CERTAINTY BIAS AS AN INDICATOR OF PROBLEMS IN CONCEPTUAL CHANGE: THE CASE OF NUMBER LINE

Kaarina Merenluoto & Erno Lehtinen,
Department of Teacher Education, University of Turku, Finland

This paper presents an analysis of interaction between levels of answers and respective certainty estimations in a number concept test participated by pupils at upper secondary school. In the analysis four different profiles of interaction were found and they are explained by various pupils' characteristics (prior knowledge, achievement level in mathematics, gender and test effort). The results refer to significant differences in the sensitivity to the need for conceptual change and tolerance for ambiguity, which seem to be essential for a conceptual change.

INTRODUCTION

The crucial idea in the theory of conceptual change is the radical reconstruction of prior knowledge which is not adequately taken into account in traditional teaching. In educational contexts, mathematics is considered to form a hierarchical structure in which all new concepts logically follow from prior ones, which allows students to enrich their knowledge step by step. The transitions from the domain of natural numbers to the domains of more advanced ones, are often treated as a continuous growth of knowledge. From a cognitive point of view, however, they are better described as a radical reconstruction, because every enlargement requires fundamental changes in the previous thinking of numbers.

In previous research (Merenluoto & Lehtinen, in press; Merenluoto, 2001) about conceptual change needed in enlargements of the number concept, the majority of upper secondary school pupils had not restructured their prior beliefs on numbers in order to understand the concepts of rational or irrational numbers even on the preliminary level. The results refer to a mistaken transfer by the pupils from natural numbers to the domains of more advanced numbers. This suggests a low sensitivity by most of the pupils to the needed change in thinking the numbers. We explained these results with the theories on conceptual change (see Carey, 1985; Chi, Slotta, & de Leeuw, 1994; Vosniadou, 1994; 1999; Duit, 1995), which consider the relationship between the learners' prior knowledge and information to be learned as one of the most crucial factors in determining the quality of learning. In previous studies it has also been found that conceptual change involves not only change in specific beliefs and presumptions but also development of metaconceptual awareness and consideration of metacognitive effects (Vosniadou, Ioannides, Dimitrakopoulou & Papademetriou, 2001; Limòn, 2001).

Hence in the case of numbers, the major obstacle in understanding the advanced concepts is the quality of students' prior knowledge and beliefs on numbers. But there seem to be three distinctly different components in this obstacle, where the first component is cognitive, but the two others clearly metacognitive (see Flavell, 1987) and motivational (Pintrich, 1999).

From the cognitive viewpoint, the enlargements of the number concept require drastic changes in the very thinking of numbers. For example, the fundamental ideas of natural numbers, as the concept of a successor, are necessary for learning the notion of natural numbers. But in the domains of rational and real numbers the principle of a successor is not defined, but infinite successive division is possible. Thus some of these basic concepts are in serious conflict with the very character of both rational and real numbers (Sowder, 1992). Therefore in order to understand these concepts, a very profound change in thinking of numbers is necessary.

Secondly, small natural numbers and the concept of a successor seem to be among those special concepts which have a high unconditional certainty attached to them. This kind of certainty seems to be a result from innate cognitive mechanism relate to numeral reasoning principles (Gallistel, & Gelman, 1992) but also from the everyday experiences and the linguistic operations (Wittgenstein, 1969). It has a subjective nature with the feeling of self-evidence. Because these concepts seem self-evident, self-justifiable or self-explanatory, they easily lead to overconfidence (Fischbein, 1987). As such they might act as an obstacle for conceptual change or lead to mistakes and misunderstandings on more advanced domains of numbers. This is the case, when it does not even occur to the pupils that they need to rethink their knowledge and logic on numbers even though it would be necessary. In other words, the self-evident nature of this kind of certainty means that the students might have a low sensitivity to the need for a change.

Thirdly, because of the drastic nature of the needed change, the process seems to be related to pupils subjective experiences with mathematics (Merenluoto, 2001) and to their tolerance of ambiguity (Lehtinen, 1984; Stark, Mandl, Gruber & Renkl, 2002). Experiences in mathematics have been studied from many different viewpoints: In the analysis of *feeling of difficulty* in mathematics (Efklides, Akilina & Petropoulou, 1999) the results suggest that these kind of feelings form a system of their own, which is mainly influenced by performance and cognitive ability rather than affective factors. The certainty estimated by the boys were more typical than those of the girls, which seemed to me more context-related. Certainty experiences in mathematics have been studied also from the viewpoint of *self-efficacy, self-awareness, self-regulation and math anxiety* (Pajares, 1996; Schoenfeld, 1987). These studies are based on the assumptions that the personal confidence, which has earlier been experienced in mathematics has causal effects on the performance or certainty feelings later.

Thus we have a hypotheses that in process of a radical change in the thinking of numbers the students are forced to tolerance the ambiguity which comes from newly learned operations and characteristics of numbers while they do not yet fully understand the concepts.

The aim of this paper is to analyse the data from our previous research to find the factors referring to the sensitivity to the needed change in thinking of the numbers and to find the factors related to the tolerance of ambiguity.

METHOD

Subjects and procedure: The data used is from a number concept test, given to 537 students (mean age 17.2 years) from 24 randomly selected Finnish upper secondary schools (see Merenluoto & Lehtinen, in press). The students, who participated in the test after their first calculus course, were asked to estimate their certainty while answering the questions. In this estimation they were asked to use a scale from 1 to 5, where 1 meant that their answer was a wild guess, and 5 that they were absolutely sure, as sure as they know that 1+1=2. In this paper only the critical questions (table 1) pertaining to the density of the number line are discussed.

Scoring and variables: The performance in the tasks was measured with a 5-point scale, from 0 (no answer) to 4 (Table 1). The certainty scores were multiplied with 4/5 in order to set them to the same scale. *Certainty bias:* the task scores were subtracted from the certainty scores in each task. The negative values on this variable respond to uncertainty, the positive ones to overconfidence. *Test effort:* a percentage of answered items was calculated for every student. *Achievement level in mathematics:* students mark in mathematics was calculated as percentages from maximum. *Group position:* the group mean of achievement level in mathematics was subtracted from the respective pupils' mark in mathematics. The negative values refer to a group position below average, positive values to a position above average. *Gender:* there were 335 boys (62.4%) and 202 (37.6%)girls.

RESULTS

Identification of profiles: On the basis of interaction of the task scores (Table 1) with the respective certainty estimations, a cluster analysis was used and four different profiles were found. The general difference in the quality of conceptual change is obvious in the profiles of the answers (Fig. 1), where there is a significant difference between the tasks 2 - 3 compared to the tasks 4 - 5. This difference is due to the quality of conceptual change. Although there were question about the concept of infinite divisibility (and limit) in all the tasks, it is possible to answer at a high level without making any notable change in thinking of numbers in tasks 2-3. Whereas it requires a radical change in order to answer, that the "next" or "closest" is not defined in Q or R. The different level of task scores was obvious in the profiles (Fig. 1) but the appearance of the difference was clearer in the certainty bias profiles (Fig. 2).

TABLE 1. The critical questions pertaining to the number line and the scoring based on the level of answers.

The critical questions	Incorrect[1] (scored 1)	Superficial (scored 2-3)[2]	Correct (scored 4)
1. **Interval.** Define on which interval on the number line is the number, which has the approximate value: 5,01.	5 – 6	5.005-5.01499…	[5.005; 5.015[
2. **Density in Q.** How many rational numbers there are on the number line between the numbers 3/5 and 5/6? Why?	6	"several" "many" "infinite" [1]with or no explanations	There are infinite number of rational/ real numbers, It is always possible to add numbers between any two.
3. **Density in R.** How many real numbers there are on the number line between the numbers 0,99 and 1,00? Why?	None		
4. **Limit in Q.** Which fraction is the "next" after 3/5? Why?	4/5	"none" "all of them" [1]with or no explanations	The "next" or "the closest" number are not defined in Q or R, it's always possible to find numbers that a closer .
5. **Limit in R.** Which real number is "the closest" to 1,00? Why?	0,999…		

[1]The answer is based on the logic of whole numbers.
[2]The answers with no or meaningless explanations was scored as 2, answers with explanations like *"infinite, because it is possible to make the numbers more exact"* etc. were scored as 3.

<u>The factors behind the profiles</u> (table 2) indicate in the profiles the difference in pupils' prior achievement level in mathematics, his/her group position, test effort and certainty bias was significant. Whereas gender difference was significant between profiles 2 and 3 ($\chi 2$ (1) =14.34; p<.001).

TABLE 2. The means of variables behind the profiles and means of certainty bias and test effort.

Profile	N	Mark[1]	Group position[2]	Gender % of girls	Gender % of boys	Test effort[3]	Cert. bias[4]
Profile 1	135	54%	-.49	31	21	54%	-.06
Profile 2	161	61%	-.06	37	26	72%	.27
Profile 3	188	64%	.13	25	41	82%	.84
Profile 4	54	78%	1.08	7	12	94%	.34
All	538	62%	.01	100	100	74%	.40

Significant ($p<.00$) difference between profiles: [1]$F(3, 525)=17.5$, n.s. between profiles 2 and 3; [2]$F(3, 525)=16.2$, n.s. between profiles 2 and 3; [3]$F(3, 534)=112.9$; $p<.001$; [4]$F(3, 534)=53.2$;n.s. between profiles 2 and 4.

For the pupils in profile 1 (n =135; 25.1%) the quality of answers and the estimated certainty level was low. They identified numbers only by their superficial features as whole numbers, fractions and decimal numbers. The profile is characterised by answers based on the logic of whole numbers, where there was a negative bias in certainty except for the tasks 2 and 4 where the word “fraction” was used instead of “rational number”. Their low level of answers suggest a low level of sensitivity for a need for change in thinking, which seems to be due to the serious deficiency in their prior knowledge about numbers. Their low level of test effort (Table 2) together with low estimations of certainty and low achievement level in mathematics suggest a low tolerance of ambiguity.

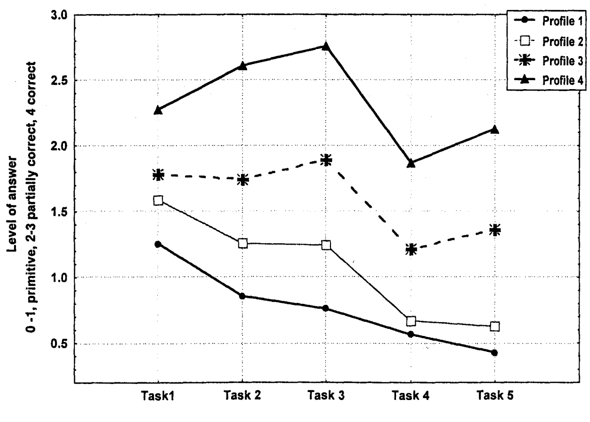

FIGURE 1. Profiles of the task scores, cluster means.

The profile 2 (n = 161; 29.9%), is characterised by a positive certainty bias in tasks which seemed familiar (tasks 1-2 and 4), but negative bias in task 3, where the set of numbers was referred to as "real numbers". They mainly used the logic of whole numbers in their answers and were sensitive to the difference in tasks 2-3 compared to the tasks 4-5 (Fig. 1), where the questions were very different (Table 1) compared to the ordinary questions they had been used to in school. This suggests a context-related tolerance of ambiguity. The level of answers refer to a low sensitivity to a needed conceptual change.

The profile 3 (n = 188; 34.9%), is characterised with systematic over-estimation of certainty. The levels of answers were significantly better than for the students in profile 2, but the difference in the certainty bias was still higher. This refers to a high context-independent tolerance of ambiguity, which is suggested also by their achievement level in mathematics and high test effort (Table 2). Their answers to the tasks, though they were better than in the previous profile, however, refer to a superficial level of conceptual change: to an enrichment kind of learning which in turn refers to a superficial level of sensitivity to the needed change. The difference between the profiles 2 and 3 has also a significant reference to the gender of the pupils.

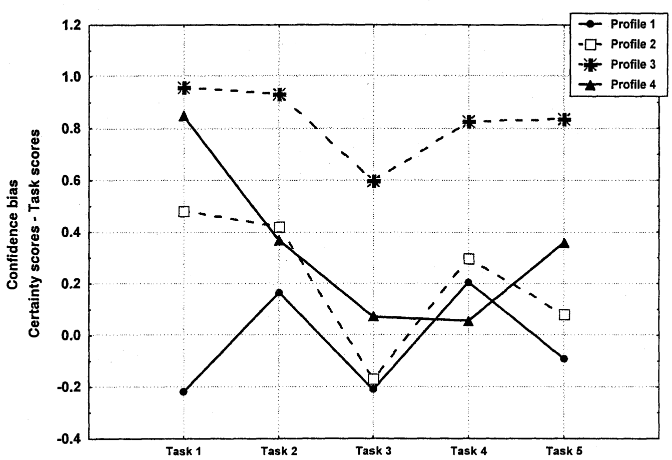

FIGURE 2. Certainty bias profiles in the tasks, cluster means.

The pupils in profile 4 (n = 54), who gave the highest level of answers to the questions, had a positive certainty bias in seemingly familiar tasks (2-3 and 5). But they were sensitive in their certainty estimations in more difficult tasks (4-5), where their explanations clearly referred to a more radical change in their thinking of numbers and the density of number line, compared to the pupils in the other clusters. This kind of change seemed to yield to a very significant difference in the certainty bias profiles between the profiles 4 and 3. The best answers had significantly lower certainty estimations because of the novelty of the ideas and radical nature of the change experienced. These students had a high sensitivity to the needed change, which was obvious in the quality of their explanations. Their achievement level in mathematics, their group position and high test effort refers to a high tolerance of ambiguity.

CONCLUSION

The results give a suggestion that the conceptual change is related to metacognitive and motivational aspects, which still needs further studies. The students in profile four had high sensitivity to the needed change combined with high tolerance of uncertainty, a combination which seems to be optimal for a conceptual change. These pupils had a quite high level of understanding of the density of the number line. Whereas the superficial level of sensitivity combined with the high tolerance of ambiguity seem to be restrictive to a more radical change and deeper understanding of the concepts. Their thinking of the density of the number line was based on operational thinking (making numbers more exact, adding decimals, etc.) without any references to the structural differences between the numbers. These students have an illusion of understanding and do not necessarily see any reason to strengthen their metaconceptual thinking. The majority of students (profiles 1 and 2) had serious problems in their prior formal understanding of numbers, which is an obstacle for their conceptual change also. Their thinking of the density of the number line was more or less based on thinking of whole numbers.

These results refer to the necessity to consider the metacognitive and motivational aspects in future research on conceptual change. They also suggest that process of conceptual change, altogether, is a complex and a gradual affair, which needs to be taken care of also when planning learning environments which support conceptual change.

References

Chi, M. T. H., Slotta, J. D. & de Leeuw, N. (1994). From things to process: A theory of conceptual change for learning science concepts. *Learning and Instruction*, 4, 27-43.

Duit, R. (1995). The constructivist view: A fashionable and fruitful paradigm for science education research and practice. In L. P. Steffe & J. Gale (Eds.) *Constructivism in education.* NJ, Hillsdale: Lawrence Erlbaum Associates.

Efklides, A., Akilina, S. & Petropoulou, M. (1999). Feeling of difficulty: An aspect of monitoring that influences control. *European Journal of Psychology of education*, Vol XIV (4), 461-476.

Fischbein, E. (1987). *Intuition in science and mathematics, An educational approach.* Dordrecht: D. Reidel Publishing Company.

Flavell, J.H. (1987). Speculations about the nature and development of metacognition. in F. E. Weinert & R. H. Kluve (Eds.). *Metacognition, motivation and understanding.* London: Lawrence Erlbaum, 21-29.

Gallistel, G. R. & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition,* 44, 43-74.

Lehtinen, E. (1984). *Motivating students towards planning and controlling of thinking activities. A constructivistic approach to learning*. Publications of Turku University, B. 164.

Merenluoto, K. 2001. Lukiolaisen reaaliluku. Lukualueen laajentaminen käsitteellisenä muutoksena matematiikassa. *[Students' real numbers. Enlarging the number concept as a conceptual change in mathematics. A doctoral dissertation. Includes an English summary].* Publications of Turku University C 176.

Merenluoto, K. & Lehtinen, E. (in press). Conceptual change in mathematics: understanding the real numbers. In M. Limón & L. Mason (Eds.) *Reconsidering conceptual change. Issues in theory and practice.* Dordrecht: Kluwer Academic publishers.

Pajares, F. (1996). Self-efficacy beliefs and mathematical problem-solving of gifted students. *Contemporary Educational Psychology*, 21, 325-344.

Pintrich, P. (1999). Motivational beliefs as resources for and constraints on conceptual change. In W. Schnotz, S. Vosniadou & M. Caterro (Eds.). *New perspectives on conceptual change.* Killington, Oxford: Elsevier Science, 33-50.

Schoenfeld, A. (1987). What's all the fuss about metacognition? In A. Schoenfeld (Ed.) *Cognitive science and mathematics education.* Hillsdale NJ: Lawrence Erlbaum Associates, 191 –215.

Sowder, J. T. 1992. Making sense of numbers in school mathematics. In G. Leinhardt, Putnam, R. & Hattrup, R. (Eds.) Analysis of arithmetic for mathematics teaching. *NJ: Lawrence Erlbaum Associates, 1 - 46.*

Stark, R., Mandl, H. Gruber, H. & Renkl, A. (2002). Conditions and effects on example elaboration. *Learning and Instruction*, 12 (1), 39-60.

Vosniadou, S. (1994). Capturing and modelling the process of conceptual change. *Learning and Instruction*, 4, 45-69.

Vosniadou, S. (1999). Conceptual change research: state of art and future directions. In W. Schnotz, S. Vosniadou, & M. Carretero (Eds.) *New perspectives on conceptual change*. Killington, Oxford: Elsevier Science, 3-14.

Vosniadou, S., Ioannides, C., Dimitrakopoulou, A. & Papademetriou. (2001). Designing learning environments to promote conceptual change in science. *Learning and Instruction*, 11 (4-5), 381-419.

Wittgenstein, L. (1969). *Über gewissheit / On certainty*. Oxford. Basil Blackwell.