

SUPPORTING TEACHERS ON DESIGNING PROBLEM-POSING TASKS AS A TOOL OF ASSESSMENT TO UNDERSTAND STUDENTS' MATHEMATICAL LEARNING

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The study was designed to support teachers on designing problem-posing tasks to understand students' mathematical learning. Seven classroom teachers and the researcher collaboratively set up a school-based team participating in an assessment project that assists teachers in implementing assessment integral to instruction. Four categories that the teachers generated problem-posing tasks included number sentences, pictorial representations or drawings, mathematics languages, and students' solutions collected in a class. Insight into teachers' understanding of students' learning was identified through students' responses to the problem-posing tasks. The tasks that students engaged in generating problems were a useful tool for assessing students' understanding and then informing teachers making further instructions.

Key words: assessment tasks, problem posing, assessment integral to instruction.

INTRODUCTION

The reformed curriculum suggested that every instructional activity is an assessment opportunity for teachers and a learning opportunity for students (NCTM, 2000). The movement emphasized classroom assessment in gathering information on which teachers can inform their further instruction (NCTM, 1995). Assessment integral to instruction contributes significantly to all students' mathematics learning. The new vision of assessment suggested that knowing how these assessment processes take place should become a focus of teacher education programs.

Problem-posing task referred to in the study was that the task teachers designed requires students to generate one or more word problems. The professional standards suggested that teachers could use task selection and analysis as foci for thinking about instruction and assessment. According to De Lange (1995), a task that is open for students' process and solution is a way of stimulating students' high quality thinking. Training teachers in designing and using assessment tasks has also been proposed as a means of improving the quality of assessments (Clarke, 1996). However, the design of open-ended tasks is a complex and challenging work for the teachers who are used to the traditional test. Thus, the tasks involving in the study were considered as an informal way of assessing what and how individual student learned from everyday lesson. Thus, the preparation of the tasks involving in this study was not prior to instruction; rather, teachers generated them from the activities in which students engaged in everyday lesson. The mathematics contents covered in the textbooks were a dimension of the assessment framework of the study.

The reformed curriculum calls for an increased emphasis on teachers'

responsibility for the quality of the tasks in which students engaged. The high quality of tasks should help students clarifying thinking and developing deeper understanding through the process of formulating problem, communicating, and reasoning (MET, 2000). Thus, these cognitive processes were the other dimension of the assessment framework. The tasks teachers designed in the study were to assess students' problem posing, communicating, and reasoning. Due to the limitation of space, this paper is primarily concerned with the problem-posing tasks.

Problem-posing is recognized as an important component in the nature of mathematical thinking (Kilpatrick, 1987). More recently, there is an increased emphasis on giving students opportunities with problem posing in mathematics classroom (English & Hoalford, 1995; Stoyanova, 1998). These research has shown that instructional activities as having students generate problems as a means of improving ability of problem solving and their attitude toward mathematics (Winograd, 1991). Nevertheless, such reform requires first a commitment to creating an environment in which problem posing is a natural process of mathematics learning. Second, it requires teachers figure out the strategies for helping students posing meaningful and enticing problems. Thus, there is a need to support teachers with a collaborative team whose students engage in problem-posing activities. This can only be achieved by establishing an assessment team who support mutually by providing them with dialogues on critical assessment issues related to instruction.

Problem-posing involves generating new problems and reformatting a given problems (Silver, 1994). Generating new problems is not on the solution but on creating a new problem. The quality of problems in which students generated depends on the given tasks (Leung & Silver, 1997). Research on problem posing has increased attention to the effect of problem posing on students' mathematical ability and the effect of task formats on problem posing (Leung & Silver, 1997). Such problem-posing tasks that situations were presented in a story form were created by researcher rather than by classroom teachers. Moreover, there is a little research on teachers' responsibility for the variety and the quality of the problem-posing tasks. The way in which teachers explored to create tasks for students generating problems from a contrived situation was investigated in the present study.

THE ASSESSMENT PRACTICES IN MATHEMATICS CLASSROOM PROJECT

The Assessment Practices in Mathematics Classroom (APMC) project funded by the National Science Council was designed to develop a teacher program in which supports teachers on practicing assessment integral to instruction. An aim of the project was to assist teachers to recognize how students develop their learning with understanding, and how this can be supported through the program. To reach the aim, encouraging teachers used mathematical journal as an informal way of gathering the information about students' thinking processes, strategies, and their developing mathematical understanding to assess individual entire learning process by writing about mathematics. Assessment tasks as the prompts of mathematical journal that was served to establish a better means of communication among students, parents, and

teachers about mathematics learning taking place in classrooms.

Supporting teachers on generating mathematical tasks was the kernel part of the APMC project, the concerns included that: 1) supports a method of assessment that allows students to demonstrate their strengths; 2) stimulates students to make connections between mathematical ideas; 3) promotes high quality of problem-posing, communicating, and justifying one's way of thinking; 4) generates creative tasks that do not separate mathematical processes from mathematical concepts; 5) generates the tasks for assessing what and how students learned from a lesson. To generate the high quality of the tasks from everyday lesson, the tasks covered in each journal including one or two problems were reasonable.

The rationale of the APMC project was a social constructivist's view of learning. Learning is viewed as the product of social interaction in a professional community (Vygotsky, 1978). Therefore, activities related to generating assessment tasks were structured to ensure that knowledge was actively developed by the teachers, not imposed by the researcher. The generation of assessment tasks as the part of practices of assessment integrated into instruction was initiated from teachers' everyday instruction and modified by professional dialogues. Thus, the teachers frequently observed, discussed, and reflected to the quality of assessment tasks altogether.

The study reported in this paper was focused on how a school-based professional development program supported teachers on designing problem-posing tasks as a tool of assessing students' mathematical understanding. Two research questions require to be answered: What supports did teachers need when they created assessment tasks initiated from instruction? What kinds of problem-posing tasks did teachers generate from everyday instruction for assessing students' understanding?

METHOD

To achieve the goal of the study, a school-based assessment team consisting of the researcher and seven teachers was set up to discuss the assessment issues which occurred in a classroom by comparing to others'. Because the same mathematical content lent itself to a focus and similar pedagogical issues addressed drew attention from each teacher, leading to in-depth discussions, the seven teachers were recruited from two successive grade levels. The second- and third-grade classrooms were the primary contexts for teachers learning to generate tasks. Participations in the regular weekly meetings were the other primary context for the teachers improving the quality of the tasks. The three second-grade teachers were P2, Q2, and R2; four third-grade teachers were A3, B3, C3, and D3. Their teaching experiences were ranged from 3 to 16 years. The role of the researcher was not to provide ready-made tasks for the teachers to use, but to create the opportunities for teachers sitting together to design creative assessment tasks for students.

The teachers had little knowledge of assessment integral to instruction, so that classroom observation was used as a means of increasing their awareness of generating tasks initiated from the lessons observed altogether. The teachers had routine weekly meetings lasting for three hours. The meetings gave teachers to share

creative tasks mutually and to rethink if the tasks gathered information of students' in-depth understanding. At the very beginning of the study, I encouraged them to create at least an assessment task each week integral to their teaching. The assessment tasks were encouraged to be open-ended questions contrived by the teachers as part of homework that students completed after school. Moreover, they required bring students' responses to the tasks for others to analyze. Besides, each teacher required reporting in the meeting what they learned from the tasks they administrated and what information they gathered from students' responses to the tasks.

Data was gathered through classroom observations, problem-posing tasks, regular weekly meetings, and students' responses to the tasks. The routine weekly meetings were audio-recorded and the lessons were video-recorded. The audio- and video-tapes were transcribed to be faithful as possible to the teachers' exact words. Students' responses to the tasks were transcribed as possible to students' exact words to produce readable English.

RESULTS AND DISCUSSION

The tasks that students generated problems from a contrived situation were categorized according to the following four elements: 1) number sentences; 2) pictorial representations; 3) mathematical languages; 4) students' various solutions collected in everyday lesson. Each of these elements is addressed below and includes the problems that students generated and the supports of assessment incorporating into instruction that the assessment team provided.

Category I: Giving a number sentence to create word problems

Designing a creative task that students generate word problems is a new experience for the teachers. A typical problem-posing task the teachers designed was given by a number sentence. The typical task was occasionally covered in the current textbooks. This was the only category that the teachers designed in the very early of the study. The task given by a number sentence was bounded to the mathematics contents that the teachers taught.

It is found that the task dealing with problems generated from a number sentence helped teachers perceived the difficulties students encountered. The teacher A3 reported that either the multiplicand 1 or 0 is particular difficulty for her to explain to students. Hence, she conducted the following Task 1 to examine if students recognized the meaning of multiplicand 1. Task 1 and three students' responses to the task are displayed as follows.

Task1: If you were a teacher, how would you give your students a problem situation represented by $1 \times 5 = ()$? Write it down in words." (A3, 10/12/2000).

Wu: There are five third-grade classes in Din-Pu School. There are clocks in each class. How many are clocks there altogether?

Hwei: There are freezers in each house. How many are freezers in the five houses?

Sue: A cow produces a bucket of milk. How many buckets do 5 cows produce totally?

After analyzing students' responses, she realized 11 of 35 students still having the difficulty with understanding the multiplicand 1. Of the responses, students were not able to distinguish the difference between the multiplicand 1 and integer 1. "A group of 5"¹ referred to in textbooks was expressed by $5 \times 1 = ()$. Wu and Hwei had the same misunderstanding with the meaning the multiplicand 1. Wu understood incorrectly "1" for $1 \times 5 = ()$ as "Each class has clocks" instead of "Each class has a clock".

One of the teachers, B3 recommended A3 bring one of the two improper problems to classroom to ask students to repair it. Next day, A3 acted as though she needed help and then asked students, "Is it wrong? [There are five third-grade classes in Din-Pu School. There are clocks in each class. How many are clocks there altogether?]. Could you help me to repair it so that it can be solved?". As observed, the majority of students devoted to repairing the improper problem.

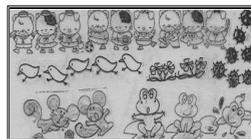
It shows that the task allowed A3 to gain insights into the way students constructed mathematical understanding. The improper problems that students generated were served as the indicator of their unclear understanding. These improper problems made profitable when asking students to repair them and informing teachers' decision-making. Thus, correcting misconceptions or repairing the improper problems that students responded to the tasks became a common activity for the teachers at the very beginning of each class.

Category II: Giving a picture or drawing to formulate word problems

With the exception of the task by a given number sentence, it is hard for the teachers to create a new task without supports. The researcher, as a learning partner of the teachers, shared the possible tasks referred to the literature of problem posing to the school-based assessment team. Interestingly, the teacher P2 created a new task by a given picture. P2 with more than six years of teaching in second grade perceived that the beginners of learning multiplication should understand clearly on the concept of multiplication. She conducted the Task 2 to assess if her students understood the meaning of multiplication.

Task 2: This is a picture about the princess and 7 dwarfs.

If you were a teacher, what word problems would like to formulate? (P2, 10/29/2000)



The problems Horng created as follows.

- 1. There are 2 mice. Each mouse has 2 legs. How many legs are totally?*
- 2. There are 4 flowers and each has 5 leaves. How many leaves are there?*
- 3. There are 7 bugs. Each bug has 6 legs. How many legs are the bugs totally?*
- 4. There are 7 dwarfs. Each dwarf has 2 eyes. How many eyes are there totally?*

The day after the lesson, the teacher P2 brought the bug problem Horng provided

¹ The textbook of my country dealing with "a set of 5 apples" as "one five" "five times one" " $5 \times 1 = ()$ " is not consistent with those of other countries. Students do not learn the multiplication until they are in the second grade.

into the classroom to ask students to solve it. During the class, Horng persistently kept an eye on seeing if his problem was solved. After grading, 8 out of 32 students still hardly made the distinction between $6 \times 7 = ()$ and $7 \times 6 = ()$ (Observation, 10/30/2000). Later on, R2 reacted to the lesson in the weekly meeting followed immediately the observation and said:

“Students did not look the problem carefully to identify which is the size of each unit or the number of unit; as a consequence, they misused the two expressions. This confusion occurred in my class as well (R2, Meeting, 10/31/2000)”.

This confusion became as a focus of the professional dialogues. B3 shared her last year experience of teaching second-graders and suggested that it is necessary to explain “ $6 \times 7 = ()$ ” meaning “6 is the size of each unit and 7 is the number of the unit”.

We noticed that the teacher P2 got the supports from the others and also supported to others of the professional team. Again, at the very beginning of the third day of the lesson, P2 made a remediation for her students to clarify the meaning of “6 sets of 7 meaning 7 times 6 and can be expressed as $7 \times 6 = ()$ ”. From P2 sharing the task, R2 got an insight and adapted it into as the Task 3.

Task 3: Using the figure to generate a word problem.



Category III: Giving a mathematical language to formulate word problems

The task 4 that P2 conducted was to assess if her students understood the of language “6 sets of 5”, “5 times 6” and connecting to $5 \times 6 = ()$ ”.

Task 4: (1) Draw a picture and create a word problem for “6 sets of 5”.

(2) Draw a picture and create a word problem for “5 sets of 6”. (P2, 11/02/2000)

P2 brought students’ responses to the task 4 to a weekly meeting. After analyzing students’ responses, we found that more than 80% of her students understood well the distinction between “6 sets of 5” and “5 sets of 6” whatever it was displayed in a picture or a word problem. Only 2 out of the 30 students still were confused with the distinction between these two terms. Based on the classroom observation and professional dialogues in the weekly meetings, the teacher D3 with only one year of experiencing second grade reflected to her last year teaching and stated that

“I finally realized why my students had the difficulty with using a number sentence to represent a multiplication problem. It is resulted from my neglect of the significance of understanding the meaning of multiplication (P2, meeting, 11/06/2000) ”.

Category IV: Displaying students’ solutions to formulate word problems

In the midterm of the first semester, we observed a lesson related to two-digit subtraction without regrouping. As observed, the teacher Q2 and students engaged in discussing three students’ solutions of the problem “Tom has 39 dollars. He needs 15 dollars to buy a sandwich. How much money does he have now?. Using mathematical expressions represents your solution”. The three solutions were:

<i>Yee:</i> $39 - 25 = ()$	<i>Mei:</i> $39 - 25 = ()$	<i>Tai:</i> $39 - 25 = ()$
$30 - 20 = 10$	$39 - 20 = 19$	$29 - 25 = 4$
$9 - 5 = 4$	$19 - 5 = 14$	$10 + 4 = 14$
$10 + 4 = 14$		

Q2 asked the three students to come to the front of the classroom to explain their solutions and justify their thinking to others. We, as observers, were surprised with students articulating their own thought so clearly. However, C3 suspected if those students who were silent in the class understood what the classroom happened. Interestingly, in the homework of the day, Q2 generated the following Task 5 including two of the solutions displayed in the classroom to examine if students understood the discussions. The task Q2 conducted was initiated from her everyday teaching.

Task 5: We solved a word problem in today lesson. The solutions were given by Yee and Mei as follows. Would you please to write possible problems they solved?

<i>Yee: $39-25=()$</i>	<i>Mei: $39-25=()$</i>
$30-20=10$	$39-20=19$
$9-5=4$	$19-5=14$
$10+4=14$	

Students' responses to the Task 5 were sorted into three categories. The first is the problem was the same as the one solved in the classroom. The second was the new problems students created correctly. The third category shows parental interventions with students' work but it is unreasonable. For instance, "*Dad gave Joe 30 chocolates. Joe ate 20 of them. Mom gave him 9 more chocolates. Joe ate 5 more. How many chocolates does Joe have now?*" and "*Dad gave Eddy 39 chocolates. Eddy ate 20 of them. A day later, he ate 5 more. How many chocolates does Eddy have now?*".

Because students' unreasonable work with parental intervention was not acceptable by the teacher, several parents therefore learned from this task about the role of working with students' assignments to be as a supporter instead of a provider.

The task 5 indicates that the teachers were available with the tasks generated from everyday teaching. The teachers participating in the study found that classroom discourses on mathematical ideas became a resource of conducting such kind of task. Through the entire year, generating assessment tasks incorporating into everyday classroom teaching became as part of the mathematical instruction. This is a natural source and never terminates to initiate assessment tasks.

DISCUSSION

This study supported the teachers on creating an environment in which problem posing is a natural process of mathematics learning. The supports included the strategies of helping them in designing problem-posing tasks integral to instruction and the variety of tasks that situations require students formulate problem. As a result, the supports contributed the teachers to optimizing the quality of assessment and instruction, and thereby optimized the learning of the students.

For teachers, the problem-posing tasks allowed them to gain insight into the way students constructing mathematical understanding and served to be a useful assessment tool. As an assessing tool, the tasks incorporating into everyday instruction, decisions about task appropriateness were often related to students' communication of their thinking, or the students' problem-solving strategies

displayed in classroom. The mathematics concepts to be taught at a grade level became as an elementary element of designing assessment tasks integrated into instruction. Other decisions concerning the appropriateness of a task were relevant with teaching events students encountered in everyday lesson.

The assessment task integral to instruction referred to in the study was characterized by the tasks conducted by the teachers collaborating with the researcher. The tasks created by the teachers and modified by the assessment team are more likely to improve the quality of the tasks. The result supports Clarke's claim (Clarke, 1996). The intervention of the researcher contributed more to the theoretical perspectives of problem posing, while the involvement of the teachers devoted to their classroom assessment practice. Comparing to the assessment tasks generated by individual, sharing multiple perspectives of appropriateness of task in a school-based assessment team was likely to achieve the purpose of task and the variety of task.

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