DIDACTICAL REFLECTIONS ON PROPORTIONALITY IN THE CABRI ENVIRONMENT BASED ON A PREVIOUS EXPERIENCE WITH BASIC EDUCATION STUDENTS

Elena Fabiola Ruiz Ledesma
Cinvestav del IPN, Mexico
leslieruiz2000@hotmail.com

José Luis Lupiáñez Gómez
Universidad de Cantabria, Spain
lupi@matesco.unican.es

Marta Valdermoros Alvarez
Cinvestav del IPN, Mexico
mvaldemo@mail.cinvestav.mx

Abstract

In this paper, the authors propose constructions in the Cabri-Géomètre environment to approach activities of proportionality. The proposal builds upon a prior experience with students in México who had completed elementary education. Those students worked on the topics of ratio and proportion through teaching models that had been designed based on recognizing some cognitive components. We attempt to translate our reflections to the role that dynamical geometry can play in the teaching of notions such as ratio and proportion.

Introduction: Proportionality in basic education research

The themes of ratio and proportion are fundamental in school teaching. Hence, these topics have been subjected to study by different educational researchers. In spite of advances in this area of educational research, it is necessary to go deeply into its study, especially in relation to the implementation of new technologies in the mathematics classroom.

Piaget explored proportional thinking. He conducted longitudinal studies of the stages of cognitive development through the stage of *formal operations*. Specifically, Piaget's findings led to understanding the foundations of the treatment of the ratio and proportion themes. Piaget (1978) pointed out that individuals could construct the scheme of qualitative proportionality when they understand that an increment in an independent variable yields the same result as a decrement in the dependent variable. That is, when subjects realize that an element of counterbalance is required.

Other research studies focused on the needs of instruction. For instance, Hart (1988) pointed out that proportional thinking is present in the adolescent and that its most advanced level is accomplished once the adolescent has constructed certain concepts. For Hart, some levels of generalization, such as the handling of ratios or ways of generating equivalences, occur when multiplicative strategies are used. Moreover, Freudenthal (1983) and Streefland (1984, 1990 and 1991) combined didactical aspects with the mathematical reflection about ratio and proportion. These researcher reports allow us to contemplate their contributions to these fields. Based on the cited studies, it is possible to create an instruction design under a constructivist approach.

Proportionality and curriculum: The contribution of the new technologies

It is necessary to analyze and classify the areas of knowledge, mathematical and transversal to the discipline, on which the understanding of the concepts of ratio and proportion exercises a direct or indirect influence. Thus, it is very important to focus the theoretical reflections about those concepts from a curricular and formative viewpoint. In the light of that type of analysis, we can justify the importance of teaching the topics of ratio and proportion by reasons such as the following (Fiol & Fortuny, 1990):

1. From the intermediate courses of elementary education (grades 4-6) throughout lower secondary education (grades 7-9), proportionality is the core for the unification of trends of notions in the learning of mathematics. Such notions include, among others: fraction and rational number, changing units of measurement and scale, distribution problems, percents, probabilities, graphs of linear functions, geometric theorems and similarity of figures, and the number π and the golden ratio.
2. Many concepts in physics and chemistry are associated to relations of proportionality. These concepts include nouns such as velocity, acceleration, density, and dilatation, and the formulation of laws such as Ohm's law, Hooke's law, or Proust's law.
3. The notion of proportionality is also present at the end of elementary school in the programs of social sciences or geography under the form of population density, birthrate, reading of maps, and so on.
4. Proportionality also appears, as we have cited above, outside the realm of the sciences in specialties such as genetic epistemology (Inhelder & Piaget, 1955) and in the studies related to cognitive development.

In this same trend, other researchers such as Lesh, Post, and Behr (1988) emphasized that the teaching of ratio and proportion starts in elementary school and serves as a basis for the understanding of other concepts. Very often, it is necessary to resort to the recognition of similar patterns or structural similarity in different situations. Many of those later concepts are critical in fundamental knowledge of the sciences and mathematics, as well as for the solution of daily life problems.

The increment of the number of educative projects that include the use of a technological component is impressive. This situation motivates teachers and researchers to include new technological devices in their teaching activities. Hence, the translation of our reflection to the role that these devices can play in the teaching of notions such as ratio and proportion is immediate (Lupiáñez & Moreno, 2001). In this paper, we introduce some activities in the Cabri-Géomètre environment of dynamical geometry. The designing of these activities was based on observations in a previous experience (which is described below) with students of elementary education.

A previous experience with elementary school students

The didactical program of the prior educational experiment initiated in January and finished in June of 2000. The problem situations that formed part of this proposal were associated to teaching model[1] (Ruiz, 2000), which were experimented at different moments, as required by the development itself of the research study

[1] Figueras, Filloy, & Valdemoros (1987). A teaching model comprises meanings—in the technical as well as in common language—, didactical treatments, specific modes of representation, and the relationships among them.

implementation. From two to seven working sessions were carried out per month, depending on the progress shown by the students and on factors external to the program implemented by the researcher. Each session lasted from one hour and a half to three hours; including the complementary activities, 25 sessions were necessary. The scheduling of the sessions is shown in Table 1 below.

Table 1. Schedule for the work sessions of the teaching program.

Month	Model(s) used	No. of sessions per model or complementary activity	Total of sessions per month
January	"Designing dance halls" "The world of Snow White and the seven dwarfs"	2 2	4
February	Complementary activity 1: "Size and shape" "The world of Snow White and the seven dwarfs" Complementary activity 2: "Double and half"	1 5 1	7
March	"Making photo frames" "The great problem of the footprint" Complementary activity 3: "Equivalent fractions"	2 1 1	4
April	"Establishing proportions" "Making photo frames" and "The world of Snow White and the seven dwarfs" Complementary activity 4: "Review of the concept of ratio"	3 2	5
May	"Soccer competition" "Building your own football soccer field"	2 1	3
June	"The picture of your team"	2	2
Total of sessions		25	25

The teaching proposal departed from qualitative aspects. Qualitative aspects are those based on linguistic recognitions for the elaboration of comparison categories, such as "big" or "small." The intuitive component, which is the piece of information based strongly on experience, on practice, on the senses, is a part of the qualitative aspects. This approach was used so that the students could give meaning or signification to the concepts of ratio and proportion, and could make the transition from the qualitative thinking to the quantitative thinking gradually, without completely abandoning the qualitative aspects. This transition process was traversed by means of different situations and at different moments. Similarly, Kieren (1988) emphasized the transition from the concrete to the abstract: he pointed out that the intuitive component is not completely abandoned.

After that, an ordering appeared: when students made comparisons, they used the phrases "greater than" and "less than." In the matter, Piaget (1978) pointed out that when the idea of order appears in the transition from the qualitative to the quantitative thinking, the concept of quantity is not yet present—what he called *intensive quantifications.* Later, the pupils used measurements when doing comparisons. First, they contrasted parts of the body with objects, they superimposed one figure over another, and then they used a measuring instrument (conventional or unconventional). In terms of Freudenthal (1983), comparers can be classified in two modalities: direct or indirect. The direct modality of comparing occurs when an object is superimposed over other object, while the indirect modality occurs when there are two objects (A and B) and a third element (C) to make the comparison.

Thus, a great step forward to come close to quantification was that students started to measure using natural numbers when doing comparisons. Measuring became a significant prerequisite for the use of multiplicative operators.

At a later moment in the development of the didactical experience, students established relations among magnitudes, worked in the set of natural numbers, and used fractional expressions as well. Thus, they started to work in the field of rational numbers in an elementary way. Eventually, students characterized "ratio" as the relation between two quantities, expressed as the quotient of one divided by the other, and "proportion," as the equality between two ratios. This outcome coincides with the definitions given by Hart (1988).

Independently of the model under treatment, we gave special attention to the different modes of representation that students used when they were confronted to the proposed situations. We attempted that students could use any of the following three modes of representation indistinctively: pictorial, tabular, and numerical. For the purpose of illustration, in the following section we describe one of the models used; it was composed of two activities and was developed through several sessions with a sixth grade class of elementary education.

Activities of the model: "Soccer competition" and "Building your own football soccer field"

The first activity is to determine the ratios of different measurements of three football soccer fields in relation to the authorized measurements of a football soccer field, some values being given. That is, in the activity of the *Soccer competition,* students have to find out the measurements of three soccer fields given the length of one of them and the measurements for an official one, so that the four fields are proportional to each other.

A maxima problem is posed in the second activity: to construct the greatest possible soccer field in the playground of the students' school so that it is proportional to an official soccer field. For this activity, *Building your own football soccer field,* students know the measurements of an official football soccer field and the actual measurements of the playground in their school.

In the two tasks, students determine proportions by working with the ratios they find. The dynamics of work varies as each activity develops: students are required to work individually or collectively. However, they always have worksheets where to make computations and write down their results.

The analysis of the activities allowed observing that the students could remember the authorized length and width measurements of a soccer field. They used a process of halving these measurements, wrote the obtained quantities on the corresponding sides of rectangles they had drawn, and tabulated those same values. By contrasting with the measurements of their playground, they realized that it was possible to lay out on it different soccer fields proportional to an official one. Then they read the directions for this activity carefully and noticed that in addition to being proportional to an official soccer field, the one to be built in the playground of their school should be the greatest possible. They knew that the required values could not be greater than the measurements of the playground. After several trials, they found out that the greatest soccer field would be one with dimensions equal to the fifth part of an official one.

For their arguments, students made use of the three registers of representation mentioned above: they placed the obtained data in tabular form and drew rectangles. Students asserted that the rectangles they drew were similar because the ratios of their measurements were proportional. They handled ratio as a relation between magnitudes, using fractional expressions, and handled proportions as equality relations of ratios. They also used equivalent fractions to determine proportionality. They did not use the rule of 3: this rule was not taught by the researcher during the didactical sequence, neither by the teacher because it is not included in the official program of school mathematics.

Activities of proportionality in Cabri-Géomètre

Students in the last grades of elementary school have potentiality for the acquisition and handling of the notions of ratio and proportion, in a context coherent with their prior mathematical knowledge. This potentiality showed clearly in the light of results obtained throughout the development of the teaching proposal. Our proposal provides the setting for the activities described in the next paragraphs. Our aim is that students complement their learning with the observation and manipulation of representations made available by the dynamical geometry environment of Cabri-Géomètre. However, some of the tasks are better adapted for a continuation of work at the beginning of lower secondary education (grades 7-9), not for the level of elementary education (grades 1-6). We assume that students in both levels of education have acquired the basic skills for handling this software.

Recognition of patterns. In this first activity, students are shown a family of rectangles. These rectangles can be moved and superposed without deformation in the screen. Students are asked to pair rectangles of the "same shape," no matter if these are of different size. Although this activity is clearly related to similarity of figures, the term "similarity" is not used.

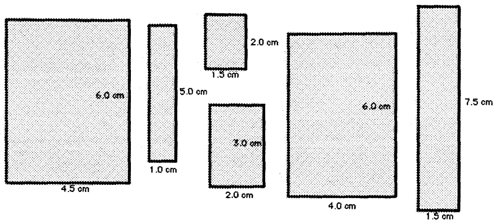

Figure 1.

The objective is that students arrange the rectangles according to their visual perception, and once they have done this, the measurements of the rectangles can be shown. After discussing whether there is a relation in the simple cases, such a relation can be generalized to all cases.

First measures. Computation of ratios. The objective of this activity is that students get to know the basic tools of the Cabri-Géomètre software and use them to work with ratios. For this purpose, students can draw line segments, measure them, and find out the ratios between their dimensions. Starting with these simple manipulations, important aspects of the system are brought into operation: taking measurements, manipulating units of measurement, and the possibilities of computation.

Construction of proportional and unproportional figures. The purpose of this activity is to introduce a criterion for the construction of rectangles proportional to each other. The rectangles must be constructed so that they share one vertex and their diagonals through it lie on the same straight line (Figure 2). If we move the vertex F on that straight line and keep record of the different measurements in a table, we can verify that the resulting proportion does not change, obtaining thus a whole family of similar rectangles. Likewise, in Figure 3 we can check that by adding the same constant to the dimensions of the sides of a rectangle, we do not necessarily obtain a rectangle proportional to the original one. After developing this task, it is now possible to confront the activity of the *Soccer competition* by implementing these criteria.

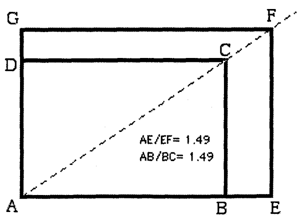

Figure 2.

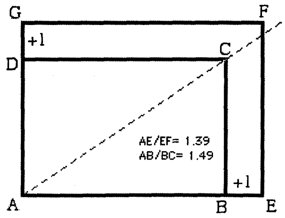

Figure 3.

Another interesting method for finding a rectangle proportional to a given one is by using the center and the diagonals of the latter one. If the new rectangle has the same center and its produced diagonals coincide with the extended diagonals of the first one, both rectangles are similar, that is, the sizes of their sides are proportional.

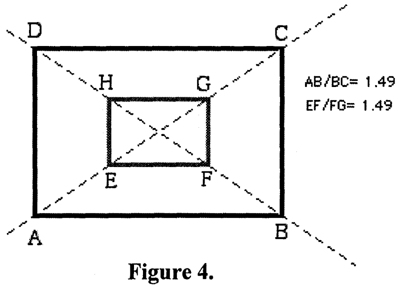

Figure 4.

Moreover, this type of constructions yields a variety of figures that remain equal to each other although proportions change, and many new possibilities of study arise.

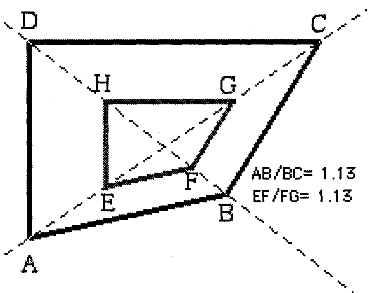

Figure 5.

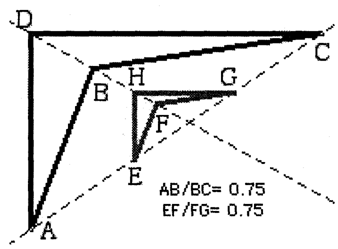

Figure 6.

Similarity and homothesis. In the first courses of lower secondary education, it is possible to introduce the invariance of similarity of figures under homothesis. This topic becomes especially easy and at the same time very illustrative in the Cabri-Géomètre environment: it is very interesting to see how moving the point *P* generates families of similar figures (see Figure 7).

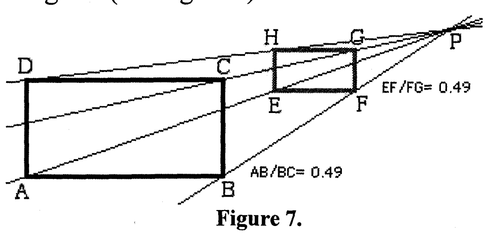

Figure 7.

Conclusions

There are different activities for enriching and widening students' knowledge about the notions of ratio and proportion. The representation and visual verification of Tales theorem, the study of linear functions, and the computation and graphic representation of constants like π or the golden ratio can be used for those purposes. Additionally, some of those activities can serve as potent elements for motivation.

References

Fiol, M.L., Fortuny, J.M. (1990). Proporcionalidad directa. La forma y el número. Madrid: Síntesis.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures.* Reidel Dordrechet. 28-33, 178-209.

Hart, K. (1988). Ratio and proportion. En: J. Hiebert y M. Behr (eds.), *Concepts and operations in the Middle Grades, 2.* Reston, Virginia: National Council of Teachers of Mathematics.

Inhelder, B., Piaget, J. (1955). *De la logique de l'enfant a la logique de l'adolescent.* París: PUF.

Kieren, T., Nelson, D. y Smith, G. (1985). Graphical Algorithms in Partitioning Tasks. *The journal of mathematical behavior 4.* 25-36: Edmonton, Alberta, Canadá.

Lesh, R., Post, T. y Behr., M. (1988). Proportional reasoning. En: J. Hiebert y M. Behr (Eds.) *Concepts and operations in the Mddle Grades, 2.* Reston,Virginia: National Council of Teachers of Mathematics. 93-139

Lupiañez, J.L., Moreno, L. (2001). *Tecnología y Representaciones Semióticas en el Aprendizaje de las Matemáticas.* En Rico, L. (Ed.*) Iniciación a la Investigación en Didáctica de la Matemática.*. Univ. de Granada: Dpto. Didáctica de la Matemática.

Piaget, J. e Inhelder, B. (1978). Las operaciones intelectuales y su desarrollo. En J. Delval (Comp.), *Lecturas en Psicología del Niño, I (70-119).* Madrid: Alianza Editorial.

Ruiz, E.F.. (2000). Study of solving strategies and a Proposal for the Teaching of ratio and proportion. Vol 2. *Psychology Mathematics Education.*

Streefland, L. (1984). Search for the roots of ratio: Some trough on the long term learning process. Part I.*Educational Studies in Mathematics, 15-3.* 327-348.

Streefland, L. (1990). Free Productions in Teaching and Learning Mathematics En: K. Gravemeijer, M. Van de Hunal y L. Streefland (Eds.), *Contexts Free Productions Tests and Geometry in Realistic Mathematics Education* (33-52). Utrecht: Researchgroup for Mathematical Educational Computer Canter State University of Utrecht, The Netherlands.

Streefland, L. (1991). *Fractions in realistic mathematics education.* Thesis doctoral Kluwer Academic Publishers. 46-134.