CONCEPTS OF OPERATIONAL SYMMETRY IN THE THINKING OF FOUR ELEMENTARY SCHOOL STUDENTS

Norma Presmeg and Jeff Barrett

Illinois State University

The research described in this poster is part of a 4-year longitudinal study of the thinking of four students who are presently in grades 4 and 5. We describe some of their concepts of symmetry, defined broadly as iteration of patterns, and involving rotation and translation as well as 'mirror reflection'. Interview tasks with these students included working in 2 and 3 dimensions. Preliminary results suggest that degrees of symmetry (in the sense of "this is more symmetrical than that") is a natural idea for these students; that moving from the metaphor of folding to point-wise operations in reflective symmetry is a complex but necessary process; that notions of rotational symmetry in two and three dimensions are less well developed but may be fruitfully taught at this age; and that all of these developments facilitate and enhance performance on tessellation tasks.

Our objective is the establishment of a cognitive basis, through literature review and clinical interviewing of students, for investigating how symmetry and pattern recognition may be used as generative processes in the teaching and learning of mathematics at various levels. Here we focus on the notions of symmetry that evolved in the four students as evidenced by data collected in the fall semester of 2001, based on interviews using the following tasks. Preliminary results are reported.

Interview tasks

1. Compare six clay solids and rank them in order of apparent symmetry: a spherical ball; a cylindrical "snake"; a coiled basket made by spiralling a long, thin, cylindrical piece; a rectangular prism; a triangular pyramid; and an irregular lump of clay.

2. Compare pair wise, four large photographs of landscapes. In each of these six comparisons, decide which picture is more symmetrical than the other, giving reasons for your choice.

3. Two-dimensional symmetry:

(a) Given a drawing of a geometrical figure, can you "make the figure symmetrical" (rotation or reflection) and draw the corresponding position of the figure?

(b) Given a drawing of a parallelogram, with a diagonal drawn, or a line through midpoints of opposite sides, explain with reasons why this is a line of symmetry, or why it is not.

4. Three-dimensional visualization task: Can you take a two-dimensional region and rotate if about an axis to generate a solid of revolution by prediction?