

THE INFLUENCE OF SOCIAL ISSUES ON THE RE-CONSTRUCTION OF MATHEMATICAL NORMS

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This report centers on the diversity of interpretations of the norms of mathematical practice. In particular, it focuses on one such norm and in how a group of students changes its interpretation after a whole class discussion in which issues of social positioning and power are clearly in play. We argue that an understanding of these social issues is necessary in order to develop learning environments that are more sensitive and hence equitable to all learners.

INTRODUCTION

The frequent lack of cultural and social affinity in the mathematics classroom requires an understanding of what factors lead the learner to re-construct his/her first interpretations of certain norms. The presence of different meanings for the classroom norms and the obstacles to negotiate these meanings are to be taken into account when analysing shifts in students' participation trajectories. An understanding of these shifts in participation seems necessary towards the development of equitable learning environments. In this report we center on one episode from a recent research (Planas, 2001) that illustrates the diversity of interpretations for the norm of the "context of a problem" in a problem-solving situation. Relevant excerpts from classroom transcripts will be used to uncover the construction of social meanings behind the multiple ways of interpreting this norm.

THEORETICAL FRAMEWORK

The idea of a system of meanings has been broadly used to characterize the so-called 'culture of the mathematics classroom' (Voigt, 1998). In particular, the normative meanings, that is the obligations and rules of how to act, conform an important part of this culture. The research of Yackel and Cobb (1996) has pointed to the diversity of interpretations of classroom norms and its impact on the teaching and learning of mathematics. However, recently some researchers have

questioned an approach essentially based on the cultural issues of the classroom. There has been a call to emphasize the 'social' in the socio-cultural paradigm of research in mathematics education (see, e.g., Lerman, 2000). As it is argued by Morgan (1998), the study of the norms from a socio-critical perspective is still in its beginning stages. There is a need to look at how norms can be interpreted differently as well as how different interpretations may be related to different values given to participants (Abreu, 2000). Abreu states that the tasks which need to be solved in the classroom can be made difficult or easy depending on how participants interpret the legitimate norms.

RESEARCH QUESTIONS AND METHOD

In this report we address the following questions:

- a) Given a norm, can it be interpreted differently by the various participants?
- b) How do different interpretations of a norm influence each other?

The research was conducted in three classes located in three urban high schools in Barcelona, Spain¹. In these schools there is an increasing number of students socially at risk. They are students belonging to ethnic minorities, first or second generation immigrants. Many local families do not want their children to attend such schools due to the high percentage of immigrant population from Magreb, Pakistan, India, South America as well as the local gypsy students. The enrollment in these schools is therefore lower than usual.

A total of 36 lessons were videotaped, tape-recorded and transcribed. Non-participant observer methods, field notes and individual interviews of students were also used. There was a triangulation of methods for obtaining the data and a triangulation of perspectives in analyzing them. The video-recordings and the interviews' transcripts were discussed by the teacher, the observer and an external researcher in regular meetings. The lesson we document here is embedded in a problem-solving unit. The nine students were asked to work in small groups. This

was followed by a whole class discussion in which the three groups shared their approaches. They were given the following problem:

Here you have the population and area of two neighbourhoods in your town. Discuss in which of these places people live more spaciouly:*

<i>Neighbourhood 1 (N1)</i>	<i>Neighbourhood 2 (N2)</i>
135,570 inhabitants	297,930 inhabitants
14 km ²	3 km ²

(*Neighbourhoods 1 and 2 were given their real names and were known to the students)

RE-CONSTRUCTIONS OF ONE MATHEMATICAL NORM

The question ‘where do they live more spaciouly’ created important differences in the interpretation of what was required to solve the problem. Some students decided that there were data missing while some others were comfortable with solving the problem as it was. This led to different interpretations of the norm “context of the problem”. One group (GA) took the problem as a typical school task and solved it in an algorithmic way. They divided the population by the area and reached the conclusion that “they live 10.25 times better at N1 than at N2.” Clara, this group’s leader said, “we’ve solved similar problems before; you have to make some divisions, that’s all. The more neighbourhoods, the more divisions.”

Another group (GB) also used the concept of density. They divided the population by the area but then expressed some concern as to whether the answers were reasonable. For example for Neighbourhood 1, the division led to 9,683.57 persons per square kilometer. They agreed that it should be 9,684 p/km² because “you cannot have 0.57 people.” This group also concluded that people would “live much better in N1” but most of their time on the problem was spent on their concern on what to do with these decimal answers for people.

The third group (GC) engaged in an animated discussion about the wording of the problem. Their concern centered on a need to know more about the living

conditions in the two neighbourhoods. Albert, one of the group members, said, “well, it doesn’t say anything about the people. We need to know if they live in flats or houses. Some of them live very spaciouly and some of them live very badly, right?” And Lordwin, another student in this group said “it depends on how many children there are in each family, then we write the number of children there, we’d better write it again.” Group C took their task as that of rewriting the problem to fit more closely their own lived experience.

All the groups offered reasonable explanations without making incorrect uses of mathematical operations or procedures. The difference was in their understanding of the norm of the context suggested by the problem. Groups A and B followed academic procedures and used the concept of density. Neither of these groups questioned the wording of the problem. Group A took it as typical school problem, while Group B showed concern for the real-world context and was aware that one cannot have decimal numbers as answers about people. Group C took the problem as a real situation for which more information was needed. These neighbourhoods were real to them. Their approach to the task reminds us of critical pedagogy contexts in which students would be encouraged to use mathematics to make sense of their world (Skovsmose, 1994). The teacher in this case, however, did not seem interested in pursuing such an approach. The other two groups were surprised at Group C’s questioning of the problem statement. The whole class discussion that followed the group work shows how the social positioning of the students may have influenced their subsequent approach to the problem.

As each group faced comments from the class and was confronted with alternative norms, first approaches were reaffirmed or changed. This allows us to study not only the various interpretations of the same norm but also how dynamic these interpretations are. We only look at the changes that occurred during the whole class discussion, though other possible meanings for the appropriate context of the problem may have emerged in the small groups. Groups A and B insisted on their

first approach. The notion of density was still considered the key factor. Group C's change was highly significant. The change was apparently radical but we wonder whether this change was more the result of pressure from members of the other groups than from having reached internal conviction that the other approaches were more valid. Group C ended up rewriting the problem as follows:

In N1, houses have gardens and swimming pools. Families have one children. In N2, there are small houses and skyscrapers. Families have many children. But the important thing is that there are 135,570 people and 14 km² in N1, and 297,930 people and 3 km² in N2. Where do people live more spaciouly?

The first part of the problem (until the sentence starting with "But") is what they had at the end of their initial group work. The second part is what they added to conform to the whole-class discussion. The critical context was somehow combined with the school context. All throughout the discussion, however, comments from this group indicate that they were not really satisfied with their revised approach. For example, Lordwin kept on mumbling that they needed to know how many children there were, and said, "now you have a number but you don't know anything about the kind of houses they live." An indication of the pressure they felt is given by Albert's comment "if we don't do as we are told, the teacher is going to get angry." The group was reluctant to give in, "we don't need to cross out our problem, do we? Just say ours is the same as Clara's (Group A leader)." In the next section we focus on the case of Albert, the leader of group C, to explore the possible role of social positioning in classroom dynamics.

EXPLORING ISSUES OF SOCIAL POSITIONING

Why did group C change their initial approach to the problem and ended up with one that somehow conformed to the expected school context? We argue that an understanding of the social positioning of students in the class may shed some light on this question. The other two groups' reluctance to accept Group C's strategy, and this group's decision to change their approach must be analysed taking into account the valorizations that were made explicit between the first attempts and the

re-constructed contexts that appeared some minutes later. This expresses not only a normative difference between the three groups, but also interpersonal relations of power between individuals. For example, if we focus on Albert, we see a student who seems concerned about the teacher's reaction ("if we don't do as we are told, the teacher is going to get angry"). While the teacher showed concern about Albert's involvement, he was also somewhat sarcastic in his reaction to that group's attempt to rewrite the problem ("how cute of you..."). We also see how other students reacted rather negatively to Albert's suggestions. For example, Clara (GA) got very upset at him, "if you don't know what to answer, just shut up!", and Bernat (GB) was extremely concerned about Albert making them waste time. At a certain point, many participants were frowning at Albert. Table 1 summarizes some of the negative valorizations given to this student as well as his reaction.

Teacher: "How cute of you! If someday I put you in charge of writing a problem, please, remind me of your sense of humour... You are confused about what it means to think about a problem, but don't worry, don't be discouraged about the problem, it's very normal that you find it difficult"	Clara: "You must be joking as usual! Ask for help if you have no idea of how to solve the problem... if you don't know what to answer, just shut up!"	Bernat: "Don't make us waste our time! Stop clowning around! You are always trying to get the teacher's attention by saying silly things! This problem has nothing to do with swimming pools and gardens!"
<p><u>Albert's responses:</u> (utterances and speech defects such as stuttering)</p> <p>(to Teacher) (stumbling) "We didn't know it was forbidden to improve the problem, we thought we had to understand the situation, if...if we had known it was forb... forbidden, we would have never tried to change it, teacher, we promise"</p> <p>(to Clara) (without looking at her eyes) "I'm not joking, I'm saying the same as you are but in a diff... different way"</p> <p>(to Bernat) (puzzled) "I'm not a clown, you are always putting... putting me down! We didn't mean to make you waste your time, don't be so hard on me!"</p>		

Table 1: Some negative valorizations given to Albert

Albert is a 16 years old student who usually interprets the problems by himself without having to wait for the teacher to interpret them. His responses during an interview that same day seem to indicate that he was not really willing to change

his group's approach but felt like he had no choice. The following excerpt from the interview shows to what extent he is aware of issues of power and status that took place in the class:

Interviewer: What do you think about the other groups' strategies?

Albert: What Clara and Bernat said is okay, but what I said is also okay.

Interviewer: Why do you think they didn't like your ideas?

Albert: I don't like looking for the most difficult way to solve a problem and they do.

Interviewer: What about the teacher? He didn't seem to like your ideas either...

Albert: I don't care if they [Clara and Bernat] know a lot of math because I also know a lot of things. But they don't want to know what I know... and neither does the teacher.

(...)

Interviewer: Did you like what they told you?

Albert: I'm sure they wouldn't talk to me like this if I was not repeating this grade for the second time.

Different people in front of similar circumstances may feel in very different ways. Individual differences are then an important issue in the understanding of certain re-construction processes of the mathematical norms. The same circumstances can encourage some students and discourage others. In other words, the processes of re-constructing normative meanings refer to both the characteristics of the context and those of the individual. Students like Albert easily feel upset and do not seem to struggle with such difficulties. During the interview, he reflects on how his role in the classroom would be different if he was not repeating the year. The data obtained reveals that we need a better understanding of the interaction of cultural, social and emotional factors.

When students participate in the mathematics classroom and get involved in certain practices, they do so in ways that other participants may perceive as appropriate or inappropriate. To what extent these perceptions of correctness influence the ongoing patterns of participation? Do some students have less opportunities to

negotiate their meanings and patterns of participation? Do unequal opportunities have to do with status as well as with the distance to the legitimate norms? Although the results of the study do not provide complete answers to these questions, they suggest the kind of research that is needed. Cases like that of Albert illustrate the need to go deeper into the 'social'. We strongly believe that if we want any educational act to be positive both for the individuals and their communities, it will be necessary to consider the impact of the classroom social context on the learning processes.

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