

STUDENTS' CONCEPTIONS OF LIMITS AND INFINITY

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The two concepts limit and infinity are crucial for mathematical analysis. Both concepts are complex but necessary for mathematics studies. The aim of this presentation is to discuss the students' explanations of their written solutions to limit tasks with special focus on infinity. The results presented are part of a larger study where the research question is: How do students deal with limits of functions at a basic mathematics course? (Juter, 2003).

Analyses of textbooks, curricula and student solutions to tasks were conducted to reveal how limits of functions and infinity were introduced to students and how the students solved problems in an analysis and algebra course at a Swedish university. The results are discussed in terms of *concept images* (Tall & Vinner, 1981).

Many students in the study calculated limits in various ways without problems. There were several tasks revealing students' reasoning. Some of the students gave correct answers with incorrect explanations to tasks. Infinity was often a reason for the

students' mistakes. One task for example was to decide if the function $f(x) = \frac{x^3 - 2}{x^3 + 1}$

can attain the limit value $\lim_{x \rightarrow \infty} \frac{x^3 - 2}{x^3 + 1}$. It can not and as many as 26 % of the students

answered "no", but with a motivation like: "x never attains ∞ ". Such situations give a false sense of security about the concept image's accuracy.

There were students showing other traces of incoherent or inadequate concept images as well. Some students were unable to solve tasks correctly since they had problems understanding the limit definition.

Textbooks used at upper secondary schools do not provide much theory or tasks about limits and infinity. Most new students at universities do not have a developed image of the concepts. The fact that textbooks do not deal with infinity in a thorough and explicit manner implies that students are expected to work with their possibly vague conceptions from childhood and school. The results of this study indicate that students need well founded conceptions of limits and infinity.

References

Juter, K. (2003). *Learning Limits of Functions, university students' development during a basic course in mathematics*. (Licentiate thesis) Luleå: Luleå University of Technology.

Tall, D. & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12, 151-169.