HOW TO PROMOTE CHILDREN'S UNDERSTANDING OF FRACTIONS? AN EXPLORATORY STUDY

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This study involves a teaching experiment aimed to improve children's understanding of fractions. Fifty seven children (8-10 years old) were divided into three groups: Experimental and Control Groups: 3rd graders with no previous instruction in fractions; and Reference Group: 4th graders who were taught in a traditional way based on a mechanistic approach to fractions and focused on application of rules. Children in the Experimental Group were given a teaching experience based on problem solving situations and on activities that enabled them to think and discuss their thought processes when solving problems involving fractions. All participants were given a pre and a post-test. The Experimental Group showed a more expressive improvement than the other groups. The results are discussed in terms of the nature of instruction in fractions understanding.

Introduction

This study is based on conceptual field theory in which the fraction is considered one out many other concepts that takes part of multiplicative conceptual field, or simply multiplicative structures, such as multiplication, division, proportion, linear function, vectorial space. According Vergnaud (1983, 1988, 1997) multiplicative conceptual field is a set of situations that requires a diversity of concepts, actions and symbolic representations consistently linked to each other.

When dealing with fractions, for instance, children have to deal with many other concepts and with a large range of situations involving part-whole relations; equal or equivalent units; exhaustive division/sharing (that is, nothing remains); fair sharing; and equality or equivalence of shares. Also, they have to deal with the fractional language and its mathematical representation (*a/b*). Thus, fraction is not an easy concept to be understood and to be meaningfully mastered by children.

Children's difficulties with fractions are well known among researchers and educators (e.g.; Kerslake, 1986; Koyama, 1997; Nunes and Bryant, 1996; Watanabe, Reynolds & Lo, 1995). They have difficulties to understand fractions as mathematical ideas (a number or a quantity) and construct meanings of fractions. According to Streefland (1991) some of the sources behind the failure of fraction instruction reside in the complexity of this concept, and, also, in the traditional approach to fractions which is formal and mechanistic from the very start. Also, the teaching is often detached from reality, consists of rigid rule-oriented instruction, and the informal knowledge children have acquired is ignored entirely.

Thus, one may ask whether children would overcome their difficulties and develop an understanding of fractions if they were given an instructional experience that emphasises realistic and meaningful problem solving situations and that allowed children to show and discuss their informal knowledge (strategies used, informal representations of fractions).

THE STUDY

This study aimed to explore the role played by an instructional experience to improve children's understanding of fractions through a variety of problem solving situations. The idea was that third grade children who were taught about fractions for the first time would benefit more from this alternative teaching programme than fourth grade children who were taught in a traditional approach to fractions. Thus, a study was carried out in which a pre and a post-test were applied to children who were divided into three groups: Experimental Group (alternative instructional approach to fractions); Control Group (pre and post test only); and Reference Group (traditional approach to fractions).

Participants and groups

The participants were 57 low-class children aged 8 to 10 years old attending a public school in São Paulo, Brazil. These were divided into three groups:

Experimental Group (EG): 19 third graders (mean age: 8 years old) with no previous instruction in fractions were given a teaching experience based on a sequence of instructional activities that enabled them to think, discuss and communicate verbally their thought processes when solving problems involving fractions

Control Group (CG): 20 third graders (mean age: 8 years old) with no previous instruction in fractions were given a pre and a post-test only.

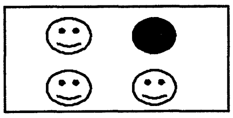
Reference Group (RG): 18 fourth graders with previous formal instruction in fractions were taught about fractions in a traditional way based on a mechanistic approach to fractions, detached from realistic situations and focusing on rigid application of rules.

Pre and the post-test

The pre and post-test consisted of a series of 15 items involving fractions related to continuous and discontinuous quantities (Figure 1).

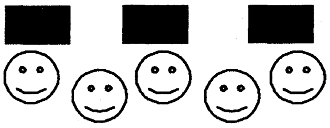
Figure 1: Examples of some of the items in the pre and in the post-test.

João has painted one of the four faces in a set. How can you represent the number of faces he had painted in relation to the total number of faces in the set?



Response:

Could you divide equally 4 bars of chocolate between 5 children? How much chocolate does each child get?

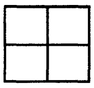


Response:

Make a circle around one third of the collection of hearts bellow. How can you represent the number of hearts you circled in relation to the total number of hearts in the collection?

Response:

Could you paint half of the figure bellow? How can you represent the number of parts painted in the figure bellow?



Response:

How can you represent the amount of parts painted in the figure bellow?

Response:

Some of these items involved part-whole relations and others involved sharing. In these items children had to solve the problem and then numerically represent its result in a form of a fraction (a/b). Both tests had the same level of difficulty and were mathematically equivalent. They were applied to all participants and solved by means of pencil and paper.

Overview to the teaching experiment

The teaching experiment offered to third grade children in the Experimental Group comprised ten 2-hours sessions, two sessions per week over five weeks. The sessions involved whole class activities focusing on a problem task and follow-up questions and discussion posed by the researcher who acted as a teacher according to a previously planned didactical sequence. Children also worked in pairs and in small groups. The role of the researcher was to provide assistance to the students, understand and clarify their thinking (insights and difficulties) and, at the end of each session, offered them a conclusion about the whole activity and its solution. In this instructional experience the approach to fractions was based on (a) realistic and meaningful problem solving situations; (b) student's own fragmentary and informal knowledge; (c) interactive situations: discussion and collaborative work; and (d) student's thinking about their own thought processes when solving problems and communicating verbally their strategies and ideas. Diagrams, manipulative and concrete materials were widely used in a large range of situations involving continuous and discontinuous quantities in which fractions were taught through partitioning and part-whole relations.

RESULTS

The results in the pre and in the post-test, in the three groups of children, were analysed according to the number of correct responses and according to the types of errors children made.

Correct responses

Table 1 presents the general scores in this study. It can be noted that no significant differences were found among the three groups in the pre-test. This result was confirmed by means of Mann-Whitney test (EG vs. CG: $p = 0.6080$; EG vs. RG: $p = 0.7330$; and CG vs. RG: $p = 0.7502$). However, there were marked difference between groups in the post-test (Man-Whitney - EG vs. CG: $p = 0.0000$; EG vs. RG: $p = 0.0000$; and CG vs. RG: $p = 0.0000$). This was due to children in the Experimental Group and in the Reference Group being more successful than children in the Control Group; and children in the Experimental Group performing better than those in the Reference Group.

Table 1: Number and percentage of correct responses.

	Experimental Group (n = 285)	Control Group (n = 300)	Reference Group (n = 270)
Pre-test	29 (10.2%)	36 (12%)	29 (10.7%)
Post-test	199 (69.8%)	37 (12.3%)	86 (31.8%)

The performance of each group in the pre and post-test was compared by Wilcoxon test. This revealed that there was no significant difference between pre and post-test for the children in CG ($p = 0.5014$). In contrast, EG and RG children performed significantly better in the post-test than in the pre-test ($p = 0.001$ and $p = 0.003$, respectively). This suggests that both groups of children improved their performance after instruction. However, it is important to know whether the progression children had was similar in both groups.

Looking at each child individually it was possible to identify that there were children who showed a progression from pre to post-test, while others showed a regression in their scores or did not showed any change in their performance (stability over tests). Table 2 and Table 3 presents the incidence of children in each group according to their progression, stability over tests and regression.

Table 2: Number and percentage of children in each group according to their improvement, stability and regression from pre to post-test.

	Experimental Group (n=19)	Control Group (n=20)	Reference Group (n=18)
Improvement	19 (100%)	8 (45%)	17 (94%)
Stability	0	4 (20%)	1 (6%)
Regression	0	7 (35%)	0

Children in the Experimental Group and in the Reference Group were able to benefit from the instruction given, independently of whether the instruction was a traditional way of teaching (RG) or an alternative one (EG). Regression in performance was only to be found among children in the Control Group. However, when we look at the improvement observed in more details, we can verify that the level of improvement was higher among children in the Experimental Group than among those in the Reference Group, as shown in Table 3.

It can be seen that children who took part in the teaching experiment had a greater improvement than those who were taught about fractions in a traditional way. This was particularly so in relation to the number of children who had Level 3 improvement (EG: 52.6% and RG: 0%).

Table 3: Number and percentage of children who progressed from pre to post-test.

Level of Improvement	Experimental Group (n=19)	Control Group (n=8)	Reference Group (n=17)
Level 1 (1 – 30%)	0	8 (100%)	12 (70.6%)
Level 2 (31 – 60%)	9 (47.4%)	0	5 (29.4%)
Level 3 (61 – 100%)	10 (52.6%)	0	0

Types of errors

Three main types of errors were identified in the pre and in the post test[1]:

Type 1 – children interpret fractions as natural numbers when symbolic representing their answers. This error was more often when discontinuous quantities were involved.

Type 2 – children interpret fractions as natural numbers when solving the item, during the solution procedure. This error was more often when discontinuous quantities were involved.

Type 3 – children do not recognise the need of having equal shares when solving the items that required the division of the whole into parts. This error occurred when continuous quantities were involved.

Type 1 errors were related to the representation children used when writing down the results of the items; while Type 2 and Type 3 were related to the solution procedure adopted. On the other hand, Type 1 and Type 2 errors expressed their difficulty to discriminate fractions from natural numbers; while Type 3 expressed their difficulty to divide the parts into equal shares. These errors were reported in previous studies in the literature. Table 4 shows the frequency of these errors in the Experimental Group and in the Reference Group in the pre and in the post-test.

Table 4: Incidence of each type of error.

Types of error	Experimental Group		Reference Group	
	Pre-test	Pos-test	Pre-test	Post-test
Type 1	159	29	128	49
Type 2	32	29	23	51
Type 3	18	5	24	26

[1] Children in the Control Group had the same high incidence of errors in the pre and post-test.

As a whole, the main difficulties children had in the pre-test decreased in the post-test. However, children in the Experimental Group showed a greater decrease of errors than those in the Reference Group. This was particularly so in relation to Type 1 and Type 3 errors. Thus, even though both groups seem to have overcame some of their initial difficulties, children who took part in the teaching experiment had a greater improvement than children who were taught in a traditional way. It is noteworthy to stress that children in the Experimental Group were younger and were attending a more elementary grade than children in the Reference Group.

CONCLUSIONS AND DISCUSSION

The results in this study revealed that the teaching experiment improved children´s understanding about fractions. This intervention led third graders to have a better performance than (a) children of the same age and grade who did not receive any instruction on fraction; and (b) children who were older and attending a higher grade of schooling. It is interesting to mention that performance among groups was much the same in the pre-test. This means that children who were not instructed about fractions (third graders in the Experimental and in the Control Groups) showed a similar understanding of fractions as those who already had received instruction on fractions in a traditional way (fourth graders in the Reference Group). After instruction, children who took part in the teaching experiment increased the number of correct response and overcame the initial difficulties they had in the pre-test. Also the number of children who progressed from pre to post-test was greater among them.

The conclusion that can be drawn from the results was that even though both groups benefited from instruction, the alternative approach to fractions given to children in the Experimental Group was likely to be more successful than the traditional approach given to children in the Reference Group.

Another aspect that should be stressed was that a short period of teaching intervention (10 sessions) proved to be beneficial. The teaching appears to have achieved some success. However, one must be cautious about the fact that the teaching proposed here did not explore all the features and situations that are relevant for the understanding of fractions in a broader sense.

To conclude, it is crucial to find ways to reduce some of the difficulties children encounter when dealing with fractions. Also, it is important to encourage classroom discussion: talk to children about their errors, ideas and the strategies they use (independently of whether they are correct or not); and listen to children by encouraging them to communicate verbally their ways of thinking and their interpretation of fractions. Actually, teachers and researchers should, in collaboration, raise and test hypotheses about how to promote children´s understanding of fractions.

Further intervention studies should be conducted with elementary school children. It would be of interest, for instance, to develop a teaching experience in which students would be given the opportunity to explore fractions in a wider context, in different situations, using different forms of representation and in connection with other mathematical concepts. Fractions should be introduced and explored through a large variety of situations (including 'part of a whole model' since it showed to be useful to establish some of the basic ideas about fractions) that allowed children to understand a fraction as a number, to relate it to division and to recognise the equivalence between fractions.

REFERENCES

Koyama, M. (1997). Students'representations of fractions in a regular elementary school mathematics classroom. *Proceedings of the 21st PME Conference*, Lahti, Finland, 3, 160-167.

Kerslake, D. (1986). *Fractions: Children's Strategies and errors: A report of the Strategies and Errors in Secondary Mathematics Project.* London: NFER-NELSON.

Nunes,T. & Bryant, P. (1996). *Children doing Mathematics*. Oxford: Blackwells.

Vergnaud, G. (1983). Multiplicative Structures. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 127-174). New York: Academic Press.

Vergnaud, G. (1988) Multiplicative Structures. In J. Hilbert, J & Bert, M. (Eds.), *Number concepts and operations in the middle grades* (vol.2, pp. 141-161). Hillsdale, NJ: Lawrence Earlbaum Associates Inc.

Vergnaud, G. (1997). The nature of mathematical concepts. In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 5-28). Hove: Psychology Press.

Streefland (1991). *Fractions in realistic mathematics education: A paradigm of developmental research.* Dordrecht: Kluwer Academic Publishers.

Watanabe, T.; Reynolds, A. & Lo, J.J. (1995). A fifth grader's understanding of fractions and proportions. *Proceedings of the 19th PME Conference*, Recife, Brazil, 3, 200-207.