Connecting Children's In-School With Out-school Mathematics By Using Mathematical Writings

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Abstract

The purpose of this study was to explore the process of mathematical enculturation in children's cultural activities through the use of mathematical writings. There were ninety-nine second graders participating in the study. Three stages were developed in the use of mathematical journal writings. The results indicated that (1) the daily activities mathematics embedded into that were perceived by second-graders included drastic disasters, causal accidents, and surrounding environments. (2) The quality of children's mathematical writing was improved by evaluating each other on their writing journals. (3) Five commentary levels of mathematical writing were characterized. They were moved toward advanced commentary level. (4) Children became flexibly and efficiently in resolving the mathematical problems they posed.

Introduction

School mathematics experiences at all levels should include opportunities to learn about mathematics by working on problems arising in contexts out of school of mathematics (NCTM, 2000). Therefore, the opportunity for students to experience mathematics in variety of contexts is important. In recent years, many studies have focused on mathematical cognition relating to individual competence in daily life context (Bishop & Abreu, 1991; Carraher, 1988; Lave, 1988; Saxe, 1991; Tsai & Post, 1999). A review of children's out-of-school mathematics raises critical questions about how children come to understand mathematics and how they connect informal knowledge out of school with formal knowledge in school (Hibert & Carpenter, 1992; Millory, 1994; Resnick, 1987). In keeping with the connection between in- and out-school mathematics for children, the researcher has developed a teaching model called the Cultural Conceptual Learning-Teaching Model (CCLT) (Tsai, 1996) that attempts to combine individuals, activities, concepts, and culture together. The previous studies focusing on CCLT suggest that the linkage from children's cultural activities to school mathematics contributes to children's better performance in school mathematics and their abilities in solving problems in daily life (Tsai & Post, 1999; Tsai, 2000; Tsai 2001). However, An important question is how children applied school mathematics they learned in the CCLT model into real life situation. Thus, this study intended to provide children an opportunity with the mathematical journal writing to help them make sense

school mathematics implicitly embedded in everyday activities.

Mathematical writing is considered as a tool of assisting children in making sense of, formulating, and solving mathematics problems in daily activities and since it provides an opportunity to construct ones own knowledge (Countryman, 1992). This process is part of mathematical enculturation (Bishop, 1991).

The Cultural Conceptual Learning Teaching Model (CCLT)

The CCLT (see Figure 1) contains three learning environments and six learning stages. Play Stage provides children with cultural activities. In this stage, children share, negotiate, and construct their immediate experiences to achieve the emergent goals of arithmetic problems with peers and more advanced children (the expert children). In the Construction Stage, the teacher designs a worksheet that has structural objectives those need to be accomplished by students. In the Connection Stage, based on children's experiences or strategies, the teacher tries to help children construct a connection between their experiences and concrete materials like ten-based blocks or mathematical symbols and procedures. In the Reapplication Stage, the teacher provides another similar or same cultural-conceptual activity for children to reapply to the learned mathematical concept. In the Practice Stage, children try to practice school mathematics in everyday situations by using opportunities provided for them. In the Reflection Stage, children are trained to monitor their thinking and to be aware of where and how they can apply school mathematics in everyday activities. The CCLT model contains two parts: play stage, construction stage, and connection stage are the first part of the area of cultural mathematization; Reapplication stage, practice stage, and reflection stage are the second part of the area of mathematical enculturation.

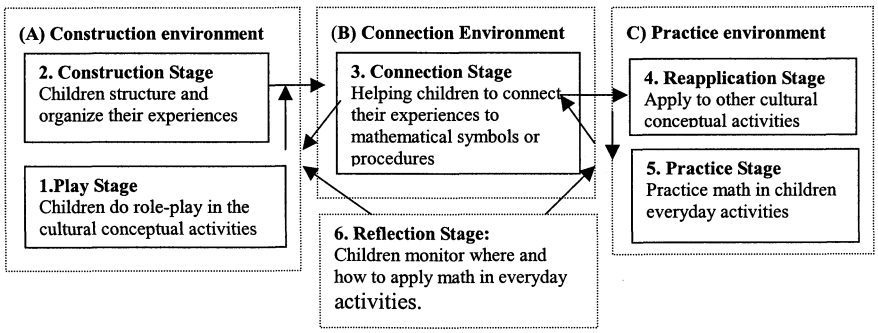


Figure 1: The Cultural Conceptual Learning Teaching Model

In the first part, several studies show that learning arithmetic through children's cultural activities based on the CCLT model not only affect children learning of school mathematics but also improve their ability to solve task problems (Tsai &

Post, 1999; Tsai, 2000; Tsai 2001). This study focused on the second part that created a learning environment by using mathematical journal writing for children to perceive school mathematics in which of daily life cultural activities.

Methodology

The ninety-nine participants were second grade students from three classes in one school. The class size includes thirty-three students. Limited space prevents to report how the students make sense of mathematics in daily settings from each class. An illustration of the effect of journal writing on children perceiving mathematics in daily settings is only taken from one class.

To improve second-grade students' abilities to formulate and solve problems, three stages were considered in the use of mathematical journal writing. First, children were expected to be aware of mathematics in which of activities or events in daily cultural activities. They were asked to answer the question: What are activities or events you encountered in daily situation involved in mathematics? Second, the students were asked to write down or draw a picture to represent the mathematical problem embedded in daily activities or events. Finally, the students were asked to solve the mathematical problems that students created in the first two steps. Between the second and the third stage, students were administrated with a critical task that mutually makes comments and criticizes on the problems students created. The first two stages were undergone at the beginning of the study. At the period of the second stages including critical task, each student was asked to write mathematics journal entries twice for every month and class discussion about students' commentary once for every month. One circulation from first stage to the third stage took two months to accomplish. Four circulations of using mathematical writings have been carried out throughout the entire school year.

In terms of the critical task, the classroom teacher attached all students' journal writings on blackboard and asked them to chose one or two journal entries in which they valued good or bad, followed by making comments on the 76.2cmx76.2cm Post-it-Notes and attaching the Post-it-Notes on the journal writing. Likewise, the student who wrote the journal entries was asked to give follow-up responses whether s/he agreed with the comments from others. The responses are written on different color Post-it-Notes. The critical task provided students the opportunity of mutual support with learning from other students how mathematics is relevant with daily cultural activities. Besides, the critical task was designed to improve the quality of making comments for second-grade students.

Results

The results indicated that not only students improved their writing journal but also progressed on the critical thinking on the writing comments. Several narratives delineate how students perceived, formulated, and solved mathematics problems in everyday settings. The comments and follow-up responses children made are described in the second part. The development of quality of comments will be described in the third part.

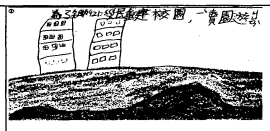
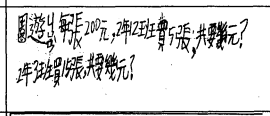
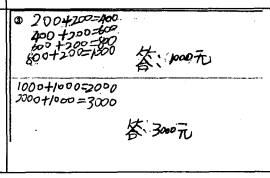
Cases analysis in writing journal

The results showed that the students used more efficient and flexible methods to solve the problems they formulated. The use of writing journals contributed to deepening and connecting the concepts what they learned in classrooms.

(1) Mathematics embedded in an accident event

The data of Table 1 portrays that S1 drew a picture representing the situation of the 921 earthquake happened in Taiwan. To assisting the people ruined in the disaster in rebuilding their houses, S1's school had an activity for collecting money. He posed two mathematics problems about buying the tickets. When he solved the first problem described in Table 1, he counted the money one by one. But in the second problem, he used a unit of five-tickets to calculate the answer. Therefore, he used a flexible strategy to solve a complicated problem.

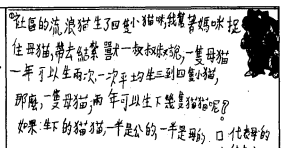
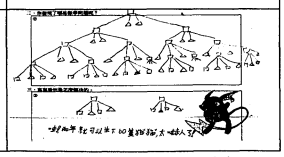
Table 1: S1 used mathematics in an accident event

Setting: To assisting the people ruined in the disaster in rebuilding their houses, the school held an activity of selling tickets for collecting money.	
Problems: (1) One carnival ticket is 200 dollars. Class A bought 5 tickets. How much are there altogether? (2) Class B bought 15 tickets. How much are there altogether?	
Solutions: (1) 200+200=400 400+200=600 600+200=800 800+200=1000 Answer: 1000 元 (2) 1000+1000=2000 2000+1000=3000 Answer: 3000 元	

(2) Mathematics embedded in a casual accident

The data of the Table 2 shows that S2 and his mother caught a cat with 4 kittens. S2 and his mother go to find Doctor for help.

Table 2: S2 used mathematics in a casual accident

Setting and problems: Mother and I caught a cat with 4 kittens in our community. Mother and I bring the cat to zoo hospital for treatment. The doctor said that a cat gave birth to 4 kittens in a year, how many kittens would be after two years? If half of cats are female.	
Solutions: S3 used the tree diagram to solve the problem, and then she felt surprise and said " Whoa! After two year, the cat would give birth to 60 kittens. It is amazing!"	

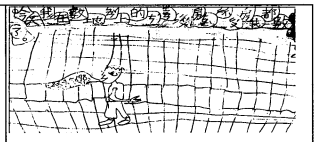
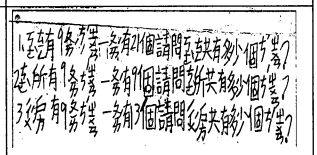
Doctor gave him a mathematical problem to solve: A cat gave birth to 4 kittens in a year, how many kittens would be after two years? If half of cats are female.

S2's solution was represented with a tree diagram in which square represents female cats and triangle represents male cats. S2 posed a complex problem but used an elegant method to represent his way of thinking. The second-graders have not been taught the tree diagram. He used an efficient way to solve mathematical problems in his familiar situations.

(3) Mathematics embedded in surrounding environments

The data of Table 3 shows that S3 was aware of the amounts of bricks on the floor equivalent to the area of the floor in each living room, kitchen, and restroom. The second-graders did not learn the formula of an area until the fourth grade. However, S3 has extended what he learned the fundamental meaning of area into a familiar situation to her. Moreover, she did not learn the multiplication with two-digit numbers until she entered into third grade. In accordance with her solutions, she was able to extend the operation of multiplication with one-digit into two-digit numbers. The result indicates that mathematical journal writing not only connected the mathematics students learned in school with daily situations but also made connections within mathematics.

Table3: S3 used mathematic in her house

Setting: S3 described how to count the bricks in living room, kitchen, and restroom. He tried to understand the amount of bricks in each row multiplying by the amounts of rows in each room.	
Problems: 1. In the living room, there are 9 rows and each row has 21 bricks, how many bricks have totally? 2. In the restroom, there are 9 rows and each row has 9 bricks, how many bricks have totally? 3. In the kitchen, there are 9 rows and each row has 13 bricks, how many bricks have totally?	
Solutions: 1、21x9=(189) 21x1=21,21x2=42,21x3=63,21x4=84,21x5=105,21x6=126,21x7=147, 21x8=168,21x9=189.There are(189)bricks. 2、9x9=(81) 9x1=9,9x2=18,9x3=27,9x4=36,9x5=45,9x6=54,9x7=63,9x8=72,9x9= 81.Restroom has (81) bricks. 3、13x9=（117） 13x1=13,13x2=26,13x3=39,13x4=52,13x5=65,13x6=78,13x7=91,13x 8=104,13x9=117, kitchen has（117）bricks.	

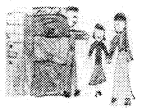
Improving children's journal writing by evaluating each other

The exchange of points of view was an essential part in using journal writings. The result indicates that it was a powerful way to evaluate journal entries each other for improving the quality of journal writing.

The data of table 4 shows that the writer of the journal and the commentator interacted frequently when they inspected the journals. S4 would teach the new idea to the commentator. S5 accepted the commentator's suggestion and applied

the suggestion to the next journal. S6 explained the process of solution to commentator. S7 and S8 asked the commentator to indicate which part was good or clear. In the increasing use of journal writings, the students were required to write the journal or comment as clear as they could; otherwise, other students would criticize on unclear parts.

Table 4: Cases of children's comments and responses.

Setting	Posing problem	Solution	Comment	Response
S4 	Mother buys 6 hot dogs, 20 dollars per hot dog. Mother brings 150 dollars, how much left does mother have?	20×6= (120) 150 - 120 30	Wu! We did not learn multiplying with 20 yet, so I do not understand your calculation, ok!	My mother taught me, if you like, I can teach you, ok!
S5 	One egg is 5 dollars, how much does it cost for 10 eggs?	5+5+5+5+5+5+5+5+5+5= (50)	Your problem is too easy!	Thanks! I will change at next time!
S6 	There are two skirts, two trousers, and one T-shirt, how many clothes do I have?	2+2+1=5 2+2=4 4+1=5	Your problem is clear, but I do not understand your solution.	2+2+1=5 2+2=4+1=5 include skirts, trousers, and T-shirt.
S7 	There are 22 apples on the tree and 19 apples drop, how many apples are still on the tree?	22-19=(3) 20-10=10 10+2=12 12-9=(3)	Tang! Your solution is very good!	Thanks your comment, but can you tell me where is good?
S8 	Father gives me 60 dollars. Mother gives me 50 dollars. How much money do I have in total?	60+50=110 60 + 50 110	Ya-tang! You need to work hard and I will give you 100 scores.	Chung! Can you tell me what the problem is? I do not know what you mean? Next time, please tell me clearly, OK?

Five levels of comments were identified in this study

Five levels of comments were identified in this study, when analyzing students' comments on Post-it Notes: unrelated comment, general comment, question comment, suggestion comment, and evaluation comment. (1) **Unrelated comment** is that students gave comments irrelevant with the problem. For instance, "Your paper is very clean!" "You need to write the date". "If you change your temper you will write better". (2) **General comment** is that students gave general comments but did not indicate what is or where is clear. For example, "I do not understand what you say" "Your problem has a contradiction" "Your problem cannot be solved". Those comments do not specify the clear part of the problem. (3) **Question comment** is that students gave comments that pointed out the problematic parts but did not give further suggestions. For example, "You do not write the price of apple so how you can calculate the answer" "39+52=81 is wrong". (4) **Suggestion comment** is that students gave some suggestions but did not illustrate the reasons. For example, "You can use multiplication to do it". When a student calculates with 30+50=80, 100-80=20, A suggestion from others is that "You can calculate it by 100-30=70, 70-50=20". (5) **Evaluation comment** is that students identified the problematic parts and also gave some suggestions. For example, "Your answer 71 is wrong, you should write 43+26=69".

Those five levels of comments are characterized as from the lowest level to highest level. Suggestion level is higher than question level. The students who were able to identify the problematic parts moved toward the advanced level that gave further suggestions without illustration. Evaluation comment contains the questions addressed and suggestions.

Developing the levels of comments in this study

From Figure 2, initially, students didn't know how to write comments when they inspected others' journals. The number of students' comments stayed at unrelated comment, then decreased from general comment to question comment. In the second month and the third month, students' comments decreased the unrelated level and increased to question level and suggestion level. In the fourth month, students' unrelated comments disappeared and moved toward the evaluation level.

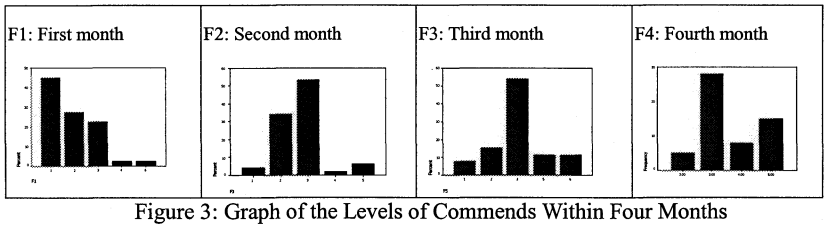


Figure 3: Graph of the Levels of Commends Within Four Months

Conclusion and Discussions

The result indicated that most students performed well in solving the problems they created from daily activities by using mathematical journal writings. The daily activities mathematics embedded into that were perceived by the students included drastic disasters, causal accidents, and surrounding environments. It appears that children could use flexible and efficient strategies to solve the problems relevant with familiar situations. Nevertheless, the strategies were been not taught in school for the second graders. One possible interpretation was that the mathematical problems they formulated were based on their familiar situations; as a result the solutions were the ways of sense-making to them. The other interpretation was that there was a close relationship among the three steps so that students understood the relationships among setting, problem, and its solution. A significant finding was that the mathematics concepts students learned were easily extended and connected by the use of mathematical journal writings.

The critical task involving the study provided the opportunities for children to take-and-share perspectives and sequentially resulted in the improvement of writings and comments. The students evaluated each other and discussed in class about the way of writing journals and what comments are valuable for journal writers. The higher quality of comments brought about the higher quality of mathematical writings. Finally, children were aware of the mathematics they learned in school embedded in various cultural settings and daily activities through the use of journal writings. As a result, their abilities in formulating and solving the mathematics problems were improved.

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