SEVEN YEAR OLDS NEGOTIATING SPATIAL CONCEPTS AND REPRESENTATIONS TO CONSTRUCT A MAP

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We report on-going research on the spatial – geometrical meanings generated by two groups of seven-year olds collaborating to construct a map of the area surrounding their school. The learning environment involved the use of a G.P.S. device dynamically linked to cartography software and was designed so that each group worked in a different external representations environment and was reliant only on verbal communication with the other in order to co-construct meanings which would enable them to make the map. Results showed student negotiations about concepts of motion, location and orientation to include instances where they dissociated from the software representations and made choices from a variety of geometrical reference systems.

FRAMEWORK

We report on-going research on the spatial – geometrical meanings generated by two groups of seven year olds collaborating to construct a map of the area surrounding their school. The learning environment was designed so that each group worked in a different external representations environment and was only able to communicate with the other group verbally in order to co-construct meanings which would enable them to make the map. One group physically roamed the area carrying a Global Positioning System device. The other worked indoors using cartography software[1] where they could see an icon of the G.P.S. carrier on the screen dynamically moving and turning in the corresponding direction in synchrony to the carriers' movement. Both groups could communicate verbally with walkie-talkies. We studied their actions and talk while they made discriminations between their different environments (screen representation and actual space) and attempted to develop some joint understanding about position and orientation in order to construct the map in the context of making a treasure hunt game.

Related research has highlighted the dual nature of the map as representational medium and as embodiment of spatial concepts i.e. scale and absolute (Cartesian, polar) and relative (Euclidean) orientation, position and motion (Liben &Yekel 1996). In that context, map *construction* may provide an environment for creating and understanding *representations* of such spatial concepts in order to use them for negotiating about the concepts themselves. As Kaput et al (2001) put it:

> 'People need to represent for themselves how things work, what makes systems fail and what would be needed to correct them. This kind of knowledge is increasingly important;

it is knowledge that potentially unlocks the mathematics that is wrapped invisibly into the systems we now use and yet we understand so little of' (p. 16).

This research draws from two perspectives in analyzing the problematic in students' understandings of spatial concepts and representations. One has to do with the symbol–referent relationship and specifically what we could refer to as the *distance* between the two, i.e. the level of abstraction needed to relate surface structure (signifier) and deep structure (signified). In discussing algebraic notation, Kaput (1987) stressed the importance of students' abilities in choosing, building and interpreting mathematical representations and their difficulty in relating these to the represented concepts, rather than merely focusing on the surface rules of the representations themselves. Research with younger children involved attempts to narrow the distance between symbol and referent by means of using tangible objects to represent real objects in space (e.g. a model of a classroom and the objects inside it, Lieben and Yekel, 1996). DeLoache et al, (1999), however, found that the children have difficulty dissociating from the tangible properties of the representational objects in order to relate them to the represented ones and displayed better understanding of iconic representations which were more distant to the referent space. DiSessa studied primary students' bottom-up methods of building abstract representations (graphs) in order to negotiate and explain experiences with physical phenomena (diSessa, 2000). On the other hand, however, Nemirovsky et al (1998) suggested that fusion between signified and signifier can be a source of intuition for understanding representations, in environments designed to encourage linking kinesthetic experiences with their immediate iconic representation on a computer screen. In our study we had two groups of students negotiating to understand and build on representations of space by providing one group with tools to interpret and create iconic representations and by placing the other in an environment constituting the referent space itself.

The other research perspective relevant to our study has to do with the spatial – geometrical concepts embedded in map construction and students' related intuitions. Even though Papert coined the term body-syntonicity (1980) in describing students' use of their intuitions about their own movement in order to understand motion and orientation on the plane, subsequent studies revealed different sets of intuitions employed in settings involving a different system of reference to the plane, such as the Cartesian (Lawler, 1985). Students find it particularly difficult to relate their intuitions about Topological, Cartesian and Intrinsic systems of reference (Kynigos, 1992). Most research, however, is based on learning environments focusing on the use of one system at a time and on its use for the understanding of geometrical figures and properties, rather than studying the ways in and the reasons for which students choose between systems to understand concepts related to location, and orientation. In this research, we focused on the students' respective choices in a setting where these reference systems were simultaneously available since they could choose to refer to external objects, to bodily movements and to relative positioning of external objects.

SETTING

The activities

Two pairs of seven year-old pupils collaborated for the joint construction of a map. One pair was the "base team", situated indoors in front of the computer screen. The other was the "roamers", following the teacher who carried the bulky G.P.S. Whole group discussions took place before and after each research session, involving setting out goals and experience exchanges. The two groups exchanged roles when a part of the activity was completed. The activity involved creating a map of the area around the school and then using it to play a treasure hunt game. That area had a number of identifiable objects, such as trees, water taps, waste bins, lamp - posts, the flag. The two groups had to communicate about the objects in the area so that the base team could place ready made or child - created icons on the corresponding locations on the screen. The rationale underlying the deliberate differentiation of pupils' perspectives while working on the same task was to pivot the activities around language exchanges (Nemirovsky et al, 1998). Before the map construction activity, the students engaged in two dot-to-dot type tasks. The first involved rubber cones placed on the playground and numbered icons of these on corresponding positions on the screen. The base team had to drive the roamer team from cone to cone in the right sequence (the roamers could not see the numbers) to form a star shape. The second involved the base team instructing the roamer team to place the cones in the corresponding positions to form the shape of a fish.

The computer environment

From a technical point of view, the environment was based on synergy between high accuracy G.P.S. (around 1 meter) and corresponding cartography software. The G.P.S. continually fed data to the computer via mobile phone so that the effect would be a dynamic synchronous correspondence between G.P.S. motion and orientation change and the respective changes of the roamer icon on the screen. The software involved a combination of a G.P.S. receiver component with a map and an icon editor component ('E-slate' software, see endnote). It was thus designed to embody a scalable representation of physical motion on the plane (the roamer icon moved leaving a linear trace and turned in the direction of the motion) in combination with the students placing pictures or self – constructed icons of objects in positions and orientations decided after negotiation with the roamers (see picture next to excerpt 1). The teachers or authors of the software configured the scale and orientation of the simulated plane so that it matched the base teams' experience of the physical space outside the classroom.

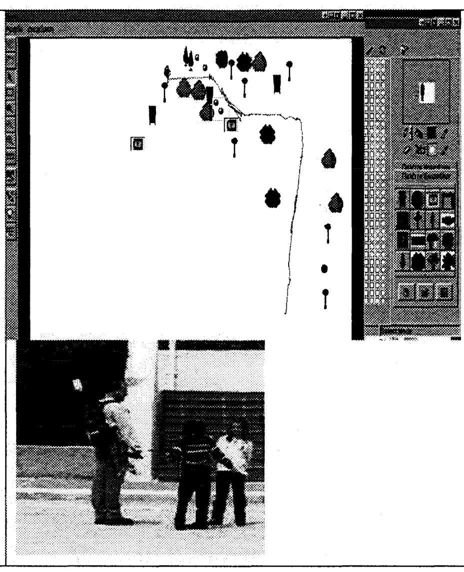
RESEARCH METHOD

The study lasted for 16 hourly sessions with the same seven year-old students, 3 girls and 1 boy. They were selected in collaboration with their teachers so that they ranked at the middle of their class with respect to their classroom general abilities, knowledge and willingness to collaborate and communicate. We adopted a generative methodological stance, attempting to gain some detailed insight into student - generated meanings. Both groups were videotaped simultaneously throughout all the sessions of the research. For the data collection we engaged in participant observation intending to stimulate students to make their thinking explicit as well as to challenge their actions and explanations. During the observations we focused on a) the verbal exchanges between groups and among the members of each group separately b) students' motions and gestures and c) the data captured on the screen. In our analysis we consider the exchanges between the groups in relation to the exchanges within each group. The discourse analysis focused on our subjective interpretation of the students' communicational intent and meanings rather than on some pre-defined 'objective' methodological construct.

USING THE REPRESENTATION AND ACTUAL SPACE AS RESOURCE

One of the key features of the learning environment requiring rigour in the children's exchanges was that one group used representation on the computer screen as resource for these exchanges while the other used their direct experience of the area in question. During the activity, neither team had access to the others' experience apart from being able to talk about it. The base team could also "see" the roamers' position and heading represented on the software. In accordance with our theoretical framework, its not surprising that at the beginning, the base team seemed bound to the representations on the screen, not being able to discriminate which of these were dependent on its orientation and which could be directly associated with the roamers' physical experience. This was manifested in the language they used in their attempts to communicate as shown in the following excerpt. S1, S2 are the base team, S3, S4 are the roamers, and wt stands for talk into the walkie-talkie. S2 used the word "downwards" signifying the literal orientation of the screen. S1 asks S2 to clarify some end point as direction finder, in effect requesting for an external reference point (line 2). The roamers' also take S2's comment literally thinking she's asking them to sit on the ground. However, since this request did not seem reasonable, the need to re-negotiate words and meanings arose. S1 tries a different idea, linking the requested heading change to the roamers' present one, in effect, using an intrinsic frame of reference by taking the roamers' point of view (line 7). From a mathematical point of view, this is correct. S2 cannot see that, bound to the upright orientation of the screen. S1 tries to mediate between the two parties by eliminating the word "downwards" which was explicitly causing confusion. However, she uses the word "diagonally" indicating that she too is bound to the screen representation, even though in her previous comment she seemed to have disentangled from the screen addressing the roamers' heading alone.

1. S2wt: As you are [means keep the heading you have] move downwards a bit.
2. S1: Where to?
3. S4: Should we sit on the ground?
4. S3: What did she say?
5. S4: As you are move down a bit.
6. S4wt: What do you mean? How? Towards the ground? [waiting]
7. S1: Move a bit towards the direction you are looking at.
8. S2wt: Move left, diagonally down.
9. S4wt: What do you mean move down?
10. S1wt: Move left diagonally.



Excerpt 1

At another session, this issue arose again, provoking more articulate language in the children's attempt to negotiate a request for a roamer move (excerpt 2). S3 indicated the roamers' position in space in terms of her perspective (line 11). S2 appears to try to relate the referent space with the next target point on the screen and to make an inference on what the change of heading should be (line 13). S1, on the other hand seemed to prefer to use external points of reference (using the word "towards" and pointing, line 12) but with no indication that she was thinking about the external space and not just the screen. However, her adamant comment that "down" was not communicable (line 16), brought S2's comment on the walkie-talkie, requesting a change in direction and position using intrinsic terms of reference associated with the roamers' perspective (line 19).

11. S3wt: We are looking towards the flag
12. S1 They should go towards there [indicates the point on the screen with her finger]
13. S2: So if they are heading to the flag they should move with their left hand [he means towards their left hand side] down
14. S1: Yes
15. S2: What?
16. S1: Down, they won't understand
17. S2: So, turn on your left
18. S1: And move a bit

19. S2wt: Turn left and walk four steps

Excerpt 2

The specific episodes we presented here along with the body of our data indicate that there were instances where students attempted to relate symbols and referents and vice versa even though they initially referred to spatial concepts with expressions bound to the specific resources (line 4). Even though the students had to grapple with a set of representations with varying distances between symbol and referent, the dynamic link between physical motion and its representation and the continual negotiations to relate symbol and referent provided them with rich opportunity to develop rigorous communication and generate corresponding meanings.

USING MULTIPLE REFERENCE SYSTEMS

As mentioned before, the environment was set up so that the children would have opportunities to use concepts which we may assign to three different representational systems, the projective or intrinsic, the Euclidean and the topological. In line with previous research, we were expecting the children to use concepts from all three, but were interested to investigate whether they might show preference in using one system over the others and in the case where a child would use concepts from more than one, what was it that triggered their choice. Furthermore, we were interested to study the ways in which they made relations between concepts belonging to different systems, having in mind that the previously mentioned research has shown this to be particularly difficult. Our data indicates that children consistently used concepts from all three systems. The excerpts we present here illustrate this use. In the following excerpt for example, S2 seems to employ a combination of intrinsic and Euclidean concepts, since she uses the term "left" and the term "diagonally". With respect to the latter, however, the dialogue indicates that it was the tangible size of the screen representation which enabled some logical connection between the two concepts. It may have well been that her intention was to find words for the concept of a 45 degree angle turn, and since the children did not know about degrees or angle size yet, it was the next best way to convey that meaning. S3 seems to consider the terms "left" and "diagonally" as contradictory, having a notion of left as a right angle turn and not being able to see himself walking on the edge of an area so that its diagonal would have meaning. However, that specific moment would have been an opportunity to help the children focus on the meaning of turn size and of the scale of the area they were mapping. It important to consider this, since the whole activity was replete with such events.

20. S2: Go left diagonally. Over
21. R2: Look at the agent. Ok? Look also at the agent.
22. S3: Left. Diagonally. I didn't understand a thing.

Excerpt 3

So, this use of concepts from different systems in an ad hoc fashion brought about the need for rigour in the children's communication, since the result of what they jointly understood was immediately visible on the screen. In amongst the occasions where a combination of concepts were used, there were instances of expression of the respective mathematical meaning. The following excerpt is such an example. In excerpt 4, S3 used a combination of an intrinsic ("move left") and a Topological ("not the first but the second cone") notion. He does not use some screen – bound representation and addresses the cones (which a mathematician would perceive as standing for points on the Euclidean plane) as a means to specify the direction he wanted S2 to turn towards. In doing that he makes a relative use of two cones.

23. S3wt: Move left towards not the first but the second cone
24. S2: Left, which means this way [shows left hand]

Excerpt 4

Finally, during the map construction activities, there were many instances where topological relations between objects came into play (Excerpt 5).

25. S3 wt What is next?
26. S1 What did we tell them before?
27. S2 About the water taps...
28. S1 They have to put the trash can and the lamp post
29. S2wt: ... In front of the water-taps there is a trash-can and further ahead there is a lamp –post.

Excerpt 5

In fact, S2's comment is quite difficult to disentangle, since she uses the water –taps as a central reference point to place the other two objects taking for granted that her own position and heading are clear and that the base group can associate her comment with that. This may mean that it was hard for S2 to conceptualise the topological relation between the objects she could see in conjunction to her own intrinsic position and heading. The excerpts in this section, however, indicate that the environment provided rich opportunity for the use of concepts from all three reference systems and in this way invited appropriate didactical engineering for the children to create relations amongst these.

CONCLUSION

The learning environment seemed to provide rich opportunity for the children to negotiate spatial concepts during the co-construction of the map. With respect to the kinds of intuitions employed for referring to position and orientation changes and to objects and the relative positions, we did not detect some preference with respect to system of reference. It would be interesting, however, to study over more time perhaps what kinds of strategies and connections children might make amongst concepts associated with the three systems. The interdependency between symbols and referents inherent in the learning environment played an important role in students' understandings of relationships between the two. In this sense, relating symbol and referent in meaningful settings with carefully designed dynamic representational tools may play an equally if not more important role in understanding and creating representations to the distance between these and the corresponding concepts. This may provide some explanation of the seemingly disparate views of di Sessa and Nemirovsky et al described earlier.

NOTES

C Cube: Children in Choros and Chronos, European Commission, Esprit LTR, Experimental School Environments, #29346, 1999-2000. The cartography software was co-developed by the authors with the E-slate software (http://e-slate.cti.gr). In a version for high school students, the object icons can be linked to information on a student created database and scaling of building perimeters can be implemented through variable Logo procedures dynamically manipulated with a variation tool.

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