INVESTIGATING COGNITIVE AND COMMUNICATIVE PROCESSES THROUGH CHILDREN'S HANDLING OF SOLIDS

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The article presents the results of an on-going longitudinal research project which was begun in 1993. Initial goals for the research are defined and suitable research tools were determined. Four tasks based on the constructivist approach to teaching were chosen as the tools and these were given to young children, then student-teachers in the Czech Republic and finally to children in the UK. Analysis of pupils solutions of the tasks allowed us to identify and define cognitive processes on a theoretical level and apply these to particular pupils´ solutions. The importance of the communicative processes emerged whilst analysing the early experimental work and the goals and tasks modified to take account of this.

INTRODUCTION

This longitudinal research project began in 1993. It was designed to undertaken by a relatively small number of pupils whose work would be subject to a deep qualitative analysis. The first goal was to investigate the contribution of tactile and visual perception on the building of geometrical images. The second goal was to identify cognitive phenomena which occur in school communication when pupils work with solids then characterise and classify them. Early experiments showed that communication was essential in identifying the cognitive processes and a third goal was added namely the investigation of communicative phenomena (Jirotková, Swoboda, 2001).

The goals relate to the writings of J. A. Comenius (Analytical Didactics, Principle 55) 'the greater the number of senses which are involved in putting an image into the mind, the more familiar we are with the image and the more likely we are to retain it.' This means that the pupils who only use visual perception to learn about solids have a lower quality of understanding of the solids than those who employ tactile perception and even manipulate them (building, cutting etc.). Although this is generally known we find that in the classroom insufficient attention is paid to this fact by teachers. The didactics of mathematics should contribute to the changes of this situation by investigating the mechanisms of particular perception processes. Hence the goals which were chosen for this research.

TOOLS AND METHODOLOGY

To help us to determine the tools for our research we set ourselves a series of questions to set the parameters for this work.

· Are there pupils who prefer learning about solids through a tactile perception rather than a visual one and does the reverse apply?

· Is it possible to gain evidence of the extent to which a pupil's memory can store images of solids which were perceived by tactilely or visually?
· Is it possible to set up a situation from which we can gain evidence that pupils perceive certain spatial phenomenon better tactilely rather than visually?
· Can a situation be set up which gives evidence that a pupil has a precise understanding of certain geometrical phenomena but is unable to express this verbally?
· Is it possible to determine the preciseness of the pupil's understanding of certain geometrical phenomena?

From the consideration of these questions and the researchers' experience, the following four tasks were devised which fulfilled the goals which were set. The latest modification of the tasks done in 2001 are presented here.

In the four tasks a total of 16 solids were used in varying combinations. The solids used were: 1 Cube; 2 Square prism; 3 Large rectangular prism; 4 Small rectangular prism; 5 Right-angled triangular prism; 6 Right-angled isosceles triangular prism; 7 Non-convex pentagonal prism; 8 Hexagonal prism; 9 Tetrahedron; 10 Square-based pyramid; 11 Truncated rectangular-based pyramid (x2); 12 Non-convex pentagonal based pyramid; 13 Sphere; 14 Cylinder; 15 Cone; 16 Truncated cone.

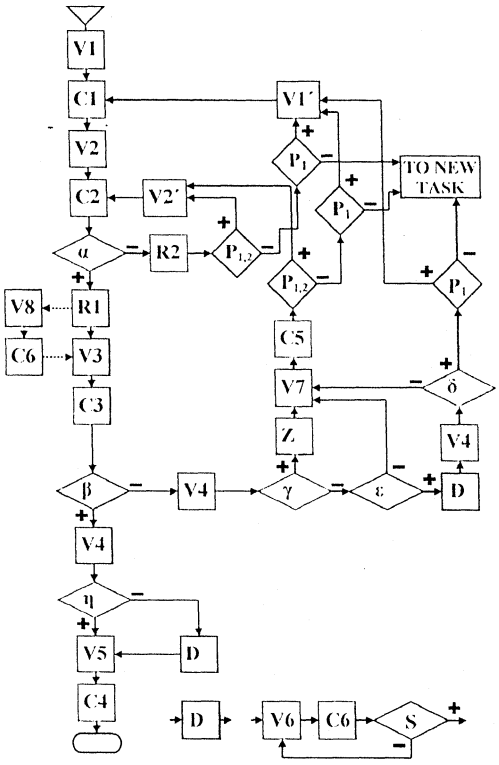
Task 1. Identification

Materials: Two cloth bags A, and B, bag A contains solids 1, 2, 5, 6, 9-16. Bag B contains solid 11.
Instruction: Dip one hand into bag A and feel the solid. Now take your hand out of bag A and dip the same hand into bag B. Try to find the same solid. Tell me if you can find one. Before you take it out give reason(s) why you think it is the same and what is interesting about the solid.
Presentation: Experimenter shows the bags, explains the tasks in language suitable for the age of the pupil, checks to make sure that the pupil understands what s/he has to do. If the pupil does not understand properly, the experimenter will rephrase the task.
Scenario: Individual pupils given task. There was no time limit given for the solution.

It took several experiments before we were able to standardised Task 1 and to represent it in the form of a flowchart. The development of this chart illustrates that although the task appears simple, as soon as you start to analyse the possible scenarios its complex nature is clearly seen. The chart is in an evolutionary state and the latest modification was made in 2001 when the communicative element was added, indicated by the dotted lines.

Legend

Experimenter boxes
a) Instructions
V1 Dip one hand into bag B and feel the solid.
V1' Dip one hand again into bag B and feel the solid.



V2 Dip the same hand into bag A and find the same solid.
V2' Dip the same hand again into bag A and find the same solid.
V3 Take the solid out of bag A and check visually if your choice was right.
V4 Do you think your solution was correct?
V5 Take the solid out of bag B and compare both solids.
V6 Give reasons for your uncertainty about your choice.
V7 Put the solid back into bag A.
V8 Give reasons for your choice and say what is interesting about the solid.
b) *Others*
Z Experimenter doubting asks 'Are you really sure?'
β Experimenter decides if the solution is correct or not.
ε Experimenter decides if discussion needed to make pupil aware of mistake and to learn the reason for the mistake (+) or if a second attempt is necessary (-).
S Experimenter decides if the reasoning was sufficient or not.

Pupil's boxes
a) Activities
C1 Pupil perceives solid and makes mental evidence of the solid.
C2 Pupil is searching for the solid and compares the previous and current tactile perception.
C3 Pupil compares current visual perception with the previous tactile one.
C4 Pupil compares current visual perceptions of both solids and confirms the choice was correct.
C5 Pupil replaces the solid in bag A.
C6 Pupil gives reason for the choice of solid.
R1 Pupil stated that the solid was found.
R2 Pupil stated the solid was not in bag A.
b) Decision boxes
α Pupil decides if the solid from B is(+), is not (-) present in bag A.

γ, δ Pupil decides that the choice was correct (+) or incorrect (-).
η Pupil is (+) is not (-) sure about his correct choice.

Administrative boxes
P_1 (abacus) After first entry, exit-path is (+), after second entry exit-path is (-).
$P_{1,2}$ (abacus) After first/second entry, exit-path is (+), after third entry exit-path is (-).
The explanation of the transfer to another task will be given orally.

Task 2 - Selection

Materials: One bag containing solids 1, 2, 6, 9, 10, 11, 16.
Instruction: Put one hand in the bag and from the solids which are there choose one which you think is different from the others. Before you take the solid you have chosen from the bag, tell me why you think it is different.
Presentation: Experimenter shows the bag, explains the task in language suitable for the age of the pupil, checks to make sure that the pupil understands what s/he has to do. If the pupil does not understand properly, the experimenter will rephrase the task until understanding is satisfactory.
Scenario: Individual pupils attempting task. No time limit given.

Task 3 - Classification

Materials: One bag containing solids 1, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15.
Instruction: Both hands may be put in the bag. Arrange the solids into two groups, one group having common attribute(s) which you choose, the solids in the other group will not have those attribute(s). Before you take your two groups of solids out of the bag say what are the common attribute(s) or how they differ. Take out your groups and say whether you are happy with your classification. If not change them and say why you have made the changes.
Presentation & Scenario: As for Task 2.

Task 4 - The game OWL

Materials: The following solids were placed on a table : 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.
Rules: One player to choose a solid mentally. The other player is to discover the chosen solid by asking questions which can only be answered by either YES or NO.
Presentation: Experimenter explains the rules of the game; makes sure everything is clearly understood. If necessary the experimenter amends the rules during the course of the game, to allow only questions that refer to geometrical attributes of the solids.
Scenario: The players may both be pupils or a pupil could play with the experimenter. No time limit is set. The experimenter avoids influencing the game.

RESEARCH SAMPLE

In 1993 ten pupils with ages in the range 7 years to 10 years, were observed. They undertook all the tasks including multiple games of OWL. All verbal communications

were tape-recorded and then transcribed in the form of protocols. An observer was always present to make notes of non-verbal actions and communications. In some cases photographic evidence was taken. The protocols were analysed. We found the tasks, especially the game OWL appropriate for use with University student-teachers and it is now an integral part of the syllabus. From this work we found that the majority of information regarding cognitive processes can be obtained from the analysis of communication of the pupils/students. Hence the tasks were slightly modified to challenge the pupils to verbalise their perceptions as much as possible. This form of the tasks was given to two nine-year old pupils in England in 2001. The analysis of this latest work and our research experience enabled us to construct the mental mechanisms of some cognitive processes which were involved in solving the tasks. These were the *mechanism of tactile selection* - how the process of choice of a particular solid is made by tactile perception only; the *mechanism of tactile-visual verification of the selection* - how the process of verification their tactile choice is made visually and tactilely; the *mechanism of tactile classification* - how the process of classification of the solids into two separate groups is made without visual perception; and the *mechanism of tactile-visual verification of the classification* - how the process of tactile classification is verified using visual-tactile perceptions. These theoretical descriptions of the mechanisms were applied to the actual solutions of the tasks undertaken by the pupils. In the next paragraph we will describe the mechanism of tactile selection and apply it to the solution of the first task by two pupils. For the description of these mechanisms we used the methodology of analysis elaborated by M. Hejný (Hejný, Michalcová, 2001).

MECHANISM OF TACTILE SELECTION

The process of selection is found in the decision box a in the diagram above. In our research we identified three types of the process and these differ according to the extent the pupil is familiar with the perceived solid.

1. The perceived solid is entirely new for the pupil and s/he does not have any experience of committing tactile images to the short-term spatial memory, that is the pupil has not been challenged to verbalise tactile perceptions. When the pupil is challenged to describe verbally the spatial phenomena perceived tactilely, s/he must perceive the solid analytically that is s/he needs to perceive vertices, edges, shape of face etc. In this case the pupil perceives the solid as a whole, that is s/he perceives its *gestalt* and not its individual analytical characteristics (Van Hiele, 1986). On the pupil's first attempt to find the identical solid in bag A s/he does not know which solids are hidden in the bag, therefore s/he does not know on which attribute to focus his/her attention. The pupil will try to sort the solid from bag A by using his/her global perception first and if not successful attempts to recall and use at least one analytical characteristic of the perceived solid in bag B.

2. The perceived solid is entirely new for the pupil but the pupil has some experience of committing tactile perceptions to the short-term memory. In this case the pupil tries to commit as many dominant characteristics of the solid to his/her short-term spatial memory as possible. When the pupil is then sorting the identical solid from bag A s/he attempts to find the characteristics, which s/he had put to the short-term memory, on each of the shapes in turn.

3. The pupil is familiar with the perceived solid and it is correctly committed to his long-term spatial memory. When the pupil is sorting the identical solid from bag A, the pupil is led by the good image committed to his/her memory.

The above three types of mechanism of tactile selection are described theoretically. The boundaries between them are not well defined and the reality is often on the overlap of them. The images from the long and short-term memory are mutually penetrating. We gain information about how the short-term memory works either from non-verbal expressions (gestures) or from verbal expressions (pupils generate new expressions eg. 'squangle' for square).

APPLICATION OF MECHANISM OF TACTILE SELECTION

The first pupil we wish to discuss is Jan, a Czech boy aged 10 who volunteered to take part in the research. We worked with him in November 1993. Jan had to find the truncated pyramid in bag A but his first choice was solid 2, the square-based prism. He was not certain about this choice and when he took it out he asked 'Is it right?' Then he perceived the solid in bag B again and he realised that his first choice was incorrect. Nevertheless on the second attempt he again made an incorrect choice by choosing the cube. Jan concentrated a lot during his third attempt and his choice was correct. We explain his mistakes by applying the mechanism of tactile selection. We believe that Jan was working according to the first type of mechanism as defined above. When he perceived the solids in bag B only a global perception was committed to the short-term spatial memory. When he tried to find the correct solid he did not find any global characteristic to help him select the solid. He then tried to recall some analytical characteristic of the solid and he found one characteristic which we could call 'four-sidedness'. This idea is supported by two arguments. There are only three solids in bag B which could be characterised by 'four-sidedness'. Jan chose these one-by-one. The second argument is that when he solved the other tasks he expressed that 'being quadrilateral' is, according to him, the dominant characteristic of a solid. For his second choice he was led by the perception of four right angles. This characteristic was perceived when he felt the solid in bag B again. Putting his hand into bag A again the first solid he met which had this characteristic was the cube which he chose without any further reference to any other shapes. Visual perception on taking the cube out of the bag gave him quick information about the metric characteristics of the cube. His third attempt was successful. It should be noted that our second mechanism listed

above - mechanism of tactile-visual verification of selection - was also used in this process.

The next example is a good illustration of a pupil perceiving the solid as a gestalt and not its separate properties on meeting it for the first time. Jill was a nine year old English girl when we worked her on the tasks in September 2001. Her teacher chose her for the research. Jill showed unusual mathematical culture and perfect insight into the geometrical world when solving all the tasks. She also had a remarkable mathematical vocabulary, which she used with understanding. When she found the identical solid in bag B, (task 1) before she took it out she was challenged to say why she thought she had made the correct choice. The following is an fragment of the transcript of the subsequent dialogue with her. J4 represents the fourth comment by Jill and E5 the fifth by the experimenter.

J4:	Because it feels like the first one.
E5:	So what do you think was special about it?
J5:	*It was small, it has the same amount of sides.* (She meant faces.)
E6:	*How* many*?*
J6:	(A short pause whilst she checked the held solid.) *Six.*
E7:	*So you think it has six sides, do you think you have seen a shape like this one before?*
J7:	*No.*

Both used words 'feel' and 'small' indicate that Jill perceived the solid as a gestalt. The second part of J5 was surprising because it showed that she tried to give an analytical description of the solid and the use of the incorrect terminology, 'sides' instead of 'faces', which was an unusual occurrence in all her communication, was evidence of the demanding nature of the transfer from global to analytical descriptions (Pegg, Baker, 1999). The drain of energy, which this transfer caused, was the reason she chose the incorrect word for 'face'. J5 shows unusually sophisticated mathematical thinking for a nine-year old girl. She was aware that the global characteristic of a solid was not the perfect answer to the task, hence she felt the need to define the solid by analytical characteristics. We did not meet any similar case of such thinking in our research. This is a good example of the second type of mechanism of tactile selection.

The analysis of our research did not produce any interesting results related to the third type of tactile selection mechanism.

Jill's use of correct mathematical language and her knowledge of the characteristics of both solid and plane shapes can be seen in the following examples. When she was asked by the experimenter at the end of Task 1 what solid did she think would be created if the faces of the truncated pyramid were extended to a point, she immediately replied 'a square-based pyramid'. In the third task she was asked to say what was different about the solids that she had chosen. Her response was 'They all have at least one face which is a quadrilateral in 2D'. The experimenter made sure that Jill fully understood what she was saying by both 'at least' and 'quadrilateral'.

CONCLUSION

We believe that little research has been carried out and written up this area. Our research has produced tools which we consider are beneficial for investigating cognitive and communicative processes. They could be used to advantage in the normal classroom in two ways, the first being that the methodology of the tasks is very much in the format of constructivist teaching strategies (Noddings, 1990) and so would help the children to learn about solids in a concrete, meaningful and understandable way. The second is that all the tasks, develop the pupil's ability to communicate mathematically which we find is lacking in school text-books. These books get the pupils to construct shapes or solve riders with no opportunity to explain or discuss their work and their understanding of geometrical concepts verbally. In particular we found that the game OWL, which can be modified for all age groups, not only helps the communicative ability of the pupils and develops their mathematical vocabulary it can also be used to quickly diagnose deficiencies in these areas (Jirotková, 2001).

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