SECONDARY SCHOOL MATHEMATICS PRESERVICE TEACHERS' PROBABILISTIC REASONING IN INDIVIDUAL AND PAIR SETTINGS

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Fairly large amount of research on students' conceptions of probability has been reported in the literature. Most of this work represents school students' reasoning of probability in individual setting. This study provides 8 secondary school mathematics preservice teachers' reasoning of probability in both individual and pair settings. Consistent with previous studies the findings of this study indicate that preservice teachers' conceptions of probability are often different across settings. However, a few preservice teachers who strongly believed in formal reasoning provided consistent argument across all settings if similar tasks were presented. It is suggested that the teaching of probability should provide an opportunity for students to interact in both individual and group settings.

INTRODUCTION AND THEORETICAL FRAMEWORK

Fairly large amount of research on student conceptions of probability has been reported in the literature. Several researchers have explored K-12 and first-year college students' conceptions of probability and have reported that students' conceptions of probability are often different from those presented in schools (Kahneman & Tversky, 1972; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; Shaughnessy, 1992; Tversky & Kahneman, 1982; Van Dooren, De Bock, Depaepe, Janssens, & Verschaffel, 2002). A few studies have also explored preservice teacher's understanding of probability and have reported that similar to students at K-12 and first-year college, preservice teachers' conceptions were often informal and intuitive particularly when the probability tasks were novel and non-routine (Bramald, 1994; Koirala, 1999, 2002).

Most of these research reported in this field have focused on students' conceptions of probability in individual settings. Because researchers have emphasized the importance of interactions generated from collaborative and cooperative learning (Carvalho & César, 2001; Kieran & Dreyfus, 1998; Mueller & Fleming, 2001), it is important to investigate how people reason similarly or differently in individual and group situations. The purpose of this paper is to provide some insight into this area of individual versus pair problem solving in the context of probability problems. This study sought to answer the following questions:

> What kinds of probabilistic reasoning do secondary school mathematics preservice teachers demonstrate in individual and pair settings?
>
> In what ways do preservice teachers' reasoning of probability vary across individual and pair settings?

How do preservice teachers perceive the individual and pair problem solving in probabilistic situations?

RESEARCH METHODS

Participants

This is a part of larger study in which 40 secondary school mathematics preservice teachers participated in solving 15 probability problems in written, pair, and/or individual interview settings. This paper focuses on 8 secondary school preservice teachers, who had taken at least 10 mathematics undergraduate courses, including a probability and statistics course. These preservice teachers were paired into four groups based on their responses to a written questionnaire and a probability problem. Each pair consisted of preservice teachers with diverse backgrounds and opinions so as to produce interactions that would help broaden the exploration of students' conceptualization of probability.

Tasks

This study is based on data collected from three tasks, one in the written setting, one in the pair setting, and one in the individual interview setting. Although the participants were asked to solve several other problems, only three are selected for this study because these three tasks are almost identical and allowed the researcher to track participants' views in individual and pair settings.

The task in the written setting was called the birth sequence task, and asked participants to select the most likely sequence of births in a family. In the pair setting, the participants were given the head-tail sequence task, wherein participants were asked to determine the most likely sequence of alternatives that might result from flipping a fair coin six times. The head-tail sequence task was repeated in the individual interviews to determine whether or not the participants transferred their knowledge from the pair setting to the individual setting. The detail of each task is provided below.

Written Task

1. Birth sequence task

A couple plans to have six children. Which of the following orderly sequence of birth is most likely? (Boys = B, Girls = G).

(a) BGBBBB

(b) BBBGGG

(c) GBGBBG

(d) BGBGBG

(e) All four sequences are equally likely.

Give reasons for your choice.

(Modified from Kahneman & Tversky, 1972 and Konold et al., 1993)

Pair Task

1. Head-tail sequence task

Which of the following orderly sequences is most likely to result from flipping a fair coin six times? (Heads = H, Tails = T).

a) HTHHHH

b) HHHTTT

c) THTHHT

d) HTHTHT

e) All four sequences are equally likely.

Give reasons for your choice.

(Modified from Kahneman & Tversky, 1972 and Konold et al., 1993)

Interview Task

1. Head-tail sequence task

Same from the pair task

At the final stage of the interview, the participants were asked to determine if the orderly sequences would be still equally likely if the number of trials was increased from 6 to 20.

DATA COLLECTION AND ANALYSIS PROCEDURES

The written component of this study was presented in a test-like situation and participants' work was collected on the same day. Their written work was analyzed and the participants were paired after two weeks. In the pair-problem solving, participants were asked to discuss possible solutions to the task. That is, pairs discussed and attempted to solve the task together. The investigator observed the pairs' work and made notes. Only in certain circumstances, for example if pairs stopped talking or reached an agreement very quickly, did the investigator provide prompts. The discussions were audiotaped. The participants were interviewed in the individual setting after a week. They were given the same head-tail sequence task from the pair setting in this interview. Each interview was audio-taped and then transcribed.

The researcher coded, analyzed, and categorized each response from written, pair, and interview settings using qualitative data analysis methods, in particular the constant comparative method (Guba & Lincoln, 1989) and the interactive model (Huberman & Miles, 1994). Their responses were categorized in three different ways based on their use of formal or informal probabilistic reasoning, their change of reasoning from one setting to the other, and their perception of solving problems in individual and pair settings.

RESULTS AND DISCUSSION

While attempting to solve probability problems, the preservice teachers used both formal and informal probabilistic reasoning. Their formal probability was often based on university courses and informal probability was based on their everyday intuitions and experiences. Four out of 8 preservice teachers provided formal reasoning in the birth sequence task, which was presented in a test like situation in the written setting. In the birth sequence task of the written component they reasoned that all four sequences are equally likely because "the probability of having a boy or a girl does not depend on the previous child's sex." Three out of these 8 preservice teachers reasoned that since there were six births the probability of each sequence is $\_^{6}$. The participants who provided informal reasoning in the written task demonstrated some elements of representativeness heuristic (Kahneman & Tversky, 1972). According to this heuristic: (1) the outcome in the sample should look like its parent population and (2) events in an outcome should appear random. Four preservice teachers provided informal and inconsistent reasoning in their written responses. For example, in the birth sequence task two participants selected that the sequence GBGBBG would be most likely because they had the same number of boys and girls and the sequence also appeared more random.

In the pair setting, it was clear that all preservice teachers wanted to solve the tasks using their mathematical knowledge from a formal perspective. In particular, all four preservice teachers who provided formal reasoning in the written setting also provided formal reasoning in the pair-problem setting. For example, a formal thinker stated:

> It's equally likely because the probability of having a head doesn't depend on ... a previous throw. The probability of getting a tail is not going to be greater if you had a head in the previous throw. If you had a head in the first time, the probability of getting a head in the second time is still one half. Doesn't matter if you've 10 heads in a row, the probability of getting the eleventh head is still one-half and the probability of getting the eleventh tail is still one-half.... What happened in the past doesn't influence the present throw.

Similarly, another preservice teacher, who provided formal reasoning in the written setting argued that all the four sequences in the head-tail sequence task of the pair setting are equally likely. According to this preservice teacher,

> ... no matter how many times you flip the coin it's a 50/50 chance of whether or not you're getting a heads or a tails and there is no order to it. Every time you do it doesn't depend on the time before. If I get a head this time ... there is no effect whether or not I get a head or tail the next time.

In all the pairs the formal thinkers started the conversation and provided formal reasoning as above. When the informal thinkers in the pair settings heard this kind of formal reasoning, they often quickly agreed with the formal thinkers and stated that “even if it’s a tail or a head for the first time the probability of having a tail or a head for the second time would be still 50%.” In two cases the informal thinkers resisted initially but changed their mind when further convinced by their partners. However, the interactions between them were positive and no tensions were noticed.

In the interview setting, all the preservice teachers recognized that the head-tail sequence task was repeated from the pair setting and all of them initially stated that all sequences are equally likely. Clearly the informal thinkers learned from their interactions in the pair setting and provided formal answers in the interview setting. In further probing, one preservice teacher who had provided informal reasoning in the written setting and had shown some initial resistant in the pair setting was not convinced in the interview setting that all sequences were equally likely. This participant stated:

> Okay, first of all I thought that all four outcomes are possible and then I realize that ... it also deals with order. When you say for example b) is HHHTTT... the first one of course is half/half, either half head or tail. Then if you want to produce a second head I think the probability of producing a head and a tail would be different because since the first time

> is not a tail so I think increases the probability of having a tail instead of a head. That's why I'm thinking that the ... four outcomes will not be equally likely. You have to look at the order in order to decide which one is most likely. ... Here the first one is head and then tail then following four heads. I guess when I think about it ... I'm trying to think about ... a possible answer and also ... I try to use my common sense to decide. I think the first two are not most likely. c) and d) are probably likely in the sense they're more even like three tails and three heads but the order is different. It looks that c) is more random than this one [d]. So I feel somehow that the random is more likely.

Clearly the pair interaction was not sufficient for this preservice teacher to change her mind. The above response shows an important element of representativeness heuristic as described by Kahneman and Tversky. Interestingly, this preservice teacher has also shown the same kind of reasoning in the written setting.

Furthermore, the preservice teacher responses differed when the interview task was modified by increasing the number of trials from 6 to 20. Three of the formal thinkers who had used a mathematical procedure in the written and pair settings still provided formal reasoning even when the task was modified. For the modified head-tail sequence task with they were quick to say that the probability of getting a sequence in the task was $(\frac{1}{2})^{20}$. But one of the four formal thinkers, who had not used a mathematical procedure as this one, was not sure if all of the sequences would be equally likely if the number of trials was increased up to 20. None of the four preservice teachers who had provided informal reasoning in the written setting were ready to agree that all four sequences are equally likely if the number of trails was 20. One of these preservice teachers clear stated that when the right answer should be (*e*) wherein all sequences are equally likely especially because of the relatively small sample size. If the sample size was large enough, she would think that a sequence with nearly an equal number of heads and tails would be most likely. Preservice teachers who provided inconsistent reasoning across various settings, was because of the conflict between their informal beliefs and their mathematical knowledge of probability.

With regard to their perceptions on individual versus pair problem solving, they reported that they felt comfortable in the pair setting because solving the given problem was a shared responsibility of both participants. The preservice teachers were asking questions and providing answers to each other without hesitation, even though the investigator was present during the pair problem solving. They all thought that they had fun solving problems in a pair situation. The informal thinkers, in particular, thought that they learned important probabilistic reasoning from their peers even though this learning was not transferred in a more difficult probabilistic situation.

CONCLUSIONS AND IMPLICATIONS

The secondary school mathematics preservice teachers demonstrated both formal and informal reasoning in this study. Three of 8 preservice teachers consistently used a formal mathematical procedure in all the settings. The informal thinkers who could not use a mathematical procedure were the ones who demonstrated inconsistent informal reasoning in various settings. Despite this inconsistency they reported that solving problems in pairs was helpful for them both socially and academically. They felt comfortable and also demonstrated some formal probabilistic reasoning in the interview setting when the same task was presented. The findings of this study are similar to those found by Carvalho & César (2001) and Kieran and Dreyfus (1998) that the participants benefited from peer interactions that took place in pair problem solving.

This has an important implication for teaching probability. Because many students and preservice teachers find probability to be difficult and confusing (Bramald, 1994; Kahneman & Tversky, 1972; Koirala, 1999, 2002; Konold et al., 1993; Shaughnessy, 1992; Tversky & Kahneman, 1982; Van Dooren, et al., 2002), it is important that they have opportunity to share their thinking with their peers and learn from those social interactions. It has to be noted, however, that the informal thinkers carried their learning to the interview setting only when the same task was presented. This might be because the participants had only a small amount of time to interact in the pair setting. It is critical that they have more opportunity to participate in various types of problem solving in pairs or groups to increase the likelihood of their ability to transfer their learning in a different situation.

In all settings of this study, formal probabilistic reasoning was valued by the preservice teachers. Regardless of the number of mathematics course taken, a preservice teacher who used formal probability usually led the discussion in the pair problem solving. However, the formal thinkers not necessarily took the largest number of mathematics and probability courses. There was one preservice teacher in particular who had taken only 10 mathematics courses, including a probability and statistics course, provided a strong formal reasoning than any of the other preservice teachers who participated in this study. She led the discussion even though she was paired with a student who had completed an undergraduate major in mathematics with 16 courses.

This raises a question about the purpose of a probability course taught in school or university. Is the purpose of a school or university probability course to enhance students' formal probabilistic thinking? If so, why do preservice teachers who have taken a large number of university courses do not show formal reasoning while attempting to solve probability problems? If the secondary school mathematics preservice teachers with a reasonably strong background in mathematics do not show a formal probabilistic reasoning, then how can school students with a comparatively weak background in mathematics be expected to show a formal reasoning? These questions raise important issues about what, why, and how we should teach probability in schools, colleges, and universities.

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