

DIFFICULTIES ARISE WITH IDEA OF COMPLEMENT
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On the basis of experiences from long-term experimental teaching in a primary school and other experiments M. Hejný suggested that further research on the phenomenon of a complement with pupils needed to be carried out. The aim of my poster presentation is:

1. To show several concrete examples where the phenomenon occurs.
2. To outline prepared research.

Illustration of the Idea of Complement

Many areas of mathematics need the idea of the complement to reduce the work involve in the normal arithmetic or measuring circumstance. I list below three such possibilities.

A. Pupils are given Fig. 1 and they have to find the number of triangles. The pupils count it in the following way: $2 \times 5 = 10$. Then when given Fig. 2 they have to find the number of triangles as well. They count each triangle separately and the black dot is disregarded. Most pupils at 8 years do not discover the idea of complement, that is to find the number of all the triangles and then to subtract the one black dot.

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Fig. 1

Fig. 2

B. Most pupils do not use the idea of complement in the processes of addition and subtraction even though they are aware it. For example the task $14 + 29$ using the idea of complement could be solved as $14 + (30 - 1)$ but pupils usually solve these tasks using decomposition of the second number as in this example $14 + (20 + 9)$ or even $14 + (20 + 6 + 3)$.

C. The idea of complement is also present in the finding the area of triangles on squared paper. Pupils getting Figure 3 find the area by a cutting method – two ‘half square’ triangles give one square and one plus one square gives two squares. However with Figure 4 pupils see that the cutting method does not work and they usually struggle to find a solution. It is difficult for them to discover the idea of complement – the triangle lies in a 2×2 square and when we cut the marked triangle from this square pupils can more easily find the area of remaining parts. They then subtract these from area of the 2×2 square to get the required area of the triangle.



Fig. 3



Fig. 4

I intend to develop the research which I have already undertaken involving the Abracadabra problem (Polya, 1966). This research did not involve the phenomena of the complement. To be able to research ideas of the complement it will be necessary to reconstruct the research tool so that it is suitable for pupils in the age range 9 to 14 years.

Reference:

Polya, G., 1966: ‘Mathematical Discovery’. John Wiley & Sons Inc. USA

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