

THE INFLUENCE OF SUBJECT MATTER ON TEACHERS' PRACTICES: TWO CASE STUDIES.

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Abstract

In this article, the general principles and practices adopted as guidelines to a graduate course for secondary Mathematics teachers are discussed. We argue that once the curriculum is designed both to complement the teachers' subject content knowledge and to provide activities promoting links between their conceptual knowledge and their actual practices, it can have a strong positive influence in teachers' subsequent practices. Two teachers who completed the course were subsequently followed during a two year period. They were interviewed and their practices were observed. We analyse their reflections on the main differences in their practices before and after attending the course.

Introduction

The need to create opportunities for regular in-service training courses for secondary Mathematics teachers in our geographical area led the team in charge of the undergraduate teacher-training course in our University (the oldest established in Brazil) to create a two year graduate course. The basic principles underlining the present versions of the graduate and the undergraduate courses is that the sum of solid mathematical formation with basic pedagogical and didactical knowledge is essential in training teachers, but it cannot be considered enough. Both courses should also include activities especially planned to allow the trainees to reflect upon the importance for their classroom practices of establishing connections between their own learning experiences in Mathematics at an advanced level and their practice as secondary Mathematics teachers.

Specific disciplines aiming at establishing these connections have been included in our undergraduate teacher-training curriculum few years ago. The experience on writing materials and developing one of these disciplines (Algebra and Arithmetic) is reported in Belfort, Guimarães and Barbastefano (2001a). This kind of experience has also reflected on the development of the curriculum for the graduate course.

We discuss in this article the general principles and practices adopted when designing the curriculum for the graduate course, and their consequences for the teachers' subsequent practices. The main aims of the course are to complement the subject content knowledge preparation of the teachers and to associate this

knowledge with a series of activities specially prepared to “close the gap”^{*} between the Mathematics they are learning and the Mathematics they teach as secondary teachers.

In order to obtain data on how these experiences changed the practices of the teachers involved in the program, two teachers who completed the course were followed during a two year period. During this time interval, we had plenty of opportunities to observe their practices and to interview them. We discuss here their reflections on the differences in their practices before and after attending the course.

Background

There were many reasons leading us to conclude that it was necessary to include experiences specially designed to highlight the connections between the mathematical disciplines developed for the trainees in the program and the subject matter to be taught at secondary schools. There are indicators from research that a solid subject content knowledge may be essential for a successful teacher (for examples, see Ball and Feiman-Nemser, 1988; Grossman et al., 1989; Ball, 1991; Leinhardt et al., 1991). Nevertheless, Ball and McDiarmid had already taken one step further: they began to investigate the special qualities prospective teachers' actual subject content knowledge might lack in order to foster good classroom practices. Ball (1988) stated that "...[secondary teacher candidates' additional studies] do not seem to afford them substantial advantage in explaining and connecting underlying concepts, principles, and meanings" (pg. 24), while McDiarmid (1992) argued that "elementary and secondary teachers frequently lack connected, conceptual understandings of the subject matters they are expected to teach" (pg. 1).

Ma (1999) discusses the importance of the “*profound understanding of fundamental mathematics* (PUFM)” demonstrated by some elementary teachers. She defines this concept as "an understanding of the terrain of fundamental mathematics that is deep, broad, and thorough" (pg. 120). As a consequence of PUFM, some elementary teachers "do not invent connections between and among mathematical ideas, but reveal and represent them in terms of mathematics teaching and learning" (pg. 122). According to her, the work of these teachers displays the following characteristics: connectedness, multiple perspectives, awareness of the basic ideas, and longitudinal coherence.

Our practice in teaching subject matter in undergraduate and graduate courses directed at secondary Mathematics teachers suggests that the difficulties our trainees face with subject content are of a similar nature (Belfort, Guimarães & Barbastefano, 2001a, 2001 b). If we add our experiences to other researchers' descriptions, we can

^{*} The notion of 'gaps' between different learning stages can be found in Hart (1987), where she discusses the use of concrete materials in elementary education.

at least conjecture that this may be a widespread problem.

If we attempt to transpose Ma's ideas to the situation of the study of Mathematics at secondary level, we are faced with the twofold connection of this concepts: to the previous knowledge of elementary Mathematics the students would have and, in the other hand, to the mathematical structures studied in their undergraduate course. Issues such as Cognitive Units (Barnard & Tall, 2001) and Flexibility (Gray & Tall, 1991) also have to be taken into consideration, as explanations for and justifications of some of the mathematical procedures studied at secondary school may be provided only if one resorts to knowledge of structures and properties studied in advanced Mathematics (a simple example: several of the divisibility criteria often taught in the initial years of secondary school depend for their justification on the properties of the integer numbers as an integrity domain, and of the general properties of congruence in this set).

So it seems that, if the teaching of secondary Mathematics is to display characteristics of connectedness, multiple perspectives, awareness of the basic ideas and longitudinal coherence, the teacher would have not only to know Mathematics at an advanced level, but also to be able to reflect on its connections with the contents of secondary school. Such were the premises guiding the design of the curriculum for the graduate course.

Curriculum Guidelines

Several teachers enrolled in the graduate course were previously involved in a summer in-service course by our Institute. So, when designing the curriculum, we considered not only the research-based reflections mentioned above but also the difficulties these teachers seemed to face when dealing with secondary mathematics. Our experiences with these teachers had confirmed the lack of connected and conceptual understandings of the subject matters commented by McDiarmid (1992). These teachers also demonstrated different levels of mathematical reasoning (for example: inductive reasoning, incomplete and/or misleading arguments were found among their justifications to mathematical secondary results).

The curriculum included disciplines to allow the teachers to reflect upon results coming from Mathematics Education research and pedagogical matters. A discipline aimed at fostering the use of technology for teaching Mathematics was also included. But the majority of the disciplines offered these teachers opportunities to review, from an advanced standpoint, the contents they have to teach (algebra, geometry, number structures, calculus and statistics). Based on the principles discussed above, the development of all disciplines included activities designed to allow teachers to make connections between their learning experiences and their practices. We can summarise the general guidelines for developing the courses as:

- we offer the teachers several opportunities to work with (and reflect upon) the basic properties of the mathematical objects;
- we punctuate the courses with commentaries on mathematical reasoning and justification: what it means to show that a result is false (use of counter-examples) and different methods of proof;
- formal proofs are often accompanied by a previous discussion of the need for it and of the strategy in searching for it;
- we encourage the teachers in establishing connections between the topic under study and examples of actual practice in the secondary school classroom;
- every mathematical concept is correctly defined, and possible difficulties in its construction by secondary students are often discussed;
- we furnish ample opportunity for the critical analysis of texts and questions found in Brazilian textbooks;
- we present several examples of wrong and/or incomplete problem solutions to be analysed; and
- we often ask the teachers to present more than one solution to a problem, as a way to encourage flexible and consistent mathematical reasoning.

We expected the course to have a strong positive influence in teachers' future practices. In order to verify this hypothesis, it was necessary to follow some of them in the subsequent years. In fact, it was not a difficult task to select a sample for the case studies. A partnership was established with many of the teachers engaged in the graduate course. They keep coming back to discuss their practices, to ask for comments on their work, to use the library or simply to have a chat.

Teachers' Reflections on their Practices

We selected two teachers, with very different backgrounds, to be followed during a two year period after finishing the course. Their practices were observed and they were interviewed several times. The main aims of the interviews were to ask these teachers the reasons behind their teaching decisions and to verify how these differed from the ones they used to make before attending the course. Observation was used as a mean to verify the conformity between discourse and practice.

Ada is a very experienced and enthusiastic secondary school teacher, working in schools in Rio de Janeiro for the last 20 years. She had a secondary level training as a primary teacher and an undergraduate teacher training course for secondary Mathematics in a private college in Rio de Janeiro. Even though she had been enrolled in several in-service courses before joining our course, she had to work hard

to overcome several difficulties with subject content matters during the graduate course.

Ricardo is also an experienced teacher, working in secondary schools in smallish town near Rio de Janeiro for 12 years. Ricardo did his teacher training course in a private college at his home town. He also had been enrolled in several in-service courses, and joined the one we offered in the summer of 1997. The year after that, he enrolled for the graduate course. He commuted to and from his home town every week for the duration of the course.

There were several changes in their careers in the last two years, and both declared them to be related to the course. Ada still works at a state school in Rio, as she has been doing for almost 20 years. She works in adult education during the evening period in this school. She got a new job in a highly respected experimental school in Rio, and became Head of the Mathematics Department there. She is also in charge of introducing the use of computers in Mathematics lessons. Apart from that, she is working as a trainer in in-service courses for primary teachers, and she was recently invited to lecture didactics of mathematics for prospective primary teachers at a private college in Rio. Ricardo also has changed jobs in his home town. He is now working in one the most prestigious private schools in the city and became the Pedagogical Head of this institution. He is in charge of preparing secondary students for the universities' entry exams and he is also training the school's primary teachers at Mathematics.

Both teachers report several changes in their practices. For instance, Ricardo has changed the school adopted Mathematics textbook. He says:

"The old textbook had many exercises to train the kids, but did not teach them mathematical concepts... and most teachers usually only teach what is in the book. I know that because I used to do it myself (laughs). I used the textbook as my only source of study. I usually prepared all the exercises beforehand and, whenever I could not do one of the exercises, I took it out of the list I proposed to the kids. I never discussed solutions of the exercises with other teachers. I guess I felt too insecure to do that ... anyway ... we had a meeting to discuss the textbook. It was not easy to convince the other teachers to change the book ... probably because they were used to the old one, and they did not feel secure adopting the new one. It took me a while to convince them that we would be working as a team, and that we would be discussing concepts and exercises. I guess they finally believed me because I was already bothering them during the breaks to help me with difficult exercises (laughs)."

Ricardo's school accepts students for the last three years in secondary school. Ricardo says that the group of students joining the school this year came from different backgrounds, and most of them demonstrated gaps in their previous

mathematical preparation. He has implanted a “Saturday morning of work”. He is very enthusiastic about this experiment:

“All pupils in the group are volunteers ... well, sort of! (laughs). In fact, I asked them to solve selected exercises during an interview. It was easy to convince them that it would be a good idea to take time to review some of the concepts they were expected to know at this level. I have been planning these activities carefully, using different sources of materials, because I want the kids to enjoy each lesson and to come back for more the following week. Lots of games and competitions - they work in small groups most of the time. Whenever they finish an activity, they tell the others what they did. It is very noisy, but it is amazing to see this kids discussing their different solutions. This is also the moment of the lesson in which I reinforce the main results and generalise the mathematical concepts. The experience is still at its very beginning, but I believe that these kids would do well in their regular Maths lessons ... and they keep telling the others that the “Saturday lessons” are great fun.”

Ada not only changed the textbook she adopted but also she radically changed the way she uses the book. She usually gives the students a series of activities she prepared herself before she asks them to read and to do the exercises in the book.

“This is a much better textbook than the one I used to adopt. Nevertheless, the pupils do not enjoy using it all the time ... also, I believe that different experiences are necessary to promote mathematical learning. So, I usually get them started with a group activity. I use different sources to prepare these tasks, so they would have a great variety of experiences. This strategy also keeps them interested and it gives me the chance to complement the viewpoint presented in the book ... it is very helpful for the students.”

Ada declares that the course was a turning point in her career. She says:

“I had been enrolled in several in-service courses before, and I was almost giving up! It was a terrible experience to listen to the trainers statements: you should do this and you should construct that!... But I did not know how to do it and they most certainly did not teach me. I realise now that my mathematical knowledge was mostly instrumental, and I had little conceptual understanding. So how could I possible teach to promote it? ... I even had difficulties understanding the textbook I adopt now, and several activities proposed there were meaningless to me ... so, I avoided it - and every other one of the kind. After these in-service courses I kept doing instrumental teaching, relying on a bad textbook - and feeling very uncomfortable.”

Ada's comments about her activities as a primary teacher trainer also show the importance she is now attributing to conceptual understanding:

“As a trainer in in-service courses I realised that most primary teachers could not justify their work... they know how to do it, but they don't know why they

do it. I guess this is probably the way they were taught - and it is also the way they teach the kids! Mathematics has become a series of meaningless rules for them. On the other hand, it is interesting to observe their reactions to a series of activities aimed at constructing the concepts instead of instrumental Mathematics or Pedagogical discussions detached from content matters. Once they overcome their initial surprise, it seems that they start learning ... at least, they work hard and enjoy the experience! It is gratifying to see how quickly they adapt most of the activities of the course into activities for their pupils. Lots of good ideas discussed in the class! It feels like starting a machine: you turn on a button, and it goes on and on ... but then again, I believe it was the same process with me! (laughs)”

Comments and Reflections

Cooney (1988) described teaching as “an interactive process”, in which conscious decision making is needed not only during the planning of the lessons but also “on the stage”. Models to explain the role of the teacher usually consider the interaction between teacher’s knowledge and beliefs as the basis for their decisions (see, for instance, Fennema et al., 1989). The data from the case studies described above point to the strong influence of subject content knowledge in this process.

The case studies suggest that there were substantive changes in the ways these teachers work. Both are now in charge of special programs of study in their schools. Both have changed not only the adopted textbooks but also the ways they use this resource. They are designing complementary materials, which suggests they are much more confident about making decisions about their pupils’ learning experiences. As a consequence, the textbook is used as a helpful source of reading and activities, but it is no more determinant of the curriculum developed in their classrooms.

Their reflections on their work before attending the course are also notable. Because of the relationship developed during the course, they are very open to discuss their previous difficulties. Ricardo made clear that his previous difficulties were not related to class management or pedagogical issues, but were due to a lack of content knowledge. Similarly, Ada was almost giving up on in-service courses. Even though she was not able to make these reflections at the time, it seems that she felt uneasy about learning methodological issues she could not apply, mainly because they should be associated to a conceptual content knowledge she did not possess.

We can say that our main goal was to convince these teachers that subject content matter knowledge is at the very root of their jobs - and that it is also the most powerful of all tools they can apply as mathematics teachers. To use it effectively, it is necessary to develop a deep understanding of the mathematics they teach, making

it more flexible and establishing connections among different mathematical topics. We hope to have made them aware that this is a never-ending job.

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