TEACHERS' QUESTIONS AND THEIR ROLE IN HELPING STUDENTS BUILD AN UNDERSTANDING OF DIVISION OF FRACTIONS

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The purpose of this paper is to share some of the results of a year-long teaching experiment in which fourth grade students were provided with opportunities to develop an understanding of fraction concepts before the introduction of formal algorithms. In particular, we will address the role of the teacher/researcher's questions in prompting students to build these ideas. Our results indicate that the students were successful in building models to explain division of fractions.

Introduction and Theoretical Framework:

This research examines one portion of a teaching experiment[1] that focused on the development of fraction knowledge in a class of fourth grade students. In the sections that follow, we will briefly discuss the role that the teacher can play in helping students deepen their understanding of this content, and several instances that document one teacher/researcher's skillful use of questions to help children overcome difficulties with a problem dealing with division of fractions.

Many students have experienced difficulty in solving problems involving fractions (Tzur, 1999; Davis, Hunting, and, Pearn, 1993; Davis, Alston, and Maher, 1991; Steffe, von Glasersfeld, Richards and Cobb, 1983; and Steffe, Cobb and von Glasersfeld, 1988). When considering these difficulties, Towers (1998) states that traditionally, the teacher has been seen as separated from the student, and that teaching and learning have been regarded as discrete entities. Tower's research examines the role of teacher interventions in the development of students' mathematical understanding, and her findings suggest that children can overcome some difficulties traditionally related to fractions, when appropriate conditions are in place. Other researchers have also documented how teachers can help in facilitating the development of ideas relating to fractions (c.f. Steencken, 2001; Steencken and Maher, in press; Ma, 1999; Cobb, Boufi, McClain and Whitenack, 1997).

The role of the teacher in helping children develop insight into ideas relating to fractions was a central topic of the year-long teaching experiment that is the focus of this paper. The fundamental premise was that in order to help students build a conceptual basis for considering these ideas, teachers must move away from more

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teacher-centered instructional approaches which emphasize rote memorization and the execution of rules and procedures, and move toward instructional practices which are more student centered and provide them with the opportunity to build concepts and ideas as they are engaged in mathematical activities that promote understanding (Davis & Maher, 1997; Maher, 1998; Cobb, Wood, Yackel & McNeal,1993; NCTM, 2000; Klein and Tirosh, 2000; Schorr, 2000; Schorr and Lesh, 2001). A more student-centered classroom requires teachers who listen to the explanations of their students, probe them for justifications, encourage them to share their solutions with their peers as they work together to refine, revise and extend their solutions.

Teacher questioning is a critical component of a more student-centered classroom, and the topic of this paper. International studies have documented the effects of teacher questioning in helping children advance their mathematical thinking (Klinzing, Klinzing-Eurich and Teicher, 1985; Sullivan and Clarke, 1992; Martino and Maher, 1999). In each of these studies, it was found that asking more "open-ended questions aimed at conceptual knowledge and problem-solving strategies can contribute to the construction of more sophisticated mathematical knowledge by students" (p. 55, Martino and Maher, 1999). In addition, Cobb, Boufi, McClain & Whitenack (1997) propose that when students are engaged in discourse with peers, there is growth and development of mathematical ideas. In this study, the researcher used responsive questioning to elicit explanations, to help students develop appropriate justifications and to redirect them when they were engaged in faulty reasoning. In the sections that follow, we will describe how questions were used in this study, and the types of student responses that were elicited.

Methods and Procedures:

Background, Setting and Subjects: This research took place in a small suburban NJ district, over the course of a year, as part of a teaching experiment. The focus of this intervention was to investigate the development of children's mathematical ideas about fractions. A 4th grade class was chosen because these students had not been introduced to the formal algorithms usually associated with learning fractions. This particular class consisted of 25 students, 14 girls and 11 boys. Each of the approximately 50 classroom sessions lasted approximately sixty to ninety minutes. This paper reports on the 21st session of the study.

One of the goals of this teaching experiment was to create a classroom community in which student inquiry and discovery were of paramount importance. Teachers and researchers did not tell students that their work was correct or incorrect. Rather, students were questioned and encouraged to justify their solutions so that they could develop their own sense of accuracy, not based on the approval of an outside authority. The overarching perspective was that if students were invited to work together and conduct thoughtful investigations with appropriate materials, they would be able to build mathematical ideas relating to fractions (Maher, Martino, Davis, 1994). Throughout this experiment, the teacher/researchers worked to promote a classroom culture that supported children as they explained, explored, and reflected upon mathematical ideas. They were always invited to talk about their thinking, and they were challenged to defend and justify their ideas. The expectation was that each child, or group of children, would be able to build a model or representation of their idea, talk about their idea, and share their ideas with the teacher and their classmates. The children were always encouraged to build models of their solutions and share them with each other and the class. In discussing their solutions, children listened to each other and developed convincing arguments to support their ideas. They often raised questions that triggered further exploration. They drew pictures, labeled the pictures and eventually developed notations for their ideas. Steencken (2001) studied the earliest sessions of this project in depth, and documented the emergence of sophisticated mathematical thinking about fractions.

Data: All sessions were videotaped using two or three video cameras, one or two of which attempted to capture students' work and classroom interactions. An additional camera was frequently set up to record explanations by students, which were presented at the overhead projector. Students' written work and Teacher/Researchers' notes were also carefully collected. Transcriptions and detailed narratives of the data were recorded.

Coding: A coding scheme was designed to flag elements for study. The four classifications of codes used were intended to record teacher interventions, ideas expressed, representations used by students, and justification and reasoning by students. In all, there were twenty teacher intervention codes that fall into six categories. Codes relating to teacher's intervention included: T_I) giving information, T_R) rephrasing or requesting that another student rephrase what a student said, T_Q) asking questions, T_J) requesting justification, T_D) directing students to construct a representation, consider an existing representation, or to the work or ideas of others, to discussion with others or towards previously done work; and, T_N) seeming to do nothing. In this research, we will focus on these teacher codes as well as the outcome of these interventions in eliciting students' justification and reasoning.

Results and Discussion:

The episodes that are portrayed here occurred as students participated in an activity called, "Holiday Bows" (c.f. Bellesio, 1999), created to provide an opportunity for students to consider ideas relating to division of a natural number by a fraction. Questions that Teacher/Researchers asked were designed to be responsive to what students were doing and to help them to clarify and express their thinking by justifying their solutions.

In the first episode, the teacher intervention occurs when a Teacher/Researcher joins two boys who are working on a problem involving 2 ÷ 2/3. The Teacher/Researcher employs various types of interventions: questions, reiterating what the students say, requesting justification and explanations. When the students do not know how to begin, she asks, "What do you think you know?" In the excerpt that follows, the teacher is working with one student, Andrew. Her previous question has invited him to look at the ribbon and begin to "imagine dividing" it into three pieces.

Andrew: I think it's three umm because when you divide thirds up it's...it's ummm... 1/3, 2/3, 3/3 *[places his finger on the ribbon which is lying on top of the meter stick indicating approximately where 1/3 m, 2/3 m and 3/3 m would be]* so it would be like this would be one ribbon if you made it into a ribbon *[places his finger at approximately the 1/3 m mark]*

Teacher/Researcher: uh huh

Andrew: this would be another ribbon *[indicating approximately 2/3 meter]*

Teacher/Researcher: uh huh

Andrew: and this would be another ribbon *[indicating end of meter stick]* so if you divide it into thirds, you would have 3 *[inaudible]* bows, but we want to make sure.

By reminding the students to use what "you think you know" the teacher has offered that the students consider a technique that they had used before in similar types of problems. When the Teacher/Researcher posed this question, the student, Andrew, spontaneously solved the problem of finding out how many bows, each 1/3 meter in length, could be made from one meter of ribbon.

The Teacher/Researcher then asked the students in the group to justify their solutions. Justification of solutions often drives students to examine the validity of their own ideas. In this excerpt, one student, James, believes that the solution to the problem is 4 bows. At this point, the Teacher/Researcher asks: "Why are you going to get 4 bows, cause I'm...I'm confused...convince me." As the children consider their justification, she requests that they clarify what appear to be conflicting statements and provide valid mathematical evidence for them. While working on the problem, how many bows, each 1/2 meter in length, can be made from a blue piece of ribbon which is two meters long, students confuse the idea of dividing two by one-half with taking one-half of two. Questioning by the Teacher/Researcher helped the students to examine their own and each other's ideas.

Teacher/Researcher: What's... What's...Ok, Let... Let me go back because I think you're telling me a lot of things and I just want to be sure I understand. Um...

If you have the blue ribbon and we want to figure out how many bows we could make and each one is 1/2 meter how did you figure out, first you told me it was 2 and then you said no, it's 4.

James: It's 4 because…

Andrew: It could be either way.

Teacher/Researcher: It could be either? What do you think about that, James? Could it be either?

At this point, the Teacher/Researcher uses the strategy of rephrasing in order to help James consider the ideas that Andrew is referring to. She attempts to refocus the discussion back to James, the student who has correctly stated that there would be four bows.

James: I don't know. [*Inaudible*] 1/2 of a meter.

Teacher/Researcher: Right,

James: Not 1/2 of two meters, so…

Andrew: He's saying 1/2 of two meters not 1/2 of one meter

It appears that James is now considering what Andrew has said and he verbalizes that there is a distinction between taking 1/2 of a meter and 1/2 of the entire ribbon, which is 2 meters long. Andrew states that he thinks that James is referring to 1/2 of the ribbon, not 1/2 of a meter, which would be the length of one of the bows.

Teacher/Researcher: Ok. I think what it's saying is the ribbon length of the bow is a 1/2 a meter so each ribbon is going to be a 1/2 a meter long; OK, and the question is how many bows can you get out of that blue ribbon if each bow is going to be a 1/2 a meter long?

James: Four 'cause this [*holding up blue ribbon*] is two meters long

Andrew: It's saying 1/2 of two meters

Teacher/Researcher: Ok, Let me see if I can understand what James is saying.

Andrew: It's saying it's a 1/2 of 2 meters because the blue ribbon here is 2 meters, is the whole

Here Andrew has indicated that he believes the problem is to find out what 1/2 of 2 meters would be. The Teacher/Researcher again directs the attention to James, who is interpreting the problem correctly.

Andrew: So then this is a 1/2 of 2 meters… that would only be 2

Teacher/Researcher: Ok. What does this [*points to the paper with the problem on it*] say to you? What is a 1/2 m?

Andrew: Half of this [*holds up white ribbon which is one meter long*]

Here, the Teacher/Researcher's question helps Andrew to focus on the fact that the 1/2 in the problem refers to 1/2 meter in length, which would be the length of each bow. Andrew indicates his understanding of this by holding up the white ribbon, which is one meter in length and saying that each bow would be half that length.

James: Yes. So that would be four (referring to the blue ribbon, which is 2 meters long) and this would be 6 (referring to 3 m gold ribbon in the next problem)

Andrew: And so this [the white ribbon] is 2 and both of these [white ribbons]...[*inaudible*] would be 2 halves and another one we'd put on then that would be 4.

Andrew uses the white ribbon as a metaphor for 2 bows. He communicates his representation of two bows being made from each one-meter piece of white ribbon. The teacher's interventions invite him to clarify the question and to formulate and explain his metaphor, the white ribbon, for 2 bows. Her manner of questioning encourages the boys to talk not only to her, but also to each other. When it appears that Andrew believes there are two possible solutions, she probes further to encourage him to state that both his answer and James's answer are correct but that they are answering different questions. Andrew ultimately explains the difference between finding out how many bows, each 1/2 m in length can be made from 2 m of ribbon and finding 1/2 of 2 m of ribbon.

Conclusions

The preceding episodes were intended to highlight how one teacher/researcher helped students build a solution to a task involving division of fractions. Her interventions invited the students to listen to each other, to consider each other's arguments, to express their ideas and to create a representation to help them solve the problem. In this case, the children did not rely exclusively upon the authority to supply affirmation for their work or to impart information which they were expected to acquire without understanding. Rather, the students built a solution that made sense and they did this by exchanging information with each other and with the teacher/researcher. The task of monitoring children's construction of ideas, and posing timely questions is a challenge to teachers. Polya (1985) states, "This task is not quite easy; it demands time, practice, devotion, and sound principles." (p.1). The above episodes are instances of how, in a research setting, attention can be directed to the students' thinking. The interventions and questions were designed to engage the students in working out a model that made sense to them. Questions can become a catalyst for urging learners to justify their ideas and explain them to each other. This, in turn, has the effect fostering deeper thinking about the ideas involved in the problem situations.

Davis advocated a learning environment for the teaching of mathematics, which fosters the connection between the representations in the mind of the teacher and the mind of the student. This implies that a teacher makes sense of the developing ideas of students by the representations they build, explanations they give, and arguments they make, and, as the late Bob Davis suggested, "taking their ideas seriously". Davis (1992, p.349)

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