MULTIPLICATIVE STRATEGIES OF NEW ZEALAND SECONDARY SCHOOL STUDENTS[1]

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Secondary school students' use of multiplicative strategies in an exploratory New Zealand Numeracy Project was examined. This Numeracy Project enabled teachers to interview each of their students concerning their mathematical knowledge and strategies. The percentage of students who used multiplicative strategies increased from initial to final assessment. However, the percentage of the students from two low socio-economic schools was significantly lower, both at the initial and final assessment, than that from two upper socio-economic schools. It is suggested that it may be inappropriate to expect secondary school students to repeat the progress through additive stages that Piaget reports for younger children. Instead, it may be better to move them directly to multiplicative thinking.

BACKGROUND

Gelman (1999) suggested that multiplicative concepts are not among the naïve mathematical concepts learned by all. Tirosh and Graeber (1990) and others have written about the difficulties that pre-service teachers have with mathematical concepts that involve multiplicative thinking. Yet many activities, including operating with rates and fractions, require the flexible use of multiplicative procedures. This paper discusses a project intended to help students develop multiplicative strategies.

The development of multiplicative concepts have been widely discussed by mathematics education researchers (e.g. Harel & Confrey, 1994). A quick review of the incomplete set of PME proceedings on my shelves shows that in 1988 there were working groups on rational numbers, in 1990 and 1992 on ratio and proportion, and in 1999 on multiplicative processes. The difficulty of acquiring such concepts is well known to researchers, but not necessarily to teachers.

For the purposes of this paper, multiplicative concepts are defined as any concept that requires considering groups of numbers as a single unit. Piaget (1985/1987) discussed multiplication as more complex than addition, as it involves implicit quantification. Students who operate multiplicatively know that there is a certain quantity in each of the numbers multiplied, but do not need to refer to the individual items or numbers in a group. He describes several stages that young children go through as they develop this understanding, with Stage IIB and III being truly multiplicative.

Mulligan and Mitchelmore (1997) also described a developmental model for young children's approaches to multiplication problems. Their model showed multiplicative concepts to arise out of additive ones. The developmental pattern that they described is

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similar to that used in the project described here and is similar to that commonly used in New Zealand schools. In this model, children move from direct counting to rhythmic counting, skip counting, additive calculation, and finally to multiplicative calculation.

Both Mulligan and Mitchelmore and Piaget describe the nature of multiplicative thinking used by young children, aged 7 – 10. Yet, as indicated above, many adults and older students fail to develop multiplicative thinking. Students continue to use additive calculation or repeated addition and do not move to multiplicative strategies. While using addition appropriately may give accurate answers, it is time consuming for more than the simplest problems and does not permit children to understand the more complex activities of finding a fraction of a number, working with rates, or linear algebra. These students fail to move to the stage of implicit quantification that Piaget refers to as seen in much younger children.

Despite researchers knowing that multiplicative concepts are difficult to teach and learn, New Zealand secondary school teachers discovered this anew in 2001 and 2002. Their discovery was the result of the introduction of a Numeracy Project (New Zealand Ministry of Education, 2002) for children from the ages 5 through 14. A major aim of this project was to alert teachers to the numerical strategies that their students used so that they could help them develop more advanced strategies and related numerical knowledge. While the project is generally accepted in elementary schools, its use in secondary schools is experimental. This paper examines the proportion of New Zealand students, in their first year of high school (age 14), who used multiplicative strategies when appropriate, both at the initial and final stages of the project, and the percentage of students who advanced to the use of more advanced strategies during the project. Factors that contribute to this change are given in comments by teachers, school administrators, and project facilitators. This paper discusses only a small portion of the data from this project.

METHOD

Participants

The secondary schools that participated in this project were those that either expressed an interest in being included or were asked by the Ministry of Education if they would be willing to be included. Results reported here come from four schools that were in the project for the second year in 2002. The students attended two schools in low socio-economic areas (N=189) and two in relatively high socio-economic areas (N=225). Schools in New Zealand are given a decile ranking based on the socio-economic background of the parents, with Decile 1 being the lowest and Decile 10 the highest ranking. Ethnicity of the students varied with decile ranking, with more students of Maori or Pacific Island ethnicity in the lower decile schools and more students of a European background in the upper decile schools.

The Numeracy Project

In the Numeracy Project all teachers assessed each of their students individually, using an assessment that took about 20 minutes each. (The Ministry of Education paid for other teachers to take their classes while they did this assessment.) The assessment covered strategies used for doing addition, multiplication, and proportion problems mentally, and knowledge of the number sequence, base 10 grouping, fractions and decimals. Stages for doing multiplication problems were: counting, skip counting, repeated addition, deriving answers from known multiplication facts ("early multiplicative"), and using a range of mental multiplicative strategies ("advanced multiplicative" or "proportional"). There were suggested procedures for helping students to move from their diagnosed level of knowledge or strategy to higher levels. A facilitator was provided for each school who explained the framework of the project, demonstrated the interviews, and helped teachers with their planning based on the results of the assessment. This facilitator also taught sample lessons in each class and watched the teaching of the classroom teachers, praising what was working well and making suggestions for ways to help students advance their strategies or knowledge.

Problems

Only the percentage of students demonstrating multiplicative thinking on the scales of multiplicative and proportional items are presented here.

Problems used in the assessment of multiplication were, in brief: given a grid of eight rows of five trees, how many rows would be added if 15 more trees were planted; if 3x20=60, what would 3x18 be; if 8x5=40, what would 16x5 be; how many muffins would there be in 6 baskets if there were 24 in each basket; and how many cars could be fitted out with 72 wheels. Proportional problems were: what is 1/4 and 3/4 of 28; 3/5 of 35; if 10 balls of wool made 15 beanies how many balls would be needed for 6 beanies; and what percentage of a class were boys if there were 21 boys and 14 girls. All problems were to be solved mentally.

Teachers scored items by the strategies that students used, as described in the previous section. For example, if students used a combination of multiplying and adding they were considered "early multiplicative" and if they found 3/5 of 35 by dividing and then multiplying would be considered advanced multiplicative.

Data gathering

Each school entered codes for mathematical strategies that individual students used on each scale on a national database. For this paper, I have analysed the strategies used on multiplication tasks and proportion tasks for the two top decile and two bottom decile schools involved. I also interviewed a sample of teachers, heads of mathematics departments, principals, and all facilitators.

Data from both 2001 and 2002 showed that students at a higher grade level started well below the level reached by students in the lower grade by the end of the year, so gains in strategies could not be attributed to maturity or existing tuition.

RESULTS

One facilitator reported that the main effect of the project was "teacher awareness". When the program was first introduced in 2001, teachers were shocked at the low level of achievement of their students.

> Principal: Some of the findings blew me out of the water. Place value, … we had taken for granted. Students had a veneer of knowledge…. Schools have to respond to where students are.

Perhaps the biggest shock in 2001 was that 43% of students from Decile 1 to 4 schools were unable to find 1/3 of 24, even if given counters. They did not appear to know what was requested of them. Only 8% of the students from the higher decile schools were unable to do this initially.

The full results for 2001 have been reported in Irwin and Niederer (2002).

The following table shows the initial and final percentage of Year 9 students, in lower and upper economic groups, who used multiplicative strategies for problems designated as multiplication and proportion.

Decile 1 schools N=189		Decile 8,9 schools N =225	
Initial	Final	Initial	Final
24%	34%	66%	83%

Table 1. Percentage of students who used multiplicative strategies from schools rated as of lower and upper socio-ecnomic status at initial and final periods of a numeracy project in 2002.

Statistical analysis (Newcombe, 1998) showed that a significantly smaller proportion of students in Decile 1 schools used multiplicative strategies, both at the start and finish, than did students in the Decile 8 or 9 schools ($p<.01$). An increased number of students from both groups came to use multiplicative strategies, but by following the recommendations of the project to teach the next higher stage, developmentally, students from lower decile schools had much less opportunity to become multiplicative thinkers because they started at lower stages.

	Decile 1	Decile 8,9
Students on ceiling initially	5%	37%
Total students gaining	40%	49%
Students gaining within additive strategies	23%	7%
Students moving from additive to multiplicative strategies	11%	25%
Students who gained within multiplicative strategies	5%	17%

Table 2. Percentage of students gaining at least one stage on the numeracy framework. All percentages are based on the number of students not already at ceiling.

In terms of the stages provided in the project, 40% of the students in Decile 1 schools, not already at the ceiling level, improved, and 49% of the students from the Decile 8 and 9 schools improved. Table 2 shows that their improvement was at different levels.

These data show that students did move up stages according to the hierarchy assumed by the project, a hierarchy also proposed by Mulligan and Mitchelmore (1997) for young children. However, adopting this progression left the students from lower economic areas still well behind their peers from more affluent areas. In accordance with the directions of the project, most low decile students worked on additive strategies, whereas most upper decile students worked on multiplicative strategies. With this emphasis, it is not surprising that more than twice the percentage of upper decile students progressed from additive to multiplicative thinking.

DISCUSSION

The main questions raised by these data are: (1) what brought about the increased use of multiplicative strategies, and (2) does this project, which emphasises methods used by much younger children, disadvantage lower decile students.

What brought about the change? Teachers reported that there had been major changes in their knowledge of individual students, and in their teaching. Teaching was different in each of the schools despite the suggestions from the project. Some reported a change from their existing pattern of whole-class teaching, usually using a textbook, to teaching skills and strategies that they had not previously taught, and to teaching in groups. Others reported adding an initial portion to their lessons on number sense, working from their students' known levels. None of these schools abandoned their usual curriculum, but they did give more time to numeracy than previously. Comments included:

> Teacher: They are finding the work within their means, so I can actually sit down with one or two or three students. It is that that is reaping the benefits. I am able to listen to them and hear what is going on in their heads and help them with the best strategy for them rather than doing one thing for the whole class.
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> Head of a mathematics department: Most people would say that their classes are happier. That doesn't mean that they are more saintly but certainly they are happier because they have things that they can do. The kids in the bottom group are much happier. It has been most successful for them.
>
> Facilitator: They are listening to their students, and moving from there.

Listening to students has been seen as essential to good teaching from Plato through to current educators. Constructivist classes are characterised by teachers listening to students and students listening to one another (e.g. Kamii & Warrington, 1997). Yet these secondary teachers had possibly been preoccupied by their own teaching agenda and not had the time to listen to their students. The interviews gave them the initial opportunity to listen, and facilitators helped them to continue to listen while in the project.

Does the project continue to disadvantage lower socio-economic students by encouraging them to move up through a framework developed for young children? This is a serious concern, especially as one hope was that the experimental project would prove to be remedial for this group. However, in using a developmental framework appropriate for young children the project developers apparently expected older children to move through the same stages. These students may have only two more years of schooling and are unlikely to spend much more time on numerical concepts. This suggests that the majority will leave school as additive thinkers. It might be more appropriate to introduce them directly to thinking about groups of numbers as units, with inherent quantification. One Head of Mathematics from a Decile 1 school commented that these students are overly dependent on algorithms.

> Head of Mathematics: We need to teach them to go back to skip counting. They see a hard multiplication problem and want to do it with the algorithm rather than seeing that they could multiply it by a larger number and subtract.

Many teachers have commented that when elementary school students have been through the project, this problem will not be seen in secondary schools. It seems unlikely that this problem will go away that easily. It would seem more important to introduce these secondary school students directly to thinking of nested quantities, as in Piaget's Levels IIB and III (Piaget 1983/1987). This would be more in the spirit of remedial programs for adults such as that introduced by Triesman (Mathematics Department, University of Illinois, 2002). Engaging the students in the value and power of multiplicative thinking as young adults could be more beneficial than expecting them to move up through the stages of young children.

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