

RF05: RESEARCHING MATHEMATICS EDUCATION IN MULTILINGUAL CONTEXTS: THEORY, METHODOLOGY AND THE TEACHING OF MATHEMATICS

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INTRODUCTION

Multilingualism is prevalent in classrooms worldwide. In most mathematics classrooms, however, only dominant regional or national languages are used, often for practical or political reasons. Multilingual contexts may include the presence of:

- migrant communities (e.g. Vietnamese speakers in Australia);
- indigenous communities (e.g. Navajo speakers in the USA);
- historically multilingual communities (e.g. South Africa, Singapore);
- immersion schooling (e.g. English-medium education in Hungary).

Given the prevalence of multilingualism, it is important for researchers in mathematics education to consider the consequences of multilingualism for their research, even if multilingual issues are not the primary focus of their research. In this forum we draw on the increasing amount of research being conducted within the PME community into the teaching and learning of mathematics in different multilingual contexts (most recently, Adler, 2001; Barwell, 2001, 2003a; Khisty, 2001; Moschkovich, 2000; Setati, 2003).

Conducting research in multilingual contexts leads to a number of theoretical and methodological challenges. Classical research methods may be hard to apply, leading to the development of original approaches to research. In particular, issues arise concerning validity, interpretation and the relationship between language, mathematics and mental processes. To tackle these issues, researchers in this field have drawn widely on theories from a range of disciplines, including psychology, linguistics, anthropology and sociology, as well as education. A further challenge for researchers is to draw on their work to inform the practice of teaching mathematics. Our main aim in this Research Forum is to explore the impact of multilingualism on three inter-related issues mentioned above: theory, methodology and teaching mathematics in multilingual contexts.

THEMES

The first theme concerns *the role of theory* in research in this area. Researching multilingualism in mathematics education is by its nature inter-disciplinary. Research into multilingualism within mathematics education has drawn on a variety of theoretical perspectives, including: bilingual education; discursive theories of cognition; discursive approaches to socio-linguistics; Vygotskian approaches to

teaching and learning. This inter-disciplinarity leads to a number of questions. What theories are relevant to work in mathematics education? How might these theories be applied in mathematics education? What are the challenges which arise from working with theories from other disciplines? A basis for exploring these issues is provided by Hoffmanova, Novotna and Moschkovich, whose paper also provides the theoretical backdrop for the Forum.

The second theme concerns *implications for mathematics teaching* arising from recent research. Although research has focused on the role of the teacher in supporting mathematics learning in multilingual mathematics classrooms, these classrooms are located within a wide range of different linguistic contexts. Whilst Adler's (2001) research, for example, raises important issues or dilemmas for teachers, these are issues which arise in multilingual South Africa. This context is different from the multilingualism found in Europe, Australasia or Asia. How are such contexts different? And what do any differences imply for the teaching of mathematics? A discussion of these questions is offered by Clarkson and by Halai.

The final theme concerns *methodological issues* thrown up by research in multilingual classrooms, whether or not multilingualism is a focus of the research. Research is necessarily mediated by language. When participants are speakers or learners of several languages, languages which may not be shared with the researcher, many challenges arise for the researcher. Going beyond the basic challenges of collecting and preparing multilingual data are the more complex issues of interpretation. One challenge, for example, concerns the visibility of mathematics in linguistic analyses of mathematics classroom interaction. How can language and mathematics both be kept in focus? Linguistic anthropologists deal with such issues as a central part of their work. How do they deal with these issues? These issues are considered by Staats and by Barwell.

ORGANISATION

Each of the three themes will be introduced by short presentations by the relevant contributors, whose papers follow this introduction. The presenters for each theme will conclude their presentations with a key question, which will form the starting point for focused small-group discussion of that theme. Following consideration of the three themes, the forum will continue with an extended plenary discussion, with the opportunity to raise issues arising from the earlier discussions. The forum will be concluded by Mamokgethi Setati in the role of discussant.

WORKING WITH THEORIES FROM OUTSIDE MATHEMATICS EDUCATION

Marie Hofmannová, Jarmila Novotná, Charles University, Faculty of Education

Judit Moschkovich, University of California, Santa Cruz

We describe why research in mathematics education should consider theoretical views and empirical findings from research on language to provide an accurate picture of the complexity of learning and teaching mathematics in multicultural and multilingual settings. We believe that knowledge of language learning is essential to further progress in understanding the connections between language and the process of learning-teaching mathematics, especially in classrooms where students are bilingual, multilingual, or learning an additional language.

INTRODUCTION

Many of the classrooms where we teach and conduct research include students who speak two or more languages or are learning an additional language. The first part of the paper provides an overview by presenting a brief account of the main theories related to the area of second language learning and acquisition. Special attention is paid to those aspects of the theories and findings relevant to the interaction of mathematics learning and teaching and the teaching of English as a second or foreign language. The second part of the paper describes how a sociocultural and situated framework can be used to frame analyses of mathematical discussions that include more than one language and involve bilingual or multilingual learners. This framework expands “what counts” as the mathematical competence to include the voices of bilingual students and those who are learning English.

THEORIES AND FINDINGS RELATED TO SECOND LANGUAGE LEARNING / ACQUISITION

Although everyone agrees that thought and language are related, the nature of the relationship remains controversial. Traditionally, linguists have studied only the natural languages used by members of human communities to communicate with each other. This, however, leaves out wider senses of communication, e.g. mathematical and logical codes that can be used to transmit messages.

Theories about how we initially acquire language rely on psychological theories of learning in general. They have influenced each other over time. Moreover, different authors bring different models of L2 [1] learning (Ellis, 1994, table 10.2). The very distinction between learning and second language acquisition (SLA) is controversial. We have therefore decided to adopt an eclectic approach to be able to cover the most influential theories.

The Behaviorist Approach

Behaviorists regard language learning as habit formation, as a result of connecting responses to stimuli. Children learn to speak because they are reinforced for doing so.

Correct responses lead to good habits, errors are perceived as bad habits. The negative effect of mother tongue (L1) on students' production of L2, causing errors through analogy with L1, was described as a Contrastive Analysis Hypothesis (Lado, 1964, in Brown, 1993). Critics of the behaviorist position claim that although this view may have an intuitive appeal it provides only a partial explanation of children's early language learning.

The Cognitive Approach

Children do not simply imitate the language they hear, but rather learn to construct grammatically correct sentences they have never heard before by generalizing about language. There appears to be a critical period of language acquisition when SLA can take place naturally and effortlessly (Lenneberg, 1967, in Brown, 1993). From a cognitive perspective, language acquisition occurs in increasingly complex stages as children actively seek ways to express themselves (Brown, 1993). The sequence appears to be universal.

One example of work from this perspective is the psycholinguistic studies comparing monolinguals and bilinguals when doing arithmetic operations (Magiste, 1980; Marsh & Maki, 1976; McLain & Huang, 1982; Tamamaki, 1993). All we can safely conclude from that research at this time is that "retrieval times for arithmetic facts may be slower for bilinguals than monolinguals" (Bialystok, 2001, p. 203). It is not clear whether these reported differences in response to time and accuracy between adult monolinguals and bilinguals during experiments also exist for young learners or would be evident in classrooms.

Such an emphasis on the deficits of bilingual learners or second language learners is described as a cognitive deficit model of learning in L2. As a contrast, other psycholinguistic research has shown that while bilinguals and second language learners may face some disadvantages, they also display some important cognitive advantages over monolinguals. Bialystok (2001) concluded that bilinguals develop an "enhanced ability to selectively attend to information and inhibit misleading cues" (p. 245) [2]. This conclusion is based, in part, on the advantage reported in one study that included a proportional reasoning task (Bialystok & Majunder, 1998) and another using a sorting and classification task (Bialystok, 1999). These results would seem to be closely related to mathematical problem solving.

Linguistic Universals

Universality is one of the most fascinating characteristics of language. Children in all cultures appear predisposed to acquire language through almost the same phases, and may be born with an innate mechanism to learn language – Language Acquisition Device (LAD). Mentalist/nativist theories state that there seems to be one best type of grammatical analysis that all of us are programmed to develop and it is universal to all languages, using the same grammatical forms and relations or linguistic universals, which were later applied to SLA (Chomsky, 1965). We are not completely sure that this so-called universal grammar is accessible to adult learners. After a

certain age we are still able to learn a language using such other mental faculties as the logical and the mathematical. The learning of mathematics can be seen as a process parallel to the way children acquire language skills, developing structure in oral ability prior to the more symbolic abilities with writing and reading (Gardella & Tong, 1999).

Social Models

Social models of language acquisition consider that social factors have an indirect effect on all mental processes including SLA. These theories examine linguistic variability rather than universality and claim that children may develop more than one grammar depending on particular situational contexts. A complex view of L2 learning called The Socio-Educational Model explains how individual factors and general features of society interact in L2 learning. The Acculturation Model (Schuman, 1978, in Brown, 1993) suggests that successful learning means “acculturation” – becoming part of the target culture. Learning takes place in society and depends on motivation and aptitude.

The Humanist Approach

The Humanist Approach differs from others in that it focuses on the affective components of learning. For a long time the relationship between cognition and emotion has been a controversial issue. Increasingly, we are becoming aware that cognition, emotion and personality are not entirely independent (Crowl *et al.*, 1997). The success of the humanist approach towards teaching depends on the extent to which the teacher caters to learners’ affective domain. Critics have a variety of objections, but it would appear that many humanist programs have not been evaluated properly to determine their effectiveness.

Creative Construction Theory

Creative Construction Theory was first developed and described as The Monitor Model (Krashen, 1977, in Ellis, 1994) and later as Creative Construction Theory (Dulay & Burt, 1982, in Ellis, 1994). The theory brings together research findings from different domains. According to Krashen, SLA is subconscious and equals LAD, contrary to Chomsky for whom LAD is but one of various mental organs, a construct that describes the child’s initial state. More recently, Chomsky’s statements seem more compatible with Krashen’s argument that adults and children have access to the same LAD. These ideas have provoked strong criticism. Empirical research studies have shown that the development of L2 is a process in which varying degrees of learning and acquisition can be beneficial. No input is acquired as new language without conscious awareness. Swain emphasizes the role of output in SLA (see Ellis, 1997).

Interlanguage Theory

Interlanguage (IL) is a term introduced to refer to the developing competence of L2 learners, from an initial stage of very limited knowledge about the new language, to a

final stage of almost complete fluency. The concept was coined to describe the kind of language that is independent of both the learner's L1 and L2. Recent developments in this area of research try to answer questions concerning the role of L1 (IL is influenced by L1 but the influence is not always predictable), the acquisition of IL (form-function relationship), and the systematicity and variability of IL. The results of experiments provide evidence that mistakes made during bilingual education are both intralingual (within L2) and interlingual (between L1 and L2). Nowadays, IL is considered to be the central concept in SLA (Ellis, 1997). He identifies many external and internal factors that account for why learners acquire an L2 in the way they do.

A SOCIOCULTURAL AND SITUATED THEORETICAL FRAMEWORK INFORMED BY SOCIOLINGUISTICS

Work in sociolinguistics has informed the study of mathematics learning and teaching in multilingual classrooms. This work has contributed theoretical frameworks for studying discourse in general, methodologies (e.g. Gee, 1996), concepts such as register (Halliday, 1978), and perspectives on classroom discourse (e.g. Cazden, 1986; Mehan, 1979). It also provides theories, concepts, and empirical results in second language acquisition, bilingualism, and biliteracy (Bialystok, 2001; Hakuta & Cancino, 1977; Valdés-Fallis, 1976, 1978; Zentella, 1997). This work has provided crucial concepts necessary for studying mathematics learning in multi-language classrooms, such as code switching, as well as important distinctions for example between national and social languages, or among different types of code switching, in different cultural settings such as South Africa (Adler, 2001; Setati, 1998; Setati & Adler, 2001) and in bilingual classrooms in the USA (Moschkovich 1999, 2002).

Psycholinguistics and sociolinguistics differ both in how they explain and explore language practices. While the sociolinguistic perspective stresses the social nature of language and its use in varying contexts, the psycholinguistic perspective has been limited to an individual view of performance in experimental settings. According to the sociolinguistic perspective, psycholinguistics experiments provide only limited knowledge about speakers' competence or how people use language:

The speaker's competence is multifaceted: How a person uses language will depend on what is understood to be appropriate in a given social setting, and as such, linguistic knowledge is situated not in the individual psyche but in a group's collective linguistic norms. (Hakuta & McLaughlin, 1996)

Code switching has been largely used in sociolinguistics to refer to the use of more than one language in the course of a single communicative episode. In contrast, research that looks at bilingual performance from a psycholinguistic perspective sometimes uses the term 'language switching' to refer to a cognitive phenomenon, the act of switching from a second language to a first language as the language of thinking when a bilingual person is individually engaged in a mathematical task rather than in a conversation. While work from a sociolinguistic perspective also distinguishes between language choice, code switching, and code mixing, sociolinguistics assumes that all of these phenomena are social rather than individual

in nature and function.

These two perspectives see bilingualism itself in different ways. From a psycholinguistic perspective we might define a 'bilingual' as any individual who is in some way proficient in more than one language. This definition might include a native English speaker who has learned a second language in school with some level of proficiency but does not participate in a bilingual community. In contrast, a sociolinguistic definition of a bilingual would be someone who participates in multiple language communities and is "the product of a specific linguistic community that uses one of its languages for certain functions and the other for other functions or situations" (Valdés-Fallis, 1978). This definition describes bilingualism not as an individual but also a social and cultural phenomenon that involves participation in language practices and communities.

Research in mathematics education should address the relationship between language and mathematics learning from a theoretical perspective that combines current perspectives of mathematics learning and classroom discourse with current perspectives on language, second language acquisition, and bilingual learners. In this section we consider how the situated and sociocultural perspective proposed in Moschkovich (2002) can inform our understanding of the processes underlying learning mathematics when learners speak more than one language.

Moschkovich's (2002) approach combines a situated perspective of learning mathematics and the notion of Discourses (Gee, 1996). This perspective implies that learning mathematics is viewed as a discursive activity. Learning mathematics is seen as participation in a community of practice (Forman, 1996; Lave & Wenger, 1991; Nasir, 2002), developing classroom socio-mathematical norms (Cobb et al., 1993), and using multiple materials, linguistic, and social resources. This perspective assumes that learning is inherently social and cultural "whether or not it occurs in an overtly social context" (Forman, 1996, p. 117), that participants bring multiple views to a situation, that representations have multiple meanings for participants, and that these multiple meanings for representations and inscriptions are negotiated. Learning mathematics is seen as participation in a community where students learn to mathematize situations, communicate about these situations, and use resources for mathematizing and communicating. From this perspective, learning to communicate mathematically involves using social, linguistic, and material resources to participate in mathematical practices.

This approach also draws on Gee (1996), who defines Discourses as more than sequential speech or writing:

A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and 'artefacts,' of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or 'social network,' or to signal (that one is playing) a socially meaningful role. (p. 131)

Discourses involve more than the use of technical language, they also involve points

of view, communities, and values. Mathematical Discourses (in Gee’s sense) include ways of talking, acting, interacting, thinking, believing, reading, and writing, but also mathematical values, beliefs, and points of view of a situation. Gee emphasizes that such interactional and non-language symbol systems, should be included in Discourse analysis. Thus, we should consider the importance of gestures, artifacts, practices, beliefs, values, and communities in communicating mathematically. Participating in classroom mathematical Discourse practices can be understood in general as talking and acting in the ways that mathematically competent people talk and act when discussing mathematics.

A situated/sociocultural perspective focusing on participation in mathematical Discourse practices generates particular questions when analysing mathematical discussions. For example:

1. What are the situated meanings of some of the words and phrases that seem important in the situation?
2. What are the multiple resources students use to communicate mathematically? What sign systems are relevant in the situation (speech, writing, images, and gestures)? In particular, how is “stuff” other than language relevant?
3. What Discourses are involved? What Discourse practices are students participating in that are relevant in mathematical communities or that reflect mathematical competence?

This situated and sociocultural perspective complicates our view of the relationship between language and learning mathematics. A crucial consequence is that it allows us to replace deficit models of bilingual mathematics learners with a focus on describing the resources that students use to communicate mathematically.

We would like to share a word of caution. There are dangers in borrowing isolated concepts while leaving behind the theoretical framework. It is not enough to borrow an isolated concept. If a concept is not connected to the theoretical framework that generated the concept, it can easily become an idea that bears little resemblance to the original idea. For example, we might borrow the concept of “code switching” from its sociolinguistics framework that assumes that language is a social phenomenon but neglect to take the sociocultural view of language along with it. If we do this, we would be reducing code switching to an individual phenomenon. Similarly, if we use “register,” a term framed by a sociolinguistic view of language, to mean “lexicon”, which unlike register is independent of the social context, we are removing “register” from its sociocultural framework and replacing that framework with an individual view of language.

CONCLUDING REMARKS

Focusing on mathematics is our job as researchers in mathematics education. But focusing on mathematics also has consequences for how we portray students’ mathematical competence. Teaching and research are framed by theories of learning

in general, theories of mathematics learning and, in this context, theories of SLA. Whether we are teaching or analyzing a lesson we need to consider the theoretical framework and the assumptions that we bring to our work. We believe that theories and empirical results from linguistics, cognitive psychology, and sociolinguistics have laid the groundwork for the study of mathematics learning as it occurs in the context of learning an additional language.

ENDNOTES

1. L1 – the mother tongue, L2 – the target language.
2. The cognitive advantages of bilingualism seem to depend on some level of proficiency in both languages and “the extent to which an individual is fully bilingual is instrumental in mediating the effect on cognitive performance”. (Bialystok, 2001, p. 205).

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MULTILINGUAL CONTEXTS FOR TEACHING MATHEMATICS

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Attention is drawn to aspects of teaching that inevitably rely on deep communication and various multilingual contexts for teaching. Examples are given of how different teachers and societies have dealt with these contexts. Little research has been completed in this area. The few studies that are available suggest that informal or exploratory talk in students' first languages is vital before moving to formal mathematical language. In multilingual situations the exploratory talk may be a situation of broken communication, but this may not be recognised by participants.

The great majority of teachers throughout the world now work in classrooms that are multilingual. However it has only been recently recognised that the linguistic backgrounds of students has an impact on students' learning of mathematics, and hence on the teaching of mathematics. For many monolingual teachers the problem of students who have a first language different to theirs has become a reality. Students' language backgrounds have been seen as one factor that means, in the view of some such teachers, that these students are unavailable to learn, or to learn in the way such teachers expect. Other teachers are exploring what tools and strategies they can use to face this growing and complex challenge of changing classroom settings.

Contexts of mathematics classrooms vary enormously, influenced in part by the reactions of individual teachers through to societal judgments having a mixture of impacts. One of the factors in this variation is the different languages of communication that might be present, as well as how both teacher and students view and use them. It is useful to begin by reviewing some of the research that has particularly looked at the teacher's role in situations when there has been more than one language in the mix. In scanning four journals dedicated to research in mathematics education from 2000 to 2003 (*Journal for Research in Mathematics Education*, *Educational Studies in Mathematics*, *For the Learning of Mathematics*, and *Mathematics Education Research Journal*), at least in English written contributions (about 300 articles) there are very few articles that focused on the teacher's role in such situations. There are far more, and perhaps understandably so, that have students as the central foci. Hence it may be that more research is needed to clarify the roles that the teacher may play. Three studies seem relevant.

Setati and Adler (2000) discussed the language practices of teachers in some primary schools in South Africa where students' normal out-of-class talk is in a non-English language, but the official teaching language is English. They were interested in the code-switching behaviour of teachers. Although they suggest that it makes a lot of sense for teachers to encourage students to code-switch, and use this as a teaching strategy too, there are challenges in this practice that can not be overlooked. At times it seems that teacher talk is down-played in some curriculum reforms, and yet it is teacher talk they suggest that often illuminates ideas for students. Types of discourse,

such as informal talk in students' first language leading to more formal mathematical talk in English, are also critical paths to trace carefully in such complex multilingual situations.

Gorgorio & Planas (2001) were working in classrooms where the teaching language was Catalan. Students were a mix of Catalan students plus immigrant students who spoke a variety of languages at home. The authors suggest it is hard to separate the social, cultural and linguistics aspects of mathematics teaching and learning. Indeed they took the view that it was better to think of broader communication within the classroom than a narrow linguistic one, although language aspects cannot be ignored. In particular they note that in their classrooms, the informal or exploratory talk can often be 'broken communication', particularly for the teacher, since this inevitably occurs in the students' first languages. Therefore helping students to move to the more formal mathematical talking and writing, which often involves a switch to the language of the classroom, can be fraught with unknown linguistic set-backs.

Khisty & Chval (2002) contrast the teaching styles of two teachers who were teaching groups of Latino students in the USA. The two classes were of different levels in English proficiency, and hence there was more frequent use of English in one classroom than in the other by the bilingual teachers. The authors write that a critical issue was the way one teacher used precise and extended mathematical language in her verbal discourse with her class and promoted an expectation that the students would also use such language. The results of the investigation suggested that students did in the end use the formal mathematical language promoted by this teacher. The underlying emphasis is that bilingual students will not learn this type of English, unless they are witnesses to deliberate examples of such discourse.

The above three studies emphasise that the issues of teaching in multilingual contexts are not straightforward. The teachers need to cope in situations where they will not have full management of the discourse, unless they too are proficient in the students' language(s), as well as the teaching language. However the flow from exploratory verbalizing of ideas through to their formalising in a rich mathematical language, both verbal and written, seems to be a given across the contexts. How to manage the flow is an issue that needs further research. What are the teaching strategies that teachers can employ with good effect to this end? In the next section, I consider several examples of the current challenges faced by mathematics teachers in different parts of the world.

CHALLENGES

Various multilingual mathematics classroom contexts can be generated by considering three of the possible interacting sources of language: the students' language or languages, the teacher's language or languages, and the official teaching language (and less often languages). The snippets below discuss various multilingual situations, highlighting the wide range of possible contexts.

In Papua New Guinea, students in a typical classroom will speak a common

language, although they may well speak a number of other languages too. The teacher may speak the common student language if s/he comes from the same region, but will also be multilingual. Up to year 3, schools can choose which teaching language they will use, but from year 3 the official teaching language is English, although teachers are encouraged to use a mixture of languages if possible through years 3, 4 and even 5 (Clarkson *et al.*, 2001). Classroom observations suggest, however, that teachers seem to prefer English only when teaching mathematics. It seems that dealing with mathematical concepts is difficult in a vernacular or Melanesian Pidgin. This raises an interesting question. Are crucial nuances lost in translating terms into English, with embedded cultural meanings being marginalized? Should the rule of using English be sidestepped so that the cultural meanings can be explored?

In urban Australian schools, many monolingual teachers teach a mix of multilingual students, many of who are from migrant families, although the migrant community to which they belong may have been in Australia for a number of years. It would seem, however, that few teachers realise the role that a first language plays for these students. This is summed up by the surprise of a primary school teacher, who had recently completed graduate studies in Teaching English as a Second Language, when she discovered how often her year 4 Vietnamese students were switching languages when doing mathematics in her class (Clarkson, 2002). In some European countries too, teachers are faced with teaching many migrants. It would appear that in the main the reaction of teachers has been one of holding a line of orthodoxy. That is, that 'newcomers' should learn the ways of the dominant society and integrate with it, including learning the use of the main language in the classroom as soon as possible. But this new context is challenging other teachers to think deeply about their use of language in teaching mathematics.

In Malaysia, at the beginning of the 1970s in Malay schools, the teaching language was changed from English to Bahasa Malay. This was mainly for political purposes to emphasize the unity of the relatively new political amalgamation of historically different kingdoms and states. However from 2003 due to a rather rapid political decision, although the main teaching language remained Bahasa Malay, the teaching of mathematics and science reverted to English. This has interesting ramifications for teachers of mathematics.

In New Zealand, the indigenous Maori peoples have developed a small system of schools where only Maori is used for all communication while present in the school, though both students and teachers live in a dominant English speaking community. Mathematics is taught in Maori. Further, the mathematics curriculum has been translated into Maori, with some changes to include some specific Maori mathematics. In some areas of the Northern Territory in Australia a different strategy has been employed by indigenous communities. Through the 1980s and early 1990s there was political support for the use of the people's first language to be used as the teaching language. Further, there was insightful curriculum work carried out to devise mathematical curricula that commenced in the early years of schooling with

Aboriginal ideas. Hence in one area in the desert, indigenous spatial ideas became the basis of the early years curriculum, whereas on the north coast the notions of relationships were used as the key framework concept. In these instances not only were the teaching languages changed to that of the students and community, but the mathematics curriculum too was transformed.

CONCLUDING REMARKS

The actions of individual teachers, as well as societies, will be influenced by deep-seated beliefs, which in turn may be manifest in the language(s) of communication in classrooms. A teacher who is a member of the dominant society and who believes that teaching should be in the dominant language will have little inclination to explore any other language options. The perception of mathematics that is held will also have an influence. If mathematics is conceived as a 'language free zone', then the teacher who takes this view will be less inclined to think about the role that the teaching language, or any other, has on the learning of mathematics. If, on the other hand, the teacher accepts that not only does the teaching language impact on the learning of mathematics, but so too may the students' first languages, they may consider which languages can be used in their classrooms and even of what mathematics can be taught.

To address some of the questions raised in this paper, however, a far more detailed meta-analysis of the relevant literature is needed, the state of which is only hinted at in this paper, with the multilingual context of the teaching as a crucial aspect of such an analysis. Such research may allow useful commonalities in teaching mathematics across multilingual contexts to emerge. At the same time, notions and practices that should be seen as context specific may also be identified.

TEACHING MATHEMATICS IN MULTILINGUAL CLASSROOMS

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In this paper, arising from my doctoral research, I highlight issues that emerge for teaching and learning mathematics in multilingual mathematics classrooms. These are classrooms where the language of instruction is not the first, often not even the second language of the learners.

Research about teaching and learning mathematics suggests that the dynamics of teaching and learning mathematics in multilingual contexts take on an added complexity, giving rise to a number of issues and dilemmas. There is need for acknowledging this added complexity and to understand the factors that lead to it. Some of the dilemmas are well recorded in Adler (2001) who highlights three:

- The dilemma of code switching, when learners and/or teachers switch from language of instruction to the first language
- The dilemma of mediation when teachers move towards the learners preferred language
- The dilemma of transparency when the teacher spends time explicitly teaching mathematical language (Adler, 2001, p.)

While Adler's context is multilingual South Africa, in today's increasingly connected world, multilingual classrooms are the norm. Hence, it is important to recognise the centrality of multilingual mathematics classrooms so that reform efforts may take them into account.

My doctoral research (Halai, 2001) involved a study of reform in learning and teaching mathematics. Of particular interest to me was the role of social interactions in students' learning of mathematics. The study was based in two classrooms (henceforth, classroom A and B) in Karachi, Pakistan. These were classrooms where the teachers were using reform-oriented teaching approaches i.e. students were typically set mathematics problems which were open ended in nature and were situated in everyday world contexts. A small group of students (10-12 yr.) doing mathematics was observed in each classroom. Both schools were English medium schools. This meant that the official medium of instruction including the textbooks used and the tasks set in the class were in the English language. In both classrooms, the instruction was almost entirely in English but during group work students spoke in a mixture of Urdu and English. The teachers also reverted to Urdu when they went to the various groups. In reporting the group work to the whole class the students spoke in English with a smattering of Urdu. At times students *took permission* from the teacher to report in Urdu. However, the mathematical terms used in this mixture were invariably in English because these terms came from the textbook, which was in the English language.

The data was qualitative in nature and came from classroom observations and interviews with students and teachers.

ISSUES IN TEACHING MATHEMATICS IN MULTILINGUAL CLASSROOMS

A number of issues pertaining to the processes of teaching and learning arose because of the language of instruction being different from the first language of the learners. These included:

- Understanding the language to make sense of the mathematics;
- Use of everyday language and mathematics learning;
- Using own language to express mathematical thinking;
- Language of the textbook.

Understanding the language to make sense of the mathematics

As students worked at mathematical problems it appeared that their understanding of the problem statement required interpretation at least at two levels. At one level the students appeared to make sense of the language in which the mathematics problem was coded. This involved making sense of the grammar and usage of words. And at another level they appeared to make sense of the mathematics involved. For example, in one problem task students were given the statement “*Sara will be 28 year old after 9 years. Find her present age*”. Their task was to form an equation for the given statement and to solve it.

Analysis of classroom interactions showed that understanding how the word “will” was used was crucial to the students’ successfully doing the task. Knowing that ‘will’ is future tense had major implications for how the problem statement is converted into a mathematical equation and then a solution is sought. There were other examples which showed that the understanding of the specific structures and usage of words in the English language was important for the students to make sense of the mathematics. For example, in her introductory lesson on ratios the teacher used the task of mixing water and orange juice concentrate to make drinks which were “stronger than”, the drink shown as a sample. However, students’ interaction showed that one student in the group translated and explained to the others that stronger means ‘ziada’ which is the Urdu word meaning “more”. The result was that the students categorised as stronger the juice that was ‘more in quantity’ and not as the one, which was more concentrated and hence stronger. This and other similar examples through the research provide vivid evidence of issues that students face in interpreting mathematical tasks that require an understanding of the grammar and usage of words in a second or third language. Questions arise for the teaching and learning processes in the multilingual mathematics classrooms. Was the teacher aware of these language issues arising in the classroom? What could the teacher have done to facilitate students’ learning? How else could the teacher have organized his/her classroom so that issues such as those above have been acknowledged and

addressed?

Use of everyday language and mathematics learning.

In the classroom that I observed, teachers had prepared tasks that were set in everyday life contexts and made use of everyday usage words. There appeared to be an assumption that using everyday contents and language would facilitate learning. However, these words of everyday usage were in English language and therefore raised questions about assuming that students would be familiar and would be able to make a link with the mathematical concepts embedded in those words. My observation in the classroom showed that the teacher's use of everyday words for mathematical concepts led to difficulties for the students. For example, the teacher used "fair share" for proportional division but students appeared to think that the word "fair" meant "divided equally" or "divided easily". This had implications for how they worked on other related tasks.

A point that I am trying to make is that using discursive strategies to teach mathematics where students are expected to build on their knowledge of the everyday context and language takes on an added complexity in a multilingual context. This complexity arises because of possibly unquestioned assumptions regarding students' familiarity and understanding of the language of the language of instruction.

Students expressing mathematical thinking in their own language

A pattern in the classroom interactions was that the students and the teacher moved back and forth in the use of language. While there is evidence that students change languages, understanding why they do so would be important to making a difference in the way teaching is organised in multilingual mathematics classrooms. Furthermore, changing from one language to the other raises the issue of translation from one language to the other. Now, translation is a nuanced and complex process. In mathematics classrooms translation of key words and phrases would require that the mathematical meaning is also kept intact. Examples quoted in this paper show that students did not always translate in a mathematically appropriate manner. Indeed, on occasions the issue was that there did not exist an appropriate equivalent translation of the key word or translation being used.

Issues pertaining to the status of language also arises (Haque, 1993). Why did the students need to take permission in order to report their work in Urdu? Issues of power and politics of language emerge. Why did students use a mixture of Urdu and English instead of resorting to Urdu only? Is it that they saw English as a more powerful and therefore wanted to belong to the community of English speakers?

Language of the textbook

The teachers and the students in the classrooms where I did my research were all expected to follow prescribed textbooks. The textbook was used as a guide for the subject content to be taught and for providing exercises for practice. Each teacher emphasised to me that if the material provided in the textbook was not covered they

would be accountable to the head teacher. This emphasis on the textbook raised issues pertaining to the language being used by the students and that being used in the textbook. Language in the textbook used formal mathematical terminology coded in English. While teachers were using everyday words in English and the students translated these everyday words to Urdu. This rather complex scenario compounded the issue of transfer from the everyday language to mathematical language.

CONCLUDING REFLECTIONS

To conclude, classroom data shared above shows that in the course of teaching and learning mathematics in multilingual classrooms, dilemmas and issues arise, similar to those discussed in Adler (2001). This is because students and teachers in these multilingual classes switch back and forth from one language to the other. This switching requires translation from one language to the other which is complex and not always possible. Furthermore, politics and power of the language of instruction and the students' language also gives rise to issues.

From the discussion so far certain questions arise for the academic and practitioner communities. I will end with some such questions:

- How can teachers organise their teaching to address the issues and dilemmas that arise in a multilingual mathematics classroom?
- In what ways can teacher education prepare teachers for multilingual mathematics classrooms?
- How can research inform practice in mathematics teacher education/ mathematics teaching in multilingual classrooms?

QUESTIONS OF VISIBILITY

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In this paper I explore this issue of the visibility of mathematics when multilingual mathematics classroom interaction is examined from discursive perspectives. The use of such perspectives is related to the linguistic concerns of research in this area. It can lead, however, to a critique that argues that such analyses are insufficiently revealing of the mathematics taking place. I draw on an example from UK data to explore the relationship between mathematics and discursive practice.

The issue of visibility arises from a view that a focus on language issues in research in mathematics classrooms, while interesting, often omits a suitable focus on the mathematics taking place. This view is similar to that encountered in critiques of much socially or politically oriented research in mathematics education: by focusing on the social activity of a mathematics classroom, mathematics ‘slips from view’ [1]. At the heart of such critiques is an implicit position on what mathematics is. This position is perhaps motivated by the (realist) idea that mathematics exists outside of the social, discursive or political world. In the context of multilingualism (including bilingualism), this position entails mathematics existing somehow outside of language, so that research which explores language practices in multilingual mathematics classrooms is not seen as exploring mathematics. The linguistic, social or discursive practices involved in doing, teaching or learning mathematics are not quite the same as mathematics.

Visibility becomes an issue for me, with the adoption of particular linguistic perspectives on mathematics classroom activity arising from my interest in linguistic phenomena. I conceptualise mathematics classrooms in terms of linguistic practice because my questions concern language. One approach, therefore, is to find ways of keeping my focus on questions of language, and my perspective of linguistic practice, whilst relating what I see to the practices and ideas of the mathematical community. This approach might be termed ‘language to mathematics’. An alternative is to focus on mathematical practices and ideas and seek to relate what I see to language-related issues, an approach that might be termed ‘mathematics to language’. The danger of the first approach is that mathematics is not seen as sufficiently visible. The danger of the second approach is that issues of language are not seen as sufficiently visible. The challenge is to work with both language and mathematics and keep them both in view. I will use aspects of my own research to exemplify and explore some of these issues.

MY OWN RESEARCH

In research into the participation of 9-10-year-old learners of English as an additional language (EAL) in the UK, I analysed transcribed audio-recordings of students working together. The students are working on the task of writing and solving arithmetic word problems. To analyse the transcripts, I developed ideas from discursive psychology (Edwards, 1997) and conversation analysis (Sacks, 1992). In

particular, I drew on the notion of ‘participants attention’ (Sacks, Schegloff and Jefferson, 1974), seen as a feature of interaction, rather than a form of mental activity. My analysis, a form of the ‘language to mathematics’ approach, entailed the identification of patterns in students’ attention (see Barwell, 2003b). I then explored how attention was used by students as part of the social activity of thinking. The following brief extract and commentary illustrates this approach to analysis [2]. The extract involves Cynthia, who arrived in the UK from Hong Kong, 18 months before this recording. Cynthia speaks Cantonese and some Mandarin. She is working with Helena, who is a monolingual English-speaker. They are starting to write a new word problem, which Helena suggests could involve a character called ‘Cynthia’ [3]:

- H Cynthia has thirty pounds for/
 C no/ not for her her mum/ if (I bought)/ for my mum
 H for her mum’s present
 C if give my mum thirty pound I bought nothing from her/ that not make sense
 H no/ I won’t writing for you mother/ I said Cynthia has thirty pounds for her mother’s present
 C thirty pound/ I gave thirty pound for my mum present
 H no/ I didn’t say give it to her
 C then how why you
 H you **have** thirty pounds [for your mum’s present
 C [no
 but/ I think this make sense/ Cynthia has thirty/ pound/ thirty pound/ she bought err something something something/ it’s cost something something/ from her mum present/ and how much she left?/ is that make sense little bit

In this extract, my interest is in how the students discursively manage their attention. Different areas or patterns of attention are apparent. The two students attend to the word problem genre, for example, as in Helena’s opening suggestion of the opening words of the problem and Cynthia’s implicit acceptance of them. This pattern of attention is also apparent at then end of the extract in Cynthia’s exposition of a standardised word problem with blanks for the numbers. At other times, attention is on what I will call ‘narrative experience’, the use of narrative accounts or reasoning (Bruner, 1990) to make sense of the interaction. Cynthia uses attention to narrative experience when she expresses concern that ‘if give my mum thirty pound I bought nothing from her’. She is using narrative reasoning to support her claim that the problem does not make sense. Superficially, neither of these two patterns concerns ‘mathematics’. They are used, however, by the two students, to make sense of their word problem. Cynthia, for example, gives an interpretation of the opening ‘Cynthia has thirty pounds for her mum’s present’ as meaning that the Cynthia in the problem gave her mum thirty pounds, which for Cynthia, is not a present. A present should be

having money to by a present means, what is done with the money and so on. The solution ends with attention shifting back to genre, ‘how much she spent’, the two students thus relating their calculations to their problem. Cynthia’s ‘three oh oh oh’ are not isolated, abstract digits, but a link in a chain (Bakhtin, 1986) of meaning-making. Where, then, is the mathematics? Only at the end of the process outlined above? Or throughout the process? For me, the whole process of developing the word problem is implicated in its solving. It is difficult to draw a line between ‘mathematics’ and ‘other practices’ within the discourse of school mathematics.

CONCLUDING REMARKS

My approach to researching multilingualism in mathematics classrooms focuses primarily on social, discursive practices, with the aim of linking this analysis to practices established within a broader mathematical community. The case of Cynthia and Helena shows how such practices are central in mathematical meaning-making. This position, however, is based on a broad notion of what constitutes mathematical practice.

NOTES

1. The comment that mathematics had ‘slipped out of view’ was made in a review of an earlier version of Barwell (2003b).
2. The interaction between Cynthia and Helena is more thoroughly presented in Barwell (2003b).
3. Transcription conventions: / for a short pause, // for a longer pause, [for overlapping speech, () enclose uncertain transcription, (...) for inaudible speech.

MATHEMATICS DISCOURSE AS PERFORMANCE: PERSPECTIVES FROM LINGUISTIC ANTHROPOLOGY

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This paper draws on perspectives from linguistic anthropology to look at mathematics classroom discourse. In particular, the paper introduces the notion of performance. An illustrative analysis of a mathematics classroom discussion is presented.

While the folk performances that linguistic anthropologists study might seem at first to be markedly different from speech in mathematics classrooms, the two are indeed linked in a fundamental way. In both cases, discourse brings pre-existing knowledge into the social world, often with personal improvisation, presenting it for the evaluation of others who decide whether it was a successful performance or not. This shared process of learning and presenting anew means that in some respects, the analytical tools of linguistic anthropology and folklore are as relevant to education research as the methods of discourse analysis. A great deal of scholarly effort in folklore since the mid 1970s has focused on the concept of performance. In this paper, I outline the dimensions of performance and illustrate how these ideas can be used to examine mathematics classroom interaction.

PERFORMANCE

Richard Bauman writes that performance is a way of speaking that is characterized by “the assumption of responsibility to an audience for a display of communicative competence...highlighting the way in which verbal communication is carried out above and beyond its referential content” (1993, p. 182). Bauman continues “[f]rom the point of view of the audience, the act of expression on the part of the performer is thus laid open to evaluation for the way it is done, for the relative skill and effectiveness of the performer’s display” (p. 183). Formal discursive features distinguish performance speech from ordinary, factual, referential speech, for example, opening phrases like “once upon a time” or vocal qualities like the sonorous harangue of a legislator. Still, performance can occur in ordinary, even conversational contexts (Duranti, 1997, p. 16; Silverstein, 1984). Because speakers possess different levels of competence and willingness to perform, performance is an emergent aspect of speech: speakers can achieve varying degrees of performance (Bauman, 1977). Overall, the key attributes of performance are communicative competence, accepting responsibility for a competent expression, highlighting the communicative event as different from ordinary discourse, opening the speech event up for audience evaluation, and the emergent quality of performance.

To what extent do these attributes occur in mathematics classroom discourse? The issue of communicative competence (Hymes’ critical response to Chomsky’s linguistic competence) and the audience evaluation of competency are clearly typical components of classroom speech. In a traditional classroom that relies primarily on the “Triadic Dialogue” (Lemke, 1990) (the discourse pattern of teacher question,

student answer and teacher response), the teacher has the major evaluative role, but in many US reform classrooms, student evaluation of competence is prominent. The question of whether mathematics discourse is distinguished from ordinary speech requires more analysis than is possible in this format, but I can at least note that in folk performance, cross-culturally, one of the most common means of highlighting performance discourse is through the use of specialized vocabulary, as in the mathematics register (Pimm, 1987).

The association of performance and responsibility is most apparent when speakers deny their ability to perform, as in “I don’t really know how to tell jokes, but I heard one that went like this” or in a mathematics classroom, when a student addresses the teacher, “I’m not sure, but the book says...” These are ways for a speaker to give a report of previous knowledge rather than take responsibility for a full performance. Still, discourse that opens as a report can nonetheless develop into a more confident portrayal of the speaker’s mastery of a topic. Judging from work on the emergent qualities of folkloric performances, several discourse features are likely to indicate that students’ speech is a performance of mathematical knowledge rather than simply a report:

- Use of the mathematics register;
- Configuration of speech to control audience critique;
- Use of indexical language to orient the audience to particular aspects of the context;
- Semantic and syntactic parallelism or patterning.

A performance-centered approach to mathematics discourse allows researchers to track the emerging confidence of students’ mathematical speech beyond a simple assessment of whether a given statement is factually correct. In the next section, I provide an example of how the above ideas can be used to examine mathematics classroom interaction.

PERFORMANCE IN THE MATHEMATICS CLASSROOM

Performance-centered approaches to discourse have been successful in revealing communicative principles in many folkloric genres and in many languages primarily because they offer formal features that arise across languages. Gee’s definition of discourse is relevant:

A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and ‘artefacts’, of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or ‘social network’, or to signal (that one is playing) a socially meaningful ‘role’ (Gee, 1996, p. 131).

It is not always easy to appreciate the way a “role” is constructed in multiple

languages without the sorts of formal features that performance-centered approaches emphasize.

A technique commonly used in folklore and linguistic anthropology for revealing the orderliness and beauty of verbal art is to render discourse in poetic lines. Poetry, rather than a dramatic script, becomes the model for representation of discourse. Parsing sentences into lines can be based on many different features, including breath pauses, intonation curves or parallelism, so that the same text can be divided into lines differently according to the analysis at hand (Tedlock, 1983). Take, for example, a passage of discourse in which Spanish-speaking third graders compare a parallelogram and a trapezoid (reproduced from Moschkovich, 1999, p. 16):

[Julian and Andres have several shapes on their table: a rectangle, a trapezoid and a parallelogram]

Julian: *Porque si. Nomás estas* (Because...Just these) sides get together [runs his fingers along the two non-parallel sides of the trapezoid...] *pero de este* (but on this side only). [runs his fingers along the base and top parallel sides of the trapezoid]

Mario: *Y este lado no* (And not this side)

Andres: *No porque mira, aqui tiene un lado chico* (No because, look, here it has a small side) [points to the two non-parallel sides of the trapezoid] *y un lado grande y tiene cuatro esquinas* (and a large side and it has four corners).

Julian See? They get together, *pero acá no* (but not here). [runs his fingers along the base and top parallel sides of the trapezoid]

Andres: *Acá no*

Recomposing this scene in poetic lines, with stanzas representing different speakers, we have:

Porque si. Nomás estas sides get together pero
de este

Y este lado
no

No
porque mira,
aqui tiene un lado chico
y un lado grande
y tiene cuatro esquinas

See? They get together,
pero acá no
Acá no

Here the lines are indented to display the syntactic parallelism. For example, the line *porque si* is echoed in *porque mira*. Semantic parallelism is present too, in, for example, the phrases *un lado chico...un lado grande*. In this representation of the transcript, the lines are also segmented into stanzas to indicate units of syntactically parallel lines each. This representation of the passage shows that there are more ways to analyze bilingual discourse than simply through code-switching. It demonstrates that performance, along with mathematics understanding, can emerge as a collective achievement of several speakers. A high degree of parallelism develops between the speakers as they repeat each others' sentence structure and word choice. The students developed parallel structures, including shared parallel structures, in both English and Spanish, as Julian's repetition of "They get together" blends into an echo of "pero acá...acá no." The students' language "got together," not just the sides of the figures!

The main advantage of the performance concept for mathematics education research is that it expresses a great deal of what we want our students to achieve. Our pedagogies should foster a student's ability, as Hymes put it, to "breakthrough into performance" (1981), to juggle mathematical ideas even when their level of mastery is tentative and incomplete. When students perform mathematics, we know that they are intellectually engaged.

CONCLUDING REMARKS

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Philip Clarkson, Australian Catholic University, Victoria, Australia

The ideas presented in this research forum have, we hope, served to raise issues and questions concerning the role of multilingualism in research in mathematics education. We have provided an opportunity to explore three aspects of this topic.

On *theory*, the forum has included an introduction to a range of theories of language, of language learning and of language acquisition. Whilst such theories form the basis for an entire field of applied linguistics in their own right, participants may now at least be aware of key reference points and have some indication of where to look further.

On *mathematics teaching*, the forum has highlighted the wide range of multilingual contexts in which mathematics classrooms are situated. Such diversity makes it difficult to make general claims concerning teaching or learning in multilingual contexts. We have, however, seen some general questions which arise more widely, such as the issues of interpretation across languages arising in Pakistan in Halai's research.

On *methodology*, the forum has focused on the issues of relating mathematics to language practices, with the perspective of linguistic anthropology offering an alternative light on the examination of mathematics classroom interaction.

An important motivation for this forum is the awareness that multilingualism is prevalent in mathematics classrooms around the world, yet rarely mentioned in research in mathematics education. It is clear from the exploratory nature of the work presented in this forum, that much remains to be done to take account of multilingualism at substantive, methodological or theoretical levels of our research. This observation leads us to see two clear areas in which PME research needs to take greater interest:

- There is a need for more research specifically focused on the role of multilingualism in mathematics education;
- There is a need for all research to acknowledge and take greater account of the multilingual contexts in which it is frequently situated.

We hope this forum has provided encouragement and a starting point.

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