# THE CONSTRUCT VALIDITY OF AN INVENTORY FOR THE MEASUREMENT OF YOUNG PUPILS´ METACOGNITIVE ABILITIES IN MATHEMATICS

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*In recent years metacognition has been receiving increased attention in mathematics education. Special attention has been focused on metacognition and its essential role in achievement settings. The basic difficulty of the study on the field of metacognition is to develop and use valid tasks in order to measure metacognitive ability especially for young pupils. The present study is a part of a larger research on the development of young pupils´ metacognitive ability in mathematics. It represents the initial phase of an instrument development and the examination of its construct validity. The confirmatory factor analysis confirms the existence of a second order structure representing metacognition and two basic first order factors indicating metacognitive knowledge and metacognitive regulation.*

## THE TWO DIMENSIONS OF METACOGNITION: METACOGNITIVE KNOWLEDGE AND SELF-REGULATION

Although there is growing evidence that metacognition is an important component of intelligence and cognition, there is confusion on the conceptual definition of the term. There are many different kinds of knowledge and processes subsumed under the term metacognition. According to Campione (1987) this term "has been used by different authors to mean different things" (p.119). In modern psychological literature, the term "metacognition" has been used to refer to knowledge about cognition and regulation of cognition.

Flavell (1976), who was the first to introduce the term "metamemory" into the literature, used the term "meta" in order to indicate the second order knowledge or function, in the sense the predicate is used in a variety of cases such as metatheory. Then the term metamemory was expanded in order to include the metacognitive knowledge and regulation of cognition (Flavell, 1979). On the other hand Demetriou (1993) rejected the term "metacognition" and uses the term "hypercognitive system", asserting that "meta" indicates an order process that follows all the cognitive processes and has a higher order role. The present study uses the term "metacognition" referring to the awareness and monitoring of one's own cognitive system and it's functioning. It accepts that although metacognition is a multidimensional construct, knowledge of cognition or metacognitive knowledge and self-regulation of cognition are its two basic dimensions.

Metacognitive knowledge is "knowledge or beliefs about what factors or variables act and interact in what ways to affect the course and outcome of cognitive enterprises" (Flavell, 1999, p.4). The major categories of these factors or variables are: person, task and strategy. The person category encompasses everything that a person believes about the nature of him/herself and other people as cognitive processors; it refers to the kind of acquired knowledge and beliefs that concern what human beings are like as cognizing organisms. The task category concerns the information about the object available to a person during a cognitive enterprise. Thinkers must recognize that different tasks entail different mental operations (Demetriou, 2000). The strategy category includes a great deal of knowledge that can be acquired concerning what type of action are likely to be effective in achieving what goals and in what sort of cognitive undertakings. Actually, metacognitive knowledge is knowledge that people have about their cognitive abilities (I have a bad memory), about cognitive strategies (to remember a phone number I should rehearse it) and about tasks (categorized items are easier to recall).

Whereas Flavell uses the person-task-strategy taxonomy to define metacognitive knowledge, Brown (1987) has categorized metacognitive knowledge based on a person's awareness of this knowledge: declarative, procedural and conditional knowledge. Declarative knowledge is propositional knowledge which refers to "knowing what", procedural knowledge refers to "knowing how" and conditional knowledge refers to "knowing why and when".

The second dimension of metacognition, metacognitive regulation refers to processes that coordinate cognition. It is the ability to use metacognitive knowledge strategically to achieve cognitive goals, especially in cases that someone needs to overcome cognitive obstacles. It has become clear that one of the most important issues in self regulated learning is the students´ ability to select, combine and coordinate strategies in an affective way (Boekaerts, 1999). It is essential to the development of students´ ability to learn cognitive strategies, such as self-questioning, widen the application of these strategies and gain conscious control over them. Self-regulation strategies play an important role on the learning process. Successful learners are able to swiftly transfer the knowledge and strategies acquired in one situation to new situations, modifying and extending these strategies on the way. Self-regulatory behavior in mathematics includes clarifying problem goals, understanding concepts, applying knowledge to each goal and monitoring progress toward a solution.

Metacognitive regulation and metacognitive knowledge are interdependent constructs. For example, awareness that one is not very good at a certain task would lead him/her to monitor his/her processes more carefully. On the other hand, if one monitors his actions and detects a lot of errors, he/she may conclude that the task is difficult.

For the solution of any complex problem-solving task a variety of metacognitive processes is necessary because problem solving is a complex interplay between cognition and metacognition. It is known that individuals with higher levels of metacognitive ability perform better in problem solving tasks (Artzt & Armour-Thomas, 1992). Some of the programs that were proposed in order to develop pupils' metacognition by teaching metacognitive skills focused either on a wide range of processes or on only one of them. Improving either metacognitive knowledge or metacognitive regulation improves learning. Especially according to Flavell (1979) metacognition improves by practicing it or by practicing other processes, which are not metacognitive, themselves but which indirectly promote metacognitive ability.

The development of metacognition occurs in the same manner as cognition with children learning more about the structure and the function of their cognitive system as they get older and they have a variety of metacognitive experiences. Actually the development of metacognition is the product and a producer of the cognitive development. Metacognitive theory has not focused on the development of metacognition mainly because researchers encounter serious methodological problems in their attempt to develop valid instruments measuring metacognition.

## THE DIFFICULTIES ON THE MEASUREMENT OF METACOGNITION

One of the basic problems of the study on the field of metacognition is to develop and use valid tasks measuring metacognitive ability. Brown (1987) believes that using the term metacognition to refer to two distinct areas of research makes the research procedure more difficult and creates confusion that clouds any interpretation of research findings. Although several methods of measuring metacognition have been implemented each of these methods has advantages and disadvantages. For example, one of the most popular approaches for assessing both metacognitive knowledge and control is to ask students to explain directly about what they know or what they do. For assessing metacognitive regulation, participants may be asked to think aloud about what they are doing and thinking as they solve a problem. Nevertheless verbal reports are subject to many constraints and limitations (Baker & Cerro, 2000). Asking children, particularly young children about their cognitive processing, poses some special problems. Answers may reflect not what the child respondents know or do not know, but rather what he/she can or cannot tell to the interviewer. On the other hand, metacognition is rather cognitive in nature than behavioural and consequently, self-report inventories are, in some ways, the least problematic technique to measure metacognitive ability (Sperling, Howard, Miller & Murphy, 2002).

Schraw and Sperling-Denisson (1994) developed a 52-item Likert-type self-report inventory for adults (MAI), which measured both knowledge of cognition and regulation of cognition. They set out to confirm the existence of eight factors, from which three related to knowledge of cognition and five related to regulation of cognition. The final factor structure was best represented by two main factors. Post-hoc content analysis confirmed that these factors were ended as knowledge of cognition and regulation of cognition. Sperling et al. (2002) used the idea of the MAI inventory and developed two inventories for the use with younger learners, the Jr MAI, version A and B. Their results indicated that this inventory revealed two distinct factors, accounting for 64% and 56% of the sample variance for versions A and B, respectively.

## THE RESEARCH

The present study is a part of a bigger research on the development of cognitive and metacognitive abilities in mathematics. The major purpose was to develop an inventory based on the idea of MAI (1994) and Jr MAI (2002) for the measurement of young pupils´ metacognitive ability in mathematics. As Baker and Cerro (2000) argue it is time to focus on more valid approaches. Firstly we wanted to examine statistically the construct validity of the inventory we have developed for the measurement of young pupils´ metacognitive ability in mathematics. The second aim of the study, in our attempt to avoid a big inventory, which is inappropriate for young pupils, was to choose statistically the least number of items that could constitute an inventory for the measurement of metacognition.

Sample: Participants included all 246 children in grades four through six (about 8 to 11 years old) of an elementary school (74 were 4th graders, 81 were 5th graders and 91 were 6th graders).

Procedure: The questionnaire was consisted of two basic parts. The first part measured metacognitive abilities in mathematics. Pupils were instructed to read 30 items and for each item circle the answer that best described their thoughts when solve a problem they might see in a math class (1=never, 2= seldom, 3=sometimes, 4=often, 5=always). Examples of items are the following: "When I encounter a difficulty that confuses me in my attempt to solve a problem I try again", "After I finish my work I know how well I performed on it". The second part was about their cognitive ability in problem solving. Pupils were instructed to read a problem and answer a set of questions before and after their attempt to solve it. In all classes the questionnaire was administered as part of normal class procedure.

## RESULTS

In this paper we present only the results of the analysis of the first part of the inventory. The 30 items were checked with respect to skewness of kurtosis and all items were found within normality criteria. The inventory demonstrated an overall high reliability (Cronbachs' alpha 0.8298). Firstly we conducted an exploratory factor analysis and we choose the 25 items that after a content analysis indicated to load at nine factors which were in relation to metacognitive knowledge and regulation (KMO=0,807, p=0,000). Then we conducted a confirmatory factor analysis to explore the hypothesized structure the inventory examined. Actually we used confirmatory factor analysis and structural equation modeling to test our hypothesis on the existence of the two first order factors and a second order factor. The a priori model hypothesized that: a) responses to the inventory could be explained by two first order factors (metacognitive knowledge and self-regulation) and a second order factor (metacognition), b) each item would have a nonzero loading on the factor it was designed to measure, c) measurements errors would be uncorrelated.

Analysis was conducted using the EQS program and maximum likehood estimation procedures. Multiple criteria were used in the assessment of the model fit. In a Confirmatory Factor Analysis study the parameters typically consist of factor loadings, factor variances, covariances and measured variables. In the present study 25 indicators were hypothesized to represent the model. After conducting the LMTEST (Bentler, 1993) we arrived at an elaborated model in which the goodness-of-fit index was good in relation to typical standards. Fifteen items were dropped depending on their low loadings on the hypothesized factors (<0,300) and three items were connected with both the factors. The different models that were examined in the attempt to decrease the number of items with their goodness of fit index were as follows:

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| *First model with two first order factors:*<br>F1: 10 items, F2 : 15 items<br>$X^2(274)=497{,}313$ p=0,000<br>$X^2/df = 1{,}815$ CFI= 0,764, GFI=0,857<br>AGFI = 0,830 RMSEA = 0,058 | *Second model with two first order factors*<br>F1: 11 items, F2 : 13 items<br>$X^2(181)=326{,}741$ p=0,000<br>$X^2/df = 1{,}805$ CFI= 0,828 GFI=0,884<br>AGFI = 0,852 RMSEA = 0,057 |
| Third model with two first order factors<br><br><br>F1: 8 items, F2 : 10 items<br>$X^2(79)=119{,}128$ p=0,002<br>$X^2/df = 1{,}507$ CFI= 0,925, GFI=0,940<br>AGFI = 0,909 RMSEA = 0,046 | Final model with two first order factors and a second order factor (3 items connected with both the factors)<br>F1: 8 items, F2 : 10 items<br>$X^2(77)=119{,}128$ p=0,0014<br>$X^2/df = 1{,}547$ CFI= 0,925, GFI=0,940<br>AGFI = 0,907 RMSEA = 0,047 |

F1: metacognitive knowledge, F2: metacognitive regulation

Table 1: Models examined and their goodness of fit

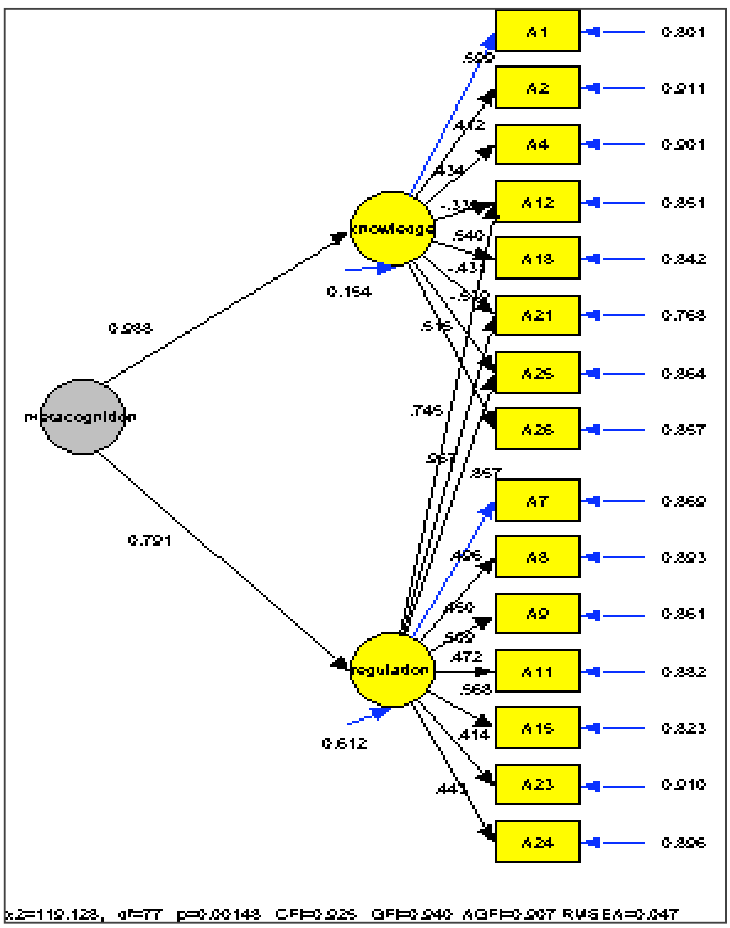


Figure 1: The final model for the measurement of metacognition

The final model proposed that five items were indicators of the metacognitive knowledge, seven items were indicators of the metacognitive regulation and three items split at both the dimensions of metacognition. The loadings of the two first order factors were loaded high on the second order factor (0,982 and 0,778). The parameter estimates of the model are shown above in Figure 1.

The fit of the final model was excellent and the values of the estimates were satisfactory in all cases. It is noted that to have a good fit to the data a model must have a fit index, such as the Comparative fit index (CFI) higher than 0,9. Also, ideally the p- value for the $X^2$ must be higher than 0,05 indicating that the model does not differ significantly from the data. When this is not possible because sample size is large, as at the present study, a $X^2$/df criterion of less than 1,96 is considered satisfactory.

The findings of the structural analysis presented above suggest that all items were used at the inventory split at two basic factors, which stood up as first order factors in the model that found to fit our data. The finding that three items loaded on both the factors is likely to due to the high correlation between knowledge and regulation of cognition. Very important is the high loadings of the two first order factors on the second order factor, confirming the existence of the two basic dimensions of metacognition through the items of the specific inventory.

## DISCUSSION

This study presents the initial phase of an examination of the validity of an instrument development for the measurement of metacognition in mathematics education appropriate for pupils in primary education. Undoubtedly it is not easy for young pupils to express their thoughts about their cognitive system and their cognitive abilities. According to Schraw (1998) promoting metacognition begins with building awareness among learners that metacognition exists, differs from cognition and increases academic success. We believe that it is possible in an attempt to measure metacognitive abilities to interfere to the metacognitive processes while pupils are obliged to think about their cognitive system. Those thoughts are metacognitive by their own nature.

The purpose of the present study was to test for the validity of a higher order factorial structure of the inventory we have developed based on the analysis of data within the framework of a confirmatory factor analytic model findings yielded support from the hypothesized second order structure. A first order factor contained items for the knowledge of cognition and a different first order factor contained items for the regulation of cognition. The existence of the three common items for both the factors indicated the high correlations between the two factors because of the high correlation between the two basic dimensions of metacognition: knowledge of cognition and regulation of cognition. The present study is in line with the results of previous studies indicating factors of metacognition and their high relationships. Especially findings are consistent with the work with the MAI (Schraw & Sperling-Dennison, 1994) and Jr MAI (Sperling et al, 2002). We believe that the final set of the15 items consist an appropriate valid inventory for the measurement of young pupils´ metacognitive abilities in mathematics (see Appendix).

Results of this research are expected to be of substantial interest to researchers whose concerns focus on the measurement of metacognition. Being able to measure metacognition provides the educational system with plenty of tools to help pupils develop their metacognitive abilities.

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APPENDIX

The final inventory that is proposed

1. I know how well I have understood a subject I have studied (A1).
2. My performance depends on my will and my effort (A2).
3. I can learn more about a subject on which I have previous knowledge (A4).
4. I define specific goals before my attempt to learn something (A7).
5. I examine my own performance while I am studying a new subject (A8).
6. When I finish my work I wonder whether I have learned new important things (A9).
7. After I finish my work I repeat the most important points in order to be sure that I have learned them (A11).
8. I use different ways to learn something according to the subject (A12).
9. I know ways to remember knowledge I have learned in Mathematics (A15).
10. I understand a problem better if I write down its data (A18).
11. When I try to solve a problem I pose questions to myself in order to concentrate my attention on it (A21).
12. When I encounter a difficulty that confuse me in my attempt to solve a problem I try to solve it again (A23).
13. While I am solving a problem I wonder whether I answer its major question (A24).
14. Before I present the final solution of a problem, I try to find some other solutions as well (A25).
15. After I finish my work I know how well I performed on it (A26).