DESCRIBING THE PRACTICE OF EFFECTIVE TEACHERS OF MATHEMATICS IN THE EARLY YEARS

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Within a major numeracy project, there were marked differences in achievement between classes, which, as reported in Sullivan and McDonough (2002), we attribute to the teachers. To gain insights into the practices of effective teachers, six case studies of highly effective teachers at the Prep (first year of school) to Grade 2 level were conducted. This paper shares the research methodology and insights into the practices of the case study teachers. It suggests that the description of the practices of effective teachers can inform the work of those involved in teacher education.

INTRODUCTION

Few would dispute that the knowledge, beliefs and practice of the teacher are major influences on learning in the mathematics classroom. As a result, those responsible for teacher education (both preservice and inservice) seek to assist preservice and inservice teachers to develop the knowledge and skills likely to increase their effectiveness. But what is it that effective teachers do? This paper shares insights from intensive case studies of six highly effective teachers. These teachers were participating in a large scale numeracy project, focusing on the first three years of school, and they were chosen because of the particularly impressive growth in their students' understanding of mathematics, as revealed by a one-to-one interview. We believe that descriptions of the practice of these particularly effective teachers that arose from these case studies provide a framework that will be useful to all those involved in mathematics teacher education.

CHARACTERISTICS OF EFFECTIVE TEACHERS

Classrooms and schools are complex places. To gain insights into what goes on in effective classrooms, that is, how teachers might best assist children to grow in mathematical understandings in these environments, it is of value to study the practices of those who are effective.

For many years researchers have sought to describe teacher behaviours that correlate positively with growth in student achievement. For example, as early as the 1970s and 1980s, the so-called *process-product* research sought to describe behaviours that correlated positively with student achievement (see, e.g., Brophy & Good, 1986). More recent research has provided a range of insights into the practices of effective teachers.

Within the large scale numeracy project drawn upon for the present paper, a study was conducted of growth in understandings related to Length for children in their first year of schooling (Sullivan & McDonough, 2002). It revealed marked differences in achievement between classes, irrespective of geographic or socioeconomic variables. Interviews with the most effective teachers revealed that they had a clear vision of the mathematical experiences needed, were able to engage the students, and were prepared to probe the thinking and understanding of the children.

In a major study of effective primary school mathematics teaching in the United Kingdom, Askew, Brown, Rhodes, Johnson and Wiliam (1997) studied the practices of a number of different teachers, with varying levels of effectiveness. Using mean class gains on a test involving aspects of the number system, computation, and problem solving, aurally administered by teachers with children writing their answers, teachers were grouped into broad categories according to effectiveness. Insights into the practices of effective teachers came primarily from focus schools and case studies of individual teachers, six of whom were identified as highly effective.

The teaching practices of the highly effective teachers

- connected different ideas of mathematics and different representations of each idea by means of a variety of words, symbols and diagrams;
- encouraged students to describe their methods and their reasoning, and used these descriptions as a way of developing understanding through establishing and emphasising connections;
- emphasised the importance of using whatever mental, written or electronic methods are most efficient for the problem at hand; and
- particularly emphasised the development of mental skills.

Brown, Askew, Baker, and Millett (1998, p. 373) noted that international observational studies "seem to show some agreement on some of the aspects of teacher quality which correlate with attainment. These included the use of higher order questions, statements and tasks which require thought rather than practice; emphasis on establishing, through dialogue, meanings and connections between different mathematical ideas and contexts; collaborative problem solving in class and small group settings; and more autonomy for students to develop and discuss their own methods and ideas."

Reflecting on a range of research studies, Brown (1999) noted also that "quality of teaching is more important than class organisation. … it's not whether it's whole-class, small group or individual teaching but rather what you teach and how you interact mathematically with children which seems to count" (p. 7).

The six intensive case studies discussed in this paper provide much support for the findings of others, while adding new insights to the discussion.

THE EARLY NUMERACY RESEARCH PROJECT

To explore the features of highly effective teaching of mathematics, we draw on results from the Early Numeracy Research Project (ENRP)[1] that investigated mathematics teaching and learning in the first three years of schooling, involving teachers and children in 35 project ("trial") schools and 35 control ("reference") schools (for details see Clarke, 2001; Clarke, McDonough, & Sullivan, 2002).

There were three key components within the ENRP:

- the development of a research-based framework of "growth points" in young children's mathematical learning (in Number, Measurement and Geometry);
- a 40-minute, one-on-one interview, used by all teachers to assess aspects of the mathematical knowledge of all children at the beginning and end of the school year (February/March and November respectively);
- extensive professional development at central, regional and school levels, for teachers, coordinators, and principals.

The ENRP framework of growth points in young children's mathematical learning encompassed nine mathematical domains within three strands: Number (Counting, Place value, Addition and subtraction strategies, Multiplication and division strategies); Measurement (Length, Mass, Time); and Space (Properties of shape, Visualisation orientation).

Within each domain typically five or six growth points were stated with brief descriptors for each. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (with nine objects screened and another four objects). Many young children "count-all" to find the total ("1, 2, 3, . . . , 11, 12, 13"), even once they are aware that there are nine objects in one set and four in the other. Other children realise that by starting at 9 and counting on ("10, 11, 12, 13"), they can solve the problem in an easier way. Counting All and Counting On are therefore two important growth points in children's developing understanding of Addition. The ENRP Growth Points informed the creation of assessment items for the one-to-one interview, and the recording, scoring and subsequent analysis. For examples of ENRP growth points and interview tasks within the domain of Multiplication and division strategies, see Sullivan, Clarke, Cheeseman, and Mulligan (2001) and within the domain of Length see Sullivan and McDonough (2002).

Interviews were conducted by the classroom teachers, who were trained in all aspects of interviewing and recording. The processes for assuring reliability of scoring and coding are outlined in Rowley and Horne (2000).

The data from the ENRP arise from intensive interviews with large numbers of children, with trained interviewers, and experienced coders, with double data entry, and using a framework for learning based on interpretation of research. We argue that these data provide a reliable measure of learning, and a further perspective on previous research on the ways that teachers influence student learning.

IDENTIFYING HIGHLY EFFECTIVE ENRP TEACHERS

While the three key components of the ENRP, as listed above, informed, involved, and potentially empowered the project teachers, it was the teachers, professional learning teams, and schools who ultimately made the decisions of whether and how the information and experiences provided within the project would impact upon their classroom practice. The approach taken fits with Doyle's (1990) Reflective Professional paradigm according to which both preservice and inservice teacher education should

> foster capacities of observation, analysis, interpretation, and decision making. … Within this framework, research and theory do not produce rules or prescriptions for classroom application but rather knowledge of methods of inquiry useful in deliberating about teaching problems and practices. (Doyle, 1990, p. 6)

Rather than a recipe, the notion of rich ingredients that are combined to meet the needs of individual children, the mathematics and the teaching context, using the professional judgement of teachers, was the approach taken. For this reason, the practices of effective teachers were not determined by the researchers and could not be anticipated.

The key criterion for selection of highly effective teachers, that is, student growth, was ascertained from interview results for 1999 and 2000 showing children's mathematical growth across the nine ERNP framework domains. The six case study teachers with high student growth were chosen to represent a cross section of grades with one teacher from

each of Grade Prep (first year of school in Victoria), Grade Prep/1 Grade 1, Grade 1/2 and Grade 2. One highly effective teacher of Prep children from predominantly non-English speaking backgrounds was selected also for study. The case study teachers had taught within the ENRP and at the same level for the three years of the project, and represented a cross-section of situations such as school location and school socio-economic profile. The teachers are later referred to by grade level (e.g., Ms Grade 1), except for the teacher of the non-English speaking background children: Ms NESB.

Being mindful of the need to avoid spurious conclusions, the case study methodology incorporated corroboratory and alternative sources of data (LeCompte & Goetz, 1982). The six teachers were studied intensively through use of the following data sources:

- five lesson observations by teams of two researchers (three consecutive days in the middle of the school year, and two consecutive days a couple of months later), incorporating detailed observer field notes, photographs of lessons and collection of artefacts (e.g., worksheets, student work samples, lesson plans);
- teacher interviews following the lessons (audiotaped and transcribed) to discuss the teacher intentions for the lesson, and what transpired;
- teacher questionnaires completed through the duration of the project; and
- teacher responses to other relevant questions and tasks posed to them.

Decisions needed to be made on the kinds of notes that would be taken in the lesson observations. The decision was taken to attempt to note as much as possible of what transpired in the lesson in a relatively "free" form. We were guided in this decision by the experience of others. For example, Stigler and Baranes (1988) conducted mathematics classroom observations in three countries using two methods. In the first, a structured coding scheme was used with an elaborate time sampling plan. It was therefore possible to obtain estimates of the percentage time given to various classroom activities. In the second study, the researchers "decided to trade the greater reliability of an objective coding scheme, for the inherent richness of detailed narrative descriptions of mathematics lessons" (p. 294). The ENRP research team made the same decision.

However, the research team did agree on a broad framework for the observations, interviews and analysis made up of nine categories: Mathematical focus; Features of tasks; Materials, tools and representations; Adaptions/connections/ links; Organisational style(s), teaching approaches; Learning community and classroom interaction; Expectations; Reflection; and Assessment methods. This framework was chosen to be quite broad, to avoid constraining our observations because of what we may have hoped to see. Our aim was to describe the practice of demonstrably effective teachers and to look ultimately for common themes, not to judge.

Many steps were involved in the case study data collection and analysis, beginning with detailed observer notes of each lesson using a laptop to record as much as possible of what was said and what happened, without interpretation. After each of the lesson observations by two observer/researchers, independent analysis of the lesson was carried out according to the categories agreed upon by the team and listed above.

Following the first three lessons each observer/researcher team worked together to produce a summary statement for the teacher. These statements were shared verbally with the research team in a meeting in which two "critical friends", not involved in the

research, then provided feedback on the kinds of themes they were hearing. This process occurred again after five lessons had been observed for each teacher.

	Effective teachers of Prep to Grade 2 mathematics . . .
Mathematical focus	• focus on important mathematical ideas • make the mathematical focus clear to the children
Features of tasks	• structure purposeful tasks that enable different possibilities, strategies and products to emerge • choose tasks that engage children and maintain involvement
Materials, tools and representations	• use a range of materials/representations/contexts for the same concept
Adaptions/ connections/ links	• use teachable moments as they occur • make connections to mathematical ideas from previous lessons or experiences
Organisational style(s), teaching approaches	• engage and focus children's mathematical thinking through an introductory, whole group activity • choose from a variety of individual and group structures and teacher roles within the major part of the lesson
Learning community and classroom interaction	• use a range of question types to probe and challenge children's thinking and reasoning • hold back from telling children everything • encourage children to explain their mathematical thinking/ideas • encourage children to listen and evaluate others' mathematical thinking/ideas, and help with methods and understanding • listen attentively to individual children • build on children's mathematical ideas and strategies
Expectations	• have high but realistic mathematical expectations of all children • promote and value effort, persistence and concentration
Reflection	• draw out key mathematical ideas during and/or towards the end of the lesson • after the lesson, reflect on children's responses and learning, together with activities and lesson content
Assessment methods	• collect data by observation and/or listening to children, taking notes as appropriate • use a variety of assessment methods • modify planning as a result of assessment
Personal attributes of the teacher	• believe that mathematics learning can and should be enjoyable • are confident in their own knowledge of mathematics at the level they are teaching • show pride and pleasure in individuals' success

Figure 1. Common themes emerging from ENRP case studies of effective teachers.

Following the lesson observations, interviews and meetings with the "critical friends", it was decided to use the original framework to describe the practices of effective teachers. For each of the nine categories, one researcher was responsible for identifying common themes within summary documents, and summarizing within tables. These tables were checked by the teams of researcher/observers to ensure accurate interpretation of teacher summaries and that all themes were substantiated by data collected within the study. The tables were also cross-referenced with reports from the "critical friends". From this

process a list of 25 practices of effective teachers evolved. It was agreed to list common elements where evidence was available for at least four of the six teachers.

THE PRACTICES OF HIGHLY EFFECTIVE TEACHERS

The description of effective teachers, as revealed in this study, is given in Figure 1 above. Because of their relative brevity as compared to field notes, a selection of classroom vignette summaries, teacher interview statements and observer/researcher summaries are shared below to illustrate practices of these teachers.

ILLUSTRATING EFFECTIVE PRACTICES

Two of the 25 effective teacher themes are illustrated below. The first is "Effective early numeracy teachers hold back from telling children everything."

In a lesson observed in Ms NESB's Prep class, the teacher worked with a group making two digit numbers. Each child chose a card with a two digit number written on. (The teacher had removed the 'teen' and 'decade' numbers.) The children then used base 10 blocks (tens and ones) to build a model of their selected number. Ms NESB watched the students build their models, which most children did correctly. Ahmed had the number 52, but he built a model of 25. The teacher watched, but did not intervene. After a few minutes, the teacher asked if everyone had finished building. She then told them each child would explain their model.

As Ms NESB gave these instructions, Ahmed looked at the other children's models, and changed his to correctly show 52. The teacher noticed, but said nothing. Several children were asked to explain their models. While this happened, Ahmed changed his model back to show 25. The teacher looked at his model and said, "Let's come back to you, Ahmed."

Several other children explained their models, and Ahmed again changed his model to correctly show 52. The teacher then asked the other children: "Is Ahmed's correct now?" The children all affirmed this and Ahmed was able to explain his model.

Ms NESB explained later why she did not draw the child's attention to the error. She believed Ahmed could correctly model 52, and that he would eventually correct this himself by comparing to other children's models. "Ahmed knew he'd made a mistake, it just took him a while, as soon as he started counting he realised he'd made a mistake." She involved the other children in confirming Ahmed's thinking.

A second theme is "Effective early numeracy teachers listen attentively to individual children."

This feature of Ms Prep's teaching was observed repeatedly over the classroom observations. Ms Prep seemed to talk directly to individuals when interacting with them. These discussions had the characteristics of an individual conversation, even when student initiated. The research/observer summary report following the first three visits included the following comment:

> [The teacher] was very positive towards the children. She praised them for their thinking. She showed caring and respect for individual children, giving them her attention even when within a group situation. She answered individual children's questions and followed their train of thinking, unless it was totally off the topic. In an interview following a lesson observation she

explained that if a child asks a question she will answer "even if … the rest of the class is rolling around the floor. I think, quick, I have got to explain that with this child so that they go away knowing or understanding what they have done."

Summary statements written at the end of the data collection by the researcher/observers for two other case study teachers included the following:

Ms Grade 1/2 was a caring teacher, who listened carefully to children and their strategies in solving problems ... She endeavored to find out where the children were at, and to extend and challenge them appropriately.

Ms Grade 2's classroom was a place where learning mathematics was taken seriously. It was also infused with enjoyment, success and appreciation. Expectations were very high in terms of both content and mathematical behaviours. Learning opportunities were carefully and thoughtfully structured for her children. Fundamentally each child was required to engage with a specific mathematical concept in a lesson and given some flexibility, choice or options within which to work. … Ms Grade 2 communicated with children in a highly personalised way and knew about the thinking of individuals.

A common element within these reports is the attention given by teachers to individual children and their learning through one-to-one interaction and focused listening, which commonly informed planning of appropriate learning experiences. This interaction was not dependent on class organization strategies, with, for example, Ms Grade 1/2 frequently working with a small group and Ms Grade 2 always observed to work with her class as a whole. It appears that it was the interaction features illustrated here, combined with others practices from the list of 25 provided in Figure 1 that enabled these teachers to be highly effective in their teaching of mathematics.

CONCLUSION

It is interesting to consider the extent to which the list of 25 teacher behaviours and characteristics presented in Figure 1 has application to other grade levels. We believe that similar research in Grades 4-12 (and possibly beyond) would yield many elements in common with these. As suggested earlier in this paper, there is potential for teacher education courses to be informed by the findings from this research on highly effective teachers of mathematics. If further research reveals such applicability then this has implications for all educators of preservice and inservice teachers of mathematics.

1The Early Numeracy Research Project (ENRP) was a collaborative venture between Australian Catholic University, Monash University, the Victorian Department of Education and Training, the Catholic Education Office (Melbourne), and the Association of Independent Schools Victoria. The project was funded in 35 project ("trial") schools and 35 control ("reference") schools. The views presented here are those of the authors.

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