

A MEASURE OF RULERS - THE IMPORTANCE OF UNITS IN A MEASURE

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Recent publications continue to show that significant numbers of students in junior grades, while competent in basic ruler skills, seem not to have acquired important concepts about how rulers work and units of length. This paper reports on the results of a set of tasks completed by students from Grades 5 and 6. The results show that many students at the end of their primary schooling are unable to identify the unit of measure for length on a ruler or on the commonly used one centimeter cube. It is suggested that early measurement activities include explicit instruction in the relationships between informal units and the construction of scales on rulers.

Introduction

In recent years increasing attention has been paid to the teaching of measurement in mathematics lessons. Analyses of data gathered through regular large-scale testing by The National Assessment of Educational Progress (NAEP) show that while students have shown steady overall improvement in basic measurement skills and concepts since 1990 there appear to be significant gaps in student understanding of how scales on formal measuring tools work (Strutchens, Martin and Kenney, 2003). This becomes apparent when students are asked to measure lengths not aligned to zero or when the scale to be used has no numbers on it. Students seem not to have constructed adequate understandings of the property of length (Wilson and Rowland, 1993) and of the linear nature of units of measure (Bragg and Outhred, 2001). It is also apparent that while most students by Grade 5 appear competent with basic paper and pencil measurement and construction tasks many students are also unable to indicate *what is being counted* in the measurement process (Bragg and Outhred, 2000a).

Hiebert (1984) suggested that the discrepancy between procedural knowledge and conceptual knowledge may lie in the student's failure to link classroom experiences with the formal symbols. This may occur, for example, at the point where understandings about units of measure become represented in the markings and numerals on a scale (Stephan and Clements, 2003). "The hash marks and numerals on a ruler therefore, represent the result of iterating 12 inch-sized units" (p.4). Many students, however, may come to understand measuring length solely as an exercise in applying some rules for the alignment of an object and the reading of a number (Bragg and Outhred, 2001).

Bragg and Outhred (2001) showed that significant numbers of students in Grades 3-5 are unclear about what is being counted when they use one-centimetre cubes to measure length even though they were able to align and count them correctly. This is important because students in Grade 3 use the same cubes to measure area, perimeter

and then volume. This confusion is also apparent when students are asked to indicate which feature of the scale on a ruler is counted when measuring a length. Younger students were more likely to colour in the spaces between the unit markers while older students were more likely to count the unit markers (hash marks) themselves. For these students "...the marks on the ruler "mask" the intended conceptual understanding involved in measurement" (Stephan and Clements, 2003, p.5). Some students simply did not believe that anything was counted at all, believing instead that the number at the end of the object was the measure.

The process of *iteration* is a fundamental concept that must be learned early in the measurement curriculum (Barrett, Jones, Thornton, and Dickson, 2003). Once a unit has been selected the measure obtained by counting tells how many of these units, placed end to end, are used to cover the length of the object. The tendency to count unit markers when the scale is unfamiliar would seem to indicate that students may have connected the iteration of informal units with the most prominent feature of on the ruler, the unit marker, even though the unit markers are at right angles to the *length* of the object or line being measured. This indicates that some students may understand the measurement of length using informal units and applying a ruler to be two separate skills. The first skill involves the correct use of a 'count-the-object or -action' process to determine a length while the second skill uses rules about the correct alignment and reading to obtain a measure with a ruler. It has therefore been suggested that teachers should not rely on paper-and-pencil tests of measuring as an indication that students have acquired a deep understanding of units and scales (Bragg and Outhred, 2000b).

Results of both NAEP and TIMMS (Lokan, Ford and Greenwood, 1996) indicate that the difficulties continue into the high-school years. Since the results previously reported by Bragg and Outhred (2000b, 2001) covered Grades 1-5 it was apparent that information was needed for Grade 6 students who had completed their foundational instruction for the measurement curriculum. An investigation of this type is timely as research is beginning to point to the need for teachers to focus on the meaning of the numerals on scales (Clements, 1999), and the conceptualisation of a length as a movement in space (Lakoff and Nunez, 2000) away from a point of origin that becomes zero on a scale (Lehrer, Jaslow and Curtis, 2003).

Methodology

Two studies are reported in this paper. A comparison is made of the results for Grade 5 students in the first study with the results of Grade 6 students using 5 of the tasks from the original study. The Grade 5 data form part of a larger study involving 120 students from Grades 1-5 (aged 6-10 years) from three state primary schools in a medium to low socio-economic area of Sydney. The second study involved 89 Grade 6 students (aged 11-13 years) from a non-selective private girls school in northern Sydney. Following the survey, the Grade 6 students were interviewed in small groups of 5 to 7 to try to ascertain what the students understood about the concepts that were tested. The first researcher interviewed all students towards the end of the school year when they had completed the year's instruction in measurement.

Tasks 1 and 2 were designed to force students to apply their knowledge of rulers in unfamiliar contexts. Task 1 was an ‘offset- ruler’ question requiring the students to state the length of a shoe printed above a ruler between the 3 and 8 cm unit marks. Task 2 asked the students to measure an 11cm line printed above a ruler without numerals. Tasks 3 and 4 required the students to mark centimetre units on a ruler or centimeter cube. Task 5 asked students to draw what they thought one centimetre would look like if they could see it between the forefinger and thumb drawn on the page.

The tasks have been grouped below to emphasise their conceptual similarities. They were presented in the same order to all students but were not presented in the order seen below.

Table 1 Tasks involving a scale

Task	Description	Knowledge
1	Measure object above a ruler printed between the 3 and 8cm marks.	Length may be measured by counting spaces on a unit scale. A numeric scale can be applied to a congruent set of marks.
2	Measure a line using a ruler with unit markers but no numerals.	As for Task 3.

Table 2 Tasks involving identification of linear units

Task	Description	Knowledge
3	Draw the linear unit on a picture depicting a familiar representation of a centimetre: thumb and forefinger placed 1cm apart.	Identification of the linear unit in a pictorial representation.
4	Identify the linear units on a ruler for a given measure.	Linear units are separated by marks. A numeric scale aligned with marks gives the number of linear units from the origin.
5	State what part of a 1cm cube is used when measuring a length.	The length of an object gives the measurement unit (its area and volume are irrelevant).

Results and Discussion

The results for Task 1 show that the number of students able to state the correct length improved in Grade 6 (69%) compared with the Grade 5 students (50%). Almost a quarter of the Grade 6 students (24%) gave the measure as the numeral aligned with the end of the object. In the small group discussions students were asked why they used this strategy. Ellie’s response was typical of the 21 in this group: “You just read the number at the end.” Suzie continued, “That’s because you can’t move the ruler.” When challenged to re-measure the shoe with their own ruler, most were able

to see why their original measures were incorrect. Several, however, remained unconvinced, unsure whether to measure from one.

Unlike the Grade 5 students who were more likely to count the spaces, the successful Grade 6 students counted the unit markers from zero. There was also an increase in the number of students who used the most sophisticated strategy of finding the length by subtraction; (12% versus 3%). Few students use this method even though it is used to find a remainder in word problems. Students fail to recognize the ‘offset-ruler’ task as belonging to the class of ‘difference’ problems. The length of the object ‘offset’ on a ruler may be thought of a subset of the length between zero and the end of the object when aligned with a ruler. Classroom tasks explore the concept of additivity, so it would seem to be important for teachers to include tasks that require students to find measures by counting units on scales not aligned with zero. Students need explicit instruction in the use of the ‘rename as zero’ or the counting of the spaces between unit markers provided that students understand that the measure itself counted linear subunits. A number of students stated that there was “...nothing on the edge of a ruler” and that the numbers “pointed to the lines” [unit markers].

Table 3 Results as percentages correct for Tasks 1 to 5

Task	Name	Grade 5	Grade 6	Comments for Grade 6
1	The Offset Ruler	50%	69%	Measure as the end of the object 24%
2	The Ruler Without Numbers	54%	37%	Count unit marks from one 53%
3	The Finger-Thumb Picture	54%	74%	Draw cube 7%, square 6%, ruler features 11% (e.g. little unit markers or numbers)
4	Units on a Ruler	42%	6%	Indicated unit markers 69%, 21% coloured spaces
5	A Centimetre on a Cube	52%	71%	Indicated square 20%, cube 8%

The results for Task 2 were unexpected. The number of students giving the correct measure fell to 37% in Grade 6, with just over half of this group (53%) counting the unit markers from ‘1’. This is compared with 54% of the Grade 5 students. In the discussion groups the majority of the students quickly saw that their measure was incorrect when they checked their answers with a ruler. Many of them noted that they should have counted the ‘first little mark as zero’. As with Task 1 it seems that the older students were more likely to count unit markers than the spaces between the markers. Esther noted that she “just forgot about the zero”. The students were asked why they thought they got confused about where to measure from on a ruler. The majority commented that the measuring they did in the early grades only involved counting from one. Georgia said that the teacher in her previous school told them that “When you use a ruler you don’t have to count anything. You just remember to line

up the start with zero and read the number at the other end. Mrs. Z. used to get a red marker pen and draw over the nought so we would remember. I always got it right then.”

In Task 2, the unexpected drop in numbers of correct answers in Grade 6 may be due to the fact that greater emphasis is being placed on more complex measurement applications with little time being spent on revision of basic concepts. These results reflect the way students understand the use of a ruler and the cues used for correct alignment. The zero printed at the first unit marker may serve only to direct alignment, while the number corresponding to the end of the object may be seen as the ‘measure’ rather than the count of linear sub-units. Indeed, some students stated that the number in a measure only told “...where the line ended’. The absence of a zero as a cue therefore elicited a counting-from-one action of the unit markers.

In Task 3, (the Finger-thumb drawing) 74% of the sixth graders (54% in Grade 5) were able to represent a centimetre as a linear unit. It is interesting to note that almost all of the centimetre lines were drawn with a unit marker at either end. The prevalence of these features may be partly due to their prominence on rulers and the emphasis that is placed on them as students learn to mark off the iteration of informal units in early measuring activities. Those who were not successful were almost as likely to draw a square or a cube (13%) as they were to produce a drawing with ruler-like features (11%). The confusion with squares and cubes may have resulted from the use of cubes for area and volume measurement without explicit reference to the feature of the cubes that is used for different measures. The most common ruler-like features were the numbers to 10 or a set of 8-10 tiny unit markers. The belief that the unit marker is the unit of measure was also seen in Task 4 where students had to indicate the centimeters on a ruler. Five students (6%) in Grade 6 correctly drew over the linear units at the edge of the ruler while 61 (68%) marked the unit markers and 19 (21%) coloured in the spaces.

This confusion was also seen in the results for Task 5 (identifying a centimeter on a cube). While the results also show an improvement from Grade 5 (52%) to Grade 6 (71%), it is significant that almost 30% of these students about to enter high school remained confused about the property applied in different measuring contexts where cubes are used. In the small groups almost all the students who drew or coloured squares or cubes referred to the use of the small cubes used in the measurement activities. “Well, it doesn’t really make a difference ‘cause you just count the cubes” (Sarah, Grade 6). “You don’t count the edges unless the question is about ‘edges and faces’...Oh, I know, what about the pictures in Signpost, [referring to the student text] we did it in Ms L.’s class. We lined up the shorts and counted them” (Emma, Grade 6). Teaching guidelines state that students should have a clear understanding of the attribute that is being measured (Outhred, Mitchelmore, McPhail and Gould, 2003). If ‘shorts’ are used for all for length, area and volume, then it is important for teachers to ensure that students understand which feature is counted for each measure.

Conclusion and implications for teaching linear measurement

Research into effective classroom instruction over the last 30 years has yielded important information into fundamental understandings about measurement. The findings reported here confirm that the sequence of early measurement tasks needs to be carefully considered in relation to what students must know about how a scale works and its construction. Of particular relevance is the students' belief that the unit markers are the measures. These markers are, however, at right angles to the length of the object or line being measured and should only be understood as the feature that marks the end of each unit. It would, therefore, seem critical that young students should learn that a length is a linear entity that can be defined. The point of origin can therefore be identified and written as zero. The current practice of filling the space between the endpoints of an object with a line of informal objects called the 'units of measure' may encourage students to consider these objects as they would for exercises in the counting of discrete objects. The continuous nature of measures calls for a counting action that does not use a 'point-count' action but rather a counting action reflecting the essential movement from the point of origin. This is only possible if students are able to see or visualise the linear units and that the count is made by moving a finger for example, along the units counting as they reach the end of the unit. The purpose of the unit marker may be more easily seen as the point where each unit starts and ends. The unit markers on rulers are particularly prominent features that, in the absence of careful instruction about their function, may become the focus of a student's counting.

A line used to represent a length also satisfies the requirement that students should learn to identify the attribute (Outhred et al. 2003) and helps students discriminate it from the units used to measure length and volume. The consequence of using a concrete representation of a length is that the unit of measure will be defined by the length of object or action chosen as the informal unit. A line of objects, for example paper clips, allows the students to draw the unit markers and when the clips are removed the linear units thus created are counted from zero not the unit markers or the informal units. An analogous relationship is created that has a closer fit to the form of a scale. This relationship may also be satisfied when students iterate a single unit or use an action such as hand spans. Early measuring activities should emphasise the measuring process as having three steps: 1) Defining the length and assigning zero, 2) iterating the unit and 3) counting the linear units created. Teachers should also be aware that counting skills and strategies used to find a measure should be introduced and discussed with the students. Since a correct measure can be found by counting the spaces or unit markers it is critical that students understand what the measure obtained actually counts.

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