FACILITATING THE TEACHING OF SPACE MATHEMATICS: AN EVALUATION

Kay Owens - Charles Sturt University; Cathy Reddacliff – University of Western Sydney; Diane McPhail - NSW Department of Education and Training

The evaluation of an implementation of a NSW teacher development program considered whether a system-led curriculum change for the teaching of the space (pre-geometry) strand of mathematics changed teachers' knowledge about space mathematics and how to teach it and confidence about the teaching of space mathematics. The study identified the role of school facilitators and the comprehensive support package as effective features of the teachers' professional development. The package included the purpose for teaching space written in terms of students' expected learning, background theoretical notes, assessment tasks, lesson plans, and supporting videotapes.

OVERVIEW OF TEACHER PROFESSIONAL DEVELOPMENT

In the late 1980s professional development changed from providing teachers with programs focused on content to focus more on teachers as reflective professionals (Clark, 1992). Although a system may be providing strong leadership and support for a specific change, nevertheless the framework for teacher development needs to take account of a teachers' purposes, a teacher as a person, the real world context in which the teacher works, and the working relationships that teachers have with their colleagues (Hargreaves & Fullan, 1992). Teacher development should be a collaborative partnership which is ongoing within the school (Stoll, 1992) and transformative of the school's education. Craft (1996) suggested that the program needs to produce the necessary information, to be acceptable, and to be available within time and resource constraints. Skills need to be developed and practised within the classroom setting, and structures must provide for facilitating and structuring collaborative relationships enabling teachers to solve implementation problems (Dean, 1991; Joyce & Showers, 1995). Empowerment for ongoing self-development rather than dependency on a facilitator is a hallmark of a good teacher development program (Bell & Gilbert, 1996).

THE PROFESSIONAL DEVELOPMENT MATERIALS

NSW Department of Education and Training (DET) developed a program, *Count Me Into Space* to improve the quality of teaching space mathematics in K-2 classroom. The package was based on research into spatial thinking and visualisation of 2D and 3D shapes (e.g., Owens & Clements, 1998; Presmeg, 1997). The materials were initially developed by the first author in consultation with NSW mathematics consultants. The challenge was to incorporate a large body of research on the use of imagery into effective learning experiences for students through the provision of teacher development.

A framework of space mathematics was central to the program. It identified two key learning areas in space mathematics: (a) part-whole relationships and (b) orientation and motion. The first area concentrated on how shapes are made of parts and how these interrelate to form a shape classification with links to other shapes. A key aspect of learning about the shapes is the actual noticing of parts, that is the disembedding of parts and embedding of parts into the whole shape or configuration of shapes. The second learning area deals with the importance of movement of whole shapes and parts of shapes

to create changing patterns and relationships. It also deals with 3D shapes, their nets, names, and alternative perspectives. Within each area students are expected to develop

- emerging strategies as they start engaging in learning,
- perceptual strategies requiring hands-on materials,
- preliminary imagery strategies that are pictorial, static and limited,
- more advanced imagery associated with pattern and dynamic changes, and finally
- efficient strategies that incorporated in-depth knowledge and visual imagery.

Pirie and Kieran (1991) had identified "primitive knowing, image making and imaging having" as the initial steps in conceptual development. Properties of the images could be noticed, and structures and concepts developed.

Teachers were encouraged to enhance students: (a) investigating and visualising, and (b) describing and classifying. In order to assist teachers to become familiar with the framework, teachers in Kindergarten and Year 1 were allocated part-whole relationships and teachers in Year 2 pursued orientation and motion.

The NSW Department of Education and Training provided the schools with the theoretical framework, assessment tasks, lesson plans, blackline masters for cardboard equipment, background information and videotapes specifically made to introduce the ideas to teachers.

IMPLEMENTING THE PROGRAM

Following the successful implementation of *Count Me Into Space* using district mathematics consultants in five schools (Owens, Reddacliff, Gould & McPhail, 2001), the implementation in the following year used a school-based facilitator. In the second term, 15 schools were involved, and in fourth term 16 schools. The total number of teachers involved was 124. Additional lessons, the videotapes, and minor revisions to the assessment tasks were the main differences in the materials between the implementation with consultants and the current study. Further changes were made for the second group of schools in this study. These included additional lessons and the grouping of lessons according to the strategies mentioned in the learning framework.

The Department provided for a facilitator-teacher to undertake training in Sydney on the key ideas of the program, the assessment tasks, and the types of lessons. Each school facilitator committed to train four teachers and to provide on-going lesson support for ten lessons over a six to ten week period. Each teacher was required to assess six students individually before and after the lessons, keep a lesson register, meet with colleagues and answer evaluation questionnaires. These experiences provided teachers with a realisation of the needs of students, and opportunities for reflection as well as a means by which the program could be evaluated. Each school was provided with a grant equivalent to 13 teacher relief days to assist with implementation of the project.

RESEARCH QUESTIONS AND METHOD OF EVALUATION

An evaluation of the program was made by assessing whether the planned changes were being experienced by students and resulting in increased student learning (cf. Joyce & Showers, 1995). Based on the literature on teacher professional development, we asked whether teachers were holistically involved in the program in the sense of being empowered by increased understanding, values and skills. Was the collaborative support generated by the facilitator model transforming the school's education? In other words,

we were asking whether the program was appropriate for the students and teachers, and how a system-led innovation might lead to effective teacher change and empowerment.

The students' learning was assessed by an analysis of pre- and post-implementation responses of a sample of students from each class to five task-based interview items. The teachers selected six students (two from each of the middle, the bottom, and top of the class but not the highest or lowest achieving students).

The extent of implementation in classrooms was assessed from teachers' lesson registers and their responses to questionnaires. For each lesson, teachers answered three questions:

> What did students' learn in terms of the framework?
>
> What did you do to facilitate this?
>
> Other comments (e.g. what you will need to follow-up, what would have improved the lesson, suitability).

The extent to which teachers' knowledge, values and skills changed was assessed mainly through responses to the questionnaires. Summaries of teachers' meetings, notes on the facilitator's telephone conversations with the Department project officer, and observations of seven classrooms were made. These were analysed qualitatively for themes and interrogated with the aid of Nvivo and other computer tools. Support through the triangulation of data from several sources and several kinds was possible.

RESULTS

Students' Responses to the Assessment/Observation Tasks

The percentage of students who improved on each task and in three or more tasks is presented in Table 1. The results from the facilitator schools indicate that for part-whole relationships, between a half and two-thirds of students improved on each task with two-thirds and three-quarters of the students (first and second groups respectively) increasing on three or more tasks and 14% and 21% respectively improving on all five tasks. For the orientation and motion tasks, about half the students improved on each task with over half improving on three or more tasks and 12% improving on all five tasks. The increased percentages for the second group of students probably reflects the improvements made to the program and tasks between implementations as well as variability in individual teacher's motivation and skills.

Table 1. Student Improvement on Assessment Tasks

Part-Whole Relationships	**Number (%) who improved**		**Orientation and Motion**	**Number (%) who improved**	
	Group 1	**Group 2**		**Group 1**	**Group 2**
	N = 135	N = 193		N = 136	N = 160
Task 1 - shapes	89 (66)	129 (67)	Task 1A - flip tile	57 (43)	73 (46)
Task 2 - tiles	63 (47)	130 (67)	Task 1B - jigsaw	63 (49)	98 (61)
Task 3 - part hidden	74 (55)	113 (59)	Task 2 - rotate angle	41 (31)	69 (43)
Task 4A – making with sticks	95 (70)	131 (68)	Task 3 - make triangles	73 (54)	94 (59)
Task 4B – seeing shape in design	73 (54)	122 (64)	Task 4 - fold net	72 (53)	81 (51)
			Task 5 - turn pyramid	66 (49)	80 (50)
Three or more tasks	79 (64)	141 (73)	Three or more tasks	70 (53)	103 (66)
All tasks	17 (14)	40 (21)	All tasks	16 (12)	19 (12)

Teachers' Implementation of the Intended Program

Teachers' lesson registers indicated that 95% of classes received ten lessons. Ten teachers appear to have not read the materials or viewed all the videotapes and relied on the facilitator for direction and information. Some teachers said they had not changed their teaching approaches but most of these were already using hands-on materials, group work and class discussions. Overall, teachers found the lessons enjoyable and appropriate with a few lessons too hard for a particular class.

Efficacy for Teaching Space

Teachers were asked to select whether they *strongly disagree, disagree, agree, strongly agree* to nine statements on knowledge of the terminology and the teaching approaches used in the program, and on their confidence and attitudes to teaching space mathematics. The results in Table 2 show that the program has had significant effects on teachers' knowledge and confidence in teaching space mathematics. Teachers may have been unclear of the meaning behind the items on visualisation or they were already familiar with the ideas before completing the pre-intervention questionnaire as a result of the *Count Me in Too* program. Responses to open-ended questions, staff meetings, telephone conversations and class observations suggested that teachers knew about visualisation but are now appreciating the deeper theoretical aspects of extending imagery by changing aspects of their teaching to involve more hands-on experiences, questioning, and predicting.

Table 2. Percentages of Teachers who Agree or Strongly Agree with the Item Before and After the Intervention

Item	Group 1 N= 60		Group 2 N=46, 65	
	Pre	Post	Pre	Post
1. I know a lot about how children learn about Space	10	78	15	90
2. The class spends time looking at shapes in our environment	73	96	65	94
3. We devote less than 3 in 10 maths lessons to Space	76	33	75	36
4. We devote more than 3 in 10 maths lessons to Space	20	55	22	64
5. We make a lot of equipment for teaching Space	10	33	24	48
6. I am confident about teaching Space mathematics	51	84	59	92
7. I think students need to "see" the parts embedded in the rest of the shape in order to learn about properties	91	100	100	95
8. I think visual imagery involves moving and patterned images	86	100	96	94
9. I am pleased with my teaching of Space mathematics	34	93	43	88

EFFECTIVE FEATURES OF THE IMPLEMENTATION

The final open-ended questionnaire was intended to elicit what teachers learnt about the framework but also what needed to be improved with the materials. So positive feedback was not expected and if it was given, it is noteworthy.

Materials provided for the teachers. As one teacher recorded, the materials "highlighted different ways of learning, investigating, importance of language and strategies used by students in space maths." For many teachers, their knowledge of what space mathematics is and how students learn was greatly enhanced by the materials. A large number of teachers referred to the background information and the framework giving purpose and an explanation about how students learn space mathematics (30% of teachers in the

second group). Having the outcomes written out in full and next to lessons (an innovation for the second group) made the links to the framework clearer. In the past, purpose for space lessons seemed to be a problem. In the second group, 60% of teachers commented favourably on the global set of outcomes in comparison to smaller dot-pointed indicators or objectives. A common response was that they provided for flexibility in teaching.

The videotapes were seen as enhancing the materials, so teachers could see what was meant in action. The quality of the videotapes was noted by teachers. Overwhelmingly, 80% of teachers in the second group referred positively to the lesson notes. Most of these commented on the sequence of lessons (this was an improvement made for the second group) or the sequence of steps within a lesson. They commented on the teaching points and suggested questions as well as the clear and precise instructions. Teachers also valued the large number of creative ideas embedded in the lessons. Teachers from a couple of schools noted that certain lessons were "open to interpretation".

The role of the facilitator. In response to the question about whether the materials could be used without the assistance of the facilitator, responses fell into three groups. First, some (e.g., 14 teachers in the second group) felt the notes and videotapes were adequate to motivate and get one started. The second kind of response referred to the systemic support needed for implementing new teaching approaches and the value of the facilitator's team teaching and supporting role. Two-thirds of teachers, however, felt the facilitator was necessary to provide personal encouragement, to answer questions, to get teachers to reflect on their current teaching, to organise the teachers to participate despite their busy schedules, to help with explaining assessment tasks and teaching, to summarise the materials drawing out the key aspects when there was so much new terminology and information, and to encourage professional conversations. Using a teacher as facilitator provided a very effective implementation of the program.

Teachers' efficacy to teach space mathematics. A typical comment was "I now enjoy teaching space maths. I also use more groups and more equipment and more investigating. I challenge the children more than I did. It's definitely improved my skills." A small number commented that they were still teaching the same way with subtle changes such as more variety of resources. Many teachers noted that they understood students' conceptual development better, that they were questioning better to draw out understanding and language, that they were clearer about the purpose of space lessons, and they taught space more often or spent more time on each lesson. Teachers mentioned that lessons were more enjoyable, intensive, structured and guided (due to "good lesson notes, not as generalised as maths syllabus"). There was more involvement of students, better modelling, more equipment, more drawing, more integration with other Key Learning Areas (KLAs), and more use of assessment embedded in activities with greater concentration on students' skills. Teachers were able to focus on students' understanding of part-whole relationships. Initially some found that students had fewer skills and understanding of shapes than expected. Students were more aware of size and orientation of shapes and seeing shapes in their environment.

Over a third of teachers mentioned the biggest change was to their questioning. Teachers were drawing out discussion and descriptions about shapes from the students rather than the students just giving drilled properties like "a square has 4 sides". This teaching

strategy was often linked to the sequence of steps in the lessons with activities and whole class discussions and to having a purpose as set out in the framework. "I now feel more confident to teach space lessons - I think I understand their purpose and I now know what students need if they can't do a particular thing. Before I was only able to assess if they could/couldn't do something. I also see how questioning can be used to assess students understanding or why they have a particular understanding"

Continuing the intended curriculum for space mathematics. When asked what needed to happen in the future to continue student learning, teachers mentioned that they would continue with hands-on experiences, language and visualisation, and be more challenging. They would extend activities to include more nets, slices, and surfaces. They would budget for more materials, implement the lessons over a longer period of time, adapt to higher stages, and consolidate ideas and link them to other concepts like area and angles. One teacher commented she would be changing to incorporate the excellent activities having seen the results in action and one teacher said, "I have ditched the textbook." Others said they would encourage their colleagues to use the materials. Teachers made minor suggestions to improve a lesson plan or told us of ways they had extended the lesson idea into new lessons, lesson breaks, and other KLAs. This was particularly pleasing as it indicated teachers were gaining a sense of ownership of the lessons and were able to develop their own.

Teachers' use of the terminology and framework in describing student learning. When talking about what students had learnt, it was pleasing that only four teachers in the second group mentioned activities per se like cutting up larger shapes into smaller ones. By contrast, two-thirds of the teachers referred to students' processing like looking, listening, experimenting, trialling, discussing, reporting, comparing, testing, making mental images, and visualising before trying. "Students are disembedding shapes looking at properties, categorising the same shapes under the one heading, looking at size and orientation e.g. triangles". Many teachers were using terms which were made familiar by the framework. However, this was one area in which facilitators and teachers needed more time to familiarise themselves. This recommendation was taken up in the next implementation by the Department.

Teachers mentioned that all students progressed at least within the strategy band if not to the next band of strategies. Teachers mentioned students excitement, enjoyment, confidence, and interest. "Everyday is like a new adventure," said one teacher. Teachers, especially with the older students, noted students were "more able to explore possibilities like changing a square into a rectangle" or "visualising shapes and movement of shapes". Over a third of the teachers referred to the more precise use of language, discussion of parts, and use of words like rhombus, and flip, slide and turn actions. The classes who were observed showed that teachers implemented the lessons with questioning and encouraging visualisation through prediction.

CONCLUSION

Having knowledge of how students learn in space mathematics and having a clearer and more extensive purpose for teaching space lessons has been internalised by the majority of participating teachers as a result of classroom implementation and the support of the program (cf. Dean, 1991; Joyce & Showers, 1995). The lesson plans challenged and

assisted teachers to cater for hands-on activities in large classes, and allowed teachers to see students' learning according to the framework. A few teachers were ready to develop their own lessons based on the framework. Clark (1992) had earlier said that a good professional development approach encouraged teachers to develop their own professional development.

The program provided necessary information that was generally acceptable to the teachers and manageable within the constraints as recommended by Craft (1996). The study supported the importance of structuring collaborative relationships to overcome implementation problems (Dean, 1991). We can say that in terms of the framework for evaluation suggested by Joyce and Showers (1995) that the teacher development did provide students with the intended curriculum in nearly all cases and that the students' learning was enhanced. Teachers intend to involve other teachers and this is necessary for real change across the school (Stoll, 1992).

References

Bell, B., & Gilbert, J. (1996). *Teacher development: A model from science education.* London: Falmer.

Clark, C. M. (1992). Teachers as designers in self-directed professional development. In A. Hargreaves and M. Fullan (Eds.), *Understanding teacher development* (pp. 75-84). New York: Teachers College Press.

Craft, A. (1996). C*ontinuing professional development: A practical guide for teachers and schools*. London: Routledge with Open University Press.

Dean, J. (1991). *Professional development in school.* Milton Keynes: Open University Press.

Hargreaves, A., & Fullan, M. (1992). Teacher development and educational change. In M. Fullan & A. Hargreaves (Eds.), *Teacher development and educational change.* (pp. 1-9). London: The Falmer Press.

Joyce, B., & Showers, B. (1995). S*tudent achievement through staff development: Fundamentals of school renewal.* New York: Longman Publishers.

Owens, K. (1999). The role of visualisation in young students' learning. In O. Zaskavsky (Ed.), *Proceedings of 23rd PME* (Vol. 1, pp. 220-224). Haifa: PME.

Owens, K., & Clements, M. A. (1998). Representations used in spatial problem solving in the classroom, *Journal of Mathematical Behavior 17* (2), 197-218.

Owens, K., Reddacliff, C., Gould, P., & McPhail, D. (2001). Changing the teaching of space mathematics. In J. Bobis, B. Perry, & M. Mitchelmore, *Numeracy and beyond* (Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia, Sydney, pp. 402-409). Sydney: MERGA.

Pirie, S., & Kieren, T. (1991). Folding back: Dynamics in the growth of mathematical understanding. In F. Furinghetti (Ed.), *Proceedings of 15th PME* (Vol. 3, pp. 169-176). Italy: PME.

Presmeg, N. (1997). Reasoning with metaphors and metonymies. In L. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images* (pp. 267-279). Hillsdale, NJ: Lawrence Erlbaum.

Stoll, L. (1992). Teacher growth in the effective school. In M. Fullan & A. Hargreaves (Eds.), *Teacher development and educational change* (pp. 104-121). London: The Falmer Press.