A CASE STUDY OF A UNIVERSITY STUDENT'S WORK ANALYSED AT THREE DIFFERENT LEVELS

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An on-going study of a long-term one-to-one work with a future mathematics teacher will be presented. The framework for a threefold analysis of data is suggested. The first level of the analysis of data yielded several settings which will be described by some variables (e.g. didactic contract), the second level of analysis concentrates on different situations within the settings and the third level of analysis investigates communication patterns. The first results of our analyses are briefly given. Finally, further research is proposed.

In 1998, we started a research project focused on the building of internal mathematical structures [1]. A series of semi-structured interviews was conducted with several future mathematics teachers who volunteered to be part of the research. One of them, Molly, who was in her first year of study then, was very excited about the mathematical topic she was working on – so called restricted arithmetic RA [2]. She studied it at home and often came back to the experimenter (the author of this paper) with new suggestions. Very soon the previously formal interviews transformed into a qualitatively different setting in which the research purposes grew less and less important, while the teaching-learning purposes became prominent. Molly continued investigating RA with growing autonomy, formulated hypotheses, definitions, tentative theorems, meeting regularly with the experimenter to discuss her work. Her 'co-operation' with the experimenter spanned all five years of her study and culminated in her writing a two-hundred-page diploma thesis on the topic.

What contributed to Molly's enthusiasm and endurance? Why did she do so much additional work in her free time? Why did she feel such a need to explore RA as much as possible? Why did she go on with the work while the other students were content with several interviews and stopped? What teaching-learning settings did she go through during her interaction with the experimenter and how can they be characterised? We do not know the answers to these questions yet but we have tried to find a methodology of how they might be found. This will be the focus of this paper.

THEORETICAL BACKGROUND

At the university level, many types of teaching-learning situations can be determined, some similar to those in elementary or secondary schools, others specific to university teaching: tutorials, lectures, seminars, individual instruction, demonstration, class discussion, home study, office hours, etc., among non-standard teaching-learning situations we have, for instance, scientific debate (Alibert, Thomas, 1991), and using constructive, interactive methods involving computers and co-operative learning (Leron, Dubinsky, 1995). Some attention has been paid to individual types of

situations, e.g. tutorials (Nardi, 1996, Jaworski, 2001), lectures (Boero, Dapueto, Parenti, 1996), however, the author does not know of any research which focuses specifically on situations at the university level and determine their invariants and specifics.

We will use a qualitative research paradigm and our considerations will be grounded in the theory of constructivism and its basic tenets that knowledge cannot be transmitted but must be constructed by the learner (von Glasersfeld, ed., 1996) and that it is necessary to create contexts which stimulate creativity (Hejný, Kurina, 2001).

METHODOLOGY

As stated above, the object of our research will be our work with Molly. For the present purposes, we will focus mainly on interactional and emotional matters, neglecting the cognitive point of view (i.e. what knowledge is being learned) which will be the topic of subsequent studies.

The data consist of tape recordings of interviews, protocols, Molly's work from the interviews, the experimenter's field notes, Molly's solutions to mathematical problems, subsequent versions of Molly's 'mathematical' text (mathematical description of RA), her concept map of RA and the protocol of her description of it, the protocol and recording of Molly's teaching experiment and subsequent versions of her analysis of the experiment, and finally subsequent versions of her diploma thesis. The framework for the interpretation of data we want to propose here is based on a threefold analysis.

First level of analysis

First, seven settings S1-S7 [3] will be distinguished in the course of our work with Molly and characterised by variables. For our considerations at this level of analysis, we will borrow a general term of the *definition of situation* from interactionism and a more specific term of *didactic contract* (Brousseau, 1997).

Every situation can be viewed as a communication situation in which its participants play some role. Each participant defines the situation on the basis of his/her prior experience with similar situations. Their definitions of situation come to the fore when they are not compatible and when, for instance, some participants' expectations are violated. When defining a situation, a person assesses (McHugh, 1968) the theme and its elaboration during the interaction, how individual parts of the interaction fit the theme, if the instance in the interaction is typical or likely, if it is given by some previous event, etc.

It is not easy to describe briefly the term didactic contract introduced by Brousseau. Bodin and Capponi (1996) give a concise explanation: Didactic contract "refers to the system of reciprocal expectations held by teachers and students, in other words all the rules, mostly *implicit*, which determine the part for which each of them, teacher and students respectively, is responsible for handling in the teacher-student relationship".

Similarly to the *definition of situation*, the *didactic contract* is being constantly redefined by the participants on the basis of their interaction.

Second level of analysis

Next, within these settings different situations will be identified. Here, we will draw on the theory of didactic situations (Brousseau, 1997): situations of action (students first attempt to solve a problem), situations of communication (students communicate the results of their work), situations of validation (the results are justified), and situations of institutionalisation (results are summarised, the 'official' terminology is used).

Third level of analysis

Last, the analysis of interaction will be made with the aim to identify communication patterns in individual situations. For the analysis of interactions between a student and the experimenter, we will use the framework presented in Dreyfus, Hershkowitz, Schwarz (in press) and used for the analysis of pair interactions. They distinguish six types of statements which establish and maintain the flow of the conversation: control statements (proposals, plans), elaborations (what is done to continue or develop an idea), explanations (comments on the results of actions), queries (which put in question previous utterance(s)), agreement (or concession) and attention. The last type of utterances only show that the person pays attention to the actions of others.

In the interactionist research, several patterns of interaction in the classroom have been identified, e.g. recitation pattern, funnel pattern and focusing pattern (Wood, 1998), repetition pattern (Cestari, 1998), various thematic patterns of interaction (Voigt, 1995, cited in Sierpinska, Lerman, 1996, p. 854). Dreyfus, Hershkowitz and Schwarz, (in press) identified four patterns of interaction which can lead to abstraction: guidance/self-explanation, symmetric argumentation, asynchronic collaboration and collaboration in parallel. Brousseau's Topaze effect and Jourdain effect (Sierpinska, Lerman, 1996) can also be seen as interactional patterns. It is important to note that the various forms of interaction "are not always consciously recognised by students, or even by teachers" (Ellerton, Clarkson, 1996).

ANALYSES AND THEIR FIRST RESULTS

The paper presents an on-going study. In this section, the process of analysing the data will be briefly presented and some preliminary results discussed.

The settings presented below can be defined by the didactical variables, and also by the psychological and sociological variables (Sierpinska, 2000), i.e. the personal characteristic of the teacher/experimenter, and those of the student. Molly has always been a diligent student. Her mathematical ability is above average (in comparison with her fellow students), her attitude towards mathematics (and German language which is the second subject she will be teaching) has always been positive. She is a conscientious worker. She often underestimates her abilities. She is communicative and willing to speak about her thoughts and feelings.

First level of analysis – identification and description of settings

So far seven settings S1–S7 which we feel are different in at least some characteristics have been identified in the course of our work with Molly. Using the comparative analysis of the settings, the following variables have been determined so far for the description of the seven settings: MR – Molly's role, MG – Molly's overall goal, ER – experimenter's role, EG – experimenter's goal, MDS – Molly's definition of situation (part of didactic contract), EDS – experimenter's definition of situation, E – emotions.

One setting will be described in more detail, the others only listed and briefly characterised. The names of the settings are only tentative. In S1-S3 and S7 there are two participants – Molly and experimenter, in the others Molly alone.

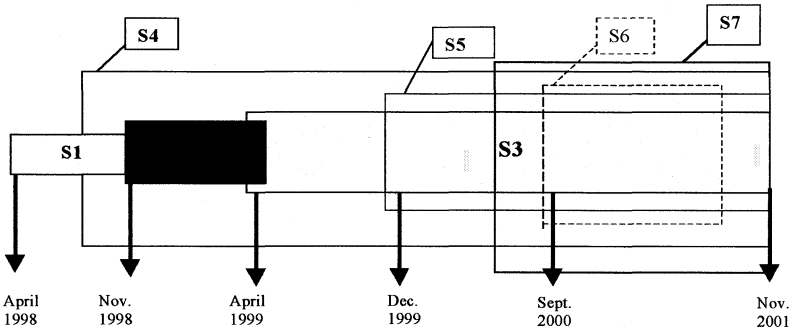
S1 – semi-structured interview

MR: pupil
MG: to try her best to solve the tasks
ER: expert
EG: to get data for the research on structuring mathematical knowledge; to motivate Molly for independent work; not to teach anything
MDS: expects to be told what to do; to be presented with some mathematical problems; maybe to be taught something
EDS: expects Molly to be a problem solver; expects Molly to communicate about her ideas and thinking; expects to be an observer
E: Molly is a bit apprehensive about the non-standard situation (being recorded), she thinks of the situation as similar to an oral examination; gradually she is more and more relaxed. Her worries whether she will be able to solve the problems diminish when the experimenter keeps encouraging her and saying that the situation is non-standard and nobody is expected to understand it straight away. Molly is more and more attracted by the mathematical tasks.

Notes: The expectations of both participants are to a certain extent violated. Molly expects more help from the experimenter. During the first session, Molly redefines the situation and starts to behave in a more independent way.

The other settings are: S2 – teaching interview (unlike in S1, the experimenter has didactic intentions and behave more like a teacher, rather than an observer), S3 – discussion (Molly continues investigating RA, the experimenter is a discussion partner, sometimes a teacher), S4 – home study (Molly is engaged in studying on her own), S5 – writing a mathematical text (Molly acts as a 'mathematician', she was asked to write in a concise way everything she had learnt so far about RA for a journal), S6 – Molly's teaching experiment (Molly acts in the role of a teacher and researcher, she prepares, carries out and analyses her own teaching experiment), S7 – spontaneous discussions (while S3 are sessions planned in advance and Molly knows that she will be expected to speak and explain work she has done at home, S7 is more like impromptu 'office hours' with Molly coming to ask a question or make an observation).

Setting S4 does not exist by itself. It is combined with S1, S2, S3, similarly S5 is combined with S3 and S7. The complex situation is best illuminated by the following figure which shows the time sequence of our work with Molly.



Molly's role has changed during the course of our co-operation. From the role of a pupil (expected to be taught) she got into the role of an independent problem-solver, autonomous learner, 'mathematician' at times, teacher and teacher researcher. We consider the process of change between the individual roles an important aspect of the whole process and will analyse it in more detail in the future.

Second level of analysis - situations

Every effort was made to suggest the "natural order" of the growth of scientific knowledge (Sierpinska, 2000) from action, formulation and validation to institutionalisation during our work with Molly. Moreover, whenever Molly validated anything it was her own conjecture or the conjecture conceived in co-operation with the experimenter, not given as a ready-made product.

Sierpinska (2000) also claims that at the university level, the last type of situation is the most frequent while the others do not appear at all or only in a degenerate form. She gives an example of proofs of theorems which are not students' own conjectures as a degenerate situation of validation; an example of a degenerate form of a situation of formulation when a teacher punishes an incorrect formulation of a definition; a degenerate form of a situation of action when a teacher gives students hints and suggestions before they tackle the problem. The first analyses of our data showed that despite the experimenter's effort on the contrary, some degenerate actions can be identified especially during the first interviews with Molly. They will be the subject of further analyses.

Third level of analysis – communication patterns

In this section, we will restrict ourselves on S1 – semi-structured interview [4]. The communication patterns can be briefly characterised as follows (E stands for the experimenter, M for Molly):

- a lot of *encouragement* from E ("Well done");
- virtually no *rhetorical questions* (i.e. utterances which only pretend to be questions);
- E's queries (the result of her need for clarification) have often the form of *repetition* of M's utterances;
- a good deal of M's *thinking aloud* (we do not classify it as explanation because it does not explain the results of action but rather M speaks as she thinks);
- there were some instances of E's utterances which we classified as *pointers* which were specific to the experimental situation (e.g. E repeats what M is writing in order to record it on the tape recording and thus make the analysis of the data later easier; or utterances like "Write the solution to a new problem on a different sheet", "Write in a different pen", "Say aloud what you are doing on the calculator"; some of them can be classified as control statements);
- E's *prompting* (e.g. "So...?"); a lot of E's *control statements* (E sets the agenda most of the time) – often it has the form of: E's control statement, E's question ("Do you understand?"), M's consent or M's *reformulation* of the task in her own words;
- E's *admittance* that she herself does not know the answer or that M found something which E did not see before (e.g. "I do not know if there is a pattern.", "Well done! I did not see this before.");
- demonstrations of *rapport* between E and M (e.g. jokes, laughs);
- M's *questions* are of a technical nature ("Shall I do it here or at home?", "Shall I finish the problem?"), she never asks for strategies; towards S2, the typical pattern is: E's question, M's explanation, E's brief consent (often non-verbal), M's explanation, E's consent, etc.

PEDAGOGICAL IMPLICATIONS

On the one hand there are indisputable advantages of the long term individual work with a student (it has a very strong and long-term influence on the individual, a student experiences 'doing' mathematics and constructivist learning and thus it is one of the ways how to get constructivist teaching to schools [5]). On the other hand, we cannot omit the fact that there was no co-operation with other students and that it is time consuming for both participants. Moreover, it is very difficult, if not impossible to reproduce a didactic situation (see obsolescence of didactic situations, Brousseau, 1997), "the practices observed are the products of multiple interactions whose elements are not always the same, even for any one teacher " (Bodin, Capponi, 1996).

PROPOSED FURTHER RESEARCH

Within the given framework of analysis, we have so far completely omitted at least three aspects which strongly contributed to the success of our work with Molly. First, the mathematics Molly was working on – RA, which is a rich context suitable for a student's independent investigation. Second, the personality of the experimenter and

her motivation. Mathematics, no matter how interesting, does not work by itself in the above way. And third, the experimenter previously investigated RA herself and was herself very excited about it. We hypothesise that if a teacher works on the same topic as his/her students, it is an important source of a positive motivational climate. All three aspects will be the focus of our analysis in the future.

It was stated at the beginning that the present study is an on-going one. Next, the analysis outlined above will be consolidated and applied to data, the cognitive aspect will be put under scrutiny and last but not least, Molly's long-term progress as a practising teacher will be followed. We hypothesise that Molly's experience with her long-term work will influence her teaching strategies towards the constructivist approaches.

In order to answer the questions stated at the beginning of the paper, a comparative analysis of interviews with other students must be made, too. Why did Molly go on with her work? We are sure that the reason must be a combination of several factors.

NOTES

1. First results were given in Stehlíková, Jirotková, 2001.

2. For its description see Stehlíková, Jirotková, 2001.

3. We will need a more convenient term 'situation' for other purposes.

4. We presume that further analysis will show that the communication patterns differ in individual settings. However, only S1 and S2 were tape recorded, in S3 and S7 our considerations can only be based on written materials, the experimenter's memory and Molly's self-reflection.

5. In view with Becker and Selter (1996), we strongly believe that simply hearing or reading about new teaching approaches is not enough. The student – future mathematics teacher – has to experience the new approaches in a way we want him/her to use at school.

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