ASSOCIATIONS BETWEEN MATHEMATICALLY INSIGHTFUL[1] COLLABORATIVE BEHAVIOUR AND POSITIVE AFFECT.

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Associations between mathematically insightful behaviour and student affect are studied. A tool is developed to identify this behaviour and study its association with positive affect. Mathematically insightful behaviour displayed by a collaborative pair of senior secondary calculus students is used to illustrate this tool and discuss a situation in which student autonomy, spontaneity and creativity accompanied positive affect. The tool builds upon the ideas of Dreyfus, Hershkowitz, and Schwarz (2001), the concept of 'flow' (Csikszentmihalyi & Csikszentmihalyi, 1992), and the identification of student engagement as they discover complexities (Williams, 2000).

INTRODUCTION

This paper illustrates associations that can exist between positive affect in students and the creative development of a new cognitive structure (a mathematically insightful process of abstraction). An analytical tool that simultaneously displays student cognition, social interaction patterns (Dreyfus, Hershkowitz et al., 2001) and affective indicators (Csikszentmihalyi & Csikszentmihalyi, 1992; Williams, 2000) was used. Attention to whether the dialectic interaction occurred in response to earlier dialogue within the interaction under study facilitated identification and analysis of student spontaneity and student autonomy. An interaction was seen to be more autonomous and spontaneous where the students within the pair responded to interaction from within rather than outside the pair. Where autonomous and spontaneous behaviours were exhibited, further analysis was undertaken to find evidence of creativity. The simultaneous display of affective indicators 'made visible' associations between engagement with the task and students' autonomy and spontaneity in the process of 'abstraction' (Dreyfus, Hershkowitz et al., 2001). It is recognised that the cognitive artefacts students assemble during the creative process could include strategies, ideas and concepts that relied upon contributions from individuals external to the group prior to this interaction. In relation to the interaction under study, these previous influences contribute to the cognitive artefacts students assemble (Dreyfus, Hershkowitz, et al., 2001). The extent to which an interaction is seen as creative in this study depends partly on the spontaneity with which students assemble these artefacts and the students' prior knowledge of their relevance.

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LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Student alienation with school is an issue of current concern (Marks, 2000) and one solution is to create enhanced learning situations where positive affect is linked with new learning (Csikszentmihalyi & Csikszentmihalyi, 1992). The presence of positive affect associated with mathematics learning has been identified in Nicholls' (1983) task-centred rather than ego-centred learning. The characteristics of students undertaking task-centred learning can be seen in instances of learning through 'discovered complexity' as identified by Williams (2000). Discovered complexity occurs during task completion where a group of problem solvers perceive intellectual and conceptual complexities not evident at the commencement of the task. When a complexity is discovered, the group spontaneously formulate a question that leads to higher level thinking (analysis, analytical-synthesis, synthesis, and evaluative-synthesis) in the domain of mathematics. The resolution of the question leads to an abstraction associated with this complexity. Both Nicholls (1983) and Williams (2000) linked student learning accompanied by positive affect to 'flow' (Csikszentmihalyi & Csikszentmihalyi, 1992). Flow is an optimal learning condition that may occur when a person works just above their present skill level on a challenge almost out of reach. Individuals or groups in flow become so engrossed with the task at hand that they lose awareness of self, time and the world.

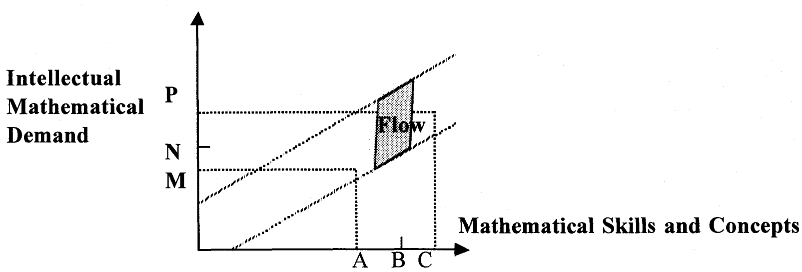


Figure 1. Representation of associations between discovered complexity and flow

Williams (2000) illustrated the fit between discovered complexity and the conditions for flow by modifying the schematic representation (Figure 1) developed by (Csikszentmihalyi & Csikszentmihalyi, 1992). A student's perceived level of skills and concepts and perceived challenge comfortably overcome are represented by **A** and **M** respectively. Students in flow are seen as working to achieve a goal represented by a point within the shaded region of flow **(B, N)**. This goal is just above their perceived skills and concepts level and involves a challenge perceived to be almost out of reach (flow). Once this goal is achieved, each student's perceived skills and concepts level is **B** and they can comfortably achieve a challenge of **N**. The shaded region representing flow is now located to the right of **B** above **N** between the parallel lines. To sustain flow would require a discovered complexity that led to a goal represented by a point like **(C, P)**.

By definition (Williams, 2000), the mathematical ideas in a discovered complexity are new to all students in the group and the teacher does not contribute new mathematical ideas during the interaction. Before the present interaction, other class members, the teacher, or another source may have contributed to the cognitive artefacts the students chose to assemble. In such situations where the student group develops new concepts through discovered complexity, the type of student response described is seen to be partially attributable to the task and the implicit pedagogical approach (Williams, 2000). The process students enact when working with discovered complexity is an insightful process of autonomous, spontaneous, and creative abstraction similar to the activity described by a research mathematician (Chick, 1998). Dreyfus, Hershkowitz and Schwarz (2001) are interested in 'abstraction'—an activity of vertical reorganisation of 'previously constructed mathematical knowledge into a new structure' (p. 377). (Vertical refers to a new mathematical structure as opposed to strengthened connection between a mathematical structure and a context ('horizontal')). They have diagrammatically represented a simultaneous display of cognitive activity and social interaction present during the abstraction process. Six categories of dialogue were used in the analysis of social interaction—control, elaboration, explanation, query, agreement and attention. Through analysis of the dialogue of participants as they undertake the social process of critical inquiry, the cognitive elements of the process of abstraction were made more visible. These nested elements of the process of abstraction as identified were: (a) 'recognising'—seeing a previously known mathematical structure within a new context or realising a previously known mathematical structure fits a new context; (b) 'building-with'—using a combination of previously generated abstracted entities in a new context; and (c) 'constructing'—use of assembled resources to vertically re-organise a mathematical structure.

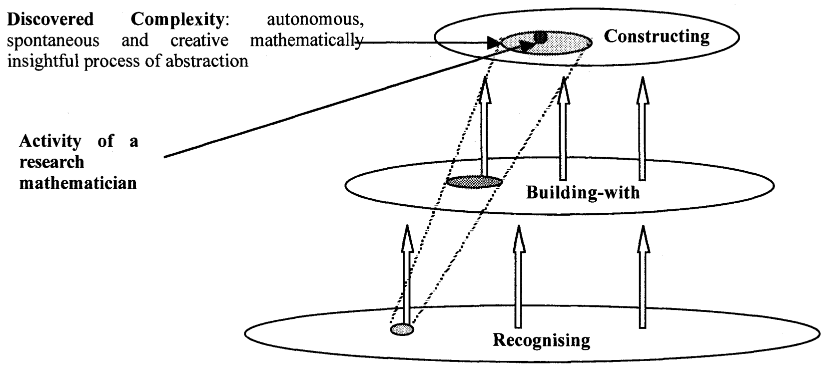


Figure 2: Relationship between nested elements of abstraction and discovered complexity

Figure 2 illustrates the relationship between discovered complexity, the activities of a research mathematician (Chick, 1998), and the epistemic elements in the abstraction process. In examining the process of creation of mathematical concepts, Chick (1998) reported the presence of positive affect as strategies, ideas and concepts were synthesised to produce a novel mathematical insight about the complexities in a mathematical structure. This is consistent with Krutetskii's (1976) description of the mental activity of combining concepts to create a new idea.

Engagement in a task was monitored by Williams (2000) through observation of video data to identify indicators of positive affect: (a) eyes on the task; (b) pens on the task page or bodies leaning in towards the task; (c) unaware of the world around; (d) participating in the interaction; (e) students building on each others' ideas (latching comments); (f) exclamations of pleasure.

Analysis of what triggered each part of the dialectic interaction and the nature and source of the conceptual artefacts upon which students draw will facilitate the identification of insightful mathematical behaviour.

BACKGROUND ORIENTATION FOR THE ILLUSTRATIVE TRANSCRIPT

Three students, William, Talei and Gerard were videotaped working as a collaborative group to solve the unfamiliar challenging problem 'Understanding the Double Derivative'. The teacher selected the groups, provided the task sheet and stated that $f''(x)$ meant the derivative of $f'(x)$. Students followed a familiar classroom-working pattern; they worked in groups for twelve minutes then made their first brief report to the class as a whole. The interaction reported occurred eleven and a half minutes into group-work and continued for one and a half minutes.

In the previous twelve months, all three students had worked briefly with sign diagrams and extensively with gradients of curves in individual and group settings through investigation and exercises. The students had not previously encountered second derivatives. Gerard had not been exposed to the term 'curvature' in a mathematical context. Talei and William were working to mathematically formulate 'curvature' in another mathematics subject studied simultaneously (with another teacher). At the time of this research, they had not achieved their goal. The teacher made no reference to the idea of curvature or to the other subject.

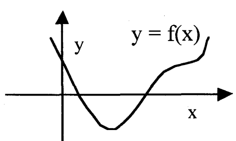


Figure 3: The shape of y = f(x) in the 'Understanding the Double Derivative Task'

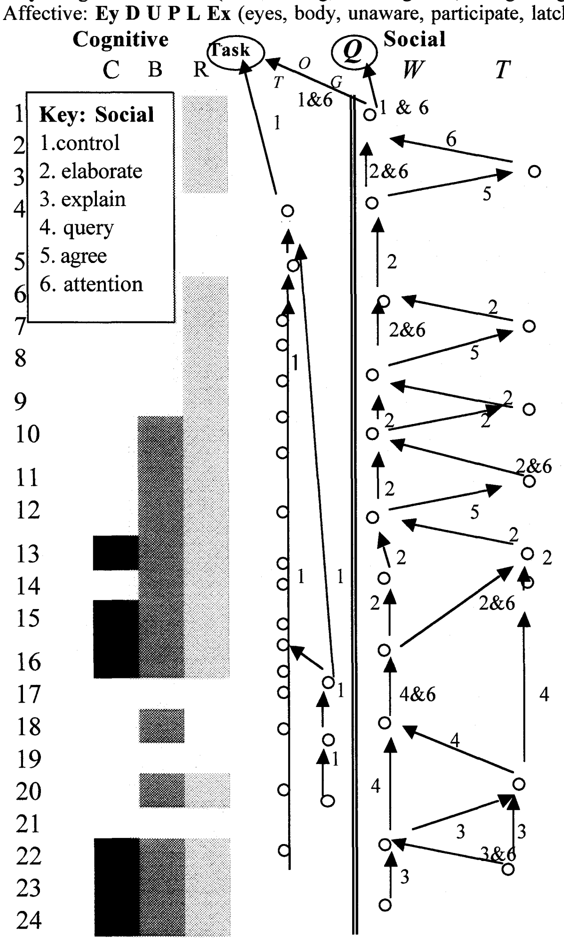
The task 'Understanding the Double Derivative' required students to use the graph (Figure 3) as a starting point. By sketching the graphs of $f'(x)$ and $f''(x)$ and the sign diagrams to the three graphs and generating other information if they chose to do so, students were required to investigate links between $f''(x)$ and $f(x)$. The task sheet contained general suggestions and questions about possible ideas to explore but these were not prescriptive and did not provide hints or directions about specific ideas to explore, strategies to use or pathways to follow. They included questions and statements like: 'What happens to key features on one graph in another graph?' 'Search for patterns and reasons why these patterns exist'. 'Predict'. 'Check'. 'Is the $f''(x)$ graph sufficient information to be able to generate the f(x) graph? Explain'.

Key to transcript: '}' one individual completed or extended the previous statement (latch); '/' one individual cuts across the statement of another (cut).

1	William:	The shape. [Time 11 min. 30 secs.]
2	Talei:	Uh hu.
3	William:	To there.
4	Buzzer:	[Denotes the end of collaboration time].
5	Teacher:	[Begins to organise the reporting phase. No mathematical talk, just organisational. Continues simultaneously with interaction to Line 24]
6	William:	Well if you look between there and there it is curving}
7	Talei:	{Yes.
8	William:	The curve is getting less and less}
9	Talei:	{Yes ... it's the gradient yes it's}
10	William:	{'till there which means it's negative ... it's steep}
11	Talei:	{Yes}
12	William:	{'till there when it starts to get more}
13	Talei:	{So when the curvature is
14	William:	and more}
15	Talei:	{smaller but that's
16	William:	but what about between there and there ... what happens? Why is that?/
17	Gerard:	/What are we going to talk about? Do you want/
18	William:	/Why is that ... a positive?/
19	Gerard:	/to talk?
20	Talei:	It's sort of ... that is ... that's true .../
21	Gerard:	/Do you want to talk?/
22	William:	/It turns there but it doesn't turn and go the other way ... it always turns the same way- THAT'S IT! [Moves his hand in the shape of a minimum turning point and watches his hand move]
23	Talei:	Yes} [Smaller hand movement in the shape of a minimum turning point without appearing to see William's hand movement]
24	William:	{It's positively turning that way. [Time 13 mins.]

By the eleventh minute, the group had found several patterns through the discovery of six successive complexities (Williams, 2001). William then commented 'there must be something more'. William's focus of attention on a level beyond searching for patterns and towards finding an over-arching reason for why these patterns occurred produced the implicit spontaneous focus question to another discovered complexity. It appears the pair were responding to an implicit question like 'why do the patterns we have found exist? Gerard did not participate in this final collaboration because his attention was drawn back to the class task (and away from the spontaneous question explored by William and Talei) by the buzzer (signalling the end of group-work) and the organisational comments of the teacher preparing students for the reporting session. For this reason, in this paper Talei and William have been described as a collaborating pair and Gerard as an outsider for this final one and a half minute interaction where 'constructing' occurred between Talei and William. William and Talei gave no indication that they were aware of the activity around them; they did not respond to the external attempts to focus their attention on the reporting process (buzzer, teacher, or Gerard).

Key: Cognitive **C B R** (constructing, building-with, recognising). Social **O W T** External // Internal. Affective: **Ey D U P L Ex** (eyes, body, unaware, participate, latch, exclaim)



Affective

	Ey	D	U	P	L	Ex
1	√	√		√		
2		√		√		
3	√	√		√		
4	√	√	√	√		
5	√	√	√	√		
6	√	√	√	√		
7	√	√	√	√	√	
8	√	√	√	√	√	
9	√	√	√	√	√	
10	√	√	√	√	√	
11	√	√	√	√	√	
12	√	√	√	√	√	
13	√	√	√	√	√	
14	√	√	√	√	√	
15	√	√	√	√	√	
16	√	√	√	√	√	
17	√	√	√			
18	√	√	√	√		
19	√	√	√			
20	√	√	√	√		
21	√	√	√	√		
22	√	√	√	√		√
23	√	√	√	√	√	
24	√	√	√	√	√	

Figure 4. Tool to aid identification of student autonomy, spontaneity, and creativity in the abstraction process (from Dreyfus, Hershkowitz & Schwarz; 2001; Williams, 2000)

DISCUSSION AND CONCLUSIONS

The analysis tool (Figure 4) used to explore associations between student affect and the creative process of abstraction is a diagrammatic representation displaying the cognitive, social, and affective features of the interaction. Numbers down the left hand side of the diagram represent the line of transcript. The shaded squares in the first column indicate the inferred presence of epistemic cognitive elements in the process of abstraction for the individual speaking in that line of transcript. The centre column contains a vertical line that separates the small circles representing those within the collaborating pair from those outside the collaborating pair. The numbered arrows indicate the individuals to whom the speaker appears to attend and the social interaction category to which those prior statements belong. The number of ticks in the grids on the right hand side provides an indication of task engagement. Arrows that cross the separating line in the centre column suggest less spontaneity and autonomy than when the arrows from members of the collaborating pair are directed to dialogue within the pair. By considering cognitive activity in conjunction with social interactions, inferences can be drawn about student creative behaviour. Where students are constructing and the social interaction pattern indicates students are responding only to sources internal to the group, the students display autonomous, spontaneous behaviour that may be creative. The pattern shown by the directions of the arrows from William and Talei (Figure 4, centre column) indicate the pair responded only to each during the interaction. William's first statement is in response to the overall task and the pair's discovered complexity within it (Q in Figure 4). They displayed numerous indicators of positive affect (Figure 4, right column) as this autonomous, spontaneous pair employed mathematically insightful behaviour whilst constructing a new mathematical insight. Evidence of creative behaviour exists and requires further exploration.

William explored the complexity by directing attention to a feature of a segment of the graph using common language to elaborate ideas (Transcript Lines 1,3,6 and 8 categorised as attention in Figure 4). The use of common language rather than precise mathematical language indicated the presence of an amorphous idea rather than a well-structured mathematical idea. Talei recognised the mathematical significance of William's comment (Transcript Line 9); she synthesised the amorphous structure of 'curvature' she recognised as applicable and built-with this as she elaborated further (Transcript Line 13 & 15). Talei and William now moved their pen along the curve undertaking evaluative-synthesis between the f(x) graph (Figure 3) and $f''(x)$ graph. Almost simultaneously they found the minimum point in f(x) (Figure 3) appeared to contradict their theory (the 'but' in Line 15 and 16). Their subsequent behaviour suggested they explored 'Why doesn't the minimum point on f(x) show as a key feature on $f''(x)$?' William's exclamation and comment framed partially in common language (Line 22) and Talei and William's hand movements (Line 23, 24) provide evidence of vertical cognitive reorganisation.

A more detailed characterisation of the factors that contributed this student interaction would require exploration of factors such as classroom culture, task characteristics, and the teacher's introduction to the task. This brief illustrative example shows how this tool can be applied to distinguish between the processes of 'abstraction' in general and a subset of that process where student behaviour is spontaneous and creative. Further work is required to extend the tool to examine the nature of dialectic intervention from an 'outsider' who stimulates rather than inhibits spontaneity and creativity. The tool presents possibilities for examining such incidents and has been designed for use with the video and interview data in the Learners' Perspective Study of Year 8 Mathematics classrooms being conducted in ten countries.

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