DEVELOPING TAKEN AS SHARED MEANINGS IN MATHEMATICS: LESSONS FROM CLASSROOMS IN PAKISTAN

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This is a study of the role of social interactions in students' learning of mathematics. The study was based in two classrooms in Karachi, Pakistan. A small group of students (10-12 yr.) doing mathematics was observed in each classroom.

Methodology used was qualitative in nature. Participant observation was the primary mode of conducting the research. To follow up on questions emerging from ongoing observations students were interviewed under stimulated recall. Analysis was through grounded theory procedures (Strauss & Corbin, 1998).

The teachers had initiated a change in the social and socio-mathematical practices in the classroom. Findings showed that the purpose of changed mathematical practices introduced by the teacher did not necessarily come to be taken as shared by the students. Developing a taken as shared meaning of the purpose of classroom practices involved mutual negotiation between the teacher and the students.

This paper reports on the findings from my doctoral research[1]. I examined the role of social interactions in students' learning of mathematics. My study was located in two secondary schools (10-15yrs.) in Karachi, Pakistan. It involved observing two small groups of students, each engaged in working at mathematics tasks set for them by their teacher.

METHODOLOGY

Methodology was qualitative in nature. Participant observation was the primary mode of conducting the research. Questions emerging from ongoing observations were followed up in stimulated recall interviews with students. Meetings were scheduled with the teachers who were seen as a significant part of the socio-cultural context. Observations were recorded on videotapes and interviews were audiotaped. Analysis was through grounded theory procedures (Strauss & Corbin, 1998).

THEORETICAL PERSPECTIVE

To study learning in the social context necessitates taking account of those shared understandings and invisible meanings that establish a classroom culture, and that provide meaning to the interpretations being made by the participants of the classroom i.e. the students and the teacher. A sociological construct used to describe

[1] Tentative findings of this work while it was in progress were presented as a short oral presentation in PME 25.

the shared understandings of a culture is 'norm' which refers to understandings or interpretations that become normative or taken-as-shared by the group (Yackel, 2001). She goes on to elaborate that these norms are inferred by identifying regularities in patterns of social interaction in the classroom. Those classroom participation structures that are stable facilitate in inferring the social norms prevalent, while the notion of taken-as-shared implies that individual interpretations fit with the purpose at hand.

The teacher necessarily represents the discipline of mathematics and the culture of the mathematics classroom, therefore, the teacher is a significant force in initiating and stabilising the social norms in the classroom (Wood, 1994). However, understandings do not become taken-as shared or normative through teacher's initiation alone. Wood (1994) suggests that one way to establish classroom norms, would be to for teachers and students to negotiate mutual expectations and obligations that are constituted in the classroom. Wood's position implies that in a classroom activity, participants in the activity would have expectations from others, and obligations from self which guide their participation. The collaboration among the various participants would then take place as a consequence of fulfilling these expectations and obligations.

From the discussion so far I deduce social norms to mean normative understanding of the ways of collaborating with others in the classroom. For example, when learning in small group settings, the understanding that students are expected to share their thinking with others in the group would be a social norm. As such, the social norms are shared understandings that could be found in any classroom irrespective of the specific discipline, and are not unique to mathematics. For the analyses of students' learning of mathematics, the construct of socio-mathematical norms is taken to mean normative aspects of mathematical discussions that are specific to students' mathematical activity (Yackel, 2001). Hence, a socio-mathematical norm would be a shared criterion of what is valuable, in a mathematical explanation or solution.

RESULTS & DISCUSSION

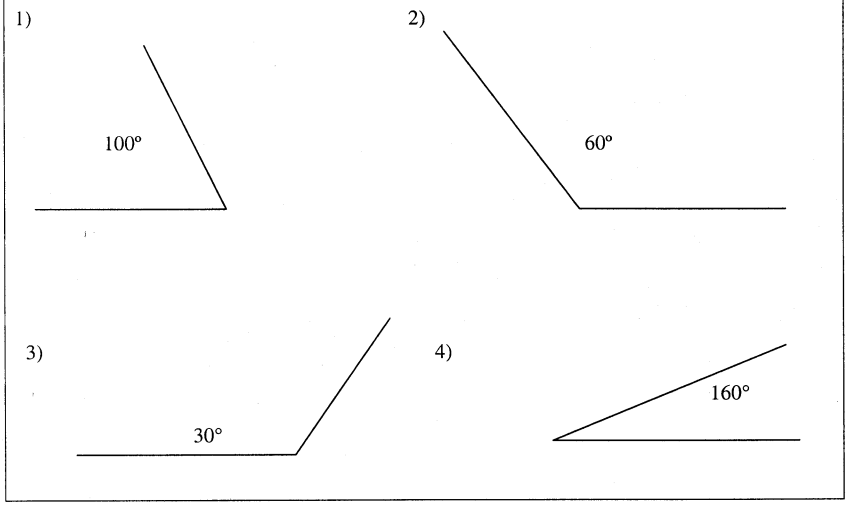
In this paper I discuss my findings from class VI (10-11 yrs) where I observed four girls, Maheen, Shabnum, Samina and Naima. The teacher Mohammad Aslam had initiated a change in the social and socio-mathematical practices in the classroom. He taught lessons that required students to work together in small groups at specially designed problem tasks. At the end of each group session the small groups were expected to report back their work to the whole class. This was a change from his previous social setting of the class. The problem tasks prepared by Mohammad Aslam were different from those that he said he had used previously. The difference was that the tasks that he prepared often had questions that were open ended in nature. For example, questions like, "can you decide the reasons for the mistakes shown?" opened up the possibility of more than one solution to be put forward.

Furthermore, students were free to use their own strategies to solve problems and share their solutions with the whole class using their own words.

From the above I inferred the teacher to have a goal that students should learn mathematics meaningfully i.e. students would rationalise their solutions and/or propositions, and at the same time the solutions and/or propositions should be acceptable mathematically. For example, enabling students to interact frequently with each other in small groups, use their own solution strategies, and explain their thinking to others meant that students had opportunities to explore their own interpretation of the mathematics embedded in the problem tasks. While, setting up the groups to report in the whole class ensured that the teacher could monitor those interpretations. However, this goal was implicit in the teacher's practices. There was very little evidence of explicit negotiation of the purpose of changed practices between the teacher and the students.

Episode: Alina's Mistake

Alina has drawn some angles in her maths book, but has measured them wrongly. Can you decide what she has done wrong?



To locate the discussion of findings in the classroom context I present an episode from my lesson observations. The students in this episode worked in groups at the task shown in the box above. Different groups shared their work with the whole class. Through this episode I illustrate the issues that arose for students' learning as they worked in the changed social setting of the classroom.

Task Interpretation: A recurring feature in the findings was that students interpreted the purpose of task based on their prior experiences in the classroom. In this case, the

students approach to the work on the task above suggested that they had interpreted the purpose of the task to measure angles. Once all angles were measured they considered their work finished and Samina read out the answers in the microphone left on their table for my data collection. This reading of answers stood out as a separate activity illustrating a pattern where one person would suggest reading the answers and the others would comply by reading the answers in turn. Moreover, every time they finished a task or a portion of a task they raised their hands to indicate to the teacher that work had finished. This implied an implicit belief in the answer as a valued product of the mathematics problem tasks, and a dependency on the teacher.

A possible reason for students' interpretation of task, as angles to be measured, could be that they were interpreting the task from their previous experiences of participating in classroom mathematics practices. The geometry curriculum in Pakistani primary and secondary schools has a heavy emphasis on Euclidean plane geometry. A consequence is that such cultural artefacts as protractors, compasses and work sheets and/or textbooks with diagrams of angles and other figures in plane geometry are common in the classroom milieu. Among other things, students are expected to measure and/or construct angles.

However, in this case the question statement "can you decide what she has done wrong?" is not clear in its intent because Alina's mistake (that she has 'measured the angles wrongly') has already been identified in the statement "had measured them wrongly". That the task requires a reason for Alina's mistake is difficult to infer from the question statement as it is phrased. Hence, the reason for student's interpretation of task as one of measuring angles could be because of the way the problem statement was phrased.

Mathematics being Learnt: The mathematics topic that was being addressed by the students was "acute angles and obtuse angles". The conversation being reported below is when Shabnum had placed a protractor on the second angle in the box above and the other three girls looked on. The numbers 60 and 120 refer to the readings on the protractor. The word 'degrees' is not used by the students it is implied

1	Shabnum:	(reading the protractor) err sixty
2	Maheen:	120 120
3	Naima:	(refers to the position of the protractor) bring it down, bring it down
4	Maheen and Naima:	120
5	Samina:	120 or 60?
6	Maheen/ Naima:	120
7	Maheen:	120 because our base line starts from here (points to the baseline on the protractor). So it is starting from here so 60 will come here (traces her finger on the protractor from 0 to where 60 degree angle comes).

8 Samina:	No because 90 is coming in between (60 and 120) and at 90 the base line ends after which the numbers on top start. (refers to the readings on the protractor)	
9 Naima:	No	
10 Shabnum:	(with emphasis) No	
11 Maheen:	No this is obtuse. It is obtuse	
12 Naima:	120	
13 Maheen:	(to Samina) accept it accept it. This is 120	
14 Samina:	(puts the protractor on the third angle)	Data item: 1

The above conversation suggests that Shabnum (line 1) thought the angle that Samina measured was of 60 degrees. Samina (line 5) was not clear whether the angle was 120 degrees or 60 degrees and raised the question. Maheen and Naima (line 4) provided their answer and Maheen (line 7) went further to elaborate why the angle was 120 degrees and not 60 degrees. Samina's subsequent response (line 8) exposed her confusion not only about acute and obtuse angles, but also about what constituted the base line in an angle and how to read the protractor. From her statement in line 8, it seemed that in measuring an obtuse angle she first read the protractor from 0 to 90 and from 90 onwards she read the readings given on the top line of the protractor. However, Maheen (line 13) urged Samina to accept her (Maheen's) answer. This urging on Maheen's part had the effect that the group moved on to the next task while Samina was left with confusions regarding the difference between acute and obtuse angles. Questions also remained about Shabnum's understanding of acute and obtuse angles. It was not clear to me whom she addressed in line 10? Was she refusing to accept Maheen's correct explanation or was she addressing Samina? The former would imply that her knowledge of acute and obtuse angles was weak.

Different Answers in Small Group & Whole Class: A pattern was that students provided a different answer to the same task in the small group and in the whole class presentation. The responses in the whole class presentation were richer in terms of quality of thinking, longer, more detailed, with examples. For example, the conversation reported below is when Maheen reported the work her group had done.

1 Maheen:	(points at the vertex of the acute angle already drawn on the black board) This is, this is, when we, err the base line is here so we show the base line here. From here we go this way

(with movement of hands shows a turn from the base line of the angle on the blackboard in an anti clockwise direction)

2 Teacher:	Inaudible

(Maheen picked up the large black board protractor that was used by the teacher in the geometry lessons, and placed it on the angle already drawn on the blackboard by Shabnum).

3 Teacher:	Yes that is right. Should I hold it? (comes forward and holds the protractor leaving Maheen free to give the explanation)

4 Maheen: First of all this is 100 and this is coming inwards.(points to the acute angle already there on the blackboard) This is an acute angle. An acute angle never goes above 90 (paused and looked at the class) and our base line is--- Data item: 2

(A disturbance in the classroom. The teacher asked Maheen to repeat herself)

Maheen's answer to the task was qualitatively different from the answer she had given in lines 2 & 4 (data item 1), during group work. Why was this expectation, not just to perceive why Alina was wrong, but to be able to explain and justify her mistake, not made evident in the group? Did the teacher's request in the plenary session to give the reason for Alina's mistake provide Maheen with the push to articulate the reason that led to Alina's mistake? Yackel (1995, p.148) identified similar occasions in the 'classroom teaching experiment' where children proposed solutions using methods during class discussions that were markedly different from those they developed during small group work. She goes on to show that this difference is due to a reconceptualisation of the problem. She suggests that the reconceptualisation occurred because the whole class session was more than a report back session. It involved discussion and questioning when students did not understand an explanation or a solution method. In the case of my research, students reported back their solutions to the whole class. However, it was rare for other children to challenge or question them (as in Yackel's work). Here, the teacher played the role of challenger and questioner. For example, in the above task 'Alina's mistake' the teacher asked the question 'what was the reason for Alina's mistake?' The question apparently led to Maheen, reconceptualising the question where she elaborated on her earlier answer in line 2 (data item 1), to a more detailed answer in line 1 & 4 (data item 2). An interpretation could be that Maheen articulated differently what she had conceptualised earlier because in line 7 (data item 1) she did give the reason for Alina's mistake. However, this explanation in line 7 was in terms of the mistake Alina might have made in selecting the wrong starting point on the base line. Whereas, in the whole class she extended her explanation of starting the reading from a wrong end of the base line to link it with her knowledge of acute and obtuse angles to explain the reason for Alina's mistake. Yet another interpretation could be that it was the social setting of whole class presentation as opposed to that of small group work that influenced students' responses. It could be that the students did not expect their peers to ask them questions and so did not feel obliged to answer them by providing a rationale for their arguments.

To get a deeper appreciation of the questions raised by these different responses, I followed this issue further in the stimulated recall interview with the students. In the following segment from the audiotape transcript Maheen reports on what she believed were her reasons for the difference in the work in the small group and whole class sessions.

1 Maheen: When I went in front of the whole class so there I had to explain all. I had to explain the mistakes in detail so it came to my mind there

2 AH:	Okay, so how did you know that in front of the class you have to give the details but not so in the group?
3 Maheen :	It is in group also, but I thought that in the group they must have understood
4 AH:	Okay
5 Maheen :	When all (questions) were answered so it was obvious that they must have understood.
6 AH :	Okay so you thought they must have understood. Would any of the group members like to say something about this point? Yes Naima?

Data item: 3

Maheen's statement (line 1) indicates that she believed that in the whole class an explanation of her answer was required. It could be that the teacher's request to provide the reason made explicit the expectation in the problem task that the students were required to give the reason for Alina's mistake and not just point out her mistakes. Maheen's subsequent remark (line 5) implied that she did not provide the explanation in group because she assumed that her peers in the group had understood the task in the worksheet. So, a consideration guiding her responses were the needs of the participants in the social interaction. She judged the need to have been met once all answers had been given. Implicit in Maheen's statement is the message that being a student she is not used to thinking critically about her peers' answers and to taking responsibility for whether the other students have understood. Hence, once she had understood and once all answers had been given she assumed that her peers had also understood. Naima on the other hand gave a slightly different perspective.

1 Naima:	err I think she(Maheen) did not tell us (the explanation) because we were thinking that we only had to measure the angles-----
2 AH:	Okay
3 Naima:	------and we had not thought that sir could ask us these questions. When sir asked these questions, strange possibilities started coming to our mind, that it could be like this, or like this. Like she told.
4 AH:	Okay

Data item: 4

Naima indicated (line 1 and 3) that it was the teacher's question that led to the re-interpretation of the problem so that different possibilities were raised for the solution. Naima's remark confirmed my own interpretation that it was the teacher's questions that led to a change in thinking about the demands of the question. Naima's remark indicated a sophisticated level of thinking as regards the dawning realisation that the ways of working in the classroom were changing. Her statement in line 3, about "these questions" leading to "strange possibilities" suggested that she recognised the teacher's questions as not being ordinary routine questions that she might have been used to encountering in her class. Rather, these were questions that led to strange possibilities, probably, those of a non-numerical verbal answer or a variety of answers to the same task. This was a significant appreciation on Naima's

part (and perhaps the others). It implied that the students saw that the rules governing the classroom participation were changing.

CONCLUDING REFLECTIONS

As students worked in the new social setting what appeared to be missing was some evidence of how the changed practices linked to the teachers' goal of learning mathematics meaningfully. Students appeared to work in groups through interpretations made from the perspective of old norms prevalent in the classroom. Hence, while ways of working in groups came to be taken as shared to some extent, mathematics rationalisation in the group did not. For example, working at specially designed problem tasks in groups and later presenting this work to the whole class appeared to be an accepted part of the classroom practice. Besides these new practices in the classroom that students appeared to regard as normative there were certain old understandings that also appeared to be taken as normative by the students. For example, a norm prevalent in the classroom was that a purpose of the mathematics problem task was to find the right answer. Hence, there appeared to be agreement among the students regarding this purpose when they engaged in mathematics tasks in the classroom. Although, wide experience of prevailing norms in Pakistani classrooms supports this interpretation I refer to these as 'old norms' because I did not have direct evidence of them. My interpretation is that two main factors led to students working through old ways in new settings. First, the teachers' communication of the purpose of change was largely implicit in his practice so that the students were expected to infer the purpose implicit in the change. Second, students' perceptions of mathematics and the purpose of classroom tasks did not support critical reasoning of mathematical ideas. Hence, classroom evidence led me to conclude that students and teacher are both participants in the classroom culture, so that meanings do not come to be taken as shared as a consequence of teacher initiation alone.

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