THE ROLE OF MATHEMATICS EXPERIENCES IN FORMING PRE-SERVICE ELEMENTARY TEACHERS' VIEWS OF MATHEMATICS

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Abstract. *The purpose of the study was to examine pre-service elementary teachers' views of mathematics and the experiences that influence their development. A model was created in order to be able to investigate students' experiences and views in a fruitful way. The study was carried out in a mathematics methods course during the academic year 1999-2000 in a Finnish university. The study shows the importance of finding out and taking into account students' views of mathematics during their mathematics studies. In addition, the data lead to the conclusion that it is possible to influence students' views of mathematics during their studies.*

Introduction

Over a period of several years pre-service elementary teachers have many experiences that are linked to mathematics and its teaching and learning. The experiences shape their views of mathematics and thus influence their ability to receive new knowledge during their studies. In addition, their views of mathematics influence their teaching at school in the future and therefore also their pupils' views of mathematics. (e.g. Ernest 1989) The results of prior research show that pre-service elementary teachers' views of mathematics are not the best possible at the beginning of their studies from the point of view of their future career. Their beliefs and knowledge about school mathematics seem to be restricted. In addition, many of the students have a poor attitude toward mathematics and some of them are even afraid of it. (e.g. Ball 1990)

This paper addresses a major problem that has been noticed in mathematics education in many countries: that mathematics education programmes appear to have only a limited affect on pre-service elementary teachers' capacity and willingness to change their teaching (e.g. Hill 2000). There have been many attempts to develop mathematics studies in order to bring about permanent changes in students' views of mathematics and their teaching practice. Some studies have paid attention to students' school memories (e.g. Trujillo & Hadfield 1999), others to increasing students' knowledge (e.g. Graeber 1999), to improving their attitudes (e.g. Ellsworth & Buss 2000) and to diversifying their beliefs (e.g. Vacc & Bright 1999). Despite all of these studies, the problem of effectiveness still remains unsolved. The study reported in this paper was designed to contribute to the solution of this problem (Pietilä 2002).

The view of mathematics

The purpose of this study was to examine pre-service elementary teachers' experiences in mathematics and their influence on students' views of mathematics. The view of mathematics develops with exposure to different experiences with mathe-

matics in interaction with affective, cognitive and conative factors (e.g. Op 't Eynde, De Corte & Verschaffel 1999). Emotions, beliefs, conceptions and attitudes work as a kind of regulating mechanism in the formation of one's view of mathematics. In addition, learning demands cognitive aptitudes, like understanding, identification, thinking, evaluation and reasoning as well as conscious goal-oriented aspiration and activities. On the other hand a student's view of mathematics also influences his or her understanding, decisions, affective reactions and actions, for example in different mathematics-related learning situations (Schoenfeld 1985). In addition, self-concept is of great significance in the formation of the view of mathematics (cf. McLeod 1992).

The view of mathematics is therefore defined here to be a combination of knowledge (e.g. Shulman 1986), beliefs and conceptions (e.g. Thompson 1992) as well as attitudes and emotions (e.g. McLeod 1992) that develop with exposure to different experiences with mathematics. The view of mathematics consists of two parts: knowledge, beliefs, conceptions, attitudes and emotions about

1) oneself as a learner and teacher of mathematics, and

2) mathematics and its teaching and learning.

Both parts can be divided into smaller parts. The second part includes, among other things, views about how teaching should be organized and what the roles of teacher and students are.

Mathematics experiences are of central importance in the formation and development of the view of mathematics. A model for personal experiential knowing made by Malinen (2000) was modified to model the formation of the view of mathematics. The view of mathematics consists of a hard core, which contains the persons' most fundamental views, and a protective belt, which contains more flexible views (cf. Green 1971: the psychological centrality of beliefs; Kaplan 1991: deep and surface beliefs). Mathematics experiences need to be penetrated to the hard core in order to change the view of mathematics in an essential way.

Former experiences (or rather the situations that are linked with them) are marked in the model so that their arrows reach the hard core of the view of mathematics (at the bottom of figure 1). This is based on the reasoning that home, school, friends, myths about mathematics, and temporary posts as teachers have influenced a student's view of mathematics before his or her studies. One example of the mechanisms of this influence is the fact that students' study of mathematics at school is usually textbook-centred. It is then natural that mathematics is merely considered to be computation. Likewise for example the myth that boys are better in mathematics may have influenced one's view of mathematics.

Experiences during studies (or rather the situations that are linked with them) are marked in the model (at the top of figure 1) so, that their arrows reach only the border of the protective belt. Student can have meaningful experiences in mathematics methods course, other studies and practice teaching. In addition, temporary posts as a

teacher can be significant even at this stage. For example, a student can receive guidance in the mathematics methods course on how to use manipulatives in his or her teaching. Students also have an opportunity to use temporary posts as a means of applying what they have learned and can reflect on its usefulness.

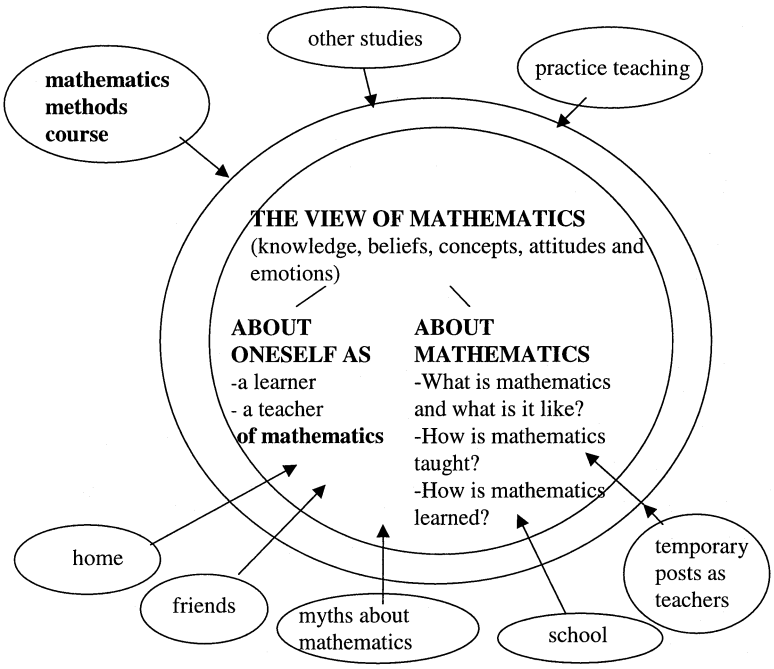

Figure 1. A model for the formation of the view of mathematics

The study reported in this paper was designed to determine 1) how pre-service elementary students viewed mathematics at the beginning of the mathematics methods course and what experiences had shaped their views, 2) what kinds of experiences students find meaningful in shaping their views of mathematics during their first year of university study, and 3) the main changes in the students' views of mathematics during their first year of study.

Attempts were made to influence students' views of mathematics using a variety of research-based methods during the mathematics methods course (4 credits). It was considered important to focus on the students' views at the beginning of their studies in order to be able to arrange the most beneficial learning environment (cf. Malinen 2000). In addition, it was thought significant that students pondered their views of mathematics, which demanded a safe atmosphere (e.g. Stuart & Thurlow 2000). The studies were designed so that students could try out as much as possible of what they had learned. It was assumed that exploring and learning the use of manipulatives would help students to understand elementary school mathematics more deeply (cf.

Quinn 1998). At the same time it was assumed that students would learn to teach for understanding (cf. Graeber 1999). In addition, combining theory and practice was thought to be able to help students to consider the learned topics useful (cf. Hill 2000).

Methodology

A research method based on phenomenological starting points was used in methodological solutions, but problems related to interpretation and understanding were also examined from the viewpoint of hermeneutics (cf. Giorgi 1997). The research material was gathered from the students (N=80) in the form of written homework as a normal part of their study. The questions were made as broad as possible and they were designed to obtain information about certain themes. Students wrote five different letters during their mathematics studies that dealt for example with their experiences of mathematics at school and at the mathematics methods course. In addition, eight students were interviewed approximately one year after the end of the mathematics methods course in order to increase the trustworthiness of the study.

The research material was analysed using following steps: 1) The material was read through twice in order to get a sense of the whole. In addition, it was important to 'bracket', which means to put aside things that we know in order to experience students' experiences freshly. 2) The texts were divided into meaning units (themes of the study). 3) The students were grouped based on their situation in the beginning of their studies. 4) The research material was analysed one research question at a time. 5) The contents of the meaning units were identified and recorded. 6) The common features and structures for different groups were recorded. (cf. Giorgi 1997)

Interviewed students read analyses of their views of mathematics at the beginning of their studies and checked their correctness (member check). An effort was made to increase the validity of the study by means of triangulation, prolonged engagement, peer debriefing and referential adequacy. (Lincoln & Guba 1985)

Results

The students were divided into four groups based on the answers given at the beginning of their studies. Students' views of themselves as learners of mathematics were considered as criteria for this grouping because they influenced very holistically both their descriptions of their experiences and their views of mathematics. The groups were categorized according to their views of mathematics as follows: 1) Mathematics is challenging problem solving (13%), 2) Mathematics is important and usually pleasant (36%), 3) Mathematics is one subject among others (20%) and 4) Mathematics is difficult and unpleasant (31%). This data led to the conclusion that only approximately half of the students are interested in and/or enthusiastic about mathematics and about studying it at beginning of their studies. Some students were even afraid of mathematics (c.f. Trujillo & Hadfield 2000).

Mathematics is a bugbear. Mathematics is just the thing that I have always felt difficult. When I was small others were afraid of bugbears while I was afraid of mathematics. That bugbear

has kept me in his possession until these days. (a letter written at the beginning of the studies, group 4)

In addition, all students but the students in group 1 had a very narrow view of mathematics and its teaching. Most of them thought that mathematics is merely computation and that it is based on rules and procedures that should be memorized (cf. Ball 1990).

I think that mathematics is a tool that gives means, formulas and instructions for solving computational problems. (a letter written at the beginning of the studies, group 2)

In addition students' knowledge of mathematics was compartmentalized, usually very superficial and not based on understanding (cf. Hill 2000). Thus it became important to find out students' views of mathematics at the beginning of their studies and to pay attention to them during the studies.

Mathematics studies helped students to question and redefine their views of mathematics and its teaching and learning. Students felt that their view of mathematics became better organized and they knew more about what they think about mathematics (cf. Llinares & al. 2000). They noticed that mathematics could be taught using many different methods that had not been used when they were in school. They realized that it is important to understand what they learn and not just to memorize. In addition, they understood that it is important to actively involve pupils in the learning process.

A new insight to teaching and learning mathematics was that one should try to figure things out by oneself instead of trying to memorize rules. (a letter written at the end of the studies, group 2)

The studies also challenged students to re-evaluate their relation to mathematics. Their views of mathematics became more positive and they became enthusiastic about teaching it (cf. Raymond & Santos 1995).

Mathematics has actually surprised me. As a subject it is really many-sided, at its best an impossibly inspiring and attractive subject. (a letter written at the end of the studies, group 3)

In addition, as they learned more about the subject and its methodology their views of mathematics became broader and more accurate. The studies gave the students an opportunity to understand elementary school mathematics more deeply, for example by using manipulatives (cf. Quinn 1998).

The belief that not everybody can understand mathematics has vanished and I noticed during the mathematics methods course that I realized many things that I had not understood at school. (a letter written at the end of the studies, group 4)

Students reported many factors that were important from the point of view of the change in their views of mathematics. The mathematics methods course, practice teaching, holding temporary teaching positions and other studies combined to make it possible for the students to consider their studies to be meaningful. Students thought that it was important that the mathematics studies gave them the opportunity to try to

explore different things by themselves. The close relationship between theory and practice helped students to see that the studies were useful (cf. Ellsworth & Buss 2000).

> *It is important that we did things instead of just talking about them. We handled the materials and tried them out. If you can't use them in the course why should you try them later? (a letter written at the end of the studies, group 3)*

Pondering and talking about own experiences and views, finding models for their own teaching and understanding the learned topics were also experienced to be important. In addition, a positive learning atmosphere was considered to be very essential (cf. Stuart & Thurlow 2000). Conversations helped students to understand their past and improve their attitude.

> *The fact that I have been obliged to ponder on my past experiences as a learner of mathematics has helped me to disprove the myth: "I don't understand anything about mathematics."... We have discussed fears and attitudes toward mathematics. It has been rich and has changed my attitudes. (a letter written at the end of the studies, group 4)*

The students had an opportunity to use practice teaching and temporary posts as a means of applying what they had learned. Students' experiences were mainly positive and they recognized the usefulness of what they had learned (cf. Stuart & Thurlow 2000). Students noticed that they could explain so that pupils understood. In addition, the pupils were enthusiastic about the new teaching methods.

> *I could test my own ability to demonstrate and concretize when I helped students. It was enlightening to realize how much concretization helped students to understand. Some of them seemed to understand some basic things for the first time. (a letter written after practice teaching, group 4)*

Other studies helped students to see mathematics studies in a larger context, although their significance for enhancing their views of mathematics seemed to be small. Students noticed during their studies that they needed many skills in order to develop into good mathematics teachers. They realized they needed knowledge about both the subject and pedagogy. On the other hand the learning of new skills and knowledge increased their self-confidence as teachers (cf. Raymond & Santos 1995).

> *I noticed during the mathematics methods course how many facilities and skills I actually needed in order to develop into a good mathematics teacher. We treated mathematics broadly during the lectures and especially during group meetings, and I became aware of many new facets. (a letter written at the end of the studies, group 3)*

Conclusion

Many positive changes took place in students' views of mathematics during their first study year. The depth and the durability of the change can still only be guessed. Some idea of this was gained through the interviews that were carried out one year after the mathematics studies had ended. Four students had participated in practice teaching during their second study year, and their views of mathematics had became

even more positive because they had become more confident as mathematics teachers. Four students had not studied anything that could be linked to mathematics. Their views had remained almost unchanged, becoming perhaps a bit more realistic.

It was important on the point of view of the stability of the change that students developed the ability to recognize and reflect on their views of mathematics (cf. Raymond & Santos 1995). In addition, the awakening of students' responsibility gave the researcher faith in their willingness to change their teaching. The students realized that most of the pupils' understanding and enthusiasm depend on teacher's teaching methods. The stability of the change could probably be increased by combining theory and practice even more (cf. Hill 2000). It would be important for the students to try out what they learn as much as possible in practice (with students) during the mathematics methods course. By doing so, they would have the opportunity to become more convinced of what they learn. In addition, the material would be easier to internalize and remember.

The model developed for researching the students' views of mathematics seemed to be functional. Based on the changes in the students' views of mathematics, it was possible to conclude that knowledge, beliefs, conceptions, attitudes and emotions affect the formation of their views of mathematics. In addition, the division of the view of mathematics into two parts was justified because students' views of themselves as teachers had such a strong influence on their views of mathematics as a whole. In addition, the graphic model of the view of mathematics seemed to be functional and useful. It provided an opportunity to portray their experiences and their views of mathematics in a compact picture.

It would be interesting to follow the students' views of mathematics when they obtain posts as teachers. In addition, it would be interesting to study what experiences have the most influence on their views of mathematics, for example, the school's curriculum, colleagues, size of the class, textbook, teaching materials, in-service training and so on.

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