Promoting students' meaningful learning in two different classroom environments

Marie Kubínová, Jarmila Novotná, Charles University, Prague

Abstract: The paper continues the research presented in (Kubínová, Mareš & Novotná, 2000) and (Kubínová & Novotná, 2001). It reports on an observational study focused on the analysis of concrete situations in two classes taught in different ways by different teachers in the past but taught by one teacher at present. The differences in students' behaviour, teacher's approaches and results achieved by students are diagnosed and illustrated by the topic Functions.

1. Theoretical framework and related research

The teaching experiment dealt with in this paper is a part of a longitudinal research focused on the transition from the traditional transmissive, instructive way of teaching to the constructive one.

In (Kubínová, Mareš & Novotná, 2000), four schemes *subject matter – teacher – students* were analysed and characterised: **IR** (instructive teaching method, direct teaching of ready information or learning from text), **ID** (instructive teaching method, attempt for students' independent transfer of acquired knowledge), **CR** (constructive teaching method, learning from text), **SC** (social constructive teaching method). The main consequences of the use of constructive teaching methods are: The subject matter becomes an intermediary which enables the development or modification of students' existing concepts and the creation of new ones; social relations among individual cognising subjects are accepted; the role of the social relationship between students and teacher are accentuated, and the social relations among students are taken into account.

In (Kubínová & Novotná, 2001), the differences in students' behaviour, teacher's approaches and results achieved by students were diagnosed by analysing concrete situations in two classes taught in different ways by different teachers in the past but taught by one teacher at present. It is shown that even if the teacher who wants to implement the change from instructive to constructive teaching, is sufficiently qualified, has long-term experiences with constructive teaching strategies and has no obvious external obstacles for implementing their plans, has to be open-minded and respect students and their prior experience.

The present paper focuses on the development of students' understanding of mathematical concepts when the constructive teaching method **SC** for 4 years had been applied. The findings are illustrated by the topic *Functions* which is commonly agreed to be one of the difficult topics of school mathematics.

Function concept

(DeMarois & Tall, 1999) undertakes the complexity of the function concept. The function concept is studied as an organising principle for algebra and beyond. The aspects studied include the function notation and the symbolic, numeric and graphic representations. The pre-procedural, procedural, process, object, and proceptual levels are studied. It is documented that for many students, the complexity of the function concept is such that the making of direct links between all the different representations is a difficult long-term task.

In (Even, 1990), ideas giving basis for the difficulty that functions represent to students are summarised: the arbitrary nature of functions; the univalence requirement; the function as a unifying, complex concept at one side and their different behavior, representations, notations on the other; alternative ways of approaching functions - point-wise, interval-wise and objects-wise; the richness and accessibility for students of the basic repertoire of functions; the importance of both procedural and conceptual knowledge and the relationship between them.

In the Czech Republic, the function is usually the first concept that students meet at the Basic school and that contains a certain dynamics, movement. This fact markedly influences the process of constructing a notion. A significant progress was reached mainly in that the curriculum pf all three valid educational programmes for the Basic school suppose that the stage of preparing the function begins in an implicit form already at the first years of schooling with a more significant use of a variety of inter-disciplinary links and with a systematic stabilisation and refreshing of the mutual connection *function – (real) story, notion*. The programme authors appreciate that in the didactic interpretation in the concrete schooling, the dynamic nature of the function enables very well among other the use of experimenting, solving problems intuitively, modelling, but also a timely preparation of further concepts.

Social interactions

Social situations exist in the school regardless of the significance we give them (Kubínová, 1999). If an opening of the space for effective teaching is to occur, it is necessary to create more natural conditions for teaching, i.e. a situation which enables this to happen. The teachers should

- admit that they are not the only source of information for students, that discussions with other people, TV programmes and Internet access significantly influence students' knowledge and way of learning,
- understand that each of their students creates their own concepts and these concepts are multileveled with respect to the student's own concepts as well as to their peer group, and that many of these concepts are not complete, sometimes even not correct but used by them for experiencing the world,
- suppose that students already created a certain concept through various resources, not only during the work in class.[1]

Steinbring (2000) has studied social discourse and a reflexive discussion, and shows by analysis of teaching episodes how individual learning strategies and social-interactive constructions of knowledge favour different forms of an epistemological

[1] Similar ideas can be found already in the work of J.A. Komenský (Comenius).

development of new mathematical knowledge. Construction of a new knowledge is not only an individual process, but collective processes make potential development of new knowledge possible.

2. Our research

When studying questions related to using constructive approach to teaching (mathematics) we use variety of methods: longitudinal evaluation of teaching effectiveness by comparison of periodic testing of parallel classes, direct observation of the milieu of the classroom and analysis of teaching strategies, the teachers accounts of their own classroom experience, analysis of audio/video recordings of lessons and of students' products, as well as the direct teaching at one Basic school. In the school year 2000-2001 we faced a singular opportunity. One of the authors of this paper taught mathematics in two parallel classes of the ninth grade (9.A and 9.E referred to below where students are age 14). In 9.A she taught the class for five years and using constructive teaching strategies (Kubínová & Novotná, 2001). The class as a whole showed very good results in mathematics, in the inquiry most students put mathematics among their favourite subjects. She has never taught the other class (9.E) before. In this class, there was never one teacher teaching longer than during one year. The class was commonly considered as average, two students showed excellent results. Only two students put mathematics on the list of their favourite subjects.

Following the direct observation, interviews with students and teachers who had taught in both classes in the previous school year and the analysis of students' written products, the input diagnosis of both classes was created. For the purposes of our paper we will focus in the further text on the phenomena related to the creation and fixing of the function concept.

INPUT DIAGNOSIS (15.9.2000)

In the previous period in the mathematics teaching

- ***the emphasis was put on***:

9.A	**9.E**
Long-term preparation of the function concept and of accompanying phenomena	Transmission of ready-made knowledge about the function – definition, graph
Constructing of the function concept	Instructive teaching strategies
Cultivation of communicative abilities including mother tongue development and the work with different function representations	Assigning and elaborating standard tasks mainly in the written form
Work with diverse information sources in and out of school	Work with textbooks and mathematical tables as the only "legal" information sources
Long-term building of the function concept in the student's cognitive structure, creation of separated and universal models	Immediate student's performance based on memorising

Work with an error as a source of cognition	Error as an indicator of the immediate student's performance
Evaluation of qualitative changes in the student's work during a certain period	Evaluation of the immediate student's performance when solving standard problems
Use of the function concept as an intermediator when using inter-disciplinary links and solving real life problems	Use of the function concept when solving standard mathematical problems
Cooperation, team work and support of social links in and out of the class	Individual students' work

- ***the teacher was apperceived by students as*:**

9.A	**9.E**
A person guiding the teaching/learning process	The only person who has the right to decide about the teaching/learning process
An authority providing enough space also for the ideas of students	An authority who does not need to provide students with space for their ideas
A facilitator and advisor (also in matters not directly connected with mathematics teaching)	A person whose duty is to transmit ready-made knowledge and instruct students what they are to do

The diagnosing tool evaluating the input analysis was the student project *What do the graphs say* (see Table 1).

What do the graphs say
Describe by another way the dependences those graph is on the figure.

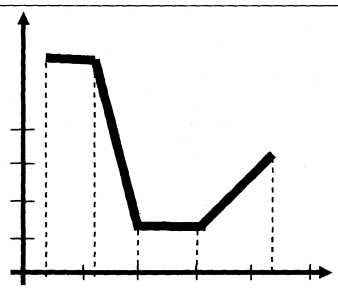


Table 1

At the end of September 2000, pairs of students in both classes elaborated the project individually outside of school (in order to be able to use various sources of information). After that period they showed written materials and presented their results in a mathematics lesson.

Working pairs in 9.A presented divers solutions (often unexpected for us) and were able to defend their results also during the following discussion about the project outputs. In Table 2, all types of 9.A students' solutions (including the incorrect ones) and some of their statements are summarised. Students' solutions (*I*), (*V*), (*T*), (*S*), (*U*) and (*R*) represent separated models of dependences, (*F*) an universal model.

What do the graphs say		
Identified dependence/ number of occurrences	***Quantities monitored***	***What happens (in the student's statements)***
Trip graphs (*I*)/7	Time, distance	• Train (car, student, dog, ...) moves between two places with different velocity or is staying • It is possible to continue the graph (time is running always).
Water in tank (*V*)/8	Time, water cubage	• In the tank there is a certain amount of water, the bleeder is opened, amount of water decreases, after some time it is closed again and then the feed-pipe with another velocity is opened and the amount of water is increasing. • Instead of filling in and out we can take out water from the tank by cans and add some using other cans. • It is possible to do it as along as the tank is empty (e.g. they forgot to close the bleeder) • Water may also flood if we let the feed-pipe opened too long.
Changes of temperature (*I*)/9	Time, temperature	• We follow the outside temperature regularly. It stabilise after a while, then descends, then is again stable and then increases. • From a digital thermometer we would not have such a nice line.
Changes of state (*S*)/3	Time, amount of heat	• We are cooling boiling water until its temperature starts to descend. When it is at zero, we still cool it until we get ice. After a while we will heat the ice and we get water that will start to heat. • Time when nothing happens cannot be extended as we want. After a certain time the state changes and the temperature starts to change as well.
Savings (*U*)/2	Time, amount of savings	• We have some savings and after a certain time we begin to spend it regularly. Then we stop it for a while and then we start to save again. • Or we have also some money and have to pay interests. After paying them, we do not do anything with our money and then we put them on an account and they give us interests, so out money begins to increase.
Changes of velocity (*R*)/1	Time, velocity	• We are at the motorway in a super-fast car with the same velocity, then we have to brake in order to move with a smaller velocity and at a certain moment we can increase the velocity again.

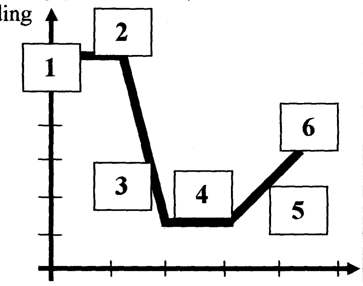
Universal description (*F*)/4	*x*, *y* not stated	1. beginning 2. nothing is happening (bigger value) 3. graph is descending 4. nothing is happening (smaller value) 5. graph is ascending 6. end 

Table 2

For our work with 9.A the following findings were most important:

- Students interpreted the open assignment of the problem in various contexts, even from non-mathematical once. Here we were mainly surprised by the physical interpretations (changes of temperature and above all changes of state depending on time).
- Students were aware of limiting conditions for individual dependence descriptions (e.g. limitations for the range given by the volume of the tank or by the size of the latent heat of the dissolution of ice).
- In the cognitive structure of our students the universal model of a continuous (discrete) function was not created yet. They used a "continuous" line for interpreting discrete actions (measurement of temperature in given intervals, saving money, supplying water in a tank using a can).
- Students reflected the existence of more precise tools for the graphic record of dependences (e.g. digital thermometer).
- Four of eleven working pairs offered, besides the separated models, also the universal one (*F*), see Table 2.

The situation in 9.E was different. Only three from twelve working pairs accepted the openness of the project assignment and presented separated models (*V*) (2 pairs) and (*T*) (3 pairs). One of these three pairs labelled the coordinate axes and presented the universal model (*F*). Four pairs did not solve the problem anyhow while they did not considered the curve to be a graph of a dependence. (it did not correspond with any of graphs they had in their register, i.e. the graph of linear or quadratic dependence or indirect proportion). The last five pairs modified the task first to a closed one (by a concrete labelling of axes) or simplified it (omitting parts of the graph) and presented the unique solution. In four cases, the dependence description was the dependence of distance on time, in one case of temperature on time.

Using the input diagnoses, teaching strategies based on constructive approaches in both classes were stated for the period of one school year. In 9.E they could not be developed to the whole versatility because students grasped only step-by-step the cooperative ways of working, learned to work with open problems, solve non-standard problems, trust their own decisions etc.

The **output diagnoses** based on participating observation and analysis of written products showed that having worked with the function concept during one school year 9.A students précised their knowledge to such extent that they

- did not failed when solving standard school problems,
- were able to work with the universal model of a dependence (function), solve non-standard problems, use various representations of the concept when solving school and practical problems,
- used systematically various information sources (also out of school),
- identified at the intuitive level important properties of functions (continuity, discreetness, monotony, extremes, ...)
- classified some classes of functions (linear, quadratic, ...).

In 9.E, the class climate changed significantly towards the cooperative one. 9.E students did not show a significant shift in grasping the function concept during the school year. We identified only the enlargement of the group of separated models and more frequent attempts to solve non-standard problems. There survived the link to the function assignment and creation of graphs of "known" functions following the rules given beforehand. The influence of the preparatory period neglect was significant.

3. Concluding remarks

In (De Corte, 2000) it is stated that: "... we should realise that powerful learning environments ... require drastic changes in the role of the teacher. Instead of being the main, if not the only source of information – as is often still the case in average educational practice – the teacher becomes a 'privileged' member of the knowledge-building community, who creates an intellectually stimulating climate, models learning and problem-solving activities, asks provoking questions, provides support to learners through coaching and guidance, and fosters students' agency over and responsibility for their own learning." From the evidence above it is clear that the teacher's role is crucial, he/she has to understand and respect the situation in each individual group of students. It is not possible to transmit the methods and forms of work which were successful with one group of students to another unmodified, however it is possible to use experiences gained with one group of students to organise work in another group.

In (Edwards & Jones, 1999), the grouped categories of students' views of learning mathematics in collaborative groups were classified. The following were clearly identifiable in our analysis of 9.A and 9.E performances in school mathematics:

benefits of working together, respecting others in the group/sharing knowledge, confidence building and speed/volume of learning. In the class where the interactive teaching strategies were newly introduced (9.E) it influenced the climate in the class first, the influence on the mathematics behaviour and knowledge was significantly milder and occurred later. To change students' gained norms of acting to a greater extent demands a long period of phased transition from transmissive teaching strategies to constructive ones (in our case, after 8 years of schooling, one school year was not sufficient).

In our experiment the role of peer interactions in the process of cognitive development was important. To profit from them needs a long experience of students in the similar activities, the enlargement of their self-confidence as well as the changes in their attitudes towards the subject. Students urgently need to see clearly mathematics as the subjects having narrow links to other subjects and mainly to the life situations.

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