THE EMERGENCE OF MATHEMATICAL GOALS IN A RECREATIONAL PRACTICE

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While the influence of individuals' participation in practices of economic subsistence on the development of mathematical knowledge has been widely reported in the literature, there are not as many studies about the relationship between cultural life and mathematical cognition in the context of recreational practices. This article reports a study on the development of arithmetical knowledge and reasoning in the context of adults' and children's engagement in a game of dominoes. The study was carried out in a small community in Northeastern Brazil, where twenty-five individuals were regular players of the game. The results show that subjects with little or no formal schooling were highly sophisticated in performing arithmetic operations, examining hypothesis, and coordinating the rules and goals of a complex game. Implications for the study of recreational practices and their interplay with school instruction are discussed.

Very many studies have discussed how the organization of specific professional practices lend support to the development of widely diverse mathematical competencies (e.g., Nunes, Schliemann and Carraher, 1993; Lave, 1988; Saxe, 1991; and many others). Much less of the psychological research on culture and cognition has focused on mathematical activities that people engage in as they participate in recreational practices. We explore in this article various aspects of a specific recreational activity which engages children and adults from a rural community in Northeastern Brazil, players of a domino game called "lustrado". In order to understand the interplay amongst engagement in such activity and the development of mathematical knowledge by players, we employed Saxe's (1991) framework of emergent goals. Saxe describes four parameters that determines the emergence of goals in a practice: 1. The activity structure, as complexes of actions that emerge repeatedly over time through a cyclic structure; 2. Social interactions, in the context of which individuals construct and share specific goals; 3. Artefacts (instrumental tools) and conventions (symbolic means) which function as the material basis for action; and 4. Prior understandings in the form of experiences and knowledge that individuals bring into a practice. Central to Saxe's framework is the notion that goals are emergent phenomena, continuously transforming and being transformed by the social and material structuring of settings. Thus, emergent goals motivate actions which in turn trigger the emergence of new goals. In an important sense, emergent goals do not "belong" to individuals. Rather, they emerge at the intersection of personal understandings, the structure of one's activity, the cultural forms or conventions (e.g., Hindu-Arabic numerals) and cultural artefacts (e.g., currency) of one's practice, and the social interactions among co-participants of the practice. Saxe's (1991) framework was used in this study to the extent that it helped us as researchers to organize, coordinate, describe and interpret the critical relations among actors' purposeful actions and the scenarios of their activities. As suggested above, the emergence of goals has a reciprocal relationship to the activities one is engaged in. Activities are realized through one's actions in specific settings. These actions involve at the same time cognitive, social, and material aspects of the phenomena under investigation in the form of prior understandings, social interactions, and the use of conventions and artefacts. Thus, the study of emergent goals is expected to contribute to our understanding of how people use and transform mediating tools in activity. The article brings about three sets of contributions: 1. An illustrative analysis of practices from the perspective of Saxe's framework; 2. An interpretive look at the relations between cultural participation and mathematical cognition in the context of a recreational practice; and 3. A discussion of how knowledge constructed outside school might relate to school knowledge.

EMPIRICAL STUDY

The game. Though played with the same pieces and most of the rules of traditional dominoes, the lustrado opens four instead of the regular two ends and the players' main goal is to play the pieces so that the sum of values in all ends is a multiple of five. The game is played by four people each of whom starts up with 6 pieces (out of the 28 set; a subset of 4 pieces is set aside to be used optionally by any of the players as needed). A game is played through several rounds and it ends when any of the players reaches 200 points. In each round players take turns to play their pieces with the goal of building a summation multiple of five (among other goals). Each round comes to an end when any of the players plays his or her last piece (or, as in the traditional domino, none of the players have pieces that match the available ends). The winner at each round gets the sum of all points he or she made during the round (the total sum of all sums multiple of five) plus the multiple of five closest to the sum of values in all pieces not played by his or her opponents. The convention [a;b] will be used to represent the values on the face of a domino. As a piece is connected to one end or extremity available in the game, "a" will stand for the value on the free side and "b", the value to be connected (for instance, [5;2] may connect to an end with the piece [2;1]). Pieces with the same value on both squares contribute to the sum with the full number of dots (for example, [5;5] counts as 10).

The players. Subjects in the study were 25 villagers in small neighboring farms surrounding a countryside town in Northeastern Brazil. The group had 15 adults (agricultural workers aged 16 through 61) and 10 of their children (aged 6 to 13). Of the fifteen adults, 14 had left school before finishing fourth grade and one was in high school (a woman who was also an elementary school teacher in a nearby village). All the children were enrolled in school at the time of the study: 8 of them in classes up to fourth grade, one in seventh and one in eighth grade. These individuals were the only players of lustrado in the community (with the exception of two adults who refused participation in the study). Therefore, nearly the entire population of players in this particular community were also the subjects of the study.

The observations and tasks. The study included open interviews with local informants, naturalistic-ethnographic observations and videography of spontaneous games, along with individual problem solving sessions. Two types of tasks were used in the problem-solving sessions: 1. A school-like task with problems about multiples (though we did not use this particular term, that is, we asked the subjects to, for instance, recite the numbers between 49 and 70 if we were counting by 7s) and arithmetic operations (addition and multiplication); 2. A game-like task which included arranged games involving multiples other than “five” and simulations of game situations frequently observed during spontaneous tournaments (the subjects were asked either to imagine or chose from available options a domino they would find most “appropriate” to play next in an ongoing fictitious game).

ANALYSIS

The analysis will be parsed into three interrelated sections, following the parameters suggested by Saxe's framework of emergent goals, though grouped in a somewhat different fashion: 1. Activity structure; 2. Social interactions, artefacts and conventions of the practice; and 3. Prior understandings and the mathematics at play.

Activity structure. This part of the analysis aimed at characterizing the game as a structured activity. Two general phases seemed important for the players themselves during the game: 1. The start of a round; and 2. Making decisions at each turn. Each phase incorporated complexities that sparked the emergence of diverse goals. At the start of a new round, each player would receive six domino pieces and analyzed the set in regard to at least the following two aspects: 1. The number of repeated values across the pieces (e.g., [1;2], [5;1] and [1;1] all have “ones”); 2. The number of dominoes with “fives” (e.g., [2;5]) and/or “blanks” (e.g., [0;6] and [5;0]). Players with many repeated values at start up were believed to have better chances of dominating one or more ends within a round. To realize this during the game, players built hypotheses about the dominoes other players could have, which in turn involved careful consideration of the pieces in one's own hand and those already played by others. Dominoes with a blank or five (in any combination) were important because scoring in the game depended on generating sums multiple of five. Dominoes with a blank allowed the “cancellation of surplus values” in a sum: for instance, if the current sum of values in all ends was 18 and one of the ends had a 3 on it, a player would score 15 points by playing the [0;3] piece at that end, or “18 - 3 + 0 = 15”. Dominoes with fives were even more valuable for they could be used to cancel surplus values and at the same time add value to a sum: for instance, if the current sum was 18 and one of the ends had a 3, a player would score 20 points by playing the [5;3] piece at that end, or “18 - 3 + 5 = 20”. [This representation of the players’ actions as arithmetical expressions is not intended as a description of the procedure effectively implemented by the players as they “read” the sum of all values at the ends, a theme discussed later.] After examining the dominoes received at the start of a round, players would then decide what piece to play first among all possible starts. For instance, say that the very first player in a round had many pieces with repeated values in his or her initial set. If he or she would chose to initiate the game with one among such pieces, the other players would rapidly loose their dominoes containing that value (for the first turn may require the other players to play up to three dominoes with, for example, a “5” on them out of the seven pieces with this value). In such cases, opponents would become vulnerable in subsequent turns since they would be more likely not to have an exemplar of that specific value. The episode below, extracted from annotations taken during observation of a spontaneous game among adults, illustrates this situation:

> Levi got the pieces [1;2], [1;5], [5;2], [3;1], [5;0], [1;1], and started off the round with [1;1]. In doing so, he caused two opponents to play dominoes with a “1” and a third opponent was made to “pass” (not being able to play any of his pieces). Thus, after the

very first turn, Levi kept for himself three out of the four dominoes with “1” still to be played. Later on in the game, he effectively gained control over a relatively large number of ends within the round.

Game participation also required many decisions on subsequent turns. In this second phase, players seemed to repeatedly analyze and build hypotheses about at least the following five factors: (1) The number and values of dominoes still in the hands of opponents, based on a count of the number and values of dominoes already disclosed; (2) The chances of missing a turn at playing; (3) The chances of having the last domino of a kind; (4) The summation of all dominoes one had at a given moment; and (5) The possible values of the four pieces left aside at the beginning of each new round. These factors contribute to the complexity of the game, specially when played by adults. As a consequence of all these factors, the decisions made by players resulted in a complex system of goals and activities which we have categorized as follows: (1) Scoring. The very basic goal of building sums multiple of five was a relatively complex one since each domino could be played in eight different ways per turn (1 domino times 2 values per piece times 4 ends); (2) Tracking opponents. Sometimes a player’s prime goal would not be scoring, but constructing hypothesis about the pieces his or her opponents could have. This action relied on careful observation of the opponents' game, specially at moments in which they avoided an extremity or missed their turn; (3) Individualizing ends. Typical among players who collected many pieces with repeated values at start up, this goal was aimed at making opponents to miss a turn and/or not score at specific ends.; and (4) Lowering deficits. As the loss of a round was inevitable, players very rapidly attempted to play their pieces, specially those with the greatest values (e.g., [5;6], [6;6] etc.) since the winner of a round added to his or her final score the multiple of five closest to the summation of all pieces not played by his or her opponents. The players' activity in each round and along the turns was structured and continuously oriented by these goals, which emerged either individually or combined. Goals emerged in different ways and combinations depending on circumstantial aspects of the activity, e.g., whether the game involved only adults or children and adults, as we will show in the next section. In sum, participation in the game involved a recurrent and cyclic structure of goals which included the summations at each turn and the examination of hypothesis about where and how to play.

Social interactions, artefacts and conventions of the practice. Our observations revealed that engagement with the game was not primarily guided by strict rules and routines of action. Instead, the players’ activity could be better described as emerging from a sophisticated set of local goals, social interactions and culturally constituted mediational artefacts. The emergence of individuals’ goals depended on the game’s structure as delineated in the previous section, but also on the nature of interactions among players (e.g., who plays against whom) as well as on the discursive conventions typical to the practice (who talks and what is said). An important outcome of the study in regard to social interactions emerged from comparative observations of spontaneous tournaments involving only adults or children with those in which adults and children played together. When playing amongst themselves, the children were a lot more concerned with scoring than with, for instance, tracking opponents or individualizing ends. The opposite was observed during spontaneous games involving only adults, as they struggled to combine scoring with the goal structure of hypothesis building and tracking of opponents as described earlier. Yet, in games involving adults and children, the former tended not to use more elaborate strategies but to direct their game towards scoring only (typical among children) while the later more frequently attempted to make use of strategies such as lowering deficits and individualizing ends (typical among adults). Adults would still win over the children in such games, but such a process of "mutual appropriation" of goals (Newman, Griffin and Cole, 1989) could certainly support the development of expertise among the younger players as they imitated the deployment of advanced strategies under adult supervision. Curiously, a different result was obtained when we compared adults' and children's performance in interviews with simulated games. The task involved the presentation of an on-going fictitious game and asked the subjects to chose amongst several options or imagine the domino piece they found most appropriate to play in the situation presented (half of the tasks showed a set of actual pieces to choose from while in the other half the subjects were asked to freely imagine "the best piece"). We then categorized the subjects' responses according to whether the chosen piece (actual or imagined) represented the best, intermediary or worst option for scoring in that particular turn (when presenting the simulation, however, we did not specify any playing goals or whatever we meant by "the best piece"). Finally, we grouped the subjects' answers in a performance scale of four levels (A through D), in which each level marked the percentage of answers in the upper category ("best options"). Thus, level A grouped the subjects who chose (or imagined) the best option in 90 to 100% of all tasks presented; level B, 80 to 89%; level C, 70 to 79%; and level D, less than 70% of all answers in the best option category. Based on this schematic view of the subjects' performance on the game-task, we found that the children chose more of the assigned best options than the adults (40% of the children's answers in level A compared to 33% of the adults' in this level). [Note that all but two players of lustrado in the community took part in the study. Since almost the entire population of players were interviewed, this sort of direct comparison of frequencies should be sufficient to determine meaningful differences between groups.] This result runs contrary, or so it seems, to the observation of spontaneous games in which adults were more successful than children. A possible answers comes from a close examination of the study itself, in particular of the research scenario it created. Indeed, the game-task presented simulations that, though representative of the actual game, were removed from the usual game context. The subjects were then asked to chose or imagine "the best piece" to continue an arranged fictitious game which was presented individually in a problem solving session. As it was, this task re-contextualized the game practice in a way that obscured the dynamics of the game itself. For instance, the strategy of tracking opponents did not immediately result in scoring: its effects could only be noticed at a later time in the case a player had become dominant at specific ends. The analysis indicates that 46.7% of the adults' answers to the game-task were attempts to deploy this strategy against only 30% of the children's answers to the same tasks. However, this strategy was really not effective (or "the best option") in the simulated games since there were no opponents to be defeated. On the other hand, scoring was the main overt strategy for the children in up to 40% of all their answers, against only 13.33% of the adults' answers. In sum, the unexpected result –more children than adults in the upper level category for the simulation task– was due precisely by an unintended effect of our re-contextualizing the game as a problem-solving task. Indeed, our initial conception of "the best piece" was limited to the goal of scoring (typical amongst the children). On the other hand, none of the simulations presented had a history of play (either past or future) or actual opponents against whom to use more elaborate strategies (typical amongst the adults). It seems though as if the adults had constructed virtual opponents and consistently attempted to employ in this new game scenario well known strategies that proved ineffective.

Prior understandings and the mathematics at play. Our first attempt to account for the mathematics at play in the lustrado prioritized the notion of multiples of a number. It is of importance that the participants themselves described their activity in the game as directed towards finding "multiples of five". Nonetheless, our observations indicated that players were most of their time performing sums which involved the values of pieces played at the ends on each turn. More importantly, players also constantly dealt with cancellation of surplus values: the process through which a player calculated the difference between the sum of all ends and the closest multiple of five, and then chose a piece-value that could null the difference and possibly add up to a multiple of five. Consider, for instance, a player's (João, aged 26) answer to what piece he would like to have in a situation with the ends [4;1], [2;2], [3;3] e [6;6] ("4 + 4 + 6 + 12 = 26"): "Uh, I wanted to have a three by four ([3;4], with the 'four' connecting on [4;1])…. I'd play here ([4;1]), I'd make 25, because look, 4 ([2;2]) and 6 ([3;3]), 10; plus 12 here ([6;6]) makes 22, right? And three by four here (on [4;1]), 25 (which is obtained by first canceling out the 4 in [4;1] by connecting the [3;4] to it, which we can represent as 26 – 4 = 22; then adding the 3 from [3;4] to the previous 'result', 22 + 3 = 25)." We observe in this episode that the player seemed to visualize the needed piece and its best position through a process involving sums and cancellations of surplus values. This process was very present and seemingly crucial to players. On the other hand, recognition of scoring values (multiples of five from 5 to 35) happened to be trivial, which run contrary to our initial hypothesis of the centrality of multiplicative operations in the game. From analyzing the players' behaviors, we suggest that they had either memorized a multiplication series with the appropriate values (5, 10 etc.) or, alternatively, that they could regenerate the entire sequence by repeated addition by 5s beginning with five. Players' ability to generate lists in such a way is shown in the following quote, registered as an adult approached a school-task which asked for multiples of 3: "3, and 3, 6, and 3, 9 and 3, 12, and 3, 15, and 3, 18, and 3, 21." Also, when asked to play the lustrado using multiples of 7 instead of the usual 5, players had no difficulties in generating the list of scoring values, as in the following quote: "it's by 7s, if it comes out 6 you don't score, it's 7, 14, 21." Further evidence that players seemed not to be aware of more elaborate notions about multiples is given in the following. When asked to spell out multiples of a number between two given values (without access to any part of the sequence of multiples) or when asked to continue a given sequence, children and adults alike (including those with higher schooling) did not do as well as in the previous situations:

Interviewer- If I were counting by 7s, what numbers would be between 49 and 70?
Isamara- 49 and 70?
Interviewer- Yeah.
Isamara- It would be 49, 52, 59/
Interviewer- We're counting by 7s.

Isamara- Oh! It's 49... 50, wait, I was confused, 49 then 56, 63, zero... 70.

In sum, we did not find strong evidence for the use of multiplicative structures among the players, beyond their knowledge of multiplication facts and repeated additions. In spite of this, participation in the game may have had a positive effect on players' performance in school-like tasks. Indeed, we observed extensive use of oral strategies of decomposition and repeated groupings (as discussed by Nunes, Schliemann and Carraher, 1993) also employed by the players during the games for calculating partial and total scores. Consider, for instance, the following strategies employed by the players when solving arithmetical operations presented in a school-like fashion: (a) 72 + 12 = ? → "70 + 10 = 80; 80 + 4 = 84"; (b) 240 x 30 = ? → "240, times three, it's 200 times three, equals 600, plus 100 from 40 times three (120), plus 20 left over (from 120), 720 (the subject solved the problem as if it were 240 x 3)". Although applied generally, these methods produced a greater number of correct responses for additions (56.7%), more often used in the game, than for multiplications (30%, considering the mean frequency of correct answers among adults and children). However, comparative analyses of the subjects' performance according to their school level suggest that school knowledge of counting, computation and multiples did not have an important contribution for performance in the game-task. Additionally, none of the three subjects in the upper school levels were regarded within the community as specially good players. In order to verify these observations, we run a multidimensional analysis that included data from all tasks (school and games-tasks), field notes and videos. Two contrasting groupings (or poles, each of which formed by matched subjects) resulted from this analysis. The groups differed in the game strategies most likely employed by its members, but no differentiation was found in regard to schooling or age. This analysis also showed that the best players (as determined by observations of spontaneous games and informal assignments from the players themselves) had a good performance in the school-task (particularly in questions about multiples) but were not in the higher grades of schooling. Seen in the light of all previous analyses, this suggests that mathematical knowledge developed within the game practice may have influenced the subjects' performance in the school-task, though we did not find evidence for reciprocal effects.

CONCLUSIONS

This study offered a set of analyses about individuals' participation in a recreational practice and their development of mathematical knowledge. Engagement in such a practice was shown to involve players with arithmetical knowledge, the exam of hypotheses and the coordination of goals of relatively high complexity. We hope thus to have contributed to a characterization of a well-structured and efficient body of knowledge developed outside schooling but reasonably distant from practices of economic subsistence, usually described in the literature. In regard to school instruction, this study draws attention to the following question. As constructivist pedagogies (in their many forms) gradually made their way into grade school, games and other sorts of artefacts were credited as providers of "concrete" scenarios for the learning of otherwise "abstract" (therefore intangible) mathematical concepts. Games have indeed been used with the goal of overcoming well-known problems of traditional instruction such as its overemphasis on memorization and algorithmic skills. Yet, we must notice the complex relations between what have been (inadequately) called "formal" and "informal" knowledge (sometime contrasted as "school" versus "everyday" knowledge). It is not uncommon to find both in research and teaching, interpretations of studies such as those by Nunes, Schliemann and Carraher (1993) and Saxe (1991) as if out-of-school practices were meaning-oriented while school activities were, by definition, meaningless. As a consequence, it is tempting to import "real-world", "everyday" activities to school as tasks. We see this false dichotomy as fetishizing the "real-world" and the "everyday" (usually taken as belonging exclusively to out-of-school places and practices), by encapsulating them in tasks that could eventually enter the classroom and replicate the system of meanings of the "same" activity as historically realized outside school. The use of money in make-belief situations as a way to teach about arithmetics is a case in point. Money, a familiar cultural artefact, and its use in pretend games of buy-and-sell within school are believed to create situations tailored to profit from the web of meanings previously lived in by the children out of school, with the effect of making "transparent" the mathematics behind the game. However, several authors have shown the complexities involved in such pedagogical practices and the ways in which children's experiences with money out of school radically differ from the ways that money gains entry in school as a knowledge domain (Meira, 2000; Brenner, 1998; Walkerdine, 1988). Thus, this sort of transfer is largely insufficient for the development of scenarios for learning within school, and usually create a whole new set of problems for mathematics instruction. Perhaps a more productive strategy would be to think of school learning environments oriented towards explicit reflection and discussion about different and contrasting ways of constructing meanings for mathematics. In arithmetics teaching, for example, the lustrado game could be used in school activities as a culturally specific mode of signifying numbers and computations, and then contrasted with other modes of arithmetical sense-making such as those developed by candy-sellers (Saxe, 1991) or professional mathematicians. Through discussion and argumentation, the game would be appropriately re-contextualized (not de-contextualized) as an acceptable (from the pupils' viewpoint) and rich artefact for the development of mathematical knowledge and discourse within school.

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