AN AVERAGE WITH UNIMAGINATIVE WEIGHTS WHEN THE WEIGHTS EQUAL THE VALUES

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The **self-weighted mean,** denoted *SW*, is a weighted mean in which the weights are the values themselves:

$$SW = \frac{x_1^2 + x_2^2 + \ldots + x_n^2}{x_1 + x_2 + \ldots + x_n}$$ where $x_1, x_2, \ldots, x_n$ are n positive numbers.

It is easy to see that *SW* is greater than the arithmetic mean (*A*), because in *SW* the larger values get larger weights. *SW* is linearly (and positively) related to the *variance* of the *n* values. Lack of distinction between the two means is sometimes referred to as "sampling bias" (Stein & Dattero, 1985).

SW is called for whenever the probability of sampling a given value is proportional to its size, that is, under self-weighted sampling (SWS), usually known as "size/length-biased sampling". SWS is encountered in diverse areas: demography, medicine, management science, and many others (Patil, Rao & Zelen, 1988). Therefore, understanding this concept, and gaining reasoning ability about the mean in SWS situations is important for both information consumers and data producers.

Our research focuses on learning and analysing people's intuitions: How do people intuitively fare in different SWS tasks (e.g. assessing the expected waiting time for a bus that arrives at varying intervals, or the mean class size obtained by questioning students)? Are there typical fallacies, and how could they be overcome? We first hypothesised that some people might fail to understand the need to weight the averaged values, thus calculating *A* instead of *SW*. This hypothesis was confirmed in our experiments. The implicit assignment of equal weights to all the values is compatible with Tversky and Kahneman's well-known description of heuristic principles that people rely on when assessing the value of an uncertain quantity. In particular, tacitly assuming uniformity is in harmony with people's predilection for symmetry and equality (Zabell, 1988). We also found that experiencing the problem's procedure – even only via a thought experiment – provides a useful corrective instrument, as does emphasising the weights by using a roulette with variable angles, and asking about the mean of these angles, for repeated turnings of the roulette.

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