INTERIORISATION OF *FORMS* OF ARGUMENTATION: A CASE STUDY

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This report deals with the interiorisation of that form of argumentation which consists in the dialogic elaboration on, and transformation of, arguments within a given theoretical framework, up to getting a contradiction. A teaching experiment (carried out in a Grade VIII classroom) concerning the mathematical modelling of the elongation of a spring, and conceived in the perspective of the 'Voices and echoes game', will be analysed in order to better understand the mechanisms of interiorisation and the potential inherent in Galileo's dialogic voice.

The development of argumentative skills is a relevant issue in Mathematics Education. Argumentative skills that are important in mathematical activities include the mastery of sophisticated forms of organisation of the discourse, as for instance hypothetical reasoning, reductio ad absurdum, etc. How can students approach and develop such skills? In an early Piagetian perspective, their roots are embedded in appropriate social peer interactions (see Piaget, 1924, chapter II). In a Vygotskian perspective, the more such skills are far from 'common' forms of reasoning, the more they need the mediation of an adult (or a more competent peer). Imitation in the Zone of Proximal Development is seen as a possible way to perform the necessary mediation. (see Vygotsky, 1978, chapter VI). A reflection about these general hypotheses brought us to consider the potential of the "Voices and echoes game" (see Boero et al, 1997) as a possible effective educational methodology (based on active imitation) to convey some *forms* of organisation of argumentation which are inherent in the scientific debate.

Galileo's dialogues offer examples of high level organisation of scientific discourse. Empirical evidence is not so frequently used (due to reasons inherent in the counter-intuitive character of the hypotheses proposed by Galileo in his dialogues). A typical *form* of Galileo's argumentation consists of the elaboration on, and transformation of, the adverse arguments *within the frame of the adverse theoretical position*, up to getting an "evident" contradiction. Mental experiments, reduction to the limit, etc. are frequently used for this purpose. We can recognise this *form* of organisation of the scientific discourse as an inner dialogue in a number of important personal mathematical activities related to checking mathematical conjectures and scientific hypotheses in order to validate them, for instance in applied mathematics, when a mathematical model is put into question: by transforming an algebraic formula, proposed as a model, it is sometimes possible to draw conclusions, which are in contradiction with some known properties of the modelled situation. In the search for counter-examples of a mathematical conjecture, examples that work for it may be transformed in such a way to hold the coherence with the constraints inherent in the hypotheses and, at the same time, to contradict some aspects of the thesis.

It is out of the scope of this report to discuss the (controversial) historical and personal sources of the *form* of Galileo's argumentation that we are considering. It is more important for us to remark that, according to our observations, this *form* of argumentation is not produced spontaneously by VIII-grade students neither in situations of interaction, nor in individual performances concerning mathematical modelling situations. But students of that age do elaborate on arguments and transform them in everyday life argumentative situations. In mathematical and scientific activities, the main difficulties in approaching the *form* of Galileo's argumentation seem to consist for students (while trying to elaborate on, and transform, the arguments) in *holding the coherence with a given theoretical framework* and also in the *complexity* of the elaboration and transformations needed.

This report will elaborate on the hypothesis that the use of Galileo's dialogues in an active imitation perspective (like the one of the "Voices and echoes game" – see the next Section) can help students to appropriate the *form* of argumentation that we are considering, and use it in mathematical and scientific activities. We will present some essential features of a classroom activity concerning the problem of the elongation of a double length spring in relation to the elongation of one single spring hanging the same weight. Students produced two hypotheses, then discussed them and discovered the right one. Then they were asked to produce, as an 'echo', a Galilean dialogue about the two hypotheses, following the model of the Galileo' dialogue about the falling body phenomenon. The discussion of collected data will lead us to consider the potential inherent in the *virtual* character of Galileo's Dialogues.

THE 'VOICES AND ECHOES GAME' (VEG)

What is the VEG? Some verbal and non-verbal expressions (especially those produced by scientists in the past) represent important steps in the evolution of mathematics and science in a rich and communicative way. We called these expressions *'voices'* (cf. Wertsch, 1991). We called *VEG* an educational situation aimed to make students produce echoes to a voice through specific tasks, for instance: *"How might Aristotle have interpreted the fact that a feather falls down at a slower speed than a stone?"*.

What are the aims of the VEG? Our general *initial* hypothesis was that the VEG might broaden students' cultural horizon, by embracing some elements of the theoretical knowledge that are difficult to construct in a constructivist approach and difficult to mediate through a traditional approach (see Boero & al, 1997). The need to exploit the potentialities that emerged in the first series of teaching experiments led us to try to *find a better characterisation for the elements of the theoretical knowledge* to be mediated through the VEG, in order to better organise and analyse how students interiorise them (see Boero & al, 1998). The research reported in Garuti et al (1999) concerned another potential of the VEG, namely the possibility of developing skills related to detecting conceptual mistakes and overcoming them by general explanation.

THE TEACHING EXPERIMENT

The teaching experiment involved one VIII-grade class with 17 students. The mathematics and science teacher has been the same since grade VI. The didactical contract included (in particular) the production of exhaustive individual verbal reports about personal problem solving strategies, the comparison of solutions, argumentation about hypotheses discussed in the classroom, etc.. Students' individual texts as well as transcripts of classroom discussions were systematically collected. This class had already been involved in two preceding teaching experiments concerning the VEG. In grade VII, the students had been asked to produce an echo to Plato's 'Menon' dialogue concerning the length of the side of a square of double area of a given square. The echo concerned a common conceptual mistake recognised and discussed in the classroom (see Garuti et al, 1999, and the end of the preceding Section). In Grade VIII, they had had to produce echoes to Aristotle's and Galileo's theories about the 'falling bodies' phenomenon, with tasks concerning the *content* of those theories (such as *"How could Aristotle have interpreted the fact that ...").*

The Choice of the 'Double length spring' Problem as a Target Problem for This Study

The 'Double length spring' problem consists in the following task (the weight of the spring is supposed to be very small in relation to the weight hung up on it):

> *"Imagine to know the elongation of a spring under a given weight and to take another spring of the same material, diameter, etc. having a double length. What can you say about its elongation? Why?".*

In previous teaching experiments concerning mathematical modelling, the 'double length spring problem' emerged as a very interesting elementary mathematical modelling problem because of the following reasons (see Boero & Garuti, 1994): it is a challenging problem, even for cultured adults; the "wrong" hypotheses rely on some principles which work very well in other situations; the "valid" hypothesis cannot be easily detected on the basis of immediate everyday life experience; the "wrong" hypotheses can be demolished through an argumentation which exploits accessible arguments in a suitable way. Our a priori analysis established some links with the falling body phenomenon. In the same way, in this case the 'wrong' hypothesis agrees with some principles, which work rather well in many other situations, and the immediate experimental evidence is not in favour of the 'good' hypothesis, but the "wrong" hypothesis can be put into question by an appropriate elaboration of accessible arguments.

Before this teaching experiment concerning the 'double length spring' problem, the students had already performed activities concerning the elementary mathematical model of the elongation of a spring. They had arrived, under the teacher's guidance, to discover that the formula $L= L_0 + KP$ is a good model for the length of a spring of initial length L_0 under the weight P, provided that the coefficient K is well chosen for that specific spring and that P takes values in a suitable interval (not too extended on the right). The activity had an experimental counterpart, allowing students to discover (in particular) that K is smaller if the spring offers a stronger resistance to the elongation. Also the proportionality $L-L_0=KP$ had been discussed.

Afterwards the 'double length spring' problem was posed. The discussion of the "equal elongation" and "double elongation" hypotheses (produced during the individual solution phase) led students (even before the experimental testing) to share the correct hypothesis of the "double elongation". In this process the students brought forward different arguments, in particular: cutting the double length, and imagining what happens under the weight, then adding these effects; or thinking about each coil, and imagining the effect of the weight on it, then the global (additive) effect on the single length spring and on the double length spring. Then some parts of Galileo's Dialogue were read and discussed. Attention was paid to the role of the three interlocutors and to some Salviati's dialogic strategies (in particular, the use and transformation of Simplicio's arguments within the frame of Aristotle's theory in order to get an evident contradiction with his own premises). Finally, the following individual task was given:

> *"Imagine to be Galileo writing a dialogue about the problem of the double length spring. The characters are: Salviati, who represents you, trying to convince Simplicio (and the reader) that the double length spring elongates the double, and to explain why; Simplicio, who supports the hypothesis that the double length spring elongates the same length, because the material is the same and the coils have the same diameter; Sagredo, the moderator".*

The students worked individually for approximately two hours. Their individual texts were analysed by us according to the criteria listed at the end of the next Section.

From Galileo's Dialogues to Some Criteria to Analyse Students' Dialogues

Here I will try to make a summary of a crucial part of Galileo's dialogue concerning the 'falling bodies' phenomenon. This part was read and discussed in the classroom with the help and under the guidance of the teacher:

> Simplicio illustrates Aristotle's theory in general and with an example. Salviati elaborates on Aristotle's theory and puts it into question by using Simplicio's example. Simplicio tries to contrast Salviati's doubt by providing an interpretation of Aristotle's words, while Sagredo takes a position that relies on experimental evidence. But the core of the debate is not the experimental evidence! Salviati wants to provide Simplicio (and the reader) with a theoretical proof within Aristotle's framework. Galileo involves Simplicio in a mental experiment which in three steps leads to a contradiction with Aristotle's theory, by argumenting within Aristotle's framework. Then Simplicio reacts by making reference to intuition, and presenting an example. Galileo answers providing another example (which takes to the extreme the kind of example proposed by Simplicio).

The analysis of this part of Galileo's 'Dialogues' suggested some criteria to analyse students' dialogues and evaluate their quality:

- existence of a real dialogic structure, in the sense that the direct questions Salviati pose to Simplicio play the role of involving him in the argumentation and getting his consensus on crucial arguments and steps of reasoning
- elaboration on, and transformation of, the adverse hypotheses through examples or analogies;

- evidence is explicitly shown for the contradiction;
- Salviati wins through the logical strength of his argumentation;
- concrete experiments deserve only the function of putting into question Aristotle's theory and justifying doubts about it;
- the final "passage to the limit" is used to win Simplicio's last resistance based on physical intuition.

During the discussion about Galileo's dialogue these criteria surfaced (under the teacher's guidance) as main characteristics of its organisation.

Some Excerpts from Students' 'Dialogues'

Sara:

Simplicio: *Two springs made of the same material and with the same coil diameter, but different initial length (one is the double of the other) elongate the same because the initial length is not influent. For instance* {HERE AND AFTER: A SUMMARY OF OMITTED PARTS: the example of a 10 cm spring and a 20 cm spring follows; the L=H+KP formula is evoked to say that K is the same}

Salviati: *I am against your supposition; I say that the initial length is influent and K varies; if your double length spring elongates 2 cm, the other one elongates 1 cm.*

Sagredo: *I am afraid, Simplicio, I made the experiment and saw that the double length spring elongates the double.*

Salviati: *We can prove it.*

Simplicio: *They must elongate the same because the weight is divided according to the number of coils. For instance let us suppose that the weight is 20 grams, if the number of coils of the shortest spring is 10, each of them elongates 20:10=2, while each of the 20 coils of the longest spring elongates 20:20=1. Summing up, we obtain the same elongation.*

Salviati: *You made a mistake, because if you take a double length spring and you divide it into two equal parts, each of them elongates the same under a given weight, so if I join them again, the elongation is double.*

Sagredo: *Salviati is right, because* {Sagredo provides an example, showing that each part of the double length spring supports the same weight}

Salviati: *Dear Simplicio, do you think that the weight hung up to the longer spring is different from the weight hung up to the first and the second **spring?***

Simplicio: *I got it!*

Salviati: *As a conclusion* {he says the valid hypothesis}

Francesco:

Simplicio: {he presents the erroneous hypothesis: general statement first, then an example, like Sara}

Salviati: *According to what you have said, it is as if you performed the experiment, but I have doubts about it*

Sagredo: *I performed the experiment and I can tell you that the double length spring elongates the double*

Salviati: *In accordance with Sagredo who performed the experiment, I can prove that a double length spring elongates the double. Do you share, Simplicio, the idea that all springs with the same length and made of the same material do elongate the same?*

Simplicio: *Certainly* {he provides an explanation for it}

Salviati: *And do you think, Simplicio, that two springs with the same length, same material, same coils when connected together elongate the same as one of them?*

Simplicio: *It seems logical to me*

Salviati: *Let us suppose that what you think is true. Let us consider two springs with the same initial length; let us hung up the same weight on each of them, then let us join the two springs keeping them elongated with the same weight: we get a double elongation, and this conclusion is against your hypothesis.*

Simplicio: *I am confused; it seems to me that if I join the two springs together, I get one spring, then I get the same elongation*

Salviati: *Here is your mistake: it is not true that if you join the two springs the elongation of one of them disappears* {still Simplicio resists; and then Salviati suggests the comparison with two springs in parallel; finally, reacting to Simplicio's scepticism, Salviati brings to the extreme Simplicio's example with a 20 cm spring and a 100 m spring}.

An Overall View of Students' Productions

8 students out of 17 produced a "dialogue" like the one presented by Sara. They show they understood some crucial theoretical arguments in favour of the double elongation hypothesis. They also keep into account the result of the experiment. But their 'dialogues' do not fit the first, third and fourth criterion (and indeed in some cases, like in Sara's dialogue, Sagredo must intervene to explain the right hypothesis). The second criterion is only partially satisfied: the dialogues refer to Simplicio's hypothesis, but Salviati's reasoning follows his own path, and there is no real interaction with Simplicio's arguments. It is as if Salviati (or Salviati and Sagredo together) illustrated his (their) own theory against Simplicio's theory. The students were able to perform a satisfactory 'content' echo to the voices, of the classroom

discussion, supporting the valid hypothesis, but not to Galileo's dialogue; in particular, their dialogues do not contain its typical *form* of argumentation (see Section 1). 9 students out of 17 produced "dialogues" with the same quality as Francesco's (or even superior). They share all the crucial characteristics of Galileo's dialogue. It is interesting to observe that there was no intermediate performance: the dialogues that did not contain Galileo's typical *form* of argumentation were the same ones that do not meet the other main requirements of Galileo's dialogue. In particular, they were either parallel developments of two monologues, or dialogues with no real interactions between the arguments brought in by Salviati and Simplicio.

DISCUSSION

Our research hypothesis was that the use of Galileo's dialogues in an active imitation perspective (like the one of the VEG) could help students appropriate the *form* of Galileo's argumentation, which consists of the elaboration on, and transformation of, the adverse arguments *within the frame of the adverse theoretical position*, up to getting an "evident" contradiction. On the basis of the available data it is not possible to prove that students learned to do it in situations that are very far from the 'falling bodies' example. And it would be even more difficult to prove that they interiorised this method in the perspective of a real individual inquiry situation that requires questioning a personal hypothesis and demolishing it by elaborating and transforming some arguments supporting it. The available data only show that about one half of the students were able to use the *form* of Galileo's argumentation in another similar situation. The analysis of students' individual productions and recordings of classroom discussions in previous problem solving situations shows that this *form* of argumentation was far from the organisation of students' argumentative performances.

In our opinion, more interesting outcomes of the reported study concern the mechanism and the conditions of the interiorisation process in relation to the *virtual* character of Galileo's dialogue. Indeed Galileo's dialogues are not real interpersonal dialogues; they were produced by a scientist who wanted to present his theory and convince the reader through a virtual debate.. As such, Galileo's dialogues are close to the inner dialogue that we, as adults, produce when we get ready for contrasting an interlocutor in a public debate (when we imagine his arguments and think about arguments which would convince him and the other listeners). They represent the inner, individual (or intra-personal) counterpart of interpersonal practices. This feature of the dialogic voices (with the inherent differences between a real debate and a virtual debate) might represent an important opportunity for the interiorisation of high level *forms* of argumentation. This specific issue is related to some ongoing research about the process of interiorisation (from interpersonal construction to intrapersonal development) (Engestrom, 1991; David'ov, 1991). In particular, considering the *"interiorisation process"*, i.e. *"the process of individual activity formation on the basis of collective activity"* Davydov wrote: *"Numerous versions of the theory... notice the fact that the structures of these two forms of activity are to a certain degree similar, but pay very little attention to their difference. But exactly the characteristics of this very difference and dissimilarity form a particular problem of* *activity theory"*. And then, referring to the need for revealing "structure and functions of the specific character of each activity form", he wrote: *"It is vital to give a more exact, certain and comprehensive description of various stages of this process* [interiorisation*] and emphasise the specific importance of the conditions of its realisation"*. A reasonable outcome of the case study reported in this paper is the hypothesis that the active imitation of a virtual dialogic voice through the VEG creates the conditions for the interiorisation of those forms of organisation of the scientific discourse, which are inherent in the voice; this is due to the fact that the voice itself represents an interiorised activity. Another related indication that emerges from the analysis of the teaching experiment is that in future experiments attention should be paid not so much to the scientific debate in the classroom about a given subject as a preliminary, interpersonal construction in the perspective of the subsequent interiorisation, as to the interpersonal reflective practices on the functioning of Galileo's dialogue before the 'echo' task. Those practices might represent a crucial condition for the development of the process of interiorisation. Indeed, the students who failed to construct a dialogue bearing that form of argumentation, which was the goal of the teaching experiment, were the same who were unable to produce a real dialogue. It was as if the lack of control on the dialogic structure of their text prevented them from producing the required argumentative form (cf. Engestroem: *"The new activity structure... requires reflective appropriation of existing culturally advanced models and tools")*.

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