# EXPLORATIONS OF RANDOM MIXTURES IN A 2D SPACE CONTINUUM

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*This paper focuses on how children express their ideas for randomness in two-dimensional continuous space, through tools for directing and redirecting the simulated movement of balls. It reports the findings of a study in which children aged between 6 and 8 years old engaged with a game-like environment to construct for themselves random behaviour by making spatial representations of sample space. In response to a range of tasks, the children manipulated the sample space in ways that generated corresponding outcomes in the game. We present some case studies of children's activities, which illustrate how the medium mediates the children's understanding of chance events.*

## INTRODUCTION

The literature on randomness overflows with references to children's and adults' incompetence in dealing with judgements of chance (for just one of many references, see Kahneman et al., 1982, who set out a series of heuristics that, because of their inherent bias, have been termed by other authors as "misconceptions"). Piaget and Inhelder's (1951, translated 1975) work represents a seminal effort to investigate the origin of chance. They devised a number of experiments, which yielded 'chance' outcomes and they used these for probing the conceptual development of children from pre-school ages to adolescence. One of their experiments concerned a tilt box, where the child was given a rectangular box, resting on a transversal pivot, allowing seesawing. They concluded that young children fail to understand the random nature of mixture. Their main argument is that a fundamental property of operations is that they are reversible, so that even when a child is a concrete operational thinker, randomness is still a strange idea that does not fit into the way they normally see the world. Further, they maintained that the idea of chance is not acquired before the stage of concrete operations.

However, Fischbein (1975) has reported how children have intuitions for relative frequency from a very young age, and this leads us to search for other evidence that even very young children have emergent cognitive resources for making sense of randomness. In research on intuitions and fairness, Pratt & Noss (1998) found out how their subjects make sense of dice situations and they show how existing intuitions about fairness, often based on actual outcomes, are co-ordinated with new meanings, derive from interacting with the microworld. Papert (1996) has argued that educators should look in pragmatic ways for connections between pieces of knowledge and that, on a theoretical level, the metaphor of learning by construction leads to a range of interesting questions about the connectivity of knowledge.

In terms of knowledge about stochastic phenomena, Pratt (2000) has also reported on how children (aged 10/11) were observed to use four separable resources for articulating randomness namely: unsteerability, irregularity, unpredictability and fairness. Children constructed meanings for randomness in the context of a computational microworld, visualising how the random behaviour of objects actually worked. In this paper we describe the activities of even younger children (aged between 6 and 8) who were expressing their ideas for randomness in two-dimensional continuous space, through tools for directing and redirecting the simulated movement of bouncing balls.

## THE SOFTWARE

The game was written in a rule-based system, called 'Pathways'[1] and was designed to afford children the opportunity to talk and think about probability in the context of quasi-concrete manipulations. The game is shown in Figure 1.

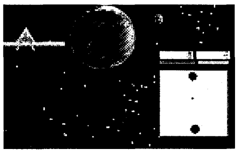

Figure 1: The game

In the right-hand corner is a square. A small ball bounces around the walls of the square and occasionally collides with and bounces off other balls. In Figure 1, one red and one blue ball is depicted (they are the other two larger balls inside the rectangle), but the child can control the number, size and position of the balls. Each time there is a collision with a red ball, one point is added to the red score (just above the rectangle), and the "space kid" (the triangular creature) moves one step up the screen. Similarly, collisions with the blue balls add one point to the blue score and move the space kid one step down the screen. Whilst individual collisions can be seen as single trials in a stochastic experiment the totality of these movements gives an aggregated view of the long-term probability. The game itself is defined in terms of simple iconically represented rules, which are designed so that the children can easily change the nature of the game itself (see Goldstein and Pratt, 2001, for a fuller description of the Pathways environment).

The task was given to the children to construct for themselves representations of the sample space. They constructed for themselves random behaviour by making spatial representations of sample space in a game-like environment. In order to

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explore children's connections between fairness and randomness, we began with a situation in which the children had to try to make the space kid move around a centre line in order to construct a 'fair sample space'. Later, we asked children to make changes to the two-dimensional continuum space to make the space kid move upwards or downwards from the centre line in ways that represented other global outcomes (such as being likely to go off the top of the screen).

## METHODOLOGY

The findings reported here are part of a broader study that adopted a strategy of iterative design, in which the computer-based tool was developed alongside the gathering of evidence for children's use of the tool. The children were interviewed before and during their interaction with the computer: all interviews of each iteration were videotaped and transcribed. The role of the researcher was that of participant observer, interacting with the children in order to probe the reasons behind their answers and their actions. In the final iteration, 22 children, aged between 6.5 and 8 years, were involved. The children worked with the software individually for between 2 and 3 hours. The semi-structured interview began with children expressing their ideas about Piaget & Inhelder's tilt task. The children were then asked to inspect the rules of the game through the Pathways tools, and finally worked with a range of game-oriented problematic situations.

## EXPRESSIONS OF RANDOM MIXTURE

The results from the interviews on the tilting task were consistent with those of Piaget and Inhelder. Most of the children responded to the task by expressing what the order of the colours should be when the balls move to the other side of the tilt box. For example, Jane said, 'The reds will move here, then the yellows, the greens here and the oranges here'. The children envisaged that the balls would finish up in an organised manner, rather than being mixed up in the tilting process.

In this context, the children were generally unable to express notions of random mixture. In contrast, they seemed able to express the notion of random mixture in two-dimensional space continuum environment. The computer environment supported several ways in which the idea of mixture could be expressed. The children changed the position, size or number of the red and blue stable balls in the sample space, or they described in words the 'uncontrolled' continual movement of the white ball. We describe four categories of children's strategies below: haphazard movement, complex movement, symmetry of placement and equal size of balls.

### Haphazard movement

The children's awareness of a lack of pattern and lack of controls over the movement within the two-dimensional space is evocative of the local resources for making sense of randomness as reported by Pratt (2000). Lucy (age 7 years and 8 months) characterised the white ball's movement and mixture as follows: 'It goes right-left, up, down and on the balls. We don't know where it goes. It moves in the yellow square, where it wants to go'. Mixture for Lucy meant to move where the ball wants to go, without any obvious pattern or pre-ordained positions. Children were using similar internal resources or intuitions as one might expect in a more conventional study of randomness, and so lends credence to the idea that these very young children were indeed constructing meaning for a random mixture, in a way that was not evident in Piaget & Inhelder's experiment.

**Complex movement**

Paul (age 6 years and 10 months) expressed his idea of random mixture in a different way. He was influenced by the manipulations of the task, and he attempted to make all the balls bounce around, collecting points.

| | |
|---|---|
| Paul: | Ok now! Ah! I know what to do! |
| | *He changes the balls and the speed again.* |
| Researcher: | The blues have more points than the reds. |
| Paul: | Do you know what to do? We can take out all the white balls and give a speed to the red and blue balls and when the blue touches to a blue or a red touches to a red ball to get two points otherwise to get one point. |

Paul gave movement to the red and blue balls and decided to change the rules under which points were scored and the mixture of the balls was made more complex.

**Symmetry of placement**

*Symmetrical teams*

Most of the children tried to achieve a fair game by placing the balls symmetrically. They didn't care so much where each ball was placed, but were concerned about the positioning of a team of the balls. This is very obvious to Brian (age 6 years and 6 months), who not only separated the two colours, but also put lines between the two teams and constructed new rules.

| | |
|---|---|
| Researcher: | Can you do changes in the lottery machine? |
| Brian: | I want to make a line... I will blow this ball... I know, I will do something (Figure 2). |
| | *He separates the two colours and he constructs a new object.* |

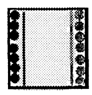

Figure 2

| | |
|---|---|
| Researcher: | What do you want to do? |
| Brian: | I need a rule 'When the ball touches me, I take all the |

messages from all the red balls and I give them to the red scorer.' I need the same to happen to the blue balls.

The children saw each colour as one team. For example, Jane (6 years and 7 months) expressed randomness by first separating the two colours to create two different teams of equal numbers.

Jane: Because I will put them near to each other in the middle. So, when the ball goes to touch the one, it will touch also the other that's near it. So, it will touch both of them and we will have equal points. We do not know where it (*the white ball*) will go, but if it touches one ball it will touch the other as well.

Researcher: That sounds a nice idea...

Jane: ...I will put the two lines in the middle. Now, I need another two balls. I'll get the magic wand to get more balls. It's the good fairy that gave it to us...

Researcher: Do you know how many balls do we have?

Jane: Yes...they are equal. I know they are equal, but I don't know how many balls I have (Figure 3).

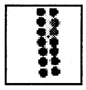

Figure 3

Researcher: How do you know that they are equal?

Jane: I copied one red and one blue each time. It doesn't matter actually how many they are. They are equal.

This excerpt emphasizes that what mattered to Jane was the equality of the teams, rather than precisely how many there were. She seemed to equate equality with fairness, and presumably fairness in some way with randomness.

*Making a pattern*

The pattern was very useful for Lucy to create a random environment.

Researcher: Can you arrange the balls in a way in order the space kid to move near the yellow line?

Lucy: I will then leave only two balls...or...I will make a pattern in order not to move so up or so much down… I will copy some more balls (Figure 4).

*She destroys balls and keeps only one red and one blue. Then she copies some to make a pattern.*

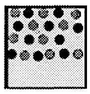

Figure 4

Researcher: How does the pattern work?

Lucy: They are going to have equal numbers. It (the white ball) will move up, on the edges…the ball will get the same points. I will also copy another white ball to move quickly…They became rows.

*She starts the game.*

Researcher: What happens? Where does the white balls go?

Lucy: It goes everywhere…around the balls. They have equal numbers now! I got one ball and another. I made a row and then another row and I made the white ball to move in a way and now they are going to have the same numbers.

The logic behind it was for one colour to be near the other, so that when the white ball was going to touch one colour it would touch the other as well. The pattern was also used as a way, as she described, to have equal number of balls, without counting.

*Making circles*

The circle was made to trap the white ball in order to touch the balls in the circle the same number of times, and sometimes it turned out to be a start for a symmetrical development, as well. Anne (age 6 years and 6 months) here started by having the white ball in a circle and then constructed another random situation by copying more balls.

Anne: …I'm going to make all the balls have the same size. I'll do another arrangement.

Researcher: So, what are you doing now? (Figure 5)

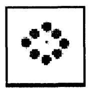

Figure 5

Anne: I'll make more copies of them.

*She switches the game on.*

Researcher: Oh...does it work?

Anne: Yeah...It keeps going up, down, up, down....

Researcher: Great!

Anne: I'll make more copies...

Researcher: What's the arrangement now?

Anne: That one (the blue ball) is facing that one (the red ball) and that one is facing that one and so on... I've got also a better idea! They (the red balls) will face a blue one. There! (Figure 6)

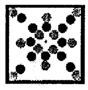

Figure 6

Researcher: What did you do?

Anne: The blue ones are facing the red ones and the red ones the blue ones.

Researcher: Ok! What number will you have here (on the scorers)?

Anne: I don't know, I'll try it out!

*She starts the game.*

Anne started here by constructing a circle and developed that into a symmetrical picture.

**Equal size of balls**

The spatial environment played a major role in helping Jane to look at whether two balls were equal in size or not. As she said, one of the balls 'is bigger and it (*the white ball*) will touch the most of the time, because the ball takes up more space in the yellow square'. She looked at the effects of the game and she used the global event, the movement of the space kid, to see whether her environment was fair.

Jane: I think the red will win.

Researcher: Why is that?

Jane: I think I made it a little bigger than the other... We can open the game and if the scorers are the same that means that they have the same size, otherwise the one is bigger than the other (Figure 7).

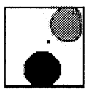

Figure 7

Researcher: What about the space kid?

Jane: If it is as now that means our balls have the same size...

Jane was making a connection between the spatial appearance of the sample space and the possible outcome from the game in the longer term.

## CONCLUSIONS

The study seems to indicate interesting differences between the way that children responded to Piaget & Inhelder's experiment and how children in this non-conventional context were able to express ideas for random mixture. A main difference from the Piagetian study was perhaps that, in this study, the children constructed randomness, not just as a cognitive act or thought experiment, but in a quasi-concrete way. Children's thinking moved from finding outcomes and describing the random behaviour to *constructing* a random behaviour. The study indicates that children have various cognitive resources for constructing randomness, beyond those that might be expected from classical experiments. We believe that a possible reason for this is that the tool offers them the opportunity to use these resources for random mixture in a two-dimensional continuum. Moreover, children's culture involves many experiences of random movement, and the context for such experiences is changing from playing board games towards playing video games. We conjecture that cognitive resources for making sense of random mixture may be more likely to find a means of expression in continuous two-dimensional movement than in more conventional contexts that involve discrete number.

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