HIGH SCHOOL STUDENTS' IDENTIFICATION OF EQUAL SLOPE AND Y-INTERCEPT IN DIFFERENT STRAIGHT LINES

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Abstract

In this study I use a semiotic perspective to analyze with three student samples from different schools identification tasks of equal slope and y-intercept in different straight lines. Some of the students strategies take into account a visual conception that lies upon Duval's vision concept (Duval, 1999). When using these strategies, the students set aside the previously known slope and y-intercept definitions and the graphical aspect of the line, and limit themselves to the use of the coordinate axes as a visual reference, serving as an anchorage for the visual orientation for the identification of the line.

Resumen

En el presente estudio utilizo una perspectiva semiótica para analizar el trabajo de identificación de la ordenada al origen y la pendiente de tres muestras de estudiantes provenientes de diferentes escuelas. Algunas de las estrategias usadas por los estudiantes tomaron en cuenta una concepción visual que se ajusta al concepto de visión de Duval (1999). Al utilizar tales estrategias, los estudiantes hacen a un lado sus conocimientos previos sobre las definiciones de ordenada al origen y pendiente de una recta y su aspecto gráfico, y se limitan al uso de los ejes de coordenadas como referencia que sirve como anclaje de la orientación visual para la identificación de la recta.

Introduction

I want to mention two remarkable misconceptions about the understanding of the straight line. The first one was mentioned by Janvier (1978), quoted by Leinhart, et al, (1990). According to Leinhart et al, Janvier says that between the findings about the construction and interpretation of a straight line are those about slope/height confusion: "students have been found to confound these two graphical features on both interpretation and construction tasks" (op. cit. 1990, p. 37). "No consensus exists regarding the cause for such errors ... A common interpretation is that students are confusing two graphical features: highest value versus slope" (op. cit., p. 39).

The second one is mentioned by Schoenfeld, Smith and Arcavi (1993). In their fine-grain analysis, they found in an interviewed subject that she had separate ideas about slope and y-intercept notions. This was evident when she had to develop construction and interpretation tasks. In their study they say that she knows that m = $\frac{y_2 - y_1}{x_2 - x_1}$, but this knowledge is nominal and although it is used to compute the line slope, this computation has no graphical entailments, and although she knows that b is the y-intercept, her understanding is nominal too and it is not tied to the underlying structure. The authors think that this is caused by the absence of what they call the Cartesian Connection, that is, "a point is on the graph of the line L if and only if its coordinate satisfy the equation of L" (Moschkovich, Schoenfeld and Arcavi , 1993, p. 73).

In regard to the above "misconceptions", I rather prefer the term "alternative students' conceptions". These conceptions appear when students interpret or read the graph. For Mavarech and Kramarsky (1997, p. 229) interpret or read the graph means the "students' ability to read a graph (or a portion thereof) and make sense or gain meaning (of) it ".

In the first "misconception" it is necessary to compare the slope and height graph; and in the second one, in order to make the Cartesian Connection, the student needs to translate between two different representations, one algebraic and other graphic.

In my research I used a questionnaire with items that require the only visual strategies. Unlike the results mentioned above, my work lies upon the visual aspects of the straight line, and the elements that take place in the treatment are the semiotic signs, their meanings, and their relations.

But, what is a graph? And what do we have to see in it so as to read it? From Bertin's (1968) perspective the visual perception (in graphs) consists of the perception of three factors: the variation of shadows and the two dimensions on the plane, regardless of the time variable.

In a similar way, Duval (1995, p. 142) claims that "a figure is an organization of sharp contrast of the brightness. It emerges from a background through the presence of 'traces' or 'spots', governed by Gestalt laws and perceptual clues".

The visual information in the graph, must be an important part of the graph comprehension. Some researchers, like Friel, Curcio and Bright (2001, p. 132), claim that "by graph comprehension we mean graph readers' abilities to derive meaning from graphs created by others or by themselves", but they do not explicitly consider, in their graph structure comprehension, the importance of the meaning of sign. They missed the consideration of the semiotic meaning of the graph.

From Duval point of view, the function of a graphical representation lies upon two figures: the ground-figure (axes, grid and orientation) and the form-figure (the graph). This relation is completely defined by this representation forming gestalt rule (Duval 1996, 2001).

From his interpretation I take the idea of representational units (Duval, 1999), as well as the gestalt relation, and I consider two of the three units: the slope and y-intercept. I am not considering the third unit, the angle with the x-axis, because it is not included in the high school math curriculum.

In my early observations of this kind of tasks that require visual interpretation, I have found that the students use some specific visual strategies. In this research, they used two different strategies to identify whether the slope and the y- interception of straight lines are the same or not, without taking into account the geometric or algebraic definition. These strategies are:

S1. The student can make a visual movement, in general translation or rotation, from one line to another. They can make a parallel translation in the case of the straight line position, they move the approximate measure of the angle related shape slope or of the position of the y-intercept respect to the horizontal axis.

S2. The student can take into account some real or imaginary tokens, or apparent relation, to help the orientation of the graphic elements, for example the quadrants that the path of the straight line observed and that make sense to the line position.

I observed that both strategies are neutral, this means that you can use them either with right or wrong results, in this way it is useful to mention Duval's idea (op.cit, p. 12) about vision and visualization concepts: vision refers to a visual perception, it gives direct access to the physical object; it consists of a simultaneous apprehension of several objects or a whole field. Visualization is based on the production of a semiotic representation. A semiotic representation shows relations between two representational units.

In my work I found that students used the mentioned visual strategies, regardless the algebraic or proportional definitions, to do some identification tasks. They used a vision oriented conception. This conception has a strong reference on the interpretation of the axes like an anchorage (Mesquita and Padilla, 1990).

Methodology

In this work I used a questionnaire answered by high school students in the 3thd. semester (17 years old). I worked with three samples of students (A,41; B,23 y C,35) in three different schools located in two industrial cities. The intention was that the samples were different only in their location because I wanted to see if this factor affected the answers.

Previous to my study, the students had some knowledge about straight line, namely, its point by point graph construction from a known equation, a slope definition coming from algebraic and proportional issues. They had some skill in calculating straight line slops and y-intercepts. They knew that between two points only on straight line can pass and they calculated slope and y-interception from visual information on a graph.

In my questionnaire I asked for the construction of different straight lines with different slopes and y-intercepts. I requested some explanation about possible changes in the construction in order to make two different lines equal. The students also needed to identify equal straight lines among different lines. In general, the students did well. Most of them showed that they can easily recognize the visual shape of slope and y-intercept.

But, in some items, I found two kinds of answers that caught my attention. In this paper I will report one of them. This result seem to show that under specific situations the student quit the definition and limits herself to consider exclusively the shape. This kind of answer were detected in two groups of exercises in which the students have to identify slope and y-intercept among several options; before they answered the questionnaire, I gave to the students a definition of slope and y-interception because I did not want this to become an obstacle in their performance. The referred items are supported on visual information only, and the indications were posed in natural language.

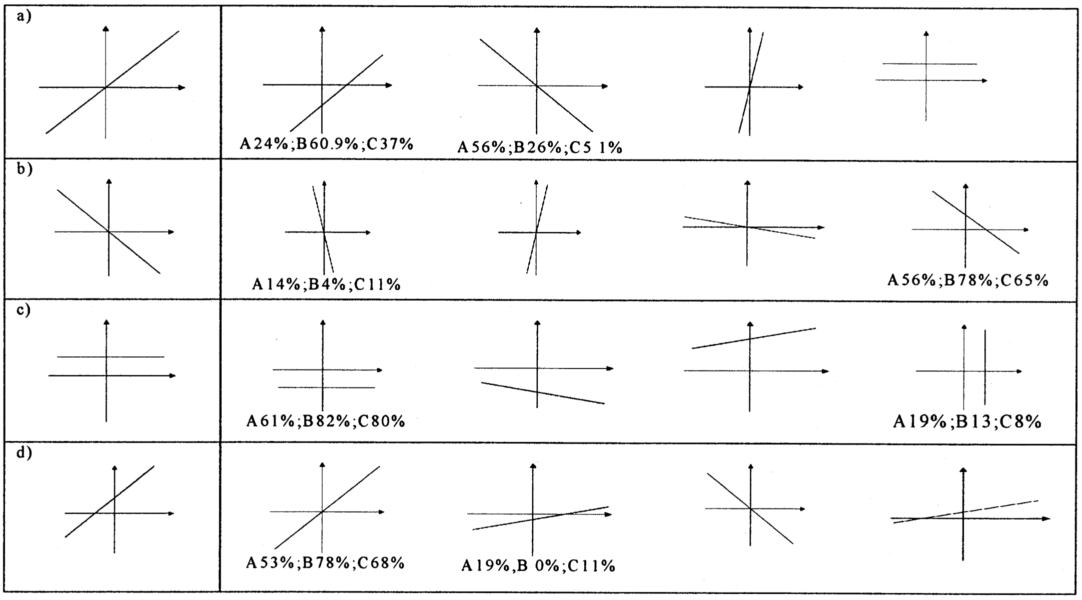
In this qualitative research I expect to give an approximate answer to the following questions: Can the student make a consistent visual identification of same line with the slope and y-interception among different straight lines?, to what kind of signs do the students pay attention to find the difference?

Question Analysis and Results

My observation is based in the following two groups of questions. The student have to choose among different graphs using visual criterions posed in natural language. In the table we have written the frequency of incidence of right answers and the some of most frequent wrong answers, in order to compare the results .

The first group of exercises (belonging to Item 9 of the questionnaire) follows:

9. For each graph on the left, mark whit with an X the graph on the right that has the same slope. The slope is the slant that the straight line has with the horizontal axis.



Frequencies of right answers of the samples A, B and C for item 9.

In Item 9(a) it seems that the term “same slope” is associated to “same angle” between the straight line and the x-axis, but in doing the identification they consider an non-oriented angle. In this case they followed strategy S2 and the tokens used are, in one hand, the x-axis and the “opening” that it forms with the line, both tokens are part of the global aspect of the graph. And in the other, there is the point (0,0) that appears as y-intercept on both lines.

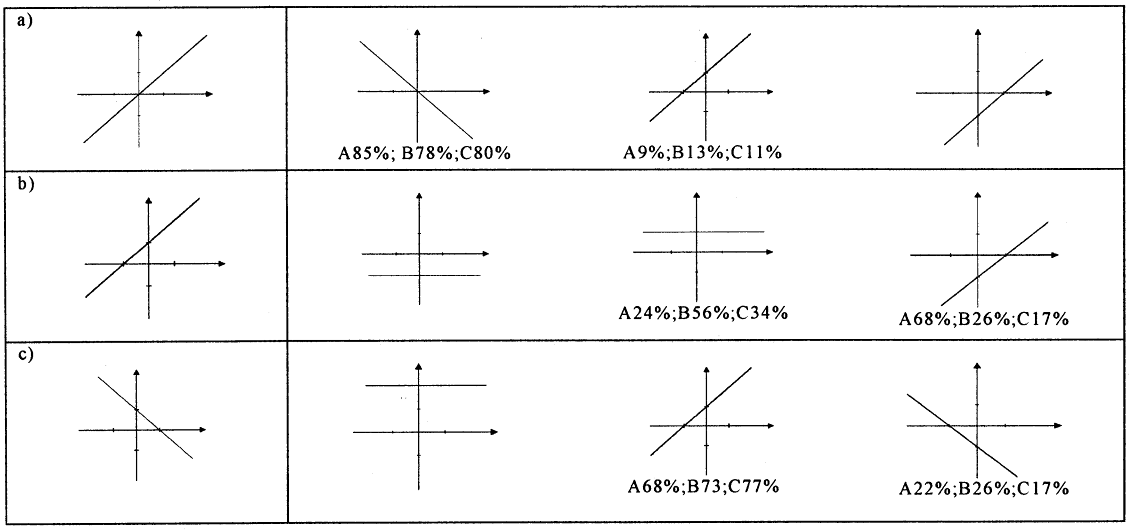
In this case, their use of S2 is oriented by the vision that refers to a visual perception, and it gives access to the physical object and give us the simultaneous apprehension of several objects or a whole field (Duval, 1999). Students focus their attention on the x-axis and the point (0,0), and these tokens are considered like an anchorage (Mesquita and Padilla 1990). Their solution does not have a semiotic treatment, that is, a treatment in which the semiotic representations show their relations between representational units, the gestalt relation is missing.

The other group of exercises (belonging to Item 10) are :

10. For each graph on the left, mark with an X the graph on the right that have equal y-intercept. The y-intercept is the point where the line crosses the y-axis (see diagram on the next page).

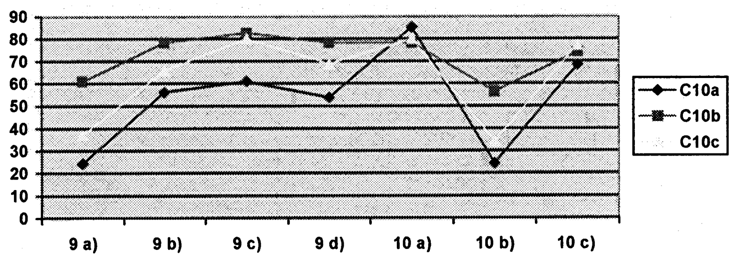
In this exercise the students’ identification is more or less successfully in 10(a) and 10(c) (A,85% and 68%; B, 78% and 73%; C 80% and 77%

respectively) nevertheless, the situation in 10(b) is quite different, this is shown in the frequencies graph that follows (see next page). The wrong answers are supported by a consideration of the global shape of the graph, oriented by the vision, instead of a visualization. It seems that a oblique line has nothing to do with a horizontal one.



Frequencies of right answers of the samples A, B and C for item 10

The following graph shows the frequencies of correct answers of the two discussed items, the frequencies are similar in the three different samples.



Frequencies of both groups of exercises

Discussion

The present research lies only on the visual treatment of a straight line graph, for this reason the students' conceptions, evident from wrong answers, are different from those that have a functional treatment like in Janvier (1978) and Schoenfeld et al (1993); the visual application of the conceptions are different too.

The students' difficulties can appear even in the visual situations when we ask the students to make graphs without explaining nor working the gestalt relations, that is, the relation between form-figure and ground-figure, or how we can plot graphs considering the spatial relation ruled by the axes, the grid and the orientation.

In my opinion this students' conceptions where the strategies S1 and S2 take place, were hidden because the visual treatment was not sufficiently considered in the school curricula.

The identification tasks about y-intercept and slope are quite far to be simple applications of a definition, the results suggest that the students can make almost always a consistent visual identification but the wrong answers in items 9(a) and 10(b) suggest that in tasks with a strong situational component, this visual identification fails, so there is evidence that students are not entirely consistent in making visual identifications in the case of y-intercept and slope.

The answers permit us to know some students' conceptions about y-intercept and slope. In fact, our students pay attention to the marks on the axes, and the axes themselves are used as important visual tokens, but in some critical situations this mechanism do not work adequately because the election is arbitrary. In these cases, the axes function like a strong anchorage in the general shape of the graph.

Finally, the students used their own "gestalt" relation that took place on the visual identification despite the previous training or definitions about slope or y-intercept, and they make their own interpretations in cases in which there was not an intentional intervention of the teacher, this interpretations are supported by their own ideas.

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