COMPLEXITY IN TEACHING AND CHILDREN'S MATHEMATICAL THINKING [1]

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In the research conducted, the relationship between teaching complexity and children's mathematical thinking was investigated in 4 'reform' classes and 1 conventional elementary class (7-8 years). Forty lessons were analyzed for the type of teaching and children's mathematical thinking revealed during class discussion. The results indicate increased complexity in teaching and level of children's thinking was highly related to the kinds of interaction that distinguished three class cultures. These findings complete a previously proposed theoretical framework that integrates teaching and learning by detailing acts of teaching in relation to complexity of children's thinking.

It is well documented that the concerted effort in the U. S. to change conventional mathematics teaching to forms of pedagogy that coincide with learning for conceptual understanding is more difficult than initially anticipated. One reason may be, in part, due to the fact that this requires the development of far more complex and sophisticated pedagogy than was understood or even known at the onset of the effort (Wood, Nelson & Warfield, 2001). Although research on learning over the past century has influenced our knowledge of learning, similar transformation in our understanding of the teaching practices has yet to occur. Educators, such as Darling-Hammond (1996) believe the challenge for education in this century is the advancement of "knowledge for a different kind of teaching . . . that goes far beyond dispensing information, giving a test, and giving a grade" (p. 7). From recent studies such as Askew et al. (1999) and Franke et al. (1998) we are beginning to understand what characterizes the complexity in new forms of teaching and how this relates to student learning. However, it still remains that "only a few studies exist which empirically examine teaching in these classes with the same detail and attention to theory building as found in the investigations of learning" (Wood, 1998, p. 193).

In previous research we have examined class cultures for differences in pedagogy and found that teaching for conceptual understanding does not consist of a singular practice, but rather varies on two dimensions—expectations for class members' participation and the breadth of pupils' thought (Wood & Turner-Vorbeck, 2001). While these two dimensions differentiate the nature of teaching in 'reform-oriented' class cultures, the relationship of teaching to children's mathematical thinking was only theoretically conjectured. Therefore, the purpose of this research report is to present the results of an investigation into the relationship between teaching and children's mathematical thinking

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in order to account empirically and theoretically for differences in the complexity of teaching. In the next section, we discuss revision of the previously proposed theoretical framework that includes the addition of results from empirical analysis of children's mathematical thinking.

THEORETICAL FRAMEWORK

In recent research, we examined the nature of children's mathematical thinking and found substantial differences existed not only between conventional and reform-oriented classes, but also among reform-oriented classes in terms of the mathematical thinking and reasoning students' revealed during class discussion (Wood, 2002; Wood, Williams & McNeal, in review). Specifically, these differences could be attributed to variations in the nature of student participation and the level of students' mathematical thinking. Although, these two dimensions, participation and thinking, continue to form the basis of the theoretical framework, the recent analysis resulted in a reconceptualization of the prior three class cultures into two, *strategy reporting* and *inquiry/argument* as shown in Figure 1.

Thinking Dimension

The *thinking dimension* of the theoretical framework represents children's increasing responsibility to engage in and reveal higher levels of thought and reasoning during class discussion. In creating this dimension, it is hypothesized that children's increasing responsibility to engage in higher levels of thinking is connected to the type of culture that exists in the class. The vertical black arrow in Figure 1 represents the axis, *Responsibility for Thinking*, which relates to students' mathematical thinking and highlights differences between reform class cultures.

Cognitive theories following constructivism but drawing specifically from work in mathematics by Krutetskii (1976) Hershkowitz, Schwartz, and Dreyfus (2001) and elaborated by Williams (2001) were used to connect the thinking dimension to types of children's mathematical thinking described in the fourth column of Figure 1. Krutetskii's descriptions of mathematical reasoning describe the mental mathematical activity that underlie the observable epistemic actions described by Dreyfus, Hershkowitz and Schwarz (2001) that occur in the process of mathematical abstraction and generalization. These levels of thinking not only indicate a deepening in thought processes but also represent a means to particular kinds of knowledge outcome that represents the hypothesized development of increasingly integrated knowledge networks or structures that differentiate the three class cultures (reform and conventional).

Conventional Class Culture

Discussion Context	Reporters	Listeners		Mathematical Thinking
	Student	Teacher	Student	
Conventional	tell right answers tell prescribed procedures	evaluate ask test questions	pay attention check answers and procedures	*Recalling* recalling answers and prescribed procedures

Reform Class Culture

Discussion Context	Explainers	Active Listeners		Mathematical Thinking
	Student(s)	Teacher	Students	
Strategy Reporting	tell different strategy or method clarify solutions	accept solutions elaborate solutions	listening to decide if own strategy is different	*Recognizing* comprehending applying *Building with* analyzing
Inquiry/ Argument	give reasons justify defend solutions	ask questions provide reasons ask for justification make challenges	ask questions for understanding & clarification disagree and give reasons make challenges & justify	*Building with* synthetic-analyzing evaluative-analyzing *Constructing* synthesizing evaluating

Responsibility for Thinking

Responsibility for Participation

Figure 1

Participation Dimension

The *participation dimension* of the theoretical framework consists of the extent to which it is possible for all students to participate actively in the class social interaction and discourse. The horizontal black arrow in Figure 1 represents the axis, *Responsibility for Participation,* which illustrates the increase in students' interaction as they take more responsibility to participate in the ongoing discussion. From these empirical findings, we linked student participation to the norms constituted in a class and the social interaction patterns that evolve. The results indicate that teachers in each culture establish different social norms for children's participation (Wood, McNeal, & Williams, in preparation).

The increasing demand on student thinking and the expectation for social interaction is the basis upon which meaning making, individual and collective, is accomplished in the class and represents a link between student cognition and classroom social processes. Individuals, in this perspective, are seen as constructing knowledge in the interpersonal activity of negotiation of meaning that leads to the attainment of mathematics that is held as common or taken-as-shared knowledge. The assumption that underlies the Responsibility for Participation axis is that the increase in responsibility to participate in the class culture is consonant with increases in student autonomy in learning (Bruner, 1981).

As shown in Figure 1, the two types of reform class cultures are considered to be 'deepening' in terms of responsibility for student thinking and for participation and, therefore, are hierarchical and nested. The main focus in a strategy-reporting class is on children's presentation of different strategies for the problem-centered tasks. Children presenting their solutions may be asked to provide more information about how they solved the problem by the teacher and sometimes by other student listeners. Classes classified as inquiry are those in which children offer different solution methods, as in the strategy reporting classes, but they provide reasons for their thinking in order to make sense to others. Student listeners and teachers in these classes may ask questions for further clarification and understanding. Argument classes contain the features of strategy reporting and inquiry but, in addition, a challenge or disagreement from student listeners or teachers initiates an exchange that prompts the thinking of justification in support of student ideas. Although inquiry and argument are distinguished by differences in interaction patterns and knowledge construction, they are better characterized as a single class culture, because the lines between reasoning and justification are somewhat blurred.

The revised theoretical framework, shown in Figure 1, presents thinking in conventional class culture along with the mathematical thinking that is revealed in reform class cultures. The categories, *recognizing*, *building with*, and *constructing,* and the types of mental activity are ordered on Figure 1 to represent increasing complexity in the thinking children revealed.

This addition of empirical results about the nature of children's mathematical thinking is a key contribution to the revision of the theoretical framework. Nevertheless with this information at hand, the relationship among teaching processes and student thinking still needs to be defined. Therefore, the purpose of this research report is to describe in greater detail the manner in which teachers promoted mathematical thinking.

METHODOLOGY AND ANALYSIS

Data Source

The data consisted of videotapes selected from 2 classes identified by their culture as strategy reporting, and 2 consisting of both inquiry and argument and 1 class, previously identified as conventional that was selected as a contrast case. This particular class was selected because it consisted of conventional textbook lessons and problem-solving lessons.

Methodology

The methodology and analysis followed a qualitative research paradigm and procedures similar to those of Strauss & Corbin (1990) in which categories were developed from the data, examined for confirming and disconfirming evidence and revised. For each of the classes, 30 lessons were selected for analysis. From each of the reform-oriented classes, a subset of 8 lessons selected as representative of each class culture was selected from the second semester for more intensive analysis of the interaction patterns that structured the class discussions. These 8 lessons (16 for each culture, strategy reporting and inquiry/argument) were the data source for the analysis of the ways teachers prompt students' collective thinking. From the conventional class, 8 lessons were selected; 2 consisted of textbook only instruction, which focused on place value and 2-digit addition and subtraction. The remaining 6 lessons consisted of both open-ended problem solving and textbook instruction. This combination provided the unique opportunity to examine textbook and problem solving lessons within conventional instruction.

Analysis

The coding scheme previously developed for investigating interaction patterns in reform-oriented mathematics class discussions was extended for the current study to include additional codes developed for conventional mathematics class instruction. Following the coding and identification of structure, the interaction patterns were further analyzed to identify consistent and repeatable patterns across the lessons, and then categorized and labeled. The transcripts of class discussion in each lesson were also used to analyze teachers' prompting questions in relation to children's thinking. Using coding categories developed for examining teacher's questions and statements each line in the transcript was coded. Finally, the analysis of the interaction patterns, teacher prompts, and children's thinking were recombined to recreate the discussion in the order the interaction patterns occurred and reanalyzed.

RESULTS AND DISCUSSION

The results of the analysis of the interaction patterns define and delineate the specific nature of each class discussion. The analysis reveals that not only does the frequency of interaction between teachers and children increase progressively across the four cultures but also the nature of interaction changes indicating increasing opportunities for student discourse and participation. The analysis also reveals that the types of interaction patterns vary across the classroom cultures with the textbook culture consisting of the fewest kinds of interaction patterns and inquiry/argument the most.

The results of the analysis revealed that the frequency and level of teacher prompting through questioning or statements varied considerably across these class cultures. Moreover, the frequency and complexity of teacher prompts for mathematical thinking progressively increased across the types of class cultures, with the conventional environment, both textbook and problem solving, being predominately situations of prompts for recall for children. Instances of teacher prompts for mathematical thinking in the conventional textbook class culture were infrequent (N=14) and consisted of recognizing (comprehending and applying) mathematical ideas as shown in Figure 2. In the strategy reporting class culture teacher prompts for mathematical thinking dominated (N=86) with a majority of questions consisting of recognizing (comprehending and applying) and building with (analysis). In inquiry/argument classes, teacher prompts for mathematical thinking were most frequent (N=177) with more questions focused on building with (synthesis analysis and evaluative analysis) see Figure 2.

Other analysis revealed that questioning in conventional teaching was directed at prompting children to give teacher expected information, while strategy reporting and inquiry/argument emphasized student exploration of methods and justification of student ideas. Comparison of strategy reporting and inquiry/argument revealed teachers' differed in the frequency of prompting for mathematical thinking during inquiry interactions and situations involving proof and justification. The resolution of differences in students' answers was dealt with differently by teachers in the two reform cultures. Teachers in strategy reporting class cultures emphasized proof of a correct answer through the use of concrete objects, while teachers in inquiry/argument relied on children's explanation and justification to resolve differences in reasoning.

Conventional Class Culture

Class Discussion	Mathematical Thinking Revealed	Teacher Prompts
Conventional	*Recalling* recalling answers and prescribed procedures	What is the answer? Two plus 3 is_____?

Reform Class Culture

Class Discussion	Mathematical Thinking Revealed	Teacher Prompts for Mathematical Thinking
Strategy Reporting	*Recognizing* comprehending applying *Build with* analyzing	I'm confused. Would you tell us again what you thought? Does this make sense (do you understand)? How did you decide this? Why would that tell you to subtract? Any comments on the answer/method? Why? Why would you do that? What is happening? Are there patterns? Is there a different way you can do this?
Inquiry Argument	*Building with* synthetic-analyzing evaluative-analyzing (warrant) *Constructing* synthesizing evaluating	How are the 2 things the same? What is the same about each method? Does this make sense (is the method reasonable)? Why not? How do you know that? Why do you think that? Can you link all the ideas you found in some overall way? Does it always work? Is it always true? Why does this happen?

Responsibility for Participation

Figure 2

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