TEACHERS' ROLE IN THE MANAGEMENT OF MATHEMATICAL KNOWLEDGE: AN ANALYSIS OF A PROBLEM SOLVING PROCESS

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The present study deals with a problem solving process in the early grades. Data analysis focuses on pupils' mathematical behaviour and the teachers' management of the mathematical knowledge during the problem solving procedure. The analysis reveals two factors that seem to play an important role in the devolution of the problem and consequently in the construction of the pupils' mathematical knowledge. The former is related with the fact that pupils control the correctness of their outcomes themselves and the second one with the fact that the teacher facilitates the pupils without reducing the mathematical meaning and the cognitive features of the task.

THEORETICAL ISSUES

Current research on teaching and learning mathematics, independently of the perspective taken, accepts the premise that pupils are not passive absorbers of information, but rather have an active part in the acquisition of knowledge. Furthermore, it emphasises the need for mathematics teaching to be much more than a study of ready-made mathematics, which is still so prevalent. Since learners always construct their own knowledge, the critical issue is the nature of the socially and culturally situated constructions (Cobb, 1994). Thus the teachers' task is to challenge pupils by introducing effective mathematical activities, and maintain a classroom culture that encourages and facilitates independent learning.

In this perspective, two are the basic issues, which are of importance for the research in mathematics education: the choice and the formation of the mathematical content and the organization of the mathematical activity within the classroom. In relation to the former, since the common consideration is that mathematical knowledge cannot be reduced to a stock of retrieval facts, but it is constructive and, consequently, is best demonstrated in situations where something new is generated (v. Glasersfeld, 1983), a considerable number of research focuses on the change of the mathematical content in order to teach mathematics as a subject to be created and explored (Cooney, 1988). This framework implies reforms in the classroom organization: a pupil to perform in a correct way on content is not enough but actually contradictory to the consideration of mathematics as a subject to be created and explored. Thus, children should be given opportunities not only to respond to, but also to pose questions, to phrase hypotheses, to explore their ideas, to check and convince others about their correctness and to re-consider their mathematical ideas and constructions (Bowers, Cobb & McClain, 1999, Jaworski, 1994).

Thus, a third issue emerges, which concerns the teacher's role in the management of the classroom as well of the mathematical knowledge. The teacher should adopt such an approach, which will allow children to develop their ideas and actions: to provide them with challenging tasks, to listen to them, to encourage them to express their thoughts, to organize the results of their work (Jaworski, 1994). At the same time, s/he avoids interventions such as providing hints, simplifying the task, giving a sing indicating the expected response. This kind of hints function as external indicators, they are often misinterpreted by the pupils, who adapt them to their existing system of knowledge as this requires less effort (Brousseau, 1977). This is possible to reduce the cognitive value of the task (Henningsen & Stein, 1997) and of the mathematical meaning of concepts and relations (Steinbring, 2001).

For teachers, the shift from familiar instructional practice to a reformed approach is not easily accomplished (Fennema & Nelson, 1997). On the one hand, this is due to the cognitive demands of challenging tasks required for the organization of the new didactical environment. And the difficulties teachers have

> "to determine the preparedness of particular students for investigatory work, and the type of support they may require to successfully engage in investigations" (Diezmann, Watters, English, 2001, p. 258).

On the other hand, the relevant difficulty can be attributed to the teachers' conception of their role with respect to the acquisition of the mathematical knowledge: they attempt to keep control of the pupils' outcomes and their validity, to maintain the students' dependence and to keep for themselves the management of mathematical knowledge (Arsac et al., 1992, Jaworski, 1994, Sakonidis et al, 2001).

In this context one of the important issues concerning the management of the mathematics' classroom is related with the "devolution of mathematical knowledge" (Brousseau, 1997, p.46).

> "the teacher must therefore arrange not the communication of knowledge, but the devolution of a good problem. If this devolution takes place, the students enter into the game and if they win learning occurs" (Brousseau, 1997, p.31)

How pupils interpret the mathematical meaning of a problem, and how the teacher reacts to the different meanings, how s/he helps the pupils to reconsider the problem, its mathematical meaning, without "telling" them the solution and without taking on him/her the mathematical knowledge are central issues for the research.

Based on the above considerations, this paper reports on the analysis of a problem solving process in the early grades (5 and 6 years old). It is a case study and the analysis focuses on the teacher's role in the management of the mathematical knowledge and tries to identify those elements, which led to the successful outcome of the process.

THE STUDY: A PROBLEM SOLVING PROCESS

The data analyzed for this paper come from a problem-solving situation tackled by pupils 5-6 years old. The problem given to the pupils was as follows: The teacher asked the class to divide in groups and each group try to share among its members in equal portions a circular cake. For this purpose, each group was given a cake with a diameter of about 25 cm, made of paper and decorated with non-symmetrical drawings. This problem was chosen because of its certain epistemological characteristics: it includes references to real life situations, it is commensurate with pupils' interests and experiences, it is challenging given that the solution is not obvious, children can apply their basic knowledge, and the material allows children to control in their own the validity of their outcomes

The pupils decided on their own to form two single gender groups, one consisted only of girls and one consisted only of boys. This resulted in two groups of different number of pupils (4 boys and 5 girls) and of different ages [1], which, as it will be seen later, influenced decisively the problem solving. Four sessions were needed all together for the completion of the task, one session per week.

Sessions were video and audio recorded. After each session, the teacher [3] completed a report with her comments. In the end of the whole problem solving procedure, a discussion was contacted with the researcher.

Data presentation

In the following, the development of the whole problem solving process is briefly presented. (numbers between brackets are referred to the transcript, which reports about the group of boys)

First session

In order to tackle the problem, children cut in a random manner the cake in pieces, each of his/her own. These pieces are of different size and a large piece of cake is left over. The children initially are worried about the left over piece and are not concerned with the unequal size of the pieces (1-3). They compare the pieces they had cut only after the teacher's intervention (4). The inequality of the pieces 'disturbs' the children only when their own piece is smaller than of someone's else (5,7). A point worth mentioning here is Costas's pointing out to the difference between senior pupil and junior pupil[2]: it is unacceptable a senior pupil to have a larger piece from a junior pupil! (10). During the process of pieces' comparison, children, while in the beginning they used to compare them by just looking at the pieces, as time went by, they started using another process of comparison, that is, by placing the one piece on the top of the other.

Second session

In the second session, the pupils tried to cut the cake using the process they adopted in the previous sessions: they would cut a piece of cake and they would place it the remaining cake to get the next piece, and so on. They chose this technique, because this way, they know beforehand, the pieces will be equal (15). Thus, the problem that children remained to solve was to cut the whole cake. In order to succeed, they cut larger pieces. However, there is still a remaining piece of cake. It has to be noted the children, in an effort to get an acceptable solution, that is, to be seen as having shared the cake in equal pieces, they suggested to the teacher to take the large piece that is left over (17).

Third session

In the third session, the children tried to solve the problem, to have no left over, using the same strategy as in the second session, but cutting this time larger pieces. However, they still do not manage to get the right size, to solve the problem. Thus, the problem remains. In order to get rid of it, Antonio puts forward the suggestion he had made in the previous session: the teacher to take the piece of cake that is left over (20). The teacher does not reject the participation in the solution of the problem, that is, to also take a piece. However, she insists that she should be treat as an equal member of the group and take a piece of cake of the same size of the others (21). Alexnadros' suggestion that the teacher takes his piece (22) is rejected by the other members of the group: although the older to receive a larger piece is acceptable by the pupils, a partner to receive a larger piece is not; the equilibrium of the system is destroyed. Thus, the pupils, although they are disappointed by the difficulty of the task (25-27) and ready to give up, they are finally forced to carry on with it and the problem is left to be tackled in the next session.

Fourth session

Accordingly to the teacher's comments, children expressed great anxiety in relation to whether they would be able to finally solve the problem.

The girls decided to drop Vaso, who is junior, in order to be as many as the boys, that is 4, and solve the problem. This action led to the formation of three groups and to the change of the situation, as one of the three groups had only two members. In this latter case, the problem was similar to the original, but much easier for the pupils, who were familiar with the concept of 'half' from their everyday life experiences (34-35). Thus, the change of the situation final led to the solution of the problem, passing through the solution of a simpler problem.

ANALYSIS OF DATA - DISCUSSION

The analysis of the data will focus on two issues: the mathematical behavior of the pupils and the management of the mathematical knowledge by the teacher.

With respect to the first, the following points are of importance: the children adopted the problem, but the attached to it a different meaning from that intended by the teacher. Thus, for example, in the first session, the pupils tried to solve the problem in such a way as to each of them take a piece, in the second session they focused on sharing in equal pieces and from the third session onwards they attempted to consider the problem as a whole. We could say that the problem the children effectively solve during each session is the following: in the 1st session the try to share the cake, in the 2nd to take an equal piece of cake, in the 3rd to solve the problem and to 'get rid of it' and in the 4th to solve finally the problem: the problem's devolution takes place during all 4 sessions. The pupils' solutions satisfied each time their own interpretation of the problem and it was only through the evaluation of the result, performed after the teacher's intervention, that the children adopted a new interpretation and attempted to approach differently the problem. The difficulties encountered by the pupils were of epistemological nature and concerned the equality of the pieces and the properties of the circle. With respect to the former, it is possible that the relation of the problem to the everyday situations functions as an obstacle. This is because, as the pupils tried to overcome the difficulty of finding a satisfactory solution, by appealing to real life rules (the youngest takes a smaller piece, the older a larger), which could make their solution acceptable (thus helping them to get rid of the problem). The final agreement that all (junior pupils, senior pupils, teacher) should receive equal pieces constitutes an epistemological advancement towards the mathematical concept of equality. As far as the properties of the circle are concerned, it could be argued that it is a lack of knowledge (Diezmann et al., 2001) and that the pupils learn about these properties (folding like a cross-symmetry) through this problematic situation.

The above analysis of the problem's devolution is related to the manner in which the teacher manages the mathematical knowledge and the mathematical learning. She intervenes by posing questions concerning whether the cake was cut in equal pieces or whether the piece that was left over is equal to the others (1,4,8,16,19), avoiding to say whether pupils' outcomes are right or wrong. Thus, her questions focus on the need for the pupils to check their results. This checking is the crucial element, which will allow them to proceed to the solution of the problem, by re-considering their ideas. This way, she facilitates the children's approach, without indicating what is right or providing hints, or simplifying the problem, but by helping them to stay focused on the problem (to not give up) and to cope with it, its cognitive demands and the mathematical knowledge it involves. In other words, she allows the devolution (Brousseau, 1997) of the mathematical knowledge. Another important point is when the pupils 'desperate' suggest that she takes the larger piece (22). She refuses to do so (if she had accepted, she would have reinforced their conceptions), but she does not refrain from participating in the solution of the problem with the condition of being treated as an equal member of the group. Thus, by 'rejecting' the role of the adult who deserves a larger piece and as teacher who does not solve the problems (they are pupils' tasks), she reinforces her facilitating role, as described earlier.

The simplification of the task (the creation of two groups of 4 pupils and one group of 2 pupils) was initiated by the pupils themselves. This seems to be a special feature of this particular case. However, if this simplification did not occur in the fourth session, the teacher could have intervened by suggesting the formation of groups of 2 children, without reducing the cognitive and mathematical benefit of the task (it was the right moment).

CONCLUDING REMARKS

Summarizing, it could be argued that there are two factors which seem to play a decisive role in the successful solution of challenging tasks: the first is related to the task itself and it is the possibility for control of the outcomes by the pupils' themselves. The second one is related to the management of the knowledge by the teacher: facilitating and supporting the pupils without reducing the cognitive value and the mathematical meaning of the problem. In this way the problem's devolution takes place and thus the related mathematical knowledge is constructed.

Transcript

First session [...]

01 Teacher: Has the cake been cut into equal pieces?

02 Dinos: There is some leftover, because there are not many of us.

03 Alex: I should have cut bigger pieces...

04 Teacher: Are all the pieces the same?

05 *Both children compare the pieces they have cut visually. Dinos is taking Alex's piece and as he puts it on the other piece of cake, he finds out that his piece is smaller and he gets angry.*

06 Dinos: I will take the big one (*he means the piece of cake that is left over*)

07 *All the others strongly disagree with his action*

08 Teacher: Has the cake been cut into equal pieces?

09 Costas: I have a smaller piece than Antonio does (Antonio is junior-pupil)

10 *Costas reaches this conclusion by comparing the two pieces via putting his piece on top of the other piece of cake.* [...]

Second session

11 Alex: We are going to cut the cake two by two pieces.
12 Teacher: Hmm, how are you going to do that?
13 Alex: I will cut one piece and I will put it on the top of the first.
14 Teacher: So, then?
15 Alex: These two pieces will be the same. [...]
16 Teacher: Is the leftover piece of cake the same as yours?
17 Antonio: You are going to eat it Miss, because it is big and you are the older.
18 Costas: Alex should have cut the cake into bigger piece [...]

Third session [...]

19 Teacher: Is this piece of cake in the middle the same size with all the others?
20 Antonio: It is big. You should eat it!
21 Teacher: I would like to have a piece of cake the same size as yours.
22 Alex: Take my piece, Miss.
23 Dinos: So, are you going to be the one who will eat the bigger? Oh, no. I don't agree with this.
24 Teacher: So, the cake has not been cut into equal pieces!
25 Pupils: It is difficult, Miss!
26 Teacher: We are going to try again!
27 Pupils: Yes, but not today! [...]

Fourth session

28 Dimitra: We would like to be four of us Miss, like the boys did!
29 Alex: Girls are jealous of us.
30 Dinos: Vaso you better cut an other cake with Mrs Evi. (*Vaso is a junior-pupil and Evi is the other classroom teacher*)
31 Teacher: I agree, if Vaso agrees too.
32 Vaso: I agree.
33 Teacher: Well, today we are three groups and we are going to cut three cakes. I am wondering which group will be the one, which will successfully cut the cake into equal pieces. I would love to hear what you are thinking about [...]
34 Dimitra: You are only two. So you are going to have half cake. This is how my mother does when the chocolate bar is too big.
35 Vicky: It is so easy! You will cut it in the middle.[...]
36 Dinos: I will fold it, as my mother does when I need a paper to draw.
37 *Dinos is folding the paper and cuts it* [...]

NOTES

1. In the early grade, which usually lasts 2 years, enroll pupils of 5 to 6 years old. Children of the first year are called junior pupils, whereas children of the second year senior pupils.
2. After this first problem was solved, four more sessions followed, where pupils engaged in sharing cakes of other shapes: square, rectangle, triangle (isosceles and equilateral).

3. There were two teachers in the classroom, exchanging roles, according to the content of the teaching.

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