LOGICAL CONSEQUENCES OF PROCEDURAL REASONING

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This presentation makes a link between the common student view of mathematics as a collection of procedures and a certain type of error in constructing or validating proofs.

It is well recognized that students commonly view mathematics as a set of procedures (Schoenfeld, 1992). These procedures typically take some mathematical object(s) as an input (e.g. a function) and return some other object(s) (e.g. the derivative of that function). In this presentation we suggest that this view contributes to student difficulty in understanding a point of logic when they are required to construct or validate proofs.

This claim is illustrated using the following task, which was given to students during a study that tracked their developing understanding of real analysis (Alcock & Simpson, 2002).

> Check this proof and make corrections to it where appropriate:
>
> Theorem: $(\sqrt{n}) \to \infty$ as $n \to \infty$.
>
> Proof: We know that $a < b \Rightarrow a^m < b^m$.
>
> So $a < b \Rightarrow \sqrt{a} < \sqrt{b}$.
>
> $n < n + 1$ so $\sqrt{n} < \sqrt{n+1}$ for all n.
>
> So $(\sqrt{n}) \to \infty$ as $n \to \infty$ as required.

The “deduction” made from the third to the fourth line effectively claims that since the sequence is increasing, it tends to infinity. This is invalid since, for example, the sequence $(-1/n)$ is increasing but does not tend to infinity.

However, *for the sequence in question*, both lines are true, and we will present an illustration in which a student appears confused as to why sequences other than the original $(\sqrt{n})$ are relevant to the proof. We will argue that the procedural view of mathematics may contribute to this since the student is accustomed to applying a series of steps to one object and not to considering other objects during this process.

References

Alcock, L.J. & Simpson, A.P., (2002), "Definitions: dealing with categories mathematically", *For the Learning of Mathematics*, (in press).

Schoenfeld, A.H., (1992), “Learning to think mathematically: problem solving, metacognition and sense making in mathematics”, in Grouws, (Ed.), *Handbook for research on mathematics teaching and learning*, New York: Macmillan, 334-370.