LEARNING MATHEMATICS FOR TEACHING: DEVELOPING CONTENT KNOWLEDGE AND PEDAGOGY IN A MATHEMATICS COURSE FOR INTENDING TEACHERS

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Abstract. *This study is situated within the context of a mathematics course for undergraduate students intending to apply to the teacher education program. The course is problem-based and designed to develop students' mathematical knowledge and address students' fears and apprehensions in studying mathematics. Analysis of data collected through student autobiographies, journals, and written feedback indicate that: 1) students developed attitudes toward learning that included an inclination to seek conceptual understanding and alternative ways of understanding and solving problems, and 2) students reflected not only on their understanding of math but also on their roles as future teachers. This study builds on our understanding of mathematics for teaching and how a mathematics content course can provide opportunities for learning content and pedagogy.*

INTRODUCTION.

Over the past decade much has been written about the mathematical preparation of teachers. There are many studies that document teachers' adequate or more often inadequate knowledge of mathematics (Ball, 1990; Borko et. al, 1992; Leinhardt & Smith, 1985; Koirala, 1998; Ma, 1999; Heaton, 2000). Moving beyond a counting of the number or type of mathematics course taken or the kind of degrees obtained as a way of defining mathematical preparation for teaching, current research attempts to understand more about the kind of mathematical knowledge teachers need in order to teach well. Questions of what mathematics content teachers need to know, how they should develop that knowledge, and how their understandings of mathematics relate to teaching practices are not easily answered. Research on professional development provided to practicing teachers which includes opportunities for teachers to examine students' thinking (Franke, Carpenter, Fennema, 2001; Vacc, Bowman, & Bright, 2000), reflect on teaching practices (Schifter, 1996), develop or discuss teaching cases (Barnett & Tyson, 1999), or participate in teacher study groups (Stigler and Heibert, 1999), indicates promise toward enhancing teacher content and pedagogical knowledge. However, we know far less about how we might support beginning or intending teachers' understandings of mathematics for teaching. This paper examines issues around the kind of mathematical content and pedagogical structures needed to help pre-service teachers' develop their knowledge of mathematics. Our study is situated within the context of an undergraduate mathematics content course

and we ask how such a course might provide opportunities for learning mathematics for teaching.

THEORETICAL FRAMEWORK

Researchers have developed various frameworks for describing the mathematics needed for teaching well. Ball (1991) suggests that teachers need not only knowledge of mathematical concepts, topics, and procedures but also about the nature and discourse of mathematical inquiry. Using her own teaching practice as a site for research, Ball concludes that teachers' subject-matter knowledge needs to be correct, connected, and meaningful. Ma (1999) extends Ball's work in her analysis of Chinese teachers' development of profound understanding of fundamental mathematics. A "Profound understanding of fundamental mathematics," suggests Ma, "goes beyond being able to compute correctly and to give a rationale for computational algorithms" (p. xxiv). With a profound understanding of mathematics, a teacher, according to Ma, "is not only aware of the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics, but is able to teach them to students" (p. xxiv). Ma explains that a profound understanding of mathematics is an understanding that is deep, broad, and thorough. Deep understanding is evident when teachers make connections to topics that have further conceptual power. A broad understanding, states Ma, involves teachers in connecting topics that have similar conceptual power. Thoroughness, as an aspect of profound understanding, involves interweaving both deep and vast understandings into a coherent whole. For example, a teacher who is able to connect the topic of subtraction with regrouping to rates of composing and decomposing (depth), can connect this topic to addition or to subtraction without regrouping (breadth), and can interweave these into a coherent whole (thoroughness) would have profound understanding of fundamental mathematics and be in a position to connect student understanding to the curriculum.

Ma found that Chinese teachers, even with less post-secondary education than American teachers, developed their profound understanding of fundamental mathematics during their years of teaching. Comparing this to American teachers and using data collected in Ball's (1990) research, Ma found that neither experienced nor beginning American teachers tended to develop such profound understanding. The Chinese teachers in Ma's study also displayed various mathematical attitudes. Chinese teachers sought to "know how to carry out an algorithm and to know why it makes sense mathematically" (Ma, 1999, p. 108). Unlike American teachers they shared a disposition to ask why and to explore the mathematical reasoning underlying mathematical procedures. Other basic attitudes of Chinese teachers include the expectation that claims be justified with mathematical arguments and that problems be approached in multiple ways. These basic attitudes play an important role in teaching. A teacher may have conceptual and procedural understanding of mathematics but may not see the importance of conveying that understanding to

students. On the other hand, a teacher may come to teaching with an expectation that learning mathematics involves a focus on proficiency with mathematical procedures over a focus on why they work. As a result teachers may not be in a position to hear or attend to the possibilities for learning or teaching displayed in student responses, problems, or solutions. Teachers' attitude, as an aspect of their understanding of the discipline, influences what mathematics they know, how they know it, and how they share their understanding with students.

An important aspect of Ma's research is the intertwining of content and pedagogy. The Chinese teachers in Ma's study developed their profound understanding of mathematics over time during their teaching careers. Teachers learn mathematics through multiple sources: examining curriculum materials while teaching, working with colleagues, learning mathematics from students, and doing mathematics themselves. Learning mathematics for teaching involves an interplay of learning mathematics content and pedagogy. One does not necessarily precede the other. Teaching practices provide opportunities for learning mathematics. Likewise undergraduate and teacher education programs are contexts for learning both content and pedagogy. However, what teachers take from their undergraduate mathematics coursework is not often of use to them in classroom practice. How we might offer mathematics courses to intending teachers, so that teachers are in a better position to develop the kind of understanding and attitudes Ma speaks of, is an important question.

RESEARCH CONTEXT AND DESIGN

The Course

The context for this study is an undergraduate mathematics course taught through the University of British Columbia's mathematics department. A condition for admittance into the university's elementary teacher education program is the successful completion of any undergraduate mathematics course. One particular course was designed by the math department to meet the unique needs of intending elementary teachers. Taught differently from most undergraduate math courses in the department, this course focuses on developing students' skill and confidence in doing mathematics and engaging in mathematical inquiry. The course is problem based; lecturing is replaced by cooperative work in small groups and whole class discussions, and reflective math journal writing is expected. The content is connected to the school curriculum and is composed of three sections: geometry, arithmetic, and combinatorics (with probability). These sections are organized around themes or conceptual anchors such as the Pythagorean theorem and scaling, exponential growth, and binomial probabilities. Moving within the themes and sections the historical development of mathematical ideas can be addressed.

Participants

Forty-six of 54 students enrolled in the mathematics content course during the summer of 2001 participated in this study. The class met everyday for two hours over a six-week period. All students enrolled in the course did so as a requirement for entry into the elementary teacher education program. Students were either nearing the completion of their undergraduate degrees or had extensive work experience and were considering a career change. Thus, the demographic blend of the students varied; the youngest were in their early 20's and the oldest in their mid 50's. Participants therefore came with varied educational backgrounds and work experiences, as well as different expectations and visions about the teaching profession. There were 11 males and 43 females in the class.

Data Collection and Analysis

For the purposes of this paper we draw upon data collected through the teaching of this mathematics course. Zahra Gooya was the instructor for the course. Data sources include: participants' autobiographies collected during the first day of the class; participants' math journal entries written throughout the course; students' written evaluations of the course collected at the end of the course; participants' responses to a course reflective feedback survey collected at the end of the course; and the instructors' reflective field notes. Student autobiographies, student daily journal entries, and their responses to the course were analyzed for common issues and themes that were raised by students in terms of their mathematical backgrounds, their initial attitudes toward mathematics, learning and teaching mathematics, and how these attitudes developed or changed by the end of the course. Using Ma's (1999) description and framing of basic attitudes for teaching mathematics our analysis involved both direct interpretation of individual student responses across the course and the aggregation of particular instances (Case, 1995).

RESULTS AND DISCUSSION

Changes in Attitudes

Student autobiographies and journal entries written at the beginning of the course indicate that most students were apprehensive about taking a mathematics course and successfully completing the course as a requirement for application to the teacher education program. More than three-quarters of the students in the class expressed negative feelings toward mathematics, writing that they were "terrified", "intimidated", "anxious" and/or "stressed" about studying it. They described their previous experiences learning mathematics as "difficult", "overwhelming" and "frustrating." Although a few students mentioned they liked the predictable and sequenced nature of solving problems, many wrote about their loss of interest in mathematics as it began to make less sense or became less meaningful to them over their years of school study. This student's comments are representative of others: "I actually liked math as a kid. It was always challenging and I liked that challenge. But

I gradually liked it less as I was not able to apply the higher level math to practical life". With apprehension in successfully completing a university level mathematics course many students expressed a desire for the course to be taught in a familiar and traditional manner. As one student wrote, "we need lecturing, solving examples, doing lots of drill and practice, and telling us what and how to do it."

The course, however, was not taught in this traditional manner. Students worked in small groups on rich problems [1]. The instructor emphasized conceptual understanding through problem solving and encouraged students to communicate their thinking orally to their peers in whole class discussion and in writing in their journals. As the course progressed, the content of students' journal entries and reflections of their work with mathematics changed. Students' journal entries and their course evaluations written toward the end of the course indicate that many students not only developed a conceptual understanding of the math content taught but also some basic attitudes they felt necessary for learning math. Many students wrote that "the course encouraged me to be more curious about math and greatly diminished my anxiety towards the subject." In addition to changes in feelings toward mathematics, students wrote about the importance of explaining mathematical ideas, about seeking understanding rather than memorizing, and about searching for different ways of solving problems. For example, this student's comments are representative of others: "I feel this course did an excellent job of easing me back into math. It made me realize that it is not sufficient just to get the answer, it is necessary to know where and how it came to be, and how the formulas are created." Other students wrote expressions similar to this student's: "I've developed not only a clearer understanding of the basic concepts of mathematics, but also an appreciation for the many different methods there are to explain one concept." Still others, such as this student, wrote about their participation in creating mathematics, "I really enjoyed learning how to make formulas and proofs, instead of being given them."

These comments indicate how students' attitudes developed throughout the course. However, such a change did not occur for all students. Eight students of the 54 in the class wrote about how the course did not meet their needs. Throughout the course during class discussions and through journal writing, these students expressed a preference for a more traditional teacher-centred approach to teaching and were angry when this approach was not adopted. Although the course instructor offered opportunities for individual instruction outside of class time to these and all students, few of these dissatisfied students accepted this as an opportunity to meet their needs.

Learning Teaching While Learning Mathematics

Although this course was a content course taught to students who were intending to but not yet enrolled in a teacher education program, students made explicit connections between their learning mathematics and their roles as future teachers. Analysis of students' journal entries and reflective course feedback indicate that

students were learning about pedagogy while they were learning about content. Many, such as this student, wrote about the importance of developing conceptual understanding of mathematics, "It isn't sufficient just to get the answer, it is necessary to know where and how it came to be, ... and how the formulas are created." They reflected on how they might engage students in learning mathematics. This student's comments are representative of others when she wrote, "I've learned to work from where students are in terms of their level of learning, learned to ask what they know and how they understand what they know." A focus on teaching strategies was not an explicit focus of the course, yet students in this course drew upon their experiences as students in the class and upon their analysis of the instruction to inform their ideas about teaching. Their comments focus on both themselves as teachers (e.g. "I understand now that we as future teachers need to understand the underlying principles of math and also how and why, ... this course is preparing us"), and on their future students (e.g. "Children will ask 'why, why is math important, why do we do that, why is it important to learn math?' and we have to be prepared to respond to that"). Using their own experiences in the course as a place of reflection, the course provided beginning opportunities for students to learn content and pedagogy.

CONCLUSION

The results of this study point to the possibilities offered in mathematics content courses for developing productive attitudes and dispositions toward learning and teaching and for developing understandings of content and pedagogy. Studies of experienced and prospective teachers' subject-matter knowledge indicate that teachers require a rich and connected understanding of the mathematics they will be teaching in order to teach well. Ma's (1999) research indicates that teachers can develop their knowledge of mathematics for teaching over their careers of teaching and that these teachers display attitudes which include an inclination to pursue a conceptual understanding of a concept, to seek alternative solutions to problems, and require mathematical reasoning to justify claims. Most students in our study, as a result of their experiences in the course, developed the desire to pursue an understanding of mathematical concepts. They also wrote about the excitement of seeing and the challenge of understanding alternative solutions to problems. Although students wrote about these attitudes and their own changes in attitudes, how students use these in their roles as beginning teachers is a question for further research.

It must be stated that not all students wrote about a desire to learn and understand the conceptual underpinnings of various mathematical principles. These students sought a more traditional form of instruction, one that provided sample problems, clear steps to follow, and problems that allowed students to practice the application of the procedures. These students, eight in total, did not feel as if the course were designed

around their learning needs. This raises questions around how we might provide instruction that meets the needs of all our students.

For a mathematics course to offer opportunities for students to learn mathematics and pedagogy it requires a structure and content unlike typical mathematics courses offered to undergraduate students. Two issues need to be considered, one issue focuses on the mathematics content and the other on the nature of teaching a mathematics course. Students in the course spoke about the need for the content they were learning to be connected to the content they will be teaching. Students who wrote and reflected on the course content, mentioned how they felt they could now engage their own students in a similar investigation of mathematics. Yet, the content for this course was only loosely structured around the elementary school math topics of arithmetic, geometry, and probability, and more closely fit the secondary school curriculum. Interestingly, those students who were dissatisfied with the course spoke of the need for the content to the same as that which elementary students would be learning. This raises an important point as to what mathematics intending teachers should study. Our study indicates that content which is aligned with school mathematics, is familiar to students, but is not necessarily the same as it can be a context for learning mathematics and engaging mathematical inquiry. A second issue focuses on pedagogy. Students need opportunities to learn mathematics with their peers, they need a chance to communicate their thinking to each other and to the instructor, they need to experience mathematical inquiry around rich problems, and their fears about studying mathematics need to be addressed. It is the modelling of good teaching that provides students with possibilities for how students might experience mathematics and how they might consider teaching it. Developing mathematics courses where students can develop their subject-matter knowledge and pedagogical knowledge requires the co-operative efforts of university mathematics and education departments, of mathematicians and mathematics educators.

This study shows that a mathematics content course can have a profound influence on students' learning in general, on their attitudes toward mathematics, in particular, and on their developing ideas for teaching. The study challenges the traditional separation of content and pedagogy, in which content is typically taught through a mathematics course and pedagogy through a methods course. Current studies suggest that methods courses can be places to learn mathematics (e.g. Tirosh, 2000), our study suggests that a content course can also be a place for learning about teaching. This study emphasizes the interconnectedness of content and pedagogy and points to the possibilities well-designed content courses can offer in helping develop teachers' mathematical knowledge, attitudes, and beginning ideas about pedagogy.

NOTES

1. An example of a problem posed to students early in the course involved squares and their roots. Moving historically from Meno to Pythagoras and one of the first recorded math lessons where Socrates helps a boy

discover how to double a square geometrically, students in the class are asked to generalize this method to find ways of producing squares equal (in area) to 5, 10, 13, 17 times a given original. In terms of side-length, this means geometrically constructing certain square roots. For example, re-arranging the area left over when four (equal) corner triangles are cut from big square. This area can form a single mid-size square or two smaller ones. This leads to the theorem of Pythagoras – merging two squares area-wise into a single one.

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