SUPPORTING TEACHERS IN BUILDING MATHEMATICAL DISCOURSE COMMUNITIES BY USING RESEARCH-BASED CASES

Pi-Jen Lin
National HsinChu Teachers College, Taiwan
linpj@mail.nhctc.edu.tw

Abstract

This study was designed to examine the effect of using research-based written cases on supporting teachers in building mathematical discourse communities. Four first-grade teachers enrolled in five workshops in which they studied the cases about first-graders' learning and teacher's roles in discourse. Through case discussion, the participants were supplied with needed experience and support for evolving their pedagogy. Asking students to explain, to clarify and justify their ideas orally, and criticizing for challenging their thinking were the three ways the participants used most frequently to encourage students to engage in discourse. The set of discussion questions in each case appeared to be an essential contributor to have a salient focus for the case discussion.

Key words: cases, narratives, mathematical communication, discourse.

Introduction

Communication is central to the current vision of desirable mathematics teaching (NCTM, 2000; MET, 1993). The process of creating mathematical discourse communities dealing with complex and multifaceted undertaking is a challenge for teachers (Lin, 2000; Silver, 1996). Teachers are challenged by the interplay between the reform vision of instruction and their own experience with more traditional tasks and pedagogy. Helping teachers toward an instruction rich in communication is likely to require new experience of learning mathematics in a manner that emphasized discourse and require needed support from collaborative communities of practice in which mathematical discourse occurs. One of the ways to meet the needed support and acquire new experience for teachers is through the use of cases that reflect others' experience on encouraging students to participate in discourse and centering discourse on mathematical ideas (Harrington, 1995). Therefore, case discussion is considered to be the kernel part of the study for helping teachers creating mathematics discourse communities.

The research-based cases involved in the study refer that the cases were constructed collaboratively by the researcher and teachers participating in a previous school-based teachers' professional program. The program is designed to help teachers keep with the tenet of 1993 vision of curriculum reform that emphasized engaging students with challenging mathematical tasks and enhancing students' levels of discourse about mathematical ideas. The effect of cases on supporting teachers' professional development has been examined (Lin, 2000). It indicates that both case discussion and case writing are two critical aspects of cases on developing teachers' thinking (Lin, 2000;

Richardson, 1993). Since participating in the research context in which cases were constructed, teachers are more likely to empower the effect of case writing and case discussion. However, it is still considerable whether the research-based cases in a written form would contribute to building discourse communities in mathematics classrooms of those who did not participate in the research context in which the cases were constructed.

Cases referred to in part of previous studies are conducted by personal practical experience (Barnett, 1998; Shulman, 1992). Thus, such kind of cases is intimately tied to personal practice and lacking of multiple levels of interpretation and analysis. As suggested by Merseth (1996), the research-based written cases referred to in the study are characterized in four essential ways. (1) Cases are based on careful research. The first-grade case-teachers participating in the previous professional program have been collaborated with the researcher to enhance their mathematical instruction through an emphasis on mathematical communication within the context of innovative curriculum. (2) Cases present reality. (3) Cases are developed to stimulate thought and debate for expanding the perspectives of users, but not to provide best practices. (4) Cases are potential to help users to recognize salient aspects launched from a set of discussion questions.

Researchers have examined how teachers interact each other through case discussion (Levin,1995; Richardson, 1993), but little empirical work describes how case discussion influences teachers creating mathematical discourse communities. Thus, this study was designed to examine what teachers learn and investigate how teachers establish mathematical discourse communities in their classrooms through the discussion of research-based written cases.

Theoretical Perspectives

The theoretical rationale for this study stems from the constructivist's perspectives. Piaget (1932) suggests that children's conflicting ideas are resulted from peer interactions. Cognitive conflicts result in an imbalance providing the internal motivation for an accommodation. Similarly, conflicts can occur for teachers, when they learn to teach individually or socially. The social interaction in a group discussion has the potential for initiating cognitive conflicts, hence to result in teachers' change. Vygotsky (1978) asserts that what is learned in the social interaction of the group is prerequisite to cognitive development. From this perspective, the interaction and the content of the group discussion are crucial to what is learned from cases. This would manifest itself in differences in understanding the issues in cases between those who do and do not participate in the process of constructing cases. In addition, his notion about "the Zone of Proximal Development" can be interpreted as suggesting that cognitive conflicts, caused by discussing, debating and negotiating in interactions between learners and more capable peers, act as a catalyst for reaching a higher development level. This indicates that case-teachers with more experience should influence the thinking of other teachers with less experience who interact with them in the case discussion. The two theoretical perspectives from development and social psychology provide a basis for helping us think about how knowledge is constructed both individually and socially. Case discussion fosters personal reflection through an external process (Shulman and Colbert, 1989). This notion suggests that social interaction during case discussion among the case-teachers and the teachers in a group seems to be likely to initiate cognitive conflicts and then contribute to reaching a higher level of psychological development.

Method

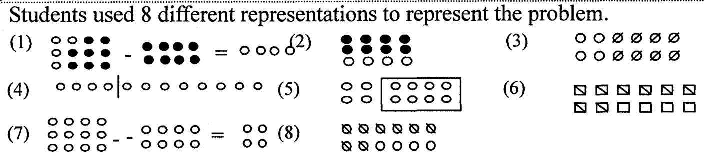
The participants in the study were four first grade teachers (Lao, Pan, Hu, and Wu) at two schools, located in a suburban area. They were experienced teachers with at least 10 years of teaching experience, but they were beginning teachers in their second year of teaching the 1993 vision of innovative curriculum. The participants were volunteers to participate in five workshops as part of a teacher education program that was designed to help teachers understand how first-graders develop mathematics concepts and identify various aspects of a teacher's role in mathematical discourse.

The research-based cases have the following major issues: (1) a mismatch between teacher's goal and objectives of a lesson, (2) students' various solutions of resolving a problem, (3) inappropriate tasks, and (4) framework of underpinning the curriculum. Through the cases, teachers are expected to deepen their understanding and expand their views of students' ways of thinking, hence to identify various situations in deciding what to pursue in depth, when to model, and how to encourage student to engage in tasks.

A set of discussion questions is one of the six components included in a case (Lin, 2000). The discussion questions", as kernel part of each case, are incorporated the reflections of case-teachers who involved in constructing the case into the major concepts to be learned of the lesson. The questions listed in cases are to stimulate teachers' rethinking about mathematical teaching and reflect to their practices. A set of focus questions of the case entitled "How do you categorize them?" serves as an example (see Figure 1).

There are 12 toys. Kei-Hua took 8 of them away. How many toys are left?

Students used 8 different representations to represent the problem.



Discussion Questions

1. How would you categorize the various representations? Why did you do so?
2. Which of the representations would be presented in your teaching?
3. Do you consider the factors including the forms, arrangements, and the order of crossing out the pictures when you categorize them?

Figure 1 Discussion Questions in a Research-Based Written Cases

As part of the workshops, the discussion of each case is used to probe for responses to the questions in the written cases. To structure case discussion, I, as the leader of the discussion, played various roles in order to guide, probe, give feedback, and observe each participant's thinking about cases. There were two common questions to be addressed across cases in the case discussion session. (1) How would you respond to each question of the "Discussion questions"? (2) What are some of the issues in this case coming up in your mind in this case?

The participants read and discussed eleven cases in the five workshops. They were observed during their mathematical instructions in the entire year. To document the efficiency of cases resulted in teachers' change, each workshop determined the time of each observation. There was around one-month difference between two workshops. The time of the four classroom observations and five workshops was depicted as O_1-W_1-W_2-O_2-W_3-W_4-O_3-W_5-O_4, O_i: the i^{th} observation, W_j: the j^{th} workshop. Available data for each participant's four curriculum units include 12 classroom observations. O_{pre} including the O_1 and O_2 observations occurred in the first school term and O_{pos} including the O_3 and O_4 observations in the second term.

The data were collected from the participants' responses to questions of each case in the five workshops and their instructions to be observed. The videos and audiotapes were recorded and transcribed verbatim. The data was analyzed by using Strauss and Corbin's grounded theory (1994). Through recursive reading the transcriptions of each video teaching, a coding scheme including ways of communication and contents of teaching was developed. There were nine ways of commutation in mathematics emerged: restating students' statements (R), asking students to explain what they understood (E), asking students to clarify and justify their ideas orally (C), criticizing for challenging students' thinking (J), giving students hints when students struggle with a difficulty (H), encouraging students to share their answers (S), asking students to model or demonstrate their understanding (D), motivating students to provide multiple ways of thinking (M), and inviting other students' follow-up ideas for making up an incomplete interpretation (I). Each lesson was categorized into various phases of teaching across mathematics contents when discussing the problem to be solved, the use of hands-on, symbols and number sentences given by students. Each sentence in the transcription under a unit of analysis was encoded and the frequencies were counted for each lesson. The constant comparative analysis was used to compare data segment, determine similarities.

Limited space prevents to report what the teachers learned from each research-based written case and how they learned about building mathematical discourse communities. An illustration of the case influencing on teachers' thinking about cases is taken from one case that deals with three levels of representation. To illustrate in-depth the effect of case on teachers' ways of communicating mathematics with students, a participant's lessons as an example will be the focus of analysis reported in this paper.

Case discussion on teachers' awareness of students' learning mathematics

Initially, the four teachers were puzzled with their students' early use of number sentences prior to teaching it. They were not aware of the importance of the development from concrete, pictorial and to symbolic representations to first-graders. The issue of representations listed in the discussion questions of a case was brought up in the second workshop to discuss as follows.

> As observed, a student wrote a number sentence 8-5=3 first, and followed by drawing the circles oooooooo - ooooo = ooo representing the given problem: *"There are 8 children and 5 presents. Each child gets a present. Are the presents enough?"* Do you accept the student's representation, oooooooo - ooooo = ooo?Why? What instructional strategy would you take to reduce the use of oooooooo - ooooo = ooo? (2nd workshop, 11/08/2000).

In the discussion, Ms. Lao and Ms. Wu reflected their students' high frequencies of using number sentence to their first year of teaching using the innovative curriculum. They faced the challenge of students' disordering learning of pictorial with symbolic representation, but they did not realize its hindrance of learning in multiplication. They perceived that their students did not make sense to the meaning of the number sentence at this time, so that the circles most first-graders drew merely meet teacher's needs rather than represent a process of student's thinking. Understanding a mathematics concept meant by them was that students are capable to use it. Thus, prior to teaching it, they accepted students' use of the number sentence 8-5=3 followed by oooooooo - ooooo = ooo no matter who they learned from. Lao suspected why students must go through the use of hands-on and drawings, since they had used 8-5=3 to solve the problem. According to Lao's response to the issue, she did not appreciate the importance of the developmental processes among the three levels of representation to the future learning.

To resolve the puzzle, Ms. Hu offered an instructional strategy in the group discussion. "Decomposing a step into sub-step" means that "a word problem is divided into several parts, one part is presented at a time and students' follows". A problem described by *"There are 8 children and 5 presents. Each child gets a present. Are the presents enough?"* is a typically complete statement. Alternatively, the problem is posed by decomposing it into three sub-steps as "*There are 8 children, representing the 8 children by chips; 5 presents, showing the presents by chips. Are they enough? (counting the amount of chips indicating the answer)*". In the discussion of the case, Hu not only introduced its meaning but also explained the merits of the alternative approach to other three teachers who have never heard this term before.

They supported mutually. Lao, Pan, and Wu learned the meaning and the strategy of "Decomposing a step into sub-step" from Hu and put it into their following lessons. Moreover, they became an opponent of the approach, since the strategy indeed resulted in the low frequencies of their students' early use of number sentence. Lao stated the efficiency of cases in the third workshop.

> The previous case discussion helped me perceive my blind spots in instruction. Reading and discussing it ahead of my teaching make significantly effectiveness of teaching the innovative curriculum in this year. I re-taught previously unsuccessful lessons by taking "Decomposing a step into sub-step" approach aiming at the use of hands-on or pictorial representation of a word problem. Through the approach, I found that first-graders' at this stage did not use the symbolic representation to solve the word problem any more. Thus,

if I can understand the main ideas of a lesson in which they are focus questions of a written case, then I would not be frustrated with students' difficulty with learning. Moreover, I could predict students' various solutions in advance and then help them have better understand (Lao, 3rd workshop, 12/06/2000).

Case discussion on supporting teacher in building mathematical discourse

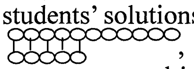
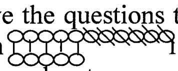
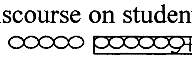
The topic of the lessons determines what and how teachers communicate mathematics with students. To be coherent the effect of cases on teachers' thinking about cases with supporting them in building mathematical discourse communities, the analysis of this section will be only focused on the lessons relating to number area. The lessons in a curricular unit is determined by each teacher's instructional activities. The textbook Lao and Pan used is different that of Hu and Wu used. The lessons of number topic included in the textbooks scheduled on the list to be observed in the study were included only in Lao's and Pan's six lessons. Lao's six lessons serve as an example shown in Table 1.

Table 1: Frequencies of Ways and Contents of teaching in Lao's Mathematical Discourses

Content \ Frequencies. \ Ways		Restate statements (R)	Give students hints (H)	Explain (E)	Model (D)	Share answers (S)	Clarify justify (C)	Criticize (J)	Multiple thinking (M)	Compensate incomplete answer (I)	**Totally**
problem posing	O_{pre}	18	4	16	2	3	20	6	5	1	**75**
	O_{pos}	0	2	31	10	13	13	13	10	8	**110**
hands-on	O_{pre}	12	1	11	11	1	19	12	0	11	**78**
	O_{pos}	7	0	23	18	3	29	33	3	18	**134**
drawings	O_{pre}	1	1	13	1	8	3	5	0	1	**33**
	O_{pos}	1	2	41	2	3	46	7	39	0	**141**
symbols	O_{pre}	3	1	19	17	9	20	26	3	4	**102**
	O_{pos}	8	0	26	25	3	14	24	13	3	**116**
Totally		**60**	**11**	**180**	**86**	**43**	**164**	**126**	**73**	**46**	**789**

$\mathbf{O_{pre}}$: observations occurred in the first school term. $\mathbf{O_{pos}}$: observations occurred in the second school term.

The data of the Table 1 shows that the most three frequencies of ways Ms. Lao used to encourage students to participate in classroom discourse were asking students to explain what they understood (E), asking students to clarify and justify their ideas orally (C), criticizing for challenging students' thinking (J). The frequencies of the three ways used by Lao were 180, 164, 126, respectively. Lao's ignorance of the importance of the pictorial representation described earlier was also evidenced by the data shown in her communication in classrooms. The data suggest that Lao did not frequently encourage students to draw circles representing the process of their thinking in the first school term (33 frequencies) as compared to the second school term (141 frequencies). The result indicates that the cases influencing on Lao's more attentions to the discussion of drawings with students. Lao improved her skills in asking students to explain (41 frequencies) and to clarify what they understood (46 frequencies) as she sought to engaging students in the meaning of drawings they drew.

As observed, Lao created a mathematical discourse community in discussing various strategies of a compare word problem. After posing the problem "*Shiao has 11 marbles. Mei has 5 marbles. How many more marbles does Shiao have than Mei? Drawing pictures represents your thinking and writing its number senescence with subtraction.*", she furnished opportunity for students to share their solutions and justify their thinking publicly. In the class discussion, Lao frequently asked students the question "Does anyone of you have different solutions?" to encourage students showing their different solutions from others'. To provoke thoughtfulness in the discourse, she asked students various questions about their answers. The questions were varied with students' solutions. For instance, after Kao-Bin offered his solution as 11-5=6 , Lao asked the questions to Kao-Bin "what do the circles mean presented in the above row?" "why did you use the way?" "What does 11 represent in 11-5=6? How about 5?How about 6?". Likewise, Lao asked Uein-Jye the questions to clarify and justify his answer, after he provided the solution  11-6=5. "Why did you write 11-6=5?" "What did the part crossed out represent?" Could you tell us which part represents Shiao's more marbles than Mei's?" "What does the difference between your drawings and Kao-Bin's?". Lao did not stop the discourse on students' various solutions until Shin-Jane presented the solution 6=11. Because Lao learned first-graders' possible solutions coming up in the problem from previous workshops, she expected the third solution showing up in the class discussion. Meanwhile, she asked a question to criticize for challenging students' thinking, "Did 5+6=11 given by Shin-Jane meet the demand of the problem in which number sentence is expressed by the subtraction?" From the above scenario, Lao clearly had progressed in satisfying the demands of listening carefully to students ideas, encouraging students' engaging in thinking about mathematical ideas, and asking students thoughtful questions to clarify and justifying their ideas orally.

Discussion

It is found that the use of research-based written cases enhanced the teachers' ability in creating mathematical discourse communities. In the discussion of cases, the teachers were supplied with new experience and needed support of mathematical discourse community from the member of the discussion group. They learned about the role of the researcher in creating mathematical discourse communities from the case discussion in which the manner is similar to that of mathematical discourse in classroom. Likewise, the researcher learned from the teachers about what they responded to the discussion questions described in each written case and how they learned to teach. The teachers supported mutually in understanding how first-graders learned mathematics by the discussion of the case. Lao's acquisition of students' various solutions from case discussion was an important contributor to her success in moving toward the creation of authentic mathematical discourse communities in her classroom. The new experience and needed support indicate that the teachers appeared to evolve their thinking and pedagogy from a traditional form toward a form of instruction centered with mathematical discourse. In Piaget's (1932) and Vygotsky's (1978) notions, the case discussion in a group created the opportunity of social interaction for the teachers. The set of discussion questions integrated with case-teachers' various perspectives are readily to initiate the teachers' reflection to their practices and cause their cognitive conflicts of mathematical teaching, hence to trigger change. The set of discussion questions are integrated with multiple perspectives of those who participated in practice-oriented research. The case-teachers embedded in research-based written cases play the significant role of more capable peers.

The research-based cases were not expected to provide best practices but to initiate the users' cognitive dissonance. Instead, the cases referred to the merits and breakdowns in classroom practices that call for solutions within the context of that practice. Thus, the use of research-based cases was more likely to support reflection-on-action and then oriented toward reflection-in-action (Schon, 1987). The discussion of written cases seemed to be a catalyst for the teachers creating mathematical discourses in classrooms. The set of discussion questions described in each written case appeared to be an essential contributor to have a salient focus for the case discussion. The set of discussion questions as a kernel part of the written cases become distinguished characteristics that are not possessed in the videotaped cases. The comparisons between the effect of cases presented in a written form and in a video form on either in-service or pre-service teachers' thinking about cases and decision-making of classroom are valuable for further investigations.

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