MATHEMATICS EDUCATION AND CRITICAL CONSCIOUSNESS

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This paper discusses mathematics education as a contribution to the development of democratic competence. As a feature of democratic competence, the concept of critical consciousness is examined and some results of a study involving students' work within a First Calculus Course are presented.

EDUCATION FOR CITIZENSHIP

As Constant (1998) states, «the job of citizen cannot be improvised». Thus the citizen must be educated. Educational institutions are most often indicated as being the place of such education, but the meaning and content of education for citizenship have been understood in different ways. For some, education for citizenship takes place in a specific academic subject, in which students learn about the rights and obligations of the citizen, the rules which establish who is authorised to take collective decisions and which processes are to be used. For others, this type of education is transversal: it is to be found across the board of academic subjects and involves the development of capacities which are indispensable to self-determination and autonomy (Hall *et al.*, 1998). The work presented in this paper falls within the latter context.

CRITICAL CONSCIOUSNESS

If the democratic citizen is a presupposition of democracy, democratic competence is also an indispensable condition for the democratic citizen to behave as such. In this paper it is considered that democratic competence has, essentially, three components: critical consciousness, sustained and sustainable action, and co-operation.

According to Freire (1975a), critical consciousness is the ability that each individual must have to «understand the reality which surrounds one and situate oneself within the social context in order to intervene in this context in a conscious and creative manner» (p.33).

It is critical consciousness that allows us to understand that the political, social and economic conditions in which we live are not immutable, and that we can participate in changing them, that we can participate in the making of history. But in order to do this, we must begin by understanding what is happening with ourselves. Thus a primary component of critical consciousness is the capacity to identify the *dispositions* (Skovsmose, 1994) which shape our convictions, our attitudes, our behaviour – it is the awareness of our own system of values, it is the capacity to find a justification for our actions and judge the rationality of them.

To recognise that systems of values and codes of conduct which we thought to be a personal creation, are, in fact, culturally induced can be liberating and can constitute a stimulus for change and a factor of understanding of others (Brookfield, 1987).

Critical consciousness involves, therefore, a reflective dimension – critical reflection – which results in a change in the way we see ourselves, the way we see the world and the way we see ourselves in the world. Or, as Freire (1975b) said:

> "(...) people begin to understand, critically, how they *are* in the world *with which* and *in which* they find themselves" (p.102).

In the words of Broughton (cited by Brookfield, 1987, p.18), imagining and exploring alternatives involves «the capacity to generate mentally a structure of possibilities standing beyond the empirically known world of the here and now». Creating and exploring alternatives is another aspect of critical consciousness that leads to methodical doubt – to «reflective scepticism» (Brookfield, 1987, p.21). It is this doubt which questions, for example, whether the fact of a given social structure having remained unaltered for a certain period of time is synonymous with its worthiness and inevitability. It is this scepticism which is revealed in the calling to question of information sources or even universal rules.

THE PEDAGOGICAL PROJECT

The pedagogical project, from which the small sample of the students' work analysed here originated, is a curricular development project whose aim is to develop democratic competence within the context of mathematics education at the same time, obviously, as acquiring mathematical knowledge and skills.

The students involved were in the first year of a degree course in Management and the work was undertaken in Calculus classes. The author of this paper was both the teacher and the researcher.

In what concerns the mathematical content it was decided to keep to the items originally stipulated in the Calculus syllabus. Mathematical applications and real problem solving were selected to provide a context in which to approach mathematical concepts. Applications and problems were chosen so as to encourage reflection and to elicit sustained opinions about topics which affect Portuguese and/or world society. Ethics and the exercising of power were, then, the backdrop to the topics chosen: ecological issues – global warming, the treatment of waste, the preservation of biodiversity; problems of personal responsibility – drug use, AIDS; population increase and the exhaustion of natural resources; problems of ethics in companies. Co-operative group work was preferred as *modus faciendi*. The students participated in the organisation of the course in the following aspects: definition of the assessment model and choice of topics for the piece of work each group had to develop and present. The students' receptivity to the type of work being developed was evaluated by gathering their impressions/opinions at three different points in the semester: at the end of the fourth, ninth and fifteenth weeks.

ABOUT THE RESEARCH PROJECT

The duration of the research project was one semester - the length of the course module in which it was undertaken. A qualitative/interpretative methodology was used. Two types of data were collected: (1) students' written reports; (2) students' impressions/opinions regarding the work they were carrying out. The work assignments collected for the sample came from five groups of students, randomly selected from the ten groups formed in the class.

The research project was developed around the following questions: (a) to what extent does mathematics education contribute to the development of a democratic competence? (b) in which ways can mathematics education and democracy be connected through mathematical contents? (c) how are the nature of problem situations and the nature of classroom culture relevant to the ways in which democratic competence is acquired?

This paper presents the analysis of a sample of data collected. It addresses a particular piece of an essay produced by a group of students on one of the activities proposed.

ANALYSING A SAMPLE OF EMPIRICAL DATA

The sample analysed here comes from an assignment which was set as follows:

> In the 1960's the population of the Antarctic Blue Whale was reduced to such a low level that the rapid extinction of the species was forecast. However, under the influence of an International Commission for the Protection of Whales, measures were taken which permitted the population to increase to around 10,000 in just 10 years. By 1994, their numbers had reached around 50,000.
>
> We assume that the mathematical model which expresses the population variation of the Antarctic Blue Whale between two consecutive years is the function h(x), analytically defined by:
>
> $$h(x)= 0{,}000002(-2x^3+303x^2-600x)$$
>
> where x represents the population of the current year, in thousands of whales [1].
>
> Write an essay analysing the problem of whale hunting in economic, ecological and political terms.

The students from group C began by carrying out an analytical study of the function h(x). They then identified the minimum viable population level, the carrying capacity and the maximum sustainable yield, thus giving ecological content to the study. Next they attempted to construct a graph and it was here that the first problem arose.

Students: To make a graph of the function, we used the Excel programme and obtained the following graph (fig.1).

Due to the scale, the graph does not show the values found in the study of the function. So we decided to construct a new graph, with the actual population varying between zero and three thousand (fig.2).

By ascribing values between zero and three to the population, the graph already shows that the function is negative between zero and two, and the minimum found for x=1 can be seen perfectly.

This critical attitude of the students regarding the product of the computer stands out; not only for the critical attitude in itself, but also for the confidence they reveal in their own work. Students often trust the efficiency of technology above their own and it is not uncommon to see them re-doing a correct piece of work so that it will agree with the results given by the computer. The students quoted above, however, are convinced of the correctness of the results of their study and try to identify the discrepancy between the graph supplied by the computer and their own expectations about the behaviour of the function.

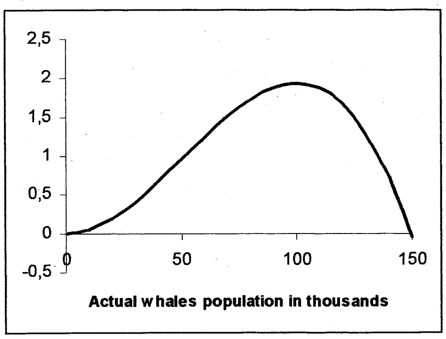

Figure 1

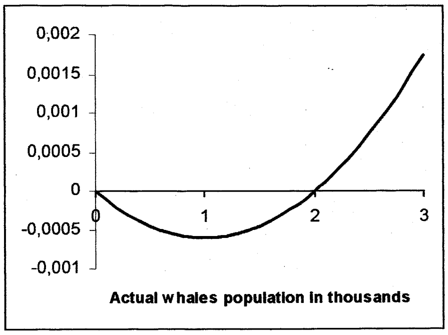

Figure 2

Further on, the students introduce new information, combining it with the values supplied about the actual population size:

Students: At the beginning of the 20th century, large regions of whales were discovered in the Antarctic. Each summer, numerous whale-hunters and factory ships headed South. Thousands of whales were killed and processed in the floating factories.

Whaling was at its height between 1925 and 1960. In the 1960's, the number of blue whales was so small (we do not know the exact number) that the International Commission for the protection of whales took its first protective measures. Due to these measures, within 10 years the population reached 10,000 and in the beginning of the 1990's it stood at 50,000, that is, in 20 years it had grown five times.

And, surprisingly, the criticism of the numbers obtained:

Students: However, if we consider the reproduction function, instead of the variation function, we do not get these results.

Let us see how they reached this conclusion:

Students: As the population variation function between two consecutive years is h(x) = f(x)-x where x is the population in a given year and f(x) is the population in the following year, we can determine f(x):f(x)=h(x)+x.

In the case of the blue whale it will be f(x) =0,000002(-2x3+303x2–600x)+x.

Reconstructing the function $f(x)$ is a simple operation. Attributing values to x and obtaining the corresponding values $f(x)$ is a trivial activity. Less so is the manipulation of the function so as to produce the values of the population in an interval of time, since this involves creating a recursive sequence in which the first term, u_1, is $f(a)$, with a chosen randomly, the second term, u_2, is $f(f(a))$, that is $f(u_1)$, … , $u_n=f(u_{n-1})$.

The results of the work of these students are shown below:

Students: The values we obtained from Excel were the following:

Year	*Actual*	*1 year later*	*Year*	*Actual*	*1 year later*
1	10	10,0446	17	10,769428	10,82179
2	10,0446	10,08963	18	10,8217927	10,8747066
3	10,0896345	10,13511	19	10,8747066	10,9281779
4	10,1351096	10,18103	20	10,9281779	10,9822153
5	10,1810318	10,22741	21	10,9822153	11,0368275

As can be seen, if the population variation function were that, the population twenty years after reaching 10 thousand (year 1) would be around 11,000 (year 21). The value of 50 thousand whales could only be reached 178 years later, as can be seen from the spreadsheet calculations:

Year	*Actual*	*1 year later*
175	46,2732692	47,11899
176	47,1189937	47,98944
177	47,9894376	48,88538
178	48,8853838	49,80763
179	49,8076271	50,75697

In assigning the piece of work on the blue whale, the researcher had been less rigorous than the students and had not proven that the true numbers concerning the blue whale populations and the values obtained with the proposed function actually coincided. Tomastik was consulted again, where it was verified that the values obtained by scientists regarding the carrying capacity and the maximum sustainable yield did indeed coincide with the values obtained by applying the function. The minimum viable level was unknown. But what had led the author to suggest a model of population variation that gave such different results to those observed empirically? The researcher had an idea of what may have happened, but she was not the only one.

In fact, during the discussion of the assignments in the classroom, and in the face of group C's discovery, a student from group G suggested that there could have been a mistake and the function should perhaps have been:

$$h(x) = 0{,}00002(-2x^3 + 303x^2 - 600x)$$

that is to say, an extra zero had been introduced in the factor in evidence.

This student thus showed a perception of the effects of multiplying by 10, the multiplicative parameter, on the behaviour of the function. Group C understood and accepted the proposed alteration and immediately went ahead with the modification of their calculations. And there, after 20 years, was the five-times-as-big population:

Year	Actual	1 year later	Year	Actual	1 year later
1	10	10,446	16	28,1191655	31,68396
2	10,446	10,93631	17	31,6839628	36,114961
3	10,9363144	11,47755	18	36,114961	41,7014054
4	11,477552	12,07765	19	41,7014054	48,8386105
5	12,0776512	12,74622	20	48,8386105	58,0473056

Having accepted the idea that the function needed to be altered, which aspects of the study of the function should be re-examined? And which characteristics maintained? The discussion was taken up again in the different groups.

Coming back to group C's report, the students also analysed the problem from the economic point of view:

Students: Whale-hunting is a highly lucrative business (...)

And they juxtaposed this with an ecological vision:

Students: Yet despite generating wealth, this activity threatens biodiversity and the balance of the ecosystem, as the extinction of the whales would break the food chains (...)

However the local situation is not forgotten:

Students: (...) in zones like the Azores, it was an activity with a long tradition and, as it was artisan in character, it would hardly have put the survival of the whale population in danger (...)

Finally, they concluded:

Students: (...) the countries which do not condemn and therefore do not apply laws and coercive measures to combat the capture of whales (...) should change their attitude, indeed they should be conscious of the true implications of this problem and transmit to their citizens the message that they should not compromise the future of generations to come.

CONCLUDING REMARKS

From the strictly mathematical point of view, the students developed skills in the analytical study of functions and acquired competences for the interpretation of reality, in view of the relevant aspects of the population variation function. But besides the skills and competences they applied in the analysis of the problem it is the actual situation – in the broad sense – which guides the students towards certain procedures, leading to a better understanding of the mathematical concepts involved. In fact there is a reciprocal influence: mathematical knowledge helps the understanding of reality, but reality 'demands' certain mathematical procedures and gives sense to the concepts.

The analysis of the problems posed by whaling cannot be reduced to the «*conscientization*» (Freire, 1975a, 1975b) of the established reality. The knowledge of the dynamics of populations constitutes a prospective knowledge which should orientate practices. These should not be subordinated to economic interests, but rather be governed by an ethic of responsibility. It is in the balanced management of ecosystems that these students focus their attention and the maintaining of biodiversity as a value in itself is as important to them as the warning of ruptures which could affect the quality of life of future generations, if not their actual survival.

It is also noteworthy that, for these students, there is no room for dogmas or for "untouchable" experts. Things are not what they are because "that's what it says in the book" or because "teachers say it is so" [2]. With all the naturalness of those who have learned to think about things, rather than submit themselves to a supposed truth, they detect a contradiction, propose an explanation and correct the author of a book and the teacher who placed blind trust in it; and they do it in an environment and context in which this has become a legitimate, permitted and even structuring process.

As regards group C's use of the computer, some observations would seem to be pertinent. The group used the computer with two different aims in mind: firstly, to create a clearer, well-presented graph of the function; secondly, to avoid having to carry out fastidious calculations themselves. The first objective was initially thwarted because the first graph did not faithfully represent the behaviour of the function. And this is, in fact, one of the negative aspects of a purely technological approach, when the computer users have no idea or expectation of what they should obtain. In the case under analysis, the students had previously carried out a complete study of the function. That is why they proceeded with a localised study to produce an amplification of a part of the curve representing the function. On the other hand, the huge potential of the computer is shown in the work that the students did to verify the affirmations about the size of the whale population. It is unthinkable that without the computer the students could have taken their work in this direction, so great being the number of operations they would have had to do. The same could be said regarding the work carried out in the classroom, i.e. the alteration of the function's parameters. Once again, it is evident that technology should be considered a vehicle of cognitive development, a tool which promotes autonomy, an instrument which can stimulate an investigative attitude (Moreira, 1989).

The «reflective scepticism» shown here in questioning the sources of information was, at other times, also evident in the contestation of universal rules.

Adopting an idea, adhering to a cause, assuming a practice is not something which comes from a spontaneous enthusiasm, but from a period of questioning, analysis and reflection during which its value or legitimacy is established.

In such social and political dimensions, mathematics education has a crucial role to assume but the conditions for the development of citizenship arising from mathematics classrooms have to be created. That is a job that cannot be improvised either.

NOTES

[1] Numerical data and function taken from Tomastik (1994).

[2] The expressions in inverted commas occur frequently, in other contexts, in the discourse of students to justify procedures or affirmations.

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