LINGUISTIC POINTERS IN YOUNG CHILDREN'S DESCRIPTIONS OF MENTAL CALCULATION

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Children's descriptions of their mental calculations can reveal more to the teacher than merely the strategy that has been used. An analysis of nearly 2000 responses given in interview by 26 children aged between 6 and 9 years old has revealed that the language they use may point to individual differences in their modes of thought. Different use of pronouns, tense and causal connectives was associated with different levels of achievement. Descriptions of calculations which accompanied correct answers were more likely to be in the present tense, to be expressed in terms of what "you" do and were more likely to include explanatory words of deduction : "if", "then", "so" and "because".

INTRODUCTION

This paper focuses on the different styles of language that young children use when describing a mental calculation they have just performed. I have previously described this language as 'image-like' (Bills and Gray, 1999) because the words used were related to previous classroom activities. I have suggested that the language could indicate the influence of these activities on the children's thinking. Analysis of the descriptions of 45 mental calculation questions performed in six interviews has also revealed that higher achieving children tended to use a language of generalisation. Lower achievers simply described the particular calculation (Bills and Gray, 2001).

The data presented in this paper suggest that there are other linguistic characteristics of descriptions of mental calculations that are associated with levels of achievement. The speech style observed in the classrooms in this study used the present tense and the pronoun "you" to describe calculation procedures. Language of deduction (causal connectives: "if", "then", "so" and "because") was used in explanations by teachers and pupils. In my interviews with children they most frequently adopted this style when describing a calculation they had performed <u>successfully</u>.

All the children in this study had the same teaching yet differences in their learning was apparent in the differences in their language. I suggest that this is evidence that commonalities in language may simply reflect the speech style of the community yet differences in the use of language may be indicative of differences in mental constructions. The children in this study had been exposed to language of procedure and explanation and they all demonstrated that they could use it in non-calculation situations. The more frequent use of this language style in association with correct answers suggests that its use was not simply a linguistic trait but that it was used when children had a successful procedure. The conclusion is drawn that teachers may learn something about learners' procedural knowledge from the language that they use. Language can act as a 'pointer' to different qualities of thinking.

LINGUISTIC POINTERS

Deictic terms: "this", "it", "you", (from the Greek 'deixis', meaning to point), serve to identify objects, people, times and places without reference to particular things. Rowland (1995) has thus suggested that the use of these words could be indicative of a generalisation. Rowland proposed as a 'deictic principle' that language is a code to express and 'point to' concepts, meanings and attitudes. So that not only deictic terms themselves but use of other words could be indicators of cognitive structures.

Rowland (2000) noted that the use of the pronoun "you" to refer to generalities, i.e. what usually happens, is common in non-mathematical situations where "you" is frequently used in place of the more formal "one". This is particularly true of children in their description of rules of games. Rowland has also suggested, however, that the use of "you" is an effective 'linguistic pointer' to a quality of thinking. For the pupil in Rowland's study the shift from "I" to "you" in a problem solving situation signified her move from working with particular numbers to expressing a generalisation.

The quality of the language used in explanations may also be an indication of the quality of understanding. Donaldson (1986) questioned whether young children's inability to give an explanation is due to poor understanding of the concept to be explained or to a lack of the linguistic ability to explain. She argued that adequate linguistic competence for explanation is demonstrated by appropriate use of the causal connectives "because" and "so". Her studies show that children do use "because" and "so" appropriately and by the age of eight years can use them in the deductive mode. She concluded that the linguistic ability of understanding causal connectives and the cognitive ability of understanding causality are interdependent.

In a study conducted by Vygotsky (1962) 80% of children in the sample, at both 7 and 9 years, were able to correctly complete sentence fragments ending in "because" when related to 'scientific concepts'. Only 60% of the 7 year-olds could do this with sentences related to 'spontaneous' everyday concepts. He attributed this to the fact that scientific concepts had been learned in collaboration with teachers. He assumed that the ability to use "because" appropriately in everyday concepts is improved by being able to do so in scientific concepts. Luria (1982) noted that children may understand the ordinary features of their familiar social experience but will need activities to provide experience of the unfamiliar scientific concepts encountered in more formal school settings. He suggested that at school children may initially use words or perform actions without knowing why. Thus children can use words and perform actions initially without having the same understandings as the teacher.

Piaget (1928) referred to Claparede's 'law of conscious realisation': the more automatically an idea is handled, the harder is the process of its conscious realisation. Piaget suggested that when asked how they had performed a calculation, after giving an answer by some automatic process, children may invent something which would give rise to the same answer. In my study (Bills and Gray, 2001) it seemed that when the high achievers performed the calculation automatically (the answer was given very quickly) they simply stated a rule that would give the answer. The lower achievers tended to describe, in detail, the particular calculation they had performed. This adds supports to Piaget's suggestions that initially children can only reason about the particular (Piaget, 1928) and also that the child only imitates that which he can understand (Piaget & Inhelder, 1971). Higher achievers may have expressed themselves in terms of a generalisation because they understood the rule in this way whilst lower achievers only gave the steps in the particular procedure performed.

Rowland, Donaldson and Piaget suggest that language may indicate individual mental constructions whilst Vygotsky and Luria maintain that language is a social construction. The data presented in this paper suggest both perspectives are valid.

METHOD

Lesson observations and pupil interviews were first conducted with two classes from Year 3 (pupils aged 7 and 8 years) in a school for children aged 5 to 11 years in a large middle-income village near Birmingham U.K., from September 1998 to July 1999. The same pupils were also observed and interviewed in the following year. The 80 children in the whole year had been placed in one of three groups for mathematics lessons based on their previous attainments. Lessons with the high attainment and the middle attainment groups were observed and a sample of 14 pupils from the first and a sample of 12 pupils from the second was interviewed in December, March and July in each year. The samples were chosen to represent the spread of attainment levels in each group.

Over the six interviews 78 questions were used. They were classified into 10 calculation types and 6 non-calculation type. Each was presented verbally and followed by the question "What was in your head when you were thinking of that?"

Type	Description	Examples of questions
1	1-digit addend	17 + 8, 17 + 9 (repeated in each interview)
2	Missing addend	13 + * = 18, 30 + * = 80, 27+*=65
3	2-digit addition	48 + 23 (repeated in each interview)
4	Addition of multiple of 10	97 + 10, 597 + 10, 1097 + 10, 1197 + 10
5	Counting	What comes before 380, 2380, 12100; after 12386
6	Rounding	Round 2462 to the nearest ten, 239 to nearest hundred
7	Recent topic	What is difference between 27 and 65, 0.6+0.7
8	Recent topic	65 subtract 29, Read time (11:40), 0.1 times by 10
9	Division and fractions	quarter of 40, third of 48, 140 divided by 3

10	Multiplication	48 multiplied by 3, 47 multiplied by 5
11	Numerical procedure	Tell me how to: add 23, find a third, times by ten,
12	Non-numerical procedure	Tell me how to: cross road, tell the time, do subtraction
13	Maths concept, first	First thing in head when I say: centimetre, three, million
14	Maths concept, more	What else about: centimetre, three, million
15	Non-Maths concept, first	First thing in head when I say: shadow, ball, adjective
16	Non-Maths concept more	What else can you tell me about: shadow, ball, adjective

Each response to "What was in your head?" was analysed for its use of pronouns, tense and causal connectives.

Responses involving "I" and "you" were categorised in to three types which conform with the 'particular', 'generic' and 'general' classifications described in Bills and Gray (2001). The examples given here are responses to the question "What was in your head?" following the mental calculation "What is 48 add 23?":

Category 1: responses related to the particular, expressions such as "I/you did this"

Bobby **I** added the 40 and the 20 to make 60 then **I** added the 8 and the 3.

John 48 add 20 comes 68 and then **you** add 3 on.

Category 2: responses related to examples, expressions such as "I/you do with these"

Simon say **I** was adding 30, **I** go 40, 50, 60

Irene **you** put the one down in the units column in your head and **you** carry one

Category 3: responses related to the general, expressions such as "I/you always do"

Ellain **I** count it all in my head, and **I** sometimes use my fingers

Elspeth **you** just take the ten of there and **you** put it back and **you** put it together

RESULTS

Linguistic characteristics of correct calculations

There were 1158 mental calculations performed in the interviews and 63% of these were correct. A comparison was made between the responses which accompanied correct mental calculations and the responses which accompanied incorrect calculations. Tables for the distribution of correct and incorrect responses involving different uses of pronoun, tense and causal connectives were compiled and chi-squared tests of significance conducted. Only tables for statistically significant results will be given here.

The following table suggests that pupils whose calculation was described exclusively using "you" were more likely to have been correct than those who used "I" exclusively or a mixture of the two. Notice that 97 responses used "you" only and

79% of these were correct. In comparison 520 responses used "I' exclusively and only 65% were correct. The differences in proportions of correct and incorrect responses was statistically significant (p<0.05):

	"I" only	"I" and "you"	"you" only
Percentage correct	65%	66%	79%
Number of responses	520	166	97

Analysis of the categories of pronoun use is also informative. When "I" was used to describe specifically what the individual had done in that particular question the answer was less likely to have been correct than if "I" indicated what the individual did in general (p<0.005):

	"I" category 1	"I" category 2	"I" category 3
Percentage correct	63%	68%	79%
Number of responses	489	47	133

By contrast the difference in the distributions for different categories of "you" was not statistically significant. These two results for categories of pronoun use suggest that the adoption of the common classroom speech style involving use of "you" may be less of an indicator of understanding than the use of the personal pronoun "I". The use of "I" may point to ownership of the rule whilst "you" may simply follow the classroom convention.

The difference between the distribution of tenses in which children chose to phrase their explanations after they had given a correct answer and after an incorrect answer was also statistically significant. It appears that exclusive use of past tense was accompanied by a lower proportion of correct answers than present tense and mixed tense expressions (p<0.05):

	past tense	present tense	mixed tenses
Percentage correct	62%	71%	71%
Number of responses	391	244	259

There is again an indication that descriptions simply of what had been done were associated with less successful calculations than descriptions phrased in the rule giving, present tense, mode.

When any of the causal connectives were used it was more likely that the correct answer had been given than those responses where no causal connective had been used. This could indicate that use of language of explanations is more frequently associated with correct mental calculations and thus with greater procedural competence.The differences in the proportions correct were all statistically significant (p<0.005):

	% correct when used	% correct when not used
"because"	79% (of 189)	59% (of 969)
"so"	72% (of 284)	59% (of 874)
"if"	82% (of 135)	60% (of 1023)
"then"	68% (of 448)	59% (of 710)

Linguistic characteristics of high and low achievers

The sample of 26 children achieved between 12 and 38 correct answers out of 45 mental calculation questions. The 13 pupils above the median were classified as 'high achievers' and the remainder as 'low achievers'. Comparisons were made between the distributions of responses involving the linguistic indicators (pronouns, tense and causal connectives) for the two groups. The differences were all statistically significant ($p<0.05$)

	Percentage of responses involving the indicator	
	Higher achievers (n=578)	Lower achievers (n=580)
"I" only	42%	49%
"you" only	11%	6%
past tense	41%	34%
"because"	20%	13%
"so"	30%	19%
"then"	45%	32%
any connective	66%	49%

The proportions of 'category 3' pronoun use were also compared. A higher proportion of the use of "I" was category 3 for the higher achievers than for the lower achievers. Similarly with use of "you":

	Percentage of responses involving the indicator	
	Higher achievers	Lower achievers
"I" category 3	32% (of 244 uses of "I")	19% (of 284 uses of "I")
"you" category 3	25% (of 157 uses of "you")	17% (of 89 uses of "you")

The differences were statistically significant ($p<0.05$). The responses to non-calculation items in the interviews were also analysed for use of the linguistic indicators. There were <u>no</u> statistically significant differences in the distributions for the two groups of children for any of these indicators in non-calculation contexts.

DISCUSSION

The fact that the two groups of children had the same linguistic characteristics in non-calculation questions suggests that the differences apparent in response to calculation questions were not simply a matter of linguistic ability. All the children were capable of using causal connectives in non-calculation context and the two groups did so in similar proportions. Their use of pronouns and tense was also similar in these non-calculation contexts. The differences between their response after calculations had been performed may thus be an indication that they had different ways of thinking about those calculations. The higher proportion of use of causal connectives to accompany correct calculations may indicate that the procedure was well understood.

The use of the pronoun "you" and present tense was common in the classrooms observed. All the children were exposed to phrases such as "you add the tens and you add the units". Some used phrases like this when describing what was in their head after performing a mental calculation. Others talked only of the particular calculation and used phrases such as "I added 40 and 20 and I added 8 and 3". Those who adopted the teachers' mode of language were the most successful. This suggests that the children were not simply copying the language. They tended to use this mode of language when they were using a procedure which was successful. Notice that my question "What **was** in **your** head?" would seem to require a response from the individual in the past tense and in the first person. The fact that they chose to use the classroom speech style is an indication that they were following a familiar procedure.

Rowland has suggested that children's use of "you" may be a pointer to their progress toward generalisation. In this study the use of "you" is an indication that a familiar procedure had been followed but rules that the individual used were also expressed in general terms using "I". Expressions involving "I" to describe what the individual **does** rather than simply what they **did** were more likely to accompany correct answers. This mode of expression could thus be a powerful indication that the child has formed their own generalisation since it is not couched in the ubiquitous classroom style using "you". The individual pupil may demonstrate their individual mental construction based on their experience of the classroom activities through their use of "I" in expressing general rules.

It is not surprising that children adopt the common classroom speech style but the results presented above seem to suggest that pupils are more likely to do so to accompany a successful calculation. The social constructivist perspective of Luria and Vygotsky could suggest that all children might simply adopt the classroom speech style. This study indicates that the pupils were most likely to do so when they were successful. There is an indication in this that the children's different language use was a pointer to their different mental constructions. The pupils were capable of simply copying their teacher's style of language but were less likely to do so when they had given an incorrect answer even though they were not told if their answer was right or wrong.

IMPLICATIONS FOR THE CLASSROOM

Use of "I" in the sense of 'what the individual does in general' was not common after mental calculations had been performed (8% of responses) but on nearly 80% of these occasions the answer was correct. Similarly exclusive use of "you" is unusual (11% of responses) and has the same high association with correct answers. The causal connectives are also strong indicators, particularly "because" and "if", with low proportions of occurrence but high associations with accuracy.

These words thus act as linguistic 'pointers' to a quality of thinking which the teacher will wish to encourage. In the classroom children's use of these linguistic pointers can alert the teacher to differences between individuals. They may also give indications of development in thinking. A single response can not, in itself, give an assessment of a pupils procedural competence but responses over a period of time can point to a changing quality of thinking. Identifying differences between pupils does nothing to improve the learning of pupils who are less successful in mathematics unless the information is used by teachers to help these pupils gain greater understanding. Identifying the pupils' differences through the use of the linguistic pointers can be a first step.

Listening to the words that children use can be more informative than listening simply to the strategy that has been employed.

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