TEACHING AND LEARNING ABOUT MEASUREMENT: RESPONSES TO THE "COUNT ME INTO MEASUREMENT" PROGRAM

Lynne Outhred

Macquarie University, Australia

A program to teach measurement of length, area, volume and mass in grades kindergarten to three is described. This program, "Count Me Into Measurement", was introduced into 38 elementary schools across New South Wales in 2000. The implementation was evaluated and in this paper the teachers' reactions to the program and their assessments of student learning are analysed. Overall, the teacher's reactions were positive; they found the Program easy to use, they particularly liked the expanded lesson notes and they felt the Program improved their teaching of measurement. Teachers' assessments of student learning showed increased knowledge of specific outcomes as a result of using the Program.

INTRODUCTION

Although measurement is a fundamental aspect of everyday mathematics, there is some evidence that measurement concepts are not well understood by students (Bragg & Outhred, 2001; Hart, 1989; Lokan, Ford & Greenwood, 1996; Outhred, 1993). For example, an analysis of elementary school students' knowledge of length measurement found that many students measured from 1 on a ruler or counted marks or spaces; few appeared to perceive the units as linear (Bragg & Outhred, 2001).

Students may not realise that there are major differences between measurement and number concepts, in particular, that measurement involves subdivision of a continuous quantity and is inexact, and that the same quantity can be measured using different units, both informal and formal. For the quantities, length, area and volume, spatial organization of these units is crucial, while for quantities, such as mass, time and temperature, spatial organization is not important.

However, students may consider that measurement is essentially a process of counting or calculating units, and may not realise how the actions, of exhaustively aligning, covering or packing lengths, areas and volumes respectively, are related to subdivision of quantity. The spatial structure of the iteration of identical units for these quantities does not seem to be emphasised in textbooks and curriculum documents, yet it would seem to be central to understanding and linking these two processes of unit enumeration and subdivision of quantity.

Emphasising the spatial structure of the unit iteration may assist students to link concrete, pictorial and symbolic representations of measurement concepts. For example, moving one length unit and marking the end of each move (without gaps or overlap) creates a linear scale analogous to that of a ruler. Although many students can use rulers, few of them understand how the scale is constructed (Bragg & Outhred, 2000) and many of them do not seem to understand linear measurement in such a way that they could generalize the procedures to practical problems. One possible explanation for students' reliance on procedures might be that teachers rely largely on worksheets and textbook exercises that focus on techniques of measuring. Thus, abstract concepts, such as knowledge of linear scales and the relationships between quantities and also measurement units are not developed.

Studies by Outhred and Mitchelmore (2000) and Battista, Clements Arnoff, Battista, and Borrow (1998) have found that the structure of a unit covering of squares is not well understood by students. Their findings suggest that students will not be able to subdivide a region into equal parts if they cannot visualize the unit structure for area—a rectangular array (tessellation) of squares, partitioned into rows and columns with equal numbers of units in each row and in each column. However, teachers may not emphasise the tessellation structure because they conceive area measurement to be a process of covering and counting, rather than of subdividing of a region (Outhred & McPhail, 2000). In their descriptions of teaching area measurement, the teachers did not mention any structural features of a covering (e.g., no gaps or overlaps, congruent units, rows and columns). An emphasis on counting suggests that area is a set of discrete points rather than a region and may hinder students' development of abstract representations of area units, in particular, rows and columns as composite units.

Structural knowledge of unit packings of rectangular containers has also found to be difficult for students, especially linking the structure of packed cubic units to the dimensions of the container (Battista, 1999; Hart, 1989). Early experiences of volume are often limited to filling containers with water (capacity). Such an approach avoids completely the structural problems of packing. The two situations appear to be very different, and students may not see that both involve measurement of volume. As with area, volume measurement seems to involve the construction of composite units (in this case, layers or sections), but the process is more complex because students have to coordinate three dimensions and diagrams cannot show the layer structure clearly. Campbell, Watson, and Collis (1990) found that elementary students counted the number of individual cubes in pictured regular rectangular prisms and that many students counted only the visible cubes.

The Count Me Into Measurement Program

Many teachers may not be familiar with or confident of fundamental measurement processes and how these can be designed as sequences of instructional activities. To assist them, the New South Wales Department of Education and Training (NSW DET) has commissioned the "Count Me Into Measurement" (CMIM) project (Outhred and McPhail, 2000) to develop materials to support the teaching of measurement concepts in the elementary school. The CMIM materials are based on the assumption that students' understanding of the spatial organization of the units develop from practical activities involving estimation and measurement, followed by discussion or recording of the measurement process. The materials provide a conceptual framework to elaborate significant measurement concepts and processes and to provide a sequence of similar conceptual levels for length, area, and volume. The first three levels are:

- **Level 1:** Identification of the attribute includes directly comparing and ordering quantities.
- **Level 2:** Informal measurement includes finding the number of units to cover, pack, or fill a given quantity without overlapping or leaving gaps; knowing that the number of units used gives a measurement of quantity; using these measurements to compare quantities and realizing that a quantity is unchanged if it is rearranged (the principle of conservation)
- **Level 3:** Unit structure includes replicating a single unit to cover, pack, or fill a given quantity, either by drawing or visualizing the unit structure; and realizing that the larger the unit, the fewer units will be needed.

While the first two levels are common to other suggested measurement teaching sequences, the third level does not seem to have been included previously. These levels provide a conceptual sequence for teaching the topics, length, area and volume—students are expected to progress through these levels for each topic but they are not expected to be at the same level in each topic. A sequence for mass that partially fits the framework has also been devised. Formal units are introduced in Level 3 and the Program is currently being extended to include additional levels for Grades 4 to 6. Accompanying the framework are suggested activities that are sufficiently open-ended to promote different approaches to measuring. Teachers can exploit this diversity of approaches to focus student attention on fundamental measurement principles and processes. Example lesson plans for some activities are also provided to scaffold teachers' knowledge of key questions they might ask to develop students' knowledge of measurement processes.

This paper presents the results of an evaluation of the introduction of the first three levels of the CMIM Program in the second half of the year 2000.

METHODOLOGY

Each of the 40 districts in NSW was invited to participate in the initial trial of CMIM and 38 agreed to take part. In each district, the mathematics consultant nominated one teacher who would act as a facilitator for their school to implement the CMIM Program in Kindergarten to Grade 3. These facilitators all attended one training day in Sydney. Facilitators were provided with eleven release days to train and assist a minimum of three teachers from their schools to trial the Program with their students. In addition to teaching ten lessons across two of the four topics (length, area, volume and mass), each teacher was asked to complete an open-ended questionnaire about their responses to the CMIM materials and to assess five students before and after teaching each topic. Facilitators also completed the questionnaire, which sought feedback about:

- The CMIM document itself.
- The facilitators' and teachers' approach to teaching measurement.

The final sample comprised 36 facilitators and 118 teachers, 154 in all. The teachers who returned completed questionnaires included 33 Kindergarten or Year K/1, 44 Year 1 or Year 1/2, 30 Year 2 or Year 2/3, 11 Year 3 or Year 3/4.

The questionnaire responses were scanned for commonly occurring themes, issues and concerns. These responses were formulated into categories and a coding system was devised for each question; these categories were then coded. A response to a question might encompass one or more categories. The written responses were also analysed to identify any insights, issues or suggestions.

The student assessments were complex to analyse and only assessments for which common outcomes had been assessed at both the beginning and end of the implementation were included. Thus, the numbers of students varied for each topic and level. Although, the data for all the classes were combined for a topic, the results for Level 1 mainly included Kindergarten and Grade 1 students while Level 3 mainly comprised students from Grades 2 and 3. Fewer students were assessed on Level 3 than on the earlier levels.

RESULTS AND DISCUSSION

The responses of both facilitators and teachers have been combined in reporting the results in this paper and for simplicity both are referred to as teachers. In general, the facilitators' responses were more positive and detailed than those of the teachers, perhaps because the facilitators were more familiar with the document and had a broader perspective as a result of their professional development role.

The CMIM program

The responses clearly indicated that both facilitators and teachers were satisfied with the CMIM document. The majority of the respondents (57%) commented positively on the organisation (clearly set out, well-organised, sequential) of the document and many (42%) on its ease of use (easy to follow, user friendly). However, simply providing the lesson ideas and the knowledge and strategies that teachers should look for did not seem to be sufficient. Teachers valued the expanded lesson notes that gave examples of questions that teachers might ask—about a quarter of both facilitators and teachers commented on the helpfulness of these: "Within the expanded lesson notes, the section entitled "Questioning, Comments and Discussion" is important as it is through this and teacher knowledge that the children can potentially think on a deeper level and in addition become more aware of, and able to use and understand specific mathematical terminology" (School 17).

The expanded lesson notes also emphasised the mathematical language associated with measurement and some teachers had not realised that their students did not know this, "The language or lack of it was an eye opener and emphasised the necessity of doing these lessons" (School 10). Since many measurement terms (e.g., length, area, row, column) are commonly used in everyday life, teachers may assume that students understand the mathematical meaning of the words. Often misunderstandings of the terms only become apparent when teachers ask students to explain their reasoning.

Requests for more expanded lesson notes and lesson ideas were common responses from teachers, presumably because of time constraints—teachers expressed a need for a "pick-up and teach" document (School 5)—but also because the expanded lesson notes were seen to be a "good model" (School 38) and provided guidance about the mathematical concepts underpinning the activities. About a fifth of the teachers mentioned that using the document had encouraged them to reflect on how to teach measurement or had clarified their own concepts of measurement: "Yes, it gave measurement a sequence. Something that I didn't understand." (School 38); "Yes, taught concepts I'm not confident in teaching. Made me use the correct vocabulary" (School 20).

Even teachers who are confident about teaching measurement may have limited knowledge of basic measurement processes. In the small-scale study mentioned earlier (Outhred & McPhail, 2000) in which teachers described their teaching of area measurement, even when teachers were specifically asked about skills and understandings, they did not mention important area concepts. Most teachers construed area to be a covering. Few used the term "surface" and only one mentioned that the same units should be used for comparison and that an appropriate sized unit should be considered.

The main change that teachers recommended to improve the CMIM document was to clarify the student assessment. Teachers suggested the inclusion of either proformas or work samples to provide explicit assessment guidelines. They thought that annotated work samples would help to clarify differences between the levels, as well emphasising what they should focus on in their assessments.

Approaches to teaching measurement

Almost all respondents (96%) agreed that the document had helped them to teach measurement. The majority of responses reinforced comments that had been made about the strengths of the document—the teachers approved of structured, sequenced material that provided information about what they were expected to teach and that students were expected to learn. For example, "The clear progression of activities, the activities themselves, the framework seems so logical although I'd never really thought out those divisions of learning." (School 34); "Yes, it made me realise that I take some things for granted. For example, I may have assumed that a particular child knew something that they didn't yet, or I may have underestimated some children's abilities and been surprised with their response or interpretation" (School 23).

Student engagement would seem to be fundamental to learning and many teachers (21%) mentioned student interest in the activities, often associated with the use of concrete materials. About a third of all respondents referred positively to the "hands-on" activities—although some teachers said they taught measurement using such an approach, others did not. The responses of students to the CMIM activities appeared to prompt several teachers to reconsider their methods of teaching while others mentioned that it increased their confidence in teaching measurement: “It made the class teacher reflect on the present style of teaching concepts and gave them practical examples of how to achieve new concepts in a motivational way.” (School 27); “Yes, it gave us a different view of teaching measurement, more learning by doing with concrete materials and students were accountable for learning records.” (School 21)

A number of teachers (14%) commented that they liked the Program’s emphasis on students discussing, drawing and writing explanations of their mathematical thinking. Such recording appeared new to many teachers and they seemed to find this feature of the document informative: "I especially liked the idea of students recording what they had learnt so (1) they thought about it and (2) I was aware of their strengths and needs" (School 21). One teacher’s assessment of student writing made her aware that "the lower ability children were always focussing on irrelevant factors when writing about their experience. Eg It was fun, I used pink counters, Pink is my favourite colour" (School 15). Teachers who are aware of the problems of such students would seem to be in a good position to offer assistance.

The main concerns raised by teachers involved resource and time issues because the activities were designed as small group ones in which the students undertook practical tasks using a variety of equipment. As one teacher commented "practical groupings could be difficult without assistance if I had to teach a larger group as there is a lot of equipment involved and many children need guidance" (School 3). Others were concerned about the preparation of resources; “My only concern would be the time factor. Whilst these lessons were great, in reality if a teacher had to prepare all the resources before teaching especially volume/mass strands I don't think many would or could afford the time” (School 1).

The time and resource issues are important ones to address because many teachers will not continue to incorporate practical measurement activities into their classrooms if they find the management and organization of groups and materials too demanding.

Student learning

The results indicated a large change in student achievement over the period of the project. The only length outcome that more than 50% of students had mastered at the beginning of the study was to directly compare and order lengths (Level 1); only about 10% could replicate a given unit to measure a line (Level 3). By the end of the project more than 80% of students appeared to have mastered the first two levels, that is, they could measure, compare and order lengths using informal units, as well as by direct comparison. The results for Level 3 had also increased dramatically—about 50% of the students assessed at Level 3 were considered to have mastered the outcomes for this level.

The results for area indicated that area concepts were much less well known than those for length. Only about 20% of the students assessed could directly compare and order areas, as well as systematically cover areas with identical units and compare them. The results for Level 3 were similar to those for length; few students appeared to have any knowledge of the structure of a rectangular tessellation at the beginning of the project. After the CMIM lessons the assessments show increased performance on all levels with teachers reporting that about half the students perceived the structure of a unit tessellation in terms of rows and columns. The results for volume were consistent with length and area—teacher reports of students' learning increased markedly.

However, these results would have to interpreted with caution, as the teachers were likely to be biased towards finding an improvement in student learning because they had spent much time and effort in implementing the program. Nevertheless, the data on student learning provided some evidence that the sequencing of the levels and activities was sound.

CONCLUSION

Overall, the first implementation of the CMIM Program appeared to have been successful. Teachers approved of the Program and felt that it assisted their teaching of measurement. They liked the open-ended, practical nature of the activities although there was evidence that they may not continue such teaching without support because of the time and effort that was required in terms of resources, time and organization.

A number of teachers reported that they did not feel confident about the measurement concepts they were expected to teach, nor the sequence in which these concepts should be taught. Many teachers appeared to rely on commonly published textbooks for teaching measurement. In effect, publishers are interpreting the mathematics curriculum for teachers. In textbooks, concepts are often fragmented, for example, a worksheet on length may be followed by ones on area, mass, and volume, interspersed with pages presenting number and space concepts. Several teachers commented that they found that teaching a sequence of five lessons on one topic as part the CMIM Program consolidated students' learning.

Teachers may not realise the extent to which the complexity of measurement increases with the dimensionality of the units. Anecdotal comments on the training day indicated that some teachers considered volume concepts to be easier than length or area ones, suggesting that these teachers may have focussed on filling containers with liquid (capacity) rather than volume as the packing of an interior space, or as displacement. There was also some evidence from work samples that were sent back with the questionnaires that a few teachers may not have realised the importance of constructing the unit iteration to reinforce the structural properties of measurement to students.

Building connections between a linear scale, two-dimensional array structure, and three-dimensional packing are important if students are to understand relationships among measurement concepts. These concepts are also closely linked to repeated addition and array multiplication. The difficulty of applying multiplication skills in a meaningful way has been documented for concepts of area and volume (Hart, 1989).

She found that many students continue until well into secondary school to determine both area and volume measures by counting, rather than by multiplying. Counting does not generalise to fractional dimensions or to formulae. Students require instructional tasks that assist them to construct links between the measurement unit and the spatial structure of the unit iteration, as well as to repeated addition and multiplication. Teachers' knowledge of measurement processes may also need to be developed so that they realise the fundamental differences between the enumeration of discrete and continuous quantities.

REFERENCES

Battista, M. (1999) Fifth Graders' enumeration of cubes in 3D arrays: Conceptual progress in an inquiry-based classroom. *Journal for Research in Mathematics Education, Vol 30*, No. 4, 417-448.

Battista, M. T., & Clements, D. H. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. *Journal for Research in Mathematical Education, 27*, 258 - 292.

Battista, M. T., Clements, D. H., Arnoff, J., Battista, K., & Borrow, C. V. A. (1998). Students' spatial structuring of 2D arrays of squares. *Journal for Research in Mathematics Education, 29*, 503-532.

Bragg, P. & Outhred, L. (2000). Student's knowledge of length units: Do they know more than rules about rulers? In T. Nakahara & M. Koyama (Eds.) *Proceedings of the 24th. Conference of the International Group for the Psychology of Mathematics Education. Vol. 2*, 97-104. Hiroshima University.

Campbell, Jennifer, Jane Watson, & Kevin Collis. (1990). Volume Measurement and Intellectual Development. *Paper presented at the 13th Annual Conference of the Mathematics Education Research Group of Australasia Conference*, Hobart, Australia.

Hart, K. (1989). Volume of a Cuboid. In Hart, K., Johnson, D., Brown, M., Dickson, L., & Clarkson, R (Eds.) *Children's Mathematical Frameworks 8-13: A Study of Classroom Teaching*, pp. 126-150. London: NFER-Nelson.

Lokan, J., Ford, P. and Greenwood, L. (1996). *Mathematics and Science on the Line: Australian Junior Secondary Students' Performance—Third International Mathematics and Science Study (TIMSS).* Melbourne, Australia: Australian Council for Educational Research.

Outhred, L. & D. McPhail (2000). A Framework for Teaching Early Measurement. In J. Bana & A. Chapman (Eds.) *Mathematics Education Beyond 2000.* Fremantle, WA, MERGA Program Committee.

Outhred, L. and Mitchelmore, M. (2000). Young Children's Intuitive Understanding of Area Measurement. *Journal for Research in Mathematical Education 31* (2000): 144-167.

Outhred, L. (1993). *The Development in Young Children of Concepts of Rectangular Area Measurement.* Ph.D. diss., Macquarie University, Australia.