THE ACCURACY OF MATHEMATICAL DIAGRAMS IN CURRICULUM MATERIALS

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In this paper, two diagrams are analysed, for accuracy, using a set of principles generated from a review of the research literature from the fields of mathematics education, cognitive science, computer-aided learning, computer graphic design and semiotics. The diagrams are typical of those found in an Integrated Learning Systems (ILSs) evaluated in Queensland schools and a student workbook. Using this set of principles, it is explained why many of the diagrams contained within these curriculum materials do not facilitate the construction of mathematical knowledge.

BACKGROUND

The general aim of this pilot study is to investigate the accuracy of mathematical and scientific diagrams available in curriculum materials. The focus of this paper is on the mathematical components of the study. The study is timely in four ways. First, it focuses on mathematics and science education that are disciplines crucial for economic developments for Australia. Secondly, it integrates science and mathematics. Making connections across the curriculum has been recognised as an important learning outcome (International Society for Technology in Education, 1999; National Council of Teachers of Mathematics [NCTM], 1989; National Research Council, 1996). Third, it concentrates on diagrams, important in our iconic world. Finally, it considers an area with scant literature concerning diagrams in curriculum materials. No other studies have investigated science and mathematics coupled with an accuracy analysis of diagrams contained in curriculum materials. Mukherjee and Edmonds (1994) made the observation that diagrams in many Integrated Learning Systems (ILS's) often seem to have been developed in a vacuum by individuals or teams that have no background in graphic design or visual literacy. Diagrams therefore have the potential to be erroneous and misleading.

Curriculum materials. Most curriculum materials suffer from a lack of coherence and focus (Schmidt, McKnight & Raizen, 1997). They do little to promote critical thinking about mathematics (Risner, Skeel & Nicholson, 1992). Many teachers' lack of confidence and knowledge in relation to mathematics teaching is a world-wide problem with most attempts to remedy the problem achieving limited success (Peacock, 2001). As a consequence, teachers throughout the world rely on diagrams in both print and electronic based curriculum materials to provide the knowledge, and techniques of mathematical ideas to their students, and so such diagrams greatly influence the content of lessons. The only way to gain information relating to the suitability of instructional material is through an evaluation of the instructional materials available. However, teachers have neither the confidence nor competence to make these decisions in mathematics (Peacock, 2001).

Teachers may not realise that diagrammatic errors are present in some curriculum materials (Kidman, 2000). They reason that if several materials use a similar diagram to teach the same concept in exactly the same way, how could all those materials and diagrams be wrong? (Beaty, 1996). While publishers and other curriculum material developers are eager to claim that their materials have content accuracy, few schools and individual teachers are able to devote the time and resources necessary to judge the accuracy of scientific diagrammatic content themselves.

Relations between diagram and text. Diagrams are frequently used in mathematics curriculum materials at every academic level. Most texts have several diagrams on each page (Iding, 2000). Diagrams are included for two purposes: that of instruction and that of decorative purposes. High quality diagrams enhance instruction by encoding new information not referred to in the text, thus compensating for text deficiencies, and also by verifying each clause in the text, to develop an initial representation (Hegarty & Just, 1989). Hunter, Crismore and Pearson (1987, p. 122) identified five possible relations a diagram has in relation to the text it accompanies:

1. *Embellish* - provides completely new information not discussed in the text.
2. *Reinforce* - repeats all information presented in the text.
3. *Elaborate* - partly repeats and partly adds to information presented in text.
4. *Summarize* - provides a broad overview of text, much like an advance organizer.
5. *Compare* - provides information intended to be compared with a previous graphic.

This categorization, and the one that follows, are particularly appropriate for investigating the accuracy of mathematical diagrams because they focus on the relationship between textual and diagrammatic information and on the typical diagram types that are reasonably representative in mathematics curriculum.

Role of diagrams. A review of the research literature indicates that diagrams can play at least 4 different but interrelated roles in learning and instruction:

1. *Identification* - Diagrams that point out or identify parts of things (Cook & Mayer, 1988, Charles & Nason, 2001). For example, most students are familiar with geometry diagrams of the circle upon which the names of the diameter, radius and circumference are labelled (Iding, 2000; Kidman, 2000; Lowe, 1993).
2. *Comparison* - Diagrams that compare one kind of thing to another (Lemke, 1999; Hunter, Crismore & Pearson, 1987; Winn, 1989). For example, two different kinds of time-pieces, analogue and digital, might appear next to each other.
3. *Sequence* - Diagrams that point out stages in a chain of events (Cook & Mayer, 1988; Hunter, Crismore & Pearson, 1987). Typical examples would be diagrams of a sporting race showing the ordinal finish.
4. *Combination* - Possibly the most frequently occurring type of diagram, particularly in scientific texts, is one that combines two or more of the above

functions (Cook & Mayer, 1988; Hunter, Crismore & Pearson, 1987; Iding, 2000; Lowe, 1993; Winn, 1989). For example, a diagram of a scaled map can provide more than one view, for example, an additional cross-section or enlarged view (i.e., comparison). Aspects of the map can be labelled (i.e., identification) and the pathway to follow can be indicated via the use of arrows (i.e., sequence).

Accuracy of diagrams. Diagrams contained in curriculum material, with the purpose of instruction rather than decoration, need to be evaluated as to how well they address instructional criteria. Instructional criteria can be arranged in seven broad principles (see below), and are consistent with effective mathematics learning and teaching found in the National Council of Teachers of Mathematics current standards (NCTM, 2000).

1. Diagrams should help students to *recall knowledge* and *skills*, and *make connections* between prior knowledge and new situations (Charles & Nason, 2001; Derrori & Lemut, 1995; Gentner, 1982; Janvier, Girardon, & Morand, 1992; Kidman, 2000, 2001; Kidman & Nason, 2000; Lowe, 1993).
2. Diagrams should *introduce terms* and *procedures*; *represent ideas accurately*, and *demonstrate/model procedures* (Gentner, 1982; Kidman & Nason, 2000).
3. Diagrams should provide *conceptual links* within the representation, and allow the learner to *abstract and understand the important notion* underlying the diagram (Fish & Scrivener, 1990; Kidman & Nason, 2000).
4. Diagrams should *reduce the working memory demands* of the problem solving process (Fish & Scrivener, 1990; Kidman & Nason, 2000).
5. Diagrams should *allow for exploration* of ideas and understandings not possible *from natural language* (Lemke, 1999; Kidman, 2001; Kidman & Nason, 2000).
6. Diagrams should contain elements of natural language *facilitating links between* scientific and mathematical *expressions and natural language* (Lemke, 1999).
7. Diagrams should allow the learner to *interpret underlying scientific and mathematical notions*, and allow the learner to participate in an *expressive learning* activity (Gordin, Edelson, & Pea, 1996; Kidman & Nason, 2000).

DATA ANALYSIS

Data comprised the diagrams. The relations between the diagram and the text, and role of the diagram were considered. The analysis of accuracy of a diagram was informed by a set of seven principles (above) for analysing diagrams within mathematical curriculum materials (Kidman & Nason, 2000).

RESULTS AND DISCUSSION

Two examples are presented for discussion. One example is drawn from an Integrated Learning System (ILS) (Computer Curriculum Corporation, 1996) (a collection of electronic worksheets, divided into a range of strands (e.g., fractions)). 550 instructional diagrams were sampled from the ILS and evaluated. 220 of these diagrams were found to be deficient in at least five of the seven principles. A second example is drawn from a Year 5 student workbook (Boswell, 1998). Of the 501 instructional diagrams found in the workbook, 38 were found to be deficient in at least five of the seven principles. Both the ILS and workbook series are found in many classrooms throughout Queensland, Australia.

In order to illustrate the nature of the diagrams found in the ILS and workbook, a detailed analysis of the two diagrams is now provided. The deficiencies identified in the ILS example are typical of those found throughout the 220 diagrams (e.g., font problems, unclear purposes and layouts, forced formatting and poor links between natural language and visual representations) as is the case for the workbook.

The diagram presented in Figure 1 comes from the repertoire of exercises contained in the 'Fraction' strand of the ILS. The relation between the diagram and text in Figure 1 is that of elaboration. The diagram partially repeats text information in the form of the triangle having a base (b) and height (h), and partly adds the information that height is perpendicular height. The role of Diagram 1 is one of identification as it identifies where the base and height are to be found on the triangle. The intended skill of this exercise is the multiplication of fractions (see upper left hand corner of Fig. 1: fr840 = Fractions, Year 8, 40 percent of the way through Year 8).

The diagram presented in Figure 2 comes from the exercises contained in the 'Number' strand of the workbook. The relation between the diagram and text in Figure 2 is that of embellishment. The diagram provides completely new information not covered in the text. The role of Diagram 2 is one of identification as it identifies the spatial relationships between five cities. The intended skill of the exercise associated with Figure 2 is subtraction "from four-digit numbers, with regrouping" (Boswell, 1998, p. 93). The text accompanying Figure 2 is "Here is a chart showing some airports and their distances from one another. Use this information to write three subtraction problems" (Boswell, 1998, p. 93). To the right of the diagram is a highly structured area for the student to write the three problems.

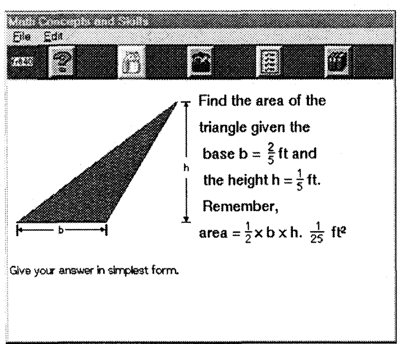

Figure 1. ILS Example

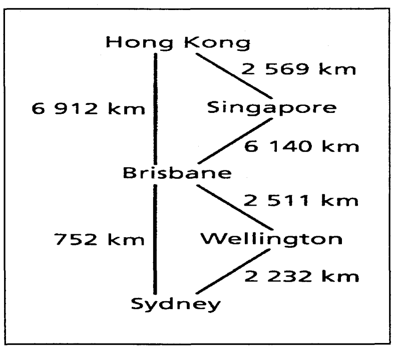

Figure 2. Workbook Example

The analysis of the diagrams in Figures 1 and 2 are presented in Table 1. Both of the diagrams analysed in this paper are clear and relatively uncluttered. However, neither of the two diagrams facilitates meaning-making.

Table 1 Analysis of Figures 1 & 2

Principle	Compliance	Commentary
1	Low	The font and the layout of both diagrams are clear to read. A major problem found in both diagrams making understanding difficult is that neither diagram is drawn to scale. Understanding of the diagram in Figure 1 is hindered by the text leading the reader to believe the exercise is on area measurement and not on fraction concepts. Understanding of the diagram in Figure 2 is hindered as the reader may be led to believe that a distance of 6912 km is the same as 752 km on the chart.
2	Low	The diagram In Figure 1 does not relate the area of the triangle to the notion of the unit (i.e., a 1x1 square), so it does not enable students to focus on deep structural knowledge. It does not encourage the use of intuitive knowledge about fractions. The diagram in Figure 2 does not relate the notion of the different distances between cities and geographical locations, so it does not enable students to focus on deep structural knowledge. It also does not encourage the use of intuitive knowledge about subtraction with regrouping. Neither diagram encourages spatial awareness.
3	Low	An important notion in Figure 1 that needs to be abstracted from the diagram is that 1/2 x 2/5 x 1/5 is equivalent to 1/25 because if two fifths is halved, then a fifth is generated; and if a fifth of a fifth is then found, then a twenty fifth is generated. Because the diagram does not relate the area of the triangle to this, the "Correct" answer is not conceptually linked back to diagram. An important notion in Figure 2 that needs to be abstracted from the diagram is that in subtraction algorithms, the bottom number needs to be subtracted from the top number, even when at first sight it cannot be done. Because the diagram does not relate this notion, the "Correct" answers written by the child are not conceptually linked back to diagram. An adequate environment for learners to abstract and understand this notion is not provided in either diagram.
4	Low (Fig 1) High (Fig 2)	Students are unable to add notes to the diagram in Figure 1. Unless they are instructed to make notes on paper much information has to be memorised potentially overloading the working memory capacity. Students are able to add notes to the diagram in Figure 2, and a highly structured area is supplied for the student's working. Very little information has to be memorised, freeing up the working memory capacity.
5	Low	It is very difficult to adequately represent, in natural language, the important notions noted in Principle 3 above for both diagrams. Neither diagram enables students to explore these notions. Therefore, students probably will not construct the iconic

		understandings of the relationship between 1 whole, 1/5, and 1/25 from Diagram 1, nor understandings of subtraction with regrouping from Diagram 2.
6	Low	The diagram in Figure 1 contains no natural language, and because the diagram in Figure 2 contains only a little natural language (but as it is not to scale - even remotely), neither facilitate links to natural language.
7	Low	The diagrams allow for some interpretation but because of the lack of compliance with the majority of the above Principles, only low levels of interpretation can occur. The diagrams cannot be used for expressive learning activity.

For example, neither diagram:

1. highlights the relationships between the problem information or prior knowledge and skills;
2. enables the reader to focus beyond the surface level aspects of the task., and
3. provide students with the means to construct a deeper appreciation of the concepts beyond that which can be achieved through the semantics of natural language.

Because of these limitations, it is highly unlikely that either of the visual representations would do much to facilitate the construction of deep-level, principled knowledge about fractions or numeration.

However, the effects of these limitations may be more serious than this. The covert geometry curriculum presented in both these diagrams is not given the attention it deserves. The barriers to the development of spatial intuition created by these two diagrams are of concern due to the lack of attention to scale. The development of spatial awareness is informal. "It is the use of space, shape and form at an intuitive, personal and unstructured level such as interpreting a map" (Booker, Bond, Briggs, & Davey, 1997, p. 270). Geometry allows many ideas to be pictured and thus, facilitates problem-solving, therefore placing geometry in a unique position in relation to other branches of mathematics.

Both diagrams unnecessarily lack an attention to scale. In Figure 1, the base of the triangle ought to be twice that of the height. This is not even approximated on the diagram. Figure 2 is a map supposedly showing airports. It actually shows only two airports (Brisbane and Wellington) and three cities (the names of these three cities airports differ to their city names). The map indicates that Wellington is somewhere between Brisbane and Sydney. This is not correct, it is geographically south east of Sydney, in a different country to Sydney! The lack of attention to these details, and many others in Figure 2 are inexcusable. Present curriculum documents like those produced by the National Council of Teachers of Mathematics (2000) call for integrated links between mathematical topics and concepts. This is clearly not being done in the diagrams assessed in this and other studies (Kidman, 2000; Kidman & Nason, 2000).

CONCLUSION

The findings from this study and the previous studies by Kidman and Nason indicate that instructional diagrams are not facilitating mean-making. The studies have shown that a significant proportion of diagrams are not facilitating the construction of mathematical knowledge. This study has shown that while the diagrams may be attractive, and possibly attract and maintain a student's attention it is highly unlikely that either of the diagrams would do much to facilitate the construction of deep-level, principled knowledge about fractions or numeration. The lack of attention to scale in both diagrams is a problem for the development of spatial awareness.

The National Council of Teachers of Mathematics (2000) has called for a coherent curriculum where mathematical ideas are linked and built on one another. It argued that this would facilitate understanding, deepen knowledge and expand application. It is evident that this coherence is not present in these diagrams.

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