Mathematical and Pedagogical Knowledge of Pre-and In-Service Elementary Teachers Before and After Experience In Proportional Reasoning Activities

David Ben-Chaim
Oranim, University of Haifa

Bat-Sheva Ilany
Beit- Berl College

Yaffa Keret
Israeli Science Teaching Center

Abstract - *In this research, we investigated the cognitive domain of proportional reasoning mathematical knowledge of pre-and in-service elementary mathematics teachers – including its three components: the intuitive' the algorithmic and the formal. This investigation was carried out by assessment of change in participants' procedures while they were presenting possible solutions to authentic problems and revealing their reasoning in dealing with ratio and proportion problems. The affective/behavioral domain was also investigated by measuring change in the attitudes toward teaching proportional reasoning following exposure to and experience of investigative proportional reasoning activities.*

Theoretical Background

Recent research findings all over the world have indicated many gaps in the content knowledge of pre-and in-service teachers in mathematical subjects taught in the elementary school, including the topics of ratio and proportion. Frequently, existing knowledge is technical, schematic, unconnected and incoherent and, as a result, difficulties arise which are evidence of the pre-service teachers' lack of understanding of mathematical concepts and feeling that they are incapable both, coping with the material and teaching it. This is usually based upon a negative attitude toward the specific material (Keret, 1999; Fischbein, Jehiam and Cohen, 1994; Tirosh and Graeber, 1990).

A review of the literature about proportional reasoning by Tourniaire and Pulos (1985) indicates a number of factors relating to the context and number structure of proportional reasoning problems that are responsible for much of the variability and difficulties in the performance of both students and teachers. The sources of the difficulties are mainly cognitive and they are consequences of the operational scheme's second order status – meaning that to understand proportion one needs to make a comparison between two rates, each of which needs to be able to coordinate between two variables (Inhelder & Piaget, 1958).

Findings of many research studies indicate that appropriate practice, which includes realistic contextual problems that are interesting and challenging and which encourage students to construct their own conceptual and procedural knowledge, can help them build their proportional scheme (Harel and Confrey, 1994). The American Connected Mathematics Project (CMP) has developed learning material which incorporate these ideas for grades 6-8. The findings indicate that such students are capable of developing their own repertoire of sense-making tools to help them produce solutions and explanations (Ben-Chaim et al., 1996a). Findings of research studies with pre-service mathematics teachers indicate similar results (Keret, 1999).

The conclusion reached from these research studies is that, in order to improve the situation, we need to implement the research findings in teaching ratio and proportion topics in the elementary school and, in parallel, to deal with these topics differently in in-service teachers education. The in-service training should include three central components: the cognitive component which includes the mathematical content knowledge and the pedagogical-didactical knowledge; the affective component which includes attitudes, beliefs and feelings of being capable (that may prevent or encourage the readiness to teach the subject); and the component of teacher behavior which one can see in the teacher's readiness to cope with constructing teaching units and planning the teaching of the subject. In this research, after exposing the pre-and in-service elementary mathematics teachers to investigative and authentic activities in ratio and proportion, we followed the way these teachers changed in regard to the above three components.

Methodology

Sample: 49 pre-service teachers from Israeli teachers colleges as part of their training to teach mathematics in elementary schools and 14 in-service mathematics teachers in elementary schools. The study was conducted during the academic year 2000/2001.

Research Instruments: 1. A proportional reasoning questionnaire that was administered twice: first at the outset of the course and second after gaining experience with investigative activities (19 activities in 10-12 teaching sessions). There were 3 versions of the questionnaires each consisted four parts:

Five rate and density problems. Problems 1&2 deal with unit price- where problem 1 needs numerical comparison and problem 2 has missing value. Problems 3&4 deal with proportional relations between distance, time and velocity – both using numerical comparison and the main difference between them is in their numerical structure. One problem includes only integers and the other includes fractions and decimals to present the factor of time. The fifth problem deals with information of population density, also of numerical comparison type, but with relatively larger numbers. In all numerical comparison problems, the ratios are not equal and in this case the problems are generally considered to be more difficult compared to those with equal ratios (Karplus et al., 1983a, b).

Five ratio problems. The first two problems deal with finding the ratios between the sizes of two groups of children. There is no need to solve the problems, but to specify as many different ways as possible to find the ratio between the sizes of the groups. The responses for these problems will be analyzed in a separate section. The third problem deals with comparison between two ratios. The fourth problem deals with the comparison made between different types of ratio representation. In the fifth problem there is a need to find the whole value by using a given ratio.

Five scaling problems. All five problems relate to finding a ratio and its application after enlarging or reducing the dimensions of pictures. Each problem refers to a different aspect; the first one deals with finding the scaling factor; the second one deals with comparing two ratios; the third one is of a missing value type; in the fourth

one there is a need to relate to a quadratic enlargement (areas); and the fifth problem relates to a situation with 2 consecutive enlargements.

Six exercises that include work with fractions. This part consists of six exercises with plain numbers/fractions/decimals similar to those included in the rate, ratio and scaling problems.

The questionnaires are based on versions administered to large populations in U.S.A. and that were translated into Hebrew (Ben-Chaim et al., 1998) with some correspondence of units, and local situations.

2. Attitude questionnaire. The attitude questionnaire includes 22 items and 3 open questions (more details are given in the results section).

3. Authentic investigative proportional reasoning activities. Nineteen activities were developed in order to assess the influence of exposing pre-service teachers to authentic investigative proportional reasoning activities. The development of the activities was based on research findings from around the world (Ben-Chaim et al., 1998; Keret, 1999). The activities were conducted in one of the pre-service mathematics education courses in each of the two participating teachers colleges.

4. Observations. These were aimed at the formative evaluation of the activities created and at a follow up of the learners' procedures.

5. Personal Interviews. These aimed at in-depth and comprehensive examinations of the questionnaire's findings. For this purpose, a representative sample of 14 participants was interviewed. The interviews served as a validation of the findings achieved from the observations and the questionnaires.

Results and Discussion

The layout of the research is one group pre-test – post-test. The questionnaires were administered before and after conducting the ratio and proportional reasoning activities. A special rating form was created to analyze and score the students' responses. Three major response categories were identified and each of them was accordingly subdivided into subcategories (see Table 1).

As indicated before, six exercises in fractions were included to examine if the students had difficulties in comparing fractions and/or acting with naked numbers/fractions similar to those that appear within the verbal problems. In this part, there were no differences found between pre and post testing results. The students had no special difficulties in solving the fractions exercises.

The following report relates to the performance of the pre-service teachers on 15 problems, before and after exposing them to a process of solving authentic problems in ratio and proportion.

It should be noted that the assessment tasks are different from those that appear in standard tests. They stem from familiar situations such as: buying soft drinks, riding bicycles, population density, bussing to school and a visit to the photo shop.

Moreover, the problems and the situations are different from those presented within the investigative activities during the teaching sessions.

Table 1: Overall Pre-Post Results on 13 Problems (5 Rate, 3 Ratio, 5 Scaling) N = 32 Pre-service Elementary Mathematics Teachers

		Correct answer			Incorrect answer			Blank
		only the correct answer	correct support work	incorrect support work	only incorrect answer	correct thinking but wrong conclusion	correct support work	
Rate	Pre	1	56	3	2	11	8	19
	post	-	86	3	-	9	1	1
Ratio	Pre	-	38	10	-	6	7	39
	post	1	81	6	-	10	-	2
Scaling	Pre	1	51	3	-	8	18	19
	post	1	70	3	-	15	8	3

* **All of the numbers in the table are percentages.**

Table 1 presents the overall results of the pre-service teachers' performance, before and after exposure to the authentic activities. It can be seen that, before exposure to the activities, 56% (Rate), 38% (Ratio) and 51% (Scaling) of the pre-service teachers correctly responded with correct support work. After the exposure to the activities, the performance of the students improved dramatically, to 86% (Rate), 81% (Ratio) and 70% (Scaling) of the pre-service teachers answering correctly with full support work. Furthermore before the experience with the proportional reasoning activities, 27% (Rate), 46% (Ratio) and 37% (Scaling) of the pre-service teachers did not attempt the problems or responded incorrectly using incorrect thinking. In contrast, the percentages after the exposure to the activities were only 2% (Rate), 2% (Ratio) and 11% (Scaling). The majority of the participants attempted to answer the problems with support work, both before and after the exposure to the activities. If we ignore the subcategories "only correct answer" and "only incorrect answer" and the category "Blank" then before the exposure: 78%, 61% and 80% of the participants and after the exposure: 99%, 97% and 96% of the participants provided support work for their answers. Hence, when examinees are asked to "show your work", or "how do you know", or "explain" they do add written support for their answers. Yet, the quality of writing is important. Analysis of the written support work clearly showed that after the exposure – during which the participants were guided to explain and discuss their reasoning and ideas, the arguments were more detailed, clearer and of a better quality.

The subcategory of "correct answer with incorrect support work" showed up before exposure to the activities 3% (Rate), 10% (Ratio) and 3% (Scaling). Thus, on a typical mathematics exam that does not ask for justification of answers, such

examinees would be marked correct and their misconceptions would be unrecognized and thus uncorrected. On the other hand, within the subcategory of "incorrect answer with partial understanding/thinking" there were 11% (Rate), 6% (Ratio) and 5% (Scaling) before the exposure, and 9% (Rate), 10% (Ratio) and 10% (Scaling) after the exposure. This subcategory of response often occurred when the problem called for calculations followed by reasoning, as is the case for most of the problems presented. The examinees can think correctly, but draw the calculations incorrectly, or they might get the correct number and information, but draw an incorrect conclusion about measurement units. It can be assumed for students in this subcategory that they are beginning to understand the content, but their understanding is still shaky. Certainly in this case, complementary teaching/learning is needed. All of the above is true and characteristic for each of the problems.

As indicated for the first two ratio problems, the participants were asked to specify different ways to present ratios and compare between them. The results show, that, before the exposure 32% (prob.1) and 50% (prob.2) of the pre-service teachers did not succeed in finding even one way to present ratios for problems 1 and 2 compared to only 3% (prob. 1) and 3% (prob. 2) after the exposure. Before the exposure, 68% (prob. 1) and 50% (prob. 2) of the examinees successfully solved problems 1 and 2 in at least one way; after the exposure there was a visible gain in success, while 97% of the participants correctly solved the problems using at least one way with the majority of them using more than two different ways.

Regarding scaling problems, the findings indicate that the pre-service mathematics teachers had difficulties especially in solving problem # 4 in which they were required to deal with quadratic enlargement – area enlargement. Before and after the exposure to the activities, the rate of success was 40%. Obviously, the quadratic enlargement here was not assimilated successfully. This is in agreement with findings of previous studies regarding pre-service teachers (Keret, 1998), as well as elementary school students (Ben-Chaim et al., 1996b). In contrast, the performance on the fifth scaling problem was much better one – 65.5% before the exposure and 80% after. In this problem, the examinees were asked to relate to double enlargement. This problem was presented in two versions: one version included a figure of segments (no numbers) and the problem was to assess the overall enlargement; the second version included a description of a machine that enlarges double and, then triples the size and the problem was to find the resulting enlargement. In both versions the rate of success was almost the same.

As indicated before, the attitudes toward the topic of ratio and proportion were also evaluated in order to detect changes that took place between pre and post exposure to the activities. The assessment was conducted by a Likert attitude questionnaire with a scale of 1-5. The questionnaire included items dealing with attitudes toward: mathematics teaching in general, confidence in the ability to deal with ratio and proportion, difficulties in teaching ratio and proportion, and the importance of teaching ratio and proportion.

Table 2 indicates that pre-service teachers' attitudes toward teaching mathematics in general, before the exposure to activities, are quite positive - they like and enjoy teaching mathematics. Following the exposure, they were even more positive. The standard deviation measure of post testing indicated that all the participants enjoyed teaching mathematics. Another finding shows that the pre-service teachers are aware of the need for mathematics teacher to have much more content knowledge than the material taught in school. This finding is supported by the interviews, as one of the interviewees noted: " I would like to go deeper, there are additional subjects to learn, such as inverse ratio; in my opinion, teachers should know much more than pupils in order to teach". A positive change occurred regarding the feeling of being capable of dealing with ratio and proportion. After the exposure the participants felt that they were more confident in their capability to teach the subject and felt capable of planning and creating relevant mathematics activities.

Table 2: Summary of Attitudes Toward Ratio and Proportion: Scale (1-5)

# Of Items		Mean Before Exposure N = 49	Mean After Exposure N = 12
4	**Items relating to attitudes toward teaching mathematics in general**	4.127	4.777
7	**Items relating to confidence in ability to deal with ratio and proportion**	2.948	3.746
3	**Items relating to attitudes toward difficulties in teaching ratio and proportion**	2.993	3.666
6	**Items relating to attitudes toward the importance of teaching ratio and proportion**	3.691	4.241

To those who were exposed to the activities, teaching ratio and proportion seemed to be more complicated than before the exposure. The interviews strengthened this finding, for example: " I thought that the topic of ratio and proportion was easy and that I knew how to teach it, but today after I have learned, I realize that it is very complicated and that I still lack of knowledge"; or another quotation: "It exposed me to a topic about which I did not know how much I don't know and how unready I am to teach it".

Another change that occurred following the exposure to ratio and proportion activities is connected to the need to include ratio and proportion as part of pre- and in-service teacher training. Before the exposure, not all the examinees thought it was very important to include a course related to ratio and proportion; however, after the exposure, almost all the participants thought that it was very important to include it. As one of the interviewees said: "It is very important to teach this topic of ratio and proportion in college since it is the A B C of mathematics. If you are not familiar with this topic, it is impossible to teach the other topics". These findings indicate a significant improvement in the participants' attitudes toward the overall components and aspects of ratio and proportion topics. The observations during teaching the

activities and the interviews conducted after the teaching, clearly indicated that the teaching style caused the participants to construct their understanding of the concepts of ratio and proportion through an enjoyable and efficient process; as one said: "Before, I learned the topic by formula and I could never remember which formula to use. This time the inquiry enabled me to develop it in my own way regarding the topics of ratio and proportion ...". Another one related to the authenticity of the activities: "The topic of the presented problems was new to me and topic that I did not like, but it brought it closer to me. I especially liked the recipes enlargement and reduction of a drawing, and the cutest of all was solving the detective's puzzle – such activities raise curiosity and after you solve them, you suddenly know that you have advanced in learning ratios. It is wonderful". Another one related to teaching in the elementary school: "The activities are close to our world and it is possible to also fit them into the world of children and I am sure that they will also enjoy solving them".

Conclusions

The findings of this study indicate several crucial points:

1. Authentic activities are essential for teaching the topic of ratio and proportion in pre-and in-service elementary teacher education.
2. The type of activities developed and used in this study were found to be suitable, effective and contributory to pre-and in-service teachers for understanding the topics of ratio and proportion.
3. After exposure to the investigative ratio and proportion activities, the pre-and in-service teachers were more successful in solving the problems presented to them in the knowledge questionnaire. It is important to note that the problems and situations included in the questionnaires were different from those presented during the teaching cycle.
4. The majority of the students provided support work and explanations for their solutions before and after the teaching. Thus, when examinees are asked to add arguments and explanations most of them generally meet the requirement.
5. After exposure to the activities, the participants succeeded in finding more methods to solve the problems and their explanations for their methods were of a better quality.
6. The request to add support work is justified, since a correct answer with explanations, frequently identifies an examinee who has a lack of understanding and who would otherwise be considered to be successful. Furthermore, an incorrect answer accompanied by support work can reveal an examinee that already has partial or unstable understanding and just needs complementary tutoring.
7. The pre-and in-service teachers do not have difficulties in solving exercises with plain numbers/fractions and decimals.

8. Following learning ratio and proportion through the authentic activities, the pre- and in-service teachers improved their attitudes toward mathematics in general and all the components and aspects of ratio and proportion.

In summary, the concepts of ratio and proportion are basic concepts in mathematics. In addition they are important for other disciplines in which different phenomena are described as ratio and proportions. Conceptualization of the ratio and proportion concepts and the ability to implement them in problem solving arising from different disciplines can assist learners to construct their mathematical knowledge. More important however is that it will lead to the development of the ability to apply the proportion scheme – meaning proportional reasoning, something that is vital for developing analytical and mathematical thinking.

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