

# STUDENTS' CLASSROOM NOTEBOOKS AND THE BLURRING OF PUBLIC AND PRIVATE DOMAINS

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*Students' notebooks are frequently the source of data concerning students' mathematical thinking and practice. But the part the notebook itself plays in the mathematics classroom and the students' mathematical life has rarely been the subject of research in mathematics education. This paper presents some observations showing how notebooks tend to be of an exclusively public nature, and it explores theoretical implications of this tendency. A brief description is also given of the Learners Perspective Study in which these observations were made. That study is an ongoing international project concerned with understanding school mathematics practice from the students' point of view.*

## INTRODUCTION

Among the accoutrements of mathematics learners, hardly any could be more universal than notebooks. Yet, while student notebooks are a never neglected source of data about students' mathematical work and thinking, the place of the notebook itself in students' school mathematical life seems to have been little studied. In considering the notebook and its significance for the student, one question that arises is to what extent the notebook is a private or public object. In this paper, we shall explore this question and present some observations made in an eighth grade algebra class. These observations indicate that, in the classroom, the mathematics notebook takes on an utterly public character, a character unchallenged by the student and unwittingly encouraged by the teacher. We believe that this has practical and theoretical implications which touch on such issues as the importance of communication in the mathematics classroom (as emphasized in NCTM, 2000), of reflection, and of writing in the development of mathematical thinking.

We shall proceed in a somewhat unorthodox fashion, beginning with the research setting in which we made our observations about notebooks, namely, the Learners' Perspective Study, going on from there to the observations themselves, and only ending with the theoretical background and implications. In this way, we believe, the paper will better reflect the process by which we came to ask our questions, make our observations, and draw our conclusions concerning student notebooks.

## THE SETTING: THE LEARNERS' PERSPECTIVE PROJECT

Our observations concerning students' notebooks in the mathematics classroom were made in the course of a more extensive, and still ongoing, study of the students' point of view, called the *Learners' Perspective Study* (Clarke, 1998, 2000). This project, which involves eight countries including Australia, Germany, U.S.A, Hong Kong,

Japan, Israel, Sweden, and South Africa, seeks to explore a number of questions concerning the way students conceive mathematics classroom practice and mathematics learning. The project arose out of the Third International Mathematics and Science Study (TIMSS). As is well known, the TIMSS study not only established national profiles of student achievement, but also sought to identify national norms for teaching practice that might account for “poor” or “high” achievement scores by videotaping and analyzing a statistically representative sample of eighth-grade mathematics classes in Japan, Germany and the USA (Fernandez, C. et al., 1997). Although this component of the TIMSS study was impressive and unprecedented in international comparative studies, the validity of the “effective scripts” discerned in the TIMSS videos was widely debated and not universally accepted (e.g. Keitel & Kilpatrick, 1999; Stigler & Hiebert, 1997, 1998, 1999). Among the major objections was that the TIMSS Video Study focused exclusively on the teacher and ignored the important role students have in the learning process. The present project, accordingly, expands on the work done in the TIMSS study by focusing on *student actions* within the context of whole-class mathematics practice and by adopting a methodology whereby student reconstructions and reflections are considered in a substantial number of videotaped mathematics lessons.

The particular case on which we focus in this paper was a sequence of 15 lessons on systems of linear equations taught by a dedicated and experienced teacher, whom we shall call Danit. Danit’s 8<sup>th</sup> grade class is heterogeneous regarding level; it consists of 38 students, mostly native born Israelis, but also new immigrants from the former Soviet Union and one new immigrant from Ethiopia. As specified in Clark (2000), classroom sessions were videotaped using an integrated system of three video cameras, one viewing the class as a whole, one on the teacher, and one on a “focus group” of two or three students. Following each lesson, the students in the focus group were interviewed, and their notebooks, containing the notes for that particular lesson, were photocopied. Moreover, once a week Danit herself was interviewed. Although we had a basic set of questions for both the student interviews and the teacher interview, we allowed the interview protocol to remain flexible so that we could freely pursue particular classroom events; in this respect, our interview methodology was along the lines of Ginsburg (1997). An important aspect of the interviews was that in the course of them the students could view and react to the videotape of the lesson for which they were the focus group. Needless to say, the interviews themselves were also videotaped.

### **DANIT’S LESSONS AND NOTEBOOKS IN DANIT’S CLASSROOM**

The sequence of lessons viewed in Danit’s classroom began with the idea of an equation in two variables, went on to the graphical representation of linear equations, to the graphical solution of a system of two linear equations, and, finally, to the algebraic solution of such systems. The lessons followed a fairly consistent pattern: a frontal lesson for roughly 10 minutes, then independent work on exercises by the

students for the remaining 35 minutes of the class period. On days when there were two consecutive lessons, the work period was often extended through the second class period. The students worked in groups of two or three while Danit went from group to group, checking the students' work, asking questions, and helping with difficulties.

In each meeting, we noted that all of the students opened their notebooks only after Danit finished the frontal lesson; at the beginning of the class period, when she presented new material, gave examples, went over exercises from the previous lesson, the students' notebooks lay closed on their desks (there was one meeting that deviated from this pattern; it is the first incident recounted below). We found this curious: did the students not find Danit's examples and lessons worth recording? Did Danit not think that the students ought to write down her remarks and examples so that they would have them to think about and to refer to later? In several of the interviews, we asked the students about this. They replied that Danit allows them to choose whether they use their notebooks or not. In fact, however, the use of notebooks among the students bore no individual stamp; it was determined by Danit, as the two following incidents show. The first incident occurred when Danit began her lessons on the graphical representation of equations. In that lesson, she not only told the students to take out their notebooks and copy the coordinate system, but she also told them what heading to write, and she also told them to use a ruler in setting out the axes of the coordinate system. Afterwards, in the lessons involving graphs, the students produced coordinate systems just as they had done in this particular lesson. The second incident occurred in the meeting following the one above. In that meeting, Danit began immediately with exercises and opened the lesson by saying "Take out your notebooks—I want to see that you're doing the work correctly." The students showed no expression of surprise; this was not an odd request.

Both these incidents, which, we remark, are not at all unusual in Israeli schools, show Danit's control over what the students write in their notebooks and when. The second, however, was striking because it was in such blatant contradiction with the students' claim that Danit lets them choose whether or not they write in their notebooks. Indeed, if they know that Danit will eventually inspect their work, surely they must weigh how Danit might react if nothing were written, or if what was written was written badly, or wrongly. Far from being a matter of choice, then, three basic rules of notebook use seem to be followed in Danit's class:

- 1) During frontal lessons, students only listen; their notebooks are closed.
- 2) When given exercises, notebooks are to be opened immediately, and all the exercises are to be recorded neatly (the students should use rulers, for example) in them.
- 3) The teacher will occasionally look at the notebooks, so these rules must be followed.

The knowledge that the students' notebooks are to be open for inspection, in particular, means that they are a public matter—they are not to be a record of the students' private thoughts about what they are learning: desultory reflections; false starts; mistaken conclusions and their, perhaps embarrassing, corrections. In fact, to their chagrin, Israeli teachers commonly find their students carrying out preliminary work for exercises, calculations for example, not in their notebooks, but on their desks! What students seem to learn is that the notebook must contain finished work that can be confidently held open to view (see the example reproduced in the appendix). The notebook is, in this respect, a sort of rehearsal for that part of school life in which the students' work is inspected most closely, the examination. Thus, students emphasize the use of notebooks to record the sort of exercises that will appear on examinations rather than the explanations behind them:

Interviewer [referring to examples Danit had written on the blackboard]: Did you write these exercises in your notebook?

Moshe/Sharon: Yes.

Interviewer: Why did you write them?

Sharon: So we won't be confused or something...for example, I try to solve...

Moshe: She also told us to write in our notebooks.

Interviewer: And if she didn't tell you?

Moshe: We write.

Interviewer: Write...

Moshe [without letting the interviewer finish her remark]: This is all material for the exam, so it can help.

Interviewer: Do you also write Danit's explanations?

Moshe: You mean her words of explanation? No...I don't anyway.

These students, and others in the class, say that Danit explains her lessons well, but these explanations are not written in the notebooks, only what the teacher might see on the students' examinations sheets. In this connection, it is worth noting that one of the only signs of individuality we could discern in Danit's students' algebra notebooks was the decoration that often adorned the headings, decoration that one often finds adorning their exam sheets as well; but are not such decorations also an expression of the notebooks' public nature, as something to be seen by others?

The public character of the students' notebooks is, interestingly enough, mirrored in the character of Danit's own lesson book. During our first interview with her, we asked if we could see her own notebook. She said that she does not have a detailed lesson book for her 8<sup>th</sup> grade class since, having taught that grade often, she no longer needs a lesson book, however, she let us see her lesson book for her 9<sup>th</sup> grade class. The lessons on geometry consisted of pages of solved problems and those on algebra,

pages of numbered exercises. We could see no stated goal, no exposition of the material, no distinctions between major and minor examples, and no lesson structure. She also told us that she brings her lesson book to class and allows, even encourages, the students to compare their notes with hers (as we later confirmed in the lessons following this interview); this way, she said, the students can check and see if they have missed anything. We asked her if she hopes their notebooks will look like hers. She said yes. Thus, there is a notable consistency here between teacher and student practice, namely, that 1) a notebook is the place to record exercises only and 2) a notebook may be inspected at any time, that a notebook is always public.

## **PUBLIC AND PRIVATE FUNCTIONS OF NOTEBOOKS AND WRITING**

To gain some insight on the significance of the observation concerning the public nature of notebooks in Danit's classroom, we need to remember that questions about the importance and function of notebooks in students' mathematical life are closely, if not intimately, related to questions about writing. Considerations of writing, on the other hand, suggest that notebooks, in principle, have a place in *both* private and public domains, and not, as we observed in the use of notebooks in Danit's classroom, in the latter domain alone. In fact, it can be said that writing mediates between the two domains. Take its most perspicuous public function, that of being a form of communication.

Though one is tempted to say that, as a means of communication, one writes only to be read, it is clear that the act of writing is not completely separate from the thinking that goes on before it. That this is true of creative mathematicians was long ago recorded by Hadamard in his classic work on mathematical invention (Hadamard, 1949, chap. V). But it is no less true for young students of mathematics. Thus, in the communication standard for grades 6 to 8 of the NCTM Principles and Standards, it is stated that "To help students reflect on their learning, teachers can ask them to write commentaries on what they learned in lesson or a series of lessons and on what remains unclear to them" (NCTM, 2000, p.271). Carpenter and Lehrer (1999) are even more explicit in connecting internal mental activity and mathematical understanding to external expression. They write:

Articulation involves the communication of one's knowledge, either verbally, in writing or through some other means like pictures, diagrams, or models. Articulation requires reflection in that it involves lifting out the critical ideas of an activity so that the essence of the activity can be communicated. In the process, the activity becomes an object of thought. In other words, in order to articulate our ideas, we must reflect on them in order to identify and describe critical elements. Articulation requires reflection, and, in fact, articulation can be thought of as a public form of reflection" (Carpenter & Lehrer, 1999, p.22).

The argument has been extended by some relatively new research by Lillie R. Albert (Albert, 2000). Albert highlights the mediating function of writing by seeing it as a bridge between Vygotsky's socially dependent zone of proximate development (ZPD) and a zone of learning in which the student is independent and self-regulating, a zone of learning she calls the zone of proximate practice (ZPP). The question Albert addresses is, of course, the one Vygotsky himself saw as "an important concern of psychological research," namely, "to show how external knowledge and abilities in children become internalized" (Vygotsky, 1978, p.91). For Albert, the ZPP "is the result of the students' transformation from the interpsychological to the intrapsychological plane of functioning," and adds that "A basic assumption that underpins the ZPP construct is that the writing students do, as shaped by the collective practice in which they engage, determines how they independently think about mathematical ideas or concepts" (Albert, 2000, p.117).

Whether it is understood as an activity of clarification in the course of articulation or of reflection and internalization following social interaction and collaborative work, writing is seen to come between communication, which is fundamentally public, and reflection, which belongs to a private domain. To the extent that it is connected to communication, surely, writing must be clear and organized, and to the extent that it is connected to reflection, it must be free and exploratory. As an illustration of the latter, consider this description by Thomas Mann concerning his work habits:

For a longer book I usual have a heap of preliminary papers close at hand during the writing; scribbled notes, memory props, in part purely objective—external details, colorful odds and ends—or else psychological formulations, fragmentary inspirations, which I use in their proper place (quoted in John-Steiner, 1985, p.76).

Since students' notebooks are their *own* notebooks, their own possession, it would seem that they are the proper loci for writing serving reflection, that is, writing in which thoughts are worked out and developed, and not only writing in which expected and taught solutions to given problems are recorded. In any case, since the notebook is the chief place in which students do their writing, the research by Albert and others alluded to above suggests strongly that notebooks lose an important function if they do not contain such writing serving reflection.

## CONCLUSION

In Danit's classroom, student notebooks are exclusively public: they contain finished work only and may be inspected at any time. They contain no exploratory work, no false starts, alternative strategies, or random reflections on material being taught to them. Since their notebooks contain no writing of this kind, the ability for writing to mediate between the students' learning on a interpersonal plane and on a intrapersonal plane becomes seriously limited.

We would like to conjecture that a lack of a private domain in learning—and, surely, the notebook is the most natural place for such a domain—influences students' very ability to grasp that mathematical ideas are the sort of thing requiring reflection. Our reason for making this conjecture is that in the course of our interviews with students, we found that they had great difficulty understanding and appreciating the connection between “doing mathematics in your head” and writing mathematics down; on the one hand, “doing mathematics in your head” lacked legitimacy, and, on the other hand, writing mathematics down is done only to show the teacher that “you understand the material, that you didn't just guess.” What we are conjecturing is if students' notebooks were allowed to be a private domain for the students, a place for written reflection, they might have the opportunity to see how writing mathematics down and “doing it in your head” can be complementary and mutually enriching.

If this conjecture is true, it would support the practical suggestion, such as that in the NCTM Principles and Standards (NCTM 2000), that students keep a mathematical journal together with an exercise notebook. We would stress, however, that in order for the benefits of a mathematical journal to be fully realized it ought to remain strictly within the student's private domain; their journal must be the place for truly *uninhibited* reflection. For this reason, we differ with the NCTM's recommendation that the journal play a part in assessment. But, be that as it may, we believe that, in general, our observations about notebooks point to a need for educators to recognize where the lines are drawn in mathematics classrooms between private and public domains, and to recognize that finding a balance between these domains may have serious implications for the students ability to reflect on mathematical ideas.

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