

GRAPPLING WITH COMPLEXITY: CO-LEARNING IN INQUIRY COMMUNITIES IN MATHEMATICS TEACHING DEVELOPMENT

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The complexity of mathematics teaching involving both cognitive and sociosystemic factors makes development problematic. Examples from research into teaching reveal factors in complexity and ways in which inquiry between teachers and didacticians can foster deeper ways of knowing in the developmental community. Research is seen as the basis of an inquiry process which develops inquiry in mathematics teaching as both a tool for teaching and a way of being for all learners. Theoretical discussion on the concepts of inquiry and community and their relation to development leads to consideration of approaches in which mathematics teaching development is the object of a design process. Tensions between learning theory and proposals for developmental practice are revealed and addressed. Research that is designed to create and study inquiry communities is introduced.

INTRODUCTION

The question of how mathematics teaching does or can develop and how that development is sustainable is at the centre of this paper. Research is revealing increasingly valuable insights to mathematical learning, both in terms of mathematical content and of processes in constructing mathematical ideas. We see also more sophisticated curriculum construction, some of it taking into account research findings. Yet, we find there are still serious issues in students' achievement in mathematics. Teaching is not achieving the widespread mathematical know-how that society would like to see. So, we question teaching and the education of teachers. Research here has revealed many factors that seem important to teacher and teaching development. Yet, despite a theoretical knowledge of such factors and a sincere desire to foster students' mathematical competence, we experience activity in classrooms that does not seem to foster learning. There are many sources that address issues involved here, particularly issues I might call 'sociosystemic'. I will start with just three references.

In 1987, Desforges and Cockburn, writing about sincere teachers who were aware of the importance of developing students' higher order skills and able to do so in subjects other than mathematics, suggested that "classrooms as presently conceived and resourced are simply not good places in which to expect the development of higher order skills currently desired from a mathematics curriculum" (p. 139). They wrote further, "the teacher's job is more complex than that assumed by those who advise them on how to teach mathematics ... [indeed] the job is more difficult than even the teachers realize"; "constraining classroom forces operate in concert and ...

teachers' necessary management strategies exacerbate the problems of developing children's thinking" (p. 155).

In 2002, Despina Potari and I discussed episodes from the work of a teacher concerned to offer *mathematical challenge* to students in her class. While some episodes provided clear evidence of challenge, there were others in which challenge was lacking, in which the teacher answered her own questions and offered her own explanations in response to students' apparent inability to do so. Discussion with the teacher revealed a complexity of factors militating in this and other situations against teaching that would provide appropriate challenge (Potari & Jaworki, 2001). It was not a case of saying what *should* or *could* have been done, but of addressing the sociosystemic demands on the teacher. I shall return to this research.

In 2002, Razia Mohammad reported research with mathematics teachers who had followed an eight week university course addressing mathematics and the learning and teaching of mathematics from a perspective of developing teaching. She found that the teaching she observed conformed largely to traditional school practices with little evidence of the course having made a difference. Sociosystemic factors were evident – physical conditions, authority structures, attitudes, teacher-pupil relationships, text books, examinations, and time. One teacher challenged the researcher as follows:

Is it all applicable in this situation? If you were allowed to work here would you be able to maintain the quality of thinking and work you all do at the [university]. (p. 112)

This challenge from a teacher to a researcher/educator captured a gulf between the thinking and conditions at the university and those in the school. The course had developed a rapport between teachers and educators. The closeness of relationship was still evident when the researcher worked with teachers in the school context. But the teacher knew that the researcher's knowledge did not encompass the same understandings of school conditions as the teacher's knowledge – what it was like to work as a teacher, deeply embedded in the social milieu of the school system.

I used here the word "knowledge", but an alternative term which captures multiple forms of knowledge relating to situation and context is "ways of knowing" (Belenky, Clinchy, Goldberger, & Tarule, 1986). The educator in the example above had been for many years a teacher in the same kind of school system, but now her ways of knowing, developed within the university context, differed from those of teachers trying to implement university-knowing in their school context. However close this educator came to understanding the teaching context – *knowing* it from her previous experience and knowing *about* it now – she could not experience it as a teacher now.

In addressing teaching development it seems essential to address the ways of knowing of those who contribute to development which includes teachers and educators. The word "educator" can be seen as divisive: teachers are also educators. Mathematics educators in a university setting are *didacticians* of mathematics – they have a responsibility to conceptualise and theorise learning and teaching of

mathematics, to develop knowledge in these areas, which is different from teaching mathematics per se (although they might do this as well). In most cases, they do not teach mathematics in schools and their ways of knowing school culture are different from those of teachers. They might also be teacher-educators with responsibility to teach teachers. These distinctions are important to the discussion that follows.

In my introduction above I used the pronoun “we”. For example, I said “we experience activity in classrooms that does not seem to foster learning”. For inclusivity of teachers in conceptualizing teaching development it needs to encompass both teachers and didacticians. It is a challenge for both groups to achieve ways of working together that draws on all their ways of knowing in mutually fruitful ways. Sandy Dawson has written about an “in-service culture” which assumes that “there is something wrong with mathematics teaching world-wide, and that *we*, as mathematics educators, must fix it” (1999, p. 148, my emphasis). It becomes more and more obvious to me that didacticians can theorise and suggest, and work to understand how theories and suggestions can be realised the school culture, but they cannot “fix it”. How much more powerful might it be if theories and suggestions were to come also from *within* the school culture – from teachers? I have been working on this question for many years (e.g., Jaworski, 1998).

It seems important here to recognise that

- a) there are many issues relating to mathematics learning and teaching in schools that need to be addressed and that didacticians bring ideas and concepts that can be explored in such contexts;
- b) teachers’ ways of knowing mathematics learning and teaching are largely school bound, and often school cultures militate against theories and suggestions from outside the school context;
- c) didacticians’ ways of knowing mathematics learning and teaching are largely theory based and, although many have been teachers formerly, it is rare for such theoretical knowing to be embedded in a school context.

My focus in this paper lies in how to draw fruitfully on both kinds of knowing for developing practice in the learning and teaching of mathematics. I first offer some examples to illustrate complexity in teaching and teaching development, with teachers and didacticians engaging together in inquiry to improve mathematical learning. I then discuss theory relating to inquiry communities in mathematics teaching development, leading to some discussion of developmental theory and practice. Finally, I introduce a current research project that is rooted in these ideas.

EXAMPLES OF COMPLEXITY IN MATHEMATICS TEACHING

Management of Learning in a Vectors Lesson

Ben’s Year 10 class (ages 14-15) was working on vectors. They had considered the idea of a vector \underline{AB} (\underline{v}) as a journey from A to B, and had moved on to considering $2\underline{v}$ and $3\underline{v}$. I had observed the lesson and was talking with Ben about it afterwards.

1. BJ Oh, another thing I recall now, do you remember when you'd got three-AB up there, six, six? (Ben said 'yeah') And you turned round and you asked Luke. And my understanding of that was, Luke's not paying attention. You're checking that he knows what's going on. And you asked him to explain that. And he clearly hadn't listened at all, but he comes up with an alternative correct representation.
2. Ben But that's Luke. That's the sort of person he is, isn't it? I think.
3. BJ I mean, I was quite surprised not to / for you not *then* to make the link, but you decided to go on and ...
4. Ben I felt there was so much around, that I had to sort of / it's these judgments again isn't it? You make these judgments all the time. (Jaworski, 1994, p. 191)

Discussion was about the vector $(6, 6)$ which had emerged from three-times the vector \underline{v} ($3\underline{v}$) where $\underline{v} = (2, 2)$. The class had many questions which were being asked and discussed. For example, if the vector AB is a journey from A to B, what is the journey related to three times this vector – where is B in $3AB$? Students asked their questions vociferously and were answered, equally vociferously, by others. Ben, the teacher, was one voice among many as he responded to and managed the discussion. Luke looked as if he was not attending and Ben addressed him directly. I had expected Luke not to know what was being discussed, but quick as a flash he suggested $(6, 6)$ was $2AB$ plus AB ($2\underline{v} + \underline{v}$). This seemed to me like a new way of seeing $(6, 6)$ and I was surprised that Ben did not emphasise it to the class.

In whole class mode, Ben was managing a complex interchange of questions and answers. In the middle of it all, he checked up on Luke. He then returned his attention to other students. Contrary to my expectations he did not take Luke's contribution further. When we discussed this later, he referred to "judgments" which had been a topic of discussion between us many times. What are the factors contributing to each *judgment* a teacher makes? How can the teacher manage his attention to such factors? Can he be aware enough of factors to have the option to deal with explicit choices at the moment they arise? John Mason (2001) talks about "noticing in the moment" and Donald Schön (1987) about "reflecting-in-action". The theory is that the teacher is sufficiently aware of the choices to be made, and possibly the issues involved, that he can act knowledgeably at the moment of choice.

Michael Eraut (1995), in a critique of Schön, suggests that teaching is too complex for reflecting-in-action to be a serious option for most teachers. He suggests that "reflection-in-action involves thinking at a meta level about the process in which one is engaged"; involving "a ladder of reflection, where people move up a rung to reflect at a meta level on what they have been doing then down again to take consequent action". He emphasises "the effect on the mode of cognition of the time available for thinking", recognising that "a teacher has to be constantly assessing the situation, responding to incidents, deciding whether to change the activity, alert for opportunities to tackle difficult issues". This suggests that time in teaching decisions is too short to support the metacognitive activity required; that teaching, is too

demanding to allow for noticing, and acting knowledgeably, in the moment (c.f., Desforges & Cockburn, above). Yet, I know from my own experience and reports from teachers (Jaworski, 1994) that reflection-in-action *does* happen with consequences for immediate teaching action.

Ben was engaged in a complex management of the learning environment in the situation described, captured by the term ‘management of learning’, a theoretical construct I have used as part of a triad, *the teaching triad*. This also includes constructs of *sensitivity to students* and *mathematical challenge*. The three constructs, ML, SS and MC, are deeply linked to account for teaching situations. According to Ben, he saw SS and MC to be subsumed within ML (Jaworski, 1994, p. 144). In the episode, Ben’s management of learning included handling the whole class discussion, deciding how best to respond to students’ questions, injecting new ideas, and checking up on Luke’s involvement. He noticed that Luke did not seem to be attending, so he acted to check on Luke. He said afterwards that he had not followed up Luke’s input because there was “so much around”: that is, so many factors of which he was aware. Further discussion revealed Ben’s attention to students who were struggling with vector concepts, gender issues in trying to include some quieter girls in the dominating questions from other students, and the noise levels in the room which were sometimes unacceptable for thinking and interaction. The choice, a) to value Luke’s input overtly and b) to offer the alternative way of seeing the vector $3\mathbf{v}$ to others in the class, was something I saw, as an observer without teaching responsibilities. Ben was focusing on multiple concerns. As we grappled together with such issues and inquired into teaching processes and tensions, our knowing developed relative to our particular contexts: Ben’s in terms of his making of judgments, mine in a developing awareness of factors of complexity.

Some further examples

In Potari and Jaworski (2001), we described research which explored the use of the teaching triad as a developmental and analytical tool. We wrote about the teacher Jeanette who wanted her Year 9 class to appreciate relationships between volume and surface area of cuboids and wanted to *challenge* pupils fruitfully (i.e., mathematical challenge directed at conceptual learning outcomes). We point to an episode in which two boys seemed to be developing strong concepts (they used the term ‘compact’ to describe minimum surface area for a given volume), and were able therefore to react well to the teacher’s challenges. There seemed to be *harmony* between sensitivity and challenge: Jeanette’s in-the-moment decisions there seemed appropriate in her management of the learning situation. However, later, under the stress of a Friday afternoon lesson, students’ unwillingness or inability to offer explanations, and time factors in finishing an activity, this same teacher entered a funnelling process in which she herself explained the concepts she wanted students to address. She was aware of the conflict between her aims and actions, but she needed a closure to current activity and, in the moment, no other actions were obvious. In reflecting *on* the activity later, she explained that what she would have done, ideally,

did not fit with time factors and the mood and behaviour of students. This discussion in our research team led to an elucidation of socio-systemic factors that have to be considered in the teachers' design of teaching (p. 372/3).

In an episode we are writing about in another paper currently, two girls had not done their homework. Their teacher, Sam, had asked them to look up the meanings of "mode", "median" and "mean" in a dictionary for homework. They said they thought they needed a *French* dictionary, and did not have one. They had been unable to make sense of his task, and had avoided the necessity to do so. Many students had not engaged with the homework task. Sam remonstrated, students grumbled and the atmosphere became unpleasant. Sam was unable to engage with his planned activity for the lesson. He was irritable; changing his plans on the spot (finding a way to deal with students' difficulties, avoidance of work and current disruptive attitudes) challenged his teaching. At the same time, he experienced a growing awareness of the inappropriateness of the challenge in his task for the students. In our research, he had become aware of his tendency to offer mathematical challenge without attending to the sensitivities involved. He had set himself the task of paying greater attention to his sensitivity to students. In this case, as he worked with the students to overcome the unpleasantness in the classroom, analysis shows how successive interactions addressed students' cognitive and emotional needs and that learning outcomes were more fruitful than might have been expected. Again, as we talked about this together we grappled with complexities in teaching and how design of teaching, both before and in a lesson, could account for all that was 'around'. The teaching triad played an important role in these analyses.

Key factors in complexity and development

These examples just start to sketch the kinds of complexity I see in trying to develop teaching. They include dealing with in-the-moment decisions involving cognitive and sociosystemic factors relating to the diverse needs of pupils in class and beyond: time factors, syllabus demand, mathematical or didactical beliefs, emotions of teachers and pupils and more. Teachers tried to balance challenge and sensitivity within a management of learning that was both inclusive of students (sensitive to their thinking and needs) and focused on deep consideration and development of mathematical concepts. Line by line analyses of classroom dialogue provided a fine-grained insight to a complexity of relationships between challenge and sensitivity.

As we talked about who the teacher attended to at certain times in the classroom, how he or she steered the mathematical discussion, what sociosystemic factors influenced decision-making and so on, we explored issues and recognised complexity for teachers. Our awarenesses of the relatedness of theory and practice, and the corresponding tensions in dealing simultaneously with theory and practice, led to a powerful form of *co-learning* in which *inquiry* was a central element. Seth Chaiklin has written, of social situations where research contributes to development,

Social science research has the potential to illuminate and clarify the practices we are studying as well as the possibility to be *incorporated into the very practices being investigated*. (Chaiklin, 1996, p. 394. My emphasis.)

My focus on teaching development, considering ways of knowing of both teachers and didacticians in developmental practice, looks into how research itself is a major factor in enabling growth. I offered a framework for analysing the qualities of such research (in Jaworski, 2003) and have taken “inquiry” as a unifying factor between research and the learning and teaching development on which research has focused.

RESEARCH AND DEVELOPMENT – INQUIRY AND CO-LEARNING

So far, I have emphasised aspects of complexity in developing mathematics teaching, the differential ways of knowing of teachers and didacticians, and the centrality of research in teaching development. I want now to explore further the relationship between research and development, linking to notions of inquiry and co-learning.

With reference to her work on “reciprocal teaching”, Ann Brown (1992) recalls the significance of her work being dismissed as “only the Hawthorn effect”, which claims “... the mere presence of a research team will lead to enhanced performance because of the motivational effects of the attention received by the “subjects””(p. 163). She suggests that the Hawthorne Effect, far from being a factor to be wary of in educational research, is actually one to be valued and built on to enhance knowledge and promote improved practices. These days, we might talk of “participants” rather than subjects: however, I want to go further. In the examples above, teachers are not just participants in empirical research; they are *partners in developmental* research. In the research with Ben, in which I set out to do an ethnographic study of his teaching, the relationship soon developed a mutuality in which learning was reciprocal. He became far more than a “subject” of this research. However, the learning resulted from there *being* a research project.

What do I mean when I say that Ben was “far more” than the subject of the research? Put simply, I claim that he became a partner in the research because he engaged in inquiry too, for example, into the question of “judgments”. His inquiry was different from mine. He was much less interested than I was in generalised research knowledge, and had no wish to write research papers. However, he was very interested in thinking about teaching and exploring ways of enhancing learning. Thus his design of teaching, my analytic observations of his teaching and our subsequent (lengthy) discussions served both our purposes, and moreover our learning was mutually dependent – we learned from each other’s activity and expression. This has been true in subsequent projects in which I have worked with didacticians and teachers. The act of engaging together in research has meant that we are all inquiring into the learning and teaching processes in which we have differing roles and goals. The mutuality of inquiring together leads to clearer understandings - *co-learning* - for both partners.

Thus *inquiry* provides a theoretical basis for seeing research as a developmental tool. Chambers' English Dictionary (McDonald, 1977) suggests that *to inquire* means: to ask a question; to make an investigation; to acquire information; to search for knowledge. Wells (1999) sees dialogic inquiry as

a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them (p. 122).

Wells' "to ask questions ... and ... attempt to make answers to them" is one way of interpreting Chambers' "search for knowledge". This search for knowledge and its relation to learning, 'coming to know', forms the essence of the inquiry process.

Of course, inquiry has a long history in mathematics education. I think of inquiry as being at the roots of the problem-solving movement, deriving from Dewey and Polya, and promoted by John Mason and Alan Schoenfeld and others in mathematics education (see references for key sources). Inquiry in (school) mathematics can be seen to follow the activity of research mathematicians, and lead to recognition of the value of processes such as specializing and generalizing, conjecturing, convincing and proving (e.g., Mason, Burton & Stacey, 1982). Involvement in questioning and investigating focuses minds on aspects of mathematics and generates further questions and lines of inquiry, seeking answers and supporting learners in coming to know. For example, in their further work on vectors, Ben asked pupils to draw their own vectors and find their lengths. In addressing *what vectors can we draw*, students recognized what seemed like negative or zero vectors and had to resolve these apparent inconsistencies with the idea of a vector being a journey (Jaworski, 1994). Cognition could be seen to develop through tackling such inconsistencies and arguing them out in class. *Viability* (Glaserfeld, 1995) of constructed knowledge suggested that inconsistency was inappropriate and some resolution had to be found. As students argued and explored, results (like the length of a vector being positive even if the vector seemed to be negative) emerged and were seen to make sense. There was evidence of pupils' growth in mathematical knowledge.

It seems to me that inquiry in mathematics, as a mode of activity for pupils learning mathematics, has processes in common with both inquiry in developing mathematics teaching and inquiry in the research process. Indeed, the research with Ben and other teachers began as a study of investigative mathematics teaching: exploring the practices and issues arising from working in an investigative way with pupils in mathematics classrooms. Investigation was a mode of learning, a way of designing activity for pupils and a way of developing teaching. Thus I see inquiry in three mutually embedded forms or layers:

- *Inquiry in mathematics*: Pupils in schools learning mathematics through exploration in tasks and problems in classrooms;
- *Inquiry in teaching mathematics*: Teachers using inquiry to explore the design and implementation of tasks, problems and activity in classrooms;

- *Inquiry in research which results in developing the teaching of mathematics:* Teachers and didacticists researching the processes of using inquiry in mathematics and in the teaching of mathematics.

In each of these layers we have people as individuals and people as groups inquiring into mathematics, mathematics teaching or into the contribution of research to teaching development. The individual-and-social nature of the processes involved is central to what I see as being the way ahead for teaching development. Jon Wagner talks of “co-learning” in research partnerships, writing

In a co-learning agreement, researchers and practitioners are both participants in processes of education and systems of schooling. Both are engaged in action and reflection. By working together, each might learn something about the world of the other. Of equal importance, however, each may learn something more about his or her own world and its connections to institutions and schooling. (Wagner, 1997, p 16)

We are all deeply embedded in social and cultural worlds (including political, economic, religious and systemic factors). Knowing can be seen both as situated in the context, community and practices in which we engage and as distributed within a community of practice (Cole & Engeström, 1993). Individual construction of understanding occurs within a ‘community of practice’ and is rooted in the norms of activity within that practice. Learning is in dialogue in the social plane before being internalized to the mental plane through inner speech (Vygotsky, 1978). Wenger (1998) has emphasised the production of *identity* through participation in a community of practice. Learning is presented as a “process of becoming”. Wenger states, “It is in that formation of identity that learning can become a source of meaningfulness and of personal and social energy” (p. 215). He speaks of “modes of belonging”, including *engagement*, *imagination* and *alignment*. We engage with ideas through communicative practice, develop those ideas through exercising imagination and align ourselves, critically, “with respect to a broad and rich picture of the world” (p. 218). I believe we can conceptualise inquiry learning in such terms.

I have struggled with the individual/social tension in a shift over the years from a constructivist position on knowing and learning to a more overt recognition of the social embeddedness of learning as expressed briefly above. The commensurability of these positions has been both a source of contention and an inspiration to seek some resolution between them, since both are essential (e.g., Bruner, 1997). Two factors, however, were always clear to me: (1) the power of inquiry in processes of learning; (2) the importance of dialogue in coming to know. Theoretically, I believe that a shift from ‘community of practice’ to ‘community of inquiry’ provides a perspective in which reflective development of teaching by individual teachers results in a developing community (Wells, 1999). In a community of inquiry, inquiry is more than the practice of a community of practice: teachers, develop inquiry approaches to their practice and together use inquiry approaches to develop their practice. This indicates a reflexive relationship between inquiry and development (where development implies learning and deeper knowing). Wells describes teachers

as “attempting to develop such communities of inquiry and simultaneously making their attempts the objects of their own inquiries” (1999, p. 124). A feature of a community of inquiry that distinguishes it from a community of practice, according to Wells (fitting well with references to Mason and Schön on *reflection* earlier) is

the importance attached to *meta-knowing* through reflecting on what is being or has been constructed and on the tools and practices involved in the process’ (page 124, my emphasis).

He adds, ‘the construction of understanding is a collaborative enterprise’ (p. 125).

Such a model is an individual process and a community process: as part of a community of inquiry, individuals are encouraged to look critically at their own practices and to modify these through their own learning-in-practice. Developments within the community result from rationalisations, implicit and overt, between ongoing practices. Participants grow into and contribute to continual reconstitution of the community through critical reflection; inquiry develops as one of the norms of practice and individual identity develops through reflective inquiry.

In my view, inquiry is both a *tool* and a *way of being*. In constructivist terms, it can be seen to stimulate accommodation of meanings central to individual growth. In sociocultural terms it is a way of acting together that is inclusive of the distributed ways of knowing in a community. The notion of “way of being” reflects Wenger’s concepts of becoming and belonging. When different communities interact in a mode of inquiry, meta-knowing that results through inquiry processes allows understandings that cross community barriers (c.f., Wagner, above). It is within this theoretical frame that teachers and didacticians collaborate for mutual learning. This view accords with the idea of ‘inquiry as stance’ introduced by Marilyn Cochran Smith and Susan Lytle (1999). Teachers taking an inquiry stance “[raise] questions about what counts as teaching and learning in classrooms” and “critique and seek to alter” systemic norms and relationships; further, they suggest, “the work of inquiry communities is both social and political”, aiming to bring about change in traditional ideas of knowledge and develop richer conceptions of practice (p. 289).

However, there is a fundamental tension in addressing teaching complexity through inquiry communities that I will try to capture before going further. The theory expressed above articulates a concept of *learning* through inquiry in communities in which teachers and didacticians are *learners*. The communities both support the inquiry and grow through the inquiry. However, so far, the role of a teacher or teacher educator in these learning processes is hidden. Consider again my three levels: at Level 1 we might expect a teacher to contribute fruitfully to students’ learning of mathematics and at Level 2, a teacher educator might contribute similarly to a teacher’s learning of teaching. Indeed systemic requirements and social expectations demand that teachers and teacher educators have goals for the learning of their students. Certain complexities of teaching arise from trying to reconcile developing teaching through a community of inquiry with expecting that teachers will

have clear goals for their students' learning – *the inquiry/goals tension*. My next section will start to address these issues.

DEVELOPING TEACHING: DESIGN, INNOVATION AND INQUIRY

Learning Study (LS)

Inquiry as *a way of being* is fruitful for development, as experience and research show (e.g., Schoenfeld, 1996; Wells, 1999). I see that inquiry as a tool is valuable to induce inquiry as a way of being. The tool needs to be used purposefully. Ference Marton and colleagues in Sweden and Hong Kong (Marton, Tsui, Chik, Ko, Lo, Mok, Ng, Pang, Pong, & Runesson, 2004) have used inquiry as a tool in a developmental process they call “Learning Study”. Developing from Japanese *Lesson Study* (e.g., Stigler & Hiebert, 1999), Learning Study encompasses elements of inquiry, design and innovation. Marton et al. write, “Students’ learning should not be accidental ...” (p. 331). They add, “Teachers’ opportunities to learn are a key factor affecting classroom practice ...” (p. 332); and “Intervention studies must change what teachers do ... in order to affect student learning.” (p. 333). It seems to me that, in developing teaching through inquiry, teachers’ learning is not accidental; and in good research, researchers’ learning is not accidental. This does not mean that we cannot learn what we did not set out to learn, but rather that, in purposeful activity, we have goals for learning; and moreover, it is problematic if learning does not accord with declared goals. But, how do we achieve our goals? These statements about goals speak directly to the tension outlined at the end of the last section, especially if our goals are for the learning of some person other than ourselves. Is a student (or teacher) in a position of deficit with respect to a teacher’s (or teacher educator’s) goals? Is this tension instrumental in complexities observed?

In learning study (LS) a group of teachers designs innovative classroom activity, based on agreed theoretical principles, and explores the consequent teaching. *Design* and *innovation* offer purposeful directions. Teachers use inquiry as a tool to explore teaching, alongside didacticians who offer theoretical ideas and practical support and who research the processes of teaching development. Teachers develop their thinking and practice through successive cycles of inquiry. They each work in their own classroom, interpreting a design they have produced jointly. Observation of each other’s teaching and group reflections lead to building of group and individual awareness through which *inquiry as a way of being* develops.

LS goes beyond *lesson study* in two major respects. The first is its theoretical basis. Design is based on *variation* theory (Marton et al, 2004). Didacticians and teachers work together to establish a theoretical basis for joint inquiry. The second is its purposeful nature in terms of pupil learning. LS conducts research into pupils’ attitudes and understandings throughout the developmental process. Thus teachers use variation theory to design activity related to curricular topics such as *subtraction* or *fractions*, and tests are applied before and after classroom activity to find out what students have learned. Marton et al acknowledge their use of “design” as being in

accord with a paradigm becoming known as “design research”: this, I believe, both comes up against *and* offers ways to address the inquiry/goals tension.

Design Research

The design research paradigm in education, developing from the work of Ann Brown and colleagues, uses *design* as a developmental tool. According to Anthony Kelly (2003), design research

attempts to support arguments constructed around the results of active innovation and intervention in classrooms. The operative grammar, which draws upon models from design and engineering, is generative and transformative. It is directed primarily at understanding learning and teaching processes when the researcher is active as an educator. (p. 3)

If we see *educator* here to refer to teachers and didacticians, both of whom are also researchers, this definition applies well to LS. However, we need clearer distinction on the activity of these partners since it is likely that neither their roles nor their goals in research are the same. I will come back to this.

According to Paul Cobb and colleagues, *design experiments* offer a means of addressing complexity. They result in an understanding of a *learning ecology* in which “designed contexts are conceptualized as interacting systems rather than as either a collection of activities or a list of separate factors influencing learning” (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p. 9). A learning ecology typically includes

the tasks or problems that students are asked to solve, the kinds of discourse that are encouraged, the norms of participation that are established, the tools and related material means provided, and the practical means by which classroom teachers can orchestrate relations among these elements (Cobb, et al., 2003, p. 9).

Taken from a special issue of *Educational Researcher* which focused on design research at an abstract level, these papers say little about the roles and involvements of teachers. In most cases, although talking of collaboration with teachers, they seem to suggest that design is the province of didacticians, and that teachers in some way that is not explicit implement such design. Predating this writing, Erich Wittmann (1998), writing about the importance of design in teaching development is more explicit: “Teachers need to be trained and regarded as partners in research and development and not as mere recipients of results” (p. 95). Despite the word *partners*, these words suggest that agency in such partnership rests with the designers who are not teachers. Indeed Wittmann says that design “cannot be left to teachers” (p. 96). “The teacher can be compared more to a conductor than to a composer, or perhaps better to a director ... than to a writer of a play” (p.96).

So, an issue for design research, as I see it, is how it conceptualizes the activity of teachers with respect to design and implementation. What kind of teacher agency is evident in the design process? In LS, it is teachers who design classroom activity

based on theoretical understandings nurtured by their didactician colleagues and supported by these colleagues. The learning of pupils is a clear goal, but this is expressed directly in terms of the required curriculum, not in terms of learning through inquiry. Thus, the inquiry/goals tension is not so evident in LS.

I will conclude this paper with reference to a research project that aims to build on ideas from learning study and from design research while keeping inquiry as its central theoretical focus. Research is designed to look carefully at the building of communities in which inquiry is used and developed and at learning and teaching goals that are addressed. The inquiry/goals tension is an explicit focus of research.

A RESEARCH APPROACH BASED ON INQUIRY IN LEARNING COMMUNITIES

This project, *Learning Communities in Mathematics (LCM)*¹, is designed to build communities of inquiry involving teachers and didacticians to develop teaching and enhance learning of mathematics. The theoretical basis of the project is inquiry as an approach to learning mathematics, to teaching mathematics and to researching the processes and practices of building inquiry communities to develop teaching. The project aims to use inquiry as a tool to develop inquiry as a way of being in developing teaching and studying related classroom activity and learning of pupils.

We are establishing agreements with 7 schools, from early years to upper secondary, each with a teacher group of at least 3 teachers committed to the project. Teacher groups in schools will focus on design of classroom activity that both builds in ideas of inquiry and addresses systemic requirements, including the goals of the school and educational system. It is the teachers who will design classroom activity based on inquiry as a tool for learning mathematics.

At the beginning, the role of didacticians is to draw teachers into inquiry in a variety of ways: firstly through workshops (at the college) in which we work together on what inquiry means for us all with respect to mathematics learning. Didacticians design workshops to create opportunities to do mathematics together in inquiry mode. Teachers and didacticians together will inquire into what inquiry looks like in mathematics learning. The role of teachers is to work on developing ideas of inquiry in relation to their own knowledge of mathematics, pupils and schooling, and take ideas back for further development in the school context.

In school, during the same time period as the workshop activity, teachers in each school form an inquiry group to think about what their teaching might look like from an inquiry perspective and to plan classroom activity. Within their own social setting – of curricula, programmes of study and school milieu – teacher groups will design innovative classroom activity that encourages pupils to get involved in inquiry in mathematics. Didacticians will support teachers in thinking about the nature of

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inquiry, drawing on experience and literature, getting involved in discussion of mathematical topics and examination questions, providing readings, software, advice on using software, access to mathematics and so on: responding to needs rather than imposing directions.

Didacticians study the design activity and the processes that emerge from implementing design; this includes both the design of our project and teachers' design of classroom activity. Here we expect to address the inquiry/goals tension at a number of levels, and to study how the use of inquiry as a tool in design, and in the tasks designed, promotes inquiry as a way of being. What kinds of interactions take place between didacticians and teachers? How do we address issues and concerns? What is needed at practical levels of ideas and resources? How does the thinking of all of us develop through our joint activity?

Data in the above will be collected through audio recordings and hand written notes from meetings and from personal reflections of the people involved. We shall video-record workshops and classrooms. We are recognising complex decisions in choice of methods and use of technology in data capture and analysis, aware that sophistication introduces its own problems. We expect to have a lot of data, so we have to think carefully about data reduction processes, how we shall recognize and validate significance; how our grain of analysis can be judged to capture elements of the delicate "process of becoming", of "formation of identity [in which] learning can become a source of meaningfulness and of personal and social energy", of "modes of belonging", including *engagement*, *imagination* and *alignment* (Wenger, 1998, p. 215). These theoretical issues are central to our inquiry process.

Although our study of interactions within the project will be ongoing (over a 4-year period), we expect to have two phases of data collection in which we video-record classroom interactions, and audio-record conversations with teachers and pupils individually or in groups. Here we shall be looking at the outcomes of the design process, gaining insight to the thinking of pupils, teachers and didacticians, and teasing out key issues in our developmental process. Classroom data will be related to data from the design process, to explore relationships between design and activity. Between these two phases we shall focus on learning in the project so far, ways of being that we can see developing and issues for dissemination and sustainability.

Ultimately we are looking for inquiry models that have a practical foundation in terms of the reality of schools, classrooms and teachers' lives. The communities that develop should be sustainable beyond the life of the project because the people involved have developed ways of being. As we talk with teachers and negotiate delicately the early stages of our relationship, the inquiry-goals tension is already evident. Teachers, enthusiastic to take part in the project, are wary that it may take time from necessary curriculum planning, or require classroom activity that does not address curriculum goals. While excited by the possibilities the project offers, some overtly air their concern that project activity will demand different kinds of planning space and different goals. Shifts in planning and goals are a focus of our study.

The current challenge for didacticians in these early talks with teachers, is how we get jointly to seeing this as a project in which the concerns are shared; in which teachers are not just responding to the ideas and desires of didacticians, but themselves taking on the mantle of the project – with ownership of its goals for learning and teaching within their own sociosystemic setting – and grappling with the tensions and issues that arise. We shall be reporting further on our progress in this and other aspects of the project in the coming years. We welcome interest from, and cooperation with colleagues in other parts of Norway and around the world.

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COMPEX DILEMMAS CONCERNING INCLUSION AND DIVERSITY IN MATHEMATICS EDUCATION RESEARCH WITH TEACHERS.

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Those invited to respond to the plenary papers at PME28 have been given a brief that asks them to bring a new perspective to the theme offered by the plenary speaker, or oppose the focus given. The invitation suggests that this might be done by presenting alternative perspectives or by suggesting dilemmas arising from the perspective put by the speaker. The main aim is to fit in with the theme of Inclusion and Diversity by stimulating subsequent debate on the ideas presented.

Barbara's plenary paper gave me a great deal of food for thought. She has a track record of really trying to work with practicing teachers for the improvement of classroom practice and we have seen and attended the contributions of many of her students here at PME over the years. So her ideas are well developed and have a wealth of thought and experience behind them. I applaud the work that she has been involved with and trust that this new project will be rewarding.

You will gather from this that I do not intend to take up the option of opposing the focus that she has given. Instead I plan to respond by using Barbara's paper and the issues that she has raised as a springboard for raising some of the current unresolved dilemmas that I am having to face in my own work with teachers and their research into their own practice. In sharing these personal dilemmas I hope to stimulate debate on Barbara's paper.

I have been working on teaching for the past 30 years from my various positions as teacher, didactitian and Director of an in-service provider. In addition, for the past five years I have offered a taught Masters module at my university which draws on the work of Davis (1996), Maturana and Varela (1986) for its enactivist approach to understanding learning, and on Depraz, Varela and Vermersch (2003) and Mason (2001) for its techniques on approaches to becoming more aware of one's own practice. The first students using this module as a foundation for their dissertations are in the process of graduating with what I consider to be exciting work. I have also used the above course as a foundation for another set of courses on Complexity and Diversity that I have been running for the past three years at UCT's Graduate School of Business, where my starting point again comes from an enactivist position but also draws on the work of Capra (1997, 2002) and business theorists such as Stacey (1996), and Lissack and Roos (1999).

Using this background I am going to draw on three different sources as a backdrop to my response. The first of these flows from my understanding of Complexity Theory and enactivism.

Complexity theorists draw a distinction between the descriptors complicated and complex. This new interdisciplinary field begins by rejecting the modernist tendency to use machine-based metaphors in characterising and analysing most phenomena. Machines, however complicated, are always reducible to the sum of their respective parts, whereas complex systems - such as human beings or human communities - in contrast, are more dynamic, more unpredictable, more alive. (Davis and Sumara 1997, 117)

Boundaries that currently define schools and universities should be blurred ... so that the relations between that which we call teacher education needs to move away from a model that focuses on mastery of classroom procedures and toward a more deliberate study of culture making. (Davis and Sumara 1997, 123)

In such a (diverse) community information and ideas flow freely through the entire network, and the diversity of interpretations and learning styles – even the diversity of the mistakes – will enrich the entire community. (Capra 1996, 295)

I also want to locate myself within the themes of the conference of Diversity and Inclusion and in addressing this I have been influenced by the following comments which were posted on the conference web page.

While celebrating diversity ... it is vital to develop criteria for centrality.
(John Mason, Oct 22 2003)

I would like to reverse the phrase “inclusion and diversity” to “diversity and inclusion” (in order to) bring our focus towards enquiring structures of power inherited...
Sikunder Baber (Nov. 3 2003)

The term “inclusion” in the title “inclusion and diversity” is a recognition of (the) presence of dominant structure, which has the power to “include”, and therefore “exclude”. Therefore the retention of the term “inclusion” in the theme title is an implicit celebration of the power of dominant structure, an act, inherently counter-productive in the equation of intercultural relationships, and therefore of “diversity”.
Al-Karim Datoo (Nov. 21 2003)

Finally, I have for a long time been interested in the field of Teachers as Researchers.

... the essence of the Teacher Research movement came from the dissonance and unease that it caused in its quest to improve the education system... The teacher-research movement can assist by causing dissonance and trouble. Trouble that comes from conviction based on evidence drawn from research by those in the field who know that we haven't got education right and who are prepared to put their energies into getting something changed. The minute teacher research becomes comfortable, someone else needs to take over. (Breen 2003, 541).

Lewin and Regine (1996) maintain that the main entry into a complex view of the world depends on the value we attach to the stories we tell and the way in which they are listened to. My response at the conference in July will largely take the form of a collection of personal stories. The problem with these stories is that they are

inevitably situated within a specific time, context and current interpretation, and as I sit at this keyboard in May, I cannot know what particular form of story I will want to tell at Bergen in July. However there are three main stories which fill the menu at present.

Story One occurs when, as a didactitian at the time, I was privileged enough to be asked to allow my teaching to be used as a research site by another didactitian. The ensuing interaction gave me some insights into issues for teachers working with didactitians.

Story Two centres around a request to me as didactitian to work with two teachers to assist them 'work on their own practice'. We have written about this elsewhere (Breen, Agherdien and Lebethe 2003), and the issues raised at the time were complex.

Story Three involves three postgraduate students registered for the Masters in Teaching, who attempted to research aspects of their own practice for their dissertations and the challenges this faced for them (as it took them at times in directions in opposition to the academy in general) and for me as supervisor.

In all three of these stories I can most accurately be scripted as a troubled man faced with problems of identity and uncomfortable choices. The issues contributing to my dis-ease have to do with:

- Who initiates the 'project'?
- Whose questions are privileged?
- Whose theories are foregrounded?
- How do participants cope with different agendas?
- What do we learn from each other?
- Who is in control of the process?

These questions are not exactly the same ones that Barbara has raised but they are the ones that come back to me as I think about the dilemmas of a didactitian as s/he tries to set up a project where teachers are included in a community of inquiry. I hope that those in the PME audience when I respond will find some resonance with her paper. These issues are (obviously) crucial for me and they are the largely unresolved questions that I have to live with as I work with teachers and their work in classrooms. In a sense I am reassured by the understanding that I am working in that complex place that is also known as the 'edge of chaos' or 'border of disorder' and that all that I can do if follow Rilke's exhortation to 'live your questions now' (Rilke 1986, 45)!

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