PURPOSE AND UTILITY IN PEDAGOGIC TASK DESIGN

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In England and Wales, the introduction of a National Numeracy Framework for the teaching of mathematics from ages 4 to 13 has placed a very strong emphasis on teachers' planning of objectives. By looking retrospectively at the design of computer-based tasks that have underpinned our research for many years, we recognise a theme of purposeful activity, leading to a planned appreciation of utilities for certain mathematical concepts. We discuss how the identification of objectives needs to go hand-in-hand with careful consideration of the planning of tasks, and propose two constructs to guide that planning.

BACKGROUND

The National Numeracy Framework (NNF) (DfEE, 1999a) was introduced to England and Wales in 1999. By defining how teachers should plan, assess and teach mathematics, this initiative extends the previously established National Curriculum for Mathematics (DfEE, 1999b), which merely set out to define a syllabus of content. The NNF is organised around sets of yearly objectives based on the content of the National Curriculum. The NNF also emphasises short-term planning based around tasks and activities:

> short-term plans: weekly or fortnightly notes on tasks, activities, exercises, key questions and teaching points for 5 to 10 lessons, including how pupils will be grouped, which of them you will work with, and how you will use any support. (p 41)

For clarity, within this theoretical paper, we use the term "task" for what is set by the teacher, and reserve the term "activity" for what subsequently takes place in the classroom setting. The planning emphasis in the NNF makes it opportune to reflect on the complexity of connecting objectives to the design of tasks. We believe that, though this reflection has stemmed from a phase in the development of curricula in England and Wales, in fact the arguments and principles elaborated below will have relevance to mathematics educators and teachers across international boundaries. We begin by problematising the procedure of setting objectives in mathematics lessons, and later we look retrospectively at the design of tasks that have played a significant part in our own research in order to propose two constructs that can guide the connecting of objectives to the design of tasks.

THE PLANNING PARADOX

We begin with a statement of what we call *the planning paradox*.

> If teachers plan from objectives, the tasks they set are likely to be unrewarding for the children and mathematically impoverished. But if teachers plan from tasks, the children's activity is likely to be unfocussed and learning difficult to assess.

To elaborate a little, we offer two contrasting examples. The teacher begins the lesson by saying, "Today, we are going to work on adding two-digit numbers together". The lesson proceeds with some explanation and then practice. The task has been determined by the objective in a narrow and constrained way. Such a teacher has fallen foul of the first part of the planning paradox. Now consider a teacher who, as the focus of the mathematics lesson, asks the children to design their ideal bedroom. The children may become highly engaged in a meaningful activity, but the teacher may find it difficult to monitor any mathematical thinking.

TASKS INSIDE AND OUTSIDE THE CLASSROOM

In recent years, mathematics educators have taken a great interest in situated cognition research (for example, Lave, 1988, and Nunes et al, 1993). Such studies argue that everyday tasks (*street* mathematics) lend an authenticity to activity that provides not only purpose but also meaning. A possible implication is that we should attempt to offer such authenticity to children in classrooms. Whilst we would agree with Schliemann (1995) that 'for meaningful mathematical learning to take place in the classroom, reflection upon mathematical relations must be embedded in meaningful socially relevant situations', we see the provision of 'authentic' tasks as inherently problematic.

Teachers often provide children with tasks that may superficially offer authenticity. For example, a teacher of young children may set up a play shop in the corner of the classroom to encourage some mathematical learning. However, the structuring resources provided by this situation will be very different from those offered when the child is really shopping with a parent. For example, in the play-shop task, emphasis may be placed on number, and so the prices of items on sale are simplified to an extent that even young children will recognise as unrealistic. Even with an element of role-play, the social interactions of the play-shop will not provide the structure and constraints experienced in a real shopping trip.

Situated cognition research has typically studied the activities of master and apprentice. In such situations, the master and the apprentice have a common goal, which in the short-term is typically to make some product (so the tailor and his apprentice may aim to make a waistcoat), and in the longer-term is to make a profit. We question whether teachers and children can have common goals. The teacher's agenda must be to focus on their pupils' learning (and the children know this), whereas the children's agenda will be to complete the task, hopefully to the satisfaction of the teacher. This leads us to question what kinds of products teachers and children might make together. We now consider approaches that do place product creation at the forefront of children's activity, though in ways which differ significantly from the tailoring workshop.

Our focus over many years has been on the use of technology in the learning of mathematics, and so it is natural for us to look towards the literature in that field for inspiration.

The constructionist movement (Harel & Papert, 1991a) has proposed that tasks in which children make products, generally through programming computers, are particularly conducive to learning. Thus, in one sense the constructionists replace the "waistcoat" with a product that is programmed by the child into the computer. In our experience (for example, as reported in Pratt & Ainley, 1997, and Ainley et al, 2000), such programming tasks can generate activity that has some of the characteristics of everyday activity studied in the research of situated cognitionists. For example, teachers often become engaged in working with the child to make the virtual product, reminiscent of the tailor and apprentice collaborating to make the waistcoat. The task, rather than the externally set objectives, takes on the role of being the arbiter of what counts as progress. Thirdly, any mathematical learning that takes place is contextualised within the activity of making the product, which provides meaning for the mathematics, but perhaps limits its apparent range of applicability.

Harel and Papert (1991b) also recognise the connection between constructionism and situated cognition, and at the same time signal some differences in emphasis.

> We see several trends in contemporary educational discussion such as 'situated learning' and 'apprenticeship learning'… as being convergent with our approach, but different in other respects… our emphasis (is) on developing new kinds of activities in which children can exercise their doing/ learning/ thinking…(and) on project activity which is self-directed by the student... (p42)

We believe that the constructionist approach recognises that the classroom is not the market place, and does not attempt to place emphasis on authenticity, but, by placing emphasis on the creation of products, it positions consideration of meaningfulness and motivation high on the agenda for the design of tasks that are likely to promote mathematical learning. Our aim is to draw out from these ideas constructs that inform the teachers' problem of connecting objectives to task design (the planning paradox).

CONNECTING OBJECTIVES AND TASK DESIGN

Schliemann (1995) concludes a discussion of the problems of bringing everyday mathematics into the classroom with the statement that 'we need school situations that are as challenging and relevant for school children as getting the correct amount of change is for the street seller and his customers'. In considering both constructionist approaches, and 'authentic' settings for learning mathematics, we identify a common feature, which may provide this challenge and relevance, as the *purposeful* nature of the learners' activity. We see this feature of purpose for the learner, *within the classroom environment*, as one key construct informing pedagogic task design. It is important to note that *purpose*, as we use the term here, is not necessarily linked to 'real world' uses of mathematics. Indeed, there is considerable evidence of the problematic nature of pedagogic materials which contextualise mathematics in supposedly real-world settings, but fail to provide purpose (see for example Ainley, 2001, Cooper and Dunne, 2000). We define a purposeful task as one which has a meaningful outcome for the learner, in terms of an actual or virtual product, or the solution of an engaging problem.

Thus the purpose of a task, as perceived by the learner, may be quite distinct from any objectives identified by the teacher. In a classroom situation, this maybe true in a trivial sense: learners may construct the purpose of any task in ways other than those intended by the teacher. However, we are saying something more than this: within our framework for task design, *purpose* is a distinct element that needs to be considered separately from, but in parallel with, objectives.

However, a focus on purpose in isolation may produce tasks which are rich and motivating, but fall foul of the second part of our planning paradox, by lacking mathematical focus. We therefore introduce into our framework a second construct of *utility*. Within pedagogic tasks that are designed to have purpose for learners, we have found that it is possible to plan for opportunities for learners to appreciate the *utility* of mathematical concepts and techniques. Whilst engaged in a purposeful task, learners may learn to use a particular mathematical idea in ways that allow them to understand not simply how to carry out a technique, but how and why that idea is useful, by applying it in that purposeful context. This parallels closely the way in which mathematical ideas are learnt in out-of-school settings.

A TASK DESIGNED WITH PURPOSE AND UTILITY

An example may help to clarify the related constructs of purpose and utility. A task which we have used (and written about) on a number of occasions, is that of designing a paper spinner, or 'helicopter' (see Ainley et al, 2000). In this task the purpose for the learners is clear: to make a spinner that will stay in the air for as long as possible. In investigating aspects of the design, for example by changing the length of the wings, children record results of test flights on a spreadsheet, (for example, the wing length and time of flight). Their activity offers opportunities to use a number of mathematical ideas, including measurement of length and time, decimal notation, graphing. From these we now describe two examples of *utility*.

Initially it is difficult for children to see patterns in the numerical data on the spreadsheet, partly because the data is not usually collected in a systematic way, and partly because of experimental inaccuracy. Using a scatter graph to display the results at intervals during the experiment makes it easier to see emerging patterns in the time of flight as the wing length varies. Information from the scatter graph is used to make conjectures about the effects of changing the wing length, and which spinners will prove most efficient, and also to identify further areas for experimental investigation. Using a scatter graph in this purposeful way offers opportunities to learn about the conventions of this particular graph, but also to understand that graphing is an analytical tool, which can inform the process of doing an experiment: that is, the children are given clear opportunities to construct a *utility* of graphing.

Discussion of the experimental inaccuracies in the activity leads us to introduce the idea of taking the mean value of several experiments with each wing length to produce a 'better' graph. This can be done quickly and easily using the AVERAGE function of the spreadsheet. Children are able to use their everyday knowledge of the meaning of 'average' to understand enough about this process to appreciate a *utility* of average (which does indeed produce a clearer graph), even though they do not know the detail of how the mean was calculated.

These two instances of introducing the utility of mathematical ideas both involve situations in which the use of technology means that mathematical ideas (graphing, average) can be *used* without children having learned the skills and techniques that underpin them (constructing a graph, calculating the mean). This is not co-incidental, and we return to further discussion of this point later in the paper. What we wish to emphasise here is that the opportunity to understand the utility of these ideas arises because of the purposeful nature of the task set, and of the learners' activity in response to these tasks. Without the underlying purpose of producing an efficient spinner, graphing experimental results and using average values could only have been introduced as isolated techniques. Their usefulness might be described through imagined applications, but could not be experienced in ways that allowed learners to construct rich meanings for the mathematical ideas.

RESOLVING THE PLANNING PARADOX

The two constructs of purpose and utility offer a framework for task design that may resolve the planning paradox. Designing tasks that are purposeful for learners ensures that the activity will be rich and motivating. Such purposeful tasks provide opportunities to learn about the utility of particular ideas, which will give the focus that may otherwise be absent.

It is widely recognised that constructing meaning for a mathematical idea involves many related elements. The distinction is often made between those elements relating to procedures or techniques, and those concerned with conceptual or relational understanding. We propose here a third cluster of elements: those relating to the utility of an idea. A rich understanding of a mathematical idea involves procedural and conceptual elements, but also understanding why that idea is useful, how it can be used and what it can be used for. We conjecture that understanding mathematical ideas without an understanding of their utility leads to significantly impoverished learning. Unlike 'street' mathematics, ideas in school mathematics are frequently learnt in contexts where they are divorced from aspects of utility. Within the classroom, opportunities to understand utility can only be provided through purposeful tasks.

However, the design of tasks that offer both purpose and utility is challenging. It requires the teacher to imagine the trajectory of a learner's activity, taking both a mathematical and a learner-centred perspective. In order to tease out aspects of the design of such tasks, we will discuss our own struggle to create a task with purpose and utility in a particular environment (for an extended discussion of this task see Pratt and Ainley, 1997).

As part of a long-term project, a group of children in our research school had been given access to dynamic geometry software (the original version of Cabri), and had explored its use as a drawing package with little intervention from their teacher. The children had explored many of the features of the software, and produced impressive drawings, but had actually made no use of construction. Their (self-selected) tasks were purposeful, with clear end products (drawings of a football pitch, a clown, and motorcycle and rider), but we felt that the children were not learning any mathematics.

We set out to design tasks that would be equally purposeful for the children, but also introduce the idea of geometric construction in a way that allowed children to understand its utility. This proved problematic. It was easy to design tasks that involved construction (such as creating a square which couldn't be 'messed up') but such tasks had no purpose for the children – except to satisfy the teacher. We knew that the children found creating drawings purposeful, but for such tasks placing points by eye, rather than constructing them, was perfectly satisfactory.

Eventually, we came upon the idea of harnessing the children's interest in drawing by setting them the task of producing a 'drawing kit' for younger children to use. This entailed them writing macros to produce a range of basic shapes (triangles, squares, diamonds etc.) from which young children could create their own pictures. For this task, there was a real purpose behind constructing a square, which could be reproduced many times and manipulated without being messed up by the younger children. As the children worked on making their drawing kits, the utility of construction for producing 'perfect' robust shapes became clear.

The design of the drawing kit task embodies some elements which we offer here as heuristics for creating purposeful tasks.

i) It has **an explicit end product** that the children cared about (a feature in common with the 'spinner' task described earlier).

ii) It involves **making something for other children** to use. In this case, the fact that the children for whom the product is intended were younger added an implicit further dimension of teaching other children.

iii) It was well focussed, but still contained **opportunities for children to make meaningful decisions**: although most of the drawing kits produced contain a similar set of basic shapes, many groups added their own designs, such as wheels or roofs.

To these, we add two other heuristics, which we know to be effective.

iv) Purposeful tasks are often based on **an intriguing question.** The spinners task may be seen as an example: how does wing length affect the spinner? Questions like this, which are solved through optimisation, seem to be particularly rich.

v) Tasks which involve **children arguing from a particular point of view** can engage children purposefully in contexts which may be unfamiliar (see for example McClain and Cobb, 2001, Ben-Zvi and Arcavi, 2001).

A SHIFT IN EMPHASIS: INVERTING A PEDAGOGIC TRADITION

The two examples given above of tasks designed using the constructs of purpose and utility exemplify the role of technology in supporting a shift in pedagogic emphasis, which we see as lying at the heart of opportunity for powerful mathematical learning offered by this approach. Mathematical ideas (such as average, graphing or geometrical construction) are rich and complex, composed of different elements, which here we categorise very roughly as procedures (techniques and algorithms, specific rules of formulae), relationships (links within mathematics, internal structure and consistency), and utilities (why, how and when the idea may be useful). As a learner constructs meaning for a new mathematical idea, mental connections will be made with existing knowledge, but the pedagogic emphasis placed on the different elements will affect the ways in which those links are made.

It is generally acknowledged that pedagogic approaches that focus mainly, or exclusively, on procedures will result in impoverished learning. However, even approaches that emphasise relationships tend to give little attention to utilities. The pedagogic tradition, embodied in textbooks around the world, is to begin with procedures and relationships, and to address utilities as the final stage in the pedagogic sequence (if at all). We suggest that this results in mathematical knowledge becoming isolated as weak connections are made to the learner's existing knowledge of the contexts in which it may be usefully applied.

In learning mathematics in out-of-school contexts, we believe that immediate connections between the learner's existing knowledge and the utilities of the new idea are established in ways which enrich mathematical learning. In school mathematics, the initial links are generally made to procedures and relationships. Pedagogic design based on the framework of purpose and utility, with the support of technology, inverts the pedagogic tradition of school mathematics by placing the emphasis primarily on the utilities of a new mathematical idea. Thus the learner is able to construct meanings that are shaped by strong connections to the application of that idea.

This inversion is made possible largely (though not exclusively) by the power of technology to offer opportunities for *using* a mathematical idea before you learn about its procedures and relationships. Technology affords the possibility of pursuing purposeful tasks by working with mathematical tools, instantiated on the screen, whilst simultaneously coming to appreciate the utility of those tools, in ways which lead to powerful mathematical learning. Ongoing research is developing and refining this framework for the design of pedagogic tasks in various areas of the mathematics curriculum.

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