TEACHER HEARING STUDENTS

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This study examines elementary school teachers' "hearing" students, i.e., understanding what they are saying, showing, feeling and doing. In the larger study there are 25 elementary school teachers who participated in a weekly four-month long inservice workshop. In this paper we analyze the case of Ruth. We identify four types of teacher interpretations of students' talk and action: describing, explaining, criticism and justifying. We illustrate the nature of hearing characteristics, and examine two possible sources for Ruth's overhearing and under-hearing.

INTRODUCTION

It is widely accepted today that teachers should be aware of and knowledgeable about their students' thinking and understanding in order to make appropriate decisions for helping and guiding students in their knowledge construction (e.g. Ball, 1997; Even & Tirosh, in press; NCTM, 1991; Shulman, 1986). Teachers who understand where students are can challenge and extend their thinking; respond to their questions, remarks and ideas in meaningful ways; conduct discussions; promote discourse; and modify or develop appropriate activities for them.

However, assessing students' knowledge, understandings and ways of knowing, learning and thinking is not an easy task. Understanding what students are saying, showing, feeling and doing—what Ball (1997) calls "hearing students"—is complex. Teacher interpretations of students' talk and action are closely connected to cognitive, social and affective aspects. For example, to teachers' own ways of understanding the subject matter and their commitment to students (Ball, 1997).

Currently, research that explores teacher "hearing" students is scarce. The study reported in this article is an initial investigation of this issue. It is part of a larger study that examines elementary school teachers' interpretations of students' talk and action when students are engaged in mathematics problem solving. In this paper we focus on Ruth's (pseudonym) hearing two of her students. We analyze her interpretations of the students' work and their ways of thinking, both with regard to the nature of the interpretations and to their possible sources.

DESCRIPTION OF THE STUDY

Participants

There are 25 elementary school teachers from several schools in a southern town in Israel in the large study. The teachers participated in a weekly four-month long inservice workshop, led by the first author at a regional teacher center. The group was heterogeneous with regard to the grade level taught, teaching experience and age. Most of the teachers were teacher-college graduates, and so was Ruth, the teacher who is the focus of this paper. Ruth had 11 years of teaching experience in

elementary upper grades. In the year of the study she taught in fourth grade. Like most elementary school teachers In Israel, she taught all subjects, among them mathematics.

The workshop

The workshop's main component centered on work on several mathematics problems taken from Wallach and Regev (1999). The main characteristics of these problems, as indicated by a field-test conducted with elementary school students, are: challenging; require long time to solve; facilitate different ways and different strategies of solving; do not have an immediate solution; encourage mathematical thinking such as: reasoning, conjecturing, explaining, justifying, etc; encourage cooperative work and discussions; integrate different mathematical topics and concepts from the school curriculum and encourage reflective thinking.

At the beginning of the workshop, the participating teachers were introduced to these problems. They solved several problems and then discussed their solutions in small groups. Later, each teacher chose one of the problems and presented it to a pair of students from her own class. The teachers observed and videotaped the students n working on the solution of the problem. The teachers were instructed not to intervene by suggesting comments, hints or advice to the students. Then, each teacher summarized and reflected on her observation in writing and met with the workshop leader to discuss her interpretations of the students' work and ways of thinking while focusing on one episode from the students' video-tape chosen by the teacher.

Data collection

Data collection includes:

- Observations of all workshop sessions. The sessions were video-recorded. The first author kept a journal where she documented her ideas and reflections after each session.

- Written work prepared by the teachers, such as reflections on their students' problem solving.

- Videotapes of the pairs of students' problem solving sessions.

- Individual interviews conducted with each teacher for about 45 minutes. These interviews centered on one episode that the teacher chose from the videotape of her students.

- Focus-group interviews were conducted twice. Once at the beginning of the workshop to discuss the teachers' own solutions of the mathematics problems and how they expect their students to solve them (about one hour). Another focus-group interview was conducted toward the end of the workshop to discuss the work of an unfamiliar pair of students who worked on the same problem that the teachers chose for their students (about one hour).

Data analysis

The analysis is based on the Grounded Theory method (Glaser & Strauss, 1967). We code the data using utterances as the unit of analysis and generate initial categories. We constantly compare new data with current categories, refine them and identify core categories, looking for integration and hierarchy among the categories, and using them as source for theoretical constructs.

RUTH'S HEARING AND INTERPRETATIONS

The "Shirts and Numbers" problem

After solving several problems at the workshop, Ruth chose to present two of her fourth grade students—Sigal and Ore—with the "Shirts and Numbers" problem (Figure 1). This problem deals with sport-team players who wear numbered shirts. The students are asked to divide the team into two groups, according to different criteria that are indicated in the problem. For example, in one part of the problem, they are asked to divide a team of 15 players so that in one group there are 5 players less than in the other group. In another part, they are asked to divide a team of 12 players so that in one group there is one half of the number of players in the second group. The part of the problem, on which Ruth chose to focus at her individual interview, deals with a task that has no solution, as is described below:

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The task in front of you does not have a solution.

Divide a team of 15 players, so that in one group there are 4 players less than in the other group.

a) Explain why such a division is impossible:_____

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b) Change the number of players, so that there will be a solution.

Demonstrate the solution:

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Figure1. The "Shirts and Numbers" problem

Sigal and Ore's solution

The video recording of the students' work on the problem shows that immediately after reading the instructions Sigal says: "Because the number 15 is odd and the number 4 is even, then it's impossible". Then Sigal and Ore start trying to change the number of the team's players. During about three minutes they try to find a suitable number by thinking, reading again the problem and examining different divisions of the shirts that are drawn on their worksheet. Sigal and Ore suggest the numbers 16 and 12, but do not pursue these possibilities. After three more minutes of trying to find two groups that would satisfy the conditions, Sigal says: "Here are seven children [Ore confirms by saying "yes"], and here are three [pointing to the appropriate places on the worksheet drawing, see Figure 1]...Up to here it's seven. And three, so it's ten [players]. Four, seven minus four is three. Then here it is. Here are four children [players] less. Do you understand? [Sigal asks Ore] so ten children [players]". They both agree on ten players divided into one group of seven and one of three players. They make appropriate drawings on their worksheets. Below (Figure 2) is Sigal's drawing:
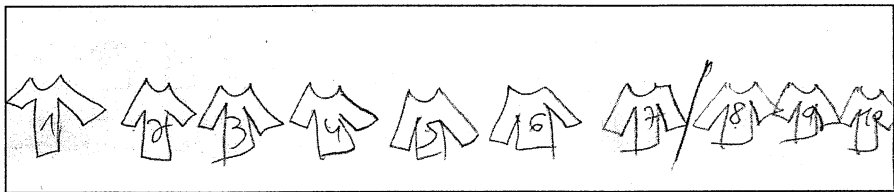

Figure 2. Sigal's drawing

Ruth's types of interpretation

We identified four types in Ruth's interpretations of Sigal and Ore's talk and action: Describing, Explaining, Criticism and Justifying. Following is an elaboration of each.

- Describing—the teacher describes students' talk and action by direct (or almost direct) "quotation" or portrayal.

- Explaining—the teacher explains students' talk or action. This includes ideas about students' thoughts, reasoning, knowledge, and assumptions. The explanation is the meaning the teacher attributes to students' talks and actions.

- Criticism—the teacher criticizes students' talk and action, based on the "Meaning" she assigns to them.

- Justifying—the teacher justifies her criticism of, or the meaning she attributes to, students' talk and action. This type of interpretation is different from the other three types, because here the teacher reflects on her own interpretation and not on the students' talk and action.

The following two excerpts from Ruth's interview illustrate these different types of interpretations. The first excerpt demonstrates the first three, and the second excerpt demonstrates the last two types.

First excerpt:

> Teacher: She says, let's take 4, she marked 4. Now she wants to divide it by 2 (Describing). Because she forgot that it is impossible with an odd number (Explaining). She forgot again. She tried to counterbalance the 4 (Explaining). This is not a method at all (Criticism).

Second excerpt:

> Teacher: They got mixed-up when working on this (Criticism).
>
> Interviewer: What would you expect them to do?
>
> Teacher: Frankly, I expected them to work more, a lot faster and a lot more easily (Justifying).

Ruth's hearing her students

Ruth's interpretations of Sigal and Ore's talk and action are reflections of her hearing Sigal and Ore, of her understanding what they are saying, showing, feeling and doing. Below we illustrate the complexity of hearing students, what might be entailed in interpreting students' talk and action, and suggest possible sources for Ruth's hearing.

Overhearing. We start with an example that portrays how even the relatively simple type of interpretation—describing—is not as simple as might first meet the eye. When Sigal and Ore attempted to solve the "Shirts and Players" problem, Sigal said, "Because the number 15 is odd and the number 4 is even, then it's impossible". Before we look at Ruth's interpretation, let us examine this statement. What does Sigal mean?

One possible interpretation is that Sigal knows that to solve the problem she can remove 4 players from the entire team and then divide the reminder players into two equal size groups. Finally, adding the four players to one of the groups would give the solution of the problem. However, Sigal understands that in this specific case the partition is impossible because 15 is odd, and when she subtracts 4 from 15 the difference is 11, which is an odd number, and an odd number cannot be divided by two. Consequently, Sigal claims: "it's impossible". It might even be that Sigal understands the generalization of this method, that the partition is possible only when the number of the players is even. Because only in this case the removal of 4 players would leave an even number of players, which can then be divided into two equal size groups.

Another possible interpretation is that Sigal knows that the partition is impossible because this is stated in the text "The task in front of you does not have a solution". She also knows and identifies that 15 is odd and 4 is even. So she connects these two

pieces of information, and without any logical argument states that "15 is odd, 4 is even, then it's impossible". Of course, there may be other possible interpretations.

Let us examine now Ruth's interpretation of Sigal's statement. During the interview, on two different occasions, Ruth "quotes" Sigal's explanation (Describing type) as "...If you take away an even number from an odd number, you are left with an odd number, which you cannot divide by 2". In another occasion she says " They said: Oh, for sure it's impossible. Odd minus even, it's impossible to divide it by two". However, Sigal did not say all that. She refers only to the two numbers, 15 and 4 and indicates that they are odd and even respectively. She does not mention any operation on these numbers or on the difference between them. Still, what Ruth hears is different. Ruth hears Sigal talking in generalizations, without mentioning any specific number. She hears her discussing odd and even numbers, the difference between an even number and an odd number, and of its divisibility by 2. Even when the interviewer tries to shake Ruth's confidence about her interpretation and asks: "Perhaps here Sigal did not understand what you think that she understood" Ruth remains confident and responds without hesitation "Then what, she just said it? Like a parrot?" and " It seemed to me real understanding".

This short episode illustrates that Ruth "hears" things that were not said by the students. We define this characteristic *overhearing*. Interestingly, such overhearing is reflected even in the relatively simple type of interpretation—describing—which naturally would be expected to be an accurate duplicate of what the students said. The discrepancy between what students say or mean and what the teacher hears could be significant to teacher decision-making.

Under-hearing. When later Sigal suggests the solution of 10 players, Ruth does not seem to believe that there is any substantial reasoning underlying this solution. Several times throughout the interview Ruth is asked to explain how the students reached this solution. Her responses (Explaining type) did not focus on the students' reasoning. For example, "The solution just came out of the blue", or " She just said 10 off the top of her head". On another occasion Ruth says: "Suddenly, she had a flash of the 3 and the 7. OK, 10 shirts [players]... Fine, she got it, she reached an even number" or " It seems to me that she just got it". These explanations do not seem to get into the details of the students' reasoning. Ruth does not refer to the process in which Sigal builds the two groups, by circling first 4 shirts followed by creating two groups accompanied with her explanation: "Here are 7 players and here are 3 [pause] so [pause] 10 players. 7 and 3 makes 10. And 7 minus 3 is 4, so 10 players". Ruth "ignores" some of the things said or done by the students. We define this characteristic as *under-hearing*.

SOURCES OF RUTH'S HEARING

Why does Ruth "hear" what was not said or done by the students? Why does she "mishear" what was said or done by them? Possible sources of teacher hearing their students are teacher own knowledge of mathematics, conception of the solution of

the problem at stake, beliefs about the nature of mathematics learning and knowing, understanding of the nature of mathematics teaching, disposition towards the teacher role, and so on. In the following we analyze Ruth's concern for her students' success and Ruth's own conception of the solution to the "Shirts and Numbers" problem. Then we examine the possibility of explaining Ruth's hearing Sigal and Ore on the basis of these two sources.

Ruth's concern for her students' success

Throughout the interview Ruth expresses clearly her desire to see Sigal and Ore succeed in solving the problem. While watching the video-recording of them working, she looks happy and satisfied when they say what she considered as good and upset and disappointed when it seems to her that they are not on the right track. She says, for example, "I really enjoyed it", referring to Sigal's statement that 15 is odd and therefore it is impossible, or "I enjoyed it very much, I was astonished by the way she came up with it immediately", etc. This concern to see her students succeed may emerge simply from her liking them. It may also be related to her role as their teacher, where Sigal and Ore's success may be regarded as her own success.

Ruth's conception of the "Shirts and Numbers" problem's solution

When solving the problem by herself, to show that there is no solution to the "Shirts and Numbers" problem when the group-size is 15 (part (a) of the problem), Ruth subtracts 4 from 15 and receives 11. Then she points that the division of the result by 2 does not give a whole number, which means that the problem has no solution. Throughout the workshop, when discussing hers and her colleagues' solutions, Ruth emphasizes that it is enough to state a "short version" of this solution, that "15 is odd and 4 is even. It is not possible to divide 11 by 2". This short version includes a claim about the impossibility of solving a more general problem when the group size is odd and the difference between the sizes of the two teams is even. When discussing the solution with her colleagues, Ruth is willing to accept even a shorter version, where only two components are mentioned: "15 is odd and 4 is even". For her, such a statement represents the more elaborated solution.

When solving part (b) of the problem, Ruth changes the number of players to 14. Analysis of the data suggests that Ruth's solution strategy is to *remove* a *minimal* number of players to reach an even number of players. For example, in her interview Ruth says: "...It bothered me that they didn't remove a shirt immediately, [pause] and to reach an even number. She [Sigal] knows that it [the number order] goes even, odd, even, odd. So, remove a shirt and then get an even number". On another occasion Ruth says: "I was really surprised that they [Sigal and Ore] changed to 10, [that they] removed 5 shirts. Remove 1[pause]... I don't know, it seemed to me that you need to remove 1 and try".

Hearing through

Ruth's concern for her students' success and her own conception of the problem's solution seem to contribute to Ruth's overhearing and under-hearing Sigal and Ore.

Wanting to see them succeed, Ruth is receptive to interpret their talk and action in this way. When combined with her own conception of the problem solution, Ruth overhears in Sigal and Ore's statement that "15 is odd and 4 is even" the answer she expects. However, when Sigal and Ore propose the solution of 10 players that they found by actually *constructing* two teams in which one has 4 players more than the other (3 and 7), Ruth's under-hears them. She is unable to figure out how they reached this solution because their *constructing* strategy is different from her *removing the minimum possible to reach an even number* strategy. Even after watching this part of the video recording several times, Ruth under-hears her students and claims that " The solution [10] just came out of the blue", or that " She just said 10, from the top of her head". Ruth knows that 10 is correct but because both the strategy of solution and the resulting number are different, Ruth mishears her students.

FINAL REMARKS

In our investigation of Ruth's hearing her students, of her understanding what Sigal and Ore were saying, showing, feeling and doing, we identified four types of teacher interpretations of students' talk and action: describing, explaining, criticism and justifying. We illustrated the complexity of hearing students, even in the relatively simple case of describing, and examined two possible sources for Ruth's overhearing and under-hearing: Ruth's concern for her students' success and Ruth's conception of the problem's solution. Ruth's case is an initial investigation of teacher hearing students that raises interesting questions, such as, Are there additional interpretation types? Are they interrelated? How? These and other questions are the focus of the next stage of our research.

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