

THE USEFULNESS AND LIMITATIONS OF THE NOTION OF INCOMMENSURABILITY IN ANALYZING 8TH GRADE STUDENTS' UNDERSTANDING OF ALGEBRA

Miriam Amit & Michael N. Fried, Ben-Gurion University of the Negev

The findings to be presented here arose out of an intensive investigation of one 8th grade classroom studying systems of linear equations. The investigation was part of an ongoing international research program called the *Learners' Perspective Study*. That study, as its name suggests, aims to explore the doings of a mathematics lesson from the students' point of view (see Fried & Amit, 2002). Indeed, despite the apparent concurrence of students and teachers with respect to the aims and content of mathematics lessons, in fact, a profound divide often exists between them.

In the 8th grade class examined, a divide such as this was observed between the, sometimes surprising, ways the students understood specific mathematical concepts—'unknown', 'equation', and 'system of equations', among others—and the understanding their teacher assumed or tried to convey. For example, when asked by the researchers what the word 'system' refers to in the phrase 'system of equations', more than one student replied that it referred to the coordinate system, which the teacher had used initially to explain the graphic solution method for linear systems.

What was striking was that teacher seemed unaware of these misapprehensions, while students, for their part, seemed unaware that they had in any way misunderstood the teacher's goals. For this reason the divide between teachers and students brings to mind the phenomenon spoken of in the history and philosophy of science known as *incommensurability*—the phenomenon in which people use words such as "time" and "energy" but in completely different conceptual frameworks; they think they understand one another but, in fact, they are worlds apart. In considering the level of the students' conceptual knowledge, this analogy was found to be useful. No less useful, however, was thinking about its limitations; for the notion of incommensurability assumes that the conceptual frameworks are individually coherent, and such is not clear in the case of the students.

References

- Fried, M. N. & Amit, M. (2002). Students' Classroom Notebooks and the Blurring of Private and Public Domains. In A. D. Cockburn, E. Nardi, (Eds.), *PME26: Proceedings of the 26th Annual Conference 2*, (pp. 376-383). Norwich, UK: University of East Anglia