EXPLOITING CLASSROOM CONNECTIVITY BY AGGREGATING STUDENT CONSTRUCTIONS TO CREATE NEW LEARNING OPPORTUNITIES[1]

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Abstract: We examine new activity structures that involve aggregating personal mathematical constructions built on hand-held devices such as graphing calculators on larger computers in publicly shared displays. Among the many possible applications of classroom connectivity, we focus on situations engineered to require students to coordinate mathematical ideas and representations within systematically varying families of functions.

INTRODUCTION: THE CONTEXT AND GOALS FOR THE STUDY

Data, Subjects, Course-Context, and Technology

The context for the study is the same as reported in (Hegedus & Kaput, this PME). In particular, video, field notes and student work in a course for academically weak university freshmen were analyzed. The 12 students reported on here, averaging 19 years of age, were mathematically similar to 14-16 year olds. The three 1-hour sessions from which the observations were made followed those reported on in the accompanying paper, and occurred slightly past the midway point of a 15 class SimCalc exploratory teaching experiment. Students sat at tables in groups of 3-5 in a crowded space, where each used a TI-83Plus graphing calculator running MathWorlds software. Calculators could be connected, by 2m wire, to a hub that could serve up to 4 wires "quadrapus"-style, where different calculators might be interchanged to use a single wire if needed. The hub, in turn, could wirelessly send to and receive data from an external server. This was a prototype version of the TI-Navigator™ system. At the front of the class, in control of the teacher and his assistant, was a computer with display that could upload and aggregate (as described below) student work from the server while running a computer version of MathWorlds. The actual mechanics of the prototype system are ignored for the purposes of this description since they are subject to change as the system evolves.

Goals of the Study

Our aim was to understand the affordances of this level of classroom connectivity, both in terms of the new kinds of activity structures it could support as well as the teaching and learning opportunities it might support, both planned and spontaneous.

COORDINATING MULTIPLE DESCRIPTIONS OF SITUATIONS IN SIMCALC CLASSROOMS

Essential Goals of the SimCalc Project

The SimCalc Project, underway for almost a decade as of this writing, seeks to democratize access, beginning at as early an age as possible, to the core ideas of the mathematics of change and variation, especially the ideas underlying calculus (Kaput, 1994; Kaput & Roschelle, 1997; Kaput, Roschelle & Stroup, 2000). The essential means for the effort are a combination of representational and curricular strategies beginning with two foundational representational strategies that change the representational infrastructure embodying the ideas, from algebraic to visual:

(1) To use interactive simulations hot-linked to new forms of visually editable graphs and visualization tools, and

(2) To build the fundamental relationships between rates and accumulations (what is normally referred to as the "Fundamental Theorem of Calculus") into both the software and associated curriculum.

Much of the student work using these representational strategies takes the form of coordinating multiple descriptions of the physical or simulation phenomena involved. For example, relating to #1, students must connect time-based coordinate graphs of position or velocity functions to screen animations driven by those functions, e.g., understanding that where a velocity graph is flat, the object must be moving at constant velocity, and when a position graph is flat, the object must be stationary. Relating to #2, they must learn to coordinate velocity and position descriptions of the same motions (or rate-accumulation relationships for other change-phenomena), e.g., understanding that a horizontal velocity graph corresponds to a straight-line position function, and vice-versa.

Further, students must also coordinate multiple representations of the same functions, e.g., algebraic formulas and coordinate graphs. Not only is the coordination among different descriptions and representations an essential part of the SimCalc strategy, it is also a more general goal of mathematics education extending well beyond the mathematics of change and variation.

Exploiting Diverse Devices and Classroom Connectivity

Technologically, the SimCalc Project has built software to support the above representational strategies, first for computers, and more recently for hand-helds, particularly the popular TI-83Plus graphing calculator (see www.simcalc.umassd.edu for downloadable software and curriculum materials for each). Most recently, we have begun development and extension of technologies that attempt to exploit the strengths of these different kinds of hardware platforms by using them in combination, particularly where each student has the personal at-handedness of a graphing calculator, and where the teacher has available the processing power and high resolution/color display of a larger computer. Further, we attempt to exploit the newly available classroom connectivity as sketched above. Other work exploiting classroom connectivity and diverse hardware platforms is underway by Stroup and Wilensky (Wilensky & Stroup, 2000) and Resnick and Collela (Collela, 1998), where the focus is also on integrating individual student constructions into larger classroom structures, especially participatory simulations, where each student plays the role of an agent in a larger system with emergent behavior.

The Set-Up: Unique Two-Digit Student Identifiers to Support Aggregation

We used the same three levels of activity-organization as described in Hegedus and Kaput (this PME). Individual, small-group (students sharing a hub, so we refer to these as "hub-groups"), and whole-class. Each student in the class has a unique 2-digit identifier rather arbitrarily defined as follows. The first digit is determined by their hub-group number (which ranged from 1 to 3 for these sessions), and the second digit is determined by simply counting-off in each hub-group beginning with 1. The 3 respective hub-groups had 5, 3 and 4 members for these sessions, so the identifiers ranged from 11 to 34 and were fixed across the three sessions because the students stayed in the same groups, with the same count-off numbers.

The Two-Prior Aggregation Activities

In Hegedus and Kaput (this PME), we described two initial aggregation activities using the 2-digit identifiers to create parametrically varying families of linear position functions where the variation depends directly on the students' identifiers, which in turn means that the variation depends on the students themselves. The *Staggered Start, Staggered Finish* activity involved a simulated "race" with a Y=2X target position function controlling the motion of one screen object ("***A***"), where each students created, in Y=mX+b form, a linear function controlling a second screen object ("***B***") where their second object ***B*** started at their count-off number but traveled at the same rate as ***A***. Thus they finish at the same distance apart as they were when they started. Within a hub-group each student's function was different from each other student's function, varying in the Y-intercept, so each was parallel to ***A***'s Y=2X function. Each student then sent their position function for ***B*** to the teacher where it was aggregated with all the other students' position functions. While each student's object ***B*** was represented as a dot in the aggregated collection of objects, there were only 5 different (parallel) position functions because there were only 5 count-off numbers. Part of the whole-class discussion involved identifying each student's dot and function-graph, where as many as three students' position graphs overlapped.

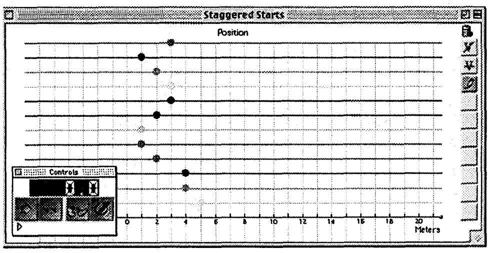
The next aggregation activity was a more challenging variation, the *Staggered Start, <u>Simultaneous</u> Finish* activity, where each student started at 3 times their count-off number and was to finish in a tie with ***A***. After aggregation in this case, the motion showed a series of dots starting in staggered positions and traveling at constant velocities which depended on how far they needed to travel in the given 6 seconds. And the slopes of their respective Position functions likewise varied depending on the starting positions (Y-intercepts) since all the graphs "focus" in to intersect at the common (6, 12) endpoint, reflecting that the race must end in a tie.

Of special interest in this paper is how we are able to exploit further the students' personal connection with their constructions in the aggregated publicly displayed set of student constructions in order to serve developing the two kinds of critically important coordination skills described above.

AGGREGATION-BASED ACTIVITY STRUCTURES: WHERE ARE YOU?

New Student Coordination Activities: Where Are You?—Example 1

These activities move to a higher level of complexity by involving both the student's count-off number and their hub-group number in their individual mathematical constructions. Each activity was designed to put the student in the position of needing to coordinate important information about their mathematical construction in order to identify either their motion-object, their function, or a closely related function in the aggregated set in the classroom display – to "find yourself." The first case involves coordinating position graph information with motion information.



Example 1: Make a Y=mX+b Position function formula for a 5-second motion for **B**, where your starting point is your count-off number and your slope is your group number. Then send it up and we will examine where you are.

Figure 1: Initial Positions Equal to Count-off Numbers at Time = 0 Sec

In *Figure 1*, is what the students saw when the functions were first aggregated. In particular, note that we deliberately did not show their function graphs, which they had already seen on their calculators, and, in fact could—and did—refer to as the discussion proceeded. In response to the teacher's question "Where are you," there followed an animated 30 minute discussion that occurred in 3 stages, the latter two of which were based on revealing, respectively, additional information when a consensus was formed that the information was needed: (a) initial position as shown in *Figure 1*, (b) approximate velocities and ending points, and (c) exact velocities as shown by the "Marks" in *Figure 2*. Also revealed in stage (b) was the fact that one student had created a 6-second motion and that one student had entered an incorrect slope (2, which should have been 3). Space limitations prevent inclusion of the transcript of the extremely rich classroom discussion (a full account is in progress), although a few observations are central to understanding the activity design and how the students responded.

In stage (a) students recognized those dots that might represent them but decided that they could not exactly identify themselves, except to know that their companions on the "starting-line" all shared their count-off number. The one exception was Clive, who was the lone member of the class with count-off number equal to 5. He appears in the bottom position, and, when he identified himself, seemed to stimulate and consolidate the consensus that more information was needed by the remainder of the class. Interestingly, Clive had been among the quietest students in the class and barely spoke publicly during the first 6 weeks of the course. By the end of the 3rd class where he was the only student with 5 as a count-off number, he was acknowledged as "famous" by another student and frequently contributed to classroom discussions.

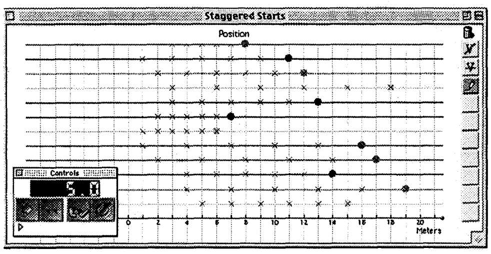


Figure 2: Ending Positions with Marks Shown

In stage (b), as the class came to recognize that some groups would be faster than others and hence would yield greater total distances traveled, they were able to make progress in identifying themselves, although the errors mentioned above added sufficient uncertainty, as did the fact that the starting positions disappear when the animation is run. The need for specific identification forced a move from qualitative analyses (e.g., "He went the farthest so is probably in Group 3") to quantitative analysis.

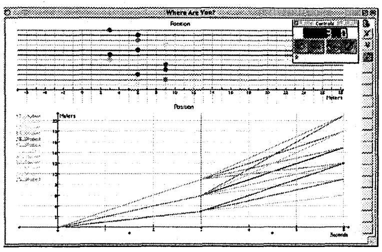
An important visualization feature built into MathWorlds is to have the moving objects "drop marks" (location-traces) for specified time intervals as they move, with the default being 1 second. The students had come to use this feature regularly when the velocity of an object was in question. Hence they called for marks to be dropped. This resulted in *Figure 2*, which shows the positions of each object at 1-second intervals from the initial position to the ending position.

Since the velocities are constant in this case, we can now read off the respective velocities of each object and coordinate it with the starting position to uniquely identify the identity of each dot.

Reflections on Example 1: Using Personal Identity to Coordinate Linear Function & Motion Information, Especially X-Coefficient & Rate-of-Change

Example 1 indicates how, by selectively hiding certain information, in this case the coordinate graphs for the algebraically defined linear functions, and revealing additional information about the associated motion in a pedagogically functional way, students' personal identities act to motivate and focus attention on key features of a display. Exactly the kinds of cognitive things we want to happen in traditional motion-representation coordination activities at the heart of SimCalc representation strategy #1 identified above (e.g., identify the motion that goes with this formula, or make a formula that matches this motion) can occur very naturally when the activity is of the *Where Are You?* type. Far less obvious in this sketchy, abbreviated and static-medium account is the high level of personal engagement of the students, and, more importantly, how that engagement helped structure the coordination process.

Example 2: Coordinating Velocity & Position Descriptions of Motion—Part 1



Example 2: Make a 2-step velocity function, each step 3 seconds long, where your velocity for the 1st segment is your Group # and your velocity for the 2nd segment is your Count-off # in the group. Everyone starts at 0. Send it up and we will look at the position functions and examine where you are.

Figure 3: Branching Position Graphs – Group # = First Velocity

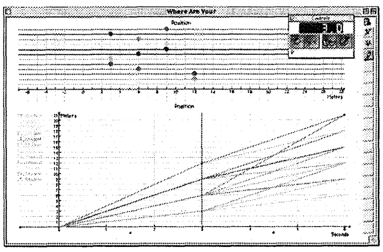
MathWorlds on the calculator (as on the computer) enables students to make step-wise varying graphs directly by systematically adding and manipulating segments. We displayed the dots and position graphs for the whole class and then asked "Where Are You?" The goal here was to coordinate velocity and position descriptions of a motion—using the ideas that velocity is slope of position graphs and that area under velocity graph segments gives the position-change during those segments. In this case the geometry of the configuration comes to play an interesting role relative to the motion.

Note that *Figure 3* is a cleaned-up version of the initial one that appeared—one student sent up a constant velocity graph steeper than the rest and was discovered to be in error. The set of position graphs consist of a 3-branched tree from the origin, one branch for each of the 3 groups, each of which then branches for each of the counting numbers in the group, which varied from 3 to 5. The discussion leading to the identification of individuals was extraordinarily rich, and filled with excitement as the students gradually recognized the two separate roles for their two numbers in the shape of the graphs and the resulting motions. Interestingly, as can be seen from *Figure 3*, all members of a group travel together, side-by-side, for the first 3 seconds, at which time they diverge to travel at their different count-off number velocities, a fact noted by the students.

Note that Clive, the only "5" was an outlier, and went the fastest at the end of the trip, tied for the longest distance, etc. One student observed that those people with straight graphs and constant motions were those whose count-off number equalled their group number. They also kept their velocity graphs available for reference in the coordination and used the motion information to anchor the two descriptions, exactly as we use it in our traditional SimCalc activities, most of which take the form of matching activities (e.g., make a position function for B to match A's motion exactly, where A's motion is given by the given velocity graph).

Example 3: Coordinating Velocity & Position Descriptions of Motion—Part 2.

Here, members of a group split apart at the outset, where student with the same count-off number were side-by side. This version involved a very different configuration of graphs and motions appearing—see *Figure 4*—except, of course, for those students whose Group # = their Count-off #. Of interest is how group identity was vocalized in this case, as each group "broke apart" in the 1st 3 seconds and never came together, despite the fact that they all traveled at the same velocity for the last 3 seconds. It appears that these kinds of activities also provide a prime context for purposeful and logical problem solving, because both their and their classmates' identities are involved. In this case, there was someone unaccounted for and that person was identified by elimination that involved coordination between the slope-information on the position graphs and the velocity information.

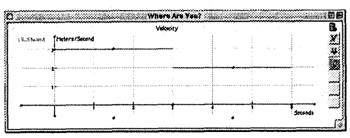


Example 3: Make a 2-step velocity function, each step 3 seconds long, where your velocity for the 1st segment is your Count-off # and your velocity for the 2nd segment is your Group #. Everyone starts at 0. Send it up and we will look at the position functions and examine where you are.

Figure 4: Position Graphs With Count-off #'s as 1st Slope.

Example 4: Velocity Graphs Provide No Position Information.

In this case, we included the example to drive home the point, which had arisen previously, that velocity information tells us "how fast" but not "where." This shows up quite dramatically where we showed what appears to be a single velocity graph (they all overlapped—See *Figure 5*) and asked "Where are you?" We temporarily hid the dots, which would have given away the position information. Of course, it is still inconclusive in this case even when coupled with the motion shown—where all the dots move at the same velocity, and end in the same configuration in which they started.



Example 4: Make a 2-part position function, where you start at the sum of your count-off and group numbers, and each part is 3 seconds long. The slope of the first segment should be 1 and the slope of the second should be 2. Send it up and we will look at the velocity functions and examine where you are.

Figure 5: Velocity Functions for 12 Students—all overlapping

REFLECTIONS ON THE EXAMPLES: USING TECHNOLOGICAL CONNECTIVITY TO GENERATE PERSONAL CONNECTIVITY

These early examples only scratch the surface of what we believe to be possible in exploiting new classroom connectivity for educational purposes by tapping into personal identity as a resource for focusing attention and generating engagement in complex mathematical activity. Many more examples are being examined, and the reader could surely generate more. For example, we are currently studying the use of CBR-based data, where groups separately create motions based on their own physical motion, which are then aggregated in a class to form dances and marches and then re-animated on the computer screen. Again, the kind of thinking needed to plan out a dance that, of course, is executed simultaneously, but produced serially, is exactly the kind that we would want to produce by traditional means.

As pointed out by Donald (2001), humans accomplish extraordinarily complex tasks of management of their mental resources and communication in everyday social contexts using the cultural tools of language and, as needed, other representational tools. Most mathematical activity ignores these resources despite the unanimity of recognition of the power of classroom talk and norms that support inquiry and purposeful discussion. The kinds of aggregation activity structures described above deliberately build systematic mathematical variation, personal identity and ownership into a functional classroom role by making them an intrinsic part of the activity structure itself at one or another levels of group organization. Future work will continue to explore and map out this extremely rich opportunity space.

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