FACILITATING STUDENT LEARNING THROUGH MATH JOURNALS

Hari P. Koirala
Eastern Connecticut State University

Abstract. *In the last two decades, mathematics teachers have shown a great deal of interest in how students learn mathematics through journal writing. I have also used journals in my mathematics classes for the last five years. Based on the analysis of more than 1800 journal entries written by approximately 200 students in the last five years, I conclude that journal writing have potential to aid in student mathematical learning even though teachers need a large amount of time to examine student journals and provide feedback. Math journals not only help instructors in understanding students' feelings, likes, and dislikes about classes but also help students to demonstrate their mathematical thinking processes and understanding.*

Introduction and Theoretical Framework

The National Council of Teachers of Mathematics (NCTM, 1989, 2000) and the American Mathematical Association of Two Year Colleges (AMATYC, 1995) emphasize that students should be able to communicate mathematically, both in written and oral forms, using mathematical vocabulary and notations. NCTM (2000) argues that "reflection and communication are intertwined processes in mathematics learning" (p. 61) and writing in mathematics provides opportunity for students to express their thinking. Teachers also benefit from student writing by getting access to student thinking, which can be used to improve instruction. Following these recommendations, many teachers have used journals as a regular feature of their school and college mathematics classrooms and report that journal writing helps students reflect and learn math concepts (Burns & Silbey, 2001; Chapman, 1996; Dougherty, 1996; McIntosh & Draper, 2001; Pugalee, DiBiase, & Wood, 1999).

The use of journal writing in a mathematics classroom is not only being supported from the experience of classroom teachers but also from research. Qualitative researchers working from a constructivist framework often use journals to inquire and understand participants' mode of thinking (Kroll & LaBoskey, 1996; Mewborn, 1999; Miller, 1992; Pugalee, 2001). Because the construction of knowledge is a continuous process, it must be regularly communicated and reflected. Mewborn uses journals as tools for reflection. Miller points out to the benefits that teachers get from reading students' writing. Pugalee claims that writing is a necessary part of metacognitive thinking, which helps in constructing mathematical knowledge. The present study focused on the role of journals in mathematics content course designed for prospective elementary school teachers. The main purpose of the study was to

explore the ways in which math journals influence the teaching of mathematics to prospective teachers. Consistent with previous studies, this study claims that math journals help preservice teachers in their learning of mathematics as they express their thinking and get feedback from the instructor. Furthermore the instructor of the course consistently valued student math journals as they provided helpful hints on how to modify and improve classroom instruction.

Research Methods

The setting for this study was the course "Number Systems," which is a required mathematics content course for prospective elementary school teachers at a North Eastern university in the United States. Most of the students in this course intend to become future elementary school teachers. The main goal of the course has been to encourage students to embrace the challenge of learning mathematics through inquiry and exploration. In the past five years of my teaching, the course has taken a problem-solving approach in which students were required to monitor their thinking process while solving mathematical problems. Students monitor their thinking through math journals, which is a necessary component of this course. So far more than 200 students have participated in journal writing.

In the first two years of teaching, the course required all students (approximately 100 students) to write weekly journals, 1 to 3 pages long. Each student wrote a total of twelve entries for the course during the semester, totaling approximately 1200 journal entries. Because of the large volume of journals in those two years, the frequency of journal writing was reduced to bi-weekly in the last three years, which generated approximately 600 journal entries. All 1800 entries were carefully analyzed in this study.

The students wrote journals in a variety of forms. In some entries they simply reflected on their excitement or frustration regarding their learning of a particular mathematics concept that was discussed in the classroom. For example, they might describe a new mathematics concept they learned that week or reflect on the confusion caused by a new concept. For other entries, the instructor provided them with specific questions or scenarios for response. For example, in one of the entries students were asked to respond to the following scenario developed by this researcher:

> Senator *X*, an expert mathematician, proposes a new number system to be adopted in the United States. She said that the new system will have only two numerals 0 and 1. According to her these two numerals can be used to represent any numbers of the present Hindu-Arabic numeration system. "If we can do all the work with two numerals, why bother using the Hindu-Arabic system with ten numerals", argued senator *X*. Other senators, however, are totally confused with the senator *X*'s proposal. Senator *X* further argued that adults are confused with the new system

because they have used the Hindu-Arabic system for a lifetime. "But look at the future of the millions of children who will learn only two numerals to represent all numbers," she contended.

The Senate is now interested in getting responses from public. Write a letter to the Senate describing your position on whether or not the new system should be adopted in the US. Remember that most of the senators have difficulty understanding this system and you should clearly describe what the system is and the advantages and disadvantages of the new system over the Hindu-Arabic system. Please provide examples to demonstrate your understanding of the new system in your letter.

Additionally some selected students were interviewed about their experience in journal writing. All the students interviewed valued the role of journals in their mathematical learning. Because of the space limitation, only the analysis of journals is provided in this paper.

Data Analysis

The researcher/instructor read all the responses from student journals and responded to them during teaching. In addition to reading and providing feedback to students, the researcher also coded, analyzed, and categorized each journal using qualitative data analysis methods, in particular, the constant comparative method (Guba & Lincoln, 1989) and the interactive model (Huberman & Miles, 1994). The researcher also made an overall reflection about student journals at the end of each semester.

Two kinds of questions were considered in the analysis. First, what were students' responses in a particular week's journal? For example, what were various responses provided by students to the entry related to Senator *X*'s proposal of the binary system of numeration? Second, how did journals helped students for their growth of mathematical understanding? These questions led the analysis mainly in two categories: Journals that only described students' attitudes and feelings and the journals that demonstrated student mathematical thinking. Because the constant comparative method of data analysis was used students' responses were checked for consistency and patterns.

Results and Discussion

In response to open topics related to course materials, more than 80 percent of the students simply chose to express their beliefs about mathematics. Although they were asked to demonstrate their mathematical understanding in their journals no such understanding was noted. They made statements like "I liked the last week's class because it was fun" or "I did not understand what you were trying to do in the last class." Moreover, the journals were rather brief and did not demonstrate any in-depth reflection from students. I continuously encouraged them to express their thinking

and problem-solving methods in their journals. Sometimes, I asked them to read specific materials and at other times I asked them to reflect on particular concepts that we discussed in class. These responses encouraged them to think at a deeper level. As the course progressed, the students developed sufficient confidence to express their concerns, difficulties, frustrations, and excitement in their journals. They also began to identify mathematics concepts in their journals. Some of the examples from students are provided below.

> I wish to begin this journal by reflecting on my frustration that I have experienced during this first week of class. The two words "Problem Solving" frighten me, as I have never been successful at solving any math problems (word problems) ever.
>
> I was frustrated this week when we started discussing the binary system of numeration. I liked the Dienes blocks and Unifix blocks [cubes] that we were using and I understood the Base-10 number system (probably because I've been using it for years!), but the binary system was confusing to me when we were using the Unifix blocks. I wasn't sure what you meant by "trade" this one for that one.
>
> This week in math we did more fun things. The first thing we did was the chart to find your age. ... When we first did this I didn't understand how you were getting the answer. I thought that you were looking at every number in each of the columns but then I found out that you add all the numbers together from the top row to get the answer. ... I tried it on some of my friends and they enjoyed the game also.

While the information obtained from students' general entries were helpful to address their problems and concerns, it was difficult to explore their mathematical thinking. Students' mathematical thinking was more evident on specific journal entries, such as those in the senator *X*'s proposal even though there was still a significant proportion of students (about 25%) who provided very general response to this scenario. A student who was struggling to understand the materials in the course responded to the journal as follows:

> Having only two numerals, 0 and 1 would lead to much confusion. As an example telephone numbers, bank accounts, social security numbers etc., would be impossible to distinguish for all citizens in the United States.

This response does not provide good evidence that the student understood the binary system of numeration. It is so general that anyone can state this without actually understanding the system. Some other students, however, wrote what they understood. For example, one student responded like this.

> This is how the senator feels this new system should work. .000000001 would represent 1. After all the zero's are represented by 1, the next 1 will be represented on the other side of the decimal point. This is what the system would like:

.000000000=0
.000000001=1
.000000011=2
.000000111=3
... ...
.111111111=9
1.000000000=10
11.000000000=20 and so on.

The student knew that the binary system of numeration has only two numerals 0 and 1, but did not know how these numerals can be combined to write numbers. The student was simply adding the digits to represent the base-10 numbers. But what was important in the journal was that the student clearly communicated what she understood about the system.

There were some students who clearly demonstrated their understanding. For example, one student responded to the proposal as follows:

> Ladies and gentleman of the Senate, this new system of numbers is very disturbing to me. Let us say we want to call someone. Their phone number is currently 555-2233. To fit this new system we will have to change the number to 101 101 101-10 10 11 11. What kind of phone number is that? Are you really going to remember that kind of a number? It's hard enough to remember a 7-digit code not including the area code. It would be impossible to remember that number.... How old are you? Not 33 anymore. Now the person would be 100001 years old. I am sorry but there is no need to have an age that looks like that. Imagine trying to write that on job applications. If you live at 5 Main street, you now live at 101 Main street. Whoever lives at 101 Main street now lives at 1100101 Main street. Does anyone want to write that for an address? ... Ever buy a computer? The one I am using costs about 1899.00 dollars. In the new system it would be approximately $11101101011. How can we change all of our monetary units to match this system?

This student clearly understood how to convert the base-10 numbers to the base-2 and wrote the journal in a powerful way. The journal was particularly powerful because all the examples chosen are from real life contexts and there are no mathematical errors in the journal. There were other journals in which students demonstrated an excellent understanding of the binary system. For example, let us look at an excerpt from one student below.

> Senator *X* proposes changing our current numeration system (Hindu-Arabic), which is base 10 to a Base 2 system, in which the only numerals used would be 0 and 1. Let me first identify some properties of this Binary System.

1. Remember that all values can be represented, but numbers other than 0 and 1 do not exist as symbols.
2. Hence, the first 10 numbers in the Binary System that Senator X proposes would be represented as: 0, 1, 10, 11, 100, 101, 110, 111, 1000, 1101.
3. For your understanding let's use our (Base 10) number 27 as an example. Instead of a two digit number to represent the value of 27, the binary representation would be 11011! Imagine the length of the number used to represent larger values!

This student demonstrated that she understood not only how to convert the base-10 numbers to the base-2 but also how to perform operations such as addition and subtraction in the new system. The journals that were more specific like the above ones allowed the instructor to explore students' understanding of mathematical concepts required for the course. Moreover, students themselves found that journals were powerful tools for them to express their concerns. As one student commented, "I like the journals because they give me time to think and reflect about what I learned and ask questions as they come up, and not have to wait until class-time." Another student noted:

> I've never really liked math before, but I did in this class. I feel like I had a chance to go back to basics, and find out why everything works the way it does. All through my schooling in mathematics, teachers have said, do this, do this, do that, and never really given a reason, how we know why things really work. Now we know why things really work. I really loved all of the specific journals we did. They really made us think.

The case was similar when students had to reflect on why minus times minus is plus. Many students really had a hard time providing a logical explanation. They knew the rules but they did not know how the rules worked. Following are some typical responses:

> I must be honest about this journal entry. I've never encountered this question. My teachers always told me the facts, such as positive plus positive equals positive or negative times negative equals positive, but never why. I was told to memorize the equations but never understood why. "That's just the way it is;" that's what I was told.
>
> For this week's journal our assignment is to explain why a negative multiplied by a negative equals a positive number. I had a very hard time finding a way to explain why two negatives multiplied together equal a positive because I never questioned why this was true.
>
> It is very strange how we are taught things in school and we just think that this is how something is done without even asking why. I'm sure this occurs in many other

subjects besides math and I think maybe we should ask more questions, and be more critical and make people explain what they mean.

The above responses made it clear to me that many of our students who intend to be elementary school teachers in future never had the opportunity to learn mathematics in a meaningful way. They were taught to accept rules without asking why those rules worked. The responses from students provided me with an opportunity to discuss the importance of understanding mathematics conceptually.

Conclusions and Implications

The use of journals in this course helped the instructor improve his teaching of mathematics in two ways. First math journals were effective in soliciting students' mathematical thinking, both cognitive and affective. In this case, math journals were serving as tools for thinking. Second, the instructor was able to understand students' mathematical thinking and was able to respond to their concerns. As a result of this interaction, students' mathematical understanding was improved.

Despite the roles journals played in students' mathematical growth of understanding, it was clear that only journals entries such as senator *X*'s proposal solicited student's specific mathematical thinking. The non-specific entries provided information about students' beliefs and attitudes, which were also helpful for the course. Because of these positive atmosphere created by journals, the author of this paper believes that journals are important tools for student growth of mathematical understanding despite the fact that the use of journals demands substantial amount of time on the part of the instructor.

The findings of this study are similar to those found by Miller (1992) and Pugalle (2001) that journal writing is useful in mathematics class to improve instruction and student learning. In spite of substantial amount of time I spent in examining student journals, I find that they are powerful means of communication and reflection. It is refreshing that many students who hesitate to speak in the class express themselves well in their journals. This study implies that writing about mathematics improves student mathematical reasoning. This is an important area that needs further exploration by researchers who are interested in student thinking.

Another implication of this study is that student responses in journals depend mostly on the tasks assigned. It the tasks are general such as "write what you think" the students' responses are also bound to be general, in which it is hard to find evidence of their mathematical thinking. These general responses are usually affective, which are related to student feelings and attitudes. Although these affective domains contribute to teaching, there needs to be some specific mathematics tasks such as Senator *X*'s proposal to generate specific mathematical thinking from students.

References

American Mathematical Association of Two-Year College (1995). *Crossroads in mathematics: Standards for introductory college mathematics before calculus*. Memphis, TN: Author.

Burns, M., & Silbey, R. (2001). Math journals boost real learning. *Instructor, 110*(7), 18-20

Chapman, K. P. (1996). Journals: Pathways to thinking in second-year algebra. *The Mathematics Teacher, 89* (7), 588–590.

Dougherty, B. J. (1996). The write way: A look at journal writing in first-year algebra. *The Mathematics Teacher, 89* (7), 556–560.

Guba, E. G., & Lincoln, Y. S. (1989). *Fourth generation evaluation*. Newbury Park, CA: Sage Publications.

Huberman, A. M., & Miles, M. B. (1994). Data management and analysis methods. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research* (pp. 428-444). Thousand Oaks, CA: Sage Publications.

Kroll, L. R., & LaBoskey, V. K. (1996). Practicing what we preach: Constructivism in a teacher education program. *Action in Teacher Education, 18*(2), 63-72.

McIntosh, M. E., & Draper, R. J. (2001). Using learning Logs in mathematics: Writing to learn. *Mathematics Teacher, 94*(7), 554-557.

Mewborn, D. S. (1999). Reflective thinking among preservice elementary mathematics teachers. *Journal for Research in Mathematics Education, 30*(3), 316-341.

Miller, L. D. (1992). Teacher benefits from using impromptu writing in algebra classes. *Journal for Research in Mathematics Education, 23*(4), 329-340.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Pugalee, D. K. (2001). Writing, mathematics, and metacognition: Looking for connections through students' work in mathematical problem solving. *School Science and Mathematics, 101*(5), 236-245.

Pugalee, D. K., DiBiase, W. J., & Wood, K. D. (1999). Writing and the development of problem solving in mathematics and science. *Middle School Journal, 30*(5), 48-52.