THE ROLE OF INSCRIPTIONAL PRACTICES IN THE DEVELOPMENT OF MATHEMATICAL IDEAS IN A FIFTH GRADE CLASSROOM

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The purpose of this report is to examine the role of classroom practices that emerged within a fifth grade classroom teaching experiment that helped coordinate and support student understanding of the Quotient construct through the invention, modification, and interpretation of inscriptions. Two types of inscriptions supported classroom discourse about quotients: Introduced inscriptions (those drawings and written marks offered to the class by the teacher or text materials as culturally appropriate forms of representing and communicating); and Ad-hoc inscriptions (those marks developed *in situ* by the students to convey their thinking). Through the use of public depictions of thinking, the classroom coordinated its practices by modifying Introduced forms to fit immediate demands of problem solving, and adopted the ad-hoc contributions of students to form a collective inscription retaining surface features and underlying meanings of both.

Theoretical Framework

The acts of doing mathematics and making sense of mathematical ideas takes place within the microecology (Lemke, 1997) of the classroom. They are reflected in the products created and used within the classroom microculture (Cobb et, al. 1997).

Most research in the domain of rational numbers has been conducted from an individual psychological perspective, whereas most classroom instruction is done in a social system. The analysis conducted in this paper attempts to account for the developments that occurred in a fifth grade classroom during mathematics instruction over a five week period. Our reasons for conducting the analysis was motivated by understanding how students co-constructed a unified understanding of the quotient construct and also the mediational means through which knowledge was constructed within the microculture of the classroom. Therefore, the practices and the products that were created and used as representations of mathematical ideas within the context of the classroom were examined. The emergent perspective takes into account that mathematical learning is a process of active social construction and a process of enculturation (Cobb et al, 1997).

A representation is an idea, experience, or object symbolized in another form. Because mental operations take place inside the mind, it is difficult to see or understand another individual's thoughts without some sort of mediational process (Davis, Hunting, & Pearn, 1993). Therefore, individuals need to explicate their thinking using external forms of representations such as through *inscriptions* (e.g., concrete objects, pictures, and symbols (Roth & McGinn, 1998). These external signifiers (Whitson, 1997) serve as cultural models through which communities of individuals can coordinate thinking (Ball, 1993; Brown, Collins & Duguid, 1989, Greeno, 1991, Kaput 1994).

Representations not only serve as tools for thinking but also for communicating thinking. When a base meaning has been negotiated for a particular inscription, then this taken as shared meaning serves as a "boundary object" to coordinate activities and divergent view points of several individuals, thereby facilitating the flow of resources (such as information, practices and materials) among the variety of individuals in a social setting (Roth & McGinn, 1998).

This means that as students think about a particular concept, they use various forms of inscriptions to record pertinent aspects of their thinking, and then use the set of inscriptions themselves as proxies for operating on the concept. Much of modern mathematics is dependent upon the systematization of inscriptions. In particular, when solving algebraic equations it is necessary to manipulate inscriptions and then retranslate the inscriptions back into conceptual forms is necessary (Kaput, 1999). New ideas are generated when new patterns of inscriptions signify different features of a concept than previously encountered (Ball, 1993).

Method

A five-week whole class teaching experiment was conducted in a fifth grade classroom in an urban school district in the Southwestern United States. Nineteen students and the classroom teacher participated in the study. An instructional unit based on Toluk's (1999) model of students' development of Quotient understanding was developed by the researchers and utilized as part of the instructional sequences. An Anchored Instruction (CTGV reference here) video was also developed as part of the instructional unit. The teaching experiment consisted of the following phases: classroom observation, clinical interview, teaching, analysis and hypothesis testing.

During the *classroom observation* process, whole group development and individual development were examined. The purpose of the classroom observation phase was to establish the physical, social, and cultural context of the classroom as a contextual referent within which lesson activities and thinking took place. The *clinical interviews* of the teacher and selected students provided additional information on the cultural context of the classroom. The *teaching phase* consisted of the actual implementation of the instructional unit in the classroom over a period of four weeks. These lessons were videotaped. The researcher collaborated with the participating teacher informally to modify and discuss the progression of lessons immediately preceding and following each teaching episode. The researcher also served as a recorder of information by making field notes during the teaching phase.

During the *analysis* and *hypothesis testing phases,* the researcher analyzed the data collected from the classroom observations and student work. Student work was analyzed and coded. The videotapes were transcribed. The classroom discourse was analyzed in relation to the activity of creating and translating inscriptions to make meaning of the quotient construct. The hypothetical learning trajectory of the whole class was examined. The teacher and the researcher formally met once a week to discuss the findings and make modifications to the instructional unit based on the findings. The researcher identified the "essential mistakes" students made during the lessons from the analyzed data and used it as a basis for discussion with the teacher. The teacher thought about the essential mistakes students made, and reflected on how she could help her students to overcome them. These reflective sessions resulted in the teacher modifying her teaching practices and the researcher modifying the instructional unit.

Both whole group interactions and a focus group of 4 individual students were videotaped. Digital images of all marks made in students' notebooks were recorded. In the analysis, the creation and modification of inscriptions were used as a focus for examining the connections between students' individual knowledge construction, their contribution to the knowledge of the collective, and the ways in which the practices of the collective constrained those of the individual students. For example, students' written marks in their notebooks often were different than those depicted during whole group discussion. When a student chose to annotate or alter a mark at the board, we cited this as evidence of individual contributions to the collective inscription. The relationship between individual conceptual development and collective development was made by comparing these marks and how they changed over time. For example, when students' annotations in their individual notebooks changed following a whole group discussion, we cited this as evidence of the collective practices constraining those of the individual. In recording the changes in inscriptions and the discourse that accompanied these changes, we were able to abstract a learning trajectory for the whole class, and note the individual inscriptions and thinking in the hypothetical learning trajectories of the individuals within the class. Normative inscriptional practices for creating and translating inscriptions within were noted during the teaching episodes as students engaged in whole-group discussion on a flip chart (all pages of the flip chart were displayed on the board throughout the 5-week teaching experiment).

Results

Students engaged in the practices of creating and translating inscriptions to make mathematical sense of the relationship between fractions and division with regard to the quotient construct. The whole-class learning trajectory emerged out of these practices. First the emergent collective learning trajectory will be briefly described, and then the inscriptional practices through which this learning trajectory emerged will be explained. Discussion will center on how these inscriptional practices became part of the classroom norms.

The learning trajectory established in the classroom corresponded to key uses of drawings and written marks. These changes roughly followed three Phases: In Phase I, students pictorially represented and solved fair sharing problems. They partitioned the unit into equal sized pieces and distributed them into groups to make equal shares. Initially students represented the answer as a whole number indicating the "number of pieces that made up each share. Then when asked "how much" the number of pieces represented in relation to the whole, students were able to symbolize the answer as a fraction. In Phase II, students focused on understanding that a fair-sharing problem represented a division relationship between two extensive quantities. They began to realize that this division relationship could be symbolized using the conventional notation of the fraction bar or a division symbol (÷, /).

In Phase III, students began to symbolically represent the fair-sharing problem and solution and started to think about the relationship between the division problem and solution in terms of quantities. Prior to this phase, students had been solving the fair-sharing problems using pictorial representations and then symbolizing the answers. Students first represented the answer of the fair-sharing problem as a fraction or a whole number and fraction on the right side of the number sentence. Then they proceeded to figure out how to symbolically represent the left side of the number sentence containing the division relationship between the quantities of the fair-sharing problem. Therefore, students treated symbolizing the number sentence as a two-step process where they first symbolized the answer and then the division relationship. They thought about each side of the equation as quantities, but they did not focus on the relationship between the fair-sharing problem and answer as a number sentence.

Focusing Attention to the Immediate Problem Context

Students and teacher generated inscriptions on chart paper during whole group discussion to make thoughts and ideas explicit. The whole class focused their attention on the representational context of the person creating an inscription as he or she drew pictures, symbols, or wrote words. The person usually explained what the inscription represented and verbally recounted a running narrative of what he or she was producing. Simultaneously that person used gestures such as pointing and engaged in the act of writing or drawing on the chart paper to draw the whole class' attention to the particular thought or idea being communicated.

The act of creating an inscription and providing an explanation coordinated the group's attention to the immediate problem context. Therefore, the act of creating the inscription and the whole class's focus on this action was situated. Even though there were other inscriptions (other parts of the same inscription) that had already been created and were visibly displayed, the whole class focused their attention on the part of the inscription that was in the process of being created.

Record of Distributed Thinking

Once an inscription was created during whole group discussions, then it became a historical record of the dialog that took place and hence, could be recalled as a common exemplar to which all participants could attach meaning. Furthermore, the inscription also represented multiple viewpoints of several different students within the group—i.e., such an inscription represented the distributed expertise of the group.

The types of socio-mathematical norms that emerged included: 1) proving or defending one's idea in relation to another student's verification; 2) model competition, and 3) explaining and clarifying another student's thinking (e.g., perspective taking).

Inscription Became a Tool to Coordinate Student Attention to the Problem Context

Inscriptions were used as tools to focus student attention to the problem context. The inscription was used as a reference point to re-create previous conversations and actions. Most whole group mathematical discussions that took place focused on uncovering "essential mistakes" or misconceptions that student were experiencing during that day's lesson or the previous day's activities.

The teacher influenced the focus and direction of the whole group conversation, which became part of the inscriptional practices that took place. In other words, the topic of conversation and the nature of the inscriptions were influenced by the actions of the teacher. The teacher also played an instrumental role through her questioning and statements to facilitate the coordination of inscriptions and dialog to make mathematical meaning.

Analyzing Another Student's Thinking About a "Taken-as-Shared" Inscription

Analyzing another student's thinking during the whole group discussions became another mathematical norm. In doing so, students were forced to actively think about another student's strategy and reasoning and relate it to how they thought about the problem.

Continuous Reconstruction of Meaning

Ideas recorded in inscriptions were continually re-examined throughout the teaching episodes and reconstructed in the process of developing meaning. Inscriptions were initially interpreted on a surface level. This meant that the students were focusing on examining what the immediate problem represented and what was required to solve the problem. A deeper level of understanding was reached when the students were able to connect their understanding of the immediate problem to the mathematical concepts that could be abstracted to another situation.

Discussion

In this paper, we have documented the role of inscriptional practices in the development of student understanding of the quotient construct over a five week period. In doing so, we have pointed out that the inscriptional practices that took place mediated the creation and translation of inscriptions which ultimately influenced the type of knowledge constructed as reflected in the learning trajectory that emerged. The inscriptional practices that took place were part of the classroom microculture and became part of the classroom norms. Therefore these practices must be taken into consideration when examining how knowledge is constructed in the classroom.

Agents and cultural tools mediate human action Wertsch (1998). The agents were the students and the teacher and the inscriptions represented the cultural tools produced through interactions between the agents and the tools within the classroom. These interactions of creating and translating inscriptions to make sense of the quotient construct became the mediational means through which the learning trajectory emerged.

The inscriptional practices that are described in this paper also illustrate how ideas that were recorded as inscriptions were adapted and modified through time. Inscriptions became used as tools. The nature of a tool has meaning and significance based on the context in which it is used. A tool by itself does not have any significance. The use of the tool changes the situation and perception of the user (Wertsch, 1998). In this whole class teaching experiment, students created inscriptions as tools to make their thinking explicit and communicate their thinking to others. They translated inscriptions by examining them to make meaning. Inscriptional practices that took place represented individual contributions as well as the distributed thinking of the group. The usefulness and the meaning of these inscriptional practices must be understood with regard to the learning trajectory that emerged.

Lastly, the inscriptional practices described in this paper, because they packaged a complex discourse into relatively few retrievable exemplars whose structure embodied key features of quotients, afforded students the opportunity to revisit and build upon ideas efficiently to make sense of the quotient construct. The purpose of making sense of the division and fraction relationship to understand the concepts guided the meaning making that took place within the inscriptional practices. There was a continuity of ideas that were revisited and build over time through these inscriptional practices, as opposed to the inscriptions being generated as a series of disconnected activities (e.g., as "representations"—temporally static pictures with a fixed meaning). Therefore the nature of the inscriptional practices in a classroom can either afford or constrain concept building that takes place in the classroom.

Further investigation is needed to fully understand the role of inscriptional practices within the classroom. How do inscriptional practices get established as classroom norms in the classroom? What elements of the inscriptional practices can afford or constrain thinking?

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