APPROACHING GRAPHS WITH MOTION EXPERIENCES

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ABSTRACT

In this paper we report on a study about students' activity in interpreting their body motion graphs on a symbolic-graphic calculator connected with a motion sensor. This is done in the context of introducing functions, graphs and modelling at secondary school level, in order to teach algebra with a semantic approach (in which symbols are used not only in a pure syntactic way). We analyse the ways in which students build a meaning for the graphs, comparing them with the ways used by students who did not take part in the same experience. Finally, we try to interpret these differences, by comparing language and gestures of the two students' groups.

INTRODUCTION

In literature it is well-known that students have many difficulties in interpreting graphs, particularly those in which one variable is time-dependent, as for example space-time or velocity-time graphs. Researchers have identified some ways in which students misinterpret graphs. One is the *graph-as-picture* interpretation, in which students expect the graph to be a picture of the phenomenon described. In kinematics, this can result in the student interpreting a graph of space versus time as if it were a road map, with the horizontal axis representing one direction of the motion rather than representing the passage of time (Berg & Phillips, 1994; Clement, 1989). Another common misinterpretation is the *slope/height* confusion, in which students use the height of the graph at one point, when they should use the slope of the line tangent to the graph at a point, and vice versa (Thornton & Sokoloff, 1990).

Mathematics and Physics Education research tells us that microcomputer-based laboratory instruction with motion sensors is quite successful in supporting students to learn to interpret graphs, because of the help of the real-time display of graphs, together with the kinaesthetic experience of motion, that give students control over the data. Generally, all the experiences in which students can interact with a tool to create phenomena, help them to understand the mathematics connected with those phenomena (see e.g. Noble & Nemirowsky, 1995).

The ongoing research project we are involved in, is concerned with the students' cognitive behaviour when using a symbolic-graphic calculator, connected with a motion sensor. Secondary school students are first engaged with experiencing various body motions. Second, they are asked to interpret the graphs and tables containing the data related to their motion, using natural language first, then in terms of increasing/decreasing functions, and finally in terms of slope, calculated in some points (Arzarello & Robutti, 2001). The general research problem is the study of the students' transitions from the perceptual level [1] to the theoretical one and backwards and the mediation of technology in these transitions.

In our investigation, we obtained the same results witnessed in literature, as regards the comparison of students who experience body motion with sensors and calculators (we will call them the *experimental group* of students) with students who do not participate in the experience and have to solve the same problem situation (we will call them the *control group* of students). Generally the students of the experimental group interpret the motion graph correctly, while the students of the control group reveal the kind of misinterpretations described in the introduction.

Our attention is focused on the *ways* the students of the experimental group construct the meaning of the graph they observe on the screen after the motion experience. In doing so, we analyse the students' language and gestures, which provide information about the students' cognitive activities, and compare them with those of the students of the control group, who observe the graph only on the paper, after a brief introduction made by the teacher. A crucial point seems to be the transition *static-dynamic* and backwards, namely when the graph is seen as a pure graphical sign, or is conceived as a time-dependent representation [2].

Our hypothesis is that, for the construction of a correct meaning for a graph, it is fundamental to make some transitions from a static to a dynamic interpretation of it and backwards. Two main theoretical ingredients are at the basis of this study, namely: the transformational reasoning (Simon, 1996) and the mental times (Arzarello, et al., in press).

METHODOLOGY

The activity described in this paper is part of a long-term teaching experiment carried out in a classroom of 25 first year students (the first set) of a scientifically oriented high school, [3] (9^{th} grade) during the school-year 2000-01 in Italy. This activity lasted three hours, in which the students worked in *small groups* (three-four pupils each). Each small group used two technological tools: the CBR (Calculator Based Ranger, a motion detector which collects space-time data in real time) and the symbolic-graphic calculator TI92, for realising a motion in front of the sensor and then interpreting the space-time graph on the calculator. The small group activity was followed by a final classroom discussion, directed by one of the authors. Previous activities, specifically oriented to make students familiarise with the motion sensor and the gathering of the data in the calculator, have been submitted to the students belonging to the experimental group. At the moment of the experiment, the pupils did not know any algebra rules, the linear function and its representation, the equation of uniform or accelerated motions and the definition of velocity (in a few words, they had not been taught algebra, nor yet kinematics).

The students of the control group are those of another classroom (20 pupils in the same initial conditions as the first set), in the same school. Working in *small groups*, they do the same kind of activity as the experimental group of students, without making the motion experiment and without using technology. They have to analyse the space-time graph of a body motion printed on a sheet of paper, and to answer the same questions as those of the first set, after an explanation led by the teacher about the way this graph has been obtained.

During the activity of both groups (experimental and control), four people were present in the classroom: two teachers (one of mathematics and one of physics), to observe the small groups; one University student, to videotape one small group and the final discussion, and one researcher, to observe the groups and guide the final discussion. The data consist of video-tapes and some written notes.

THE OBSERVED STUDENTS

In the experimental group of students we observed a small group of four pupils, three boys and one girl: Filippo, Gabriele, Fabio and Giulia. They are all medium achievers, but with different features: Filippo is a reserved, studious and thoughtful pupil, Gabriele is a discontinuous student, who prefers to go on with a personal rhythm, Fabio is a clever and intuitive boy, while Giulia is a determinate and studious girl, active in a collaborative group.

In the control group of students we observed a small group of three boys: Filippo, Francesco and Giorgio. Filippo is a meticolous and respectful pupil who obtains good results, Francesco is an extrovert and expansive boy who overcame some initial difficulties with good will, Giorgio is a student with discontinuous results.

THE ACTIVITY

The activity proposed to the experimental group of students is the following [4]:

> *Walk or run in the corridor in order to make a uniform motion; when you arrive at the red line, come back with the same motion. The CBR will record your position with respect to time and will collect the data in a graph and in a table. The data are expressed in seconds [s] and in meters [m] respectively. Each 1/10 s a couple of data (time and position) are collected.*
>
> *Describe the kind of motion you made in the corridor.* Using the graph and the table, describe how space changes with respect to time (increase, decrease, ...). Analyse the graph (Is it like a line? Is it like a curve? Does that curve increase? Does that curve decrease?). Consider the ratio $m = (s_2 - s_1) / (t_2 - t_1)$ and use it to describe mathematically the graph of your motion (t_1 and t_2 are two subsequent time data and s_1 and s_2 are two subsequent position data).

PROTOCOLS

Experimental group of students

In the experimental group of students, each small group makes the experience of motion in the corridor. In the observed small group, the student who is running in the corridor (from the sensor to a red line and back) is Fabio, while the other members are looking at the calculator. After the motion, they are all around a desk, to answer the questions. In this first excerpt, the students are exploring and describing the shape of the graph (Fig. 1) and trying to connect it with the motion.

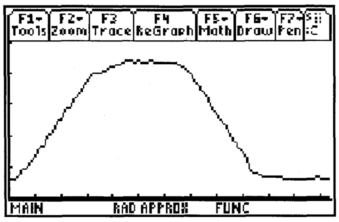

Fig. 1 - The graph obtained by the observed small group in the motion experience.

74	**Gabriele:**	*So, we started from a point and we made a motion...* [...]
78	**Gabriele:**	*And then, at the end, this line* [he is moving his pen on the horizontal stretch of the graph from left to right]
79	**Fabio:**	*Here* [he is pointing at the starting point of the horizontal stretch], *here is when I got to the red line* [...]
81	**Gabriele:**	*Here you stopped* [he is marking the same point on the graph with his pen]
82	**Giulia:**	*...You came back* [she is pointing at the negative slope stretch] *and then you stopped again* [...]
91	**Fabio:**	*The one who ran, uhm...* [...]
94	**Fabio:**	*...He tried to take the steps always in the same...in the same time interval* [...]
112	**Giulia:**	*He tried to keep the same pace there and back* [...]
116	**Gabriele:**	*Walking always the same distance*
117	**Fabio:**	*...During the whole, during the whole time in which I walked, I tried to, to make, to keep always the same velocity* [...]
123	**Giulia:**	*Constant velocity!* [she is writing it]

The students go back and forth from the graph on the screen to the description of the motion experience. The way pupils are looking at the graph changes over time. At the beginning, they use the *deictic* function of language (Radford, 2000), to indicate some parts of the graph (e.g. #78 "*this line*"). This kind of language reveals a static approach to the graph. Two observations are important now: first, the students observe the graph dividing it into different parts, as already noticed (e.g. Monk & Nemirovsky, 1994), and they try to interpret these parts first separately and then together. Second, the *cognitive pivot* (Arzarello & Robutti, 2001) to interpret the graph is given by an event of the motion experience: Fabio's arrival to the red line (#79). So, the deictic function is used together with the *generative action* function of the language (Radford, 2000), in order to link the graph with the motion experience (#79 "*when I got*"; #81 "*Here you stopped*"; #82 "*You came back and then you stopped again*"; ...). The generative action function, together with some adverbs (e.g. "*when*") reveals a dynamic interpretation of the graph. This evolution from static to dynamic indicates the presence of a transformational reasoning: "... the ability to consider, not a static state, but a dynamic process by which a new state or a continuum of states are generated" (Simon, 1996, p. 201). At #91 a new phase starts, with new generative action functions. There is a mathematisation process, in which the language evolves and becomes more specific through consecutive refinements. The steps taken in the same time interval (#94), in fact, become, for Giulia (#112) "*the same pace there and back*" and then, for Gabriele, "*the same distance*" (#116). The students find a sort of motion rule: the presence of the adverb "*always*" (#94, #116, #117) reveals the generalisation of this regularity in the sentences listed above. This word, as observed by (Radford, 2001): "underpin the *generative functions of language*, that is, the functions that make it possible to describe procedures and actions that potentially can be carried out reiteratively" (p. 83).

In our opinion, there is something more in the information carried out by this term: the transition from the timed experience to a never-ending process, through the de-timed rule describing the graph. So, there is a new transition, from a dynamic to a static interpretation of the graph, to produce a time-independent rule. This new static viewpoint is not to be intended at the same level as the first static one, but at a higher one, due to the presence of a new concept: velocity. So, the uniform motion becomes a motion with "*constant velocity*" (#123). The basis of this transition is a transformational reasoning, as in the previous excerpt. The mathematisation process in the interpretation of the graph evolves towards correct ideas, as noticed by other researchers (e.g. Monk & Nemirovsky, 1994). This evolution, in our opinion, passes from static to dynamic stages and backwards. All the previous observations refer to the *physical time* (Varela, 1999), while also the students' *inner times* (Guala & Boero, 1999) are involved in this evolution. In fact, the students act in *the time of past experience*, when they recall the kind of motion, and in the *contemporaneity time*, when they connect the motion with the graph (Arzarello et al., in press). Here the inner times of the students are very closed together, so the interaction among them is more productive than the situation of different inner times.

439	**Fabio:**	*...I stopped and when I stopped...*
440	**Giulia:**	*It goes on* [she is pointing at the curve], *doesn't it?*
445	**Gabriele:**	*If you always go on at the same velocity, they* [time and space] *both increase* [...]
448	**Gabriele:**	*Yeah, it* [the curve] *goes on in this way and then it goes down* [with his pen he traces the consecutive slanted sides of an "open triangle"]
449	**Filippo:**	*Yeah, it is not a straight line. They are two half lines* [...]
452	**Giulia:**	*Because if you did not, if you did not lose, let's say, time for turning on yourself* [...]
459	**Fabio:**	*I should have walked in a more uniform way, I did not walk...*

In this excerpt, initially, the word "*when*" and the generative action function mark again a series of timed sentences (#439). Then, at #440, we can observe a dynamic reading of the graph (when Giulia says that the curve "*goes on*" although Fabio stopped), which marks a transformational reasoning. At this point a new function of language enters the scene (#445, #452): it is the *logic function*, which appears in hypothetical expressions of the form: "if... then..." and reveals a conjecture or a deduction. The pupils evolve toward a more mathematical description of the graph in terms of increasing and decreasing functions. In doing so, the motion experience is linked to the shape of the graph (#452, #459): the students are in the time of past experience.

Control group of students

The excerpts listed below refer to the observed small group belonging to the control group of students. They have to describe the motion, referring to the graph of Fig. 1.

11	**Francesco:**	*It* [the graph] *is uphill, therefore he is accelerating* [...]
19	**Giorgio:**	*However he is going on at the same velocity* [...]
22	**Francesco:**	*Then he suddenly slows down* [he is moving his finger twice on the second slanted stretch] *and...* [...]
26	**Francesco:**	*Let's see: there is an increase of velocity and then, I don't know*
36	**Francesco:**	*There is a moment* [he is pointing at the horizontal stretch] *in which the velocity is constant* [...]
42	**Francesco:**	*Sorry, but here* [he is pointing at the starting point of the decreasing slanted stretch] *is he going fast?* [he is moving his finger on this stretch] [...]
46	**Giorgio:**	*Then here* [he points at the first slanted stretch several times] *he is covering more space*
47	**Filippo:**	*But here too* [with his pen he is pointing at the second slanted stretch] *theoretically* [...]
59	**Filippo:**	*And then here* [at the starting point of the second slanted stretch] *he is stopping in theory, because, hum...*
60	**Giorgio:**	*No* [he is pointing at the higher horizontal stretch], *he continues to go on at the same velocity* [...]
64	**Francesco:**	*It is here* [he points at the starting point of the graph]. *Namely he starts here and then he comes back here* [at the final point of the graph] [...]
88	**Giorgio:**	*This* [the first slanted stretch] *and this* [the first horizontal stretch] *are a straight line, all equal straight lines!* [...]
96	**Francesco:**	*It is a curve* [Filippo is writing it] *which starts increasing and ends decreasing*
97	**Filippo:**	*Increasing...then acceleration* [...]
99	**Francesco:**	*...And ends decreasing*
100	**Filippo:**	*Slowing down!*

The students misinterpret the graph, with a confusion between distance and velocity (slope/height mistake). For example, at #11 they think that the ascending part means an increasing velocity; at #19 the first horizontal part represents a constant velocity. From #42 on, the misinterpretation continues: for example at #46, where pupils refer to a velocity. In all the dialogue there are many deictic functions of language and many pointing gestures, with the role of indicators of the graph (e.g. the word "*here*", often used with a gesture, in #42, #46, #47, #59, #64, #88). Here, language and gestures are a kind of communication indicating a static interpretation of the graph which does not become dynamic. These students do not come to a correct interpretation of the motion represented in the graph, because they are not able to link the graph itself to a real motion. We think that they do not make a transformational reasoning, because they did not experience the motion, nor use the technology. Furthermore, these students' inner times are not so closed together, as those of the previous small group.

CONCLUSIONS

It would be too simple concluding that the motion experience, together with the use of a proper technology offering a powerful mediation for learning, is sufficient to construct a right meaning for a graph. The two observed small groups of students offer a significantly different approach to the activity, but it is also important to describe the general approaches of the two classrooms (the experimental group and the control group). In the first classroom, all the small groups were able to describe the graph in terms of motion, while in the second classroom, only one small group reached a correct interpretation, while all the others (5 small groups, for a total of 16 students) produced the misinterpretations described in the literature [5].

Our suggestion is that both the motion experiment and the mediation of technology (calculator and CBR), together offer a support to make the transitions from a static to a dynamic interpretation of the graph and backwards. These transitions help the students to evolve towards a correct meaning for the graph, linking it to the phenomenon which it describes. This fact appears crucial, insofar the technology used in this experiment supports both the use of transformational reasoning and the synchronisation of students' inner times. Along with Nemirowsky, when he says that "learning graphing entails the enrichment of a broad range of experimental domains, involving the refinement of visual, kinaesthetic and narrative resources" (Nemirovsky et al., 1998), we believe that this kind of activities is important for Mathematics Education research and practice, because they foster the passage from perceptual to theoretical thinking.

NOTES

1. As perceptual level, we mean using perception (e.g. seeing and touching, while moving). As theoretical level, we mean producing a conjecture in a conditional form and validating it with a proof.
2. It is important to underline that not only a graph of a time-dependent phenomenon, but also a graph of time-independent one, or a geometric figure, or a function and so on, can be conceived in a dynamical way.

3. It is a Liceo Scientifico. These students attend five mathematics classes and three physics classes per week.
4. The words written in italic type are not present in the activity of the control group of students: they are substituted by an oral description of the motion.
5. It was only after a discussion led by one of us, during which the actual motion was carried out, that these students were able to understand the shape of the graph.

REFERENCES

Arzarello, F. & Robutti, O. (2001). From Body Motion to Algebra through Graphing. *ICMI Studies*, Melbourne, Australia, December 2001, vol. 1, 33-40.

Arzarello, F., Bartolini Bussi, M. & Robutti, O. (in press). Time(s) in Didactics of Mathematics. A Methodological Challenge. In: Lyn English, M. Bartolini Bussi, G. Jones, R. Lesh e D. Tirosh (ed.), *Handbook of International Research in Mathematics Education* (LEA, USA);

Berg, C.A. & Phillips, D.G. (1994). An investigation of the relationship between logical thinking structures and the ability to construct and interpret line graphs. *Journal of Research in Science Teaching*, 27, 803-815.

Clement, J. (1989). The Concept of Variation and Misconceptions in Cartesian graphing. *Focus on Learning Problems in Mathematics*, 11(1-2), 77-87.

Guala, E. & Boero, P. (1999). Time complexity and Learning. *Annals of the New York Academy of Sciences*, June 30, 1999, 164-7.

Monk, S. & Nemirovsky, R. (1994). The Case of Dan: Student Construction of a Functional Situation through Visual Attributes. *CBMS Issues in Mathematics Education*, vol. 4, 139-168.

Nemirovsky, R., Tierney, C. & Wright, T. (1998). Body Motion and Graphing. *Cognition and Instruction*, 16 (2), 119-172.

Noble, T. & Nemirovsky, R. (1995). Graphs that Go Backwards. *Proceedings of the 19th PME Conference*, Brazil, vol. 2, 256-263.

Radford, L. (2000). Signs and Meanings in Students' Emergent Algebraic Thinking: A Semiotic Analysis. *Educational Studies in Mathematics*, vol.42, 3, 237-268.

Radford, L. (2001). Factual, Contextual and Symbolic Generalisation in Algebra. *Proceedings of the 25th PME Conference*, The Netherlands, vol. 4, 81-88.

Simon, M. (1996). Beyond Inductive and Deductive Reasoning: the Search for a Sense of Knowing. *Educational Studies in mathematics,* 30, 197-210.

Thornton, R.K., & Sokoloff, D.R. (1990). Learning motion concept using real-time microcomputer-based laboratory tools. *American Journal of Physics*, 58, 858-867.

Varela, F.J. (1999). 'A Dimly Perceived Horizon: The Complex Meeting Ground between Physical and Inner Time'. *Annals of the New York Academy of Sciences*, vol. 879, 143-153.