Gender and Written Mathematical Communication

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Abstract

This research study deals with the connection between two central topics in mathematics: communication and gender. The study examines the connection between gender and achievement, communication types and communication quality. The findings are based on analysis of 164 9th grade students' answers to problems that demanded communication of reasoning, explanations and justifications. The results of the study challenge pervious research, as not only were no gender differences were found in achievement, but also no tendency was found for girls to communicate verbally more than boys. These findings demand further study on the value of verbalized communication, particularly before policy makers modify high stake tests that are meant to be in the interests of implementing affirmative action for the advancement of girls in mathematics education.

Background

In research literature dealing with gender differences in mathematics, it is agreed that no gap exists in achievement at the elementary school level. However, gaps in favor of boys, begin to manifest in middle school. These gaps grow as the students grow older and are most prominent at the very high levels of achievement (Leder, 1990). In Israel, gender differences in mathematical achievement have been found at all ages (Amit & Movshovitz-Hadar, 1989; Ministry of Education, 1996;TIMSS-R, 2000).

The source of differences in test results is not unequivocal and depends on the content and structure of the exam and type of items. It has been found that boys surpass girls in exam items having to do with geometry and problem solving, particularly those given within the context of everyday life. Boys have higher achievement when the problem solving demands strategies that have not been taught in school. The gaps, in favor of the boys, become more significant as the degree of difficulty in the exam items is greater. The girls surpass the boys' in exam items dealing with calculations, algebraic manipulations and in problems having to do with abstract mathematics. Girls have higher achievement also when solving exam items similar to those appearing in text books (Seegers & Boekaers, 1996; Wang & Lane, 1996; O'Neil & McPeek, 1993). Despite the above, in an analysis by Hyde, Fennema & Lamon (1990), no gender differences were found at middle school classes in anything relating to verbal problem-solving.

In psychology, differences between boys and girls in verbal ability are regarded as an established fact. For example, in tests for verbal ability, it was found that already at the age of 10-11, the girls reached higher achievement than the boys. Those gaps were found in the varying aspects of verbal ability: vocabulary, reading comprehension, writing and speech flow (Halpern, 1986). Despite the above, in an analysis by Hyde & Linn, (1988), it was found that the intensity of the differences in verbal ability was so small that it may be considered nil.

Pomplun & Capps, (1999) argue that some of the gender differences stem from the test structure itself. Hence, boys have higher achievement on multiple-choice tests while girls have higher achievement in open-ended problems. What makes this argument stronger is that the American SAT exams where there has been a gap for many years in favor of the boys, but when a verbal element of essay writing was added to the test, the gap between the boys and the girls decreased significantly (Willington & Cole, 1997).

Mathematical tasks based on open-ended problems have the potential for the students to show a variety of mathematical comprehension and communication levels. Even regular problems, taken from textbooks, may "turn into" open-ended problems by adding a demand to explain and justify the answer, and therefore cause the students to develop and use mathematical communication. (Cai, Jakabcsin & Lane, 1996). The information obtained from the process of writing and its product comprise a diagnostic tool for the teacher and makes it possible to identify misunderstandings and assessing the degree of the student's comprehension of materials that have been studied (Pugalee, 1998).

We assumed that verbal ability may improve mathematical communication and problem solving, particularly where verbal reasoning is required. Based on this assumption and on research literature, we hypothesized that we shall find significant difference in achievement and communication between boys and girls; this hypothesis, however, has not been confirmed.

Methodology

This research study examined gender differences in achievement and in mathematical communication modes which students used in their answers to problems that required explanation, argument or justification. In particular, we examined the connection between gender and:

1) level of achievement (correctness of answers);
2) the type of written mathematical communication;
3) the quality of written mathematical communication

The type of written communication referred to the representation through which the students chose to explain their answers. This is based on the perception that the mode of representation is the external expression that reflects the solution processes of mathematical thinking. The quality of communication referred to soundness of the mathematical justification, that is, whether the justification is based on correct mathematical argumentation and whether it consistently and completely supports the solution presented by the student (Cia, Magone, Wang & Lane, 1996).

Setting and instruments

The research instrument comprised of three problems from a regional test in mathematics to given 5,928 ninth graders in 73 schools in the southern district of Israel in February 2000. The test items were put together by the Ministry of

Education – as part of evaluation of a 3-year intervention project. The students being tested were asked to answer 46 problems referring to varying areas within the ninth grade mathematics curriculum. About half of the items were multiple-choice or short constructed response ones, and half of them were open-ended problems (to different degrees of "openness").

For this research, three problems, whose solutions the students were asked to explain and/or justify, were chosen. This requirement obligated the students to select a communication method suitable for expressing mathematical ideas and arguments. In the instructions on how to answer, the students were told they could use any method they wished for justifying their answers.

The first problem dealt with an optimization situation: finding the most worthwhile condition for purchasing a product offered by two companies. The students were asked to mark the correct answer and then to justify their choice. The problem was a non-routine one and was new to the students.

The second problem dealt with rate of change. This problem seems to be a routine open-ended problem, like those appearing in ninth grade textbooks. But it was new to the students to be asked not only to give a verbal answer, but also to explain their answer.

The third problem was in geometry and dealt with the relation between area and circumference. This problem was hypothetical – the students were asked to deal not only with the given situation, but also with a hypothetical change, a process that requires deep understanding of the subject. The problem was formulated as a multiple-choice question in which the students were to choose the correct answer and, afterward, to justify their choice. The integration between the closed items and the demand to explain and justify the answers, turns the problems into open-ended problems and makes it possible to examine mathematical communication.

It is important to note that neither the teacher nor the students being knew, at the time of the exam, that part of the tests would be researched. This fact increases the authenticity of the answers.

Method of Analysis

The analysis of the students' answers was according to three criteria: the correctness of the answer, the type of communication and the quality of mathematical communication.

- The correctness of answers: The students' answers were checked with regard to their correctness, regardless of the type of explanation or its quality. The scale included: right answer, wrong answer and no answer.
- Types of communication which the students chose, were sorted according to their representations: verbal, numerical, algebraic, diagramatic.
- Quality of mathematical communication: This ranking reflected the quality of the explanation or reasoning given for the answer. The score was on a scale of: 0 = no explanation up to 3 – complete explanation that communicates effectively

and presents supporting arguments which are logically sound and complete. This ranking method is based on Cai, Jakabcsin and Lane (1996).

The above method was tried out in a pilot study, and underwent refinement. The emphasis on checking communication quality was on the nature and quality of mathematical communication and not on the verbal/linguistic ability of the student. There are likely to be situations in which the explanation is impressively formulated but not supported by mathematical knowledge. In such a situation, the communication quality will be ranked as low quality. On the other hand, it is likely that a mathematical explanation is correct while not formulated properly because of spelling mistakes and/or language errors. In this case, the communication quality will be ranked as high.

Results

Comparison of Test Scores

For the sake of generalization, the test scores of the study population were compared with the scores of the entire examinee population, as rendered by the Ministry of Education. There was no significant difference between the mean score of the two populations; Study population mean score 67.08; Examinee population mean sore 65.19; t-test result $t(163) = 0.99$

There were no significant differences between the girls' achievement and the boys' achievement in this test as indicated in Table 1.

Table 1: Gender differences in test mean scores

	Boys N = 83	Girls N = 81	
Mean score in entire test	67.02	67.11	$t(164) = -0.02^a$
Mean score in test (without the 3 problems being analyzed)	67.90	68.20	$t(164) = -0.07^a$

[a] n.s.

Gender and correctness of answers:

Answer correctness was divided into three categories: correct answer, wrong answer and no answer. An χ^2 test was implemented to the connection examine between gender and correctness of answers: No significant connection was found in any of the problems between gender and answer correctness. The distribution of answer correctness and the results of the χ^2 test are presented in Table 2.

Table 2: Distribution of answer correctness in each problem by gender:

	Optimization Problem		Rate of change Problem		Area-circumference Problem	
	Boys	Girls	Boys	Girls	Boys	Girls
Correct Answer	40	36	64	58	19	17
Wrong answer	27	24	6	4	44	50
No answer	16	21	13	19	20	14
	$\chi^2(2) = 1.041^a$		$\chi^2(2) = 1.805^a$		$\chi^2(2) = 1.543^a$	

[a] n.s.

Gender and type of written communication:

In the optimization and in the rate of change problems, no significant connection was found between the type of communication mode and gender. In the problems of area-circumference, the distribution of the different communication types and the few students who used algebraic representations, statistically prevented the use of a χ^2 test, but it may be seen that the distributions of communication types for boys and girls are alike. The distribution and the result of the χ^2 test are presented in Table 3.

Table 3: Distribution of communication types by gender

	Optimization Problem		Rate of change Problem		Area-circumference Problem	
	Boys	Girls	Boys	Girls	Boys	Girls
Numerical	41	35	27	23	3	3
Algebraic	8	10	7	8	3	2
Verbal	15	10	31	26	33	38
Diagramatic					14	12
	$X^2(2) = 1.022^a$ N= 120		$X^2(2) = 0.301^a$ N= 122		N= 108	

[a] n.s.

Gender and the quality of written communication

In the problem of optimization and in problem on area-circumference, there was no significant connection between gender and the quality of mathematical communication. In the problem on the rate of change, there was a significant connection, favoring girls, between gender and the quality of mathematical communication. The distribution of the quality of written mathematical communication by gender and the results of the χ^2 test are presented in Table 4.

Table 4: Distribution of communication qualities by gender:

	Optimization Problem		Rate of change Problem		Area-circumference Problem	
	Boys	Girls	Boys	Girls	Boys	Girls
Good Quality	26	26	30	40	4	6
Medium Quality	27	21	30	10	32	35
Poor Quality	11	9	9	5	17	14
	$\chi^2(2) = 0.42^a$ N= 120		$X^2(2) = 11.71^*$ N= 122		$\chi^2(2) = 0.67^a$ N= 108	

[a] n.s. $^*p < 0.003$

Discussion and Recommendations

Reform in math education puts emphasis on writing in mathematics as a communication instrument meant to develop mathematical reasoning and for assessment (NCTM, 1989, 2000). This study examined the quality and type of written mathematical communication modes of 164 ninth-grade students from five high schools in southern Israel. The analysis was derived from answers to problems within a regional exam. The first problem was a non-routine optimization problem. The second was a routine rate of change problem in which a requirement for justification was added. The third problem was a non-routine one in geometry, in which one chosen answer had to be justified. We selected open-ended problems, requiring explanation and justification, because they enable students to manifest a variety of levels in mathematical comprehension and communication.

Specifically, in this paper we present connections between gender and (1) correctness of answers, (2) type of mathematical communication that referred to representations and (3) quality of mathematical communication that referred to reasoning.

Some of the results of this study go in line with previous research findings, while some challenge previous ones. In the current study, there were no gender differences found at the achievement level on the entire exam, nor in answer correctness to problems that were investigated and analyzed. These findings go well together with the general trends closing the gender gap (Hyde & Linn, 1988), but contradict other studies where gender differences were indeed found (O'Neil & McPeek, 1993; TIMSS-R, 2000). A possible explanation for this result may be attributed to the nature of the population being studied. The gender gaps reported in the research literature are not only age-dependent, but are also dependent on learning level: Among college bound students, gaps appear already in the 7^{th} grade, while among students who are not college bound, these gaps appear in the 11^{th} grade (Hyde et al, 1990). Students in this study were 9^{th}-graders learning in heterogeneous classes, of which about 40% will be bound for academic studies (college bound) and about 60% for non-academic studies. Based on previous research, it is reasonable to assume that among the majority of the study population, the differences will arise at a later stage.

Surprisingly, in the current research, no connection was found between gender and the type of communication mode, particularly in the verbal mode. Yet in previous research as well as tests, that explicitly required verbalization, a verbal advantage had been found among girls (Amit & Movshovitz-Hadar, 1989; Leder, 1990; Halpern, 1986; Amit, 1988; O'Neil & McPeek, 1993;). It may have been expected that these differences would be reflected in the choice of communication type, and that girls would tend to use verbal representations more than boys. However, in the three problems investigated, no connection was found between gender and type of representation that were chosen as a means of communication.

A possible explanation for this rests on a trend of closing the gap in verbal ability in favor of the girls, to the point where it may be said that it does not exist (Hyde & Linn, 1988). However, if verbal ability indeed still exists, another possible explanation is that despite there being girls with higher verbal ability, they do not tend to use this ability when there is no explicit demand for it. That is, girls do not naturally transfer the relative advantage they have in verbal ability for the area of mathematical learning.

This assumption was further supported by the analysis results, where the girls surpassed the boys in the quality of communication for the rate of change problem. In the optimization and area-circumference problems, no connections were found between the quality of written mathematical communication and gender. The rate of change problem was distinct from the other two, in that it was completely open-ended, without an interim state of multiple-choice or constructed response that was in the others. In order to be ranked into "communication of good quality", the students were asked to answer the problem fully verbally, to reach verbal conclusions and then to justify them well. It is likely that in a problem of this type, where there is an explicit demand for verbalization, the girls' advantage in verbal ability was expressed. Girls produced fuller explanations of their reasoning, and therefore their quality of communication was graded higher. These findings are similar to the ones reported by Lane, Wang & Magone (1996).

Closing Recommendation

This research study challenges a number of conventions regarding the gender differences in mathematics education. No evidence was found in support of previous widespread reports, that at the age of 15, boys' achievement surpasses that of girls. This study reinforces some others that claim the closing of the gender gap. However, the intriguing finding is that no tendency for girls to choose communication modes based on verbal representations was found, although girls have an advantage over boys in verbal ability.

Those who determine education policy, who are interested in advancing girls' achievement in mathematics education, tend to integrate more and more verbal communication opportunities into the tests with the assumption that this will comprise a reformation preference for girls. This assumption, which finds no support in the above study, needs to be considered cautiously, and further research in this direction is required.

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