From the Lotto Game to Subtracting Two-Digit Numbers in First-Graders.

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The purpose of this study was to design a context for the subtraction of two-digit numbers that would survive the transfer into the classroom situation, make sense to the children and thus have the power to elicit their outside-school intuition and would lead to the invention of an efficient computational method. A modified version of the Lotto Game was found to be such a context. All the first-graders who participated in this study invented and used the "Overshoot-and-Come-Back" method in the context of the game. Moreover, this method was used by the children also when later presented with abstract subtraction expressions where the unit digits of the subtrahend were larger than the unit digits of the minuend.

Background

It has been found that grade 2 and 3 students who use their own invented-strategies to solve multidigit subtraction make significantly fewer errors than students who were taught to use the standard subtraction algorithm (Carpenter et al, 1998). Moreover children who use invented-strategies develop knowledge of base-ten number concepts earlier (Hiebert & Carpenter, 1992) and gain higher levels of understanding (Hiebert & Wearne, 1996) than students who were taught from the beginning to use the standard algorithm. First grade students, however, usually do not demonstrate ability to invent algorithms for subtraction of two-digit numbers, although they are able to for addition (Carpenter et al, 1998) even though they had been previously provided with opportunities to solve both addition and subtraction word problems that involved two-digit numbers and solved them meaningfully.

Typically, pupils who are encouraged to develop computational strategies learn in mathematics classrooms that are characterized by instructional designs that afford the construction of mathematical knowledge via solution of context problems (word-problem situations). Well-chosen context problems provide the students with opportunities to develop specific-situation solution strategies. When these informal strategies do develop, they can then be modeled. These models can then be generalized to develop into entities that may become models for mathematical reasoning (Gravemeijer & Doorman, 1999).

Study

Following the above-reported results that indicate that invented strategies develop students' understanding and result in more correct solutions, it would seem disadvantageous to teach first-grade students algorithms for two-digit subtraction expressions. However, since situations that involve subtraction of two-digit numbers are meaningful to them (and they are able to solve them albeit in primitive basic methods like one-digit back counting or modeling with unit counters) it seems challenging and advantageous to look for situations that are likely to provide new affordances for fostering higher invented strategies (Schwarz & Linchevski, 2002). For instance, children have been party to situations where they paid for an item out of a given amount of money they had in their possession. For example, a child might have 64 cents in his or her pocket – 6 dimes and 4 one-cent coins – and buy candy for 28 cents. Thus, if it were possible to "import" such a situation into the classroom in a context that would evoke in the children their outside-school intuition (situated intuition), an invented computational strategy might emerge and then be developed into more formal mathematical thinking.

Two questions arise:

1) Is it possible to design a context also in the classroom that would afford the development of an invented computational strategy for the subtraction of two-digit numbers in first graders?
2) If so: a) What strategies would be invented for this context?
   b) Could the model of this situation become a model for abstract subtraction problems?

Our hypothesis was that

a) A " buying-selling" situation could be a context for the developing of invented strategies for the subtraction of two-digit numbers.

b) Since invented subtraction strategies generally start with the larger unit (Hiebert & Wearne, 1996; Fuson et al. 1997), we expected two possible invented strategies: i) Decompose-Tens-and-Ones; ii) Overshoot-and-Come-Back. For example, in strategy i) the children would first pay 2 tens out of the 6 tens, change another ten to 10 ones and then pay 8 ones out of the 14 ones leaving 6 ones. Answer: 36. In strategy ii) the children would pay the 28 with 3 tens, get 2 ones back, so they are left with 34 + 2. Answer: 36. This strategy we coined the "Change" method (sometimes referred to as "Compensation" or "Overshoot-and-Come-Back").

Students who would develop strategy (i) could be guided to model it mathematically as, for example: 60-20=40, 10(out of the 40)+4=14, 14-8=6, 30+6=36. Students who would invent strategy (ii) could be guided to model it mathematically as, for example: 64-30=34, 30-28=2, 34+2=36.

Study Design: The study involved two sets of teaching episodes in heterogeneous groups in a cooperative learning environment (Linchevski & Kutscher, 1998). The children were first-graders who had not yet received any instruction in subtracting two-digit numbers, although they were acquainted with addition and subtraction of one-digit numbers. One of the researchers was both teacher and interviewer. All the meetings were videotaped to allow further analysis. The sequence of episodes was designed a) to elicit out-of-school spontaneous strategies for the subtraction of two-digit numbers, b) to allow the shift of the spontaneous strategies from intuitive meanings to mathematical meanings expressed in mathematical sentences (Linchevski & Williams, 1999).

Research population: An average, suburban first-grade class was chosen for our research population. Students who participated in the study were those who on a pretest were able to a) read/identify two-digit numbers b) decide which of two given two-digit numbers is the larger c) understand that a two-digit number like 27 is 20 plus 7. The two top students of the class were excluded. 20 students participated in our research.

Teaching episodes: During the first set of teaching episodes a "buying-selling" game – a modification of the well-known children Lotto Game - was played by the groups. Each of the participants was given a different lotto-board that had 9 picture squares drawn on it. For each lotto-board there were 9 square picture-cards identical to the ones drawn on the board that the players had to accumulate in order to cover the pictures on their lotto-board and thus win. We modified the game by writing prices of two-digit numbers on the pictures; their values were between 11 to 45. Each child started off with the same amount of money – 77 IS - 7 strips with division lines, each strip representing a 10 IS note and 7 squares representing the ones. Each child had an empty 200-number-board on which he or she could store and arrange their money. There was also a communal bank of money where there were ample tens and units that the students could change from tens to units and vice versa, so that they could apply any strategy they wished. The cards were distributed equally among the children. If a player did not have a picture-card for his or her lotto-board they had to buy it from another player in the group according to the "price" of the picture-card. This "buying" process laid the base for the subtraction.

During the second set of episodes the children reflected on concrete cases that they experienced during their game and were offered ways of translating the intuitive strategies they used during the game to mathematical sentences. The translation was done in four stages:

a) A verbal formulation of the solving strategy in shared spoken-language terms for concrete cases from their game. The verbal formulation was based on the shared verbal communication they had developed during the game.

b) (i) A written account of the solving process for concrete cases using the verbal formulation.

(ii) Solving expressions written in verbal formulation of hypothetical cases that could occur in a game.

c) (i) A written representation in mathematical symbols for concrete cases.

(ii) Solving expressions written in mathematical symbols of hypothetical cases that could occur in a game

d) Solving written abstract subtraction expressions.

Results

First set of teaching episodes: The lotto game.

When the children were faced with the problem of having to pay money for a picture-card, having enough money on the board, but not enough ones, **all** the children who participated in this study used the "change" method to solve their problem.

The process of the invention of the "change" strategy: 1) Identification of the problem 2) Solution of the problem.

1) Two patterns of identification were recognized:

a) Identification of the problem after trying to pay the exact amount:

Y wishes to buy a flag. Its price is 17. She takes off 1 ten-strip while she counts the units. She takes off money of value 16. There are no more units on the board. She immediately takes off another ten-strip – the buds of the "change" strategy.

b) Identification of the problem before paying by comparing the number of units on the board to the number of units that need to be paid.

K wants to buy a car. Its price is 29. *"I don't have".*
Interviewer: *What do you mean when you say 'I don't have'?*
T (another player in this group): *That he doesn't have the number.*

Interviewer: But you have even more (than you need).
T: *That we don't have like these;* and he points at the units.

2) The solution - a joint learning effort: Report and Analysis.

The following is representative of how the groups constructed cooperatively their strategy:

B is interested in buying a sofa. Its price is 47. He removes 4 ten-strips and says *"I don't have".*
Interviewer: *Does anyone have an idea?*
A: *To give one like this.* And she points at one of B's ten-strips.
B removes 1 ten-strip from his 200-board.
Interviewer (to V, the owner of the sofa-card): *What will you do?*
V: *I'll give* change.
Interviewer: *How much change? You were supposed to get 47 and you got 50.*
V and A announce a few numbers: 2, 5, 7. And A shouts: *"Three!"* This answer is approved by all.

Every member of this group was involved in the construction of the solution. B identified the problem, A suggested the need to "overshoot", V the need to "come back" and A found the amount you "come back" with.

As reported earlier, **all** children adopted the "change" method in the game. But none found it necessary to find, after a transaction, the amount of money left on his or her 200-board: the answer to the multidigit subtraction problem. Their behavior corresponded to the situation that elicited their spontaneous strategy: in a buying-selling situation one does not usually calculate the amount left in the wallet – unless there is some concern that the amount will not suffice. Thus the children's disregard of the amount left on their 200-board is not surprising. The teacher would call their attention both to the initial amount on the 200-board and to the amount left after each transaction in the second set of teaching episodes, when the transactions would be translated to mathematical subtraction- expressions.

When encountering the problem of not having enough units to pay, in no instance did the pupils suggest that the buyer exchange a ten-strip for ten ones albeit the 'bank' was on the table. The decomposition of tens strategy did surface once, when the seller did not have enough change:

V is interested in buying a flute. Its price is 16. She removes 2 ten-strips and gives B. B doesn't have change of 4 ones on her board .
Interviewer: *Does anyone have an idea? V doesn't have 6 units and B doesn't have 4 units to give her change?*

A: *We'll change this* (points to a ten-strip on B's board) *to small ones.* B removes a ten-strip, puts it in the 'bank' and takes 10 unit squares and sticks them on his board on the row that was vacated when he removed the ten-strip. Only then does he give V 4 square-units.

The preceding vignette allows us to observe the powerful effect of situated-intuition. Why is it that here "decomposition-of tens-and-units" was the spontaneous reaction and that in all the other instances only the "change" strategy was evoked? It might be that in 'real' life experiences, it is usually the responsibility of the cashier to change the "big" money into smaller denominations in order to be able to give change to the buyer. Furthermore, just as was observed in B's behavior, in real life when the seller changes money into smaller denominations he places it in the till and only then gives the buyer his or her change. Thus, when both the "buyer" and the "seller" did not have the necessary units to complete the transaction, the solution that was offered in the game, mirrored a real-life problem-solving strategy. However, this decompose-tens strategy although understood and approved by the group was not used by the students later on.

<u>Second set of teaching episodes: Translation of spontaneous strategies from intuitive meanings to mathematical meanings expressed in mathematical sentences:</u>

This teaching episode had 3 parts:

Part 1) The children played a shortened version of lotto in the presence of the interviewer who documented their transactions using their shared language. This time the game ran very smoothly: all children applied the change method to complete their transactions. At the game's completion the interviewer reconstructed with the children the game's transactions and translated them first into (i) shared verbal formulation they developed and then (ii) into written spoken-language. The translation was from the viewpoint of the buyer as this was the language the children had used during the game. The transaction of money from the buyer was coined "I gave". The transaction of money to the buyer was coined "I received". Reference to the amount of money left on the 200-board was coined "I am left with". The reconstruction was done in 3 stages: a) The interviewer recalled a transaction from the game: "*B had 68 and she had to give 38.*" b) Each child reconstructed this by first sticking 68-worth of strips and/or squares on the board, then "gave" 38 and reported the money that "I am left with". c) The children documented the transaction using a combination of numbers and spoken-language stickers. This was done first with "simple" transactions such as 56-32 that was translated to:

56 [I gave] 32 [I am left with] 24

A "complex" transaction (needing change) such as "There were 74 and one had to give 38" was reconstructed and executed via the 200-board and translated in 2 stages:

a) verbal spoken language: "I had 74, I gave 40, I received 2, I am left with 36",

b) written spoken-language (the expressions "I gave", "I received", "I am left with" were written on stickers to facilitate the first-graders' 'writing'): 74-38= ;

74 [I gave] 40 [I am left with] 34; 34 [I received] 2 [I am left with] 36

The children seemed to be able to reconstruct the transaction and follow the corresponding steps in writing quite easily. But despite the fact that they counted the strips and/or squares left on the board, when the interviewer summarized: *"There were 74, I gave 38, how many are left?"* the children could not answer the question immediately. This difficulty repeated itself a few times in each group. Some children re - counted the strips and/or squares on the 200-board, some recalled the end result on their board and some re-looked at what was written on the paper.

Part 2: Each child was given "Work-Sheet1" with written abstract subtraction problems such as "63 [I gave] 26= " Some children disregarded the equal sign completely, verbalizing the expressions in shared spoken-language as "I had 63, I gave 26" and put a sticker " I am left with" next to the "= " sign: 63 [I gave] 26 [I am left with] 27. It seems that they perceived the equal sign as a "start-working" instruction. For these children the mathematical expression seemed to be part and parcel of the game situation. The children learned to document each part of the process in written spoken-language in the same steps as is reported in *Part 1*.

There were some children who developed dependence on "reconstruction" as a model for documenting the solution. They would solve the entire problem with strips and /or squares, then verbalize these steps and only then use stickers to write all the steps of the solution.

Part 3: The spoken-language expressions "I gave", "I received", "I am left with" were replaced by corresponding mathematical symbols -, +, =. Thereafter the interviewer reconstructed with the children two transactions from the lotto game they had previously played, using the strips and/or squares to calculate the result. Following each game move, she translated it with the children into mathematical sentences using only mathematical symbols, in the same steps as had been previously made with the spoken-language stickers. Each child was then given Work-Sheet2; they were asked to solve written abstract subtraction expressions such as "57-13=" and "42-36=". All the children solved these problems correctly using their strips and/or squares. However, they found the documenting process to be a burden as it broke their line of thought. Thus most children preferred to write only the numerical answer to the problem. Two children did document, but with the aid of stickers. One child documented using only mathematical symbols.

Discussion

The results of this study suggest that first grade students are able to solve two-digit subtraction problems. We infer that the reason our first-graders were able to invent quite a sophisticated computational method for the subtraction of two-digit numbers, despite the fact that children of this age and mathematical experience, are generally not at the appropriate stage of number-concept development, is because of the specific context-design and documenting device. The buying-selling situation - embodied in a modified game of lotto - survived the transfer into the classroom context, made sense to the children and thus had the power to elicit the outside-school intuition. This led to the invention of a computational method - "overshoot-and-come-back" - for subtraction problems where the unit digits of the subtrahend were larger than the unit digits of the minuend. Moreover this computational method was used by the children also when later presented with abstract subtraction expressions.

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