GEOMETRIC SIGNS AND STUDENTS' VERBAL REPORTS: THE CASE OF THE GEOMETRIC MODEL OF SUN SHADOWS

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I analysed a large amount of written verbal reports, produced by Grade IV and Grade VI students as a response to the task of communicating their knowledge about sun shadows. The texts were produced immediately before and soon after the introduction of the elementary geometric model of sun shadows by the teacher. Some relevant changes were detected; they concerned a much more frequent production of hypothetical and causal sentences expressing geometrical links in the 'after' texts. A discussion about related cognitive, cultural and educational issues is sketched.

INTRODUCTION

In a Vygotskian perspective (see Vygotsky, 1978, Chapters I and VI), important changes can intervene in students' thinking strategies when the teacher introduces some peculiar signs as tools to solve problems. In particular, the signs introduced may allow students (by themselves, or with the help of more competent peers, or under the guidance of the teacher) to solve previously inaccessible problems. Still in a Vygotskian perspective (see Vygotskij, 1990, Chapter VI), spontaneous students' conceptions can develop towards scientific conceptions when scientific conceptions are made accessible to them by the teacher through appropriate (external) representations. In the study reported in this paper I am interested in the effects of the introduction by the teacher of a specific sign (the elementary local geometric model of sun shadows – shortly, ELGMS) on the *ways of thinking* about the sun shadow phenomenon.

Fig. 1: The ELGMS

As reported in Boero et al (1995), and as we will see in details in the next Subsection, the ELGMS is not spontaneously produced by students neither in Grade IV, nor in Grade VI . Moreover, it is very far from the graphic representations of the sun shadow phenomenon, which are spontaneously produced by most of them (see later). And the ELGMS appeared relatively late in the historical development of human cultures as a relevant invention, whose cultural implications were very rich (see Serres, 1993). The research hypothesis underlying this study is that *the appropriation of the ELGMS as a tool to solve elementary geometrical modelling problems concerning the sun shadow phenomenon can deeply change the way of thinking about this phenomenon.* Some experimental evidence will be provided to support this hypothesis. In particular, by analysing students' verbal reports concerning the sun shadow phenomenon we will see that an important change can be traced in students' reports after the introduction of

the ELGMS: explicit hypothetical and causal links between the height of the sun and the lengths of the cast shadows become much more frequent. The final Discussion Section will elaborate on this result. Some cultural and educational implications will be discussed (in particular, as concerns the meaning of the ELGMS as a prototypical thinking tool belonging to the 'rationality' of the Western civilisation). In general, my hypothesis (together with the related cultural issues) agrees with recent developments of research in the Vygotskian perspective. Stetsenko (1995) wrote:

> *"The originality of the Vygotskian approach to children's drawings is primarily that it addresses and clarifies the functional role of drawings in the overall development of the child – that is, in the entirely of cognitive, emotional, communicative and other aspects of this development." (page 147). "What the cultural-historical theory strives at is a precise specification of the unique ways making and looking at pictures help a child both to understand the world and come to terms with it"(page 148).*

METHOD

The Students' Educational Background

Since the second half of the 70s, both Genoa Group Projects for mathematics and science education in primary school (6-11) and lower secondary school (11-14) have devoted a wide interest to the sun shadow phenomenon: from early non-geometric conceptions to local geometric modelisation, till global considerations of the phenomenon on the sun system scale. The level of sophistication of the mathematical tools introduced and the difficulty of the mathematical problem situations tackled at the end of the activities is obviously different in the two Projects. Most of 'our' primary school students do not join classes that adopt the Lower Secondary School Project and most of 'our' lower secondary school students do not come from classes involved the Primary School Project, so the initial steps in the approach to the sun shadow phenomenon are the same in both Projects (even if they are introduced at a different pace). The activities concerning sun shadows take place over a very long period of time in primary school (from the second half of Grade III to the beginning of Grade V). They take place over the whole school year in Grade VI. In both cases, the activities start by provoking students to write and draw what they think about sun shadows; a number of games played in the courtyard follows; then the 'shade space' (between the object and the cast shadow) is discovered; finally, more and more systematic observations are organised (at different times of the same day) and verbally reported by students. In particular, these are common initial steps:

- standardised questions about the sun shadow phenomenon, and discussion about the answers. In particular, the following question is posed both in grade III and at the beginning of grade VI :

> *«Have you ever noticed that when you are walking in a sunny place your body casts a shadow on the ground?* (YES / NOT option). *Is your shadow longer at 9 a.m. or at noon ?* (9 a.m. / NOON option). *Why ?»* .

It is interesting to remark that the majority of students in Grade III, and still more than 40% in Grade VI, chooses the 'NOON' option *"because the sun is stronger"*, *"because I see it better"*, etc. (for details, see Boero, 1999)

- production of drawings representing the sun shadow phenomenon. We can notice that most of these drawings (both in Grade III and in Grade VI) are very far from the ELGMS and correspond to non-geometric conceptions. Here we present only some examples; for further details, see Boero, 1999.

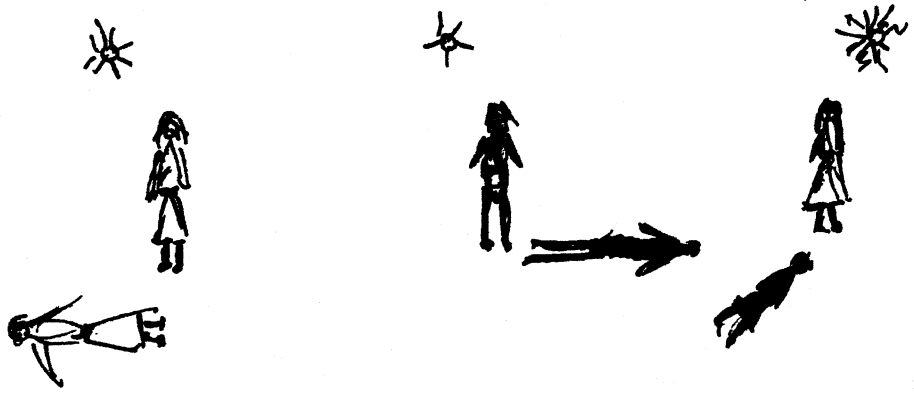

Fig.2: Three examples of students' initial drawings

As remarked in Boero et al (1995) (for further details, see Boero, 1999), the ELGMS is spontaneously produced by very few students, both in grade III or IV and in grade VI, even after all the previously mentioned activities; and it does not spread spontaneously across the classroom. It can be observed that this fact is in accordance with the importance attributed to the invention of the ELGMS by historians of Science: the first traces of the model go back to the late developments of Babylonian and Egyptian civilisations and the early developments of Greek geometry, between the VII and the V century b.c. (Serres, 1993). In the Primary School Project classes, the teacher introduces the ELGMS at the beginning of Grade IV (after 3-4 months of activities in Grade III); in the case of the Lower Secondary School Project classes the introduction of the ELGMS takes place one month after the activities described above. The teaching strategies adopted by teachers to introduce the ELGMS may vary according to personal preferences, theoretical motivations and occasional circumstances: in some cases the teacher exploits the proto-geometrical drawings produced by some students, asking the other students to use them to solve other problems (but it can happen that no student produces an 'exploitable' drawing!); in some cases the teacher introduces the ELGMS as a device to solve problems; in other cases the teacher tries to guide the production of the sign by the students through suitable observations (e.g. the visualisation of the upper border of the shade space, and the task of drawing such situation), then some applications follow (for a discussion about these educational strategies see Scali, 1998).

Available Data and Selected Data

The fact that the sun shadow phenomenon was a crucial subject for both Projects for more than twenty years, together with some methodological choices (in particular, the systematic practice of written verbal reporting for all activities since the end of grade I, and the practice of frequent classroom discussions guided by the teacher), offered a big amount of interesting materials from the classroom: rich individual texts, videotapes and recordings of classroom discussions, etc., concerning the activities described in the previous Subsection. In particular, 24 teachers gathered individual students' texts from 81 classes with detailed information about the activities performed! Given that I am interested in the consequences of the introduction of the ELGMS on students' ways of thinking the sun shadow phenomenon, I looked at the texts produced immediately before, and/or soon after, the introduction of the ELGMS. I considered only the texts produced as a response to a standardised task:

> *«Write a letter to a friend of yours in order to explain him what you know about sun shadows at this moment».*

I took into account the classroom activities between the introduction of the ELGMS and the production of the 'after' text, as reported by the teachers. My preliminary choice was to consider only texts coming from classes who had worked *individually* in that period on two or three applications of the ELGMS over a period of no more than two weeks. According to the collected information, in all these classes, after the introduction of the ELGMS, the students could look at it on the walls and also in their personal copybooks. Then only texts including explicit reference both to the sun and to the shadows were considered (about 60% in grade IV and 68% in grade VI).

Some preliminary analysis performed on relatively small samples of students (two or three classes in each case) had shown that:

- the way chosen by the teacher to introduce the ELGMS does not seem to affect the quality of the 'after' texts;

- concerning the use of texts produced by the same students or not (before and after the introduction of the ELGMS), the only difference was that the second production of the same student was in some cases less rich and less suitable to use than the first one (the repetition of the identical task at the distance of 10-15 days did not help motivation!). So I decided to select a rather big number of classes who had written *only one* report, immediately before *or* soon after the introduction of the ELMGS, for a large scale comparison between 'before' and 'after' texts. Selection was made in order to get similar social environments between the different groups of students. I have considered 202 'before' texts and 206 'after' texts for grade IV; and 182 'before' texts and 170 'after' texts for grade VI. I performed a more detailed analysis on other two smaller groups of students (80 for grade IV and 66 for grade VI) who had produced *both* the first *and* the second text, in order to trace the personal evolution of their performances (see table 3, concerning grade IV).

Criteria for Classifying Students' Reports

- **Conditional** texts: at least once in the report, the student expresses conditional links between the height of the sun and the length of the sun shadows, e.g.: *«If the sun is high the shadows are short».*

- **Causal** texts: at least once in the report, the student expresses causal links between the height of the sun and the length of the sun shadows*: «At noon the shadows are shorter than at nine because the sun is higher»; «At noon the sun is higher than at nine, and so the shadows are shorter; "At noon the shadows are short because the sun is high*", etc.

- **Descriptive** texts: all the other reports. The student reports what he/she saw at different times of the day, with no explicit 'conditional' or causal link between the height of the sun and the length of the sun shadows: *«At nine a.m. the sun is low and the shadows are long; at noon the sun is high and the shadows are short».*

Some comments about the proposed classification follow.

- Some reports can belong both to the first and the second category. We can remark that causality is very close to 'conditionality' in many situations of communication. In particular, in everyday life situations people frequently use the clauses *«If B, then A», «B, and so A»* and *«A because B»* as if they were equivalent. I preferred to make a distinction between Conditional reports and Causal reports because I had observed that in our specific situation Conditional reports are usually produced by students in order to express a general 'conditional' link (*«if the sun is high the shadows are short»*), while most of the causal reports refer to a specific situation (*«at noon shadows are short because the sun is high»*).

- In Italian, like in other languages, the *«A and B»* clause can be used to suggest the idea of a causal link between A and B. In the sentence *«The driver was running very fast and the car got out of the road»* we can see the intention of implicitly stating a cause-effect relationship. As a consequence, part of the Descriptive reports (before as well after the introduction of the ELMGS) may have been produced by students who were thinking about a causal relationship between the height of the sun and the length of the shadows.

- There would be some reasons for the inclusion of reports containing a 'when' clause *(«when the sun is high, the shadow is short»)* in the category of Conditional reports: the *'A when B'* and the *'if B, then A'* clauses often have similar uses in everyday language; and the 'conditional' clause seems to represent a de-timing of the 'when' clause (see Arzarello, 2000). But in most of students' reports presenting only the 'when' clause the whole linguistic context gives the impression that the 'when' clause merely expresses a temporal coincidence with no 'conditional' link between the information about the position of the sun and the information about the length of the cast shadow. Some interviews (performed immediately after the production of a written report containing one 'when' clause) confirmed this impression.

SOME RESULTS

The following tables display some outcomes of the analysis performed according to the criteria listed in the previous Subsection.

	Total number	Conditional	Causal	Descriptive
'Before' texts	202	38 (19%)	28 (14%)	146 (72%)
'After' texts	206	72 (35%)	75 (36%)	81 (39%)

Table 1: 'Before' texts and 'after' texts of two different groups of Grade IV students

	Total number	Conditional	Causal	Descriptive
'Before' texts	182	40 (22%)	27 (15%)	127 (70%)
'After' texts	170	56 (33%)	70 (41%)	64 (37%)

Table 2: 'Before' texts and 'after' texts of two different groups of Grade VI students

	'Before'texts	Conditional	Causal	Descriptive
'After' texts	80	26	28	34
Conditional	15	8	5	2
Causal	9	1	7	1
Descriptive	59	17	16	31

Table 3: 'Before' and 'after' texts of the same 80 IV-grade students

Table 3 provides information about the evolution of students' productions within the same group of students. For instance, let us consider the last row. 59 students had produced Descriptive texts 'before' the introduction of the ELGMS. 'After' the introduction of the ELGMS, 17 out of them produced a Conditional text; 16 produced a Causal text; while 31 still produced a Descriptive text. By considering these numbers on the last row we get 64 texts: it means that 5 texts became "Conditional"*and* "Causal".

Data concerning the comparison of 'before' and 'after' texts of the group of 66 VI-grade students were similar to those displayed in Table 3.

All tables show a common trend: a statistically relevant increase in the percentage of both Conditional and Causal texts between the 'before' texts and the 'after' texts. The increase is more relevant for Causal text (more than 100% increase in all tables).

DISCUSSION

By analysing students' verbal reports concerning the sun shadow phenomenon we have seen that, after the introduction of the ELGMS, an important change happened in students' reports: explicit hypothetical and causal links between the height of the sun and the length of the cast shadows became much more frequent. The fact that this change happened over a very short period of time (no more than two weeks), both in grade IV and in grade VI, suggests that spontaneous maturation cannot explain the change. The fact that no *collective* classroom activity intervened between the introduction of the ELMGS by the teacher and the second verbal report seems to exclude the possibility of the adoption of those *«modes of saying»* by imitation. And the very nature of the change suggests that the introduction of the ELGMS affects not only the *way of describing* the sun shadow phenomenon, but also the *way of thinking* about it (although in general it is difficult to infer the underlying thinking processes from verbal traces: cf Ericsson and Simon, 1981). It is interesting to examine this *way of thinking* more closely. Its importance does not seem to rely much on the fact that the ELGMS fits better with the sun shadow phenomenon in everyday life experiences, in comparison with other ways of conceiving it. Let us consider an example taken from some observations performed by Claudia Costa, a teacher working in the Italian school in Asmara (Eritrea) (see Boero, 1999). From individual interviews and classroom discussions it emerged than half of the students of two VII-grade classes thought that the shadows were longer in the early morning and in the late afternoon because then the Sun was less strong (or less bright), and so the shadow (a manifestation of the darkness, the opposite 'entity') succeeded in being longer. This non-geometrical conception fits rather well with many everyday life experiences! The cultural importance of the ELGMS relies on the fact that the 'rationality' inherent in it is different from the 'rationality' inherent in the non-geometrical conceptions that agree with empirical evidence. In order to understand this difference let us consider the following statements, produced by one of 'our'students and by an Asmara student:

> *"At noon the shadow is shorter than in the early morning because the sun is higher";*
>
> *"At noon the shadow is shorter than at 9 because the sun is brighter and beats the darkness"*

The validity of the first statement relies on geometrical necessity, while the validity of the second statement relies on the consideration of the increasing strength of the light, which changes the 'strength equilibrium' with the opposite 'entity'. The introduction of the ELGMS (a product of the cultural evolution) brings in a new kind of 'necessity' in the way of thinking about nature: a geometrical necessity. Western 'rationality' has strongly developed towards this direction over the last twenty five centuries, not only as concerns the geometrical models in astronomy, but also as regards for example the differential models, the stochastic models, etc.: it is like if 'something' happens, and will always happen in the future, because the inherent variables are constrained according to a given mathematical model. This provides us with a very efficient tool to solve quantitative problems, forecast the evolution of many phenomena, etc.

At the beginning of the activities concerning sun shadows, many students of 'our' classes produce 'causal' texts when they describe the sun shadows phenomenon, but causality does not concern the links between the height of the sun and the length of the cast shadows (e.g.: *"The shadow is longer at noon, because the sun is stronger"; or "The shadow is longer at noon, because I see it better"*. The contradiction with empirical evidence and systematic observations gradually brings most of them to write that *"At noon the shadow is short and the sun is high"*. This could be explained well in terms of the Piagetian "adaptation" of mental representations (see Piaget, 1926). Afterwards, students use the ELGMS (introduced by the teacher) in few problem situations. This is sufficient to change the quality of the verbal reports of many of them (see our data). The following activities reinforce this trend. An entirely different situation happens with the Asmara students: even the Asmara students wrote (in their individual initial texts) a lot of rather complex causal and hypothetical sentences. The difference is that they produced those sentences coherently with a way of thinking about the sun shadow phenomenon, which seems to be strongly related to the environmental culture (and not in immediate contradiction with empirical evidence). This means there is a potential richness that schools should not waste. Why, and especially how, to introduce another way of thinking (the one based on the ELGMS) in such a situation, without destroying the existing one, remains a difficult challenge for nowadays intercultural educational perspectives! (cf Barton, 1996)

REFERENCES

Arzarello, F.: 2000, 'Inside and Outside: Spaces, Times and Language in Proof Production', *Proc. of PME-XXIV,* Hiroshima, vol. I, pp. 23-38.

Barton, B.: 1996, 'Anthropological Perspectives on Mathematics and Mathematics Education', in A. Bishop et al. (Eds.), *International Handbook of Mathematics Education,* Kluwer A.P., Dordrecht, vol. II, pp. 1035-1053.

Boero, P.; Garuti, R.; Lemut, E.; Gazzolo, T. & Lladò, C.: 1995, 'Some Aspects of the Construction of the Geometrical Conception of the Phenomenon of the Sun's Shadow', *Proc. of PME-XIX,* Recife, vol. 3, pp. 3-10.

Boero, P.: 1999, 'Elementary Geometrical Modelling of Sun Shadows in Different Cultures', in A. Rogerson (Ed.), *Proc. of the International Conference on Mathematics Education in the XXI Century,* Cairo, vol. II, pp. 9-17.

Ericsson, K.A. & Simon, H.A: 1980, 'Verbal Reports as Data', *Psychological Review,* 87, 215-251.

Piaget, J.: 1926, *La représentation du monde chez l'enfant,* P.U.F., Paris

Scali, E.: 1998, 'Choix des tâches et organisation des intéractions dans la modélisation géométrique élémentaire', *Proc. of CIEAEM 49,* Setubal, pp. 186-194.

Serres, M.: 1993, *Les origines de la géométrie,* Flammarion, Paris

Stetsenko, A.: 1995, 'The Psychological Function of Children Drawing: a Vygotskian Perspective', in G. Thomas and C. Lange (Eds), *Looking and Drawing: Theoretical Perspectives,* Simon & Schuster, London, pp. 147-160.

Vygotsky, L. S.: 1978, *Mind in Society*, Harvard University Press, Cambridge MA.

Vygotskij, L. S.: 1990, *Pensiero e linguaggio* (Italian critical edition), Laterza, Bari