TEACHER INVESTIGATIONS OF STUDENTS' WORK: MEETING THE CHALLENGE OF ATTENDING TO STUDENTS' THINKING

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The purpose of this research report is to describe some of the findings from a study of teacher investigations of students' work. The intent of the teacher investigations was for teachers to interpret their students' thinking as revealed on non-routine, thought-revealing mathematical tasks, known as Case Studies for Kids (Lesh, Hoover, Hole, Kelly, & Post, 2000)[1]. This research report focuses on instances during the investigations when the teachers engaged in what the author has termed 'mini-inquiries', discussions during which the teachers addressed why their students thought about the associated case studies as they did or the teachers addressed the underlying mathematical complexities associated with the case studies. During these mini-inquiries, the teachers met some of the challenges of attending to students' thinking that Confrey (1993) and Schifter (2001) have identified.

RATIONALE FOR THE STUDY

The latest reform efforts in mathematics education in the United States stress the importance of teachers attending to and understanding their students' mathematical thinking. The National Council of Teachers of Mathematics (NCTM, 2000) states, "Effective teaching involves observing students, listening carefully to their ideas and explanations, having mathematical goals, and using the information to make instructional decisions" (p. 19). This view of effective teaching has specific grounding in several research projects, including Cognitively Guided Instruction (Fennema, et al., 1996; Franke & Kazemi, 2001), the Purdue Problem-Centered Mathematics Project (Cobb, et al., 1991), SummerMath (Simon & Schifter, 1991), the Kenilworth Project (Maher, Davis, & Alston, 1992; Maher & Martino, 1992), the Mathematics Case Methods Project (Barnett, 1998), and the work of Putnam and Reineke (1993). Collectively, these research projects have found that when teachers attend to their students' mathematical thinking, potential benefits include:

- The ability on the part of teachers to construct or select appropriate, worthwhile mathematical tasks;
- A shift from teacher-centered didactical instruction to student-centered problem-solving instruction;
- Higher levels of conceptual understandings by students without compromises in their computational performances; and
- More positive beliefs of teachers and students toward mathematics.

Despite these benefits that may occur when teachers attend to their students' thinking, Ball (1997a; 2001), Confrey (1993), and Schifter (2001) point out that focusing on

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[1] Case Studies for Kids are also known as Model-Eliciting Activities.

students' thinking can prove challenging for several reasons. First, students do not always express their thinking in ways that are logical to adults. Students often present unconventional and multiple representations for thinking about a given mathematical problem. Second, although students may appear not to understand a particular mathematical concept, there may be sense in their thinking. Teachers have to de-center from their own perspective and imagine what the view of the students might be like. Third, identifying the conceptual issue that a student is currently trying to understand can be difficult when the students' thinking is illogical. Teachers need to identify the concept in order to help move the student forward in his or her understanding. Finally, teachers often tend to focus on the pedagogical aspects of a learning situation rather than focusing on the mathematics expressed by students. Paying attention to a myriad of pedagogical aspects often causes teachers to lose sight of the mathematical ideas that their students are expressing.

To address these difficulties, Ball (1997a) describes three approaches with "promise for equipping teachers with the intellectual resources likely to be helpful in navigating the uncertainties of interpreting student thinking" (p. 808). One of these approaches is investigating artifacts of teaching and learning, such as students' written work. Several other sources support Ball's suggestion for teachers to investigate students' work (Allen, 1998; Blythe, Allen, & Powell, 1999; Driscoll & Moyer, 2001; NCTM, 2001), and several teacher development projects (Katims & Tolbert, 1998; Kelemanik, Janssen, Miller & Ransick, 1997; Saxe, Gearhart, & Nasir, 2001; Schorr & Lesh, in press) have found that when teachers engage in investigations of students' work, they have the potential to gain several benefits including:

- An expanding conception of what students are able to do mathematically;
- The realization that although students' methods may appear different from a teacher's approach, students' methods may still be valid; and
- The development of abilities to interpret students' thinking in class and to make appropriate future instructional decisions.

Despite these results, what is still missing is an in-depth investigation of how the examination of students' written work influences teachers' interpretations of students' thinking. Specifically, how the individual activity of examining student work coupled with the collective interpretation of this work influences teachers' development. Therefore, this study examined a particular instantiation of teachers investigating students' work. In particular, the purpose of the study was to closely examine (a) the teachers' collective interpretations of their students' thinking and (b) the social processes (patterns of interaction and norms for interaction) that occurred during the investigations.

THEORETICAL FRAMEWORK (ORIENTATION)

The theoretical perspective for the study was micro-sociology, which focuses on the face-to-face interaction of individuals and how these individuals act in relation with one another in everyday life (Blau, 1987; Charon, 1999; Gerstein, 1987). Micro-sociologists believe the social structure for these social interactions is composed of normative interaction and discourse patterns (Berger, 1963; Cicourel, 1974; Goffman, 1967; Gumperz, 1983). The research tradition for the study was ethnography of communication

(Hymes, 1986; Saville-Troike, 1989). Ethnographers of communication strive to describe the many different ways of communicating which exist within a community.

METHODOLOGY AND ANALYSIS

Seven middle grade teachers (of students aged 12-14 years) participated in the study by engaging in five investigations of their students' work, which occurred during the 2001-2002 school year. The purpose of each investigation was to interpret students' mathematical thinking as revealed in their students' work on a Case Study for Kids.

Mathematics Tasks

Case Studies for Kids are explicitly designed to help middle school students develop conceptual foundations for deeper and higher order ideas in pre-college mathematics (Lesh, et al., 2000). The tasks are non-routine because each task asks students to mathematically interpret a complex real-world situation and requires them to formulate a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because groups of students are producing a description, procedure, or method, students' solutions to the task reveal explicitly how they are thinking about the given situation (Lesh, Cramer, Doerr, Post, & Zawojewski, in press).

Procedure

For each of the five teacher investigations, the teachers attended two teacher workshops. At the Introductory Workshop, the teachers completed the Case Study for Kids and discussed the mathematics inherent in the activity, expected students' responses, and implementation issues. Then, the teachers implemented the case study within their own classrooms. After implementation, the teachers attended the Follow-Up Workshop, where they discussed their interpretations of their students' mathematical thinking and ultimately developed a Consensus Students' Thinking Sheet. The sheet synthesized the students' ways of thinking into three or four primary solution strategies, included examples of students' work, described the mathematics that the teachers believed the students used while invoking the solution strategies, and outlined the teachers' perception of the efficiency and the effectiveness of each of the solution strategies. Requiring the teachers to create Consensus Students' Thinking Sheets provided the opportunity to study the teachers' *collective* interpretations of their students' thinking and the *social processes* that occurred.

Analysis

To capture the teachers' collective interpretations and social processes, the data sources consisted of transcripts from the videotapes recorded during the teacher workshops and the teachers' synthesis of their students' solution strategies recorded in the Consensus Students' Thinking Sheets. For the analysis of the data, a 'grounded theory' approach was used, as described by Strauss and Corbin (1998). Specifically, the procedures used were open coding, the process of naming concepts in the data, defining categories, and developing categories in terms of their properties and dimensions, and axial coding, the process of relating categories by identifying which categories are subcategories of other categories. Initial analyses have been conducted on the data from the first, third, and fifth teacher investigations. Analyses on the data from the second and fourth teacher investigations are in progress.

PRELIMINARY RESULTS: MEETING THE CHALLENGE OF INTERPRETING STUDENTS' THINKING

Throughout the first, third, and fifth teacher investigations, the teachers engaged in 17 patterns of interaction considered to be *mini-inquiries*. Specifically, the teachers engaged in inquiry discussions during which they addressed rationales for why their students thought about or interpreted the case studies as they did or they addressed the underlying mathematical complexities of the case studies. For 16 of these 17 mini-inquiries, the teachers met some of the challenges of attending to students' thinking, as identified by Confrey (1993) and Schifter (2001). Specifically, for seven of the teachers' mini-inquiries, the teachers de-centered from their own perspective of the case study and considered how their students viewed the case study. For three of these seven instances, the teachers not only de-centered from their own perspective and considered the view of the students; they also closely looked for sense in their students' thinking when the students' thinking did not appear entirely logical. Finally, for nine of the teachers' other mini-inquiries, the teachers identified the mathematical conceptual issue with which the students were struggling or were using to approach the case study.

The following excerpt illustrates the pattern of interaction and discourse that occurred during one of the teachers' mini-inquiries. This occurred during the first investigation when the teachers were examining their students' work from the Summer Jobs Case Study for Kids. For this task, students are to develop a procedure that will enable a concessions vendor to rehire the six most productive employees from last year's nine employees. Students are provided with data for each employee about the hours worked and the money made during the months of June, July, and August for the busy times at the park, the steady times, and the slow times. During this interaction, the teachers are observed to de-center from their own perspective to consider why their students chose to average some of the data provided with the case study[2].

69 Author: Okay, any other? [The teachers are discussing the mathematics associated with one of the students' solution strategies. This question is asking the teachers if they feel there are any other mathematical skills or concepts associated with this particular solution strategy.]

70 Lauren: I think when the kids, I hate to bring this up, but I think when the kids found the averages, it seemed more realistic to them because the numbers were more, the numbers were smaller and they seemed just realistic, but while that's not a gigantic amount of money, it's just a monthly, you know amount of money, it seemed

71 Author: So, you're . . .

72 Jim: Well, I think it shortened the categories

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[2] Pseudonyms are used for the three of the seven teachers that participated in this particular interaction: Lauren, Jim, and Tom. 'Author' refers to the author of this proposal, who served as the facilitator of the teacher investigations and as the researcher.

73 Lauren: Right

74 Jim: All of a sudden you've got all these different groups of hours, different groups of money, but when you have an average, all of a sudden you've got one number.

75 Lauren: Right.

76 Jim: I think it made the data more manageable.

77 Author: Well, Lauren is touching upon something cause like on my own spreadsheet when I played with it, I just added up all the money for say Maria and all the hours for Maria and then I divided the two, so what I had was dollars per hour kind of in the sense of across the whole summer; whereas if the kids found the average per month, they'd be finding the dollars per hour per month and maybe it's easier for them to think about having dollars per hour within a month?

78 Tom: A shorter amount of time (he's shaking his head to agree).

79 Author: You know, instead of the overall amount? Maybe that's what makes it [easier]. Okay, did we get most of the math?

As illustrated by this excerpt, for most of these mini-inquiries, the teachers engaged in a particular pattern of interaction. The pattern began whenever a teacher offered an insightful comment about (a) the mathematical skills needed for a particular solution strategy, (b) an error some of the students made while using a particular solution strategy, (c) why the students thought about the case study as they did or used a particular computation (as in line 70 above), or (d) how the students interpreted information provided in the problem statement. Most of these comments were prompted by one of two things. One type of prompt (for 5 out of the 16 mini-inquiries) was when I asked the teachers if they had anything further to add to our discussions, such as whether they felt there were any remaining solution strategies or mathematics associated with the solution strategies (as in line 69 above). The other prompt (for 10 out of the 16 mini-inquiries) was the topic under discussion. Frequently, the topic under discussion reminded the teachers of something that they had observed and thereby led them to share their observation with the group.

Once the pattern began, the initial sharing of the insightful comment was typically followed by additional comments or sharing from the other teachers. Sometimes the additional comments simply provided support for the original comment, and the teachers moved on to discussing another topic. However, more commonly, the teachers made several additional comments and therefore contributed to the original comment (as in lines 72, 74, 76, and 78 above). Thus, the pattern of interaction typically consisted of successive comments by the teachers in which they built on the original comment about how the students thought about the associated case study or about the underlying complexities of the case study, frequently providing more insight into the students' thinking. The pattern usually ended when either I encouraged the teachers to return to the task of creating the Consensus Students' Thinking Sheet (as in line 79 above), when a teacher made a comment about a new idea, or when a teacher offered a comment that resolved an issue under discussion about what the students did to solve the case study or how they thought about the case study.

During this pattern of interaction, a powerful norm for interaction appeared to guide the teachers' behavior. Specifically, an expectation seemed to exist that the teachers should consider rationales for their students' thinking. In other words, the teachers appeared guided by a norm that moved them beyond explaining how the students solved the associated case study to considering possible rationales for why the students solved the case studies as they did. Additional information and detail will be provided during the research session about the teachers' mini-inquiries and findings from the analyses in progress of the second and fourth investigations.

CONCLUSIONS AND IMPLICATIONS FOR EDUCATION

The mini-inquiries allowed the teachers to address why their students thought about the associated case studies as they did and therefore to address underlying mathematical complexities associated with the case studies. Thus, the mini-inquiries engaged the teachers in a deeper level of analysis of their students' thinking than simply reporting their students' solution strategies. In addition, while engaging in the mini-inquiries, the teachers met three of the challenges of interpreting students' thinking. First, they were able to de-center from their own perspective and to consider their students' view of the case studies. Second, they were able to seek sense in their students' thinking, even when the students' thinking was not entirely logical. Third, they were able to identify the conceptual issues with which the students were struggling or were using to solve the associated case study. The findings from this study provide initial support that engaging teachers in investigations of students' work holds promise for assisting teachers with the challenges of attending to students' thinking.

However, the teachers only engaged in these mini-inquiries intermittently throughout the teacher investigations and as the facilitator of these investigations, I did not recognize these mini-inquiries during 'real-time', thereby missing the opportunity to take advantage of these occasions. Thus, some questions remain about how to increase these occasions and their power for assisting teachers with interpreting students' thinking. For example, can teacher educators facilitate teacher investigations in such a way that teachers will be more likely to engage in mini-inquiries? If so, how? What pedagogical content knowledge is necessary for teacher educators to recognize powerful interactions such as these and to use them as starting points to further teachers' insightful examinations of students' work?

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