HOW MUCH DOES CABRI DO THE WORK FOR THE STUDENTS?

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ABSTRACT

This report will discuss potentialities and pitfalls of a piece of technology (Cabri-Géomètre), as used by secondary school students in open geometry problems. A case study will illustrate in what way and how much the software can do the work for the students. The conflict between a perceptual-numerical way of using the software and a more theoretical-general one can be solved thanks to the intervention of the teacher, who leads the students to different schemes of use of the technology involved.

INTRODUCTION

"The computer by itself cannot fundamentally change either what is learned or how, and issues of learning and teaching are dependent on more than the simple presence of the computer in the learning situation"(Noss & Hoyles, 1996, p.52). This means that using new technologies in the classroom implies the redefinition of content, of methods and of the role of the teacher (Bottino & Chiappini, 1995).

Our research is focused on what the computer makes possible for mathematical meaning-making. This issue can be investigated from different perspectives, taking into account the interaction between the students and the technology, and the role of the teacher in this interaction. In relation to this, many recent works in the literature highlight a crucial question: what does it mean to say that a technology does the work for the students? (Hershkowitz & Kieran, 2001).

In this paper we will tackle this question as regards one particular piece of technology, that is Cabri-Géomètre. Cabri is a dynamic geometry microworld generally recognised as a powerful technological tool, which provides a useful mediation in many different directions(Arcavi & Hadas, 2000; Arzarello et al., 1998; Gardiner et al., 1999; Healy, 2000; Laborde, 1998; Mariotti, in press; Olivero & Robutti, 2001). This report, starting from the potentialities and pitfalls of using Cabri in open geometry problems (Arsac et al., 1988), first analyses in which way and how much this software can work for the students, and then points out the fundamental role of the teacher in the process of meaning construction.

In doing so, we take on board the perspective of the instrumental approach(Verillion & Rabardel, 1995), as elaborated by (Mariotti, in press). According to this, any technical device has a double interpretation: on the one hand it has been constructed according to a specific knowledge which assures the accomplishment of specific goals, and on the other hand, there is a user who makes his/her own use of the device.

In this perspective, it is important to highlight the distinction between *artefact*, which is "the particular object with its intrinsic characteristics, designed and realised for purpose of accomplishing a particular task" (Mariotti, in press), and *instrument*, that is "the artefact and the modalities of its use, as elaborated by a particular user" (Mariotti, in press). "For a given individual, the artefact at the outset, does not have an instrumental value. It becomes an instrument through a process, or genesis, by the construction of *personal schemes*" (Artigue, 2001, p.4, *italics is ours*), or *schemes of use*, which function as organisers of the activity of the user. As different and co-ordinated schemes of use are successively elaborated, the relationship between user and artefact evolves, in a process called *instrumental genesis*, which can be directed either towards the artefact or towards the subject. In the second case, the genesis "leads to the development or the appropriation of schemes of instrumented action which progressively constitute into techniques which permit an effective response to given tasks" (Artigue, 2001, p.4).

AIM

We consider Cabri as an artefact, which students use to solve a particular task. In using this artefact, an instrumental genesis takes place, and students construct meanings from the activity they carry out with the software. However the meanings constructed may not be the useful ones directed towards the solution of the given problem. "Meanings are rooted in the phenomenological experience (actions of the user and feedback of the environment, of which the artefact is a component) but their evolution is achieved by means of social construction in the classroom, under the guidance of the teacher." (Mariotti, in press).

This *evolution* may take different forms, according to the educational aims which are at stake. An evolution from a "mechanistic-algorithmic" way of using the technology, to one that is "led by students' search for meaning" has been observed(Hershkowitz & Kieran, 2001) in another environment, i.e. the symbolic-graphic calculators. In our case, concerning geometry problem solving with Cabri, this evolution can be described in terms of a shift from a *perceptual-numerical* use to a *theoretical-general* one. A *perceptual-numerical* use of the software involves reading either qualitative or quantitative information about the figures on the screen, based on perception only, doing manipulations and taking measurements. A *theoretical-general* use implies a (re)-interpretation of what happens on the screen in terms of a theory. These two ways of approaching the software can be considered two different *schemes of use*.

In the next sections, we will present and discuss a situation of geometry problem solving in Cabri, in which the students use Cabri in the two ways mentioned above and are able to evolve from the one to the other thanks to the teacher's intervention in the small group discussion.

METHODOLOGY

The example we discuss below is taken from a research project which aimed to develop and implement an integrated approach to new technologies in the classroom. Teachers and researchers developed and implemented classroom activities in an attempt to integrate geometry teaching and Cabri at secondary school level (15-18 year old students), with a particular focus on conjecturing and proving. All the classroom teachers involved in the project participate in a Mathematics Education Research Group [1] at the University of Turin as *teachers-researchers* (in the sense of Arzarello & Bartolini Bussi, 1998).

In the classroom sessions the students were presented with an open problem and were asked to work in pairs at the computer (with Cabri), trying to formulate conjectures and proofs, in a 2-hour session. Classroom observations were carried out. Two observers were usually present in the different classrooms and observed one pair of students. The data collected are field-notes and videotapes of the work of this pair, both in Cabri and with paper and pencil.

The observed students, in the protocol presented below, are 15 years old and attend 4 mathematics classes per week. They are medium achievers; they have used Cabri some times over the year before this activity [2].

"WHY AREN'T YOU PARALLEL?"

The problem given to the students is "The axis of a quadrilateral":

You are given a quadrilateral ABCD. Construct the perpendicular bisectors of its sides: a of AB, b of BC, c of CD, d of DA. H is the intersection point of a and b, M of b and c, L of c and d, K of a and d. Investigate how HMLK changes in relation to ABCD. Prove your conjectures.

The students draw a quadrilateral and they drag it until they get a parallelogram.

[...]

542 Paola: (looking at Figure 1) so if ABCD is a parallelogram, HKLM is a parallelogram. **Hypothesis**: AB parallel and congruent to DC and AD parallel and congruent to BC. **Thesis**: KL parallel and congruent to HM, KH parallel and congruent to LM.

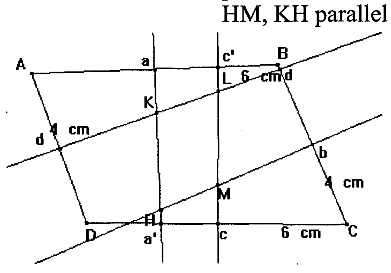

Figure 1

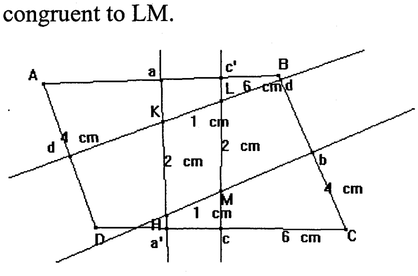

Figure 2

In the first phase, the students observe the Cabri figure while dragging it and formulate a conjecture. The use of Cabri, at this stage, is perceptual, because the students get the idea of the parallelism of some sides of the quadrilaterals only by observing the constructed figure.

543 Paola: but you can see that it's... wrong.

544 Giulia: no, it's right, try with measures!...you must define the segments.

545 Paola takes measurements of the sides of HKLM (Figure 2).

546 Paola: **look at the measurements, it's right! End of the story!** The proof is finished!

At this point the students search for a validation of their conjecture within Cabri, through the use of measures (544) [3]. According to Paola, once they have checked that the measures of the sides of HKLM correspond to (one of) the requirements of the thesis (which was previously formulated), the proof is finished (546). Cabri is now used in a numerical way, as the students focus on some quantities related to the figure. A first scheme of use appears: the perceptual-numerical one.

547 Giulia: well...in order to prove they are congruent we need to prove that the figures... [...]

549 Giulia: are congruent!...of course...and then parallel, I have no idea how we can prove they are parallel! [...]

551 Paola uses 'parallel line' and constructs the line parallel to d and going through M.

552 Paola: they are not parallel...ok, they are not parallel.

553 Paola deletes the line.

554 Paola: **why aren't you parallel?...**let's see if the sides of the initial figure (ABCD) are parallel...I'm worried about that!

Now the students want to prove the conjecture (547-549). The construction of the proof shows a strong link with the Cabri environment. Paola uses the construction of parallel lines in order to check whether HKLM is a parallelogram, that is in order to check if the thesis is correct (551). This test tells her that lines d and b are not parallel. In this case the checking does not support the conjecture previously formulated within Cabri. This provokes Paola to ask "Why aren't you parallel?"[4] (554). She seems to rely on her conjecture and try to understand the answer given by the software, which contradicts what previously seen and formulated. This episode provokes a chain reaction: since the thesis appears to be wrong, the students go and check if the hypothesis is correct, i.e. if ABCD is a parallelogram (554). Their reasoning is mathematically correct, but it is not supported by a correct interpretation of the particular construction in Cabri. The way they use Cabri is perceptual-numerical, not connected with the theory: as a consequence, the conflict grows in the students' minds.

555 Paola draws the line parallel to AD and going through B.

556 Paola: actually...it's parallel...

557 Giulia: well, but the other one as well...

558 Paola draws the line parallel to AB and going through C.

559 Paola: look. **It's not...parallel**

560 Giulia: **then there's nothing to prove...it's all wrong!**

561 Paola: I have no idea!

562 Giulia: neither do I!

563 Paola: those are not parallel, the sides are not parallel...there is still one thing missing: the angles! Let's measure the angles!

564 Paola measures the angles A, B, C and D.

565 Paola: **They are not congruent...it's all wrong!**

566 Paola deletes measures.

They check the parallelism of the opposite sides of ABCD (i.e. the hypothesis), and, by looking at the Cabri drawing, they realise the sides are not parallel, so they conclude that neither the hypothesis holds. For them the end of the story is that "it's all wrong" (560). As a last resort, they check the congruence of the angles of ABCD, that is they check another property that should hold for ABCD to be a parallelogram. Using Cabri in a numerical way, they get the same result: "they are not congruent" (565), because their measures are different. The students trust constructions (parallel sides) and measures (equal angles), so they get to the conclusion that their conjecture must be wrong: "It's all wrong!" (565). The Cabri figure does not correspond to the geometric properties a parallelogram should have from a theoretical point of view. In this case the use of measures and constructions in Cabri works against the validation of the conjecture. At this point the students are stuck, because they are presented with a conflict between a conjecture, discovered in Cabri, and a refutation of this conjecture, discovered in Cabri too. Will the software work for the students and help them to solve the conflict?

WHAT'S NEXT?

The answer to the previous question seems to be negative. The conflict can be solved only by a change in the way of looking at the situation. This change rarely happens if the students are not supported by the teacher, as happens in this case. In fact, the students call the teacher to understand how to deal with a figure that is supposed to be a parallelogram, but it is not a parallelogram in Cabri.

567 Giulia: miss! [...]

575 Teacher: you had a conjecture, didn't you? Your conjecture was: if ABCD is a parallelogram, then HKLM...

576 Giulia: but the problem is that ABCD is not a parallelogram!

577 Teacher: but can you transform ABCD into a parallelogram?

578 Giulia: I need to make the opposite sides parallel to each other!

579 Teacher: can you do that?

580 Paola: yes...but...it is no more the initial figure

581 Giulia: you must construct another figure!

582	Paola:	I mean, you need to move this line parallel to this (BC)
583	Teacher:	(she takes the mouse) you get a figure which has got parallel sides...this is OK!
584	Paola:	but they are not parallel...shall we **do as if they were parallel**?
585	Teacher:	yes. You formulated a conjecture saying that **if** ABCD is a parallelogram **then** HKLM is a parallelogram. OK? Now, can you prove that? If you can construct a proof, then what you're saying must be true.
586	Paola:	eh...we can try... **if we do as if that was a parallelogram, we'll make it!**

The figure observed by the students is still the same, but it changes status and it is now seen as a *generic object* (in the sense of Balacheff, 1999), which has the mathematical properties of a parallelogram, even if on the screen it does not look like a parallelogram: "shall we do as if they were parallel?" (584). The students need to construct a proof, as the teacher says: "If you can construct a proof, then what you're saying must be true" (585). A new scheme of use is developed through the intervention of the teacher: the use of Cabri in a theoretical-general way. While in the previous excerpts the words written in bold indicated facts (equal, parallel), in this last part of the protocol the words in bold indicate theoretical assumptions (do as if they were parallel; if ... then; as if that was a parallelogram). These words are indicators of a change in the scheme of use.

FINAL CONSIDERATIONS

How much did Cabri work for the students? In the first part of the protocol, the students stick to Cabri wanting to understand what is happening: *why aren't you parallel?* (554). They search for a 'unity': they want all the answers gathered in different ways (conjecturing, measuring the sides, constructing parallel lines) be the same and coherent. This solution path presents many similarities with others in the literature, for example with what is shown by (Hershkowitz & Kieran, 2001) in a problem situation involving the use of graphic calculators. The students want to make the technology work for them and do not abandon the technological device even if the figure produced cannot be justified mathematically. They develop a first scheme of use, which relies on a perceptual-numerical interpretation of the software's feedback. The artefact (Cabri) is transformed into an instrument, which however is not the one which serves to accomplish the goal of the problem situation.

The technology works for the students if there is an evolution from a perceptual-numerical use to a theoretical-general one, that is if there is a change in terms of schemes of use. A new instrument needs to be constructed by the students, based on a theoretical-general way of 'reading' the Cabri figures. The role of the teacher is crucial in developing this new scheme of use. On the one hand, the students use the artefact in order to accomplish a task and some meanings emerge; on the other hand the teacher uses the artefact to direct the students in the construction of meanings which are mathematically consistent. Further research will analyse the role of the teacher in the framework of *semiotic mediation* as presented by (Mariotti, in press).

From a didactical point of view, this classroom situation gives the opportunity to introduce the concept of *generic object*, by reproducing in a concrete case what happened in history. In fact, the dialectic between perceptual-numerical and theoretical-general approaches has a long history. For example,Locke considered the generic triangle as the one which is not oblique, nor rectangle, nor equilateral, nor isosceles, nor scalene. Berkeley defined the generic triangle as the one which has no particular properties and he does not assume any particular hypothesis about it. And he specifies that even if the drawing of a particular triangle is used when constructing a proof, its particular properties should not be used in the construction of the proof. Kant solved the debate assuming that the generic triangle is a triangle which is constructed by imagination in the pure intuition, whom a drawing is a representation. From the didactical point of view, it is crucial that the teacher shows how to "see that it is not" (Lolli, p. 139), that is to see that a proof does not depend on the particular graphical representations being used. In this respect, Cabri may support students because of the multitude of representations it produces. However, students need to be able to give those representations the correct theoretical status (as either hypothesis or thesis) in order to fulfil the aim of proving. The software does not work for the students if a particular *scheme of use* is not developed by students, through the interaction with the teacher.

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1 In this group teachers and researchers collaborate in the development, implementation and evaluation of new materials for the classroom.

2 They use Cabri I-7, because this was the version available in the school at the moment of the experiment (2000).

3 The numbers refer to the lines in the transcript.

4 This situation is very similar to other situations reported in the literature, as for example *"So why aren't they meeting?"*, reported by (Hershkowitz & Kieran, 2001), and commented with: "They now pay attention to the hard numerical evidence that there was a single point of intersection. Their inability to match this evidence and the graphical representatives they had obtained became very clear and waited to be resolved and explained" (p.103).

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