# “SPONTANEOUS” MENTAL COMPUTATION STRATEGIES

Ann M. Heirdsfield
Centre for Mathematics and Science Education, QUT, Australia

*The focus of this study was to investigate mental computation conceptual frameworks that Heirdsfield (2001c) formulated to explain the difference between proficient (accurate and flexible) mental computers and accurate (but not flexible) mental computers. A further aim was to explore the potential for students' developing efficient mental strategies.*

## INTRODUCTION

Mental computation is defined as “the process of carrying out arithmetic calculations without the aid of external devices” (Sowder, 1988, p. 182). Literature at national and international levels argues the importance of including mental computation in a mathematics curriculum that promotes number sense (e.g., Maclellan, 2001; McIntosh, 1998; Reys, Reys, Nohda, & Emori, 1995). International research (e.g., Blöte, Klein, & Beishuizen, 2000; Buzeika, 1999; Hedrén, 1999; Kamii & Dominick, 1998) has focussed on children formulating their own mental computation strategies in the belief that when children are encouraged to do so, they learn how numbers work, gain a richer experience in dealing with numbers, develop number sense, and develop confidence in their ability to make sense of number operations. A common thread to this research has been valuing students' strategies, promoting strategic flexibility, and encouraging student discussion. One difference between the European (in particular, Dutch and German work) and American and New Zealand work is that models (e.g., Empty Number Line) are used as representations for mental computation in European classrooms. These do not feature as much in the other classrooms (although Thornton, Jones, & Neal (1995) advocated the use of the hundreds chart for supporting mental computation). While these studies are supported by a constructivist approach, there is some support for a behaviorist approach to teaching mental computation (e.g., Morgan, 2000). Morgan suggested teaching mental computation strategies in a sequential fashion. However, the sequence does not take into consideration number combinations, merely strategies. That is, a sequence of strategies is introduced over the seven years of primary school, regardless of the numbers involved. Some of this sequencing is based on the sequential teaching of written algorithms; however, this sequence is not theoretically based. Although, Morgan (2000) does conclude, “The emphasis needs to remain on students exploring, discussing, and justifying their mental strategies, as well as their solutions.” Currently, in Queensland (Australia), whether children should be taught computational strategies or whether they should develop their own is being addressed while the new curriculum is being developed.

In Australia, the inclusion of mental computation in the curriculum is a recent phenomenon. In the Queensland context, there has been some research into mental computation, for example, a five-year longitudinal study identified children's mental computation strategies, and tracked changes in strategy use (e.g., Cooper, Heirdsfield, & Irons, 1996; Heirdsfield, Cooper, Mulligan, & Irons, 1999; Heirdsfield, 1999). Further research (Heirdsfield, 1996) identified some cognitive factors that were associated with proficient mental computation (flexible use of efficient strategies and accuracy) in Year 4 children (approximately 9 years old): proficient number facts (speedy recall and efficient number fact strategies) and proficient estimation. This study raised further questions about other factors that appeared to be associated with mental computation. Thus, the focus of a further study was the identification of cognitive, metacognitive, and affective factors that might be associated with mental computation (Heirdsfield, 1998, 2001a, 2001b, 2001c; Heirdsfield & Cooper, 2002). For the purposes of identifying flexibility, mental computation strategies were classified using a scheme (based on Beishuizen, 1993; Cooper, Heirdsfield, & Irons, 1996; Reys, Reys, Nohda, & Emori, 1995) that divided strategies into the following categories: (1) *separation* (2) *aggregation* (3) *wholistic* and (4) *mental image of pen and paper algorithm* (see Table 1).

| Strategy | | Example |
|---|---|---|
| *Separation* | *right to left (u-1010)* | 28+35: 8+5=13, 20+30=50, 63<br>52-24: 12-4=8, 40-20=20, 28 (subtractive); 4+8=12, 20+20=40, 28 (additive) |
| | *left to right (1010)* | 28+35: 20+30=50, 8+5=13, 63<br>52-24: 40-20=20, 12-4=8, 28 (subtractive); 20+20=40, 4+8=12, 28 (additive) |
| | *cumulative sum or difference* | 28+35: 20+30=50, 50+8=58, 58+5=63<br>52-24: 50-20=30, 30+2=32, 32-4=28 |
| *Aggregation* | *right to left (u-N10)* | 28+35: 28+5=33, 33+30=63<br>52-24: 52-4=48, 48-20=28 (subtractive); 24+8=32, 32+ 20=52, 28 (additive) |
| | *left to right (N10)* | 28+35: 28+30=58, 58+5=63<br>52-24: 52-20=32, 32-4=28 (subtractive); 24+20=44, 44+8=52, 28 (additive) |
| *Wholistic* | *compensation* | 28+35: 30+35=65, 65-2=63<br>52-24: 52-30=22, 22+6=28(subtractive); 24+26=50, 50+2=52, 26+2=28 (additive) |
| | *levelling* | 28+35: 30+33=63<br>52-24: 58-30=28 (subtractive); 22+28=50, 28 (additive) |
| *Mental image of pen and paper algorithm* | | Student reports using the method taught in class, placing numbers under each other, as on paper, and carrying out the operation, right to left. |

Table 1. Mental Strategies for Addition and Subtraction

Conceptual frameworks were developed to explain the differences in particular types of mental computers (Heirdsfield, 2001a, c). The findings of this study showed that Year 3 students who were proficient in mental computation (accurate and flexible) exhibited strategic flexibility, dependant on the number combinations of the problems. It was posited that an integrated understanding of mental strategies, number facts, numeration, and effect of operation on number supported strategic flexibility (and accuracy). Moreover, this cohort of students also exhibited some metacognitive strategies, possessed reasonable short-term memory and executive functioning, and held strong beliefs about their self developed strategies. Blöte, Klein, and Beishuizen (2000) also considered associated cognitive, metacognitive, and affective factors in their research into mental computation and conceptual understanding. Further, Maclellan (2001) posited that mental computation was situated in a richly connected web.

Where students exhibited less knowledge and fewer connections between knowledge, Heirdsfield (2001c) found that students compensated in different ways, depending on their beliefs and what knowledge they possessed. For instance, students who had sufficient knowledge to support the ability to compute mentally (although not necessarily efficiently) generally held strong beliefs about teacher taught strategies, and used these strategies to successfully obtain answers to mental computations. These students were identified as being inflexible, that is, they employed a single strategy, *mental image of pen and paper algorithm*.

It has been argued elsewhere (Brown & Palincsar, 1989) that an aspect of a study of "knowing" should address Vygotsky's zone of proximal development (ZPD) (Vygotsky, 1978). Vygotsky claimed that a child's level of development cannot be understood unless both the child's actual developmental level (determined by independent activity) and potential developmental level (determined by guidance provided to the child) were established. The zone of proximal development is the "distance between the actual developmental level … and the level of potential development" (Vygotsky, 1978, p. 86). Children at the same actual level of development may have different zones of proximal development. Van der Heijden (1994) used a Vygotskian approach to investigate mental addition and subtraction of primary school children. Vygotsky's ZPD was considered an important aspect of qualitative assessment of children's mental addition and subtraction proficiency, defined by speed, accuracy and efficient strategy use. Pre-determined scaffolding questions were presented to children who did not employ what was considered efficient mental procedures. Results indicated that students possessed a considerable potential for efficient strategies, they generally agreed that the efficient strategy was easier.

In Heirdsfield's study (2001c), it was found that most students possessed the potential to use efficient strategies, as evidenced by their ability to access alternative strategies (although not always through to successful completion). This concurred with the findings of Van der Heijden (1994), but the finding of students in Heirdsfield's study preferring their first strategy (not always the more efficient strategy they accessed) was in contrast to that of Van der Heijden.

Another factor in mental computation research and teaching is how to assess mental computation. Some researchers and teachers accept that mental computation is important in the curriculum, but fail to see it in the bigger sense – as a means to develop number sense by actively engaging in the construction of efficient and economical strategies, which make use of number understanding. If the goal of involving students in mental computation is to improve their reasoning and thinking, then traditional tests cannot assess students' understanding, merely whether they can calculate in their heads. It has been shown that there are students who possess little number sense, yet they are "successful" (in terms of arriving at the correct answer) on mental computation tests (e.g., Heirdsfield, 1996, 2001b, c; Heirdsfield & Cooper, 2002; McIntosh & Dole, 2000). These tests often take the form of directing students to solve problems mentally and write down the answer or say the answer without explaining their strategies. Unfortunately, some teachers in Australia have mistaken the term, *mental computation* for an out dated term used in the sixties (and before), *mental arithmetic* (Morgan, 1999). Lessons in mental arithmetic were "characterised by a series of short, low-level unrelated questions to which answers are quickly calculated, recorded, and marked." (Morgan 2000). Thus, the emphasis in mental arithmetic was testing, rather than teaching/learning.

The focus of the study reported here was to further investigate the conceptual frameworks that Heirdsfield (2001c) developed for accurate mental computation, both flexible and inflexible (cognitive, metacognitive and affective factors) in Years 3 and 4 students (8, 9, and 10 year olds), and to further explore the potential of students' developing more efficient mental strategies.

## METHOD

The research project was essentially *qualitative* in nature, with a focus on developing case studies (Denzin & Lincoln, 2000). One-on-one structured and semi-structured clinical interviews were used to explore flexibility, identify associated factors, and probe the potential for students to develop efficient mental strategies.

### Participants

The participants were eight Year 3 students (8 and 9 year olds) and eight Year 4 students (9 and 10 year olds) who attended a Brisbane school that served a middle socioeconomic area. The students were selected from a cohort of forty-one Year 3 students (4 classes) and thirty-three Year 4 students (3 classes) (selected by teachers as being reasonably proficient in mathematics), on the basis of accuracy in structured selection mental computation interviews. They were able to complete successfully at least 80% of the addition tasks in the selection interview (subtraction examples were generally less successfully solved than addition examples). In Year 4, four flexible students and four inflexible students were selected for further indepth interviews; while in Year 3, six flexible students and two inflexible students were selected for indepth interviews (only 2 inflexible and accurate students could be identified in Year 3 – all other flexible students were flexible).

### Instruments

The instruments were adapted from previously developed instruments (Heirdsfield, 2001c), and then modified and extended for the two year levels (previously the instruments addressed Year 3 only). The instruments consisted of: *a structured selection interview* - one-, two-, and three-digit addition and subtraction mental computation items, presented in picture form, while the question is verbally presented to the student (e.g., "What is the total cost of the two computer games?" - $68 and $31); *a series of semi-structured indepth interviews to investigate factors associated with mental computation* – focusing on strategies for mental addition and subtraction (different from but similar to the selection items), number facts, numeration, effect of operation on number, computational estimation, metacognition, affects, and classroom context. While many of the tasks for Year 3 were repeated for Year 4 students, some were made more appropriate for Year 4 students by increasing the complexity of the numbers involved (e.g., 107-15 for Year 3 was replaced by 127-35 for Year 4).

## Procedure

The students were withdrawn, individually, from class to a quiet room in the school for all interviews. The indepth interviews consisted of three sessions of videotaped interviews: (i) a number facts test and mental computation interview; (ii) computational estimation interview and numeration interview; and (iii) effect of operation on number interview. Within each set of indepth interviews, further questions to probe for evidence of metacognition and affects were posed. Of particular interest here, are the questions that were asked during the indepth mental computation interviews. Following Van der Heijden (1994), predetermined scaffolding questions were presented to students who did not employ what was considered an efficient mental strategy (or where a more efficient strategy might be used). These were: (1) Can you think of another way of solving the problem? (2) What is (e.g., 99) close to? (3) Can you work with this number? (4) What can you do now? If the student accessed a more efficient strategy (whether resulting in a correct answer or not), he/she was then asked which strategy was preferred and why.

## Analysis

For the purposes of identifying flexibility in mental computation, mental computation strategies were identified using a previously developed categorisation scheme (see Table 1). Mental computation responses were analysed for strategy choice, flexibility, and accuracy. Evidence of each student's number sense (understanding of the effects of operation on number, numeration, computational estimation, and number facts) was also sought. Analysis of the interviews investigating these individual factors was undertaken, with the intention of exploring connections with mental computation. Students' responses were also analysed for metacognition and affects (although this was not investigated in depth). Each student's results for aspects of number sense, metacognition, and affects were summarised.

The findings of the present study were compared with the frameworks developed by Heirdsfield (2001a, c) for accurate mental computers. Individual student's knowledge structures, metacognition and affects were analysed to explain the effect on both selection and implementation of mental strategies.

Whether individual students could access more efficient mental computation strategies was noted. Success or otherwise was analysed in relation to individual student's knowledge and understanding within the conceptual frameworks for mental computation.

# FINDINGS

## Mental computation strategies

Although all students in the present study were reasonably accurate mental computers, not all these students employed what could be considered efficient mental strategies. Students who were considered flexible employed a variety of mental computation strategies, including *separation – left to right, right to left*, *cumulative sum/difference*; *aggregation*; *wholistic*. *Aggregation* was used rarely (5 students used the strategy, only 1 student used the strategy more than once). Students did not necessarily solve very similar examples using the same strategy at different times, for instance, one student used the following strategies:

| Selection interview | Indepth interview |
|---|---|
| 148+99: 147+100=247 | 246+199: 200+200=400, 46+100=146, 446-1=445 |
| 165-99: 100-99=1, 1+65=66 | 234-99: 234-100=134, 134+1=135 |

Those who employed *wholistic* spontaneously stated that they often used the strategy in class to solve written algorithms, although they had not been taught to use it, and that the teachers probably did not know they were using it. Overall, the students improved their performance from the selection interview to the indepth interview, both in accuracy and in flexibility (this is discussed below).

## Number fact knowledge

Year 3 students tended to be slower than Year 4 students, although just as accurate. Most number facts were solved using *derived facts strategies*. In both groups of students, more subtraction examples (than addition examples) were solved using a count strategy. This was more prevalent with Year 3 students than with Year 4 students. Students who were fast and accurate, and solved number facts by *recall* or *derived facts strategies* tended to be more proficient mental computers (accurate and used a variety of efficient mental strategies). Students who were slow and used count strategies to solve number facts tended to be the students who employed *mental image of pen and paper algorithm* to solve the mental computation tasks.

## Numeration

All students required MAB material to regroup/rename 2-, 3-, and/or 4-digit numbers (e.g., "tell me about 209 in as many ways as you can"). Many students renamed numbers as if using a numeral expander (e.g., 1634 = 16x100 + 3x10 + 4x1). When it was suggested that MAB might be helpful, one student (inflexible mental computer) regrouped to make 15 hundreds, 13 tens and 4 ones, but she had to count the number of hundreds and the number of tens, as she was simply manipulating material, rather than understanding what she was doing. In conversation with the teachers, one teacher questioned the reason to be able to regroup in such a fashion. From that conversation, it was inferred that students were not encouraged to think of numbers in more than one way (in this school), resulting in inflexibility in numeration.

## Effect of operation on number

The effect of operation on number, particularly the effect of changing the minuend, was not well understood by any student, particularly the students who used *mental image of pen and paper algorithm*. However, many students were able to use the concept in the mental computation indepth interviews, for instance, to solve 234-99 and 53-29 using *wholistic*.

## Computational estimation

Overall, computational estimation was poorly understood, particularly by students who used *mental image of pen and paper algorithm* to solve the mental computation tasks. Many students simply guessed answers or employed a rounding strategy whether it was appropriate or not. However, there was evidence of the more proficient mental computers checking their working and solutions in the mental computation tasks, for instance, "No, that can't be right. It's too big."

## Metacognition, affects, and classroom context

Although this study did not have a strong focus on metacognition, there was evidence of metacognitive strategies being used by the proficient mental computers to make sense of their calculations. In contrast, the inflexible students were not concerned that some of their answers were unreasonable. All students said that they valued mathematics and they thought it was important to calculate in their heads. Further, they all believed that they were capable of solving the examples.

What were of interest though were the insights into the classroom through he students' eyes. One Year 4 student stated that "mentals are done in class, like 31+12. But we don't discuss the strategies." Another student stated that he "used to do sums in my head in class lessons [presumably mental arithmetic], but the teacher stopped me because he/she realised that I was too good"! Finally, another student who preferred to use *mental image of pen and paper algorithm* throughout, said that 300-298 could not be solved, as she had difficulty with the regrouping (not realising that she could have counted!).

### Potential for accessing efficient mental strategies and factors that supported this

All students were scaffolded at least once in the indepth mental computation interviews, but levels of scaffolding differed for individual students, from question 1 (see procedure above) – "Can you think of another way of solving the problem?" to question 4 – "What can you do now?" As a result of scaffolding, all students accessed *wholistic* for such examples as 56+39, 246+100, 63-29, and 234-99, but with varying degrees of success. Failure was generally a result of a lack of understanding of the effect of operation on number, and these students were generally those who employed *mental image of pen and paper algorithm*. However, in many instances, students who were unsuccessful attempting to solve these more complex examples in the selection interview or even in the indepth mental computation interview were successful when they used *wholistic*. In particular, a Year 4 student who employed the "buggy algorithm" of "take smaller from bigger" in the selection interview for 265-99, "spontaneously" employed *wholistic* successfully for 234-99, possibly as a result of being prompted to find a more efficient strategy to solve 80-49. Another Year 4 student who employed *mental image of pen and paper algorithm* in the selection interview, with limited scaffolding ("What is close to?"), solved 56+39 (55+40) and 246+199 (245+200) (*wholistic leveling*). Other students, who accessed wholistic with scaffolding, stated that they started to use this method, as it "is easier". Many students started to use *wholistic* as the first strategy choice for solving examples that could easily be solved using the strategy. In general, the employment of *wholistic* resulted in improved accuracy. Further, most students stated that they found this method easier than their previous strategies.

## CONCLUSION AND DISCUSSION

In general, the results of this study confirmed the conceptual frameworks for the accurate mental computers (Heirdsfield, 2001c); however, numeration understanding and understanding of the effect of operation on number were not robust. It can be said, though, that the flexible students exhibited better understanding of these two factors than the inflexible students. Further, flexible students employed more efficient number facts strategies than the inflexible students. They employed metacognitive strategies, while the inflexible students did not. Thus, the flexible students had more integrated and extensive conceptual structures to support flexible mental computation (c.f., Blöte, Klein, & Beishuizen, 2000). However, most students (flexible and inflexible) were able to successfully access more efficient mental strategies with prompting and/or scaffolding, and all but one student agreed that the accessed strategies were "easier" (concurring with Van der Heijden, 1994).

No student in the present study had been taught mental computation strategies, nor had they been taught to calculate using a number line, empty number line or 99/100 chart. The only representation they had access to was MAB. However, students successfully employed efficient mental computation strategies (with and without scaffolding), probably unknown to the teachers. Therefore, it is posited that students do not need to be taught these strategies, merely encouraged to develop and use efficient strategies (c.f., Morgan, 2000).

It is also interesting to note that most of the Year 3 accurate mental computers were flexible, while only half the accurate Year 4 students were flexible. In other words, accuracy at Year 3 level was a result of self-developed strategies – they could solve the examples without the taught strategies (c.f., Heirdsfield, 2001c); while accuracy at Year 4 level resulted from both the taught strategies and self-developed strategies. This begs the question, why are students taught computational procedures if they can already successfully and efficiently use their own strategies?

The findings of this study add further support to students' developing their own mental computation strategies by valuing students' strategies, promoting strategic flexibility, and encouraging student discussion. Further, "Focus is needed, both in classroom and in research, on the teacher's role in promoting pupil's thinking at a metacognitive level to gain efficiency with understanding" (Beishuizen, 1998).

**References:**


Beishuizen, M. (1993). Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second grades. *Journal for Research in Mathematics Education, 24*(4), 294-323.

Beishuizen, M. (1998). Which mental strategies in the early number curriculum? A comparison of British and Dutch views. *British Education Research Journal, 24*(5), 519-539.

Blöte, A. W., Klein, A. S., & Beishuizen, M. (2000). Mental computation and conceptual understanding. *Learning and Instruction, 10*, 221-247.

Brown, A. L., & Palincsar, A. S. (1989). Guided, cooperative learning and individual knowledge acquisition. In L. B. Resnick (Ed.), *Knowing, learning and instruction. Essays in honor of Robert Glaser* (pp. 393-451). Hillsdale, NJ: Erlbaum.

Buzeika, A. (1999). Invented algorithms: Teachers face the challenge. In J. M. Truran & K. M. Truran (Eds.), *Making the difference* (pp. 128-134). Sydney: MERGA.

Cooper, T. J., Heirdsfield, A. M., & Irons, C. J. (1996). Children's mental strategies for addition and subtraction word problems. In J. Mulligan & M. Mitchelmore (Eds.), *Children's number learning* (pp. 147-162). Adelaide, Australia: AAMT.

Denzin, N. K., & Lincoln, Y. S. (2000). *Handbook of qualitative research, 2nd edition.* California: Sage Publications.

Hedrén, R. (1999). The teaching of traditional standard algorithms for the four arithmetic operations versus the use of pupils' own methods. In I. Schwank (Ed.), *European research in mathematics education 1* (pp. 233-244). [http://www.fmd.uni-osnabrueck.de/ebooks/erme/cerme1-proceedings/ cerme1-proceedings.html]

Heirdsfield, A. M. (1996). *Mental computation, computational estimation, and number fact knowledge for addition and subtraction in year 4 children*. Unpublished Master's thesis, QUT, Australia.

Heirdsfield, A. M. (1998). Flexible/inflexible: Clare and Mandy's story. In C. Kanes, M. Goos, & E. Warren. (Eds.), *Teaching mathematics in new times* (pp. 241-249). Gold Coast, Australia: MERGA.

Heirdsfield, A. M. (1999). Mental addition and subtraction strategies: Two case studies. In J. M. Truran & K. M. Truran (Eds.), *Making the difference* (pp. 253-260). Adelaide: MERGA.

Heirdsfield, A. M. (2001a). Integration, compensation and memory in mental addition and subtraction. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education* (Vol 3, pp. 129-136). The Netherlands: Drukkerij Wilco.

Heirdsfield, A. M. (2001b). Integration and compensation in accurate mental computation. In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and beyond* (Vol 2, pp. 273-280). Sydney: MERGA.

Heirdsfield, A. M. (2001c). *Mental computation: The identification of associated cognitive, metacognitive and affective factors*. Unpublished doctoral thesis, QUT, Brisbane.

Heirdsfield, A., & Cooper, T. J. (2002). Flexibility and inflexibility in accurate mental computation: Two case studies. *Journal of Mathematical Behavior, 21*, 57-74.

Heirdsfield, A. M., Cooper, T. J., Mulligan, J., & Irons, C. J. (1999). Children's mental multiplication and division strategies. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education.* (Vol. 3, pp. 89-96). Israel: Technion.

Kamii, C., & Dominick, A. (1998). The harmful effects of algorithms in grades 1-4. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics, 1998 yearbook* (pp. 130-140). Reston, VA: NCTM.

Maclellan, E. (2001). Mental calculation: Its place in the development of numeracy. *Westminster Studies in Education, 24*(2), 145-154.

McIntosh, A. (1998). Teaching mental algorithms constructively. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics, 1998 yearbook* (pp. 44-48). Reston, VA: NCTM.

McIntosh, A., & Dole, S. (2000). Mental computation, number sense and general mathematics ability: Are they linked? In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000* (pp. 401-408). Perth: MERGA.

Morgan, G. R. (2000). Methods of calculation in primary school mathematics - adjusting the balance. *Teaching Mathematics, 25*(2), 5-10.

Reys, R. E., Reys, B. J., Nohda, N., & Emori, H. (1995). Mental computation performance and strategy use of Japanese students in grades 2, 4, 6, and 8. *Journal for Research in Mathematics Education, 26*(4), 304-326.

Sowder, J. (1988). Mental computation and number comparisons: Their roles in the development of number sense and computational estimation. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 182-197). Hillsdale: NJ: Lawrence Erlbaum Associates.

Thornton, C. A., Jones, G. A., & Neal, J. L. (1995). The 100s chart: A stepping stone to mental mathematics. *Teaching Children Mathematics, April,* 480-483.

Van der Heijden, M. K. (1994). A Vygotskian approach on mental addition and subtraction up to hundred; Handy arithmetic for primary school children. In J. Van Luit (Ed.), *Research on learning and instruction of mathematics in kindergarten and primary school* (pp. 108-124). The Netherlands: Graviant Publishing Company.

Vygotsky, L. S. (1978). *Mind in society – The development of higher psychological processes*. Cambridge, MA: Harvard University Press.