BEYOND DISCOURSE: A MULTIMODAL PERSPECTIVE OF LEARNING MATHEMATICS IN A MULTILINGUAL CONTEXT

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This paper presents the idea of multimodal teaching and learning and discusses how this perspective can help better understand the learning of students. The discussion is based on data gathered in a qualitative study of a fifth-grade bilingual classroom where at-risk students were successful in mathematics. We report on one class episode and one student as a case study for understanding multimodal learning. Analyses focus on how students use various texts such as speech and calculator keystrokes as resources to create meaning. This work suggests that a broader perspective and use of modes can support learning and provide students, especially those at-risk, with greater access to mathematics.

Research in mathematics education over the last several years has included communication such as speech and written texts as important factors in learning (e.g., Steinbring, Bartolini Bussi, and Sierpinska, 1998). However, there also has been a tendency to assume that learning is primarily a linguistic accomplishment. Such thinking has obscured the fact that there is actually a multiplicity of modes of communication by teachers and students, including such modes as gestures and actions (Nunez, 2000), and that all of these modes express meaning and contribute to learning. Ignoring these other modes of communication or not bringing them to the forefront of consideration in the learning process, can blind us to key elements of the meaning making process. Moreover it can cause educators to overlook the full range of various resources students draw on to create mathematical meanings. The purpose of this discussion is to consider the idea of multimodal teaching and learning and to present a particular classroom episode to demonstrate how students fluidly and simultaneously use multiple texts to create meaning of a geometric problem.

MULTIMODAL TEACHING AND LEARNING

Our discussion and analyses draw on the work of Kress and his colleagues (e.g., Kress, Jewitt, Ogborn, and Tsatsarelis, 2001). In this work, communication refers to all meaning-making systems, or "modes"; these are organized, regular, and socially specific means of representation (Jewitt, Kress, Ogborn, and Tsatsarelis, 2001). Learning can be seen as a "…process in which pupils are involved in actively 'remaking' the information and messages (or complexes of 'signs') which teachers communicate in the classroom. In this way learning…is the pupils' 'reshaping' of meaning (signs) to create new meanings (signs)" (Jewitt et al. 2001, p.6). Learning, therefore, is rooted in a dynamic process of sign making, but one which is devised, organized, and used according to social needs and practice (Halliday, 1985). Inherent in this conception of learning is that "…meaning arises as a consequence of choice and that meaning is multiple (Jewitt et al. 2001, p.6). In other words, when we make meaning, we choose to speak, use gestures, make drawings, move our bodies, or use whatever resource available to communicate or represent our meaning, and we use multiple modes simultaneously.

In classrooms, teachers use a complex of signs or an ensemble of communicative actions to convey specific meanings. Students remake these signs to convey their own meanings or messages of what they have learned. The richer the complex of signs, the more resources students can select and use to make new meanings. These resources or signs are integrated with those that the student already has from other experiences and knowledge including those from outside of school. Therefore, the representational resources available play a key role in mediating the meanings a student produces (Jewitt et al. 2001). Students' text, be it spoken or written, can be considered as 'signs' or evidence of their thinking or meaning making or choices they make in an active and complex transformation. In essence, a multimodal perspective provides a conceptual resource for observing crucial relationships among situated, embodied, and connectionist aspects within emergent forms of cognition.

We apply these ideas to one students' writing, which was part of a lesson solving a geometric problem. We first present a context that includes the problem that had to be solved, another student's oral presentation of a solution, and then the writing. We analyze the writing by examining the connections and selection of resources this student made to convey his meaning. Our intention is to understand the thinking process the text represents in order to better be able to understand multiple modes of communication. We conclude with the implications of a multimodal perspective for learning and for teaching.

CONTEXT OF THE STUDY AND THE DATA

The data were gathered as part of a qualitative study of a bilingual fifth-grade teacher's classroom (Chval, 2001). This teacher had a reputation for implementing standards-based mathematics and for consistently having high achievement gains in mathematics for a majority of her students. Unlike most classrooms in this school and neighborhood, students regularly used TI-30 calculators. In addition, students developed skills in speaking, reading, and writing calculator keystrokes. They used keystrokes to analyze the strategies of others, and to create and articulate reasoning to justify their solutions.

$\boxed{100}\ \boxed{\div}\ \boxed{3}\ \boxed{=}\ \boxed{\times}\ \boxed{4}\ \boxed{=}\ \boxed{\div}\ \boxed{\pi}\ \boxed{=}$

$\boxed{\sqrt{x}}\ \boxed{\times}\ \boxed{2}\ \boxed{=}\ \boxed{\text{STO}}\ \boxed{\times}\ \boxed{\pi}\ \boxed{\div}\ \boxed{4}\ \boxed{\times}$

$\boxed{3}\ \boxed{=}\ \boxed{\text{SUM}}\ \boxed{\text{EXC}}$

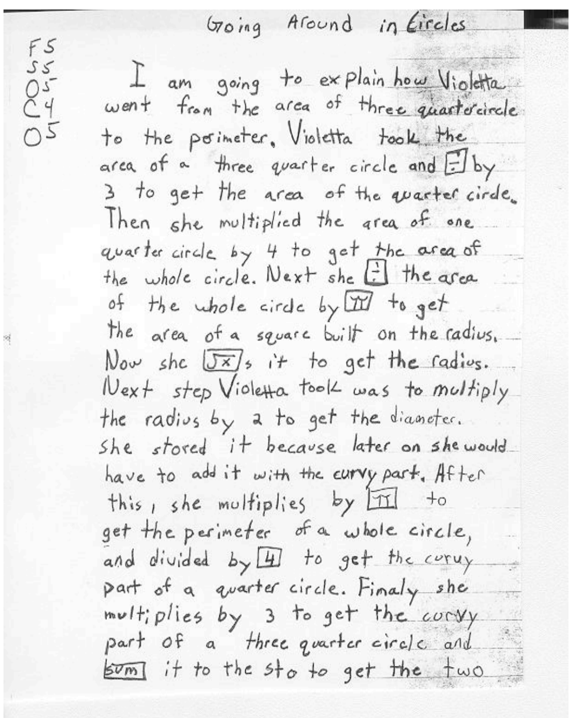
Figure 1: Violetta's calculator keystrokes for problem 1.

Late in the school year, the students were given two problems one of which was: "A three-quarter circle has an area of 100 square centimeters. Calculate the perimeter of the three-quarter circle." Students had already solved similar problems involving full circles; however, the three-quarter element was completely new. Students worked individually and collaboratively to solve this problem. All of them solved the first problem, but only one student, Violetta, solved the second problem. As a result, Violetta volunteered to present the solutions for both problems at the board to the class. Violetta drew a sketch of a three-quarter circle on the board and then wrote the calculator keystrokes for her solution to the first problem (see Figure 1). As she wrote each keystroke, she explained the meaning behind it.

The following is a transcription of Violetta's presentation of the first problem:

01: Violetta: We are going to find the perimeter of the three quarter-circle. The area of the… The area of the three quarter-circle are 100 square centimeters. Now, we are going to go backward from the area to the perimeter. One hundred divided by three equals the area of one quarter-circle. Multiply by four to get the area of the whole circle. Divide by pi to get the area of the square built on the radius. You take the square root to get the radius. And then you multiply by two to get the diameter. Then we store it. [A reference to the student's calculator.] Then we multiply by pi to get the circumference of the circle. Then we divide it by four to get the quarter-circle. Then we multiply by three to get the curvy part of the three quarter-circle. Then we sum it, sum it to memory. [Another reference to the calculator.] So we can get the circumference, the perimeter, of the three quarter-circle.

After Violetta's presentation and the resulting class discussion, students were asked to write about Violetta's explanation to the problem. Juan's final draft follows.

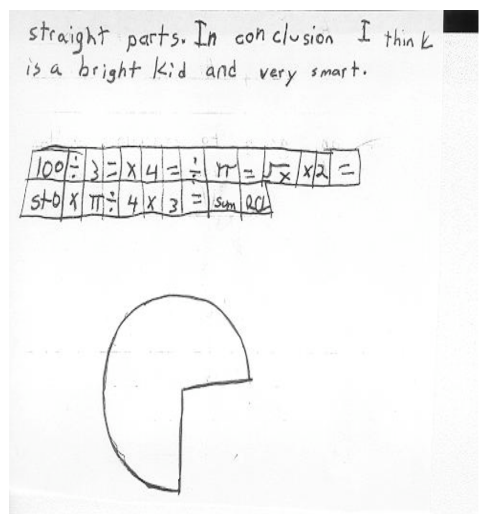


Going Around in Circles

FS SS OS C4 OS

I am going to explain how Violetta went from the area of three quarter circle to the perimeter. Violetta took the area of a three quarter circle and $\boxed{\div}$ by 3 to get the area of the quarter circle. Then she multiplied the area of one quarter circle by 4 to get the area of the whole circle. Next she $\boxed{\div}$ the area of the whole circle by $\boxed{\pi}$ to get the area of a square built on the radius. Now she $\boxed{\sqrt{x}}$s it to get the radius. Next step Violetta took was to multiply the radius by 2 to get the diameter. She stored it because later on she would have to add it with the curvy part. After this, she multiplies by $\boxed{\pi}$ to get the perimeter of a whole circle, and divided by $\boxed{4}$ to get the curvy part of a quarter circle. Finaly she multiplies by 3 to get the curvy part of a three quarter circle and $\boxed{\text{sum}}$ it to the sto to get the two

Figure 2.



straight parts. In conclusion I think is a bright kid and very smart.

Figure 3.

A MULTIMODAL ANALYSIS OF JUAN'S WRITING

Juan's writing and expression of his mathematical understanding began when the problem was assigned and he went to work to solve it. Violetta's talk contributed to his understanding as well. However, dialogue or talk can be considered only one part of a broader range of modes that contributes to Juan's learning. For our purposes, we concentrate on understanding the interrelationships specifically between four modes of semiotic mediation: Juan's mathematical writing (Written Text), the calculator keystrokes (Mathematical Symbols), Violetta's presentation to the class regarding the same problem (Written & Spoken Text), and the three-quarter circle drawing (Geometric Figure).

We suggest that meanings are being constructed as he moves continuously from the written text to Violetta's spoken text to the geometric figure to the calculator keystrokes in no predetermined order. In this fluid process, Juan uses a variety of signs to create his written text. Each mode shapes his meanings as he constructs his text, but likewise, the written text becomes a mode itself that in turn shapes new meanings for him. To write his text he has to construct meanings regarding how these three modes of signs (Violetta's talk, the geometric figure, and the keystrokes) relate to each other to form a coherent idea.

We partitioned Juan's writing into three sections: finding the area of the whole circle (Table 1); finding the diameter of the circle (Table 2); and finding the perimeter of the three-quarter circle (Table 3). This organizes the data and identifies the various modes of communication that were available to Juan as he wrote his text and the kind of meanings he constructed in this multimodal process. Each table contains Violetta's talk in regards to her presentation to the class, the keystrokes that both Violetta and Juan wrote down either on the board or in Juan's writing, and the geometric images of the three-quarter circle that Juan and Violetta refer to in their explanations.

PART 1: FINDING THE AREA OF THE WHOLE CIRCLE

We begin by looking at Juan's written text regarding finding the area of the whole circle. Juan took what Violetta said, "one hundred divided by three equals the area of one quarter-circle" and changed it to "Violetta took the area of a three quarter circle and $\boxed{\div}$ by three to get the area of the quarter circle." For Juan the 100 (what Violetta said) and the area of the three-quarter circle (what Juan writes) have the same meaning. Juan also included the keystroke symbol $\boxed{\div}$ to help him express his mathematical thinking. His choice to use the "divide" keystroke suggests he equates the word "divides" with the $\boxed{\div}$ keystroke. Altogether, this leads us to believe that, in fact, one can map the calculator keystrokes to Juan's written text (see Table 1). For example, every time Juan uses the expression "to get" in his text, this relates to the keystroke symbol "=". Reciprocally, one may assume that Juan gave the equal symbol the meaning "to get." The use of "to get" also suggests a connection between the keystroke steps and the mathematical concept the steps represent.

Although Juan only included the picture of the three-quarter circle, we wondered what role the images of the three-quarter circle, one-quarter circle, and the whole circle played in the kinds of meanings Juan constructed in this part of his discussion. Over the course of the school year, the students had worked so much with circles in various forms that they seem to be able to visualize the image of a one-quarter circle. In essence, the students have a strong conceptual history for solving the current problem, and this helps them in the current context. Therefore when Violetta says, “multiply by 4 to get the area of whole circle,” she is actually guiding her peers, including Juan, in forming an image of four quarter-circles forming a whole circle. In Table 1, we see that only the first geometric figure (the three-quarter circle) was evident in Juan’s writing, but clearly the other two images of circles were evident in Juan’s thinking.

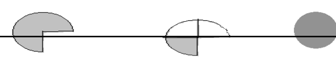
Violetta’s Talk on her pr to the class	*“The area of the three quarter circle are 100 square centimeter are going to go backward from the area to the perimeter. On divided by three equals the area of one quarter-circle. Multiply get the area of the whole circle.”*
Keystrokes	*100* | ÷ | *3* | = | *X* | ***4*** | =
Geometric Figures	 *to* *to*
Juan’s Writing	*I am going to explain how Violetta went from the area of the thr circle to the perimeter. Violetta took the area of a three quarter* ÷ *by three to get the area of the quarter circle. Then she multip area of one quarter circle by 4 to get the area of the whole circle.*

Table 1. Finding the area of the whole circle.

PART 2: FINDING THE DIAMETER OF THE CIRCLE

After Juan found the area of the circle, he divided by pi. In an earlier lesson, the students were given a circle with a diameter of 10 centimeters inscribed in a square with a side length of 10 centimeters. The square was partitioned into four equal squares built along the radius of the circle. Through various classroom activities students discovered that four squares built on the radius overestimated the area of the circle and three squares underestimated the area. As a result, students thought about building a square on the radius and multiplying by pi to calculate the area of circles. Thus, dividing by pi for Juan meant geometrically that he was finding the area of the square built on the radius (see Table 2). Again the historical context plays a critical role in Juan’s understanding.

Also in Table 2, notice that in Juan’s writing he embeds the keystroke symbols (e.g., 4, π, SUM) inside his narrative in a way that fits grammatically and syntactically with his explanation. We also can see that Juan uses different tenses (e.g., “multiplies,” “divided,” and “would have to add”) throughout his narrative. Writing is the most complex mental function, and it is misleading to think of writing as simply symbolic representation of speech (Wells, 1999) although there is a relationship between the two modes of language.

Both speech and writing express the same underlying meanings; however, writing is inherently a problem-solving situation fostered by attempts to create a visual representation of the meanings communicated in speech (Wells, 1999). Writing requires extraordinary deliberate analytical action which we see as Juan balances his written words ("area of a square built on a radius") with Violetta's spoken words, the keystrokes, and the geometric image of a quarter circle, all of which represent the area of a square built on a radius.

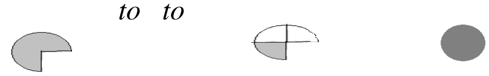
Violetta's Talk on her pr to the class	*"The area of the three quarter circle are 100 square centimeter are going to go backward from the area to the perimeter. On divided by three equals the area of one quarter-circle. Multiply get the area of the whole circle."*
Keystrokes	*100* ÷ *3* = *X* ***4*** =
Geometric Figures	*From to to* 
Juan's Writing	*I am going to explain how Violetta went from the area of the thr circle to the perimeter. Violetta took the area of a three quarter* ÷ *by three to get the area of the quarter circle. Then she multip area of one quarter circle by 4 to get the area of the whole circle.*

Table 2. Finding the diameter of the circle.

PART 3: FINDING THE PERIMETER OF THE THREE-QUARTER CIRCLE

Finally, Juan and Violetta took the circumference of the whole circle and divided by four to find the length of one-fourth the circumference, which Violetta calls the "curvy part." Then both students multiplied by three to get the "curvy part" of the three-quarter circle. The "curvy part" is represented geometrically in Table 3. Juan and Violetta used four keystrokes related to the calculator's memory system, namely "SUM," "STO," "EXC," and "RCL." The "STO" key was used by both students to save the value of the diameter of the circle. Juan added his curvy part "to the STO" and Violetta added hers "to memory." In both cases, the students used the "SUM" key in their keystroke text to add the value of the diameter to the value of the "curvy part," but recovered the value of the diameter in slightly different ways as their keystroke text demonstrates in Figures 1 and 2. Juan used "RCL" and Violetta used the "EXC" key, both of which give the same results. This is very important because it is evidence that Juan and Violetta are not simply copying text.

Also each student used different geometric images for combining the curve of the circle and the perpendicular radii, and represented the process in different linguistic styles. Juan combined the "curvy part of the three-quarter circle…and the two straight parts" (radii). Violetta, on the other hand, combined "…the curvy part of the three-quarter circle. Then we sum it, sum it to memory." The memory she is referring to is the value of the diameter of the circle that she stored earlier. Even though they expressed the three-quarter circle differently, their keystrokes were the same for both of them with the exception of the last keystrokes (RCL and EXC). Clearly both Juan and Violetta seem to understand that the diameter is two times the radius and can be shaped geometrically by perpendicular radii to connect the three-quarter circle. Juan, therefore, is able to move between different ways of representing the diameter to make sense of the problem at hand and additionally balance spoken text, keystrokes, geometric images, and his own text.

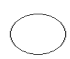
Violetta's Talk on her pr to the class	*Then we multiply by pi to get the circumference of the circle. divide it by four to get the quarter circle. Then we multiply by th the curvy part of the three quarter-circle. Then we sum it, sum it t [Another reference to the calculator.] So we can get the circumfe perimeter, of the three quarter-circle.*
Keystrokes	$\boxed{\times}$ $\boxed{\pi}$ $\boxed{\div}$ $\boxed{4}$ $\boxed{\times}$ $\boxed{3}$ $\boxed{=}$ $\boxed{SUM}$ $\boxed{RCL}$
Geometric Figures	 
Juan's Writing	*After this, she multiplies by* $\boxed{\pi}$ *to get the perimeter of a whole and divided by* $\boxed{4}$ *to get the curvy part of a quarter circle. Fi multiplies by 3 to get the curvy part of a three quarter circle and to the sto to get the two straight parts.*

Table 3. Finding the perimeter of the three-quarter circle.

CONCLUSION

Our purpose has been to draw attention to the idea of multimodal teaching and learning in mathematics. The case we discussed presents an example of how students use many modes of signs as resources for learning or meaning making. This suggests that educators need to utilize multiple modes of communication and encourage students to utilize them also. Unfortunately, there is a tendency to inadvertently limit students' access to and use of multiple modes as evidenced by the infrequent use of calculators, drawings, manipulatives, enactment, or even student presentations. For second language learners, this infrequent use of multiple modes of communication actually limits their means to express their thinking and denies them more than one way to access mathematics.

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