

WHAT A SIMPLE TASK CAN SHOW: TEACHERS EXPLORE THE COMPLEXITY OF CHILDREN'S THINKING

Cynthia Nicol, Heather Kelleher and Carole Saundry

University of British Columbia, Canada

The Early Numeracy Project provided the opportunity for teachers to engage over a three year period in collaboratively developing resources to support early numeracy. Teachers field tested assessment items, and found one particular item to be particularly revealing about children's understanding of number. This paper discusses what it is about the task that captured teachers' interest. Task complexity, representational level, use of imagery and spatial reasoning, and the importance of disposition are described as possible ways to expand our lens beyond strategy use to more fully appreciate the complexity of children's thinking.

Research provides us with a variety of frameworks for considering children's development of progressively more sophisticated and complex understanding of number. Carpenter and Moser's (1984) 3 year longitudinal study of children's solution strategies for addition and subtraction, resulted in a framework for analyzing children's understanding of number, which provided the basis for Cognitively Guided Instruction (Carpenter & Fenemma, 1999). Steffe and Cobb's (1988) work on children's arithmetical meanings and strategies provided a framework that has been widely used, and is the basis for a number of subsequent projects related to children's mathematical thinking, such as the number framework developed by Wright (1998). The Early Numeracy Research Project (Clarke et al, 2001) built on some of this earlier work to provide sets of growth points for considering the development of children's mathematical ability. The British Columbia Early Numeracy Project [ENP] drew on these resources to shape an early numeracy assessment and follow up instructional resources.

The British Columbia Early Numeracy Project (Ministry of Education, 2003) was a three year collaborative initiative involving the B.C. Ministry of Education, University of British Columbia mathematics teacher educators, and teachers from six school districts in the province. The project's focus was to learn more about the ways to best assess and support the development of numeracy in the early grades. Project goals include involving teachers and researchers in: 1) the creation and use of performance-based tasks most appropriate for assessing numeracy in young learners; 2) the development and refinement of instructional strategies to support numeracy in school and at home; and 3) the development of reference standards on key assessment items that provide a portrait of young students' mathematical thinking. Sixteen project teachers extensively field-tested the ENP items while many teachers across the province field-tested items in their own classrooms as part of their school district's professional development initiatives on early numeracy. In the course of developing an early numeracy assessment with a focus on at-risk students, over 200

teachers field tested the assessment items. In professional development feedback sessions, teachers met to discuss the items, tried them with their students, then met again to analyze the results. Video tapes of children doing the items were used as the basis for these discussions, and were used in conjunction with a research based scheme to develop interpretation and scoring frames of reference (Kelleher, 1996; Nicol & Kelleher, 2003). Teachers were asked to comment on which ENP assessment items were most and least useful to them as teachers. In written and verbal feedback, 70% of the teachers ranked the Build and Change item above the 16 other assessment items. This frequent response prompted us to explore what it was about the task that teachers found so illuminating, and what aspects of mathematical activity were revealed by the task.

The “Build and Change” task involves students building a set of blocks, then changing the number as directed by the teacher, but first telling what they have to do to get the new number. Warm up examples are provided to ensure children understand the directions, and to ease them into predicting ahead of actually making the change. Directions are as follows:

Provide blocks or unifix cubes, and give students two warm-up examples:

“Show me 5 blocks. Now change it to 3 blocks.

What did you do?” (e.g., I had to take away two blocks.)

“This time tell me first...(fold your arms or sit on your hands)... how can you change your 3 blocks to 6 blocks? What will you need to do?”

You are working towards having the child tell you first what needs to be done. Provide another example if needed.

Examples for scoring:

- a) Change 6 to 4
- b) Change 4 to 8
- c) Change 8 to 5
- d) Change 5 to 12

Can the child predict what to do?

"Tell me first. What do you have to do to change your 6 blocks to 4 blocks?" You are looking to see whether and how the child says the set will need to be changed ahead of actually doing it.

Can the child solve the problem, and if so, how?

"Check to see if you are right."

Or, if there is no prediction, ask, "Show me how you can find out."

You are looking to see what the child does to actually make the change. You may need to ask, "How did you figure it out?"

The task generates a great deal of observable information and the teacher can ask questions for further information. For examples involving a decrease in the number (changing 6 to 4 and changing 8 to 5), it is possible to predict how to create the lesser number simply by visually analyzing the display of blocks. For examples where the number increases (changing 4 to 8 and changing 5 to 12), the child needs some way to construct the missing part, making these two examples even more challenging. Both addition and subtraction examples require an understanding of the question, the focus to attend to the given starting quantity, and the ability and inclination to respond verbally. If any of these were absent, the child was unable to complete the problems.

WHAT MAKES THE TASK SO INTRIGUING TO TEACHERS?

Using field notes collected during 15 professional development sessions and 140 written comment from teachers we analyzed what teachers thought of the Build and Change task and why they thought it was so intriguing.

Complexity of the task.

Many teachers who field-tested the Build and Change considered relatively simple yet were surprised by which students could and could not complete the task. Teachers found, for instance, that some children could not grasp the idea even after the two warm-up examples (in these cases the teacher would not continue). Some children, such as Josh (6 years, 1 month), successfully used the change idea in the scaffolded warm-up examples, but could not apply the idea when working independently without teacher help. Josh was successful with the warm-up examples but reverted to disregarding the starting sets. For example, in changing 6 to 4, he did not see how he could work with the existing 6 blocks, and instead said he needed to get 4 new blocks. He proceeded to build a set of four beside the starting group of 6, showing his four, and ignoring the 6 that remained. For changing 4 to 8, said he had to get rid of the four, moved them out of the way, and then brought back 8 new blocks. He was unable to build on to the given part for addition examples, and unable to see the part to remove in examples involving reducing the set. However, he was able to construct a requested set from scratch.

Variations in representation.

Teachers reported that they were surprised with how dependent many children were on sensory information to complete the task. They found that some children were unable to predict, and needed to actually move the blocks to make the changes. These children did not state ahead of time whether adding or removing blocks was required, even when asked that question. These children appeared to have no access to mental imagery for number, no personal strategies, and were unwilling to make an estimate. However, with the blocks and guided questioning, they were able to complete the changes. These variations in level of representation captured the interest of teachers.

Interviewers were interested in whether children who confidently completed the task using a direct modeling approach might be able to predict ahead of time what to do. Teachers, therefore, encouraged children to fold their arms beforehand to limit the sensory information. Mary (6 years, 4 months), for instance, in working on additive changing for the task, recounted the starting set then counted on blocks to reach the new total. When her interviewer asked her to fold her arms in an effort to see if she could predict how to change 8 to 5 before doing it, she complied but then proceeded to use her elbows to try to move the blocks. In subsequent examples she was able to imagine the change and describe it if it only involved a change of one or two.

Teachers were intrigued by the unique ways children supported their own reasoning. Some children appeared to be in the process of developing their own way of predicting changes by using a combination of manipulating materials, using visual aids, and attempting to access mental imagery. Adam (5 years, 4 months) in changing 6 to 4, easily analyzed the part/whole relationship and predicted that 2 needed to be removed. For the example of changing 4 to 8, Adam tried valiantly to “see” in his head the blocks that were needed. He was obviously trying to count on the needed blocks, but was unable to keep track of how many. He looked at the blocks, moved two, looked up and thought about it as he quietly counted, then added two more blocks and recounted. His actions showed he was beginning to mentally represent number, and had some imagery for counting on, but no tallying method to keep track of what he was adding other than using blocks to keep track of the change.

Use of strategies.

Estimating. Children’s strategy choices were of great interest to teachers. They observed examples of estimating, counting on or back, or using known facts and grouping, and were intrigued to see the connection of representation level to strategy use. Sam (6 years, 3 months), did not have a way to predict the change accurately, but was willing to make an estimate, then check by making the change. Shane used an estimate for every example he was given, saying “Oh I’d say you need about five more.” His estimates were usually close to the actual number needed, and sometimes were right on, but he had no way of checking other than actually moving the blocks. He required concrete models for all number work and did not appear to

have access to mental representation of number. He used direct modeling and counting all for working out each example, never used counting on, and always counted from one to establish how many after a perceptual change. However, his use of estimates showed a ball park understanding of when to add or subtract, a grasp of the part whole relationships involved in the task, and the confidence to take a chance with his responses.

Counting. Many of the grade one children used counting on or counting back to make their predictions, and all of these examples were accompanied by a variety of kinesthetic, verbal, or visual cues. Some children used taps or fingers or head nods to tally the count, some used self talk such as “6 to 5 is one, 5 to 4 is 2, you need to take away two.” Alex, when given “change 3 to 6” looked up at the clock and after a moment said “put three more”. When asked how he knew, he said he “counts on the clock”. His process of keeping track of the count involved very subtle body “nods” to accompany the count.

Grouping. Teachers were surprised by the children who didn’t count by ones, but rather used known groupings to figure out the change. Chantelle, when asked to explain her reasoning for “change 3 to 6” said, “Remember our doubles? I did like...I pretended these were like these (made a column of 3) and I put three more in” (making 2 columns of 3, a 3×2 array with the blocks). She drew on her knowledge of the known combination $3+3=6$ and the imagined arrangement of 3×2 to determine the change needed. Chris (6 years, 8 months) connected every example to familiar groupings, and carefully arranged the blocks into arrays of twos, threes, or fours. For changing 4 to 8, she said “put two more is six, and another two is eight”. For changing 5 to 12, looking at the arrangement of 3 and 2 and using her hands to show how to add columns of three, said “ make three, three, three, and three, four threes make 12”. She then carefully added one block to the two to make a second set of three, and two more sets of three to get a 3×4 array. Her ability to see groupings and think in groupings rather than using counting by ones was apparent in all tasks in the assessment. She appeared to have a very strong visual spatial sense for quantity, and manipulated spatial groupings and mental groupings, with confidence.

DISCUSSION

Throughout the early numeracy project the participating teachers developed a keen interest in the importance of understanding children’s thinking and how best to assess children’s thinking. Teachers reported that their opportunities to try out items like the Build and Change task and then collaboratively discuss their results with colleagues was key to broadening their interpretive frameworks for making sense of student thinking. Listening to what other teachers found or could see in their students’ work was an invaluable aspect of the project. Teachers also spoke about how seeing mathematical situations through the eyes of a child can be challenging. This is understandable. Once our adult familiarity with number concepts is established, and our skills become automatic, it is often difficult to reconstruct how a beginner might interpret and respond

to number situations. Yet, teachers stated that opportunities to explore students' thinking was intriguing and fascinating.

Field-testing teachers were surprised that not all children used counting strategies to solve the Build and Change task, and were impressed with the range of strategies children used. They reported being particularly intrigued by the variations in sensory information that children seemed to depend on to support their thinking. Field notes and written feedback indicate that some teachers described these aspects of representational preference in terms of increasing competence, with more visual, auditory and tactile/kinesthetic involvement at early levels, and a gradual process of internalizing reasoning as children developed a greater understanding of number and a greater ability to work mentally.

Teachers were also intrigued by the number of children who appeared to rely on visual spatial reasoning in thinking about quantities. Many children would arrange the blocks into arrays and appeared to use the rows and columns as scaffolds to thinking about groupings and part/whole relationships. This reinforced for them the value of using graphic organizers such as ten frames and arrays for number so that children could capitalize on their spatial reasoning and subitizing capacity.

They recognized the importance of attention, organization, and the ability to keep track of more than one aspect of a problem. They were fascinated by the importance of mathematical disposition and how it related to success with the tasks, particularly perseverance, curiosity, and confidence. Teachers noted how these dispositional and procedural aspects of competence are ones that appeared to have an important impact on performance.

The Build and Change task challenged teachers' preconceived ideas about what mathematics young students can do and the strategies they might use. They reported that this task provided an efficient yet powerful portrait of their students' thinking. Counting has been accepted as children's primary means for making sense of number, and the majority of work on early number learning has attended to the development of counting approaches. However through trying the Build and Change (and other ENP tasks) teachers found that some children, particularly those with a strong visual spatial sense or poor auditory-verbal skills, preferred other pathways to making sense of number. In particular, it highlighted representational level as it relates to strategy use, visual spatial reasoning, and dispositional and procedural aspects of competence. These are all areas in need of closer scrutiny in order to enhance our ability to better understand the development of children's understanding of number. The Build and Change task captured the complexity of children's mathematical activity related to early number, and expanded teachers' thinking beyond a focus on counting to more fully appreciate children's ways of knowing about number.

REFERENCES:

Carpenter, T.P., & Fennema, E., (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T.P., & Moser, J.M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education*, 15, 179-202.

Clarke, D. M., Sullivan, P., Cheeseman, J., & Clarke, B. A. (2000). The early numeracy research project; Developing a framework for describing early numeracy learning. In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000: Proceedings of the 23rd Annual Conference of the Mathematics Education Research Group of Australia*, (pp. 180-187). Fremantle, Western Australia: MERGA.

Kelleher, H. (1996). Making sense of number: A study of developing competence. Unpublished dissertation, University of British Columbia.

Ministry of Education (2003). *Early Numeracy Project Resources*. Province of British Columbia, Canada: The Author.

Nicol, C. & Kelleher, H. (2003). Learning to learn from students: Teacher learning in the British Columbia Early Numeracy Project. In N. Pateman, B. Dougherty, and J. Zilliox (Eds.), *Proceedings of the 2003 Joint Meeting of the International Group for the Psychology of Mathematics Education and North American Chapter of the Psychology of Mathematics Education (Vol 1 pp. 297)*. Hawaii: University of Hawaii.

Steffe, L. & Cobb, P. (1988). *Construction of arithmetical meanings and strategies*. New York: Springer-Verlag.

Wright, R. (1998). An overview of a research-based framework for assessing and teaching early number learning. In C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching mathematics in new times: Proceedings of the 21st Annual Conference of the Mathematics Education Research Group of Australia*, (pp.701-708). Brisbane: MERGA.

