DEVELOPMENT OF 10-YEAR-OLDS' MATHEMATICAL MODELLING

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This paper addresses the developments of a class of fifth-grade children as they worked modelling problems during the first year of a 3-year longitudinal study. In contrast to usual classroom problems where students find a brief answer to a particular question, modelling activities involve students in authentic case studies that require them to create a system of relationships that is generalisable and re-usable. The present study shows how 10-year-olds, who had not experienced modelling before, used their existing informal mathematical knowledge to generate new ideas and relationships, and how these developments were fuelled by significant social interactions within small group settings.

INTRODUCTION

Our ever-changing global market is making increased demands for workers who possess more flexible, creative, and future-oriented mathematical and technological skills (Clayton, 1999). Of importance here is the ability to make sense of complex systems (or models), examples of which appear regularly in the media (e.g., sophisticated buying, leasing, and loan plans). Being able to interpret and work with such systems involves important mathematical processes that are under-represented in the mathematics curriculum, such as constructing, describing, explaining, predicting, and representing, together with quantifying, coordinating, and organising data. Dealing with systems also requires the ability to work collaboratively on multi-component projects in which planning, monitoring, and communicating results are essential to success (Lesh & Byrne, in press).

Given these societal and workplace requirements, it is imperative that we rethink the nature of the mathematical problem experiences we provide our students— in terms of content covered, approaches to learning, ways of assessing learning, and ways of increasing students' access to quality learning. This paper reports on one approach to addressing this issue within the primary school curriculum, namely, through mathematical modelling activities. Although these activities provide all students with rich learning opportunities, their use with younger children has received limited attention.

MATHEMATICAL MODELLING FOR CHILDREN

Problem solving over the past couple of decades has typically engaged children in problems where the "givens," the "goals," and the "legal" solution steps have been specified clearly; that is, the interpretation processes for the student have been minimalized or eliminated. The difficulty for the solver is simply working out how to get from the given state to the goal state. The solutions to these problems are usually brief answers obtained from applying a previously taught solution strategy, such as "guess and check," or "draw a diagram." Furthermore, although these problems may refer to real-life situations, the mathematics involved in solving them is often not real world and rarely do the problems provide explicit opportunities for learners to generalize and re-apply their learning (English & Lesh, in press). While not denying the importance of these problem experiences, they do not address adequately the knowledge, processes, and social developments that students require in dealing with the increasingly sophisticated systems of our society. Mathematical modelling activities, in the form of meaningful case studies for children, provide one way in which we can overcome this inadequacy.

As used here, models are systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behaviour of some other experienced system (Doerr & English, 2001). The modelling activities of the present study engage small groups of children in challenging but meaningful problem situations that encourage multiple solution approaches and multifaceted products. Key mathematical constructs are embedded within the problem context (which takes the form of a case study) and are elicited by the children as they work the problem. In contrast to typical classroom problems, these case studies require children not only to work out how to reach the goal state but also to interpret the goal and the given information along with permissible solution steps. Each of these components might be incomplete, ambiguous, or undefined; furthermore, there might be too much data, or too little data, and visual representations might be difficult to interpret. When presented with information of this nature, children might make unwarranted assumptions or might impose inappropriate constraints on the products they are to develop. This is where the input from group members plays a powerful role.

Unlike traditional non-routine problems, modelling activities are inherently social experiences (Zawojewski, Lesh, & English, in press). Their design demands the work of small teams of students, who must develop a product that is explicitly sharable. Team efforts are required to generate the multifaceted products that involve descriptions, explanations, justifications, various mathematical representations, and frequently, media presentations. Numerous questions, issues, conflicts, revisions, and resolutions arise as students develop, assess, and prepare to communicate their products. Because the products are to be shared with and used by others, they must hold up under the scrutiny of the team (Zawojewski, Lesh, & English, in press).

DESCRIPTION OF THE STUDY

The present study involves a class of 30 ten-year-olds and their teachers, who are participating in a 3-year longitudinal study of children's developments in mathematical modelling. The children are from a co-educational private school that caters for preschool through to Year 12. Drawing upon the multitiered teaching experiments of Lesh and Kelly (2000), the study has adopted a four-tiered collaborative model that addresses the simultaneous development of researchers, classroom teachers, preservice teachers and classroom students (English, submitted). This paper, however, is confined to the developments of the children.

The data reported here are from the first year of the study (2001) and are drawn from two of the activities the children completed towards the end of the program. The program commenced in June 2001 and continued until November 2001, with a month's break between September and October. The children had not experienced modelling activities prior to the program. The classroom teacher implemented each of the activities. Preparatory meetings and feedback sessions were conducted with the classroom teacher and with the preservice teachers. The children usually worked the modelling activities twice a week, with each session lasting around 80 minutes.

Description of the Modelling Activities

The program commenced with preparatory experiences where the children expressed their feelings about, and their perspectives on, mathematics and mathematical problem solving and posing as they occurred in and out of school. The children also explored a range of non-routine problems, where they analyzed the mathematical structures, identified similar structures, discussed ways they would approach the problems, and shared their solutions. These experiences were followed by introductory modelling activities (from the "Packets Investigators"; ETS, 1997), with more advanced modelling being introduced next, as indicated below.

The activities that elicited the present data are the *Aussie Lawn Mowing Problem* and the *Christmas Holidays Jobs Problem* (adapted from Hjalmarson, 2000, and Lesh & Lehrer, 2000). Both problems involve interpreting and dealing with multiple tables of data, exploring relationships among data, using proportional reasoning and the notion of rate, and representing findings in visual and written forms. The Aussie Lawn Mowing Problem involves three components: (a) a warm-up task comprising a mathematically rich "newspaper article" designed to familiarize the children with the context of the modelling activity, (b) questions to be answered about the article, and (c) the model-eliciting activity. Excerpts from the third component appear as an appendix. The Christmas Holidays Job serves as a model application activity where children apply their learning within a new context (selecting part-time and full-time vendors for an amusement park, based on their performance in the previous Summer). The children spent two sessions in completing each of the problems. After the groups had developed their models, they presented their work to the class for questioning and constructive feedback. Next, a class discussion focused on the key mathematical ideas and relationships that developed.

Data Sources and Analysis

The data sources included audio- and video-tapes of the children's responses to the problem activities, together with their work sheets and final reports detailing their models and how they developed them. Field notes were also taken. The data were analyzed for evidence of children's mathematical and social developments over the course of the activities. In the next section, consideration is given to the progress of groups of children across the two problems. There were six groups of mixed achievement levels. The children's developments are reported in terms of cycles of increasing sophistication of mathematical thinking, with each cycle representing a shift in thinking (Doerr & English, 2001). The cycles also display significant social interactions that impacted on the children's modelling developments.

RESULTS

Cycle 1: Focusing on Subsets of Information

Each of the groups commenced the Aussie Lawn Mowing Problem by scanning the tables of data to find employees who scored highly in one or more of the categories (i.e., hrs worked, no. of lawns mowed etc.). Limited mathematical thinking was displayed in this unsystematic approach, as evident in Gavin's (Group 1) comments: "*Also, I think Jonathon is good because he works top hours and doesn't drive much. Also mows quite a lot of lawns and makes a bit of money....*"

While most groups initially used this approach, Group 4 decided to choose employees "with different specialities" and remained with this decision in developing their model: "*We'll get Travis to work at the shop selling fertilizers and all that—from 8 to 5—that's about 9 hours. He earns the most money from the info we've got, so if we have our best worker at the shop they make the most money.... What's Matthew's speciality? He loves big lawns. Matthew could work all the time from 9 to 5 because mowing is a lot...what's Jonathon like? Jonathon likes small lawns. He could do the small lawns.*" Because the Lawn Mowing Problem lacked some information, the groups frequently brought in additional ideas and assumptions based on their real-world knowledge (e.g., hours the garden shop should open; how much customers should be charged; how much the employees should be paid).

With the exception of one group, none of the groups commenced the problem by considering whether some items of information were more important than others or whether some information might be irrelevant. The children did, however, engage in heated debates over how to interpret "kilometres driven" and whether more kilometres driven indicated a more desirable employee. Again, the children used their informal knowledge to make a number of conjectures and justify their claims:

Tim: No, wait a second. We're looking at how many kilometres you drive **in a truck that's owned by them; that's bad.**

Samantha: No, it's good if you drive a lot because that means that you're not a slacker, not lazy, and you're willing to go and drive over to someone's.

Ben: Isn't it social, like they're just going out to buy some beer or something.

Tim: **No, Company truck. It costs a lot of money to have company trucks.**

Because most of the groups did not use any systematic approach to tackle the problem initially, they frequently argued over which employees should be chosen. This led them to see the need to mathematize, in some way, their employee selection. The groups began to use two main mathematical operations to aggregate the data for each employee, namely, (a) simply totalling the amounts in each category (hours worked, kilometres driven etc.), and (b) finding the average for each category.

Cycle 2: Using Mathematical Operations

The need to mathematize their procedures was initiated in Group 1 by Joanne, who challenged Gavin over his unsystematic approach: "*Gavin, not necessarily he [Jonathon] mows the most lawns..... How about we work out the hours they work. How about we work out their average*." She justified her decision by explaining, "*Well, it's kind of difficult working out how much they worked each month. Sometimes they worked less and sometimes more*." Gavin and Alison remained unconvinced, however, so Joanne, along with Mindy, proceeded to work out the average number of hours worked, lawns mowed (treating, 'big,' 'medium,' and 'small' separately), kilometres driven, and money from products sold for each employee listed in the tables. At the same time, substantial discussion and argumentation took place when the group members tried to convince Alison that she was misinterpreting the table of money from products sold (interpreting it as the amount of money the company *paid to the employees*.) Despite repeated explanations (e.g., "*Average money per week from **products sold**. They **don't give** them this.*"), Alison would not accept their arguments. In fact the group became quite bogged down as Gavin and Alison continued to argue over the meaning of the table.

Joanne and Mindy, however, remained very much on task and frequently had to remind Gavin and Alison that they had to apply a systematic method: "*You have to work it out properly. Like, you can't just get someone you like and say, 'O.K., I like this one'...I don't think that works. You have to work out like Joanne's doing—working out the average —and then understanding what they are and everything*."

Because Alison (and at times, Gavin) insisted on using an unsystematic approach, Joanne and Mindy frequently reminded them of the problem goal and clashed over their interpretations of "best people." For example, when Gavin and Alison were insisting that they include a person "who isn't *that* bad," Joanne and Mindy insisted, "*But there are four better people. We need the **best** people*." Alison retorted, "*It* [the problem statement] *doesn't say **good** employees.*"

The group continued to be divided over their approaches to solving the problem. While Joanne was working out all the averages, Mindy was repeatedly reminding Alison and Gavin of the need for a method: "*Some structurable thing*." Mindy also challenged the selections of Gavin and Alison:

Mindy: Alison, how do you know that? (that Mathew is one of the best)

Gavin: Look at all the stats!

Mindy: So, just on that information, you can't just say, "O.K., we want him." This is pulling people out of the hat again.

Alison: We're not pulling people out of the hat. We're just compromising.

Mindy: Where's the structure for that?

Once all the averages had been found, the group did not progress further. They selected those employees who scored high averages across all categories, explaining in their report: "*Well, we worked out the average for average money per week from the products sold and looked for the 4 highest and did the same for the hrs worked.*"

Cycle 3: Identifying Trends and Relationships

Two of the groups progressed to looking for trends and relationships to help them choose the employees for the Lawn Mowing Problem. Group 4, for example, explored trends within categories (e.g., "*Kim is always gaining... 200, 250, 256*" [in the money category]). This led the group to compare trends across categories: "*So Travis should be our first guy. He may have done 5 less hours than Jonathon, but he did more jobs.*" Group 4 did not progress to the notion of rate, however, in part because they kept conjecturing about why the trends occurred (e.g., "*With the lawns mowed, they hand them out maybe, but then if they hand them out, he* [Aaron] *might not have been able to get them because someone else got them*").

On the applied problem (*Christmas Holidays Job*) all of the groups transferred their learning and frequently referred to what they had done in the previous problem (which they had solved 5 weeks earlier). Furthermore, 4 of the 6 groups extended their understanding by exploring relationships between hours worked (by vendors in a previous Summer) and money collected (for busy, steady, and slow periods). The groups used these relationships as their basis for deciding whom to employ full and part time in the next Summer. In Group 2, for example, Marianne was recapping on what she thought was the group's decision: "*I thought the person who works the less or the least but makes the most money we were going to employ because that would work.*" To counter some disagreement and misunderstanding that followed (e.g., "*No, because if they work tons of time they earn tons of money*"), Marianne provided a concrete example: "*Because, like someone might work for say, 10 days and make $5000, and somebody might work for 5 days and they get $10,000.*"

Likewise, Group 5 spent considerable time debating the relationship between hours worked and money earned, which led to observations such as, "*Chad works 20 hours. Will works 19. Chad earns $1031. Will earns $1034. Will works one hour less, and he makes $3 more.*" Earlier, Group 5 had been working out averages, but once they had explored these relationships, a couple of the group members noted, "*Why did we have to use averages? Why in the world did we use averages? That doesn't make sense!*" Using their understanding of kilometres per hour, two of the groups progressed to calculating rates (money per hour) in developing their model.

CONCLUDING POINTS

The first year of this study has shown how one class of 10-year-olds was able to work successfully with mathematical modelling problems when presented as meaningful, real-world case studies. On the problems addressed here, the children progressed from focusing on isolated subsets of information to applying mathematical operations that helped them aggregate the given data. Moreover, some children displayed an explicit awareness that they needed to adopt a structured approach to developing their final model. While some groups remained with averages, other groups moved on to discover relationships and trends in the data, and applied this learning to the second problem. These developments took place in the absence of any formal instruction, and involved the children in describing, constructing, explaining, justifying, checking, and communicating their ideas. Of significance in these developments are the social interactions that took place naturally within the groups. These interactions engaged the children in planning and revising courses of action, challenging one another's assumptions and claims, asking for clarification and justification, monitoring progress, and ensuring the group worked as a team. Few traditional problems generate learning of this nature.

REFERENCES

Clayton, M. (1999). What skills does mathematics education need to provide? In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the mathematics curriculum* (pp. 22-28). London: Falmer Press.

Doerr, H., & English, L. D. (2001). A modelling perspective on students' learning through data analysis. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the International Group for the Psychology of Mathematics Education* (pp. 361-368). Utrecht University.

Educational Testing Service (1997). Packets Investigators. Princeton, NJ: ETS.

English, L. D. (Submitted). Reconciling theory, research, and practice: Multilevel collaboration.

English, L. D., & Lesh. R. A. (2001). Ends-in-view problems. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: A models and modelling perspective on teaching, learning, and problem solving in mathematics education.* Mahwah, NJ: Erlbaum.

Lesh, R. A., & Byrne, S. (In press). Technology and distance learning in education: Focusing on complexity, collaboration, and communication rather than transmission of information. In T. Duffy (Ed.), *Web-based learning environments to support learning at a distance*. Hillsdale, NJ: Lawrence Erlbaum.

Lesh, R. A., & Lehrer, R. (2000). Iterative refinement cycles for videotape analyses of conceptual change. In R. A. Lesh & A. E. Kelly (Eds.), *Handbook of research design in mathematics and science education* (pp. 665-708). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R. A., & Kelly, A. E. (2000). Multi-tiered teaching experiments. In R. A. Lesh & A.E. Kelly (Eds.), *Handbook of research design in mathematics and science education* (pp. 197-230). Mahwah, NJ: Lawrence Erlbaum.

Hjalmarson, M. (2000). *Green Thumbs Garden to Open Soon*. Unpublished Modelling Problem. Purdue University.

Zawojewski, J. S., Lesh, R. A., & English, L. D. (2001). A models and modelling perspective on the role of small group learning. In R. A. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: A models and modelling perspective on teaching, learning, and problem solving in mathematics education.* Mahwah, NJ: Lawrence Erlbaum.

APPENDIX

Aussie Lawn Mowing Problem: Green Thumbs Garden to Open Soon

Part (c): The model-eliciting activity

Background Information: At Green Thumb Gardens, James Sullivan will provide lawn-mowing service for his customers. Another local landscaping service has closed, so he has offered to hire 4 of their former employees in addition to taking on some of their former clients. He has received information from the other landscaping business about the employee schedules during December, January, and February of last year. The employees were responsible for mowing lawns and selling other yard products like fertilizer, weed killer, and bug spray. The other business recorded how many hours each employee worked each month, the number of lawns each employee mowed, and how much money they made selling other products. The lawns mowed are divided into big, medium, and small jobs. Big jobs may have larger lawns or additional work than medium or small jobs. Some lawns may be small, but may have many obstacles for the mower to get around or there may be different kinds of edging or trimming to be done which determine the size of the job. They also recorded the kilometres driven to clients in one of the green company trucks during each month.

Problem: James needs to decide which four employees he wants to hire from the old business for this summer. Using the information provided, help him decide which four people he should hire. Write him a letter explaining the method you used to make your decision so that he can use your method for hiring new employees each summer. (The following **tables** were supplied [data for 5 of the children have been omitted here].

Hours Worked			
Employee	**Dec.**	**Jan.**	**February**
Jonathan	80	80	80
Cynthia	75	65	70
Jack	66	64	63
Kayla	45	50	55
Tim	67	70	79

Kilometres Driven			
Employee	**Dec.**	**Jan.**	**Feb.**
Jonathan	198	200	201
Cynthia	199	201	198
Jack	197	199	198
Kayla	201	203	199
Tim	200	199	200

Total Number of Lawns Mowed									
	December			**January**			**February**		
Employee	**Big**	**Medium**	**Small**	**Big**	**Medium**	**Small**	**Big**	**Medium**	**Small**
Jonathan	15	12	30	16	14	34	16	15	35
Cynthia	18	10	35	19	12	35	14	16	36
Jack	14	16	22	15	16	22	13	16	22
Kayla	15	13	15	14	13	17	15	12	18
Tim	20	12	14	22	14	16	20	13	25

Average Money Per Week from Products Sold			
Employee	**December**	**January**	**February**
Jonathan	$150	$175	$170
Cynthia	$75	$80	$80
Jack	$125	$150	$150
Kayla	$80.	$72	$65
Tim	$135	$130	$125