PLENARY PANEL[1]

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TEACHERS WHO NAVIGATE BETWEEN THEIR RESEARCH AND THEIR PRACTICE

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TEACHERS INVOLVED IN RESEARCH

Teacher research represents a broad and very live topic not only in the field of mathematics education. But what is meant by teacher research? In (Anderson & Herr, 1999), the following characterisation is given: "By practitioner[2] research we refer to a broad-based movement among school professionals to legitimate knowledge produced out of their own lived realities as professionals. This includes an ongoing struggle to articulate an epistemology of practice that includes experiences with reflective practice, action research, teacher study groups, and teacher narratives". The role and status of teacher research is an object of sharp and vivid debate not only in the field of mathematics education – see for example (Anderson, 2002), (Metz & Page, 2002).

Breen (2003) presents the contrasting views on the contributions that teachers are making to the field of mathematics education: "On the one hand, there is a growing movement for more teachers to become involved in a critical exploration of their practice through such methods as critical reflection, action research, and lesson studies. The contrasting position makes the claim that these activities have done little to add to the body of knowledge on mathematics education."

In the following text we do not continue the above mentioned discussions. Our objective is to present on one hand the differences between the roles of teachers and researchers and on the other hand the advantages of the links between both activities. "The skills and knowledge we have learned through conducting research figured in both our administrative and teaching roles in our programs and in our accounts. Without our full-time research lives, we would have been very different practitioners and very different authors." (Metz & Page, 2002).

PANEL OVERVIEW

J. Novotná

The aim of the panel is to present several models of the navigation of teachers of mathematics between theory and practice. Each of the panellists will present a different view on the problematic. From the context of various teaching/research situations the following questions will be discussed:

1. How do panel members connect their roles of teacher and researcher?

[2] Practitioner research (USA), action research (Great Britain). In our text (except the direct citations) we will use the term teacher research.

2. How does panel members' own research influence their work as a teacher and vice versa?
3. How do panel members' teacher/researcher efforts inform the larger educational community?

Jarmila Novotná illustrates the differences in the two roles – a researcher and a teacher. COREM, one of the successful projects of co-habitation of research and teaching practice, is presented as an example.

Agatha Lebethe illustrates her own reasons to become a teacher-researcher; her searching for a suitable theoretical background, and her development in the researcher role. She illustrates the conflict between her results as a researcher and the official programme-based teaching strategies required by the standard curriculum. The way of implementing her research results is illustrated in her approach to mathematics teacher training.

Vicki Zack addresses the intimate dialectic relationship between practice and theory as she speaks about the teacher research she has done in the elementary classroom for the past twelve years. She shows that researching from the inside has been transformative and immensely fulfilling, but also emphasizes how demanding and exhausting the work can be. Where Breen (2003) suggests that in most instances teachers are not at the centre of the research project, in Vicki's situation, she sets the agenda, and seeks allegiances with (university) colleagues when the need arises.

Gershon Rosen is a full-time secondary teacher committed to improving mathematical practice in schools. In his contribution he shows the use of one theoretical approach in concrete school mathematics situations. Besides giving details of his method and the related personal growth as a teacher-researcher, he also describes how his research results are disseminated in the school milieu.

SOME QUESTIONS RELATED TO THE PANEL TOPIC

- Is practitioner research really research?
- Why do practitioner research?
- Should all teachers do practitioner research?
- Should faculties of education prepare practitioners to do education research?
- What is the impact of teacher research on the larger community?
- Should teacher research be included in the same category as traditional academic research knowledge?
- Are there differences in the research results if the direction is teacher teacher researcher or researcher teacher? If yes, what are the main differences?

THE TEACHER/RESEARCHER ROLES

Jarmila Novotná

The differences and similarities in school teaching and research practice are described by Brousseau (2002): "When I am acting as a *researcher*, the interpretation of each step of teaching begins with a systematic questioning of everything, a complex work of a priori analyses, of comparisons of various aspects of the contingencies, of observations first envisaged and then rejected, etc. How to distinguish what is relevant but inadequate, adequate but unsuitable, appropriate but inconsistent is not clear, nor is the transformation of appearances and certainties into falsifiable questions, etc. When I am a *teacher*, I have to take a number of instantaneous decisions in every moment based on the real information received in the same moment. I can use only very few of the subtle conclusions of my work as researcher and I have to fight with starting to pose myself questions which are not compatible with the time that I have, and that finally have the chance to be inappropriate for the given moment. I react with my experience, with my knowledge of my pupils, with my knowledge of a teacher of mathematics which I am treating. All these things are not to be known by the researcher … The advantage of a teacher over a researcher is that they can correct an infelicitous decision with a converse decision and this with another one. The most difficult situation for me is after the lesson. The researcher (and me) have all the tools and all the time, after, but too late, to perceive bad decisions, all types of errors, the inability of the mediocre teacher that I am … The way my knowledge of didactics can help the teacher that I am, is much more delicate, complex and indirect. And I have to have the same cautious awareness of my influence on other teachers. The "didactisme" is a deviation of the didactics similarly as the "scientisme" is the deviation of the science."

We illustrate differences in teacher and researcher roles by an example of a Czech teacher-researcher Jana Hanu_ová. Jana is a full-time teacher of mathematics at an 8-year general secondary school (students aged 12-19) with more than 25 years of teaching experience. For the last 8 years she has been cooperating in research with the Department of Mathematics and Mathematical Education at the Faculty of Education of Charles University in Prague, for the last four years having been a part-time PhD student of Didactics of Mathematics. She represents both – a teacher (we will label this role of Jana as *Jana-teacher*) and a researcher (*Jana-researcher*) in one person. The following episode from her professional life is intended to illustrate the differences in her two roles.

The topic dealt with is Trigonometric functions with the 17-year old students. The long-term practical experience of *Jana-teacher* confirmed by her discussions with other teachers signaled the didactical demands of the topic for students. The main difficulty diagnosed was that the students' perception of the function ~~sinus~~ (sine) is limited to the letters mostly used to label the triangle sides. *Jana-teacher* tried to develop new teaching strategies to help her students to overcome this obstacle, but with a very little success.

Jana-researcher tried to find help in the ideas from scientific didactics of mathematics. She consulted and critically evaluated several theoretical works concerning educational strategies. She decided to apply a constructivist approach. She found a problem as a

starting-point that she used as an activity for her students. (Hejn_ & Jirotková, 1999, p. 58):

They are given points O[0,0], *P*[5,0] *and points A*[2,1], *B*[5,2], *C*[7,4], *D*[16,6], *E*[22,11] *and F*[101,50]. *Arrange the angles* α = ∠*AOP*, β = ∠*BOP*, γ = ∠*COP*, δ = ∠*DOP*, ε = ∠*EOP*, φ = ∠*FOP according to size*.

When *Jana-teacher* used the problem for the first time she was happy with the activities in her class. She saw that students discovered themselves that the size of an angle can be expressed using a ratio. They discovered the pre-conception of the sine function not via a triangle but in the environment of a Cartesian grid.

Jana-researcher analysed her experiment and discovered the following drawback in *Jana-teacher*'s activities: She did not have a sufficiently detailed documentation of students' solutions and ideas. *Jana-researcher* decided to repeat the experiment with its more detailed recording. She prepared a lesson plan for *Jana-teacher* very carefully.

In the new experiment *Jana-teacher* explained to her students what they were supposed to do and asked them to record everything on either a sheet of white or grid paper, separately their own ideas and the ideas born when discussing with other students. During the individual and group work, *Jana-teacher* observed the students and their work and completed the information on the sheets when necessary.

After the lessons *Jana-researcher* compared her expectations with the reality in the classroom, analysed the records and the whole experiment and students' records. *Jana-researcher* with *Jana-teacher* tried to explain the reasons for the differences between the expected outcomes and the reality and discovered mistakes. In the same symbiosis of roles she modified the next lessons based on her practical and theoretical experiences.

Jana-researcher wrote an article about the experiment to a journal.

FURTHER THOUGHTS ABOUT THE COOPERATION OF TEACHERS AND RESEARCHERS

Going deeper into the teacher/researcher issue, let us detach for a while from our topic – a teacher and researcher as one person – and observe them more generally. The question we are dealing with is: What are the benefits obtained from the close cooperation between teachers and researchers? This more general view deepens the understanding of the issue of a teacher-researcher as one person. We will try to answer three sub-questions:

1. *Does the teacher need the direct presence of a researcher during his teaching?* Common school practice shows that this is not true. Good teachers do their important work excellently without such a close collaboration. The answers to theoretical research questions do not have a direct impact on the daily work of the teacher. The teacher cannot use them in the concrete situations in the classroom in a concrete situation that happens. (In our example, the proposals of *Jana-researcher* were applied by *Jana-teacher* later, with another class, in another school year …). See also (Brousseau, 1989)
2. *What are the possible benefits for the teacher of a teacher and researcher in direct cooperation?* At first sight, the answer would be that there are only advantages – the teacher can find the answers to questions which are faced in their everyday teaching in

the researcher's results and then implement them in their teaching. But this simplified view does not correspond with reality. The research results should not only offer the teacher ideas for solving the problems they face in the work in classrooms, but also provide inspiration for further elaboration. In the real situations teacher's reactions are answers to the concrete situation where the immediate decision can be influenced by the theoretical results but it is always fully "in the hands" of the teacher. Many examples from reality could be shown to illustrate the dangers of the blind application of research results in teaching.

3. *Does the researcher in education need the direct cooperation with one or more teachers?*[3] Our answer to this question is yes. It is the researcher who needs the teacher for finding answers to their research questions. Without close contact between researchers and teachers, the danger of producing superficial answers to research questions, results in not having "real roots" and significantly, there is a doubtful applicability in the school reality. *To find answers to research questions, the researcher needs direct contact with teachers and genuine access to the reality of teaching.*

The following example represents good practice between teachers and researchers.

THE COLLABORATION BETWEEN RESEARCHERS AND TEACHERS – COREM

Brousseau's ideas were successfully implemented in COREM, Le Centre d'observation et de recherche sur l'enseignement des mathématiques (school Jules Michelet, Talence, France). COREM was created in 19734 with the following objectives (Salin & Greslard-Nédélec, 1999):

> To achieve the research necessary for advancement of knowledge of the mathematics education phenomena.
> To conceive and study new educational situations enabling a better acquirement of mathematics by pupils.
> To develop in this way a corpus of knowledge necessary for teacher training.

In COREM a close collaboration of researchers from university teacher trainers, elementary school teachers (pupils aged 3-11), school psychologists and students of didactics of mathematics took place. Its existence allowed the constitution of two resources of data: a long-life collection of qualitative and quantitative information about the teaching of mathematics at the elementary level and two types of observations – these destined for finding and explaining phenomena of didactics referring to teaching and those for research.

Michelet School consisted of four kindergarten and ten elementary school classes. The school was not selective; pupils represented a very heterogeneous population. The programmes in all subjects were those valid for all other schools.

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[3] For a nice example of a close cooperation of teachers and researchers see e.g. Newsletter #104 of V.M. Warfield, University of Washington, Seattle, http://www.math.washington.edu/~warfield/news/.

[4] COREM in the described form worked until 1999. From my experience when visiting Jules Michelet, I can confirm that the atmosphere created there during the 26 years of COREM in Jules Michelet has not disappeared.

In Michelet, the teaching staff were ordinary teachers without any special training. Their task was to teach, not to do research. They worked in teams, three teachers for two classes. One third of their working hours were devoted to COREM. This time consisted of four types of activities: Coordinating and preparing in common the ordinary work of the pupils and discussing all the problems of the school (educational, administrative, social and so on), observing directly the work in the classroom, for research or as well as for a normal feedback, participating with the researchers at the conception of the session to be observed and collecting data about pupils' comportment in mathematics, permanent education in the form of a weekly seminar on the subjects asked by the teachers.

The daily mathematics activities were designed in collaboration with one teacher trainer from IUFM (Institut Universitaire pour la Formation des Maîtres) who monitored the mathematics during the whole school year and was a guide for mathematics content and guaranteed that the research did not impair the normal educational activities of the school. The interactions of researchers with the observed class were institutionally adjusted.

There was one important rule in the decision making process in the team. The teacher had the final say about what was done, if the team did not succeed to find a consensus. The detailed analyses of the teaching units were done by the whole team including the teachers.

The observations were of two types:

a) Those of sequences prepared together with a researcher

In this case, the researcher was responsible for elaborating the project of the teaching sequences. They presented the project to the teachers including the knowledge presumed to be attained at the end of the teaching sequences, the problems to be presented to pupils, and the register of the expected pupils' strategies. When the project was accepted by the team, the next step was the elaboration of teaching sequences. The ideal situation was if the teacher was able to accept the scenario of the lesson with pupils directly from the project. If this was not the case, the extensional questions were discussed, as for example: What vocabulary should be used in each phase and how? If and how should the teacher intervene in pupils' validating of strategies? What decisions should be made if pupils do not behave as presumed? Are the applied exercises necessary? The result of the collective preparation was a written description of the session that played the role of a guide and was distributed to the observers in advance.

The teacher was completely responsible for what happened in the classroom. It included the right to make decisions different from those presumed.

After the observed sequence, its immediate first analysis was done. In this analysis, all participants reconstructed as precisely as possible all the events of the session. The analyses of events during the lessons had the prescribed order: First the teacher summarized what was or was not good and why from their point of view. The team discussed the issues explainable by the conditions, and for those that were singular, looked for phenomena. In such a way the observation had the character of involvement.

The discussions provided the researcher with a considerable amount of additional information.

b) Those prepared by the team of teachers themselves

Regular weekly observation of a series of "ordinary" lessons, i.e. of lessons that had not been prepared with a researcher, served to find and explain the contingent decisions (both good and bad) of "all" teachers. The researcher who was interested in continuous observing of teaching of mathematics during a certain period did organize their observations individually.

Teachers and researchers were members of one team at least in the preparatory phase. Their roles were different. But the teacher always made the last decision. In the class, the teacher had the responsibility for pupils. Various distortions could happen, e.g. the researcher had not formulated their expectations adequately, or the teacher did not grasp them correctly. Sometimes, the teacher had to make important decisions in order to reach the presumed conclusions.

The successful functioning of COREM depended on the collaboration of all participating persons as well as much administrative and managerial work. The results were disseminated in various ways; from allowing interested persons to participate in the whole process, to presenting the organization, functioning and results at conferences and symposia in France and abroad. But the teaching processes prepared for observation have never been published or given as a model for use in ordinary classroom conditions. It is important to remark that to be a teacher or researcher are different functions but not a definitive and personal status. In the COREM some persons were both, but never in the same time or on the same activity.

A MOMENT IN TIME, TWISTED BETWEEN THEORY AND PRACTICE

Agatha Lebethe

I work for the Schools Development Unit at the University of Cape Town as a teacher educator who supports teachers. This paper is about how I live in the space of the ecotone. The ecotone is the place where two habitats meet, a place of tension. The habitats which occupy my space, which cause the tensions, are theory and practice. The ecotone is fertile and abounding. This is due to the overlapping of the habitats. This paper is the description of the productive tensions that exist, the complexity and the intensity that is experienced in my attempt to understand my space. In this attempt to understand, I describe how I exist with theory and practice.

I hope that the reader will get a glimpse into how I live between the chaos of my experience and trying to create a linear and tidy narrative. So if you find yourself entangled in the text, welcome to my ecotone. I also need to acknowledge here, that I come to this paper with a pastiche of beliefs about theory and practice. There are moments of naiveté and blatant forms of tacit knowledge.

The blend of experience that I have with theory has some of its roots in my work place where theory cannot be criticised in a discursive critique. The critique itself often becomes theoretical and self-recursive. I don't have a position with this, but the word 'theory' in my workplace has denoted prestige and therefore exclusion. So any romance with practice in this academic site is seen as frivolous and simply not sufficiently academic. It seemed to me as if theory had cast a spell on the academics and so to question their attachment to theory felt slightly dangerous and likely to be interpreted as a display of sheer ignorance on my part. Thomas (2002: 420) says that theory is used:

To designate high-order generalisations, or strong declarations of basic beliefs, or programmatic statements of political or economic agendas, or descriptions of underlying assumptions.

Often I have felt inadequate and thought; 'What's the hype about, in terms of devotion to only theory?' Thomas suggests that this devotion to theory undeniably exists and sees the discovery of theory as a major task. He also mentions grounded theory as one of the characteristics of naturalistic inquiry. Through this experience I learnt that theory sure sounds a whole lot better than saying 'I think that'; or I was often told 'Substantiate your point Agatha", and "What are you drawing on?' Among the education academics generalisability is of great importance. Thomas says that the goal of research is to provide information that is true and 'of considerable importance here is the question of how, as researchers and readers we are able to generalise from findings about particular situations studied to conclusions that have such general relevance' (Thomas, 2002: 426). To be recognised as a scholar or an inquirer emphasis is placed on validity, reliability and prediction.

I must admit that there were/are times that I have anointed my every day beliefs, descriptions and generalisations as theory. Yet, often I'd catch myself saying to myself that descriptions are just that, descriptions. I remain caught in this web. As a teacher

researcher I have accorded special status to the integrity and validity of my own interpretations. In the academic environment in which I work, I have maintained and regarded my interpretations as valid in their own right. I have in my practice taken and used my local interpretation to influence my own practice. Yet I have also used theory in this respect to account for what I do and to account for what is described.

I have to admit that there were times when I felt pulled in two directions – the need for generalisibity because of the emphasis in my work place and my own desire to foreground the significance of my interpretation. As a teacher and practitioner this is the kind of tension I find myself caught up in.

Confused? Well I am.

And so I bow humbly in my knowledge about theory and its relationship to/with practice. I take from the local, let it influence the local and thereby influence my own practice (Thomas, 2002). I make no claims to generalisibility or do I? I don't know.

Caught up and twisted in a veil of tension once again!

As a teacher educator teaching teachers, my practice has often been constructed for me. Course content is sometimes prescribed and so have been the models of delivery that I have been told to follow.

For example, during the last two years I have found myself strangled and twisted in a thread of tension. The Department of Education in South Africa embarked on a national strategy to train and equip mathematics, science and technology teachers. They developed a five-year programme to train a substantial number of educators in each of our provinces. The programme targeted Intermediate Phase (Grade 4 to 6) and Senior Phase (Grade 7 to 9) teachers to ensure an early and solid foundation for learners at higher levels. The intention was that teachers will emerge with an Advanced Certificate in Education (ACE). The National Education Department set out the following outcomes for the programme and for the institutions that would deliver the programme:

- A progressive through-put if well-trained mathematics, science and technology educators per province, who can:
    - demonstrate competence and confidence in classroom practice;
    - assess teaching and learning in line with curriculum stipulations;
    - demonstrate understanding of policy imperatives impacting on teacher development, and
    - become professionally qualified educators with an ACE.

The South African Government has made teacher education one of their biggest priorities and has put forward tenders to the education institutions to start a national mobilisation for education and training as stated by Professor Kader Asmal, MP, Minister of Education. The Minister in 1999 sent out an urgent "Call for Action". After close study of the condition of education and training he assessed the state of affairs and isolated nine areas for priority attention. The fifth priority was the development of the professional quality of the teaching force in South Africa.

The Western Cape Education Department issued tenders and the Schools Development Unit at the University of Cape Town applied for and won the tender. Hence we now teach

on a programme called the Advanced Certificate in Education (Mathematics) or as we refer to it, the ACE.

The course has strong characteristics of being designed by a technical-rationalist who sees the curriculum through the metaphor of a delivery system. The teachers are simply operatives in the education's factory and knowledge is seen as a commodity. This commodity metaphor I believe has become a way in which we then describe education, teaching and the learning process. This knowledge packaging finds expression in the modular courses we offer on the course.

THE STRANGLING THREAD OF TENSION

My teaching on the ACE programme has meant that my practice has *become* the national agenda to train the teachers in my Province, the Western Cape. I have found myself caught in this national agenda of the Education Department. My practice and the theories that I draw on that have acted as support agents in the professional development of teachers. Some of the threads of tension that have arisen are that I was not able to exist comfortably with how I understood my practice and how I chose to live with both theory and practice. The ACE programme has a pre-packaged content, which has resulted in an efficient means of delivery. The course attempted to integrate theory and practice but at a very superficial level. My concerns were that as teacher educators:

- We need to think very carefully what kind of theory is most useful and how we should teach this theory so that teachers can use it to deepen their understanding of educational processes.
- We also need to consider the educative role played by experience.
- And, how exactly should theory and practice be related when the Education authorities want well-trained maths educators. (Gultig, 1999).

The experience of the course felt tight and constraining and definitely not true to my nature, especially when the focus seemed to be more on delivery than on learning. To **navigate** between Theory and Practice suggests to me that they are two separate entities and that one can move from the one to the other. Navigating ***between*** (the emphasis is my own) theory and practice puts forward that they can be taken up separately or avoided. I believe that the ACE programme treated theory and practice as two separate entities in the same way. I choose to use theory as a tool to interrogate my practice. I do not ditch theory for practice or practice for theory. You see, as I walk as a teacher educator, theory and practice walk ***with*** me. There are times that I choose to stress theory and times that the practice is stressed or ignored. Often struggling, I attempt to stress both. Moments exist when there is some observable practice and non-observable theory to someone who watches me teaching. However in my mind, the mind of the practitioner, theory and practice live together as an intertwined entity.

Theories will die if they remain disconnected from me (my practice) and my practice would be lifeless if not inspired by theory.

My experience with practice has included researching my own practice. To distil the tensions I embarked on a research process that allowed me to probe my assumptions which influenced the ACE course. I tried to pay attention to the voices of some of my students from the course so that this knowledge could be shared with colleagues and so

reshape the ACE programme and contribute to our understanding of professional development and teacher education. The purpose of the research was to find out from the teachers what it meant to a mathematics teacher in their everyday, lived situations.

The data was collected during a conversation with the four teachers from the ACE Programme. Varella, Thompson and Rosch (1991) use the term conversation to refer to the interlacing of the co-ordination of consensual behaviour and emotion that occur in living together in language. Basically this means that all human life occurs in conversations, and that human existence takes place in the continuous flow of language and emotioning. I chose to have a conversation with the teachers because my practice is grounded in the belief that stories express a kind of knowledge that describes human experience.
The way in which the research was constructed was largely influenced by my practice. For example, the way I chose to collect the data was a method that I have consistently employed in my daily work with teachers.

I am still in the process of analysing the data but this experience of reflecting on my teaching and engaging in practitioner research has made me aware of the perpetual tension of the elevation and retrieval of theory and practice. I am in the midst of probing the legitimacy of conversation as a form of research that can be used a mechanism for critical inquiry. My researching is about searching, returning to the texts again and again and again … The research becomes my practice, actually it is my practice.

In probing the legitimacy of using conversation I am stressing theory ***in*** the practice.

The research will not hide my interpretations and will not seek to disembody my voice from the text and so the research will at times be written in the first person and by doing so I am taking responsibility for my statements or opinions. I do this in my practice and therefore in my research.

I do have a slight problem. I am not sure about the role that generalisability will play in the research. At this stage I remain undecided whether to use the stories (the teachers and mine) to assist further reflection on the ways that individuals and institutions construct courses in teacher education in South Africa.

I collect old leather suitcases. When inquiring and reflecting on my experience I have used the well-weathered metaphor of a journey. As always I never leave behind my suitcases. In a suitcase you will find my theories packed. Sometimes they're neatly folded and at other times just jumbled and I have to search for them. There were times where the theories developed out of my practice and influenced the nature of some of the research I engaged in, and moments existed when I was introduced to a new theory that I found I could relate to. The theories that are discussed below are just a few examples of those theories that caused conflict with my practice while teaching on the ACE. I struggled to live these particular theories.

Let me unpack some of them. The theories might look as if they are practices rather than theories. – my ignorance here? Be warned I make no excuses for the ones that are creased! (theories?)

Well at least **narrative inquiry** is still neatly folded.

I have used narrative inquiry (Connelly & Clandinin, 1990), (Clandinin & Connelly, 1991), (Clandinin, 1992) as a research methodology and in my work with teachers. Narrative inquiry forms the source of the information through story telling as well as the method of interpretation and reinterpretation. My work with teachers is shaped by the belief that it is through stories that a narrative authority is developed and involves both voice and action:

> Our narrative authority develops through experience made manifest in relationships with others. Because the narrative version of knowledge is transactional, authority comes from experience and is integral as each person shapes his or her own knowledge and is shaped by the knowledge of others. Thus narrative authority becomes the expression and enactment of a person's personal practical knowledge that develops as individuals learn to authorize meaning. (Olson & Craig, 2001)

I have tried really hard to make my teaching a safe space for the stories of the teachers to be articulated, heard and examined. The thread of tension on the ACE programme was that I could not create a formalised safe space for teachers to develop knowledge communities as defined by Olson and Craig. The curriculum on the ACE is not negotiable and so the teachers are given more content knowledge of mathematics and more knowledge of teaching methodology (Breen, 1997). The outcomes stated by the National Education Department have to be met. Olson and Craig say that knowledge communities take shape around common places of experiences as opposed to around bureaucratic and hierarchical relations that declare who knows, and what should be known.

Right, now it is time to unfold **enactivism** from the suitcase. I discovered enactivism while trying to find a theory that reflected my experiences in supporting teachers and while undergoing my own profession development (see Breen, Agherdien and Lebethe, 2003). While doing school and classroom based support I am concerned with the belief that it is through interaction that I am shaped, that I learn and the same happens to those whose space I have occupied. Enactivists believe that one is shaped by the location and the location is also shaped by one's presence. Man does not develop in isolation, but through co-emergence: that which is created or co-evolves in the interactional space between an individual, the environment and others. Maturana & Varela (1980), developers of autopoietic theory, view cognition as action that is embodied and embedded in the lived fabric of one's life. I understand this as: knowing is no longer separable from doing.

During the many times that I have supported teachers in the classroom I have drawn on Davis's (1996) understanding of listening. It is listening by attending to the person's action and situation, and not just to his or her voice that one comes to know the other. Davis does not mean to look, but to listen, to hear what a person is doing, to what a person is also hearing. I have used this understanding of listening in my research as a data collecting method to help me understand what I pay attention to in the Mathematics classroom and what is it that I ignore.

Enactivism assumes complexity and my interest in the emerging theory on learning is the focus on how learning affects the entire web of being, and it follows that what one knows, what one does, and who or what one is cannot be separated (Davis, 1996).

In trying to meet the National agenda of the Education Department I struggled to help the teachers locate themselves within the complex web of relationships to enable them to see their decisions and actions as being constrained and influenced by all nodes of the web. My classroom was not an enactive environment as described by Dawson (1999):

> an open system in which students, through interaction with peers and parents, teachers and technology, create order – make sense of disorder□. Viable pathways which do not exist within classrooms may or may not exist.

The predetermined nature of the course meant that teaching was about telling, the learning was orchestrated. The knowledge gained on the course could be tested and the teachers' representations of that knowledge could be matched against this external standard (Dawson, 1999). One of the thrusts of the enactivist work is not to link the experience of learners to external representation of the curriculum, but to view the curriculum as being occasioned by the learners' experiences in their school environment.

I hear Chris Breen saying: Agatha why do you want to correct the chaos? Learn to live with it.

Where is the **Discipline of Noticing** in this suitcase? It sure looks pretty well-worn.

I have worked with the Discipline of Noticing (see Mason, 1997, 2002) for many years to inquire into and study my own experience. The Discipline of Noticing has been beneficial in allowing me to employ my own will to juxtapose past and present experiences in order to learn from them.

During the conversation (of the research) moments were collected (data) and recorded as brief-but-vivid accounts. Within the Discipline of Noticing data arises from the making of observations and the collection of it constitutes the first level of abstraction from the phenomenon studied. Mason suggests that when recording the brief-but-vivid accounts it is best to write them as giving an account of rather than accounting for. This brief-but-vivid account enables re-entry into the moment. Brevity and vividness help to make descriptions of the incidents recognisable to others.

This form of researching experience and a theory presents me with the opportunity and tools to live the research in everyday practice, and research the living in practice everyday as well as practise the living in everyday research.

CONCLUSION

Curriculum changes in South Africa and teaching on the ACE have opened up a moment in time where I have been forced to navigate between theory and practice as two separate entities. My reality was reconstructed. This moment in time did not reflect theory and practising as my lived experience.

Living with theory and practice causes uncertainty and confusion for me. A very messy situation, but it is one in which I choose to live because I am starting to feel comfortable with it.

Theory and practice can exist separately and they can belong to the same world.

> People do not stay neatly in role: at times, setting aside the role of practitioner of theorizing, the educational theorist is a practitioner of education (a teacher); at times the teacher (as educational practitioner) is a theorist. (Carr, 1995)

This is based on my understanding that there is no single picture that is all encompassing which can capture the world as a whole; that is without horizons. (Gam, 2002).

SO DO I MAKE CLAIMS TO GENERALISIBILITY?

I hope that the reader can see that although I live with the confusion and the tension that I have been describing, I feel comfortable as I walk with my suitcase jumbled with theory and practice. The issue of generalisability is not something that I worry too much about at the moment. I am much more interested in the pursuit of illumination and concealment as I go about my business of teaching and learning. And I'm far less ready to separate theory and practice into artificial approximations of the truth!

NAVIGATING BETWEEN RESEARCH AND PRACTICE: FINDING MY OWN WAY

Vicki Zack

In this paper I will speak to why I do teacher research, what drives me and what I have gained. I will also deal with the constraints of being a researcher in the elementary classroom for the past twelve years, for while, as I will show, researching from the inside has been generative and transformative, it has at the same time been very demanding of time and energy. Some have spoken of the uniqueness of teacher research, the insider status of the teacher-researcher, the requirement of spiralling self-reflection on action, and the intimate dialectical relationship of research to practice (Anderson & Herr, 1999, p. 12), noting that practitioner research has its own unique set of epistemological, methodological, political and ethical dilemmas (Anderson, 2002, p. 24; see also Clandinin & Connelly, 1995; Cochran Smith & Lytle, 1993; Goswami & Stillman, 1987). They have suggested that the teacher doing research from the inside can do what no other can. I have for the past twelve years been working to define who I am and why I do teach-research as I do.

In speaking about navigating between research and practice, I will start where I am grounded, in the classroom, and will show how integral research and theory has been to my practice. My journey as I navigate between research and practice is the process of my making meaning, making the ideas of others (theoretical ideas, research literature ideas) my own. My personal focus is on my own learning, on improving my practice, on the role research activity plays in my personal and professional growth. In pursuing my own questions, I search with curiosity, and out of need. Some academics might ask me: "What did you prove?", or "What can **you** as a (lowly) teacher teach **us** (Gussin Paley, 1999)?" I ask: "What did I learn?"

Some insist that teacher research is about change, that as educators we must be thoroughly committed to improving our practice and the conditions in which practice takes place. However, it may just as likely entail a deliberate attempt to make more visible what is going on (Cochran Smith & Lytle, 1993, p. 52). My point of departure has been to study the children's mathematical thinking. As I attempt to make more visible what is going on, I come to understand the mathematics better, and to better understand the children's thinking, and this in turn affects my practice.

BACKGROUND

Let me say a little about my background, and the school and classroom environment first. I will then speak about the research work, and some of the findings which emerged due to the research activity.

I returned to the classroom in 1989 after completing my doctoral work, and after working at the university level in a faculty of education for a number of years, in order to research from the inside, in the changing ecologies of reform-oriented classrooms (literature-based approaches in reading and problem-solving approaches in mathematics). The school in which I work is a problem-solving culture in which the students are expected to support their positions and present arguments for their point of view in most areas of the curriculum. In my fifth grade classroom (10-11-year-olds) we use an inquiry-

based approach in which we – students and teacher alike – often pursue questions of import and of interest to us (see for example, Borasi, 1992). In the case of the task I have chosen for this paper, we, teacher and students, explored some of the surprises and puzzlements together. There have been numerous instances, including the episode described herein, in which I have learned something significant about the mathematics due to the children's questions and investigations and this has changed my understanding in substantive ways (Zack, 1997a, 1997b). My background in formal mathematics is weak and in regards to personal identity I have seen myself as a "literature" person for much of my life; I came to a love of mathematics in my late 30's. My insecurity is perhaps in line with the perception of many (self-aware) teachers who find themselves wanting in regard to knowledge about subject matter, about children's thinking, about pedagogy. I have worked for the most part alone, always following my own agenda, posing and pursuing questions of interest to me; as some have noted, in teacher research (and in qualitative research generally), the path is laid in walking. At various points when the need arose I enlisted the help of others whose fund of knowledge was far greater than mine. I will show here for example how David Reid's help was invaluable, and how our collaborative work together evolved. Other academics/ friends/ colleagues to whom I owe a debt are Barbara Graves, Mary Maguire, and Laurinda Brown. I can well attest to the importance of connectedness in research relationships. Stating my position as the initiator of the investigations is important, however, since in reports about teacher-research, there is often talk of issues of power, of who's in charge (i.e. whose agenda); teachers involved in classroom research are at times co-opted into investigating topics of another person's choosing when working with university faculty (Breen, in press).

I have been a member of PME and PME-NA since 1987, and PME and other conferences have served, among other purposes, as a source for networking for me. The PME research sessions offered reports of recent research, and in addition I pushed myself to write and submit papers. The PME Teacher-as-Researcher Working Group (which met from 1988-1996) provided a forum for talking about the teacher-research process itself. Whereas for academics the impetus to write often has to do with "publish or perish", for me the impetus to write and to attend PME and other conferences was related to (1) pushing myself to formulate my ideas and explain them to others, and (2) to engage in discussion with others who might then push my thinking. I have spoken elsewhere (Zack, 1997b) of the joy and benefits of initiating, designing and directing the research in my classroom and of doing so as a life-long commitment (as opposed to fulfilling the requirements of a degree program for example). However I have also shown that it has been so demanding, this life doing two jobs, that while I could not <u>not</u> do research while teaching, I could see that I might not be able to continue at the pace which leaves me with no quality of life outside of the classroom (Zack, 1997b). Few have dealt with this issue of time constraints and energy drain in teacher research**.** Of the ones who have noted it, Zeni recently issued the strongest statement yet on the ethics of including "protection for the long-term health and sanity of teacher-researchers" (2001, pp. 151-152).

THE INTIMATE DIALECTIC BETWEEN RESEARCH AND PRACTICE

In examining my own assumptions and attempting to find my own way, my own voice, I have read widely. Others have pointed out that this is common, that practitioner researchers are "likely to seek out research done by outsiders and to become critical

consumers of this research" (Anderson, 2002, p. 24; see also Huberman, 1996, p. 131). My questions emanate from neither theory nor practice alone but from the juxtaposition of the two, and from critical reflection on the intersection between the two (Cochran Smith & Lytle, 1993, p. 15) in areas which are of intense and enduring interest to me. There is recursiveness in the process, wherein questions are continuously reformulated, extended, re-visited, methods are revised and analysis is on-going. I have felt joy in what has transpired, and what I have been able to explore. And yet a great deal of what I have learned is not in my writings, not as yet consciously conceptualized – it resides still in the realm of what Polanyi calls "tacit knowledge". I recognize the value of practical knowledge, and also respect the place research can hold in informing practice. However, I emphasize the challenge involved in understanding others' ideas. Bakhtin has made mention of the difficulty of the process, and how one's construction is half one's own, half someone else's (1981, pp. 293-294). Each person appropriates, reworks, re-accentuates while making their own way (Zack & Graves, 2001). Bakhtin's conceptualization is important to me as it relates both to my work in making meaning of the research and theoretical issues and seeing what it might mean to my work as a teacher, and to the children making meaning of the mathematics as they work together with me and their peers in the classroom, and at times, their parents at home.

Finding my individual voice happens due to dialogue with others, those immediately there – my students, my colleagues in Canada, my friends and colleagues here at PME –, and those long gone or those whose ideas I encounter in books and research papers. In mentioning some of the theorists, and some of the researchers in mathematics education who have strongly influenced my work, I will highlight theorists Vygotsky and Bakhtin, Bruner, Dewey and Piaget, and in regard to researchers will mention the debt I owe to mathematics educators such as Paul Cobb, Erna Yackel and Terry Wood and their colleagues, Carolyn Maher, Alice Alston, Roberta Schorr and other members of the Rutgers team as well as to numerous others who have influenced my thinking.

My goal has been to study how learning is interactively accomplished. Vygotsky's (1978) theory that the thoughts and practices of others become integrated in one's own, and Bakhtin's (1986) theory of the dialogic nature of learning have been fundamental to my work. I have been particularly intrigued by Vygotsky's notions of the ZPD, and everyday and scientific language. My focus when I began was on explanations – How is mathematical meaning shared? and I then extended my search to explore arguments, and the children's notions of convincing and proving. In a setting in which children have received no instruction in 'formal' approaches to reasoning or proving, I asked: How do they proceed when asked to 'prove' that they are correct? What do they consider valid arguments which will prove their case and convince others? What language do they use to express their arguments? What kinds of reasoning do they use: inductive, deductive, other? In pursuing these questions, other questions arose. There is much I have learned to date and have shared various components in some detail in papers I have written.

In order to give a few examples of the paths I have pursued, I have chosen to present here a task which as it evolved over the years offered surprise after surprise. And I am still learning. The investigation deepened my understandings, as the findings were richer and more complex than what I had anticipated.

The task is one I have called Count the Squares (a variation of the Chessboard problem) and the first activity is one in which I asked the children how many squares of varied sizes there are in a four by four grid. The task is a deceptively simple one (first introduced to the children in April 1994). I only discovered years later that this was a rich mathematical problem. My goal was to observe if the children could see a pattern and generalize it. The children introduced me to patterns I had not anticipated, and over the years I extended and nudged the children further. Interesting developments emerged when I posed the question: What if it were a 60 by 60 square? (April 1996). My original challenge to the students was that they construct a general procedure, which some succeeded in doing. In the midst of seeking to encode the general procedure into an algebraic expression, the children and I were blocked. I was shocked to find that we could not construct an algebraic expression for this 'sums of squares' problem. I sought out in a journal and offered the students a 'non-obvious expression' which worked –a $n(n+1)(2n+1) \div 6$ (Anderson, 1996). The children, in 1996, in turn raised the bar: they saw *that* it worked but asked *why* it worked as it did (Zack, 1997a), a question I could not answer. This led me to enlist a longstanding friend and colleague, David Reid's (Acadia University) help, as the scope of his mathematical knowledge was far greater than mine. David in turn searched for an explanation which made sense to fifth graders, in answer their need to understand *why* the non-obvious expression worked as it did. With David's guidance, the students over the years during one-week periods in May (1996, 1998-2001) constructed various algebraic expressions which were similar/equivalent to the Johnston Anderson one. It was constantly a delight to see how from the activities which began in April the children moved from the starting point of the four by four grid to endeavouring to understand David's personal adaptation of the visual proof offered by Nelsen (1993) – with varying degrees of understanding of different aspects of course. David and I studied the children's notions of proof (Zack & Reid, 2001). However, at the moment perhaps of most interest to us is our deliberation about thinking. Looking closely at the tapes of David working with the students and the students working together during those one week periods in May over a number of years led us to think about how we – children and adults alike – come to understand complex ideas. Learning mathematics is often portrayed as sequential; complete understandings of underlying concepts is assumed to be necessary before new concepts can be learned. Our data led us to suggest otherwise. Learners work with "good enough" ideas as placeholders; when confronted by many complex ideas, learners keep diverse and at times incongruent possibilities in the air, waiting at times to the end to make sense (Zack & Reid, 2002). When I read years ago that teachers who are researchers "become theorists … testing their assumptions and finding connections with practice" (Goswami & Stillman, 1987, preface), I remember thinking –Who, me? And yet here I am, theorizing, albeit twelve years into the process of teacher researching.

It has to date been an eventful journey for me. My first paper on the chessboard task (Zack, 1997a) detailed the patterns seen, and showed how three students used their knowledge of the patterns and generalizations to construct three counterarguments to refute the position of another pair of students. I then looked closely at the structure and language the three students used for the three counterarguments and was able to show how the students moved between everyday and mathematical language (Zack, 1999). The

students' talk is embedded in what sounds like everyday conversation, but at the same time revealed a complex mathematical structure. My analysis of the talk led me to look closely again at Vygotsky's notion of everyday and scientific thinking. Whereas a common interpretation of Vygotsky's theory of everyday and schooled language is that he is speaking of a move upward from the everyday to scientific language, with scientific replacing spontaneous concepts, I have come to see it as more of a to and fro movement, and contend that everyday language should always maintain a place. As a result I have been explicit in the classroom in encouraging my students to always press for explanations, to constantly strive to keep in touch with personal ways of knowing. Indeed if one looks back to Vygotsky's (1987) own original work, one sees that he too spoke of spontaneous and scientific concepts as closely connected processes which continually influence each other. I feel, however, that Vygotsky underestimated how hard it is to align everyday and scientific concepts. I was delighted to find that Van der Veer (1998) concurred. Van der Veer spoke of the tension and challenge of connecting the personal with the schooled:

> Ideally, genuine conceptual knowledge is based on the combined strengths of everyday and scientific concepts. That is, children should be able to give the formal definition of a concept and point out its link with other related concepts. ... Moreover, the concept should come to life for the students by being confronted with their everyday understanding of the subject. It is clear that such genuine conceptual knowledge is the ideal – an ideal that even by adults is achieved only in some specific domains. (p. 91)

This is one example of how I needed to be critical in relation to common interpretations of Vygotsky's theory, needed to return to what Vygotsky actually wrote, and then decided that I disagreed in part with Vygotsky, feeling he did not acknowledge the challenge that was entailed in linking everyday and schooled concepts. For me, ultimately, conceptual knowledge means understanding (complex) ideas in such a way that one can express them in one's own words. This led me in turn to encourage my students to be proactive throughout their school life and lifespan, to keep pressing when ideas do not make sense.

Another of Vygotsky's (1978) concepts, his notion of the Zone of Proximal Development (ZPD), has been a fundamental anchor for me in my search to see how learning is interactively accomplished. His original formulation of the ZPD, pivotal for my doctoral work in the 1980's, was "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (1978, p. 86). The ZPD is often presented in the research literature as a site with fixed boundaries which the teacher knows, and that the teacher provides activities which are within the child's range. Over time due to my work with the children my notion of the zone has evolved, as I have seen that (1) I as teacher cannot predict with certainty the outer bound, the upper limits of knowing, and that (2) in this intellectual space, created in the moment, it is not only the children who learn but also the teacher (Zack & Graves, 2001). And so, what does it mean to redefine teaching as inquiry across the professional life span? I can attest to the extensive benefits as well as to the constraints of being a teacher-researcher, and suggest that each teacher must choose for herself/himself whether to embark upon this exhilarating but demanding journey.

WITH LESS DO MORE

Gershon Rosen

> Teachers who engage in self-directed inquiry into their own work in classrooms find the process intellectually satisfying; they testify to the power of their own research to help them better understand and ultimately transform their teaching practices (Cochran-Smith & Lytle, 1993, p. 19).

I am a teacher-researcher whose research emerged from working both with students who were frustrated by not progressing, meeting the same equations year after year after year and with teachers of such students who were equally frustrated at their inability to help their students progress. The status quo had us as teachers breaking up the content up into little steps, so that both we and the students lost sight of the whole. I will put forth in this paper my theory of "WITH LESS DO MORE" which initially examines a more global aspect of the mathematics to be studied. It takes into account any knowledge and techniques retained by the pupil, however elementary, to examine as much of the material as possible and gradually modifies that knowledge and techniques as we progress.

I am full time teacher working a full teaching load in a Secondary Comprehensive School in Israel, teaching mathematics, sometimes computer programming and IT and occasionally physics. This I have been doing for nigh on 40 years, the first ten of which were in England and the rest in Israel. Four months after my arrival in Israel I started teaching middle and high school students up to and including the advanced level matriculation. Within a year I was working with teachers too. It was in these initial years, with poor command of the Hebrew language, that I began to research and develop what I call my "global" approaches to the learning of mathematics. I came to realise that my not knowing "how to say it" was not necessarily a handicap in my students knowing "how to do it". Seeing more or less the whole picture put many learning difficulties in perspective.

Broadly speaking there are two different and complementary ways of processing information - a linear step by step method that analyses parts that make up a concept and a spatial or global approach which enables freedom to focus on Parts of the whole (Rosen, 1977) - sometimes called serial vs. simultaneous processing.

My global model in this research was like that of finding one's way around an exhibition or museum. Without a map we have to see the exhibits in the order that they are set out. We sometimes come upon what seems to be a brick wall or restricted entry with no way through and no way to get to the interesting exhibits on the other side. We can't get into the inner sanctum. It can be frustrating knowing that there are others, the privileged, who have found their way there. However, if we have a map we can see it globally. Before we start we can choose which exhibits to study, which catch our interest etc. We are often able to see where we are heading and even how to navigate the obstacles or even to avoid them altogether. I am of course referring to obstacles such as: Number facts, fractions, decimals, directed numbers, formal use of letters at the beginnings of algebra, equations in one unknown, etc. generally anything that has to be memorised whether it be multiplication tables, names and properties of the different geometrical figures and their components, or various algorithms which hide the essential mathematics. I use a global approach and the maxim: "WITH LESS DO MORE", i.e. in the understanding of

mathematics a little knowledge can be a very powerful tool. We can, in many cases, achieve an understanding of the tasks before us using more primitive methods than the text books prescribe and thus get to that inner sanctum. Once there it is very often possible to see how to arrive much more efficiently i.e. finding the key to that hidden door. GLOBALLY we look at a world we are about to explore mathematically. WITH LESS we find an elementary technique with which to explore that world and with DO MORE we explore as much as that world as possible with that elementary technique. If we find a situation that the technique cannot handle we by-pass it for the moment until we come across similar situations that justify either developing an additional technique or a modification of the existing technique that might prove more efficient.

Children up to early primary school learn to explore their world globally and then slowly try to make more sense of various parts of it by examining those parts more closely. Progress is not linear. Any toddler when presented with a plate of biscuits will not be satisfied until she has one in each hand. She doesn't know the word “two” in any language but she has a strong understanding of what more than one is. She also matches a prism's cross section with the appropriate hole in the post box toy without knowing the names of the shapes. She recognises certain properties of the objects, their symmetries (or non-symmetries) without being able to verbalise them. Teachers from all levels, from Kindergarten to 12 grade, when asked to describe what the child is doing, describe a property of the shape instead of the shape itself saying things like: “She took the circle or the pentagon”, when in fact she took the cylinder or the pentagonal prism. (Rosen, 2001). As the child progresses through the school she learns that a linear approach is the accepted norm. Work has to be set out in a particular way. The teacher chooses which problems are to be answered and in which order. Uniformity and efficiency are the order of the day. Students who cannot adapt to this linear approach are in many cases put into the lower ability levels

My research is into how and where can the global approach aid those who are failing in their learning of mathematics, in particular what material from the standard curriculum is more easily internalised linearly and what globally. Also how to devise learning materials and situations based on “WITH LESS DO MORE” which enable students to explore see the topic seen from a global perspective.

> All this may seem more reasonable if we again remember that neither scientists nor laymen learn to see the world piecemeal or item by item. Except when all the conceptual and manipulative categories are prepared in advance ... both scientists and laymen sort out whole areas together from the flux of experience. The child who transfers the word “mama” from all humans, to all females and then to his mother is not just learning what “mama” means or who his mother is. Simultaneously he is learning some of the differences between males and females as well as something about the way in which all but one female will behave toward him. (Kuhn, 1962)

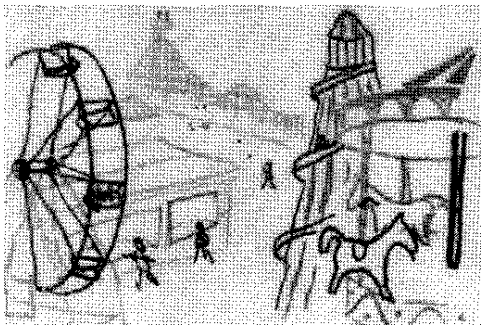
AN EXAMPLE OF LINEAR APPROACH TO TRIGONOMETRY.

Students in Israel are presented in the 10^{th} or 11^{th} grade by a series of trigonometrical ratios: tangent, sine and cosine (and sometimes also the cotangent). Each ratio is presented separately and each of the special applications is explained followed by a series of exercises. Weaker students falling by the wayside as we trudge through the chapter.

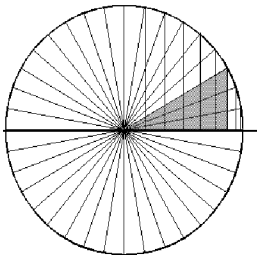
More falling when we come to solving problems where the appropriate ratio has to be chosen. This is a very linear approach and for the so called "weaker students" the process is broken down into ever smaller and simpler steps.

AN EXAMPLE OF A MY GLOBAL APPROACH

Applying "with less do more."



With the help of a picture of a Luna park, a large hard cardboard disc (which first represents a carousel and later a Ferris wheel), a couple of pencils and some imagination we can get a global picture of the material that we are about to study. Globally, using the cardboard Ferris wheel, the students learn to model a point on a revolving circle (or a point revolving on a circle) and can say something about the height of the point above the centre of the circle, at various stages of the rotation. A closer look at a drawing enables them to "see" a small set of triangles (LESS) and use them to calculate lengths and angles of MORE interesting triangles.



> (In addition, revisiting the Luna park, which is now familiar territory makes studying graphs of the family of sine functions easier) (Rosen 1997).

When the disc is a carousel the discussion follows something like:

> Question: "Suppose this pencil represents your little brother or sister on the carousel" (Rotates the disc in horizontal plane), "Does it matter where you stand in order to get the best photograph?"
>
> Discussion: "No it doesn't matter because she is rotating at constant speed." "Yes I would photograph when she is nearest to me." "Yes I would photograph her when she is coming towards me."

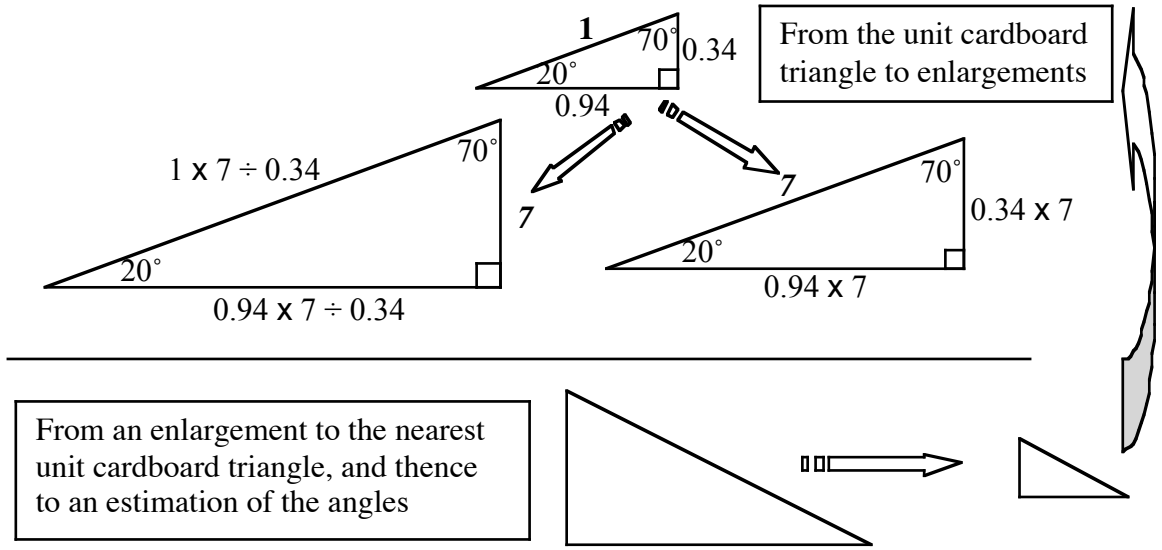
When the disc represents a Ferris wheel we talk about the motion of a point on its circumference, its height above and below the central spindle, which points are at the same height in relation to the amount turned. The students draw and measure the heights on a 1 dm. radius disc for various angles of rotation and discussion ensues as to what the heights would be if the radius was 1 m. 10 m. 15 m etc. Drawing the heights helps pick out the right-angle triangles within the circle.

At the WITH LESS stage we begin by constructing from thin cardboard a set of eight right triangles with hypotenuse of unit length (1 dm., angles 10^o to 80^o). We measure the lengths of sides opposite the angles and write these lengths as a decimal fraction of 1 dm. Along the appropriate sides (including the hypotenuse which is opposite the right angle). Handling the triangles leads to the observation that it was really only necessary to construct the first four (10^o, 20^o, 30^o, 40^o) because the other four are duplicates. Note: the word sine is not mentioned nor needed at this stage. Nor are any of the other trigonometrical ratios.

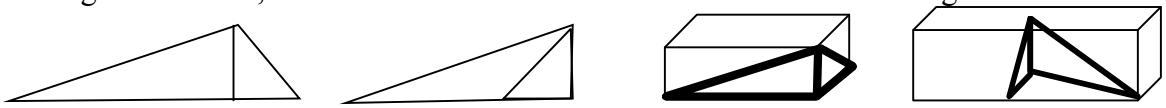
The essence of the DO MORE being that with just the set of cardboard triangles it is possible to solve the majority of trigonometry problems in two and three dimensions. Including acute and obtuse angled triangles. Any right triangle is an "enlargement" or "reduction" of its appropriate cardboard cut out triangle.

Results for triangles with angles other than these are estimated (interpolated). Each problem chosen is solved for <u>all</u> its angles and <u>all</u> its sides with respect to the data and so there is less need to solve a large number different problems. Without pages and pages of practice the pupils develop the ability to discern four important general facts:

- When to multiply and when to divide by the calculated scale factor i.e. when we expect a larger or smaller solution. (No formal equations to solve).
- Which of all the obtained results are required for the particular problem being solved
- How to proceed when we find situations which are not solvable with the help of the four cardboard triangles. (i.e. only the right-triangle with only the two sides adjacent to the right angle known).
- Calculating <u>all</u> angles and <u>all</u> sides enables a check whether the results are logical for example whether the result for the hypotenuse is larger than those of the other two sides even though the length of the hypotenuse may not be required for that particular question.



More interesting diagrams involving two or more triangles involve moving from one triangle to another, with situations in two and three dimensions with diagrams such as:



These are the same problem in different settings - "Seen one – seen 'em all". With increased confidence some of the pupils "see" where to add the line that converts a scalene triangle into two right-angle triangles. No need for the sine or cosine rules either. Thank goodness for the calculator which helps us speedily calculate with awkward numbers and leaves our brains free to roam at will.

What was particularly exciting was the fact that this global approach with minimum knowledge enabled the students to solve more complicated diagrams in two and three dimensions. Even so-called "weaker" students, who couldn't handle any but the simplest

of equations and with no formal prior knowledge of similar triangles or ratios, helped their more advanced friends to cope with such problems.

Teachers however react in one of two ways.

In a workshop situation the more conservative dominate with comments like “it might work with the brighter ones but then they don't need it. And anyway, we can't ‘waste’ our time giving them the whole picture we have to give them more practice”. Another complaint is that this approach does not look good on the page. It is messy. Some others, not so vocal in public and because the approach is new to them, recognise that they have students for whom the linear approach described above does not work. Many try out this global approach with their students and generally report back that they and their students enjoyed the lessons, and progressed onto more interesting questions in two and three dimensions. Not all were prepared to adopt the unit circle definition of the sine ratio. They did however like the idea of solving all aspects of the triangle i.e. seeing the whole picture instead of just the lengths or angles required to answer the specific question using only the conventional sine ratio definition. This, they commented, cut down drastically the amount of different problems the students needed to solve - a valuable saving in time both for the teacher and pupil not to mention the ability to actually do homework.

Apparently my approach was sufficiently refreshing so that in a very short time I was invited to start conveying the application of my approach in other areas of the mathematics school curriculum, to teachers who keep coming back for more.

A teacher has to “get through” the syllabus and prepare the students for internal and more important, external exams which affect the students future. It is natural for us to prefer to stick closely to the order in the text-books, afraid that, if we miss something, the students will not be able to answer all the questions in the exam. A researcher comes into the classroom maybe once or twice a week, sets a task to a small group of students or to the whole class, and records the responses. He then goes away to assess the results before coming back for another session. Some researchers, who do not have to compete with large classes and a full teaching workload with all that goes with it, come to the teacher with remarks like: “*In my research I have shown that children do not know that a square is a special case of a rectangle*” and continue with “*you must do something about it*!” a conclusion that every teacher with any experience at all intuitively knows about, but the text book has no new suggestions as to how to treat the problem.

I am a teacher who does practical research, finger on the pulse, relevant to the particular topic that is to be taught and being adapted on a day to day, if not moment to moment, basis. I look for situations where standard text book linear methods are not effective with certain classes. I have to be ready to drop one approach and change it for another, observing, reflecting, interacting all the time. Building up a store of experience where the result of the research is how the students respond in situations not necessarily covered by the material taught. When trying to convey my ideas to others, I am not looked upon as an outsider. I am part of the community. I confront and have to handle, on a daily basis, the same problems as the teachers I work with: lessons lost to school activities, interruptions from this or that, lessons at the end of a hot day, absentees, discipline problems, many classes most of which are not with the quickest learners, the list goes on

and on.

I have two strategies of face to face contact with teachers. Firstly, I actively work with them (and their students), before, during and after their classes (no sit back and take notes for me). The teacher and I are a team in the lesson with the teacher calling the tune. Secondly, I give in-service workshops for teachers either with a group of teachers in the same school or from several schools in a particular area.

By presenting a topic in mathematics globally I am able to get the students to see connections with various parts of the topics and thus get them to provide their own verbalisation. This enabled them to then read the text-books and fill in the gaps or ask me to fill in the gaps for them. Thus expanding their vocabulary. It empowered them to take charge of their own learning. Over the years I have set many regional Exams and results have shown that students prefer and have more success in situations that they do not recognise as questions that they have failed at in the past. These are investigative type questions that the teachers haven't practiced with the students even if they appear in the text books. The students, like the teacher in the vignette, cannot call on algorithms and standard methods, which they have no understanding of, so they have to resort to more basic approaches that are more accessible.

Effective teaching involves listening to your students and helping them to build their mathematics using their initially their own vocabulary rather than forcing the students to listen to the teacher. Similarly, I have found that effective teacher education involves sensing where the teachers are in their development and helping them to build their mathematics using their vocabulary. Many teachers, when presented with a topic globally, come to understand where they are meant to be heading. Teachers that resort to ever smaller and smaller steps and more and more exercises to practice on never seem to get on to the interesting questions with their pupils. No wonder they never get anywhere. The linear ever-smaller steps approach always reminds me of Achilles and the tortoise, he never caught up until he came to the conclusion that he had to bend the rules. The tortoise beat the hare in that race too.

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