THE EFFECT OF A SIMCALC CONNECTED CLASSROOM ON STUDENTS' ALGEBRAIC THINKING

Stephen J. Hegedus
University of Massachusetts Dartmouth

Jim Kaput
University of Massachusetts Dartmouth

We report the findings of an empirical analysis of the performance of a group of middle and high school students before and after an after-school algebra enrichment program using the SimCalc software incorporating classroom networks. The results highlight statistically significant gains in their learning and briefly outline contributing factors of the innovation that gave rise to such improvement.

CONTEXT

The long-term goal of the SimCalc project has been to democratize access to the Mathematics of Change and Variation (MCV) (Kaput, 1994) especially algebraic ideas underlying calculus (Kaput & Roschelle, 1998) using a combination of new representations, links to simulations and new curriculum materials for grades 6-13. The software enables students to interact with animated objects whose motion is controlled by visually editable piece-wise or algebraically defined position and velocity functions. One form of the software has been developed for the TI-83+ (Calculator MathWorlds) and the other is a cross-platform Java application (Java MathWorlds) which exploits higher screen resolution, with the ability to pass MathWorlds documents between the two platforms – see http://www.simcalc.umassd.edu for further details.

Present work has studied the integration of various kinds of connectivity with the SimCalc environment that enable new and intense forms of social interaction and learning possibilities. Recent studies (Kaput & Hegedus, 2002) reported the affordances of newly emerging connectivity technology that allows teachers to distribute and collect students' work across diverse platforms from hand-held devices, such as the TI-83+ graphing calculator, to desktop computers. Since this earlier work, we have expanded the design space of classroom connectivity to include the passing and sharing of students' individual constructions between computers using standard Internet protocols. The development of a dedicated Java MathWorlds (JMW) server has allowed students to send their individual work from within JMW to the teacher who can aggregate and then display their work on a single coordinate system.

Utilizing this new connectivity/aggregation ingredient, we developed a 5-week after-school algebra enrichment program for local middle and high school students in a standard school computer lab. This paper examines their mathematical performance gains.

THEORETICAL BACKGROUND

The capabilities of classroom connectivity has not merely enhanced the management of information flow in the classroom, but more powerfully, changed the nature of MCV learning activities. We have begun to document activity structures (Hegedus & Kaput, 2002) that give rise to mathematically deep and socially intense learning where students' personal constructions become part of shared mathematical objects as their work visibly participates in those aggregated objects. The social structure of the classroom plays a direct role in the structuring of mathematical activities, and vice-versa in a dialectical fashion. Students, organized into groups, build functions that vary parametrically across the groups as well as within groups, yielding structured families of functions reflecting the directly experienced social structure of the classroom. This epistemologically elevates the organizational structure of the mathematical objects, from functions to families of functions. In doing this, students construct parts of a mathematical whole and so the focus of their attention is on the relations between their individual contribution and the whole. Thus, students' personal identities are intimately involved in their building and sharing of mathematical objects in the public space of the classroom. The main aim of this paper is to show that classroom connectivity not only offers new and exciting pedagogical opportunities for teachers but it can significantly improve students' performance in core algebra topics over a short period of time.

THE SET-UP OF OUR CONNECTED SIMCALC CLASSROOM

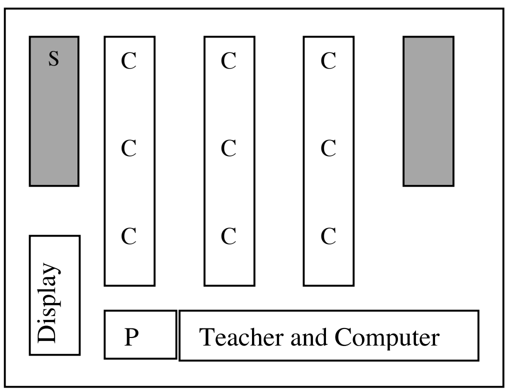

Figure 1. Classroom Set-up

During the course of our intervention, the class met after school in a dedicated computer lab. Figure 1 illustrates schematically the classroom set-up with a large amount of activity occurring around the whiteboard (Display) where the teacher computer was displayed. Two rows of computers (shaded) were used for other school purposes except for one computer (S) dedicated to running the SimCalc Server application. Four rows of computers (16 computers) were used, usually with 2 students to a computer (C). The classroom set-up was traditional in its layout with rows of computers, making interaction and classroom discussion logistically difficult in contrast to networks of personal hand-helds. The focus of our attention here is on the students rather than the teacher – a novice SimCalc teacher who received regular direction from us.

Our key innovation was to incorporate a unique identifier for each student into the new connected classroom activities. Each student was assigned a 4-digit number, which resonated the physical group set-up of the classroom. The first two digits specified their group number (established by which row of computers they were in for example) and the last two digits specified their count-off number in their group. This 4-digit number served as a natural variant, which mapped to the parametric variation within the mathematical activities. For example, a now standard introductory task in our work requires each student to construct a motion by visually or algebraically editing position-time graph segments to make their screen object travel at 3 feet per second for 5 seconds but *starting at their group number*. When each student's work is aggregated, the teacher can display a naturally emerging family of functions (a set of parallel functions – see Figure 2) and begin to discuss the variation across the graphs in terms of the variation across the groups.

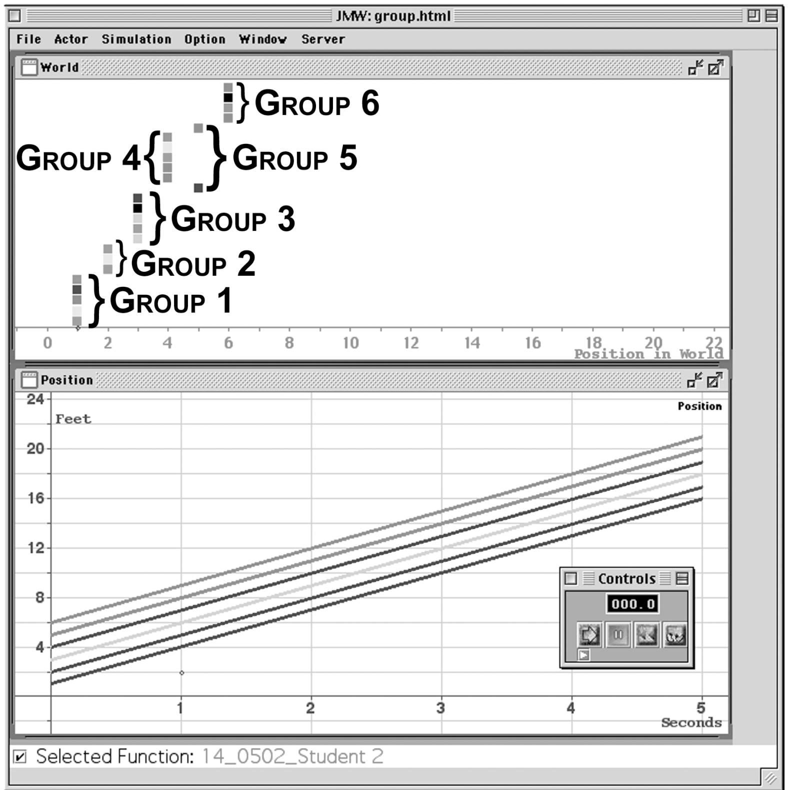

Figure 2. Class Aggregation in Java MathWorlds

Overlap occurs in the position-time graphs since each group contains several students, but each student is distinguished by a unique square dot in the upper half of the screen ("the World"). Clicking on a dot displays the student's name in the lower left part of the window (this can be deselected to provide anonymity). Animating the group motion leads to all the dots moving synchronously in the world, but offset in side-by-side groups as in a parade. Based on projection of the teacher's computer onto a classroom display, class discussion centers on who is where and why, and why the motion and graph configurations appear as they do. Students are also asked to produce a generic formula for the entire group, for this example, Y=3X+B where B is the group number. An additional feature of the SimCalc software is a Matrix window, which displays all work retrieved from the students and allows the teacher to hide/show students' graphs, dots and other representational features of the motion. In addition, and more importantly, the teacher can sort dots by group or count-off number, thus displaying groups of dots which naturally correspond to groups of students. Here, the students' identities are projected into an organized aggregate structure, which resonates with the structure of the mathematics. In addition, the dynamic feature of the SimCalc environment enables a motion-representation of the generic formula and the role of the parameters M and B – via the parallel motion and offset starting positions. The parallelism of the linear functions is represented in the *parade-like motion* of the group!

In later activities, we systematically vary the role of *both* group and count-off number in increasingly more rich and complex tasks dealing with core algebra topics, such as slope as rate, linearity, parametric variation and systems of equations. A more detailed account of our intervention triangulating our empirical work with our observational data can be found in Hegedus & Kaput (in review). Following our intervention fusing the SimCalc environment and classroom connectivity with these and many more related activities, we saw significant gains in student performance. We now report our testing methodology and empirical results.

PRE-POST TEST METHODOLOGY

In order to measure changes in students' understanding of the core algebra ideas attended to in our intervention, we administered a 20-item pre-post test comprised of 12 questions from the 10^{th} grade 2001 state (Massachusetts) exams, 1 Advanced Placement Calculus item and 7 questions selected from a pool of items developed by the SimCalc Project and refined over several years. Fourteen of the questions were multiple choice and the remaining items were short answer or open-response. Some of the questions were not directly addressed in our intervention and served as face-validity items that assessed whether more general algebraic skills were developed during the intervention. Twenty-five students from our original group of 38 completed the course. The scores of 24 of these students were used in our statistical analysis, as one student did not complete the pre-test. Those students not completing the course were not statistically different (mean=0.430, variance=0.019, n=13) in their achievement on the pre-test from the final sample used for pre-post test comparison (mean=0.427, variance=0.019, n=24).

We adopted the rubric for the MCAS open-response item (4 points) and combined it with the other multiple-choice questions and SimCalc items (scored between 2 and 4 points) to total a test score of 31 points. The test items were comparable in difficulty and passed a test for internal consistency reliability (□=0.71) for its use as a pre-post-test measure. The pre and post-tests were scored by two markers. A high interrater coefficient (r = 0.80) was obtained and a third marker was used in collaboration with the other two markers to obtain the final results.

DATA RESULTS AND ANALYSIS

We aimed to show that any change in student's performance was mainly caused by our intervention of the connected SimCalc classroom and not necessarily the social or academic demographics of the participants or other indirect variables. To this aim, we present our results, which separate out the performances of 2 different subgroups and the performance on individual items combining various statistical procedures and measures of gain. Our middle school students were higher achievers (as described in their mean scores). Our high school students were low achievers, with low proficiency levels (average 218) on recent 8^{th} grade state examinations. The five-week teaching experiment had a positive effect on the mathematical behavior of both groups of students as presented in Table 1.

We used a paired Student's t-Test to measure the significance of the difference in the groups mean scores. A paired test was suitable given a high correlation coefficient between their pre/post-test scores (0.78) as well as the two groups being identical and the same test being used to measure the effect of the intervention.

Group	**n**	**Pre Test**		**Post Test**		**Cohen's *d* effect**	***Hake's Gain***	***p***
		Mean	**SD**	**Mean**	**SD**			
All	24	**42.7%**	0.141	**65.9%**	0.149	1.60	0.42	0.0001
7th & 8th	10	**52.2%**	0.092	**76.8%**	0.123	1.78	0.5	0.0001
9th	14	**37.7%**	0.158	**62.0%**	0.136	1.91	0.36	0.0001

Table 1. Pre- and Post-Test Results

In addition, a parametric test was used given that the students were selected from a large population and had varying mathematical abilities. This was confirmed using a normality test[1]. The results show a statistically significant increase ($p<0.05$) in both the mean scores as a group, as well as by age group. Even though the group of grade 7 & 8 students ($n=10$) showed higher test averages than the grade 9 students ($n=14$) the latter group yielded a higher effect size of 1.91 standard deviations. In both groups, the effect size was extremely high. This illustrates that while the 5-week session had a very positive effect on both groups it appears that there was a more positive effect on the 9th grade students. Our concern was with the difference in both age groups both in background and prior knowledge. How much of this gain was due to prior knowledge? To attend to this question, we used Hake's gain statistic – an average normalized gain – which related mean gain relative to original performance, i.e. Gain = <Post> – <Pre> / 1 – <Pre> where the angled brackets represent mean scores. Hake studied over 6000 diagnostic tests of physics undergraduates in reform- vs. traditional-based classrooms (Hake, 1998) observing higher gain scores (>0.4) for reform-based classrooms. Hake's work and other studies indicate that this statistic is related to students' growth in a more cognitive sense (McGowen & Davis, 2001; Hake, 1998). Table 1 highlights how, while our group of 9th graders had a greater effect size, our group of 7th and 8th graders had a greater gain (0.5) relative to their performance on the pre-test. We calculated an individual Hake's gain statistic for each student for two purposes. First, we calculated how their increase in performance from pre to post test related to their prior knowledge by correlating their individual Hake's gain statistic with their pre-test scores. There was insignificant correlation for both the middle school students ($r=0.09$) and our high school group ($r=0.12$) highlighting that instruction was at the right level for students who have an average or little prior knowledge of the subject as judged by the pre-test score. This was an important find for us in establishing that gain in our non-standard classroom was mainly based on our intervention rather than student background given that we had a mixture of students of varying educational performance, of varying exposure to the core mathematical ideas we were attending to, as well as ages.

[1] To test whether the samples were from a Gaussian distribution we used the method of Kolmogorov and Smirnov.

Test Item	Type	Pre-Test		Post-Test		*p*
		Mea	SD	Mea	SD	
1	Direct linear extrapolation. Interpreting straightness qualitatively. Unit conversion	0.333	0.482	0.667	0.482	0.0195
2	Modeling. Reading graphs with zero starting	0.917	0.282	0.875	0.338	>0.999
3	Algebraic modeling of linear quantities with non-zero starting values	0.833	0.380	1	0	n/a
4	Concept of averaging. Unit conversion. End point analysis	0.333	0.482	0.417	0.504	0.547
5	Graphical interpretation. Comparing starting & ending point differences.	0.458	0.509	0.917	0.282	0.001
6	Co-linearity. Interpretation as slope	0.333	0.482	0.458	0.509	0.375
7	Linear extrapolation. Multiplicative reasoning	0.792	0.415	0.833	0.381	0.813
8	Attention to labels and scales. Unit conversion.	0.667	0.482	0.875	0.338	0.109
9	Reading graphs	0.750	0.442	0.875	0.338	0.313
10	Interpreting Pi as a coefficient	0.333	0.482	0.875	0.338	0.0002
11	Point-wise interpretation of graph to obtain two varying quantities. Graphical construction	0.583	0.504	0.625	0.495	0.820
12	Slope as rate. Interpretation of velocity (SC; OR4pt)	1.917	0.583	2.958	1.367	0.002
13	Slope as rate, Change of sign (slope) (AP)	0.542	0.509	0.583	0.504	0.813
14	Concept of rate. Multiplicative reasoning (SC, SA2pt)	0.750	0.676	1.792	0.588	<0.001
15	Slope as rate. Average. (SC, SA2pt)	0.125	0.338	0.208	0.509	0.491
16	Slope as rate attending to scales (SC)	0.333	0.482	0.833	0.381	0.0005
17	Concept of rate with divisors. Non-standard. (SC)	0.417	0.504	0.625	0.495	0.206
18	Algebraic expression of real-life varying quantities. Interpreting the y-intercept	0.750	0.847	2.042	1.233	<0.001
19	Determining slope from value. Concept of families of functions (parametric variation) (SC, OR4pt)	0.625	0.824	1.167	0.637	0.0012
20	Algebraic and geometric reasoning. Interpretation of real-life scenarios. Systems of equations.(MCAS,OR4pts)	1.458	1.141	1.792	1.103	0.148

Table 2. Item-by-item analysis

We also conducted an item-by-item analysis to assess which questions and corresponding content and skills contributed to the increase in performance for the group. Table 2 highlights the mean scores from pre- to post-test as well as content areas for each question, where "SA" denotes short answer and "OR", open-response questions. The remainder are multiple-choice. All questions are from the 2001 State examination unless stated; SimCalc (SC), Advanced Placement (AP). We conducted a Wilcoxon matched-pairs test for multiple-choice items (binary response) and paired Student's t-test for short answer and open response questions. We highlight 8 items of statistically significant gain ($p<0.05$). One ceiling-effect item (3) could not be tested.

The highlighted items in Table 2 outline a genre of skills, which were mainly consonant with our original aims for the course. They involve interpretation of graphs, understanding of slope as rate, interpreting Y=MX+B and how varying the parameters M and B vary graphical views, linearity, interpreting slope in real-life situations (e.g., (constant) velocity as slope of linear position-time graphs), and, most significantly, to generate and interpret *families* of linear functions from parametrically varying M and B. Curiously, we had not originally intended to concentrate on graphical interpretation, especially for non-motion based scenarios, and we believed our intervention had not concentrated on such a skill, yet the overall performance of the group on such tasks (items 1, 5, 10, 12) led us to believe that there was something implicit in our instruction and the connected classroom set-up that enabled students to continually reflect on the graphs they created (both algebraically and visually). We cannot include the whole test but a full version is available on-line at the SimCalc website[2], however we wish to present two of these questions which serve as face-validity items to strengthen our claim that a connected SimCalc classroom has a positive effect on students' learning. The students scored significantly higher on the post-test for item 5. Their pre-test performance was very similar to the mean scores for the high school (45.0%) the previous year as well as the State overall mean score (46.0%). This item required students to interpret and compare non-linear graphs of varying enrollment figures of three clubs over time. Here students must determine end-point differences and interpret the results as quantities.

5. Based on the graph, which organization showed the most growth in membership over the 10-year period?

A. The Math Club

B. The Hiking Club

C. The Drama Club

D. The Drama Club and the Hiking Club are tied for the most growth.

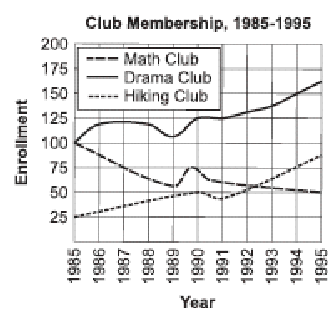

Figure 3. Item 5

The second face-validity item we wish to highlight is item 10, which shows the most significant increase given that on the pre-test the students performed the same as the high school (34%) and the state (33%), which was quite poor, and excelled on the post test (87.5%). Furthermore, the question required the students to interpret a non-standard algebraic relationship of two geometric quantities. Our intervention had predominantly used standard algebraic notation (i.e. Y=MX+B) or some incorporation of identifiers (e.g. count-off/group numbers) into the parameters M and/or B.

10. The circumference, C, of a circle is found by using the formula $C = \pi d$, where d is the diameter. Which graph **best** shows the relationship between the diameter of a circle and its circumference?

[2] http://www.simcalc.umassd.edu/NewWebsite/pretest.html

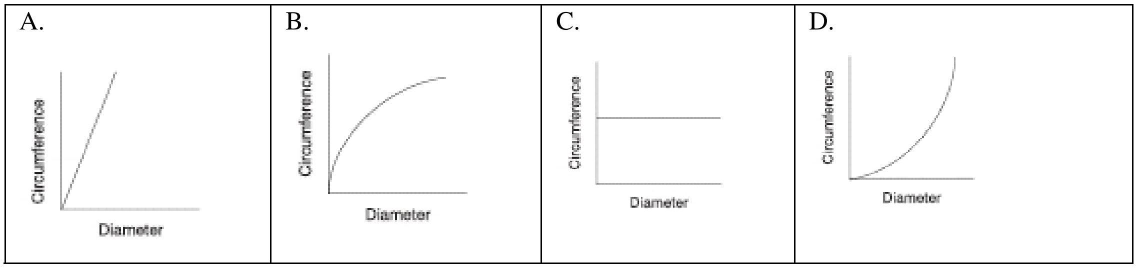

Figure 4. Item 10

CONCLUSION

It is our primary claim that in combining the dynamic SimCalc environment with classroom connectivity we can significantly improve students' performance on 10^{th} grade MCAS algebra-related questions in a short period of time. Even though we had a non-standard mixture of students, our analysis has shown that all our students performed better relative to their prior knowledge, which in some cases was little or none, on questions involving core algebraic ideas. We believe that classrooms which integrate dynamic software environments with connectivity technology can dramatically enhance students' engagement with core mathematics beyond what we thought possible in the absence of such support. Further work is needed, both to explain such enhancement and to exploit it.

References

Hake, R. R. (1998) Interactive-engagement vs traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. *American Journal of Physics*, 66, 64-74.

Hegedus, S. & Kaput, J. (2002). Exploring the phenomena of classroom connectivity. In D. Mewborn & L. Hatfield (Eds.), *The Proceedings of the 24rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Vol. 3)*. Columbus, OH: The ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Hegedus, S. & Kaput, J. (in review). Improving algebraic thinking through a connected SimCalc classroom. Submitted to *Journal for Research in Mathematic Education.*

Kaput, J. J. (1994). Democratizing access to Calculus: New routes using old routes. In A. Schoenfeld (Ed.). *Mathematical thinking and problem solving* (pp. 77-156). Hillsdale, NJ: Lawrence Erlbaum.

Kaput, J. J. & Roschelle, J. (1998). The mathematics of change and variation from a millennial perspective: New content, new context. In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the mathematics curriculum* (pp. 155–170). London: Springer-Verlag.

Kaput, J. & Hegedus, S. (2002). Exploiting classroom connectivity by aggregating student constructions to create new learning opportunities. In A. D. Cockburn, & E. Nardi (Eds.) *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education*. Vol3, pp.177-184

McGowen, M. and Davis, G. (2001). What mathematical knowledge do pre-service elementary teachers value and remember? In R. Speiser, C.A. Maher & C. N. Walter (Eds.) *Proceedings of the XXIII Annual Meeting of the North American Chapter of International Group for the Psychology of Mathematics Education, Vol. 2, 875-884.*