DIFFERENTIAL EFFECTS OF DYNAMIC GEOMETRY SOFTWARE ON THE ACQUISITION OF BASIC GEOMETRIC NOTIONS

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Introduction

The various capabilities of Dynamic Geometry (DG) have caused an ever-growing interest during the last decade. But many publications in this area might as well bear the generic title „The Impact of Computer Technology (on the Mathematics Classroom)", as they focus mainly on *technology* and its really promising potential, but neglect somewhat the *impact* caused by it (and all too often only allude to the *classroom*).[1]

Nevertheless, the ever-growing availability of computers in the classroom and the public expectations concerning their use entail administrative measures towards regular use of DGS, as nowadays can be readily noticed in Germany. Therefore it seems appropriate to investigate in *regular* classrooms the impact of DGS on students' achievements, conceptions and attitudes. In this paper, we consider mainly differential effects on students' achievement.

Related research

Only few concepts of computer application in the mathematics classroom have been evaluated on a scientific basis. Moreover, a detailed meta-analysis by Ruthven (1997) concluded that only a small number of studies (regarding CAS) had an acceptable design (i.e. experimental and control groups, pre- and post-tests). Concerning DGS, Hölzl (1996, 2000) meticulously performed a diversity of qualitative case studies revealing epistemological shiftings and increased cognitive challenges as unwanted, but perhaps unavoidable side effects of applying DGS.

> His results show that the interactive aspects of the medium motivate the students to develop individual interests and stamina – But there are also subtle interactions between the implementation of geometry by the software and the students' understanding of geometry. This shows clearly the double-edged character of computer application: on the one hand the heuristic potential yields a – mathematically extremely attractive – extension of scope, but on the other hand interactivity entails also the danger of "degoaling" (Hoyles und Sutherland 1989).

Thus a "computational transposition" (Balacheff 1993) seems inevitable also in the realm of DGS. In order to minimize its effect, a thoroughly and didactically reflected teaching concept is required.

Also, from an instructional psychologist's point of view, the prospective value of DGS has to be appraised retentively, since it is usually taken for granted that the choice of medium is far less important than the nature of the treatment[2]. But

[1] Hölzl (2000)

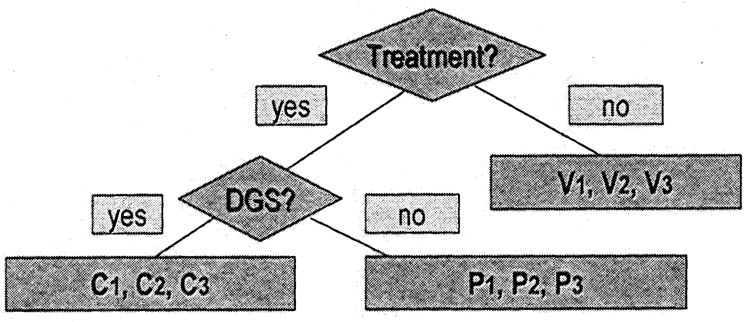
[2] Salomon (1978) and Clark (1983)

since the genuine possibilities of DGS cannot be reduced to just changing the medium, a comparative study seems nonetheless appropriate.

There are only few studies on the impact of *highly interactive* computer usage. Lewalter (1997) investigated cognitive information processing when learners utilized animated vs. static computer illustrations. Though the possibilities to interact with the software are far more restricted than with DGS, Lewalter's results shed some light on the core problem: even the restricted possibilities of the dynamic presentation are not really used by learners for the purpose of elaboration. Rather, her analysis reveals that dynamic visualizations of kinematical processes seemed so easily understandable to the learners that they all to early got the false impression to have grasped the whole issue. This result seems to be in accordance with the "degoaling" phenomena described by Hölzl (1996).

Method

Procedure The study was performed in nine grade 7 classes (N=214) of three senior high schools. At each school, three classes were assigned to the **C**(omputer), **P**(aper) und **V**(control) group yielding the following hierarchical design:



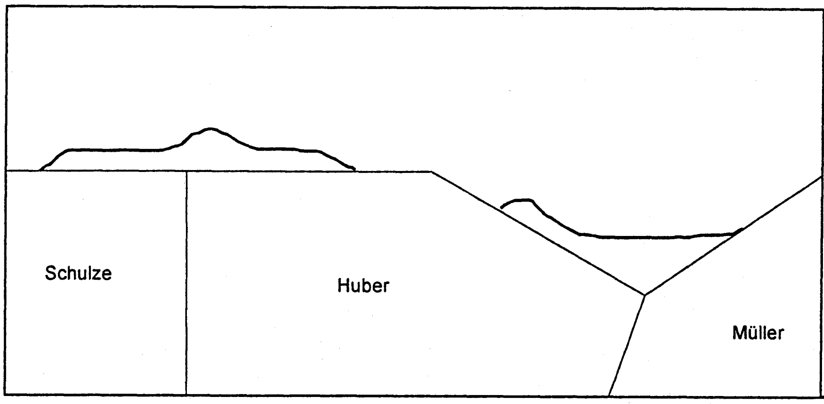
Pre-and post-tests were performed in all classes (the pre-test surveyed the prior geometric knowledge of the students as well as their attainment levels). Meanwhile, the treatment was implemented in the experimental groups using the respective learning environment (i.e. paper and pencil in P resp. DGS in C). For the 12 lessons given, teacher students that had in advance been trained intensively on our courseware replaced the usual teachers. At every school, the C and P classes were taught by the same teacher student. In the V classes, ordinary geometry lessons took place. After six months, a follow-up post-test was performed in all classes.

Treatment The treatment covered perpendicular and angular bisector, the circumcentre and the circumcircle, the incentre and the incircle. In C, we used interactive electronic work sheets[3] that focus on *investigating given* figures rather than *constructing new* ones. As an extra dimension, we added embeddings of geometric problems into problem contexts: for instance the distribution of soil was posed as a problem (compare fig.1) to introduce a small modelling process

[3] compare Elschenbroich (2001)

ending up with the necessity to find a construction for bisecting angles – thus demonstrating the utilizability of mathematical tools in "real life".

Measurement The tests can be divided into procedural and explanatory parts, so the pre-post-difference D splits into a procedural component D_p and an explanatory component D_e : $D = D_p + D_e$.



Find out how the new land can be distributed to the three peasants according to the "Roman law" (remember: new land belongs to the one, whose lands

Fig. 1

Hypotheses

1) The problem-based approach influences significantly the achievement[4].
2) The use of DGS does not influence significantly the achievement[5].

Results

The box plot of the pre-post-difference D and its explanatory component D_e in fig. 2 reveals:

- Overall outcomes were about equal in P and C and considerably higher than in V, with smaller variance in P.
- In the explanatory parts, students in P did somewhat better than in C and V, with considerably greater variance in V.
- All in all, C lay consequently a little behind (regarding both measures).

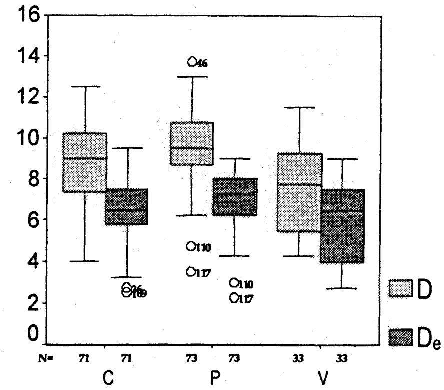


Fig. 2

Inference statistics yields somewhat heterogeneous results: In the whole population we find for D that μ (P) >> μ (C) $\approx$ μ (V), μ (P $\cup$ C) >> μ (V), so 1) can be confirmed, 2) has to be rejected – but this result is slightly obscured by the fact that the means of P and C differ only by .75. This difference is *significant* (α=.05), but is less than the *insignificant* difference 1.00 of C and V. So one might suspect a statistical artefact due to the diverging group sizes (the drop-out rate was –not uncommonly– significantly higher in V).

An explorative data analysis is thus in order. For practical purposes, we restricted ourselves to factors that are easily accessible to teachers: Besides gender, we distinguished higher and lower achievers by splitting at the median

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[4] measured as pre-post-difference, which were checked against mean posttest achievements adjusted by pretest.

[5] but the chosen strategies!

(that has also statistical advantages). Here, we have to restrict to display some tendencies in the data (fig. 3-5) [6]:

- ➢ except for girls at public schools, in C classes higher achiever profit more than lower achievers, vice versa in P,
- ➢ while in public schools P and C scores are about equal, for girls in the private schools P was (significantly) superior to C,
- ➢ for girls at public schools, C is slightly more effective than P, irrespective of achievement level.

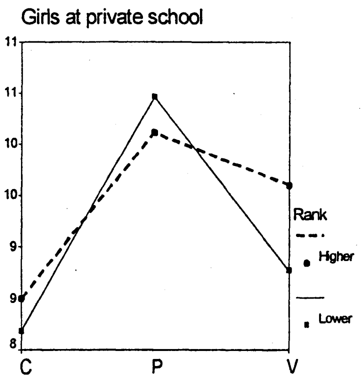

Fig. 3

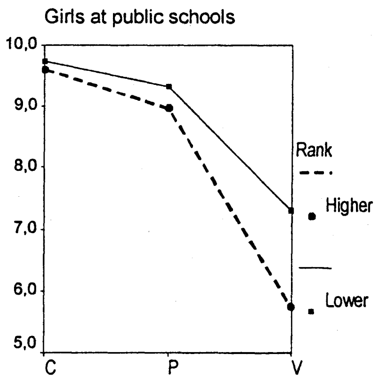

Fig. 4

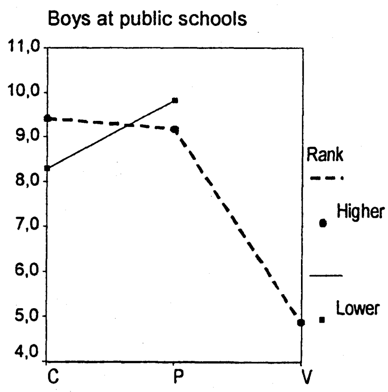

Fig. 5

Also, differences in achievements and strategies between P and C occurred with "dynamic" problems[7]: Namely, the post-test contained one especially „dynamical“ task – the students were given fig. 6 and had to answer the following questions:

1. This is a campground from above. The friends Anton, Bert and Charley want to have breakfast together. Can you place a table on the campground in equal distance to their tents?
2. What happens if Charley moves his tent along the sea to find the best fish ground? Why?
3. If he chooses a certain place for the tents, a problem will occur to the three friends. Can you figure it out?

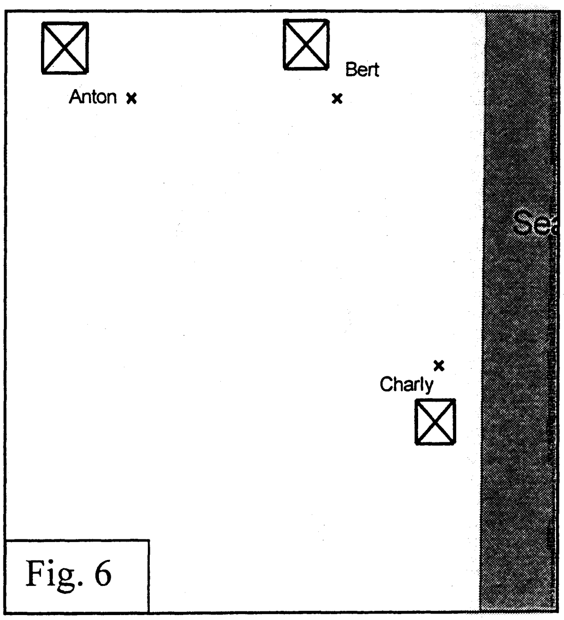

Fig. 6

1. involves of course the utilization of the circumcentre in a problem context. And to see, whether DGS was helpful in developing dynamic mental images they were asked 2. This made it also possible to check by 3. whether students

[6] See Gawlick (2001a, 2002) for more details.

[7] but surprisingly also with some "static" ones, compare Gawlick (2002).

realized the limitations of such a mathematical modelization. All in all, the "dynamic task" 2. is certainly apt to show differences in outcome if DGS is really more than "just another medium". But again the results are rather ambiguous: The total score for the "dynamic task" does not show any significant differences. But if you consider only, their hypothesis what happens to the table when Charley moves his tent, C is clearly ahead of P – but vice versa if you consider their reasons, *why* this will happen. These adverse effects nearly cancel each other. A possible interpretation would be: DGS is helpful to generate hypotheses, but not to find arguments supporting them. This would fit neatly to the .„degoaling"–effects observed by Hölzl (1996).

On the other hand, one of the rather procedural tasks was to explain the construction of the middle perpendicular. But in this rather "static" task the C groups did astonishingly somewhat better than the P groups. This may be some evidence for a reinforcement of mental images by DGS that can even strengthen students' ability to scrutinize the rationale of the said construction.

Discussion

Of course, the findings should not be overestimated – they are based on a relatively small sample and difficult to generalize due to their heterogeneity. The inconclusive statistical results are in accordance with media and ATI research. But the interaction of environmental and gender factors seems to be new in the context of DGS and – though statistically insignificant – deserves further attention[8]. It seems that in coeducative classes, girls profit from DGS treatment, but possibly at the cost of lower achieving boys. Therefore, special care should be taken of these in order to prevent them from using DGS as a plaything.

Hopefully the evaluation of the follow-up test (work in progress) will shed some light on these and eventually additional other differential factors.[9] Also, the analysis of students' strategies in the "dynamic" task may yield some further evidence how to ameliorate the disappointing outcome in this task.

Finally, it should also be noted that the preliminary evaluation of the follow-up data gives some evidence toward the conjecture that *in the long run* DGS as an additional tool for the acquisition of geometric notions may be superior to the paper-and-pencil-only approach.[10]

Concluding Remarks

It seems at present that when dealing with fairly standard examples, the benefits of dynamic exploration can even in a carefully designed course far too easily be outweighed by the extra costs of DGS. So we strongly confirm that *dynamics is not a didactical advantage per se* (Hölzl 2000) – the use of DGS should

[8] The deviating results in the private school seems difficult to interpret, but should be kept in mind when discussing the issue of coeducation concerning DGS.

9 Eventually, evidence on this will be presented orally at the conference.

[10] Again we have to refer the reader to the oral presentation and/or electronically obtainable information.

therefore be preceded by thorough consideration. It will be most favorable when an objective requirement for the tool meets an advanced mathematical experience. Therefore, the design of teaching units and learning environments that make the most out of the computer's heuristic and computational capabilities continues to be one of the most challenging objectives of nowadays mathematics education.

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