PROMOTING STUDENTS' AWARENESS IN APPLYING BIJECTIONS IN ENUMERATION TASKS

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Much mathematical knowledge is retained in the mind in a factual mode that is not in the form that can be readily accessed for problem solving activities. This was noted as far back as 1929 when A. N. Whitehead talked about 'inert knowledge'. We conjecture that this phenomenon may occur for some very basic notions. In this communication, we will specialize in one such notion. A 1:1 and onto function cognitively corresponds to the pairing off of each element of the domain with one of the co-domain such that no elements are left unmatched after the process ends, so bijections preserve set order. (Here the more overtly definitional case of infinite sets is not considered.) However, if students are not aware of this knowledge as a potential problem- solving tool, they could not avail themselves of approaches involving the construction of bijections in enumeration tasks.

The study has two aims. The first is to provide some evidence that indeed the knowledge of the preservation of set order under bijective correspondence is inert. The second is to test whether the above situation may be ameliorated through a particular teaching approach. What motivates the teaching approach is the following. To try to make the students more aware of applications of bijections in enumeration tasks, we propose a general framework about developing techniques to address inert knowledge. A technique lays down several standard stages to achieve, but their accomplishment may require substantial problem solving. An important factor in a technique is a 'cue', i.e. a description of the circumstances where the technique may be regarded as being worth considering. The rationale behind the cue corresponds to the 'releasing' of some inert knowledge. When the understanding of why the technique works is transparent, the awareness of the technique depends mostly on the familiarization with its cue. The idea of the teaching approach is to prompt the students' attention to the cue (in its contextual form), and after this to (largely) leave the students to carry out the application itself. In our case, the cue would either be two sets whose orders we wish to compare, or one set whose order we wish to determine by identifying another (better understood) set for a correspondence.

In the presentation, we shall briefly indicate how the above aims were implemented in fieldwork conducted on college students, and shall outline the results and conclusions reached.