HAVE ALL CHILDREN BENEFITED?

A Perspective on Curriculum Initiatives and Low Achievers

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Abstract

This paper considers whether or not curriculum initiatives within England are in fact leading to qualitatively improved levels of thinking amongst children identified as lower achievers in arithmetic. Revisiting data from two earlier studies and introducing current data, the paper draws comparisons in the strategies that 8-year-old children use to solve a range of addition and subtraction combinations on numbers to 20. It considers the outcomes in the context of current theories of concept development and national requirements of expected levels of achievement for such children. The results suggest that these requirements and the way in which they are implemented may seriously impede these children's cognitive development in elementary arithmetic.

Introducing a Perspective

The analysis of the strategies young children use to solve simple arithmetical problems can provide a framework to study the development of their arithmetical skills and the level of sophistication they employ to engage in numerical activity (Gray, Pinto, Pitta, & Tall 1999). It is the purpose of this paper to consider these notions in the light of initiatives implemented within English primary schools over the past decade. In doing so we focus particularly on children who, despite these initiatives, have difficulty recalling number combinations to 20. The evidence suggests that children within the lower quartile of arithmetical achievement are continuing to operate at a procedural level in the sense that they attempt to use a step by step routine such as counting to obtain solutions to elementary number bonds.

In the context of elementary arithmetic the initiatives have placed an emphasis on the ability to recall basic facts and use them to derive others. The latter feature lies at the heart of a strong emphasis on the development of flexible mental approaches to the calculation of two and three digit number combinations. Within this paper we present comparative evidence to illustrate that difficulties remain for some children. Fusing evidence drawn from three distinct studies over the past ten years we consider cognitive and pedagogic reasons as to why the situation has not changed. In doing so we are aware of social and cultural factors that may have a strong influence on the mathematical behaviour of the children we are considering (see for example, Cobb, 1987; Gruszczk-Koczynska & Semadini, 1988) but that is for a bigger story.

Theory and Practice

The construction of mathematical knowledge is a central platform from which the considerations within this paper are developed. It takes the view that hypothesised notions of development may not fit reality, particularly if children at the lower extreme of achievement are considered.

The development of early numerical concepts is heavily associated with physical activity and the pervading belief is that these concepts evolve from an interaction with the environment (Tall, 1995 *after* Piaget, 1973). Actions on physical objects can lead to the development of procedures through which processes are named symbolised and conceptualised. Establishing an appropriate conceptualisation is seen as the product of a suitable form of abstraction (Piaget 1985, Dubinsky, 1991). However, the knowledge and beliefs that learners bring to a given learning situation can influence the meanings that they construct in that situation. This would suggest that learners might select different aspects of an activity to focus upon which in turn leads to different forms of abstraction. When the latter is based upon perception of physical objects it is termed *empirical.* Alternatively such abstraction may not be based on the perception of objects but on the common feature of a series of actions. In such a case it is termed *pseudo-empirical.* The repeated practice of counting leading to the concept of number provides an example of this form of abstraction. The metamorphosis of actions with physical objects through a variety of increasingly abstract representations to form numerical concepts is outlined by Steffe, von Glaserfeld, Richards and Cobb (1983). Through such transformations it becomes possible to act upon the results of carrying out processes without bothering about the processes themselves.

Stages in the process of pseudo-empirical abstraction are frequently discussed in the context of process/object theories (see for example Sfard 1991; Cotterill et al, 1996) and they form the basis for the encapsulation or reification of new objects. There is much to be gained from such a move. Cognitive strain is reduced if it is possible to think of a concept as a single object rather than a lengthy process. However, it is difficult (Sfard, 1991) and its difficulty can lead to qualitatively different outcomes (Gray & Tall, 1994) which can be associated with success or failure.

The observation that some individuals are more successful than others in mathematics has been evident for generations. Piaget provided a novel method of interpreting empirical evidence by hypothesising that all individuals pass through the same cognitive stages but at different paces. Such a foundation underlies the National Curriculum (DfEE, 1999a[1]) with its sequence of levels

[1] Although only the most recent is referred to within this paper, The National Curriculum has been through three transformations since its implementation in 1991. It outlines both legal requirements (Attainment Targets) and also provides information to help teachers implement these requirements with in schools (Programmes of Study)

through which all children should pass at an appropriate age, some progressing further than others during the period of compulsory education.

Within England and Wales the process of improving standards in numeracy — some aspects of which are seen to improve "confidence and competence with numbers" requiring "an inclination and ability to solve number problems in a variety of contexts" (DfEE, 1999b, p 4) — are specified within two documents. The first, the National Curriculum for Mathematics (DfEE, 1999a), identifies the contents of the mathematics curriculum that should be taught during Key Stage 1 (KS1, age 5 to 7) and Key Stage 2 (KS2, age 8 to 11). The second, the National Numeracy Strategy (NNS) (DfEE, 1999b), identifies, almost through a process of genetic decomposition, how and what should be taught to instill a sense of being numerate. The teaching requires that "teachers will teach the whole class for a high proportion of the time and that oral and mental work will feature strongly in each lesson" (ibid. p.2).

Since the inception of the National Curriculum, formal assessment of the level of achievement of each child against the specified targets has been made through Standard Assessment Tasks, which are taken at the end of KS1 (children 7+, the average to achieve level 2) and KS2 (children 11+, the average to achieve level 4). Since this report is looking at early number development it is appropriate to indicate that for a Level 2 attainment a child would need to demonstrate mental recall of addition and subtraction facts to 10, and recall addition and subtraction facts to 20 for Level 3 (age 8/9). It should be noted that the programme of study for KS1 indicates that 7+ children should be using known facts to 10 to derive facts with totals to 20.

Both the programmes of study and the NNS are fully aligned, the latter providing a detailed basis for implementing the statutory requirements. The development of the early number requirements of the National Curriculum are identified within the NNS by an approach that sees a transition from the use of perceptual items to "more sophisticated mental counting strategies" finally sublimated by the acquisition of basic number facts that are taught. By and large we may see a curriculum that can be compared favourably with the Steffe et al (1983) model

To achieve these ends the NNS strategy suggests that a daily mathematics lesson is appropriate for almost all pupils. Individual needs do not necessarily warrant individual attention although the needs of particular children may be met through differentiated work and other teaching strategies.

Comparing Approaches

The question arises as to whether or not the initiatives within the UK have made substantive differences to the quality of thinking of those who are at the lower end of the spectrum of mathematical achievement. Is it possible to detect qualitative changes in the way in which they interpret mathematical symbolism by comparing the general use of strategies by children who have participated within the NNS with those who have not? If the initiative has been advantageous we would expect to see that there is a greater evidence of the ability to recall basic facts and evidence that the children use these facts to establish the facts not known.

To begin to consider a response we draw upon and compare three pieces of evidence indicating the strategies used by 8/9 year-old children (Y4):

1. The classified strategy responses reported in Gray (1991)
2. Classified strategy responses reported in Pitta (1997)
3. Recent data obtained as part of an investigation into children's participation within the oral phase of mathematics lessons conducted in accordance with the NNS (2001).

Four different schools are represented within the analysis and these range in type from two small town schools (1991), a small suburban school (1997) and large inner city school (2001). Data collection methods for each of the three samples were essentially similar and included semi-structured clinical interviewing, video recording of these interviews and subsequent analysis of the responses.

Children within each sample represented the lower end of the spectrum of achievement. The 1991 sample, in the class teacher's view were representative of the lowest quartile of achievement within the year group. In the second and third cases children were chosen from within the lowest 20% of scores within the KS1 Standard Assessment Tasks. Within the chosen year group (Y4) the average child would be expected to recall all number combinations to 20. Schools from which the two most recent samples had been drawn have subject to recent external inspection by the Office for Standards in Education (Ofsted). Mathematics teaching within both was identified as "mostly satisfactory".

Here we report on strategies used to solve the questions that were common to all children, namely 0+2, 6+3, 3+5, 14+4, 3+16, 4+7, 9+8, 3-3, 6-3, 9-8, 15-9, 13-8, 12-8, 16-10 and 17-13.

	Addition Combinations to 10		
	1991 (n=4)	1997(n=4)	2001 (n=16)
Known Facts	33	42	31
Derived Facts	0	0	2
Count-on	25	50	48
Count-all	42	0	8
Error	0	8	10
	100	100	100

	Addition Combinations to 20		
	1991 (n=4)	1997(n=4)	2001 (n=16)
Known Facts	0	13	5
Derived Facts	0	13	6
Count-on	75	75	53
Count-all	25	0	14
Error	0	0	22
	100	100	100

	Subtraction Combinations to 10		
	1991 (n=4)	1997(n=4)	2001 (n=16)
Known Facts	8	50	29
Derived Facts	0	8	8
Count back	8	17	17
Take-away	67	25	17
Error	17	0	29
	100	100	100

	Subtraction Combinations to 20		
	1991 (n=4)	1997(n=4)	2001 (n=16)
Known Facts	0	19	2
Derived Facts	0	6	3
Count back	6	31	36
Take-away	56	31	0
Error	38	13	59
	100	100	100

Table 1: Comparative responses to elementary addition and subtraction combinations

Figure 1 indicates the cumulative percentage responses of the approaches used by children within each sample. The numbers of children within each sample are given by "n".

Differences between each year group can clearly be identified, for example:

- Highest use of known facts by the 1997 group, particularly with subtraction combinations to 10
- Evidence of the use of derived facts by the 1997 group
- Highest number of errors by the 2001 group

However, what is more in evidence is the remarkable similarity between the children considered within each year. This can more clearly be seen in Table 2 which identifies distinctions, through the use of percentages outlining the use of processes to do (count-on, count-all, count back, count back to, take away etc) and concepts to know (known fact, derived fact). Essential differences in process driven approaches between the 1991 children, interviewed before the introduction of the initiatives, and the 2001 children are largely accounted for by the errors, mostly procedural, of the 2001 children.

	ADDITION					
	To Ten			To Twenty		
	1991	1997	2001	1991	1997	2001
Concept driven	33	42	33		25	11
Process driven	67	50	56	100	75	67
Errors		8	10			22

	SUBTRACTION					
	To ten			To twenty		
	1991	1997	2001	1991	1997	2001
Concept driven	8	58	38		25	5
Process driven	75	42	33	63	63	36
Errors	17		29	38	13	59

Table 2: Comparative Process/Concept distinctions in elementary arithmetic

The evidence suggests that subtraction combinations to 20 are more difficult for these children than the other combinations. In every case apart from the 1996 group's response to subtraction combinations to 10, the use of processes, always counting, is higher than the use of concept driven approaches. Of course, it is difficult to identify whether or not knowing facts is concept based. A clearer confirmation could be obtained if those facts that are known are used to establish those that are not known. This feature is a specified aspect of the programs of study for KS 1 within the National Curriculum and there is a strong emphasis on this aspect of numeracy within the NNS. The indicators suggest that though we may identify differences between process driven and concept driven strategies, for these children those facts that are known are not used to establish those that are not known. The consequence is that we would see these as isolated pieces of knowledge. Reconsidering Table 1 would suggest that for children at the lower end of the achievement spectrum initiatives developed to improve the sophistication of their thinking have hardly had the desired outcome.

The application of a fixed routine procedure was a feature of the approach used by the children if they failed to recall a fact. In some instances these began to emerge very early within the interviews for example in combinations to ten and in the excessive use of fingers. The application of difficult almost nongeneralisable procedures became common when children attempted to apply the same procedure for subtraction combinations to 20. Count-back dominated as a procedure and this caused excessive difficulties with 17-13. Indeed, the greater proportion of errors within all groups is largely accounted for by procedural difficulties associated with counting. Children who successfully used fingers for counting small numbers had difficulty with quantities greater than ten. In some instances this was recognised. For example 17-13 was "*too tricky to count*" (1997 child). In others children attempted to obtain a solution but miscounted.

"9 add 8. .... It's hard. " (1997 child)

"1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ..12, 13, 14,15" (2001 child)

Others simply had no effective procedure and tried to keep a mental check (format)

"9 add 8—I ran out of hands"

"1, 2, 3, 4, 5, 6, 7, 8, 9, 10..., 11, 12, 13, 14, 15, 16" (2001 child)

This child's second and third attempts gave the answers 17 and 20.

Thus, though in some instances no suitable counting procedure was identified, in others, particularly when attempting the subtraction combinations, difficulties associated with double counting procedures caused problems:

"I just counted 1,2,3,4... 16, 17 then I went 17,16,15,14,13 and said 5" (1997 child)

Discussion

We are drawing our evidence from those having the greatest difficulty in responding to the new initiatives at an elementary level. It is most unlikely that they will be in a position to respond to the initiatives at subsequent levels of difficulty. However, our evidence should not be used to imply that such children will forever be confined to the use of inefficient procedural approaches. Neither do we wish to place an artificial ceiling on their ultimate performance. We do recognise however that should they persist with what is essentially a fragmented knowledge structure, which in some instances is procedurally inefficient and in others is insufficiently compressed, they may be forced into less powerful strategies through trying to cope with too much information.

In one sense both the requirements of the National Curriculum and the pedagogic approaches associated with the National Numeracy Strategy may be attempting to overcome these problems by firstly giving children too many options to consider and then by focusing on rote-learnt procedures to perform as sequential action schemas. If successful, such knowledge can be used to solve routine problems, as we can see where children are successful in their counting, but it occurs in time and may not be suitable to support thinking about the whole entity, which is formed as a result of encapsulating the process.

By their very nature, the National Curriculum and the National Numeracy Strategy are offering a sequence of activities that teachers have to follow. They are grounded within a stage theory that we would suggest is flawed from the very outset. In addition there is also the sense that the teaching strategy has been identified through a form of genetic decomposition (Cotterill et al, 1996) which relies on the teaching of partial structures that may be qualitatively different from the way the whole is to be understood. Resnick & Ford (1981) questioned the reliability of such an approach, but given the Ofsted measures of mathematics teaching within the schools we would suggest that not only can the children not see this whole structure but it is strongly likely that neither can the teachers. Even if we assume that the NNS contains perfectly constructed, sequenced activities it is evident that the children are not seeing things this way. We have to look at the greater number of errors reported within the 2001 sample. Obviously these children have not interiorised (in the sense of Sfard 1991) an appropriate counting procedure to deal with combinations to 20.

Pitta (1999) suggests that children who have difficulty with elementary arithmetic are either unable to, or simply choose not to, see through actions and objects to embrace more abstract qualities. Their disposition has tended to lead them towards an empirical form of abstraction. Developing suitable stages in the process of pseudo-empirical abstraction in the context of counting actions and the development of number concepts has been one of the functions of teaching. The occurrence of a greater number of errors within the 2001 sample would suggest that the considerable emphasis now given to oral and mental strategies and whole class teaching has limited the time available to these children to interiorise and condense appropriate counting actions in order to make the abstraction. Perhaps teachers do not realise its fundamental importance. In the past we have often heard children told to "count" if they had difficulty.

We would suggest that the emphasis on effective mental approaches and institutional and national demands to achieve them have in fact removed the necessary experiences that these children require in order to make appropriate pseudo-empirical abstractions in the field of elementary number. Their focus is very much on physical objects to support counting. In the absence of aids, such as counters, they search for alternatives. The action is supplementary and we would suggest a secondary focus which has not been effectively interiorised (particularly for those who display errors), nor as our few examples illustrate, condensed. Some children now seem to be seriously disadvantaged. Not only are they demonstrating evidence that they have not encapsulated a counting process but they are also demonstrating that they have not interiorised the procedure.

The cumulative evidence is that if they know facts they fail to use them in the way that the initiatives suggest they should. Some reasons for this have already been discussed elsewhere (Gray & Tall, 1994). Now however we seem to see a higher proportion of children who do not seem to have the procedural competence to consider an alternative approach with confidence. It seems that what may truly be a positive initiative for a great number of children may not be so very positive for some of those it was designed to help.

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