

Levels of understanding of letters in the word problem solving processes

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In (Arcavi, 1994) the deep analysis of manipulations with symbols is being discussed: “Even those students who manage to handle algebraic techniques successfully, often fail to see algebra as a tool for understanding, expressing and communicating generalisations, for revealing structure, and”

In this contribution, we are comparing the levels of dealing with letters during the period of grasping the meaning of the assignment (encoding phase), and the period of mathematisation of the solving process (transformation and calculation phase). The use of letters in students’ written records are analysed and classified in two concrete situations.

Situation 1 – The assignment does not contain algebraic elements (Novotná, 1997): The following four stages of the transition from an arithmetical to an algebraic way of using letters in the written record of assigned information were identified:

- 1a) Solvers use one letter for labelling several values, the letter is a symbol of a general unknown for them.
- 1b) Solvers use one or more letters in the encoding stage without working with them in the transformation stage, the unknown is only used as a label for something that is to be found.
- 1c) Solvers consciously use letters for labelling required values and for describing assigned relationships, arithmetical models are more important and thus the arithmetical solution is used.
- 1d) Solvers use letters for labelling the values and algebraic operations are carried out and solved. The conditions for the successful use of algebraic methods have already been created.

Situation 2 – The assignment contains algebraic elements (Novotná – Kubínová, 2001): The following four stages of dealing with the assignment were identified:

- 2a) Solvers ignore data which are not assigned as concrete numbers, their ability to work with algebraic representations is not developed.
- 2b) Solvers use letters only as labels for something that is to be found by calculations, the value assigned by a letter is handled as an unknown.
- 2c) Solvers are aware of the nature of data assigned as letters; by substituting a concrete number a letter, they change the problem into a pure arithmetical one. The symbolic algebraic description of the situation is not yet fixed in their knowledge structure.
- 2d) Solvers are able to work successfully with data assigned in both arithmetical and algebraic languages.

The correspondence between the two situations will be documented by concrete examples of students’ solutions.

References

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