THE ROLE OF CALCULATORS IN INSTRUMENTAL GENESIS: THE CASE OF NICOLAS AND FACTORS AND DIVISORS

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We present in this paper a case study of instrumental genesis involving a 14-year-old pupil, the multi-line-screened TI-73 calculator, and a family of numerical tasks related to notions of factors and divisors. This study sheds further light on the interaction among technique, task, and theory in the instrumentation process by showing the crucial importance of the role played by the pupil's need for control of the mathematical situation.

INTRODUCTION

In research on the use of technology in the teaching and learning of mathematics, increasing mention is being made of two distinct ways of viewing tools, according to a theory elaborated by Verillon and Rabardel (1995), that is, as artefacts and as instruments. For example, Mariotti (in press, p. 10) states:

> It is useful to make the following distinction; on the one hand there is the *artefact*, that is, the particular object with its intrinsic characteristics, designed and realised for the purpose of accomplishing a particular task, and on the other hand there is an *instrument*, that is, the artefact and the modalities of its use, as are elaborated by a particular user.

In accord with this theoretical approach, remark that it is the subject who transforms the artefact into an instrument by means of precise and well-organized actions, identified as "utilization schemes." Thus, a finer definition of instrument can be formulated:

> An *instrument* is an internal construction, the development of which is a long-term process; that means that at different moments, different *instruments* are concerned, although the same artefact is actually used. (*ibid*)

According to Verillon and Rabardel (1995), the transformation of artefact to instrument is a complex construction:

> A machine or a technical system does not immediately constitute a tool for the subject. Even explicitly constructed as a tool, it is not, as such, an instrument for the subject. It becomes so when the subject has been able to appropriate it for himself--has been able to subordinate it as a means to his end--and, in this respect, has integrated it with his activity. (p. 85)

Trouche (2000, p. 242) has described instrumental genesis as a combination of two processes: "A process of instrumentation by which the subject adapts self to the tool and a process of instrumentalization by which the subject adapts the tool to self." In the former process, "instrumental genesis is directed towards the subject, and leads to the development or appropriation of schemes of instrumented action which progressively constitute into techniques which permit an effective response to given tasks" (Artigue, 2001, p. 5). Recently, researchers (e.g., Guin & Trouche, 1999; Lagrange, 2000) have attempted to focus on related factors involved in the instrumentation process by which techniques are constituted. They point, for example, to the role played by technique and the task itself in the development of mathematical theory by the student. In fact, according to Lagrange (in press, p. 2), it is helpful to view technique as a link between tasks and conceptual reflection:

> A technique is generally a mixture of routine and reflection. It plays a pragmatic role when the important thing is to do the task or when the task is a routine part of another task. It plays also an epistemic role contributing to an understanding of the objects that it handles particularly during its elaboration. It offers also an object for conceptual reflection when comparing with other techniques or discussing its consistency.

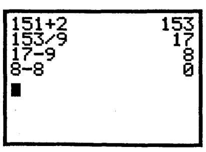
Within the framework of the above research studies, we present in this paper a case study of instrumental genesis involving a 14-year-old pupil, the multi-line-screened TI-73 calculator, and a family of numerical tasks related to notions of factors and divisors. This study sheds further light on the interaction among technique, task, and theory in the instrumentation process by showing the crucial importance of the role played by the pupil's need for control of the mathematical situation.

THE RESEARCH STUDY

GENERAL BACKGROUND

The research project consisted of two phases--the first in 2000 and the second in 2001. In each phase, six classes of Secondary 1, 2, and 3 students (12 to 15 years of age) participated, from schools in Cuernavaca and Montreal. The same basic problem situation was used in both phases, and the technological tools were similar (the TI-83 Plus in the first phase and the TI-73 in the second). The general aim of the research project was to investigate the nature of the mathematical strategies that emerged, as well as their evolution, in the context of particular calculator activities both within and across grade levels. The experience of the first phase led to three main adjustments in the second phase: a modification of some of the tasks; a refinement of the theoretical framework, along the lines described above; and the introduction of the classroom view-screen as a methodological tool that provided a trace of all the calculator attempts of certain students for a given task. The basic problem situation, "Five steps to zero," was as follows:

"Take any whole number from 1 to 999 and try to get it down to zero in five steps or less, using only the whole numbers 1 to 9 and the four basic operations +, -, x, ÷. You may use the same number in your operations more than once." (based on Williams & Stephens, 1992)



A set of 10 worksheets were developed to accompany the basic problem, with questions such as, "Here is a way of getting 432 to zero (a six-step method was given); show how you would do it in fewer steps; explain your strategy"; "The number 266 has as divisors 2, 7, and 19; what would you say is the best strategy for getting 266 to zero in the fewest number of steps; why?"; "Pick a number (less than 1000) that you think would be a hard one for the other group to get to zero in five steps or less; say why you think it is a hard number." Students were also given a brief pretest inquiring into their: (i) use of and attitudes toward calculators; (ii) knowledge of divisors, multiples, and primes. An individual interview with four students from each class was carried out at the end of the week-long activities.

The research was carried out in each class during five consecutive mathematics periods (50 minutes each). The students worked in groups, each student having her/his own calculator. While students were busy with their worksheets, there was always one student at the front of the class doing his/her work with a calculator hooked up to the view-screen that was video-taped (Phase 2). As well, the ensuing classroom discussions and student demonstrations of their strategies were videotaped.

The analysis of student strategies that emerged at each grade level and the way in which they evolved over the course of the first phase of the study was reported at PME last year. For the second phase of the study, rather than focusing on global trends, we decided to concentrate our analysis on the development in the thinking of individual students in the participating classes and to situate that analysis within the theory of instrumentation. Of the series of case studies emerging from this phase of the research, we present in this paper the story of Nicolas--a rather bright, Secondary 2 student, who seemed to enjoy mathematics.

THE CASE STUDY

Nicolas's initial strategy and its evolution

From the beginning, Nicolas's strategy was to find the largest divisor possible for the given number, and if it was not immediately divisible to then use trial-and-error until he obtained a number that would be divisible. However, by the third worksheet, this method was proving itself to be rather unsatisfactory to him (see Figure 1). The given number was 732. The way in which his first two attempts are crossed out--with large Xs and scribbling out of some of the steps--suggests the frustration that might have been mounting. Even though Nicolas's third attempt with 732 was successful in the required five steps or less, we notice that he continued in his efforts. In the fourth attempt, there seems to be no trial-and-error, as would be evidenced by mid-step additions/subtractions, and he arrives at zero in only four steps. One wonders whether his strategy is changing, but nothing to that effect has been made explicit.

It was not until the sixth worksheet where the technique that was hinted at, on the fourth attempt with 732 of Worksheet 3, was stated explicitly. On Worksheet 6, when he was asked explicitly for his strategy at getting 731 down to zero in fewer than 5 steps, he wrote (translated from the Spanish text): "First I multiplied 9x9x9 and I obtained 729. After that I noticed that 731-2 was 729. So I did the same operation, but in reverse."

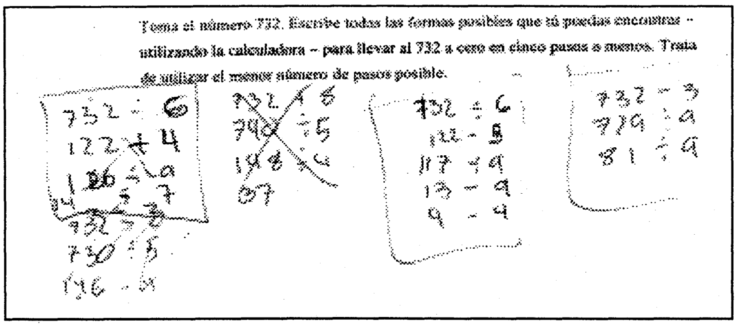


Figure 1: Worksheet 3 of Nicolas

The interview with Nicolas during the following week

It was during the individual interview with Nicolas that we were able to explore further Nicolas's use of his new, instrumented scheme, which had progressively become constituted as the product-of-factors technique. At a certain moment during the early part of the interview, Nicolas was asked what he would do if the given number was not divisible by a number between 2 and 9 on the first step (I = Interviewer; N = Nicolas), to which he said that he would add or subtract:

Verbatim	Comments
31. I: How do you know if you have to do an addition?	
32. N: Because ... Well, I also have a "technique" that I use. First, I do a multiplication, say, 9x9x3 or something like that to arrive at another number, and I look at that number.	He doesn't really answer the question; rather, he gives another way to attack the problem, using the word "technique."
33. I: Let's see, repeat that for me one more time.	Just to be sure about what he has just said
34. N: For example, if I have the number 571 and I multiply 9x9, it gives 81	Here he has chosen his own starting number.
35. I: Let us say that I give you the number 431	The interviewer wishes to offer one for which Nicolas may not have a ready-made solution.
36. N: OK, so I go: L1: 9×9 81 L2: ans×3 243 L3: 9×9×4 324 ~~L4: 9~~ L5: 9×9×5 405 So, like that, I arrive more quickly L6: 9×9×5 405	We notice here that Nicolas generates three potential factors. Then he systematically adjusts one of them. Legend for calculator screen transcription: Ln--refers to line on the screen (note that the TI-73 can display seven lines at a time, and then scrolls); A screen-line that is crossed out is one that the student has deleted from the screen.

37. I: But I said 431. With this strategy that you have just described, how do you begin?	An attempt to get him back to the given number 431.
38. N: First, 9×9×something, no? Until arriving close to the number. For example, L7: 9×8×6 432	He now controls the last two factors at the same time. The 9 is reduced to 8 while the 5 is increased to 6. Quite astonishing!
39. I: Yes, I told you 431	Again, to bring the task to its conclusion
44. N: So, 431 plus 1, divided by 6, divided by 8, and so on.	An oral solution, dividing first by the last factor of 9x8x6, and then moving toward the right, inversing each operation, until arriving at 9 which is handled by a final subtraction.
45. I: Let's see.	An invitation to show it on the calculator
46. N : L8: 431 + 1 432 L9: 432/6 72 L10: ans/8 9 L11: ans – 9 0 And there it is!	He enters his solution into the calculator.

Our analysis of the technique developed by Nicolas suggests that, in lines L2, L3, and L5, he may have been thinking about 81 as the basic unit and realized that 4 groups of 81 (9x9x4) would yield a product that is 81 units larger than 9x9x3. This would have given him rapid approximations of the increasing order of magnitude of the successive products. However, it is not at all obvious when he reached 9x9x5 (yielding 405) why he decided to decrease the second 9 to 8 and increase the 5 to 6 as a means of reaching a number close to 431. Did he "see" that the difference between 9x5 groups of 9 and 8x6 groups of 9 was 3 groups of 9, thus increasing the product of 405 by 27? However, he never did elaborate on how he was able to arrive so quickly at the appropriate factors.

As well, we never witnessed him using this technique with numbers larger than 738, which is 9 more than the product 9x9x9, and thus solvable in four steps. We are of the opinion that, while he could control three factors, four were beyond his range. Thus, by the end of the study, Nicolas appeared to possess the following hierarchical array of instrumented techniques for the tasks of the given family of situations: i) See if the given number is immediately divisible by a largish number, such as 9,8,7,or 6--the division technique; ii) If not, and if it is less that 739, use the product-of-factors technique that he had seemed to generate within the course of the research study; iii) Resort to various trial-and-error combinations of addition/subtraction and division.

DISCUSSION

USE OF SEVERAL INSTRUMENTS FOR THE SAME ARTEFACT IN THE COURSE OF A GIVEN SITUATION

Within the context of this study, there are two issues that arise in the process of instrumentation. One is related to the development of new instrumentation schemes, and the other to their use in given situations. Treating the latter first, we refer to a comment made earlier: "At different moments, different instruments are concerned, although the same artefact is actually used" (Mariotti, in press, p. 10). The Instrumented Activity Situations model as developed by Verillon and Rabardel "does not cover all the characteristics of situations where activity is instrumented: for instance, the fact that a same subject may use several different instruments in the course of a complex action" (1995, p. 85). Thus, a question of interest in our analysis was whether the subject was aware of his/her use of several instruments within a given task and what might be the mechanism driving their differentiated use.

As will be seen from the following protocol extract, Nicolas was quite aware of the two schemes of instrumented action that he was using (see especially the episode of line 64, along with the follow-up in line 66), as well as his reasons for trying the division technique before going on to use the product-of-factors technique. It was simply a question of speed and efficiency (line 48).

Verbatim	Comments
47. I: If the given number is divisible by a number between 2 and 9, do you always begin with a division?	
48. N: Yes, because I think it is faster if you do a division than if you solve it with another technique. So, first divide, and if that is not possible, then add or subtract.	Being able to solve a problem by the quickest method possible seems important for him.
54. N: Yes, add or subtract to obtain another number that can be divided.	
55. I: By what number?	
56. N: Say 8 or 9. The thing is to look for the largest divisor. Since you can add or subtract 9, you obtain many numbers.	His technique is aimed at finding the largest divisors possible.
63. I: Good, how about 362?	
64. N: Alright L1: 36 L2: 362/9 40.2222 L3: 362/8 45.25 L4: 362/7 51.7142 L5: 9×9×4 324 L6: 9×9×5 405 L7: 9×8×5 360 ~~L1, L2, L3, L4, L5, L6, L7~~ L8: 362 – 2 360 L9: 360/9 40 L10: 40/8 5 L11: 5 – 5 0	He begins with his first strategy (L2-L4), that is, see if the numbers 9, 8, or 7 are divisors of 362. He clearly does not know the rule for divisibility by 9. From L5, he uses his product-of-factors technique. In L6 and L7, he controls one factor at a time, but notice the shift from the last factor to the second-last factor in L7. He thus finds the factors of a number in the immediate vicinity of 362. He seems quite aware of the two schemes of instrumented action that he has used. From L8 to L11, after clearing the screen of his calculator, he applies his technique in reverse, using the largest factors, so as to provide a solution to the problem.
65. I: Very good. Now do the same with 323.	
66. N: OK L1: 9×9×4 324 L2: 323 + 1 324 L3: ans/9 36 L4: 9 L5: ans/9 4 L6: 4 – 4 0	Drawing on the information that his product-of-factors technique had produced for the previous problem (L5 of 64), he does a quick check (L1). From L2, he applies the results of the technique.

NEED FOR MATHEMATICAL CONTROL AS A MOTIVATION FOR THE DEVELOPMENT OF NEW INSTRUMENTATION SCHEMES

The driving force behind Nicolas's development of new schemes of instrumented action is conjectured to be the need to use the calculator artefact in the search for problem solutions in the most efficient manner possible. The scribbled-out work of his third worksheet, included above in Figure 1, suggested a distaste for his initial technique, one that combined division and repeated trial-and-error. This technique did not always allow him to advance in a clear way towards a solution. Quite quickly (by Worksheet 3), he seemed to tire of the trial-and-error component and want to have more control of the problem situation. We postulate that this mathematical need was the basic mechanism underlying the generation of his new instrumented scheme, a scheme oriented toward working in the opposite direction by means of multiplication of factors.

Obviously, Nicolas's technique of manipulating sequences of three factors was presently limited, within the context of the given family of tasks, to whole numbers less than 739. We have little doubt that, had the research activities continued over a longer period of time, his mathematical needs would have pushed him to extend the utilization scheme that he had developed so as to handle larger numbers. "An instrument is rarely definitively constructed; schemes evolve" (Trouche, 2000,p243).

ROLE OF THE TOOL IN THE CONSTITUTION OF TECHNIQUES

A thornier issue concerns a subtler question, that of the role played by the technological tool, in this case the multi-line calculator, in the development of the product-of-factors instrumentation scheme. When we consider the triad, technique-theory-task, it seems likely that, for Nicolas, the technique was constituted in response to the task and that the technique contributed to his further understanding (the theory) of the mathematical object--in this case, the interplay between divisors and factors. What is less evident is the extent to which the constitution of the technique was provoked by the affordances and constraints of the artefact. Clearly, the task was one that lent itself to the use of the calculator artefact. And perhaps the speed with which the calculator is able to carry out the demands of the user leads him/her to think about developing new approaches that would not have occurred in a paper-and-pencil environment. However, this factor is much harder to access. Certainly, Nicolas was NOT of the opinion that the calculator "helped him to think." At the end of the interview, he was asked what role the calculator had played for him in these problems:

N: Well, for the operations, I did them more rapidly and I knew if the numbers were divisible or not. The other way [i.e., with paper and pencil] I would have to do each operation: divide like we do normally and that would take me much more time in everything that we have just done.

I: Do you believe that the calculator helped you to think?

N: No

I: When you told me: 'First I look at the number, I do multiplications by 9, and I see if I am approaching the number.' Doesn't that mean that it helps you to think?

N: In my opinion, no. Because I think first about a procedure and how I could do it, and only afterwards I enter the numbers into the calculator and see the results.

Despite his claim that he thinks first before using the calculator, there were rarely any noticeable pauses on his part while using the calculator during his interview--even when he was manipulating various factor combinations. Even though the result presented on the calculator screen served as a basis for his next thought, Nicolas was unwilling to give it any more status than the result he might have generated by paper and pencil. Thus, the use of the calculator as a "tool to think with", in tasks of a numerical nature, is a concept that certain calculator users seem unwilling to entertain.

It was pointed out earlier that, according to Varillon and Rabardel (1995, p. 85), a tool becomes an instrument for the subject when "the subject has been able to appropriate it for himself--has been able to subordinate it as a means to his end--and, in this respect, has integrated it with his activity." Nicolas had certainly subordinated the calculator as a means to his end, and it had thereby become an instrument for him. However, the instrumented scheme he developed, which became constituted as the product-of-factors technique, did not--in his opinion--include explicit ties to the artefact, despite the fact that the tasks were calculator-based to begin with. Nicolas seemed to view his <u>need to control the mathematics of the situation</u> (the *task*) <u>in a 'thinking' way</u> (the *theory*) as the basis of his new *technique*.

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