Teacher Behaviours that Influence Young Children's Reasoning

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Abstract: Children are expected to develop the habit of reasoning from their earliest years at school. However, there has been limited emphasis on strategies that teachers can use to support young children's reasoning. We report on a case study of four teachers who implemented mathematical investigations that provided reasoning opportunities for their classes of seven-to-eight year olds. An analysis of teacher behaviours and their students' responses suggests that reasoning is influenced by (1) teachers' expectations of reasoned actions and responses, (2) instruction in reasoning as systematised thinking, and (3) authentic opportunities for reasoning. These influences are discussed together with examples of how these teacher behaviours support or inhibit reasoning.

Reasoning is central to mathematics as a discipline (Steen, 1999) and underpins mathematical learning (Russell, 1999). As the National Council of Teachers of Mathematics (2000) emphasizes, the ability to reason is essential to mathematical understanding, and should be a primary goal in mathematics education: "By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas and — with different expectations of sophistication — at all grade levels, students should see and expect that mathematics makes sense (p. 56). While the NCTM recommends that students from their earliest years at school should engage in reasoning activities, there are conflicting viewpoints about young children's capacity to reason. Some have proposed (e.g., Piaget & Inhelder, 1969) that there are developmental constraints to young children's reasoning. However these assumptions have been strongly challenged. For example, Tang and Ginsburg (1999) have argued that poor mathematical performance is a consequence of educational failure rather than children's inability to reason. Two alternative constraints to young children's reasoning have been proposed. First, young children's ability to reason is limited by their knowledge (Brown & Campione, 1994; Metz, 1997). Thus, reasoning is adversely affected by a weak knowledge base. Second, young children's reasoning can be inhibited by teacher competence (Brown & Campione, 1994). Metz (1997) argues persuasively that if the constraints to young children's knowledge and skill could be overcome then with judicious scaffolding, they should be able to engage in abstract reasoning. This hypothesis was supported in a study of young children's reasoning in mathematics and science that involved conjecture, argumentation, and evaluation of evidence (Watters & Diezmann, 1998).

Given that teacher competence can be a constraint to reasoning and the NCTM recommendations, the purpose of this paper is to identify the teacher behaviours that influence young children's reasoning. Teachers' competence in supporting young children's reasoning appears to be related to the nature of the instructional tasks, and the classroom culture.

Challenging Instructional Tasks
The teacher plays a crucial role in supporting the development of students' reasoning through the selection and implementation of cognitively challenging mathematics tasks (Henningsen & Stein, 1997). Hiebert et al. (1996) explain that the cognitive value of a task resides in the opportunity that it provides for students to explore and solve a problem. Teachers need to maintain the cognitive challenge in a task by proactively supporting students' cognitive activity without unnecessarily reducing the complexity of the task (Henningsen & Stein, 1997). While some tasks provide limited opportunities for reasoning, mathematical investigations are ideal tasks due to their scope for cognitive challenge (Boldt & Levine, 1999; Diezmann, Watters, & English, 2001). Investigations involve solving open-ended problems in which students make and test conjectures, engage in logical thinking, seek patterns and relationships, and explain and convince others of their viewpoints (e.g., Greenes, 1996).

Reasoning in the Classroom
Reasoning in the classroom is affected by teacher expectations, the classroom discourse, opportunities to make sense of mathematics through different types of reasoning and more general conditions such as a supportive climate. Gravemeijer, Cobb, Bowers, and Whitenack (2000) argue that in supportive classrooms: "(1), The students would explain and justify their thinking when contributing to whole-class discussions; (2) The students would listen to the contributions made by their classmates; (and) (3) The students would indicate when they did not understand a classmate's explanation or contribution and ask clarifying questions" (p. 251). These expectations encourage student participation, and a quest for understanding. They also highlight the importance of productive discourse in mathematics whereby students explain their ideas, build on others' ideas, and generalize beyond a specific example (Sherin, Mendez, & Louis, 2000). Mathematical investigations involve the exploration of a particular line of thinking in which students use resources to test conjectures, and hence, engage in transformational reasoning. According to Simon (1996) transformational reasoning is a dynamic process in which the conclusion is reached by running physical or thought experiments. Investigations also provide opportunities for inductive and deductive reasoning when students generalize from their experiments or explain the chain of thinking that led them to particular conclusions.

While sound reasoning is ultimately the goal, instances of flawed reasoning are inevitable in classrooms where students are novice reasoners. Russell (1999) argues that flawed reasoning plays a dual role in the classroom. First, students need to practice the various components of reasoning (e.g. conjecturing). They also need opportunities to justify and critique their own and others' reasoning, and respond to challenges to their reasoning. Second, flawed reasoning may also highlight mathematical issues that are relevant to the whole class and need to be addressed. For example, flawed reasoning may reveal that students hold the erroneous view that it is acceptable to reach different solutions in a convergent problem if different solution processes have been used. Thus, teachers should be alert to flawed reasoning and capitalise on the learning opportunities they it presents.

Design and Methods

This research adopts a case study design (Yin, 1994) in which a teaching experiment was conducted with the goal of supporting the development of investigatory abilities in young children. The study was implemented in four Year 3 classes with a total of 95 children (ages 7-8 years) in a parochial school in a relatively affluent outer suburb of a major city. Three of the teachers had in excess of ten years teaching experience while the fourth was a first-year, albeit mature age, teacher. Class sizes ranged from 18 to 26 students with the smallest class containing two special needs students. Students engaged in a 90-minute session each week for 10 weeks focussing on mathematical investigations (Diezmann et al., 2001) that was taught by their classroom teachers. The teachers were provided with ongoing professional development about investigatory approaches and reasoning, and teachers and researchers met regularly during the ten-week period to debrief, plan, and evaluate the program. The case study database comprised achievement tests, teacher and student interviews, student work samples, teacher and researcher notes, and photographs of teachers' and students' classroom work. Additionally, two researchers (CMD, JJW), who were non-participant observers, captured salient events on video. An assistant videoed the whole class. Data were analysed using constant comparative strategies (Glaser & Strauss, 1967) to identify emerging patterns, themes, and issues related to young children's reasoning and investigatory abilities while being sensitive to the existence of conflicting data to disconfirm the analysis. In this paper, we report on those data that inform us about how teachers can facilitate or inhibit reasoning behaviour in young children.

Results

Analysis of teacher behaviours revealed three patterns of interaction that influenced students' reasoning. First, by providing modelling and timely intervention, teachers developed the perception that reasoning involves justifiable actions. Second, teachers built on this perception by encouraging students to engage in systematic thinking using the language, and strategies associated with reasoning. Third, teachers capitalised on situations and conversation to create opportunities for reasoning.

1. Reasoned Actions and Responses

Teachers developed an expectation of sense making in mathematics through their patterns of discourse with students and by modelling sense making. These behaviours included asking students to justify their responses and actions; encouraging students to use "because" in sharing information; pressing students to explain unclear or incomplete responses; rephrasing students' words to enhance clarity; explicitly connecting ideas from different students; drawing students' attention to critical aspects of tasks; negotiating step-by-step guidelines for accomplishing challenging tasks; and by justifying their own actions — as if thinking aloud. The following examples illustrate how teachers influenced reasoned actions and responses from the students. In the first example, the teacher set up a clear expectation of reasoning to which the students responded. In contrast, in the second example, the teacher's failure to address flawed reasoning inhibited reasoning.

Smartie Can Example: Students were working in small groups to predict how many Smarties (sweets) were in small opaque sealed canisters. In one group, Melissa predicted that there were 27 Smarties. She reached this prediction by establishing that the canister had a length and circumference of three Smarties and nine Smarties respectively and computing the product of these figures. After the teacher instructed the class to justify their conclusions, Melissa repeatedly resorted to demonstration and explanation to convince her group that 27 was a sound prediction. While the group readily accepted the plausibility of Melissa's approach, they did not accept her prediction immediately. Melissa's persuasive argumentation and use of evidence led to a vigorous group discussion, which culminated in unanimous support for her conclusion. Thus, the teacher's clear expectation of reasoning resulted in this group striving to make sense of the task and each other's responses.

Card Game Example: During a place value card game, the teacher generally prompted students to justify their actions and encouraged them to monitor and challenge each other's actions. However, not all students' reasoning was subject to the same level of scrutiny. Michael, a mathematically gifted student, twice engaged in flawed reasoning, which went unchallenged by either teacher or peers. For example, small groups of students were dealt three cards to represent a three-digit number (e.g., $\boxed{2}\,\boxed{4}\,\boxed{3}$). They then took it in turns to draw a card from the pack (e.g., $\boxed{8}$) and used it to make the highest possible number by replacing an existing card in the hundred's, ten's or one's position (e.g., $\boxed{8}\,\boxed{4}\,\boxed{3}$). When all cards had been used the person with the highest three-digit number was declared the winner. Michael had seven turns during this game and demonstrated flawed reasoning on his second and sixth turns. The teacher queried his reasoning on the second turn but did not press Michael for an explanation: "Why? Why didn't you put it (the $\boxed{1}$) on the one's"? Michael shrugged and the teacher commented: "Now you have made it 30 smaller not just four smaller." Michael again made an error on his sixth turn but his peers did not query his placement of cards, though they queried each other's actions during the game.

(Dealt) $\boxed{3}\boxed{4}\boxed{5} \rightarrow \boxed{\mathbf{8}}\boxed{4}\boxed{5} \rightarrow \boxed{8}\boxed{\mathbf{1}}\boxed{5} \rightarrow \boxed{8}\boxed{\mathbf{3}}\boxed{5} \rightarrow \boxed{8}\boxed{\mathbf{6}}\boxed{5} \rightarrow \boxed{8}\boxed{6}\boxed{\mathbf{1}} \rightarrow \boxed{8}\boxed{6}\boxed{\mathbf{8}} \rightarrow \boxed{8}\boxed{6}\boxed{\mathbf{3}}$

Michael's exemption from justifying his actions is a concern. His flawed reasoning could be explained by disinterest if the game was insufficiently challenging. However, given that he was a keen participant in this game, was highly competitive, was playing with another capable student, and was observed to engage in flawed reasoning at other times, it seems more likely that Michael was either impulsive or did not fully understand the game. Thus, while the learning environment provided support for other students to account for their reasoning, Michael was not afforded this same opportunity. Other students' reasoning can also be inhibited if they unquestioningly accept responses from students, such as Michael, whom they somehow regard as infallible. Teachers also inhibited reasoned actions and responses through their use of recall-oriented questioning, limited wait time, and through inadequate responses to requests for assistance.

2. Reasoning as Systematised Thinking

These students were generally unfamiliar with the culture of reasoning. Teachers supported their enculturation into the practices of reasoning by developing relevant vocabulary; scaffolding students to make and test conjectures; challenging students' assumptions; focusing students' attention on the available evidence; promoting logical thinking; encouraging students to present their ideas as a chain of reasoning; and highlighting argumentation as a tool for evaluating alternative conclusions. The following examples illustrate the importance of discourse in developing students' understanding that reasoning is systematised thinking. In the first example, the teacher's interaction and use of mathematical language supported the students to think about the unanticipated result. In the second example, the teacher failed to adequately address the student's incorrect use of everyday language, thus limiting the development of his knowledge that reasoning is systematised thinking.

Speed Investigation: During a class sharing session, one group reported on an investigation in which they explored whether the speed at which a Smartie rolls down a slope is affected by its colour. The group had predicted that colour would have no effect on speed. However, after completing their investigation, this group recorded that "coulor efex speed. We though it would'nt efet but it did [Sic]". The class reaction to the group's conclusion about colour being a critical variable for speed was mixed. The teacher worked from the students' comments to draw attention to the ways in which the Smarties might have differed from each other apart from colour, thereby challenging the conclusion that colour alone affected speed. She also proposed how disagreements could be resolved through further investigation.

Mark	It might just be their weight or something.
Teacher	So you think actually that probably some of the (differently coloured) Smarties were **a little bigger** than some of the other Smarties. And you think that is a better reason (than colour alone) for them sliding faster?
Andrew	It could have been the way they were made.
Teacher	Yes. You think they had **different sizes and shapes**?
Andrew	We do think colour affected it as well.
Amanda	I don't think colour (alone) would affect it.
Teacher	Why don't you think colour would affect it?
Amanda	Because, it doesn't matter what colour it is. Colour is just to make it look better (on the surface).
Teacher	Look different? ...So that is a very interesting point isn't it. **We probably would have to investigate that a bit more before we come to some opinion that we all shared.**

Colour Example: The class had been investigating the frequency of particular colours in boxes of Smarties. Michael's report on his Smartie box was marked by idiosyncratic use of the terms "all" and "except".

Michael	We had **all** of them (colours) **except** green
Teacher	You had **every** colour **except** green? Where there **any** white ones?
Michael	No.
Teacher	Were there **any** black ones?
Michael	No.
Teacher	Oh, **any** browns?
Michael	Yes.

Though the teacher challenged Michael's statement twice, this was insufficient to prompt him to revise his statement that "We had all of them except green". Thus, there was a need for the teacher to pursue this inconsistency and encourage Michael to reflect on his reasoning. This example illustrates the need to emphasise the correct use of everyday words that indicate class inclusion and exclusion, such as "all", "every", "any", and "except". The student in this example, Michael, was the gifted student also featured in the card game example. Thus, although Michael had a heightened capacity for reasoning, there were shortcomings in his reasoning, which needed to be addressed.

3. **Authentic Opportunities for Reasoning**

The use of intrinsically interesting situations can enhance young children's motivation and commitment to reasoning. The following two examples illustrated how teachers promoted and limited opportunities for reasoning in these situations.

Story Example: One teacher promoted reasoning about quantity through her implementation of the story, "The Doorbell Rang" (Hutchins, 1986). This story involves the repeated occurrence of children sharing out a batch of 12 cookies but before they can eat the cookies the doorbell rings and they also have to share their cookies with the new arrivals. Thus, over time, each child receives fewer and fewer cookies. The teacher capitalised on the reasoning opportunities in this story through her organisation of and interaction with the students. She seated the class around a batch of cookies and assigned students to be the story characters. After the actors depicted the arrival of each new group of children in the story and shared the cookies out, the teacher asked questions, such as "Have they got the same amount of cookies? How do we know? How else could we know?" Through the enactment of the story and the teacher's encouragement to reflect on what was happening, the class developed a clear understanding of the inverse relationship between the number of children who were to share the cookies and the quantity they would receive. Later in the story, the students were also able to ascertain the number of cookies each child would receive after Grandma arrived at the door with another batch of 12 cookies. Some students continued to use this experience as a referent. For example, over 12 weeks later, Alice reasoned from cookies when the outcome of a division was not a whole number: "And if we had an odd number of cookies we had to do something … like break them in half." Hence, the implementation of the story provided a memorable referent from which the students could reason about quantity.

Dissolving Smarties: Students were using Smarties to explore an aspect of mathematics. One group decided to explore what happened to Smarties after they had been dropped into hot water. This investigation provided opportunities for reasoning, about time, temperature, volume, and quantity. However during the investigation and in the subsequent sharing session, the teacher emphasised the "fun" element of the activity to the extent that the mathematical opportunities were overlooked. Thus, while it can be motivating for students to engage in interesting activities, the teacher needs to preserve and foster the mathematical value of the activities, and support students' reasoning within the particular context.

Conclusion

Young children's reasoning can be enhanced or inhibited by teachers' behaviour through their discourse, the type of support they provide for their class, and how they implement mathematical tasks. Reasoning was promoted when teachers clearly valued reasoning, modelled reasoned actions and responses, and held high expectations of the class. The children responded to high expectations with continual attempts to engage in sense making in their actions and discourse, and challenges for their peers to do likewise. While these children were generally unfamiliar with reasoning as a systematic way of thinking, with ongoing teacher guidance and scaffolding, most children began to utilise at least some of the language, rules, and strategies associated with reasoning. Learning experiences that provided rich opportunities for students to develop and practice this more "formal" reasoning were those where the teacher focussed strongly on the mathematics in the situation and used the task to create a forum for reasoning about topics of interest.

These insights provide the base for us to extend our studies in developing more comprehensive documentation of the teacher behaviours that support and inhibit reasoning. If reasoning mathematically is to become "a habit of mind" (NCTM, 2000, p. 56) from the earliest years of schooling, action needs to be taken at preservice and inservice levels to develop the appropriate pedagogical content knowledge.

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