DEVELOPING CONCEPTUAL UNDERSTANDING: THE ROLE OF THE TASK IN COMMUNITIES OF MATHEMATICAL INQUIRY

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As part of a study of mathematics classrooms functioning as communities of inquiry, video and other data were collected in ten randomly selected Victorian grade 3 and 4 classrooms and two classes at the Japanese School of Melbourne. In this paper, two lessons — one Victorian and one Japanese — are analysed in terms of the conceptual focus and cognitive demands of the instructional tasks and the opportunities these afford for advancing students' conceptual understanding. The stark contrast between the two lessons suggests that, in Australia at least, insufficient attention is being paid to the critical role of the development of conceptually focussed, robust tasks which can be used to support the development of sophisticated mathematical thinking.

INTRODUCTION

> A community of inquiry is not just ... a certain kind of social, interpersonal or ethical environment. It is an environment in which inquiry occurs, by which is meant, among other things, a focus on key concepts and complex thinking. (Splitter, 2000, p. 14)

The *Mathematics classrooms functioning as communities of inquiry: Models of primary practice*[1] project was based on the notion of communities of inquiry, which underpins the *Philosophy for Children* movement (see, for example, Splitter & Sharp, 1995; Splitter, 2000). One of Splitter and Sharp's (1995) conditions for dialogue, which is the cornerstone of a community of inquiry, is that "conversation is structured by being focused on a topic or question which is problematic or contestable" (p. 34). Thus, in a community of inquiry, participants are engaged in confronting problematic situations and participating in dialogue and argumentation (in the sense of Krummheuer, 1995).

Classroom discourse and the sociomathematical norms associated with achieving quality dialogue have received considerable attention (see, for example, Yackel & Cobb, 1996; Kazemi, 1998; Groves & Doig, 1998). However, frameworks for effective teaching to support children's conceptual understanding also emphasise the need for tasks which are mathematically challenging and significant (Askew, Brown, Denvir & Rhodes, 2000; Fraivillig, 2001). In particular, the critical role of the design and enactment of instructional tasks in the development of students' increasingly

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sophisticated mathematical reasoning and understanding is a key component of much of work related to mathematical communities of inquiry (see, for example, Cobb, Yackel & Wood, 1995; Gravemeijer, McClain & Stephan, 1998).

The purpose of our project was to examine current models of Australian mathematics practice in order to investigate the extent to which these support or hinder mathematics classrooms functioning as communities of inquiry, and to determine the extent to which the wider local education community endorse the goal of mathematics classrooms functioning as communities of inquiry. Analysis of focus group responses from principals, teachers and mathematics educators showed a high level of support for mathematics classrooms functioning as communities of inquiry, together with a realisation that current Australian practice falls far short of this goal (Groves, Doig & Splitter, 2000). Principals and mathematics educators rated the cognitive demands of typical lessons as low to very low and not challenging children, while teachers saw the cognitive demand as being determined by the tasks (Doig, Groves & Splitter, 2001).

This paper compares two videotaped lessons from the project in terms of the conceptual focus and cognitive demands of the instructional tasks and the opportunities afforded for advancing students' conceptual understanding.

METHODOLOGY

One mathematics lesson of approximately one hour's duration was videotaped in a stratified random sample of ten year 3 and 4 classrooms in the state of Victoria. A lesson observation schedule was also used to record detailed field-notes of each lesson. Teachers' lesson plans, as well as copies of any work-sheets used by the children, were also collected. In addition, each teacher was asked to complete a written questionnaire providing background information, information related to the specific lesson (e.g. the main topic of the lesson, the aims, resources used, the purpose of any group work and whole class discussion, and the most important part for children's learning), together with information on their mathematics teaching in general. In keeping with Hiebert and Stigler's (2000) view that one way to imagine alternative models is to "step outside the … culture and look at how others handle similar issues" and that "Japan provides an eye-opening contrast" (p. 10), similar data were collected from the year 3 and the year 4 classes at the Japanese School of Melbourne.

An analysis of the videotapes was carried out, using a framework based on that developed by Schmidt *et al* (1996), who use the term "characteristic pedagogical flow" to describe recurrent patterns of observable characteristics in a set of lessons. Each lesson was analysed in terms of its structure, organisation, interactions, cognitive demand, and teacher actions. Based on this analysis and the written questionnaires, data from two lessons — one Victorian and one Japanese — were selected for a more detailed re-analysis in terms of the conceptual focus and demands of the tasks and the opportunities afforded for advancing students' conceptual understanding. These lessons are described and discussed below.

THE CHANCE AND DATA LESSON

This lesson, which occurred in a grade 4 class of 30 children, in a middle class suburb of Melbourne, focussed on aspects of Chance and Data and lasted for about one hour. Children were involved in three different tasks: using sales catalogue prices to spend $1 million while satisfying certain constraints; tossing three coins to investigate the outcomes; and estimating the amount of money in a jar of coins.

In her responses to the written questionnaire and the lesson plan provided, the teacher, Ms E, identified the main mathematics topic of the lesson as being "The chance of winning one million dollars" and listed the above three tasks.

During the lesson, children worked in two groups — one for each of the first two tasks — with the money estimation happening simultaneously as children took the jar of coins around the room and collected estimates. Ms E stated that the two groups would swap tasks in the following lesson.

For reasons of space and the emphasis in this paper on conceptual focus, only the coin tossing task will be discussed here. Ms E said that the aims of the lesson were stated in her lesson plan, where she described the coin tossing task as follows:

> Three coins to be tossed in the air at the same time. What are the possible outcomes? (These are to be listed.) Look at the pattern after 5 throws. BEFORE the 6th throw predict. RECORD what you think the outcome will be. Toss. Record. Repeat activity — compare results. What do you conclude?

This outline closely follows an activity from the *Mathematics: Course advice* (Directorate of School Education, 1996) on which Ms E based her lesson. The Course Advice version includes questions such as: "Did anyone predict the correct answer for any of the throws?" and "Is it possible that all coins could land on the same side?" It also suggests that results should be combined with those of another class leading to "Which results were most common?" and "How do these compare with predictions?"

Ms E identified the outcomes from this activity as being ones related to Level 3 of the *Curriculum and Standards Framework (CSF)* (Board of Studies, 1995), namely that children will be able to record and identify all possible outcomes arising from simple chance experiments; identify outcomes as being equally likely; and make predictions. (In fact, the Level 3 *CSF* learning outcomes state "identify *some* outcomes as being equally likely" [emphasis added] and the only mention of predictions is at Level 4 "make simple predictive statements about everyday events ...", which suggests a quite different context from the one in this lesson.)

In the lesson, children were told to use any three plastic coins from a collection of coins of different denominations and devise their own method of recording. Some children used three coins of the same denomination and others two or three different denominations. Children recorded their throws typically as either HTH or 2H 1T, but it was never clear whether or not HTH was considered different from HHT and there was no discussion of this at any stage.

While the children worked on their tasks, Ms E moved from group to group, discussing progress and generally praising children's efforts. Typical comments from Ms E include:

> What do you think the next [toss] will be? Have a guess. ... Have you got your estimate?
>
> Compare your results with those of ... same or totally different? [Two girls carefully compare their results line by line, with obvious enjoyment but without any comment or follow-up.]

After 17 minutes of coin tossing, Ms E called the group together to ask what they had found out. Children's comments included: "More heads than tails", "Tails, heads, tails the most — five times", "Three heads most", "Normally get two tails and a head". The discussion lasted a total of five minutes and continued as follows:

> Ms E: Can you tell me how many different combinations you can have and what they are?
>
> John: Five.
>
> Ms E: Who agrees? [Three children raise hands.] What are they?
>
> John: TTH, THH, TTT, HHH, [inaudible].
>
> Ms E: Oh right. Has anybody got something to say about that?
>
> Mary: I had tails.
>
> Ms E: What is your most common combination? ... And how many did you have altogether Neil?
>
> Neil: Two.
>
> Ms E: Two! ... You just had two for all those throws!
>
> Sally: I got the same.
>
> Jim: I had five.
>
> Ms E: And John had five. Did anyone else have five? [No hands.] Who had four combinations? How many did you have Helen?
>
> Helen: Two.
>
> Ms E: Two! Oh goodness! ... [You had] quite a few throws and still had only two different combinations every time, Goodness gracious! I think you can put these in your maths folders now.

According to Ms E's questionnaire responses, the most important part of the lesson from the perspective of children's learning was:

> Being aware of the price of homes, cars and holidays. The exchange of views between individuals. Thinking it would be easy to spend one million and finding the opposite.

A number of things are clear from this lesson. Firstly, the coin tossing task is severely flawed — both as recommended and as implemented. Is it intended that there are eight possible (equally likely) outcomes (i.e. are the coins intended to be treated as different) or only four? Secondly, what concepts are intended to be developed and what possible place does the prediction (far less "estimation") of outcomes have in such a development? Thirdly, there clearly needs to be some agreement about the possible outcomes and how these are recorded before any comparison can be made of the results of the experiment. But perhaps most importantly, the discussion at the end of the lesson segment proceeded no further than eliciting results, with questions all being of the form "Who got this?", "Who agrees?" or "Who got something different?". Unfortunately in our experience this type of discussion is not uncommon. What is clearly lacking in this lesson is any serious consideration of the mathematical concepts that might be developed through the use of the activity, the types of responses which children might give and how the discussion could be used to focus on the conceptual aspects of the task.

THE CIRCLE LESSON

This lesson occurred in a grade 3 class of 8 children at the Japanese School of Melbourne. The school teaches the Japanese curriculum, in Japanese, to children whose parents are in Australia for periods of one or two years, with teachers usually coming to Australia for three years. An interpreter was present throughout the lesson and assisted with completion of the written questionnaire. According to the teacher, Mr J, the main mathematics topic for the lesson was "the concept of a circle".

The lesson began with Mr J producing a pole for a game of quoits. Mr J placed the pole in the centre of an open space in the classroom and asked three children (referred to here as B1, G1 and B2) to stand at three marked places on a red line along one side of the room (see Figure 1). Children expressed concern that the game would be unfair, but mainly focussed on the distance between children on the red line. After a discussion on how to measure distances and children using a metre ruler to measure children's distances from the pole, Mr J put out pre-cut coloured strips of paper, as shown in Figure 1, and held them up to show that the distances were different. He then defined the problem as being: "How can we make the game fair?" It is only after five more minutes, during which children continued to try to find points to stand on the red line, that B2 came up with a way that two people can be the same distance from the pole — he moved the yellow strip so that it went from the pole to the point B2* on Figure 1. Mr J then gave all the

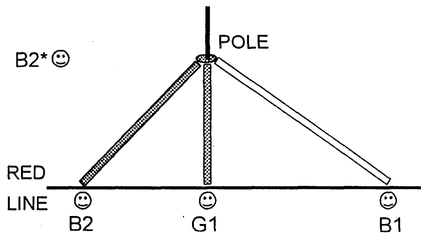

Figure 1: B2's solution for making the quoits game fair

children a yellow strip and asked them to "think for themselves" and find somewhere to stand so that everyone was the same distance from the pole. Children were excited that now would be at a disadvantage. This segment took 20 minutes in the 45 minute lesson.

Mr J then reproduced the situation on a large sheet of paper on the board. He stuck a miniature pole on the paper and asks children to use sticky yellow paper strips and dots to represent their positions (see Figure 2).

Mr J: Look at the different positions — what do you notice?

G2: It's like a round circle [makes circle shape with hands].

G3: No — it's like a flower.

G1: If you follow the end of each yellow strip it will become a circle [traces large circle on the desk with her finger].

Mr J: What if every student in the school took part? [adds more strips] ...

B3: If there are many students standing round, maybe it's a circle.

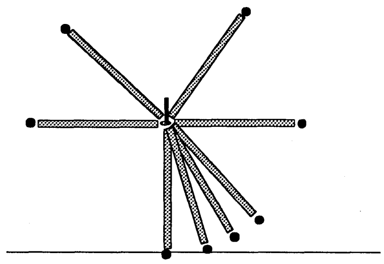

Figure 2: Paper representation of children's solution

Mr J removed the pole and put another sheet of paper over the first with a circle drawn where the dots were and asked "How many yellow points would we need? 100? 1 000 000?" He then put the word circle on the paper and elicited names for the centre, radius and diameter from the children.

The remaining 15 minutes of the lesson were taken up with the children working in pairs drawing circles. Initially many children chose to use a compass, even though Mr J told them that they had not yet learnt how to use one and encouraged one girl, who said that she could use a yellow strip of paper or a plastic circle to trace round, to show him how. After about 7 minutes, Mr J asked children to find a way to draw a circle without a compass. A few minutes later Mr J said "Now everyone is tracing — is there another way?" Children tried various ways, while Mr J pivoted one of the yellow strips of paper around an end held by his finger. B2 excitedly cried out that he could do it and demonstrated drawing a circle by holding the middle of one end of his pencil case and tracing a circle with his finger in the hole at the other end. Children applauded and Mr J demonstrated B2's method at the front of the class. The lesson finished with a few minutes of suggestions from children how to fix one end, culminating in the use of a drawing pin. Mr J summed up by saying: "As you suggested, there are other ways of drawing a circle than just using a compass".

In contrast to Ms E in the chance and data lesson, Mr J highlighted the conceptual aspects, both in the lesson and in his responses to the questionnaire. He stated that his aims were that "children have the *concept* of a circle and find *real* circular objects" [emphasis in original]. According to Mr J, the most important aspect of the lesson in terms of children's learning was that children understand that the circle is a locus. The purpose of working in groups (in this case pairs) was "to facilitate discussions while working", while the purpose of the whole class discussion was for children to "share ideas and strategies for solutions [demonstrating that] there are many different ways of thinking which reach the same conclusions". Mr J further described his mathematics lessons as follows:

> Introductory lessons [to a topic] use materials. So this was typical. The introduction is very important and takes a lot of time. After that there is much practice, then we go to calculations — a series of 3 or 4 lessons [per topic].

Mr J concluded his questionnaire with the comment that "Mathematics should be part of children's daily lives". In the 20 minute quoit activity, Mr J embedded the concept of a circle in a rich, intriguing, intrinsically motivating, problematic framework, by asking: "How can we make the game fair?"

CONCLUSION

The two lessons above provide a stark contrast in terms of the potential offered by the tasks on which they were based to advance children's conceptual understanding.

Kazemi (1998) argues that in his two classroom examples, where teachers use the same tasks and establish similar social norms, high press for conceptual thinking was created by the establishment of appropriate sociomathematical norms. While we would not wish to disagree with this, we contend that, in general, insufficient attention is being paid to the critical role of the development of conceptually focussed, robust tasks which can be used to support the development of sophisticated mathematical thinking. Moreover, teachers themselves need to engage in the detailed planning of how such tasks can be implemented in their lessons and this requires a deep engagement with the concepts involved, as well as an anticipation of likely student responses. The focus for teachers needs to be on all three of the key components of communities of inquiry: the social norms, the sociomathematical norms, and the conceptually complex, intriguing and problematic tasks.

While Mr J attributed the source for his lesson as being himself and the textbook, we believe it is no accident that while at ICME9 in Japan, we saw photos and descriptions of a number of similar lessons on the concept of a circle at a display by a local teachers' association. We concur with Hiebert and Stigler's (2000) view that the Japanese process of lesson study, where groups of teachers engage in an iterative process of refining a small number of lessons, warrants consideration as a procedure for reform of teaching. As they argue (p. 16), "Improving teaching does not depend on eventually perfecting 182 lessons but rather on engaging intensively with the issues involved in teaching any lesson".

REFERENCES

Askew, M., Brown, M., Denvir, H. & Rhodes, V. (2000). Describing primary mathematics lessons observed in the Leverhulme Numeracy Research Programme: A qualitative framework. In T. Nakahara & M. Koyama (Eds.), *Proceedings of the Twenty-fourth Conference of the International Group for the Psychology of Mathematics Education.* (Vol. II, pp. 17–24). Hiroshima, Japan: Hiroshima University.

Board of Studies (1995). *Curriculum and standards framework: Mathematics.* Carlton, Vic: Board of Studies, Victoria.

Cobb, P., Yackel, E. & Wood, T. (1995). The teaching experiment classroom. In P. Cobb & H. Bauersfeld (Eds.). *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 17–24). Hillsdale, NJ: Lawrence Erlbaum.

Directorate of School Education (1996). *Mathematics: Course advice*. Melbourne: Community Information Service, Directorate of School Education, Victoria.

Doig, B., Groves, S. & Splitter, L. (2001). Primary mathematics practice: The Victorian position. In J. Bobis, B. Perry & M. Mitchelmore (Eds.). *Numeracy and Beyond.* (Proceedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia, Vol. I, pp. 178–185). Sydney: Mathematics Education Research Group of Australasia.

Gravemeijer, K., McClain, K. & Stephan, M. (1998). Supporting students' construction of increasingly sophisticated ways of reasoning through problem solving. In A. Olivier & K. Newstead (Eds.) *Proceedings of the Twenty-second Conference of the International Group for the Psychology of Mathematics Education.* (Vol. I, pp. 194–209). Stellenbosch, South Africa: University of Stellenbosch.

Groves, S., Doig, B. & Splitter, L. (2000). Mathematics classrooms functioning as communities of inquiry: Possibilities and constraints for changing practice. In T. Nakahara & M. Koyama (Eds.), *Proceedings of the Twenty-fourth Conference of the International Group for the Psychology of Mathematics Education.* (Vol. III, pp. 1–8). Hiroshima, Japan: Hiroshima University.

Groves, S. & Doig, B. (1998). *The nature and role of discussion in mathematics: Three elementary teachers' beliefs and practice.* In A. Olivier & K. Newstead (Eds.), *Proceedings of the Twenty-second Conference of the International Group for the Psychology of Mathematics Education.* (Vol. III, pp. 17–24). Stellenbosch, South Africa: University of Stellenbosch.

Fraivillig, J. (2001). Strategies for advancing children's mathematical thinking. *Teaching Children Mathematics, 7* (8), 454–459.

Hiebert, J. & Stigler, J. W. (2000). A proposal for improving classroom teaching: Lessons from the TIMSS video study. *The Elementary School Journal, 101* (1), 1–20.

Kazemi, E. (1998). Discourse that promotes conceptual understanding. *Teaching Children Mathematics, 4* (7), 410–414.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.). *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 229–269). Hillsdale, NJ: Lawrence Erlbaum.

Schmidt, W. H., Jorde, D., Cogan, L. S., Barrier, E., Gonzalo, I., Moser, U., Shimizu, K., Sawada, T., Valverde, G. A., McKnight, C., Prawat, R. S., Wiley, D. E., Raizen, S. A., Britton, E. D. & Wolfe, R. G. (1996). *Characterizing pedagogical flow: An investigation of mathematics and science teaching in six countries.* Dordrecht: Kluwer Academic.

Splitter, L. J. (2000). Concepts, communities and the tools of good thinking. *Inquiry: Critical Thinking Across the Disciplines, 19* (2), 11–26.

Splitter, L. J. & Sharp, A. M. (1995). *Teaching for better thinking: The classroom community of inquiry.* Melbourne: The Australian Council for Educational Research.

Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation and autonomy in mathematics. *Journal for Research in Mathematics Education, 27,* 458–477.