

**A STUDY ON THE ROLES OF “OTHERS” IN LEARNING:  
FROM THE CASE OF CHARACTERS METHOD,  
A NEW WAY TO LEARN MATHEMATICS**

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*In this paper, the social aspect of mathematics learning, especially the concept of “Others” is examined. The three types of “Others” in the learning environment are identified, and a framework of learning, “Reflexive Writing Activity”, which consciously emphasizes the existence of “others”, is presented. Moreover, one type of learning method in this framework, “Characters Method” has put into practice for some implications. In conclusion, Reflexive Writing Activity is not only the effective learning method, but also the aims of mathematics learning which is helpful for both knowledge-understanding and the rich-fruitful learning environment.*

## **INTRODUCTION**

From the script of TIMSS video study, one characteristic of Japanese mathematics lessons is described as follows.

Problem solving comes first, followed by a time in which students share the solution methods they have generated, and jointly work to develop explicit understandings of the underlying mathematical concepts.

In this script, we can find the importance of “students’ sharing” or “jointly work”, which might be the same ideas to social affairs or so-called Cooperative Learning. In the previous studies, Bishop(1985) points out the importance of “social construction of meaning”. NCTM(1989) asserts “Mathematics as Communication”, and Artzt & Newman(1990) examines “Cooperative Learning”. Yackel et al.(1990) indicates the “Student-Student Interaction”. It seems that mathematics learning is not just personal affairs any more, but the social activities which are conducted not only by the learner her/himself but with others. In this paper, social aspect of mathematics learning is focused on, and the roles of “others” in the learning environment is examined. In order that, one of learning methods, “Characters Method”, which makes students much easier to be aware of “others”, has put into practice, and some implications are discussed.

## **THEORETICAL FRAMEWORK**

Lave & Wenger(1991) regards learning as “increasing participation in communities of practice”, or “participation in social practice”(p.49). It says;

Activities, tasks, functions, and understandings do not exist in isolation; they are part of broader systems of relations in which they have meaning. These systems of relations

arise out of and are reproduced and developed within social communities, which are in part system of relations among persons. The person is defined by as well as defines these relations. (p.53)

Legitimate peripheral participation is far more than just a process of learning on the part of newcomers. It is a reciprocal relation between persons and practice. This means that the move of learners toward full participation in a community of practice does not take place in a static context. The practice itself is in motion. Since activity and the participation of individuals involved in it, their knowledge, and their perspectives are mutually constructive, change is a fundamental property of communities of practice and their activities. (pp.116-117)

From the same standing point, Saeki(1995) mentions the concept of *others*. It defines two kinds of *others*. One is the *others* who relate to the person her/himself just as mere classmates. In this case, the *others*' learning never affect to the person her/himself. Of course they might be friends, but learning mathematics is totally the personal affair and nobody never relate to others' learning. Saeki(1995) describes such learning environment as "**They-World**", because every relation between classmates in their learning is the third personal. She/he is in the same classroom, but what I am learning is not her/his business and what she/he is learning is not my business. In this paper, such kind of *others* are regarded as "**Third Personal Others**". Students tend to be *Third Personal Others* under the environment of knowledge-transmit type classes.

The other concept of "others" is in the environment of situated learning. As Lave & Wenger(1991) mentions; "*Activities, tasks, functions, and understandings arise out of and are reproduced and developed within social communities, which are in part system of relations among persons*", *others* in such learning environment play very important roles to the learning of a person her/himself. Saeki(1995) describes such learning environment as "**You-World**", because every relation between students in their learning is the second personal. What you learn is very important for my learning, and what I learn is also important for your learning. Everyone is in the same learning community and appreciates each other. Student-student interaction, or cooperative learning will be the integral part of such learning community. In this paper, such kind of *others* are regarded as "**Second Personal Others**".

Further more, Hirabayashi & Shigematsu(1987) explains the concept of metacognition on the analogy of person, "Inner Teacher", which means it works as if there is a teacher who warn to the student within the inside of her/himself. This analogy seems to be very effective to understand such internal operations. Not only metacognition but reflective thinking, metaknowledge, or affective issue are also essential internal operations which play important roles in learning mathematics. Since such internal operations can be regarded as a person who is in the inside of the learner her/himself, the term of "**First Personal Other**" is provided in this paper. Another way to say, *First Personal Other* are another self, who watches the learning activities of the self and makes some internal operations.

It is obvious that the ideal learning environment makes classmates to be the *Second Personal Others* rather than the *Third Personal Others*. It is also important to notice that the *Second Personal Others* are not just classmates but the essential source of each student's learning. Moreover, *First Personal Other* should also play an important role in the ideal learning activities. We need to pay more attention to such internal operations. After all, it is crucial that learners should be conscious of both *Second Personal Others* and *First Personal Other*, and put them to practical use for fruitful mathematics learning.

## METHOD

Ninomiya(2001) proposes a framework of *Reflexive Writing Activities*, which is a kind of writing that has reflexive interaction with both learner and class activities. Student writes *Reflexive Writing* from the viewpoint of either her/himself, *Second Personal Others* or *First Personal Other*. Although *Reflexive Writing* is basically the reflection of learner's own learning, she/he can never stop writing just the answer or her/his own solution. Students need to write more. Since students need to reflect their own solutions, some reflective *internal operations* are needed and students are encouraged to show them. However, sometimes it is hard for students to distinguish metacognitive or other internal affairs from cognitive operations, so the concept of *First Personal Other* is introduced to the students as *another self*. Students are encouraged to watch their own learning processes from the view of *another self*, and have *another self* make some comments toward their own learning as if they were told by teacher. Moreover, the ideas or comments from *Second Personal Others* are also important for students' learning. Because such ideas or comments are integral part of learning, students are also encouraged to show them.

In this way, *Reflexive Writing* is formed with (1)student's own answer or solution, (2)ideas or comments from *Second Personal Others*, and (3)comments from *First Personal Other*. The important point is that *Reflexive Writing* never ends with only one single statement. For example, when a student writes her/his own solution, she/he may also add to write some other ideas from *Second Personal Others* afterward, and compare with his/her own. Making comparison, she/he may be aware of something which are metacognitive or other internal affairs', then she/he may be able to add some more comments from *First Personal Other*. Further, because of such comments, she/he can foster her/his own idea, and may get another solution. In such way, writing activity and student's learning may develop their mutual interaction, and the nature of their relation is reflexive. Moreover, since every student is an autonomous participant in each class activity, her/his description is not a copy of the blackboard but her/his own learning process. However, watching the student's *Reflexive Writing*, we can figure out not only how she/he promote her/his own learning, but also how the whole class activity progresses. *Reflexive Writing* is a reproduction of the class activity, which is produced from each student's viewpoint. From this point of view, *Reflexive Writing* is the reflection of

each class activity, and the nature of their relation is also reflexive. Summarizing, there are reflexive natures both in the relation between *Reflexive Writing* vs. student's own learning, and *Reflexive Writing* vs. the whole class activity.

One of the best ways to promote *Reflexive Writing Activities* is "*Characters Method*". In this method, some characters such as persons, animals, etc. are used on purpose, in order to let students be aware of the existence of *Second Personal Others* and *First Personal Other*. Each character becomes either student her/himself, *Second Personal Others*, or *First Personal Other* (another self). Because students can directly see the subjects of each comment, or they can distinguish who is talking which comment, "*others*" are easily intervened into the students' own learning environment. During the instruction of *Characters Method*, teacher never force students to use characters, but just show how to use characters on her/his blackboard writing. Most of the students spontaneously imitate their teacher's way, because they love to study in such a way. Although there is not special instruction for "*Others*", students learn the importance of *Others* by themselves. The example of *Reflexive Writing* is shown as Fig.3 in the following paragraph.

## RESULT

In this paper, 4<sup>th</sup> grade class (18 boys and 19 girls), in which students have been encouraged to learn with *Characters Method*, is investigated, and a case of one hour period class is examined. The topic of the class is "folding a piece of paper". The outline of the class is shown in Fig.1, and the description on the blackboard is as in Fig.2. Also, two typical cases of *Reflexive Writing* are shown as in Fig.3 and Fig.4.

In this class period, the time of *Reflexive Writing* is set up twice, as after the Introduction and after the Wrapping up. After the Introduction, students are encouraged to write the task and the first impression. Then, they start their own investigation, or problem solving. Solving the task, they also write some *Reflexive Writing* for the reflexive interaction with *First Personal Other*. When the class discussion starts, they are encouraged to write some *Reflexive Writing* for the reflexive interaction with *Second Personal Others*, as well as with *First Personal Other*. During this period, they are expected to present their own ideas, to discuss each other, to write some *Reflexive Writing* with both *First* and *Second Personal Others*, and think again through their *Reflexive Writings*. Finally, they are encouraged to write some *Final Remarks* after the Wrapping up. Although there is very little time to devote only to *Reflexive Writing Activity*, students may write anything they want at any time during the class, and their *Reflexive Writing* become so fruitful.

## DISCUSSION

First of all, a brief description of the students, Miku and Yoji, is presented, and their *Reflexive Writings* are investigated based on their learning behaviors.

**Introduction (5 min.)** presenting the task  
 Teacher picked up a piece of paper and begin to fold it.  
 T: How many lines are there when I fold once?  
 S: One line.  
 T: How about twice?  
 S: Three lines.  
 T: How about 3 times?  
 S: Seven.  
 T: 4 times, 5 times, 6 times, 7 times-----, so how many lines will be there when I fold 7 times?

**Reflexive Writing activity (2 min.)**  
 Writing the perspective of solving the task

**Problem solving by each students (10 min.)**  
 Each student try folding a piece of paper.

**Confirming the task (6 min.)**  
 T: Don't you have any trouble?  
 S: It has become harder to fold. It is impossible to fold 7 times.  
 T: How can we manage it?

**Presenting Yoiji's idea (6 min.)**

# of folding:	1	2	3	4	5	6	7
# of lines:	1	3	7	15	31	63	127
		2	4	8	16	32	64

Yoiji: Adding the next number to the #of lines, we get the next # of lines. Added number have become twice as big as previous one.

**Presenting Yuri's idea (4 min.)**  
 T: Where are these added numbers come from?  
 Yuri: They are the numbers of small rectangles (rooms) when folging.

**Comparison their ideas (5 min.)**  
 T: How about you, Yoiji? Is this same to what you have thought?  
 Yoiji: No, this is different. I have folded the paper till I have got 16. Then, I found the number become twice as big as previous one when I fold one more time.

**Presenting Ayu's idea (2 min.)**  
 Ayu: The number of lines is just 1 smaller than the number of the room.

**Summarizing students' ideas (5 min.)**  
 T: What are the numbers which have become twice as big?  
 S: Number of the rooms. The # of the room at the previous stage.  
 T: # of rooms (previous stage)  $\times 2 - 1 =$  # of lines

**Wrapping up (3 min.)**  
 Teacher briefly summarized today's class, again.

**Reflexive Writing activity (4 min.)**

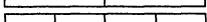
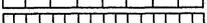
**Presenting several student's writing (1min)**  
 Two students read their own brief remarks.

Fig.1 The Outline of the Class



How many lines will be there when I fold 7 times?

Less than 30 lines-----rest of the students  
 More than 30----- 7 students

	Folding once, then 1 line
	twice 3 lines
	3 times 7 lines
	4 times 15 lines



It has become harder to fold !!

**Yoiji:**

# of folding:	1	2	3	4	5	6	7
# of lines:	1	3	7	15	31	63	127
		2	4	8	16	32	64

Answer: 127 lines

Yuri: +2 +4 +8 +16  
 the number of the room  
 Yoiji: maybe twice as big as previous  
 +16 (twice)  $\rightarrow$  +32

**Ayu:**

	# of the rooms	# of the lines
once	$1*2=2$	1
twice	$2*2=4$	3
3 times	$4*2=8$	7
4 times	$8*2=16$	15
5 times	$16*2=32$	31
6 times	$32*2=64$	63
7 times	$64*2=128$	127

# of the rooms -1 = # of the lines

# of rooms (previous stage)  $\times 2 - 1 =$  # of lines

Fig.2 The Description on the Blackboard

How many lines will be there when I fold 7 times?

about 15 lines }  
about 30 lines }  
more than 30 lines-----7 students

Mm, I totally can not figure out. After all, how many are there???

1	2	folding once	1 line			
1	2	3	4	5	folding twice	3 lines
8	folding 3 times	7 lines				
16	folding 4 times	15 lines				

Oh! Now it is so hard to fold!

**Yoji's idea**

# of folding:	1	2	3	4	5	6	7
# of lines:	1	3	7	15	31	63	127
	2	4	8	16	32	64	

Answer: 127 lines

I still can not understand

**Yuri:** +2 +4 +8 +16  
the number of the room

**Yoji:** twice as big as previous  
+16 → +32

Yuri's idea and Yoji's idea are different. But, it seems something relates.

**Ayu's idea**

# of the rooms	# of the lines	
once	1*2=2	1
twice	2*2=4	3
3 times	4*2=8	7
4 times	8*2=16	15
5 times	16*2=32	31
6 times	32*2=64	63
7 times	64*2=128	127

# of the rooms -1 = # of the lines

# of rooms (previous stage) × 2 - 1 = # of lines

**Final remarks**  
First, I tried to fold 7 times, but it was so hard to do. However, when I saw Yoji and Ayu's ideas, I found out it's gonna be twice as big. I really could understand very well. Math is a bit difficult----

Fig.3 Miku's Reflexive Writing

I raise my hand for "over 30 lines", but how can I figure-out?

# of folding:	1	2	3	4	5	6	7
# of lines:	1	3	7	15	31	63	127
	2	4	8	16	32	64	

127 lines

**Final remarks**  
Though I did not mention in my explanation, I have also found that it becomes twice because the paper is piling on. Maybe the number of the lines will also become twice.

Fig.4 Yoji's Reflexive Writing

Miku is a student whose math ability is not so high. She used to have no confidence for mathematics, but when she knows Reflexive Writing Activity and start learning with this method, she has become to be confident of. Because she surely

understood the importance of Reflexive Writing Activity and started learning consciously with this method, she has been able to understand mathematics. Although she sometimes has hard time to understand classes, she has flexible and accepting attitude toward *Others*, and try to learn from every classmate.

In contrast, Yoji is a student whose math ability is very high and has strong confidence in his math ability. However, he believes that the value of learning is only the “correct answer”, and he never appreciates the “process” of the problem solving. He also values his rapidity of solving, and believes that the rapidity shows his high quality of math ability. Therefore, he understands neither the importance of Reflexive Writing Activity nor the importance of interactions with classmates. He is a type of the student who rejects “*Others*” in his learning process, because he seems to believe that he is the smartest student in his class and his solution is the best.

The biggest difference between Miku’s and Yoji’s writing is whether there are ideas or comments from *Second Personal Others*. This is due to the difference of how they appreciate Reflexive Writing Activity. Miku tries to deepen her ideas or understanding through the class discussion, by means of her Reflexive Writing. As mentioned above, her math ability is not high enough to solve the task in her alone. In fact, she made a mistake in her diagram of “folding twice”, and her First Personal Other expresses that she could not understand Yoji’s idea. However, her Reflexive Writing promotes her understanding, and it indicates the progress of her learning until she finally understands the idea and solves the problem. She could learn from Yoji’s, Yuri’s, and Ayu’s ideas through her own Reflexive descriptions. The comments from First Personal Other arrange her process of learning, and they clarify her own understanding. Finally, her learning becomes so rich and fruitful.

In contrast, Yoji indicates only his own ideas or comments, and never mention *others*; which means there is no comment from Second Personal Others and almost nothing from First Personal Other. Even though there are some characters in his writing, these comments are regarded as his own. Because of his rejecting attitude toward *Others*, he has missed some important ideas, although he was the first student who could get the correct answer in this class. He found the pattern in the number of *lines*, as the difference of sequence is powers of 2. He could get the right answer so quick, but because he values only the “correct answer” and never appreciates the “process”, he has not deepened his investigation more. He persisted in his own idea of “the powers of 2”. Because this is a very strong strategy for solving this task, he was very satisfied with getting this strategy and never examined the reason why “the powers of 2”. Because of his rejecting attitude, he could not accept Yuri’s idea and he missed the meaning of *the difference of sequence*, even though there was a description on the blackboard.

Comparing Miku’s and Yoji’s learning, there might be little difference on understanding the problems; however, the results of their learning are so different. Even though Reflexive Writing Activity is one of the learning methods, Reflexive Writing itself must be regarded also as the aim of math learning. Compare to Yoji,

Miku has greater ability to express in Reflexive ways. Although it may seem that their difference is whether they know how to use Reflexive Writing, we ought to regard it also as the “difference of the learning abilities”, which means that this is not only the evidence of her power in representation but the evidence of her learning ability. Because Miku has improved on her ability to express in Reflexive ways since she began to learn in Characters Method, her knowledge or understanding of mathematics have been also improved, shown as in Table 1.

	Miku's score	Mean	SD	Miku's Standard score
1 <sup>st</sup> semester	87.5	71.5	22.1	0.72
2 <sup>nd</sup> semester	92.0	79.8	10.7	1.14

Table 1 Miku's Achievement in “Knowledge-Understanding”

Table 1 indicates not only Miku's achievement but also the development of whole class learning environment since they started learning with Characters Method. The change of mean is the improvement of their knowledge-understanding score, whereas the change of standard deviation(SD) implies that those who was low score could improve their score a lot. It might be true that Characters Method is helpful in the improvement of knowledge or understanding, as well as in the rich and fruitful learning environment.

In conclusion, *Reflexive Writing Activity* is not only the effective learning method, but also the aims of mathematics learning which is helpful for both knowledge-understanding and the rich-fruitful learning environment.

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