AN INTERPRETING GAME IN A THIRD GRADE CLASSROOM

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The paper analyzes the dialogical interaction between a group of third graders and their teacher in one classroom episode. The semiotic and cognitive activity of the dialogical interaction is considered to be an "interpreting game". The episode exemplifies how in dialogical interaction the back-and-forth loops of interpretation, intention, and linguistic expression contributed to the emergence of meanings carry out by the equal sign. Dialogical interaction is considered to be an interpreting games in the sense of being a playful semiotic and cognitive activity between teacher and students constituted by "interpreting cycles" as their most elementary units of analysis. The interpreting game analyzed here is constituted by four interpreting cycles.

INTRODUCTION

The purpose of the paper is to analyze the dialogical interaction between teacher and students as the students interpret a number sentence in which a number is missing. This dialogical interaction is analyzed here as an "interpreting game". It is labeled a "game" because of its elements of playfulness, creativity, surprise, and unpredictability. The dialogical interaction is considered to be an "interpreting game" because the interpretation of the participants stimulates and promotes the dialogue. In interpreting games, the intentions of the participants (to either communicate, guide, explain, justify, or convince, etc.) are anchored in former interpretations. The interpretations, intentions, and linguistic expressions of the third classroom participants mingled with one another in an interrelation characterized by mutual influence and transformation from which the construction and refinements of meanings of the equal sign emerged.

THEORETICAL RATIONALE

Like any other discourse, discourse in mathematics classrooms is essentially a semiotic activity that shapes and is shaped by the cognitive activity of the participants— the teacher and the students. According to Wood, "Communicating in or about the field of mathematics involves taking part in mathematical discourse, whether by reading, writing, listening or speaking. Discourse is a broader concept than language because it also involves all the activities and practices that are used to make meaning in a particular profession" (2000, p. 1).

Discourse in mathematics classrooms inherits, in a certain way, those characteristics of written discourse in textbooks. This discourse is first and foremost constituted by two symbolic systems that are mainly conveyed through verbal and/or written means: natural language and mathematical notation. Most of the time, students interpret mathematical meanings from written mathematical records. Initially, mathematical notations appear to the students only as physical marks or as empty things on a writing surface. However, through processes of interpretation, those written marks cease to be perceived as empty things when they begin to acquire limited meanings. They slowly attain the status of symbols with more generalized meanings and uses. Those emergent symbols carry only those mathematical meanings abstracted by the students from different meaning-giving contexts in which the marks are used. Unfortunately, complete meanings cannot be transmitted directly from the teacher to the student because, as Hersh (1979) points out, physical marks do not embody completely all the mathematical meanings they are intended to convey. Therefore, in the classroom, the construction of mathematical meanings from written notation is an interpreting process. Students engage in this process with the guidance of the teacher who presents appropriate meaning-giving contexts to illustrate mathematical meanings denoted by the written marks.

The interpretation of mathematical notation is a personal process. Bauersfeld (1995) points to the fact that students in the classroom are usually left alone in their own constructive acts of interpreting, reflecting, integrating, and understanding and some students are not able to go beyond the physical characteristics of mathematical notation.

The semiotic and cognitive experience of teacher and students and their classroom discourse is carried out by their interpretation of all manner of signs—social, cultural, linguistic, and mathematical in nature. Or as van Oers (2001) puts Bakhtin's ideas, "people's utterances in a communication process are not only regulated by the processes that occur in direct interaction, but also by the historically developed style of communicating in that particular community of practice" (p. 68). Mathematical symbols are special kind of signs that in addition to their physical shapes, they have names and variations of meanings according to the context in which they are used. Adding to this complexity is the variety of descriptions in natural language of the mathematical meanings that symbols are intended to convey (Sáenz-Ludlow, 2002). These peculiarities of mathematics discourse make the interpretation and abstraction of mathematical meanings a sophisticated semiotic, cognitive, and social activity.

Interpreting games are in essence sign activity, namely the sign activity of the initiator (teacher/student) and the interpreter (student/teacher). Interpretation, intention, and expression are the basic elements of interpreting games and therefore the elements of "interpreting cycles", which are considered to be the most elementary units of interpreting games. Let us consider a cycle in which the teacher is the first acting participant. What the teacher intends to convey to the student is intimately related to the teacher's interpretation of a mathematical concept. In turn, the student's interpretation, of the deliberate action of the teacher, generates an intended response (to communicate, explain, justify, convince, etc.) on the part of the student(s). The cycle continues when the teacher interprets the intended response of the student(s) and generates another intended response that is again interpreted by the student(s) who in turn generates another intended response. And so on, the cycle continues until some kind of interim closure is arrived at. Interpreting cycles remain open to possible refinements of meaning. In each interpreting cycle, every intention is anchored in some prior interpretation within a particular mathematical context. That is, in interpreting games a sequence of interpreting cycles lead to an eventual convergence of the meanings; meanings that are co-constructed by the students and the teacher (Sáenz-Ludlow, 2002). Through interpreting games, the written marks of mathematical notation undergo a sequence of interpretations from which mathematical meanings emerge and become more structured and decontextualized. As Peirce (1931-1948) puts it, "every symbol is a living thing in every strict sense…the body of the symbol changes slowly, but its meaning inevitably grows, incorporates new elements and throws off old ones" (2.222).

METHODOLOGY

A group of third graders (6 girls and 8 boys) from an at-risk school participated in a yearlong teaching experiment. The teaching-experiment methodology predominantly focuses on students' conceptual constructions over long periods of time. The teacher concentrates on the students' mathematical constructions as indicated by what they say and do as they engage in mathematical activity (Cobb and Steffe, 1983; Steffe, 1983; Steffe and Thompson, 2000). On the one hand, the interaction between teacher and students dialectically determine and orient students' mathematical thinking. On the other hand, the teacher should constantly make hypotheses of students' cognitive paths, and also interpret and assess students' mathematical actions (Steffe and Thompson, 2000). The teacher's process of making hypotheses, interpreting, and assessing students' mathematical actions provide ample opportunities to vary questions and arithmetic tasks according to the students' ways of conceptualizing and interacting to foster and sustain the evolution of their understanding. The teaching-experiment in which the third grade participated focused on students' sign-interpreting, sign-making, and sign-using processes as they re-conceptualized number, place-value, and operations with numbers in collaboration with the teacher and the other students.

To analyze the evolving classroom arithmetic activity, the changes in students' cognition, and their sign-making, sign-using, and sign-interpreting processes, the lessons were videotaped daily and field notes were kept; also, the task pages and students' scrap papers were collected and chronologically filed.

Instructional tasks were specially generated for the teaching experiment and they underwent changes according to the cognitive and arithmetic needs of the students. New arithmetic tasks were generated as a result of students' interactions in teaching episodes. In general, the generation of instructional tasks and the research activity co-evolved in a synergistic manner.

ANALYSIS

The task analyzed in this paper was posed to the students when the teacher experiment was in its fifth month. At this time, students had achieved a certain degree of flexibility to add numbers mentally.

The teacher wrote the question on the board, gave the students time to think, and then started a whole class discussion that is analyzed here as an interpreting game. Teacher and students engaged in an interpreting game when they interpreted each other intended verbal expressions. In this game, the teacher pursued students' interpretations and their lines of reasoning to help them construct a new meaning for the equal sign emerging from their own cognitive activity. In this process, one of the students started to have an insight into the commutative property of addition.

In the following dialogue T stands for teacher and the other abbreviations for the names of the students. Such abbreviations are italicized in the body of the paper.

Which number will make the number sentence true? 246 + 14 = _____ + 246

1 T: Da, please read the question on the board.
2 Da: Which number will make the number sentence true?
3 T: All right Da. Now read the number sentence for me.
4 Da: Two-hundred forty-six plus fourteen equals…
5 T: …something…
6 Da: plus two-hundred forty-six.
7 T: Kr, what does that equal sign mean?
8 Kr: Equals…it equals something?
9 T: Sh, what does "equals something" mean?
10 Sh: It's…it's when you add something. The equal sign is there so you can put the answer by the equal sign.
11 T: So, are you telling me that on the other side of the equal sign you have to have the answer?
12 Sh: Well yeah, because the equal sign is like when you add something up and the equal is there so you can put the answer down.
13 T: Okay. Does anyone else have an explanation?
14 Ka: The equal sign is the sum. It's like if you add two-hundred forty-six plus fourteen the sum is two-hundred sixty.
15 T: Mmm hmm…So, is that what the equal symbol means here?

Interpreting Cycle #1. The teacher interprets, through the interactions with the students, how they interpret the equal sign in the given equality. Lines 4-5-6 indicate the teacher's interpretation of Da's difficulty in reading an empty space in the context of arithmetic. Instead, lines 7-14 show how the teacher guides the dialogue using the interpretation of each student to pose questions. The teacher's intentional questioning leads her to understand that the students are far away from interpreting that the equal symbol in the given equality stands for the fact that the order of the addends is not significant in the result of the addition. The teacher closes the cycle (line 15) by reassuring herself that the students interpret the equal sign as a command to find the answer and the blank space as the place to "put the answer down".

Two questions come to mind. How will the teacher contribute to the evolution of the students' interpretations? Will the students progressively influence each other and finally arrive to a consensus on the meaning of the equal sign in this context? The following cycle in the interpreting game indicates how the teacher skillfully interpreted the students' interpretations and intentionally used them to pose questions and to involve other students in the discussion.

16 T: Da, you want to say something, what is it?
17 Da: Umm, I think that the equal sign is asking you something like what is six plus six.
18 T: What if I say six plus six equals six plus six? Is this a true sentence?
19 Sh: No.
20 T: So, six plus six does not equal six plus six!
21 Da: Actually it does. It's kind of the same.
22 T: Kind of the same!

23 Ka: It does. I can prove it because that's how much it equals up to. Six plus six equals twelve and you could say that six plus six equals six plus six because they both equal the same amount.

24 T: Teacher writes on the board 6 + 6 = 6 + 6.

25 Ka: And that equals the same thing.

26 Sh: I disagree.

27 T: Tell me why Sh.

28 Sh: Because equal doesn't mean you put six plus six again. You're supposed to add the numbers up and put the answer down. That's what equal means.

29 T: Okay, so are you saying that equals means you have to have an answer on the other side?
So, six plus six does not equal six plus six.

30 Sh: Yeah.

31 Ka: Yes, it does because both sides equal the same amount.

32 T: Can I write 6 + 6 = 6 + 6?

33 Ka: Yes.

34 Mi: Yes

35 Sh: No. Six plus six equals twelve.

36 T: Sh, six plus six is twelve (covering the left side of the equality). What is this six plus six (covering the right side of the equality)?

37 Sh: Twelve.

38 T: (Teacher writes on the board) 6 + 6 = 6 + 6 12 12
Are you telling me that twelve does not equal twelve?

39 Sh: Yes … no. I don't get it. That's equal…But how do you do the six plus six? Six plus six equals six plus six? You can't do that cause six plus six equals twelve. If you write six plus six you're just repeating over six plus six again.

Interpreting Cycle #2. The teacher modifies the example introduced by Da and puts it into the context of the initial question (line 18). Sh's negative answer and Da's answer in uncertain terms is argued by Ka with her own numerical "proof" (lines 23, 25, 31). Ka is able to see the validity of the equality 6+6 = 6+6 keeping in mind the result of the addition and without the need to see the result written down. Regardless of Ka's numerical argument (line 23), Sh continues struggling with her own dilemma (line 28). For Sh repeating the numbers is not the same as performing the addition and writing down the answer. The teacher interprets Sh's dilemma and uses the argument given by Ka to try to convince Sh of the truth of the equality 6+6 =6+6 (line 32). Once the teacher interpreted the cognitive need of Sh, she involves her as a participant in the argument that could have been presented by the teacher in a unilateral manner (lines 35-38). As a result Sh comes to doubt her own interpretations although her own dilemma is not resolved yet (line 39).

One question comes to mind. Will Ka or another student come up with a reasoning that will help Sh and other students to determine the missing number and the truth of the number sentence? The following cycle indicates that Ka is able to use an argument by contradiction to try to convince her classmates that the number on the blank space should be 14.

40 T: What is the number that will make true the equality 246+14 = ____+ 246?

41 Ka: (Ka goes to the board) You could put the answer right here (Ka writes 260 on

the blank space of the original equality) 246 + 14 = 260 + 246. Now, it would not be the same on both sides of the equal symbol because two-hundred sixty plus two-hundred forty-six is not the same as two-hundred forty-six plus fourteen. But if instead of 260 you write 14 then that would be the same thing.

42 T: So, is two-hundred forty-six plus fourteen equals two-hundred sixty plus two-hundred forty-six a true number sentence?

43 Ss: No. That's not a true statement.

44 T: Well, how can we make this (the equality written by Ka on the board) a true statement?

(T erases the 260 that Ka wrote on the blank space)

45 Sh: By putting two-hundred forty-six again or fourteen either one.

46 T: Why?

47 Sh: Six plus six equals six plus six. Two-hundred forty-six plus fourteen equals two-hundred forty-six plus two-hundred forty-six.

48 Ka: But if you put 246 in the blank space, then 246 + 14 = 246 + 246. If you put these two together (she refers to the numbers on the right side of the equality) then it's going to be four-hundred ninety-two.

49 Sh: You don't add them!

50 T: Yes you do; it says plus. The left side is two-hundred sixty; we know that. Ka says that the right side is four-hundred ninety-two. Is this a true number sentence?
246 + 14 = 246 + 246
492

51 Ke: Can I show you something?

52 T: Uhh huh.

53 Ke: (Ke goes to the board and erases the 246 in the blank space) All you're doing is to equal up to two-hundred sixty to make this one (Ke is referring to the ight side of the equation) equal up to two-hundred sixty. Then, all you're doing is just putting 14 backwards (Ke writes 14 in the blank space) 246 + 14 = 14 + 246

54 T: So, now you're trying to tell us that two-hundred forty-six plus fourteen is the same as fourteen plus two-hundred forty-six?

55 Ke: Yes.

Interpreting Cycle #3. The teacher, aware of the students' interpretations, intentionally turns the attention of the class to the initial question. Ka comes up with a numerical argument by contradiction (line 41). This argument was a tremendous insight considering that children are never presented with this type of logical argument. We also have to consider the progress in Ka's thinking as we compare this intervention with her initial interpretation (line 14). In line 42, the teacher again uses Ka's new argument to pose questions to the students in order to sustain the dialogue. Through questioning, the teacher comes to realize that Sh has over-generalized the number sentence 6+6 = 6+6 (lines 45 and 47). However, it is Ka that deals with Sh's over-generalization. Ka, with her natural ability to tinker with arguments by contradiction, takes on Sh's interpretation of repeating one of the numbers and recreates again her argument by contradiction (line 48). This time she is a bit more explicit. In line 50, the teacher uses Ka's argument with the intention to make the contradiction even more explicit (line 50). Sh made no immediate intervention. In line 53, Ke, a student who has no intervened in the discussion up to now, comes with the idea of using 14 in the blank space to "equal up" the result of the addition of the left side of the equality; this, for him, is the same as writing the same addends on the right side of the equality "backwards". Up to this moment, Ke is the only student that comes to see simultaneously the change in order of the addends, the actual addition, and the quantitative balance between the two sides of the equality. In line 54, the teacher closes the cycle by verbally summarizing Ke's argument.

In this cycle we not only see the progressive interpretations of Ka to counter Sh's cognitive dilemma but also the interpretation of Ke who had remained quiet during the discussion up to this moment. It is also apparent that the sophistication of Ka's logical argument by contradiction and Ke's comprehensive interpretation of the equal sign would have not been possible without the interpretations (valid or invalid) of the other students. In other words, progress was the product of dialogical interaction.

Two questions still linger in our minds. Will Sh modify her interpretation of the equal sign as a command to perform an operation to represent the preservation of a quantitative balance while changing the order of the addends? The other question is whether or not there were other students actively making their own interpretations although they have not intervened in the dialogue up to this point. The answers to these questions are in the positive as indicated in the following dialogue.

56 T: (The teacher sees Me raising her hand) Let's see what Me has to say.

57 Me: I think this. Two-hundred forty-six plus fourteen equals fourteen plus two-hundred forty-six. So, I say the same as Ke. It is fourteen.

58 T: Why do you think it is fourteen? You are the third person who says that. Three people said fourteen and two people said two-hundred sixty.

59 Me: Well other people think that it's two-hundred sixty. Umm…I don't mean to disagree but I disagree.

60 T: Why? Why do you disagree?

61 Me: Well, what I think you guys are thinking is that when you guys put these two together

(Me is referring to the numbers on the left side of the equal symbol) it's two-hundred sixty; so you guys think you put two-hundred sixty right here (referring to the blank space on the right side of the equal symbol) and then two-hundred sixty plus two-hundred forty-six will be two-hundred sixty. That's what I think some of you guys are thinking. But I think that fourteen should be in the blank space.

62 Sh: May I say something?

63 T: Huh uhh.

64 Sh: (Sh goes to the board) It's like this. Two-hundred forty-six plus fourteen is two-hundred sixty. If we put two-hundred sixty right here (Sh is referring to the blank space) then we have to plus two-hundred-sixty and two-hundred forty-six and that would be five-hundred six. Like this

246 + 14 = 260 + 246
260 506

65 T: So do you think this is a true statement? Will you put two-hundred sixty in the blank space?

66 Sh: I don't agree with that. It's kind of like (Sh erases 260 and replaces it with 14). It's kind of like the equal sign is down here and you put it right here. It's kind of like you're just separating this

246 + 14 = 14 + 246 = 260
260 260

67 T: All right. She has an interesting idea because she wants to see what she considers to be the answer.

68 Sh: Yeah, I like to see the answer.

Interpreting Cycle #4. One could have assumed that the interpreting game had ended in line 55 given that the answer to the question posed by the teacher was attained. However, the teacher continued another cycle because some other students were still willing to participate. Me not only agrees with Ke's conclusion but she also makes her own interpretation of the interpretations of other students (lines 57 and 61). Sh also wanted to participate again and what a rewarding surprise it was for the teacher. Sh modified her initial interpretation and now she starts assuming that the number in the blank space is 260 and recreates Ka's argument by contradiction. Finally, Sh concludes that the number in the blank space should be 14 (lines 64 and 66). Sh even goes a step further and creates a chain of equalities. Such a chain indicates that Sh has come around to solve her cognitive dilemma. Sh adds the numbers to be consistent with her initial interpretation of the equal sign as a command to "find and write the answer down" and she also uses the equal sign to symbolize a quantitative balance.

It is important to note here that a first analysis of this teaching episode made the research team aware of the need to generate sequences of instructional tasks to consolidate students' extended meaning of the equal sign to symbolize quantitative balance and the commutative properties of addition and multiplication as special cases. Space restrictions do not permit the analysis of such sequences and they will be analyzed elsewhere.

CONCLUSION

The analysis indicates that these students' initial interpretation of the equal sign was formed through their initial use of it in the performance of simple arithmetical operations. The students' struggles and successes in finding the number that would make the number sentence true indicate that the initial meaning of the sign as a command to perform an operation needs to take on a meaning of quantitative balance while still keeping implicit the former meaning.

The extension in meaning of the equal sign was mediated by the dialogical interaction between teacher and students and it necessitated the explicit communication of students' interpretations and the teacher's interpretation of the students' interpretations to allow the teacher to guide the dialogical interaction in accordance to the needs of the students.

The analysis of the dialogical interaction in this episode as an interpreting game constituted by interpreting cycles in which interpretations, intentions, and linguistic expressions intermingle and influence one another allows us to see how dialogue plays an important role as a mediational means in the dynamic transformation of the written mark "=" into a mathematical symbol in the mind of the students. In this episode only one student indicated that he has started to decontextualize the equal sign to interpret the commutative property of addition that was implicitly expressed in the number sentence.

References [A reference list will be provided by the presenter at the session. It will also be available from "Saenz-ludlow, Adalira" <sae@email.uncc.edu>]