IMPLICATIONS OF COMPETING INTERPRETATIONS OF PRACTICE TO RESEARCH AND THEORY IN MATHEMATICS EDUCATION

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In this article we explore the issue of interdependency of theory and research findings in the context of research on the practice of mathematics teaching and learning at school. We exemplify how analyses of a lesson by using two different theoretical perspectives lead to different interpretations and understandings of the same lesson, and discuss the implications of competing interpretations of practice to research and theory in mathematics education.

INTRODUCTION

In their chapter on 'Competing paradigms in qualitative research', Guba and Lincoln (1994) point to an interesting, sometimes unforeseen, connection between the theoretical framework used by the researcher and the findings of the research. They claim that "theories and facts are quite *interdependent* – that is, that facts are facts only within some theoretical framework" (p. 107). Moreover, they assert, "Not only are facts determined by the theory window through which one looks for them, but different theory windows might be equally well supported by the same set of 'facts'." (p. 107). In this article, we explore this issue of interdependency of theory and research findings in the context of research on the practice of mathematics teaching and learning at school. We exemplify how analyses of a piece of practice – a lesson – by using two different theoretical perspectives lead to different interpretations and understandings of the same lesson, and we discuss the implications of competing interpretations of practice to research and theory in mathematics education.

BACKGROUND

The lesson to be analyzed was part of an innovative yearlong introductory course on functions for ninth-grade. The curriculum developers aimed for the students to investigate problem situations with computerized tools, raise hypotheses, collaborate on problem solving, explain and discuss their solutions, and reflect on their learning in individual or collective written reports. After a successful implementation of the course in high-achieving classes (A-level), the function course is tried in lower-achieving classes (B-level).

The teacher of this trial has been a central member of the curriculum development team. She has decades of experience in the dual role of high-school mathematics teacher and curriculum designer. In both roles she regularly sought for innovations in content and ways of teaching, and systematically reflected on her own teaching and the learning processes of her students. The year preceding the present experiment, she successfully taught the function course to an A-level class in the high school where she regularly teaches mathematics – a religious all-girl academic oriented school. The experimental B-level class belonged to the same school.

The lesson we analyse in this article took place rather early in the school year. It centred on the "Fence Problem", displayed in Figure 1.

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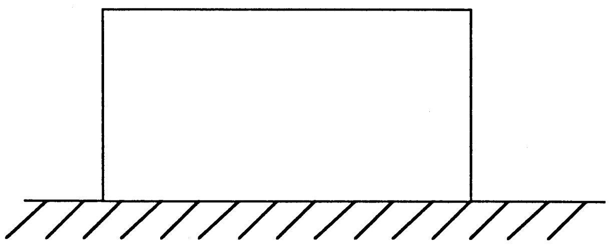
The Fence Problem

Oranim school received a 30m long wire to fence a rectangular vegetable garden lot. The lot is adjacent to the school wall, so that the fence has three sides only.



a. Find four possible dimensions for the lot and their corresponding areas.

b. For which dimensions does the lot have the largest area?

c. If one of the dimensions is 11m, what is the area of the lot? Can you find another lot with the same area? If you can, find its dimensions; if not, explain.

d. How many lots with the following areas are there: $80m^2$? $150m^2$?

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Figure 1. The Fence Problem

The work on the Fence Problem was spread over several lessons. One of them is the "Windows lesson" which is the focus of this study. The Windows lesson was designed to enhance student learning of the concept of function, their understanding about different representations of functions and how to use the graphic calculator to solve problems that require the passage from one representation to another (mainly, between algebraic and graphic representations).

The following two sections present analyses of the 50-minute Windows lesson from two different theoretical perspectives. We first analyse the lesson from a cognitive perspective and then from a socio-cultural perspective.

VERBAL ANALYSIS: A CLASSIC COGNITIVE APPROACH

Cognitive Science focuses on processes and representations. The research question of the cognitive component of the study concerns with the extent to which students, during the Windows lesson, conceive the passage to a new (especially graphical) representation of a function as a problem solving *method*. We use verbal analysis (Chi,

1997) as a representative of Cognitive Science approach. The data for the verbal analysis is the protocol derived from the video recording of the three whole-group discussions that comprised about one-half of the class time (the rest of the lesson comprised of small-group student work). The whole-group discussions provided rich and coherent verbal data, enabling us to consider the whole group of students as an entity interacting with the teacher.

Segmenting the protocol

In line with the research question focus, we chose the passage from one representation to another, as well as the passage within the same representation – between referring to it as a display or an action representation (Kaput, 1992) – as a natural boundary for segments. The different representations of function in the Windows lesson were algebraic, tabular, graphic, and verbal. We identified 19 segments in the reduced protocol.

Coding

For each segment we identify who initiated the passage to a new representation (the teacher or the students), who was the trigger for this passage, and what was the nature of the interlocutor's response (if any). We define five kinds of responsiveness. *Accompanying* talk refers to talk in which the interlocutor attends to the other's talk without elaboration, typically acknowledging that the interlocutor follows the other's talk. *Elaborating* talk refers to talk in which the interlocutor elaborates utterances and expresses deeper cognitive involvement. *Opposition* refers to talk in which the interlocutor explicitly expresses disagreement and objection. *Puzzlement* points to talk expressing confusion, perplexity or bewildering. Finally, *non-responsiveness* refers to the absence of talk subsequent to the initiator's talk. We also distinguish between a passage to a different representation that is embedded in the context of the Fence problem situation and a passage to a different representation that is disconnected from a situational context. Moreover, as we assume that the nature of the class discourse is related to student understanding, we characterise the types of utterances at each segment according to the following categories: Presentation (Pr), Short Questions (SQ), Extended Questions (EQ), Short Answers (SA), Extended Answers (EA), Rephrasing/re-voicing (R), Objection (Ob).

Depicting the coded data

Depicting the coded data is conducted in two ways. One way is displayed in Figure 2. The segments of the lesson are marked in the first row (segments I, S and V indicate the small-group work parts of the lesson which are not part of the analysis). Each of the following four rows designate one of the four representations to which the class talk refers when the context concerns the Fence situation. Sit designates segments in which participants focus specifically on objects and events of the Fence task; Gr, Tab, and Al designate segments in which the talk refers to graph, table and algebra/formula (respectively). The last three rows similarly designate the representational systems to which the talk refers, but here, with no connection to the Fence situational context but rather related to a formal task.

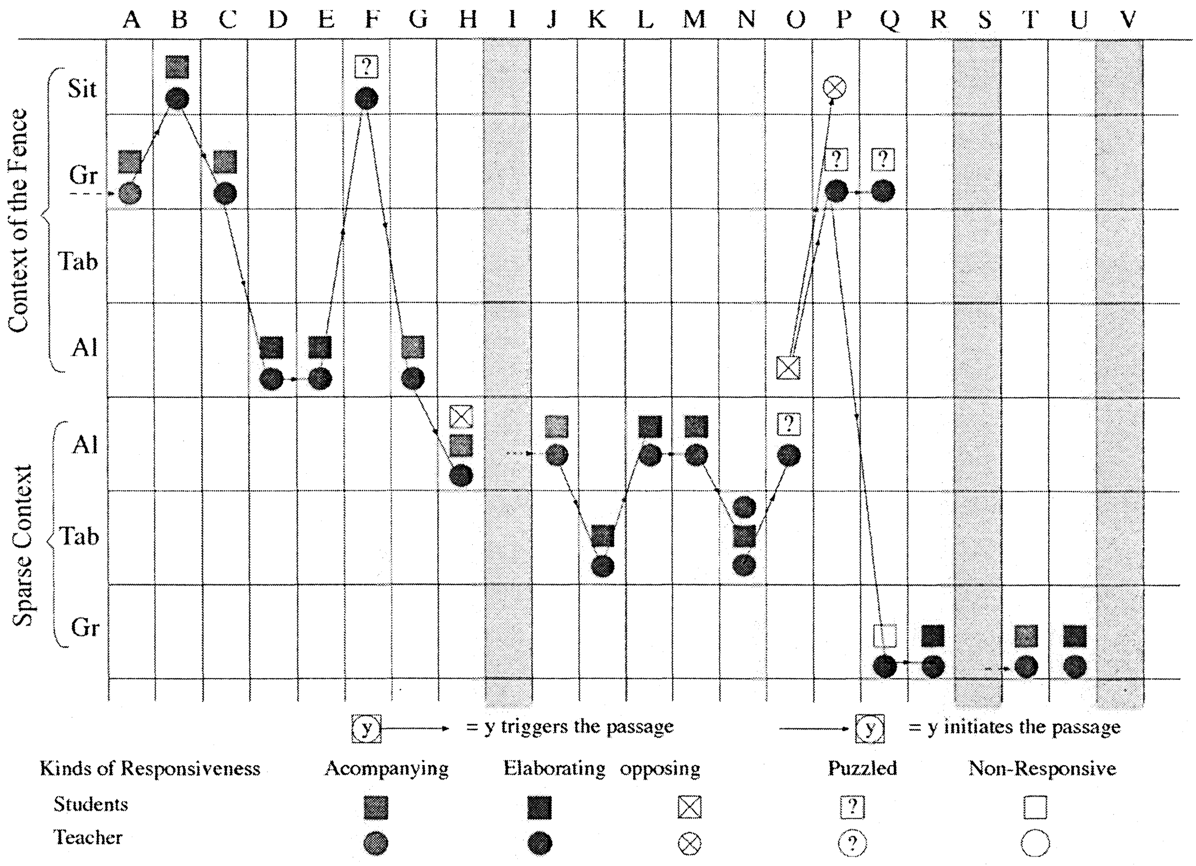

Figure 2. Schematic description of the Windows lesson

Circles represent the teacher and squares, the students. For each segment we mark who initiates the passage to a new representation or the change in the way the representation is used (display or action) by an arrow targeted to the initiator. The trigger for the passage to a different representation is marked at the beginning of the arrow. The nature of responsiveness is marked as following: gray represents *accompanying* talk, black – *elaborating* talk, inscribed question mark – *puzzlement*, crossed – *opposition*, and white – *non-responsiveness.*

Figure 2 displays a comprehensive picture about initiatives and the nature of responsiveness regarding the passing from one representation to another. But it does not show information on the nature of the utterances that comprised the talk. Another method we use to depict the coded data focuses on the categories of utterances. Table 1 presents the percentages of the following kinds of utterance: Presentation (Pr), Short questions (SQ), extended questions (EQ), short answers (SA), extended answers (EA), rephrasing or re-voicing (R), and Objection (Ob).

Table 1. Percentages of utterance kinds

	Pr	SQ	EQ	SA	EA	R	Ob
Teacher (68%)	27%	42%	7%	4%	2%	19%	
Students (32%)		9%	6%	76%	6%		3%

Seeking patterns and coherence in the depicted data

Figure 2 shows that the teacher has a major role in initiating the passage to a new representation throughout the lesson while the students generally only respond to the teacher's initiative. An examination of the protocol indicates that even when students react to the teacher's initiatives with elaborating utterances it is often only few specific students who do it.

Table 1 that shows the nature of <u>all</u> the utterances during class talk (and not only those related to the passage between representations) confirms the central role played by the teacher. About two-thirds of the utterances are the teacher's and she is the only one to make Presentations. When students do participate in the class talk, they typically respond with Short Answers to teacher's Short Questions. Students almost never ask questions. Throughout the lesson both students' and teacher's extended Questions as well as Extended Answers are rare.

Interpreting the data

A reasonable interpretation of the findings is that for the students to use multiple representations to solve problems is a cognitive obstacle. Detailed analysis of different parts of the Windows lesson supports and verifies this interpretation. This conclusion fits with findings of other cognitive studies where it was found that interpreting information from one representation to another is cognitively difficult (e.g., Even, 1998; Schwarz & Dreyfus, 1995). While it is frequently argued that the multiplicity of representations facilitates the learning of concepts as it helps in the integration of perspectives, the multiplicity of representations could also be a *cognitive load* that hinder learning processes.

ACTIVITY THEORY APPROACH

Activity theory (Leont'ev, 1981) takes into account social origins of cognitive processes. The unit of analysis is the human *activity*. Need is always an essential part of an activity. Activities are chains of actions related by the same *object* and *motive*. *Actions* are the basic "components" of human activities that translate them into reality and can be understood only within the activity in which they are embedded. The actions that constitute an activity are energized by its motive, and are directed toward conscious *goals*. *Operations* are the means by which an action is carried out. They depend directly on the conditions under which a concrete goal is attained.

The research question guiding the analysis from an activity theory perspective is: What is the nature of the activity in which the teacher and the students participate during the Windows lesson? Data sources for the analysis include the video recording of the lesson, classroom observation, the protocol used for the cognitive analysis, the two ways of depicting the coded data (Figure 2 and Table 1), and numerous formal and informal discussions with the teacher and the curriculum development team.

The Teacher

Examination of the data for identification of the teacher's motive, actions, goals, and operations indicates that the teacher's overall motive when teaching mathematics is that her students understand and learn mathematics in meaningful ways. More specifically in the Windows lesson her motive is that her students learn about different representations of functions and understand how to use the graphic calculator to solve problems that require the passage from one representation to another. The teacher's actions during the lesson aim at helping the students learn and understand the above and at creating a need to use the graphic calculator as a problem solving tool and to choose cleverly a Range that would enable them to see the relevant part (window) of the graph. Throughout the lesson, the teacher's operations reflect her desire to engage her students in the activity while being attentive to their understandings. The operations are aimed to help the students become involved, understand, and develop a sense of shared ownership of the activity's motive as well as the actions' goals. The teacher's actions and operations suit the teacher's general approach to learning and her belief that students learn by constructing their knowledge through active participation in mathematical investigations.

The Students

Analysis of the lesson observations and of the video recording indicates that many students in the class appear uninvolved and uninterested in the intellectual challenges presented by the teacher to the whole class. Only a small number of students participate in the whole class discussions that the teacher strives to conduct. Even then, as Table 1 and Figure 2 also show, their participation is rather shallow and of low-level, characterized by short answers to teacher's short questions. The majority of the students become involved in the mathematics tasks only during the small group work parts. But also then, most of the students are not involved in exploration, problem solving, debates among themselves, and the like, as was common to the A-level students the teacher has taught in a previous year. Rather, many of the current students tend to yell for teacher help after minimal attempts to solve the tasks, asking her to tell them what the correct answers are. In general, students seem satisfied when they quickly reach the correct final answers and frustrated when they do not. These students, who have a history of learning difficulties and low achievement in years of traditional mathematics teaching, behave according to common beliefs about school mathematics, adapting social and socio-mathematical norms prevalent in traditional mathematics classes. Thus we may conclude that the motive of many students is *surviving* the lesson.

Same Lesson But Different Activities

The analysis of the lesson from an activity theory perspective suggests that the teacher and the students in the Windows lesson participate indeed in the same lesson but in different activities. Their motives are different and consequently the different actions that constitute the activity and are energized by its motive are different. The teacher's actions during the lesson are derived from her overall and specific motives. Her actions are well connected, one leading to another, creating a complete whole. However, beauty is in the eye of the beholder – what the teacher sees is not necessarily seen by the other lesson participants – the students. Many students do not make connections between the different parts of the lesson. Students' goals are different from those of the teacher's – theirs usually centre on obtaining correct final answers to the problems assigned by the teacher, not on making sense of the mathematical situation nor on using connections between different representations of functions as a tool to solve problems. As such, the nature of the students' participation in the activity is very different from what the teacher had wished for and does not contribute to the development of the activity as designed by the teacher. While this may seem odd when the teacher's design of the activity is considered, it makes perfectly sense when *surviving* is taken as the student's motive.

CONCLUSION

Both the cognitive verbal analysis and the activity theory based analysis of the Windows lesson indicate that things are not going smoothly in this lesson. Both analyses show that students do not behave mathematically as desired by the teacher. However, the two approaches suggest different interpretations of the situation and of the sources of the problems observed. The verbal analysis points to students' cognitive difficulties taken as independent of context. The activity theory based analysis suggests that the teacher and the students participate in the same lesson but in different activities, where different motives, goals, beliefs and norms regarding school mathematics drive and guide them. As a result, students' ways of participation in the lesson are different from what the teacher had wished for.

The discrepancy between the two interpretations is disturbing, as it seems to suggest that different methodologies yield different explanations of the same phenomenon. An immediate response is that the two analyses are guided by two different research questions. Naturally, answers to different questions may not coincide. Still, another question emerges: Are the two answers provided by the two perspectives compatible?

For several decades mathematics education research used to focus on cognitive development of mathematical concepts. Recently, the focus of research in mathematics education has extended from the individual student's cognition and knowledge to contextual, socio-cultural and situated aspects of mathematics learning and knowing.

The practices and culture of the classroom community (e.g., the nature of social engagements and norms) have become an important factor in studying learning processes, and mathematics education researchers started to incorporate the two perspectives – cognitive and socio-cultural – into a complex view of mathematics learning. This new focus signals a shift from examining human mental functioning in isolation to considering cultural, social, institutional and historical factors. The mathematics education community increasingly embraces the view that cultural and social processes are integral components of mathematics learning and knowing. In line with the current trend in the field of research in mathematics education we respond by proposing that a lesson, which is part of the practice of teaching and learning mathematics, is too complex to be understood by only one perspective. Consequently, a more complete understanding of the complicated practice of teaching and learning mathematics requires the use of both cognitive and socio-cultural perspectives.

This suggestion settles what at first seemed to be two conflicting interpretations of the same phenomenon. However, such resolution raises an epistemological issue that is crucial to the advancement of research in general and to our domain of mathematics education. The two interpretations of the Windows lesson and their harmonisation fit well with Guba and Lincoln's (1994) reflections on the relations between theory and research findings mentioned at the beginning. To a large extent, research findings could be explained by the theories they rely on, and confirm what the theories already could have predicted. Theory and research are then trapped in a vicious circle, which might not be productive. The effort to harmonise the two interpretations is in itself laudable, but is it legitimate?

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