INTERACTIVE WHITEBOARDS AND THE CONSTRUCTION OF DEFINITIONS FOR THE KITE

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This paper reports early work from a project examining the affordances offered by Interactive Whiteboards (IWBs). Here the focus is on the teaching of the definition of quadrilaterals through the use of Cabri Géomètre. We discuss the work of two 11-year-old children, who are exploring the kite. The protocol highlights the complexities inherent in understanding definitions of quadrilaterals, as reported widely by other researchers. We conjecture that a change of emphasis in the use of the IWB may encourage figural concepts (Fischbein, 1993) to be controlled by conceptual rather than visual components.

INTRODUCTION

The interactive whiteboard (IWB) is a combination of a projection system and a touch sensitive whiteboard, which allows the teacher (or children) to manipulate the computer screen through physical interaction with the whiteboard. Some IWBs require a special pen; others such as the one in this study can be operated by any device including the human finger. Unlike almost any other new technology introduced to education, IWBs have generally been welcomed by teachers, who seem to find their use enhances their normal conventional teaching style. Levy (2002), Glover and Miller (2001), and Smith (2001) all investigated the use of IWBs in one or a few British schools. They report generally on keenness to use the technology.

This study represents the second phase (first iteration) in a broader study in which we are teasing out the affordances of an interactive whiteboard in whole class teaching. In the first phase, we interviewed 14 teachers, most of who were identified by the ICT Coordinator in the school as being enthusiastic users of this technology. Two teachers in particular had used the technology but infrequently. These interviews generally supported the positive responses found in the previous research. Our analysis of their responses seemed to indicate two specific affordances, what we have come to call *visual* and *kinaesthetic*. The former relates to the size, clarity and colourful impact of the computer graphics, writ large on the whiteboard. The latter relates to the potential impact of dynamically manipulating the screen in such a way that the teacher's (or child's) agency in the process is far more impressive than merely following a small mouse arrow. The interviews suggested that the visual affordances of IWBs often impact through carefully prepared material by the teacher, but this material rarely exploited the kinaesthetic affordances.

It is nevertheless unclear how such affordances might shape children's learning. There is no guarantee that, when teachers talked to us about the technology, they were tuning into learning. It is entirely plausible that their responses were in fact guided by feeling in control of the class, by being the focus of attention at the IWB or by the excitement of using such impressive new technology. In order to focus on learning, we decided to examine teaching and learning in a context where it was reasonable to suppose that visual and kinaesthetic affordances would have particular potential.

The Royal Society and JMC (Royal Society, 2001) specifically advocated the teaching of geometry using a dynamic geometry software package (DGS) on an IWB. There is clearly a danger here of what Papert (1991) has referred to as technocentrism in focusing on a piece of hardware such as IWBs. It would be naïve in the extreme to claim special properties in a piece of hardware, or software for that matter, that mystically transforms the learning process. We aspire to relate the affordances of the IWB not only to the use of DGS but also to the teaching process itself with particular emphasis on the tasks and the teacher/class discourse. The focus of this paper is on work related to the construction and definition of quadrilaterals.

Tall & Vinner (1981) distinguish between what they term concept image and concept definition. Whereas the concept definition is seen as a form of words used to specify the concept, the concept image is used "… to describe the total cognitive structure that is associated with the concept" (p.152). The concept definition may be that offered by the teacher and used in rote fashion or it may be a personal construction of what has been offered and in fact departs quite significantly from the teacher's version. Classroom practice may have focussed on certain aspects of the concept definition, resulting in a concept image that scarcely incorporates the teacher's concept definition but instead emphasises certain particularities and a rather more limited personal concept definition.

Whereas Tall and Vinner's use of this terminology is intended to apply rather widely to a range of types of concepts, Fischbein (1993) has focused on a specific knowledge domain. Fischbein analyses geometrical reasoning and points out that such reasoning involves the manipulation of what he refers to as figural concepts. Such concepts fuse (at least ideally) sensorial spatial imaginings (visualisations) with conceptual attributes or properties. In fact, this ideal of fusion is modified in practice. According to Fischbein, "the figural component is usually influenced by figural-Gestalt forces and the conceptual components may be affected by logical fallacies. With age, and as an effect of instruction… the fusion between the figural and the conceptual facets improve." (p145).

Ideally the conceptual aspect should control the figural concept. In practice, children typically allow the visual component a controlling role even when they know the definition of the figure. Thus, a student who knows the definition of a parallelogram may nevertheless find difficulty in recognising the various shapes that correspond to that definition. An oblique parallelogram, a square and a rectangle are so figurally different that the unifying effect of the common concept simply vanishes.

Mariotti (1994) used Fischbein's theory of figural concepts to examine the interplay between the figural level ("observing the object as it appears", p234) and the conceptual level ("relating to the properties which characterise the geometrical figure, embodied by the object", p234) in a teaching experiment. She concludes from this episode that there is often a conflict between the figural, which stresses the differences in perception, and the conceptual, which attempts to emphasis the similarities within a classification. Mariotti places some emphasis on the role of the teacher to raise this conflict, arguing that it is only by making the tension between the figural and the conceptual explicit that it can disappear.

The importance of the visual aspect of figural concepts is made quite apparent in a review by Clement and Battista (1992). They highlight the findings by Herschkowitz (1989) who showed how visualisation was a crucial element in the process by which a student constructed geometric ideas. Furthermore, she connected that process to Van Hiele levels 1 and 2 (1986). Thus, the student in an initial stage of development will typically refer to a protypical visual image. At level 1, any figures offered to the student will be compared to the student's prototype as a critical part of the identification of the shape. According to Herschkowitz, as the student moves towards level 2, she will begin to discriminate critical aspects of the prototype and use these as part of the judgement. When the student has reached level 2, she will be able to use those attributes independently of the prototype and the figures will be seen as mere instances of the shape.

Herschkowitz pointed out that the use of prototypes at levels 1 and 2 could often be limiting as the student may be tied too closely to that single image, preventing the discrimination of the critical attributes. The use of the concept might be quite limited because the student relies too heavily on the prototypical image.

We can conclude from this literature that children narrowly using prototypical thinking are using the visual rather than the conceptual aspects of the figural concept, and that this unsophisticated concept image may at least be in part due to pedagogical circumstances.

Some researchers have found evidence that the use of software in carefully designed teaching experiments might support a move from Van Hiele level 1 to level 2. Markopoulos & Potari (1996) discuss their work with 11-year-old children using specially designed software to investigate children's construction of the concept of geometric shapes. The authors claim that, through the teaching experiment, the children began to develop connections between the figures and their properties and formed hierarchical relationships between classes of shapes. A study by Jones (2000) suggested that a teaching programme based on a DGS package enabled some children to move from Van Hiele level 1 to level 2.

De Villiers (1998) is interested in how we might encourage children to appreciate the process of defining. In a teaching experiment, De Villiers focussed on what he calls *descriptive* defining in which the current definition is improved by the selection of appropriate subsets of properties from which the other properties can be deduced. The experiment aimed to support children's appreciation of the goal of achieving "economic" definitions. De Villiers speculates that dynamic geometry software may have a special role to play in supporting the construction of hierarchical classifying by children.

There is at face value a possible connection between the two aspects of figural concepts with the two affordances of IWBs, namely the visual and the kinaesthetic. It seemed plausible that the IWB could support the enhancement of children's concept image for quadrilaterals by supporting the figural component through the visual impact of the IWB whilst encouraging the conceptual component through appropriate use of the kinaesthetic affordance. The latter type of support was the more problematic, since, although we could envisage the teacher exploiting the dynamic nature of the whiteboard to examine many non-prototypical configurations of a quadrilateral, it was far from clear to us what sort of activity might support the fusion between these two components as described by Fischbein.

METHODOLOGY

The first iteration of the second phase of this study involved a class of 11/12-year-old children at a secondary school. Neither they, nor the teacher, who was an experienced head of mathematics department, had any previous experience of DGS. The teacher's experience of using an IWB was also limited.

The researchers constructed a set of Cabri-based tasks, which we saw as possible resources for lessons. In this first iteration, our intention was not to guide the lesson planning too strongly as we were interested in how the teacher herself would employ (or not) these resources in planning a sequence of 8 Cabri lessons, which focussed on quadrilaterals, triangles, reflection and translation. It is also true to say that we did not feel ourselves to be experts who could offer ready-made lessons, but rather that we hoped the research would enable us to be better informed to plan the use of the IWB more effectively in the second iteration. The intention was that each lesson would have an introduction and a final plenary involving the use of the IWB, and that the children, working in pairs, would use PCs for personal exploration in the middle of each lesson.

3 pairs of students were chosen by the class teacher on the basis that they generally worked well together and had good attendance. These pairs were interviewed for about 30 minutes before the programme started to gain some insight into their attitudes to the school, computers and mathematics. In addition, some of their relevant mathematical ideas were briefly explored. During most of the lessons, the IWB work and the computer work of the 3 pairs of children were recorded[1]. At the end of the program, the same pairs of pupils were interviewed again. They were briefly questioned about their attitudes to the lessons.

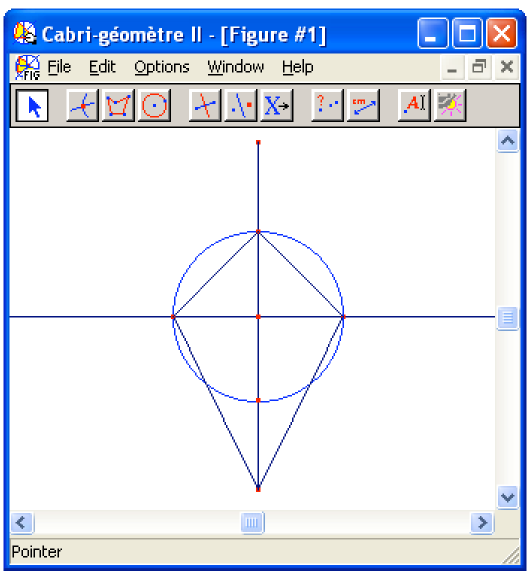

Fig. 1: The girls construct their kite.

Their geometrical reasoning was explored by asking them to work on tasks using Cabri Géomètre. In this paper, we focus on part of that final interview where two girls, whom we shall call Christine and Michelle, consider which shapes can be formed by dragging a prepared construction.

From this range of lessons, perhaps the most relevant to the episode below were lessons 3 and 4. Lesson 3 began with an introduction by the teacher in which a rhombus was made by folding a piece of paper. In this whole class work, the idea that a square is a special type of rhombus was raised in the final plenary on the IWB, as was the idea that a rhombus was a special type of kite. There was however no attempt to reinforce these ideas through further examples, probably because of time constraints. In lesson 4, there was initial discussion on the IWB about the properties of a parallelogram. In the main part of the lesson, the children attempted to construct quadrilaterals from properties based on the

[1] This was achieved by use of Camtasia software, which captured the screen images and recorded mouse 'clicks' as well as capturing speech via attached microphones. Camtasia is produced by TechSmith Corporation, 1780 E. Grand River Ave. East Lansing, MI 48823 USA http://www/techsmith.com/products/camtasia/camtasia.asp

diagonals. Christine and Michelle managed to construct a square with some help from the teacher. After one or two false starts, they also constructed a kite (Figure 1) by drawing a vertical line segment and a perpendicular bisector to the segment. They placed a point on the bisector and constructed a circle through this point, centre at the intersection of the two lines. The kite was formed by constructing a polygon through the three points of intersection between the circle and the lines, and the end of the line segment. It is worth pointing out that this kite is not general since the angle between the horizontal diagonal and either of the upper sides is fixed at 45°. Interestingly, the construction disappears when the lowest point is dragged upwards beyond the point on the circle where the figure becomes a square.

AN ILLUSTRATIVE VIGNETTE

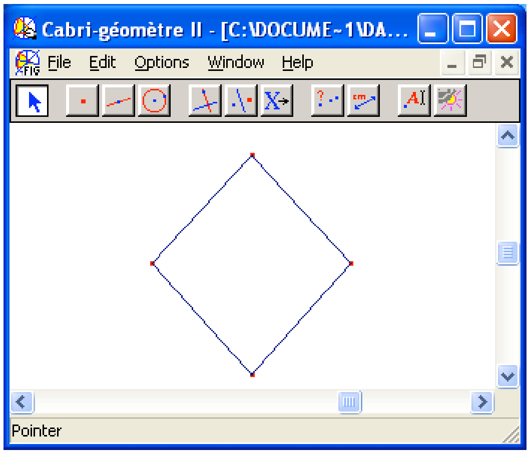

Fig. 2: The initial "rhombus" kite

During the interview, which took place after the sequence of lessons, we presented Christine and Michelle with the construction in Figure 2. This figure is constructed more generally than the one that Christine and Michelle constructed in Lesson 4. All 4 vertices can be dragged , allowing the figure to appear as a rhombus, square or kite. This figure can also be dragged into a deltoid shape.

The following discussion took place before the girls attempted to drag the figure.

M: So you can look at it like a kite. It's like a rhombus, but turned.
C: Mmm. Yes it is. Aren't kites meant to be longer at the bottom than at the top. It's like a square, but it's changed shape.
M: Yeah. If it's like that, then it's like a rhombus.
C: Yes
M: If you look at it that way, it's like a rhombus. I don't know if it is this way. Sort of like a rhombus, but upside down, to the side.
C: Yes, it's slanted.
M: So you can look at it like a kite. It's like a rhombus, but turned.
C: Yes, it's a special kind of a rhombus.

Both girls are at this stage attending to the visual aspects. The task has directed them towards the visual as they have not been asked to interact with the figure other than through their perceptions. There is evidence that both girls seem to refer to prototypical thinking as in Van Hiele level 1. Michelle seems to have in mind a prototype rhombus with its base horizontal and is able to imagine turning the rhombus from its current heading to that of her prototype (lines 1, 5, & 7). It is unclear what she means when she refers to a kite (lines 1 & 7) but Christine clearly imagines a prototypical kite in which two adjacent equal sides are longer than the other two adjacent but equal sides (line 2). It seems that this figure does not conform to her concept definition of a kite.

In the next stage, we encouraged the girls to interact with the shape by dragging it to see whether it could be "messed up" (Noss & Hoyles, 1994). They began by rotating the figure into a position that was close to the prototypical rhombus described above:

M: That's a rhombus.
I: Did you say it was a rhombus before?
M: Yes, but it didn't look so much a rhombus. [laugh] It was sort of turned.

To begin with they only dragged points, which changed the orientation. Then they picked up a point that altered the figure so it was clearly no longer a rhombus. They recognized the kite but were confused because they felt the shape had been constructed to be a rhombus and dragging certain points was consistent with the notion that the rhombus could not be messed up, whereas dragging other points did mess up their rhombus.

The girls now had a dilemma, since shapes that were not constructed were drawn. This shape could be messed up and so was not constructed. Yet, they felt that there was some sense of construction involved.

C: Hey! [laugh] I don't get that.
M: That was a kite, look that goes along there… I don't get how it can be constructed and drawn?
I: Ahh! Just say what you mean there?
M: Well, those 2 points change…{yeah} and they're constructed… And the other corner…{yeah} those 2 just...don't work [embarrassed laugh]… They like aren't constructed…they can mess it up…
C: Yeah, they can mess it up
M: It's not a messed up kite, but it is a messed up rhombus. {Yeah}
C: No it's not, because it's a square that's changed, which is…
M: I don't think it's a rhombus any more.
I: You don't think it's a rhombus anymore? {No} So what do you think it is?
M: A kite, or a quadrilateral [quadrilateral said together].

Michelle at least had recognized that the notion of messing up needed to be related to the figure supposedly constructed (line 17). They could regard the figure as the construction of a kite but not of a rhombus. This understanding was far from stable.

I: So can you mess the kite up?
C: That doesn't look like a kite any more. But if you look like that…
M: That one looks longer than that one. But sometimes it looks like a kite, and sometimes it doesn't.

The girls were still struggling with the idea that the kite sometimes looked like a rhombus. A resolution to this paradox would have been to incorporate rhombuses into their definition of kites i.e. to see rhombuses as special types of kites. It seemed though that the partitional type of definition was closer to the prototypical visual-based component of the figural concept. The pedagogical challenge here was clearly to find a means of helping the girls to focus more on defining the figure in terms of its properties. The dragging movement was raising the conflict but the resolution was not apparent to them. Their confusion was intensified when moments later yet another shape (the deltoid, or "pointy shape" as Michelle calls it in line 28) appeared:

I: Can you get it when it doesn't look like a kite?
C: Yeah, that one.
I: What would you call that?
M: Um, um… A pointy shape.

DISCUSSION

The episode illustrates the problem to which Herschkowitz (1989), Fischbein (1993) and Tall and Vinner (1981) have all alluded. Michelle and Christine clearly are constrained by referring rather infrequently to the properties of the figure. This tendency to refer to prototypes was evident despite some attention on the IWB in the teaching programme to the way in which quadrilaterals can be rotated. Their prototypes are useful resources for simple manipulations of orientation but do not support hierarchical inclusive definitions. In the teaching programme, squares were seen quite explicitly as special types of rhombi but these experiences, it seems, were insufficient to encourage Michelle and Christine to refer consistently to conceptual criteria.

It is reasonable to suppose that the figural component might have been reinforced by the visual affordance of the IWB. It is clear that task design and teacher-focus are crucial if the kinaesthetic affordance (of both the DGS and the IWB) is to be harnessed in support of the conceptual aspect of the figural concept. We do not believe that the kinaesthetic affordance was properly exploited in the first iteration of this study. We now ask ourselves whether it is reasonable to expect such children to understand the definition of a kite when they do not appreciate the utility (Ainley & Pratt, 2002) of a definition. We therefore are attracted to the approach advocated by De Villiers (1998) in which we put children in the position of being definers themselves.

We want to place emphasis on the dilemma presented by different Cabri constructions of the same (apparently) mathematical object through a task such as the following:

> Here are three figures constructed in different ways in Cabri. Which provides the best definition of a kite?

One construction might be based on Michelle and Christine's approach, which is restricted in so far as the figure disappears when the lowest point (as depicted in Figure 1) is dragged inside the circle. Another construction (as used in our interviews) might allow deltoids as well as kites. A third could be constructed in such a way that the figure disappears just as the kite is about to transform into a deltoid. All three constructions should have the property that in certain configurations the figure looks like a rhombus and in others like a square. The issue of whether a partitioned definition is better than a hierarchical definition would be raised. The same issue could be raised using a similar task based on rectangles, squares and oblongs[2].

The task would be introduced on the IWB, followed by small group work on PCs. The group work would enable children to act as definers drawing personal conclusions. The IWB would act as a public forum for opinions to be shared, conflicts raised and hopefully awareness that different definitions are possible and that utilities are attached to each possible definition.

In summary, two conjectures have emerged from this initial study and they will be tested during Spring, 2003:

(i) The visual and kinaesthetic affordances of the IWB are insufficient to encourage the fusion of conceptual and visual aspects of children's figural concepts when these

[2] In primary schools in England and Wales, "oblongs" are often referred to as shapes whose length is greater than their width i.e. rectangles which are not squares.

affordances are embodied in tasks that simply focus on the visual transformation of geometric figures, and

(ii) The kinaesthetic affordances of the IWB need to be embodied in tasks based on the utilities of contrasting definitions that draw attention to the conceptual aspect.

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