TOWARDS UNDERSTANDING TEACHERS' PHILOSOPHICAL BELIEFS ABOUT MATHEMATICS

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The aim of this study was to examine teachers' philosophical beliefs (PBs) about mathematics, the factors influencing the development of these beliefs, and their relation to teachers' beliefs and practices about teaching and learning mathematics. Data were collected through 229 questionnaires and five interviews. Analysis revealed a five-factor model, representing combinations of the three dimensions of a model proposed by Ernest (1991). Four homogeneous groups of teachers according to their perceived importance were identified. A relative consistency was also found between teachers' PBs and their beliefs regarding teaching and learning Mathematics. However, inconsistencies between PBs and teaching practices emerged, which could be partially attributed to the factors influencing the development of PBs. Implications for the development of teachers' training programs regarding the affective domain and suggestions for further research are drawn.

INTRODUCTION

Teachers' philosophical beliefs (PBs) are considered as the cornerstone of their teaching practises and their beliefs concerning teaching and learning. Thompson (1992) defines PBs as "teachers' conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics" (p.132). Hersh (1998) asserts that these beliefs affect teachers' conception of how mathematics should be presented, since "the issue is not what is the best way to teach mathematics but what is mathematics really all about" (p.13). Ernest (1991) concludes that the research literature indicates that mathematics teaching depends fundamentally on the teacher's belief system and particularly on his/her conceptions of the nature and meaning of mathematics.

The review of the literature (Raymond, 1997; Thompson, 1992; Roulet, 2000) suggests that many factors influence the development of PBs, and particularly teachers' school experiences as mathematics students, that constitute their early systematic contact with mathematics. However, PBs can be considered as a dynamic body of beliefs influenced by many other factors, including, early family experiences, teachers' education programs, social and educational characteristics and school environment. The socio-cultural environment is also considered to exert influence on the development of PBs.

Teachers' beliefs may develop into a coherent philosophical system that directly influences their overall classroom behaviour. A teacher's own philosophy is thought to function as a filter influencing decisions and actions made before, during and after instruction (Philippou & Christou, 1997). Class organization, the choice of learning activities, the questions posed by teachers, and the homework that teachers assign to students are likely to be influenced by teachers' PBs (Stipek, Givvin, Salmon, & Mac Gyvers, 2001). Moreover, PBs were found to have impact on students' achievement, teachers' attitudes about the effectiveness of various teaching methods, innovations, curricula, textbooks and software material (Ernest, 1991; Philippou & Christou, 1997; Roulet, 2000). A number of researches, however, point out that there are inconsistencies between expressed PBs and actual teaching practices (Raymond, 1997; Thompson, 1992). The subconscious character of PBs and the influence of school environment on the development of these beliefs can justify these inconsistencies (Ernest, 1991). Evidently, teachers' training programs should help prospective teachers develop and become aware of these beliefs.

Despite of the excessive research in this domain, no coherent way to measure PBs has been so far reported, due to the acknowledged complexity of PBs. The review of the relevant literature reveals a number of models that have been proposed in order to study these beliefs (e.g., Lerman's two-dimensional model, Ernest's three-dimensional model, Perry's four-dimensional model, and Raymond's five-dimensional model). We adopted Ernest's model, since it is based on three philosophical ideas, derive from the history of mathematics evolution. First, the platonist view considers mathematics as an a priori static unified body of knowledge which exists out there and is discovered, neither invented nor created. Second, the instrumentalist view regards mathematics as a set of unrelated but utilitarian rules and facts; a viewpoint that can be linked with formalism in mathematics (Roulet, 2000). Thirdly, there is the experimental–constructivist view that attributes a prominent role in problem solving and regards mathematics as a dynamic, continually expanding field of human creation and invention. Teachers holding this point of view consider mathematics as a cultural product the results of which remain open to revision.

However, we are not aware of any systematic attempt to verify whether the three aforementioned viewpoints can satisfactorily describe teachers' PBs. Thus, the main purpose of this study was to collect empirical data in order to examine the efficiency of Ernest' s model in describing teachers' PBs. The study also aimed to provide evidence regarding the factors influencing the development of PBs and the consistency between these beliefs and teachers' beliefs and practices related to teaching and learning mathematics.

METHODS

A three dimensional questionnaire was developed based on statements found in the literature (e.g. Ernest, 1991; Thompson, 1992). The first part included 17 five-point Likert-type items (1=Strongly disagree, 5=Strongly agree) reflecting teachers' PBs along the three dimensions of Ernest's model. The second part included four items regarding teachers' beliefs about teaching and learning mathematics, namely it included questions related to the characteristics of a good teacher, a good student and indications of learning and learning goals in mathematics. For each item, teachers were provided with six statements, which corresponded to the three levels of Ernest's model and were instructed to put them into a hierarchical order that best described their own beliefs about mathematics. The third part of the questionnaire included open-ended questions related to teachers' practices. Of the 345 Cypriot teachers approached, 229 responded (192 primary and 37 secondary teachers), a response rate of 66.4%. The reliability of the scale, which measured teachers' PBs, was calculated using Cronbach's Alpha (α=.69).

Semi-structured interviews with four primary teachers and one secondary teacher were also conducted. The purposive sampling technique was used. More specifically, the primary teachers belonged to each of the four cluster groups that emerged from analysing teachers' responses to the questionnaire, while the secondary teacher belonged to the second cluster group. Teachers were asked to reply to open-ended questions, which could help us identify their conceptions about mathematics, such as "*What is the first thought that comes to your mind when you think of mathematics?*" "*How would you define mathematics if you were asked to do so by one of your students*". Furthermore, teachers were prompted to mention factors influencing the development of their PBs as well as domains that could be influenced by these beliefs. Specifically, teachers were urged to mention their school and university experiences in mathematics and their daily experiences with students, colleagues and headmasters. They were also requested to describe teaching goals, how they introduce a new concept, the structure and the context of their tests and their favourite activities. The constant comparative method (Denzin & Lincoln, 1998) was used to analyse the qualitative data emerged from the interviews.

FINDINGS

Since, a large number of significant correlations among teachers' responses concerning their PBs were identified, factor analysis was used to identify underlying "factors" that explain these correlations. Moreover, direct oblimin rotation was used, since the literature review underlines the correlations among clusters of beliefs and thereby relations between the expected factors. Table 1 illustrates the five-factor solution, which explains 56.3% of the variance and can be considered as the most appropriate solution in isolating teachers' PBs since all loadings are relatively large and statistically significant. The first factor F_1 can be seen as a combination of platonist-formalist views of mathematics and explains 20.6% of the variance, while the second factor F_2 explains 13.1% of the variance and reflects the instrumental-formalist views, which are close to the second dimension of Ernest's model. The third factor F_3 refers to the experimental-formalist belief of mathematics and explains 9.1% of the variance, the fourth factor F_4 combines experimental-instrumental views of mathematics and explains 7.2% of the total variance, while the fifth factor F_5 that explains 6.3% of the variance, is a rather pure platonist approach.

Since the five factors which emerged from our study represented combinations of the three dimensions proposed by Ernest, it was considered important to examine teachers' philosophical profiles, that is groups of teachers according to their scores views in each of the above five factors.

No	Items related to PBs	FACTORS I	II	III	IV	V	h^2
1	M consists of exact results and correct procedures	.77	.10	-.13	-.01	.11	.60
2	Mathematical ideas are eternally truth	.71	.07	-.12	0.6	.17	.54
3	M consists of well defined concepts and well structured procedures	.67	.23	.10	-.04	.11	.47
4	M is a set of rules and theorems	.59	.36	-.45	-.14	.13	.62
5	M exists irrespective of time and place	.47	.21	.42	-.26	.20	.50
6	M is important because it's useful for people	.20	.72	.29	.06	.06	.60
7	M derived from social needs	-.09	.65	-.16	.41	.13	.61
8	M promotes operations, skills and procedures useful for daily needs	.01	.65	.15	.26	.13	.47
9	M is a set of algorithms and procedures	.47	.64	-.19	-.16	.16	.62
10	M is a dynamic, continually expanding domain	-.04	.23	.68	.12	.11	.51
11	M is a formal way of representing real world	.03	.23	.50	.47	.32	.52
12	Mathematical ideas should be formally expressed	.38	.37	.49	.08	.22	.51
13	M is constructed through experimentation and research	-.02	.13	-.05	.80	.16	.66
14	M serves human needs	.23	.33	.18	.59	-.05	.51
15	Mathematical ideas pre-exist in human minds	-.12	.11	-.05	.08	.74	.63
16	M is discovered	.34	.04	.26	.24	.64	.58
17	Mathematical ideas exist irrespective of the learner	.48	.18	.10	-.17	.64	.59
	Eigenvalue	3.50	2.22	1.55	1.23	1.07	
	Percentage of Variance	20.56	13.06	9.10	7.21	6.31	
	Cumulative Percentage of Variance	20.56	33.61	42.72	49.93	56.24	

Viewpoints of mathematics: I: Platonist-Formalist, II: Instrumentalist-Formalist, III: Experimentalist-Formalist, IV: Experimentalist-Instrumentalist, V: Platonist

Table 1: Factor Loadings of the Five Factors Related to Teachers' PBs as concerns Mathematics (M) Derived From a Direct Oblimin Rotation Procedure.

Though factor scores can be obtained in a number of ways, the calculation of the sum of variables which load most highly on each factor can be seen as an appropriate method of estimating factor scores (Kline, 1994) since it correlates highly, in most cases, with more elaborated procedures in which multiple regressions of all the variables on to the factors are computed. However, the relatively high values of the standard deviations, which emerged from calculating factor scores, revealed a variation among Cypriot teachers about the extent to which each factor is seen as representative of their PBs. Given that the 17 items can be classified into five broader categories, teachers' PBs were examined further by using Ward's clustering method. Cluster analysis revealed four relatively homogeneous groups of teachers, representing four philosophical profiles. The four-cluster solution is justified since the Agglomeration schedule shows a fairly large increase in the value of the distance measure from a four-cluster to a three-cluster solution, and the standard deviations of the four groups are much smaller than those of the whole group of Cypriot teachers.

Cluster Groups	F_1		F_2		F_3		F_4		F_5	
	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD
Group 1 (n=44)	3.14	.74	3.90	.44	3.48	.42	4.09	.55	2.58	.48
Group 2 (n=79)	4.13	.52	4.18	.54	3.94	.57	4.35	.64	4.03	.50
Group 3 (n=87)	3.30	.45	3.75	.53	3.64	.46	3.98	.42	3.79	.47
Group 4 (n=19)	3.37	.39	3.45	.69	3.38	.47	2.44	.60	3.46	.47

*1= Strongly disagree, 5= Strongly agree

Table 2: Means and Standard Deviations of the Factor Scores of the Four Groups Produced by Cluster Analysis on the Five Factors of Teachers' PBs.

Table 2 shows the means and the standard deviations of each group in each factor. It is clear that teachers of Group 1 (G_1) hold positive beliefs in four of the five factors. Taking into account the relatively high mean on F_4, and their negative attitudes on F_5, one can claim that G_1 teachers hold experimentalist-instrumentalist views. Group 2 (G_2) teachers were found to hold positive high views in all five factors and likewise the mean scores of Group 3 (G_3) in all factors were positive though less high. Thus, these two groups of teachers can be considered as holding a mixture of beliefs. Group 4 (G_4) teachers support the platonist-formalist views but disagree with the experimental-instrumental views. We can finally observe that the majority of teachers (166 out of 229) had complex PBs and only 63 can be positioned at the two ends of a spectrum describing PBs.

Kendall Coefficient of Concordance was calculated to examine the degree of consensus among teachers of each group in the ranking of the relative importance of the statements of the second part of the questionnaire. The analysis revealed a significant level of agreement ($p<.05$) amongst teachers of each group. Based on the mean ranks, a consistent pattern of responses was identified, in the sense that G_1 and G_3 teachers considered the statements reflecting experimental-constructivist ideas of teaching and learning mathematics as more important than platonist-formalist ideas. On the other hand G_2 and G_4 teachers responded on a reverse way, judging the platonist and formalist ideas as more important than the other ones. This pattern emerged for all the questions (i.e. characteristics of a good student and a good teacher of mathematics, evidence of learning and learning goals in mathematics).

The responses of these four groups on the third part of the questionnaire were found to be inconsistent to their PBs and teaching practices in three ways. First, a high percentage of teachers in all groups stated that they would like to teach every mathematical concept, to help students attend mathematics in the following years. They pointed out that parents, headteachers, inspectors and even colleagues exert pressure on teachers to cover all the prescribed content. Second, a high percentage of teachers, irrespective of the group, described a traditional approach in teaching mathematics (i.e., presenting the new concept on the blackboard and providing students with exercises for practice). However, there was a group of teachers who claimed that they use a dynamic investigative approach in teaching mathematics (experimentation, construction of meaning etc.) Though most of them belonged to G_1, there were teachers of the three other groups who taught mathematics in that way. Thirdly, teachers in all groups complained that the new book series does not provide for enough practice in the four basic operations, and most of them did not seem to recognize the scope of the innovation introduced. Moreover, teachers considered that the books are appropriate for the most able students.

Analysis of the interviews

The interviews produced additional evidence on some features of teachers' PBs, clarification of the factors influencing the development of PBs and how they can influence teaching beliefs and practices. First, it was clear that most teachers never took time to reflect on their PBs, as one of them put it "*I never thought of issues related to the nature of mathematics, although I teach mathematics for many years*". Second, teachers' answers revealed that their PBs were frequently inconsistent. For instance, while their responses concerning the nature of mathematics reflected the beliefs of the group they belonged to, their responses to questions regarding the evolution of the discipline and mathematical truth showed inconsistencies. Even though all the interviewees replied that mathematics derived of social-practical needs and it develops to serve social needs, almost all teachers, including those in G_1, pointed out that mathematical ideas and procedures are regarded as valid only if there is an acceptable proof for their validity.

Teachers' responses were also informative of the factors influencing their beliefs. All the participants mentioned the role played by their teachers and their school experiences on their mathematical beliefs. Some indicative extracts "*my attitude towards mathematics can be attributed to my mathematics teacher, who tried to teach in a way that mathematics lessons were never boring*" and another "*my past positive school experiences formed the way I am perceiving mathematics and learning in mathematics*". On the other side, the university tutors did not seem to make any real difference, since "*university lessons paid more attention to knowledge and ways of teaching mathematics than issues related to the affective domain*". The interviewees attributed a very restricted parental influence on their PBs, but they valued the interactions with colleagues. As one of them put it "*I had a very good group of colleagues and we exchanged ideas about mathematics and ways of teaching specific* *mathematical concepts*". These ideas and practices seemed to be more influential during their first years of employment.

The interviews seemed to reveal a marked inconsistency between teaching practices and PBs. All primary teachers, irrespective of their initial PBs, paid lip service to experimental approaches; they acknowledged the role of activities that offer students opportunities to construct knowledge. However, their favourite activities and their assessment practices echoed a rather traditional-formalist approach. Only the teacher of G_1 claimed that he developed activities that enable students to construct the meaning of mathematical ideas. On the other hand, the secondary teacher, although belonging to G_2, adopted a more formalistic approach in teaching mathematics; as she stated, "*it is important to provide students with ample opportunities for practicing the algorithms presented by the teacher*". Proofs were a basic feature in her teaching, while learning theorems and definitions were considered as very important. Thus, the secondary teacher mentioned that her tests are based on exercises that require the presentation of definitions of mathematical concepts.

DISCUSSION

The results support previous studies, which refer to the complexity of teachers' PBs and their subconscious character (Raymond, 1997; Stipek et al, 2001). The majority of the subjects in our study were classified into the two groups reflecting teachers with a mixture of PBs. The interviewees did not seem to have thought seriously about their conceptions about mathematics and its' teaching; they were unaware of their own philosophy. The complexity of beliefs was indicated by teachers' replies during the interviews concerning different issues related to the nature of mathematics and that is why the data failed to verify Ernest's model of PBs. However, the model that emerged from this study can be considered as representative of teachers' PBs, since it covers a wide spectrum of beliefs with teachers of G_1 and G_4 occupying the opposite poles of the scale. Teachers of G_1 can be considered as holding a more dynamic view of mathematics, also accepting its usefulness. On the other hand, teachers of G_4 regard mathematics as a static unified body of knowledge, consisting of facts, rules and skills that students have to acquire.

The interviews showed that that school experience plays the most prominent role in influencing the development of teachers' PBs. Some previous teachers are seen as models and influence both beliefs and practices; this and the collegiate effect accounts for the appearance of traditional beliefs, even among younger teachers. University education did not seem to influence significantly teachers' PBs. This finding is in alignment with Thompson (1992) claim that it is not possible to alter teachers' PBs during two or three university courses.

The results of this study indicate that PBs might influence teachers' beliefs about teaching and learning in mathematics as well as their teaching practices. Yet, the inconsistencies among beliefs and teaching practices found in this study show that the domain of beliefs is a complex one. Thus, it seems more appropriate to refer to intercorellations between the two variables instead of domains influenced by beliefs, since we found evidence of the reverse direction of influence i.e., previous practices influence teachers' PBs. Consequently, our study provides support to arguments about the dialectic relation between beliefs and teaching practices (Raymond, 1997; Thompson, 1992).

Finally, the present study underlines the importance of prompting teachers to reflect upon and examine their own beliefs systems. Since it is acknowledged that affective competencies can be learned and consequently be taught (Goldin, 1998), institutions that are responsible for teacher training, should also pay attention to the affective domain, providing teachers with opportunities to get aware of their own beliefs, the ways these beliefs are formed, and their dialectic relation to other domains e.g., their teaching practices. Further studies based on nonparticipant observations would be useful in shedding more light in the issues related to the inconsistencies witnessed between PBs and teaching practices.

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