what is taken as given and what is sought.

Wandering dragging: describes the often random dragging that is used when exploring a construction, seeing the dynamic relationships and searching for an interesting feature. In this mode there is only one direction of progress: from visual/ physical to conceptual; and successful use puts a high premium on what students perceive and how they interpret it.

The dragging test: dragging part of a construction to verify that a desired regularity is maintained. This powerful and direct Cabri representation of the mathematical idea of generality is the most clearly related to inductive and deductive geometric reasoning: affording both empirical justification and an sequence of actions that expresses relationships between the initial objects and the conclusion. However, the necessary co-ordination of a set of figures with an appropriate independent variable and a construction sequence leading to an interpretable phenomenon is often a complex task.

Lieu muet *dragging*. Whereas the dragging test starts with a set of conditions on which to verify a hypothesis, *lieu muet* dragging seeks to establish a locus for which the hypothesis is satisfied. Arzarello (1998a) relates this mode of dragging to logical *abduction*: seeing what rule you have the case of. This mode of dragging is popular when students learn with Cabri (Jahn, 2000): it is a heuristic that is suggested by the medium rather than a representation of a traditional geometry heuristic.

Bounded dragging: dragging a point along a perceived trajectory to explore what happens to a feature of interest. The difference from the dragging test is that the trajectory is not actually constructed, so the domain of the dragable point remains implicit and is independent of the history of construction.

Use of dragging within constructions

The dragging test has a natural mathematical status in verifying a conjecture formed as an if-then statement. Students, however, are found to prefer the other forms of dragging, and may rarely perform a drag test without the mediation of other modes. Hölzl (1996) first describes the common heuristic of dropping a construction constraint then varying by dragging until a visual solution is found. Working on a task without simultaneously concentrating on all conditions allows the user to construct representations of the constraints in the most natural way without considering the order of dependence necessary for a linked construction. Healy (2000) identifies similar heuristics in what she calls *soft constructions*. Both Hölzl and Healy show how students' soft methods could have led to complete geometric solutions, but raise the question of how to support students in this final stage of reasoning. The starting point for this study is Healy's finding that students had difficulty in making connections between deductive reasoning and apparently equivalent Cabri construction sequences, and her recommendation for more consideration of soft construction methods in task design.

Such research has been started by Arzarello (1998a, 2000) in the context of students who have had explicit instruction in modes of dragging. He traces their shifts from exploration to explanation, or from *ascending* to *descending* control, and finds that dragging, especially *lieu muet dragging* is a crucial mediator. The actions of clicking and dragging taken as non-verbal language have both a *deictic* and a *generative action* function in triggering and supporting the students' transition to abstract generalisation. Students' verbal reasoning changes from broken oral narrative associated with exploration using wandering dragging, through abductions describing *lieu muet* dragging, to the more sequential linear sentences of canonical mathematical proof, which may still use the vocabulary of Cabri actions. One crucial transition appears to be between seeing a *lieu muet* locus as a dynamic trajectory resulting from dragging to a geometric object that could be constructed in it own right (Jahn, 2000).

In summary, distinctions between the geometry of the curriculum and Cabri geometry become evident in differing approaches to the activities and logic of problem solving. Difficulties such as functional dependence, and the co-ordination of several constraints arise in both geometries, but Cabri can offer modes of action and of expressing meaning that are intermediary in finding a solution. Tasks have been most successful when learners have a clear idea of the type of endpoint required, either in Cabri or in geometrical terms, and have explicitly been taught some Cabri heuristics. The focus of study should be the interplay between the actions and words accompanying a task, which characterises the modes of dragging, and then between these modes and developing control of the problem solving and proving strategies.

METHODOLOGY

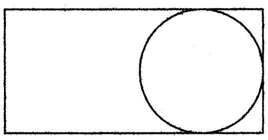
The study includes task-based interviews with two contrasting subjects: a six-year old, Daisy, with no experience of either Cabri or Euclidean geometry, and a nineteen year old, Helen, working on a task developed during a mathematical reasoning course using Cabri. Daisy's 45 minute interview was interactive; in Helen's case I took the role of non-participant observer for most of the hour interview, asking her to type responses into comment boxes onscreen, but finally asked her to talk through her progress and intentions. The data collected were recordings of what was said or written annotated with the simultaneous Cabri actions and gestures, and the screen's appearance. This data was analysed using the classifications of dragging and reasoning modes as described above.

Daisy's task – Circles and Rectangles

The aim of this interview was to investigate designing a Cabri task involving only elementary properties of shape but accessible to different modes of dragging and reasoning. Daisy played with Cabri circles and triangles, and I then gave her a previously constructed rectangle, which she dragged, from tall/thin to short/fat, then aligned horizontally with traditional proportions.

Cathy: If you have a rectangle, can you make a circle that touches all four of its sides?

Daisy constructs a circle, placing the centre near the centre point of the rectangle and pulling out the radius. "No – it would have to be an oval," stops dragging and draws an ellipse in the air. Changes the rectangle to look square, "Now you can..." and fits her circle to it.



"And if I pull it now, it won't be touching ": she extends the square horizontally into a rectangle.

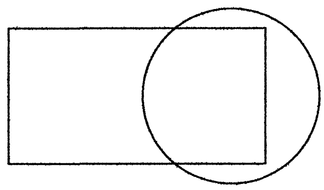
"If we were trying to move it across we would have to move it up too because if we didn't move up it would change into an oval. Say we had a square the sides of the square are all the same size. If we had a rectangle they are not all the same size. If we have to pull it across ... to be a circle it has to be the same length everywhere ... and those were against the edge so if we move it up it's going to go off the top. "

Daisy's initial response is visual – she imagines a curved shape inside a rectangle and recognises that it would be an oval i.e. not a circle. Her focus on the special case of a square appears to be motivated by getting some solution on screen. She then starts to reason about continuity from a square via one-way enlargement, a mode of reasoning afforded by my prior construction of the dragable rectangle. She makes her argument verbal, firstly in transformational terms, describing the effects of hypothetically dragging the circle ("if we were trying to move it .."), and finally by relating the geometric properties of a rectangle, square and circle (" it has to have the same length everywhere"). Her use of "Say we had... If we had..." indicates that this is an abstract generalisation, although still phrased in concrete dynamic terms ("go off the top").

Cathy: Can you make a circle that touches all four corners?

Daisy: "No, only if it's a square". She drags back to a square and pulls the circle out to touch all four corners. "It's on the outside now".

She drags out a rectangle and starts to make the circle bigger. Pauses. She moves the centre of the circle fairly haphazardly and reattaches the two right hand corners. She increases the radius again, and again moves the centre to attach the corners. Eventually: "Yes I think you can ... its got to be in the middle."



She drags the centre of the circle onto the middle of the rectangle, adjusts the radius, points and checks: "One, two, three, four corners, yes you can."

Daisy initially intended to use her previous heuristic of arguing from a square to a rectangle to demonstrate impossibility, but the perceptual results of dragging intervene to change her reasoning. She realises that she no longer has to keep the circle inside the horizontal and vertical dimensions of the rectangle. Hancock's description of *opacity* seems apt as Daisy struggles for another way to relate the two shapes. She has lost the *naïve transparency* of her earlier interactions with the geometric objects, and no longer uses any of her knowledge of horizontal and vertical symmetry. Eventually by repeating the actions of varying the centre she establishes a horizontal locus for the centre of the circle - *lieu muet* dragging to meet relaxed constraints - and this suggests the centre of the rectangle. Finally she notices as a geometric phenomenon that it is equidistant from all four corners.

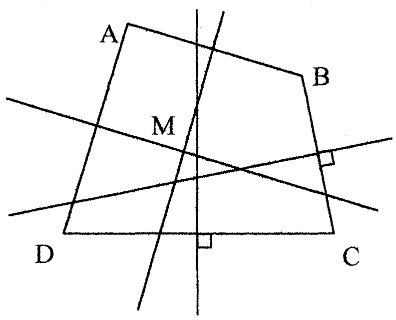
Helen's task - Perpendicular bisectors

Draw a quadrilateral ABCD. Construct the perpendicular bisectors of each side. Label the four points (M, N, P, Q, say) at the intersection of the perpendicular bisectors from adjacent sides. If you move ABCD, what happens to the inner quadrilateral MNPQ?

1. Investigate the relationship between the internal angle at A and the internal angle at M.

2. For what types of external quadrilateral is MNPQ a parallelogram?

3. Find a quadrilateral ABCD that is similar to its inner quadrilateral MNPQ (i.e. an enlargement/reduction).



This construction is also described in Arzarello (2000) with students exploring the degenerate case, but this version starts with angle relationships which are familiar to UK students, and has structured goals that leave open further enquiry. Helen started the task by *wandering* dragging, noticing that one inner point was invariant when she dragged A. She used the causal metaphor of dragging to describe this geometrical relationship: changing A changes only two outer sides, which changes only two inner sides, so only three of the inner points. She stated that the inner angles at A and M add to 180.

Helen's next step was intuitive (and she couldn't later explain it)- she constructed the diagonals of the inner and outer quadrilaterals, and it helped her see a way to make MNPQ a parallelogram: "It would be a parallelogram on the outside because the intersections of the diagonals must be at the same point. That's the only way; if I move A away, the centres don't coincide." Her reasoning accompanying this *lieu muet* dragging seems to be purely inductive; arising from the observation that as she dragged ABCD to move the intersections of each pair of diagonals together, both MNPQ and ABCD became parallelograms.

Helen again moved on without explanation to the third question. Like many of the students she did not use the geometric meaning of "similar", but also tried to give ABCD and MNPQ the same orientation: "Can't find exactly similar – must be a rotation, because to get the same angles … need a square or a rectangle .. and then it disappears." She spent some time explaining to herself why this degeneracy happened. In doing so she went back to the relationship of the angles at A and M, and wrote a justification of this invoking the two right angles in the quadrilateral with diagonal AM. Although the degenerate case was a deviation from the set questions, considering the role of the perpendicular bisectors helped to give insight into the whole task. Helen then wrote a deductive explanation of her second result: "For MNPQ to be a parallelogram, its opposite sides must be parallel. The sides of MNPQ are the perpendicular bisectors of the sides of ABCD so for them to be parallel, the sides of ABCD must be parallel". Helen then tried to manipulate the quadrilateral ABCD while keeping it a parallelogram, an example of *bounded* dragging, but she had great difficulty co-ordinating this and finally gave up. While her perceptual aim was clear in this episode, she was less clear in her purpose within the task: "I'm looking for it to be the same … similar".

Helen's task gives an example of progress towards a deductive argument in a more challenging geometric context. Although the task was structured to build up a sequence of reasoning, she actually synthesised all her reasoning at the end of the sequence of exploratory tasks. Unlike Daisy, Helen's developing *reasoning* was not immediately associated with her concurrent screen actions, which led her simply to visual answers, and so it is less clear whether dragging played a specific mediating role or that she simply had inspiration. However it was noticeable that she built up a body of empirical knowledge through the soft dragging modes before attempting any explanations at all. The construction of the diagonals, for example, was a temporary stage of her reasoning and after making her explanation she later deleted them. It was nevertheless important because it gave her *wandering dragging* a clear visual endpoint, and allowed her to progress to finding the *lieu muet* that achieved this condition.

CONCLUSIONS

My aim was to design tasks that asked for elements of deductive geometric reasoning but were accessible to students with little experience of such problems. I followed the suggestions of Healy (2000) that task design in this context should give equal consideration to soft construction methods, and of Arzarello (2000) that the focus of observation should be the interplay between the physical/visual aspects of different modes of dragging and the language of argument. Both Daisy and Helen spontaneously made purposeful use of the *wandering, lieu muet,* and *bounded* dragging modes that are characteristic of soft constructions. These modes were always associated with the purpose of exploring the next situation, finding the next answer and making progress with the task. Equally it is clear that they accompanied inductive reasoning and making abductions, i.e. establishing and expressing necessary conditions for a solution. In Daisy's case these lead to a contradiction and she finds a deductive argument, but Helen's explanation remains at the stage of finding what is necessary. This is what Arzarello characterises as gaining *ascending* control of the problem. Helen does start to move towards *descending* control, trying to express the reverse argument by varying a general parallelogram ABCD to see what happens. This suggests that her reasoning at the end of her investigation is still closely related to her perception and expression in Cabri – she does not look to see that the logical connectives are all equivalences, but starts another dynamic experiment.

In the final stages of their task both subjects carry out painfully slow dragging as they try to co-ordinate their physical actions with a number of perceptual constraints, and the experience is frustrating. Much of this could be avoided by using construction e.g. fixing the radial point as a rectangle vertex, or ABCD as a parallelogram, but Helen and Daisy did not attempt to do so. It seems that the most important heuristic is finding the methods that allow us to control the variation that we want to perceive. The soft dragging methods above have the benefit of being very accessible, allowing the user at the same time to explore the task and start to make sense of it, but they also have their limitations when the user has more sophisticated experiments in mind. It is perhaps at this stage that Arzarello's students benefited from their teaching in Cabri techniques, and were ready to start again with a fresh construction that would enable them to vary what was now of interest. *Lieu muet* dragging was found to phenomenalise a locus of conditions, thereby mediating the transition from ascending to descending control, but the next stage would be to make explicit this locus as the starting point for experiments and the given for deductive reasoning. This study suggests that it may be helpful not just to exploit the potential of soft dragging methods in making sense of and establishing a context for a theorem, but also to include support for reorganising the problem and expressing the context as a construction which can be dragged in the desired way.

REFERENCES

Arzarello, F., Micheletti, C., Olivero, F., Paola, D. & Robutti, O. (1998a) 'A model for analysing the transition to formal proof in geometry' Proceedings of the 22th conference of the International group for the Psychology of Mathematics Education, Stellenbosch, 2: 24-31.

Arzarello, F., Micheletti, C., Olivero, F. & Robutti, O. (1998b) 'Dragging in Cabri and modalities of transition from conjectures to proofs in geometry' Proceedings of the 22th conference of the International group for the Psychology of Mathematics Education, Stellenbosch, 2: 32-39.

Arzarello, F. (2000) 'Inside and outside: spaces, times and language in proof production' Proceedings of the 24th conference of the International group for the Psychology of Mathematics Education, Hiroshima, 1: 23-38.

Hancock, C. (1995) 'The medium and the curriculum; reflections on transparent tools and tacit mathematics' in diSessa, A., Hoyles, C. & Noss, R. (Eds) Computers and exploratory learning. Berlin: Springer-Verlag, pp. 221-240.

Healy, L. (2000) 'Identifying and explaining geometrical relationship: interactions with robust and soft cabri constructions' Proceedings of the 24th conference of the International group for the Psychology of Mathematics Education, Hiroshima, 1: 103-105.

Hölzl, R. (1996) 'How does dragging affect the learning of geometry?' International Journal of Computers for Mathematical Learning, 1(2): 169-187.

Hoyles, C. (1995) 'Exploratory software, exploratory cultures?' in diSessa, A., Hoyles, C. & Noss, R. (Eds) Computers and exploratory learning. Berlin: NATO pp. 199-220.

Jahn, A. (2000) 'New tools, new attitudes to knowledge: the case of geometric loci and transformations in dynamic geometry environments' Proceedings of the 24th conference of the International group for the Psychology of Mathematics Education, Hiroshima, 1: 91-101.

Noss, R., Healy, L. & Hoyles, C. (1997) 'The construction of mathematical meanings: connecting the visual to the symbolic' Educational Studies in Mathematics, 33(2): 203-233.