METAPHORS AS VEHICLES OF KNOWLEDGE: AN EXPLORATORY ANALYSIS

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The paper considers a teaching experiment carried out with secondary school students (9th grade), who face modelling tasks to approach some basic concepts of algebra and early calculus. The focus is on an embodied analysis of students' cognitive processes. The analysis highlights the use of metaphors as a means of sharing knowledge.

L'article présente une ingénierie menée au niveau de l'enseignement secondaire où les élèves (14-15 ans) ont été confrontés à des activités de modélisation pour l'introduction de concepts basics de l'algèbre et de premiers éléments de l'analyse. L'objectif est l'analyse des processus cognitifs des élèves en termes de 'embodied cognition'. Celle-ci met l'accent sur l'utilisation des métaphores comme moyen pour partager de connaissances.

INTRODUCTION

This paper focuses on the genesis of metaphorical thinking to talk about mathematical objects within a long-term *teaching experiment*. The experiment is concerned with the problem of introducing algebra at the beginning of secondary school level. From the didactical point of view this is a crucial issue in teaching and learning algebra in order to overcome some cognitive difficulties related to the differences between algebra and analysis. Particularly, the experiment is based on the concept of function and its multiple representations, like graphs, and number tables. The students participated in activities based around modelling mathematical or physical situations (for example motion), using both paper and pencil and technological environments. The experiment shares the Vygotskian perspective of the social construction of knowledge through *discussion* as a means of communication (Bartolini Bussi, 1998). The *social interaction* arises from two subsequent periods of each activity. Firstly, students work in groups, sharing their ideas with their peers; secondly, a mathematical discussion led by the teacher involves the entire class. An embodied approach is used to analyse students' cognitive behaviours. In particular, attention is drawn to the use of *metaphors* in the evolution of mathematical understanding.

THEORETICAL FRAMEWORK

"Any function in the child's cultural development appears twice, or on two planes. First it appears on the social plane, and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category" (Vygotsky, 1981, p.163). Vygotsky (ibid.) states the importance of a social nature of human learning. An interaction between individuals takes part in every kind of discussion; hence, an interpersonal activity comes before an intrapersonal interiorisation. According to this perspective, more recent research studies the role of the *mathematical discussion* in the ways individuals construct meanings (see e.g. Bartolini Bussi, 1998). A mathematical discussion is defined as "a polyphony of articulated voices on a mathematical object (either concept or problem or procedure or belief) that is one of the motives of the teaching-learning activity" (Bartolini Bussi, ibid.). Within this frame, "communication plays a central role. But communication is not seen as a disinterested communication. The individuals communicate between themselves to carry out goal-oriented activities having culturally motivated goals" (Radford, 2001). The teaching experiment taken into account in this paper is an example of such a situation: the problems are planned to favour communication processes through social interaction. Another component of the framework is represented by the *embodied* nature of mathematical ideas, that comes from cognitive linguistics and the theory of *embodiment* (Lakoff & Johnson, 1980; Lakoff & Nùñez, 2000). Such researches argue that human thought processes are deeply metaphorical and metaphors "are not random but instead form coherent systems in terms of which we conceptualize our experience" (Lakoff & Johnson, ibid.). Lakoff & Nùñez (ibid.) work within the mathematics field, asserting that abstract notions are conceptualised in terms of concrete notions through precise inferential structures: the *conceptual metaphors*. Metaphors project the inferential structure of a source domain onto a target domain, which can be both mathematical or not. There are different kinds of metaphors: in particular, when the source domain is outside mathematics, we speak of *grounding metaphors*. Such an approach pursues further connections to other fields which study the behaviour and the structure of the brain; these studies emphasise the fact that neuro-biological constraints seem to affect mathematical knowledge (see e.g. Berthoz, 1997; Dehaene, 1997).

THE RESEARCH PROJECT

The teaching experiment is part of an ongoing research project I am involved in. The project is focused on a *meaningful* approach to some basic topics in calculus (functions, limits, derivatives, integrals) and involves secondary school students, from the 9th grade onwards. From the didactical point of view, meaningful refers to the hypothesis that the use of devices and manipulative materials within different *fields of experience* (Boero et al., 1995) can support pupils in the construction of knowledge (see e.g. Arzarello, 2000). On one hand, devices and materials allow students to touch, perceive and observe changes as consequence of their actions, to feel things about the activities they face. The body as a whole, senses, intuitions and gestures are involved in the activities. On the other hand, devices and materials are *artefacts* (Verillon & Rabardel, 1995). According to an instrumental approach, artefacts are particular objects with their intrinsic characteristics and can become *instruments* if considered with a well-adapted use (Mariotti, 2002). A cultural artefact is important because it "becomes efficacious and transparent, by its use in the context of specific types of social interactions and in relation to the transformations it undergoes in the hands of user" (Meira, 1995). In this perspective, the research considers the transition from perceptual-experimental level to conceptualisation, from a cognitive point of view, and studies the ways in which students construct meanings in a social context.

THE TEACHING EXPERIMENT

The teaching experiment discussed in this paper involved 25 first year students (14-15 years old) of an Italian scientifically oriented high school (Ferrara & Robutti, 2002) [1], during the school year 2000-01. It lasted 30 hours; in the classroom sessions, first the students worked in *small groups* (of two-three-four pupils) and then participated in a *classroom discussion* (featured by the institutionalisation phase).

Activities

Some paper and pencil activities preceded other activities in which technological tools (sensors and symbolic-graphic calculators) were used. The first kind of problems has the aim of gathering and interpreting measurement data. The second one, related to the modelling of motion, features the core of the experiment. The students (of each group) moved in front of a sensor (CBR) to reproduce various kinds of motion (uniform, accelerated, periodic…), using their body or toy objects, e.g. a bouncing ball. The CBR functions as a motion detector: each tenth of a second, its inner chronometer sends an impulse towards the moving object and registers its position. Pupils observe the building of the graph (representing the time law) in real time on the screen of a calculator connected to the sonar. Time and position data of motion are stored in the memory of the calculator. The students may organise this data in a number table and in a graph. Pupils are asked to describe the movement both in a qualitative and quantitative way. Firstly they use informal language to explain the movement and the graph; secondly they are asked to interpret the shape of the graph from a global point of view. The third step is represented by a local interpretation through the calculation of the graph's slope at different points. These are not traditional problems; their didactical effectiveness lies in the possibility of relating the data of a phenomenon with the function that describes it. Generally, all the experiments in which students can interact with a tool to create phenomena, help them to understand the mathematics connected with those phenomena (see e.g. Nemirovsky et al., 1998). After these activities (which focus on the construction of a model starting from motion), the third phase of the experiment is related to the inverse passage, from models to motions. A paper and pencil activity represents the final phase; it has the purpose of finding the model of two different mathematical tasks.

Methodology

During the experiment, four people were present in the classroom: the two teachers (one of mathematics and one of physics), a university student and a researcher, who had planned the activities together. A video-camera recorded group and classroom discussion. The analysis was conducted using the video-tapes and the written notes of students. Therefore, pupils' actions, gestures and language are the ingredients by which their cognitive learning processes are studied.

EXAMPLES OF GROUNDING METAPHORS IN THE PROTOCOLS

The analysis discusses excerpts of the following situations (two letters are used as reference in the excerpts of protocols. As appeared below, they respect time order) [2]:

- *paper and pencil activities*: interpreting a graph of temperature measurement during the day (TR);
- *activities with technology*: (a) modelling uniform motion of a student (UM); (b) modelling accelerated motion of a student (AM); (c) modelling periodic motion of a student (PM); (d) reproducing a given time law with a movement (RA).

Gestures and words show a richness of different grounding metaphors in students communication. I group examples from the problems considered above, through the qualities of each metaphor.

A first case: the *Temperature is a Moving Object* metaphor

From TR (working group: C, I, S)

Tr17 S: In which time interval of one hour has the maximum decrease in temperature measured? [she is reading the statement of the problem] (…)

Tr21 I: Temperature went down more.

Tr22 C: Hmm, did temperature go down more? (…)

Tr28 I: For example, if it has gone down more between ten and eleven o'clock. (…)

Tr31 I: This! (…)

Tr39 I: …Because here it goes up [she is pointing to the part of the graph in which temperature increases]…and here it goes down. (…)

Tr47 I: Then that is the maximum decrease we were looking for.

Tr48 C: Listen, let's look at the temperature!

Tr49 I: But, it is clear…because on this side [she is pointing to the two ends of the vertical change with her fingers] did it go down from...? [she is positioning the ruler on the graph, parallel to the horizontal axis, in order to check the temperature in the two points considered]

The three girls are observing a graph of measurements of the temperature of a room, at different times during the day. They are trying to find the time interval corresponding to the maximum decrease in temperature (*Tr*17). Most of their sentences highlight the richness of verbs, like "to go up" and "to go down", to indicate a change in the values of temperature data (*Tr*21-*Tr*28; *Tr*39; *Tr*49). "Temperature" is the subject of these sentences. The subject is implicit, when "it" is used, but deictic gestures indicate the girls refer to temperature (*Tr*39; *Tr*49). Actually, the students are thinking of temperature in terms of the phenomenon they are used to experiment. This phenomenon is represented by the mercury column of the thermometer, which moves up or down with respect to a previous level, when the temperature of the room changes. This change is an increase or a decrease; therefore, it acquires spatial features (referred to the mercury level), to be described. On one side, these features are static if referred to the spatial structure of the mercury column (for example, level, height). On the other side, they are dynamic, when temperature is conceived of as a moving object (with respect to the movement of the mercury, up or down). As a consequence, we may state the existence of a metaphor. A (non mathematical) source domain (temperature) is thought of in terms of a (mathematical) target domain (space). For this reason, we can define the metaphor Temperature is a Moving Object. This is the "cognitive dimension" of the metaphor. Moreover, the metaphor has also a "social dimension" in the dialogue. From *Tr*21 on, it becomes a means of communication between I and C. Soon, the language introduced by I is acquired and used by the group mate C, first in order to understand which the question is, then to interpret the graph (the sign).

The *Time is a Moving Object* metaphor

From UM (working group: F1, F2, G1, G2)

*Um*155 G1: Time is increasing, space is also increasing. (…)

*Um*177 G1: If you look at seven [he is considering the first seven time values]…time is always increasing, all right…space: 42, 43,50. [he is reading some digits of space values on the table] (…)

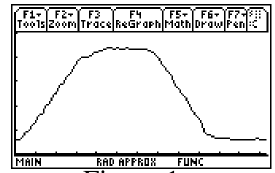

Figure 1.

*Um*296 F1: Time is always increasing, but not until a certain point.

*Um*297 G2: …Anyway, it is an observation…Space…

*Um*298 G1: But they increased together until a certain point!

*Um*299 G2: Then space started to decrease, while time continued to increase. (…)

*Um*439 F1: …I stopped and when I stopped…

*Um*440 G2: It goes on [she is pointing at the curve], doesn't it?

*Um*441 G1: Because if you consider a straight line [he is raising his pencil in a vertical position]...if time is increasing [he is raising his pen further]...if time is increasing and space is increasing too [again his pencil up]...we cannot get a curve [he is drawing a curve in the air]...for me it is a straight line. (...)

*Um*443 G1: Because they both are increasing. (...)

*Um*445 G1: If you always go on at the same velocity, they [time and space] both increase.

<u>From AM (working group: F1, F2, G1, G2)</u>

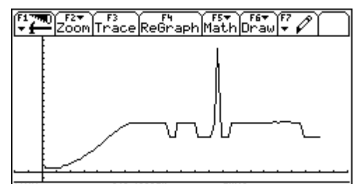

Figure 2.

*Am*102 G1: Time is increasing, space is increasing.

*Am*103 F1: Yeah!

*Am*104 G1: Both…

*Am*105 G2: They are increasing in a progressive way.

*Am*106 G1: Both space and time.

In UM and AM, the students are engaged with the CBR. They are asked to reproduce a back and forth uniform motion and an accelerated motion respectively. The excerpts show examples of a behaviour which characterises the entire experiment. In both situations, the students have to find the qualitative relationship between space and time. In the first case, pupils use both the number table and the graph (fig.1). The interesting point is their pervasive use of the verb "to increase" both for space and time data. Such a verb is properly used for a changing quantity. This is the nature of an object with spatial features. For example, its length or height may increase or decrease. Actually time passes or goes on, it does not increase. Instead, G1 early conceives of time as an increasing quantity (*Um*155, *Um*177). The same language is acquired by the other group mates (*Um*296-*Um*299). It features the discussion up to the end, even when the students have to interpret the shape of the graph (*Um*441-*Um*445). Time and space seem to behave similarly, as if they were the two domains of a metaphor. Time (as the target domain of the metaphor) is grounded on the structure of space (which becomes the source domain). The metaphor has both a social and a cognitive dimension. The students focus on the changes of a quantity; from their point of view, the "spatial features of time" (like length) are static, because they refer to the structure of the space. However, time can also have dynamic features, like in the sentences "We are approaching to Christmas" or "Christmas is approaching". In both cases, we think that a temporal period (Christmas) moves, as pointed out by the *Time is a Moving Object* metaphor (Lakoff & Johnson, 1980). The excerpt from AM shows the same cognitive behaviour in a different situation. In this situation the students only take into account the graph (fig.2). There is a second remark to do on UM; at *Um*440 the curve that "goes on" indicates the presence of a *fictive motion*: "a line is the motion of a traveller tracing that line" (Núñez et. al., 1999). G2 conceives the graph (a static sign) in a dynamic way. Hence, fictive motion allows a line to be thought in terms of motion. We can find examples in everyday or mathematical language: "the path goes across the wood", "the graph riches a minimum", and so on.

Towards a *blended space*?

From PM (working group: F1, F2, G1, G2).

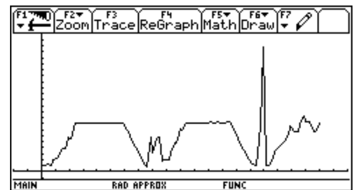

Figure 3.

*Pm*77 F1: How many times did I cover this? [he is referring to back and forth run]

*Pm*78 F2: How does it [the graph] travel?...Look.

*Pm*79 F1: It is uniform more than one time.

From RA (classroom discussion: T=teacher, A)

*Ra*81 A: It [m] is not a constant.

*Ra*82 T: It is not a constant. According to you does it change positively or negatively?

*Ra*83 A: In theory...

*Ra*84 T: I mean, does it increase or decrease?

*Ra*85 A: In theory, it increases.

*Ra*86 T: Why in theory? (...)

*Ra*89 A: It has to slow down, I mean it has to, I mean it does not continue to accelerate [he is drawing a parabolic curve in the air], then it suddenly stops [he is drawing the horizontal stretch], it has to slow down a little [he is drawing like a hunch]... If we consider that rate, I mean that rate does not increase where it slows down.

PM refers to the activity in which the students are asked to reproduce a periodic motion (fig.3 shows the graph). Instead, in RA pupils have to realise a motion represented by a given model. In the first situation the students are explaining the shape of the graph; in the second one they are discussing with the teacher about the changes of the slope (m) in a particular time interval (from six to eight seconds). In both excerpts, it is suggestive to note the trend of pupils towards linking the graph with motion (*Pm*78, *Pm*79, *Ra*89). They think of the graph in terms of the experiment they lived, when they moved in front of the CBR. A metaphor can describe the situation: we may call it the *Graph is a Moving People* metaphor. The source domain is given by the motion experiment, while the graph becomes the target domain. However, there is something more in the information embedded in this case (also with respect to the previous situations). The students seem to identify the graph and the motion. This cognitive behaviour is marked by the use of verbs of motion ("to travel", "to be uniform" "to slow down", "to accelerate") when talking about the graph ("it"; also A's gestures refer to a curve). We can interpret this process as the constuction of a *blended space* (Lakoff & Núñez, 2000). In this space the features of the two domains (motion and graph) are merged in a unique new sign: the *motion-graph*.

CONCLUSIONS

The exploratory analysis points out that metaphors seem to support the students and show an evolution in their conceptualisation processes. Moreover, metaphors appear fruitful to construct a shared language within a group. Further research in the field is needed: recent trends are going to study how a technological artefact can affect the use of metaphors (see, e.g. Robutti, accepted) to build meanings for advanced mathematical concepts. An interesting open problem is the dialectic between metaphors and symbolic manipulation, in activities in which signs are deeply involved, as, for example in the most recent works by Tall (see e.g. Tall, 2002).

Notes

1. The school is a Liceo Scientifico, in which students attend five mathematics classes and three physics classes per week.
2. I do not present the entire problems because they are not so important for the analysis.

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