PRIMARY TEACHERS' CONCEPTIONS ABOUT THE CONCEPT OF VOLUME: THE CASE OF VOLUME–MEASURABLE OBJECTS

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In this paper part of the results obtained by a research project called "Primary teachers' thinking about the concept of volume and its teaching", performed from 1997 to 2001, are reported. This paper focuses in one of the two main objectives of the aforementioned research: To describe the mental object volume of the participant teachers. The experience took place in a five-session workshop directed to in–service teachers and mathematics advisors of the Public Educational Bureau (Secretaría de Educación Pública). The sessions were audio and video taped and a set of questionnaires was posed in the first session. The transcriptions of these recordings, as well as the answers obtained by the questionnaires, were analyzed in order to attain the planned objectives.

The literature related to teachers' thinking and mathematics education is extensive since the 1990's decade. Researchers that also have under their charge the education of future teachers have developed a great part of these studies. Between the results obtained some indicate that teachers' conceptions, as well as their experiences when they were school–aged children, influence their professional practice (see Civil, 1996; Thompson, 1992).

In accordance to the aforementioned results, the importance of studying teacher's conceptions and beliefs about the different elements in the schools' curriculum is manifested. If some of their conceptions are wrong or incomplete, they will teach them to their pupils in the same way. This paper is focused mainly on teachers' conceptions about the concept of volume, and the results presented are some of those obtained by the general research.

In the mathematics' teaching research literature volume is a scarcely studied concept; it appears more frequently in sciences' teaching literature. Although, some researches centered in teachers' conceptions about the concept of volume exist (see Enochs & Gabel, 1984), anyway most of these studies focus their attention in children .

The first results related to the learning processes of the mathematical concept of volume are, probably, Piaget's quantity and matter's conservation studies (Piaget, Inhelder & Szeminska, 1970). Some others of further researchers followed them. In any case, as Vergnaud (1983) observes, the volume's conservation studies do not accomplish the volume subject matter. He, in collaboration with other researchers developed a study where the central point is volume's arithmetical processes. From other point of view, Janvier (1997) among others, obtained interesting outcomes about the teaching of volume.

Due to the problematic involved in the research that origins this paper, a particular mention is deserved for Potari & Spiolotopoulou's (1996) study in which attention is focused on the influence that certain objects' characteristics –both geometrical and physical– have on children's conceptions about the concept of volume. In preliminary studies and in the theoretical frameworks' construction, the influence that the meaning deserved by people to the term volume had on their personal domain set for the function volume (the set of things they consider as volume–measurable objects) was perceived.

In this paper, geometrical aspects are considered to be those elements, processes and procedures that take into account the shape of objects, for example their concavity or convexity. Numerical or arithmetical aspects are those centered in measurements, size and numerical operations, while physical aspects are considered to be those related to matter's condition such as solidity or density, for example.

Authors like Rouche (1992) had paid attention to volume–measurable objects from a didactical point of view. Nevertheless, we consider that, because of the consequences it may have on the teaching and learning processes of the concept of volume, one of the main contributions of the general research "Primary teachers' thinking about the mathematical concept of volume and its teaching", from which this paper is a fragment, is the call on attention about people's volume conceptions influence on their perception of objects to be volume–measured.

According to Freudenthal (1983), concept's construction should not be the main teaching purpose, even when they are the "backbone of our cognitive structures" (op. cit., p. x). He affirms that children learn what things are, for example a chair or a table, without teaching them the concept of chair or the concept of table.

To understand the difference Freudenthal established (1983) between the expressions concept and mental object we may consider that a mathematical concept is used in a variety of contexts. For instance, the concept of volume may be comprehended as the amount of space claimed by a solid, or as a free space inside a closed surface, or as the amount of material that fills a recipient or like a number of cubic units, or as the space displaced by a solid when placed into a liquid and in some other ways. The usage's rules of volume in all these cases are different. The totality of these usages, in all contexts, is what may correspond to the concept of volume, or using Puig's (1994) terms: "the semantic field of volume" or "the encyclopedia knowledge of volume".

> Now, the subject reading a text or interpreting a message does not operates in the set of the encyclopedia– that is to say, the totality of usage produced in a culture or episteme– but in a personal semantic field, which has been elaborated producing senses –senses that become meanings if the interpretation is adequate– in situations or contexts that demand new uses [of the mathematical concept of interest] (Puig, 1994, p. 9).

The scrutiny of the literature related to teachers' conceptions and beliefs showed that it is not easy to obtain agreements respect to the use of the terms conceptions and beliefs (see Thompson, 1992; Törner, 2000 for example). Recently, Pehkonen & Furinghetti (2001) pointed out that a great amount of the encountered discussion in the reviewed papers would be avoided if in mathematics a distinction were made between concepts and conceptions.

> The word 'concept' (sometimes replaced by '*notion'*) will be mentioned whenever a mathematical idea is concerned in its 'official' form, as a theoretical construct within the 'formal universe of ideal knowledge'. [Whereas] the whole cluster of internal representations and associations evoked by the concept –the counterpart of the concept in

the internal, subjective 'universe of human knowing'– will be referred to as 'conception' (Sfard, 1991 in Pehkonen & Furinghetti, 2001, p. 649).

As it can be seen, Freudenthal's characterization of the expressions *concept* and *mental object* coincide with Sfard's given meanings of *concept* and *conception* respectively. In this manner, one of the main objectives of the general research can be posed in these terms: to characterize *volume mental objects* of the observed teachers. In this paper a part of the results related to this objective are presented.

THEORETICAL AND METHODOLOGICAL FRAMEWORK

The theoretical framework was constructed using the guidelines of the Local Theoretical Frameworks Theory of Filloy (1999). This theory, is described as a "theoretical and methodological framework for the experimental observation in mathematics education" (Filloy, 1999, p. 1). In it "the object of study is focused by means of four related components: (1) teaching models, (2) cognitive processes models, (3) formal competence models and (4) communication models" (Filloy, 1999, p. 4). For the research project herein reported the theoretical framework has been partially exposed in Sáiz & Figueras (2000); its detailed description appears in Sáiz (2002). In the next paragraphs a general description of each of the components of the local theoretical framework is presented.

The formal competence models component puts together the elements that allow to describe and decode what is observed, in the case of the research herein reported, this component contains all the knowledge that an ideal user should have in order to understand and decode the messages' exchange that takes place when volume related tasks are performed. It is worth saying that this component does not correspond to the concept in the sense formerly explained, but that it conforms the observer's mental object, which is, somehow, a mental object between the concept and the teachers' personal semantic fields, for whom a characterization is pretended.

The construction of this component followed two axes. The first one consisted in a documents scrutiny in mathematics' history books in order to identify the historical process of development of the mathematical knowledge related to the concept of volume. The second one took as its initial point the phenomenological analysis applied to the area and volume concepts by Freudenthal (1983) albeit, not every aspect of his analysis was taken into account. As he states, the approaches to the mental objects area and volume constitute a large variety and the analysis required to decide if they form a consistent whole or not, and if all of them lead to the same result "*require efforts that surpass anything that can be asked or realised, say, at the highest secondary grades...* " (Ibid. p. 381). Nevertheless, the analysis has been extended to other considerations such as the relation of volume with other magnitudes like capacity and weight; the role volume plays in physics science and the geometrical and physical characteristics of bodies when they are considered as volume–measurable objects. This component was a theoretical and methodological framework not only for the observation's organization but for the data analysis as well.

For the construction of the teaching models component, a scrutiny and a comparative analysis of volume teaching models used in Mexico in a period of one hundred years (1898–1997) was applied; in this process the national educational program for the primary school, in use since 1993, was considered as the “pattern” to compare with the different volume teaching models. The formal competence models component and some specific guidelines for the teaching of volume sustained by experts of different countries were useful too.

Some empirical experiences with in service teachers in proficiency programs revealed evidence, which showed that –frequently– teachers have difficulties in solving volume problems and that they possess misconceptions related to the concept of volume. The former facts, between others, explain the need of the cognitive processes models component, performed with results derived from research linked to children’s cognitive difficulties related to the concept of volume (see Piaget, 1960; Vergnaud, 1983).

DATA ACHIEVEMENT

The communication models component is related to the data’s attainment processes. For the general research project, the challenge was to provoke a discourse that reveal information about what was being investigated, in the particular case of what is herein reported: the teachers’ mental object volume.

It was essential to design a communication process that was not a knowledge test; there were evidences that this kind of examinations make teachers feel nervous and assaulted. It was necessary to obtain a free and spontaneous talk. Hence, the application of some carefully designed questionnaires and the design of activities and problems for a workshop related to volume and its teaching, directed to in–service primary teachers, was the way that allowed the obtaining of appropriate data (see Sáiz, 2000).

Twenty-two primary teachers participated in the experience, some of them were in service teachers and others were assessors in “teachers’ centers” (locations where primary and secondary level teachers, from 1st to 9th level, may go to find books, didactic materials and assistance). All the sessions were audio and videotaped. The transcriptions of the recordings and the set of answered questionnaires contained the whole information for the analysis.

ANALYSIS METHODOLOGY

The data obtained by means of the communication processes are the teachers’ ideas expressions in the appearance of words, phrases, sentences, paragraphs and other fragments of their discourse. This discourse is expressed by means of a mathematical system of signs (Filloy, 1999) and, in its usage, signification and sense production processes are manifest. The analysis must recover these meanings in order to get elements allowing the teachers’ mental–object volume description.

Briefly, it can be said that the initial analysis categories are naturally inferred from the local theoretical framework’s components. In a first moment data was classified in three large categories: cognitive processes, teaching models and formal competence; in a later analysis, data was sub classified in different sub categories, for example, qualitative aspects and quantitative aspects. The different versions of analysis categories were modified by the scrutiny and analysis accompanying the reiterated lectures of the data. These processes show the great amount of data obtained and the difficulties to work on it;

in the last stage of the analysis a software tool was used, and a definitive analysis categories' taxonomy was obtained.

The results of the analysis are organized in different ways; teachers' mental object volume may be described by exhibiting different classes of results related to: 1) the set of objects considered as volume–measurable by the participant teachers; 2) the procedures used by them for measuring, comparing and obtaining volume; and 3) a set of meanings and results obtained by the measurement and calculus processes performed by the teachers On the next paragraph, examples of the aforementioned classes of results are presented.

DISCUSSION AND RESULTS

1. Volume–measurable objects are those for which three lengths can be obtained.

Maybe teachers are thinking in the measuring of large, wide and height. With these measurements, they say, that volume may be calculated. (In this and the next presented results, an example of the evidence that sustains each conclusion follows; the first number represents the row in the document where the evidence appears; letters M and V, followed by a number, are codes for teacher identification).

(Answers to the question: Can you obtain the volume of an auditorium?)

64 M2: [Yes, because] we can see three dimensions.

65 M4: An auditorium. Yes. It is measurable.

117 V5: An auditorium. Yes, generally they have three linear magnitudes and with them you can obtain it.

2. Bodies perceived by teachers as surfaces are not considered volume–measurable objects.

This result is related to the former one. Since teachers perceive thin objects such as paper sheets and handkerchiefs, as surfaces, they think that there's no width to measure; there is a missing data in order to obtain volume by multiplying three numbers.

(Answers to: Do you think that you can obtain volume of the following things?)

69 M4: A handkerchief. No. It has a minimal third dimension.
97 M9: A sheet of paper. No. It is an area measure.
120 M14: A handkerchief. No. It is planar.
144 V10: A sheet of paper. No. It has area and perimeter; it's a planar figure.

3. Teachers do not consider some daily things as volume–measurable objects, due to their irregular shape.

It seems that, from the participant teachers perspective, objects that are not identifiable with scholar geometric solids (cube, prism, pyramid) cannot be volume–measurable objects. Maybe the absence of a specific formula, or the lack of imagination to apply break-make transformation (Freudenthal, 1983), or few experience in using measuring tools conduct to the belief mentioned in result number 3.

(Answers to: Do you think that you can obtain volume of the following things?)
115 V5: A chair. No, because of its shape, it is not just one shape it has several ones.
119 V5: A female screw: No, because its irregular shape and tiny dimensions.
125 V5: A spinning top: No, it's difficult because of its irregular shape.

4. In non-scholar situations teachers manifest the usage of different meanings related to the word *volume*; the dominant one is that of volume as a number.

In the research herein reported some different meanings related to the word *volume* were established, some of them arose during the construction of the theoretical framework. For example, volume as 'internal volume' or the number of cubic units that constitute a body, was taken from Piaget *et al* (1970); volume as 'capacity' came into view from capacity's meaning as the volume of matter that fills a container; volume as 'enclosed volume' or the free space enclosed in a closed surface emerged from the analysis of the data; teachers mentioned and distinguished it from the 'internal volume' aforementioned. Another meaning is 'volume as a number', the one that was expected to emerge more frequently; although statistics performed by the software did not point out this meaning as the more frequently found in the data, anyway, it appears in an implicit manner in many situations. In fact, the previous cited results (1 to 3) show an implicit 'number' meaning associated to the term *volume*; that's why teachers cannot see some objects as volume-measurable, because their shape, or size, gives place to an "incomplete" numerical information, and so volume (a number) cannot be obtained.

In one of the workshop's sessions, a list of objects was presented to the teachers; just two of the elements in the list were selected by all of them as volume–measurable objects: an orange, associated to a sphere, and a tank related to a cylinder shape. Even those who select thin or tiny objects as volume–measurable, when asked about the process they will use to obtain such a measurement answered that they will measure three lengths and then multiply them. For instance consider the paragraph that contains the responses of three teachers to the questions: "Can you obtain the volume of...?" and "How will you do it?"

190 V5: ...a sheet of paper? Yes, obtaining the area and multiplying by the sheet of paper's width.

192 V9: ...a pond? Yes it has a geometrical body shape, by formula.

193 V10: ...an auditorium? Yes, by multiplying length, width and height.

194 V10: ... a pond? Yes, by multiplying length, width and height.

5. Some teachers have constituted a mental object associated with the bodies' characteristic of 'having three dimensions'.

In teacher's answers to the question: "What means volume for you?" a definition – included in textbooks of the decade of the sixties and former ones –apparently emerges: "Volume is everything that can be measured in three dimensions: large, width and height" (Sánchez, 1960, p. 26).

(Answers to: What is volume?)

13 M4: ...it's the third dimension.

16 M11: ...it's the bodies' characteristic of having three dimensions.

At the present time, the importance of giving sense to concepts such as length, area and volume and not to center measure teaching in formulas is emphasized in national and official programs and textbooks used in Mexico. Anyway, teachers' conceptions and the influence of former teaching models may continue to obstruct these tendencies. Although teachers know, repeat and, even, agree with the new educational discourse, their conceptions and beliefs betray them. Sometimes teachers have difficulties solving volume related problems and misconceptions linked to this concept. That's why teacher's education and proficiency programs must consider measuring contents in order to improve these subjects teaching, particularly in the case of volume.

It is worth mentioning that the acquired results are connected with the case of the observed teachers, and albeit they are announced as general results it is not our pretension to consider them representative from a statistical point of view. We think that, even though applying questionnaires and to analyze the performance of teachers does not constitute determinant evidence to know how volume is being taught in classrooms, the results here exposed, as well as those of the general research project, may be considered as a first approach to what happens in school in the case of the volume concept.

References

Civil, M. (1996). Pensando sobre las matemáticas y su enseñanza: Una experiencia con estudiantes para profesores de primaria en Giménez, J., Llinares, S. y Sánchez, V. (Eds.). El *Proceso de llegar a ser un profesor de primaria, cuestiones desde la educación matemática,* págs. 175-198. Granada, España: Mathema.

Enochs L. G. y Gabel, L.D. (1984). Preservice Elementary Teachers' Conceptions of Volume. *School Science and Mathematics*. Vol 84(8).

Filloy, E. (1999). *Aspectos Teóricos del Álgebra educativa*. México: Grupo Editorial Iberoamérica.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Holanda: Reidel Pub. Co.

Janvier, C. (1997). Grandeur et mesure: la place des formules à partir de l'exemple du volume. Bulletin AMQ. Vol XXVII. No. 3. Quebec. Canadá.

Pehkonen, E. Y Furinghetti, F. (2001). An attempt to clarify definitions of the basic concepts: belief, conception, knowledge. En Speiser, R., Maher, C.A., y Walter, C.N. (Eds.). *Proceedings of the twenty third annual meeting of the North American Chapter of the Group for the Psychology of Mathematics Education*. Vol. 2: 647-656.

Piaget, J., Inhelder, B. & Szeminska, A. (1970). *The child's conception of geometry,* London, GB, Routledge and Kegan Paul.

Potari, D. & Spiliotopoulou, V. (1996), Children´s approaches to the concept of volume en *Science Education* 80(3): 341-360.

Puig, L. (1997). Análisis fenomenológico, en Rico, Luis (compilador): *La educación matemática en la enseñanza secundaria*. Barcelona, España: Horsori.

Rouche, N. (1992). *Le sens de la mesure*. Bruselas, Bélgica: DIDIER HATIER.

Sáiz, M & Figueras, O. (2000). Some primary teachers' conceptions about the mathematical concept of volume en Fernández, M.L. (Ed) *Proceedings of the XXIIth Annual Meeting of the PME-NA,* Arizona, EUA.

Sáiz, M. (2002) El pensamiento del maestro de primaria acerca del concepto volumen y de su enseñanza. Tesis doctoral. Cinvestav. México.

Sánchez, Virgen. (1960). *Mi libro de 4º año. Aritmética y Geometría*. México: SEP, Comisión Nacional de los Libros de Texto Gratuitos.

Thompson, A. G. (1992). Teacher's beliefs and conceptions: a synthesis of the research, en Grows, Douglas A. (ed.), *Handbook of research on mathematics teaching and learning*, NY, MacMillan Pub. Co. Pp. 127-146.

Törner, G., (2000). Structuring Mathematical Belief Structures: Some Theoretical Considerations on Beliefs, Some Research Questions and Some Phenomenological Observations en *Proceedings of the Twenty-Second Annual Meeting of the PME-NA*. Vol. 2: 499-508.

Vergnaud, G. (1983). Didactique du concept de volume. *Recherches en Didactique des Mathématiques*. Vol. 4. No. 1: 9-25.