

EQUATIONS AS AN EXAMPLE OF A “PROCEPT”

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Gray and Tall coined the expression “procept” as an amalgam of process and concept (Gray 1994). This amalgamation reflects a flexible way of representing mathematical notions and is characteristic of students’ thinking. Sfard writes also about mathematical notions being at the same time a process and an object (Sfard 1991).

Piaget’s theory about acquiring a concept and the connections between the concept and the actions, which are linked to this concept, are applicable also to mathematics (Piaget 1972). According to Piaget the abstraction (or conceptualization) is drawn not from the object that is acted upon, but from the action itself.

In my research I chose the concepts of “a root of an equation” and “equivalent equations.” Both concepts have a formal definition but are naturally linked to a process, the process of solving equations. The main activity in algebra lessons is dedicated to solving equations. My research question was: After several years of practicing this action, are students capable of conceptualizing these mathematical concepts in a mental way or are they still bound to the process, thinking only through algebraic manipulations?

One hundred and ten 10th graders were given a set of seven pairs of equations and had to decide which of the pairs were equivalent. This was done after a discussion in class about equivalent equations.

The results show that when the answer was the outcome of standard algebraic manipulations, most of them answered correctly. But when there was no clear process to connect the two equations and one has to go back to the formal definition, students had difficulties. In this case there was no real interiorization in Piaget’s sense. There is, of course, a certain justification to this behavior. In these cases the correct answer has to do with a semantically abstract meaning of the concept and is rather detached from the “real life” meaning.

References

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- Sfard, A. (1991). “On the Dual Nature of Mathematical Conceptions: Reflection on Process and Objects as Different Sides of the Same Coin.” *Educational Studies in Mathematics*, 22, 1–36.