

# AN INTRODUCTION TO THE PROFOUND POTENTIAL OF CONNECTED ALGEBRA ACTIVITIES: ISSUES OF REPRESENTATION, ENGAGEMENT AND PEDAGOGY<sup>1</sup>

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*We present two vignettes of classroom episodes that exemplify new activity structures for introducing core algebra ideas such as linear functions, slope as rate and parametric variation within a new educational technology environment that combines two kinds of classroom technology affordances, one based in dynamic representation and the other based in connectivity. These descriptions of how mathematical and social structures interact in the classroom help account for significant algebra learning gains in recent SimCalc teaching experiments among 13-16 year old students.*

A long-term goal of the SimCalc Project (Kaput, 1994) has been to exploit technology's capacity for interactive visualization tools and simulations linked to mathematical representations to provide an alternative to the algebraically based prerequisite structure of topics such as calculus to avoid the algebra bottleneck and democratize access to big mathematical ideas that are now inaccessible to the great majority of students due to the algebra barrier. But another shorter-term objective has emerged, partly as a result of the need to work within the existing structures and capacities of curricula and schools, and partly in response to the need to serve today's students, who cannot wait for long term strategies to take hold, no matter how promising.

Hence within the past five years, the SimCalc Project has developed strategies that use the interactive representational affordances of technology (visualization, linking representations to each other and to simulations, importing physical data into the mathematical realm in active ways, graphically editing piecewise-defined functions, etc.), to energize and experientially contextualize existing algebra courses, and to do so in ways that lay the base for more advanced mathematics, particularly calculus. Recently, we have been studying the profound potential of combining the representational innovations of the computational medium (Kaput & Roschelle, 1998) with the new connectivity affordances of increasingly robust and inexpensive hand-held devices in wireless networks (Roschelle & Pea, 2002) linked to larger computers (Kaput, 2002; Kaput & Hegedus, 2002).

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## EMERGING THEORETICAL COMMITMENTS

We have come to see classroom connectivity (CC) as a critical means to unleash the long-unrealized potential of computational media in education, because its potential impacts are direct and at the communicative heart of everyday classroom instruction – more so than internet connectivity. We are now beginning to build insight into how those new ingredients, in combination, may provide the concrete means by which that potential may be realized, because they may, in fact, help constitute the *first truly educational technologies*, intimately situated within the fundamental acts of active teaching and active learning. This embeddedness may indeed be more profound than we initially recognized, because these ingredients resonate deeply with broader views of learning as participation (Lave & Wenger, 1991) and no longer fit within a “learning in relation to a machine” (large or small, in the lab, classroom or even your hand) view of educational technology. Indeed, the paradigm is shifting towards one where the technology serves *not* primarily as a cognitive interaction medium for individuals, but rather as a much more pervasive medium in which teaching and learning are instantiated in the social space of the classroom (Cobb, 1994). We deliberately choose “instantiated” ahead of “situated” (Kirshner & Whitson, 1997) because we have repeatedly seen mathematical experience emerge from the distributed interactions enabled by the mobility and shareability of representations. The student experience of “being mathematical” becomes a joint experience, shared in the social space of the classroom in new ways as student constructions are aggregated in common representations – in ways reminiscent of, but distinct from those of a participatory simulation (Stroup, 2003; Wilensky & Stroup, 1999). This epistemological shift in the place of technology in classrooms is fundamental to our theoretical perspective.

## CONTEXTUALIZATION OF VIGNETTES IN PRIOR SUCCESS STORIES

The empirical work behind this report investigated the impact of our constellation of technological, curricular and pedagogical innovations on student learning, *especially as measured by independent standard test items* on a pre/post-test basis. They include intense teaching experiments aimed at core algebra topics in middle and high-school classrooms in both Massachusetts and California. Results demonstrate comparably positive outcomes under substantially different instructional and technological conditions, somewhat different curricular targets, and different student demographics (Tatar, et al, in submission).

Pre/post-test measures (see <http://www.simcalc.umassd.edu/>) triangulated with observational video and field-note data in these and other instructional situations in undergraduate classrooms provide evidence of significant improvements in students’ algebraic thinking as measured by students’ performance. For example, in our after-school intervention (n=25), students performance was significantly better on post tests ( $p < 0.001$ ), with high effect sizes (Cohen’s  $d = 1.80sd$ ) and strong gains occurring across disjoint populations (middle and high school) with relative statistical independence of prior knowledge, based on Hake’s gain statistic (Hake, 1998) –

( $\langle \text{Post} \rangle - \langle \text{Pre} \rangle$ )/( $1 - \langle \text{Pre} \rangle$ ), reflecting strong impact of the intervention itself as exploiting very different kinds of prior knowledge in the two subpopulations (see Hegedus & Kaput, 2003; under review, for further details).

## **PARTICIPATORY AGGREGATION OF STUDENT CONSTRUCTIONS TO A COMMON PUBLIC DISPLAY**

In a connected algebra environment the class is typically subdivided into numbered groups, where the size and number of groups fit both the given size of the class and the mathematical activity (ranging from the whole class to pairs). Groups are often, although not necessarily, defined “geographically” – students who are physically near each other. The students usually also “count-off” inside the group, so that each student then has a two-number identity that can then serve as “personal parameters,” a Group Number and a Count-Off Number. Students then create mathematical objects – in the cases discussed here, linear Position vs. Time functions that drive animated screen objects. The functions depend in some critical way on students’ respective personal parameters either on a hand-held device or on a computer. SimCalc MathWorlds runs on the TI-83Plus as a Flash Application (Calculator MathWorlds - CMW), and on desktop computers as a Java Application (Java MathWorlds - JMW). When using CMW or JMW (the scenario for this report), students’ constructions are uploaded to the teacher where they are aggregated, organized and selectively displayed using JMW and discussed.

Staggered Start, Staggered Finish ( $Y=mX+b$ ): The simplest case is to produce families of functions defined by a single parameter, such as the “b” in  $Y=2X+b$ , where b varies according to, say, Group Number. Then each student in a group produces the same position function ( $Y=2X+$  Group Number) on his/her own device, so a given group’s linear graphs all overlap (same Y-intercept and same slope = 2), and the different groups’ graphs are parallel. When animated, the screen objects “representing” the members of a given group move alongside each other “as a group,” while the different groups move at the same velocity but are offset by their initial positions (see Link 1 at <http://www.simcalc.umassd.edu/PME04.htm>). Here, the group provides mutual support, and it is of special interest that the superposition of graphs provides a strong visual realization of function equality.

By clicking on the overlapping graphs, we can bring different graphs of a group to the front. Further, the graphs and their corresponding objects (“Dots” in “Dots World”) are color-coordinated, so the sequence of graph-colors that appear matches the sequence of colors of the objects of the given group. In addition, with the View Matrix (see Link 2 at <http://www.simcalc.umassd.edu/PME04.htm>), the teacher has virtually full control of which graphs and objects are displayed, and how they are organized or colored – e.g., by Group Number or by Count-Off Number.

With this simple example in mind, we present two vignettes that correspond to some of the learning gains demonstrated in our interventions.

## Vignette 1: Varying M systematically – Slope as Rate both positive and negative

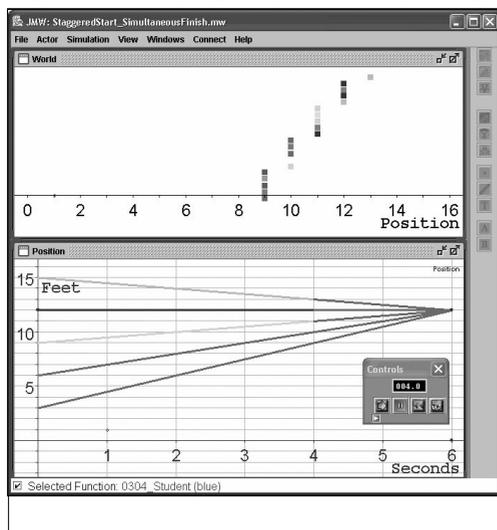


Figure 1: Simultaneous Finish

The Staggered Start – Simultaneous Finish activity is more complex than the former and requires the students to start at 3 times their Count-Off number but “end the race in a tie” with the object controlled by the target function  $Y=2X$  (so the target racer moved at 2 feet per second for 6 seconds and started at zero – see the bottom graph in Figure 1). Students now need to calculate how fast they have to go to end the race in a tie. And since they start at different positions, the slope of their graphs changes depending on where they start, which in turn depends on their personal Count-Off Number. Each group is limited to 5 people and while the group number does not

affect their constructions it gives rise to a smaller, more manageable set of functions to discuss (see Link 3 at <http://www.simcalc.umassd.edu/PME04.htm>). Secondly, and more importantly, the Count-Off Numbers 4 and 5 give rise to two important slopes. The person with Count-Off Number 4 has a graph with constant slope,  $Y=0X+12$ , since he starts at 12 ft, which is the finish line, so he does not have to move! The person with Count-Off Number 5, starts beyond the finish line (15 ft) and so has to run backwards, thus forcing the student to calculate a negative slope. We have observed students in a variety of settings using various strategies including: numerical trial-and-error, inputting values into an X-coefficient field in the algebra window (a feature of the software for algebraic editing), slope-based analysis (“What slope do I need to connect my starting position to (6, 12)?”), velocity-based analysis (“How fast do I need to move get to 12 at 6 seconds?”) and, by an implicit parametric-variation strategy, comparing what others had done in their group and averaging (i.e., observing that the slope of each person’s graph changed by 0.5 as the Count-Off Number increases by 1).

Organizing and displaying student work is a strategic pedagogical decision, e.g., in focusing student attention on the underlying mathematical structure. An important question to ask before animating the motions for the whole class is “What will the race look like?”

The aggregate motion in the classroom display becomes a personal reality through the personal link with Count-Off Number. By aggregating and displaying the class

work, students can observe how their personal construction fits into the “race” and allows them to note how people from other groups had constructed identical motions because of their Count-Off Number. In addition the shape of the graph and the parity of the slope for those who had to start past the finishing line (i.e. run backwards) was made more realistic and understandable in this motion-based scenario. In discussing different strategies and making these explicit in publicly examining outliers (incorrect answers) and natural outliers (motions for students with Count-Off Numbers 4 and 5) we have some evidence to why students began to improve on items of the pre-post test which involved slope analysis.

### **Vignette 2: Using both Group Number and Count-Off Number – creating “fans” to address the idea of slope-as-velocity**

In this vignette we discuss an activity structure where we systematically increase the complexity of the variation by using the other part of the unique identifier – the group number. Students now have to create functions so that their object will travel at a *velocity equal to their Count-Off Number* but *start at their Group Number*. We highlight here the move from a personal individual construction to a significant group structure to a class aggregation. Each group creates a family of functions, which is similar in shape (“a fan”) to every other group but offset by starting position.

Students anticipate the visual form of the aggregation as is highlighted in the transcript, where the class was asked what the collection of graphs would look like.

- J            It’s gonna start at two, and it’s gonna end at five, and... it’s gonna look kinda like a fan. And, they’re all going to start at the same place. <SNIP>
- T            So, he’s saying they’re gonna start there, and then it’s gonna kinda look like a fan?
- J            Yeah they’re gonna... *{spreads out fingers wide on one hand}* like that.
- T            ... You like that? *{hands up for more than half the class}*  
*{Clapping and a few ‘yeahs’, as the results are displayed}*

Here, John (J), indicates to the class physically with his hands (where his fingers resemble individual graphs) that the aggregation will resemble “fans” how these groups of functions will be displayed relative to each other. But although John knew where his graph was in the aggregate, he could not explain why in technical, slope terms. Here we highlight the social dynamic enabled by such a public event, which allowed two other students Alison (A) and Robert (R) to explain why John’s graph (with slope 3) is the one highlighted by linking its slope to the velocity.

- A            Go by velocity... however many... what number in a group you’re in... how many increments he goes.
- T            Okay. So, he’s the third member of the group. So..
- A            So he can go three times every second. Up three every second.
- T            ... How can we determine, Robert?

- R See how far it goes... look between zero seconds and one second?
- T You want to come show us?
- R Okay. {Robert goes to the display and inscribes the first one-second segment of the graph with a 1-wide by 3-tall rectangle}

We observed a high level of engagement by the class and reaction to the animation of the aggregate, which resulted in a wide distribution of the actors by the end of the animation. This was an opportunity to highlight how some people finished at the same place and time but followed different motions, without delving into the algebraic detail of why this occurred. This activity intends to provide an experience embodying parametric variation across both the graphical representation (see figure 2) and the motion-based gestalt of the animated objects – just as the collection of graphs has a gestalt-shape captured in the fan metaphor, the collective motion has a gestalt (see (see Link 4 at <http://www.simcalc.umassd.edu/PME04.htm>) for a sense of it.

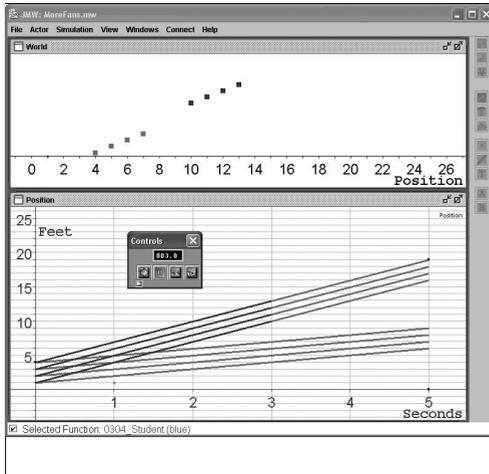


Figure 2: Making “fans” from two groups embodied in class norms of “correctness” and coherence.

At this stage, differences in students’ work are based on inter-group variation (similar fans but off-set) and intra-group variation (the establishment of one fan where individual Count-Off Number varies slope). To further build meaning of how variation in their identifiers leads to variation in their corresponding graphs, we reversed the roles of Group Number and Count-Off Number: We asked students to repeat the task but now construct a motion where they travel at a **velocity equal to your Group Number** for 5 seconds, and **start at your Count-Off Number**. Here students will produce a visually similar class aggregate, but their personal graph is now part of a fan constructed by members of other groups. In fact, their own group

We also highlighted and examined outliers, making a conscious decision in this case not to hide the names (or identifiers) of the students, although we note the issue of student anonymity in class (Scott, 1999). The software includes the ability for the teacher to hide names of selected functions if desired (identifying information appears in the lower left hand corner of the screen). Nevertheless, the class often took the initiative to determine who the outlier was, what the underlying error was, and then collectively correcting the mistake. In effect, the mathematical criteria of consistency comes to be socially

now constructs families of parallel lines, and, of course, their motions differ accordingly from the previous case.

## CONCLUSION: ADDRESSING NEW PEDAGOGICAL DECISIONS

The connected SimCalc algebra classroom opens up a new learning environment for students, with increased intensity, structures and levels of participation. In presenting these brief vignettes outlining the student activity and decisions the teacher made during the post-aggregation phase of the activity (space prevents descriptions of within-group interactions), we have begun to describe how students begin to develop an understanding of one of the core ideas of high school algebra, slope-as-rate-of-change. Through such activities, students have both an individual “mathematical responsibility” to either their group or to the larger class via construction or interpretation of shared mathematical objects, as well as a vicarious participation in the joint construction upon aggregation. With careful pedagogical decision-making by the teacher students’ attention is now moved along a trajectory from static, inert representations, to dynamic personally indexed constructions in the SimCalc environment on their own device, to parametrically defined aggregations of functions, organized and displayed for discussion in the public workspace.

Substantial teacher knowledge, a deep composite of content and pedagogy, is needed to facilitate movement along this trajectory and focus public mathematical dialogue on critical features of these visually shared objects to develop meaning. This is hinted at in the last illustration, where Count-Off and Group Numbers are interchanged. In effect, connectivity supports the pedagogical manipulation of student’s focus of attention. But the teacher knowledge needed to take advantage of a connected classroom requires extended development.

How do your -		
Motion(s)		An Individual vs. the Group
Graphs	Look Different as	An Individual vs. the Class
Formula	Look the Same as	The Group vs. the Class
Tables		

Table 1: Constructing pedagogical actions

Table 1 outlines a simple structure, which can guide the teacher’s inquiry. Choosing one item from each column leads to a particular question that can be addressed to individual students, groups or the whole class. Successively using this outline might assist teachers in moving students along suitable learning trajectories in this environment and elevating mathematical attention.

In our later activities in the interventions, aggregation was used as a means for generalization and abstraction. As more work is conducted publicly, learning increasingly occurs in the social space to complement the individual device-interaction space. Encouraging students to make sense of their constructions

publicly, annotating their graphs on the display space, and systematically highlighting parametric differences leads to more powerful understandings.

Having highlighted the benefits of social engagement in these activities, we also need to investigate in more detail the potential for negative social implications in some of these pedagogical actions and classroom decisions (e.g. issues of privacy, social embarrassment). We are confident, however, that by combining the two key ingredients, dynamic representations and connectivity technology, students can better understand fundamental, core algebra ideas by forming new, personal identity-relationships with the mathematical objects that they construct individually and collaboratively with their peers.

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