STUDENTS AND TEACHERS LISTENING TO THEMSELVES: LANGUAGE AWARENESS IN THE MATHEMATICS CLASSROOM

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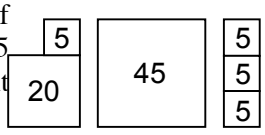
Taking seriously the recent call for critical language awareness in mathematics classrooms, I propose that students engage with their teachers in the analysis of the discourse in their classrooms. To explore the potential for increased mathematical understanding, I analyze transcripts of students and a teacher listening to and responding to their own dialogue in the context of student work on a mathematical investigation. The linguistic features analyzed include hedges, deixis and politeness.

BACKGROUND

A complex web of relationships among students, their teacher and powerful traditions emerges in every classroom, yet the discourse that forms the connections in this web is typically left unquestioned. Morgan (1998), in her call for language awareness in the mathematics classroom, promotes the direction of students' attention to language features of the genres in which they are beginning to participate. In this report, I consider what a language awareness program in a mathematics classroom might look like. How might students' mathematical learning benefit as they analyze the discourse in their classroom?

To frame this exploration, I analyze student and teacher interpretations of a particular classroom interaction involving grade 10 students (15 years old) in a pure mathematics class. I interviewed student groups and their teachers after audiotaping the students working on an investigation I had designed to relate to their curriculum (see below).

The 45 cm^2 square is the exact same height as the two stacks of squares beside it. The squares in the stack on the left have areas of 5 cm^2 and 20 cm^2. Each of the three squares in the stack on the right has an area of 5 cm^2.



a. Find stacks of squares that would be the exact same height as a square of area 72 cm^2.
b. Explain how to find the stacks that would match any other given square in height.

The transcripts in this report are drawn from two interviews, one with a group of three students and the other with their teacher, whom I call Mr. Penner. In each interview, I played an audio recording of an interaction between the students and their teacher during their work on the investigation one week earlier. This interaction, in which Mr. Penner happened upon the group some twelve minutes into their work, is represented in the following transcript. (Names of participants are also given as pseudonyms.)

A1 Natalie: Okay. I have a question. What we're trying to do right here, right? You find the area and all the lines, right. But instead of 45 we're finding 72, right?

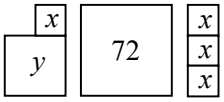
A2 Mr. P.: Well, sort of.

A3 Natalie: But, don't we need to know the ratio between 20 and 45, and then if this is 72 what would be the ratio then?

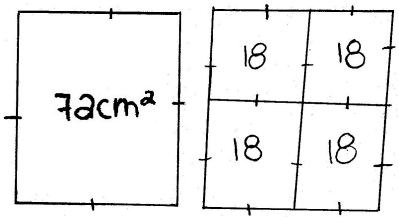
A4 Mr. P.: Let me show you one thing. If I wanted to find that. …

Natalie was thinking about ratios as a way of addressing the problem. She seemed to have something like the following calculation in mind:

$5 : 20 : 45 = x : y : 72$ ⟹



Mr. Penner was thinking about an approach that would use techniques he had taught this class one month earlier. He was thinking about square roots to express the heights of squares and addition sentences to represent stacks: $\sqrt{5} + \sqrt{20} = \sqrt{45}$ _ $\sqrt{8} + \sqrt{32} = \sqrt{72}$, for instance. The dialogue continued with Mr. Penner directing Natalie's group to look at heights using square roots. After this interaction with their teacher, Natalie and her group seemed to have forgotten about their ratio approach, which did not involve finding the heights. Though they lost the ratio idea, the group did find yet another approach that Mr. Penner had not considered before. They divided the 72-square up into smaller squares to construct stacks as required (see part of their work at right). The 72-square has the exact same height as two 18-squares stacked.



PARTICIPANTS LISTENING TO THEIR OWN DISCOURSE

I am interested in the benefits of having students and their teacher look back at their discourse. One result of exposing teachers and students to records of their classroom interaction is that it prompts them to reflect on their pedagogical and mathematical choices. It affords them the opportunity to consider what they might have done differently. There is also value in looking at language features in discourse.

What might Mr. Penner and his students have learned about mathematics learning if they were to look at their language practices? An analysis of the following interview transcripts uncovers potential directions that are worth considering for a language awareness program in which teachers and students analyze their own discourse. In the interest of self-similarity, I have chosen to analyze the interview transcripts in a manner that I foresee being used by language aware students and teachers. However, the objects of their analysis would be significantly different from mine here. Their analysis would focus on their mathematical dialogue. Here I analyze interviews to see what can happen when participants listen to their own discourse.

The first interview transcript is excerpted from my interview with Mr. Penner. In our interview, I introduced the audio recording saying, "I am going to play a bit of tape from Natalie's group." He began responding to the episode after listening for about ten seconds. He spoke while the tape was playing.

B1 Mr. P: I think, I think this is where I help them too much.
B2 I: Should I stop here? You know what you're …
B3 Mr. P: Yeah, I know what's going on here. I can picture it. Yeah. So, [*I stop the tape*]
B4 I: Yeah, so you helped them too much. Obviously you decided to help them.
B5 Mr. P: Yeah.
B6 I: So, just tell me how you feel about that …
B7 Mr. P: about helping?
B8 I: Yeah.

B9 Mr. P: I think that was after about half an hour. It was after a while anyway. I could see they really weren't getting the idea, so I gave them an example. There was an example on the page, but I gave them an example more directly, with the roots. So, I kind of opened up that whole subject to them, and then I think they kind of got a few after that point. So, I mean I really led them into it, but, [*laughs*] I mean, that group is a group that struggles in math. Out of the three people there's only one that's passing.

B10 I: I think I heard them say their highest mark was 52 in the group.

B11 Mr. P: Yeah, that's right. I guess I could have left them alone and they would never have got it. [*pause*] Possibly. With the roots. But, I think, was that the group that came up with some other creative method? I think they did actually [*referring to their work, which is partially reproduced above*], so maybe if I wouldn't have helped them along they would have come up with something.

Hedges: Mr. Penner's response to the audiotape began with a hedge: "I think, I think …" (turn B1). Rowland (2000) analyzes students' use of hedges when they are involved in mathematical conjecturing and reasoning. He describes hedges as "words which have the effect of blurring category boundaries…[and which] hedge the commitment of the speaker to that which s/he asserts" (p. 58), words such as 'sort of', 'I think', 'maybe', and 'perhaps'. Hedges are language devices that protect us from being proven wrong.

In this transcript, Mr. Penner hedged repeatedly, saying 'I think' five times, 'kind of' twice, 'maybe' and 'possibly', all in a short time span. By contrast, his students in the next transcript did not hedge at all. From whom was Mr. Penner hedging himself? Was he worried about my opinion (or judgement) of him? Indeed, we find ourselves in an uncommon predicament when confronted with records of our activity, not unlike a court where the defendant is confronted with artefacts or transcripts from prior testimony. Though I can imagine why Mr. Penner would be defensive in this provocative situation, his hedges may simply reflect his awareness that his recollections of the event were mere reconstructions. They may demonstrate his recognition of the complexity of reflection.

In a language aware mathematics classroom, teachers and students can be directed to become aware of linguistic hedges. In such a context, participants would be analyzing mathematical discourse instead of interview data. Their data would more resemble the transcripts in Rowland's work on hedges. Classroom participants' language awareness would afford them the opportunity to reflect on possible explanations for the hedging they find. For example, Natalie and Mr. Penner might wonder why he said "Well, sort of" (turn A2) instead of saying "No, you're wrong." Was his expressed uncertainty mathematically significant as well as pedagogically significant?

I suggest that there is no need to pinpoint the speaker's intentions in such a setting. Instead, the students and teacher might reflect on the importance of being unsure in mathematics and in learning. Though certainty is highly valued in mathematics, new ideas require space for investigation. Rowland (1997) names such a discourse space the 'zone of conjectural neutrality'. In various contexts, Rowland encourages teachers to provide such spaces for students, but students may also benefit from being aware themselves of the importance of uncertainty and vagueness in their mathematics practice and in their thinking about their practice. However, students need to be confronted with records of their mathematical activity in a way that minimizes the sense of confrontation.

Deixis: Following Mr. Penner's initial hedging words, he 'pointed' with the word 'this': "This is where I help them …" (turn B1). Such pointing with language is called deixis. Rowland and others (e.g. Pimm, 1987) have studied deixis in mathematics practice.

As mentioned above, Mr. Penner found himself in an odd position for this interview. How should he have pointed to himself? To what extent is/was the Mr. Penner on the audiotape the same person as the Mr. Penner in the interview? He initially pointed with the word 'this' which suggests a sense of proximity more than 'that' would suggest. He also spoke of the audio-taped episode in the present tense, though it had occurred a week earlier. The present tense suggests proximity as well.

In my interview with Mr. Penner, he switched from using present to past tense before turn B9. At the same time, he distanced himself by using the distal pointer 'that' instead of the proximal pointer 'this' to point to the event (turn B9). If he and I were working together in a context in which language awareness was part of the agenda, I could ask him why he might have switched tenses. The purpose of this question would not be to know what his intentions and feelings 'really' were during the interview. Those experiences are lost. Rather, the value in such a question would lie elsewhere. As we talked about possible reasons for him to distance himself from the event, we might, for example, learn more about how practice can be informed by revisiting past experiences.

The transcript below is from my interview with Mr. Penner's students who responded to the same audio segment, in which they were recorded interacting with Mr. Penner. The excerpt begins immediately after the point at which I stopped playing the audiotape.

C1 I: So what was he doing there?
C2 Janet: He was getting us to talk about it and then like …
C3 Natalie: Trying to solve it ourselves, like he's trying to give us hints
C4 Janet: Tell us if what we're doing, … if what we think is right.
C5 I: Did he, did he pay any attention to Natalie's question about whether ratios …
C6 Natalie: No.
C7 Janet: Not really [*all laugh*]
C8 I: You know what?
C9 Janet: 'Cause that was wrong right?
C10 Natalie: He didn't even hear me. [*simultaneous*]
C11 I: Noooo. In fact I thought of it when I was listening to it…I never thought of using ratios and I tested it in a whole bunch of ways and it is pretty interesting actually, it would work well. I thought it would work well.
C12 Natalie: Right!
C13 I: So, I played it for Mr. Penner too. So, you know, he had mixed feelings about it. So, you would have got something,
C14 Natalie: Yeah.
C15 I: something good anyway.
C16 Natalie: We needed more time on that project.

I am captivated by the deictic flow between these students and me, their interviewer. I began the interview with the distal pointer and the past tense: "What was he doing there?" (turn C1). Alternatively, I could have suggested proximity by starting with, "What *is* he doing *here*?". Janet replied using the past tense (turn C2), perhaps following

my lead. Natalie followed Janet with a present tense utterance; perhaps resisting the kind of pointing I subconsciously chose to structure the conversation. Janet followed Natalie's proximal pointing, using the present tense (turn C4). I countered with the past tense (turn C5), and the rest of the conversation was in the past tense, at least where tense was clear. Natalie closed the conversation mirroring the distal pointing I started it with, using the past tense and referring to *that* project (turn C16).

In this conversation, I used distal pointing throughout, Janet's deixis resembled that of whomever she followed, and Natalie once tried proximal pointing, but seemed to acquiesce in my language structure. Mr. Penner's interview was similar in that he also appears to have followed my lead in terms of the proximity of our deixis. He started the conversation with proximal pointing and I turned it to distal pointing in turn B4.

As with the analysis of hedges, discussion of deixis in a language aware mathematics class would focus on mathematics discourse, not interview data. Spatial and temporal deixis present significant tensions in mathematics, similar to the tensions described above. How do students (or mathematicians) point to abstract, mathematical objects, to which they have no direct access? Surely students would benefit from discussing this difficulty. The alternative is for each student to just suffer through the difficulty, unaware that he or she is the only one struggling to communicate. Scholars are increasingly becoming aware of the significance of representation in mathematics and mathematics learning. (There has been a semiotics discussion group at the last two international PME conferences.) Students may also find such discussion important.

Tensions of temporal deixis are related to those of spatial deixis. Mathematicians have asked: Where is the mathematics? (e.g. Mason and Muller, 2001). We might also ask: When is the mathematics? Is a proof or geometric construction an artefact of a person's mathematical thinking or do such mathematical objects contain the mathematics in themselves? In other words, to what extent does mathematics exist independent from human agency? The way we talk about mathematics can tell us something about our answers to this question. Do we talk with the past tense about the mathematical choices made by a mathematician or a student, or do we talk in the present tense as if it does not matter who wrote or spoke the mathematics? Chris Bills (2002) has identified significant disparity between higher and lower achieving mathematics students in their use of pronouns and verb tenses when referring (pointing) to their mathematical thinking.

Politeness: Another feature of language studied by linguists is politeness. Liz Bills (2000) has drawn on this scholarship to investigate ways in which mathematics students and teachers save face and allow the others in their classroom to save face. Power and control are important aspects of politeness.

The above analysis of two transcripts uncovers features of politeness in the interview discourse in which I engaged my research participants. In terms of the proximity of deixis, both the students and the teacher followed my lead in the interviews. When I moved Mr. Penner's interview to the past tense, for example, it was like me saying, "You're wrong to talk about the past as if it were present." His transition to the past tense was like saying, "Okay, I'll use the past tense then." Though I did not foresee the extent to which the participants in my research would follow my lead, I am not surprised by it

because they agreed in advance to cooperate with my research agenda. They could be expected to follow my structuring, not unlike students following their teacher.

My new-found awareness of interview politeness prompts me to be more critical when I read transcripts of interviews. To what extent do interviewees say what the interviewer wants to hear? School children are encultured to give their teachers what they want. They learn to say what adults in schools want to hear, but there are tensions when another adult is introduced into a setting of mathematics learning. In Bills' (2002) interviews, high achievers tended to follow their teachers' language structure and resisted the interviewer's language structure.

Besides informing the interpretation of interview-based research reports, this experience of interview politeness raises questions about any discourse. To what extent do participants merely follow the structuring of the person in power? In the mathematics investigation described and discussed here, for instance, Mr. Penner wanted to free his students to structure their own mathematics. They, however, were searching for the 'right' answer, the mathematics that would fit his expectations. In my interviews with Mr. Penner, he revealed his struggles with exercising intervention (or power), but his students seemed oblivious to the tensions their teacher felt in his pedagogical choices.

Why not open up this tension for classroom discussion? When students and their teacher revisit their dialogue, they can become aware of their politeness practices. Perhaps this awareness could free them to take more initiative in their mathematical practice.

DIRECTING AWARENESS IN THE MATHEMATICS CLASSROOM

In the above analysis of interview transcripts, I have pointed out ways in which awareness of language practices can inform mathematical learning. But, how can students and their teachers be directed to become language aware? Should they rely on outsiders' analysis of mathematics classroom discourse, or should they do analysis themselves? And, what particular texts would serve them best as exemplars for language awareness?

First, who should be involved in analysis? Current calls for language awareness assume that students ought to be the ones becoming more aware. These calls cast teachers as experts who can tell their students how to use language well in mathematics. However, the interrogation of stereotypical classroom roles, in which students are the only learners, has contributed to revelations of classroom realities that have allowed researchers and teachers to see the classroom in new ways (e.g. Matos, van Dormolen, Groves and Zan, 2002). Healthy language awareness programs need to identify all participants as learners who can both contribute unique insights and benefit from increased awareness.

My second question relates to the object of analysis. To uncover some of the language tensions that are at the heart of mathematics, I suggest that it would be most appropriate for participants to analyze their own discursive practice, not only that of others.

There is, however, awareness-building value in the analysis of texts generated outside the participants' first-hand experience. Video-taped mathematics classroom episodes have been played for teachers and students to research their perceptions (e.g. Ainley, 1988). Others have used videotapes to help mathematics teachers become aware of alternative possibilities for practice (e.g. Pimm, 1993; Sáenz-Ludlow and Perlwitz, 1994). In such

research and teacher-development, viewers typically watch video episodes in which they themselves do not appear, and it is typically assumed that teachers and researchers, not students, have the potential to become more aware of mathematics classroom discourse.

To identify the 'practical rationality of mathematics teaching', Herbst and Chazan (in press) listened to teacher responses to videotaped mathematics classroom episodes. In their analysis of the teachers' responses to other teachers' practice, they note how a video is both record and artefact. As an artefact, the video-framed episode itself is analyzed by teachers who view it. As a record, the episode reminds the teachers of similar events and their contexts. This mix of subjective interpretation with objectification, which is inherent in any experience of artefacts, is an issue for any analysis of discourse, even if the interpreters are blind to the issue.

The conflict here relates to the nature of language. To the extent that words and speech acts have meaning in themselves, utterances can be analyzed without thought of personal experience. With this view, text extracts are mere artefacts. Though few current discourse scholars would espouse such a representationist view of language, it is difficult to hold a completely relativist view of language when we consider authentic texts as records of events that are situated in particular cultural contexts. Though a particular speech act in its context may mean many things, there are many more things that it cannot mean.

The classroom transcript above exemplifies the divergence of meaning different people can find in a particular discursive event. Mr. Penner appears not to have been listening to Natalie because her words bore little resemblance to his expectations. Instead of *listening to* her with an open mind, he was *listening for* his expected response. However, the nature of language makes it impossible for a teacher to listen with no preconceived notions.

Because of their intimate acquaintance with its context, participants in a language aware mathematics classroom ought to have an easier time of being aware of semiotic tensions in language when they analyze their own discourse. Such awareness can be healthy in mathematics classrooms, in which Sáenz-Ludlow (2001) notes a tendency to believe that meaning rests on symbols, independent of context. Awareness of this semiotic tension might help students understand how, as Duval (1999) puts it, "there is no direct access to mathematical objects but only to their representations" (p. 24). Such awareness would move beyond a mere increase in language power, which seems to be the aim of Morgan (2002): "Greater awareness … may help mathematics teachers and students to develop more purposeful and hence more effective use of language" (p. 17).

CONCLUSION

The analysis of language patterns in mathematics classrooms has provided researchers with insight into classroom culture and opened up opportunities for us to share this insight with teachers. I have described here how students may also benefit from the analysis of their own discursive practice.

Though there is pedagogical potential in directing mathematics students to language awareness, there is work to be done to find viable models for such cooperative analysis. I suggest that this would be best worked out in the context of a class committed to exploring the possibilities of language awareness structures in the classroom. There is even more research possibility here. We may gain new insights into mathematics learning

if we listen to the insiders, students and their teachers, listening to themselves and analyzing their discourse. In my report, I will discuss these possibilities more fully.

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