



# PROCEEDINGS

of the

35<sup>th</sup> Conference of the International  
Group for the Psychology  
of Mathematics Education

## **Developing Mathematical Thinking**

**Editor: Behiye UBUZ**

### Volume 1

Plenary Addresses, Plenary Panel, Research Forums,  
Working Sessions, Discussion Groups, National Presentations,  
Short Oral Communications, Poster Presentations

Orta Doğu Teknik Üniversitesi [Middle East Technical University]  
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# PREFACE

It is a great pleasure to welcome you to the 35th Annual Conference of the International Group for the Psychology of Mathematics Education, which is held in Ankara, at the Orta Doğu Teknik Üniversitesi [Middle East Technical University].

Ankara is the capital of Turkey located in the centre of Anatolia province and the country's second largest city after Istanbul. The city has a mean elevation of 850 metres, and has a population around 4 million. Ankara is an ideal environment with its cultural and historical richness as well as its social opportunities. Having a long historical background as a city Ankara hosts many architectural and historical findings in its museums. Many cinemas, theaters, painting and art galleries are in the service of art lovers in Ankara. In addition to these, it is possible to watch the ballet and opera shown by "State Opera & Ballet", listen to concerts of the Classical Turkish Music Choir and the Presidential Symphony Orchestra and watch the performance of the State Folk Dance Ensemble.

Middle East Technical University (METU) is one of Turkey's most competitive universities. METU's 11000 acres main campus is located 5 kilometres the center of Ankara. The campus is conveniently served by a variety of buses and minibuses and is readily accessible by car or taxi. Today, METU's modern campus equipped with the most advanced scientific and technical facilities. I have appreciated the support from the Administrative Board of Middle East Technical University as well as the Board of the Faculty of Education and Faculty of Business and Administration.

After 35 years, PME is for the first time in Turkey. It is important that PME is taking place in Turkey because growing number of mathematics educators in Turkey will have the opportunity to make a real contribution to mathematics education based on our needs and desire and they will have a chance to meet with such a distinguished group of mathematics educators and teachers from across the world.

PME 35 will be a great opportunity for mathematics educators around the world to share the most up-to-date information regarding mathematics education generally and developing mathematical thinking specifically. At this year's conference, by understanding how we can develop mathematical thinking, we can take wiser, more effective actions for the future mathematics education. We have an exciting scientific and social programme lined up for you.

All our efforts as the PME35 organizing committee were to make the participants experience and live one of the best PMEs. Hope that all the efforts will be visible to the participants.

**Behiye UBUZ, Conference Chair**

## **SPONSORS**

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Middle East Technical University

Gaziantep University

Gazi University

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# THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION (PME)

## History and Aims of PME

The International Group for the Psychology of Mathematics Education (PME) is an autonomous body, governed as provided for in the constitution. It is an official subgroup of the International Commission for Mathematical Instruction (ICMI) and came into existence at the Third International Congress on Mathematics Education (ICME3) held in Karlsruhe, Germany in 1976.

Its former presidents have been:

Efraim Fischbein, Israel  
Richard R. Skemp, UK  
Gerard Vergnaud, France  
Kevin F. Collis, Australia  
Pearla Neshet, Israel  
Nicolas Balacheff, France  
Kathleen Hart, UK

Carolyn Kieran, Canada  
Stephen Lerman, UK  
Gilah Leder, Australia  
Rina Hershkowitz, Israel  
Chris Breen, South Africa  
Fou-Lai Lin, Taiwan

The present president is João Filipe Matos, Portugal.

## The Constitution of PME

The constitution of PME was adopted by the Annual General Meeting on the 17th of August, 1980 and changed by the Annual General Meetings on the 24th of July, 1987, on the 10th of August, 1992, on the 2nd of August, 1994, on the 18th of July, 1997 and on the 14th of July, 2005. We print here only two parts of the constitution. As members it is important that you are aware of your rights. The group has the name “International Group for the Psychology of Mathematics Education”, abbreviated to PME. The major goals of the Group are:

- to promote international contact and exchange of scientific information in the field of mathematical education;
- to promote and stimulate interdisciplinary research in the aforesaid area; and
- to further a deeper and more correct understanding of the psychological and other aspects of teaching and learning mathematics and the implications thereof.

## PME Membership and Other Information

Membership is open to people involved in active research consistent with aims of PME, or professionally interested in the results of such research. Membership is on an annual basis and depends on payment of the membership fees. PME has between 700 and 800 members from about 60 countries all over the world.

The main activity of PME is its yearly conference of about 5 days, during which members have the opportunity to communicate personally with each other about their

special and general interest. There are plenary lectures, research paper presentations, working groups, poster sessions and many other activities. Every year the conference is held in a different country.

There is limited financial assistance for attending conferences available through the Richard Skemp Memorial Support Fund.

A PME Newsletter is issued twice a year, and can be found on the IGPME website.

Occasionally PME issues a scientific publication, for example the result of research done in group activities.

### **Website of PME**

All information concerning PME and its constitution can be found at the PME Website: <http://igpme.gandi-site.net/>

### **Honorary Members of PME**

Efraim Fischbein (Deceased)

Hans Freudenthal (Deceased)

Joop Van Dormolen (Retired)

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# 35<sup>th</sup> CONFERENCE OF THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION (PME35)

Two committees are responsible for the organization of the PME35 Conference.

## **The International Program Committee (IPC)**

João Filipe de Matos	University of Lisbon (Portugal), President of PME
Behiye Ubuz	Middle East Technical University (Turkey), Chair of PME 35
Hatice Akkoç	Marmara University (Turkey)
İbrahim Bayazıt	Erciyes University (Turkey)
Aiso Heinze	Leibniz Institute for Science Education (Germany)
Mehmet Fatih Özmantar	Gaziantep University (Turkey)
Tim Rowland	Cambridge University (United Kingdom)
Tai-Yih Tso	National Taiwan Normal University (Taiwan)

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Erdoğan Çakıroğlu	METU
Bülent Çetinkaya	METU
Ayhan Kürşat Erbaş	METU
Çiğdem Haser	METU
Mine Işıksal	METU

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Zülbiye Toluk Uçar	Abant İzzet Baysal University
Aysun Umay	Hacettepe University
Sibel Yeşildere	Dokuz Eylül University

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**Conference Administrative Secretariat :**

For matters related to the administrative issues of the conference (accommodation, excursion, travels, etc) please contact:



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## PROCEEDINGS OF PREVIOUS PME CONFERENCES

The table includes the ERIC numbers and/or the e-address of the websites of the past conferences.

### PME International

No.	Year	Place	ERIC number and/or URL
1	1977	Utrecht	The Netherlands Not available in ERIC
2	1978	Osnabrück	Germany ED226945
3	1979	Warwick	United Kingdom ED226956
4	1980	Berkeley	USA ED250186
5	1981	Grenoble	France ED225809
6	1982	Antwerp	Belgium ED226943
7	1983	Shoresh	Israel ED241295
8	1984	Sydney	Australia ED306127
9	1985	Noordwijkerhout	Netherlands ED411130 (vol.1), ED411131 (vol.2)
10	1986	London	United Kingdom ED287715
11	1987	Montréal	Canada ED383532
12	1988	Veszprém	Hungary ED411128 (vol.1), ED411129 (vol.2)
13	1989	Paris	France ED411140 (vol.1), ED411141 (vol.2), ED411142 (vol.3)
14	1990	Oaxtepec	Mexico ED411137 (vol.1), ED411138 (vol.2), ED411139 (vol.3)
15	1991	Assisi	Italy ED413162 (vol.1), ED413163 (vol.2), ED413164 (vol.3)
16	1992	Durham	USA ED383538
17	1993	Tsukuba	Japan ED383536
18	1994	Lisbon	Portugal ED383537
19	1995	Recife	Brazil ED411134 (vol.1), ED411135 (vol.2), ED411136 (vol.3)
20	1996	Valencia	Spain ED453070 (vol.1), ED453071 (vol.2), ED453072 (vol.3), ED453073 (vol.4), ED453074 (addendum)
21	1997	Lahti	Finland ED416082 (vol.1), ED416083 (vol.2), ED416084 (vol.3), ED416085 (vol.4)
22	1998	Stellenbosch	South Africa ED427969 (vol.1), ED427970 (vol.2), ED427971 (vol.3), ED427972 (vol.4)
23	1999	Haifa	Israel ED436403
24	2000	Hiroshima	Japan ED452301 (vol.1), ED452302 (vol.2), ED452303 (vol.3), ED452304 (vol.4)
25	2001	Utrecht	The Netherlands ED466950

26	2002	Norwich	United Kingdom ED476065
27	2003	Hawai'i	USA <a href="http://onlinedb.terc.edu">http://onlinedb.terc.edu</a>
28	2004	Bergen	Norway <a href="http://emis.de/proceedings/PME28/">http://emis.de/proceedings/PME28/</a>
29	2005	Melbourne	Australia <a href="http://staff.edfac.unimelb.edu.au/~chick/PME29/">http://staff.edfac.unimelb.edu.au/~chick/PME29/</a>
30	2006	Prague	Czech Republic <a href="http://class.pedf.cuni.cz/pme30">http://class.pedf.cuni.cz/pme30</a>
31	2007	Seoul	Korea
32	2008	Morelia	Mexico, <a href="http://www.pme32-na30.org.mx/">http://www.pme32-na30.org.mx/</a>
33	2009	Thessaloniki	Greece, <a href="http://www.pme33.eu">http://www.pme33.eu</a>
34	2010	Belo Horizonte	Brazil, <a href="http://pme34.lcc.ufmg.br/">http://pme34.lcc.ufmg.br/</a>

Copies of some previous PME Conference Proceedings are still available for sale. See the IGPME website at <http://igpme.org/publications/procee.html> or contact the proceedings manager Dr. Peter Gates, PME Proceedings, University of Nottingham, School of Education, Jubilee Campus, Wollaton Road, Nottingham NG8 1 BB, UNITED KINGDOM, Telephone work: +44-115-951-4432; fax: +44-115-846-6600; e-mail: [peter.gates@nottingham.ac.uk](mailto:peter.gates@nottingham.ac.uk)

## **PME-NA**

No.	Year	Place	ERIC number and/or URL
1	1979	Evanston, Illinois	
2	1980	Berkeley, California	(with PME2) ED250186
3	1981	Minnesota	ED223449
4	1982	Georgia	ED226957
5	1983	Montreal, Canada	ED289688
6	1984	Wisconsin	ED253432
7	1985	Ohio	ED411127
8	1986	Michigan	ED301443
9	1987	Montreal, Canada	(with PME11) ED383532
10	1988	Illinois	ED411126
11	1989	New Jersey	ED411132 (vol.1), ED411133 (vol.2)
12	1990	Oaxtepec, Morelos, Mexico	(with PME14) ED411137 (vol.1), ED411138 (vol.2), ED411139 (vol.3)
13	1991	Virginia	ED352274
14	1992	Durham, New Hampshire	(with PME16) ED383538
15	1993	California	ED372917
16	1994	Louisiana	ED383533 (vol.1), ED383534 (vol.2)
17	1995	Ohio	ED389534
18	1996	Panama City, Florida	ED400178

19	1997	Illinois	ED420494 (vol.1), ED420495 (vol.2)
20	1998	Raleigh, North Carolina	ED430775 (vol.1), ED430776 (vol.2)
21	1999	Cuernavaca, Morelos, Mexico	ED433998
22	2000	Tucson, Arizona	ED446945
23	2001	Snowbird, Utah	SE065231 (vol.1), SE065232 (vol.2)
24	2002	Athens, Georgia	SE066887 (vol.1), SE066888 (vol.2), SE066889 (vol.3), SE066880 (vol.4)
25	2003	Hawai'i	(with PME27) ED500857 (vol.1), ED500859 (vol.2), ED500858 (vol.3), ED500860 (vol.4)
26	2004	Toronto, Notario	<a href="http://www.pmena.org/2004/">http://www.pmena.org/2004/</a>
27	2005	Roanoke, Virginia	<a href="http://www.pmena.org/2005/">http://www.pmena.org/2005/</a>
28	2006	Merida, Yucatan, Mexico	<a href="http://www.pmena.org/2006/">http://www.pmena.org/2006/</a>
29	2007	Lake Tahoe, Nevada	<a href="http://www.pmena.org/2007/">http://www.pmena.org/2007/</a>
30	2008	Morelia, Mexico	(with PME32) <a href="http://www.pmena.org/2008/">http://www.pmena.org/2008/</a>
31	2009	Atlanta, Georgia	<a href="http://www.pmena.org/2009/">http://www.pmena.org/2009/</a>
32	2010	Columbus, Ohio	<a href="http://www.pmena.org/2010/">http://www.pmena.org/2010/</a>

Abstracts from some articles can be inspected on the ERIC website (<http://www.eric.ed.gov/>) and on the website of ZDM/MATHDI (<http://www.emis.de/MATH/DI.html>). Many proceedings are included in ERIC: type the ERIC number in the search field without spaces or enter other information (author, title, keyword). Some of the contents of the proceedings can be downloaded from this site. MATHDI is the web version of the Zentralblatt für Didaktik der Mathematik (ZDM, English subtitle: International Reviews on Mathematical Education). For more information on ZDM/MATHDI and its prices or assistance regarding consortia contact Gerhard König, managing.

## REVIEW PROCESS OF PME35

**Research Forums.** The international Programme Committee accepted 2 out of the 2 submitted RF proposals. For each one, the proposed structure, the contents, the contributors, and their role were reviewed and agreed by the members of International Program Committee (IPC).

**Working Sessions and Discussion Groups.** There were 5 Working Session (WS) and 10 Discussion Group (DG) Proposals. The abstracts were all read and commented on by the International Program Committee, and 8 DG and all WS were accepted. The rejection of DG's were made on the basis that the proposed work did not fulfill the respective PME regulation requiring the participants of a DG to be engaged collaboratively in a joint activity.

**Research Reports (RR).** The IPC received 308 RR proposals. Each paper was blind-reviewed by three peer reviewers. The experienced reviewers contacted for this purpose were not, however, enough. Thus, more reviews were asked from all the reviewers. The majority of the connected PME members responded to the request and contributed decisively to the successful completion of this crucial task.

Reviewers received proposals for review according to the research categories indicated in their Reviewer Information Form. The proposals were sent to reviewers according to the research categories marked by the author(s).

All papers with two or three acceptances were accepted. The members of the IPC reexamined all the proposals with one acceptance and two rejections. For the proposals that were finally accepted, a fourth review was added to the existing three ones. For the remaining papers, the IPC offered a Short Oral Communication (SO) or a Poster Presentation (PP) or agreed that the paper should be rejected. Finally, 161 proposals were accepted, 93 were recommended as SOs, 31 as PPs and the remaining ones were rejected.

**Short Oral Communications (SO) and Poster Presentations (PP).** The IPC initially received and reviewed 194 SOs and 95 PPs proposals, 146 and 73 of which were accepted respectively. In addition, 49 SOs proposals were re-submitted from RRs and 22 PPs were re-submitted from RRs.

The reviewing process was completed during the 2nd Meeting of the International Programme Committee around the end of March 2011.

## LIST OF PME35 REVIEWERS

The PME35 Program Committee thanks the following people for their help in the review process:

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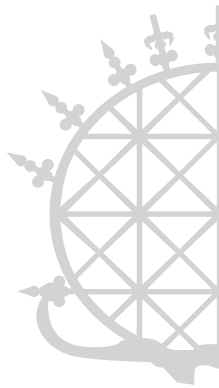
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# **PLENARY ADDRESS 1**

*Developing purposeful mathematical thinking: A curious Tale of apple trees*

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# DEVELOPING PURPOSEFUL MATHEMATICAL THINKING: A CURIOUS TALE OF APPLE TREES

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*In this paper I explore aspects of the ways in which school mathematics relates to the 'real' world, and argue that this relationship is an uneasy one. Through exploring the causes of this unease, I aim to expose some problems in the ways in which context is used within mathematics education. I present and discuss a framework for task design that adopts a different perspective on mathematical understanding, and on purposeful mathematical thinking.*

## OVERVIEW: INTRODUCING PURPOSE

My theme is *purpose*. I want to approach this at (at least) three levels: at the level of the intended curriculum, from the perspective of teachers, and through the experiences of children. An over-arching curriculum-level question might be: what is the purpose of teaching mathematics? There are many kinds of possible answers to this question:

- our economy depends on people with mathematical skills to work in science, technology, engineering, business and economics,
- mathematics is a logical discipline which trains the mind,
- mathematics is an enjoyable activity and part of our cultural heritage,
- mathematics is important for understanding the world, and for everyday life.

The last of these is generally foregrounded in curriculum and policy statements. An examination of the aims stated in curriculum documents from a range of countries reveals a fairly consistent message about the importance given to the role of mathematics in enabling learners to relate to the world beyond the classroom.

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase (National Council for Teachers of Mathematics, USA, 2004)

Mathematics education aims to ... develop in students the skills, concepts, understandings, and attitudes which will enable them to cope confidently with the mathematics of everyday life (Ministry of Education, New Zealand, 2008).

Mathematics education aims to enable students to ... acquire the necessary mathematical concepts and skills for everyday life (Ministry of Education, Singapore, 2006).

Mathematics introduces children to concepts, skills and thinking strategies that are useful in everyday life (Qualifications and Curriculum Authority, England, 2008)

Being mathematically literate enables persons to contribute to and participate with confidence in society (Department of Education, South Africa, 2002)

I want to explore the implications of this purpose for teaching mathematics, and how the content of school mathematics is shaped by it. I shall focus on the ways in which pedagogic tasks and artefacts are used by teachers in response to this need for everyday relevance. As a starting point, I want to take a fresh look at the curriculum artefacts that most clearly embody the desire to make school mathematics relevant to everyday life: contextualised word problems.

Despite the high level of agreement about policy level views of the purpose of teaching mathematics, we know that this does not necessarily carry through to the experiences of learners in the classroom. Attempts to identify a core of mathematical knowledge that everyone needs for everyday life are doomed to failure, not least because the needs of everyday life change, both for individuals and for societies. I shall argue that the use of contextualised problems is inherently problematic, and explore some of the reasons why developing purposeful mathematical thinking in the classroom that makes effective connections to everyday life is difficult.

Finally, I want to draw on themes from my own research to propose a different perspective of the idea of purpose in school mathematics.

## **A CURIOUS TALE OF APPLE TREES**

I am going to base my exploration of contextualised word problems two examples drawn from very different sources. The first is from a textbook published in 1887: *The Problematic Arithmetic for the Seven Standards*. The second comes from a very different source, though by serendipity, it is also about apple trees: an assessment item taken from the Programme for International Student Assessment, in 2006. The 1887 example is typical of a genre that has proved remarkably resilient to change.

A gardener gathered 7008 apples from twelve trees and each tree produced the same number. How many from each tree?

There is a substantial literature within mathematics education which explores and critiques many aspects the use and construction of such problems (see, for example, Verschaffel, Greer & Torbeyns (2006)). I do not wish to engage directly with this literature, but rather to consider two questions in relation to word problems that take a somewhat different perspective from those of previous researchers:

- What purpose did the author have for writing the problem in this way?
- What is the purpose for which the problem is intended to be used in the classroom?

What was the author's purpose? We might suppose that the author chose this context because it appeared a familiar 'real' situation, but it is less clear why he (and I assume it was *he*) did not choose a problem within that context in which the same

division calculation could be modelled without attributing obviously unrealistic properties to the apple trees. For example:

A gardener gathered 7008 apples and then packed them into twelve boxes with the same number in each. How many in each box?

Or perhaps with a little less contrivance:

A gardener gathered 7008 apples and then packed them into boxes each holding twelve apples. How many boxes?

It is, of course, impossible to reconstruct the reasons for the author's choices, but it does seem safe to say that a concern with accurately reflecting real life was not the main priority in the composition of this, and other, problems. The choice of an apparently meaningful context of apples and trees is sufficient for the author's main purpose, which is the teaching of mathematics. I shall pick up this point later.

What is the purpose for which the problem is intended to be used in the classroom? We might consider whether it is intended as a teaching resource, to be used to support children in thinking about the process of division, or as part of an assessment to see whether children can apply their knowledge of division to a 'real' context. As I shall argue, the purpose for which a problem is intended to be used might offer different perspectives on how we consider its value as a problem.

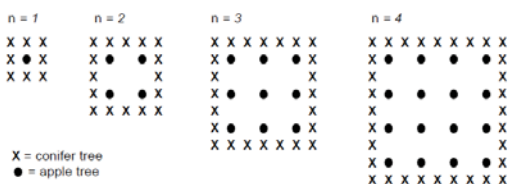
My second group of apple trees appears in the PISA materials produced by the Organisation for Economic Co-operation and Development (OECD). PISA is designed to assess mathematical literacy, which is defined as follows.

Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2006a)

For the problem I have chosen to focus on, the context is as follows.

A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number ( $n$ ) of rows of apple trees:



Three questions then follow.

Question 1:	Question 2:	Question 3:
Complete the table	There are two formulae you	Suppose the farmer wants

n	Number of apple trees	Number of conifer trees
1	1	8
2	4	
3		
4		
5		

can use to calculate the number of apple trees and the number of conifer trees for the pattern described above:

Number of apple trees =  $n^2$

Number of conifer trees =  $8n$

where  $n$  is the number of rows of apple trees.

There is a value of  $n$  for which the number of apple trees equals the number of conifer trees. Find the value of  $n$  and show your method of calculating this.

to make a much larger orchard with many rows of trees.

As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifer trees?

Explain how you found your answer.

OECD (2006b)

Although the real world context here is more elaborated, it is no less contrived than that in the earlier problem. The idea that the farmer restricts himself to square orchards mirrors the regularity of the trees that magically bear the same number of fruit. The suggestion that he is willing to plant eight conifers to protect a single apple tree stretches credulity in other ways. As part of an assessment of mathematical literacy, it seems an odd choice. I pose the same two questions.

What purpose did the author have for writing the problem in this way? A striking feature of the problem is that, because of the visual presentation, it would work perfectly well if the real world context were removed altogether, and Questions 1 – 3 were asked simply about the arrays of crosses and circles. The choice of an elaborated ‘real’ context must then relate to the stated aim of PISA to assess capacity to understand the role mathematics plays in the world. Questions 1 and 3 within the problem might, with a little imagination, be seen as something the farmer would need to work out: for any particular size of field, how many of each type of tree will be needed and how would the proportions change. Question 2, however, moves far beyond any realistic use of mathematics. The question may be interesting mathematically, but it is not clear why anyone would need to know the answer.

What is the purpose for which the problem is intended to be used in the classroom? The answer to this is clear: it is part of a written test, intended to assess ‘*capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics*’ in ways appropriate to adult life (OECD, 2006a). As outlined above, I question how effectively the problem achieves that aim, though of course it is only one problem from a larger collection. The problem has not been designed as a teaching resource, to support pupils’ understanding of the mathematics involved, although it could be used in that way

with a little adaptation. For example, in a classroom situation, a teacher might ask pupils to find the formulae used in Question 2 for themselves rather than providing them, or extend Question 3 by including rectangular fields with different proportions. This leads to more questions about the relationship between school mathematics and the real world that will illuminate why that relationship is so uneasy.

## WHY CONTEXTUALISE SCHOOL MATHEMATICS?

Clearly one purpose is to address the curriculum aims quoted earlier: to help children to understand how and why mathematics is used in everyday life, to support their understanding of the world, and to equip them to participate confidently as citizens. There is an implicit expectation that word problems can do this, by acting as ‘boundary objects’ that ‘inhabit ... intersecting worlds and satisfy the informational requirements of each of them’ (Star & Griesemer, 1989) and thus bridge between the classroom and everyday life. We have ample evidence that this expectation is not fulfilled. Word problems generally do not function in this way: they fail as boundary objects because they do not satisfy the informational requirements of everyday life.

In a wide ranging study of children’s responses to contextualised problems in assessment, Cooper and Dunne (2000) have provided revealing evidence of a difficulty some children, and particularly those from lower socio-economic groups, have in responding ‘correctly’. This difficulty is not caused by a lack of understanding of the mathematics, or of the context being used, as has been the case in some other studies, and it highlights one of the ways in which word problems fail as boundary objects. The issue which Cooper and Dunne identified is children’s failure to understand the implicit rules of the pedagogic context about how to make use of everyday knowledge. They observed children who approached solving contextualised problems by drawing on aspects of their everyday knowledge in way which were not intended, such as using the actual price that they have recently paid for a canned drink, rather than using information given in the problem to work it out.

In contrast, children who have learned the rules of the pedagogic game ‘*perceive school word problems as artificial, routine-based tasks which are unrelated to the real world*’ (Verschaffel et al., 2006, p.60) and manage to answer word problems ‘correctly’ precisely because they attend mainly to the form of the question, and largely ignore the context. Approaches adopted by teachers, and enshrined in teaching resources, involve strategies for identifying key features of the written problem (the numbers, and words such as ‘altogether’ which signal the operation to be used) that strip away the specifics of context, which are seen as a distraction. Gerofsky (1996) paraphrases this approach in the following instructions:

I am to ignore ... any story elements of this problem, use the math we have just learned to transform ... [it]... into correct arithmetic or algebraic form, solve the problem to find one correct answer... (p. 39)

Such strategies prove very effective in terms of operating successfully in the classroom, but clearly work against the espoused curriculum aim of learning to use

mathematics to make sense of real world situations. Word problems are ostensibly used to address the curriculum aims of learning the mathematical skills relevant to everyday life, and yet research and established pedagogic practice point to ignoring the context of problems as the best way for children to succeed in solving them.

Further, Verschaffel et al. (2006) discuss a wide range of research, over a long period, which has sought to explore the difficulties which students experience in making the transformation from a 'story' format into a mathematical one, and the pedagogic challenges of supporting them to do this. In the face of such evidence of the difficulties that students and teachers experience with word problems, I am led to question why it is that the writers of pedagogic materials and mathematics tests appear as committed as ever to their value in the curriculum.

### **Some reasons for the continued use of contextualised problems**

A somewhat cynical answer to the question of the continued popularity of contextualised problems is that contexts are used to 'dress up' the mathematics in order to interest and engage learners. The tales of apples trees we have already examined might not appear to be particularly likely to excite school children, but since, as we have seen, the contexts are only laid loosely onto the mathematics, we could easily replace them with something more superficially interesting or relevant: a football crowd being seated in a stadium in which the rows each hold twelve people, a supermarket selling chocolate bars which are delivered in boxes of twelve, the placing of chairs around tables in a school dining room.

I have suggested previously (Ainley, 1997) that there may be two other answers to this question, and exploring them exposes one area of confusion which contributes to the unease in the relationship between school mathematics and the 'real' world. The two answers are encapsulated in the traditional format used for school text and resources. This format is, of course, not universal, but it is common enough to reflect, and be reflected in, perceptions about the role of context.

At the beginning, the mathematical topic is introduced and explained. Here it is common for a real world context to be used, partly to provide interest and engagement, but more importantly to support the pupils' understanding of the mathematical ideas. The next section may then contain exercises to practice the ideas and procedures that have been introduced, without the support of contextual examples. Later further contextualised problems are given in which the mathematics that has been practised is applied to 'real' examples. Later still, contextual problems may be used to test whether the mathematical ideas have been properly understood.

Two contradictory ideas are at work here. At first, context is used to support the understanding of a piece of mathematics, by relating it to a familiar situation in the 'real' world, and thus offering a model for thinking about the mathematical structures. For example, arranging chairs in rows for a meeting in the school hall might provide a model for multiplication, and a context for the multiplication of relatively large numbers. This is considered to make the introduction of the

mathematical ideas easier. This notion is at the heart of the *Realistic Mathematics Education* approach.

[context is] a characteristic of a task presented to the students: referring either to the words and pictures that help the students to understand the task, or concerning the situation or event in which the task is situated. (Van den Heuvel-Panhuizen, 2005 p. 2)

Later in the text, contextualised problems are used to check that the mathematical ideas have been understood. Being able to solve these problems successfully is an indication that pupils have ‘really understood’ the new mathematical ideas, rather than just being able to perform calculations. The underlying assumption here is that context makes the mathematics harder, and there is evidence, for example from the work of Cooper and Dunne (2000) that in test situations, pupils perform better on context-free questions than on contextualised ones. This view is not uncontested. Within the RME approach, carefully designed context in assessment problems is seen as:

enhancing the accessibility of problems, contributing to the transparency ..of problems, and suggesting solution strategies (Van den Heuvel-Panhuizen, 2005 p. 2)

A distinction is being made here by Van den Heuvel-Panhuizen between word problems, ‘in which the context is not very essential – it can be exchanged for another without substantially altering the problem’ (Van den Heuvel-Panhuizen, 2005 p. 5), and ‘context problems’, in which there is a more intimate connection between the context and the mathematics. I do not find this distinction so clear-cut, perhaps because of my limited experience of RME. However the fact that context can, with careful design, clearly be used to make assessment problems more accessible does not alter the fact that contextualised problems are often used in assessment as being a better way to test ‘real understanding’ than straightforward calculations.

Thus we have two more answers to the question of the purpose for contextualising school mathematics, and they are contradictory. On the one hand, mathematics is contextualised to make it easier for children to understand, and on the other it is contextualised to make it harder, and test whether they have understood it. Of course this is a somewhat simplistic perspective, since the pedagogic contexts, and particular teacher input, will be very different in teaching situations, where ideas are being introduced, and in testing situations. Nevertheless, I believe that there is a double-think at the heart of our uses of context which are symptomatic of the uneasy relationship between school mathematics and the ‘real’ world.

### **A further confusion**

In her plenary lecture at the Conference of the European Society for Research in Mathematics Education (CERME7) earlier this year, Anna Serpinska described a difference between real mathematics and primary school mathematics as follows. (Real) mathematicians model bits of the world in mathematical terms; in primary mathematics we look for aspects of the real world that we can use to model bits of mathematics. This comment is insightful, but, I think, does not quite capture the

whole picture. First, this is not only true of primary school mathematics; I think it is true of all of school mathematics. Perhaps more importantly, there is an implicit assumption that these two activities are closely related, if not simply two aspects of the same approach, and that having bits of mathematics modelled in real world contexts will enable children to use mathematics to understand the world.

Returning to national curriculum documents we can find reflections of a two-way relationship. For example, in the English National Curriculum documents there is a claim that: *'Mathematics helps children make sense of the numbers, patterns and shapes they see in the world around them'* (QCA, 2008). In contrast in other equivalent documents, this example is from Singapore, we find claims that opportunities to make connections between mathematics and everyday life *'help students make sense of what they learn in mathematics'* (MOE Singapore, 2006).

The important difference here is one of purpose. When the mathematician uses mathematics to model the world, their purpose is to understand the world better; perhaps to be able to predict earthquakes or stock market crashes, to design efficient traffic systems or bridges. When a teacher chooses a real world context to model a bit of mathematics, their purpose is to teach mathematics; to help children understand the mathematics ideas, or to test that they have understood and can apply them.

I suggest that this is not a trivial difference, and failing to recognise it is potentially problematic. When we select real world contexts to model bits of mathematics, we look for simple structures that match those of the mathematical ideas we want to teach or to test. This is what leads us into the magic orchards where trees bear the same numbers of apples, or grow only in rigid square arrays. However, if rather than looking at the two orchards as problems intended to make connections between the classroom and the 'real' world, we look them as examples of *using the real world to understand mathematics*, they seem less problematic. The first does offer a simple model of division; we can imagine the apples distributed across the twelve trees, and the large pile of apples once they have been harvested. The second presents a related pair of patterns that can be expressed algebraically.

### **An uneasy relationship**

To summarise, I have argued that there are two important, but generally unrecognised, ways in which our thinking about the relationship between school mathematics and the 'real' world is muddled. In different ways, they both concern the purposes of the curriculum, and of teachers, for contextualising mathematics. The first is that we use contextualised problems both to make mathematical ideas easier to understand, and to make them harder, to test that children have understood them. The second is that whilst we intend to use contextualised problems with the purpose of helping children to make sense of the world, the purpose behind the design of those problems is often to use the world to make sense of the mathematics. If we look at them from this perspective, the continuing widespread use of word problems is both more understandable, and less worrying: they can provide good models for thinking



about mathematical ideas. The danger lies in thinking that having used problems in which the world is used to make sense of mathematics, we have also achieved the aim of giving children opportunities to see how mathematics can be used to make sense of the world. The result is that we often achieve the opposite, leaving children with a view that the context in questions is there to complicate the situation, and so best ignored, and a view of mathematics as irrelevant and purposeless.

... it was as if there were a kind of check-in desk just outside the classroom door labelled 'common sense', and as the pupils filed into the classroom, they left their common sense at the check-in desk saying 'Well we won't be needing this in here'. (Wiliam, 1992, p. 3)

## USING MATHEMATICS TO MAKE SENSE OF THE WORLD

I now want to shift attention to the ways in which the purposes for learning and doing mathematics may be understood by learners. So far I have suggested that real world contexts are used in school mathematics mainly for the purpose of modelling mathematical ideas, and this is, of course, somewhat simplistic. There are many resources which have been developed to link parts of the mathematics curriculum directly to 'everyday' uses of mathematics that are likely to be relevant to children's present or future lives. Whilst this may have some success, there seem to me to be a number of problems likely to occur when we try to present the purpose for learning mathematics in this way. What appears to be a logical pedagogic approach may not match the actual, and often more complex, experiences of children outside school. In a study about how children in Hawai'i learn about money Brenner (1998) illustrated this very strikingly: in school they begin by learning about cents and gradually work up to using dollars, while in their real experience in local shops, dollars are what count, and children often discard cents as useless. This is one reason why children may not recognise the purpose of learning even those aspects of mathematics that might be regarded as most closely related to everyday life, such as measurement, when they learn them in classroom contexts (Ainley, 1991).

Furthermore, in the transition into the classroom real situations get tidied up and simplified, so that they may become the equivalent of the magic orchards. Even situations such as paying tax or buying on hire purchase, which may be concerns in children's future adult lives, are unlikely to generate real engagement and interest for children who are still in school.

Research which has drawn on the perspective of situated cognition to explore 'street mathematics' may appear to offer a different approach. Lave and Wenger (1991) claim that in out-of-school learning contexts, 'learners, as peripheral participants, can develop a view of what the whole enterprise is about'. Such opportunities seem to be relatively rare in mathematics classrooms. Whilst I would support Schliemann's (1995) claim that 'for meaningful mathematical learning to take place in the classroom, reflection upon mathematical relations must be embedded in meaningful socially relevant situations', the provision of such experiences in the classroom is

inherently problematic if the context alone is transferred from the real world, but not the purpose for using mathematics which makes sense to learners.

### **Creating opportunities for purposeful mathematical thinking**

I have argued so far that the traditional approach of using contextualised word problems, and indeed other more sophisticated approaches which rely on using aspects of the real world to help children to understand specific mathematical ideas, can have only limited success in supporting children to develop the kinds of mathematical thinking that is needed to make sense of the world. I want now to try to address the more difficult question of what else might be required to embed an understanding of mathematics in meaningful socially relevant situations.

I start by looking not at school mathematics, but at ‘real world’ mathematics. We know quite a lot about the mathematical thinking that is developed, even by those with little formal mathematical education, in out-of-school contexts. For example, the methods used are often idiosyncratic, and linked to specific contexts and resources. However, an over-riding feature of ‘real world’ mathematics is that people use it for a clear purpose, to get things done. The people using the mathematics understand why and how it is being used, and it really matters to everyone involved that the answers that are produced are correct.

In my own research, largely in collaboration with Dave Pratt, I have focussed on the design of pedagogic tasks that attempt to replicate, in the classroom, this kind of context for mathematical thinking. We have developed a framework for this design, which has two dimensions (Ainley & Pratt, 2002, Ainley, Pratt & Hansen, 2006). The first dimension focuses on the learners, and is concerned with creating tasks that have a clear ***purpose for learners***, within the classroom. Here we are concerned with purpose seen from the perspective of the learner, and the activity that is taking place during the lesson. It is not (necessarily) linked to any specific application outside the classroom, and indeed may be about a situation that is clearly not ‘real’, in the everyday sense. In order to create this sense of purpose for the learner, our tasks typically have an end-product, which might be a real object (such as an efficient paper plane, or a puzzle for other children to solve) or a virtual object (such as a method for scaling dolls’ house furniture, or dynamic geometry macros which act as a drawing kit for younger children). In other cases, the end-product might be the solution to an intriguing problem, such as understanding the behaviour of an unusual die, or the height of a giant who has left a particular hand-print. In the design of these tasks we also aim to leave flexibility for learners to make real decisions about how they structure their activity, as this supports their engagement and ownership of the outcomes. An important feature of such tasks, which mirrors the use of mathematics in out-of-school contexts, is that the purpose of the task, rather than the teacher, becomes the source of feedback about progress.

We see the design of purposeful tasks as important, but only part of the design challenge. In creating such tasks our purpose, as teachers, is to introduce particular

mathematical ideas, and provide opportunities for children to make sense of them. Rather than focussing on the development of procedures (techniques and algorithms, specific rules or formulae) and relationships (links within mathematics, internal structure and consistency), the second dimension of our framework is to build into task design the need for learners to use mathematical ideas in ways that will allow them to recognise what we call their *utility*. By this we mean how and why the mathematics is useful to get things done. Again, from the learner's perspective this does not (necessarily) refer to usefulness in the 'real world': our concern is initially with usefulness within the particular task. The aim of our task design is for children to be able to construct a sense of the kinds of situations in which a particular mathematical idea can be used, and the power that it offers.

We argue that mathematical ideas are complex, and composed of different elements, which here we might categorise as procedures, relationships, and *utilities* (why, how and when the idea may be useful). As children construct meaning for a new mathematical idea, connections will be made with existing knowledge, but the pedagogic emphasis placed on the different elements will affect the ways in which those links are made. As I have already argued, traditional attempts to contextualise mathematics generally fail to give real emphasis to utilities. Just as pedagogic approaches that focus mainly, or exclusively, on procedures will result in limited understanding, we suggest that approaches which do not emphasise utilities will also result in impoverished learning in which mathematical knowledge becomes isolated as weak connections are made to existing knowledge of the contexts in which it may be usefully applied.

### **Some examples of purposeful task design**

The following examples are selected from work over two decades, to give an overview of ways in which the purpose and utility framework has been applied in a variety of projects covering different areas of mathematics. The first is taken from the Primary Laptop Project in the mid 1990's when Dave Pratt and I introduced the first Mac Powerbooks to a primary school. Amongst the software that we provided was an early dynamic geometry package. The 9-10 year-olds enjoyed exploring this to draw pictures, but showed no interest when we tried to introduce geometric construction by challenging them to draw squares that could not be 'pulled-apart'. We then exploited the partnership the children had with a much younger class in the school to design a task that involved the children in making some drawing tools for their younger partners to use. This involved creating screen objects which could be reproduced many times, and which would stay the same shape when they were dragged to change their position and size. The children engaged enthusiastically with the purpose of making a drawing environment for their younger friends, and as they created sets of shapes (squares, rectangles, different kinds of triangles, roofs, wheels) they experienced the utility of geometrical construction, and of particular geometrical relationships (Pratt & Ainley, 1997).

My next example comes from the Purposeful Algebraic Activity project in which Liz Bills, Kirsty Wilson and I used the purpose and utility framework to explore the use of spreadsheets in the introduction of algebraic notation. We developed a series of tasks for children in the first year of secondary school aiming to give opportunities for them to experience utilities of algebraic notation. The activity I describe here is based on exploring a hundred square, taking a 3 by 3 cross, and finding the total of the horizontal and vertical arms. Our focus was on the utility of algebraic notation to show structure (Ainley, Bills & Wilson, 2005). The first stage of the tasks was for the children to create a ‘testing cross’, highlighted in Figure 1, in which they used formulae to complete the whole cross when a number was entered in the central square (see Figure 2) and to find the totals. The second part of the task presented a story about a teacher who had used this activity for several years with her primary school class, but was bored with the cross shape. The children were challenged to design a new shape that could be used for the same sort of activity.

	A	B	C	D	E	F	G	H	I	J
1	1	2	3	4	5	6	7	8	9	10
2	11	12	13	14	15	16	17	18	19	20
3	21	22	23	24	25	26	27	28	29	30
4	31	32	33	34	35	36	37	38	39	40
5	41	42	43	44	45	46	47	48	49	50
6	51	52	53	54	55	56	57	58	59	60
7	61	62	63	64	65	66	67	68	69	70
8	71	72	73	74	75	76	77	78	79	80
9	81	82	83	84	85	86	87	88	89	90
10	91	92	93	94	95	96	97	98	99	100
11										
12										
13										
14			6							
15		15		17						
16			26							
17	Column	48								
18	Row	48								

Figure 1: the hundred ‘square’ and testing cross

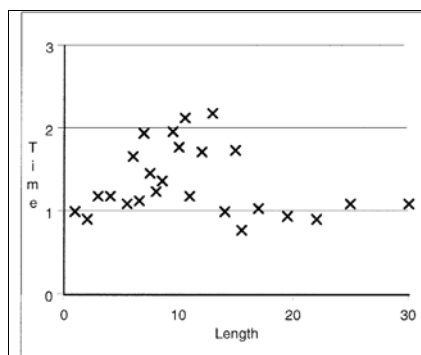
	B14 - 10	
B14 - 1		B14 + 1
	B 14 + 10	

Figure 2: formulae in the testing cross

Using formulae in spreadsheet notations to create the testing cross focussed attention on the structure of the hundred square, which remained the same wherever the cross was placed. Groups of children used this experience to develop new shapes which also produced interesting relationships when numbers in them were added in particular ways: larger crosses, diagonal crosses, L and H shapes. Their discussion revealed their appreciation of symmetry, and the place-value structure in the square.

Much of my research has been in the area of statistics education, and the comparison between the concerns of mathematics and statistics education is often interesting. In one sense the use of context is rather different in statistics, since the use of context is much more closely linked to learning and teaching of statistics ideas than is often the case in mathematics. Embedding the teaching of statistical ideas within a problem-solving cycle is a well-established approach (see for example Ben-Zvi & Garfield, 2004). The relationship between understanding of statistical ideas, and understanding of context can be complex, as Carlos Monteiro’s work has suggested (Monteiro & Ainley, 2004). However concerns about how children perceive the purpose of learning statistical ideas are just as real.

In a project which focussed on supporting children's understanding of graphing, which Dave Pratt and I developed with Elena Nardi, we used the purpose and utility framework in combination with an approach we called 'Active Graphing', in which tasks required children to make use of graphs as analytic tools during a practical activity, rather than only to present results at the end (Ainley, Nardi & Pratt, 1998). A series of tasks were designed to encourage a focus on the utility of displaying results graphically, in order to look for patterns, and make decisions about further data to be collected. In their activity in response to these tasks, 8-9 year old children developed increasingly sophisticated ways of talking about their graphs that move seamlessly between references to the graph and to the context, indicating the transparency (Meira, 1998) that the graphs have for them (Ainley, 2001). The example in Figure 3 comes from a task in which children are testing the effects of changing wing length on the time of flight for paper spinners. This is one of many examples in which we saw children beginning to gain a sense of the importance of looking beyond individual data points to identify trends (Ainley, Nardi & Pratt, 2000, Ainley, Pratt & Nardi, 2001). In some cases, appropriate intervention by the teacher extended children's thinking to include opportunities to consider the utility of repeating experiments and using average values to produce a clearer graph which helped them to identify the optimum wing length (Ainley & Pratt, 2010).



*Tom:* I think the [ones] under 10, 15, under 15 are going are the best.

*Researcher:* Okay, so less than 15 and more than . . . what is this?

*Tom:* Actually 12 and 9 they are all coming up.

*Researcher:* Yes, and then what's happening?

*Tom:* Over 12 they are all going down.

*Researcher:* They are going down . . .

*Tom:* They are just falling instead of spinning.

Figure 3: scattergraph of the flight times for spinner with different wing lengths, and Tom's commentary

### A perspective from beyond the mathematics classroom

My current research is providing a new perspective on the issue of purpose, at all three levels of curriculum, teachers and learners, by taking me beyond the mathematics classroom in a European cross-curricular project about Inquiry-Based Science and Mathematics Education. In our part of the Fibonacci Project, I am working with my science colleagues, Tina Jarvis and Frankie McKeon, to develop an approach in which the teaching and learning of mathematics and science are

integrated through an inquiry approach (Ainley, Jarvis & McKeon, forthcoming). What is emerging from this, apart from rich opportunities for the purposeful use of statistical ideas in the course of scientific experiments, is the explanatory power of mathematics in understanding scientific concepts, and explaining phenomena, within the school curriculum. For example, an understanding of ratio and proportion is important in contexts as diverse as the flight of a paper spinner, why some types of sugar dissolve more quickly than others, the need for a small child to wear more layers of warm clothing than her mother on a cold day, and why elephants have big ears. Working across this curriculum boundary is providing new challenges, and opportunities to extend the framework for task design which focuses on the utility of mathematical ideas by using tasks that are purposeful for children.

## REVISITING THE ORCHARD

I began this paper by looking at two examples of attempts to address the purpose for learning mathematics which is expressed on curriculum documents: to enable children to use mathematics to make sense of the world, and to function effectively in it. Through these two tales of apple trees, I have highlighted limitations that they share despite their very different origins, and in particular identified two areas of confusion about the purposes for which contexts are used in school mathematics: to make mathematics easier, or harder, to help explain the world, or to help explain mathematics. In the last part of the paper I have introduced the idea of utility as a third dimension of understanding in mathematics, which can be made available through tasks which are purposeful for children within the classroom. In conclusion, I want to argue that stronger connections between school mathematics and the ‘real’ world can be made by attending to why and how mathematical ideas are powerful, so that children have an opportunity to see what the whole enterprise of growing apples is about.

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# ON THE MATHEMATICAL IDEAS UTILITY: THE DESIGN OF SCHOOL TASKS

A reaction to Janet Ainley's plenary lecture  
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*Janet's proposal of a purposeful and utility framework provides novel elements for the design of tasks in mathematics education and open new paths to research on the relationship 'mathematics - real world' in the classroom. Whilst recognizing the contribution of the notions she introduces, I argue that the qualities attributed to such notions have a relative character in their potential application to task design: a) for different school levels; b) when using a variety of technology learning environments; c) when bearing in mind long-term didactic purposes.*

## AN ALTERNATIVE VIEW OF THE MATHEMATICS – 'REAL WORLD' RELATIONSHIP IN SCHOOL MATHEMATICS

Janet Ainley begins her article with a critique of how word problems are generally used in the teaching of mathematics, emphasizing just how unsatisfactory their role as "boundary objects" has been. The position adopted by Janet on this topic and on contextualization of mathematics concepts in order for them to be taught is very clear. Her position points out an intrinsic contradiction in the dual use of contextualization: "contextualize to make it easier to understand mathematics" / contextualized problems to verify (in more complex situations) mathematical understanding."

The author also refers to the points made by Anna Sierpiska concerning the difference between the use made of mathematics by professional mathematicians in order to understand the world and the use made of "real" contexts in education so as to facilitate the understanding of mathematics. Janet makes the additional point that in education there is a (false) implicit supposition that these two aspects are related, that the latter implies the former of the two.

In subsequent sections of the article Ainley takes an alternative perspective, focusing on the *purpose for the learner* of dealing with a contextualized problem or task. And it is here on this point that her contribution is revealed: the issue is to design tasks that have a clear purpose for the learner in the classroom and to build, within the task itself, the need for learners to use mathematics ideas in such a way that they recognize their *utility*. She then immediately goes into the meaning that she attributes to the term *utility*, in which the aspect that is of the greatest interest to me is its local character. Janet says: "... from the learner's perspective it – *the utility*- does not (necessarily) refer to usefulness in the 'real world': our concern is initially with

usefulness within the particular task.” The examples that she later presents are fairly convincing from the standpoint of this notion of (local) *utility* and of *purposeful mathematical thinking tasks* (i.e. from the point of view of the purposeful and utility framework, just as she defines it). However the topic does not end there; based on that alternative view and on the associated examples, it becomes worthwhile to raise in-depth issues as food for thought. Below is a brief discussion of some of those issues.

## **ON THE RELATIVE CHARACTER OF THE PURPOSEFUL AND UTILITY FRAMEWORK**

Whilst not detracting from the recognition due to Ainley in view of the contribution of the notions she introduced, which help to advance our knowledge of the intricate relationship between contexts and concepts in classroom mathematics, I would like to refer to the relative character of the qualities attributed to those notions. That relative quality will become patent when one thinks of its application in designing tasks in which sundry technology environments (designed and developed for different purposes) are used; when it is a matter of making the long-term didactic purposes of those tasks explicit; and when it comes to applying them while designing tasks aimed at learners of varying school levels.

### **The local utility of mathematics in purposeful tasks, and long term purposes.**

In the example discussed by Janet, entitled “the hundred square and testing cross”, the immediate purpose of the task for secondary school learners (the school level in which the experimental work was carried out) is transparently established. By the same token use of Spreadsheets is clearly delimited in terms of that immediate purpose: to generate in an automated manner (by way of building general formulae) numerical outcomes that have certain restrictions. Even the didactic purpose is explicitly established, namely to make evident “...the utility of algebraic notation in order to show the structure.” Nonetheless at least in this article of Ainley’s there is no discussion of how the fact that the first of the two purposes is achieved (the one focused on the learner, circumscribed to the task itself) implies achievement of the second (which is more general and more ambitious). In my experience with use of Spreadsheets to represent and solve arithmetic-algebraic problems (Sutherland & Rojano, 1993) one of the major issues that always arose in discussions with academics and specialists in algebraic thought was the lack of clarity concerning the long-term goals (the purposes) of the design of those tasks, which enabled the students to see the utility of having representations that relate the simultaneous variation of the different elements of a problem by way of general Excel formulae. That concern is derived from the fact that even though software code is similar to algebraic code, the difference between the two in terms of manipulative and transformational aspects is enormous and, hence, the transition from the first to the second is not a trivial matter. That is to say that in this case, the immediate purpose seemed to be very distant (almost extraneous to) from the aspiration of having the

students consciously use the potentiality of algebraic symbolism in order to represent and solve word problems, in other words the potentiality of the Cartesian method.

On this point, the thoughts are along the heading of asking oneself to what extent over-emphasizing the design of purposeful tasks with an immediatistic sense of the mathematics utility can neglect, in the long-run, long-term didactic goals.

### **The local utility in purposeful tasks for college - university level**

Now with respect to designing this type of task aimed at, say, college - university students, one would have to ask just how advisable it is to maintain the purpose at the local level of the task itself. I would once again like to refer to my work with Rosamund Sutherland in the Anglo-Mexican project *The Role of Spreadsheets within the School-based Mathematical Practices* [1] (Molyneux, Rojano, et al, 1999). In the foregoing project, we designed activities where the potentiality of Spreadsheets for modeling, exploring and analyzing phenomena of the physical world was intentionally made explicit for the learners, while at the same time the set of activities designed was aimed at communicating powerful mathematics ideas, such as the different types of variation (polynomial, exponential) and the notion of parameter, which encompasses the duality of being constant and variable. In other words, in that study we worked under the hypothesis that a task or set of tasks in which parameterized modeling is used can have a dual purpose. On the one hand, it can provide students with tools to deepen their knowledge of sciences (biology, chemistry and physics) with the focus that Jon Ogborn calls “learning with artificial worlds” (Ogborn, 1994). While on the other, it can provide students with the opportunity to experiment with different types of variation (polynomial, exponential, continuous, discontinuous, discrete) through different representation systems (Excel formulae, graphics, and numerical variation tables) that are dynamically linked to one another. It is noteworthy to point out that in this case, when working with university students it is feasible to reconcile use of mathematics to model and understand bits of the “real world” with use of aspects of the “real world” to model bits of mathematical knowledge (uses that, according to J. Ainley, A. Sierpinska corresponds to the domains of ‘real mathematics’ and ‘primary mathematics’, respectively).

On this other point, the thoughts are along the line of the relative character of the notions of utility and mathematical purposeful tasks, according to the school level in which one is attempting to implement them. The example of parameterized modeling suggests that some adaptation or reformulation of said notions would be desirable. In Janet’s current project, in which the teaching of sciences and mathematics are integrated, we will probably see fresh and updated versions of the ideas that she is now presenting.

### **Developing purposeful mathematical thinking within technology learning environments.**

I would lastly like to refer to the role of technology learning environments and their potential as gateways to powerful ideas in mathematics and in sciences (Rojano,

2008). The points I have made in previous paragraphs implicitly refer to that potentiality. The specialized literature also contains endless examples and experimental works that report both the affordances and the limitations of the technology environments in terms of their use in mathematics education. As those environments began to be more intensively used in teaching, it became clear that the design of the task was crucial in making its alleged potentiality effective given that in the majority of cases its spontaneous use failed to lead to the intended learning (Healy, Hoelzl, et al, 1994). In this context, the ideas raised by Janet are highly relevant since they suggest that if tasks are designed with an explicit mathematical utility within the very tasks then achieving their purpose becomes feasible.

## A FINAL REMARK

Both Janet's critical thoughts concerning the role of using 'real contexts' in the teaching of mathematics and her alternative view of the relationship between those two domains provide novel elements that are relevant to the design of tasks and that open new paths to research on the valued mathematics – 'real world' relationship in the classroom. In particular terms the theoretical elements and empirical work presented by the author may be very significant in the design of tasks to be implemented in dynamic and interactive technology environments, some of which have proved their potential to show the learners the utility of mathematical modeling in order to understand phenomena of the physical world, of the 'real' world.

## Notes

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## **PLENARY ADDRESS 2**

*Morals of an Anecdote as Starting Point of a Lecture in Mathematics*

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# MORALS OF AN ANECDOTE AS STARTING POINT OF A LECTURE IN MATHEMATICS

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*The paper is on using jokes to introduce certain mathematical concepts. The main point is attracting the student to a discussion which is initiated with an anecdote. On the one hand some very common and familiar concepts are used to approach a problem, on the other hand it is pointed out that one can criticize almost everything mathematically. Some examples of jokes and their mathematical relations are also given.*

An efficient way of introducing mathematical concepts is to provoke the student to ask critical questions about the topic to himself. In this case, everyday life experiments and familiar concepts turn out to be very useful to initiate a thinking process.

The author is a mathematician in a department of mathematics and he is not a specialist in maths education. However, he has experienced various interesting ways to make maths teaching a little bit tasty.

Using common jokes is found to be an effective method to start a discussion in the classroom. Instead of introducing some mathematical concepts, methods or problems directly, starting with a joke attracts the attention of the student. Then the everyday language is translated to mathematics and the joke is rephrased. Trying to settle the bridges between the mathematics and the anecdote helps the student to force himself for a better and steady understanding.

Another outcome is pointing out that we can find mathematics everywhere around us and it is possible to criticize almost everything from a mathematical point of view.

We have to mention the main character of the jokes. The people of eastern Black Sea region in Turkey, namely Laz People, are known to be very clever. They are also famous for their unexpected responses. Temel is one of the common names in that region and he is the hero of all Laz jokes. When you think of Turkish humor, besides Nasreddin Hodja, Temel is the first people you are thinking of. His wife is Fadime and he has very close friends Dursun and İdris. Our jokes are all about Temel. The first impression which follows a Temel joke is a doubt about the mind of him. But a critical analysis shows how clever he is.

As a word, the meaning of ‘Temel’, is ‘Basic’. We call the part of lectures devoted to Temel jokes as ‘Temel Matematik’ which means ‘Basic Mathematics’.

Below we give four examples of Temel jokes with their mathematical interpretations. The first joke is a warm-up joke which leads to an obvious problem of elementary level.

### **PARABLE 1**

Temel, with his car on the intercity highway, encounters with a signboard written ‘SLOW 50’. He sets his speed and keeps driving. After a while, he encounters with another signboard: ‘SLOW 40’ and he once again reduces his speed. Then he encounters signboards written ‘SLOW 30’ and then ‘SLOW 20’, and he reduces his speed fairly... Finally, he encounters with the signboard: ‘SLOW 10’. He keeps driving after completely slowing down his speed.

An hour later, he encounters a signboard written “WELCOME TO SLOW”

#### **Problem**

Find the average speed between the first signboard which Temel encounters and the town SLOW.

The second joke refreshes the concept of function and helps to introduce the concepts of involutions and derangements. It also leads to a problem which has no definite solution in which case we have to consider probabilistic solutions. The discussion also includes developing a measure of satisfaction.

### **PARABLE 2**

In medical school, the professor examines Temel:

- Two babies who were born at the same time are mixed up. What would you do to give each baby to her real mother?
- Professor, babies would not mix up. As soon as the birth is realized, a label has to be attached to their wrists.
- The nurse might forget to put the label.
- Still that would never happen, because babies would be brought to their mothers as soon as possible.
- They might not be.
- A sign might be put on their feet.
- It is also forgotten.
- Ok, then babies are mixed up for sure?
- Yes.
- Therefore, I would mix them up once again.
- !...



## Discussion

Let us denote the mothers by  $A$  and  $B$  and their babies, respectively by  $a$  and  $b$ . Assigning a baby to each of the mothers is a function from the set  $\{A, B\}$  of mothers to the set  $\{a, b\}$  of babies. Since each mother is given a baby, we are talking about one to one functions. Such a function is also called a bijection.

Each of the given sets has two elements and consequently there are only two bijections between these sets:

$$f_1: A \mapsto a, B \mapsto b \quad (\text{The identity function}),$$

$$f_2: A \mapsto b, B \mapsto a.$$

To bring an ease, we can denote these functions by ordered pairs  $(a, b)$  and  $(b, a)$  where the first component is the image of  $A$  and the second one is the image of  $B$ . In this way, we can regard the functions as permutations. For example, if  $X = (x, y, z, t)$  and  $Y = (a, b, c, d)$ , then a permutation, say  $cdba$  of elements of the set  $Y$  corresponds to a function which maps  $x, y, z$  and  $t$  respectively to  $c, d, b$  and  $a$ .

When professor says that the babies are mixed up, she actually means that one of these functions is used for assignment of babies to mothers without knowing which one is chosen. When Temel makes it certain (!) that the babies are mixed up for sure, he concludes that the chosen function is  $(b, a)$ . Fortunately, inverse of this function is itself and applying it once more he solves the problem.

A function which is inverse of itself is called an involution. In other words, if the composition of a function with itself gives the identity function, it is called an *involution*.

**Exercise.** The number of involutions of a set with two elements is 2. What is the number of involutions of a set with 3 elements?

Now we return back to the joke with a slight generalization: How would the problem be solved with three mothers?

If we had three mothers with three babies (no twins are assumed), there are six possible assignments (functions) of the babies to mothers:

$$(a, b, c), (a, c, b), (b, a, c), (b, c, a), (c, a, b), (c, b, a).$$

If we are given the clue that all the babies are completely mixed up, the number of functions reduces to two:

$$(b, c, a) \text{ and } (c, a, b).$$

Such a permutation (with no element at its original position) is called a *derangement*. So far we have seen that a two element set has only one derangement and a three element set has three derangements.

**Exercise:** Find the number of possible ways of assigning  $n$  babies to  $n$  mothers so that no baby is given to her mother. In other words, find the number of derangements of a set with  $n$  elements.

If the joke was settled for three mothers, Temel would only know that one of the above functions is used to assign the babies. Now we write these functions explicitly for further observations:

$$f : \begin{cases} A \mapsto b \\ B \mapsto c \\ C \mapsto a \end{cases} \text{ and } g : \begin{cases} A \mapsto c \\ B \mapsto a \\ C \mapsto b \end{cases}.$$

Identifying the set of babies with the set of mothers, we can rewrite these functions as

$$f : \begin{cases} a \mapsto b \\ b \mapsto c \\ c \mapsto a \end{cases} \text{ and } g : \begin{cases} a \mapsto c \\ b \mapsto a \\ c \mapsto b \end{cases}.$$

One can easily observe that  $f \circ f = g$ ,  $g \circ g = f$ ,  $f^{-1} = g$  and  $g^{-1} = f$ . It is seen that if the function  $f$  is used for the assignment, we have to apply  $g$  to solve the problem and vice versa. But we are not certain about the chosen function.

As the problem has no definite solution for three babies, we can not generalize the joke to three mothers to come up with a certain solution. But we can still continue to develop some solutions.

### Probabilistic approach and an attempt of defining a measure of satisfaction.

#### First approach:

If we leave one of the mothers and change the other two mothers' babies, a mother would meet with her own baby; the rest would meet with wrong babies. We could not know which mother is met with her own baby. Since the probability of meeting with her own baby for each of the mother is  $1/3$ , the total "degree of satisfaction" is  $1$ .

#### Second approach:

If we give the baby with  $A$  to  $B$ ,  $B$  to  $C$ , and  $C$  to  $A$ , with probability  $1/2$ , all babies would meet their real mothers. And this time with probability  $1/2$ , once again the baby would meet with wrong mother.

If the order is  $(b, c, a)$ , after the intervention the right order of  $(a, b, c)$  would be obtained; if the order is  $(c, a, b)$ , this time the wrong order of  $(b, c, a)$  would be obtained. Since the probability of meeting with her own baby is  $1/2$  for each mother, the total 'degree of satisfaction' is  $3/2$ .

Although both approaches do not guarantee a certain solution, the second one is preferable since the degree of satisfaction is larger.

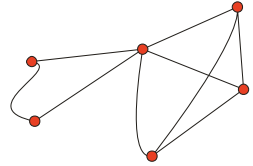
The next joke describes a particularly useful method for certain counting problems. Here we mention only one of these problems which is related with the graph theory.

## Preliminary concepts

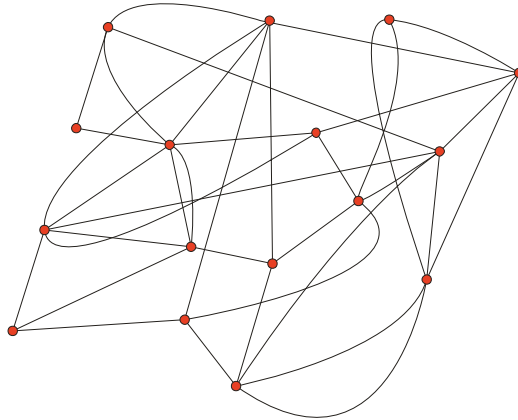
### (Graph Theory)

A set of points, together with a set of lines gathering some pairs of these points defines a graph.

The points are called the vertices of the graph, the lines joining them are called the edges. The number of the vertices is denoted by  $v$ ; the number of the edges is denoted by  $e$ . For the graph in the figure, we have  $v=6$  and  $e=9$ . The number of edges incident to a vertex is called local degree at that vertex. The local degree at the vertex  $p$  is denoted by  $d(p)$ .



The graph below has 16 vertices. How can we find the number of edges?



Before developing a method to count the edges, we refer to Temel.

### PARABLE 3

While travelling by train, Temel watches outside and whispers by himself. After a while, the passenger next to him could not stand the situation and asks him...

- Excuse me, you are whispering something since we start travelling...
- No, I'm not whispering, I am counting...
- What are you counting?
- I count whatever I saw at that moment... I am counting houses, posts, trees, cows, sheep...
- How could you count the trees at the forest, the sheep within the flock while the train goes that much fast? I believe, you just guess...
- No, I am counting.

- I do not believe that you can count. In an hour, we will reach our town. You will see one of the flocks which belongs to me before reaching the town. Let's see if you could know how many sheep are there?

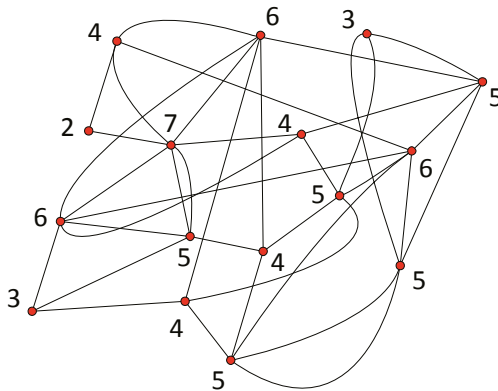
After a while, they come close to the town and the other passenger shows Temel his own flock. Temel's lips moves faster and faster. Then he lean backwards and affirms the result quite self-reliantly:

- There are three hundred eighty-four. And a dog.
- Oh my God! That is true. How could you count?
- Of course I have my own trick.
- What is it?
- I am not counting the sheep directly...
- So?
- I do count their legs and divide the result by four.
- ?...

To count the edges of a graph, we will follow Temel's method. Let's count the legs rather than counting the sheep. Here, the sheep are the edges of the graph. Each edge has two feet (end points) and they touch the vertices on these feet. Therefore, we can find the total number of the feet by summing up the local degrees (feet). Then, the number of the edges is twice the sum of the local degrees. So we easily obtain the following formula:

$$2e = \sum_{i=1}^v d(p_i).$$

If we write down the local degrees of the above graph, we see that the sum is 74.



It follows that the number of edges is 37.

The last example is a joke which rises an important problem in informatics. The problem is easy to state, hard to solve. The joke leads to a discussion about the measure of information. It can be used also when the concept of expected value is considered.

#### PARABLE 4 (Informatics)

Little Temel, did not study for the exam and when he sits for the exam, he finds out that it is a multiple-choice one with five choices. He takes out coin and starts to flip the coin to decide the ‘correct’ choice. Two hours passed. Almost all the class completed the exam, but Temel is still flipping the coin. The invigilator asks:

- It is obvious that you came to exam without studying. You did not even take a look at the questions. You are replying them by flipping a coin. What takes your time this much?
- I am already done with answering the questions. Now, I am checking my answers.

#### Problem

On the average, how many times the coin is flipped per minute?

#### Discussion

In a true/ false exam, we may let ‘TAILS’ to stand for TRUE and ‘HEADS’ to stand for FALSE to make a choice out of two ‘equally probable’ ones.

In a multiple choice exam with four choices, one may flip the coin two times for each question. First flip eliminates either the pair (A-B) or (C-D) and the second flip decides one of the remaining two choices.

It is not an easy task to make a choice out of five alternatives by just flipping a coin. Of course, if we desire each of the choices is selected with equal probability.

Each question has five choices. This means that, there is an amount of uncertainty for each question. On the other hand, each flip of the coin removes a portion of this uncertainty.

If an event has  $n$  equally probable outcomes, the *measure of uncertainty* is given by  $\log n$ . Since the answer of each question is hidden within five choices in our story, the uncertainty degree of each question is  $\log 5$ . The operation to remove this uncertainty is flipping a coin. Each flip enables us to make a decision out of two choices. Thus, every time we flip the coin, uncertainty reduces by  $\log 2$ .

If we wish to remove the uncertainty completely, we have to flip the coin  $k$  times

such that  $k \log 2 \geq \log 5$  or  $k \geq \log 5 \frac{1}{\log 2} \approx 2,322$ . This means that, Temel should flip the coin at least  $\lceil 2,322 \rceil = 3$  times to remove the uncertainty in each question. Hence, by flipping the coin 150 times, he could be able to answer 50 questions.

He could try alternative methods instead of answering the questions one by one. When all the 50 questions are taken together, the total degree of uncertainty is  $50 \log 5$

and the number of flips required is  $\left\lceil 50 \log \frac{5}{\log 2} \right\rceil = 117$ . So, he might accomplish the exam by flipping the coin 117 times by using the most economic method planned cleverly (it might be quite complex).

What would happen when it comes to check the answers? At the first try, it is expected that 10 questions are verified. It is, therefore, required to solve the rest for the second time. So, by flipping the coin 300 times he makes 10 questions certain. After this, by flipping the coin 240 times he makes another 8 questions certain and it continues like that. Finally, he succeeded the exam by flipping the coin approximately 1500 times and all of the questions are checked. (In a computer simulation we obtained an average of 1497,32 flips.) This means 12,5 flips per minute.

When the smartest method is used, it is required to flip the coin 1180 times and in two hours of time, which means approximately 10 flips per minute.

### Conclusion

The approach of using jokes in maths lectures is not a part of scientific activity. It is merely an experiment of the author for the sake of attracting the attention of students. The outcome has never measured in details. However, observation of the author indicates a positive effect on teaching and learning. First of all, everyone likes listening jokes. After telling a joke, when the students are asked what makes that joke a joke, they can easily and eagerly pulled in a mathematical discussion. Another observation is that, when they discuss among themselves they refer to the jokes for recalling the concepts.

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## **PLENARY ADDRESS 3**

*Children's Informal Reasoning: Concerns and Contradictions*

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# CHILDREN'S INFORMAL REASONING: CONCERNS AND CONTRADICTIONS

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*In this paper an attempt is made to answer two of Steen's questions about students' mathematical reasoning. Definitions of informal reasoning are provided, and from these bases, some examples of informal reasoning in action are given. These examples come from both mathematics and science, and from a range of prior-to-school and school settings. It is argued that these examples support both an affirmative and a negative response to these questions respectively.*

## Introduction

In this paper we look at children's informal reasoning, and therefore this will not concern ourselves with the more formal aspects that lead to formal ideas of logic and proof. Thus, the learning and practice of logic and proof, although interesting, will be left to other, more erudite, authors.

In terms of reasoning *per se*, it has been said that '[R]easoning is an ancient subject but an everyday practice' and that '[W]e are all able to reason.' (Munson, Conway, & Black, 2004 p. 1). Scriven suggested, somewhat presciently, that the importance of reasoning lies in the fact that '[R]easoning is the only ability that makes it possible for humans to rule the earth and to ruin it' (1976, p. 2), and at a practical level, that 'reasoning helps you to work out correct answers for yourself' (p. 3).

In the present case, therefore, it is not the 'correct answer', to the likes of the following problem, which concerns us:

1. Babies are illogical.
2. Nobody is despised who can manage a crocodile.
3. Illogical persons are despised.

(Carroll L. (Charles Lutwidge Dodgson), 1939, p. 1119)

(The solution to this problem is at the end of the references).

This paper will explore how informal reasoning is exhibited by children, and draws on both research and the author's own experiences, particularly in mathematics. While considering a range of sources, the main intention is to re-visit two of Steen's (1999) twenty questions about mathematical reasoning in general: Question 19, Is mathematical reasoning innate? and, Question 20, Is school too late?

## Why reasoning?

Historically, Plato's notion of knowledge, as justified belief, endorses the primacy of reasoning in the acquisition of knowledge in general. More recently, the importance of reasoning in mathematics was highlighted in *Adding it up: Helping Children Learn Mathematics*, the report to the United States National Academies (Kilpatrick, Swafford, & Findell, 2001). This report stated that 'In a recent study comparing schools participating in state initiatives in mathematics and science with schools not involved in such initiatives, elementary school teachers in the initiatives schools spent significantly more time than their counterparts on reasoning and problem-solving activities' (p, 45).

Consequently, Kilpatrick and his colleagues placed reasoning as one of their five strands of mathematical proficiency, defining *adaptive reasoning* as the capacity for logical thought, reflection, explanation, and justification (p, 5). This, in turn, influenced the authors of the proposed Australian Curriculum, in mathematics, to define four proficiencies: *Understanding*, *Fluency*, *Problem solving*, and *Reasoning*. Of these four proficiencies, *Reasoning* is described as 'the capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying, and generalising' (p.6). Further, it is argued that '[E]ngaging students in reasoning and thinking about solutions to problems, and the strategies needed to find these solutions, are core parts of the mathematics curriculum' (p. 9). However, at the prior-to-school and primary school levels of mathematical education, *explanation* appears to be the major aspect of this proficiency, leaving the remaining aspects to the later years of school.

The Victorian state Early Years Learning and Development Framework (VEYLDF) in the prior-to-school years (Department of Education and Early Childhood Development, 2009), argues for strengthening learning opportunities for young children, particularly in terms of explaining and reasoning, by emphasizing that '[W]ith support, children expect to learn and ... begin to develop simple explanations for observed phenomena' (p. 25).

This resonates with Piaget's '[E]xperience fashions reason, and reason fashions experience. Thus between the real and the rational there is a mutual dependence joined to a relative independence, and the problem is a singularly arduous one to know how much of the growth and elaboration of knowledge is due to the pressure of external things, and how much to the exigencies of the mind' (1930, p. 301). This relative independence was, perhaps, more succinctly put by Eliot (1925) as 'Between the idea and the reality ... falls the shadow'.

Mason (1998) argues from a self-development point of view, rather than linking the usefulness of reasoning only to the natural sciences when he says that '[T]o develop a sense of self, however socially embedded, it must surely be of assistance to have discovered that confidence can come from personal reasoning and not just from some higher authority. It is important therefore that students have experience of domains in which ... there are general sources and principles rather than a maze of particulars

from which truth can be decided. Mathematics provides an ideal domain for such discovery' (p. 4).

Which brings us to Steen's questions.

### **Is reasoning innate?**

To answer Steen's Question 19, Is mathematical reasoning innate? this section offers some examples of children's informal reasoning that I believe support the notion that reasoning may be an innate capability.

Hunting and Mousley, reporting on some of the findings of the *Mathematical thinking of preschool children in rural and regional Australia: Research and practice* project (Hunting et al., 2008) reported that 'a total of 58 of the practitioners (88%) thought that mathematical thinking starts before the age of 3, and many identified mathematical activity in babies and toddlers' This is confirmed by Hughes (2009).

Hughes records an instance of informal reasoning in the home. In this case, a toddler (usually about 2 to 3 years of age), planned a series of related moves to achieve his goal. Hughes recalls:

'I once observed a toddler named Tom, who wanted a cup from a high shelf in the kitchen. His mother was not readily available so Tom dragged a chair towards the worktop and proceeded to clamber on to the chair. He then climbed on to the surface of the worktop, from where he could now successfully pick up his self-chosen cup. He then put the cup on to the worktop and reversed the climbing process till he was back on the floor. He could now reach the cup and looked deeply satisfied as he took it to his mother.' (p. 14).

Tom's reasoning was, in part physical, yet it is undeniable that the plan was a very clever combination of understanding of basic measurement and informal logic. However, this type of creative activity is denied to many children, who are kept 'safe' by ever-watchful parents, perhaps to the child's mathematical detriment.

A more formal set of observations have been made by Doig and Ompok (2010) using the Gumnut Game. Gumnuts are the seed-pods of the eucalyptus tree, and provide cheap and easily found 'counters': other suitable counting objects would be shells, buttons or pebbles. The game is played as shown in Figure 1. The Gumnut Game is reproduced here so that interested researchers may trial the game for themselves, and report on their experiences with it.

Note that although the game is ostensibly about counting, to play one must be able to decompose numbers mentally. The reasoning aspect is mental too, as the *How do you know?* part of the game shows (see Figure 1 below).

This, and other, games have been used with prior-to-school aged children in Hungary, as well as Australia, and more recently, in Sabah, Malaysian Borneo.

### The Gumnut Game

If the child is very young try two gumnuts (counters) to start, but experience suggests that three is an ideal starting point from two-year-olds and upwards.

#### Part 1

Show the gumnuts (counters) to the child.

Can you tell me how many (counters) I have?

If the response is not three, ask them to count. If the child cannot agree that there are three (counters), then either try two (counters), or abandon this task.

#### Part 2

I am going to hide the (counters) in my hands.

Do this behind your back, or by turning away from the child.

Place some (counters) in each hand.

Show your closed hands to the child.

Which hand do you want to see (open)? Open that hand.

How many (counters) are there? Wait for their response.

How many (counters) are there in my other hand? After the child's response, open that hand.

Were you correct?

Repeat the process, changing the number of (counters).

#### Part 3

Now it is your turn. Don't let me see you hide the (counters).

After your say how many in each hand, ask:

Was I correct?

How do you know?

And so on ...

Figure 1: The Gumnut Game

In the Gumnut Game, the child needs to know either a number fact, for example that 5 is 2 and 3, or must reason that starting with 5, seeing 2, leaves 3 still hidden. Seven 3-year-olds could play this game with up to 5 gumnuts, while three 4-year-olds could play with up to 10 gumnuts. None of these children had attended any formal schooling.



Figure 2: Playing the Gumnut Game in Sabah

An example drawn from science is the response given to the question on Figure 3 below, taken from Tapping Students' Science Beliefs (TSSB) (B Doig & Adams, 1993). Students were required to write a reason for agreeing, or disagreeing, with Jenny's statement that we are animals.

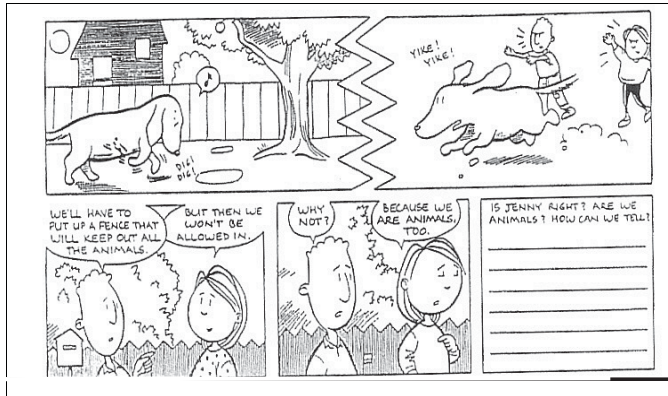


Figure 3: A question from Tapping Students' Science Beliefs

A number of students responded that we are animals because we *look* like animals. Visual reasons were also encountered in trialling other items for the TSSB. One item, that showed a drawing of a bird sitting on an electricity transmission wire asked why the bird was not electrocuted. Responses included that birds have 'special feet', which indeed they do, in contrast to their otherwise feathered bodies. Apparently, bird feet tend to look different, and to those who know about such things, insulated by rubbery looking material.

Some years ago I observed some Year 3 children (about 8 years-old) complete a short mathematics assessment. I was standing beside Alice, who was attempting to solve the following:

Tom had 23 cents and Jo had 58 cents.  
How much did they have altogether?

Like others of her classmates, Alice used tally marks to calculate her answer, and began her calculation by drawing a series of pencil strokes as shown below.



Figure 4: Alice's original tally marks

But before she had drawn 23 strokes, she murmured aloud "I can't do that", erased the strokes, and commenced to replace them with circles labelled with 1, representing one-cent coins as shown.



Figure 5: Alice's preferred tally marks

Alice's reasoning was that "they had to be like cents to work".

When Alice's teacher was informed of this, she replied that many of the children use tally marks for calculations. She did not attach any importance to the circular 'tally' marks, but regarded them as equivalent to lines. I believe that this incident suggests that Alice's reasoning shows that her mathematical development was not at the same level as her classmates, and that she was connected strongly to visual inscriptions that looked like the real object. How, then, could she work confidently with numbers represented by digits?

The Victorian Early Years Learning Development Framework (VEYLDF) emphasizes the importance of reasoning in the prior-to-school years insisting that '[C]hildren need many opportunities to generate and discuss ideas ... reflect and give reasons for their choices' (p. 25). Clearly, these opportunities, either did not occur for Alice, or she was not able to grasp them. In either case, the VEYLDF statement remains relevant.

A concern here is that a curriculum that does not address reasoning does not provide the support needed by children to develop their informal reasoning skills, skills necessary for developing other mathematical knowledge.

These examples demonstrate the types of informal reasoning that occurs before reaching school, and continues informally unless the curriculum attends to its development, as in the case of Alice and the TSSB 'animals' question.

### Is school too late?

While reasoning, both informal and formal has been the subject of educational research for many years (for example, Hughes, 2009; Voss, 1988; Voss, Perkins, & Segal, 1991) the assessment of reasoning reveals the difficulties that this proficiency causes within education. Yet, there do exist successful implementations.

Reasoning, at the school level, appears, often, in the guise of 'explanation'. Doig, Groves and Fujii (2011) reported an example of reasoning in a first year of school class. The class of five- and six-year-old children had heard the story of *Snow White and the Seven Dwarfs*. The teacher said that Snow White always sat at the head of the table, while the dwarfs sat at the two long sides, with a different number of dwarfs sitting on each side each day. The teacher asked the children 'How might the 7 dwarfs sit at the table?' The children had counters to represent the dwarfs and re-arranged these around a coloured paper rectangle representing the table.

Children were told that their job was to find as many ways seating the dwarfs as possible. The children worked on this problem for several minutes, and then, after asking for several children's answers, the teacher asked 'Have we found all the

ways?’ to which the children chorused ‘Yes!’ The teacher responded to this by asking ‘How do we know we’ve got all the ways?’ After some (to the teacher) unsatisfactory attempts, a boy said: ‘We’ve used all the numbers’.

This example demonstrates how very young children can engage with mathematical reasoning, in this case by using a pattern shown by the use of concrete materials (the counters).

Open-ended questions have become popular in both teaching and assessment, and these often require students to give their reasoning for their response. For example, Stacey and her colleagues (Stacey, Groves, Bourke, & Doig, 1993) developed a test of problem-solving, *Profiles of Problem Solving* (POPS) that contained a sub-scale, ‘quality of explanation’. This was an early attempt to both quantify and promote reasoning as an aspect of mathematics learning.

In the POPS *Profile Level Descriptions* three levels are specified. Students are categorized as being either:

1. beginning problem solvers, who give unclear explanations using numbers but not words. In this case, reasoning has to be inferred from symbols only;
2. developing problem solvers, who give explanations of only part of their reasoning (e.g. only numerical). In this case, the student’s reasoning is incomplete; or
3. advanced problem solvers, who give clear and complete explanations using both words and numbers. In this case, the student’s reasoning is made visible.


In the POPS item *Ladders*, students are asked to find the pattern for the number of matches needed to make ‘ladders’ of varying number of steps (see Figure 6).

In the ultimate step, students needed to explain, that is, give their reasoning, for the number of matches required for a ladder with a 1000 rungs. The example, given for a high quality explanation for this part of the Ladders problem, is a ‘Clear explanation of the pattern used — e.g. *there are 1000 rungs and 1001 matches down each side.*’ The reasoning displayed by this level of explanation is clearly well-developed.


**QUESTION 6**

**LADDERS**


With 8 matches, I can make a ladder with 2 rungs or steps like this: →



←



With 11 matches I can make this ladder with 3 rungs.



How many matches are needed to make the same sort of ladder with 4 rungs?  
(You can draw it here if you need to.)

↓

Answer 1

How many matches would be needed to make a ladder with 5 rungs?

Answer 2

Figure 6: POPS Ladders problem

Explanations were assessed, also, in the Third International Mathematics and Science Study (TIMSS), where open-ended questions generally asked for a solution followed by an explanation of how this solution was correct. However, despite the application of partial credit scoring and the so-called Viking rubrics (see Dossey, Jones, & Martin, 2002), for these solutions, the published TIMSS results were based on a simple dichotomous (correct or incorrect) scoring basis (Martin, Gregory, & Stemler, 2000). This was surely a lost opportunity.

Doig and Groves (2008) reported on upper primary and junior secondary students' explanations to mathematics questions in a large-scale project in Victoria, Australia<sup>1</sup> and some of these questions were drawn from TIMSS. These questions asked students to write, or draw, reasons for their answers. These questions were believed to be susceptible to analysis with the tools described by researchers such as Draper (1988) and Doig and Groves (2007).

An example of one of these items is the Odds and Evens item shown below.

<sup>1</sup> *Improving Middle Years Mathematics and Science: The role of subject cultures in school and teacher change* (IMYMS) is funded by an Australian Research Council Linkage Grant, with Industry Partner the Victorian Department of Education and Training. The Chief Investigators are Russell Tytler, Susie Groves and Annette Gough, and Associate Investigator Brian Doig.



The following diagram represents the first seven positive whole numbers:

1	2	3	4	5	6	7
•	• •	•• •	•• ••	••• ••	••• •••	•••• •••

Amy says that the sum of any two ODD numbers is always an EVEN number.

Is Amy correct?  
Explain how you know.

Figure 7: The open-response item Odds and Evens

While responses varied, many showed ingenious use of reasoning based on a visual approach. For example, one child wrote that:

‘An odd number always has one number that is left out, so if you add 2 together, the two numbers that are left out will join into a pair. This creates an even number’

The ‘one number left out’ refers to the fact that a common approach to learning small numbers uses the Montessori visual arrangement of quantities as pairs of objects. On the other hand, there were responses that simply appealed to a higher authority, such as: ‘Because it is a maths rule’.

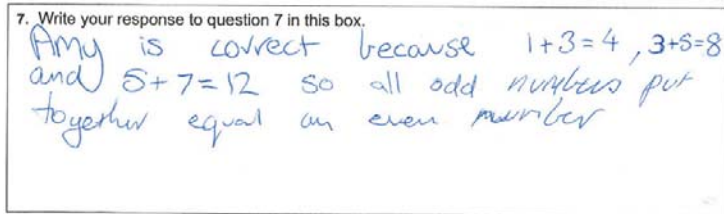


Figure 8: Example response to the Odds and Evens item

Figure 8 shows the use of demonstration as the basis of reasoning, supporting Hersh’s view that ‘convincing is no problem. Students are all too easily convinced. Two special cases will do it. (1993, p. 396). In this instance, convincing is taken to be a synonym for reasoning.

These examples show that, whether taught or caught, students possess a range of reasoning skills. As Carlson (2009) points out, in a paper considering the use of inscriptions, that ‘they [inscriptions] provide foci for the successive coordination [sic] of reasoning’ (p. 56).

Further, Carlson’s research shows that it is possible to foster students’ mathematical reasoning through the use of inscriptions as ‘Inscriptions, when integrated into the flow of activities, thus contribute to the structuring the ways users will reason about a problem’ (p. 58), and that it is ‘[A]lso evident ... that the students literally are reasoning through the inscriptions’ (p. 65).

In answer to the question, Is school too late? the best answer is ‘No’, but schools need to introduce more opportunities for students to use their reasoning from the earliest years. That is, to build on children’s informal reasoning and guide them through to ideas of formal justification and proof.

### **Discussion**

The examples described in this paper illustrate a range of issues. Some of which are concerns, others of which are contradictions. The concerns, I believe are that the implemented curriculum in mathematics, by-and-large, does not include reasoning, except incidentally, which means that the reasoning is essentially students’ informal reasoning. This is in contradiction to many mathematics curricula, which include reasoning (and many of its synonyms) in their statements of mathematical skills to be developed.

A further concern is that students come to prior-to-school, and school settings, with reasoning already in their ‘tool kit’. It does not seem sensible to me that we ignore this fact, and let the informal reasoning remain as the only tool available, when it would seem a simple matter to employ tasks and questions, such as the Snow White and the Seven Dwarfs example, from the beginning of school. As this example showed, clearly, informal mathematical reasoning was part of the children’s mathematical tool kit on entry to school.

Steen’s two questions, therefore could be answered by ‘Yes’ there is an innate reasoning capability that children appear to have, and to use, and ‘No’ school is not too late. At the moment, it can be argued that schools provide too little, too late, but if educators do not take this capability as important, and attempt to develop it from the informal to the formal, then we could claim, reasonably, that school is too late!

I present two examples that show why it is important to develop mathematical, and general, reasoning skills. The first of these concerns a common, in Australia, experiment conducted in prior-to-school centres called ‘floating and sinking’. My pre-school teacher trainees tell me that this experience is fun and children find it highly enjoyable. So, what does this experience involve, and what are the outcomes? Objects of different mass (weight) are placed in a tub of water and which objects float and which sink are noted. The outcome of this experience for children and for my teacher trainees is identical: light objects float and heavy objects sink. If I object to this finding, I am accused of being unreasonable: commonly I hear ‘We saw it happen’. Usually it takes counter-examples, such as: How does a large, metal ship float, while a small, comparatively light, orange sinks?, to disturb the teachers’ perspective. Some teachers then remember ‘density’ from secondary school, but the damage, to the children, has been done.

The second example is familiar to most of us: the longer the decimal the larger the number. That is. 0.1234 is larger than 0.2. This can be reasoned from the fact that

longer whole numbers are larger than smaller whole numbers. Clearly, this is a case of fallacious reasoning based on stretching an analogy beyond its limits.

A school curriculum that included reasoning would, one would hope, raise teacher and student awareness of looking beyond simple observations, of looking for counter-examples, of being cautious in using analogies and so on. The place that this aspect of mathematics (and science?) should occupy in the curriculum must be central. Eradicating misconceptions based on poor reasoning requires good reasoning skills, which can develop from students' prior-to-school informal reasoning.

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The answer from Lewis Carroll is: Babies cannot manage crocodiles.

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## **PLENARY ADDRESS 4**

*Teachers as Stakeholders in Mathematics Education Research*

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# TEACHERS AS STAKEHOLDERS IN MATHEMATICS EDUCATION RESEARCH

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*This paper is a kind of self-reflection on the goals and actions of the scientific community of mathematics education researchers, in particular in relation to mathematics teachers. What can be done to improve research and teaching? What role do we ascribe to teachers? To which extent is it meaningful to regard and support them as researchers? Who is responsible for “knowledge production” for mathematics teaching and learning? Has the dominant view of knowledge transfer from researchers to teachers in the sense of “technical rationality” to be substituted by “reflective rationality”? Given the fact that a Japanese colleague is asked to comment on this plenary talk, the case of Japanese “lesson study” and attempts to transfer it to other countries is taken as an example to reflect on some selected issues.*

## INTRODUCTION

The topic of this conference is “*Developing mathematical thinking*”. Different people and environments contribute to learners’ mathematical thinking. *Most important are the learners themselves*. The impact of any learning opportunity and support depends on what learners bring with them and how interested they are to invest time and effort. This holds true for all kinds of learners: students, parents, student teachers, teachers, teacher educators etc. Focusing on mathematics learning at school level, *mathematics teachers* have the *strongest influence* on student learning because they are directly working with students. In general, teachers are regarded as *key persons of educational change* (e.g., Fullan, 1993). For example, a meta-analysis on student learning (Hattie, 2003) found that teachers’ impact on students’ learning is high: Identified factors that contribute to major sources of variation in student performance include the students (50%) and teachers (30%) as the most important factors, whereas home, schools, principals, peer effects (altogether 20%) play a less important role. Of course, teachers’ high impact on students’ learning varies considerably. Some teachers are more effective than others in promoting students’ knowledge, their beliefs and their educational decisions (e.g., to do extra homework, to switch to a mathematical branch of a school, to join a preparation course for the mathematics olympiad, to choose mathematics-related study at university). The diversity in teachers’ knowledge, beliefs and practices is nothing specific to the teaching profession. It also holds true for doctors, lawyers, politicians, mathematics educators or researchers. However, educational policy and educational research often seem to *focus primarily on teachers’ weaknesses*. For example, often the immediate reaction to bad results in comparative studies is to start professional development

initiatives for teachers as if it were the teachers only who need to change. Less attention is paid to the efficacy of the support system for schools, to the teacher education system, to teachers' general conditions and reputation etc. Such reactions indirectly blame teachers and – at the same time – they are unsatisfactory starting points for reform initiatives. If educational policy does not admit that the *whole system* (including educational policy) needs to change, the phrase “teachers are key agents of change” is a threat rather than an indication of their important role.

Research can have such hindering or demotivating effects as well. For example, studies in mathematics education (or in other domains) which emphasize on teachers' low mathematical competencies, their “monoculture” regarding teaching methods or their unwillingness to inform themselves about new research results, have effects that should not be underestimated. It seems more viable to highlight the complexity of teachers' task and also to report strengths and opportunities. It should also be taken into account that *teacher educators are co-producers* of this lamentable situation by being role models in teacher education courses. Likewise researchers are responsible for offering viable opportunities that encourage teachers to get interested in research.

From this point of view, the development of mathematical thinking – in addition to students' and parents' specific contribution – needs *good interrelations* between educational policy (and administration), educational practice (teaching, but also teacher education) as well as research (and development). The triangle *policy – practice – research* should build as much as possible on both mutual trust and critical (self-)reflection. This paper is an attempt to focus in an exemplary way (and concentrated on specific issues) on such a *(self-)critical reflection on mathematics education research* and its interrelation with mathematics teachers.

## TEACHERS AS STAKEHOLDERS

Given that teachers have a key role in the further development of teaching, which role do researchers ascribe to teachers in *mathematics education research*?

There is a tradition to view teachers as *experts* (e.g., Bromme, 1992). In particular, they are regarded as *researchers* (e.g., Stenhouse, 1975; Altrichter & Posch, 1990 – English version: Altrichter, Feldman, Posch, & Somekh, 2008; Elliott, 1991; Crawford & Adler, 1996) and *reflective practitioners* (e.g., Schön, 1983). *Intervention research* with teachers as partners and *action research* by teachers or teacher educators is becoming more prominent in mathematics teacher education (see e.g., editorials and papers in JMTE 6(2) and 9(3), and in ESM 54(2-3); Llinares & Krainer, 2006). Lesson study, a teacher-led approach to further develop teaching, has a long tradition in Japan and has begun to be widespread in other countries during the last decade (see e.g., Ohtani, 2009; Hart, Alston, & Murata, 2011).

It is an *ethical responsibility* of a scientific community and at the same time a *wise strategy* to raise questions like: How does the scientific community's knowledge get



known, used and reflected by relevant people and institutions? What can be done by researchers apart from writing papers and giving talks (predominantly within the scientific community), and from teaching classes of student teachers and offering professional development courses? It cannot be taken for granted that possible addressees of research automatically read the enormously increasing amount of research papers and that traditional teacher education is a viable means to link research results with the challenges of practice.

There have been efforts by individual researchers and groups to address this issue, for example in a conference on “Systematic cooperation between theory and practice in mathematics education” (Bazzini, 1994), in papers like the “Dialogue between theory and practice in mathematics education” (Steinbring, 1994), in the book “Making sense of mathematics teacher education” (Lin & Cooney, 2001), in the chapter “Mathematics teacher education” (Krainer & Llinares, 2010) in the International Encyclopedia of Education, or in the chapter “Linking research to practice” (Kieran, Krainer, & Shaughnessy, in preparation) in the upcoming “Third International Handbook of Mathematics Education”. Despite these efforts and continuous claims of how important teacher-researcher collaboration role is, teachers are most often seen as more or less *passive recipients of researchers’ knowledge production* and sometimes as a *means* (e.g., as data supplier) to produce knowledge.

What is missing, in particular, is a *systematic effort* by the scientific community (societies, commissions, universities, research groups, etc.) to analyse the potential role of teachers in research and its benefit for teachers and researchers. The question is urgent since the increasing economic pressure on research accompanied by a citation-index-driven accountability will lead to an intensified focus on paper production (in high ranked journals); less efforts are made to “go the complex way” by writing papers together with teachers (in equally important but – from an internal promotion view within the scientific community – less valued sources). To *build the bridge* from the teaching profession to the scientific community, well-developed organizations like NCTM with high-quality publications (e.g., Handbooks, NCTM-standards) are important. Another strategy is that teacher unions (like the LCH in Switzerland) employ scientists in order to build bridges to the scientific community.

In the 1980’s, an interesting change of paradigm was taking place in conceptualizing the role of management with respect to its environment (in particular in the USA). The traditional view was the *shareholder approach* which regarded it the duty of management to fulfil the interests of the shareholder only. Basically in order to prevent having poor social performance hurt the company financially, the management aimed at satisfying clients, consumers, society etc. by specific strategies (e.g., public relations). In contrast, Freeman (1984) and others developed a *stakeholder approach*, defining “stakeholder” as “any group or individual that can affect or is affected by the achievement of a corporation’s purpose” (Freeman, 2004, p. 229). The approach dealt with the practical concerns of managers – “how could they be more effective in identifying, analyzing and negotiating with key stakeholder

groups?” (p. 230). The stakeholder idea is connected to ethics and values, which are regarded as equally important as the business itself.

Regarding researchers as those having most expertise in research (theory, methodology etc.) and thus heavily setting the trajectories of research, they nevertheless are assumed to form their decisions not only for the sake of the scientific community but more broadly for *society* as well. Of course, other persons, groups and social systems also have a stake in the development of students’ knowledge: for example, parents, principals, superintendents, mathematicians, teacher educators, educational publishers, test developers, companies, (education) policy-makers, and even the whole society can be regarded as “stakeholders” of the (joint societal) “enterprise” to promote students’ mathematical knowledge. They all have effects on students’ knowledge and at the same time they are affected by their knowledge or lack of knowledge.

Research shows that the “myopic institutions theory” – claiming that companies that invest in stakeholder management will be penalized by investors who are only interested in financial returns – gets little support (Freeman, 2004, p. 237). In other words, *it does not hurt a company to look beyond shareholders*. There are several approaches that are close to the ideas (and partially might be regarded as precursors) of the stakeholder approach. For example, Hansen, Bode, and Moosmayer (2004) refer to approaches like systems theory, coalition theory, discourse ethics, conflict ethics, and rational ethics. There is an ongoing discussion on how the “Corporate Social Responsibility” approach – implying that companies take responsibility for their actions by considering the consequences for others who are affected – correlates with the stakeholder approach. In any case, there is much overlapping, but the main message is that *looking at the whole system* (of interests) is beneficial for all parts of a system aiming at sustainable development and peace. In order to reduce complexity, in the following, mainly the *interrelations between researchers and teachers* (and partially educational policy) are focused on.

How might a scientific community reflect on its *relationship to practitioners*? One strategy is to reflect upon the relationship to those practitioners (teachers) interested in research who do not participate in key meetings of the scientific community (like PME). A second strategy is to reflect upon situations where members of this community act themselves as practitioners (teacher educators). A third strategy is to reflect upon situations where teachers themselves investigate their practice. In the following sections, an emphasis is laid on the third strategy, in particular focusing on the question of *knowledge production*.

## TECHNICAL RATIONALITY VERSUS REFLECTIVE RATIONALITY

One important issue concerning researchers’ and teachers’ *production of knowledge* is the question of how researchers’ and teachers’ knowledge is interrelated and exchanged. Some researchers claim that they learned enormously from teachers, and

even reflect this in papers. And there is an increase in studies on teacher educators' learning as practitioners (see e.g., Jaworski & Wood, 2008).

However, there are also *critical assessments* of research in mathematics education with regard to practitioners' learning. For example, Ponte (2009, p. 102) indicates the "view of the 'deficient' teacher, so common in the research literature". In contrast to teachers' lack of knowledge etc., often researchers are seen as the ones where the knowledge is situated. This characterizes a view where knowledge transfer is a *one-way street* from researchers to teachers. To put it more crudely: *Teachers have problems, researchers have solutions*; and the latter (and we might include educational policy and administration people) also know the way(s) to disseminate innovations to teachers by means of curricula, standards, tests, material, lectures, seminars etc. This is the classical *Research-Development-Dissemination* (RDD) *model of innovation* whose limitations are shown all over the world.

Schön (1983) introduced the term *Technical Rationality* into the educational discourse. It follows *three basic assumptions* (an extensive discussion with regard to the teaching profession can be found in Altrichter et al., 2008, p. 270):

- There are general solutions to practical problems.
- These solutions can be developed outside practical situations (in research or administrative centres).
- The solutions can be translated into practitioners' actions by means of publications, training, administrative orders, etc.

For teachers it sounds like an irony when such approaches are accompanied by slogans stressing how important teachers are. They can't but regard such strategies as holding shareholder views (by education policy and research) and definitely not as stakeholder views really taking teachers seriously as persons with rich knowledge. For this view it makes more sense to use the term "teacher training", in contrast to "teacher education" which includes dimensions like autonomy, self-reflection and critical citizenship, all being standards of a well-developed profession.

Technical Rationality causes a hierarchy of credibility, expressing a genuine *mistrust* of practitioners: Teachers work at a "low level of theoretical knowledge and are merely applying what has been predefined in the academic and administrative power structure above them" (Altrichter et al., 2008, p. 270). In turn, this evokes *resistance* by teachers, opposition against reform and a genuine mistrust of researchers (and of education policy and administration people). It is a *vicious circle*.

In contrast to Technical Rationality, *Reflective Rationality* (see e.g., Posch, 1996; Altrichter et al., 2008, p. 270) follows three very different assumptions:

- Complex practical problems require particular solutions.
- These solutions can only be developed inside the context in which the problem arises and in which the practitioner is a crucial and determining element.

- The solutions can only rarely be successfully applied to other contexts, but they can be made accessible to other practitioners as hypotheses to be tested in practice.

These assumptions imply *new types of communication* among practitioners and new types of communication between practitioners and researchers (some people speak of theorists). The new communication needs to be built on symmetry rather than on hierarchy – both teachers and researchers have problems (some prefer the term *challenges*); and both need to find solutions, internal to their practice, but a *critical stance and external views* can be of support in defining the problem, in finding solutions or better ways to cope with the situation.

Reflective Rationality is an expression of an emerging understanding that teaching is a *highly complex endeavour*. Schön's (1983) account of 'reflective practice' is based on an analysis of practice in a number of different professions. He formulated three types of professional action: Tacit knowing-in-action, Reflection-in-action and Reflection-on-action. Professionals need to be competent in all three action types.

Reflective Rationality regards teachers (practitioners) as important producers of knowledge and "practical theories" (see Altrichter et al. 2008, p. 64-72). This production of knowledge can be done *with or without external interventions*. Regarding the *latter option*, teachers investigate their own practice in order to improve it (in the sense of action research, see e.g., Altrichter et al., 2008). Teachers doing action research might be supported by other persons, but it is the teachers who decide which problem is chosen, which data are gathered, which interpretations and decisions are taken etc. Action research challenges the assumption that knowledge is separate from and superior to practice. The production of "local knowledge" is seen as equally important as general knowledge, "particularization" (e.g., understanding a specific student's mathematical thinking) as equally important as "generalization" (e.g., working out a classification of typical errors).

The *first option* means that external experts (being invited or accepted on the basis of a proposal) come to the field and produce knowledge about it, mainly based on their research questions. This is a type of research that *combines intervention and research*, and thus might be called "intervention research" (see e.g., Krainer, 2003). Intervention research does not only apply knowledge that has been generated within the university, but much more, it generates "local knowledge" that could not be generated outside the practice. Thus, this kind of research is mostly process-oriented and context-bounded, generated through continuous interaction and communication with practice. Intervention research tries to overcome the institutionalised division of labour between science and practice. It aims both at balancing the interests in developing and understanding, and at balancing the wish to particularize and generalize. Action research may be regraded as intervention research done by practitioners themselves. However, action research can also be a part of an intervention research project. For example, an adequate division of labour is that

practitioners investigate their own practice (*first order* action research), and that academic researchers analyse teachers' findings (e.g., a comparative study with regard to teachers' case studies). In addition, academic researchers gather and analyse further data (e.g., a sample of case studies about teachers' professional development or a questionnaire for all participants, the principal, students, etc.) or investigate their own intervention practice (*second order* action research, see e.g., Elliott, 1991).

Worldwide, there is an increasing number of initiatives in mathematics education based on action research or intervention research (see e.g., Crawford & Adler, 1996; papers in JMTE 6(2) and 9(3); Benke, Hospesová & Tichá, 2008; Kieran, Krainer, & Shaughnessy, in preparation). In some cases, even the traditional role names (teachers versus researchers) are changed in order to express that both, individual learning and knowledge production for the field, are a two-way street. For example, in the Norwegian Learning Communities in Mathematics (LCM) project (Jaworski, Fuglestad, Bjuland, Breiteig, Goodchild, & Grevholm, 2007), the team decided to replace “researchers and practitioners” with “teachers and educators” (“both of whom are also researchers”). The motivating principle on which didacticians and teachers agreed to work together was the desire to develop better learning environments for mathematics students. There are a lot of projects in which teachers document their (evidence-based) experiences in reflecting papers. In Austria, for example, nearly 1000 papers – written by teachers for teachers – have been gathered since the 1980s within the context of programmes like PFL (see e.g., Krainer, 1998) and IMST (Pegg & Krainer, 2008), and can be searched by key word in an internet database (<http://imst.ac.at>). The most extensive and nationally widespread version of action research by teachers is practiced in Japan within the framework of “lesson study”.

## THE JAPANESE “LESSON STUDY” APPROACH

In their brief history section on Japanese lesson study, Fernandez and Yoshida (2009) indicate that the origins can be traced back to the early 1900s. In the 1960s, teachers started combining lesson study (*jugyokenkyu*) and school-based in-service professional development (*konaikenshu*). Recognizing the value of *konaikenshu*, the Japanese government started supporting these grassroots activities in the 1970s. This support – small financial and other incentives – still exists today. Lesson study is by far the most common *konaikenshu* activity.

There are manifold versions and various scopes of Japanese lesson study. They range from small-scale in-school initiatives with 4-6 teachers to large-scale national-level ones with hundreds of participants, travelling long distances.

However, a *typical lesson study process* (Fernandez & Yoshida, 2009, p. 7-8; see also Hart, Alston & Murata, 2011, p. 2-5) contains 4-6 *steps*; the centrepiece of lesson study (*jugyokenkyu*) is a study lesson (*kenkyujugyo*) where a particular lesson is studied in depth:

- Step 1: Collaboratively planning the study lesson
- Step 2: Seeing the study lesson in action
- Step 3: Discussing the study lesson
- Step 4: Revising the lesson (optional)
- Step 5: Teaching the new version of the lesson (optional)
- Step 6: Sharing reflections about the new versions of the lesson

Many schools solicit the support of an *external advisor* (most often instructional superintendents, sometimes experienced teachers on leave, or university staff). Schools often organize their *konaikenshu* work around a lesson study *open house* (*kokaijugyo*). Here well-developed ideas are shared with visitors (mostly teachers and other educators from neighboring schools). When distinguished guests (e.g., an external advisor) take part, considerable attention is paid to their reactions. In many cases, lesson study open houses are followed by a joint celebration in the evening (with a mixture of relaxed socializing and exchanging opinions not articulated at the formal meeting). Some schools even produce written reports about their work (*kenkyukiyo no matome*). For example, in the early 1990s, the National Institute for Educational Research compiled every year over 4.000 reports written by teachers (Fernandez & Yoshida, 2009, p. 213, referring to Sato, 1992).

The vast majority of elementary schools and many middle schools in Japan conduct *konaikenshu* in all subjects. In contrast, very few high schools are engaged. Fernandez and Yoshida (2009, p. 16) highlight that, in principle, there is no reason for regarding lesson study less suitable for the high school level; however, they indicate that Japanese high school teachers focus a great deal of their attention on preparing students for college entrance exams.

In principle, *konaikenshu* activities are voluntary; in reality however, they are regarded as quasi-required. This socio-cultural commitment or pressure has been expressed by a principal from an elementary school as follows:

For whatever reason, almost all schools around here are conducting *konaikenshu*. As you said, none of the laws say that we [schools] must conduct *konaikenshu*, but it is highly recommended. Also, because almost all schools are doing *konaikenshu*, we feel we have to do so as well. (Fernandez & Yoshida, 2009, p. 16)

Other reasons include financial support and information about other schools' experiences and ideas. However, most important of all, many teachers find *konaikenshu*, in particular lesson study, highly beneficial. Three mathematics teachers' opinions might give a flavour of their high regard of lesson studies:

Developing a great lesson is an ideal thing but I think the best thing about the lesson study experience is that it gives you a chance to reflect about and rethink your own teaching. ... I think even it is a short period of time, having a place where everybody gets together and discusses instruction very seriously is an extremely valuable experience. ... Anyway, lesson study can help teachers develop strong relationships, something I think is really important for all teachers." (Fernandez & Yoshida, 2009, p. 17)

It is common for individual teachers to belong to more than one lesson study group. In addition to within-school lesson study groups, also autonomous *cross-school study groups* (regional study groups and teacher clubs) are organized by teachers or unions (in most cases membership fees are collected). A system of regular teacher rotations between schools allows lesson study groups to learn from each other.

Lesson study is a special kind of *action research*. Teachers participating in a lesson study are collaborative researchers who collect data, interpret them and write down their experiences in papers and books. In many cases, in order to increase the validity of research, the effectiveness of outcomes, the dissemination of knowledge etc., experienced others are invited. Their role varies enormously. They might participate in order to observe (primarily as learners), to give occasional feedback, to present an invited reaction, to give input, to (co-)investigate students' growth, or to (co-)investigate the lesson study participants' growth. However, in general, lesson study in Japan is initiated, done, reflected and transferred to written artefacts by teachers for teachers, in an *investigative attitude* towards their own practice.

## ON THE SUCCESS OF JAPANESE LESSON STUDY

Recently, there have been some attempts to describe lesson study from different *theoretical perspectives*: for example, Minoru Ohtani (2009) uses Cultural-Historical Activity Theory, Asami-Johansson (in press) the Anthropological Theory of Didactics, and Corcoran (in press) the Community of Practice approach (including the term of "boundary object"). These attempts are highly fruitful for focusing on certain elements of a concrete lesson study activity and for bringing additional lenses – and thus new explanations, interpretations and hypotheses – into play. However, when regarding lesson study as an object in general, we need to take into account that it has developed from a grassroots movement and developed its own language; thus we need to be cautious not to impose an external frame on the *culturally-situated approach*.

Therefore, reflecting on the success of Japanese lesson study, a rather *open approach* is taken. It builds on general dimensions that help to describe lesson study using its *own language*. There are – at least – *three decisive dimensions* regarding teachers' learning – *content*, *community* and *context* (three Co's) (see e.g., Krainer, 2006).

*Content*: Meaningful mathematics-related activities and reflections on these activities support prospective and practising teachers' growth. There is a high level and a good balance of subject-related action and reflection (action and reflection).

*Community*: Mathematics teachers share experiences, ideas, beliefs, competencies, challenges and needs in order to learn autonomously but also to support others' learning. There is a high level and a good balance of individual and social activities (autonomy and networking).

*Context*: Mathematics teacher education needs conducive general conditions. There is a high level and a good balance of (internal and external) support.



In the following, the *success* of Japanese lesson study is reflected by means of these three Co's.

*Content:* Murata (2011, p. 10) highlights five key characteristics of lesson study from which four can be regarded as success factors belonging to the content dimension: Lesson study is centred around teachers' interests, is student focused, has a research lesson (observed in real time and experienced holistically) and is a reflective process. Murata (p. 4) highlights that teachers learn to listen to their students' ideas and to see student development; teaching is regarded as a complex and profound enterprise, being a two-way integration of student ideas and content exploration.

*Community:* Murata's (2011, p. 10) fifth key characteristic of lesson study is that it is collaborative. In addition, she indicates that novice teachers who experience the lesson with experienced teachers are apprenticed into the profession through participation (p. 5). This community aspect goes beyond the idea of collaboration among individual persons. It is about a way of further developing a profession (by engaging novices into serious academic activity and thus fostering identity building). The lesson study movement was successful in creating a language (a practical theory), indicating the status of a well-developed profession. This language is not only communicated orally (via processes), but also in written form (via products like lesson plans and books), indicating a rich body of knowledge, constructed by the profession for the profession. In addition, community building is not confined to teachers, but extends also to students: the vision of Japanese teaching is organizing collective thinking (*shudeanshiko no soshikika*), focusing on student presentations and discussions (Fernandez & Yoshida, 2009, p. 228). The succession of *single work*, *group work*, *plenary discussion in classrooms* and *teachers' comments* helps to balance individual and social learning.

*Context:* Murata (2011, p. 5) indicates that sometimes knowledgeable others from outside the lesson study group are invited, for example, to present observations or links to research or theories. They are paid with the help of external small grants. It is common that teachers are internally supported by principals and get release time by hiring substitute teachers. Lesson study groups are supported by townships, boards of education, the ministry etc.

These features of the three dimensions are *relatively stable* in Japan, also from a time perspective. Lesson study started as a grassroots initiative but got and gets support from research and educational policy, and because of its success it also influences these latter domains, in turn. Research, educational policy and teachers seem to build a *learning system* that regards lesson study as a *joint means* of further development. In particular the financial investment by boards and the ministry are an expression of trust in teachers and of believing in school-based professional development. It makes sense to speak of a lesson study "*culture*", genuine to Japan, geographically and historically situated.



Yet despite its long history, the approach still seems open to learning from outside, in particular from Western countries. It has deep roots in movements like child-centred and problem-based learning, in particular referring to John Dewey. Constructivist thinking, system theory, action research etc. have been developed in Western countries. However, they are probably better implemented on a wider scale in Japan. Similar observations can be made with regard to industry. For example, related to the fabrication of cameras, Western enterprises never assumed that Japan would be able to reach Western quality standards and to become a market leader in this area.

A common Japanese proverb (see e.g., Fernandez & Yoshida, 2009, p. 230) says “Improve yourself by looking at others” (*Hito no furi mite waga furi naose*) which indicates a *culture of life-long effort and strong will*. In a school context, this is probably best articulated by a lesson study teacher who stressed: “There is no end to improving teaching” (p. 230).

In contrast to isolated thinking, in English the saying “Two heads are better than one” is used; this is surpassed by the Japanese “Three heads together produce *Manjusri*” indicating Buddhist enlightenment and wisdom (Yoshida & Jackson, 2011, p. 286).

## ON TRANSFERRING LESSON STUDY TO OTHER COUNTRIES

It is not by chance that the lesson study movement became popular internationally. Achievement studies like TIMSS (and later PISA) attracted attention to Japan’s school system. Probably more influential within the scientific community was the TIMSS-Video Study which uncovered a huge difference between Japanese mathematics lessons and those from Germany and the USA.

Analysing those differences, the book “The teaching gap: Best ideas from the world’s teachers for improving education in the classroom” by Stigler and Hiebert (1999) was a breakthrough. It included a chapter on lesson study, based on Yoshida’s (1999) dissertation on mathematics lesson study in a Japanese elementary school. However, research on the effects of lesson study have not developed with the same speed. Drawing on examples of Japanese and USA lesson study, Lewis, Perry and Murata (2006, p. 3) propose that three types of research are needed “if lesson study is to avoid the fate of so many other once-promising reforms that were discarded before being fully understood or well implemented”: a) Expansion of the descriptive knowledge base on Japanese and USA lesson study; b) Explication of the innovation mechanism; c) Design-based research cycles.

The most recent book “Lesson study research and practice in mathematics education. Learning together” by Hart, Alston and Murata (2011) addresses research and practice in 16 different locations (mainly in the USA, but also in Australia, Canada and Ireland). Due to the lack of experienced lesson study teachers and teacher educators (and the whole culture of *konaikenshu* activities etc.), teacher educators acted as initiators of lesson studies. This is in contrast to the Japanese lesson study

approach where teachers themselves are the initiators. The „transfer“ of Japanese lesson study so far shows external experts as initiators and co-learners who support teachers or student teachers in practicing lesson study. The external experts also write the research papers (partially with teachers as co-authors), which in all cases have shown some success. At the 7th Congress of the European Society for Research in Mathematics Education (CERME7, 2011), three research papers dealt with lesson study. For example, Back and Joubert (in press) analysed the different experiences of three lesson study teacher groups (supported by UK researchers). The study brought to light two conducive (hindering) factors: the (lack of) commitment of the schools (supporting teachers in taking part in lesson study), and that of the teachers (adopting the ideas of lesson study).

Concerning the three dimensions *content*, *community* and *context*, the *content* dimension is the one where hardly any differences between Japanese and other countries' lesson study approaches were apparent. With regard to features like the planning of lessons and their reflection, the focus on students, or on problem solving, a “transfer” seems easy to realize. Related to *community*, this holds true also for the focus on sharing ideas; however, given the culture of professional exchange of experiences in Japan and the predominant situation of lone fighters in many Western countries, there is a big difference to be taken into account. The most eminent contrast is provided by the *context*. It makes a big difference whether a practice is a grassroots culture in a country (teachers have experience, a support system) or whether external experts introduce teachers to lesson study and investigate their professional growth. With regard to this issue, one may question whether one can speak of a “transfer” of Japanese lesson study, or of a “transformation”.

All in all, attempts to “transfer” lesson study to other countries have been reported as *successful*; however, it is not clear whether this was really due to the fact that the teacher education activities had elements of lesson study or just were well designed and carried out. Whether efforts in other countries may be regarded as variations of Japanese “lesson study” depends on the question whether one regards it more as a *method* or way of professional development, or as a *culture*, deeply rooted in Japanese beliefs and way of life.

More reflection is needed on what it means that a *deeply situated cultural approach* (like Japanese lesson study) is used in other cultural contexts. Of course, the initiating and supporting role of external experts in lesson study has advantages (e.g., input of new research results or new instruments and methods). However, it may also have obstacles (e.g., what teachers may feel about being assessed whether they grow and being instrumentalised as a means of publishing results). Therefore, it seems essential that teacher educators initiating lesson study in a non-Japanese context see themselves as reflective practitioners (who try out a cross-cultural transfer) and as co-learners and reflect that in their papers, if possible with teachers as co-writers. There is the issue again to which extent teachers are regarded as stakeholders in research.

## INSTEAD OF CONCLUSIONS

This paper, after relating to the conference theme, started with the following two sentences: Different people and environments contribute to learners' mathematical thinking. *Most important are the learners themselves.* This claim holds true for students, but also for *teachers as learners*. Researchers (and administrators) can and should support teachers' knowledge production at different levels. However, they *can't transmit knowledge or theories directly to the practitioners*; they can only offer them environments in which they are able to further develop their existing knowledge and belief system. Researchers and teachers' knowledge and (practical) theories can have nearly no overlap, or alternatively, a large one (with many positions in between).

I would like to end with *three hypotheses*:

- The more researchers regard practitioners as stakeholders of research, the more their knowledge and theory bases will overlap.
- Good collaboration and mutual trust between them increases the further development of both parties.
- Since researchers are better internationally and thematically organized (in particular based on written artefacts and conferences), they should assume the responsibility of taking serious steps to promote the negotiation process (e.g., Krainer, 2008), based on both parties' strengths and as a two-way-street.

The ideal way would be to regard *researchers as key stakeholders in practice*, and *teachers as key stakeholders in research*.

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# TEACHERS' LEARNING AND LESSON STUDY: CONTENT, COMMUNITY, AND CONTEXT

A reaction to Konrad Krainer's plenary lecture  
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*This paper is reaction to the plenary address by Professor Krainer on "Teachers as Stakeholder in Mathematics Education Research". The reaction starts general remark on critical issues in the teacher's role in mathematics educational research which is followed by discussion about the three hypotheses with respect to the three decisive dimensions regarding teachers' learning. The discussion is situated in personal reflection on Japanese lesson study. The paper concludes with some recent international trends in teachers' learning and lesson study.*

## ON CRITICAL REFLECTION

Nobody would disagree with the statement that "mathematics teachers have the strongest influence on student learning and are regarded as key agents for educational change". However, if such statement is posed as slogan in reaction to comparative studies on students' performance gap and teachers' teaching and preparation gaps, it can have demotivating effects and cause genuine mistrust on the part of the teachers.

Possible factor for such hindering effects lies in lack of wise strategy to establish transparent context between researcher and practitioner. Researchers represent teaching practice in terms of theoretical concepts which often gain primacy over vernacular representation (e.g., Mehan, 1993). A hierarchy among modes of representations is formed and academic speech genre is privileged over practical speech genre. We have experienced terrible devastations that took place in Tohoku this March. Not only mega quake and unbelievable Tsunami destroyed our life, but also nuclear power plants have collapsed. Suffered people have stayed calm and disciplined with genuine mistrust from government and electric power company in that they have been predominantly regarded as passive receptor of misplaced admonition, ambiguous academic explanation that were only understandable to specialists.

The author totally agreed with Krainer's strong assertions that researchers are responsible for taking a step to get scientific community's knowledge known, used and reflected by relevant people and institutions and that researchers should highlight teachers' reflective and creative practice and offer viable opportunities that encourage teachers to get interested in research. It is crucial for both researchers and practitioners to go beyond stockholder approach and establish new types of communication so as to regard teachers as key stakeholders in research, and researchers as key stakeholders in practice. Otherwise, with traditional hierarchical relationship, practitioners are constrained by "technical rationality" which imposes



double task for them: understand theory and translate into practice. There are some case studies which rethink boundaries between theory and practice and the relative role of researchers and teachers (e.g., Chazan, Callis, & Lehman, 2008). As extensive and nationwide action research, Japanese lesson study also assumes “reflective rationality” and establishes symmetrical communication between practitioners and researchers.

## CONTENT, COMMUNITY, AND CONTEXT

The Stakeholder approach implies multidimensional system which consists in composite characteristics. This approach ask of teacher’s learning is not *what* but *where* it is. Teacher’s learning occurs not in the individual mind, but it is situated in interaction between individual agent and sociocultural context. Krainer’s “three Co’s” (2006) seems to coincide with the idea of “analysis by unit” (Vygotskii, 1982: 15) in Sociocultural approach. As a unit, we take into account of balance among all the basic characteristics of the whole system.

The author would like to add some additional characteristics of Japanese lesson study practice to the “three Co’s”.

**Content:** After World War II, schools devoted themselves to developing school-based curriculum. In 1958, Ministry of Education Japan issued new national curriculum which was more legally binding. This educational reform affected convert of teachers focus from curriculum to lesson (e.g., NASEM, 2009). However, investigation of instructional material (*kyouzaikennkyuu*), especially designing conceptually robust task still remains crucial characteristic of Japanese lesson study. *Kyouzaikennkyuu* is not always attended to lesson study in foreign countries (Doig, Grooves, & Fujii, 2011). Doig, Groves, and Fujii illustrate four types of task used in Japanese research lesson. Among them, there are tasks that “have been chosen based on a rigorous examination of scope and sequence” (p.185).

**Community:** First, teachers make collaborative team works through lesson study, the team is to some extent characterised by “flexible network” (e.g., Engeström, 2008). Japanese school has staffroom where all teachers have their own working desk with laptop PC, teaching materials, and several kinds of lesson documents. Teachers from the same grade level are seated beside each other for collaboration. They prepare and discuss their lesson plans when they have time to spare. In the picture, a female teacher is checking her lesson plan for later discussion with her colleagues in order to improve the study lesson. The configuration of the staffroom sets a base for collaborative professional development of teachers in Japan.





Second, Japanese teachers use didactical terms to discuss their teaching and learning in order to improve their teaching practice (e.g., Yoshida, 2010). These technical terms are used to explain and discuss classroom teaching and learning during lesson study. Here are some terms (cited from Yoshida, 2010: 231). “*Hatsumon*: a well thought-out question to stimulate student curiosity and thinking. *Kikan-Junshi*: purposeful scanning or monitoring of student thinking and learning when students are engaged in individual or group problem-solving processes. *Neriage*: the process of kneading and polishing student thinking by comparing, contrasting, and discussing student-presented solution ideas during problem-solving lessons. *Matome*: highlighting and summarizing the major points of the lesson during and at the end of the lesson. *Bansho*: chalk board writing and organization.” Such didactical terms mediate researchers and teachers and pedagogical content knowledge is co-constructed.

**Context:** At the end of the school year, the local education board announces personnel changes of every school’s staff. Part of the school staff, including principal and vice-principal, goes out and comes in. Such personnel relocation or reshuffling makes further collaboration among school staff essential. In order to attain educational objectives and to cover the school-based curriculum with new members, it is crucial for the school to establish a community of practice. Such socio-institutional conditions require lesson study. School as a communal activity system is not stable but evolves through a systemic lesson study approach. Teachers’ build their identity and team-efficacy through recurring collaboration with others. Thus, Japanese lesson study is deeply socio-institutionally situated. Even after mega quake disaster in March, a prefectural board of education decided regular personnel change for new school year (starts in April).

## RECENT INTERNATIONAL DEVELOPMENT

Lesson study practice is rapidly expanding internationally. In 2007, World Association of Lesson Study was established and more than twenty countries are engaging in lesson study projects (e.g., Akita & Lewis, 2008). In US, Japanese researchers have organized, implemented, and sustained lesson study community (Takahashi & Yoshida, 2004). University of Tsukuba Japan and Khon Kane University Thailand had been jointly organized international conferences of lesson study for APEC economies in which external experts acted as initiators of lesson study (Imprasitha, Isoda, & Yeap, 2011).

There are many things that Japanese researchers and teachers can learn from international experiences. Since 1999, US teachers have been intensely interested in lesson study and actively organized several kind of lesson study group up to university level (e.g., Akita & Lewis, 2008). One of the promising research projects is “Lexicon Project” by International Center for Classroom Research (ICCR) at University of Melbourne (Clarke & Mesiti, 2010). This project has been undertaken to identify classroom-related didactical terms, used in non-English-speaking cultures.

Classroom lexicon will provide researchers with shared didactical terms. Sharing such didactical terms would support good collaboration and mutual trust between researchers and practitioners without losing both parties' strength.

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# PLENARY PANEL

- Olive Chapman, Convenor
- Uri Leron
- Carolyn Maher
- Gabriele Kaiser
- Frederick K.S. Leung





# **SUPPORTING THE DEVELOPMENT OF MATHEMATICAL THINKING**

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## **INTRODUCTION**

Mathematical thinking [MT] is central to doing mathematics, and thus should be central to the teaching and learning of mathematics at all levels. Mathematics educators/teachers and mathematics teacher educators should engage their students in it in an explicit way. Mathematics education researchers should explore it in an explicit way. Mathematics curricula should highlight it explicitly. This explicit focus on MT is important in order to make it a reality and potentially a “habit of mind” for teachers and students. However, there are some fundamental questions that need to be addressed to provide the basis for this reality and potential to materialize.

The most fundamental of these questions is: What is mathematical thinking? Other key questions include: How is MT similar to and different from other kinds of thinking? How does MT develop? How does MT differ across cultures? What are meaningful, useful, and effective instructional approaches and tasks to support the development of MT? What learning environment stimulates students to engage in MT? How is MT assessed? How is MT altered, enhanced, and impeded by the use of technological tools? What are the challenges in promoting MT in students? What do teachers need to know?

This plenary panel addresses aspects of such questions in the context of the panelists’ areas of research. The intent is to offer insights regarding current, relevant, and effective progress in addressing issues related to supporting the development of MT.

## **WHAT IS MATHEMATICAL THINKING?**

Simply stated, MT is the type of thinking process used in doing mathematics. However, defining the nature of this thinking is problematic given the multiple ways or perspectives in which it has been viewed or used. This is evidenced in the work of various authors in Sternberg and Ben-Zeev’s (1996) edited book: *The nature of mathematical thinking*. As Sternberg concluded, MT is more prototypical than mathematical, that is, it represents a range of different typical qualities as opposed to an equivalence class of exactly the same characteristics. In the mathematics education literature, MT has been associated with, for example: mathematical processes (Mason, Burton, & Stacey, 1982); relational understanding (Skemp, 1976); understanding (Sierpinska, 1994); conceptual thinking (Hiebert & Lefevre, 1986); mental activity (Watson & Mason, 1998); reflective abstraction (Dubinsky, 1991); mathematical sense-making (Schoenfeld, 1994; Lakatos, 1976; Polya 1954); a

mathematical point of view (Schoenfeld, 1994, 1992); mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001); problem solving (Burton, 1984; Mason, Burton, & Stacey, 1982; National Council of Teachers of Mathematics [NCTM], 2000; Polya, 1954; Schoenfeld, 1994, 1992); reasoning and proof, communication, connections, and representation (NCTM, 2000); thinking processes, knowledge, beliefs and attitudes (Charles, 1985); a dynamic process, a way of understanding yourself and the world (Mason, Burton, & Stacey, 1982); metacognition, belief, and mathematical practices (Schoenfeld, 1992); advanced thinking (Edwards, Dubinsky, & McDonald, 2005; Tall, 1992) and mathematical habits of mind (Cuoco, Goldenberg, & Mark, 1996, 2010).

Within these various conceptions, a general view of MT is that it stresses the mental activities or methods used in learning mathematics, that is, it is associated with thinking, reasoning and coming to know a concept (Mason, Burton, & Stacey, 1982). It involves using mathematically rich thinking skills to understand ideas, discover relationships among the ideas, draw or support conclusions about the ideas and their relationships, and solve problems involving the ideas (O'Daffer & Thornquist, 1993). Schoenfeld (1992) describes it as involving the development of a mathematical point of view - valuing the process of mathematization and abstraction and having the predilection to apply them; and the development of competence with the tools of the trade, and using those tools in the service of the goal of understanding structure. He also suggests that in order for students to think mathematically, they need to develop a predilection to analyze and understand, to perceive structure and structural relationship, to see how things fit together, and the ability to reason in extended chains of arguments.

Other definitions of MT have been offered to characterize it as advanced MT. For example, Edwards, Dubinsky, and McDonald (2005) define advanced MT as “thinking that requires deductive and rigorous reasoning about mathematical notions that are not entirely accessible to us through our five senses” (p. 15). Tall (1992) links his notion of advanced MT to formal/advanced mathematics and involving a focus on logical deductions of theorems based upon precise mathematical definitions. However, Harel and Sowder (2005) argued for “advanced mathematical-thinking” (i.e., mathematical thinking of an advanced nature) noting,

This change in emphasis is to argue that a student's growth in mathematical thinking is an evolving process, and that the nature of mathematical thinking should be studied so as to lead to coherent instruction aimed toward advanced mathematical-thinking. (p. 28)

MT has also been viewed as acts of participation in a variety of different socially or culturally situated mathematical practices (Saxe, Dawson, Fall, & Howard, 1996; Williams, Wood, & McNeal, 2006). Saxe et al. (1996) give special emphasis to its reliance on culture and context. Dreyfus and Eisenberg (1996) clarify the affective aspects of it, such as self-confidence and mathematical creativity.

MT, then, can take on variations to how it is conceptualized and used to make sense of supporting students' development of it. How necessary are these variations? Are they uniquely different? What challenges or opportunities do they offer?

## SUPPORTING STUDENTS' MATHEMATICAL THINKING

The mathematics education literature offers suggestions directly or indirectly related to supporting MT from many sources (e.g., Boaler, 1997; Bransford, Zech, Schwartz, Barron, & Vye (1996); Ginsburg & Klein, 1998; Harel & Sowder, 2005; Mason, Burton, & Stacey, 1982; NCTM, 1991, 2000; Polya, 1954; Schoenfeld, 1994; Watson & Mason, 1998). The NCTM (2000) process standards embody many of these suggestions that include engaging students in ways that allow them to: apply and adapt a variety of appropriate strategies to solve problems; select and use various types of reasoning and methods of proof; analyze and evaluate the mathematical thinking and strategies of others; understand how mathematical ideas interconnect; and select, apply, and translate among mathematical representations to solve problems. The NCTM (1991) standards also highlight the importance of worthwhile mathematical tasks and discourse in helping students to develop their MT. Mason, Burton, and Stacey (1982) argued that an atmosphere of questioning, challenging and reflecting supports MT while Watson and Mason (1998) discussed questioning as an important part of the teacher's ability to establish a classroom atmosphere conducive to the development of MT. Boaler (1997) showed that an exploratory, participatory learning environment made a significant difference to students' development of MT.

More recent studies that explicitly addressed MT have added to or validated these suggestions. For example, Mason (2004) demonstrated ways in which electronic screens can be used powerfully in mediating MT. Williams, Wood, and McNeal (2006) studied the relationship between normative patterns of social interaction and MT of five classes of 7- to 8-year old children. They found that increased complexity in children's expressed MT was closely related to the types of interaction patterns that differentiated class discussions among the four different classroom cultures identified for the five classes. Van Oers (2010) reported on the importance of promoting MT through supporting young children's appropriation of schematic representations and notations in the context of play. In Pape, Bell and Yetkin (2003), the focus was on contexts that support the development of MT in the middle-school mathematics classroom. They identified factors crucial to this development as: multiple representations and rich mathematical tasks; classroom discourse; environmental scaffolding of strategic behavior; and varying needs for explicitness and support.

In my work, I consider the development of MT in the context of inquiry-based pedagogy and discourse. For example, in Chapman (2009) I reported on a study of whole-class discourse and the teacher's actions that supported and promoted *learner-focusedness* and MT in learning mathematics. A learner-focused perspective based on agency, collaboration, and reflection framed the study. Findings from the study identified four categories of engaging students in self-reflection that formed a unique

and key basis of this learner-focused discourse: reflection on “real-world” experiences; reflection on conceptions; reflection on preconceptions; and reflection on thinking. These approaches and an *intersubjective stance* of the teacher’s role were shown to be important to empower “self” in the learning of mathematics and allow students to develop MT by making sense of mathematical ideas, mathematics in their lives, their world in a mathematical way, and their ways of thinking and learning.

In general, MT can be supported through an inquiry-based pedagogy that allows students’ questions and curiosities to drive curriculum, honours previous experience and knowledge, makes use of multiple ways of knowing, and allows for creation or adoption of new perspectives when exploring issues, content, and questions. In this approach, students are given the opportunity to direct their own investigations and find their own answers.

## CONTRIBUTION OF PLENARY PANEL

Given the multiplicity of perspectives involving MT, a panel on the topic of “supporting the development of MT” could be construed in different ways in terms of its focus. One approach is to present a diversity of panelists, representing multiple viewpoints, to address pressing and interesting questions regarding how to support the development of MT and to identify and characterize important recent developments, new perspectives, and emergent issues related to MT. Each panelist could focus on his or her own work relating to MT, but also addresses a common core of themes dealing with her or his perspective of MT, the relationship of this perspective to teaching and learning mathematics, and valid ways of, and challenges in, supporting the development of MT.

In this PME plenary panel, the contribution is influenced by the panelists’ choice of focus depending on their research. As a result, key questions/themes about MT and supporting its development, such as those previously noted, for the most part are not explicitly addressed in all of the papers. However, there are some indications of, or implicit connections to, how MT is interpreted and how its development could be supported. For example: Uri Leron discusses MT in terms of the relationship between intuitive thinking and analytical thinking and explicitly addresses the connection to supporting its development. In his perspective of MT, supporting the development MT is related to how the power of natural thinking can be marshalled to help students develop their analytical thinking skills and what can be done to help them create a peaceful coexistence between these two modes of thinking even when they clash. His study illustrates how designing a “new bridging task” could facilitate this. In Carolyn Maher’s work, MT can be equated to problem-solving thinking and paired with reasoning. Findings of the study imply that an instructional intervention of studying videos about children’s reasoning produces changes in a positive direction in pre and in-service teachers’ ability to identify forms of reasoning in children’s problem solving. This ability can help teachers to become more attentive to students’



reasoning and thus to support students' development of MT. Gabriele Kaiser discusses MT in terms of the underlying cognitive character taking place during the modelling process characterised by the relation and interplay between mathematics and the real world. In this perspective, one way of supporting the development MT is related to supporting students to use their metacognitive knowledge and strategies during modeling tasks. Finally, Frederick Leung discusses MT as the learning of mathematics and a product of culture. In this perspective, based on the Chinese context, one way of supporting the development of MT that he suggests is for teachers to encourage students to study hard and engage them in challenging and interesting mathematical activities.

In general, the panel does offer multiple viewpoints, address pressing and interesting questions that can be related to the development of MT, and identify important recent developments and emergent issues related to the learning and teaching of mathematics. The contribution also provides insights into future direction for research on supporting the development of MT.

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# MIND THE GAP: FROM INTUITIVE TO ANALYTICAL THINKING

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## INTRODUCTION

A fundamental aspect of mathematical thinking is the relationship between two quite different modes of thinking: Intuitive thinking (or its close relative, natural thinking) on the one hand, and analytical (or formal) thinking on the other. When we read and try to make sense of a formal piece of mathematics, we are trying to uncover the intuitive ideas behind the formal presentation; conversely, when we are exploring or discovering a piece of mathematics, we need to be able to eventually formalize the intuitive ideas that have come up in the process.

Thus, in thinking about mathematical thinking, it is important to consider the extensive experimental and theoretical research by cognitive and evolutionary psychologists on the pervasive power of natural thinking, and its uneasy relationship with analytical thinking. Natural thinking is the suit of skills that is acquired by all people spontaneously and successfully under normal developmental conditions, or put more simply, it is what all people are naturally good at. Hence, a fundamental issue in thinking about *supporting the development of mathematical thinking* is how we can marshal the power of natural thinking to help students develop their analytical thinking skills, and furthermore, how we can help them create a peaceful coexistence between these two modes of thinking even when they clash.

At the background of this paper lurks the momentous *rationality debate*: Are humans rational beings or not? Or, better, how rational are human beings? Or, still better, what kind of rationality (or irrationality) is invoked under what conditions? This question had been endlessly debated by the great philosophers through the millennia, but has become an empirical issue for cognitive psychologists in the second half of the 20<sup>th</sup> century, culminating with the 2002 Nobel Prize in economy to Daniel Kahnemann for his work with Tversky on “intuitive judgment and choice” (Kahnemann, 2002). Here, however, I focus on a narrower (and more immediately relevant) facet of the rationality debate: What is the relation between people’s natural and analytical thinking? More specifically, when do these two modes of thinking go together and when do they clash? Or, even more specifically, when can we as mathematics educators build on the strength of students’ natural thinking, and when do we need to devise ways to overcome it?

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<sup>1</sup> This paper is based on joint work with Lissner Rye Ejersbo and Abraham Arcavi

I begin with a simple puzzle: A baseball bat and ball cost together one dollar and 10 cents. The bat costs one dollar more than the ball. How much does the ball cost?

This rather trivial arithmetical puzzle would be totally devoid of interest, if it were not for the fact that many intelligent people give the answer "10 cents" (instead of the correct 5 cents): 50% of Princeton students and 56% of students at the University of Michigan gave this erroneous response (Kahnemann, 2002, p. 451). The trivial arithmetical challenge has thus turned into a non-trivial *cognitive challenge* for psychologists: What is it about the workings of our mind that causes so many intelligent people to err on such a simple problem, when they surely possess the necessary mathematical knowledge to solve it correctly?

## DUAL-PROCESS THEORY (DPT)

This challenge and many others like it have led to one of the most influential theories in current cognitive psychology, *Dual Process Theory* (DPT), roughly positing the existence of "two minds in one brain". For a concise overview of DPT see Leron and Hazzan (2006, 2009). For state of the art review of DPT – history, empirical support, applications, criticism, adaptations, new developments – see Evans and Frankish (2009). According to dual-process theory, our cognition and behavior operate in parallel in two quite different modes, called *System 1* (S1) and *System 2* (S2), roughly corresponding to our commonsense notions of intuitive and analytical thinking. These modes operate in different ways, are activated by different parts of the brain, and have different evolutionary origins. Like perceptions, S1 processes are characterized as being fast, automatic, effortless, non-conscious and inflexible (hard to change or overcome); unlike perceptions, S1 processes can be language-mediated and relate to events not in the here-and-now. S2 processes are slow, conscious, effortful, computationally expensive (drawing heavily on working memory resources), and relatively flexible. In most situations, S1 and S2 work in concert to produce adaptive responses, but in some cases the fast and automatic S1, which operates on surface features of the task (such as the saliency of the numbers one dollar and 10 cents in the bat-and-ball puzzle), may generate *non-normative* responses, while the slow and effortful S2 may or may not intervene in its role as monitor and critic to correct or override S1's response. (Kahnemann, 2002)

A corollary of particular interest for mathematics education is that many recurring and prevalent mathematical errors originate from general mechanisms of our mind and not from faulty mathematical knowledge. Significantly, such errors often result from the *strength* of our mind rather than its weakness.

## BRIDGING THE TWO MODES OF THINKING: A DESIGN PERSPECTIVE

I begin again with a well-known puzzle: Imagine you have a string tightly encircling the equator of a basketball. How much extra string would you need for it to be moved one meter from the surface at all points? Hold that thought, and now think about a string tightly encircling the Earth – making it about 40,000 km long. Same question:

how much extra string would you need for it to be moved one meter from the surface at all points? Everyone seems to feel strongly that the Earth would need *a lot* more extra string than the basketball. The surprising answer is that they both need the same amount, which is in fact quite small:  $2\pi$  or approximately 6.28 m. (If  $R$  is the radius in meters of any of them, then the extra string is calculated by the formula

$2\pi(R+1) - 2\pi R = 2\pi$ .) As with the bat-and-ball puzzle, *this surprise is what we are after*, for it tells us something important about how the mind works, which is why cognitive psychologists are so interested in such puzzles. This time, however, alongside with the cognitive challenge, there comes also an important *educational challenge*: As teachers and mathematics educators, what do we do to help students reconcile the two conflicting solutions that now inhabit their minds?

Papert (1993/1980, pp. 146-150) demonstrates a simple but ingenious answer to this educational challenge: *Just imagine a cubic earth instead of a spherical one!* Then you can actually *see* that the two strings have the same length along the sides of the square equator, and that the only additional length is at the 4 corners. Can this beautiful example be generalized? When intuitive and analytical thinking clash, can we always design such “bridging tasks” that would help draw them closer? How should theory, design and experiment collaborate in the pursuit of this goal? Inspired by Papert's example, Lissner Rye Ejersbo and I have taken up the challenge of exploring this question in the context of the more relevant, and extensively-researched task from cognitive psychology: *the Medical Diagnosis Problem*.

## MEDICAL DIAGNOSIS PROBLEM (MDP) BACKGROUND

Here is a standard formulation of the MDP task and data, taken from Samuels, Stitch, and Faucher (2004, p. 136).

Before leaving the topic of base-rate neglect, we want to offer one further example illustrating the way in which [base-rate neglect] might well have serious practical consequences. Here is a problem that Casscells et al. (1978) presented to a group of faculty, staff and fourth-year students at the Harvard Medical School.

[MDP:] If a test to detect a disease whose prevalence is 1/1000 has a false positive rate of 5%, what is the chance that a person found to have a positive result actually has the disease, assuming that you know nothing about the person's symptoms or signs? \_\_\_\_%

Under the most plausible interpretation of the problem, the correct Bayesian answer is 2%. But only eighteen percent of the Harvard audience gave an answer close to 2%. Forty-five percent of this distinguished group completely ignored the base-rate information and said that the answer was 95%.

This task is intended to test what is usually called *Bayesian thinking*: how people update their initial statistical estimates (the *base rate*) in the face of new evidence (the *diagnostic information*). In this case, the base rate is 1/1000, the diagnostic information is that the patient has tested positive, and the task is intended to discover how the subjects will update their estimate of the chance that the patient actually has

the disease. The meaning of “5% false positive rate” is that 5% of the healthy people taking the test would test positive. *Base-rate neglect* refers to the widespread tendency among subjects to ignore the base rate, instead simply subtracting the false positive rate of 5% from 100%. Indeed, it is not at all intuitively clear why the base rate should matter, and how it could be taken into the calculation.

The solution of the task is based on Bayes' theorem, but there are many complications and controversies involving mathematics, psychology and philosophy, concerning the interpretation of that theorem (Barbey & Sloman, 2007). Here is an intuitive solution for the MDP, bypassing Bayes' theorem: Assume that the population consists of 1,000 people and that all have taken the test. We know that one person will have the disease (because of the base rate) and will test positive (because no false negative rate is indicated). In addition, 5% of the remaining 999 healthy people will test false-positive – a total of 51 positive results. Thus, the probability that a person who tests positive actually has the disease is  $1/51$ , which is slightly less than 2%.

## THE EDUCATIONAL CHALLENGE AS DESIGN ISSUE

The extensive data on base rate neglect in the MDP (leading to the 95% answer) demonstrates the counter-intuitive nature of the analytical solution, as in the case of the string around the earth. As mathematics educators, we are interested in helping students build bridges between the intuitive and analytical perspectives, hopefully establishing peaceful co-existence between them. As in Papert's example, achieving such reconciliation involves a design issue: Design a new *bridging task*, which is logically equivalent to, but psychologically much easier than the given task.

From the extensive experimental and theoretical research in psychology on the MDP (see Barbey and Sloman (2007) for a comprehensive review), we were especially influenced in our design efforts by the *nested subsets hypothesis* (Evans, 2006):

All this research suggests that what makes Bayesian inference easy are problems that provide direct cues to the *nested set* relationships involved. [...] It appears that heuristic [S1] processes cannot lead to correct integration of diagnostic and base rate information, and so Bayesian problems can only be solved analytically [i.e., by S2]. This being the case, problem formats that cue construction of a single mental model that integrates the information in the form of nested sets appears to be critical. (p 391)

Based on this theoretical background, we formulated three *design criteria* for the new task (which would also serve as testable predictions).

1. *Intuitively accessible*: The bridging task will be easier (“more intuitive”) than the original MDP (i.e., significantly more people – the term is used here in a qualitative sense – will succeed in solving it correctly).
2. *Bridging function*: Significantly more people will solve the MDP correctly, without any instruction, after having solved the new task.
3. *Nested subsets hypothesis*: Base rate neglect will be significantly reduced.



Note that the first two design criteria pull the new task in opposite directions. Criterion 1 (turning the hard task into an equivalent but easier one) requires a task which is sufficiently different from the original one, while criterion 2 (the bridging function) requires a task that is sufficiently similar to the original one. The new task, then, should be an *equilibrium point* in the “design space” – sufficiently different from the original task but not *too* different.

Armed with these criteria, we set out on the search for the new bridging task. After a long process of trial and error, intermediate versions, partial successes and failures, we finally came up with the *Robot-and-Marbles Problem* (RMP), which we felt had a good chance of satisfying the design criteria and withstanding the empirical test.

**RMP:** In a box of red and green marbles, 2/1000 of the marbles are red. A robot equipped with green-marble detector with a 10% error rate (10% green marbles are identified as red), throws out all the marbles which it identifies as green, and then you are to pick a marble at random from the box. What is the probability that the marble you have picked would be red?

## THE EXPERIMENT

The participants in the experiment were 128 students studying towards an M.A. degree in Educational Psychology at a Danish university, with no special background in mathematics or statistics. All of the participants were assigned the two tasks – the medical diagnosis problem (MDP) and the robot-and-marbles problem (RMP) – and were given 5 minutes to complete each task (in a pilot experiment we found that 5 minutes were enough both for those who could solve the problem and those who couldn’t). The subjects were assigned randomly into two groups of 64 students each. The order of the tasks was MDP first and RMP second for one group (called here the *MR group*), and the reverse order for the second group (the *RM group*). The results of the RM group were clearly our main interest, the MR group serving mainly as control.

	RM	Group	MR	Group
	RMP 1 <sup>st</sup>	MDP 2 <sup>nd</sup>	MDP 1 <sup>st</sup>	RMP 2 <sup>nd</sup>
Correct	31	17	8	20
Base-rate neglect	1	12	22	4
Incorrect other	32	35	34	40
Total	64	64	64	64

Table 1: Numbers of responses in the various categories

The results are summarized in Table 1, and it can be seen that the design criteria have been validated and the predictions confirmed. Here is a brief summary of the results for the RM group (with comparative notes in parentheses).

1. The RMP has indeed been intuitively more accessible compared to the MDP: 48% (31/64) of the subjects in the RM group solved it correctly. (Compared to 18% success on the MDP in the original Harvard experiment and 12% (8/64) in our MR group.)
2. The RMP has indeed served as a bridge for the MDP: More than 25% (17/64) solved the MDP, without any instruction, when it followed the RMP. (Again, compared to 18% in the original Harvard experiment and 12% in our MR group.)
3. The notorious base-rate neglect has all but disappeared in the RMP: it was exhibited by only 1 student out of 64 in the RM group and 4 out of 64 in the MR group. (Compared to 45% on the MDP in the original Harvard group and 34% in our MR group.)
4. Remarkably, the MDP, when given first, does not at all help in solving the RMP that follows. Worse, the MDP gets in the way: The table shows 48% success on the RMP alone, vs. 31% success on the RMP when given after the MDP.
5. Even though the performance on the RMP and the MDP has greatly improved in the RM group, still the largest number of participants appear in the “incorrect other” category. This category consists of diverse errors which do not directly relate to the MDP, including (somewhat surprisingly for this population) many errors concerning misuse of percentages.

## CONCLUSION

In this article we have seen how the dual-process theory from cognitive psychology highlights and helps explain the power of intuitive thinking. We have used the Medical Diagnosis Problem to discuss the gap between intuitive thinking and analytical thinking, and to develop design principles for bridging this gap. It is my belief that bridging this gap (in research, curriculum planning, learning environments, teaching methods, work with teachers and students) could be a major step towards *supporting the development of mathematical thinking*.

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# **SUPPORTING THE DEVELOPMENT OF MATHEMATICAL THINKING THROUGH PROBLEM SOLVING AND REASONING**

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*This paper draws on longitudinal and cross-sectional studies that followed the mathematical thinking and reasoning of cohort groups of students who were thoughtfully engaged in doing mathematics in and out of classrooms, from elementary school and beyond. The earlier work produced a unique collection of video and related metadata that are being prepared for storage and study on the Video Mosaic Repository at Rutgers University<sup>1</sup>. This research examines whether pre and in-service teachers' study of video episodes of children's learning from the Repository collection can improve their performance on measures of expertise that include recognizing a variety of forms of children's arguments.*

## **INTRODUCTION**

The importance of learners building mathematical knowledge that is based on meaning and reasoning is not new. For almost two decades, various researchers and professional groups have advocated that mathematics instruction be designed so as to provide opportunities for students to build foundational understanding of mathematical ideas and procedures, in contrast to more pervasive forms of instruction that emphasize the learning and application of rules and methods to solve certain classes of problems. In 1996, Maher and Martino reported that, in a natural way, even young children could build justifications for solutions to problems with proof-like arguments. Since then, other studies have provided support for the earlier finding (Maher, 2009; Maher & Martino, 1998; Maher, Powell, & Uptegrove, 2010; Mueller & Maher, 2009). Why then, we might ask, has there been relatively little change in carrying out instruction that fails to recognize the potential and power of student reasoning? Undoubtedly, this is a complex question that requires much study. However, we begin to explore the question by first examining teacher knowledge about student reasoning. Opportunities to study children doing and talking about mathematics and providing convincing arguments for their solutions to problems might not typically be accessible to pre-service teachers, or to many in-service teachers whose approach to instruction misses opportunities to observe how children learn and do mathematics. Video, however, holds promise of providing a window into alternative classroom settings in which communication, collaboration and the

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<sup>1</sup> Video clips from the collection are available on the Video Mosaic Repository, which is located at: <http://www.video-mosaic.org/>

sharing of ideas are the norm. The use of video clips of children thoughtfully engaged in doing mathematics thus offers a new lens with which to view student learning. From this perspective, a question that guides our work is: *Does teacher study of Video Mosaic Repository videos improve their ability to recognize the variety of forms of reasoning used by the children?*

### Using the Video Collection

Earlier studies have produced a unique collection of video and related data that are available on the Video Mosaic Repository<sup>2</sup> at Rutgers University (Agnew, Mills, & Maher, 2010). The video collection includes over 4,500 hours of source videos and emanates from longitudinal and cross-sectional studies of the development of mathematical ideas and ways of reasoning of students. The earlier studies were conducted in formal and informal educational settings and were supported with National Science Foundation (NSF) funding. Synthesizing the video data from several content strands on mathematical reasoning has led to the current NSF study, the Video Mosaic Collaborative (VMC), a partnership among Rutgers University Libraries, the Robert B. Davis Institute (RBDIL), and the University of Wisconsin – Madison. The partnership enables us to leverage research-based resources, build new digital tools, and use them for research on teacher education. The VMC project has dual goals of: (1) preservation and storage of the video collection from the RBDIL in the Rutgers Community Repository from which they can be accessed, and (2) examination of how the knowledge of content and pedagogy for teaching mathematics as a thoughtful subject can be enhanced through teachers' studying video of how children reason while involved in mathematical problem solving. RBDIL's partnership with the Rutgers Digital Library group is harnessing a cyber infrastructure to facilitate the use of the video collection to conduct innovative research in teacher education contexts.

### THE INTERVENTION MODELS

The intervention models, pre-service and in-service, that were used in the studies reported here make use of video clips on children's learning mathematics for teachers to examine carefully the reasoning used by the students to justify their ideas and solutions. Two models of interventions, one for pre-service teachers and the other for in-service teachers are described. The models differ in two respects: (1) the time span for the intervention – the pre-service model occurs during a three to four week period during a regular semester, while the in-service model spanned a school year from fall

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to spring, with three cycles of interventions, and (2) the opportunity for classroom implementation – the in-service model provided for classroom implementation and study; the pre-service model did not. The conditions for the two models came about for the practical reasons of schedule, time, and access to students.

### **Model 1: Pre-Service Intervention**

The pre-service intervention occurred within an undergraduate mathematics-education course for elementary school students, over a three to four week period. Pre-service teachers worked together on mathematical tasks that called for justification of their solutions. They shared and discussed their findings and considered the variety of arguments that naturally evolved. They then viewed video clips of children working on the same problem. Together, they discussed and analysed the children's thinking and reasoning. The tasks and videos used by the instructor made up the instructor work-flow over the intervention period. The instructor's choice of tasks and videos include a variety of forms of reasoning that are used by the children.

### **Model 2: In-Service Intervention**

There were also three cycles of intervention for the in-service teacher model. However, the interventions occurred over a school year and included classroom applications. After each cycle, the teachers participated in a modified-lesson study experience in which their own students were given the same problems they worked on and that were studied from the videos. Then, with the guidance of the instructor, they discussed together the written work of their own students' justifications of the problem. For more detail, see Maher, Landis and Palius (2010).

## **METHODOLOGY**

**Subjects:** The in-service teachers consisted of twenty-two middle and special education teachers from a school district in New Jersey, USA. The regional school district is diverse in the population of students it serves. All teachers participated in a five-month long, professional-development intervention.

Thirty-five experimental pre-service teachers and twelve comparison pre-service teachers were participants in the pre-service study. They were enrolled in each of three elementary methods sections taught by the same instructor in a private university in New Jersey, USA. The pre-service comparison group worked on the same tasks and discussed various solution strategies as the experimental group, but did not view and discuss video clips about children's learning on those tasks.

**Video Assessment:** The video pre and post-assessment consisted of a 20 minute video clip that served as the contextual basis for participants to articulate what they observed about the mathematical reasoning expressed by the children working on versions of a counting problem. The episode displayed several arguments, complete and incomplete, to include cases, contradiction and induction, and a generalized argument. Patterns used by the children to make conjectures for the problem solution

were also included in the clip. A transcript of the video was provided and an open-ended prompt was given to participants who were asked to identify children's reasoning. The video clip was piloted in earlier trials and revised, as experience suggested, to include clear examples of the children's various forms of reasoning.

**Coding:** Using a rubric that was developed and tested with pilot data, the data were scored by researchers with checks for reliability that varied between 86% and 92% agreement. In cases of disagreement, the score of the researcher with the higher reliability was used

**Analysis:** Data were analysed for pre and in-service experimental and comparison groups according to the teachers' identification of partial and complete forms of arguments. In examining and interpreting growth in a teacher's ability to recognize children's reasoning, it is important to understand the relationship between the statistics reported. Specifically, we define %Pre as the percentage of study participants who on the pre-assessment described the specific argument type referred to in the table. We define %Post as the percentage of participants who described this argument type either on the post assessment or, if not on the post assessment, referred to this argument type on the pre-assessment. Finally, we define %Transit as the percent of participants whose argument descriptions improved on the post assessment. This is calculated using only those participants whose pre-assessment indicated there was potential for growth on the post-assessment, that is, for those who did not receive the maximum score on the pre-assessment. It is useful to understand the following relationships between %Pre, %Post and %Transit. Specifically,

1.  $\%Post = \%Pre + (1 - \%Pre/100) \times \%Transit$
2.  $\%Growth = \%Post - \%Pre = (1 - \%Pre/100) \times \%Transit$

From relationships (1) and (2), we see that percent growth in the recognition of an argument type on the post-assessment is measured as the product of: (a) the fraction of participants who do not describe the argument type on the pre-test (namely,  $1 - \%Pre/100$ ) and (b) the percentage of these participants who describe the argument type on the post-assessment, but failed to describe this argument type on the pre-assessment. Thus, in comparing different experimental and comparison groups where %Pre are significantly different, %Transit is a better comparative statistic in contrast to %Growth since the magnitude of this latter statistic is proportional to %Pre.

## RESULTS

Growth in the number of student argument descriptions that were provided by the study participants was compared for each of four argument types: argument-by-cases, induction, contradiction and generalization. Separate ANOVA statistic tests of hypothesis were conducted for each of the three participating groups in the study: in-service experimental, pre-service experimental, and pre-service comparison. The results found no significant difference by argument type in the percentage of participants who demonstrated some growth in the description of student arguments. The respective ANOVA test statistics are F-Ratio = 1.39,  $df = 82$ ,  $p = 0.25$ ; F-Ratio =



0.12,  $df = 130$ ,  $p = 0.95$ ; F-Ratio = 0.80,  $df = 41$ ,  $p = 0.50$ . The percentage of in-service experimental participants who demonstrated growth, on average, was 59.7%. According to argument types, 75.0% demonstrated growth in the argument by cases, 63.6% for argument by induction, 54.5% for argument by contradiction, and 45.5% offered an argument that was a generalization.

The percentage of pre-service experimental participants who demonstrated growth, on the average, was 35.8%. According to argument type, 34.5% demonstrated growth for argument by cases, 40.0% for an argument by induction, 34.3% for an argument by contradiction, and 34.3% produced a generalized argument. The percentage of pre-service comparison participants who demonstrated growth, on the average, was 4.9%. According to argument type, 11.1% demonstrated growth for argument by cases, 8.3% for an argument by induction. None of the comparison participants produced either an argument by contradiction or a generalized argument. Table 1 summarizes the percentage of participants who demonstrated growth in the recognition of four forms of student arguments.

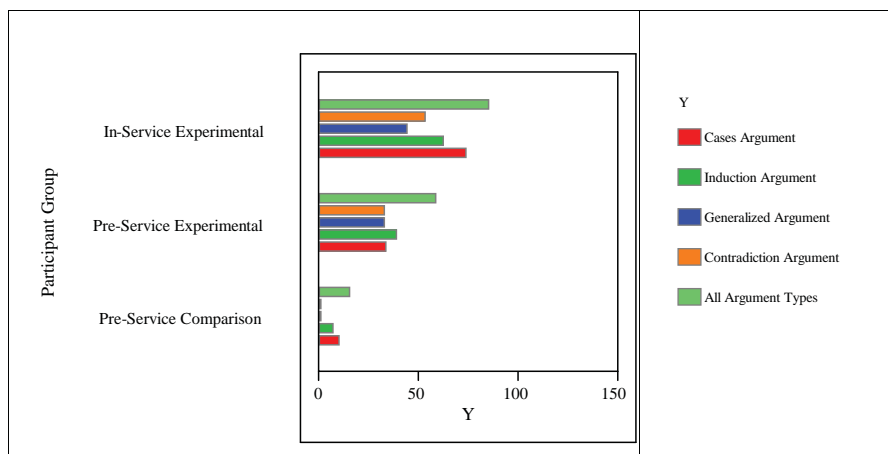


Table 1: Percent of participants who demonstrated growth on the video post-assessment

The results are generally consistent indicating that, on average, 60.0% of the in-service and 36% of the pre-service teachers in the experimental groups improved on the post-video assessment in recognizing individual forms of children's arguments including cases, induction contradiction and, generalization. In contrast, the comparison group of pre-service participants improved by 5.0%, on the average.

## CONCLUSIONS AND DISCUSSION

Analysis of data suggests that the instructional intervention of studying videos about children's reasoning produces changes in a positive direction in pre and in-service teachers' ability to identify forms of reasoning in children's problem solving. The

preliminary findings are encouraging, suggesting that it is indeed possible to provide interventions that support teachers' attention to children's thinking and reasoning. Further research on programs that help teachers learn to incorporate in their instruction emphasis on students' thinking and reasoning is recommended.

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# MATHEMATICAL THINKING WITHIN MATHEMATICAL MODELLING PROCESSES

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*Starting from various perspectives within the debate on teaching and learning of mathematical modelling, two strands of the debate on cognitive aspects within modelling processes are described. One strand of the debate analyses the individual routes of students while working through the modelling cycle. Another strand of the debate focuses, in contrast, on cognitive difficulties and mental barriers students face while going through the modelling cycle and emphasises the role of metacognition and metacognitive strategies in order to overcome these barriers.*

## VARIOUS PERSPECTIVES WITHIN THE TEACHING AND LEARNING OF MATHEMATICAL MODELLING

The relevance of including applications and mathematical modelling in mathematics education is widely accepted. For example, the PISA study emphasises developing the capacity of students to use mathematics in their present and future lives as a goal of mathematics education. This means that students should understand the relevance of mathematics in everyday life, in our environment, and for sciences. Despite the consensus of the high relevance of mathematical modelling in school and the transmission of its competencies, it is well-known, that mathematical modelling is extremely difficult for learners and that many cognitive obstacles as well as affective barriers need to be overcome, before students are effective modellers. These aspects are widely discussed (see Blum, Galbraith, Henn, & Niss, 2007) and have led, amongst other things, to different perspectives within the debate on applications and modelling in mathematics education. One proposal to distinguish different perspectives on the teaching and learning was published by Kaiser and Sriraman (2006) discriminating the different perspectives on mathematical modelling by the goals, which are pursued by them. They distinguish amongst others between realistic modelling focusing on pragmatic-utilitarian goals or educational modelling putting pedagogical or subject-related goals in the foreground. One so-called meta-perspective – called cognitive modelling – studies especially psychologically-oriented goals and pursues the following research aims:

- analysis of cognitive processes taking place during modelling processes and understanding of these cognitive processes;
- promotion of mathematical thinking processes by using models as mental images or even physical pictures or by emphasising modelling as mental process such as abstraction or generalisation.

## STUDIES ON COGNITIVE PROCESSES WITHIN MODELLING

The studies, which are taking place under this meta-perspective on cognitive processes within modelling (e.g., Borromeo Ferri, 2011; Stillman, 2011) emphasise that the nature of mathematical thinking processes and its underlying cognitive character taking place during the modelling process are especially characterised by the relation and interplay between mathematics and the real world. These aspects can be seen in different phases of the so-called mathematical modelling cycle, that is, the process, in which real world problems are tackled using mathematics. In the related debate of the last decades several types of mathematical modelling cycles have been discriminated. In principle they distinguish between mathematics and the real world or the rest of the world using a phrase by Pollak (1979). Figure 1 displays the steps, which comprise the modelling cycle and which are widely agreed upon with various nuances in other approaches:

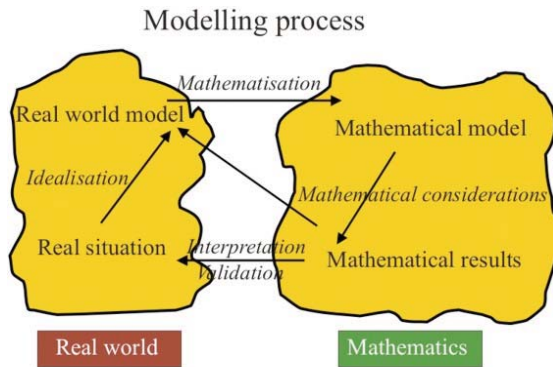


Figure 1: Modelling process (from Kaiser, 1995, p. 68 and Blum, 1996, p. 18)

Related to this notion of modelling cycles, two different strands of discussion can be distinguished: one strand of the debate analyses the individual routes of students while working through the modelling cycle in contrast to the other strand of the debate, which focuses on cognitive difficulties and mental barriers students face whilst going through the modelling cycle and emphasises the role of metacognition and metacognitive strategies in order to overcome these barriers.

The first strand of discussion is characterised by the so-called analysis of modelling routes, an approach introduced into the debate by Borromeo Ferri (2007). On the basis of cognitive-psychological work, amongst others by Sternberg or Skemp, she emphasises that students of each age do not work consecutively through the modelling cycle, but carry out special loops going backwards and forwards until they reach a satisfactory answer to the original problem. These individual modelling

processes are highly influenced by the individual modeller and differ strongly between various modellers. One special influencing factor is the preferred ways of the students to do mathematics, their so-called mathematical thinking styles. Borromeo Ferri describes in her studies how so-called analytical thinkers, who prefer to work with formulae and symbolic representations, usually proceed very fast to the mathematical model and prefer to work within the mathematical model going back to the real world model only when inevitable. Visual thinkers, who prefer to work with visual pictorial representations mostly imagine the given real world situation very vividly in pictures and images and follow the normative modelling cycle more often (Borromeo Ferri, 2010).

In order to carry out these analyses Borromeo Ferri enriches the already described modelling process through an additional differentiation, so-called mental representation of the situation, in which the students try to understand the problem and build up their own thinking about the modelling problem; extra-mathematical knowledge of the students plays an important role in this phase (see Figure 2). Borromeo Ferri (2010) refers to the work by Kintsch and Greeno (1985), which was continued by Reusser (1997), who introduced the notion of situation model. Blum and Leiss (2007) tie their studies with the work by Reusser (1997) and use the concept of situation model, which only differs in nuances and emphasises less strongly the role of the extra-mathematical knowledge. The high relevance of the students' prior knowledge for the understanding of the modelling process and especially for validation and checking of the results is emphasised in a study by Brown and Edwards (2011), especially academic and encyclopaedic knowledge seem to be of high relevance in contrast to more personal episodic knowledge.

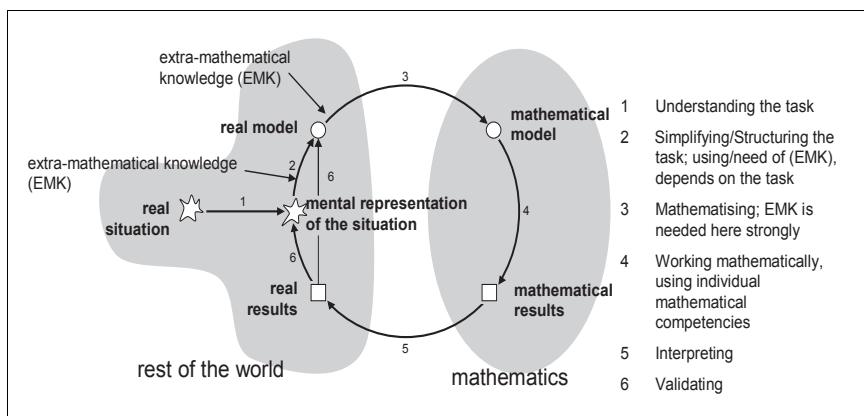


Figure 2: Modelling process under a cognitive perspective (Borromeo Ferri, 2010, p. 104)

The individual modelling routes of students are examined in other studies as well, for example, already at the end of the last century by Matos and Carreira (1995), who emphasised the links to representations within cognitive processes involved in modelling. These analyses are continued in recent work by Carreira and Baioa (2011), who emphasise the importance of practical experiments in formulating and testing hypotheses for a solution of the problem.

The second strand of the debate focuses – as already mentioned - on cognitive difficulties and mental barriers students might experience within the modelling process. Stillman (2011) in her comprehensive overview of this debate emphasises the high importance of metacognition (Maass, 2007) for mathematical modelling and analyses the role of metacognition for the transition between the phases or stages of the modelling process. Drawing on well-known approaches within this field, she defines metacognition as a complex construction of metacognitive knowledge, which comprises, amongst others, knowledge about metacognitive strategies, metacognitive experiences and metacognitive goals. Of special importance are metacognitive strategies, for example self-questioning, which are used in order to regulate and monitor cognitive processes. She analyses in detail so-called mental blockages preventing students to progress from one phase to another, referring to extensive empirical studies. Based on similar notions of modelling cycles as displayed above, Stillman (2011) points out that not all metacognitive activities are productive in helping students to overcome the barriers. However, productive metacognitive acts occur at three levels, that is, within the recognition that particular strategies are relevant, through the choice of the strategy, and its successful implementation. The first type of successful metacognitive activities draws on personal resources such as necessary knowledge and necessary competencies with respect to the real world situation. One major problem is the possibility of teachers to promote students through scaffolding while respecting their freedom within the modelling process. A powerful tool to support students is to foster them to use their metacognitive knowledge and their metacognitive strategies and point them to the lack of reflection and incorrect knowledge. This relates strongly to the proposal by Borromeo Ferri to use the already mentioned metacognitive modelling cycle as a strategic instrument to describe modelling from a meta-level and to analyse cognitive barriers, which need to be overcome for successful modelling.

Carreira (2011) points out in her summary of the most recent work in this area, that from an epistemological point of view these approaches

highlight the human nature of modelling and the conceptual and interpretative nature of mathematical models. ... All the above converge to the view that prior knowledge, experience and sense making of the task context are cornerstones in the modelling process and give mathematical modelling its subject-centred quality. (p. 160)

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# THE SOCIO-CULTURAL CONTEXT OF MATHEMATICAL THINKING

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## INTRODUCTION

Of all subjects that a child learns in school, mathematics is perhaps the one that is thought to be most “universal”, having characteristics which are independent of particular socio and cultural contexts. After all, one plus one is equal to two, no matter whether you are in a large urban classroom in central London or a small village school in Indonesia. But more and more, both from philosophical reflection and from cross cultural empirical studies, educators have begun to realize the dependent nature of mathematical thinking on the socio-cultural context (Lerman, 2000). Challenges to an absolutist conception of mathematics cast doubt to the universality of mathematics (Ernest, 1991). If mathematics is a product of fallible human beings rather than indubitable eternal truth, then clearly the socio and cultural setting have important bearing on mathematics and mathematical thinking. In addition, cross cultural anthropology studies have documented very different mathematical practices in different cultures (Saxe, 1996). It is therefore natural to assume that mathematical thinking, like other aspects of human thinking, is very much a product of culture.

Studies of the social context of mathematics education usually focus on the classroom (e.g., Wood, Williams, & McNeal, 2006), and those of the cultural level usually involved extinct cultures (such as the Incas) or aboriginal cultures (e.g., Saxe on Oksapmin of Papua New Guinea; Barton on Maui in New Zealand). But very few studies relate to a major dominant culture such as the Chinese culture. In this paper, the case of the Chinese culture will be discussed, but implications for mathematics teaching and educational policies will be drawn for other cultures as well.

## THE CHINESE CULTURE

One reason for discussing the Chinese culture is the superior performance of students from East Asian countries or systems under the influence of the Chinese culture in recent international studies in mathematics such as TIMSS and PISA. This superiority of East Asian students in mathematics is consistent with earlier studies which found that although East Asian students and American students were comparable in intelligence and in “biologically primary mathematical” domains (such as certain features of numeracy and simple arithmetic), East Asian students had a clear advantage in “biologically secondary mathematical” ones (such as algebra, calculus and problem solving) (Geary, 1996). Whereas biologically primary mathematical

abilities are pan-cultural forms of cognition, biologically secondary mathematical ones are culturally bound. Since the East Asian countries that did well in TIMSS and PISA shared a common culture (Leung, 2001), referred to as the Confucian Heritage Culture (CHC) by Biggs (1996), what characteristics of CHC may be used to account for the high achievement of East Asian students? Two aspects of CHC will be discussed here: relevant values in CHC (see Leung, 2001), and the Chinese language.

## **RELEVANT VALUES IN CHC**

### **Emphasis on the importance of education and expectation to achieve**

The Confucian culture is known to put a high emphasis on the importance of education (Sollenberger, 1968). Under this emphasis, students are expected to hold a serious attitude towards study (Garvey & Jackson, 1975), and this is reflected in a high expectation in the culture for students to achieve. This high expectation is manifested in the child-rearing behavior of the Chinese, and also in the classroom practices in East Asian countries (Leung, 2005).

### **The role of practice in learning**

Related to the high expectation to achieve is the stress on diligence and practice (Park & Leung, 2003). In CHC countries, there is a greater tendency than in other cultures to attribute success to effort rather than to innate ability (Leung, 2001). Not only is practice considered essential for success, but practice and repetition are also considered as a “route to understanding” (Hess & Azuma, 1991), and are considered to be legitimate (and probably effective) means for understanding and learning.

## **LANGUAGE AND MATHEMATICAL THINKING**

Another important feature of the Chinese culture is the Chinese language. Past studies have shown that the language one uses shapes one’s thinking (Vygotsky, 1986). This is consistent with Saussure’s idea of the signifier and the signified (Saussure, 1966), where in our case language is the signifier where the content of the mathematical thinking is signified. As Laborde (1990) observed,

The semantics of an expression are constructed by the student by means of his or her mental representations and of linguistic features of the expression. The role that natural language plays in these processes appears to be very strong. (p. 61)

There are features of the Chinese language that are thought to have effects on children’s early acquisition of numbers, their computation and conceptual processing (e.g., the regularity of its number-naming system (Miller & Paredes, 1996); the short duration of pronunciation of the monosyllabic number words (Hoosain, 1991)). Another important feature of the Chinese language is that its written form is logographic in nature. Chinese words are represented by a large number of different visual symbols known as characters which are made up of components (radicals), and have an imaginary square as a basic writing unit. Chinese characters put emphasis on the spatial layout of strokes, and the orthography of Chinese is based on the spatial

organization of the components of the characters. Kao, Leong and Gao (2002) pointed out that Chinese characters possess visual properties such as connectivity, closure, linearity and symmetry which are faster and easier to be captured by vision, and there is a close relationship between learning to write Chinese characters and the visual-spatial properties of Chinese character (Kao, 2000; Hoosain, 1991).

Lai (2008) investigated the impact of different features of the Chinese and English language on children's visual perceptual abilities. She found that, consistent with previous studies, Chinese children have higher visual perceptual and geometric skills. But the surprising finding is that Chinese children have higher visual-motor integration skills than motor-reduced visual perceptual skills.

Lai interpreted her findings by referring to Kao's idea of Geometricity, the unique visual spatial characteristics of Chinese characters due to their outward and implicit limits of frameworks, together with the geometric characteristics generated from the writer's body movement following the change in shape of the Chinese character that he/she is writing (Kao, 2000). Kao pointed out that Chinese writing is a dynamic integration of the mind and body of the writer, and the Chinese characters. Chinese character writing requires a wholehearted involvement of the writer so that one's body movement matches the character's geometric shape. Since the writer's perception, cognitive power and action are combined into one dynamic writing task, the visual spatial characteristics of Chinese characters will affect the cognitive activity in the writing process. The writer subconsciously initiates the training and refinement of the relevant visual-spatial perception capacity whenever he/she writes Chinese characters

Lai's findings highlight that Chinese writers' high visual perceptual performance originates from the motor activity during writing, and the control of motion which follows the visual spatial properties of Chinese characters may be an inducing facilitator for the refinement of one's visual perceptual abilities. When motor-reduced visual activity is carried out alone, the recognition and categorization of images cannot be carried out through stimulation of the brain due to a lack of motor coordination. This is in contrast to writing English, where the "response code" was some linguistically defined entities, for example, the letter corresponding to the sound 'h' (Van Galen & Teulings, 1991). Thus, visual skills are essential to writing Chinese words while phonological awareness is vital to writing English words. For this reason, it is not hard to appreciate the difference encountered in writing phonetic words and Chinese, and the outstanding performance of the Chinese participants in general visual perceptual abilities.

Lai's study provides strong evidence that language does affect one's skills in visual perception. In particular, the characteristics of the written language, as well as the action of writing, affect one's visual perceptual abilities. The features of Chinese characters and the motor activity of writing Chinese may have contributed to Chinese

children's strong visual-motor integrated skills. Visual perceptual abilities and visual motor integrated skills are of course related much to mathematical thinking.

## IMPLICATIONS

**Cultural Values:** Geary (1996) pointed out that “differences in secondary cognitive domains ... are more likely to reflect differences in cultural values” (p.165). The development of biologically secondary mathematical abilities requires effort, and “occurs only with sustained deliberate practice that is specifically designed to facilitate their acquisition” (p.150), and “the motivation to acquire secondary abilities is provided by the values and goals of the wider culture” (p.165). This has important implication for the debate in intrinsic versus extrinsic motivations in learning mathematics. Some educators stress the harmful effects of extrinsic motivation, but an acceptance of the social nature of learning points to the importance of extrinsic motivation in learning mathematics. Learning is not a process purely in the cognitive domain. Learning motivated intrinsically is of course desirable, but as argued by Geary above, for learning in biologically secondary domains, motivations provided by social and cultural expectations are necessary and hence legitimate. This implies that one important role of the teacher in helping students develop mathematical thinking is to encourage students to study hard, and not simply enjoy the mathematics. Motivating students to learn mathematics through introducing lively activities in the lesson has obvious merit, but the purpose of the activities should be to induce students' interest in the subject matter rather than in activities per se. Enjoyment of mathematics through hard work is more rewarding than simply enjoying the activities.

Another difference in cultural values concerns the relative valuation of achievement in mathematics (Stevenson & Stigler, 1992). As pointed out above, there is a high expectation for students to achieve in the Chinese culture. On the other hand, there have been moves in many Western countries to remove difficult content from the curriculum in order to make mathematics more accessible to students. But such repeated reduction of content difficulty is an endless retreat. On the contrary, an appropriate demand of mathematics rather than trivial mathematics will make it more challenging and hence interesting.

**Language:** As argued above, features of the language such as the structure of the number-naming system have important effects on how children learn and operate with numbers, but very often the teacher is not explicitly aware of the features of the language which she is working with and with which she is so familiar – too familiar perhaps, for “fish is the last to discover water”! To support the acquisition of numbers, efficient computation and concept processing, and to avoid misconceptions, the teacher, especially those teaching young children, need to be more reflective of the number-naming system used and the limitations associated with the system. In particular, since the Hindu-Arabic numerals are now used in all modern societies, the relationship and inconsistencies between the Arabic numerals and the structure of the

number-naming system of the culture concerned need to be explicitly identified and taken into account in the design of instructional strategies (e.g., through explicitly discussing the inconsistencies).

There is also a lesson to learn from the Geometricity of the Chinese writing. Visual and motor skills are acquired not necessarily through first perceiving the figures and then invoking eye-hand coordination in practicing. The higher visual-perceptual abilities of the Chinese tell us that the experience of the practice itself will aid the development of visual abilities. In a sense, this advantage of the Chinese language cannot be duplicated in other cultures (no serious researcher will recommend non-Chinese children to learn to write Chinese in order to improve their visual-perceptual abilities!). But perhaps the Chinese experience may hint that early exposures to drawing various shapes (a process similar to writing Chinese characters), instead of simply recognition of the shapes, may help develop visual-perceptual abilities.

**Policy Implications:** Given that mathematical thinking is socio-cultural dependent, the first task of the educator should be to understand the culture of the learner. We should take our cultural values as given, and design and improve our educational practice from this starting point. In learning from other countries, one should remember that complicated cultural factors might have affected classroom practices and student learning, and drastic changes should not be undertaken until such factors are thoroughly examined.

Simple transplant of policies and practices (e.g., importing textbooks) from other countries would not work. One cannot transplant the practice without regard to the cultural differences. Culture by definition evolves slowly and stably with the passage of long periods of time, and there is simply no quick transformation of culture. What is needed is to identify not only superficial differences in educational practice, but the intricate relationship between practice and the underlying culture. It is through studying these relationships that we understand the interaction between educational practices and culture. Through identifying commonality and differences of both educational practices and the underlying cultures, we may then determine how much can or cannot be borrowed from another culture.

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# RESEARCH FORUMS

RF1: Researching the Nature and Use of Tasks and Experiences  
for Effective Mathematics Teacher Education

Coordinators: Peter Sullivan and Orit Zaslavsky

RF2: Problem Posing in Mathematics Learning and Teaching:  
A Research Agenda

Coordinators: Florence Mihaela Singer, Nerida Ellerton,  
Jinfa Cai and Eddie Leung







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# **RESEARCH FORUM1**

*Researching the Nature and Use of Tasks and Experiences for Effective  
Mathematics Teacher Educations*

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# **RESEARCHING THE NATURE AND USE OF TASKS AND EXPERIENCES FOR EFFECTIVE MATHEMATICS TEACHER EDUCATION**

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Monash University

Orit Zaslavsky

New York University and the Technion – Israel Institute of Technology

This Research Forum aims to promote discussion on issues associated with researching the use of tasks and experiences that prompt learning by prospective and practising teachers.

There is now a widespread acceptance that one of the important components of effective teaching is choosing and using rich tasks with which students can engage. The same is true for teacher education, in that the tasks that are used with teachers are the prompt for productive activity. As Zaslavsky and Sullivan (in press) argue:

One of the goals of teacher education is to help prospective and practising teachers develop from novice possibly uncritical perspectives on teaching and learning to more knowledgeable, adaptable, judicious, insightful, resourceful, reflective and competent professionals ready to address the challenges of teaching .... These ambitious goals present great demands on teacher educators, who are responsible for facilitating learning opportunities for teachers to develop and become capable of working towards these goals. We take the stand that similarly to students, teacher learning occurs largely through engagement in effective tasks, along with reflection on the experience of working on the tasks.

The forum brings together researchers who have gathered data on the use of tasks in teacher education, to hear the results that can inform future teacher education practice, and to consider challenges experienced in designing, conducting and reporting such research.

Tasks, in this context, are the problems or challenges that are posed to prospective and practising teachers by teacher educators. Such teachers are expected to engage in these tasks actively, collaboratively, and intellectually with an open mind and an orientation to future practice. To facilitate discussion, the Research Forum categorises tasks in two ways: those that are similar to those used by classroom teachers (e.g., the analysis of a graphing problem); and those that are idiosyncratic to teacher education (e.g., critique of videotaped practice). These two categories are the basis of the sessions in the Forum. Of course these two categories are an unsuitable way of indicating the scope and potential of tasks. Zaslavsky and Sullivan (in press) provide a more nuanced set of categories of tasks for teacher education. The following is a brief summary of the categories of tasks that they use:

*Tasks that seek to develop and orientation to adaptability.* One of the goals of teacher education is the development in teachers of an orientation to being adaptable, to considering variations to questions, tasks and intended curriculum, to searching for alternatives to unsuccessful approaches, and to adapting existing resources to intended goals. Such adaptability is central to teachers making decisions about practice for themselves, an important pre-requisite for reflectivity.

*Tasks that foster awareness of similarities and differences.* Noticing similarities and differences, in the broad sense, is at the heart of learning and teaching (Mason, 1998). It is well known that the gradual process of associating concepts with categories is critical for learning. This has two aspects for teachers: one is recognising similarities and differences in mathematical concepts and representations; the other refers to similarities and differences in pedagogical situations.

*Tasks that develop awareness of conflicts, dilemmas and problem situations.* Teachers constantly face dilemmas and need to make decisions and choices under conflicting constraints, and deal with uncertainty and complexity (Sullivan & Mousley, 2001). Many of the situations that teachers face are “six of one, half a dozen of the other”, meaning that there is no one right answer, but teachers make best guesses. Awareness of this uncertainty and need for decision making is a key characteristic of learning from practice.

*Designing and solving tasks and problems for use in mathematics classroom.* Learning to attend to and enhance students' problem solving skills and strategies is a significant goal for prospective and practising mathematics teachers. Teacher education should aim at enhancing teachers' learning to incorporate in the classroom exploration of problems that have multiple solutions or solution-strategies, learning to analyse and evaluate students' problem solving strategies, learning to evaluate the degree of openness of textbook problems, and learning to use mathematical games as a problem solving context.

*Learning from the study of practice.* Teacher education is sometimes characterized by extremes. On one hand, in response to criticisms of the remoteness of the content of teacher education programs and the positive reports that graduate teachers give to their practicum experience, there are calls for more teacher education activities to take place in schools. On the other hand, an orientation to learning from practice requires much more than time spent in unreflective field based experiences, and school based programs in the absence of research-informed teacher educator perspectives. In other words, teachers who learn to study practice are able to continue ongoing learning in a sustainable way.

*Selecting and using tools and resources for teaching.* Selecting appropriate tools for mathematics teaching and using them effectively is a major challenge for teachers. Tools can be text books, additional readings, manipulatives, construction and measuring devices, transparencies, graphical calculators, and other technological environments. Making educated choices about what tools to use for certain purposes

and how to use them requires familiarity with a wide range of tools from both a learner's and a teacher's perspective. Being aware of the need for active choices on tolls and resources increase the chance of the choices being productive ones.

*Identifying and overcoming barriers to students' learning.* Education and schooling strive to redress the advantages of privilege, and create opportunities for all students, especially those who would not otherwise have those opportunities. There is a need to overcome some real, and in some cases substantial, barriers that would otherwise inhibit the realization of students' opportunities. Teachers need to be aware of possible causes of disadvantage, and of actions that they can take.

*Sharing and revealing self, peer, and student dispositions.* In the multidimensional endeavour of teaching and learning mathematics, and learning to teach mathematics, a key dimension is the disposition of the (prospective and practising) teacher as a learner, the teacher as a teacher, and the pupil as a learner. The dimension of disposition is itself multifaceted. It can include beliefs, self-regulatory behaviours, attitudes, and mathematical anxiety. Almost all aspects of teacher education have an attitudinal dimension that can be evaluated and addressed by teacher educators.

The first theme of the Research Forum focuses on tasks that can be described as the study of practice, including lesson study (e.g., Lewis, Perry, & Hurd, 2004), learning study, examination of recorded classroom exemplars (e.g., Clarke, & Hollingsworth, 2000), case methods (e.g., Merseth & Lacey, 1993), and reflection on school based practicum. These tasks emphasise aspects of practice and particularly data collection and reflection of goal directed practice. The following contributions from Ulla Runesson and her colleagues on learning study, João Pedro da Ponte on the study of exploratory tasks, and Daniel Chazan and his colleagues on using animated media for representing and studying teaching situations.

The second theme focuses on tasks for teacher education that use aspects of classroom tasks to prompt teacher learning. This can include challenges associated with the effective use of tasks such as those that prompt consideration of similarities and differences between representations and concepts (e.g., Zaslavsky, 2008), those that foster cognitive conflict (e.g., Tirosh & Graeber, 1990), those that foster an orientation to adapting classroom tasks to the level of readiness of the students (e.g., Cooney, 1994; Sullivan, Zevenbergen, & Mousley, 2006), those that use particular tools, along with consideration of particular challenges such as the use of tasks that are authentic and multidisciplinary (e.g., Peled, 2008). In other words, by studying the characteristics, opportunities for, and constraints on tasks that teachers might use in classrooms prospective and practicing teachers can learn about these critical aspects of practice. There are three contributions addressing this theme. Barbara Jaworski and her colleagues explore the way that tasks contribute to developing an orientation to inquiry, Maria Bartolini Bussi and colleagues use a particular tool, historical drawing machines, to focus attention to mathematics and the use of the

tool, and Malcolm Swan presents a typology of tasks that inform teaching and teacher education. These contributions are elaborated in the following.

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# A LEARNING STUDY OF CRITICAL FEATURES FOR LEARNING TO ADD AND SUBTRACT NEAGTIVE NUMBERS

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How could calculating with negative numbers be taught for understanding? What is necessary to be aware of in order to understand? What is critical for learning this? This was the task for a group of teachers who collaboratively used a systematic approach to investigate what it takes to learn to subtract negative numbers in grade seven and eight in a Swedish compulsory school. In this approach, student learning and their understanding of that which was taught was the main concern and in the fore of the teachers' attention. In an iterative process of planning, observing and revising a single lesson, they in depth analysed and inquired the progress of student learning before, in and after the lesson. From an analysis of students' written tests and from a close observation of video recorded lessons the teachers learned about what was critical for students' learning.

## INTRODUCTION

Ball et al. (2008) argue that "teaching involves making features of particular content visible to and learnable by students" (p 400). In order to teach, for instance how to calculate ' $-5 - (-3) =$ ', there are certain critical features that must be discerned by the learner. The awareness of what those aspects may be, we would advocate, is necessary for a teacher to take into consideration when planning for teaching and learning. However, what are those features? How might they be found? *One way* to develop knowledge about this could be made by a systematic inquiry into the teaching - learning process, preferably in a collaborative process among a group of teachers and teacher educators.

## LEARNING STUDY-A SYSTEMATIC ENQUIRY APPROACH INTO STUDENTS' LEARNING AND UNDERSTANDING

In the study presented here, a particular arrangement, Learning study (Runesson, 2008), has been used for inquiring how to enhance students' capability to calculate with negative numbers with understanding. In a Learning study, together with a researcher, a group of teachers investigate the nature of learning a particular concept or skill and how to promote learning this. The nature of the capability the students are supposed to develop *and* how they conceptualise this is in focus; hence Learning

study is both content and learner oriented. The ultimate aim of Learning study is to enhance students' learning. However, in order to provide the best learning opportunities and to make learning possible, the teachers themselves must learn about the nature of the capability they want the students to develop. For instance, if the aim is that students should learn how to calculate ' $-5-(-3)=$ ', and similar tasks with negative numbers, the teachers deeply investigate what it implies to be able to do this operation, what the learning difficulties for the students may be, and what features of this that the students must be aware of. So, Learning study is about learning on two levels; the teachers' and the students' learning.

Learning study is an iterative process of planning, observing and revising the lesson. This is similar to Lesson study (Lewis, 2002; Yoshida, 1999) but is guided by some principles from variation theory (Marton et al., 2004). Learning is seen as a process of differentiation, thus to be able to discern similarities and differences. However, what is critical for learning, to calculate ' $-5-(-3)=$ ', must include the learner and what s/he brings into the learning situation in terms of previous experiences and how s/he understands what is being learned. The way the learner perceives, understands or experiences that which is learned is due to what extent the critical aspects are discerned by the learner. A student's failure or lack of understanding can be understood in the light of un-discerned aspects. For instance, if the learner does not differentiate the double meaning of the 'minus' sign in the operation above, this feature of subtracting negative numbers are not discerned. So, the discernment of critical aspects is essential for learning. From this theoretical point of departure, in Learning study, the teachers try to find out what the critical aspects are and how they should be brought out in the learning situation in a way that makes discernment possible.

## **A LEARNING STUDY ABOUT NEGATIVE NUMBERS**

The Learning study reported here was included in a project combining school development and research. The aim of the study was to enhance students' learning how to add and subtract negative numbers. The four participating teachers were all experienced and well educated mathematics teachers and took part on a voluntary basis in the project. Three of the teachers taught grade 7, the other one grade 8 (13-14 years old). The teachers decided about the object of learning, thus what capability they wanted the students to develop, and they planned and revised the lessons mainly on their own decisions. The Learning study lasted about one semester. It started with three meetings (all audio recorded) when the object of learning was decided about, previous teaching experiences were discussed and a pre-test was designed. From the learning outcomes of the pre-test, the first lesson in the cycle was jointly designed. One of the teachers conducted the first lesson (which was video recorded), and a post-test was given. In a post-lesson meeting the results on the post-test was analysed together with the video recorded lesson. The learning outcomes were not satisfactory for certain tasks, so they decided to revise the lesson plan to the second lesson in the cycle. This lesson was taught by a new teacher (and to new students!) and the



students took a pre-test after the lesson. The same procedure (analysis of post-test and the lesson, revision of the lesson plan) was taken after the second and the third lesson. Each class had only one lesson in the Learning study cycle. It should be noted that the students had little or no experience of negative numbers before the intervention lesson. In Sweden usually this topic is taught in grade 8 as a teaching unit of about two weeks. This paper draws on data from the audio-recorded meetings with the teachers and the video recording from four lessons.

## **FINDINGS FROM THE STUDY**

Our interpretation is that the process of analysing lessons collaboratively with the aim to improve students' learning in regard to a specific object of learning, what was made possible to learn and what was actually learned was, a fruitful and rewarding experience for the teachers. They deeply investigated how the learners understand and solve tasks like  $5 - (-2) =$  or  $-3 - (-4) =$  for instance, and what it takes to learn this. They explored and identified what was critical for the students' learning. You could say that the teachers learned about the students' learning and this learning made them able to refine and develop the lesson plan in terms of how to handle the content. Throughout the process the students' learning, what they learned, what particular combinations of addition and subtraction with negative numbers they failed or succeeded with, was the main concern. In that sense, the assessment of students' learning outcomes was qualitative and formative, thus used for refining the lesson. Students' failure was never discussed in terms of attributes or shortcomings among the learners which, in our experience, is common, but rather to deficiencies in teaching. The teachers were sensitive to students' learning. However, this does not just refer to how they interacted with their students and whether they cared for them or not. Sensitivity implied to learn more about why students may have learning difficulties, how they perceive and conceptualise that which is learned and how that was related to what was made possible to learn in the classroom, thus their teaching.

The teachers had a true ambition to help the learners to understand, not just to rely on 'tricks' and rules that were meaningless to them; unfortunately a common way to teach negative numbers in Sweden. Going so deeply into how to teach and learn a topic was not just a new experience for them, it was also a challenge. Usually they rely on the text book. Here they were confronted with their own ways of teaching and what effect had had on students' learning. They had to consider their knowledge of the subject matter, their students' understandings and learning as well as of their own teaching skills. The sensitivity to the students' understanding before, in and after the lesson gave information about the way of handling the content and how that might provide possibilities for learning.

However, we noted that the teachers' ambition to facilitate learning, at some occasions, had the opposite effect. For instance, they planned deliberately to avoid negative difference and bringing out the double meaning of subtraction. This seemed to have been counter-productive for providing learning possibilities. The teachers

successively came to realising that there must be certain conditions met in the lesson in order for their students to learn to add and subtract negative numbers. Certain aspects were critical for these students' possibility to learn. Such critical aspects identified by the teachers were: *the 'minus-sign' as operational sign for subtraction compared to the 'minus' sign for a negative number, different semantic meaning of 'minus'* (e.g., subtraction as 'take away' or 'difference'), that  *$a-b$  does not equal  $b-a$* , thus the law of commutability is not valid in subtraction, and understanding *the order of negative and positive numbers* (e.g.  $-3 > -18$ ) (Kullberg, 2010). We have doubts about whether the teachers would have come to these conclusions on their own and without this systematic and cyclic approach of investigating teaching and learning.

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# **PRACTICE-BASED TEACHER EDUCATION FOCUSING ON MATHEMATICAL DISCUSSIONS**

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This paper presents a practice-based teacher education task and reflects on using with a group of experienced teachers. I begin sketching a rationale for using such tasks, then I present the context of this activity and conclude referring the value of this approach for in-service and pre-service teacher education.

## **EXPLORATORY TASKS IN THE MATHEMATICS CLASSROOM**

Classrooms in which the students do some extended work on tasks that require them to model situations and frame questions and later they present their solutions to the whole class and discuss other solutions are becoming more common in our country and perhaps elsewhere. The students usually work in pairs or in small groups. Sometimes they are asked to write a report with their strategies and solutions, in other cases they present them orally during the discussion. This requires suitable tasks to propose to students – exploratory, inquiry, or investigative tasks – that lead them to do substantial work and from which they can learn new mathematics. It also requires that teachers support students' work and conduct productive discussions during which mathematics ideas are raised, clarified, and formalized. This vision of the mathematics classroom fits with what many documents such as NCTM (2000) refer to as “reform mathematics education”.

Worthwhile mathematics tasks are critical in mathematics teaching (NCTM, 1991). However, tasks appropriate for a class may not be appropriate for another. Thus, the teacher needs to know what tasks are suitable for his/her students, in terms of structure (Ponte, 2005), degree of mathematical challenge (Potari & Jaworski, 2002), context – mathematical or non-mathematical –, and time required to complete it that may range from a few minutes to some days, weeks, months or even more (Ponte, 2005). The key role of open and challenging tasks in mathematics teaching has been recognized by mathematics educators such as Sullivan, Bourke and Scott (1997) and Skovsmose (2001) and has a significant expression in some countries (Boaler, 1998).

However, powerful mathematical tasks do not teach just by themselves. The role of the teacher in the classroom, presenting them, supporting students, and leading discussions is critical (Stein & Smith, 1998). In Portugal, a new mathematics curriculum for basic education – grades 1 to 9 – encourages teachers to propose such exploratory tasks in mathematics classrooms. It suggests that rich exploratory tasks and whole class discussions are important elements in the students' learning experiences but it leaves to the teacher to decide about the appropriate balance of classroom working modes. Such exploratory tasks are very demanding on teachers: Their selection involves a high level of understanding of the mathematics involved as well as in-depth knowledge about students' abilities and interests. In supporting

students, teachers have to restrain themselves of saying too much, at the risk of taking away the need for students' thinking. Discussions, on the other hand, require that teachers orchestrate the classroom discourse, providing opportunities for all students to intervene, stimulating moments of controversy and argumentation as well as moments of systematization and formalization of mathematical ideas.

## **TEACHER EDUCATION TO TRANSFORM CLASSROOM PRACTICE**

Research on teachers' beliefs, conceptions, and knowledge regarding mathematics and mathematics teaching stand on the implicit assumption that if these can be changed, then teachers' classroom practice may also change. However, it is becoming clear that if the goal is to have a real impact on teachers' classroom practice, then classroom practice needs to play a key role in teacher education (Ball & Cohen, 1999; Smith, 2001). This leads to the consideration of practice-based teacher education. One way of achieving this is to regard teacher education as situated in practice. That means that the materials that represent the teaching activity and their results (for example, mathematical tasks, records of students' work, classroom episodes) are used as opportunities for critique and investigation. Teachers then develop knowledge analyzing real situations that they may use later in their actual teaching practice. Teachers work with material drawn from actual classrooms that may be more or less familiar to them. This is a good choice for a small teacher education activity, when there is not much time for teachers to collect data from their classes, but that provides room to work on issues closely related to classroom events.

## **THE TEACHER EDUCATION TASK AND CONTEXT**

The example that I present here is a task for teachers based on the analysis of a classroom episode from a mathematics task for students (see below). This student task was designed and proposed to a grade 8 class by a mathematics teacher, Idália Pesquita. I found that the work that went on in the classroom is very interesting and could be used as a basis for a teacher education task with the following structure:

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Part 1. Solve the task presented to the pupils and consider:

- a) How is the task related to curriculum objectives for mathematics teaching?
- b) How this task may be used in the classroom? How to organize students? What time should they be given to solve it? And for a final discussion?
- c) What difficulties may the students feel in doing it?

Part 2. Observe the video with students discussing this task as well as the transcript.

- a) Identify and analyse then roles assumed by the teacher.
- b) Identify and analyse the interventions of the teacher.
- c) What important decisions the teacher assumes during this segment?
- d) Identify and analyse the roles assumed by students.

Part 3. Final reflection

- a) Discuss if what you saw in this episode is in line with your initial expectations.
- b) Indicate the aspects that you find important that the teacher may have into account in order that this kind of task is successful in class?

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Tasks such as this have a high potential in pre-service and in-service teacher education. In fact, the way the teachers participated in the final discussion showed

that they felt learning a lot from it. Doing this task proved to be quite successful in a number of respects, as it was apparent from the high involvement of the teachers during its realization, the frequent number of cases it was referred to in later moments, and the interest that it promoted in the participants to look at classroom situations as teacher education activities. It is more difficult to know in what measure it led these teachers to become more aware of particular issues, for example on algebraic thinking or in leading classroom discussions, but my perception is that at least it was helpful in increasing their interest for these issues.

Some conditions that seemed important for the success of this teacher education task include: (i) its clear relation to a curriculum topic (algebra) and to specific learning objectives (solving problems involving patterns), (ii) the fact that it included detailed elements about the classroom activity on the mathematical task, (iii) the teachers' perception of ecological validity in terms of the usual teaching conditions (the time available for the students to carry out the task, the number of students in the class, students' characteristics, etc.), and (iv) and the fact that the issues raised in this task resonate with broader curriculum orientations and existing literature on the topic.

The participating teachers were strongly impressed by seeing and analysing an actual mathematics classroom episode. This is very unusual in mathematics teacher education in our country. The technological apparatus (showing video excerpts of the classroom on a data projector connected to a computer, using sound columns, etc.) was intriguing. But the most important was the fact that the teachers could relate to the actual situation, had the time to discuss it in small groups with a few colleagues and finally had the opportunity to discuss it in the whole group.

This activity was carried out during 3.5 hours as planned. Some participating teachers found the time too short to do everything was asked. Some of them did not conclude working on the mathematics task and thinking about how to use it in the classroom (Part 1), some others said they needed more time to reflect on the episodes and the review the transcripts (beginning of Part 3). However, the way the task was structured, and the review that was made at the beginning of Part 2 and Part 3 helped to maintain all the teachers "on board" and enabled to spend the necessary time on Part 3, the crucial part of the activity.

## CONCLUSION

The format of this task was useful to support teachers in reflecting about issues related to exploratory algebraic tasks and classroom discussions. The Portuguese curriculum document was much referred to because it is just being introduced in schools. However, some relation to curriculum documents may assure that the task is related to significant curriculum objectives and mathematics concepts and processes.

Similar activities may be also of much value with beginning teachers and prospective mathematics teachers. Looking at actual mathematics teaching situations, especially at situations that may provide useful models for successful mathematics teaching, may help them to realize that these are not just abstract models or utopian theories

impossible to put into to practice in the classroom. However, with prospective mathematics teachers perhaps some more structure or some reading assignments could be useful to help them to deal with the complexity of the classroom situations.

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# **RICH MEDIA SUPPORTS FOR PRACTICING TEACHING: INTRODUCING ALTERNATIVES INTO A “METHODS” COURSE**

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*Our contribution to the Research Forum will articulate how rich media representations of classroom interaction (like animated cartoons or comic strips) can be resources for highlighting tactical and strategic decision making in teaching for prospective teachers. We illustrate this claim by outlining challenges around the question of calling on students to share their work publicly, particularly when students have done something that is out of the ordinary, and how those challenges can be found in the animated story “The Great Divide.” We then illustrate how a teacher educator might use this resource with prospective teachers.*

## **RICH MEDIA AS RESOURCES IN TEACHER EDUCATION**

Not only do prospective teachers need to learn what is ordinarily done in teaching, as well as what might be desirable to do instead and why, but they also need to learn how and when to carry out these actions. In Bourdieu’s (1998) words, prospective teachers need to develop “a feel for the game” (p. 25), or a sense of the rationality of the practice (see Herbst & Chazan, 2003), rather than just be the recipients of exhortations about what they should, and should not (e.g., Chazan & Ball, 1999), do.

While rich media representations of classroom interaction, like animations, do not have the authenticity of videotaped events that have actually occurred, as a result of this unreality, they have other affordances (Chazan & Herbst, 2011). Within the unreal space of an animated classroom, stories can branch, the teacher can respond differently to student ideas and each of these alternatives have the same level of reality, or unreality. Prospective teachers can thus observe the “same” teacher carry out the same lesson in different ways; and the ways in which particular teacher moves might play out differently can be the focus of conversation. This perspective builds on appreciation of the relevance of time to teaching and on research that suggests teachers’ craft knowledge is held in stories (e.g., Connelly & Clandinin, 2006).

In teacher education, similar notions have been played out around written or video cases. We suggest that rich media representations of classroom interaction are a fascinating resource because teacher educators can exercise a high degree of control over these representations and create alternative stories that highlight key tactical or strategic challenges of teaching. Once the notion that there are alternatives in teaching has been established, teacher educators can then also involve prospective teachers in creating alternatives, either as rich media artifacts (Herbst et al., 2011), or in the context of microteaching.

## CHALLENGES IN CALLING ON STUDENTS WHEN TEACHING WITH A PROBLEM

Lampert (2001) articulates that a key role of the teacher in teaching while leading whole group discussion around a problem is: “calling on students to say something that will contribute to the common experience of the class and then constructing responses to what they say” (p. 143). When students have worked in an exploratory manner and have developed different ideas, deciding on whom to call can have important tactical or strategic ramifications. A particular student’s ideas may lead the class in a direction that the teacher sees as fruitful, while a different student’s ideas might not. A particular student’s idea may be at the heart of what the teacher intends to teach, another’s may not. A particular student’s ideas may be easily understood and digested by a class, another’s may require much work in order to be understood by a class. As teachers face such decisions, one important skill is to collect relevant information in order to be able to contemplate the ramifications of different choices. For example, expert teachers use the time when students are working in an exploratory mode to come to understand how different students are thinking and to understand the dimensions of the strategic and/or tactical decision-making they will face as they manage student participation in whole group discussion.

These sorts of challenges are available for inspection around the animation, “The Great Divide,” created by the Thought Experiments in Mathematics Teaching (ThEMaT) project (A snippet from this animation can be found at <http://www.youtube.com/watch?v=lyTNP3IXwqk>). For discussion of this animation by teachers, see Chazan & Herbst, 2011; for considerations about the design of the animations see Herbst & Chazan, 2006). In this animated lesson, the teacher opens class by reminding students that they had worked on solving of equations like  $20x+5=65$  and asking them to solve the equation  $20x+5=5x+65$  that has  $x$ ’s on both sides (crossing the didactical cut articulated by Filloy & Rojano, 1989). Students work on the problem in small groups and then the teacher asks for solutions to the problem. A student, let us call him Red, is called on and goes to the board to share. As his first step, Red writes:  $4x+1=x+13$ . A different student, Blue, then indicates confusion about what Red did, why one would do that, and why Red didn’t solve the problem in a more usual way.

From that moment the story branches into two alternatives; in one of them the teacher decides to emphasize that, while mathematically correct, Red has departed from the typical way of solving equations and changes the equation to solve, proposing instead  $8x+3=5x+63$ . In the other alternative, the teacher acknowledges that what Red did is not the usual solution method, but has the class pursue the question of whether (and, if so, in what sense) Red did anything wrong by dividing by a common factor.

## THE ROLE OF AN ANIMATION IN A MICROTEACHING DESIGN

The animation described above has been used to create opportunities for prospective teachers to practice responding to students’ ideas while leading whole group



discussion. Together with the prospective teachers, the instructor had developed a rubric for assessing teacher moves in light of their potential contribution to student learning. He now sought to have prospective teachers practice responding to student ideas, in the manner of microteaching (Allen & Eve, 1968) and used “The Great Divide” first as a repository of possible student ideas and later as a repository of possible (though not necessarily exemplary) teacher moves.

The activity began with an explanation of what the class in the animation had done before. Then two prospective teachers, who would act as teachers in the ensuing interaction, were sent out of the class and the remainder of the class was divided into small groups that would represent the students in the animation. Each small group was given a synopsis of the mathematical reasoning done by that group in the animation and was tasked, if called upon by their peers who were acting as teachers, to respond with this sort of mathematical reasoning. The two prospective teachers who were acting as teachers (one in the role of the mathematics teacher and the other in the role of a special educator colleague), then came into the room and led a discussion of the problem. In doing so, they had to respond to the variety of student ideas, including Red’s idea. Subsequently, the instructor shared the animation with one of the responses of the animated teacher to Red’s work (He saved the alternative that posed a different problem for another occasion). The class then used their rubric to assess the moves of their peers, as well as those of the animated teacher. While those alternative moves could have been considered by stating them in language, as we describe them in this paper, the instructor’s decision to represent them temporally and multimodally, through microteaching and an animation, facilitated the teacher candidates’ telepresence (or immersion; Steuer, 1993) in the act of teaching.

## CONCLUDING THOUGHTS

To this point, we have emphasized the ways in which representation of classroom interaction with an animation allows for the construction of alternative teacher moves that play out in the same interactional space. The notion of alternatives that is supported in this way, then helps prospective teachers view teaching as an activity in which teachers make choices in light of their strategic or tactical goals and learn to judge teaching moves on such a basis. An additional affordance that animations have for such work in teacher education is that critiques of the work of the teacher in the animation, unlike critiques of the work of one’s peers or mentor, are not critiques of a real person that the prospective teacher knows. Our conjecture is that this dimension of unreality allows animations to overcome challenges found by facilitators of conversations about teaching when the teacher depicted in the videotape is present in the room (See Le Fevre, 2004; Sherin, 2004; van Es & Sherin, 2008).

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# USE OF TASKS IN DEVELOPING AN INQUIRY COMMUNITY FOR BETTER MATHEMATICS LEARNING AND TEACHING

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*Inquiry-based tasks were an important tool for developing an inquiry community between teachers and didacticians in the LCM Project in Norway. Teachers in eight schools from lower primary to upper secondary worked with a team of didacticians from a university. Mathematical tasks were experienced first in workshops at the university and then designed or re-designed for use with students in schools. Through the interactive process of design, use and redesign, project participants became more aware of the nature and value of tasks and of possibilities for their use in schools.*

## USE OF TASKS IN THE LCM PROJECT

***How tall a mirror must you buy if you want to be able to see your full vertical image?***

The above question (adapted from Shultz, Shultz & Brown, 2003) was presented to a group of teachers and didacticians at a workshop in the LCM Project (Learning Communities in Mathematics) in Norway, by two didacticians, doctoral students, Espen and Stig . Participants, in plenary, were given small mirrors and multilink cubes with which to experiment in discussion with their neighbours. After working on the problem for 15 minutes, Espen and Stig led a discussion on the activity which had taken place. Another didactician gave a short presentation in Cabri-geometre to illustrate angles and directions of light in forming an image in a mirror. Stig presented his thinking about the task which included ideas for introducing a task to students at differing grade levels, some of which are summarised in Figure 1 below. He had discussed all of this with members of the didactician team first by email interchange and later in a meeting of didacticians.

A decision early in the project was to use specially designed mathematical tasks as a basis for workshop activity, an aim of which was to explore the processes through which pupils, teachers and didacticians learn. Tasks were designed to contribute to the learning of students, teachers and didacticians in

- building community between teachers and didacticians through engaging in mathematics together;
- providing a basis for raising didactical and pedagogical issues related to learning mathematics and working with pupils in classrooms;
- enabling discussion on specific areas of mathematics;
- providing examples from which teachers could design their own tasks for the classroom.

Grade 1.	One student faces a mirror holding a stick (against her stomach). This student directs another, who, using a whiteboard marker, marks the mirror image the first one sees. Compare the original stick with the marks on the mirror. Try different distances from the mirror.
Grade 5.	Cut a figure from paper. Measure it and find the perimeter. Find perimeter of mirror image. Compare.
Grade 8.	Hold a cube and go close to the mirror. Draw the lines of the cube on the mirror. What do you see?
Grade 12.	How tall a mirror must you buy if you want to be able to see your full vertical image? Justify your conclusion. Try with objects with different distances from mirror. Describe ratios in the model. Use the cosine rule to derive the height of the actual figure when the height of the mirror image is known.

Figure 1: Extract from planning for the mirror task over a range of grade levels.

Tasks planned for workshops were not intended (by didacticians) as recommendations for classroom activity: project design had suggested that teachers would design their own tasks for classrooms, related to their own curriculum, with didactician support. However, teachers, from perspectives of a school community of practice and experience of teacher education events in other contexts, expected didacticians to suggest ideas for the classroom. What happened in practice was that many teachers took the workshops tasks and modified them for the classroom. This proved often to be a significant catalyst for learning.

## THEORETICAL IDEAS WITHIN THE PROJECT

Schools and university were seen as established *communities of practice* in which participants *engage* with activity, use *imagination* in interpreting their own activity and *align* with norms and expectations of the practice (Wenger, 1998). The project sought to build an *inquiry community* in which inquiry-based mathematics tasks were to be used in workshops and classrooms, teachers would engage in inquiry in exploring innovations in teaching, and didacticians and teachers together would research the processes involved in developing classroom teaching. Rogoff, Matusov and White (1996, p. 388) speak of a *learning community* in which “learning involves transformation of participation in collaborative endeavour”. The idea of *inquiry community* makes the nature of transformation more explicit: didacticians and teachers (and ultimately students) engage together *in inquiry activity* through which new ways of seeing and doing become evident and learning occurs. Thus, rather than aligning tacitly with the practices of the community of practice, members of an inquiry community *question* their participation. They seek to know more about the *hows* and *whys* of participation, create dialogue with peers to recognize and address issues in practice, and open up possibilities for changing or developing aspects of practice. We call this process *critical alignment* (Jaworski, 2006). Critical alignment was required of both teachers and didacticians in the project.

## PRACTICAL INTERPRETATION OF USE OF TASKS IN THE PROJECT

In the early days of the project, teachers and didacticians, each group aligning with their own communities of practice, conceptualised activity differently from each other. Throughout three years of fieldwork in the project, tasks were designed (mainly by didacticians), used in workshops (didacticians and teachers together) and adapted for the classroom (mainly by teachers). The nature of the tasks evolved during the three phases. At the beginning, tasks were chosen to be readily accessible to teachers of varying mathematical experience, yet with opportunity for extension to offer serious mathematical challenge for all. They were designed to demonstrate inquiry practice in learning mathematics through inquiry, not linked directly to particular areas of the curriculum. An expectation of didacticians was that teachers, becoming aware of the nature of inquiry-based tasks, would design tasks according to their own curriculum areas and students' needs. In practice, teachers focused on the *substance* of the workshop tasks, and adapted these tasks for their own classrooms. Thus students were offered adaptations of the workshop tasks, rather than new focused tasks designed by their teachers. Increasingly, particularly at the higher grade levels, teachers requested that tasks be more clearly related to a given mathematical topic such as algebra, geometry or probability, and interpreted at a level related to the age or grade of students with whom teachers worked. Through this process of use we all learned what was possible in and of our respective communities and our awareness grew of each others' contributions and needs. In a focus group interview at the end of Phase 2, one teacher, reflecting on her early experience, expressed this as,

I thought very much that you should come and tell us how we should run the mathematics teaching. This was how I thought, you are the great teachers ... now I see that my view has gradually changed because I see that you are participants in this as much as we are even though it is you that organise. Nevertheless I experience that you are participating and are just as interested as we are to solve the tasks on our level and find possibilities, find tasks that may be appropriate for the students, and that I think is very nice. So I have changed my view during this time (Daland, 2007, p. 168).

Three factors emerged as being highly significant for task design related to developmental progress: i.e., the development of knowledge and practice of teachers and didacticians through their engagement in the project. The first was that teachers were eager for tasks of an inquiry or investigative nature that they could use with students, and readily used workshop tasks in a variety of ways in classrooms. At the higher grades, they especially wanted tasks more directly related to their curriculum. Secondly, teachers' design of their own tasks in the school setting proved problematic. Teachers had problems in finding time and opportunity to meet in school during the school day. Lack of awareness as to what such tasks and their design might involve contributed to an inertia at school level. As a result of this recognition, and at teachers' request, workshop time was given to teachers' design of tasks. Teachers from different schools, in same-grade groups, designed activity for the classroom which individuals would then take further in their own contexts.

Thirdly, as teachers became confident of using new forms of activity in classrooms, they invited didacticians to video-record their work with students. Such video material became an important tool for dissemination in the project. Seeing tasks used by teachers in the realities of classrooms led to developing awareness of teachers and didacticians of students' engagement with mathematics through inquiry-based tasks and associated issues, didactical, pedagogical and systemic (For further details, see also Bjuland & Jaworski, 2009; Daland, 2007).

It became obvious to all participants that both groups brought knowledge central to the project and both learned from the project. Didacticians learned about systemic matters related to school, how these impinged on what teachers saw to be possible and how teachers thought about their design of activity for students. Teachers learned what it meant to take an inquiry approach both in the classroom and in planning for the classroom and ways in which their students responded. They gained insight into inquiry-based tasks, their nature and use. After three years of fieldwork a new project, TBM (Teaching Better Mathematics) commenced, with some schools from LCM and some schools new to inquiry-based work. Teachers from the LCM school have acted as mentors for teachers from the new schools. Here we see the growth of awareness of inquiry and its practical interpretation permeating school practice.

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## TASKS FOR TEACHERS IN THE MMLAB-ER PROJECT

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*In this contribution the authors report about tasks in a 2-year project (MMLAB-ER, Laboratories of Mathematical Machines for Emilia-Romagna, <http://www.mmlab.unimore.it/on-line/Home/ProgettoRegionaleEmiliaRomagna.html>) aiming at the construction of a) a network of mathematical laboratories in five provinces of the region; b) a network of well prepared in service teachers, able to implement in their own classrooms inquiry-based mathematics education.*

### INTRODUCTION

In the project five mathematical laboratories were constructed at teachers' centres after the model of the Laboratory of Mathematical Machines in Modena and equipped with about 80 models of working instruments; teacher training was designed and realized after the theoretical framework of semiotic mediation (Bartolini Bussi & Mariotti, 2008). Parallel teacher training programs were developed at the laboratories, interlaced with teaching activities in the classrooms. The model was sustainable (7 meeting in total for 28 hours) and based on several types of tasks (for details, see Martignone, 2010, 2011). In this contribution, we shall focus on two types of tasks: (A) a set of tasks for teachers concerning the exploration of a mathematical machine and the relationships with teaching activities; (B) a professional task concerning reflection on training.

### EXPLORATION OF A MATHEMATICAL MACHINE: FOUR KEY TASKS.

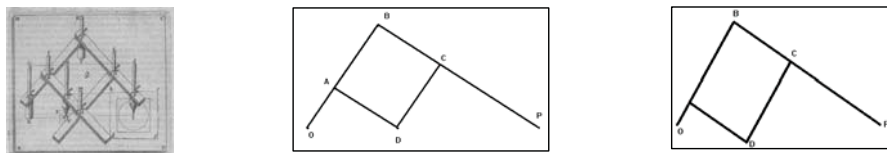


Figure 1: the Scheiner pantograph. From left to right:

A) the original drawing (1631); B) scheme of the machine; C) scheme of a not working machine.

A mathematical machine for geometry purposes is a tool that forces a point to move or to be transformed according to a given law (Maschietto & Bartolini Bussi, in press). A well known case is the pair of compasses, where the lead point is forced to move on a circle arc by the tool structure. However, in the history of mathematics, several mathematical machines have been designed and used with different purposes. For instance, a particular linkage, the Scheiner pantograph (see the fig. 1A), was introduced in the XVII century to enlarge/reduce a figure, hence paving the way towards a mechanical realization of plane similarities (drag the green point in the



java simulation at <http://www.museo.unimo.it/theatrum/macchine/simj/ml16.htm>). When the teachers are challenged with a new tool, the *teachers-as-students voice* comes out. This experience is the root of a professional reflection about the analysis of their own processes. Hence the *teachers-as-professionals voice* comes out. Moreover, teachers' beliefs and cultural perspectives emerge (Boero & Guala, 2008).

All the activities carried on with teachers in the training program of MMLAB-ER are guided by four aims 1) exploring the machine itself, 2) constructing utilization schemes and 3) rediscover mathematical meanings embodied in the machine while 4) practicing typical formats of mathematical thinking (conjecturing, arguing, proving). It is easy to acknowledge in this scheme the debts owed to Rabardel (1995) and Vygotsky as elaborated by Bartolini Bussi and Mariotti (2008). The above aims are realized by means of four standard tasks:

1. *How is the machine made?* This task foster teachers to pause over the physical structure of the machine, to detect parts and spatial relations: “*there is a point hinged to the plane*”; “*splints are forming a parallelogram*”; “*linking the tracing points to the fixed point similar triangles are shown*”, and so on. The attention is focused on the structure and diverted from the analysis of functioning.

2. *What does the machine make?* Teachers begin to construct the utilization schemes (Rabardel, 1995) of the machine: where to put leads, how to draw corresponding figures, which figures are more useful and so on. The exploration foster the production of conjectures suggested by the physical structure of the machine and by the relationships between the drawn figures: “*the points P and D are aligned with the fixed point O*”; “*there is this fixed point and similar figures, hence it might produce homothety*”; “*splints are forming a parallelogram, hence there are parallel lines with transversal lines*”; “*if I move 1 cm with P or D the other moves half or twice*”.

3. *Why does it make it?* Teachers are expected to argue the produced conjectures, up to the construction of a proof that the transformation is a homothety. Arguments usually draw on the mathematical properties embodied in the physical structure: “*the ratio between the distances of P and D from the fixed point is always the same, as they are bases of similar triangles*”. The teachers usually construct different proofs depending on the exploration and the ways of producing conjectures.

4. *What could happen if ...?* This final task fosters the solution of open-ended problems and connects problem posing and problem solving (Watson & Sullivan, 2008). Teachers are asked to explore the variations of the parts of the machine and the consequent variation of the parameters defining the homothety: *Is it possible to modify the machine in order to get a ratio of 1:3? What could happen exchanging the fixed point O with the point D?*

The analysis of teachers' logbooks about teaching activities has shown that the structure of the four key tasks has been transposed also in their own classrooms, with adaptation and integration according to the school level and the teaching aims. For instance, in a vocational school (grade 10), the teacher has organized the activity on



the Scheiner pantograph, transforming the third task above into the following task: “*Why this machine does not work?*” (see fig. 1C). The machine has still some of the features of the pantograph, but the alignment of the three points ODP fails. In this way the teacher aimed at fostering students’ description of some necessary conditions to have a plane homothety, i.e. the alignment of P and D with the fixed point O (SCK, Specialized Content Knowledge, Ball et al. 2008).

## REFLECTING ON TRAINING: A PROFESSIONAL TASK.

In the second year of the project, an e-learning platform represented a surplus value for the training and experimentation phases. One goal of that implementation was to provide a tool to accompany and support teachers and to foster the development of a collaborative work by means of the wiki tool. In particular, teachers (split into groups) were asked to *write a report for each meeting* (each group was in charge of only one report), according to the following requests: a presentation of the topic where teachers revised what they got from their viewpoints (called *situated analysis*); a reflection on what and how they made in the meeting, as well as on processes activated during the activities (analysis of shared and distributed knowledge).

The analysis of the six reports shows that the collaborative writing was realised in various ways by teacher groups. These reports give space to *teacher-as-student’s voice*, to *teacher-as-professional’s voice* and to *trainers’ voice*, that fostered teachers to do a knowledge analysis. Different importance was given by each group to the two kinds of requests: the presentation of the meeting was often prevalent over didactical reflections. In their situated analysis, the teachers present considerations about their exploration/manipulation referring to the distinction (Rabardel, 1995) between “*objects as artefacts and as instruments*” (IV meeting). Some elements on training methodology are present, for instance “*We worked to find solutions comparing each other, splitting into groups, we fostered communication into our group, we reflected on various didactical implications and further developments, starting from our geometrical constructions*” (I meeting). In their reports, elements from the analysis of processes (meta-mathematical and cognitive elements) emerge, for instance: “*We highlighted different strategies for solution, but, above all, different mathematical knowledge latent in our geometrical construction, pointing out a difficulty to communicate those strategies*” (II meeting); “*In our opinion, this experience can make us think on aesthetic value of a proof besides the formal one, but also on our pattern of thought; for each of us is easier to follow a kind of argumentation rather than another one and to consider the first one more effective*” (IV meeting). The teachers revise objectives: “*We are students and teachers at the same time, and these activities encourage us to do a meta-reflection, toward a deep reflection that support teacher to be conscious of what he knows and what he want to do*” (II meeting).

## CONCLUSION

The long term project MMLAB-ER has allowed our team to design, test and analyze a complex model of in-service teacher training, where different levels of processes

have been in the foreground and gave us elements to deepen and challenge the present literature on tasks in mathematics teacher education.

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# DESIGNING TASKS FOR TEACHER EDUCATION

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*Over the past 20 years, we have been involved in designing and providing in-service professional development (PD) courses for mathematics teachers at various stages of their careers. More recently, this has led to a recognition of the need to develop multimedia 'stand alone' resources that are designed to promote teacher-student discussion and reflection (DfES, 2005; Swan, 2008). Within these materials we provide 'genres' of tasks designed to illustrate a wide range of mathematical processes and pedagogic principles. As teachers use and reflect on these genres and design their own instantiations of them, they report changes in their views of mathematics, teaching and learning. This is also supported in many cases by classroom observation.*

## INTRODUCTION

Teacher development does not come about through persuasive argument alone, but rather through opportunities for individual teachers 'to doubt, reflect and reconstruct' their own classroom experiences in unhurried, 'safe' environments (Wilson & Cooney, 2002, p. 132). Teachers learn by taking risks, adopting new practices and reflecting on their experiences (Fullan, 1991, p. 91). In our own work, this approach is usually conducted in four stages (Swan, 2011). First, teachers are invited to reflect on the contexts in which they work, their existing values and beliefs about mathematics, teaching and learning and to describe their current classroom practices. Second, teachers analyse contrasting practices by working on classroom tasks, then watching their use on video. These provide 'tension', 'challenge' or 'conflict' when contrasted with teachers' existing practices. Third, teachers are encouraged to implement novel classroom activities using carefully designed classroom resources. Fourth, teachers meet together to share their classroom experiences, discuss the pedagogical implications and reflect on the growth of new practices and beliefs. This four-stage process is repeated, cyclically, with new pedagogical challenges presented on each occasion. Later in the process teachers are invited to create their own tasks and even provide PD for colleagues within their institutions.

This model has been used repeatedly with practising teachers on from 4 to 6 day courses spread over one year (DfES, 2005; NRDC, 2006; Swain & Swan, 2007; Swan, 2007; Swan & Green, 2002; Swan, 2005). Each time, teachers, students and independent classroom observers have reported substantial changes to beliefs about

mathematics, teaching and learning, classroom practices, and even to students' attitudes and attainments (Swan, 2006a; Swan, 2006b).

## **TASK GENRES**

Rather than offering teachers examples of isolated tasks, we design tasks that exemplify different *genres*. Each genre is related to a different combination of *learning purposes* and *pedagogical principles*. There are perhaps five distinct purposes for learning mathematics: fluency when recalling facts and performing procedures; conceptual development; strategies for investigation and problem solving; awareness of the nature and values of the educational system and appreciation of the power of mathematics in society. Although multiple purposes may be addressed in a single task, there are potential incompatibilities (for example, when a divergent 'investigation' is intended to converge on a culturally valued 'result'). Different pedagogic principles may also apply. A concept development task requires different resources and interventions to a problem solving task. While in both we may offer examples of incomplete or incorrect sample solutions for discussion, in the former these are designed to exemplify 'misconceptions' and the discussion is focused on *meaning*, in the latter they exemplify *alternative methods* and *heuristics*.

The learning theories embodied in our designs are essentially social constructivist in origin. Concepts are co-created as language and symbols are appropriated and internalised in social situations. Tasks are designed to: elicit and build on existing knowledge; focus directly on significant conceptual obstacles; create tensions and conflicts that may be resolved through discussion. They are also designed to be 'rich' in that they are accessible, extendable, and encourage decision-making, creativity and higher order questioning.

The *form* of the tasks is also regarded as critically important; they are designed to be shared and the products arising from the tasks are produced and presented collaboratively, often using posters. This produces a different learning culture to one constituted by textbooks; these tend to inhibit discussion and encourage individualism.

Table 1 exhibits examples of task genres appropriate for concept development in relation to the mental processes we seek to harness and foster. Each of these we have illustrated in mathematical topics appropriate for all ages. Room does not permit me here to include examples, but these may be found in, for example, (DfES, 2005; Swan, 2008).

## **TEACHER DEVELOPMENT**

Teachers respond to these tasks in a variety of ways. In general, our teacher questionnaires reveal that many teachers see themselves as having away from transmission beliefs and practices towards more collaborative, 'connectionist' beliefs and approaches (Swan, 2006a).

<b>Task genres</b>	<b>Description of tasks</b>
Classifying and defining	Students devise classifications for mathematical objects, and/or apply classifications devised by others. They discriminate, recognise properties and develop mathematical language and definitions.
Interpreting and translating between multiple representations	Students match cards that show different representations of mathematical objects - words, diagrams, algebraic symbols, tables, graphs. They share interpretations, compare and group the cards in ways that made connections between underlying concepts. The discussion of common 'misconceptions' is encouraged by the inclusion of distracters.
Testing and evaluating mathematical statements and conjectures	Students are given short mathematical statements or generalisations, are asked to make posters that describe their domain of validity and provide examples, counterexamples and explanations to support their decisions.
Creating and solving variants of mathematical problems	Students devise new or variants of existing problems, prepare solutions then challenge other students to solve them. They offer support when the solver becomes stuck. This promotes awareness of the structures underlying problems, and focuses attention on the doing and undoing processes in mathematics.
Analysing reasoning and solutions	Students compare different methods for doing a problem, organise solutions and/ or diagnose the causes of errors in solutions. They begin to recognise that there are alternative pathways through a problem, and develop their own chains of reasoning.

Table 1: Sample task genres for the development of mathematical concepts.

Student questionnaires reveal a less dramatic picture, but they do show a greater appreciation of discursive approaches when teachers persist in the use of our tasks. Classroom observation reveals that some teachers 'mutate' our original design intentions to fit their pre-existing belief structures. They anticipate, for example, that students will have difficulties, progress will be slow and that their planned schedule for "covering" the material will be delayed. They therefore simplify tasks and prematurely resolve discussions for students. Others, in contrast, find that the tasks give them opportunities to act out previously-held beliefs that have been compromised by the prevailing performative culture. A further group, many of whom have never before encouraged mathematical discussions, are sufficiently resourced to be able to suspend their doubts and incorporate new ways of working. As they persist with these tasks, they consistently report considerable "surprise" in their students' increased enjoyment, involvement and learning.

Recently, we have developed extensive sets of resources based on these tasks, and make them freely available to schools and colleges in England (and are beginning this in the US). Government inspections of schools have recognised their value:

These materials encouraged teachers to be more reflective and offered strategies to encourage students to think more independently. They encouraged discussion and active learning in AS, A level and GCSE lessons.... (Ofsted, 2006, paras 32, 33)

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## **RESEARCH FORUM 2**

*Problem Posing in Mathematics Learning and Teaching: A Research Agenda*

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# PROBLEM POSING IN MATHEMATICS LEARNING AND TEACHING: A RESEARCH AGENDA

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## ABOUT THIS FORUM

*The goal of this Research Forum is to present various critical perspectives on problem posing, and to engage participants in helping to provide theoretical and practical structure to this important developing field by creating an international research community.*

*In our contemporary dynamic society, exposed to unpredictable changes, problem posing becomes an educational topic with high potential for better ways to train students for a changing world. This topic has become the focus of attention of teachers from around the world and is seen as one way to address the burden of curriculum. Problem solving has a long history as part of the school mathematics curriculum and as an instructional goal. By contrast, problem posing has typically raised far less interest (Ellerton, 1986). Yet, in recent years, problem posing has received more attention in the mathematics education community – both as a means of instruction (to engage students in learning important concepts and skills and to enhance their problem-solving competence) and as an object of instruction (to develop students' proficiency in posing mathematics problems) (e.g., English, 1997a, 1997b). The question remains, however, of identifying the defining characteristics of problem posing, and of examining how the various sub-categories of problem posing might be related to each other and to problem solving in theory and in practice (Stoyanova & Ellerton, 1996).*

## OPENING QUESTIONS

Without posed problems, there are no problems to solve. Two fairly obvious questions arise. First, *who* poses mathematics problems? The equally obvious answer is that we *all* do, and the types of problems posed depend on the *contexts* in which we find ourselves, and on our *purposes* for posing the problems. The second question is *for whom* are mathematics questions posed? In this Research Forum, our focus will be to seek discussion about, and formulate structure from, more specific responses to these questions as they relate to mathematics classrooms.

Many of the problems we pose every day may have a mathematical basis, but are not directly associated with a mathematics curriculum. Such problem posing has become automated and internalised to such an extent that we scarcely think of what many of us do as problem posing. At the same time, our research writings suggest that many of us consider formal problem posing that may take place in a mathematics classroom as set apart from its seemingly more natural counterpart, problem posing in everyday life. But does the posing of mathematics problems in a mathematics classroom – whether posed by the teacher or by the student – involve fundamentally different processes from the daily posing of mathematical problems in which we all engage?

The reality is that, in everyday life, we are immersed in a network that involves extensive personal problem posing and problem solving, some of which problems are mathematical. In a mathematics classroom, we tend to be solving problems posed by someone remote from the classroom. Neither teachers nor students are routinely immersed in posing mathematics problems. Thus, both teachers and students frequently find themselves in unfamiliar territory so far as problem posing is concerned (Ellerton & Clarkson, 1996; Ellerton, 2004).

Just as there was a cry some years ago for problem solving to become an integral part of mathematics classrooms, we believe that this research forum on problem posing is needed to stimulate further discussion and research about ways in which problem posing can become a more natural and integral part of mathematics classrooms at all levels.

## **SUMMARY OF RESEARCH DIRECTIONS FOR PROBLEM POSING**

Real-world problems are not clearly defined in their initial state and they do not necessarily call for unique answers or solutions. Starting with the observation that in a traditional classroom, the teacher presents problems and students develop solutions, Brown and Walter (2005) argued that teachers should move away from this rigid format and shift their students' focus away from the so called "right ways" and "right answers". In Japan, for example, an "open approach to teaching" or teaching with "open-end or open-ended problems" was developed for designing mathematics lessons using problem posing at elementary (Hashimoto, 1997), lower secondary (Hashimoto, 1997; Sakai, 1983; Mizoguchi, 1989; Nohda, 1993), and high school levels (Imaoka, 2001; Kanno, Shimomura, & Imaoka, 2007; Saito, 1986). A less discussed topic is problem posing using computers, especially at the university level (Leung, & Poon, 2004; Shimomura, Imaoka, & Mukaidani, 2002).

Advocates for problem posing typically argue that experience with mathematical problem posing can promote students' engagement in authentic mathematical activity; allow them to encounter many problems, methods and solutions, and promote students' creativity – a disposition to look for new problems, alternate methods, and novel solutions. When problem posing has been systematically incorporated into students' mathematics instruction, even something as simple as

having students generate story problems, the reported results have generally been positive, including a positive effect on students' problem-solving achievement and/or their attitudes toward mathematics (e.g., Hashimoto, 1987; Healy, 1993; Whitin, 2004).

Within an open-ended approach, students analyze problems and problem-solving methods through a process of solving a problem in one way and then discussing and evaluating a variety of solution methods that have been developed and presented by classmates (Cifarelli, & Cai, 2005; Nohda, 1986, 1995). By looking at the initial conditions and modifying some parameters, students might be put in situations in which they develop new problems and analyze their validity (UCSMP, 2005).

A systematic training focused on problem transposition using various representations, problem extension by adding new operations or conditions, comparison of various problems in order to assess similarities and differences, or analysis of incomplete or redundant problems can raise students' awareness of meaningful problems (Singer 2001, 2004, 2009). Effective teaching should focus on representational change, within a variety of activities, which specifically address students' motor, visual, and verbal skills, as well as transfers in between them (Singer 2007a, 2008, 2010).

In problem-posing contexts, students are stimulated to make observations, experiment through varying some data and analyzing the results, and devise their own new problems that could be solved by equally using similar or different patterns. The processes by which students continue given series or patterns provide information about the cognitive approaches they use in problem solving (Singer & Voica, 2008).

Empirical research has emphasized the cognitive aspects involved in problem posing in relation to students' mathematical performance on tasks involving computation, simple problem solving, and complex problem solving (Cai, 1995), as well as to teachers' representations in mathematics instruction (Cai, 2005). Multicultural studies have shown different approaches in problem posing in classrooms in China and America (Cai & Lester, 2005), Japan (Hashimoto & Sawada, 1984), and Finland (Pehkonen, 1995). Other cross-country studies revealed teachers' perceptions on problem posing and creativity related issues (e.g. Subotnik, Singer, & Leikin, 2010). Beyond the differences, both teachers and students need to learn specific roles for the success of classroom's complex interactions.

The educational potential of integrating problem posing, along with problem solving in classrooms at various levels has been highlighted by many educators and researchers (e.g. Cai, Mamona & Weber, 2005; Silver & Cai, 1994; Singer & Moscovici, 2008). However, many aspects are still contentious and need further investigation. These are some of the issues we will address during the Research Forum.

## FRAMING THE RESEARCH FORUM

The research forum on problem posing is a continuation of the Working Sessions organized on this topic at PME 34. The two sessions focused the sustainable interest of many researchers who actively contributed to the discussions, presentations and conclusions. Moreover, this was the beginning of a collaboration that has led to the development of a Special Issue of *Educational Studies in Mathematics*, titled *Problem Posing in Mathematics Learning: Establishing a Framework for Research*, to appear in 2012.

Through the Research Forum proposed for PME 35, we intend to enlarge the research community interested on the topic of problem posing in order to give more consistency to various positions within a field that is in the process of structuring. Our explorations into this burgeoning field of mathematics education research show that it is the time for more systematic analyses for finding ways to organize both the field and its applications into the practice of teaching.

The coordinators will be helped in this enterprise by the following contributors to the Research Forum: Cinzia Bonotto, University of Padova, Italy; Mitsunori Imaoka, Hiroshima University, Japan; Barbara Georgiadou-Kabouridis, University of Thessaly, Greece; Alena Hošpesová, University of South Bohemia, Czech Republic; Boris Koichu, Technion–Institute of Technology, Israel; Ildiko Pelczer, Ecole polytechnique, Canada; Cristian Voica, University of Bucharest, Romania.

### The Structure of the Research Forum Sessions

To frame the discussion within this developing field, the coordinators will explore empirical research studies addressing specific aspects of problem posing. Short presentations by some of the coordinators will be designed to be provocative and discussion triggers rather than reporting research per se. Thus, the Forum will address questions like: How do problem posing and problem solving interact in the course of on-going mathematical exploration? How could students' learning be improved through engaging them in mathematical exploration and problem posing in open-ended problem situations?

Practical issues for using mathematical exploration and problem posing as pedagogical strategies to improve students' learning in the classroom will be examined. The connections between exploring open-ended problems and learning will be discussed, as well as the conceptual benefits/ limits/ difficulties of having students engaged in mathematical exploration.

We will also examine teachers' own decisions to focus on problem posing in order to address concerns about mathematical meaningfulness and effectiveness in their teaching practice. This will create avenues for exploring both students' mathematical knowledge and their teachers' professional development.

The coordinators will stimulate and facilitate audience interactions within the two Research Forum Sessions. To make the interactions more effective, problem-posing scenarios will be described/ acted out by the coordinators, the contributors and others as needed.

More specifically, the key categories we envisage within the first session are the following: Who is posing the problem? The teacher? (if so, what is the role of the students?) The students? (If so, what is the role of the teacher?) Do the roles change during problem posing? What criteria should be used to decide if problem posing has been involved? Does problem solving necessarily involve problem posing? What are the benefits, if any, of applying problem posing in a classroom/ in teacher training sessions? What evidence is that engaging students in problem-posing activities improves their ability to pose and solve other problems? What strategies could be used to develop problem-posing approaches in a classroom/ in teacher training sessions? While frameworks for describing key categories of problem posing will be summarized by the coordinators, we will also ask for ideas from the audience on various aspects.

An important practical aspect refers to the nature of the tasks. What types of tasks can be regarded as problem posing tasks? What sub-categories of problem posing have research and practice identified? Starting from a distributed problem-posing scenario, the participants will have the opportunity to get involved by reacting to prompts like: Is the problem posed in the given example trivial (and for whom)? Is the problem complete? What instructions about problem posing (for this scenario) had been given? Was spontaneity of problem posing involved, or had the structure been imposed on the teacher and/or the students?

Summary sheets will be distributed at the beginning, so that individuals can keep their own notes. We will ask participants to summarize their interpretations of each scenario on another sheet that will be collected at the end of the first session. The data from these summaries will allow us to collate the different interpretations of each scenario and to present this in the second session.

To get things rolling, after small group discussions across the auditorium, some groups will share brief comments with the whole audience. The aim of this segment would be to draw attention to divergent yet legitimate views of problem posing. This will open the way to another question: How can international collaborative research support the development of a network of researches on problem posing?

For the second session, new distributed scenarios with feedback forms will aim to involve the audience as much as possible in genuine attempts to move the field forward. However, rather than being a summary of recent research, this part will provide food for thought for the main conceptual directions of a future book. The last part of the Research Forum will be devoted to launching invitations for contributing chapters to a proposed book on *Problem Posing*.

## **IS PROBLEM POSING A SOLUTION FOR EFFECTIVE LEARNING IN MATHEMATICS? A COGNITIVE PERSPECTIVE**

Problem posing is a natural capacity of the human mind, essential for surviving. For example, pre-historical humans had to be aware of the need for shelter and the need for food before trying to build a house or starting to hunt. In other words they needed to pose the problem before trying to solve it. From pre-historical times until today, the needs of individuals have diversified, and our capacity for posing new and varied problems has also developed. Is this human capacity for problem posing really valued in school? However, today our basic survival needs are satisfied within the social-economical context, and from that perspective it could be argued that there is no need for special training to build an individual's problem-posing capacities. On the other hand, could problem-posing activities be organized in such a way that they could help an individual develop both creativity and effective learning?

### **Problem posing: A natural predisposition?**

Recent studies in cognitive science show that children come spontaneously to create new, previously unknown entities, starting from entities already known. They grasp context, ideas, connections; they are able to perceive an entity or its essence instantaneously, without proceeding discursively in space or time (i.e. by passing from one piece of information to another). They show a kind of readiness to start. It seems that the human mind is endowed with an innate dimension of the motivation to learn (Singer, 2009).

This implicit motivation to learn manifests itself through abilities that anticipate further development. Thus, for example, children of three or four years old begin to conserve simple number and continuous quantity transformations well before the stage of concrete operations emerges at six or seven years of age. Learning mechanisms that underlie such increases of representational power in the course of conceptual change have been called “bootstrapping processes” and have been sketched by many historians and philosophers of science, as well as cognitive scientists (e.g., Carey, 1999, 2001; Kuhn, 1977; Margolis, 1993). Bootstrapping makes use of various modeling techniques, such as creating analogies between different domains, limiting case analyses, or thought experiments. It also makes use of the human symbolic capacity to represent relationships among interrelated concepts directly while only partially interpreting each concept in terms of antecedently understood concepts (Gardner, 1983, 1993, 1999; Smith, Solomon & Carey, 2005; Singer, 2007b, 2010). Teaching for representational change (Singer, 2007a) facilitates transfer by avoiding rigid connections.

Within a society that hardly finds ways of openness, young teachers tend to reproduce habits of mind borrowed from their own schooling experience. Frequently, when they perform a teaching activity, they use an information-centered approach, although they claim that they dislike this pattern. Recent literature emphasizes possibilities of influencing students' habits of mind by the use of recurring teaching

and learning cycles within a constructivist approach (Singer & Moscovici, 2008), or by multi-representational training (Singer & Sarivan, 2009).

Studies explain the differences in students' successes and failures in problem-posing approaches in relation to the level of understanding the solution of a problem and the novelty of the posed problems. Singer, Pelczer, and Voica (2011) observed that the more the student advances in the abstract dimension of the problem and its context, the more mathematically relevant are the newly obtained versions. They found that the level of abstraction of the solution process determined the novelty of the newly posed problems and was a good predictor of the child's creative potential.

### **Report of an experimental program**

Have problem posing activities the potential to use the scaffolding tendencies of the human intellect so that learning becomes more effective? More effective in this case refers to the deep understanding of mathematical concepts. An experiment that systematically used problem-posing activities may provide at least a partial answer.

A 4-year experimental program tracked cohorts of children from grades 1 to 4 (6-7 to 10-11 years old). The program started with two classes of children from a school in Bucharest (Romania), which were taught according to an experimental methodology; in year 2, two new classes joined the program; in year 3, five classes from the Republic of Moldova also got involved into the experiment. Finally, 232 children in 9 experimental classes participated in this research.

The teachers received detailed descriptions of the tasks that they were to offer to students and the teaching periods were followed by discussions, once a week or so. The description of the learning activities is contained in four teacher's guides (Singer & Radu, 1994-1997). The main steps of the problem-posing sessions for first grade focused on the learning of addition and are briefly presented below.

During their first grade, children were systematically exposed to the following "12 steps pattern" (Singer, 2002; 2004):

1. The starting point was "drama." Simple actions were dramatized in the classroom, such as:
  - a. *Dynamic animate*. For instance: 3 girls are drawing on the blackboard; 2 boys come and start to draw. (Children act the whole scene.) The teacher asks: "How many children are drawing now?" The students in the classroom count: 1, 2, 3, 4, 5. Then, they propose other contexts for acting.
  - b. *Dynamic inanimate*. These activities take the shape of role-playing with objects, for example:

Dan: "I have two pencils."

Ann: "I give you four more. How many pencils have you got now?" (The students play the actions.)

2. *Pretend*. This prepares the background for thinking imaginatively. (The students mimic contexts in which they put objects together and count.)

The tasks pattern is then developed through gradual variation of representations given by suggestive images the children use during their activities:

3. *Explicit active dynamic union* (“How many in total?” based on drawings representing living beings that are moving in a visible way).
4. *Explicit passive dynamic union* (“How many in total?” based on drawings representing inanimate objects that are moved by others in a visible way).
5. *Explicit static union* (“How many in total?” based on drawings representing objects that are linked together in a visible way – like marbles in a necklace).
6. *Implicit static union* (“How many in total?” based on conventional representations used to visualize union, as Venn-Euler diagrams).
7. *Active dynamic union in which one of the terms is abstract* (It is similar to 3 but to solve this task category, the child must begin counting not from 1, but from one of the terms).
8. *Passive dynamic union in which one of the terms is abstract* (This is a more abstract version of 4, as well as 7 is for 3).
9. *Iconic representation* (Additions processed on the number line)
10. *Horizontal symbolic writing of the numbers involved in addition*
11. *Vertical symbolic writing of the numbers involved in addition*
12. *Mental computation with no support*.

The exposure to these categories of tasks supposes that on the one hand the teacher is to propose a variety of such problems, and on the other hand that the students are stimulated to pose their own problems in the above categories.

## Ways of training

A variety of ways for training the problem-posing capacities of first graders have been used, among which:

*Problem transposition*. The teacher or a child proposes a simple problem of addition (of the type  $a+b=x$ ,  $x$  is the unknown). The problem is reformulated keeping the same numbers (for example, by changing the position of numbers in the question). The problem is extended so that it contains two or three operations of addition, or subtraction or a combination of addition and subtraction.

*Word-problems production*. An important part of the training in this category has focused on devising new problems in various ways. This was done with starting support-points in the beginning and later, without. For example:



*With starting point in exercises.* The teacher chooses a simple exercise (for example  $5+3=?$ ) and asks for the development of similar exercises. The same exercise is transformed into a problem. The initial exercise is compared with another one (number of terms, operations) in order to determine the similarities and differences. New proposals advance exercises in which the number of terms and number of operations are increased. For each exercise, students are challenged to pose a variety of word problems that can be generated keeping the same data, but changing the context or the topic of the problem. Variations in the way the task is performed are also requested, from doing it orally, or in written form, with verbalization and without (In the last case, the child only writes or draw the problem, without any comment and the teacher checks what has been written).

*With starting point in isolated numbers.* Somebody proposes the number 4, for example, and asks students to create addition problems that involve 4 and whose answer is no more than 20. Then the teacher or somebody else asks that the same number 4 become the distance between the numbers of a sequence placed between 20 and 40, or between 60 and 80, etc. Another game might require the construction of a number in which the number 4 is a compulsory component, or requires that it is not a component. Problems are posed starting with number 4. (Start with number 4 and add its double: what is the number you get? etc.; 4 chickens... etc.; Use 4 coins to compose a given amount of money etc.). The same procedure is carried out starting from other numbers and practicing serial arrangements, comparisons, estimations, compositions and decompositions of numbers. These are practiced orally (verbal), mentally, in writing (without or with minimum verbalizing, and the result is required for checking).

*With starting point in groups of numbers.* There are requirements of the following type: "Think to 20 and 30. Write a sequence of numbers that could be constructed between them, following a certain rule. Change the rule. Compare and notice similarities and differences between these sequences." These are also to be practiced orally, mentally, and in written form, trying to reorganize information through the given perspective.

*With starting point in a word problem.* A simple problem of addition ( $a+b=x$ ) is proposed. The problem is reformulated keeping the same numbers (changing the position of the question, for example). The problem is extended so that it contains two, three or four operations of addition, later on operations of subtraction or mixed operations. The initial problem is compared with other problems in order to emphasize

possible similarities and differences. These are expressed orally, mentally, or in written form.

*With starting point in symbolic schemes.* The teacher gives the formula  $a+b=x$  and requires as many examples of exercises as possible. Then the teacher asks for the formulation of varied problems. The position of the unknown is changed ( $a+x=c$ ;  $x=a+b$ ;  $x+b=c$ ; etc.) with the same requirements (proposing various exercises and problems). The same procedure is carried out, starting from one of the models  $a-b=x$ ,  $a+b+c=x$ ,  $a-b-c=x$ , etc., or from graphical models, diagrams, tables.

Children were asked to do the tasks described above in a gradual progression of internalizing, which emphasizes recurrent cycles of understanding: orally, mentally, in writing (without or with minimum verbalization, and the result is required for checking). Letters are to be used just accidentally, or gradually, depending on the students' level and teacher's knowledge about their appropriate use. Usually, instead of  $a, b, c, x$ , other symbols – more familiar to children – were used as “boxes” or “shells” for the substitutions.

*Problem comparison.* Throughout the lessons, students share the problems they have posed in order to uncover similarities and differences.

*Analysis of incomplete or redundant problems.* Problems that were missing data or other components were deliberately proposed. First grades have analyzed if and why solving these problems was possible, and if necessary completed the text of the problem with additional information. Redundant problems were also discussed, eliminating superfluous data.

During the school year, children in the experimental classes were tested 15 times. Each test contained 10 items, amongst which 4 to 6 were open-ended questions requiring creative answers. The tests have been published as booklets (Singer & Raileanu, 1994-1997). The average success at solving creative tasks was more than 60% for students from each of the classes involved in the experiment.

A comparison can be drawn with results from the national assessment for grade 4 students. This assessment was conducted at the end of the same school year, and 18 844 students from 992 urban and rural schools were involved. In this assessment, an item requiring a non-standard creative answer was: “Devise a problem using numbers smaller than 20 that can be solved using addition, multiplication and subtraction”. This item was correctly solved by only 20% of the students (only the text of the word-problem was scored, not its solving), and almost half (46.1%) did not make any attempt. This big percentage of avoidance suggests that many 4<sup>th</sup> graders had already established a habit of mind that makes them refuse creative involvement.

## Interpretation

The methodology described above is part of the dynamic structural learning concept (Singer, 2004). The dynamic structural learning as a school pedagogy tries to move

the didactical approach from a “horizontal” way of perceiving teaching – islands of information to be transmitted – to a permanent process of “vertically” restructuring students’ knowledge by incorporating the new elements into a dynamic structure. Particularly, this implies systematically practicing some basic mental operations while creating patterns of variability.

This kind of training supposes identifying and developing optimal individual pathways in a multidimensional network of the classroom interactions. The practiced types of tasks are focusing transfers: from operating with animate (drama, role-playing) and inanimate objects to abstract thinking and coming back; from thinking aloud to “thinking in mind” and reverse, all these being practiced using a balanced dosage of conventional representations. The scale of transition from concrete to abstract grows increasingly refined.

Because learning is incremental, (Fischer & Bidell, 2006; Fischer & Rose, 1994, 2001), problem-posing activities contribute to strengthen the connections among notions and thus deepen understanding. However, these qualities are sustainable only if problem posing is practiced in the classroom on a systematic basis.

The above described successive steps are systematically followed while varying the informational content. For example, in learning the arithmetical operations, the training involves a variety of layers that incorporate stages of abstracting within passing through the following types of representations: explicit active dynamic, explicit passive dynamic, explicit static, implicit static, iconic representations, mental computing with no support (Singer, 2002). At the beginning of learning to count, acquisition is superficial: the numbers are correctly arranged in the (mental) number line and designated according to their position (the number words still play the role of labels). Two targets left to be reached: the raising of the internalized structure to the level of the formal notion of natural number, i.e. understanding the concept of (natural) number, and the transformation of the internalized structure into a dynamic one (Singer, 2003a, 2003b).

The tasks format underlies cycles of transfers in order to internalize, together with the number line sequences, a variety of representations and, what is equally important, ways to move from one representation to another one. Thus, representational change (Singer, 2007a) becomes a trustful tool for problem solving. The target is to help the child to build a dynamic mental structure that she/he can self-develop and generalize across new tasks in an adequate context.

### **Summarizing the results of the experiments**

Tracking the development of students’ abilities in a longitudinal study is a difficult task because of the multitude of variables acting and interfering in the teaching-learning process in the classroom. Within these constraints, written tests were used mostly as feedback for corrections rather than as a way to report or grade the

students. However, they give an insight into the children's progress. Two items from the 1<sup>st</sup> grade are discussed in the next paragraphs.

An item from February: "Make 12 using as many additions as you can."

Spontaneous solutions involving repeated addition (i.e. multiplication), such as  $3+3+3+3$  or  $2+2+2+2+2+2$ , frequently appeared. There were often 10 solutions per student. Some students took into consideration the addition with 0, as a distinct situation. We noticed that usually 0 does not pose difficulties anymore (unlike many classes using traditional teaching), being naturally accepted as "a starting point" in the sequence and, therefore, as a number.

Another written task, given in April: "Devise a word problem." Out of 36 students of a class, 28 solved the task correctly, proposing coherent word problems. For example, 7 students composed a problem to be solved involving one operation, 17 students composed a problem to be solved involving two operations, 4 students composed a problem to be solved involving four operations.

The results in other classes were similar, the percent of pertinent proposals of word problems being situated around 70%, with more than 50% of students choosing to use multiple operations. Some of the teachers' comments concerning the results of the experiments are included below.

**Teacher 1:** Compared to my experience with regular classes, this method saves time for the process of knowledge acquisition. This fact is pointed out by the precision of the students' answers and by their promptness to respond to any question.

**Teacher 2:** The performances obtained by the students involved in the experiment when confronted with creative tasks are much better than the results obtained in the past with the classical teaching methods.

**Teacher 3:** The students show facility in devising problems or exercises with many operations.

**Teacher 4:** Children exhibit a real pleasure in working during mathematics classes and this is maybe the most important aspect of this research. To come and attend the mathematics lessons with pleasure is usually quite unlikely.

Throughout the experiments, there was much evidence that the young participants were not only willing and able to pose problems, but in the process of doing this, they demonstrated both the breadth and depth of their cognitive structures.

Therefore, is problem posing a solution for fostering effective learning in mathematics? In the experiment described, problem posing was systematically integrated into a learning environment that was intended to be dynamic and structured (Singer, 2004). This draws attention to several new questions: How much structure is needed in order to maintain and develop the natural tendency shown by young children to search for and test hypotheses spontaneously? How much dynamism is needed in order to develop a "disciplined" (in Howard Gardner's (1999) terms) mathematical mind? These questions are still open to discussions.

## **PROBLEM POSING RESEARCH: SOME ANSWERED AND UNANSWERED QUESTIONS**

There is a long history of integrating mathematical problem solving into school curricula (Stanic & Kilpatrick, 1988). At least two commonly accepted views regarding the integration of problem solving into school mathematics have emerged. In the past several decades, there have been significant advances in the understanding of the affective, cognitive, and metacognitive aspects of problem solving in mathematics and other disciplines (e.g., Cai, 2003; Frensch & Funke, 1995; Lester, 1994; McLeod & Adams, 1989; Schoenfeld, 1985, 1992; Silver, 1985). On the other hand, problem posing research is relatively new (Kilpatrick, 1987; Silver, 1994).

Even though problem posing research is relatively new, there have been efforts to incorporate problem posing into school mathematics at different educational levels around the world (e.g., Healy, 1993; Keil, 1964/1967; Brink, 1987; Hashimoto, 1987; Leung, 2008; Chinese National Ministry of Education, 1986). Among many practitioners around the world, there has been an apparently high level of interest in making problem posing a more prominent feature of classroom instruction. This intention is understandable since problem posing as an intellectual activity has long been recognized as critically important in scientific investigation (Einstein & Infeld, 1938). According to Einstein, posing an interesting problem is more important than solving the problem.

Despite this interest in integrating mathematical problem posing into classroom, far less is known about the cognitive processes involved when solvers generate their own problems, about instructional strategies that can effectively promote productive problem posing, or about the effectiveness of engaging students in problem-posing activities. What follows is a synthesis of what we know about problem-posing research, and includes some directions for future study.

### **Who Can Pose Important Mathematical Problems?**

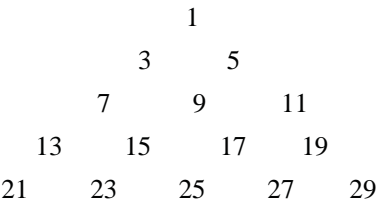
If problem posing is such an important intellectual activity, the first question we need to ask is who can pose mathematical problems. One important line of research in problem posing has been exploring what problems teachers and students can pose. In this line of research, researchers usually design a problem situation and ask subjects to pose problems, which can be solved using the information given in the situation. Figure 1 shows three sample situations.

Different types of problem situations have been used in problem posing research. Some situations are knowledge free, and other situations are knowledge rich. Some situations are quite structured (Situation 1), but others are relatively open (Situation 3). Stoyanova and Ellerton (1996) classified the designed situations into free, semi-structured, and structured situations. Some researchers have used technology to present problem situations and technology-assisted problem posing.

**Situation 1.** Ann has 34 marbles, Billy has 27 marbles, and Chris has 23 marbles. Write and solve as many problems as you can that uses this information. (This situation was used for middle school students in Silver & Cai, 2005).

**Situation 2.** Children were to pose problems based on the following statements about Rufus the dog: Rufus managed to get into the Bradley house one afternoon. He chewed up four of Amy's shoes, three of her toys, and six of her socks. He also chewed up five of Brad's shoes, seven of his toys, and two of his socks. Mrs. Smith baked two dozen biscuits. Rufus made off with twelve biscuits. He buried eight of them before Mrs. Smith discovered him. (This situation was used for elementary school students in English, 1998).

**Situation 3.**



The pattern continues. I wanted to make up some problems that used this pattern for a group of high students/college freshmen. Help me by writing as many problems as you can in the space below. (This situation was used for pre-service secondary mathematics teachers in Cai, in preparation).

Figure 1. Sample Problem Situations

School students, pre-service teachers, and in-service teachers have been used as subjects in mathematical problem-posing research. The general finding is that students and teachers are capable of posing interesting and important mathematical problems. For example, for the Sample Situation 1, students are able to pose problems such as the following (Silver & Cai, 2005):

- How many marbles do they have altogether?
- How many more marbles does Billy have than Chris?
- How many more marbles would they need to have together as many marbles as Sammy, who has 103?
- Can Ann give marbles to Billy and Chris so that they all have the same number? If so, how can this be done?

Suppose Billy gives some marbles to Chris. How many marbles should he give Chris in order for them to have the same number of marbles?

Suppose Ann gives some marbles to Chris. How many marbles should she give Chris in order for them to have the same number of marbles?

For Sample Situation 3, pre-service teachers are able to pose problems such as these:

What is the first number on the  $n^{\text{th}}$  row?

What is the number on the  $i^{\text{th}}$  row and  $j^{\text{th}}$  column?

What is the last number on the  $n^{\text{th}}$  row?

What is the sum of the numbers in the  $n^{\text{th}}$  row?

How many numbers are there in the  $n^{\text{th}}$  row?

What is the sum of the numbers in the first  $n$  rows?

What is the pattern of each of the numbers in each diagonal line?

What is the sum of  $1^3 + 2^3 + 3^3 + 4^3 + \dots + (n-1)^3 + n^3$

What is the middle number in an odd row?

### **Unanswered Question 1**

Even though research shows that students and teachers are capable of posing interesting and important mathematical problems, researchers also found that some students and teachers have posed non-mathematical problems, unsolvable problems, and irrelevant problems (e.g., Cai & Hwang, 2002; Silver et al., 1996; Silver & Cai, 1996). For example, Silver and Cai (1996) found that nearly 30% of problems posed by middle school students either are non-mathematical problems, or just statements (not problems). It is not clear why these students posed non-mathematical problems or even posed statements (even though the direction clearly asked for problems) – so these findings suggest the need to explore how students or teachers interpret a problem situation.

### **Unanswered Question 2**

Apparently, researchers have used different types of problem situations, ranging from simply deleting a question from a textbook problem to very open-ended problem situations. In mathematical problem solving research in the past several decades, researchers have explored the effect of various task variables on students' problem solving (e.g. Goldin & McClintock, 1984). However, so far, little is known about how different problem situations impact on their problem-posing responses. How do different problem situations impact on subjects' problem posing? This is another unanswered question.

### **Unanswered Question 3**

While we know that subjects are capable of posing mathematical problems, how do they pose mathematical problems based on particular situations? Previous research

does show that students tend to pose parallel problems when they are asked to pose more than one problem to a situation (Silver & Cai, 1996). Some researchers identified strategies students used to pose problems. For example, Cai and Cifarelli (2005) examined how two college students posed and solved their own problems in open-ended computer simulation task that involved the path of a Billiard Ball. They identified two different levels of reasoning strategies – hypothesis-driven and data-driven – that students appeared to incorporate in their posing and solving processes. Much more research is needed to understand students' problem-posing strategies.

## **HOW ARE PROBLEM-POSING SKILLS RELATED TO PROBLEM-SOLVING SKILLS?**

One important direction for research on problem posing is probing the links between problem posing and problem solving (see, e.g., Cai, 1998; Cai & Hwang, 2002; Ellerton, 1986; Kilpatrick, 1987; Silver & Cai, 1996). Kilpatrick (1987) provided a theoretical argument that the quality of the problems subjects pose might serve as an index of how well they can solve problems. In addition to theoretical arguments, several researchers have conducted empirical studies examining potential connections between problem posing and problem solving. Ellerton (1986) compared the mathematical problems generated by eight high-ability young children with those generated by eight low-ability young children, asking each to pose a mathematical problem that would be quite difficult for her or her friends to solve. Ellerton reported that the more able students posed problems that were more complex than those posed by less able students.

Silver and Cai (1996) analyzed the responses of more than 500 middle-school students to a task that asked them to pose three questions based on a driving situation. In particular, the student-posed problems were analyzed according to their type, solvability, and complexity. Additionally, Silver and Cai used eight open-ended tasks to measure the students' mathematical problem-solving performance. They found that students' problem-solving performance was highly correlated with their problem-posing performance. Compared to less successful problem solvers, good problem solvers generated more, and more complex, mathematical problems.

The investigation of mathematical problem solving and problem posing in Silver and Cai (1996), students' problem-solving performance was measured using tasks that were rarely related to the problem-posing tasks. In other studies, Cai and his associates (Cai, 1998; Cai & Hwang, 2002) examined Chinese and US students' problem-solving and problem-posing performances using closely related problem-posing and problem-solving tasks. Cai and Hwang (2002) found differential relationships between posing and solving for US and Chinese students. There was a stronger link between problem solving and problem posing for the Chinese sample, while the link is much weaker for the US sample. Posing a variety of problem types seems to be strongly associated with abstract strategy use in the Chinese sample. Cai



and Hwang (2002) indicated that the differential nature of the relationships for US and Chinese students does not imply a lack of generality in the links between problem solving and problem posing. Instead, the stronger link between the variety of posed problems and problem-solving success for the Chinese sample might be attributable to the fact that the US students almost never used abstract strategies.

#### **Unanswered Question 4**

Cross-national studies provide unique opportunities to understand students' mathematical thinking and reasoning. While there is a large body of literature on cross-national studies in mathematical problem solving, there have been few attempts to use problem posing in cross-national studies (e.g., Cai, 1998; Cai & Hwang, 2002; Yuan & Sriraman, 2011). The study by Cai and Hwang suggests the value of cross-national studies about problem posing. How do students in different countries pose mathematical problems? This is an unanswered question. We sincerely hope that this Research Forum will provide a venue for more researchers around the world to engage in mathematical problem-posing research in a cross-cultural context.

#### **WHAT DO WE KNOW ABOUT THE IMPACT ON STUDENT ACHIEVEMENT OF ENGAGING IN PROBLEM-POSING ACTIVITIES?**

The ultimate goal of educational research is to improve students' learning. Research on problem posing is no exception. Like problem solving, problem posing can be viewed as a classroom activity. There is increased interest in integrating mathematical problem posing into classrooms. For example, Silver (1997) proposed that engaging students in problem posing can foster their creativity. There are at least two reasons why engaging students in problem-posing activities is likely to have a positive impact on their learning. First of all, problem-posing activities are usually cognitively demanding tasks. Doyle (1983) argues that tasks with different cognitive demands are likely to induce different kinds of learning. Problem-posing activities in classrooms have the potential to provide intellectual contexts for students' rich mathematical development. Such activities can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interests and curiosity (NCTM, 1991). Second, problem-solving processes often involve the generation and solution of subsidiary problems (Polya, 1957). Previous studies (e.g., Cai & Hwang, 2002) suggest that the ability to pose complex problems would be associated with more robust problem-solving abilities. Researchers (e.g., Silver, 1994) also suggest that student-posed problems are more likely to connect mathematics to students' own interests, something that is often not the case with traditional textbook problems. Thus, encouraging students to generate problems is not only likely to foster student understanding of problem situations, but also to nurture the development of more advanced problem-solving strategies.

### Unanswered Question 5

Even though engaging students in problem-posing activities in classrooms appears to have a positive impact on students' learning and problem posing, there are no empirical studies to document any effect systematically. English (1997a) developed a problem-posing program and found in her post-interview that fifth graders engaged in the problem-posing program did, in fact, pose quantitatively more, as well as more complex, problems. Similarly, Crespo (2003) examined the changes in the problem-posing strategies of a group of elementary pre-service teachers as they posed problems to students. She found that, after teachers had engaged in problem-posing activities, they were able to pose more problems with multiple approaches and solutions, as well as pose problems that were more open-ended and exploratory, and were cognitively more complex.

Originating from an international comparative study (Cai & Hwang, 2002), Lu and Wang and their associates (Lu & Wang, 2006; Wang & Lu, 2000) launched a project on mathematical situations and problem posing. The project has two interrelated key components.

The first is the systematic development of teaching materials about mathematical situations and problem-posing tasks. Similar to the collections of open-ended mathematical problems, the teaching materials – including mathematical situations and problem-posing tasks – are not intended to replace textbooks; instead, they are used to supplement regular textbook problems.

The second is the systematic implementation of teaching materials, including mathematical situations and problem-posing tasks. By 2006, more than 300 schools in 10 provinces in China had participated in the project. Teachers received training to use mathematical situations and problem-posing tasks along with regular curriculum. Most importantly, teachers learned about how to develop mathematical situations and to pose problems (Lu & Wang, 2006). As supplementary material for the regular mathematics curriculum, a series of teaching cases was developed by mathematics educators across grade levels and across content areas. Figure 2a presents a sample teaching case for *Making a Billboard* from Lu and Wang (2006, p.359).

The teaching materials given to teachers included different problem situations (like that in Figure 2a), together with examples of problems which students might be expected to prepare. Figure 2b gives six such sample problems.

These sample problems were given to teachers as guidelines – in much the same way as worked-examples might be given in textbooks. When students were given the problem situations, they were encouraged to pose as many problems as they could.

**Mathematics content:** Linear equation with one unknown (for junior high school students).

**Situation:** A factory is planning to make a billboard. A master worker and his apprentice are employed to do the job. It will take 4 days by the master worker alone to complete the job, but it takes 6 days for the apprentice alone to complete the job.

**Students' Task:** Please create problems based on the situation. Students may add conditions for problems they create.

Figure 2a: Sample teaching case

**Problem 1.** How many days will it take the two workers to complete the job together?

**Problem 2.** If the master joins the work after the apprentice has worked for 1 day, how many additional days will it take the master and the apprentice to complete the job together?

**Problem 3.** After the master has worked for 2 days, the apprentice joins the master to complete the job. How many days in total will the master have to work to complete the job?

**Problem 4.** If the master has to leave for other business after the two workers have worked together on the job for 1 day, how many additional days will it take the apprentice to complete the remaining part of the job?

**Problem 5.** If the apprentice has to leave for other business after the two workers have worked together for 1 day, how many additional days will it take the master to complete the remaining part of the job?

**Problems 6.** The master and the apprentice are paid 450 Yuans after they completed the job. How much should the master and the apprentice each receive if each worker's payment is determined by the proportion of the job the worker completed?

Figure 2b: Examples of problems posed by students in response to task in Figure 2a

After students had posed several problems, a teacher would show students how to solve some of the problems posed. Figure 3 shows a sample solution to problem 3. Once students had solved each of the posed problems, they were encouraged to pose new problems. Additional problems posed by students are shown in Figure 4. The teacher would then show students how to solve these problems.

However, no validation or efficacy studies have been carried out to examine the effect of engaging problem-posing activities on student learning. The fifth unanswered question is: What is the impact of engaging in problem posing activities on student achievement?

Suppose the two workers worked together for  $x$  days, the master worker did  $(x+2)$  days.

$$\frac{1}{4}(x+2) + \frac{1}{6}x = 1, \text{ and } x = \frac{6}{5};$$

So the master worked:  $x + 2 = 2 + \frac{6}{5} = \frac{16}{5}$  days.

Figure 3: Solution presented by a teacher to Posed Problem 3.

**Problem 7.** The apprentice started the work by himself for 1 day, and then the master joined the effort, and they completed the remaining part of the job together. Finally, they received 490 Yuans in total for completing the job. How much should the master and the apprentice each receive if each worker's payment is determined by the proportion of the job the worker completed?

**Problem 8.** The master started the work by himself for 1 day, and then the apprentice joined the effort, and they completed the remaining part of the job together. Finally, they received 450 Yuans in total for completing the job. How much should the master and the apprentice each receive if each worker's payment is determined by the proportion of the job the worker completed?

Figure 4: Additional problems posed by students.

Research in reading showed that engaging students in problem posing could lead to significant gains in reading comprehension. The results from meta-analysis showed that the effect<sup>1</sup> sizes were 0.36 using standardized tests and 0.86 using researcher-developed tests (Rosenshine, Meister & Chapman, 1996). Although it is theoretically sound to engage students in problem-posing activities in an attempt to understand and improve their learning, more empirical studies are needed to demonstrate any actual effects. The research in reading can serve as a model for systematically investigating the effect of mathematical exploration in general and problem-posing activities in particular on students' learning of mathematics.

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<sup>1</sup> Effect sizes are statistical measures of variation. Positive effects indicate a gain, negative effects indicate a loss, and a zero effect is neutral. Generally speaking, effect sizes with absolute values around 0.20 are considered small, medium when around 0.50, and large if 0.80 or greater.

## Unanswered Question 6

Few researchers have tried to describe classroom instruction where students are engaged in problem-posing activities. This leads to our sixth unanswered question: What would a classroom look like when students are engaged in problem-posing activities? Cai (in preparation) has documented the classroom instruction of 14 pre-service teachers when they engage in the problem-posing activity shown in Figure 5.

### *Doorbell-Solving*

Sally is having a party, the first time the doorbell rings, 1 guest enters.

The second time the doorbell rings, 3 guests enter.

The third time the doorbell rings, 5 guests enter.

The fourth time the doorbell rings, 7 guests enter.

Keep on going in the same way. On the next ring a group enters that has 2 more persons than the group that entered on the previous ring.

1. How many guests will enter on the 10<sup>th</sup> ring? Explain how you found your answer.
2. In the space below, write a rule or describe in words how to find the number of guests that entered on each ring.
3. 99 guests entered on one of the rings. What ring was it? Explain or show how you found your answer.

### *Doorbell-Posing*

Sally is having a party, the first time the doorbell rings, 1 guest enters.

The second time the doorbell rings, 3 guests enter.

The third time the doorbell rings, 5 guests enter.

The fourth time the doorbell rings, 7 guests enter.

Keep on going in the same way. On the next ring a group enters that has 2 more persons than the group that entered on the previous ring.

For his student's homework, Mr. Miller wanted to make up three problems BASED ON THE ABOVE SITUATION: an easy problem, a moderate problem, and a difficult problem. These problems can be solved using the information in the situation.

Help Mr. Miller make up three problems and write these problems in the space below.

The easy problem
The moderately difficult problem
The difficult problem

Figure 5: Doorbell problem-posing and problem-solving activity

The fourteen pre-service teachers were divided into four groups and they were first given 30 minutes to pose as many problems as they could. Then the class used another 70 minutes to solve the problems these teachers posed. During the process of solving the posed problems, each pre-service teacher could pose additional problems. Pre-service teachers posed a total of 9 different mathematical problems after 30 minutes. These problems included the following: (1) How many numbers are there in the 10<sup>th</sup> row? (2) How many numbers are therein the  $n^{\text{th}}$  row? (3) What is the first number in the  $n^{\text{th}}$  row? (4) What is the last number in the  $n^{\text{th}}$  row? (5) Is the middle number in each row always a perfect square? (6) What is the number on the  $i^{\text{th}}$  row and  $j^{\text{th}}$  column? (7) How can you represent it? (8) What is the sum of the numbers in the first  $n$  rows? and (9) What is the sum of numbers on the  $n^{\text{th}}$  row?

Cai (in preparation) documented an unexpected discovery in the exploration. Two groups of students posed the same question: What is the sum of the numbers in the first  $n$  rows?

The first group of students answered the question based on the fact that the sum of the numbers in the first  $n$  rows is the “sum of the sum” of the numbers in each of the first  $n$  rows. Since the sum of the numbers in the  $n^{\text{th}}$  row is  $n^3$ . Therefore, the sum of the numbers in the first  $n$  rows should be

$$1^3 + 2^3 + 3^3 + 4^3 + \dots + (n - 1)^3 + n^3.$$

Then they posed the following question: What is the sum of

$$1^3 + 2^3 + 3^3 + 4^3 + \dots + (n - 1)^3 + n^3?$$

Another group used a different approach to answer the question. After some observations, students realized that the first row has one odd number which is 1, the second row has two odd numbers which are 3 and 5, the third row has three odd numbers which are 7, 9, and 11, and so on. The  $n^{\text{th}}$  row should have  $n$  odd numbers. Therefore, the sum of the numbers in the first  $n$  rows of the pattern should be the sum of the first  $(1 + 2 + 3 + 4 + \dots + n)$  odd numbers. Since

$$1 + 3 + 5 + \dots + (2m - 1) = m^2, \text{ the sum of the numbers in the first } n \text{ rows in the pattern should be } (1 + 2 + 3 + 4 + \dots + n)^2.$$

After the two groups of students presented their answers to the class, an unexpected finding emerged. The unexpected finding is that

$$1^3 + 2^3 + 3^3 + 4^3 + \dots + (n - 1)^3 + n^3 = [n(n+1)/2]^2 \text{ since}$$

$$1 + 2 + 3 + 4 + \dots + n = n(n+1)/2.$$

Therefore, empirical research brings evidence that collective problem posing in the classroom context can lead to surprising results. Student voices become relevant.

## STUDENT VOICES

Much of the research on problem posing reported in the literature has focused on various aspects associated with the process of posing the problems themselves, and with the forms and types of problems created. Less attention has been given to comments made by those who create problems, particularly when these are students. We need to know more about how problem posers think and react as they go about the task of posing problems.

In the section that follows, comments and reflections made by a pre-service teacher education student who was involved in problem posing as part of normal classroom activities will be given (Ellerton, in preparation). As a culminating project in a modern geometry course, students, working in pairs within table groups, were asked to prepare four non-trivial transformation tasks. Details of the project are given in Figure 6.

Each student is to create four transformation tasks (short or long answer questions which, on a Quiz, would be expected to carry 2 or 3 marks). You may include one multiple choice question among these if you wish.

Exchange the four problems with your partner, and then solve each of the 4 problems created by your partner. If you find any errors in the way the problems were written, make suggestions for improving the problem. If the problem is too trivial (or too complex), make appropriate suggestions for improving the challenge offered by the problem.

Pass your solutions (and any suggestions) back to your partner. Discuss your findings concerning the process of creating and refining these problems, and in producing well laid-out solutions.

Write up a joint reflective report on the combined set of eight problems.

Figure 6: Details of a problem-posing project set for pre-service teachers

The activities in the project were designed to match as closely as practicable the sorts of processes that those who create items for textbooks or tests might follow. The project was an attempt to move away from what can be artificial problem creation towards an activity that had genuine curriculum relevance. At the end of the semester, students submitted individual reflections. The following comments were made by one of the students about the project, and were unsolicited.

I especially loved one of the last projects we did, which was coming up with problem sets for our groups to solve. I feel I gained the most from this activity. Making problems for others to do helped me in making sure I understood the information with handling transformations. I tried to be creative, which was a little difficult, and was a main cause of some errors in my problems. However, it was fine because the

feedback I got from my group was extremely beneficial and really perfected my problems. Also, taking a look at my group member's questions and solving them gave me extra practice in solving the problems. It also gave me a look in seeing how others tried to be creative with their problem sets. The project overall was one of the most useful aspects of the course ... it will definitely help me in the future as a teacher by making me think in a different perspective.

Are we giving those who pose problems time and space to reflect on problem posing, and are we as mathematics education researchers, taking time to listen to and learn from student voices?

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# **WORKING SESSIONS**







## CREATIVITY AND MODELING

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### PROPOSED NATURE AND TOPIC OF SESSION

The motivation for this symposium comes from a similar initiative that begun in the UK in 1999 when the government there established a commission to examine the status of creativity in schools and impact on the future economy. This yielded the widely acclaimed “Robinson Report” which many businesses, educators and policy makers took serious note of. We wish to intersect this initiative with the long-term evolution of modeling research in the field of Science, Technology, Engineering, and Mathematics (STEM) education across a wide variety of content areas and age groups. The main product of this working group is to join others in developing a community of thinkers to discuss the creative enterprise of modeling and culminate activities and research on such activity.

### GOALS

- Establish a state-of-the-field discussion document of what we mean by creativity in mathematics classrooms at all age groups and how we can promote (or develop) creativity by using model-eliciting-activities
- Outline strategies that can help inform research into mathematical thinking and learning and advances into activity design with a particular focus on digital technology across the grades from little kids to graduate students
- Share and exploit results and join an established writing community in this topic

### PLANNED ACTIVITIES

- Presentations of what is creativity in learning and doing within STEM disciplines
- Work on particular model-eliciting-activities with a focus on the creative dimensions
- Generate new activities
- Shared research programs with the aim of international collaboration
- Participate in global writing project on *Creativity and Modeling* activities for parents and teachers

# EXPLORING THE TRANSITION TO AND WITHIN UNIVERSITY MATHEMATICS FROM DIFFERENT PERSPECTIVES

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In recent years transitions from school to university mathematics have been studied from several, theoretical *perspectives* – for example, see (Gueudet, 2008) for a review. Studies of transition have focused on a range of epistemological, cognitive, meta-cognitive, pedagogical, curricular, social and institutional *aspects* such as: new mathematical content; the shifts in the students' ways of thinking and communicating mathematically; the changing expectations in terms of students' study habits and skills; the differing ways in which mathematical knowledge is organised and presented to students in school and at university; and, the across-the-board differences between the institutions of school and university. Some of these aspects of transition may even arise within university studies.

In this Working Session we will examine two or three of the theoretical perspectives through engaging with a small collection of papers that we have come across through the involvement of members of our team with the work of *Working Group 14: University Mathematics Education* (in CERME7, the 7<sup>th</sup> Conference of European Researchers in Mathematics Education) and a study entitled 'Key mathematical concepts in the transition from secondary school to university', currently in preparation for ICME12 (12<sup>th</sup> International Congress of Mathematics Education) and led by Mike Thomas. Following short presentations of these papers, participants will be invited to consider brief excerpts of data in order to explore: what aspects of transition are illuminated through the use of the theoretical perspectives presented; which aspects of transition are not currently being investigated; and, how future research can do so.

We envisage the two 90' meetings as follows:

*Meeting 1.* Welcome (10'); Brief presentations of papers examining aspects of transition from different perspectives (10'); Work in small groups (40'); Discussion (30').

*Meeting 2.* Brief presentations of papers examining aspects of transition from different perspectives (10'); Work in small groups (40'); Discussion (30'); Closing (10').

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# GESTURE, MULTIMODALITY, AND EMBODIMENT IN MATHEMATICS

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The central purpose of the Working Session is look at mathematical thinking, learning and communication from a perspective that encompasses multiple modalities of expression and different semiotic systems (Radford, Edwards & Arzarello, 2009). These modalities include gesture, speech, written inscriptions, and physical and electronic artefacts. In addition to considering how mathematics is communicated, the Working Session will address the notion that, in our acts of knowing, different sensorial modalities (tactile, perceptual, kinesthetic, etc.) are integral parts of our cognitive processes. Thus, the basis for mathematical thinking, as well as language and thought in general, is found in our embodiment as physical beings (Lakoff & Núñez, 2000). In the Working Session, we will continue the process of investigating the entailments of, and evidence for, this stance.

The session will be participatory, and attendees will be asked to share videotaped data, interpretations, theories and ideas. We will meet as a whole group as well as in small groups, within which participants will be able to sign up to give brief presentations. Themes and topics addressed in previous years include:

- Gesture and semiotics
- Conceptual integration and conceptual metaphor
- Gesture and embodiment in young children and blind students
- Dynamic geometry and other computer-based tools
- Graphing and other visual modalities
- Language, culture and the body in mathematics

The session will not be limited to these topics, but will be based on the interests of the participants.

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## MATHEMATICS EDUCATION AND SOCIETY

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At PME-34 we coordinated the Discussion Group ‘Mathematics Education and Democracy’. The group ended up with the following critical question: What is democracy for you? We recognized that our discussion treated ‘democracy’ as if the term had only one meaning. After the meeting discussions followed through e-mail among some members of the group. We tried to reveal what seemed to be missing in the discussion at PME-34. We realized that we missed the point that democracy does not exist by itself, rather there has been different types of democracy, as for instance liberal (bourgeois) democracy or proletarian democracy or fundamentalist democracy and so on.

In any case, we came to agree that we are interested in the “democratic phenomenon” of the last century which we have called ‘North American democracy’. But even ‘North American democracy’ alone seems to be a complex phenomenon. For instance, it has allied itself with dictatorship regimes, as for instance in Latin America. Also, the recent events in the Middle-East are remarkable.

Our question now is how far a discussion about the meaning of democracy has reached within mathematics education research. We have suggested (Mattos & Batarce, 2010) some critical aspects for a mathematics education researcher position in such a discussion: that the historical appearance and rise of mathematics education seems to be closely linked with that phenomenon of democracy in the last century, democracy seems to be the horizon of our societies today, and democracy seems a key concept in mathematics education and society research trends.

The activities for this WS will be based on a critical discussion among participants stimulated by the coordinators from an initial review of some research on mathematics education and democracy integrated with text from outside of mathematics education, focused on democracy itself. A list of texts will be suggested in advance through a webpage <http://en.emsociedade.com.br>.

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<http://www.springerlink.com/content/m873228584277u77/>

# THE LEARNING AND DEVELOPMENT OF MATHEMATICS TEACHER EDUCATOR-RESEARCHERS

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This Working Session builds on the Discussion Group on the same topic at the PME 34 conference (Goos, Chapman, Brown, & Novotna, 2010). The aim of the Discussion Group was to explore a range of theoretical perspectives on the learning and development of university-based mathematics teacher educators, a new field of study in which there had been little research to date (Llinares & Krainer, 2006). The topic arose from interactions between the Editor and authors of two Special Issues of the *Journal of Mathematics Teacher Education* on Teacher Change (13.5 and 13.6), in which authors were also asked to include in their papers critical reflective accounts of how they changed (learned) from engaging in working with teachers.

Discussion Group participants described how they conceptualised their own learning as mathematics teacher educators working with prospective and practising teachers, and what difference this made to their practice. A number of tentative research questions and problems were identified for further investigation, and participants expressed their interest in a follow up session at the next PME conference. This Working Session will focus on emerging research proposals and projects.

In the first session, examples of planned or beginning projects will be presented and discussed in order to provide feedback on ideas and initial results. Participants will be invited to identify theoretical perspectives that inform their own work as mathematics teacher educators and how these theories might explain their own learning and development.

In the second session, outcomes of the first session will serve as the basis for choosing topics for this type of research. The coordinators of the Working Session will engage with participants in small groups, assisting them to sketch out research proposals and methodologies. It is anticipated that some of these proposals will mature into fully formed research projects that can be activated after the conference.

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# DISCUSSIONS GROUPS







# DIGITAL GAMES: NEW POTENTIAL FOR MATHEMATICS

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Digital games offer considerable potential for learning mathematics in novel and deep ways. In this discussion group, we draw on the potential of the games environment to build rich spatial understandings as well as problem-solving capacities. In his work on decision making in the games environment, Bossomaier (in press) demonstrates the sophisticated, and varied, decision-making process that take place when novices and experts play complex games. Similarly, in the digital games environment, players make critical decisions as to how to navigate through levels. Gee's (2003) work on the games environment, albeit with literacy learning, highlights the various strategies that gamers make as they move through games. He argues that learning is scaffolded so that gamers develop skills that need to be built upon, refined and made more complex as the demands of the levels change and become more complex. The problem solving involved in moving through the game not only engages the gamer but also helps to develop a rich repertoire of problem-solving strategies. Moreover, complex games create sharp changes in behaviour along a novice-expert continuum (Bossomaier, in press).

In terms of spatial understandings, Lowrie's work with Pokemon and young children has highlighted the possibilities for more complex understandings to be built by young children. The small screen of the Pokemon handset demands that only small components of the whole worlds can be displayed. This challenges players to create visual images of the worlds beyond the immediate screen so that the player can create successful routes through these worlds. These games require complex interactions between 2D and 3D space—both in terms of decoding and encoding information (Lowrie & Clancy, 2006; Jorgensen & Lowrie, under review). We will provide examples from a range of digital games and explore the (potential) mathematics sense making that occurs in these environments. Other areas of mathematics will be discussed in terms of the potential learnings for school mathematics.

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# INTUITIVE AND ANALYTICAL THINKING: WORKING TO BRIDGE THE GAP

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The interplay between intuitive thinking and analytical thinking is a fundamental aspect of mathematical thinking, accounting both for its power and for the difficulties in learning it. In the group's work we will explore this issue, starting from the vast relevant literature from cognitive psychology (culminating with the 2002 Nobel Prize to Kahnemann), and proceeding to discuss ways of helping students bridge this gap. In particular, we will discuss the design and use of *bridging tasks*.

In the first session, we will review and discuss the Dual Process Theory (DPT) from cognitive psychology, which explains the interaction between intuitive and analytical thinking by postulating (roughly) "two minds in one brain". We will briefly introduce the DPT and its theoretical and experimental basis, and then look at several case studies, including the participants' own, demonstrating its relevance to mathematics education research and practice. By way of reflection on the case studies, we will discuss the following issues, among others: Universal vs. culture-specific or individual-specific intuitions, factors that trigger analytical-system intervention, using cognitive psychology research to explain mathematical education phenomena, intuitive thinking – can we build on it or do we need to overcome it?

In the second session, we will apply the insights from the first session to discuss the crucial educational challenge of helping students build bridges between these two modes of thinking. We will present one case (possibly the notorious and extensively researched Medical Diagnosis Problem), where the intuitive and analytical solutions clash sharply, and demonstrate how carefully-designed bridging tasks can help students create peaceful coexistence between the two modes of thinking. The participants will then work in small groups on other challenging tasks and will design various means for bridging the gap.

**Acknowledgement:** This presentation is based on joint work with Abraham Arcavi.

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## MODELS AND MODELING

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### CONTEXT AND RATIONALE OF THE SESSION

The Models & Modeling Working Group has been one of PME-NA's most active working groups, beginning with the very first PME-NA that Richard Lesh held at Northwestern University in 1978. Many of its members also have been active in PME-International. During the past year, leaders in the Models & Modeling Working Group have been using videoconferencing seminars to work with more than forty faculty members and graduate students in countries ranging from Turkey, Cyprus, and Israel, to Australia and Mexico – as well as the United States. The focus has been on four areas: (a) modeling activities for primary grade children, (b) modeling activities focusing on integrated approaches to mathematics for high school students, (c) modeling-based activities for teacher development, and (d) modeling-focused design research methodologies. These also are the proposed subgroups for conducting discussions within the Models & Modeling Discussion Group.

There are two main goals of the discussion group: (i) share results and questions, (ii) establish a wider international collaboration for further work in this topic, (iii) produce ideas regarding how models & modeling perspective can be used for developing mathematics learning across a range of grade levels, mathematics teaching and research methodology.

In this context, the following activities are planned to serve for these goals:

- Presentations of how models & modeling perspective provides a grounded understanding and research approach for mathematics education,
- Discussions about model-eliciting-activities with a focus on the nature of mathematical learning and teaching they would allow,
- Analysis and critiques of various types of modeling tasks found in the literature and/or developed by the participants,
- Discussions about the use of videoconferencing seminars in this area,
- Sharing of experiences and research programs to build a network for international collaboration.

# PREPARING NOVICE RESEARCHERS TO DO HIGH-QUALITY QUALITATIVE RESEARCH IN MATHEMATICS EDUCATION

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## BACKGROUND

Qualitative research in mathematics education has grown rapidly over the last 25 years. Not surprisingly, growth in understanding the preparation of mathematics education researchers has not kept pace with the growth of qualitative research. This discussion group (DG) is intended to provide a forum for some of the needed conversation in this area. The two sessions will each focus on a particular question.

### QUESTION#1 (SESSION 1)

*What aspects of qualitative research do novice mathematics education researchers have difficulty conceptualizing and/or have difficulty engaging effectively?*

The PME community represents considerable experience in the preparation of novice researchers. The resulting knowledge has been insufficiently assembled. The preparation of researchers will be more effective if it is grounded in understanding of novice researchers thinking and development. A first step is the identification of difficulties in their development, potential foci for mentoring efforts.

### QUESTION#2 (SESSION 2)

*What abilities are involved in being able to generate a hypothesis to explain a corpus of data and how might those abilities be fostered?*

For novice researchers, hypothesis generation (or proposing an explanatory model) is an intractable aspect of doing qualitative research. Clement (2000) characterized the process as abductive and asserted,

Explanatory models ... are not merely condensed summaries of empirical observations, but, rather, are inventions that contribute new mechanisms and concepts that are part of the scientists' view of the world and that are not "given" in the data. (p. 549)

## STRUCTURE OF THE SESSIONS

Session 1 will be a facilitated discussion of Question 1. Session 2 will include two guided experiences with "data" to focus the inquiry into the subject of Question 2.

Clement, J. (2000). Analysis of clinical interviews: Foundations and model viability. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 547-590). Mahwah, NJ: Lawrence Erlbaum Associates.

# STUDENTS' CULTURAL IDENTITY AND MATHEMATICS INSTRUCTION IN GLOBAL TIMES

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In the last thirty years, ethno-mathematics and situated cognition research have put into evidence important differences between out-of-school cultural mathematical practices and school practices, and differences pertaining to their corresponding ways of mathematical enculturation. More recently, research on embodiment and multimodality have opened new perspectives on the controversial issue of the relationship between the body and the "mathematical mind", and raised questions about the variety of potential accesses to mathematical knowledge. However, the impact of these research streams and results on teachers, on teachers' educators and on mathematics curricula developers is still not evident. In our changing world, school is viewed as the main institution where selected pieces of "same" homogeneous mathematical knowledge are taught for a "standardized" abstract learner without cultural background and identity, whose performance is measured through international massive scale testings. In this context, *is it possible (and if "yes", how?) to offer mathematical instruction needed for a changing world, and at the same time to take into account the heterogeneous cultural diversity in which schools are immersed, the richness of out-of-school mathematical experiences, and the variety of personal expressions and potentials?*

The Discussion Group has two aims:

A) - to identify and discuss theoretical constructs that could help in dealing with the above question.

*In the first session*, after a short Introduction to the theme (10'), some constructs will be proposed for discussion by the presenters (40') (e.g. Vygotsky's dialectics between everyday concepts and scientific concepts adapted to mathematical knowledge, and the potential of this dialectics in taking different expressions and cultural roots of everyday concepts into account); other constructs and theoretical views are expected to be presented by the participants (40'); a synthesis will be prepared by the presenters and two other participants, and shared and discussed with all the participants *at the beginning of the second session* (35');

B) - to ascertain the participants' interest in engaging in co-operative work to tackle the above problem in the next years, and to outline and discuss related research developments (specific research questions, methodology, and possible experimental activities to be carried on in different countries).

This will be done *in the remaining time of the second session* (55'), possibly ending with the constitution of a team of five people charged to keep contacts and stimulate the follow-up of the initiative.

# TECHNOLOGY IN MATHEMATICS EDUCATION: DIFFERENT KINDS AND LEVELS OF EXPERIMENTATION

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Experimentation is one of the terms that has been used by mathematics educators and occurs in an environment where students explore patterns, make conjectures, collect data to test their conjectures, and refine their conjectures. The first question that comes to mind is how to create these kinds of rich experiential environments for the students in mathematics classes. In this technological age, different handheld or computer technologies with different capabilities offer new opportunities for teachers to create effective teaching and learning environments for experimentation in mathematics classrooms. The main goals for this discussion group are to

- refine our understanding about the roles and contributions of different technologies in helping students explore mathematics effectively and
- share our experiences and knowledge to build a community of PME members with similar interests.

## **Planned Activities:**

After brief introductions of the members of the discussion group, we will share an activity to stimulate a discussion. Depending on the size of the group, small groups will be formed to discuss significant questions listed below or others that emerge from the participants of the group. As a result of this group's discussions and activities, we will have key ideas for continued discussion or collaboration among the members of the group.

## **Questions:**

- How does experimentation take place in a technological environment?
- What are the benefits (and pitfalls) of differences among open ended environments (such as internet), semi-structured environments (dynamic software) and structured environments (animations or similar packages)?
- Is there a difference between learning with technology, learning via technology, and learning for technology?
- Are there age or content appropriate principles for using technologies in learning mathematics?
- Are certain mathematical concepts best learned by particular technologies or no technologies?
- Do specialized groups of students perform better with technology than others?
- What are the attributes, if any, that enhance learning with and by technology?

# THE CHALLENGE OF INCLUDING LEARNERS WITH SPECIAL EDUCATIONAL NEEDS IN SCHOOL MATHEMATICS

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Drawing on the fact that teaching mathematics to students in inclusive settings is an intricate process that involves the intersection of numerous and complex systems like an elaborate expert knowledge system; a multi-faceted assessment system and unique instructional designs and delivery needs for diverse students (Institute of Education Sciences, 2007) we propose in this DG to trigger a discussion about the multiple dimensions of teaching mathematics in inclusive settings.

We will discuss theoretical and methodological challenges of conducting research that investigates mathematical cognition of students with special needs and teacher preparation of mathematics teachers in inclusive settings. The DG will begin by addressing the following questions:

What are the different definitions and manifestations of “inclusive settings”? What are the classroom implications of working with student population with diverse disabilities? What is the international landscape of inclusion educational policies? (We will present cases from Brazil, USA and Spain and draw from international policy documents.) What is the role of technology in teaching and learning mathematics in inclusive settings?

During the first part of this DG and after a brief introduction, two invited speakers, Lulu Healy (UNIBAN Brazil) and Nuria Rosich (UB Spain) will present about their work raising questions on teaching and learning math for deaf and blind students and we will lead the discussion among participants.

The second part will start with an invited talk by Hanna Haydar (Brooklyn College USA) about the preparation of mathematics teachers for inclusive settings in NYC. Building on the first and second sessions' presentations we will broaden the discussion to include participants' questions and comments and conclude with recommendations for future steps. The DG will not be limited to the topics and questions raised here and will address other interests and issues that may be raised by the participants.

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# WORKING WITH LESS SUCCESSFUL TEACHERS

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## THE ISSUE

Research has demonstrated that mathematical misconceptions are rife in primary schools (Cockburn & Littler, 2008; Hansen, 2005). This has serious consequences for later educational performance (Alexander, 2010; Nunes et al, 2009) and attitudes (Brown et al, 2008; Williams, 2008). It is probable that many of these misconceptions originate in the earliest years of schooling (Alexander, 2010; Marchini et al, 2009). There are many reasons for this not least that foundation stage teachers are generalists, may have minimum qualifications and lack the knowledge and confidence to teach mathematics successfully (Williams, 2008). We are developing a comprehensive understanding of what it is to be expert teachers of mathematics (Ball et al, 2008) but very little is known about their less successful colleagues as research relies on volunteers.

**Session 1:** Presenting the issue of average and below average mathematics teachers and ongoing research funded by the Nuffield Foundation; exploring participants' observations on the extent and impact of such practitioners on children's progress.

**Session 2:** Planning for future research including how to gain access to the less successful and overcoming the ethical issues involved in potential studies.

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# **NATIONAL PRESENTATIONS**





# A SURVEY OF MATHEMATICS EDUCATION DOCTORAL PROGRAMS IN TURKEY

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*This paper aims to review the current state of doctoral programs in mathematics education in Turkish universities. In this context, we first provide some brief background information about higher education system and teacher education policies in Turkey. Then the major national initiatives towards future faculty development efforts are explained. Finally, the nature and components of mathematics education doctoral programs in Turkey is provided. In doing so, we provide information about admission procedures, requirements for coursework and dissertation, and employment opportunities for those with a doctoral degree in mathematics education.*

## INTRODUCTION

Mathematics education in Turkey is a relatively young field compared to other educational areas such as educational administration and educational measurement. In Turkish universities, undergraduate programs focusing specifically on mathematics teacher education began to appear after the establishment of *Yüksek Öğretim Kurulu* (YÖK) [The Turkish Council of Higher Education] in 1981. While founding these programs, the staff demand was met mostly by employing mathematicians, not by mathematics educators. Moreover, faculty in educational sciences has supported these programs in the pedagogical sense, without focusing much on the content (i.e., the mathematics). As Shulman (1986, 1987) and many others (e.g., Ball, Thames, & Phelps, 2008; Hill et al., 2008) have either suggested or provided evidence that simple combination of expertise in mathematics and educational sciences do not guarantee a sound base for quality instruction in mathematics. Instead, a pedagogical basis peculiar to mathematics needs to be established. Such a perspective calls for an audit of mathematics education programs and draw attention to the need for doctoral degrees in mathematics education for supplying quality staff for these programs.

Our goal in this paper is to describe the current state of the mathematics education doctoral programs in Turkish universities. In this respect, our goals include (1) reviewing background information that have impact on doctoral programs, (2) reviewing major national initiatives towards future faculty development efforts, (3) reviewing the nature and components of mathematics education doctoral programs, (4) exploring the characteristics of the faculty, students and graduates in mathematics

education doctoral programs, including issues such as gender, financial support type, employment, etc.

Every country is diverse and unique in its own way. For such a young field in Turkey, doctoral programs play a crucial role in the future of mathematics education. In fact, doctoral programs create contexts in which traditions of research, approaches to issues about school mathematics, and philosophical stands are produced and re-produced. In this sense, the nature of these programs and the discourse growing in these environments will have direct and indirect impact on cultural and political issues about mathematics education in Turkey through the graduates of these programs and work produced by the people involved. On the other hand, this paper contributes to our understanding of diversity in mathematics education doctoral programs on a global level in terms of components of these programs (e.g., program activities, students, faculty and/or staff, outcomes) and the contexts they function in various countries.

In studying doctoral programs, knowledge and research base that these programs may consider are crucial. In a report by the Association of Mathematics Teacher Educators (AMTE, 2002) in USA, some areas have been recommended as core knowledge expectations for doctorates in mathematics education. These are mathematics content; research; learning theories in connection to learning mathematics; teaching and teacher education, technology, curriculum, assessment, history of mathematics and its education. In terms of mathematics content, for example, it is emphasized that regardless of their mathematical proficiency at the beginning of the program, students should continue to study mathematics for developing broad and deep knowledge of pre-K-14 mathematics. Such focus to studying mathematics emphasizes approaches “learning/known mathematics for teaching” (Ball & Bass, 2000; Hill & Ball, 2004) and “pedagogical content knowledge” (Shulman, 1986).

As Reys (2008) have argued “while a list of ‘core knowledge’ may never be universally supported, it at least provides some talking points for those who have responsibility to develop and shape doctoral programs in mathematics education” (p. 78). On the other hand, as Reys (2008) indicated diversity in doctoral programs can be perceived both as strength and concern as different variations in doctoral preparation have been reported (Reys & Dossey, 2008; Reys & Kilpatrick, 2001). We think such a list of core knowledge areas does not necessarily weaken the diversity but serve as a guideline to establish more quality programs. The diversity can be realized in approaches and contents of the courses and forms of experiences. In doing this, the key is to design programs aligned with global understanding of mathematics education and particular needs and aims of the country, region, or program itself.

Wulff and Nerad (2006) present an alignment model consisting of five components as a framework for the success of doctoral programs: program activities, students, faculty and staff, desired outcomes, and the context (p. 85). Among these, “Program

activities in doctoral education consist of both the formal curriculum and the various activities of a doctoral program purposely designed to prepare students as intellectuals with the knowledge and skills appropriate for careers in their fields or disciplines.” (p. 85). As the second component, understanding students’ diverse backgrounds and changes that are taking in graduate population should be taken into account in assessing and improving quality of a doctoral program. For example, a report in Chronicle in Higher Education (Brady, 2009) indicates that in the US the proportion of doctorates in education awarded to women steadily increased rapidly in the last thirty years: 34.7%, 65.1%, 63.3%, and 67.4% in 1977, 1987, 1997, and 2007 respectively. On the other hand, “understanding of the varied faculty/staff experiences, commitments, and perspectives that advance or constrain efforts to achieve successful doctoral programs” (p. 89) should also be sought. As noted by AMTE (2002), a high-quality doctoral program requires more than a set of courses and a dissertation. The environment created within the institution to support preparation of doctoral students and faculty is also important. Along with physical and technological facilities and resources (e.g., financial support for full-time doctoral students), quality of mentorship, creating a supportive and respectful environment is desired.

In a study investigating the state of the mathematics education in Turkey, Ubuz and Aşkar (1999) reported that only 13 Turkish universities were offering mathematics education undergraduate programs as of 1997-1998 academic year. When 8 universities out of 13 were investigated in detail, it was noticed that only two universities (Middle East Technical University and Karadeniz Technical University) were offering Ph.D. degrees and only 6 doctoral dissertations had been completed. At that time, most of the faculty working in mathematics education programs received their masters and doctoral degrees in mathematics. Ubuz and Aşkar suggested that “the number of staff who do research on mathematics education should be increased” and “mathematics education should be considered as a discipline [*in its own right*] other than [*considering it as part of*] educational science and mathematics” (p. 99) to develop mathematics education as a discipline in Turkey. In Turkey, mathematics education research on graduate level has been conducted in various programs, such as early childhood education, primary education, and the educational sciences. However, our discussion in this paper is limited to the programs offering degrees in elementary and secondary mathematics education. A better understanding of the mathematics education doctoral programs is possible when they are situated within the larger context of higher education and teacher education policies in Turkey. Thus, we will begin with a brief background of Turkish higher education system and teacher education policies. Then we describe faculty development attempts before discussing specifically the nature doctoral programs in mathematics education.

## HIGHER EDUCATION SYSTEM AND MATHEMATICS TEACHER EDUCATION IN TURKEY

In Turkey, there is a unified system of higher education under the surveillance of the Council of Higher Education. In this system, there are 93 state and 38 private universities throughout Turkey (YÖK, n.d. -a), by 2010. Each university consists of faculties offering undergraduate programs. Admissions to these programs are centralized and based on a nation-wide examination conducted by *Ölçme, Seçme ve Yerleştirme Merkezi* [the Measurement, Selection and Placement Center]. Graduate programs for master's and doctoral degrees, on the other hand, are offered under the graduate schools in universities. According to the Council of Higher Education, in the 2004-2005 academic year, the total number of graduate students in Turkish universities was about 120 000 (YÖK, 2007). Of this number, 92 600 students were in master's and 27 400 in doctoral programs. No data were available for the graduate programs offering degrees specific to mathematics education.

Four year undergraduate mathematics teacher education programs were established within the faculties of education for initial training of teachers, since the establishment of the Council of Higher Education. Until 1998, mathematics teacher education programs trained teachers for both middle and high schools (grades 6 through 11). After that time, the programs were re-established so that the training of teachers for middle (grades 6 through 8) and high schools (grades 9 through 12) were separated. In fact the changes made in 1998 were beyond this separation, as teacher education programs in Turkey had undergone a major reform movement. The changes include, (i) shifting the focus in teacher education to the quality of teacher, (ii) focusing more on the middle and elementary grade levels, (iii) developing master's level programs for teacher education, (iv) focusing more on the methods of teaching relevant to specific subject matter, and (v) meeting the faculty shortage in these programs (Simsek & Yildirim, 2001).

Although this reform movement aimed to improve teacher education programs throughout Turkey, the faculty development aspect of it had significant impact on research and graduate programs in the coming years. Increased number of faculty members in mathematics education resulted in developments in graduate programs.

### Faculty Development

Although there have been some efforts to increase number of graduates having doctoral degrees, there is an acute shortage of faculty members for teaching and research in terms of both quality and quantity in Turkish higher education institutions and the doctoral programs is insufficient to supply that demand (YÖK, 2007, p. 132). To alleviate this shortage, bodies organizing and coordinating higher education such as the Council of Higher Education and the Ministry of National Education (MoNE) have taken certain actions to support students in graduate level studies according to Law numbers 2547 and 1416 respectively (Government of Turkey, 1981, 1929). Within this context, supporting students to study their graduate education abroad is

the most important initiative for faculty development. Following this attempt, another initiative for addressing shortage of faculty members in higher education is a doctoral scholarship program for studying in Turkish universities.

Both the Council of Higher Education and the Ministry of National Education have been sending selected students abroad mostly to get their doctoral degrees in various disciplines. Every year these institutions determine the number of doctoral degrees to be pursued abroad. Candidates are selected based on their academic records, foreign language proficiency, and the priority of the field in that year. Candidates are also required to have an acceptance from the host universities. Those students must successfully finish their studies and come back to Turkey to claim a position for which they have been sent for—a compulsory academic service. The amount of compulsory services is about two-year service for every year the scholarship had been received. In the case of no-returns or failure to finish studies successfully, the recipients must pay back the amount received with interest (Tansel & Güngör, 2003).

Between 2003 and 2008, 300 students who were sent abroad with the MoNe scholarship returned to Turkey after completing their studies successfully. Half of these students were education majors (e.g., elementary education, social sciences education, mathematics education, and science education—including physics, chemistry, biology education) and received their degrees from the universities in the United States ( $n = 106$ ), France ( $n = 21$ ), and the United Kingdom ( $n = 23$ ). Twenty-one of the education majors received their graduate degrees in mathematics education (7 from France, 6 from the United Kingdom, and 8 from the United States). The people who have completed their doctoral studies abroad are currently working in Turkish universities as faculty members. On the other hand, as of December 2008, 940 graduate students were studying abroad (mostly in the United States, 74.7%) with a scholarship provided by the MoNe. Hundred-and-twenty of those were in education related fields. Among them, about 23 to 27 of the graduate students were studying in the area of mathematics education.

Like the MoNe, between 1987 and 2008, the Council of Higher Education sent a total of 3899 students to thirty different countries for graduate education. In 1996, the Council of Higher Education has established a board called the Board for the Training of Academic Staff and Researchers to coordinate this program. A great portion of the scholarship students has been sent to the United States ( $n = 1941$ ) and the United Kingdom ( $n = 1454$ ). A great majority of these students ( $n = 2485$ ) finished their graduate education and started to serve as faculty in various Turkish universities. Among them 16 has received their doctorates in mathematics education, 7 from the United States and 9 from the United Kingdom. While 297 of the scholarship students are still in progress, the remaining students did not either finish their studies ( $n = 386$ ) due to reasons such as failure and health or return to Turkey (i.e., brain drain) and/or resigned from the scholarship ( $n = 731$ ). The statistics about the scholarships provided by the Council of Higher Education and the Ministry of National Education were obtained from these institutions through official

communication, and based on unpublished data. Some published statistics covering the years up to 2005 was provided in a report by *Türkiye Bilimler Akademisi* (TÜBA) [Turkish Academy of Sciences] (TÜBA, 2006).

The second leg of the faculty development efforts is the scholarship opportunities for seeking doctoral degrees in Turkish universities. These efforts have two legislative bases: first one is the Article 35 of the Turkish Higher Education Law No 2547 and the second one is the “Faculty Development Project.” The Article 35 of the Turkish Higher Education Law No 2547 (Government of Turkey, 1981) concerns the needs of teaching staff of the higher education institutions. According to that law, all higher education institutions, whether established or yet to be established, are responsible for educating future faculty members. Within this context, the Council of Higher Education designated more research assistantship positions to universities. In case of lack of faculty members and doctoral programs in a particular university, the assistantships in that particular university are allowed to be transferred to another university offering a doctoral program. Students who complete their doctoral studies return to their own universities to carry out compulsory service for a certain period of time.

A project called *Öğretim Üyesi Yetiştirme Programı* (ÖYP) [Faculty Development Programme] was initiated in 2001 with the financial support from *Devlet Planlama Teşkilatı* [State Planning Agency] in order to meet the growing need for quality faculty members in Turkish universities. The difference between ÖYP and article 35 of the Higher Education Law numbered 2547 is that besides getting their salaries for the assistantship positions, graduate students in ÖYP are supported financially to conduct their research and to study abroad for about two semesters. Middle East Technical University (METU) was the first to offer doctoral education within ÖYP to meet the faculty needs in other four partner universities. Until 2006, 19 other partner universities have joined the program and 562 ÖYP students were accepted for Ph.D. programs in 43 different disciplines. Currently there are eight ÖYP doctoral students studying in mathematics education at METU for different partner universities.

## DOCTORAL PROGRAMS IN MATHEMATICS EDUCATION

In this part, we will explain the nature of the mathematics education doctoral programs in Turkey. In doing so, we will explain general characteristics of the program of studies - including admission procedures and various requirements-, and employment opportunities for those with a doctoral degree in mathematics education. Describing doctoral programs throughout Turkey is a challenging task for several reasons. First of all, doctoral programs in different universities have different characteristics and expectations from students. Second, the characteristics of doctoral programs are rapidly changing and new programs are being established. Finally the information about doctoral programs is available in different levels – departments, graduate schools, the Council of Higher Education– which sometimes may not be aligned with others. Despite these difficulties, we attempt to summarize general



characteristics of doctoral programs by providing examples from the selected doctoral programs. More specifically, although our search in dissertation database of the Council of Higher Education (<http://tez2.yok.gov.tr>) has revealed that 10 universities have mathematics education doctoral programs throughout Turkey, we could collect data from the doctoral programs in 7 universities (see Table 1). Considering the number of dissertations completed and their geographical regions, we consider these programs represent the diversity in mathematics education doctoral programs in Turkish universities.

University / level focus of doctoral program	Year the doctoral program was established
9 Eylül University / Elementary	2003
Gazi University / Elementary	2005
Gazi University / Secondary	1990*
Selçuk University / Elementary	2010
Marmara University / Elementary	2003
Marmara University / Secondary	2009
Hacettepe University / Elementary	2003
Hacettepe University / Secondary	2002
Karadeniz Technical University (KTU) / Secondary	1996
Middle East Technical University (METU) / Elementary	2005
Middle East Technical University (METU) / Secondary	1986*

\* Approximate years reported by existing faculty members.

Table 1: The Universities that participated in this study, the focus level of the doctoral program, and the year doctoral program was established.

As can be seen in Table 1, some of the universities have two doctoral programs in mathematics education; one for elementary focus, the other is for secondary focus. Doctoral programs that focus on elementary level mathematics education started to appear after the year 2000. This is due to the reform efforts of teacher education in 1998, where more emphasis was given to elementary and middle level education.

The information provided in this section was obtained through the Web pages of universities and a survey sent to program chairs via e-mail and consisted of open-ended questions regarding the number of dissertations completed so far, the type of courses offered during the program, etc.

## Admissions to Doctoral Programs

In Turkey, all doctoral programs are considered under Ph.D. Students can apply to a Ph.D. program after having either a bachelor's or a master's degree. Doctoral programs do not require teaching experience prior to admission. Admission to doctoral program is based on applicants' cumulative grade point average (GPA) in undergraduate and/or the master's program (if attended), their *Akademik Lisansüstü Eğitim Sınavı* (ALES) [Academic Graduate Education Exam] scores or equivalent international examination scores, such as the USA-based Graduate Record Examination, level of English language proficiency and the evaluation of other criteria required and announced by the relevant department administration, such as recommendation letters or letter of intentions. ALES is given twice a year by the *Ölçme, Seçme ve Yerleştirme Merkezi* (ÖSYM) [Measurement, Selection and Placement Center]. This exam measures verbal reasoning and quantitative reasoning. The skills measured in verbal reasoning include the test taker's ability to analyze and evaluate written material and synthesize information obtained from it, to analyze relationships among component parts of sentences, and to recognize relationships between words and concepts. The skills measured in quantitative reasoning include the test taker's ability to understand basic concepts of arithmetic, algebra, geometry, and data analysis, to reason quantitatively, and to solve problems in a quantitative setting. Applicants' level of English proficiency is evaluated based on the result of either the university's English proficiency examination or the equivalent exams such as *Üniversitelerarası Kurul Yabancı Dil Sınavı* (ÜDS) [Interuniversity Foreign Language Examination] or Test of English as a Foreign Language (TOEFL). ÜDS is given twice a year by the ÖSYM. For admission into a graduate study program, the acceptable score on these exams is determined by the recommendation of the department administration and the acceptance of the Administrative Board of the Graduate School.

## Number of Faculty Members and Graduate Students

Institutions vary greatly in the number of faculty members as well as the number of graduate students. While Gazi University has 16 faculty members in mathematics education programs, METU and KTU have 7 and 4 faculty member respectively. The main reason for Gazi University having more faculty members is that, in their undergraduate teacher education program, they offer both mathematics and mathematics education courses, which is not the case in both METU and KTU. Therefore, the program at Gazi University employs faculty members with Ph.D.'s either in mathematics education or mathematics. As Table 2 shows, the number of young faculty members in most of the universities is in majority. Especially in the Elementary Education programs, there are only a few full professors and most of the faculty is in the position of assistant or associate professorship. For instance, in 9 Eylül, Marmara, Gazi, and Middle East Technical Universities there are no faculty members with full professorship. This is due to the impact of 1998 reform efforts, which aimed to increase number of staff in Faculties of Education in Turkey.

University	Prof.	Assoc. Prof.	Assist. Prof.	Instructor
9 Eylül University (Elementary)	-	1	4	3
Gazi University (Elementary)	-	1	9	2
Gazi University (Secondary)	3	1	2	-
Selçuk University (Elementary)	1	1	4	3
Marmara University (Elementary)	-	1	3	2
Marmara University (Secondary)	-	-	3	2
Hacettepe University (Elementary)	1	1	1	3
Hacettepe University (Secondary)	1	1	2	1
KTU (Secondary)	1	2	1	1
METU (Elementary)	-	1	2	-
METU (Secondary)	-	3	1	-

Table 2: Number of faculty members in the mathematics education doctoral programs in various universities

Currently mathematics education doctoral programs in these universities have 134 students in total (see Table 3). The distribution of genders of the current students and doctorates granted vary across universities. About half of the current doctorate students were females. Of these 134 students, 20 are in ÖYP program and 23 are benefitting from article 35. Thirty-nine of the current doctorate students have positions in the universities as research and teaching assistant. Taking together ÖYP and Article 35, forty-three doctoral students will return to different universities that have been assigned previously.

University	Assistant	Visiting Assistant (Article 35)	ÖYP	No Financial Support	Total
9 Eylül University (Elementary)	-	3	-	5	8
Gazi University (Elementary)	5	2	1	-	8
Gazi University (Secondary)	3	-	-	4	7
Selçuk University (Elementary)	-	-	-	5	5
Marmara University (Elementary)	1	1	1	12	15
Marmara University (Secondary)	-	1	-	8	9
Hacettepe University (Elementary)	-	2	3	4	9
Hacettepe University (Secondary)	3	-	-	1	4
KTU (Secondary)	18	6	-	-	24
METU (Elementary)	2	2	10	1	15
METU (Secondary)	7	6	5	12	30
Total	39	23	20	52	134

Table 3: Number of current mathematics education doctoral students and type of their financial support

The number of graduates from each doctoral program is given in Table 4. Since some of the doctoral programs are relatively new, there are only a few or no graduates at all. Among the graduates from the 7 universities, 39 were females and 32 were male, which means more than half of the Ph.D. degrees were granted to females. Gazi University, Karadeniz Technical University, and Middle East Technical University had the highest number of doctoral graduates in the last 10 years. As Table 4 shows, a considerable number of doctoral graduates ( $f=22$ ) received no financial support from their universities. Since ÖYP is a recent support program, currently there are no graduates supported through this program.

University	Number of graduates within the last 10 years.				
	Assistant	Visiting Assistant (Article 35)	ÖYP	No Financial Support	Total
9 Eylül University (Elementary)	3	-	-	1	4
Gazi University (Elementary)	-	-	-	-	-
Gazi University (Secondary)	3	7	-	8	18
Selçuk University (Elementary)	-	-	-	-	-
Marmara University (Elementary)	4	-	-	1	5
Marmara University (Secondary)	-	-	-	-	-
Hacettepe University (Elementary)	1	1	-	1	3
Hacettepe University (Secondary)	-	-	-	-	-
KTU (Secondary)	8	11	-	4	23
METU (Elementary)	1	1	-	-	2
METU (Secondary)	8	1	-	7	16
Total	28	21	-	22	71

Table 4: Number of the graduates from each mathematics education doctoral program and type of their financial support

### The Content and the Demand for Coursework

The course of studies in doctoral programs has two tracks; one for students with bachelor's degree and the other for students with a master's degree. Students who hold a master's degree must complete at least 7 courses – not being less than 21 credits in total – a doctoral qualifying examination, a dissertation proposal, and a dissertation. For those who have been accepted with a bachelor's degree, this program is comprised of a minimum of 42 credits or 14 courses, a doctoral qualifying exam, a dissertation proposal, and a dissertation. The maximum period to complete the course work is 4 semesters for students holding a master's degree, and 6 semesters for students accepted with a bachelor's degree. One credit hour for graduate courses represents an hour of lecture or two hours of laboratory work per week. Each academic year consists of two semesters. At the end of fourth or sixth semesters, CGPA must be at least 3 out of 4. In addition, dissertations need to be completed in four semesters. If necessary, students may use extra four semesters to complete their dissertations.

Consistent with the faculty in the program, the courses offered vary greatly in the range of mathematics, mathematics education and other topics (research courses and/or general education courses). While in KTU there are no compulsory courses, at METU some research courses such as statistics and research methods and at AU courses in science ethics and computer are compulsory. The rest of the courses are electives that are selected either by the students or by the recommendation of the supervisor. Mathematics education faculty at KTU and METU do not offer any mathematics courses, as the mathematics departments offer these courses. The official language in the universities apart from METU is Turkish. However, doctoral students at KTU are required to complete at least two courses conducted in English. All courses and dissertations at METU are conducted in English.

### **The Process and Nature of Doctoral Qualification Examination and Hereafter**

Upon the completion of the coursework, students in each university need to take the doctoral qualifying examination. Doctoral students holding a master's degree must take this exam within their fifth semesters, and students enrolled with a bachelor's degree within the seventh semester at the latest. The doctoral qualifying examinations committee established with five members, one being the student's dissertation supervisor, are responsible to prepare and administer the qualification examinations. Committee members are required to have a doctoral degree. The doctoral qualifying examination consists of a written and oral examination to evaluate students' skills in conducting independent research and their understanding of major concepts and issues in the field.

Doctoral students conduct their dissertation research under the supervision of a faculty member who is an assistant professor or above. If needed a co-supervisor may be appointed. A dissertation supervising committee consisting of three faculty members is also appointed upon successful completion of the doctoral qualifying examinations. Within six months after the qualification exam, each doctoral candidate needs to prepare and defend a dissertation proposal to a committee consisting of three members including the dissertation advisor himself/herself. Candidates are expected to prepare a doctoral dissertation demonstrating somehow an original contribution to the field of mathematics education.

### **Graduates with Mathematics Education Doctoral Degrees**

In Turkey, the majority of doctoral graduates in mathematics education seek positions in the higher education. For example, based on our survey, we have found out that among 71 graduates from all of the 7 universities, 56 were working as faculty members in higher education, 13 were working in the schools or Ministry of National Education and 2 were working in a non-educational business. Those employed in higher education assume a range of teaching responsibilities including teaching mathematics and/or mathematics education courses offered in the program and conduct research in the field. Considering the number of students progressing in

doctoral studies, we assume that the graduates of doctoral programs will contribute not only to higher education, but also to other institutions, such as the Ministry of National Education, schools, or private companies in the future.

## **CONCLUSION**

In Turkey, about 35 million people (about half of the population) are under the age of 28 (Türkiye İstatistik Kurumu [Turkish Statistics Institute], 2009). Considering that the population growth rate is about 1.013% per year, the need for education is a growing demand in Turkey. In recent years the government and the Council of Higher Education have a determined policy to increase the number of higher education institutions in Turkey. For instance, in 2006, fifteen new public universities were established throughout Turkey. Currently, 62 public universities and 5 private universities have faculty of education. With the trend of establishing new universities, there is an increasing need for faculty development. In this sense, the demand for staff is still evident for the coming few decades. Since some universities especially the newly founded ones are still in the process of recruiting new faculty members, most of which have the potential to employ the graduates of mathematics education doctoral programs. The statistics we provided in this paper demonstrates that, in recent years, there have been an increasing number of students pursuing doctoral degree in mathematics education. However, considering the rapid increase in the number of higher education institutions, we argue that more efforts are needed to meet the need of growing faculty need. One pathway may be to find ways to enhance the possibilities for pursuing a doctoral degree in mathematics education. Another one is to explore the benefits of doing national or international collaborations in developing new programs or improving the existing ones.

It appears that most of the mathematics education doctoral graduates in Turkey seek jobs in higher education institutions and few take jobs K-12 teaching and other areas. Considering the number of doctoral graduates and students currently progressing, the mathematics education will develop in the next decade in terms of the number graduates and the research. However, it seems that job opportunities will play critical role for the future of the doctoral programs in Turkey. Number of doctoral programs is increasing. Yet, in the future, job opportunities or further opportunities for those with Ph.D. may not increase in the same rate. The rapid increase in the demand of doctoral degrees in mathematics education will eventually reach to saturation in terms of the need of faculty in the coming decades. Therefore, we believe that, it is just the time for policy makers to think about planning the supply and demand for mathematics education doctoral degrees, by considering all employing alternatives. It is also worthwhile to state that the gender balance between the male and female students in mathematics education doctoral programs in Turkey may be taken as indication that mathematics education, at least on the research level is not a male dominated area and females are not underrepresented.

While dealing with the quantity issue, another challenge for the doctoral programs in Turkey is to maintain and improve the quality. Considering the varieties in workplace of graduates, universities should try to increase the diversity in the coursework and graduate research within the mathematics education doctoral programs. In addition, doctoral programs should also try to put more efforts in offering a program of study that reflect the nature of mathematics education in variety of ways. Finally, creating a productive research community of mathematics educators that is well integrated with their international counterparts should be a major goal for the doctoral programs.

Even the Council of Higher Education has general criteria to open graduate programs in universities (YÖK, n.d. -b), the principles to guide the design and implementation of doctoral programs in mathematics education could be established to provide a number of ideas and suggestions regarding doctoral programs in mathematics education. Besides, national conference on doctoral programs in mathematics education could be set to develop the ideas and suggestions regarding doctoral programs in mathematics education.

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# MATHEMATICAL THINKING RESEARCH BY TURKISH COMMUNITY

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*In this article we give a brief overview of the present state of research conducted by Turkish community on mathematical thinking. Particularly, the goal of this paper is to share some prototypical examples of research conducted in mathematics education by the Turkish mathematics education community, around the theme of mathematical thinking and supporting its development. The research on mathematics education is quite a recent achievement. To review the research on mathematical thinking mathematics educators' studies were researched extensively. Later a series of content analyses of the research publications were conducted to determine the contribution of Turkish researchers on mathematical thinking. Our examination of the selected research studies reveals that the Turkish researcher has contributed rarely to the process aspect of mathematical thinking and extensively to the product of this process and the issues that support or hinder the development of mathematical thinking.*

## INTRODUCTION

The systematic investigation of the articles published in academic journals and conference proceedings is important to see the present situation and future trends in mathematics education. There are, however, a few studies in Turkey investigating systematically education journals publishing mathematics education research studies (e.g. Ulutas & Ubuz, 2008). Ulutas and Ubuz (2008) conducted a series of content analyses of the articles published by *Eurasian Journal of Educational Research*, *Hacettepe University Journal of Education*, *Elementary Education Online*, and *Education and Science Journal* from 2000–2006, in terms of the language, research style, sample, research topics, subject matter, method of the study, the institution of the author, the region of the study, and the data collection procedures. According to the findings, most studies were conducted either with elementary students or pre-service teachers, in cognitive and affective domains. Additionally most studies were experimental and quantitative in nature using tests and questionnaires. Another study investigating the current state of the mathematics education community in Turkey considering their research studies and faculty (Ubuz & Aşkar, 1999) reported that the international publications have been done either by the faculty of the English medium universities or by the faculty who completed their PhD degrees in overseas universities on mathematics education. The examination of the publication titles revealed that the research areas studied were: difficulties, computers in mathematics

teaching, problem solving as a teaching method, attitudes towards mathematics, math anxiety, measurement, and so forth. As seen, mathematical thinking, mathematical processes, was not the main concern in the research studies. The mathematical processes such as modeling, inferences (e.g. analogies), conjecturing, defining, formalizing, generalizing, proving, abstracting, visual reasoning, and so forth can be considered as manifestations of mathematical thinking and relate to the learning of mathematical concepts (Ubuz, 2010). This reveals that studies focused mostly on things stem from mathematical thinking processes (e.g., misconceptions, knowledge, etc), and things that support or hinder the development of mathematical thinking (e.g., tasks, instructional strategies, teachers, etc).

The purpose of this study is to investigate the research studies in mathematics education by the Turkish community in the context of mathematical thinking in order to give a picture of different aspects of mathematical thinking. The aspects of research on mathematical thinking convey the state of research in the following areas: (i) mathematical thinking processes, (ii) things stem from mathematical thinking processes, and (iii) things that support or hinder the development of mathematical thinking. The paper does not at all cover everything on Turkish mathematics education research but share some prototypical examples.

## **RESEARCH ON MATHEMATICAL THINKING**

In order to prepare this paper, around eighty research papers published in various international and national journals as well as some conference proceedings were collected through searching Web of Science and ERIC databases. Papers were grouped in three aspects of mathematical thinking: (i) mathematical thinking processes, (ii) things stem from mathematical thinking processes and (iii) things that support or hinder the development of mathematical thinking. We will discuss these three main aspects by focusing on the concerns as well as the findings, theoretical perspectives, and methodologies used.

### **Mathematical thinking processes**

Mathematical thinking processes of various samples ranging from elementary students to pre-service mathematics teachers were the subject of several research studies. These studies focused on participants' mathematical thinking processes via the investigation of particularly identifying and defining, abstraction, generalization, reasoning and modeling. Each of these topics with the relevant studies is detailed below.

**Identifying and Defining:** Ubuz and Üstün (2004) examined, using face to face interview, the process of interaction between figural and conceptual aspects in identifying and defining process of polygons, squares, rectangles, and parallelograms. Analysis of the results revealed that (a) students often use prototypic figures but do not consider them as exclusive, (b) non critical attributes of a concept given in a figure leads to difficulties in identifying concept examples, and (c) students' own definitions include non-critical attributes of the concepts as well as their critical

attributes-the attributes that each example should have in order to belong to the category. All these mentioned above are quite prevalent among “above average”, “average”, and “below average” achievers identified by their mathematics teachers. Based on the results, authors argued that prototypical figures, assigning different names to the concepts and the non-critical attributes of the concepts play an important role in identifying and defining process.

**Abstracting:** As an important aspect of mathematical thinking process, abstraction received attention from Turkish researchers especially within the last decade. The issue of abstraction is investigated at different grade levels including primary, secondary and upper secondary levels with different mathematical topics, such as absolute value functions and greatest integer functions (Monaghan & Ozmantar, 2006; Ozmantar & Monaghan, 2007; Yeşildere & Türnüklü, 2008; Altun & Yılmaz, 2008). The researchers tend to study abstraction process via qualitative approaches and mainly with case studies in order to perform an in-depth analysis of this process. All the researchers have a unified understanding of abstraction as a process leading to the product. Abstraction process appears to have received more attention though the product aspect was not ignored. The research findings suggest that abstraction of mathematical concepts can be achieved by the recognition and use of old-knowledge structures, which then need to be put together in order to construct new ones (see also Hershkowitz et al., 2001). Monaghan and Ozmantar (2006) further added that construction of knowledge is a demanding task and immediately after construction new knowledge structures are fragile in nature and hence in need of consolidation. Therefore the studies considered abstraction as a process which includes the construction and consolidation of new mathematical structures. Only after the consolidation could the newly constructed structure be used for further abstractions. Ozmantar (2005) provided evidence that knowledge construction and consolidation process are dependent on the context which includes the learning task, student personal histories, their interaction with both the immediate participants of the instructional settings as well as the ones distant in space and time.

Altun and Yılmaz (2008) focused on students’ understanding of specific functions such as integer-value and piece-wise functions within the framework developed by Hershkowitz et al. (2001). They conducted a case study with two ninth grade students where the students were asked to solve three sequential problems which are suitable for giving them the chance to use their previous experiences and knowledge as much as possible. The first two problems were about piece-wise functions and the third problem was about integer-value functions. It was observed that the students used the knowledge they had gained from the first problem on piece-wise functions to solve the second and the third problem and they properly acquired the knowledge of piecewise continuous functions and the greatest integer functions. The findings of their study also showed that the use of real life problems can make a strong contribution to the process of abstraction in the context of integer-value functions.

**Generalization:** Kabael (in press) examined students' generalization process of functions from single variable to two-variable functions. The author conducted her study with 23 undergraduate students enrolled to Calculus II course in an elementary mathematics education program. The participants received instruction on both single and two-variable functions respectively. The author administered a test before performing single variable instruction during which function machine was employed as a cognitive root. Later two-variable functions were introduced to the students. During this process, function machine and multiple representations were used for the instruction purposes. Upon the completion of the course, the participants received another test on two-variable functions. The author also selected six students with different levels of understanding and examined them closely via in-depth interviews. The data regarding participants' generalization processes were analysed within framework of APOS theory (see Aisala et al., 1996). One of the conclusions of the study is that students did not experience considerable difficulties converting different representations of single and two variable functions; yet many students find it difficult to grasp the graphical representations of two-variable functions. The author also noted that students may show different levels of understanding depending on the number of variables in a function. It was found that especially being at the object level for the function concept is crucial to acquire process conception of two-variable functions. The research also indicates that students' comprehension of function concept and of three-dimensional space play a crucial role for the development of two-variable functions.

**Reasoning:** Visual reasoning is an important aspect of mathematical thinking process. Some of the Turkish studies focused on spatial visualization ability of students or pre-service teachers. Yeşildere and Türnüklü (2007) investigated eighth grade students' mathematical thinking and reasoning processes. Using survey method, they gathered data from 262 students who graduated from of 20 different schools. Survey included open ended problems and students' responses were analysed both quantitatively and qualitatively. Findings showed that majority of students had difficulties in solving open-ended problems. Their reasoning processes were mainly based on their personal point of views, not on given data in the problem. It was also found that students had struggled in conjecturing, connecting ideas and communicating through their reasoning.

Sevimli and Delice (2011) investigated the influence of spatial visualization ability on the use of multiple representations of definite integrals. They used the Purdue Spatial-Visualization Ability Test (PSVT) to determine the levels of 45 teacher candidates' visualization abilities. Results indicated that as spatial visualization ability increases, use of algebraic representation decreases. Teacher candidates who had low spatial visualization ability used predominantly algebraic representations and less graphical representations.

Akkuş-Çıkla and Duatepe (2002) examined pre-service elementary teachers' proportional reasoning and the strategies they used in the problems involving ratio

and proportion. 12 first year pre-service elementary teachers were interviewed for the study. Problems which were posed to the interviewees involved comparison of the relative size of two parts in a common whole, comparison of different quantities, and reasoning about similarity of figures. In the problems involving comparison of different quantities, all pre-service teachers, except one, gave correct response by either fixing given quantities at a specific value or finding unit rate. In the similarity problem, all but one participants correctly responded by using cross multiplication algorithm in the first part and unit rate in the second part. At the end of the interview, the researchers asked pre-service teachers some conceptual knowledge questions as well: “What is ratio?; What is proportion?; Are ratio and proportion the same?” They found that pre-service teachers who were able to solve the problems using cross multiplication method could not respond to these questions completely and correctly.

Duatepe et al. (2005) investigated 295 sixth through eighth grade students’ solution strategies across different problem types in the questionnaire including problems that require proportional reasoning. The problem types involved finding a missing value, quantitative comparison, qualitative comparison, non-proportional comparison and inverse relation. Analysis of students’ solutions indicated that solution strategies varied across different problem types. The most common strategies used by the students were cross multiplication algorithm, unit rate, additive strategy, and inverse proportion algorithm in the problems involving missing value, quantitative comparison, non-proportional comparison and inverse relation respectively. Although there were various strategies that students used the most in solving different problem types, the cross multiplication algorithm, which is traditionally taught in the curriculum, was found to be a common strategy across all types, except qualitative comparison problems.

Isiksal et al. (2010) examined the 8th grade students’ reasoning about measurement. 271 eighth grade students from five different schools were given a survey task in which they were asked to explore the relationship between the lateral surface area and the volume of a cylinder. Although most students successfully calculated the lateral surface areas for both cylinders by employing the formula, they mostly failed to compute the volumes of the cylinders correctly. The researchers claimed that this computational failure could be due to the lack of students’ understanding of the relationship among the geometric concepts. On the questions involved students to reason and make a generalization, the percentage of correct responses was very low.

**Modeling:** Bukova-Güzel (2010a) focused on pre-service teachers’ modelling processes in the context of problem solving. In a case study with 35 pre-service teachers taking the Mathematical Modeling course, she examined pre-service teachers’ approaches in their experiences of constructing mathematical modelling problems and the ways in which they perform the modelling process when solving the problems they construct. The results revealed various criteria used by participants when constructing modelling problems such as relevance to life, the possibility of being solved by mathematical knowledge and attracting attention. Participants were

found to be successful in understanding and simplifying the modelling process, however, they had difficulties in the interpreting and validating steps.

### **Things Stemming From Mathematical Thinking Processes**

Studies on this domain vary in terms of subjects they focused and methodological approaches they embraced as well as mathematical concepts under investigation. Methodologically, both quantitative and qualitative approaches have been adopted. Mostly, tests including open ended-questions and problems were used to investigate students', pre- or in-service teachers' understanding of mathematical concepts. Sample size ranged from 20 to 1500. Other studies used case-study method for a qualitative investigation. In terms of mathematical concepts, a close examination of the relevant literature indicates that Turkish researchers, as parallel to the literature at an international level, investigated difficulties with mathematical concepts mostly at primary and secondary levels. Among these studies students' difficulties with arithmetic concepts and algebra concepts came into prominence. Arithmetic concepts such as numbers, decimals, estimation, fractions and algebra concepts such as solving equations and inequalities, and patterns were the focus of attention. In addition to the domains of numbers and algebra, understanding of probability and geometry were also studied in Turkish context at the secondary level. At a more advanced level, mathematical concepts such as functions, radian, derivative and integral were under investigation. Below we report these studies in detail under three subheadings: number, algebra and advanced mathematical concepts.

**Numbers:** Research findings on numbers indicate that students have difficulties with constructing number sense, ordering rational or irrational numbers, performing arithmetic operations (Şandır, Ubuz, & Argün, 2007) and making estimations (Çilingir & Türnüklü, 2009). Şandır, Ubuz, and Argün's (2007) study revealed that 9th grade students made many errors in arithmetic operations and in using distributive laws, and ordering rational or irrational numbers. Another difficulty with numbers is observed in the domain of decimal numbers. Ubuz and Yayan's (2010) findings displayed that primary teachers seem to have most of the difficulties that are revealed in previous studies in scale reading, addition and subtraction of decimals and ordering decimals. For example, the findings related to difficulties in ordering decimals appear to suggest that primary teachers seem to use the rule –'select as smaller the number that has more digits in its decimals portion'–while ordering decimals having different number of decimal digits.

Making estimations is also an important aspect of number sense. Çilingir and Türnüklü (2009) investigated estimation strategies of primary school students of 6th-8th grades. The levels of estimation ability and strategies used by students were determined. Twelve strategies were estimation depending on the existing knowledge and the experience, visualizing, decomposition, comparison, estimation through experiments, rounding, accommodating, distribution, the using of front-end orders, grouping, mental calculation, and randomly made estimates.



Fraction has also attracted many mathematics educators in Turkey. Findings in this domain pointed out various difficulties with fractions such as a lack of understanding of the relationship between part and quantity (Haser & Ubuz, 2003; Zembat, 2007), difficulties in dealing with more than one unit (Haser & Ubuz, 2003), difficulties with operations with fractions and ordering them (Haser & Ubuz, 2002) and difficulties with simplification of fractions (Ozmantar & Bingolbali, 2009).

Haser and Ubuz (2003) and Zembat (2007) investigated students' and pre-service elementary teachers' conceptions of fractions in the context of word problems. Zembat's (2007) study focuses on reasoning styles of 153 pre-service elementary teachers while solving open-ended problems about division of fractions which embody several connected concepts such as division, fractions and unit. The findings suggested that pre-service elementary teachers faced serious difficulties when thinking about division of fractions. Main sources of difficulties as noted by the author were (1) participants' dependency on the part-whole meaning of fractions, (2) their oscillation between multiplication and division, (3) their rote attachment to the division procedure as "invert-and-multiply" algorithm.

Haser and Ubuz's (2003) study on fractions focused on students and investigated their conceptions of fractions in solving word problems about the part-whole concept in fractions, where part is usually given as a quantity. The test including routine and non-routine problems were administered to the students. The reasons provided for a wide range of errors were that (i) students do not understand the relationship between part and quantity, (ii) students are not aware of the resultant units of an operation, (iii) students have difficulties in dealing with more than one unit. In sum, the analysis of the difficulties met by the students evidenced the crucial role of the meanings of part and quantity, and the units in the operations. Haser and Ubuz (2002) investigated 5th grade students' performance on fractions as well as their difficulties. Students performed differently in conceptual questions depending on the fractions included. The lower performance was on integer fractions. Students' performance in subtraction and multiplication of different kinds of fractions was lower than the performance on the addition of different kinds of fractions.

**Algebra:** Another topic domain which attracted Turkish researchers with regard to misconceptions and difficulties at secondary level is algebra. Understanding of algebra concepts such as equations and inequalities were investigated by Şandır, Ubuz and Argün (2007), and Erbaş, Cetinkaya and Ersoy (2009). They both focused on ninth grade students' difficulties with solving linear equations. Erbaş, Cetinkaya and Ersoy (2009) investigated 217 ninth grade students' difficulties, errors, and misconceptions in solving simple linear algebraic equations. The students' errors were mostly attributed to algebra mal-rules which stem from some algebra misconceptions (e.g., overgeneralization of the commutative property to subtraction) that students might have. Şandır, Ubuz and Argün (2007) focused on solving inequalities as well as equations. They found that ninth grade students had difficulties

with solving multi-inequalities. Students made errors especially in adding numbers to both sides of inequalities.

Another topic of algebra which was studied by Turkish researchers is patterns. Akkuş and Çakıroğlu (2006) investigated seventh grade students' use of multiple representations in pattern related algebra problems and the reasons for their preferences for certain representations. Nine of twenty-one students preferred using equations for all problem types and only four students used tabular representations for all problem types. Seven of them preferred different modes of representations depending on the problem type. The findings indicated that students' preferences for different representation modes depend on the task as well as their perception of a representation and familiarity with the certain representation types.

In addition to the domains of numbers and algebra, understanding of probability and geometry were also studied in Turkish context at the secondary level. Memnun, Altun and Yılmaz (2010) investigated the understanding levels of the concepts related to probability. The results of the study revealed difficulties in the interpretation of some of the probability concepts in understanding and using the sample spatial concept and in doing probabilistic reasoning. Ubuz (1999a), on the other hand, focused on 10th and 11th grade students' understanding of angle concept in geometry considering their errors and misconceptions. Errors can be summarized as follows: (i) students focus on the figure itself rather than its properties; (ii) students do not know the meaning of a triangle and the properties of its exterior and interior angles.

**Advanced mathematical concepts:** Above we reported prototypical examples of studies which investigated students', teachers' or pre-service teachers' difficulties with mathematical concepts at primary and secondary levels. In what follows, we will focus on studies which investigate students and/or teachers' understanding of mathematical concepts at a more advanced level. Among these studies, mathematical concepts such as radian (Akkoç, 2008; Topçu et al., 2006), derivative (Gul & Barak, 2007; Ubuz, 1996, 1999b, 2001, 2007; Bingolbali & Monaghan, 2008, Kabaël, 2010) and integral (Ubuz, 1996) were under investigation.

Derivative concept has been a focus of attention in Turkish context. Research on derivative concept in Turkey revealed difficulties with graphical understanding of derivative (Ubuz, 1996, 1999, 2001, 2007; Gür & Barak, 2007), understanding of limit definition of derivative, finding the derivative of composite functions (Gür & Barak, 2007) and applying the chain rule (Kabaël, 2010).

Ubuz (1999, 2001, 2007) based on her Ph.D work (Ubuz, 1996) reported first year engineering students' errors and misconceptions on calculus concepts, particularly differentiation. Students had some common misconceptions as follows: (a) derivative at a point gives the function of a derivative, (b) tangent equation is the derivative function, (c) derivative at a point is the tangent equation, and (d) derivative at a point is the value of the tangent equation at that point (Ubuz, 2001). Gür and Barak's (2007) study also focus on students' errors or misconceptions of derivative. The

findings showed that students displayed difficulties in understanding the limit definition of the derivative, produced erroneous answers for the composite and trigonometric functions and find it difficult to comprehend the slope of the tangent line at a point. Kabael (2010), on the other hand, focused on understanding and development of applying the Chain Rule. The study revealed pre-service teachers' difficulties with the Chain Rule when this rule is applied to the abstract composite functions especially while taking the second order derivatives.

Bingölbali and Monaghan (2008), on the other hand, investigated undergraduate students' understanding of the derivative concept with regard to the context of learning as paramount. They examine mechanical engineering and mathematics students' conceptions of the derivative and show that mechanical engineering students' concept images developed in the direction of rate of change aspects of the derivative whilst mathematics students' concept images developed in the direction of tangent aspects.

Another concept under investigation is a specific concept in trigonometry, namely the concept of radian. Akkoç (2008) and Topçu et al. (2006) have revealed pre-service mathematics teachers' concept images of radian and possible sources of such images. In their multiple-case studies, they have found that participants did not consider the domain of trigonometric functions as real numbers, in other words in radian measure. Furthermore, due to the dominant concept images of degree measure participants had two distinct images of  $\pi$ :  $\pi$  as an angle in radian and  $\pi$  as an irrational number.

### **Supporting or Hindering the Development of Mathematical Thinking**

There are various factors that foster or inhibit the development of mathematical thinking. Some of these are relevant to learner variables and some are pertinent to instructional or teacher variables. The research on learner variables includes topics such as metacognition and individual differences (i.e., gender, prior achievement, and so on). The studies on instructional variables have a larger spectrum of topics including mathematical tasks, pedagogical approaches, including different teaching methods, and the use of technology. Another largely focused area of research is relevant to teacher variables including affective variables (i.e., efficacy, attitude, beliefs, anxiety, self-concept etc) and teacher knowledge. In these studies researchers utilized quantitative and qualitative methodologies or mixed methods. Next we briefly review selected studies relevant to each topic mentioned above.

**Metacognition:** Both cognitive processes and metacognition are important aspects of thinking mathematically (Schoenfeld, 1992). While the former refers to process of acquiring knowledge and understanding which was discussed in the earlier sections, the latter refers to learners' awareness and understanding of their own thinking processes. The research on metacognition in mathematics education research goes back to early 1980s, particularly in connection with the research on mathematical problem solving (Schoenfeld, 1992). More recently, some research conducted by the Turkish scholars focus on the issues of the interplay between metacognition and

mathematical knowledge as well as the role of metacognition in mathematical problem solving.

Regarding the relations between metacognition and geometrical knowledge, Aydın and Ubuz (2010) conducted a structural equation modelling study with 10<sup>th</sup> grade students. It was used to estimate and test the hypothesized effects of two metacognitive constructs (knowledge of cognition and regulation of cognition) on three knowledge constructs (declarative, conditional, and procedural knowledge) together with the interrelationships among these three knowledge constructs. Declarative knowledge was on measuring students' knowledge of definitions, facts, and symbols; conditional knowledge on measuring their knowledge of relational rules; and procedural knowledge on measuring their knowledge of procedures and algorithms. Major findings from the model indicated: (a) a reciprocal relationship existed among declarative, conditional, and procedural knowledge; (b) knowledge of cognition had a positive direct effect on procedural knowledge and a significant but negative direct effect on declarative knowledge; and (c) regulation of cognition had a positive direct effect on declarative knowledge and a significant but negative direct effect on procedural knowledge.

In the context of problem solving, Erbas and Okur (2010) investigated students' problem solving strategies, problem solving episodes, and metacognition and interplay of these during problem solving processes. Five high school freshmen worked on ten problems selected among mathematical literacy items used in Programme for International Student Assessment (PISA) 2003. The data were obtained from clinical interviews and a self-monitoring questionnaire filled by the participants. The results showed that successful problem solvers were the ones who could use different types of strategies efficiently. It was also found that in spite of their difficulties in every episode during problem solving, students were able to use their metacognitive skills to detect their mistakes or missing parts of the process.

**Individual Differences:** Personal variables such as gender, age, and so on are other possible factors associated with students' mathematics achievement that have been investigated by some researchers. For instance, Ubuz (2011) and Ubuz and Kırkpınar (2000) examined how the personal variables (gender, age, prior achievement and the academic major) of the students were associated with their success in calculus. More specifically, they sought answer to the following question: What is the extent to which personal variables account for student success in calculus? First study dealt with integral, sequence, series and the second study with derivative. Gender was a factor only in students' success related to Riemann Sum. That is, gender differences depend on the content and the cognitive level of the questions. Although age, academic major, and university entrance examination were not found as important factors in students' success in integral, sequence, and series, university entrance examination score was the predictor for success on the problems related to the use of definition of derivative but not on the problems related to the graphical interpretation. In both studies it was indicated that pre-test score was the best predictor of success on

the overall post-test score. This means that what the learner already knows is the important factor influencing learning

With taking account gender differences in students' answers to procedural questions on angles, the findings of Ubuz (1999a) have shown that male students were more radical compared to female students in approaching to the questions. That is, majority of male students either solved the questions correctly or left untouched. In addition female students were more successful compared to male students and there was an increase in achievement level of the students due to educational level

**Mathematical Tasks:** In developing mathematical thinking the use of tasks in mathematics instruction is of an importance as well. In this subsection we focus on examples of studies that examined the different aspects of mathematical tasks that might affect student performance, such as task format, nature of instructional tasks and problems in terms of cognitive level required.

In a cross-cultural comparative study with pre-service elementary students (Smith, Gerretson, Olkun, Akkurt, & Dogbey, 2009; Smith, Gerretson, Olkun, & Joutsenlahti, 2010) researchers investigated how mathematical word problems with spatial content in different formats improve student performance (Smith et al, 2010). The different formats are (1) standard (minimal verbiage), (2) potential causation (causal and mathematical content overlap), and (3) climax resolution (causal and mathematical content combined in a way that the outcome is perceived by the student). In one experiment in the USA and another in the USA, Finland and Turkey, undergraduate elementary education majors worked word problems in these three formats. The results indicated that using causal story elements in word problems with spatial content, written in the USA, improved performance in American and Finish students, but not Turkish students. The Turkish students did significantly better with the standard format on one of the problems. The authors conclude that these results were due to the fact that Turkish students face more standard (sterile) word problems than realistic, real life problems during their education with the influence of nationwide examination.

The school mathematics curriculum is also a possible source of various tasks that are used in the instructional settings. Within this context, Ubuz, Erbaş, Çetinkaya and Özgeldi (2010) analysed the nature of instructional tasks in the new elementary mathematics curriculum guidebook by exploring the level of cognitive demands (LCD) in the algebra tasks. The term "algebra task" refers to an activity or set of questions in the curriculum that has been written with the intent of focusing students' attention to a particular idea in algebra. The framework used classifies mathematical tasks as lower level and higher level demands. While the lower level demands are related to memorization and procedures without connections, the higher level demands are related to procedures with connections and doing mathematics. The findings revealed that 60% of algebra tasks for each grade level required higher LCD and a great majority of the remaining tasks were at the level of procedures without

connections. These findings show that the mathematical curriculum guidebook, in general, encourages the development of student reasoning, complex and non-algorithmic thinking, and problem solving skills. All the activity examples provided in the guidebook under algebra content was also classified considering proof scheme framework (Ubuz, 2009). Based on the analysis, three categories of crucial experiment and generic example: constructive, analytic, and intellectual were present in generating of conjecture and devising a justification.

**Pedagogical Approaches:** Within this topic, research mainly focuses on the effectiveness of particular ways of introducing mathematical concepts in teaching, i.e. the use of examples and analogies, and the use of different methods of teaching mathematics, such as creative drama, problem solving, etc.

Bayazit and Ubuz (2008) reported the effectiveness of analogies in the teaching and learning of the function concept by examining an experienced teacher's analogy-based teaching approach. The study was a qualitative case study using classroom observations and semi-structured interviews as the main sources of data. The findings indicate that instructional analogies cannot support students' understanding of the functions unless students are given epistemologically appropriate analogies and illustrated the structural relations between the analogies and the targeted concepts. Provision of analogies to emphasize procedures, algebraic or otherwise, may confine students to a limited way of thinking about the concept.

Ubuz, Eryılmaz, Utkun and Bayazit (2009) also responded to a need among mathematics educators for insight into the nature of analogies in function concepts and guidance on how to construct ones that are pedagogically effective by investigating (1) how the pre-service teachers manage with the analogies they introduce and (2) whether the analogies are relevant. Ubuz et al. analysed the analogical models constructed by the pre-service teachers in terms of whether the analogies constructed are epistemologically appropriate to illustrate the essence and the properties of the functions as well as the structural relations between the analogies and the targeted concepts. Similarities as well as the break down points between them are important aspects of mapping the analogies to the target concepts. The way the pre-service teachers used analogies could fall short of helping students develop epistemologically correct and conceptually rich knowledge of function due to two reasons. First, the source analogies were epistemologically inappropriate to illustrate the essence and the properties of the functions. Second, the analogies were epistemologically appropriate to illuminate the function concept, yet the teacher did not establish the mappings between the two.

In another study, Bayazit, Ubuz and Aksoy (2009) examined teachers' selection and resolution of function problems through classroom observation and document reviews and related this to their students' understanding of the concept. The results showed that procedural problems, when implemented with little connection to underlying meanings, are likely to confine students' understanding to an action

conception of function and create misconceptions. Conceptual problems that engage students with the concept of function, its properties, and related sub-ides were considered to pose cognitive demands.

As mentioned before, some studies focused on different methods of teaching mathematics. For instance, Olkun, Sahin, Akkurt, Dikkartın and Gülbağcı (2009) worked with third through fifth grade students' to support their problem solving strategies such as modelling a simple case of the problem, looking for a pattern, and generalizing to a similar problem. A pretest-posttest experimental design was used. Each of the tests involved a non-routine word problem similar in terms of structure and difficulty level. After the pretest, an intervention was conducted over a week. During the intervention, students were engaged in worksheets that involve solving three similar non-routine problems with pictorial modeling and one problem similar to the posttest item. The pretest results showed that majority of the students failed to solve the problem correctly. It was found that in general students were more successful on problems with small numbers since they used informal counting strategies. After the intervention, there was some improvement in students' correct solutions (16% correct). In particular, the fifth graders' performance was improved more than the third and fourth graders' performance. It seemed that the fifth grade students benefited further from the modeling approach introduced during the intervention.

The relative effectiveness of the Problem-Solving Method with Handout Material related to the concept of maximum-minimum in calculus on students' knowledge acquisition was reported (Ubuz & Ersoy, 1997) based on Ubuz's (1991) master thesis. The handout material was prepared by taking into account Polya's problem-solving stages (see Ubuz, 1991, 1994). It included General procedure for solving max-min word problems, Comprehension Guide, Problems with unfinished Steps. General procedure for solving max-min word problems aimed to improve general understanding of word problem-solving, but more specifically is to teach how to start to think about max-min word problems; to help students develop a general plan for attacking a max-min word problem. In solving word problems, it is necessary to focus attention on the process rather than on the answers of the problem. The comprehension guide aimed to help students understand the problem by identifying the unknowns; the data; and the conditions. Problems with unfinished steps were those problems included in the handout material with unfinished solution steps. Solution steps were filled in through discussion with students after each one's comprehension guide is discussed. Handout material contained seven problems of this type. The results showed that the Problem- Solving Method with Handout Material can significantly help students understand and learn word problems compared to Traditional Lecture Method. Mean differences in favor of the Problem Solving Group students may imply that the students taught with the Problem-Solving Method with Handout Material have successfully achieved the aforementioned steps in the problem-solving process.



Another rising topic in mathematics education research in Turkey is the role of drama-based instruction on developing mathematical knowledge and thinking. Researchers (e.g., Isler & Isiksal, 2010) studied the development of pre-service teachers' mathematical knowledge through creative drama approach and argued that the approach strengthened their mathematical content knowledge as well as pedagogical content knowledge. Moreover, Duatepe-Paksu and Ubuz (2009) investigated the effects of drama-based instruction on students' geometry achievement, geometric thinking level, attitudes toward mathematics and geometry, and retention of achievement, in comparison with traditional teaching. Multivariate analyses of covariance revealed that drama-based instruction had a significant effect on students' achievement, retention of achievement, thinking level, and attitudes, regardless of gender, mathematics grade in previous year, and prior attitudes and thinking levels. The teacher point of view about drama based mathematics instruction also confirmed the above points (Duatepe-Paksu & Ubuz, 2007). The teacher, however, pointed out that the burden drama based instruction brings to the teacher. They were preparing lesson plans and materials, arranging the classroom environment and solving less number of mathematics problems.

**Technology:** Another way to mediate student learning is to use various forms of technology in mathematics instruction. The studies we reviewed vary in their primary focus on the use of technology, such as how technology supports students' conceptual understanding or mathematical knowledge and mathematical thinking processes such as visualization.

Some of these studies focused on using web-based learning materials to promote 9<sup>th</sup> grade students' learning of mathematical concepts in their classroom-based research using both qualitative and quantitative data collection methods with students and their teachers (Baki & Güveli, 2008; Baki & Çakıroğlu, 2010). In Baki and Güveli (2008) one of the teachers taught the control and experimental groups in which web-based mathematics teaching (WBMT) materials were used. Findings suggest positive effect of WBMT on student learning of mathematical function and on attitudes towards WBMT. Although the teachers shared some concerns about the technical problems and readiness of teachers and students in using WBMT in schools, the results provide support for the use of WBMT material as a complement to traditional approaches. Similarly, Baki and Çakıroğlu (2010) investigating the use of the Learning Object Repository (LOR) by the students, their evaluations, attitudes and views towards these learning objects (LO) as well as teacher's use of them stated that students provided positive comments about the motivational and learning themes. For instance, they indicated that LOs motivated students to take more responsibility in their learning process. For the teacher, LOs provide interesting scenarios and problems helping to comprehend the concepts related to the subject, and can be used to create rich learning environments.

Some researchers focused on the effect of using dynamic geometry software on students' learning in comparison to the traditional instruction (Baki, Kosa & Güven,



2011; Ubuz, Üstün, & Erbaş, 2009). Ubuz et al. (2009) reported on 7<sup>th</sup> grade students' learning of line, angle, and polygon concepts by Geometer's Sketchpad (Jackiw, 2006). The findings of this study suggest that, if used appropriately, dynamic geometry environments can serve as an important vehicle to improve student achievement in geometry and achieve a classroom culture where conjecturing, analysing, exploring, and reasoning are daily routines. Similarly, Baki et al. (2011) found that physical manipulatives and dynamic geometry software (DGS)-based types of instruction were more effective in developing the first-year pre-service mathematics teachers' spatial visualisation skills than the traditional instruction. Moreover in this study students in the DGS-based group performed better than the physical manipulative-based group in terms of visualising a three-dimensional object from various perspectives. It was discussed that students developed their visualisation skills by constructing their geometric structure through investigation by dragging and measuring their structure.

At the college level, Ubuz (1999b, 2001, 2007) based on her Ph.D work (Ubuz, 1996) investigated whether and how computers in realistic classroom settings could influence first year engineering students' learning of derivative and integral. A variance analysis of the effects of computer and mathematics background on pupils' performance related to point of tangency, derivative at a point, and the approximate value of a function at a point (Ubuz, 1999, 2001) revealed that there was no distinguishing difference between the computer and the non-computer groups. Although no distinguishing difference was found between the computer and the non-computer groups, the results suggest that a significant relation does exist between students' performance and their perceptions of the use of computers in calculus.

**Teachers:** In this section we focus on teacher related affective variables as well as different kinds of knowledge needed by a teacher to support students' mathematical thinking. Please note that teacher difficulties and errors were mentioned in the previous section.

*Affect State.* A number of teacher related studies focused on beliefs, attitudes, efficacy, and math anxiety using interviews or measurement scales or combinations of quantitative and qualitative methods. Some of these conducted with pre-service elementary and mathematics teachers on various topics, such as mathematical understanding, mathematical problem solving, teaching and learning, self-efficacy beliefs and attitudes towards mathematics, efficacy, math anxiety and self-concept (e.g., Haser & Star, 2004a; 2004b; Akkoc & Ogan-Bekiroglu, 2006; Kayan & Cakiroglu, 2008; Cakiroglu & Isiksal, 2009; Isiksal, 2010).

In a study examining teachers' beliefs related to mathematical understanding Haser & Star (2004a, 2004b) the data yielded four components of mathematical understanding with various subcomponents: Content, reasoning, applications, and procedures. The study also revealed that participants from the most competitive high school background seemed to have richer conceptions of mathematical understanding.

Another study on teachers' beliefs about mathematical problem solving (Kayan & Cakiroglu, 2008) indicated that although pre-service teachers tended to have positive beliefs about problem solving, they believed that problem solving is primarily an application of computational skills in mathematics and is a matter of following a predetermined sequence of steps. The authors claimed that the high stakes testing in the country could possibly be the source of these traditional beliefs.

The research on the relationship between pre-service teachers' beliefs about teaching and learning and practices (Akkoç & Ogan-Bekiroğlu, 2006) pointed out the inconsistencies between belief and. It was found that some of the teachers presented different practices from their beliefs due to various reasons such as lack of subject knowledge and the complexity of classroom environment. They also believed that teaching approach should be determined on the basis of the nature of the mathematical topic. For example a pre-service teacher who held constructivist beliefs stated that functions and absolute value could be taught using discovery methods and in a technology-rich environment while polynomials and logarithmic functions could only be taught by heavy-lecturing.

Another study by Isiksal (2010) focused on the relationship among the learner related affective variables involving efficacy, mathematics anxiety and self-concept. A hypothesized structural model of learner related variables including learning mathematics anxiety, mathematics evaluation anxiety, mathematics self-concept, mathematics teaching efficacy, and mathematics teaching outcome expectancy was developed based on previous research. Through the structural equation modelling, Isiksal found consistency between the empirical relationships among the variables and those implied by the hypothesized model. The findings related to the relationship among each variable were further discussed in the context of the recent reforms in elementary mathematics curriculum as well as elementary teacher education curriculum.

Moreover, Çakiroğlu and Işıkşal (2009) reported on how gender and year of study in the program have an effect on pre-service elementary mathematics teachers' self-efficacy beliefs and attitudes toward mathematics. The findings indicated that pre-service teachers' self-efficacy scores were lower than their attitude scores which did not show any significant difference related to these factors. There was a significant effect of gender and the year of study on pre-service teachers' self-efficacy beliefs about mathematics. Furthermore, Çakiroğlu and Işıkşal argued that the increase in the mathematics efficacy scores of the pre-service teachers with the year in the program could be result of the experiences they were immersed during their teacher education programs.

In the affect state research some studies focused on in-service teachers. For instance, Duatepe-Paksu (2008) investigated teachers' beliefs about mathematics from various branch of subject, such as elementary generalist teachers, science teachers, mathematics teachers, and preschool teachers. The findings revealed that teachers,

regardless of their branch of subject, hold, what Ernest (1989) calls, instrumentalist view of mathematics. That is, mathematics is depicted as an “accumulation of unrelated facts, rules, and skills to be used by the trained expert in the search of some desired end. In other words, mathematics is a set of unrelated but useful rules and facts” (Duatepe-Paksu, 2008, p.88). To the author’s surprise this belief was more commonly held by the mathematics teachers rather than the teachers of other branches.

As a result of the current reform in school mathematics curriculum in Turkey, the use of technology in mathematics instruction came into prominence. In this context Güven et al. (2009) studied in-service mathematics teachers' beliefs about Computer Assisted Mathematics Instruction (CAMI). The teachers’ responses to the questionnaire showed that many teachers held negative opinions about CAMI in terms of learning and teaching with computers. There seems that there is a discrepancy between positive expectations from the use of computers in the curriculum and teachers' beliefs. It was also noted that most of the teachers either did not use any educational software in teaching mathematics or used them just for drill and practice.

Another study with teachers (Haser & Star, 2009) investigated the impact of Turkish national curriculum context on mathematics related (i.e. the nature of mathematics, mathematics teaching, and mathematics learning) beliefs. The findings discriminated three major factors in shaping the teachers’ beliefs, which were: the national curriculum requirements, lack of effective mentoring programs and pre-service tutoring experiences. Teachers also held both teacher- and student-centred beliefs which result in inconsistency in practices. The authors also pointed out that participants developed contextual beliefs during their material experience of teaching during their first year to deal with the difficulties faced in real classroom environments (see also Haser, 2010).

*Knowledge.* As one of the domains of teacher knowledge, pedagogical content knowledge (PCK) has attracted the interest of Turkish researchers in recent years. Some of the studies examine PCK focusing on a specific content, such as patterns (Yeşildere & Akkoç, 2009; Baş, Erbaş & Çetinkaya, 2011), derivative (Akkoç et al., 2008), number, arithmetic operations and shape (Olkun, Altun & Deryakulu, 2009), decimals (Ubuz & Yayan, 2010) and using solid objects (Bukova-Güzel, 2010b) while some focus on more general pedagogical approaches, such as pedagogical use of tools in mathematics teaching (Özdemir, 2008; Yeşildere, 2010). There is also a technology dimension in some of these and other studies, such as research on pre-service teacher preparation for successful technology integration (e.g. Baki, 2000; Bingölbali et al., 2009; Akkoç, 2011; Özmantar et al. 2010).

Studies which examine PCK focusing a specific content are mostly case studies with teachers or pre-service teachers. They used lesson plans, teaching videos and interviews as data collection tools. Yeşildere and Akkoç (2009) investigated the

development of two pre-service mathematics teachers' pedagogical content knowledge of number patterns, in particular "students' understanding of and difficulties with finding the rule of number patterns" through a school practicum course. The analysis of data indicated that the observation and discussions of number pattern lessons resulted in considerable change in the way pre-service teachers addressed student difficulties in their lessons. In another study, Baş, Erbaş and Çetinkaya (2011) investigated teachers' pedagogical content knowledge with special attention to the knowledge of students' algebraic thinking in pattern generalisation. The results showed that after examining students' work such as worksheets, teachers were able to better understand their students' way of thinking. The authors recommend that this kind of training should be used in mathematics teacher preparation programs and professional development courses.

Akkoç et al. (2008), on the other hand, focused on the topic of derivative through three components of Technological Pedagogical Content Knowledge (TPCK): PCK, TCK and TPK. The study investigated these components in a pre-service teacher lesson before and after involving a workshop on Graphic Calculus software. The data revealed the dynamics among PCK, TCK and TPK components; that is how they enrich or hinder the development of each other. The authors suggest that TPCK framework could be used in an operational way to diagnose pre-service teachers' difficulties and to identify the areas in need of development for a successful integration.

Another PCK study is Bukova-Güzel's (2010b) study which investigates PCK of using solid objects. The data revealed that pre-service teachers utilized visual and concrete materials very often to enable the students to better perceive solid objects which indicated their use of different mathematical demonstrations. On the other hand, they did not address possible student misconceptions about the topics and they did not apply effective assessment techniques to determine student learning. In light of the findings of the study, the author suggests that pre-service teachers should be helped to gain PCK in the process of teacher preparation programs.

Regarding content knowledge of teachers, Ubuz and Yayan (2010) explored the effect of educational background on teachers' performance about decimals. They reported that experience in teaching decimals was not an important factor effecting teachers' subject matter knowledge on decimals. One implication the authors emphasise is that educational attainment seemed to be an important factor affecting teachers' knowledge. Teachers who graduated from primary education are trained to be generalists and may not have an extensive knowledge of mathematics. The authors emphasise the importance of revisiting content of school mathematics in teacher preparation programs.

Some of the PCK studies focus on more general pedagogical approaches, such as pedagogical use of tools in mathematics teaching (Özdemir, 2008; Yeşildere, 2010). Yeşildere (2010) worked with pre-service elementary mathematics teachers involved

in an eight-week long workshop conducted on curriculum tools. Theoretical framework of the study was based on the instrumental approach to tool use. Results indicated that pre-service teachers had difficulty in (1) determining the appropriate use of tools, (2) making decisions about which tools to use, and (3) integrating the tools into the task. It was also found that teachers' instrumentation schemes influenced students' conceptual understanding. Considering the data obtained, the author suggested some implications about the usefulness of the instrumentation framework for effective integration of tools into mathematics teaching. Similarly, Özdemir (2008) studied pre-service elementary teachers' knowledge and skills of using manipulatives in making the mathematical concepts and relations explicit. The findings of the study showed that all the pre-service teachers demonstrated positive attitudes to the use of manipulatives while teaching mathematics. However, majority of them were not able to explain how such tools can be used to teach mathematical concepts and relations in such a way that support students to develop mathematical thinking skills.

Some of the studies related to teachers focused on pre-service teacher preparation and offered long-term courses to train prospective mathematics teachers for successful technology integration (e.g. Baki, 2000; Bingölbalı et al., 2009; Akkoç, 2010; Özmentar et al. 2010). Baki (2000) designed a two-term mandatory undergraduate course within a mathematics teacher education program and found that pre-service teachers who have adequate computer literacy successfully made the link between computer-based mathematical activities and school mathematics. The author suggested teacher educators long-term courses as they incorporate information technology into existing pre-service programs. Likewise, Akkoç and colleagues (Bingölbalı et al., 2009; Özmentar et al., 2010; Akkoç, 2011) designed a long-term teacher preparation course which aims to develop the required knowledge and pedagogical skills for successful technology integration using TPCK framework. The components of TPCK are determined as follows: (a) Technology and multiple representations (see Özmentar et al., 2010), (b) technology and students' difficulties with and misconceptions of specific concepts (see Akkoç, 2011), (c) technology and instructional strategies and methods for teaching a specific concept, (d) assessment of concepts via technology, (e) technology in the curriculum. In order to determine the influence of the programme on the participants' development, TPCK framework with its components was employed as a data analysis tool. The participants' views of the contents and delivery methods of the programme were also taken into account to shed light on the effectiveness of the programme. The findings showed that significant improvements emerged in the TPCK of the participants and that is taken as evidence for the effectiveness of the implemented programme. Alongside many other issues, authors discussed that the developed programme had an exemplary approach in terms of effective technology integration and it could be used as a course for the prospective teacher's training.

## CONCLUSION

This presentation examines the contribution of Turkish mathematics education community to the issue of mathematical thinking. This examination made it clear that Turkish researchers, though not necessarily mentioned the issue of mathematical thinking in their studies, made important contributions to this issue. They performed studies at a wide range of samples varying from elementary to pre- and in-service teachers. The researchers adopted both qualitative and quantitative approaches to the study of mathematical thinking. However in recent years there has been a considerable increase in the number of qualitative research designs.

The Turkish studies are first examined with regard to the mathematical thinking process. It appeared that the issue of abstraction, generalization, reasoning, modelling and concept-image, which are crucial constituents of mathematical thinking process, attracted the attention of Turkish mathematics educators. The examination also considers the studies which pay attention to the things stemming from the mathematical thinking process. These studies mainly considered student difficulties, misconceptions and errors as well as student understanding of different algebraic and arithmetic concepts. Finally, a majority of the research we reviewed focused on the various factors that support or hinder the development of mathematical thinking process. The research in this section was examined in three main categories: (1) studies relevant to learner variables, such as metacognition and individual differences, (2) instructional variables including mathematical tasks, pedagogical approaches, the use of technology, and (3) teacher related studies involving affective variables and teacher knowledge.

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# MATHEMATICS TEACHER EDUCATION IN TURKEY

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*In this paper, current situation of the mathematics teacher training programs will be discussed. After a brief discussion of the history of mathematics teacher education programs and policies in Turkey, the structure of the Turkish school system will be outlined and the teacher training programs for mathematics teachers who teach at different levels in the school system will be discussed in detail.*

## 1. STRUCTURE OF TURKISH SCHOOL SYSTEM

The education system in Turkey is divided into four main parts, preschool education, elementary education, secondary education, and higher education. Pre-school education is optional and begins at age 3 continues until age 6. Pre-school education institutions include independent kindergartens, nursery classes in primary schools and preparation classes. Elementary education generally comprises the education of children in the 6-14 year age group. Eight years of elementary education have been compulsory for all Turkish citizens who have reached the age of six since 1998. During the last three years (Grade 6, Grade 7, and Grade 8) of the elementary education, students have to take a comprehensive examination in order to enter a secondary school. A composite score is calculated based on the results of three exams and is used to enroll in an appropriate secondary school. However, starting in 2012 students will begin to take this test only at the end of the eighth grade. Secondary education begins at age 15 and continues until age 19. Secondary education encompasses two categories of educational institutions, namely *general high schools* and *vocational and technical high schools* (lycées) where a minimum of four years of schooling is implemented after elementary education. General high schools are educational institutions that prepare students for higher education. They implement a four-year program, and comprises students in the 15-18 year age group. Vocational and technical high schools provide specialized instruction in order to train qualified personnel for the industry. These schools also implement four-year programs. At the end of the secondary education, students have to take another important examination to be enrolled in a university. The university entrance examination is a nation-wide two-stage examination (YGS and LYS) which aims at selecting and placing students to the higher education institutions. The YGS and LYS are central university entrance examinations administered by the Student Selection and Placement Center (OSYM) every year. The center was established in 1974 and affiliated to the Council of Higher Education (YOK) in 1981 (Tekkaya, Çakıroğlu, & Ozkan, 2004). The results of the first stage of these examinations (YGS) are used both in acceptance to the 2-year post secondary vocational schools and in calculation of the total composite scores of the students in LYS which is required in admission to the undergraduate programs.

Admissions to the undergraduate higher education institutions are based on the students' composite scores, which take into account the YGS and LYS scores and the high school grade point averages (YOK, 2010).

## **MATHEMATICS TEACHER EDUCATION IN TURKEY**

In Turkey, there has been no longlasting policy for teacher training. Since 1982, the responsibility of training teachers was transferred from the Ministry of National Education to the universities (YOK, 1998). From 1982 to the 1998–1999 academic year, mathematics teachers (grades 6-11) were trained by the Science Education Departments of the Education Faculties at universities, including Physics, Biology, Chemistry, Mathematics, and Technology branches through four-year undergraduate programs offering bachelor degree. With the reform in teacher training programs in 1998, Science Education Departments were closed down. Instead, new departments such as Elementary Education, Secondary Science and Mathematics Education, and Computer Education and Instructional Technology were opened. Training mathematics teachers for elementary and secondary schools were split and the responsibility was given to the Elementary Education (grades 6-8) and the Secondary Science and Mathematics Education Departments (grades 9-12). Mathematics teachers who will teach at elementary schools were decided to be trained through four-year undergraduate programs offering Bachelor's degree. Becoming a secondary school mathematics teacher was raised to the graduate level. Two different programs were formed for being secondary school mathematics teacher: The Five-Year Integrated Programs (3.5+1.5) and Non Thesis Masters Program (4+1.5). Each year consists of two 14-week semesters. Students enter the Five-Year Integrated Programs and the elementary teacher training programs through the University Entrance Exam (YGS and LYS) as admission to higher education is centralized.

Today, most of the faculties of education in Turkey offer programs for training preschool (kindergarden) teachers, elementary teachers (for primary and middle schools), and secondary teachers who are employed by both the Ministry of Education and private schools. Right now there are 166 (104 public and 62 private) universities in Turkey (OSYM, 2011). Out of 166, 72 (65 public and 7 private) universities have faculties of education, some of which offer dual programs (both regular and evening). Although students in the evening programs are required to pay much higher tuition than the ones enrolling in the regular programs, they are admitted to the same courses of study with relatively lower scores in the University Entrance Exam than the regular ones. Elementary mathematics teacher training programs are offered under elementary education departments at 42 public and 4 private universities.

The Five-Year Integrated Programs (3.5+1.5) for secondary mathematics education are offered by 13 public and 2 private universities. One of those 13 public universities also offers evening Five-Year Integrated Programs (3.5+1.5) for

secondary mathematics education. However, at the beginning of the 2010, YOK made a policy change in mathematics teacher training for secondary schools and closed Non Thesis Masters Programs (4+1.5). After 2010, any student, who enrolled in a program in the Faculty of Arts and Science, willing to be a teacher can get a teaching certificate by satisfying required conditions determined by the YOK. This policy change has been heavily criticized by teacher educators in Turkey. It is argued that this change will cause a decrease in the quality of mathematics teachers and in turn, in the quality of mathematics education in schools. It is also argued that allowing every student to get a teaching certificate regardless of the programs they are enrolled in will harm the view of teaching as a profession.

## ELEMENTARY MATHEMATICS TEACHER EDUCATION IN TURKEY

The elementary mathematics education programs use mainly a standard curriculum, which is a result of the reform efforts in teacher education programs throughout the country since 1998 (YOK, 2005). As a result of these efforts, compared with the previous mathematics teacher-training program, more emphasis was given on the field experience, technology literacy, and methods of teaching subject matter courses. Before 2006, elementary pre-service mathematics teachers with the minor degree of science education during the typical four-year course were required to take a number of courses in the mathematics and in the different branches of science, namely physics, chemistry and biology, and several courses related to teaching profession. In 2006, elementary mathematics education programs were reorganized and the requirement of earning a minor degree in science education was ended. The current program has courses mainly in three categories: content, pedagogical and general culture courses. Distribution of the courses with respect to these categories is as follows: content courses 50-60%, pedagogical courses 25-30% and general culture courses 15-20%. Four years of coursework included overall 146 semester credit hours (Table 1) (YOK, 2007).

Courses	# of courses	T	A	C
Mathematics	19	59	10	64
Mathematics Related Pedagogical Courses	7	15	16	23
General Pedagogical Courses	6	15	0	15
Elective Courses	6	16	0	16
Others	13	25	6	28
Total	51	130	32	146

Table 1: A summary of courses offered by elementary mathematics education departments.

Note: T: Theoretical class hour A: Application class hours C: Credit



Table 1 presents the distribution of the compulsory and the elective courses by the numbers and the study hours. Each study hour is equal to 50 minutes. The organization of most of the courses includes theoretical and practical components (laboratory, practicum, presentations of students’ work). The theoretical component is delivered by the senior staff (instructor or higher such as assistant professor), and the practical component by a research assistant or the senior staff who is giving the theoretical part. General culture courses such as history, Turkish, computer literacy, and English language courses are included in “others”. In mathematics courses the students take all the basic university mathematics courses, i.e., calculus, linear algebra, analytical geometry, algebra, geometry, differential equations, number theory, probability and statistics, abstract mathematics. This group of courses also includes courses about history and philosophy of mathematics and physics. Mathematics related pedagogical courses include school experience, teaching experience, methods of mathematics teaching, assessment and evaluation, general teaching methods and principles, and instructional technology and material development. General pedagogical courses include introduction to teaching profession, educational psychology, classroom management, special education, Turkish educational system and school administration and guidance. Elective courses provide opportunities for students to advance themselves either in their mathematics background or in pedagogical background. Elective courses offered by different universities vary with respect to their academic potentials and wisdom. While some universities usually offer elective courses about mathematics related pedagogical courses such as teaching geometry, problem solving, others mostly offer mathematics courses.

THE SECONDARY SCHOOL MATHEMATICS TEACHERS TRAINING

The secondary mathematics education programs used to follow a standard curriculum, which was a result of the reform efforts in teacher education programs throughout the country since 1998 (YOK, 2005). Secondary pre-service mathematics teachers attending to The Five-Year Integrated Programs during the typical five-year course were required to take a number of courses in the mathematics and physics, and several courses related to teaching profession (Table 2). Five years of coursework included overall 174 semester credit hours.

COURSES	# of courses	T	A	C
Mathematics	24	59	48	83
Physics	2	8	4	10
Technology	5	10	10	15
Mathematics Related Pedagogical Courses	8	15	24	27
General Pedagogical Courses	4	11	2	12
Elective Courses	5	12	6	15
Others	6	12	0	12

TABLE 2. A summary of courses at The Five-Year Integrated Programs on mathematics education



Table 2 presents the distribution of the compulsory and the elective courses by the numbers and the study hours. In mathematics related courses the students take all the basic university mathematics courses, i.e., calculus, linear algebra, analytical geometry, algebra, geometry, number theory, probability and statistics, abstract mathematics, differential geometry, differential equations, topology, and complex calculus. The titles and the contexts of the general and mathematics related pedagogical courses are the same as the ones offered for elementary mathematics education programs. Elective courses aim to complete either their mathematics background or pedagogical background. Pedagogical courses start to take place in the last three semesters. The pedagogical courses in the last 1.5-year (3-semesters) part are the same for all 3.5+1.5 and 4+1.5 programs.

TABLE 3. A summary of courses at the Non Thesis Masters Program on mathematics education

COURSES	# of Courses	T	A	C
Mathematics Related Pedagogical Courses	8	15	24	27
General Pedagogical Courses	4	11	2	12
Elective Courses	2	6	0	6

Unfortunately YOK and Ministry of Education decided to require a teaching certificate for being a secondary school mathematics teacher in 2010 and this decision immediately was put into practice by the universities in the 2010-2011 academic year. Anyone enrolled in a program in the Faculty of Art and Science can get a teaching certificate offered by the Faculty of Education. While these students continue their educations in their programs, they also attend courses in the Faculty of Education in order to get a teaching certificate. The certificate program involves 10 mathematics related and general pedagogical courses. Table 4 represents these courses.

COURSES	# of Courses	T	A	C
Mathematics Related Pedagogical Courses	5	11	10	16
General Pedagogical Courses	5	10	0	10

When Table 3 and Table 4 are compared, the striking difference between the total credit hours required by both programs becomes clear. While it was required to earn 45 credit hours to become a secondary school teacher, with this new arrangement this requirement is reduced almost by half. Another striking difference between these two programs is that the number of methods of teaching secondary school mathematics and field experience courses. These courses were a year long courses in the Non Thesis Masters Program, however under teaching certificate programs, these courses are being offered only for one semester.

After graduation, teacher candidates also have to take an exam in order to be hired by the Ministry of Education. This exam is called Public Personnel Selection Exam (KPSS). KPSS is a central examination administered by the Student Selection and

Placement Center (OSYM). The number of mathematics teachers hired by the Ministry of Education varies from year to year.

## **INSERVICE TEACHER TRAINING**

In-service training of mathematics teachers is the responsibility of the Ministry of Education. Ministry of Education organizes summer programs for teachers each year. Engaging in an in-service teacher training program is usually obligatory. School administrators decide which teacher needs to attend such a program though in some cases the school administrator makes this decision based on the teachers' view. Some studies on the effectiveness of teacher training programs indicated that teachers need to attend these programs on the voluntarily and also there should be a reward for this attendance (Ozer, 2004; Bayrakci & Sungu, 2007). Teachers also report that content of in-service training programs should be adjusted to the needs of the participant teachers and they should have the freedom of choosing which programs to participate according to their needs (Boydak Ozan and Dikici, 2001). Otherwise, teachers think that attending such programs is useless and waste of time. Another important issue is that even if the teacher willingly attends an in-service teacher training program, when the school administrator has no knowledge of the content of the program, teacher may face some problems in implementing what they learnt at their school (Ersoy, 2005).

Most comprehensive study on the in-service training needs of mathematics teachers was conducted by the Department of Educational Research and Development (EARGED) in 2008. The study focused on which areas mathematics teachers need in-service training (EARGED, 2008). It was found that 60% of the teachers did not participate in any in-service training program. In addition, mathematics teachers reported that they mostly need guidance and help about how to teach effectively new school mathematics program as well as teaching students with special needs. Teacher also complained that new school mathematics program recommends a student-centered approach but it is almost impossible to use such approach in crowded classroom. Hence, they stated that they need some training on effective use of time and effective classroom management techniques.

## **CONCLUDING REMARKS**

Turkish students score behind their peers in studies on international comparisons of mathematics and science achievement such as PISA and TIMSS. Inadequate performance of Turkish students forced the policy makers to make a comprehensive curriculum reform in school mathematics. In 2005, new elementary school mathematics was put into implementation throughout the country. With implementation of the new curriculum, a need for intensive inservice training programs rise for the mathematics teachers throughout the country. It is quite clear that the quality of mathematics education depends on the quality of the mathematics teachers. Turkey took a long way in preparing qualified mathematics teachers at

universities since 1982. However, lack of stable and consistent policies on mathematics education in Turkey makes this job difficult for the universities. In addition, the frequent changes in policies bring new problems for both teacher educators and mathematics teachers.

Turkey has large population and the number of children attending schools is increasing year by year. However, physical conditions of the schools remain almost same. As a result, classrooms get crowded and this creates additional problems for the teachers in performing a teaching required by the new curriculum. In order to improve mathematics education in Turkey, it is obvious that there is a need for long term and stable policies.

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# **SHORT ORAL COMMUNICATIONS**





# TEACHERS' EXAMPLES AND THEIR EFFECTS ON CHILDREN'S UNDERSTANDING IN MATHEMATICS

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The central role of examples in teaching and learning mathematics has been long acknowledged (Zazkis & Chernoff, 2005). In order to improve and sustain pupils understanding of mathematics lesson, a teacher needs to have good knowledge on how to choose and use examples. The knowledge teachers construct with respect to their choice and treatment of examples in the mathematics classroom is constructed mostly through their practice (Zaslavsky & Zodik, 2007). Examples may differ in their nature and purpose. The specific choice of examples may facilitate or impede students learning, thus it presents the teacher with a challenge, entailing many considerations that should be weighed (Zaslavsky & Zodik, 2007).

In this case study approach, two in-service teachers were selected as participants – one experienced teacher and one novice teacher. Both were teaching two classes with pupils of similar academic achievements. The lessons were observed, videotaped and the teachers were interviewed on their selection and use of examples. A test consisting of 20 questions based on the sub-topic chosen will be given to all the pupils taught by both teachers after the study.

The basis on which examples were selected and used and the considerations made by the teachers in choosing and using the examples forms part of the data and will be analysed. Analysis was done using the four categories of examples in mathematics teaching proposed by Rowland (2008), which are *variable*, *sequencing*, *representations* and *learning objectives*. Pupils' performance in the test, coupled with interviews with the participants and document analysis will be analysed to say something about the extent to which teacher's examples effects the children's understanding.

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# erStMaL-FaSt

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This presentation draws on a longitudinal study, erStMaL-FaSt, which belongs to the erStMaL Project. The longitudinal research project erStMaL, early Steps in Mathematics Learning, relates to the investigation of mathematical cognitive development in preschool and early elementary school age from a socio-constructivist perspective. The Family Study of the erStMaL Project is named as erStMaL- FaSt. This study deals with the impact of the familial socialization on mathematics learning, in which also bilingual families are included.

From our research interest, the analysis of mathematics education in early childhood requires also to consider the familial context, which has a significant influence on the children's development in the early childhood. Thus, the family functions as an ongoing "support system" parallel to preschool, kindergarten and (primary) school for the learning of mathematics. By the term "support system" it is referred to the idea of any socio- constructivist theory, which means that the cognitive development of an individual is constitutively bound to the participation of this individual in a variety of social interactions. In the process of learning in mathematical play situations the children move on learning, while they are getting inevitably support by the members of the family. With respect of Bruner's concept of a Language Acquisition Support System (LASS), we propose a similar concept for the learning of mathematics, which we call analogically the "Mathematics Learning Support System" (MLSS) (Bruner, 1990). Especially with regard to the young age of the children of erStMaL-FaSt, it can be assumed that these forms include narrative argumentation (Krummheuer, 2009; Van Oers, 2001). Our first analyses show that in mathematical play situations with families' explanations and narrative presentations are strongly used. With regard to linguistic aspects it can be said, that these explanations and narrative presentations are main elements of MLSS. Furthermore, in bilingual families there is a distinctive form of language usage, which is called "code-switching". Also these distinctive forms have an influence on the functioning of MLSS especially in bilingual familial contexts. Through all these aspects, it will be interesting to find out the functioning of MLSSs in bilingual families.

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# AN ETHNOGRAPHIC STUDY OF MATHEMATICS TEACHERS' PROMOTION OF METACOGNITION FROM A CONSTRUCTIVIST PERSPECTIVE

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This is an ethnographic study of three teachers' promotion of metacognition - a construct widely accepted as knowledge and regulation of thinking - from a constructivist perspective, focusing on teaching practices in secondary mathematics classrooms in the UK. I adopted a constructivist approach, endeavouring to reconcile the individual knowledge construction and the socio-cultural elements. The fieldwork can be divided into two phases as a developmental process. I worked with two mathematics teachers in two schools, over two terms, followed by fieldwork with a third teacher for over another term. With each teacher, after initial observations of their teaching, I observed their use of activities that I jointly developed with them and conducted interviews regarding their teaching. I aimed at analysing and substantiating the links and parallels between the two stages of my fieldwork, providing an account of the patterns in the teachers' promotion of metacognition and the underpinning factors.

An important finding of the study was the differences in the teachers' emphasis on metacognition throughout the stages of the activities and the lessons, and during their interactions with the students of different achievement levels and progress with the activities. Another main finding was the emphasis on evaluation of thinking and mathematical work retrospectively, rather than explicitly addressing planning and monitoring skills. Constructivist foundations accompanied teachers' encouragement of students' use of metacognition. This resonates with previous claims in the field regarding the positive relationships between metacognition and a constructivist approach to teaching and learning (Thomas, 2003). I developed the Framework for Analysing Mathematics Teaching for the Advancement of Metacognition (FAMTAM), comprising four factors underpinning teachers' practices: their understanding of metacognition, their perceptions of students' features and needs, the distribution of mathematical authority and the external pressures teachers perceived. This framework can be used as a tool for teachers' reflection on their teaching practices from the perspective of fostering metacognition.

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# PURPOSELY TEACHING FOR THE PROMOTION OF CRITICAL THINKING DISPOSITION: A CASE OF “CONFIDENCE”

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Critical thinking has been investigated largely in terms of thinking skills that involve the cognitive domain. For decades, promotion of students' thinking has been the focus of educational studies and programs (Facione & Facione, 2000). Each of these programs has its own definition of thinking and/or of skills. Some use the phrase 'cognitive skills' (Ben-Chaim, Ron & Zoller, 2000) while others refer to 'thinking skills' (Barak & Dori, 2009; Zohar & Dori, 2003), but they all distinguish between higher- and lower-order skills. Resnick (1987) maintained that thinking skills resist precise forms of definition; yet, higher order thinking skills can be recognized when they occur. Thus, there seems to be no clear consensus as to what exactly critical thinking is. Some see it as simply “everyday, informal reasoning” , whereas others feel differently. Lipman considers it to be different from ordinary thinking because it is both more precise and more rigorous, as well as self-correcting. Our research is underpinned by several questions raised by Barak (2009): what do we mean by teaching a student to become a critical thinker? How can this be accomplished successfully? We hope that our research will shed some light on these questions. This research offers new possibilities of extending the use of critical thinking development programs, and for their integration into the formal high-school mathematics curriculum. we believe that these initial results are the first step in showing that it is possible to train students' critical and creativity thinking. This research shows the possibility of working in the direction of developing critical and creative thinking in the framework of the established contents of the mathematics curriculum.

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# PRE-SERVICE MATHEMATICS TEACHERS' CONCEPT IMAGES OF $\pi$

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This study investigates pre-service mathematics teachers' understanding of  $\pi$ . There is little research on how teachers or pre-service teachers conceptualise  $\pi$ . In his study, Fi (2003) found that pre-service mathematics teachers considered  $\pi$  as the unit for the radian measure. Akkoç's (2008) findings indicate that pre-service teachers' lack of understanding of the radian concept resulted in difficulties with making sense of  $\pi$  which requires to discover that there are approximately 6.28 ( $2 \times 3.14$ ) radians in a round angle.

This paper explores pre-service teachers' concept images of  $\pi$ . Tall & Vinner (1981) define concept image as 'the total cognitive structure that is associated with the concept' (p. 152). To investigate the concept images of  $\pi$ , the following question was asked to forty-eight pre-service teachers: "Are there two different  $\pi$ 's in mathematics?. Explain your answer".

To reveal participants' concept images of  $\pi$ , their written explanations were analysed. Out of forty-eight participants, twenty-three of them mentioned that there are two different  $\pi$ 's while twenty-four participants considered a unique  $\pi$  in mathematics. One participant did not respond to the question. Among those who considered two different  $\pi$ 's, twenty-one of them distinguished the number  $\pi$  (which is approximately 3.14) from the angle  $\pi$  which is equivalent to  $180^\circ$ . Even the twenty-four pre-service teachers who considered a unique  $\pi$  made similar explanations.

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# HOW DO DIFFERENT LEVELS OF CAUSATION AFFECT STUDENTS' PERFORMANCE IN WORD PROBLEM SOLVING?

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The purpose of this study was to investigate how causal and outcome related elements affect the mathematical word problem performances of middle grade students. The participants were 50 students (21 females and 29 males) from the fifth grade of an elementary school located in a middle socio-economic neighborhood in Ankara in Turkey. Students were administered the problem set during their mathematics class time. Three word problems containing some spatial content were developed by the investigators. These problems were written in three contextualized formats: standard, potential causation and climax resolution (Smith, Gerretson, Olkun & Joutsenlahti, 2010). Latin Square approximation was used to produce different versions of the problem set for the application, so that each student could see three different problems in three different formats. The different versions were assigned randomly to the students. The students were given the time they needed to solve problems and the administration was completed in about 70 minutes. The mean scores were generally quite low. Students got the highest mean score from the second problem which requires less spatial skills and can be solved with more simple processes. A significant difference was found between the formats of the same problem which had the highest mean score [ $F(2-47)=3,66$ ,  $p<.05$ ]. The scores of climax resolution and potential causation formats that combine the mathematical content with causal and outcome related elements were significantly higher than the standard format's score. These results indicate that students were affected the causal and outcome related elements in the word problems and they could create the situation model better. It is suggested that teachers should ask students different formats of the same word problems with spatial content and lead them to draw diagrams while solving problems. It may lead to more effective findings to correlate the word problem scores with different variables like learners' features.

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# RESISTANCE TO CHANGE IN TEACHER PEDAGOGY

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The National Council of Teachers of Mathematics (NCTM, 2000) has focused attention on students' conceptual understanding of mathematics suggesting that students need to be actively involved in the learning process using their experiences and prior knowledge. Such a call for change in learning and teaching was already considered by Cuban (1993) that fundamental changes needed to be applied to the classroom such that "teaching becomes structuring activities that enable students to learn subject matter, one another, and the community" (p. 4). For the changes that require students' active involvement in the construction of (mathematical) knowledge, teachers have to take a step back in controlling students' learning activities. This case study was guided by the question, "what are the challenges that a teacher has to deal with when integrating an argument-based approach?" This paper was based on one of the three videotapes captured in a mathematics classroom in which students were working on an activity related to the real numbers unit. Analyses were done according to a criteria matrix that captures three major areas in pedagogical practice: creating dialogical interaction, controlling knowledge and the problem solving process, and unit preparation and making connections. Dialogical interaction refers to teacher's questions to create student-student dialogue as well as to provoke students' alternative conceptions for the conflicts that require argumentative environment in which students engage in negotiation of meanings through small- and whole-class dialogical interaction.

The analyses in this paper suggested that the teacher, due to his didactic approach of teaching, was keen to be the center of classroom conversation rather than letting students lead any conversation/discussion or any problem solving activity. As a result of such "enthusiasm" for leading the conversation, the teacher was not able to create dialogical interaction among students. Such cases show that the teacher fluctuated between "letting go of" and taking the control of lesson to catch up with the curriculum. This resulted in taking less account of students' problem solving processes and more of having them do mechanistic parts of mathematics.

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# EXPLORING EXPECTATIONS PRIMARY SCHOOL PUPILS HAVE ABOUT TEACHING MATHEMATICS

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“Effective teaching is essentially concerned with how a teacher can successfully bring about the desired pupil learning” (Kyriacou, 1997, p.8). In other words, effective teaching is organizing the process to consider pupils’ needs and to select appropriate strategies and styles for their learning. The aim of effective teaching may “emphasize cognitive (intellectual) aspects of learning or affective (social, emotional and attitudinal) aspects of learning” (Kyriacou, 1997, p.8).

Many studies (Greenwood, 1984; Lazarus, 1974) overwhelmingly implied that mathematics learning is largely a function of mathematics teaching. If we admit that teaching and learning have precisely intertwined relations, then teachers should understand pupils’ needs and difficulties in mathematics. “Knowing students as individuals is fundamental to understanding “what works” and what “does not work” in creating motivation, interest, and a need to know in the learner” (Loughran, 2006, p.86).

The purpose of this study was to understand ‘*how pupils would like mathematics to be taught*’. Therefore, 5<sup>th</sup> grade pupils at 3 primary schools were asked to write their own expectations in a journal. Pupils were asked to answer the question ‘*Supposed that you are a teacher, how do you teach mathematics?*’ In order to analyse the data gathered, inductive analysis was used.

The results indicated that most of the pupils were not happy with their teachers’ teaching. They wrote that while teaching mathematics, they prefer games more than giving information. They stated that teaching with music could enhance their motivation. They also gave suggestions about how a teacher should interact with pupils. The pupils thought that they mostly had negative feelings towards their teachers while they were teaching mathematical operations as they believed that teachers were different in mathematics.

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# DEVELOPMENT OF KNOWLEDGE TEST ON THE HISTORY OF MATHEMATICS

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Among the elements of mathematics teachers' instructional repertory, incorporating the history of mathematics into mathematics education stands out as an alternative. Jankvist (2009) organised the arguments on this incorporation as of 'history as a tool' for helping mathematics teaching and 'history as a goal' of teaching on its own. Either way, one should initially master the historical knowledge lying behind the mathematical concept before the incorporation. On the basis of this argument, this study aimed to develop a valid and reliable test assessing prospective elementary mathematics teachers' (PEMT) history of mathematics knowledge within the borders of Turkish national elementary (K6-8) mathematics curriculum (Ministry of National Education [MNE], 2009). The test consisted of multiple-choice, short answer and true-false items which are totally 14 in number. Portraits of pioneer mathematicians, hieroglyphics from ancient cultures, old photographs and some other visual tools were utilized as forming the items. The validity of the test was ensured by the views of three experts, who are specialized in the field of elementary mathematics education, via the table of specifications composed for this test. The test was applied to 1593 Turkish PEMT (478 freshmen, 432 sophomores, 409 juniors, and 274 seniors) selected from each geographic region of Turkey by means of clustered random sampling. The reliability of the test was found as .56 by using KR-20 reliability coefficient. This low reliability which was reasonable for the study's aim may stem from the minority of the number of items in the test or the characteristics of the participants (Pınarbaşı, Canpolat, Bayrakçeken, & Geban, 2006). The item quantity could not be excessive since the national curriculum (MNE, 2009) gave limited place to the history of mathematics. Moreover, the problem originated from the sample characteristics is mostly that the PEMT's total scores were close to each other which lower the variability. It is believed that this instrument could be used to assess individuals' knowledge structures related to the history of mathematics.

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# PME INEQUALITIES PRODUCTION: A SURVEY

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Mathematical inequalities have been the subject of researches discussed on PME. As this theme involves many mathematical concepts, we decide to analyze the inequalities production since the XV until the last PME XXXIV from 2010.

Some researchers have reflected on education and learning of the concept of inequalities, for instance, Tsamir and Bazzini (2002, 2004) point out that it has a plenty call for the correct and incorrect ways to teach this concept in the Standards of the National Council Teaching Mathematics - NCTM. There they highlight the necessity of understanding the meaning of expressions equivalents forms, equations and inequalities. Tsamir and Almog (1999) say us that there is a growing interest in the last couple of years, in learning and teaching algebraic inequalities.

We decide to analyze concerns about the inequalities production in papers and works presented at PME in order to bring to the surface the researchers understanding from many countries and cultures and analyze patterns, divergences by contrast and interrelations to develop new knowledge and reflections about the concept; not only to improve inequalities teaching and learning, but also value this mathematical content in order to provide opportunity for the learning of other interrelated contents.

Our work is based on documental analysis, a kind of meta-analysis. Our research *corpus* was 17 articles. Almost all the papers had in common that they concentrate the attention in pupil behavior when are studding this concept, observe the linear and quadratic inequalities. One paper observed how the teacher worked with students' mistakes. Three of those based its investigations in undergraduate students conceptions. In the presentation we will provide our own results on the subject.

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# MATHEMATICS TEACHING AND LEARNING VIA PROBLEM SOLVING, EXPLORATION, CODING AND DECODING

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To go forward problem-solving research, researches have recommended different approaches, such as mathematical investigation, problem posing, problem solving and modeling. For example, English & Sriraman (2010) have suggested a modeling alternative. Becker & Shimada (2007) have recommended a open-ended approach. We have called for a problem exploration perspective as an alternative to existing views on problem solving. We have developed it, anchored by an intense classroom work, done with elementary school students. We titled it as “Mathematics Teaching and Learning via Problem Solving, Exploration, Coding and Decoding”. In such a proposal Problem Solving, besides being adopted as a teaching methodology, is treated in the light of a perspective of critical education, not just seen at the level of mathematical concepts and processes, but also at the level of socio-politico-cultural questions, education in general and mathematics education in particular and the classroom is observed in all its many aspects, that is, in all its multicontextuality. In developing this research, we work theoretically with the academic literature of the topics problem solving, critical education, with the pedagogy of Paulo Freire, with the psychology of Lev S. Vygotsky and Imre Lakatos' Philosophy of Mathematics. Paulo Freire was included because he provided us with the foundations of problem solving in the light of a perspective of critical education. So we can address socio-historical theory of Vygotsky and adopt then the vision of Imre Lakatos' fallibility. These theoretical studies, together with our classroom experiences intentionally materialized a part of the research's fieldwork, which was divided into two parts. In Part I, we interviewed seven math teachers from two public schools. In Part II, we worked, as a teacher and researcher, in one of the schools researched, with students in elementary school (with the same teacher) during two semesters.

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# AN ACTION PLAN FOR THE DIFFICULTIES IN GENERALISING NUMBER PATTERNS

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In this action research we present an action plan that supports making algebraic generalisation. It is aimed to determine the changes occurred in student achievement with a developed action plan and to reveal the relationships between the actions taken and the changes in generalising performance.

Action plan consists of mathematical tasks. There are two basis of task design process. The first one is the main results of research studies. We take studies' suggestions about teaching generalising number patterns into consideration. The second basis of task design process is theoretical frameworks for number pattern generalisation and task design. Algebraic pattern generalisation framework of Radford (2008) and task design framework of Zazkis et al. (2008) and Swan (2007) are utilized. After developing tasks rest on these bases, a pilot study is conducted and tasks are revised. The final version of tasks is implemented through twelve hours to 13 seventh grade students. Data resource is videos of courses. Algebraic pattern generalisation framework (Radford, 2008) is used as a theoretical tool for analysing the data.

As a result of study, it is found that tasks helped students to notice commonality to provide a direct expression for any term of the sequence. They changed their strategy from focusing on the difference between consecutive terms to focusing on the relationship between the term and term number. One another improvement is observed in the use of representation. They benefit from pictorial models effectively for algebraic pattern generalisation after task implementation. Also students begin to make algebraic generalization rather than arithmetic generalization.

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# ADDRESSING PRESERVICE TEACHERS' NEEDS OF GEOMETRY CONTENT KNOWLEDGE: A QUALITATIVE STUDY TO TRANSFORM THEORY INTO PRACTICE

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*The purpose of this presentation is to report the results of a qualitative research which in turn used to develop learning experiences for preservice teachers. The integration of qualitative results (narrative and thematic analysis) with literature on teacher knowledge (MKT model and using student work with teachers) lead to two phase learning experiences to be used in the methods course.*

Brown and Borko (1992) asserted that preservice teachers' limited mathematical content knowledge is an obstacle for their training on pedagogical knowledge. Also, the *mathematical knowledge for teaching* (MKT) model emphasizes the importance of mathematics knowledge in the teaching settings (Ball et al., 2008). The guiding research questions were as follows: (i) preservice teachers' understanding of geometry for elementary school, (ii) preservice teachers' perceptions on effective instructional strategies to promote their learning of geometry content knowledge.

Individual narrative interviews with three participants, classroom observations and classroom artifacts were analyzed by narrative analysis (Labov, 1972) and thematic analysis (Coffey & Atkinson, 1996). Narrative analysis results yield to two kinds of stories: as a learner and as a beginning teacher. The thematic analysis results are three themes: (a) history of learning geometry, (b) what is geometry? (c) experiences in methods course. These results were combined with literature on using student work, and they emerged into a two-phase protocol to address preservice teachers' needs and perceptions in order to enhance their geometry learning. The process of the development of learning experiences will be the focus of this presentation to examine ways to address preservice teachers' needs in geometry content knowledge.

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# TEACHER'S KNOWLEDGE, THINKING AND BELIEFS ABOUT DYNAMIC GEOMETRY SOFTWARE IN ELEMENTARY SCHOOL

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The Dynamic Geometry educational software have been considered a great methodological resource in the Mathematics teaching-learning process because they allow the drawing and the manipulation of the geometric objects, they facilitate the exploration of conjectures and the investigation about the relations that precedes formal reasoning. (Ponte, Brocardo & Oliveira, 2009). In addition, activities with software promote situations where the students discover new ways to solve problems also evaluate their strategies and purposes. (Van de Walle, 2009). Therefore, the teacher's knowledge is extremely necessary. Considering our theoretical framework, we created a didactic sequence about the Pythagoras Theorem using Dynamic Geometry software called GeoGebra and submitted it to evaluation (questionnaire) of 30 math teachers of public schools of João Pessoa, in Paraíba, Brazil, who work in elementary school (11-14 age). In the Activity 1, we propose the investigation of relation between the smallest squares and the biggest one, designed over the rectangle triangle sides, through the figure compositions and area calculation. In the Activity 2, different pythagorics ternary are got with "seletor" tool and from the side's measure it's possible to do new conjectures. In addition, the "Area" tool allows that the area value of each square to be shown. In the Activity 3, with "to move" tool it's easy to verify when the theorem is false moving and changing the triangles. The results of our questionnaire have put in evidence important aspects highlighted by the teachers like: the opportunity of test the geometrics proprieties (73%), the interactivity (69%), the motion (58%) and the facility to have and draw new figures (38%). Besides, they have thought constructive the resource "Area" to check the results (96%) and the "seletor" to become the activity dynamic (31%). For the teachers (96%) was essential the possibility to get dynamic samples that show the not theorem's legitimacy in order to comprehend the theorem's hypothesis. Finally, the teachers (62%) have affirmed that this didactic sequence could not be done without the software. Despite they have recognized the software's potentialities, this kind of job is still a challenge for them, 54% never used. We hope that teachers reflect about their own practices but that Brazilians projects to improve and motive the job with technologies in special with Dynamic Geometry software in elementary school.

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# AN ANALYSIS OF THE TYPES OF QUESTIONS ASKED ON HIGH-STAKES EXAMINATIONS IN TURKEY AND IRELAND

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## INTRODUCTION

Both Turkey and Ireland have high-stakes examinations at the end of secondary school that determine entry to third level education. We are investigating the effects of these examinations on the teaching and learning of mathematics by surveying students and teachers in both countries. Here we describe our efforts to compare the examinations themselves.

## EXAMINATION CLASSIFICATION SCHEMES

The QUASAR project (Stein and Smith 1998) developed a set of criteria for classifying mathematics questions called Levels of Cognitive Demand. This scheme divides questions into 4 classes: Lower-level demands (memorisation - LM); lower-level demands (one-step procedures without connection to meaning - LP); higher-level demands (procedures with connection to meaning -HP); higher-level demands (doing mathematics - HD). In the LP category the procedure to be used is evident and completion of these tasks requires limited cognitive demand. In the HP category the questions require some degree of cognitive demand and procedures cannot be followed mindlessly. Questions in the HD category involve complex and non-algorithmic thinking and considerable cognitive effort. We modified this scheme slightly to include intermediate-level demands (multi-step procedures without connection to meaning - IP) in order to be able to distinguish very simple procedures from more complicated algorithms or calculations.

## RESULTS

All three authors independently used the scheme to classify questions from the 2009 and 2010 state examinations in Turkey and Ireland (ÖSYM (2010), SEC (2010)) and any discrepancies were discussed and decided on. We found that the Turkish ÖSYM examinations contained many more higher-level questions than the corresponding Irish examinations. Classification tables will be given in the presentation.

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# TEACHERS' DEFINITIONS OF SQUARE, RECTANGLE, TRAPEZOID, AND PARALLELOGRAM

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Mathematical definitions should distinguish a concept with certainty, be minimal and elegant. Definitions of mathematical concepts and the process of defining are fundamental parts of the subject matter knowledge of mathematics teachers, because teachers' knowledge of mathematical definitions influences their curricular and pedagogical decisions. Definitions generated by teachers may serve as a tool for examining their understanding of the specific concepts involved. The purpose of this study is to investigate practicing teachers' understanding of square, rectangle, trapezoid, and parallelogram as reflected by the definitions they generate.

Thirty six practicing teachers participated in this study. Twelve of these teachers were teaching mathematics at middle school level, the rest were teaching at primary school level. Participants' teaching experience varied between 1 year and 27 years. Data were collected through a written questionnaire. Teachers were asked to write as many examples as possible for a definition of a square, rectangle, trapezoid, and parallelogram. Written responses were analyzed using a framework developed by Zaskis and Leikin (2007). This framework contains three criteria, which were (a) accessibility and correctness, (b) richness, and (c) generality. Because of the nature of the data, only the criteria of correctness and richness were used in this study. Definitions generated by the participants were coded by two researchers.

The number of definitions for each concept provided by each participant varied between one and eight. While most of the elementary school teachers provided one or two statements for each concept, majority of the middle school teachers listed at least three statements for each concept under investigation. Teachers generated total of 357 statements for the concepts of square, rectangle, trapezoid, and parallelogram. Fourteen of the statements were not geometrical. Out of the remaining 343 statements, less than half of the statements were appropriate. In addition, one third of elementary school teachers were unable to generate an appropriate definition for each of the concepts of square, rectangle, trapezoid, and parallelogram. However, more than half of the appropriate definitions were generated by the middle school teachers. In addition, only 46 definitions could be considered as rich, and almost all of these statements were produced by the middle school mathematics teachers. Almost all of the elementary school teachers' definitions attended to sides and angles of the shapes.

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# E-PORTFOLIO AS A RESOURCE TO IMPROVE LEARNING IN MATHEMATICS PRE-SERVICE TEACHER

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The innovative and challenging use of Information and Communication Technology (ICT) is a claim in mathematics education. Interactions and thinking imply a growth of understanding. One tool that can be used to promote knowledge construction in teacher education within the context of ICT is the e-Portfolio. The possibility of publication in a digital version allows teachers and students' accessibility and visibility of the creation and progress of knowledge construction using a lot of sources, links and mediating tools.

This research<sup>1</sup> is focused on pre-service teachers learning using e-Portfolios. The data came from portfolios created and reconstructed by the future mathematics teachers. The process of reviewing and republishing pages through the Internet was continually saved and reviewed by the researchers. Seven case studies were conducted.

The results highlight the importance of use of ICT with pre-service teachers (PST) as a new interface for their own learning and a strategy for them to become more reflective about their learning and to improve their knowledge. In this process we can identify discoveries and constraints. With more experience and further analysis of exemplars of existing portfolios, PST became more nuanced in their organization of their e-Portfolios, reflecting the messages they conveyed (Brandes & Boskic, 2008). Reconstructing their e-Portfolios PST applied technical sources (YouTube videos, pictures, etc.), created different types of activities (reports, curiosities, diaries, games, etc.) and posted information about themselves used by them during the course. This communication process (Sfard, 2008) improved by ICT became a useful vehicle to move away from linearity and chronology to new organizational modes that better illustrated students' cognitive processes (Brandes & Boskic, 2008).

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# EXPLORING TEACHERS' UNDERSTANDING OF PROBLEMS USING THE NORMAL DISTRIBUTION.

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This study is an exploratory one designed to gauge students' understanding of statistics problems based on the normal distribution. It was carried out with 290 in-service secondary school mathematics teachers enrolled in an upgrading or retraining program, and it focuses on one multi-part problem included in the summative assessment. In this problem students were told that a set of scores on an exam were approximately normally distributed with a given mean and a given standard deviation, and they were asked to utilize a normal distribution table to find (1) what percentage of the scores lie above or below a given score or between two given scores and (2) what score separates the top given percentage of the scores from the rest. We refer to these problems as "unknown percentage problems" and "unknown value problems" respectively.

In analyzing the students' performance we draw upon Duval's (2006) framework concerning transformations of semiotic representations. Duval asserts that two different types of transformations of semiotic representations can occur during any mathematical activity. The first type, called *treatments*, involve transformations from one semiotic representation to another within the same system. The second type, called *conversions*, involve changing the system but conserving the reference to the same objects.

In unknown percentage problems, a given value is first transformed into an associated z-score, which can be seen as a treatment by means of the processes of re-centring and rescaling. The next step, in which the student identifies the probability associated with the z-score, can also be seen as a treatment because it also involves reading values from the z-table. The final step, interpreting the probability value in the table as the answer to the question, involves a conversion because it involves working simultaneously with properties of the standard normal distribution and the properties of particular z-table values.

Preliminary findings indicate that students are generally able to carry out the treatments in unknown percentage problems, but they struggle with unknown value problems which require conversions between different systems of representation, namely the z-table and a graph of the standard normal distribution.

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# PRIMARY PUPILS' DIFFICULTIES WITH FRACTIONS: A REPRESENTATIONAL VIEW

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The misconceptions that children have about fractions relate particularly to the way in which fractions are represented as numbers, and the standard ways that we use to represent fractions in diagrams and pictures in mathematics teaching (Kerslake, 1986). In terms of representations, Hart (1981) reported that although diagrams sometimes help with the solution of fractions problems or were used as a checking procedure, the actual process that children used with diagrams did not necessarily support conceptual understanding. Behr et al. (1983) differentiated between continuous and discrete representations of fractions. However, Nunes and Bryant (1996) highlighted that pupils may have a limited range of fractions representations available to them:

“The disconnection between pupils’ understanding of division of discontinuous and continuous quantities developed out of school and their learning of fractions might come about exactly because pupils do not think of fractions as having anything to do with division, and only relate fractions to part-whole language.” (p. 228)

In this presentation, we present the findings from a small-scale study carried in England of Year 5 and Year 6 (9 to 11 year old) pupils’ understanding of fractions. As part of the study, 15 pupils completed a series of questions related to fractions. The presentation will look at the results from these questions, looking in particular at the range of representations of fractions used by the pupils in tackling the questions. The study highlights (a) pupils’ lack of understanding of symbolic representation of fractions, (b) the limitation of pupils’ fractions knowledge to part-whole representations, (c) further limitations of pupils’ knowledge within part-whole situations to particular representations such as circular pizzas, and (d) as highlighted by Nunes and Bryant (1996), not linking division situations with fractions. Based on the results, a critical look at guidance for teachers in teaching fractions in primary/elementary schools will be presented.

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# INVESTIGATING PRESERVICE MATHEMATICS TEACHERS' MODELING PROCESSES

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Models and modeling perspective (MMP) is an approach that newly characterizes mathematics problem solving, learning, and teaching in mathematics education. Rather than the mathematical understanding and abilities emphasized in traditional textbooks and tests, this approach highlights new kinds of mathematical thinking that are becoming important to be successful beyond school in the 21<sup>st</sup> century. The goal of this study is to investigate three pre-service mathematics teachers' mathematical thinking processes from the models and modeling perspective.

According to models and modeling perspective, mathematical thinking is about interpreting and describing situations mathematically (i.e., developing models) that includes expression, explanation, communication, argumentation, construction, etc. (Lesh & Doerr, 2003). This is the kind of conceptual development that is induced by some specially designed real life problems that are called model-eliciting activities (MEAs) (Lesh, Hole, Hoover, Kelly, & Post, 2000). For the purpose of the study, we used an MEA called *Summer Jobs* (Lesh & Doerr, 2003) to investigate pre-service teachers' model (conceptual) development processes. Because of the thought-revealing nature of MEAs, we expected that we could directly observe and detect significant conceptual changes in students' thinking during this single 90 minute problem solving session (Lesh et al., 2000). Participants were three pre-service mathematics teachers in their fourth years in the program. They worked on the activity collaboratively. Data were collected through audio and video records of the session as well as the worksheets they produced. Preliminary analysis of the data indicated that pre-service teachers went through a series of non-linear modeling cycles which included different ways of interpreting the problem situation.

We expect that the results would provide a window for us to look through teacher knowledge development in a new way as models are among the most important kinds of knowledge. In this context, some theoretical and practical implications of this study for pre-service teacher education will be discussed.

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# THE EFFECTS OF ELEVENTH GRADE STUDENTS' GEOMETRIC REASONING PROCESSES ON THEIR REPRESENTATIONS

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Representations are very powerful tools to explain mathematical reasoning processes and to understand relationship between any mathematical concepts in problem solving processes (Goldin, 1998; NCTM, 2000). The purpose of this study is to find out a possible relationship between eleventh grade students' representations and geometric reasoning in the geometric word problem solving process related with polygons. To do this, a qualitative design which is case study was used and three 11<sup>th</sup> Grade Anatolian High Schools students were determined as participants via results of geometry achievement exam prepared by researcher, average of written examinations which were belong to first semester, and geometry teacher's opinions about students. The data were collected using by think aloud method, two geometric word problems focused on polygons, and unstructured interviews. Each student practiced the think aloud method before solving two geometric word problems. After working on each problem separately, students were interviewed about fifteen minutes. These data were analyzed using the within-case techniques.

The findings of this study showed that when students misunderstood a geometric concept, they produced useless representations to solve problems. In addition, because of the uneasy nature of word problems, participants had some difficulties in geometric reasoning process. As a result of misunderstandings about geometric word problems, there was not any interaction between internal and external representations of participants, which revealed students' rote learning related to geometric concepts. There is no doubt to state here that it is possible to evaluate about students understanding, difficulties, and misunderstandings about geometric concepts based on geometric reasoning by means of students' internal and external representations in problem solving process. As a conclusion, geometric word problems have very important role to determine students' understandings about geometric reasoning processes in terms of their representations.

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# THE DYNAMICS OF ARGUMENTATION IN MATHEMATICS CLASSROOM: A COLLABORATIVE RESEARCH PROJECT

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In many countries, mathematics curricula underscore that mathematics must be, to students, a sense-making activity and that reasoning is a central aspect of mathematics teaching. In classes where reasoning is valued, both explanation and justification are key aspects of students' mathematical experience and, thus, an emphasis on reasoning brings to the forefront the need for their involvement in mathematical argumentation activities (Yackel & Hanna, 2003).

Learning to argue mathematically is deeply connected with a habit of mind related to the "why of things". To promote this habit, the teachers are asked to encourage and support a mathematical discourse that has certain properties; to orchestrate whole-class discussions that use students' contributions in ways that advance the mathematical learning of the whole class; to establish a *culture of argumentation*; and to pay attention both to cognition and emotions (Boavida, 2005; Lampert, 2001). This is a very complex process in which they face several dilemmas and challenges. It is important to understand this process, because teaching shapes students' conceptual understanding, their capacity of reasoning and their disposition towards mathematics.

The purpose of this presentation is to provide an overview of selected results from a two-year collaborative research project, developed by two middle-school teachers and a researcher, focused on the dynamics of argumentation in mathematics classroom (Boavida, 2005). This project is rooted in the importance of collaboration between teachers and researchers to improve teachers' practices and the knowledge about teaching (Christiansen et al., 1997). Particularly, it is intended to reveal the potentialities of the project for teachers' professional development as well as its relations with aspects that are relevant to foster mathematical argumentation.

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# THE DEVELOPMENT OF MULTIPLICATION CONCEPT IN OPEN APPROACH AND LESSON STUDY

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It is clear that failure to develop multiplicative structures in the early years impedes the general mathematical development of students into the secondary school, for example, in using algebra, functions and graphs. It appears that difficulties faced by older students can be attributed, at least in part, to the lack of development of an equal-grouping structure in early concept formation (Mulligan & Mitchelmore, 1997). In Thailand, there was an image of instruction in multiplication that the memorization in multiplication tables, very emphasized in verbal memorization and calculation (Boonlerts, 2010). Thus, the objectives of this paper were 1) to investigate how children develop a basic multiplication concepts and 2) to investigate how children develop a basic multiplication concept in Open Approach and Lesson Study context. The teaching experiment methodology was employed in this study. Data collected through 12 lesson plans by video recordings, audio recordings, interviewing, field notes, and student's written work. The research findings were found as followings;

1) Students developed multiplication concepts when they learn learning unit 1 :Introduction multiplication and learning unit 2: multiplication table of 2, 5, 3, 4. The development of multiplication concept is described in order of increasing sophistication from initial grouping and perceptual counting to abstract composite units and repeated addition and to multiplication as operation.

2) Open Approach and Lesson Study changed the way of taught from only memorize in multiplication tables and calculation to focus on equal-grouping problem, developing the new unit for counting and meaning of multiplication.

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# **A MODEL TO TEACH TEACHERS TO BECOME ONLINE TEACHERS**

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The present research on mathematics online education has focused on the possibility of teaching and learning mathematics through online environments, on the new characteristics of this modality of education, and on new challenges faced by teachers and students when education does not have the face-to-face contact. It has also been discussed how different kind of interfaces used in online discussion may change the kind of interaction. For more than ten years, as interest in on-line distance education has grown, the problem seems to recur: how to prepare teachers to proffer on-line courses? How to teach teachers, in all levels of education, to teach online? In other words: how to disseminate some of the practices of the pioneers in online teaching in order to promote, for a larger number of teachers, the ability to teach online? While these remain open questions, we present results of a research project focused on questions related to on-line distance education that emerged in online courses proffered to mathematics teachers. Specifically, we present a model for teaching teachers on-line that is based on three stages: first, future on-line teachers participate as students in a continuing education course; second, they participate as assistants to teachers who are ministering these courses; and finally, they assume command of the course under the guidance of their former teachers.

The model presented is designed to help teachers to become online teachers and may not be generalizable, especially due to the very nature of the methodology used. The model seems to work within a given context. The study used qualitative data: texts from e-mail, transcriptions from video-conferences, attachments containing solutions to problems, etc. It was based on a paradigmatic view that knowledge is contingent and is embedded with human values. The design of the study was emergent in the sense that the particular problem addressed in this paper arose during the process of investigation. This does not mean there is nothing to be learned by others who are interested in developing programs to capacitate teachers for online teaching. Researchers should adapt and develop the model presented, taking into account differences and similarities between their own contexts of teacher education and the context in which our research was developed.

# TEACHERS' CONDUCT OF PROBLEM SOLVING ACTIVITIES<sup>1</sup>

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In recent years, Turkish curricula at all levels have undergone a tremendous reform, which attaches a special importance to the development of students' problem solving (PS) skills. Teachers play a key role for the development of such skills. Hence it is important to gain insights into how teachers go about teaching problem solving as part of their instructional practices. In the field of PS, research studies have paid much to students and their problem solving processes. However, relatively less research attention has been directed to teachers' conduct of PS activities (e.g., Fai 2005) and even less to connecting teachers' conducts to the development of students' PS skills. In this study, we examine and critically reflect on teachers' conduct of PS activities and note implications for development of students' PS skills.

In our attempt, we examine 4 teachers' (two elementary teachers and two elementary mathematics teachers) instructional practices while solving mathematical problems. Our examination of these four teachers revealed that the teachers do not pay sufficient attention to the students' understanding of the problem statements; mostly ignore planning stage, starting directly with the implementation of a solution strategy; do not pay much attention to the rationale behind the selection of the solution strategy and meaningfulness of the obtained result. We also observed that control step of the problem solving often remains limited to checking the computational accuracy of the arithmetic operations, which consumes much of the time devoted to the PS.

Our findings become especially important when viewed with the result of recent studies conducted in Turkey showing that teachers tend to hold students responsible for the failures that students experience for mathematical problem solving (Esendemir et al., 2010). We certainly believe that students are not the sole actors for such failures and cannot be considered so. During the presentation, we discuss the implications of our findings regarding the development of students' PS skills as prescribed by the curriculum scripts.

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# MATHEMATICAL THINKING OF PRESERVICE ELEMENTARY MATHEMATICS TEACHERS ABOUT CONCEPT OF FUNCTIONS

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In this study, pre-service elementary mathematics teachers' (PEMT) mathematical thinking of function concept was investigated. In order to teach the concept of function effectively, a mathematics teacher should have strong knowledge about it. For this reason, PEMTs must have deep understanding about the function concept and different applications of it.

Function is one of the fundamental concepts in mathematics, yet it remains difficult to understand (Sajka, 2003). New standards and documents emphasize the use of by multiple representations such as graphs, equations, tables, and words in the development of mathematical thinking and reasoning (NCTM, 2000).

Participants of this study were 63 PEMTs in their first year in the in the program. In order to investigate mathematical thinking of PEMT's, data were gathered through open-ended questions and written responses to the questions related to the concept of functions. Also, counter-examples were used to examine participants' knowledge about functions. The data were analyzed by coding and categorizing the written data, and relevant frequencies were also developed.

According to findings of this study, PEMTs had difficulties in abstract thinking, graphing of functions, generalizing, giving counter-examples and using multiple representations of functions. One of the difficulties of PEMTs about functions was using three most important forms of representation, namely the numerical, graphical, and algebraic forms.

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# THE ROLE OF CONCRETE MATERIALS IN ARITHMETIC PROBLEM-SOLVING AT THE PRIMARY LEVEL: A MICROGENETIC ANALYSIS

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Children often use a variety of objects for which they are the only interpreters. The symbolism inherent in their relationship with the objects, which for Vygotsky are external artefacts, seems to offer interesting avenues of research to understand learning in children. In elementary school mathematics, teachers often provide students with concrete materials for them to use when solving arithmetic problems. Research confirms a higher level of success when students use concrete materials compared to a condition where they are not used. The question now is to understand in-depth, the role of these materials for solving arithmetic problems since, despite their presence in many elementary school classes, arithmetic problem-solving remains problematic.

In this study, we used a microgenetic approach (Gutierrez, 2006; Siegler et Crowley, 1991) which involves the detailed observation of student's work on a specific task over time. It is used in situations involving rapid change. The objective of the analysis is to infer the processes driving change in the role of the materials in the student's response to the problem. Theoretically, our analysis focuses on the dialectic between subject (student) and object (materials).

Six Grade 3 students took part in the study. They each solved three arithmetic word problems. Concrete materials were available if the students needed them. Each student was interviewed about (1) their solutions and (2) a video recording of them solving the problems. For this presentation we report analysis of data from three of the students. In each case, the dialectic relation subject/object is not focused on the object available, but on the object read and represented in abstract form, or through the use of images or symbols.

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# PROFESSIONAL KNOWLEDGE OF PRE-SERVICE TEACHERS ABOUT QUOTITIVE DIVISION PROBLEMS

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A relevant aspect in Mathematics Education is the identification of the necessary knowledge to teach mathematics and how the different types of knowledge are related. Ball, Thames, and Phelps (2008) made a more detailed classification of the types of knowledge proposed by Shulman (1986): common content knowledge (CCK) and specialized content knowledge (SCK) into the subject matter knowledge, and the knowledge of content and students (KCS) and the knowledge of content and teaching (KCT) into the pedagogical content knowledge. The present study focuses on the different types of knowledge that pre-service teachers used when they interpret sixth-grade pupils' answers (11-12 years old) to quotitive division word problems.

The participants in the study were 84 pre-service teachers and they answered two tasks. Task 1 consists on the resolution of 2 quotitive division word problems and task 2 on the scored of 4 primary school pupils' answers to each problem from task 1. Two of the pupils' answers were based on a division, one with a technical error; the other two are based on alternative methods (enumeration, repeated addition or subtraction or building-up strategies), one of these with a technical error. Pre-service teachers had to grade these answers with 0, 0.5 or 1 and provide justifications.

Results show that some pre-service teachers who solved the problems correctly manifested difficulties in the interpretation of some strategies based on alternative methods because they graded the answers based on a division better than the answers based on alternative methods. These results show the necessity of insisting on the knowledge related to multiplicative structure problems and particularly, on the different methods of resolution and how primary school pupils use them (SCK and KCS). On the other hand, some students that did not solve problems correctly, they identified the correct and incorrect answers. So, proposing this type of professional tasks could help pre-service teachers to acquire the different methods of resolution and different errors (SCK).

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# DEVELOPING MATHEMATICAL THINKING: CHALLENGES FOR THE PROFESSIONAL KNOWLEDGE OF TEACHERS

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Current Portuguese curriculum recommends the development of mathematical thinking for students of all ages, stressing the importance of exploratory teaching and learning based on students work with investigative tasks. This approach represents a challenge for most of the teachers, requiring them disposition to take risks and a specific professional knowledge (Ponte & Chapman, 2006; Sowder, 2007).

This presentation focuses on a study developed to understand and to contribute to foster the professional knowledge of teachers involved in the development of investigative tasks in the classroom. The researcher worked regularly in collaboration with two teachers, one of the 1<sup>st</sup> grade and another of the 2<sup>nd</sup> grade. The researcher introduced the investigative tasks to the teachers, helped them on the selection, planning and development of the investigative tasks in classroom, and encouraged teachers' reflection after the lessons. This collaboration took about six months.

We adopted the interpretative paradigm and elaborated two case studies, one of each teacher. The sources of data collection were the planning sessions, teachers' lessons, and the reflection sessions (all audio recorded and transcribed) and also the written documents produced by the teachers (lessons' plans and tasks).

The cases highlight that supporting students' mathematical thinking is a very complex teaching activity that requires the combination of teachers' mathematical knowledge and teachers' instructional process knowledge. The improvement of the adequate professional knowledge can be fostered by collaborative work that focus on teachers' practice, including the careful planning of the lessons (tasks resolution and management) and the reflection based on mathematical productions of the students.

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# ANALYSING MATHEMATICS EXAMINATION QUESTIONS

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The main purpose of the present study is to establish a model that permits to establish different categories of examinations questions complexity, and, based on that analysis to scrutinize examinations quality.

To develop the model, it was created a structure with five categories, *Prestructural*, *Unistructural*, *Multistructural*, *Relational* and *Abstract*, based on three parameters, *Knowledge*, the amount of mathematical knowledge involved in the answer; *Operations*, the type of reasoning used; and *Answers*, the kind of answers and the consistency with the conditions provided, adapted from the SOLO taxonomy.

Hypothetical answers for the examination items were considered to categorize the questions. Several exams, of different school grades (4<sup>th</sup>, 6<sup>th</sup>, 9<sup>th</sup> and 12<sup>th</sup>), were studied to test the consistency of the model. Modifications were introduced on the model.

As an example, consider the 3<sup>rd</sup> question of the 4<sup>th</sup> grade examination of 2009 (*A group of 47 children, from a holiday camp, are going to climb. Children are going by car. Each car carries 6 children. How many cars will be necessary to carry the 47 children?*). The correct answer is 8 cars - dividing the number of children, 47, by 6 (children by car) we realise that 7 cars became full and remain 5 children, which require another car. To reach the solution it is possible to use an additive or a multiplicative structure. In both cases, to answer this question it is essential to understand the meaning of the operation involved, and realise what the results mean to the proposed context.

This question can be considered a *relational* one: it is necessary to use arithmetical knowledge from the 3<sup>rd</sup> and 4<sup>th</sup> grades (DEB-ME, 2004), and to connect the mathematical solution with real context; a deductive reasoning, similar to others experienced in class, is required; the answer is closed and unique, but it is necessary to solve the inconsistency between the arithmetical result and the real world context.

This example, and others from different grades levels, seems to show that the model is consistent, and can be an important tool to analyse examination questions.

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# **PROCESS OF CONSTRUCTION OF THE KNOWLEDGE ON DIVISION TO DECIMAL PLACES AT FOURTH GRADE LEVEL**

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For a student to form mathematical knowledge in context there needs to be situations in which this knowledge answers some problem. In recent reform documents, many of the theories explaining the formation of knowledge have the idea that the student needs to form the knowledge by himself/herself as their basic philosophy. The most recent of these theories is RBC (Recognizing-Building with-Constructing), which underlines needs and stimulation (incentives) in the process of abstraction. Abstraction, one of the basic concepts used in this study, is defined in simple terms as “the process of moving from tangible to the abstract” (Hershkowitz, Schwarz and Dreyfus; 2001) This study aims to examine the process whereby a fourth grader comes to need dividing to decimal places, and the process of the formation of this knowledge. The study was conducted with the participation of a volunteering fourth grader, and uses a case study as research design. Two problems that could be solved by the existing knowledge of the student but that were designed to give rise to the need for dividing to decimal places and that are appropriate for examining the process of abstraction were used. Problems were given one by one, and the student moved on to the second question after completing his work on the first one. Both of the problems selected for this study are problems that can form the basis of an understanding of Division to Decimal Places, the reason being the consolidation of structures acquired in the first question. The process of problem solving was observed by the researcher. The meeting was videotaped, with the information and consent of the student. The verbal and non-verbal communication of the student was also observed. The study also showed that real but non-routine problems can contribute to better formation of mathematical knowledge. In learning environments, selection of proper problems is an important factor. Working on real events and problems and on their mathematization, instead of using conventional methods, can facilitate the process of abstraction.

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# MATHEMATICS TEACHERS' REALIZATION OF MODEL-ELICITING ACTIVITIES

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We are in a rapidly changeable world and will face problems much more complicated than before. Our students need to learn how to create new knowledge in order to solve tough problems in the future. Some international assessments tended to emphasize students' modeling competency, such as PISA. Therefore, the transform of mathematics instruction from lecture to modeling teaching is indispensable. We definitely need teachers who can design model-eliciting activities (MEAs, Lesh & Doerr, 2003) and teach mathematical modeling in the classroom.

This study focused on the development of mathematics teachers' realization of MEAs and the progression of designing MEAs. Data collections included the learning sheets that showed teachers' strategies of the MEAs and the results of the MEA they designed, observation journals, reflection journals, questionnaires, interview reports and video tapes of the classes.

The results showed that the key points of promoting teachers' understanding of MEAs were engaging in solving MEAs as the roles of students and implementing MEAs in their classroom. At first, Teachers regarded MEAs as open-ended problems with real-life context and more guidance, such as examples or tables. Then, they notice the necessity of newspaper article and readiness problems as warm up activities, in order to let students engage in the main problem smoothly. Finally, the experience and reflection of MEAs implementation provide more support to revise their conceptions of MEAs and models. They also mentioned the obstacles of implementing MEA and designing MEA.

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# COMPARISON BETWEEN INSTRUCTIONS OF SENIOR AND NOVICE TEACHER

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While examining teachers' classroom instruction and their expertise have long been the interest of educational researchers and psychologists albeit mainly in the West, there is a lack of systematic studies on teachers' expertise in mathematics instruction (Li & Kaiser, 2011). Thus, this study conducts comparison on instructions of two teachers, one senior and one novice (s teacher and n teacher are called accordingly below), in lessons with topic of polyhedron. The method of observation in classroom, interview after class and focus forum are employed to collect data. Yet, this article reveals only the discovery of Euler's formula in regular polyhedron.

Following are the distinctions found between the instructions of two teachers. In terms of counting number of edges, faces and vertices, n teacher tends to be result-oriented and s teacher is more course-oriented. Moreover, both conduct the lesson with table although problem posing after completion of table is different. N teacher leads students to uncover her final conclusion by feeding problem one by one. S teacher, on the other hand, poses open problem. Both teachers also adopt distinct attitude to student's discovery. N teacher responds to Euler's formula only and ignores student's other revelation and s teacher, on the contrary, encourages student's every discovery.

In General, n teacher follows teaching steps in textbook and s teacher heeds student's reaction and improvises. It is always crucial for teacher to seize the opportunity to explore further. N teacher has a great topic for the class to discuss when a student uncovers Euler's formula as well as a model which systematizes number of edges of regular polyhedron. However, she chooses to ignore the model since the student, who goes to math tuition and always makes statement ahead of the progress, has revealed her original goal, Euler's formula. Two questions are proposed for further discussion from above incident. Does student's discovery not matter as long as teacher's goal is met? If a teacher's impression towards student obstructs development and discussion of a topic? Furthermore, student's discovery may be associated with contents of problems. In this case, the additional component, shape of face, in n teacher's table may be an important element which leads student to the model. Lastly, it may be doable to supplement the material which does not appear in textbook as long as it suits student's level. Students uncover Euler's formula in this study is one good example.

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# AN ANALYSIS OF SCRIPTS OF TEACHING RELATED TO OPEN APPROACH

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Teaching is a cultural activity that is highly determined by beliefs and habits that work partly outside the realm of consciousness; if we want to change the teaching, we would begin the improvement process by becoming more aware of the cultural scripts teacher are using (Stigler & Hiebert, 1999). This study focuses on scripts of teaching based on four phases of Open Approach as a teaching approach that proposed by Inprasitha (2010).

The aim of this study was to analyse scripts of teaching related to Open Approach. The targeted group was six teachers participating in the Professional Development Project which has been implemented lesson study incorporating Open Approach. Each teacher has an experience in the traditional classroom more than 25 years. After participating in the project they changed their teaching practice based on activity under the cycle of lesson study as the followings; 1) collaboratively plan lesson 2) collaboratively do teaching the lesson 3) collaboratively reflect on the teaching approach. According to these 3 phases of lesson study, Open Approach as a teaching approach is incorporated in the second phase. The data were collected by videotaping teaching practices of one teacher including teacher's log during 2007-2010 academic year and interviewing six teachers.

The results showed that scripts of teaching related to Open Approach were as followings; the first, the teacher posed the open-ended problem by either mounting or writing the instructions on the blackboard. The second, the students involved in problem solving or doing group activities; meanwhile, the teacher walked around and observing. Moreover, the teacher kept stimulating the students to collaborate in solving the problems. The third, the teacher made the students present their work by telling what they had done. Finally, after the presentation, the teacher made summary of the current lesson taught and learnt. The findings suggested that the scripts of teaching provided teacher to notice the different from traditional classroom and challenged their traditional beliefs about mathematics teaching.

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# SIX GRADERS' GENERALIZATION PERFORMANCES OF FIGURE PATTERN

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Assignments on generalization activities could develop thinking skills and help students discover the rules; if a student can organize, conjecture, and induce data by analysing special cases and extract the rules from the patterns, the assignment could also improve his ability to solve problems. Therefore, mathematical education in schools should focus on developing reasoning skills and enable students to describe and demonstrate generalization.

Research on pattern generalization and the practice of teaching in primary school in Taiwan reveals that, aside from arithmetic, schools seldom present experience in perceptual pattern development or other solving strategies. In these schools, arithmetic is only a way to find the answer. Finding words used in mathematics classes is common; however, there is a lack of exploration regarding the design of figure patterns or student achievement in generalization. How about student performance during generalization of figure patterns? How do they generalize the problems in figure patterns? What kinds of strategies do they adopt to solve the figure pattern problems?

255 sixth graders completed 1) pattern element perception, 2) pattern combination, 3) reasoning, and 4) generalization four kind of test. The data was analysed using descriptive statistics, t test and a qualitative approach. The results show that there are significant differences among variety subtests performance, students' pattern variables recognition, regroup and reasoning performance in transparent quality figure are better than non-transparent quality figure, and according to the characters of dispense or combine structure of figure to use multiple strategies to thinking and compose patterns, students could seek the variables relationship between different stages and extend and represent figure pattern, apply visual's strategies and connect analyses methods to integrate the rule of pattern correctly. These findings can assist teachers as they engage students in opportunities to learn pattern generalization and can also be useful as teachers participate in instructional and curriculum design.

# THE STUDY OF TEACHING COMPOSITION OF GEOMETRICAL FIGURES FOR CHILDREN

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The purpose of this research is to growing competence of compose geometric shapes for young children. The goal is the creation a teaching model based on previous research of hypothetical learning trajectory, as well as instrumentation to assess levels of learning along the developmental progression underlying the model. Under following school themes, the teaching model including a sequence of tasks which was designed for children's learning by the research group. From spatial orientation to geometrical figures construction, activities and tasks provided teachers to help students learning spatial concepts and recognize geometrical figures. Then, the researcher and teachers kept tracking the children learning for modify instructions during the process. We tested both the developmental progression and the instrument through a series of studies, including formative studies (through cooperation of the researcher and 6 kindergarten teachers) and a summative study involving 80 kindergarteners' ages 5-6 years. Furthermore, the children received formative evaluation and authenticable pre-test and post-test before and after teaching. Composition geometrical figures of authentic assessment that the research group constructed and it included six tasks as the tool for using in pre-test and post-test. During the test children need to manipulate shapes as individuals to complete these tasks with tangram from one to seven pieces to make a larger square following the patterns that assessment sheets have showed and based on the shapes' attributes. Subjects were three classrooms which came from a public kindergarten within an elementary school for an intentional sample. Except tests, collecting data was through observation, individual interviews and documents focusing on research purpose. Results provide strong support for the validity of the developmental progression's levels and suggest that children more through these levels of thinking in developing the ability to compose 2-dimensional figures. From lack of competence in composing geometric shapes, they gain abilities to combine shapes—initially through trial and error and gradually by attributes—into practice, and finally synthesize combinations of shapes into a shape that the tests designed. The result found the post-test of performance in constructing shapes was better than pre-test's; it revealed the children had promoted their composition of geometric figures of abilities through the teaching model. The research suggests that teachers pay attention to the visualization of the factors impact the abilities of composing geometric shapes and give more practice on rotate and flip of skills to promote space and shapes of composition for children.

# A STUDY ON CHILDREN'S DATA VARIATION CONCEPTS USING SCENARIO-BASED INTERVIEW

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Though children have no enough knowledge in mathematics and statistics to understand variation measurement and model, but they probably can use their own experiences and knowledge in life to interpret. Many researchers suggest that statistical literacy and thinking start from being aware of data variation in the true environment. Therefore, providing children true environments with their familiar experiences in life to study their data variation concepts is necessary. In Taiwan, most statistics learning topics in mathematics curriculum of grade 1-6 have no concern of variation in dealing with data. Therefore, to explore whether students can understand data variation, how they create data variation concepts can help to have better ideas in teaching and learning statistics. This study is a three-year research project (2009-2012) sponsored by Taiwan National Science Council. An interview protocol has been devised, according to Wild & Pfannkuch's (1999) consideration of variation. The contexts for interview problems are those most students experience in life, such as shopping in the supermarket. To enable students to feel like being in the problem scenario, we went to a supermarket to shoot movies with children's dialogue to lay out the problems, and we found films on the internet to edit, such as apples screening process. Students watched these films during interview. Up to now, 60 students in grade 3-6 participated in this project. Most students (93%) recognize the omnipresence of variation in life, but, when we asked them the total weight of three apples, they (53%) used mathematical thinking to answer. Some students (72%) do not allow any slight error if there is "50g net weight" written in the candy box. The error ranges allowed by some students (33%) are influenced by rounding concepts. Most students (91%) can point out which data set in the graph is more centralized, but only few students (18%) confirm that they prefer to wait for the bus with more centralized distribution of time intervals between two successively arrival buses. An initial concept framework will be reported.

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# THE STABILITIES OF MATHEMATICS UNDERACHIEVEMENT IN THE ELEMENTARY SCHOOL

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Taiwan's math performance was ranked highly in many international mathematics competitions; however, the topic on mathematics learning difficulties was overlooked in the past. Low mathematics performance led to learning anxieties was a well-known issue, so early screening could successfully avoid academic failure from the prevention viewpoints. Many experts also suggested that examining learning difficulties from a longitudinal developmental approach could help us to determine a child's growth trend, which was fundamental for understanding learning and learning difficulties. Therefore, this study aimed to investigate the stabilities on math achievement of the underachievers in elementary school from a longitudinal developmental perspective.

There were 247 first graders and 291 third graders selected from 8 elementary schools in four different areas in Taiwan participated in this three consecutive semester study. Subjects were administrated the Mathematics Achievement Test (MAT), the Reading Comprehension Test (RCT), and the Raven's Progressive Matrices Test. Among them, those who performed lower than PR25 in MAT and IQ scores ranked above PR16 were defined as math underachievers and were further divided into two subgroups by calculating the discrepancies between math achievement and IQ scores using Z score system. All test scores were transformed into Stanine Scale and SPSS MANOVA repeated measures were used to analyse data. The main findings were: (1) although about 30% to 40% math underachievers were found across the three consecutive semesters for both graders, only one third of them continued to be classified as math underachievers; (2) IQ was the only impact factor to subjects' math performances for both graders; (3) no differences were found in math scores between two subgroup underachievers. However, from the repeated comparisons, significant differences were found between the 1<sup>st</sup> and 2<sup>nd</sup> semesters both on math achievement and reading achievement performances. Some suggestions were made in this study for future implementations.

# A FRAMEWORK FOR EVALUATING THINKING OF TEACHERS IN DESIGNING TASK SEQUENCES

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Kai-Lin Yang

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Involving teachers in design research and viewing them as active designers have become an emerging trend in teacher education. However, the ways to examine thinking in designing task sequences have not been fully investigated yet. In this regard, we formulated a framework that can be used to evaluate teachers' thinking in designing task sequences. The framework contains three critical dimensions: the theoretical perspective, the pragmatic perspective, and the practice (see Figure). The theoretical perspective in the framework was adopted from the model proposed by Ruthven, Laborde, Leach, and Tiberghien (2009), specifying the grand theory, intermediate framework, and design tools. Ruthven et al. argued that the intermediate framework and design tools can be used to contextualize the theories into practice and make connection of both. The pragmatic perspective follows the product design process from Ulrich and Eppinger (2004), which identified the idea initiation for design, ideas transformation, and the key elements embedded in the design. We hypothesize that the interplay of two perspectives constitutes the reality of designing task sequences in practice and is the key to determining the quality of the work.

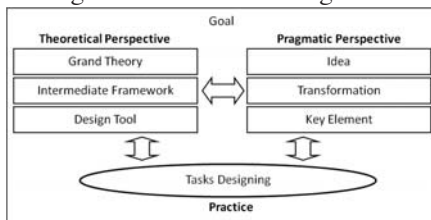


Fig. A Framework for Evaluating Thinking of Teachers in DTS

Our analysis of 15 experienced mathematics teachers participating in a professional development workshop further confirmed the utility of this framework for evaluating teachers' thinking in designing task sequences. Most mathematics teachers designed their task sequences mainly followed a pragmatic perspective rather than a theoretical one. But some mathematics teachers whose design stayed in a higher quality considered both perspectives and tried to make connection of both in their designing processes. Specifically, these teachers relied on pragmatic perspectives as means to bridge the theories into actual task design in practice rather than used intermediate framework as suggested by Ruthven et al.

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# THE PASSINGS RATE OF THE VAN HIELE LEVEL OF GEOMETRICAL REASONING

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This study presents partial results from the project “A study of the use of the van Hiele thinking level in the geometrical reasoning to the elementary school students”, funded by National Science Council of Taiwan (NSCTW, Grant No. NSC 99-2521-S-142-008) (Wu, 2010). It was undertaken to explore the van Hiele levels of geometrical reasoning of the elementary school students at the first three van Hiele levels.

There are five van Hiele levels of the geometric thinking: “visual”, “descriptive”, “theoretical”, “formal logic”, and “the nature of logical laws” (van Hiele, 1986, p. 53). The focus of this study was the first three levels.

The participants were 741 elementary school students who were randomly selected from 4 counties/cities in central Taiwan.

The instrument, developed by the authors of this study, Wu-Ma-Chen’s Test of Geometric Reasoning (WMCTGR), was specifically designed for this project. This instrument was designed base on van Hiele level descriptors and sample responses identified by Fuys, Geddes, and Tischler (1988). There are 32 multiple choice questions characterized into two types, namely *critical level* and *creative level*, based on its reasoning attributions. The reliability of Cronbach's  $\alpha$  is 0.843 ( $p < .01$ ) (Wu, 2010).

The conclusions were drawn as follows: in the critical level, the fifth-grade students have a higher pass rate in three questions (16.67%). These questions include one of two-dimensional geometry (5.56%) and two of three-dimensional geometry (11.11%). In the creative level, the fifth-grade students have a higher pass rate in three questions (21.43%). These questions are three-dimensional geometry concepts.

Keywords: elementary school, geometric, reasoning, van Hiele, thinking level

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# QUESTION-ANSWER, VALIDATION AND FOLLOW-UP IN LESSONS OF MATHEMATICS

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Schwarz, Dreyfus and Hershkowitz (2009) point to the importance of analyzing the processes involved in classroom interaction to explore what is under the shared construction of mathematical knowledge. To represent “optimal” contexts for mathematical learning, in our own research we bring up a critical perspective that aims at supporting changes in classroom cultures toward dynamics of group discussion and pair work. We assume that interaction among students is central to their mathematical learning and to the orchestration of diverse forms of “good” teaching. In her ongoing PhD work, Chico explores several classroom episodes with pair work to better understand cooperative interaction that fosters task-relevant mathematical knowledge.

Cobo and Fortuny (2000) typify several mathematical exchanges in pair interaction, some of them with evidence of cooperation and some others with students working in parallel, and not really helping each other with the mathematics. In our analysis, we pay special attention to the identification of short episodes in which students accomplish cooperative interaction. By cooperative interaction in pair work, we understand a conversation in which the two parts make progress in the discussion of a certain task as a result of continuous processes of negotiation among differing views. We search for exchanges with indicators of interaction, namely: 1) question-answer (one part answers something that has been asked by the other, and eventually introduces either new information or another question); 2) validation (one part gives effect, values or repeats what has been said by the other, and gives or asks for further argumentation); and 3) follow-up (one part states a proposition that is mathematically expanded by the other part). At present and for different episodes, we have developed tables with empirical interpretations of the indicators above as a way to illustrate equitable contribution of mathematical information on the part of each student in particular cases of pair work.

## NOTES

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# HIGH-SCHOOL MATHEMATICS TEACHER'S SPECIALIZED CONTENT KNOWLEDGE: A CASE STUDY

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No one questions the idea that what a teacher knows is one of the most important influences on what is done in classrooms and ultimately on what students learn (Fennema & Franke, 1992). Ball and Bass (2000) sought to complement the examination of curriculum and of what experienced teachers know with a mathematical analysis of core teaching activities, and sought to identify the underlying resources entailed by these teacher activities. Ball, Thames, and Phelps (2008) identified mathematical knowledge for teaching (MKT) and its structure, and Delaney, Ball, Hill, Schilling, and Zopf (2008) checked the constructs of MKT and measured MKT outside the United States. Furthermore, Learning Mathematics for Teaching Project (LMT, 2010) indicated that there is a significant, strong and positive association between levels of MKT and the mathematical quality of instruction. Therefore, it is important to explore high-school mathematics teachers' MKT, in particular, their specialized content knowledge (SCK) outside the United States.

This qualitative case study investigates one high-school mathematics teacher's SCK and its relationship to other domains of MKT in Taiwan, and revises the coding rubrics developed by LMT, to adapt to Taiwanese high-school classroom teaching. As a result, features of the participant teacher's SCK showed a fluency of cross domain interactions among SCK, knowledge of content and teaching (KCT), knowledge of content and students (KCS), and horizon content knowledge (HCK). This flow of cross domain movements in instruction seems to suggest that these 4 domains of MKT are somehow overlapped at times in the process of a teacher's classroom teaching.

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# THE INTERPLAY BETWEEN FORMAT OF PATTERN DISPLAY AND EXPRESSING GENERALITY

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Apart from student-related factors such as lack of spatial visualisation techniques, ignorance of appropriate generalising strategies and inexperience in using the highly specific mathematical language of algebra to express generality, we suspect that students' difficulty in expressing generality in some of the studies involving pattern generalisation (Hoyles & Küchemann, 2001; Cañadas, Castro & Castro, 2011) could have been influenced by certain features of the generalising tasks. However, this observation was drawn from personal insights rather than from an experimental design study in which the task features in question were systematically controlled for. Our study, therefore, takes on the challenge to determine whether certain task features could have a role in students' expression of generality.

This report presents some preliminary findings of a part of our study that investigates the potential influence of the format of pattern display on students' pattern recognition and rule construction. Data were collected through administering a test which comprised four figural generalising tasks involving linear patterns to 43 secondary school students (aged 14 years) from one school. Each task was created in two different formats: (1) a sequence of three successive diagrams, and (2) a single diagram or a sequence of two or three non-successive diagrams. The students were divided into two groups; one for each format. They had to establish the general rule underpinning each pattern. The student data were analysed and revealed some evidence of the influence of the format of pattern display on their pattern recognition and rule construction. During the oral presentation, the four linear generalising tasks used in this study and results on student performance will be presented.

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# A CASE STUDY OF METACOGNITIVE INSTRUCTION

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## INSTRUCTIONAL DILEMMA – DESCRIPTIVE OR PRESCRIPTIVE?

Metacognition refers to our knowing about our own cognitions. There is a problem, reported by Schoenfeld (1992), with any metacognitive instruction. Characterizations of metacognition tend to be *descriptive* and support recognition of strategies used but:

[do] not provide the amount of detail that would enable people who were not already familiar with the strategies to be able to implement them (p.353)

Alternatively, if they are *prescriptive* a small set of heuristics becomes an unwieldy list of skills to try and memorise. Schoenfeld's own solution was to prompt students to stop themselves while working on a problem and ask: What am I doing right now? Why am I doing it? And how does it help me? Over time they questioned themselves.

## CASE STUDY OF A RESOLUTION - EVIDENCE OF TEACHER A

The methodology in this school based UK study was enactivist. During one extended class discussion Teacher A comments to the whole class (of 11-12 year old students):

this is what mathematicians do (.) they develop their conjectures so they begin with something they believe to be true and then they might change their minds having got some results (.) ... what S5 has given us an example of (.) is where this doesn't hold to be true (.) called a counter-example ... (TA, 14-9-07 – second lesson of the school year)

Teacher A uses and expands on the word 'conjecture' at the moment when a student exhibits a behaviour that fits her notion of what it means to be a mathematician. In subsequent video recordings of Teacher A the word 'conjecture' is used less frequently. The process of looking for pattern rapidly becomes part of what students expect to do in lessons with Teacher A. 'Conjecture' comes to symbolize a complex web of ways of working in mathematics. Schoenfeld's dilemma is avoided by offering a limited set of labels (descriptions) that are allowed to accrue associated actions (prescriptions), initially through modelling by the teacher, and then through the teacher highlighting (or 'metacommenting') on particular student behaviours to the rest of the class – at the point when these behaviours are observed.

From an enactive view, developments in metacognition are linked to students' entire beings, and are not seen as an accrual of skills or knowledge. As students learn to act in new ways in TA's class they develop a new 'mathematician' aspect of themselves.

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# INTUITION AND DYNAMIC GEOMETRY IN PROVING ALGEBRAIC STATEMENTS

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The purpose of this paper is to describe the contribution of intuition and dynamic geometry in proving algebraic statements. Connection between geometry and algebra are as much historical and social as conceptual. For instance in ancient Greece, mathematicians gave many geometrical proofs of a number of algebraic formulae (Heath, 1956, p. 372). On the one hand geometry is considered to be a power tool in the development of mathematical reasoning:

“..spatial intuition or spatial perception is an enormously powerful tool and that is why geometry is actually such a powerful part of mathematics - not only for things that are obviously geometrical, but even for things that are not. We try to put them into geometrical form because that enables us to use our intuition” Atiyah (2001).

On the other hand, digital technologies make a significant contribution to the development of mathematical thinking. The software GeoGebra provides a combination of CAS (Computer Algebra System) and DGS (Dynamic Geometry Software) (Hohenwarter and Jones 2007).

In the Music School of Serres 67 students of 8<sup>th</sup> grade were asked to prove the algebraic identity  $a^2 - b^2 = (a+b)(a-b)$  geometrically, considering  $a, b$  segments of length  $a, b$ . The algebraic expression  $a^2 - b^2$  was represented by the geometrical figure 1, which is known as Gnomon (Heath, 1956, p. 370). Three students used each computer. One manipulated the computer, one took down notes and one controlled the spreadsheet. The roles were reversed periodically.

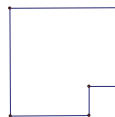


Figure 1

The findings of the study highlight the interest the students had in constructing a geometrical proof using the DGS of GeoGebra. Proofs are based on the partition of the Gnomon in already known closed figures of the plane (triangle, rectangle, parallelogram and trapezoid). In this presentation, all results and different proofs of the algebraic identity will be discussed.

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# STUDENTS' UNDERSTANDING OF THE LIMIT CONCEPT

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The aim of our research is to elucidate students' understanding of the limit concept by observing students' utterances, writing and gestures during task solving, and analysing characteristics of their mathematical thinking. Our theoretical frames are the concept image and concept definition as described by Tall & Vinner (1981) and the three worlds of mathematics (conceptual-embodied, proceptual-symbolic, axiomatic-formal) as described by Tall (2008). We aimed at revealing details of students' concept images and their consistencies with the formal concept definition, combining these with an investigation into which worlds students engage in and whether and how they make transitions between different worlds during task solving.

Our research was a qualitative study based on data collected through semi-structured interviews. Participants were postgraduate students all of whom had mathematics as their major subject in their bachelor programs. All had encountered limits at several stages in their studies and some had learnt limits more than 10 years earlier. All our candidates were unprepared for the interview and had to dig into their memories, reconstructing their concept images. We used interview tasks, gathered from three previous studies, combining different aspects of the concept such as the application of different representations and situations.

Our findings confirmed the individual nature of concept images. From a list of six concept definitions, the interviewees chose different combinations. When asked to select the best statement, not one indicated the statement that best approached the formal concept definition. Also, a student's concept image can include contradictory aspects which are stimulated by different tasks. One interviewee realised the contradiction and revised his concept image. In terms of the three mathematical worlds, none of our interviewees' understanding depicted the axiomatic-formal world. While all had been exposed to advanced mathematical lectures, the concept of limit was not retained within this world. We will present the detailed results of three cases. One student mainly stayed in the conceptual-embodied world, while we could observe with the other interviewees transitions from the conceptual-embodied to the proceptual-symbolic and back.

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# PEDAGOGICAL CONTENT KNOWLEDGE OF MATHEMATICS STUDENT TEACHERS IN HANGZHOU, CHINA

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This paper reports part of the findings from the first stage of a larger scale study on the development of pedagogical content knowledge (PCK) of mathematics student teachers from Hangzhou, a city in the coastal province of Zhejiang, China.

PCK in this study was examined under three aspects: Knowledge of Teaching (KOT), Knowledge of Students (KOS) and Mathematical Insights (MI). The first two aspects have been well defined and developed in different PCK models (Ball, Thames, & Phelps, 2008; Shulman, 1987). The third aspect MI is adapted and extended from two features of the concept of “profound understanding of the fundamental mathematics” (PUFM) by Ma (1999), namely, “connectedness” and “basic idea” (p.122). A teacher with MI should have deep and thorough understanding of the basic concepts for the topics they teach, and should be able to make flexible connections among the individual pieces of knowledge into a coherent mathematics structure. A teacher with well developed PCK should be able to scaffold students with meaningful experiences to learn different aspects of a mathematics topic with breadth, depth and thoroughness.

This presentation will report the PCK of 6 Hangzhou student teachers in teaching the topic “three term ratios”. A video clip was edited from one video in the TIMSS 1999 Video Study and used as a stimulus for exploring the PCK of the student teachers. This was followed by a semi-structured interview with questions addressing the different aspects of PCK for this topic.

Results of the study were interpreted according to the three aspects of PCK defined in this study. The results show some differences among student teachers’ responses to the interview questions. For example, four types of answers were generated by one MI question that investigates respondents’ understanding on the basic concept of ratio. The different understanding on this concept led to the varied strategies that they used for representing this topic. Results indicate that at that stage the student teachers’ KOS was limited, and their KOT was shaped by their MI. Examples of their responses on each aspect on the teaching of this topic will be shown to support these findings.

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## SCHOOL MATHEMATICS QUALIFICATIONS AND WORK

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Data from thirteen interviews with representatives of employer groups in England shows that whilst mathematics is important at work, employers have little confidence in the mathematics qualification designed to assess the curriculum of 14-16 year olds. The data, albeit limited in scope, flags the ongoing need for thinking about how school mathematics might better prepare young people for work. We will consider a) the problem solving skills that employers expect recruits to develop at work; b) ways that learning mathematics at work builds on preparation in mathematics begun at school; and c) the role of qualifications in this process.

Theoretically we locate our ideas in concepts of situated learning and expansive learning at work (e.g. Engeström, 2001). New knowledge is created through the interaction of individuals with the workplace ‘activity system’ and the crucial thing to note about expansive learning is that because genuinely new knowledge is created, it is unpredictable in advance. This suggests that the content of school mathematics might not be the ideal preparation for mathematics at work; rather it might be better to focus on the general expectation of needing to learn, and on solving problems.

Apprentices operate in contextualized situations and are frequently taken on aged 16 without having been successful at school mathematics. Industries expect to provide on the job mathematics training with a focus on application, opportunities for progression and recognition of attainment and talent where it occurs. There is a belief amongst employers that many young people are not well served by school generally; that once at work people flourish in spite of their unsuccessful educational histories. Transition to work is an important transition, and being functional with mathematics is recognised as being an important element, at all levels of operation. Faced with the employers’ apparent lack of confidence in lower level school mathematics qualifications, it is important to develop mathematics at school that signals mathematical problem solving and readiness to learn.

Essential in this process is thinking about forms of assessment that might signal more clearly the skills and mathematical understandings that can be built upon in the workplace.

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# DEVELOPING ALGEBRAIC THINKING WITH ICT

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In Portugal, official curriculum guidelines on algebra are going through significant changes, emphasizing that algebraic thinking must be developed since the elementary grades and that teachers must use adequate technology. The development of algebraic thinking requires a careful combination of tasks that enhance generalization and progressive formalization, supported by multiple representations (Blanton & Kaput, 2005). It also may involve the use of dynamic and interactive ICT features that allow, for example, manipulating objects and observing the results, stressing the construction of meaning (Ferrara, Pratt & Robutti, 2006).

Several authors outline the importance of collaborative contexts in order to develop teacher professional knowledge, promoting reflection and willingness to take risks. These contexts are adequate to achieve new curriculum demands, integrating new pedagogical tools and content knowledge (Ruthven & Goodchild, 2008).

This presentation reports a case study developed in a collaborative setting. This context involved a researcher and two middle school teachers, with grade 7 students. This paper analyses how teachers plan and conduct lessons using ICT aiming to develop algebraic thinking. The sources of data are observations of teachers' lessons and of collaborative working sessions, interviews, interactions in a Moodle platform, and other written documents.

This presentation draws on the results of the two case studies. I discuss the role of the collaborative setting to identify different uses of technology and to highlight those that promote interactions and learning. Through the use of tables and graphs and their translation from one representation to another and in natural language, students showed creative ways of thinking that go behind the teacher's expectations.

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# FRIENDSHIP GROUPS: EQUAL DISCOURSE STATUS AND EQUAL TASK-SPECIFIC EXPERTISE

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Friendship groups are often used to improve social skills rather than develop cognition in the education of 11 – 14 year olds. Yet there is a theoretical basis for arguing that these groupings can support individual cognitive development in settings which enable the equalities inherent in friendships to actively contribute to individual learning through collaborative groups. Roschelle and Teasley (1991) describe shared cognition as a “shared conceptual space constructed through shared language, situation and activity” and Dillenbourg *et al* (1995) as the “process of building and maintaining a shared conception of a problem”.

This explanatory study examines the role of student talk in maintaining a ‘shared conceptual space’ amongst groups learning mathematics in a setting which utilises learning activities which actively encourage student collaboration towards a defined goal. Achievement beyond expected norms in external examinations offers evidence of individual learning, while data from transcripts of peer group talk amongst these friends learning mathematics offers evidence to support both the definitions of ‘shared cognition’ above. The co-construction of ‘threads of thought’ and high levels of ‘talking aloud’, which appear to closely interweave thinking between group members, suggest “building and maintaining a shared conception of a problem”. The use of everyday language, particular to friends, within each group contributes to a model in which there is shared understanding of thought and knowledge.

Using this evidence, I argue that, since friendship groupings are established on a social basis, not a mathematical basis, there is likely to be some variation in ‘cognitive distance’ between at least some of the individuals in a group. The action of *established* social interactions between group participants on this difference in mathematical cognition provides a rich mathematical discourse which, in turn, enhances individual mathematical learning within the shared conceptual space. Friendship groupings may therefore provide the necessary *equal discourse status* (through social interaction) and *equal task-specific expertise* (through ‘cognitive distance’) to offer a medium for effective mathematical learning.

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# MATHS $\neq$ MATHS – TEACHERS’ CURRICULAR BELIEFS REFERRING TO DIFFERENT MATHEMATICAL DOMAINS

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Researchers in mathematics education emphasise the research of mathematics teachers’ thinking or beliefs. Rationales for this research are the high impact of teachers’ beliefs (1) on their instructional practice, (2) on students’ learning, and (3) on possibilities of changing the teachers’ instructional practice (Philipp, 2007). However, most of the increasing body of research in mathematics teachers’ beliefs do not consider that teachers’ beliefs may vary across different mathematical domains like calculus, geometry or statistics.

Thus, in a research project, we focus on differences of teachers’ beliefs referring to distinct mathematical domains (Eichler, 2011). Using half-structured interviews, the beliefs of 22 teachers (upper secondary schools) towards the teaching and learning of statistics and geometry were analysed by interpretation of interview transcripts. We regard the teachers’ beliefs concerning four different types of belief systems (Philipp, 2007), i.e. an instrumentalist, a formalist, a process oriented type, and, finally an application oriented view about the teaching and learning of mathematics.

The teachers show differences concerning the first three types. For example, the geometry teachers stress both a process oriented view emphasising problem solving, and a formalist view, but are doubtful whether geometry is an appropriate mathematical discipline to highlight an application oriented view. By contrast, the statistics teachers generally mention that a central aim of teaching statistics is to show students that mathematics facilitate to gain mathematical solutions for real situations. However they doubt that statistics is adequate to highlight a process oriented or a formalist view of mathematics. For this reason, a differentiated investigation of teachers’ beliefs referring to specific mathematical domains seems to be worthwhile to get a deeper and more valid understanding of mathematics teachers’ thinking.

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# TEACHER PRACTICES DURING CORRECTION PHASES IN ALGEBRA CLASSES

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How to teach algebra continues to be an active area of debate in the field of education. The most recent studies advocated that algebra be taught by using a pattern-based instructional approach (Radford, 2007). Even though attempts have been made to reform math education in general and the teaching of algebra more specifically, many challenges still exist and students are still facing difficulties in learning algebra. Those difficulties are well documented in the literature; however, what is still missing in the literature are relevant descriptions of teacher practices and teacher/student interactions specifically during correction phases which appear as simple segments of the lesson during which the interaction is one way between the teacher and the student. These segments actually present complex situations that may be very challenging for teachers and students alike. During these phases, teachers have to decide how to help students construct their knowledge in addition to correcting errors and validating responses. This study aims to compare and contrast teachers' ways of dealing with students during correction phases in the context of algebra in the middle school.

A compelling theoretical framework that can be used to analyse teacher practices during correction phases is that of Chevallard (1999). According to Chevallard (1999) teaching practices can be analysed based on: (1) the mathematical organization (MO) including the practice (praxis) and the discourse (logos) that describes, explains and justifies the praxis; and (2) the didactical organization (DO) which structures how the MO is constructed in the class.

Data sources for this study included official curricula in France and Lebanon, math textbooks, lesson observations and pre-lesson interviews in four classrooms. Data were analyzed using qualitative methods and a video analysis tool. The mathematical discourse during the correction process showed that teachers tended to engage students in practicing the techniques more than explaining and justifying these techniques to build conceptual understanding. Moreover, when errors appeared, teacher practices involved breaking algebraic expressions into smaller isolated parts which tended to shift the focus from understanding the task to the correctness and completeness of the answer.

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# MATHEMATICAL MODELING IN THE MIDDLE SCHOOL: STUDENTS' MODELING PROCESSES

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*The present study examines 8<sup>th</sup>-grade students' modeling and mathematization processes as they worked on a mathematical modeling problem. Results showed that students were able to successfully develop various sophisticated and efficient models.*

Recent years it has been a central aim for mathematic education to help students to develop abilities to recognize the relation between the real world and mathematics. In line with this idea, an increasing number of researchers have focused their research efforts on mathematical modeling, especially at the school level (English, 2006). According to Lesh and Doerr (2003), solving modeling problems appeared in real world situations involve mathematizing gone beyond what is taught and how in traditional mathematics classrooms. Mathematical concepts given in the context of real world situations offer rich learning opportunities for students by letting them further elaborate related concepts previously learned, encouraging the development of generalizable solutions, communicating solutions in context, and describing situations using a variety of representation media (Mousoulides, Pittalis, Christou, & Sriraman, 2010).

The purpose of this study is to examine six 8<sup>th</sup>-grade students' modeling and mathematization processes as they worked on a mathematical modeling problem. The problem addressed in this study, *The Big Foot Problem*, required students to create a tool for police detectives to use to figure out the height of a person given only their footprint. As opposed to traditional problem solving, mathematical modeling asks students to generate and develop their own mathematical ideas and processes, and to form systems of relationships that are generalizable and reusable. Preliminary results showed that students in two focus groups of three were able to successfully work with the mathematical modeling activity and developed various models which were sophisticated, useable, and shareable.

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# TYPE RELEARNING QUADRATIC EQUATIONS WITH INTERACTIVE WHITE BOARD AND A GRAPHING SOFTWARE

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The purpose of this study was to explore the effect of a technology supported learning environment utilizing Interactive White Board (IWB) and NuCalc graphing software compared to a traditional direct instruction based environment on students' achievement in understanding the graphs of quadratic equations and attitudes towards mathematics and technology. Sixty-five high school graduates attending a private education institute to study for the university entrance examination were assigned to experimental (EG) and control groups (CG). While the CG (n=33) received instruction via traditional direct teaching, the EG (n=32) was taught the same topic using IWB and NuCalc on computers. Data were collected using Graphing Achievement Test (GAT), Attitudes towards Technology Scale, and Attitudes towards Mathematics Scale (ATMS). While the GAT were administered as a pre-test, post-test and delayed post-test to both groups, ATTS and ATMS were used with EG students as pre-test and post-test only. Furthermore, semi-structured interviews were conducted with six students selected based on their pre and post scores from ATTS, ATMS, and GAT.

The results showed that EG and CG students' performances were significantly changed across time of testing. The significant interaction effect between time of testing and groups indicated that student performance across time of testing was dependent upon the presence or absence of the treatment. Contrasts suggests that although both experimental and control groups students performances increased from pretest to posttest and then decreased from posttest to retention test, the rate of decrease was about the same and the rate of increase was different with students showing a greater rate of increase in experimental group. Furthermore, the results also revealed that the treatment had positively affected students' attitudes towards technology and mathematics in EG. While the students in EG were enthusiastic about the technology use and involved actively in the lessons, as they were asking questions and making connections between what they have learned before, students in CG got bored easily after drawing couple of graphs as they thought all such graphs are nearly the same. It was also observed that their reasoning and interpretation skills did not improve as much as of those in EG. Interviews indicated that students perceived the IWB and NuCalc environment as enjoyable, interesting and more fun. Even though they were unsure or did not agree that the IWB and computers could help them to learn mathematics before the study, they expressed that it proved to be wrong. They also indicated that the use of IWB helped them to concentrate better in class.

# PRE-SERVICE TEACHERS' VIEWS ON MATHEMATICAL THINKING PROCESS

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It is known that every person whatever his/her job, needs mathematical thinking (MT) in his/her life. So it is aimed to develop an individual's MT during learning process (Alkan & Bukova Güzel, 2005). In this study which was a part of a wider research (Alkan & Ceylan, 2008), our purpose was to determine the pre-service teachers' views and suggestions on MT process. A case study design was employed in this study. At the first phase of the study, Calculus I and II courses were lectured based on MT components in a state university, six hours a week. At the second phase, the data collected from 32 pre-service teachers through four open ended questions that aimed to reveal their views on learning environment and process. In the questions, pre-service teachers were asked to criticize teaching of courses and way of assessment, to compare their thoughts at the beginning of and after the lessons and to make suggestions (if any). Data sources were pre-service teachers' written documents. The data were analysed by two researchers through content analysis and categories were determined. As a result of study it was found that most of the pre-service teachers (53%) had the view of "courses prevented memorizing and provided mastery learning". Some participants (38%) expressed that they approved the use of homework, worksheets and group presentations as assessment tools. 41% of them emphasized that although they felt anxiety and had difficulty at the beginning of the term, it decreased after a while. Besides, some participants (22%) suggested that exercises must be increased in the courses. Findings indicated that pre-service teachers found the courses which were designed to develop their MT, different and interesting that is why they expressed their difficulty to become familiar with. But findings also confirmed that mastery learning exists in this way. Although, the use of contemporary assessment tools initially forces pre-service teachers who were only assessed by computational skills until that day, this problem goes away after positive effects of this type of assessment. In order to prevent pre-service teachers to feel themselves different because of giving up previous routines suddenly, courses based on MT components can be supported by computational skills.

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# THE IMPACTS OF CONJECTURING-BASED MATHEMATICAL WRITING ACTIVITIES ON THE DEVELOPMENT OF STUDENTS' MATHEMATICAL PROFICIENCY

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Mathematical proficiency plays an essential role of mathematics teaching and learning nowadays (Kilpatrick, et al., 2001). According to Lakatos' (1976) proof analysis theory, mathematics activities include proposing a conjecture, proof, proposing counter examples, and using counter examples for re-testing the conjecture and proof iteratively. In such point of view, conjecturing is not merely the core of mathematising, but the driving force for developing mathematical proficiency. Moreover, some research indicates that engaging students in mathematical writing activities in classroom has positive effects on their mathematics learning (e.g. Ntenza, 2006). For synthesising these remarkable insights, we organise this study to examine how students develop mathematical proficiency within conjecturing-based mathematical writing activities. This study was implemented as an action research in a 7<sup>th</sup> grade class while four students with various mathematics achievements were purposely selected as research subjects. The research period was organised with three phases in one year. The first phase was focusing on having students to accommodate the conjecturing process in mathematical writing activities, such as proposing a conjecture and justifying it, discussing and making refutation. The second phase was emphasising on specialising and generalising as an ascent and descent in an ongoing process of conjecturing-based mathematical writing task. The third phase was concentrating on providing fruitful conjecturing tasks for students to experience a construction of mathematics knowledge within mathematical writing activities. Data collection mainly included student mathematical writing worksheets, video recordings of classroom activities, semi-structured interviews, and journals of teachers' reflection. These data were encoded and analysed by means of a mathematical proficiency observation table. Main research results were as follows: (1)explanatory writing tasks could foster students conceptual understand and strategic competence; (2)students had shown different levels of adaptive reasoning abilities in explanatory and probing writing tasks within the stages of justifying and refuting; (3)summarising writing tasks could foster students' conceptual understanding, strategic competence and productive disposition.

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# DISCOVERING EXPERTS' VIEWS ON TRANSITION TO SECONDARY MATHEMATICS USING DELPHI METHOD

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Transition to secondary school endangers continuity and stands as a determinant process from a lifelong learning perspective (McGee, Ward, Gibbons & Harlow, 2004). This on-going investigation, embedded in a larger project<sup>1</sup> that targets primary-secondary transition in mathematics, explores the knowledge and opinion about this issue of a group of 10 experts in mathematics education in both stages using a 3-round classical Delphi method (Skulmoski, Hartman & Krahn, 2007). This methodology, a consistent technique broadly used in various fields and also recently in mathematics educational research (Manizade & Mason, 2010), allows to offer a scenario of anonymous group decision in order to find out which factors affect students' mathematical learning in transition and which characteristics teachers should possess in order to optimally manage it.

The outcomes of the discussion show the most significant influence of teachers in the success or failure of the process, above those aspects related to changes in the educational context or even students and their development. The profiles emerged for the ideal mathematics teacher from the standpoint of continuity in transition illustrate different demands for each stage: mathematical knowledge for teaching in primary and mathematical and educational sensitivity when approaching the profession in secondary. Moreover, secondary teachers are awarded a critical role in this process and the ability of looking back in mathematics is regarded by the group as far more imperative than the ability of looking ahead.

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# INTERTEXTUALITY IN ALGEBRA. EMPIRICAL EVIDENCE CONCERNING THE SOLUTION OF WORD PROBLEMS AND OF LINEAR EQUATION SYSTEMS

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In algebra textual spaces are made up of mathematical sign systems, the codes and traditions of which are derived from the meanings attributed to them due to their social usage. Mathematical texts do not have an independent sense, rather they are intertextual. Reading/transformation processes of one text make it possible to draw relations among texts. The sense arises from the existence of a relationship between the text and all other texts previously produced and that may be resorted to for their reading/transformation. Hence the text becomes an *intertext*. This presentation intends to contribute empirical evidence regarding such assertions with respect to the productions of students who are learning algebra.

We introduce the notion of *intertext* to analyse and proffer plausible explanations for phenomena that emerge during an interview situation with 13-15 year-old students who for the first time are tackling the task of solving two-linear equation systems and word problems involving two unknown quantities. Said notion is derived from the theoretical perspective developed in Filloy, Rojano & Puig (2008, Chapters 1, 5 and 8). Our interest in this presentation is to highlight by way of two examples that the conception of a text reading/transformation act as a network of intertextual relations between a text (for instance a system of equations or a word problem) and previously produced texts makes it possible to identify crucial moments of sense production with respect to the algebraic Substitution Method (used for the solution of linear equation systems) and to the Cartesian Method (used to solve word problems).

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# THE MATHEMATICS CLASS AS OBJECT OF LEARNING

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*The objective of the study was to observe how after, four months of instruction, the educational messages of a Master in Educational Mathematics (MEM) program oriented towards professional development (PD) were being projected in the interaction between teachers, students and the knowledge in mathematics.*

Changes of the teachers' professional practice are an intrinsic need of achieving better results regarding learning accomplishments of their students. (Simon et al., 2000; Zaslavsky, 2007). The example we have selected is a 6<sup>th</sup> grade group class about symmetry conducted by teacher Carmen, she works with the students using the mathematics teaching software Cabri, Carmen selected and prepared activities leading students to express what they understand for symmetry, foresees that the difficulties that will be found by the students when solving the worksheet would be related with the figures' orientation. She also considered that they would have problems when asked to draw the image of a figure whose symmetry axis is obliquely traced. In the class, she makes sure that all students are working but to not listen students' comments hinders the mediation role to enrich learning and prepare the field that allows to improve the use of mathematical tools and the rationales of their procedures. The properties of symmetry were not explored in depth, the integration of information was not made possible.

## Discussion and results

Despite the manifest influence of the MEM program in Carmen's desires to implement new experiences, the integration of the role of its action to transcend in the achievements of a more rich mathematics activity is not yet accomplished. The teacher uses technology as a suitable and innovative resource to work geometry, but she does not take into account that the use of technology in itself does not promote the understanding, Carmen prepared all the materials and the means for explaining the use of the software, but she did not plan her role in promoting the mathematical activity i.e., there is no anticipation of her intervention as a mediator. The challenge of the MEM was to give incentive to both the knowledges useful for consolidating the particular knowledge of being a mathematics teacher, and to filter out those knowledges that obstruct mediation and communicative action in the classroom.

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# PROMOTING MATHEMATICAL UNDERSTANDING THROUGH PROBLEM SOLVING

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Problem solving has received a lot of attention in mathematics education. The NCTM Principles and Standards for School Mathematics (2000) state that by the time students complete high school, they should be able to solve problems that arise in mathematics or related disciplines, apply a variety of strategies to solve problems, and monitor and reflect on their problem solving process. In particular, the standards state that students should build new mathematical knowledge through problem solving. Yet, how to promote mathematical understanding through problem-solving remains an open question (Barbeau & Taylor, 2009). The purpose of this article is to show that strands of challenging tasks can help promote growth of students' mathematical understanding in problem-solving situations by allowing students to build isomorphisms between the problems. The isomorphisms help promote them solve the problem and build a deeper understanding of the mathematical ideas involved in the tasks. The context of the study is an NSF-funded classroom-based longitudinal study on students' development of mathematical ideas and in problem-solving contexts. Analysis of the mathematical activity of five high students ideas in a strand of combinatorial/probability tasks using Powell et al. (2003) methodology and Davis and Maher's theory of problem solving showed the students building mappings between the problems that helped the students better understand the mathematical ideas involved in the task and, address the mathematical challenges involved the problems. The analysis also suggested that the particular conditions in which the students worked on mathematical tasks in the longitudinal study helped promote the students' ability to develop isomorphisms in their problem-solving activity. These are described in the study along with the implications for teaching in mathematics classrooms.

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# MEDIATORS OF MATHEMATICS IN TASK-BASED DISCUSSION BETWEEN TEACHERS

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This presentation reports from teachers' work collaborating with two university researchers on mathematical tasks in a small group within a project workshop. The analysis seeks to expose characteristics of the teachers' engagement in mathematics related both to their own and their students' learning of mathematics. Over time, exploring data from several workshops and other collaborative work, we expect to substantiate claims for changes in teachers' practices arising from project work. Qualitative data were collected as audio-and video recordings, and written material and for all events, including workshops. Data, including transcripts, were analysed.

The project was founded on principles of community and inquiry: inquiry in learning and teaching mathematics; and inquiry in the development of teaching mathematics. Cultural-historical activity theory (CHAT) is utilised as an analytic framework (Jaworski & Goodchild, 2006) within this mediation has a central role. Rules may mediate teaching, such as an explicit requirement to set homework tasks, or an implicit rule such as 'ask questions rather than provide explanations'. The community may mediate preparation by judging what works in class and what students expect, etc. In particular, for this presentation we focus on mediation of actions by exploring the following research question: What characterises the mediation of action of teachers when they collaborate on mathematics tasks in the projects' workshops?

The workshop activity reported here was based on a task from the TIMMS video study of Japanese classrooms in which the task was to calculate an angle by drawing an auxiliary line. In discussions at both the mathematical and didactical levels, the goal of the teachers was the resolution of the task and finding alternative solutions through drawing different auxiliary lines. The interaction between participants is informative.

Teachers perceive the task through a lens of practice and the (mathematical) tools identified by rules of practice. Their initial engagement was constrained by the mathematical content of the curriculum and shared understanding the students' competencies. However, they also were provoked to step out of their practice perspective and inquire into the mathematics itself.

We learn that when teachers engage in authentic mathematical inquiry they step out of the constraints of practice and their engagement is mediated by their own mathematical knowledge.

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# SIXTH GRADE PRIMARY STUDENTS' THINKING ABOUT GEOMETRIC COMPOSITON

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When individuals start to have an interest in the things or people around them, their skills related with geometry develop through time. According to Clements, Wilson, and Samara (2004) skills related to compose shape (putting together) and decompose shape (taking apart) are important in the development of visual reasoning and in improving spatial and geometrical skills and forming geometrical ideas, even in understanding numbers (Clements et al., 1997).

While dealing with the studies of shape composition, generally young children were observed. Clements and his other friends (2004) emphasized that young children move through several distinct levels of thinking and competence in the domain of composition and decomposition of geometric shapes. Although a few studies state young children's thinking about geometric composition, there is little or no research studying with secondary school students

The aim of this study was to find out sixth grade primary students' thinking about composing shapes when they endeavored to do tasks. The case study was preferred to attain the aim. Eight students, in the context of a clinical interview, are faced 14 tasks. In these tasks students were expected to use drawings, pattern blocks or pieces of some shape. The interview took 3 hours for each student. Each interview was videotaped. These tapes were partially transcribed, coded, and analyzed.

According to the data, sixth grade students' thinking about composing shapes was described. Although the participants of this study were older than the once in Clements and his friends' studies, similar results were obtained. Regarding results of the study, it is possible to assert some thinking levels for sixth grade students in the domain of composition shapes.

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# A TASK ON SCHEINER PANTOGRAPH: A CASE STUDY

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With this teaching experiment we want to analyse the performances of two groups of students on the same task on the Scheiner pantograph. The first group was asked to solve a paper and pencil version, the second one used a computer-based version of the task built in a DGE (GeoGebra). Moreover, the first group had previously used the Scheiner pantograph in laboratory activities, while the second one had never seen a pantograph. We wanted to answer to the following questions: can the exploration performed in a Dynamic Geometry Environment (DGE), without accessing the real mathematical machine, replace the experience with the actual machine? In what ways? In which situations? Can differences (if any) in students' justifications of the answers be related to the different environments?

The teaching experiment is carried out within the MMLab-ER project (Laboratories of Mathematical Machines for Emilia Romagna) based on the theoretical framework of semiotic mediation (Bartolini Bussi & Mariotti, 2008) which describes mathematical activities in a classroom when an artifact (i.e. a mathematical machine) is introduced. Mathematical machines are tools that force a point to follow a trajectory or to be transformed in another point according to a given law. They are part of the historical phenomenology of geometry, for instance the Scheiner pantograph was introduced in the XVII century in order to enlarge or reduce a figure. The teaching experiment involved three VII-grade classes. The task was designed according to Rabardel's distinction between artifact and instrument (Rabardel, 1995) and to an exploratory scheme consisting in four standard questions: a) *How is the machine made?* b) *What does it make?* c) *Why?* d) *What would happen if...?* The first question drives the attention towards the analysis of the physical structure of the machine (artifact); the second is useful to promote its utilisation schemes (instrument); the third question concerns the mathematical meanings embodied in the machine; finally the last question fosters the solution of open-ended problems and can engage students in productive explorations. We can observe that only in the last question there are some differences, in students' behaviours, between the two environments: the exploration in a DGE of a mathematical machine can suggest variations of geometric relations between its components, not always really possible on the artifact because of physical constraints.

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# PERCEPTIONS OF RELEVANCE OF PRIOR EXPERIENCES OF MATHEMATICS IN AN ETHIOPIAN PREPARATORY SCHOOL

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This paper is a report from a pilot study of a project which aims to explore students' perceptions about the relevance of mathematics. These students are preparing for higher learning institutions (hence the school designation 'preparatory'). The principal research question is 'what are Ethiopian preparatory students' perceptions of the relevance of their prior experiences of mathematics to preparatory mathematics and how are they characterized'. It is motivated by my experience as a teacher. Students pay attention when they see that it relates to their prior experiences. Prior experiences affect students' decisions about the learning of mathematics (Reid et al, 2005) and their role in the classroom (Yackel & Cobb, 1996). They learn in a society where the teacher is referred as "father of knowledge" and mathematics is valued for economic freedom of the students and their parents. Unlike in primary schools, the textbook is in English which the students use at school only. Cultural historical activity theory (CHAT) was used as a lens. Engström's expanded mediational triangle (Cole & Engström, 1993) models activity within which perception is mediated by tools, rules, division of labour and community. Since the purpose is to see relevance through students' eyes who share similar classroom experiences, and for the sake of management, they were interviewed in groups of three. It was supported by classroom observations. Themes emerged from the narrative analysis, and were further analysed using CHAT as a frame work to identify key features. Findings indicate that students exhibited their perceptions: as an expansion from previous level; as an increment of the level of difficulty; as a reflection of a shift in their identities, and trust for the curriculum and the teacher. These perceptions are mediated by the local and school communities, the language, their role, and the school rules. Whether there is relationship between the different categories of students included in the sample (i.e., gender, grade and attainment levels, and stream) and the different characterizations of perceptions is left to the next stage.

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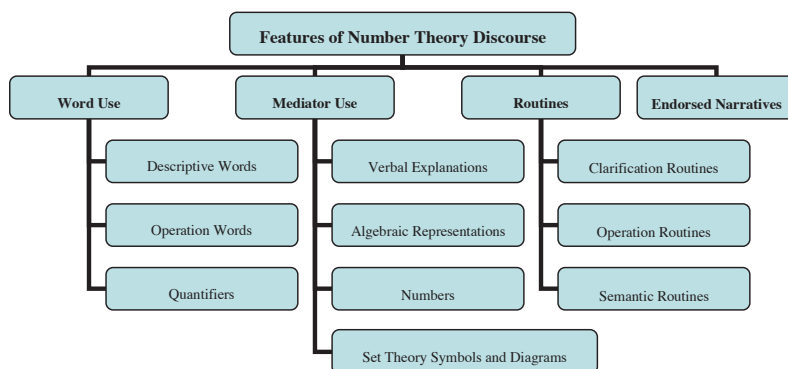
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# PROOF AS A MEANS TO COMMUNICATE MATHEMATICAL KNOWLEDGE

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This study aimed at extending the views and insights about the difficulties that pre-service elementary school teachers experience in dealing with the notion of mathematical proof. For this purpose I analysed students' discourses when they attempted to create proofs for some propositions related to elementary number theory. The communicational approach to learning is the theoretical perspective that I adopted to investigate the difficulties students experience in generating proofs. According to this perspective thinking is a special case of the activity of communication, and learning mathematics is an initiation in a certain type of discourse, which is called literate mathematical discourse. Literate mathematical discourse as the objective of school learning are distinguished from other types of communication through four criteria: (1) their special vocabulary, (2) their special mediating tools, (3) their discursive routines, and (4) their particular endorsed narratives (Ben-Yehuda, Lavy, Linchevski, & Sfard, 2005). The communicational framework provided me with a tool for analysing students' created proofs as forms of discourse. Through the analysis a further refinement of the communicational framework emerged. The profile of this refinement is presented in following figure.



The results revealed that the main difficulty that students experienced in creating a proof is that they do not know how to communicate their idea mathematically.

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# SUPPORTING ENGINEERING STUDENTS IN MATHEMATICS

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*As qualified engineers are desperately needed, the number of students giving up before graduation is alarming. MP<sup>2</sup>, a project run at Ruhr-Universität Bochum, encompasses the two interventions Math/Plus and Math/Practice and aims to remedy this problem. MP<sup>2</sup>'s basic assumptions are that engineering students often fail in mathematics either because of deficient learning strategies (Math/Plus) or because they do not see a sufficient practical relevance (Math/Practice). Our focus is on Math/Plus and deals with the question if a learning diary, especially in combination with other measures, can support students' learning efficiently.*

Math/Plus started in fall 2010 with a total of 180 (out of 1,000) students of engineering in three different project groups, Math/Practice will begin in spring 2011, each preceded by thorough preparations and followed by detailed evaluation. A rerun is planned for 2011/2012. The project consists of several core and accompanying measures and is escorted by numerous surveys.

Two thirds of the students keep a learning diary (Landmann & Schmitz, 2007). It contains pre-learning and post-learning items as well as questions on emotions and motivation. It is aimed at self-regulation and modification of behavior (Schmitz & Wiese, 2006). Additionally, one half of these students meet with a tutor once a week for a preparatory tutorial in which learning methods are presented and tested on mathematical problems. The other half attends a revision course shortly before the exam. What is more, student assistants offer to answer questions, check assignments and give tips for further work in a helpdesk. A supplementary e-learning course contains test items with immediate feedback, filing opportunities and a forum.

First results indicate that students modify their learning strategies successfully, initiated both by personal and by digital support, varying in levels of efficiency.

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# THE USE AND PRODUCTION OF DRAWINGS IN ARCHITECTURE AND MATHEMATICS CLASSROOMS

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Considering the relevance of drawing as a mediation artifact for solving design and math problems, this essay calls for the valorization of drawing production by the students in Architecture and Mathematics classroom activity. Our objective is to deepen understanding about an existing shared belief, that one can (tacitly) learn how to draw but it is not possible to (explicitly) teach how to draw (Góes and David, 2010; Veloso, 2000). To discuss this belief, we present two classroom situations, one from each context, in which there was some form of instruction (although not totally explicit) directed to the production of drawings by the students that, we believe, has contributed for some students' expansive learning (Engeström and Sannino, 2010).

Through key concepts from Activity Theory, specially the expansive learning concept, we observed some architecture classes and discuss how the teaching that took place contributed for an expansive learning regarding the architectural project produced by some students. Using the same theoretical approach in an attempt to explore a parallel to this situation in the mathematics field, we went back to some videotaped mathematics classes (part of our database) to check a special situation, of a 5<sup>th</sup> grade classroom, in which there was production of drawings of tridimensional shapes by the students. In this case, a partially tacit teaching approach also seems to have contributed for expansive learning in terms of the ability to draw and fold a cube and to use the drawing in the calculation of the volume of 3-D blocks.

Both situations reinforce the idea that through directional teaching strategies, not totally explicit, it is possible to contribute for students' expansive learning regarding the use and production of drawings according to the socially established norms in each field. In conclusion, the study gives support to the relevance of teaching drawing and to the necessity of deepening the discussion about different teaching approaches, especially considering the tacit-explicit binomial.

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# SPATIAL ABILITIES AND MATHEMATICS ACHIEVEMENT AMONG ELEMENTARY SCHOOL CHILDREN

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Spatial abilities are regarded as important prerequisites for learning mathematics. In particular, empirical findings show that spatial abilities are related positively to mathematics achievement (e.g. Fennema & Sherman, 1977). However, since most studies were conducted either with secondary school students or with adults, there is hardly any research evidence on the relationship between elementary school mathematics achievement and spatial abilities.

This research project aims to examine the relationships between mathematical achievement and spatial abilities among elementary school children. Spatial abilities are assessed by a paper-and-pencil test. Although factor analytic models of spatial abilities are often criticized, it makes sense to use the resulting dimensions as a foundation for constructing test items. The following dimensions are distinguished: Visualization, Mental Rotation and Spatial Orientation (Lohman, 1988). In order to assess mathematical abilities among elementary school children, we developed a framework for constructing test items. Basis for this framework are international studies such as TIMSS and PISA and the national educational standards.

The sample comprises 448 students of grade 4 (22 classes). The findings indicate that students with high spatial abilities have greater mathematical abilities than students with low spatial abilities ( $r = .54$ ,  $p < .001$ ). Furthermore, multilevel analyses suggest that a significant amount of variance of mathematical achievement as well as of spatial abilities can be explained at the class level. Accordingly, it seems that mathematics instruction has a significant impact especially on the development of spatial abilities. In summary, it can be concluded that a positive relationship between mathematical and spatial abilities can be confirmed already among elementary school children. Moreover, the role of mathematics instruction for the development of spatial abilities needs further research, since spatial abilities are an important factor for mathematics learning. In the presentation, further results will be discussed in detail.

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# **ELEMENTARY PRESERVICE TEACHERS CONTENT KNOWLEDGE FOR TEACHING: EXAMINING THEIR ANALYSIS OF STUDENTS ALTERNATIVE SOLUTIONS**

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Content knowledge plays a critical role in teaching mathematics, and yet teachers need more than content knowledge to be effective teachers of mathematics (Fennema & Franke, 1992). Pedagogical content knowledge (PCK), a special domain of teacher knowledge proposed by Shulman (1986), bridges content knowledge and teaching.

This qualitative study examined elementary preservice teachers' content knowledge for teaching as they analyzed students' alternative solutions to multidigit addition problems that was retrieved from the study of Ambrose, Philipp, Chauvot, and Clement (2003). The preservice teachers were also asked to choose a solution strategy for each problem that they would prefer to use in their future classrooms. Data collection includes preservice teachers' written responses and classroom observations. In this study, the framework "Mathematical Knowledge for Teaching" posited by Ball, Thames and Phelps (2008) was utilized to describe the elementary preservice teachers' knowledge for teaching.

Results of the study revealed that the preservice teachers had the knowledge and skills to solve this multidigit addition problem correctly, and that most of them preferred to use the standard algorithm for multidigit addition that is taught in our schools. We also observed that many preservice teachers were unable to provide an explanation for possible sources of student mistakes. This led us to believe that the preservice teachers had limited knowledge of content and students and relied on their learning experiences as a student to make instructional decisions.

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# STUDENT USE OF IMAGINATION AND AFFECT IN ELEMENTARY MATHEMATICS

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The degree of students can achieve depends to a large degree on their active participation. Unfortunately mathematics seems prone to a sense of disengagement especially between grade four and grade eight (Robitaille et al., 1996).

The research question at the heart of this study was “What does the use of the theory of Imaginative Education (IE) mean to children and for their engagement in elementary mathematics? Using the theory of Imaginative Education (Egan, 1997, 2005) reconceptualises education to be about the development of ‘kinds of understanding’ that enable people to make sense of the world in different ways, rather than through the development of decontextualised entities or fragments of knowledge.

Using socio-cultural methodology and qualitative case study data collection highlighting use of student voice (Fielding, 2008), the study investigated six students’ perspective of learning during a unit of geometry. An analytical model of assessing student engagement, Participatory Affective Engagement (PAE) was developed.

Results showed two themes emerging of engagement and emotions. Two students were assessed as Passive Positively Engaged (PPE) with the remaining four students assessed as Actively Positively Engaged (APE). Having the opportunity to incorporate their feelings into work, clearly gave the students an emotive humanized connection. The short oral presentation will focus on highlighting the role of affect in learning.

Listening to the students, the ones most intrinsic to education, provided a wealth of understanding about how they experienced the learning of mathematics. The results support inclusion of affective responses, and purposeful use of imagination, as valuable tools of learning.

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# DO GEOGEBRA-SOLUTIONS NEED TO BE JUSTIFIED? – TEACHERS’ LEVELS OF GUIDANCE

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Several studies have investigated how interactive software support inquiry-based learning in mathematics (e.g., Jones, 2000). In productive technology integration a teacher needs to combine his pedagogical, content and technological knowledge. This forms teacher’s technological pedagogical content knowledge – TPCK (Mishra and Koehler, 2006). Teacher’s skills in guiding students to justify their findings are essential part of TPCK. Several studies have pointed out that students need teacher’s support to transit from verifying a conjecture to explaining why the conjecture is true (e.g., Jones, 2000). In this study we use GeoGebra, which is free dynamic mathematics software.

The aim of this research is to study how prospective teachers guide students’ reasoning in GeoGebra supported inquiry tasks. Twenty prospective mathematics teachers answered to a questionnaire. The questionnaire included eight hypothetical situations where high school students present their GeoGebra supported solutions to a teacher. The prospective teachers were asked how they would react as a teacher in these situations.

We noticed the following three levels of the prospective teachers’ responses: 1) Surface level guidance which means that the teacher does not notice a certain essential aspect of the student’s solution or gives advice which is not related to the student’s solution. 2) Inactivating guidance which means that the teacher notices the essential aspect but reveals the investigations of this aspect to the student or asks for another solution method without motivating the student. 3) Activating guidance which means that the teacher notices the essential aspect and activates the student to investigate this. These kinds of essential aspects are, for example, justifying the finding, investigating deficiencies of the solution, moving from trial and error towards mathematical reasoning, generalizing and building connections.

We have used the questionnaire also as a teaching method in teacher training. Answering to the questionnaire and classifying their colleagues’ responses helps prospective teachers to reflect on their role in technology enriched learning.

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# CREATING OPPORTUNITIES FOR PROSPECTIVE TEACHERS TO REFLECT ON CONCEPTUAL RELATIONS

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This study investigates prospective teacher's reasoning on the relationships between mathematical concepts during the non-standard task of collaboratively constructing a concept map. The prospective teachers are enrolled in a teacher preparation program in mathematics for school grades 4-12. Four groups of prospective teachers with 3-4 in each group participated in the study. Each group was given a list of 19 concepts. A majority of the concepts are related to the curriculum for secondary and upper secondary school, including concepts such as equation, solution set, function, domain, continuity, limit, tangent and derivative. The prospective teachers were also asked to include concepts of their own in their concept maps as several concepts central to the definition or application of the given concepts were excluded from the list. The process of constructing a concept map was video recorded and transcribed for each group.

The findings show that the prospective teachers in their reasoning make up various relationships between non-related concepts and fail to notice fundamental relationships between concepts. The study illustrates the importance of creating situations for prospective teachers to explore and discuss their conceptual knowledge and to be challenged in their understanding of conceptual relations. Conceptual knowledge incorporates the integration of many skills, skills often taught in isolation. Compartmentalization of knowledge risks occurring when a body of knowledge splits into a larger number of isolated bits according to Chevallard's didactical transposition (1985). More concerns regarding knowledge compartmentalization are considered in students not being able to assimilate different forms of representations (e.g. Mamona-Downs & Downs, 2002), with impact on understanding, facility in manipulation, mental imagery, etc.

The activity of constructing concept maps seems to be a valuable tool in creating opportunities for prospective teachers to reflect on their conceptual knowledge and their understanding of conceptual relations. Indications of metacognitive activities related to concept mapping were also confirmed, which might initiate processes of thought that stimulate the creation of conceptual frameworks of meaningful connections.

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# ASSESSING STUDENTS' LISTENING SKILLS: A CASE STUDY

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It is commonly suggested that mathematics classrooms should be student-centred and students should work collaboratively in the classrooms. Working collaboratively requires communication and interaction between students and between a student and a teacher (Hiebert et al., 1997). For an effective communication in a mathematics classroom, students should be fluent in four modes of language skills; listening, reading, speaking, and writing. Among these language skills, listening is critical for learning as it helps students to acquire information and to succeed in communicating with others (Wallace et al., 2004). Considering the critical role of listening in learning a subject matter, the purpose of this case study is to assess the listening skills of elementary students in a mathematics classroom and to examine if and how students' performance in listening tasks and writing tasks vary.

The study was conducted in a well-equipped public elementary school. The school has come into prominence with its physical structure, achievements of students in sports, social events, and national exams. The participants of the study were nineteen 6<sup>th</sup> grade students studying from this school. The data were collected through a ten-item test, which was designed in two different forms; written test-WT and listening test-LT. Students' written responses to both tests were analyzed by a scoring guide. Results revealed that students had relatively high scores in WT compared to LT. To further analyze this difference, responses of three selected students to the same items in both LT and WT were compared. The main results that emerged from this comparison were that paying attention, taking notes, and trying not to miss what a teacher said were important for students in solving the questions correctly. The analysis of the results showed that students' listening skills should be improved through various listening activities, and there is a compelling need for the effective listening activities. Then, mathematics education community needs to work with language education specialists to develop effective listening activities to be used in mathematics classrooms.

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# USING HISTORY OF MATHEMATICS IN TEACHING MATHEMATICS: TEACHERS' VIEWS

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Despite a long debate, the interest in incorporating history of mathematics in mathematics education and the belief in its value for teaching and learning have been growing remarkably in recent years (e.g. Katz, 2006; Liu, 2003). Liu (2003) proposes five reasons for using history in teaching mathematics: (i) *History can help increase motivation and helps develop a positive attitude toward learning*, (ii) *History gives teachers a guide for teaching*, (iii) *Historical problems can help develop students' mathematical thinking*, (iv) *History reveals the humanistic facet of mathematical knowledge*, and (v) *Post obstacles in the development of mathematics can help to explain what today students finds difficult* (p. 416). However, questions such as: "What would be the educational benefits of using historical developments of mathematical concepts/topics in teaching and learning processes? What is the best way for doing it?" etc. have yet remained unanswered.

The purpose of this study was to investigate a group of secondary mathematics teachers' views about the reasons for using the history of mathematics in their instructional practices. The participants of the study were one male and four female teachers who were working in a public high school. They had ten years of experience on average. The data was obtained through semi-structured interviews. The interview protocol included questions such as: "Do you personally interested in history of mathematics? Do you think that a mathematics teacher should/must know the history of mathematics? Why or why not? How incorporating history in teaching mathematics can be beneficial for students? Please provide an example, etc." The interviews were transcribed and coded using Lui's (2003) five reasons outlined above. The results indicated that the main reason why teachers may utilize the history of mathematics was just to motivate students toward learning mathematics. Even though the textbooks that they were using included many historical anecdotes, short biographies of different mathematicians and some historical problems, the findings of the study indicated that the teachers did not incorporate them into their teaching and did not think that they might be able to develop students' mathematical thinking or deal with students' difficulties using pedagogical implications from the history of mathematics.

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# HOW IMPORTANT ARE TEACHERS IN THE DIFFUSION OF AN EDUCATIONAL INNOVATION?<sup>1</sup>

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Using survey methods, we investigated the potential sustainability and spread of an educational innovation implemented in the Scaling Up SimCalc experimental research program. This approach utilizes dynamic representational algebra software (SimCalc MathWorlds<sup>®</sup>), integrated curriculum and small professional development workshops to introduce teachers not only on how to use the software but also how to implement the package into mainstream curriculum.

In the research program, we recruited teachers to participate in experimental studies using SimCalc's integration of professional development, representational technology and paper curriculum. The experimental studies found that students learned more advanced mathematics when their teachers implemented the SimCalc approach. One year after the formal studies concluded, researchers gave teachers a survey to determine whether they were still using the materials (i.e. sustainability or what we refer to more dynamically as "stick") and sharing them with colleagues (what we refer to as "spread"). We found that 48% of respondents were still using SimCalc (stick) and 67% had shared information with a colleague about the materials (spread).

The approach we report here focuses on the attitudes and beliefs held by SimCalc teachers both about the nature and value of the intervention. We found three factors that were important: (1) teachers perceive the professional development they received as aligning with the value of the SimCalc materials in the classroom (2) teachers see the integration of software and curriculum as valuable and (3) teachers see the professional development they received as valuable to their teaching. SimCalc materials were more likely to stick and spread with teachers who reported agreement with these factors.

The main implication of these findings is to support the experimental research on scaling up, which found positive effects across a wide variety of classrooms, by showing that many teachers continue to use and spread the materials. Going beyond this implication, we see two approaches to advance sustainability: (1) increasing coherence with overall instructional goals and (2) increasing support for teachers' views of the value of these unique materials.

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# SUPPORTING PROBLEM POSING THROUGH INVESTIGATION ACTIVITIES

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Problem posing is a key element in mathematics activity and, thus, a central process in mathematics investigations. Current mathematics curriculum guidelines (NCTM, 2000) recommend the development of educational contexts to support problem posing, that is, classrooms environments where this constitutes a natural process of mathematics learning.

Mathematics investigations provide students the opportunity to experience different mathematics processes, including problem posing. Working on mathematics investigations includes looking for regularities, formulating questions for which one has no ready answer, formulating and testing conjectures, establishing plausible arguments and formal proofs to validate (or not) these conjectures, generalizing, where appropriate, and reflecting to raise new questions (Ponte et al., 2003). During an investigation, mathematics problems may arise when students pose questions concerning data, look for regularities and identify patterns from examples, formulate conjectures, test and revise them and propose alternative formulations to the initial questions, try to generalize and/or refine their conjectures to obtain a general solution and propose new questions, expanding their explorations by changing the initial conditions.

This paper aims to analyse the mathematics processes used by 2<sup>nd</sup> year university students exploring investigations, in particular, their problem posing processes. The study is based on a teaching experiment, supported by investigation activities proposed in a numerical analysis course, and follows a qualitative and interpretative research methodology. Data was collected via observation, interview, students' written reports, a questionnaire, and a researchers' journal. The results reported concern one student working on one task.

The results of this study highlight the potential of investigation tasks to promote problem posing and suggest that they may be successfully used in university mathematics courses. Thus, the students' ability to pose problems seems to be clearly stimulated by getting involved in investigation activities.

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# THE ANALYSIS OF PROSPECTIVE TEACHERS' THOUGHT PROCESSES IN THE MATHEMATICAL MODELLING THROUGH A DESIGNED FERMI PROBLEM

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Fermi problems are defined as the method of obtaining a quick approximation to a seemingly complex mathematical content by using appropriate guesses and rounded calculations (Carlson, 1997 cited in Arleback, 2009). Arleback (2009) presents that realistic Fermi problems provide good and beneficial opportunities for discussing in group works to introduce mathematical modelling at upper secondary school level. The purpose of the study is to examine upper secondary prospective mathematics teachers' thought processes in mathematical modelling while working collaboratively through a designed Fermi problem by the researchers. The participants of the study were selected from mathematics education department of the state university and took a course about mathematical modelling. The collaborative groups of 2–4 individuals were formed according to the participants' willingness and these groups worked on the Fermi problem developed by the researches. Data were obtained from the written documents of the participants' solutions, the video records including their solution processes and the observation notes taken by the researches while the participants working on the problem. While analyzing the data, categories were formed by considering mathematical modelling process. The results showed that the participants presented different and appropriate approaches in solving the Fermi problem and could successfully solve the complex problems such as Fermi problems. As results of the study, it is considered that the participants' approaches are developed working on Fermi problems and they can use this problem as a tool in their future teaching processes.

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# EFFECTIVE MATHEMATICS LEARNING IN TWO SINGAPORE PRIMARY CLASSROOMS: NEGOTIATION OF VALUES

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This paper reports the findings of the Singapore component of an on-going international project, *The Third Wave: Study of values in effective mathematics education*, which seeks to understand effective primary mathematics teaching. Two teacher participants and a focus group consisting of 4 to 5 students from each teacher participant's Grade 6 class took part in the study. Articulated ideas about the teachers' negotiation of values in relation to their students' voices about factors that contribute to their learning (see Seah & Ho, 2009) are presented.

The research questions were:

1. What convictions are co-valued in the Singapore classroom environment when primary students find themselves learning mathematics particularly well?
2. How are these values negotiated by the teacher?

Data was collected from lesson observations, post-lesson teacher and student interviews and artefacts (of photographs taken by students and journal entries of the teacher participants). 3 mathematics lessons lasting about an hour each were observed for each class over a period of 3 months. During the lessons, the student participants were each given a digital camera and requested to take photographs of moments of the lesson which they perceive to be effective mathematics learning.

One teacher decided to include recapping of the previous lesson in her subsequent lessons when she found out that her students found that useful to guide them through the lesson for the day. The other teacher, on the other hand, decided to not explain or show her solutions and answers to her students even though she knew that her students valued her explanation during the lessons. Instead, she used students' work for discussion in her lessons. Teachers' knowledge, experience and confidence might have a significant influence in the value negotiation processes.

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# DEVELOPMENT OF VISUALIZATION SKILLS IN SINGAPORE PRIMARY SCHOOL MATHEMATICS TEXTBOOKS

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This study examined how two widely-used Singapore primary school mathematics textbooks represent and teach visualization skills for the topic of volume at grade six. A study on Singapore pre-service teachers found teachers' reluctance to visualize and form a mental representation of the volume concept a possible obstacle to teaching their students for conceptual understanding (Yeap & Cheong, 2000). They also suggested that textbooks be designed to implicitly influence teachers to teach for conceptual understanding. In Singapore, the textbook is the main resource used by teachers. The importance of developing visualization skills is reemphasized in the current mathematics curriculum (MOE, 2007). Hence there is a need to examine how textbooks suggest visualization skills be taught. A framework based on Bishop's (1980, 1983) classification of visualization tasks – IFI (Interpreting Figural Information) and VP (Visual Processing) – was used to code and analyse the textbooks. The study found that both textbooks provide a good foundation for students to develop visualization skills. One of the textbooks was found to be strong in leading students to conceptual understanding of volume. The other textbook was found to be strong in higher-order thinking through multi-step application problems that require students to visualize problem situations. The study suggests including more verbal problems in the textbooks to provide students opportunities to develop their visualization skills by coming up with their own mental representations of problem situations.

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# USING METAPHORS TO ELICIT INSERVICE TEACHERS' PERSONAL BELIEFS ABOUT TEACHING AND LEARNING MATHEMATICAL LITERACY

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Large scale in-service teacher education programmes in South Africa are undertaken in order to meet the need for Mathematical Literacy teachers. Many of the participants in these programmes are mature teachers with many years classroom experience but no formal qualification in Mathematical Literacy teaching. This paper reports on the use of drawn and described personal metaphors of mathematics teaching and learning as a starting point to move teachers on one such programme to a more theorised, innovative and reflective approach to their teaching.

The use of metaphors as a tool for investigating preservice teacher thinking is well-documented (see for example Bullough, 1991; Reeder, Utley, & Cassel, 2009). In this study the focus was on surfacing the beliefs of in-service teachers and encouraging them to examine their own personal beliefs in the light of some philosophical perspectives on mathematics and some of the ideas of well known mathematics education theorists. The metaphors could be classified according to the teaching metaphors suggested by Fox (1983) and the general learning metaphors suggested by Sfard (1998), providing the lecturer/researcher with some insight into the thinking prevalent in this cohort of teachers.

Teachers were asked to draw and describe their personal metaphors for teaching and learning mathematics at the outset of the module. These were displayed on posters together with their personal mathematics stories and their colleagues wrote comments on the poster. The intention was to create a dialogue that either supported or challenged the metaphor and prompted deeper thought. Teachers were later required to discuss their metaphor in the light of the theoretical work covered in the module.

The metaphors initially drawn were predominantly of the “transfer” type often depicting feeding or watering. Learners were frequently portrayed in very passive roles. The teachers did not find it easy to reconcile their metaphors with the mathematics education ideas presented in the course, but the challenge this presented caused them to examine their personal beliefs more closely.

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# THIRD GRADERS' PERFORMANCES ON NUMBER SENTENCES AND WORD PROBLEMS OF THE CONCEPT EQUALITY

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Many research studies have shown that the concept of “equal sign” has not been clearly related to “equal in value” (Behr, Erlwanger, & Nichols, 1980; Carpenter, Franke, & Levi, 2003; Knuth, Stephens, McNeil, & Alibali, 2006). The purposes of this study were to investigate 3rd grader’s performance differences between number sentences, such as  $25+32=( )+14$ , and corresponding non-routine word problems. The content presented were partial results of the project funded by the National Science Council of the Executive Yuan (Project No.: NSC 98-2511-S-142-001-M).

The participants of this study were 812 3<sup>rd</sup> graders from 5 counties, 33 classes. The results indicated that students scored significantly better (dependent sample  $t = 21.01$ ,  $df = 828$ ,  $p < 0.001$ ) on word problems ( $M = 4.32$ ,  $SD = 2.61$ ) compared to number sentences ( $M = 2.27$ ,  $SD = 2.73$ ). In addition, students were categorized by their understandings regarding this “=” symbol. The ANOVA results were shown in the following table. In addition, even though students solved word problems correctly, only 60% of students could connect number sentences with the corresponding word problems.

	Operational ( $n = 183$ )		Relational ( $n = 41$ )		Both ( $n = 588$ )		$F$	$p$
	$M$	$SD$	$M$	$SD$	$M$	$SD$		
Total	5.82 <sup>b</sup>	4.29	6.59 <sup>ab</sup>	4.22	6.79 <sup>a</sup>	4.63	3.22	0.040
Number sentence	1.82 <sup>b</sup>	2.55	2.02 <sup>ab</sup>	2.64	2.42 <sup>a</sup>	2.78	3.56	0.029
Word problems	4.00	2.47	4.56	2.65	4.37	2.65	1.65	0.192

<sup>ab</sup> indicated the results of Scheffé post hoc test

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# PREPARING TEACHERS TO TEACH MATHEMATICS WITH ICT TECHNOLOGY BASED ON THE FRAMEWORK OF TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE

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Recently, the question of what teachers of mathematics need to know in order to appropriately integrate technology into their teaching has received much attention. The important tasks for teachers to integrate technology into mathematics classroom include the designing and implementing mathematical contents in the e-learning environment. Therefore, pre-service teachers, who have little teaching experience, need to learn how to design mathematics materials in the e-learning system. This study aims to provide a course design for e-learning environment of mathematics in the framework of technological pedagogical content knowledge (TPCK) (Mishra & Koehler, 2006), and to examine the learning performances and difficulties of pre-service teachers in this course.

The course is focused on TPCK and taught with the mentoring system. The whole time period is 17 weeks with 2 hours per week. Pre-service teachers use dynamic mathematics software GeoGebra and programming language JavaScript to design dynamic and interactive activities for mathematics learning. We reveal the true ideas and viewpoints of the pre-service teachers by observing their performances in the course, by questionnaires and by interviews.

We have two main findings: (1) Pre-service teachers gradually develop components of TPCK in different stages of the course; (2) Due to the lack of experiences in programming and integrating technology into teaching, pre-service teachers encounter difficulties in practice, but those difficulties are overcome by learning through the mentoring system.

The pre-service can understand that they need all kinds of abilities related to TPCK, but this one-semester course does not seem to offer them enough time to master all necessary knowledge. It will take a longer experience or course for them to practice, so they can manage to use what we want them to know in their teaching.

In this study most of the pre-service teachers also take the course of Mathematics Teaching Materials and Methods. Some of them can use in our course what they learn in that course, but most of them cannot integrate their knowledge learned from these two. Therefore, we believe when a pre-service teacher have some experience in learning to teach mathematics, he/she can do better in this course and in the future.

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# PROMOTING A LOW-PERFORMING ELEMENTARY STUDENT'S UNDERSTANDING OF 2D REPRESENTATION OF 3D POLYHEDRONS

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Elementary students often encounter obstacles in identifying missing vertices, faces, and edges in 2D representations of 3D polyhedrons. With the aim to address this issue, we selected a low-performing fifth-grade student, Yen, who could not identify correct number and location of edges of rectangular prism prior to the unit of polyhedrons. His prior geometrical understanding was in the *prerealist* stage (Mitchelmore, 1980). In this study, a teaching experiment was designed to understand how to better support him in progressing along the development stage. The mathematics teacher, the first author, included a hands-on activity in which the class constructed a rectangular prism using sticks and clay. We conducted pre- and post- student interviews and classroom observations to collect information about Yen's understanding of prism in particular. The use of semi-structured student interviews and teaching experiment offered an in-depth lens in students' thinking that previous studies (Mitchelmore, 1980; Wu and Ma, 2009) did not afford. The findings showed that Yen was very systematic in his construction of the prism model in class. However, in the post-interview, he still couldn't draw the edges accurately. The lack of repair after the class showed that it is insufficient to have students observe or create 3D objects. The relationship between the 3D object and 2D representation needs to be more explicit. The authors aim to extend this study to multiple case studies of students to investigate the learning progression of students along the developmental stages and discover teaching strategies that are effective in repairing students' misunderstandings of representations of polyhedrons.

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# DESIGNING A PROGRAMME FOR PRE-SERVICE ELEMENTARY TEACHERS TO DEVELOP MENTAL MATHEMATICS FOR TEACHING

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The mathematics curriculum in England and Wales has changed dramatically over the last ten years since the introduction of the National Numeracy Strategy in 1998. Mental calculation has attained a dominant position in the elementary curriculum and teacher education programmes in the UK have required adaptation to reflect these changes.

This provided the context and motivation for a design-based study which was a programme of research supporting the mental mathematics initial teacher education programme while providing a theoretically-grounded account of the means to improve practice. A review of the literature on teacher knowledge in mathematics led to the articulation of two theoretical principles for application in the design of an innovative, year-long intervention. The first principle established the nature of the intended learning outcomes of the intervention in relation to the development of mental mathematics subject knowledge for teaching. The second principle was generated by the knowledge-based theoretical position established by Shulman (1986) and Ma (1999), adapted for action within the intervention programme in the form of conceptual pedagogy. It posited that, in order to achieve the intended learning outcomes, pre-service teachers needed access to activities founded on conceptual pedagogy's connectedness, flexibility and coherence.

The process of design of activities and materials for use with a cohort of 129 elementary pre-service teachers during a year-long programme of teacher education was based on these principles. Micro-analyses of pre-service teachers' interactions with the designed activities - individually, through group work within the university and in teaching placement contexts - led to review and modification of activities while offering a theoretically-informed commentary on their effectiveness.

The study's outcomes include the illumination and exemplification of existing theoretical constructs such as coherence and connectedness over a range of mental mathematics content.

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# HISTORY, APPLICATION, AND PHILOSOPHY (HAPh) MODULES IN MATHEMATICS EDUCATION

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The short oral outlines an on-going research project, which is presently under preparation for implementation at Danish upper secondary level. One aim of the project is to investigate how the three dimensions of history, application, and philosophy of mathematics may enter into mathematics education in a meaningful way. The underlying theoretical construct for the project is the Danish KOM-report by Niss & Jensen (2002), who discuss 8 mathematical competencies and 3 types of overview and judgment, the latter referring to “‘active insights’ concerning the character of mathematics and its role in society” (p. 43).

The project includes two teaching modules, referred to as *HAPh-modules*, to be implemented in school. Each of these consists of three original, primary texts (in Danish translation) related to a chosen mathematical topic or case; one historical, one philosophical, and one on a present application of the mathematics in question. The design principles of these modules rest on *guided readings of primary historical texts* (Pengelley, 2008) and students’ *essay assignments and discussions* (Jankvist, 2010). Video recordings of students’ group discussions will be analysed by looking for *episodes of anchoring* (Jankvist, In Press), a construct drawing of Sfard (2008). The purpose of this is to see, if the students’ discussions of the more meta-perspective issues regarding the history, application, and philosophy of mathematics are somehow anchored in their actual mathematical content knowledge of the chosen cases or mathematical topics. In the talk I shall show an example of a HAPh-module including the type of activities that the students work with during these.

Yet an aim of the research project is to see if and to what extent the students’ *images of mathematics as a discipline* change during, and possibly due to, such teaching modules. Also this will be touched upon in the talk.

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# STUDENT'S AFFECTION FOR 7<sup>TH</sup> GRADE MATHEMATICS

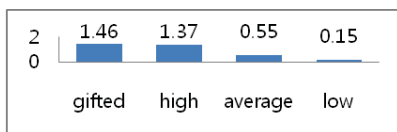
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If we consider a humanistic aim of mathematics education, an Emotional fulfilling, such as understanding mathematical beauty or having good feel in math, is also be the important purpose. From this study we want to discuss of Korea middle schooler's emotion for 7<sup>th</sup> grade mathematics. We can have some view from this study; (1) all achievement level students have good feeling in some mathematics subjects. (2) Student has own taste for mathematics. If we reflect these tastes to instruction design, we can motivate more students on.

This paper is based on survey results. We wanted to know how Korean middle schooler feels for 7th grade mathematics in concrete subject matter. Most low achiever dislike school mathematics in a naively questioned setting: 'do you like mathematics?' we assumed that we can see more about student emotion in a concrete setting questionnaire. We developed the Questionnaire of Aesthetic Mathematics (QAM) on 7-th grade mathematics. Questionnaire response has 11 Likert scales, from most dislike -5 through most like 5. Students are directed to answer to 36 question items as their emotional intensity. Questionnaire was answered by 427 students. : 103 from gifted student, 114 from high achiever, 111 average group, 64 from low achiever.


All student groups' average feeling score for 7<sup>th</sup> grade mathematics is positive number. Even a student have negative mind to holistic mathematics, they still have some items in good feel.



(Average affection score by each group.)

We analysed answered 'good feel' response by each group. When we compared top ranked items of each group, all groups have good affection for question 32 in common. Only ranked item for Average and low groups in common is question 3, 4, for high and gifted groups in common is question 7, 17, 34.

32. if we just compare 3 factors without piling up the two triangles, we can judge congruency. ex.SAS.

3.  can be '5', '1' or '0'. Number exists in our minds.

17. when I face some unknown, letting them to some characters as x, y, their perplexed relationship reveals.

We will speak more findings and student emotional taste in this session.

# THE ROLE OF THE RESEARCHER IN THE LEARNING OF IN-SERVICE MATHEMATICS TEACHERS

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The research project being reported is at its early stage and it focuses on the reflection which –under the guidance of the researcher– is made by the teacher upon the mathematical and didactic knowledge that he and other teachers carry out in an everyday class environment. That is why this project has as objectives: a) to promote the learning of mathematical and didactic knowledge which teachers bring into play in order for them to improve their teaching practice and b) to identify the impact that the help provided by the researcher has upon such learning. The conceptual framework which serves as the foundation of this research is the one referring to the mathematical and didactic praxeologies (Chevallard, 1999).

The research is of qualitative type by the use of case studies, having non-participative observation as an instrument for the gathering of data. Work will be done with four teachers who are currently teaching third-grade algebra topics in junior high school (see SEP, 2006, p. 25). In order to achieve the objectives, the research will be developed in three phases. In the first one, classes of teachers teaching the aforesaid topics will be observed. In this phase, the analysis will be presented in two stages: on the one hand, the researcher will analyze the class observation and then will choose the most representative ones; on the other hand, each teacher –under the guidance of the researcher– will analyze the chosen class observations. In the second stage, each teacher, together with the researcher, will modify the tasks which are posed in the textbook and which were worked upon at the observed classes. Once modifications are finished, classes in which teachers implement the modified tasks will be observed and analyzed again. In the third phase, based upon the results from phase two, each teacher –and accompanied by the researcher– will restructure once more the modified tasks. As final observation, teachers will be video recorded when working upon these new tasks.

Among the results attempted to be achieved stand out: to offer evidence that junior high school teachers do raise awareness of the importance of improving their teaching practice, upon learning from the mathematical and didactic knowledge that they, and other teachers, bring into play in the classroom.

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# THE EFFECT OF DYNAMIC MATHEMATICS SOFTWARE TO THE STUDENTS' BELIEFS ABOUT MATHEMATICS

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*This study aims to investigate the effect of using dynamic mathematics software to explore mathematical concepts on students' beliefs about mathematics. Students' beliefs are collected before and after an experimental period of 20 weeks by the use of a belief questionnaire.*

One of the definitions on mathematical belief is one's mathematics world view that means the perspective with his or her approaches to mathematics and mathematical tasks (Schoenfeld, 1985). By this view, one's mathematical belief may be affected by the ways of learning. Using dynamic mathematical software in the learning environment has a great potential of shifting the ways of learning mathematics from traditional to more explorative. We designed an experimental study to investigate the affect of using dynamic mathematics software to explore mathematical concepts on students' beliefs.

An elective course was conducted with 15 volunteer 11<sup>th</sup> graders from an ordinary Turkish high school. The entire course was planned as exploration task modules by using dynamic mathematics software GeoGebra and lasted 20 weeks (two hours per week). First three weeks students were given the fundamental knowledge of GeoGebra. Then, in every week students were asked to study on exploring a mathematical concept by using GeoGebra with the help of a worksheet under the guidance of their instructor. The students were in the primary role of using computer. The concepts were selected from Turkish secondary school mathematics curriculum. Most of the concepts were new for students, while some of the concepts were already taught to students by traditional ways before.

Students' mathematical beliefs were collected by a questionnaire which has three sub dimensions. These are beliefs about process of learning mathematics with 10 items, beliefs about use of mathematics with seven items, beliefs about nature of mathematics with three items. The overall reliability of the questionnaire, which has totally 20 items, was reported as .75 (Aksu, Demir, and Sumer, 2002).

We compared the students' pre and post beliefs and we will present how students' beliefs affected by the use of dynamic software to learn mathematical concepts.

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# STUDENTS' CONSTRUCTION OF LIMIT CONCEPT

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Limit as one of the difficult concepts for most students has attracted mathematics educators' attention for more than a decade. Researchers, such as Williams (1991) argue that the dynamic conception of limit is natural for students, but such notion can prevent them from developing formal understanding. Cottrill et al. (1996) emphasize this notion is a schema coordinating two processes. According to them, informal notion of the limit is necessary to construct formal understanding, and difficulties with developing formal limit conception are a result of a weak quantification schema. On the other hand, Pinto and Tall (2002) argue that APOS theory is compatible with formal thinkers' conceptions, but does not explain the cognitive strategies of natural thinkers. In his dissertation Cetin (2009) stated that there is a need to study these discrepancies between the natural and formal thinkers in the development of students' understanding of the limit concept. We aimed to investigate development of students' learning of limit concept in order to generate additional information about these discrepancies. This is an on-going study designed qualitatively in which the data was obtained from open-ended tests and clinical interviews with students taking Analysis I course in a mathematics education program. Preliminary results of this study are partially compatible with conclusions of Cottrill et al. (1996). Our results confirm that the dynamic notion of the limit concept is a schema coordinating two processes, and quantification schema is important for understanding the limit concept. Our results indicate that there is no clear cut between the third and fifth level of decomposition suggested by Cottrill et al. In fact, these two levels appear to be intertwined and followed by the remaining two levels. Therefore, our preliminary conclusion is that the genetic decomposition needs to be revised to include both natural and formal thinkers' conceptual developments providing more details to some levels.

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# THE INFLUENCE OF TEACHER'S VALUES ABOUT TEACHING MATHEMATICS IMPLEMENTING LESSON STUDY AND OPEN APPROACH ON THAI CLASSROOM

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A teacher's theory of practice is closely associated with her values and a teacher's values can influence her work life (Gultekin, 1996). It is so important to take values into account when considering research and professional development in mathematics education (Bishop, 2003). Lesson study is a system for school-based professional development of Japanese teachers (Lewis, 2002; Fernandez & Yoshida, 2004; Baba, 2007; Isoda et al, 2007; Inprasitha, 2010). Thailand has been implementing Lesson Study since 2002. An Adaptive feature of this implementation is to incorporate Open Approach as a teaching approach into Lesson Study processes (Inprasitha, 2010). The aim of this study was to analyse the influence of teacher's values about teaching mathematics implementing Lesson Study and Open Approach as a teaching approach on Thai classroom.

The research design is mainly qualitative. The targeted group was students and teachers from 4 schools participating in a teacher professional development project under the care of the Center for Research in Mathematics Education (CRME). The study was structured through a questionnaire survey of 83 teachers from 4 schools. Case studies were then conducted with 3 school mathematics teachers, involving video recorded on 3 phases of Lesson Study, lesson observations and interviews, and then interviewed 3 selected internship students, 2 selected observation teachers and 15 selected students. Data analysis is based on the idea about structure of instruction of Stigler & Hiebert (1999) and of Inprasitha (2010). The results revealed teacher's values about teaching mathematics implementing Lesson Study and Open Approach have influenced on classroom as followings: 1) form of teaching in the classroom 2) instruction sequence and time use of teachers 3) the role of teachers and teaching behavior 4) interaction in the classroom 5) atmosphere of the classroom and 6) How teachers use to evaluate students.

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# DEVELOPMENT OF STUDENT'S ABILITY TO CONSTRUCT UNIT(S) FOR TESSELLATING A PLANE WITH FIGURES

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It is an important cognitive ability in the development of geometrical thinking to construct composite units of figures and divide given figures into parts. Clements(2004) refers to “Shape” as one of Big Ideas of geometry, which is the model for understanding objects in the real world, and gives composing, decomposing, embedding and disembedding as examples of cognitive abilities related to Shape. I suppose that these abilities have students get various views of figures. The aim of this article is to understand how secondary school students before learning mathematical proof show these abilities in solving the task to tessellate a plane with given figures.

In this article, according to the qualitative research methodology, I collected and analysed three kinds of data: video data of 6 hours of 7<sup>th</sup> grade lessons about tessellation with isosceles triangle, parallelogram and arrow-shaped figure, field notes by author and individual notes by all students. The characteristic of this task is that students have to show the pattern constructed by using ruler, compass and protractor instead of manipulating materials for each figure.

The result of data analysis is that 4 stages of development of student's ability to construct various kinds of unit(s) for solving the tessellation task are identified as below(see Fig.1):

- I. Recognition of each figure as an unit
- II. Construction of composite units with figures for solving the task
- III. Construction of another composite units for solving the task efficiently
- IV. Construction of unit(s) of units

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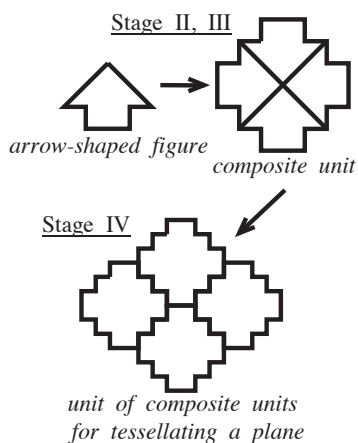


Fig 1. Units of arrow-shaped figures

# PRE-SERVICE TEACHERS SUBJECT MATTER KNOWLEDGE: THE CASE OF COMPOSITE AND INVERSE FUNCTIONS

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Many researches showed that students, pre-service teachers, and even experienced teachers have limited understanding of school mathematics, especially functions. Moreover, even if students have the enough knowledge base, they are not able to solve non-routine problems related with the same knowledge base (Selden, Selden, Hauk, and Mason, 2000). The presentation aimed to compare and analyze pre-service secondary teachers' subject matter knowledge of composite and inverse functions with respect to their performance on declarative (knowing that), conditional (knowing why), procedural (knowing how), and non-routine questions (problems which are not very similar to ones they solved before but require combination of known facts or principles). To be able to answer conditional and non-routine questions, at first pre-service teachers must have a required declarative and procedural knowledge since while solving them one needs to combine known facts and principles. The main difference of non-routine questions from the conditional questions is that former mainly asks the know facts and principles in a non-standard way, usually through use of different notation whereas in the later one the use of known facts and principles were used for explaining reasons behind the solution. As a result, having the ability to solve non-routine problems (Schroder, Schaffer, Reisch, and Donovan, 2002; Selden, Selden, Hauk, and Mason, 2000) and conditional questions can be considered as a sign for deep understanding which is considered to be important for teachers to have since they are going to transfer their knowledge.

The study was conducted with three pre-service secondary mathematics teachers. The sample was administered a function knowledge test including declarative, procedural and conditional questions and a non-routine questions task-based interview. In addition, follow-up interview were conducted based on the results of function knowledge test. The findings reveal that pre-service secondary mathematics teachers' scores on procedural questions were higher than their scores on declarative and conditional ones. Further, their lack of knowledge on declarative knowledge were led to low scores both in conditional questions and non-routine questions.

Schroder, T. L., Schaffer, C. M., Reisch, C. P., & Donovan, J. E. (2002). Preservice teachers' understanding of functions: a performance assessment based on non-routine problems analyzed in terms of versatility and adaptability. *Paper presented at the annual meeting of the American Educational Research Association* (New Orleans, April).

Selden, A., Selden, J., Hauk, S., & Mason, A. (2000). Why can't calculus students access their knowledge to solve nonroutine problems? *CBMS Issues in Mathematics Education*, 8, 128-153.

# NATURE OF IMAGES AND PICTURES IN TURKISH MATHEMATICS TEXTBOOKS

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Several factors are effective in achieving the objectives set out in education. One of them is mathematics education textbooks which are indispensable for the teaching of mathematics education. In this sense, the content and presentation of text books is very important. Textbooks should not be considered only as tools for reading contemporary and real knowledge but even should be considered in terms of visual and aesthetic values. For this reason, books should be supported by color pictures, figures, tables, charts, plans and maps to make easier understanding of subject being presented. If only these pictures should be selected in a very good way and has a very tight relationship with the text. Secondary Mathematics Education program, which came into force in 2005, in Turkey, is based on emphasis of constructivism which had tendency in increasing visual elements in textbooks compared to previous years. However, given this trend in the textbooks, brings a question to mind that how these visual elements (pictures, figures, graphics, etc.) are functional for teaching mathematics in our country? Thus, the current research was designed for the purpose of finding answers to this question. We used 9th grade mathematics textbooks which contain the concept of function in terms of images and pictures that are used for teaching that subject. Our aim is to reveal the functions of images that are used for mathematics teaching in that grade. Based on Carney and Levin's (2002) proposed functions that pictures serve in a text, this study used a similar categorization of pictures in order to examine the role of each type of pictures in students' performance, in Mathematics problem solving. Our research results show that the classification of pictures in concept of functions in 9th grade textbooks as follows: majority of these pictures do not has a direct reference in the text file that is they are not mentioned anyway. Most of these pictures are decorative that are used in textbooks. Decorative pictures do not give any actual information concerning the solution of the problem. In some books as an example they put a Picture of a group since the word "group" used in the text which has no relation with the concept that intended to be presented. As a result the writer should select the pictures and figures that are related to the subject being presented. They should give references to the Picture in the text as well. Thus the pictures should at least be organizational as well as being informational. Most of the pictures in the textbooks should not be decorative but there can be a few decorative among all the others.

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# EXPLORING COGNITIVE UNITS FOR EIGENVALUES AND EIGENVECTORS

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The purpose of this presentation is to share preliminary results of an ongoing research study that explores first-year university students' development of cognitive units (Barnard & Tall, 1997) for eigenvalues and eigenvectors in an introductory level linear algebra course. Students' development of cognitive units are investigated through analysis of two interviews of four students and course observations data by adapting the features of the Actor-Oriented Transfer (AOT) framework (Lobato, 2006), which allows researchers to take learners' perspective as learners reconstruct their prior learning experiences during interviews.

Linear algebra is one of the post-calculus mathematics courses required by other disciplines, and most of its topics are often prerequisites for many client disciplines. For example, eigenvalues and eigenvectors are revisited in quantum mechanics in physics. Earlier studies however indicate that students' have difficulties understanding eigenvalues and eigenvectors in their mathematics courses, even before using them in other courses (Stewart & Thomas, 2003). Some of these difficulties include students lacking geometric views of eigenvalues and eigenvectors, and students being unable to reason about the relationship between different representations of eigenvalues and eigenvectors (Stewart & Thomas, 2003). Since eigenvalues and eigenvectors are among the prerequisite topics that seemed to be difficult for students, educators may benefit from an investigation of the ways that students develop these concepts in order to explore at what point the difficulties start to appear.

Preliminary analysis of the on-going research study suggests that algebraic interpretations of the eigenvalue equation play an important role in participating students' cognitive units. The presentation will share more details of the preliminary analysis that attempts to describe different cognitive units.

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# INVESTIGATING PRE-SERVICE TEACHERS' UNDERSTANDING OF RATE OF CHANGE AND DERIVATIVE GRAPHS THROUGH A MODEL DEVELOPMENT SEQUENCE

Mahmut Kertil, M. Gözde Didiş, A. Kürşat Erbaş and Bülent Çetinkaya

Middle East Technical University

Robust understanding of derivative concept is necessary for understanding of further topics of calculus. Various studies related to teaching and learning of derivative concepts documented students' conceptual difficulties about rate of change and interpretation of derivative graph. This study investigated pre-service teachers' understanding of rate of change and derivative graphs as they engaged in a *model development sequence* designed by the researchers by following the *Models and Modeling Perspective* (Lesh & Doerr, 2003). The Model Eliciting Activity (MEA) adopted from the study of Yoon and others (2010), and two more follow up activities designed by the researchers are used in the study.

The participants of the study were four pre-service mathematics teachers. The data was collected through individual and group written reports, video-taped group discussion and a group interview. The findings of the study showed that pre-service teachers had difficulty in interpreting the rate of change in a non-motion context and transition from derivative graph to the original which is consistent with the findings of other recent studies (e.g., Yoon et al., 2010). Although the motion context has been the best context for the development of calculus concepts throughout the history, it is not cognitively demanding for students to think about the rate of change because of their familiarity. We conjecture that using motion context and specific terminologies as the name of derivative functions may be an obstacle for students' conceptual understanding. As observed in this study, using co-variations from different contexts such as volume-height and temperature-solubility for which the derivatives cannot be named with specific terminologies and need a verbal explanation using the language of rate of change fostered students' conceptual understanding.

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# HOW MATHEMATICS CURRICULUM MATERIALS OF KOREA AND THE USA SUPPORT REFORM

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The purpose of this study was to explore how mathematics curriculum materials support teacher learning for reform in mathematics education in two countries. For the purpose, we attempted to examine two elementary mathematics curricula developed in the USA, *Everyday Mathematics* and *Investigations in Number, Data, and Space [Investigations]*, and the 7th National mathematics curriculum of Korea in terms of how those curriculum materials support mathematics teachers to successfully implement the reform ideas suggested in the curriculum materials. To analyse the curriculum materials, we drew upon a framework from Stein & Kim's (2009) study consisting of: a) cognitive demand of instructional tasks (high-level consisting of *doing mathematics* and *procedures with connections*, low-level containing *procedures without connections* and *memorization*, see Stein, Smith, Henningsen & Silver, 2000, p. 16), b) whether they help teachers anticipate student responses or not, and c) whether or not they help teachers see the curricular developers' rationales, identified as transparency. For the analysis, we selected a topic of addition and subtraction for 3rd graders and collected the curriculum materials such as Teacher's Guides.

The results from the analyses suggested that the cognitive demand of instructional tasks in all the curriculum materials were mostly identified as in the high-level. However, the Korean mathematics curriculum material and *Everyday Mathematics* were primarily composed of *procedures with connections* tasks; in contrast, *Investigations* was mainly consisted of *doing mathematics* tasks. The results also showed that *Investigations* provided examples and information about possible student responses and the designers' rationales for a particular activity for particular mathematical topics and ideas. On the contrary, *Everyday Mathematics* and the Korean curriculum material included few examples and limited information about students' mathematical thinking and did not seem to help teachers see the rationales of the curriculum developers.

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# EVALUATING PRE-SERVICE MATHEMATICS TEACHERS' COMPREHENSION LEVEL OF GEOMETRIC CONCEPTS

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The literature indicates that learners experience major difficulties with the definitions of geometric concepts (Matos, 1994; Ubuz, 1999). Most of the studies on geometric definitions focus on identifying students' misconceptions in a specific geometry concept such as circles or polygons. We need more research on definitions of major geometric concepts. Learners' comprehension level of geometric concepts is important because conceptual knowledge helps learners use and adept their related expertise in learning new concepts. Additionally, conceptual knowledge makes it easier to move among different concepts (Hiebert & Lefevre, 1986). Here, we examine how 162 Turkish first year mathematics pre-service teachers define and draw geometric shapes. They were asked to define and draw major geometric concepts, including two and three dimensional shapes.

Data analyses indicate that most of the participants could draw two-dimensional shapes, but struggled in drawing three-dimensional shapes. They experienced relatively more problems in defining the concepts. In many cases, their definitions included inconsistent statements; for example, a number of students could define cone something as a combination of a triangle and a circle. This poor performance may be an indication of teaching geometry by introducing the concepts only with examples and simple drawings without focusing on the definitions first. Teaching geometry concepts should first begin with discussing the definition and then the learners should be encouraged to draw the shapes with the help of the definitions. Additionally, the participants' struggle in drawing and defining three-dimensional shapes could be a result of their inadequate level of spatial thinking skills. We recommend that teacher educators should pay more emphasis on geometric definitions and drawings in teacher education courses.

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# MATHEMATICAL COMMUNICATION BY GESTURE: AREA TASK

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Students confront difficulty involving area measurement and communicating idea each other in area task. Communication is one of the mathematical processes. In addition, communication is one of the basic skills that students must take along with them throughout their lives and use long after they have left school. Teachers should know how to improve their students' communication and assess their mathematical communication. The learning process was depended on basic communication with the other persons (Emori, 2005). The expressed gestures also showed the dimension of thought and language into symbolic language. Children's gestures were not only to fill the gap in children's speaking but gestures presented the valuable information of children's thought. Gesture as an action relating to the observable body, was considered normally not too much or less as the extension part including human beings' intention (Rasmussen et al., 2004). The purpose of this study was to analyse students' gestures in mathematical communication in area task. This study is a qualitative research that used video analysis from transcribing video, audio tape of four 5<sup>th</sup> grade students at Ban Bungneum Bungkrinoon School using lesson study and open approach. Data were analysed based on Emori's (2005) characteristics of mathematical communication: rigorousness, economy, and freedom. The results indicated that students used rigorousness, economy, and freedom in area task and most of students used massages communicate with concise massages.

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# CHARACTERISTICS OF AN ADVANCED MATHEMATICS COURSE FOR TEACHERS

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*This paper presents a case study to describe the course characteristics of an advanced mathematics course developed for teachers in a master's program as part of a NSF funded project.*

It has been widely accepted that mathematics teachers' content knowledge constitutes an essential part of a high-quality instruction. That is why the structure and the content of mathematics courses offered for teachers gain a particular attention. The purpose of this study is to explore whether the characteristics such as advanced mathematical knowledge, pedagogical content knowledge, real life applications of mathematics, teaching for understanding, and cultural responsive pedagogy are incorporated in an advanced mathematics course. The following research questions guide the qualitative analysis:

- How does the instructor perceive incorporating the 5 characteristics into a mathematics course for teachers?
- What is the interplay among these characteristics?

Data for the study included two observations of the course, one semi-structured interview with the instructor, and the review of course artifacts. Preliminary analyses showed that the integration of some course characteristics such as real life applications of mathematics, cultural relevance, and pedagogical content knowledge became challenging to incorporate. Based on Hill, Ball, and Schilling's (2008) domain map model that involves pedagogical content knowledge and subject matter knowledge, this study extended it by conceptualizing a new framework. In this paper, the approach taken by Zazkis (2010) on advanced mathematical knowledge is adopted and used as a substitute for teachers' subject matter knowledge. The suggested model built upon Hill, Ball and Schilling's model (2008) is a tentative one that needs to be investigated in further research.

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# INVESTIGATING PRESERVICE TEACHERS' GEOMETRIC PROBLEM SOLVING PROCESS IN DYNAMIC GEOMETRY ENVIRONMENT

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Dynamic Geometry Environments (DGE) allows geometric constructions to be explicated and contribute to the improvement of students' cognitive processes via toolbars (Scher, 2002). Students' ability to discover geometric relations by means of these software outdoes their ability in any environment without computer (Van De Walle, 2004). Therefore, the examination of students' problem solving process and students' discovering geometric relations in DGE is very important. This research examines three elementary pre-service mathematics teachers' geometric problem solving process in dynamic geometry environment. Pre-service teachers were given a task composed of 5 steps, and asked to solve the problem step by step, to discover the relations and to justify the relations they found out first in paper & pencil environment and then in DGE.

In this research, qualitative research techniques have been used for the data collection, analysis, and the discussion of the data. Results of the research indicated that all three pre-service teachers underwent similar processes and couldn't solve the problem in pencil & paper environment. However, each pre-service teacher's problem solving process was found to be different in dynamic geometry environment. It is also observed that pre-service teachers solve the problems primarily at two stages as constructions and investigation and that they did not face difficulties at constructions stage. However, at investigation stage, it is found that they underwent a helical process composing of seek for relationship, find relationship, test relationship, seek for a new relationship and justifying. Additionally types of dragging used were discussed.

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# THE CONSTRUCTIVE AND DECONSTRUCTIVE GENERALIZATIONS ON LINEAR FIGURAL PATTERNS OF PRESERVICE TEACHERS

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Studies on the generalization of patterns have been the focus of most research in recent years. In these studies, the generalization stages or levels, abilities and strategies of students in different grades were examined as different dimensions (Garcia-Cruz & Martinon, 1998; Orton & Orton, 1999; Rivera & Becker, 2008). The present study also investigated the constructive and deconstructive generalizations for linear figural patterns of the primary school preservice teachers. In order to examine this situation in detail, the research design was determined as classroom teaching experiment (Cobb, 2000). Firstly, pre-interviews were held with the primary school preservice teachers which were choose as participants in this study. Following the teaching process, post-interviews were held with them. The teaching processes were carried out in a total of four course hours - each course hour lasted one hour. The data obtained from this study were analyzed as qualitatively. At the end of the study, the preservice teachers' generalization forms of linear figural patterns were classified as the constructive standard, the constructive nonstandard and the deconstructive with respect to the strategies, which were used by them. It was found out that at the beginning, the teacher candidates predominantly preferred the numerical approach while generalizing the patterns during the teaching process and then the teacher candidates tended to move from the figural approach towards the numerical approach.

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# ASSESSMENT IN FINNISH MATHEMATICS EDUCATION: VARIOUS WAYS, VARIOUS NEEDS

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The autonomous role of Finnish teachers influences the way assessment is integrated as part of mathematics education. Recent research has focused on the shift towards internal, teacher-conducted procedures (e.g., Black & Wiliam, 2003). In Finland, both formative and summative assessment for monitoring learning outcomes are teacher-conducted, modified to the needs of teaching and school context. Applying technological solutions in existing routines can be a challenge. Teachers' beliefs, skills and knowledge-base about the utility of the assessment data and the potential to use it are essential (Webb & Jones, 2009). The overall aim of the project is to examine the design and adoption of a technological assessment tool addressing educational needs and enhancing possibilities to carry out diverse assessment processes. The assessment tool consists of a data-base of tasks from which a teacher can assemble tests and process assessment information. Design-based research approach forms the methodological framework for the study comprising of theory development, the prescriptions of successful design processes, and solutions (Design-Based Research Collective, 2003). Eight mathematics teachers in the year 2010 participated in interviews, which were elaborated through content analysis.

The results profile a diverse range of personalised assessment practices and needs of autonomous professionals. Altogether, five themes were formulated based on the data: i) assessment as part of professional development; ii) features of the available resources; iii) pedagogical aspects when applying assessment resources; iv) diverse forms of assessment; and v) special features of teaching and learning mathematics. Consequently, the assessment tool should be highly adaptable, easy to use and serve the specific demands of teachers and mathematics education. What becomes critical is the teachers' willingness to use new innovations and modify existing practice.

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# STUDENTS' CONCEPTIONS OF CONTINUITY: A CONCEPTUAL CHANGE APPROACH

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The concept of continuity is fundamental to understand many mathematical concepts such as real numbers, limit, functions, etc. While research studies highlight that students have considerable difficulties in understanding the continuity concept, more studies about the sources of these difficulties are needed. According to conceptual change theory, learning occurs as a change in prior knowledge structure either as enrichment or as reorganization. Even a reorganisation is necessary for some mathematical concepts; it is not common for students to change their prior knowledge structure. Hence, prior knowledge prevents new information to be comprehended, which results in the construction of misconceptions (Vosniadou, 1994). In this study, role of prior knowledge and experience in students' conceptions of continuity were explored based on the conceptual change approach.

The participants of this study were five undergraduate students (19 to 21 years-old), from a university in Turkey. All students had taken a calculus course; hence they were familiar with advance mathematical concepts. In order to grasp students' understanding of the continuity concept, a test was developed based on related literature. The test included open-ended definition, identification, and construction problems related with continuity concept. After completing the test, each student was interviewed. Students' written and oral responses constitute the data of this study.

The analysis of data revealed that (1) students' prior experience with the use of word "continuous" in daily life, (2) their physical intuitions derived from their everyday experience of physical reality, (3) their prior knowledge and experiences of natural number concept have a role in their conceptions of the continuity concept and these factors create obstacles in their understanding of the concept of continuity.

The findings suggest that the prior knowledge about natural numbers is needed to be reorganised for the extensions to more advanced mathematical domains. For this reason, it can be suggested that design and implementation of the learning environments for the concept of continuity should pay considerable attention to the reorganization of prior knowledge.

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# EXAMINING MATHEMATICS STUDENT TEACHERS' REPRESENTATIONS: THE CASE OF LIMIT CONCEPT

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The purpose of this case study is to examine student teachers' lessons using Knowledge Quartet's transformation unit in the context of "select appropriate forms of representation" for the limit concept (Rowland, Turner, Thwaites, & Huckstep, 2009, pp. 36). Four mathematics lessons of four participants (16 lessons in total) were observed and video-taped, and the semi-structured interviews were realized and audio-taped to discuss some of the episodes of their lessons. Six interviews were carried out, and one of them was made before the lessons, four of them were about the participants' each four lessons and the last one was about the general comments of their teaching. All video and audio records were verbatim transcribed and these transcriptions contained expressions of both the participants and their students. They also contained screen captures of the writings on the board and the participants' computer presentations. The participants' representations about the limit concept were categorized named by graphical, tabular and symbolic. The participants used graphical representations in their lessons. The usages of the graphical representations were combined as two categorizations and these were named using plane analytic geometry and real number line. The participants sometimes drew the graphics on the board, sometimes showed their examples from their computer representations, and two of the participants also used mathematical softwares to show graphics in their lessons. The tabular representations were rarely used by the participants. All of the participants utilized these representations in the first lessons for introducing the limit concept and two participants also used in their last lesson to be comprehended the infinity limit. The symbolic representations were provided by all the participants to express the symbolic representation of the limit concept, to present the properties of the limit and to solve examples. The multiple representations among the graphical tabular and symbolic were seldom used by the participants and the most transitions were realized from the graphical to the symbolic and vice versa. It was showed that the transitions from the graphical to tabular and vice versa were rarely used.

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# EXPLORING STUDENTS' LEARNING OF FRACTIONS

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Mack (1993) showed that students could run in to difficulties when using their previous understanding of fractions, when the whole is one object, to situations when the whole is a group of objects. Mack identified that “students frequently refer to a fraction such as  $\frac{3}{4}$  as ‘three pieces of a pizza or cake that is cut into four pieces’ or ‘you have three pieces of pieces of pizza and there’s four in all’ [...]. As a consequence, students’ informal strategies for rational number problems involve breaking units into parts and treating each part as though it represents an independent unit or whole number” (Mack, 1993, p. 88). Pitkethly and Hunting (1996) argues that knowledge based solely on the continuous model can lead to literal interpretations of fractions ( $\frac{1}{4}$  represents 1 of 4) that may be inadequate where discrete quantities are involved.

The present study reports on students’ learning of non-unit fractions when the whole is as a group of objects, e.g.  $\frac{2}{3}$  of 6. A pre-test and video recorded lessons showed that students in two classes (A and B) seemed to experience the numerator or the denominator as representing a number of objects. Some students used the numerator to represent the amount of objects in each part or to represent number of parts. It was also common that students used the denominator to represent the number of objects in each part. In the study both classes had two lessons about fractions, one when the whole was one object, and a second when the whole was a group of objects. Analysis of the lessons showed differences in student learning between the classes on items concerning non-unit fractions. The differences in learning outcomes between the classes are explained by how the teachers made it possible to experience the role of numerator and denominator during the lessons (Marton & Tsui, 2004). In class A many students still see the numerator or denominator as number of objects, and no improvement in students’ learning is shown. In class B, on the other hand, the students changed their ways of experiencing the denominator from seeing it as objects to seeing it as ‘piles’ or parts and students’ learning improved significant.

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# SELF-REGULATED LEARNING STRATEGIES OF PROSPECTIVE MATHEMATICS TEACHERS IN RELATION TO THEIR SELF-REFLECTIVE THOUGHTS

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Self-regulated learning (SRL) refers to self-generated thoughts, feelings, and actions intended to attain specific goals (Zimmerman, 2005). The process of learning to teach inherently involves the use of strategies for self-regulated learning, to some extent. Since the learning of teaching profession is an ongoing process after the formal university education, the theory of SRL provides an interesting and valuable lens to uncover and interpret how this learning takes place. In shaping pedagogy of mathematics teacher education, it is important to understand learning about teaching from the perspective of prospective teachers. In this sense, the goal of the present study is to explore prospective mathematics teachers' self-regulated learning (SRL) strategies based on their self-reflective thoughts within context of their teaching practices.

A total of 22 observations of teaching sessions and interviews about them were conducted with four prospective mathematics through the study. Their reflections about teaching performances after each of their teaching sessions and the observation data were analyzed by a framework adapted from Zimmerman's (2005) and Pintrich's (2005) SRL models.

The findings of the study revealed the most common self-reflections participants reported in their interviews were related to whether they were able to complete the prepared course material in the given course hour(s) or conducting the course as planned. They also mentioned about the use of the instructional time and classroom management issues by referring to the effectiveness of the lesson. Another evaluation regarding the effectiveness of the course was related to the students' learning at the end of the lesson which was perceived through the students' active participation to the lesson and responses toward questions being asked. Finally, they talked about whether they accomplished the goals identified in the pre-interviews or not.

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# ELEMENTARY MATHEMATICS TEACHERS' KNOWLEDGE OF STUDENT DIFFICULTIES IN RELATION TO GENERALISING NUMBER PATTERNS

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Pattern and generalisation are the basis of various mathematical concepts. However many student difficulties are reported in the literature concerning generalising number patterns (Stacey, 1989). This study focus on 'knowledge of students' difficulties with particular topics' component of pedagogical content knowledge (PCK) proposed by Shulman (1986). The aim of the study is to determine elementary mathematics teachers' knowledge of student difficulties in relation to the generalisation of number patterns. Baxter and Lederman (1999) described PCK as an internal construct which is held unconsciously; therefore they claim that PCK cannot be observed directly. They suggest multi-method evaluation of PCK, such as observation of teaching episodes, and interviews during which teachers reflect on their preparation and teaching. In the light of this approach 30 elementary mathematics teachers are interviewed and three of these teachers' lessons are observed after interviews during their instruction of generalising number patterns. Data source is interview records and videos of pattern lessons. Algebraic pattern generalisation framework (Radford, 2008) is used as an analytical tool for analysing the data. As a result of study, it is found that teachers' PCK fell short in knowledge of students' difficulties component. It is observed that they have similar difficulties with students in relation to the generalisation of number patterns. They tend to make arithmetic generalisation rather than algebraic generalisation. Therefore it can be said that they did not take students' possible misconceptions into consideration both in their interviews and lessons.

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# SUBJECTIFICATION WITHIN MATHEMATICS LEARNING ACTIVITY: EXAMPLES FROM WORKPLACE TRAINING

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This study draws attention to ways that individuals are changed as they participate within mathematics learning activity. Radford's (2008) socio-cultural theory of knowledge objectification (TO) serves as the basis for analysis. Central to the TO are the mutually constitutive processes of *objectification*—becoming *conscious* of and *critically conversant* with the cultural-historical logic with which mathematical and other objects have been endowed, and *subjectification*—the process of *becoming*, of *agency*, and the growth of *subjectivity*. The primary focus here is subjectification, a topic that, to date, has received little attention in the mathematics education literature.

Overt changes in the involvement of a pre-apprentice (C) and tutor (L) within a 33-minute impromptu one-on-one session are examined as the pre-apprentice learns to read fractions-of-an-inch on a measuring tape. This session takes place in a pipe-trades training program where the researcher serves as tutor for research purposes.

During the tutoring session L draw's C's attention to various aspects the fraction-of-an-inch marking pattern on the measuring tape in a systematic way and by the end C is able to read fractions to sixteenths-of-an-inch as needed for his practical work. Analysis of both the video recording and the annotated transcript of this encounter reveal changes in the patterns of C's gaze, sitting position, role in the discourse, and verbal appraisals of both his learning and sense of empowerment in relation to the measuring tape all corresponding to his objectification of it. The frequency of significant actions increases over the 33 minutes with a pronounced combined flourish in the final four minutes. At the same time, albeit in a much less dramatic manner, L's approach to reading the measuring tape also changes, from a general approach typical of school mathematics, to a more pragmatic and workplace-context specific one. These changes reflect C and L's evolving subjectivities and relationships with the measuring tape and measurement practice during the session.

These findings contribute to research by showing ways in which learners' subjectification can be reflected empirically within mathematics learning activity and grounding aspects of Radford's theory of knowledge objectification within a context of workplace training.

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# NEW CHALLENGES IN DEVELOPING DYNAMIC MATHEMATICS SOFTWARE

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During the past decades a number of different types of mathematical software have been developed. Among the most often utilised software types in education are Computer Algebra Systems (e.g. Derive, Mathematica, Maple, Maxima), Dynamic Geometry Systems (e.g. Cabri Geometry, Geometer's Sketchpad, Cinderella, GeoNEXT), Spreadsheet and Statistics Software (e.g. Excel, SPSS, Fathom, R). Most of this software has been designed by keeping in sight primarily their usability for research purposes while others were predominantly aimed for their use in teaching. In the recent years we could observe, among others, three important trends in the development of these software tools. 1) Designers of research oriented software products started to involve features and support for educational purposes; at the same time teaching oriented software have been becoming increasingly more powerful so their use in some research is increasing. 2) The distinction between different types of software has begun to blur as many products integrate features from other types of software. 3) The computer platforms are diversifying; with the appearance of smart phones, tablets, and Interactive Whiteboards (IWB) in recent years, as well as online services such as Wolfram Alpha, challenging the design and development of mathematics software. In this talk, we will outline the challenges of developing the open-source mathematical software GeoGebra and mostly concentrate on the latter two issues highlighted above.

When developing software aimed primarily for teaching, we found that we needed a simple and usable interface. While the algorithmic power could be supplied with the integration of other open-source products the ease of the use of interface should be developed in a way that reflects the need of prospective users. We will outline how we used research projects (e.g. Hohenwarter et al., 2010) and feedback from the community (e.g. Lavicza et al., 2010) to develop the CAS, Statistics, 3D and IWB extensions of the current GeoGebra software. We will highlight how the open-source community and the GeoGebra Institute network contributed to the success that GeoGebra has millions of users, covering almost every country in the world.

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## BIG IDEAS IN SMK & PCK: A RATIONALE

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In the work of the researchers of the EU-funded project ABCMaths<sup>1</sup> we have built a theoretical framework of Big Ideas in mathematical knowledge (SMK) and in pedagogic content knowledge (PCK). This framework acts as a mediating artefact structuring our developing understanding and enabling new ways of framing the objects of our activity. In this short oral we will present the rationale for the framework. Elsewhere we report on research being carried out in each of the three national settings framed by the framework (PME Research Reports submitted).

Students' learning opportunities can be conceptually rich when they address overarching mathematical concepts. On the level of SMK, prior research approaches in mathematics education have identified overarching concepts that have been labelled e.g. as fundamental ideas, universal ideas, or basic ideas. However, the existing approaches are multi-faceted and partly divergent on the one hand, and they focus on the content level of mathematics and hence only cover a part of instruction-related professional knowledge of teachers on the other hand. Consequently, there is a need for a more comprehensive way of looking at overarching concepts in both SMK and PCK. In particular, for the learning and professional development of teachers, a framework of relevant overarching concepts can provide orientation and help to develop teacher education programmes that point to relationships and connection points between overarching concepts in both. On the base of existing approaches and using a pragmatic stance, we included such overarching concepts in both SMK and PCK in the notion of Big Ideas.

We will present the table of those concepts, extend the rationale set out above, exemplify research approaches, and invite comment from colleagues in the mathematics education research community.

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# THE MATHEMATICS LEARNING JOURNEY OF A TWICE-EXCEPTIONAL STUDENT

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Asperger's Syndrome (AS) is a developmental disorder in which children have difficulty with social relationships and poor communication skills. The number of gifted learners with AS is increasing, but there has been little investigation on children with AS for their giftedness. In addition, there is little literature on the gifted students with AS concerning their mathematics learning. The research is to investigate the mathematics learning process of a mathematically gifted student with Asperger's Syndrome (AS) participated in a mathematically gifted curriculum.

From theories proposed by Neihart (2000), Attwood (1998), Grandin & Scariano (1996), there are some characteristics among gifted children with AS: excellence, or advancement, in oral expression ability; extraordinary memorization; exceptionally sensitive to numbers or texts; unusually fanatical interest in a specific topic; endless or extensive responses to questions that disturb peers; verbose or irrelevant talks; superb memorization on factual data; use diagrams, visualization and pictograms frequently.

The research subject is John who is a fourth grade mathematics-gifted student with AS. The researchers use case-study method for present research. The data is mainly collected through classroom observation, supplemented with some informal interviews and document materials. The mathematically gifted curriculum lasts for two and half years and the research period is during his fourth to sixth grade.

The results showed that John not only demonstrates an exceptional ability in geometry as stated in the literature review, he also displays superior ability in algebra, even better than other mathematics-gifted students. Regarding to express out his thoughts verbally, he is better at making himself understandable in presenting the solutions of the algebraic questions than questions that require more oral explanation. There, it seems promising that, by creating a friendly, accepting and challenging mathematics learning environment, mathematically gifted students with AS can gradually overcome their social and learning difficulty.

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# THE PROFILES OF ARITHMETIC ABILITIES BETWEEN MATH DISABILITIES WITH AND WITHOUT REDAING DISABILITIES IN SECONDARY SCHOOL

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## SUMMARY

Students with math disabilities (MD) was reported children with MD and without any remediation in primary school have difficulty with accuracy and fluency in solving single-digit arithmetic (Geary, Hamson, & Hoard, 2000; Jordan, Hanich, & Kaplan, 2003). This study aims to replicate the findings via the investigation on secondary students with MD whether or not combined with reading disabilities (RD). Four MD participants have been remediated in special educational programs from primary school until now. Two of them are MD students co-occurring with RD (MDRD), the other two are those with MD not co-occurring with RD (MD-only). They are measured by the various types of arithmetic, from simple one-digit to complicated multi-digit arithmetic, and word problems using addition or subtraction. The result differs from some studies that children with MD-only are better in accuracy but not response time while compared to children with MDRD in primary school (Geary, Hamson, & Hoard, 2000; Jordan & Montani, 1997). MDRD students perform more slowly than MD-only. The difference in processing speed between MDRD and MD-only groups is greater when they are performing multi-digit arithmetic than simple one. Furthermore, MDRD students made more errors in multi-digit arithmetic. In addition, both groups used finger and/or oral counting strategies to calculate the answers while they failed to perform math fact retrieve in simple arithmetic. They all showed low speed problems in various types of addition, subtraction and the word-problem. But MD-only students outperform MDRD students in solving speed. The result reveals the core skills of MD and which is able to be remediated in early grades, however, the arithmetic strategy of MD might affect the speed of all MD.

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# CONCEPTULIZING EDUCATIVE POWER OF RESERACHER-EDUCATOR

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For enhancing the understanding of educative power, we tried to conceptualize a framework of educative power by analogizing mathematical power developed by NAGB (2002) with the structure of a ABC-3x3 model (Lin, 2010). Herein, educative power specially means professional of researcher-educators (REs) who have experiences in reflecting on research, practice and the bidirectional transitions between research and practice. Thus, the construct of educative power indicates that REs are required to theorize in research, the production of knowledge, and to engender teachers' learning in practice, the application of knowledge. In light of this perspective, this framework, sketched in Figure, includes three critical components: communication, reasoning and connection, which support REs' research and practice within the three tiers of students, teachers and educators.

What contents and forms of the three critical components are effective to initiate and develop teachers' awareness and thinking of pedagogical challenge for REs? Under the interrelated educational settings constructed by REs, contents of communication, reasoning, and connection are identified as realistic pedagogical cases based on research, dynamic pedagogical challenge or threat in practice, and pedagogical associations between practice and research. As to forms, teachers' familiar wording is helpful to communicate research-oriented pedagogical cases. Theoretical models, e.g. the process-object duality, intuition rules, and proof schemes, are efficient to reason dynamic pedagogical phenomena on the spot. Connecting pedagogical associations is to coordinate practice in education and research on students/teachers/educators into a capsule package for easier adaptation and generalization. In sum, comprehensible communication, efficient reasoning, and coordinated connection are three critical components of educative power. Empirical data will be shown in the oral presentation.



Fig. \_ Educative Power of RE

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# FACTORS CONTRIBUTING TO FUTURE TEACHERS' MPCK: EVIDENCE FROM TEDS-M2008

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*The study was to examine the effect of the contents taught in teacher preparation on MPCK. 923 future primary teachers of Taiwan participated in the TEDS-M2008 study. The results indicate that some topics of university-level mathematics, mathematics education, and education were contributing factors to future teachers' MPCK, but all topics of school-level mathematics were significant factors.*

The superior achievement of East Asian in international comparative studies such as TIMSS and PISA has attracted attention to explore the related factors (Leung & Li, 2010). Two most important factors are the kind of mathematics student learn and the quality of the teachers who teach the mathematics. However, the teacher education in the East Asian countries is still less known to the international community, until IEA launches the study of Teacher Education and Development Study in Mathematics (TEDS-M2008) (Tatto, et al., 2009).

One debating issue was that the discontinuity between the mathematics learned in university and that which will be taught in school. However, no attempt is made to measure the relationship of the contents taught in the TP programs with FTs' knowledge. The current study examines the effect of the contents taught in the TP programs on the FTs' mathematics pedagogical content knowledge (MPCK).

The 923 FTs attended the practicum in TP institutes identified by the TEDS-M2008 consortium. The items of university and school level mathematics, mathematics education, and general education were included in the OTL questionnaire. MPCK includes two domains: knowledge of mathematical curricula and planning for mathematics teaching and knowledge of enacting mathematics for teaching and learning. (Tatto, et al., 2009).

The result shows that the superior performance of MPCK was resulted from some of the contents taught in the TP programs. The contents included the topics at university level including geometry, discrete structure & logic, continuity & function, the topics at school level including number, geometry, measurement, probability, and statistics, the topics of instruction in mathematics education, and the topics of application in education. However, the probability at university level, the foundation in mathematics education, and the social science in education were not significant factors.

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# DEVELOPING AN INSTRUMENT TO CAPTURE HIGH SCHOOL MATHEMATICS TEACHERS' SPECIALIZED CONTENT KNOWLEDGE: AN EXPLORATORY STUDY

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The study aims to share efforts in designing and testing a set of items to assess high school mathematics teachers' specialized content knowledge (SCK). We framed the items in terms of three elements of SCK (Hill, Ball, & Schilling, 2008): (1) *explanation* stands for "how to provide mathematical explanations for common rule and procedures"; (2) *representation* indicates "how to choose, make and use mathematical representations effectively and accurately"; (3) *justification* describes "how to explain and justify one's mathematical ideas". Item development was carried out in three phases. **Initial Item Construction** was to collect and modify few usable SCK items by analysing the classroom practices of three experienced high school mathematics teachers. In **Item Refinement**, two experts (a tertiary mathematics teacher educator and a retired high school mathematics teacher) reviewed the initially constructed items and provided suggestions for revising the items, eliminating ambiguous items, or adding useful items. **Pilot Test** consisted of the following three steps: (1) distributing the items to a small sample of high school mathematics teachers (11 teachers); (2) determining whether SCK was actually measured by the items and to learn more about the knowledge used to answer the items, in-depth and face-to-face interviews were then followed; (3) further discussing and refining the items within the research group, which includes a retired teacher, a mathematics teacher educator, a postdoctoral fellow, two Ph.D. students, and two graduate students. The following table shows some of the sample items corresponding to each SCK element.

SCK Elements	Example Items
<i>Explanation</i>	Problem: "Find a point on a plane, the sum of whose distances to two given points A and B on the same side of the plane is the minimum" How might a teacher explain the solving procedure of "finding the reflection of A over the plane" to students?
<i>Justification</i>	A student asserted that he would only need two points (A and B) to establish a plane: "the perpendicular bisecting plane of the line segment joining points A and B" How to explain and justify his mathematical idea?
<i>Representation</i>	A teacher drew an incorrect diagram to represent orthographic projection and did not solve the problem effectively. Could the respondent identify it and provide another effective solving strategy based on the correct one?

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# EMERGENT UNDERSTANDINGS OF EQUIVALENCE

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Students' difficulties with the equals sign in statements of equivalence are well documented and many researchers have treated students' difficulties as a dichotomy between a correct structural understanding of the equals sign, as representing equivalence, and an incorrect procedural understanding, as an instruction to carry out a computation. Recently, researchers have suggested that structural awareness will never be complete but will always be open to further generalisation (Mason, Stephens, & Watson, 2009; Mowat & Davis, 2010). Mowat and Davis (2010) consider mathematical thinking from a network theory perspective that supports a multidimensional conception of equivalence. They suggest that networks of increasingly abstract concepts, skills and knowledge form from experiences in a variety of contexts. From this perspective, generalised understandings of equivalence will become increasingly interconnected with other mathematical ideas.

In this study, students' responses to additive missing number sentence problems were interpreted using considerations of structural awareness. A representative sample of 421 New Zealand Year 4 students was given missing number sentences to complete. Only 13% of the students were able to demonstrate a strong understanding of equivalence by answering correctly at least four of the six problems. A typical problem was  $\square + 1 = 3 + 2$ , where a correct response of 4 was given by 24% of students. However, an incorrect response of 2 was given by 42% of students, suggesting that they computed  $2 + 1 = 3$  and may have been combining two numbers, attending to the operators and equals sign but not attending to all numbers in the sentence. An incorrect response of 6 was given by 15 % of students, suggesting that they computed  $6 = 1 + 3 + 2$  and may have been combining all numbers, attending to the function of operators and equals sign but not attending to the meaning of the location of the operators and equals sign. It was not possible to characterise all the erroneous responses as students treating the equal sign as a signal to carry out a computation rather than treating the equals sign as signifying equivalence. We also suggest that many students who gave no numerical response to problems may have done so, not because they could not compute an answer, but because there was conflict between particular understandings of equivalence. Most students' responses indicated fuzzy thinking and limited generalisation of concepts of equivalence.

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# MATHEMATICS AS AN ART—A LIBERAL ART APPROACH

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Mathematics and art are traditionally treated as two extremes in which the former is logical following rational paths and the latter is emotional requiring resourceful imaginations. Late distinguished mathematician Paul Halmos viewed mathematics as a creative art because mathematicians create beautiful concepts and mathematicians think like artists (Halmos, 1993). In Halmos' eyes, mathematics is much more like a discipline with an art form. On the other hand, Bruter (2002) studied various mathematical visualizations in art, emphasizing the role of mathematics in the construction of artistic works. These views, however, might be insufficient if we take a historical retrospect of the two disciplines. The present report holds a view that mathematics and art though originated independently, they not only possess similar characteristics but also play a part in each other's development.

Based upon aforementioned view, a liberal art course titled "mathematics and art" were administered to 60 Taiwanese college students and a qualitative survey was conducted to investigate in what way and to what extent the students perceived the connection between the two. For revealing the relationship between the two disciplines, "mathematics in artistic thinking" (MAT) and "thinking art of mathematics" (TAM) are two major components of this course and each component contains three branches. MAT presented the role of mathematics in painting, music, and architecture/sculpture; TAM demonstrated how inductive approaches, abstract deductions, and creative thinking are skilfully employed in mathematical problem solving. A qualitative belief questionnaire, follow-up interviews, students' in-class reports, and a web-based forum were sources used to investigate the development of students' views regarding the connection between mathematics and art. Data indicated that students' instrumentalist views of mathematics showed minor change, yet they did exhibit a certain degree of sophisticated understanding about the comparable nature of mathematics and art. For instance, some students claimed that abstraction is essential for mathematics and art since it reflects individual styles as well as enables mathematicians and artists to simplify complicated objects. Furthermore, some professed that albeit the beauty of mathematics might not be realized via senses of sight and hearing, it could be understood by our minds, which is an alternative form of art. Nevertheless, it was also found that this liberal art approach may not be able to help students overcome their fear of mathematics as manifested by a female student's feedback that, "*The assertion that everyone is scared of math is much closer to the truth than any mathematical identity*".

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# DESIGNING THE TEACHING PROGRAM ON “ESTIMATES” FOR IMPROVING 4<sup>TH</sup> GRADERS’ COMPUTATION ABILITY

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NCTM (2000) indicated that instructional programs from preK-12 should enable all students to compute fluently and make reasonable estimates, but according to the official report, there were only about 57% of sixth-graders in Taipei assessed to be able to compute fluently. Thus, designing a teaching program on the topic of “estimates” for helping students improve their computation ability seems important and worthwhile.

The study adopted ADDIE (Analysis, Design, Development, Implementation, Evaluation) Model (Gagné, Wager, Golas & Keller, 2005) and the method of design-based research [DBR] (Reeves, 2006). The structure of DBR consists of the analysis of the practical problems encountered by researchers and practitioners in collaboration, the development of solutions by referring to literature and technological innovations, the iterative cycles of testing and modifying solutions in practice and reflecting to produce design principles and enhance solution implementation. The conceptual framework of the program was based on students’ experience of the real world, and the program provided open-end tasks to activate students’ computation ability. The researchers designed the first draft of the program, which was then modified by the researchers and members of an in-service teacher’s group. Afterwards, the modified program was implemented in a 4th grade classroom, and re-modified according to the realistic teaching situations and the teacher’s suggestion who taught the program. Then, the re-modified program was implemented by another teacher and modified for the third time. The outcomes and effectiveness of the program were investigated by means of video records of teaching and interviews with the two teachers who helped teach the program.

The research findings showed that there were four types of difficulties encountered while designing this teaching program: the designer’s conventional impression of textbooks could influence the instructional design, the choosing of the situations of the questions, the lack of the ability of multimedia design and the understanding of the learner’s language. As to the effect on the teaching of the program, learners could comprehend the concept of number, shorten the counting process with skilful use of approximate number, choose some approximate numbers to deal with problems, communicate their findings and make reasonable explanations with others, and understand the tight connections between daily life and mathematics.

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# PROSPECTIVE K-8 TEACHERS' RELATIONAL THINKING IN THE CONTEXT OF ARITHMETIC AND ALGEBRA-BASED TASKS

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Past research has revealed strong links between students' learning of algebra and their ability to think relationally about equality, that is, to analyze expressions and equations *as a whole* using the properties of numbers, operations and equality rather than as a *sequence* of steps or procedures (Carpenter, Levi, Franke, & Zeringue, 2005; Warren 2003). Little attention, however, has been paid to teachers' relational thinking. This study examined K-8 pre-service teachers' (N=32) inclination to think relationally about equality in the context of arithmetic and algebra-based tasks prior to the instruction on the role of relational thinking in algebra learning.

Ten tasks, each requesting two solution approaches, were used to study pre-service teachers' ability and inclination to think relationally. The analysis of 283 responses revealed that the pre-service teachers were able to solve 68% of the problems using a relational strategy. However, they chose to use a relational thinking strategy as their first (spontaneous) strategy on only 39% of the problems. The pre-service teachers used spontaneous relational thinking significantly more frequently to solve arithmetic tasks (67%) than algebra-based tasks (17%);  $z=8.41, p<0.001$ . We found similar results for selected subgroups of tasks. For example, on tasks that elicited the use of the additive property of equality, pre-service teachers spontaneously engaged in relational thinking significantly more often on arithmetic tasks (45%) than algebra-based tasks (22%);  $z=1.69, p<0.05$ . For tasks that elicited the consideration of differences in the magnitude of numbers, the pre-service teachers engaged in spontaneous relational thinking significantly more often on arithmetic tasks (74%) than algebra-based tasks (16%);  $z=8.56, p<0.001$ . The results suggest that pre-service teachers who have the ability to think relationally may not consider relational thinking to be a desirable solution strategy. Consequently, programs concerned with preparing teachers for the challenges of early algebra instruction need to increase pre-service teachers' awareness of the role relational thinking has in algebra learning. Without such awareness, their inclination to use procedural strategies might adversely affect their ability to foster relational thinking in their students.

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# FOSTERING THE ADAPTIVE STRATEGY USE OF GERMAN 3<sup>RD</sup>-GRADERS: THE CASE OF INDIRECT ADDITION

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Indirect addition is an efficient strategy for solving subtraction tasks with a small difference between minuend and subtrahend, e. g. 304-297. However, only a minority of primary school students uses indirect addition for these specific subtraction problems (e.g., Torbeyns et al., 2009). We assume that recognizing task characteristics supports the interpretation of subtraction in these specific problems by a difference model instead of a take away model and, therefore, promotes the consideration of the indirect addition strategy instead of the straightforward application of a direct subtraction strategy. Accordingly, the question arises whether students' adaptive use of the indirect addition strategy can be fostered by an intervention study teaching the analysis of task characteristics as a criterion for strategy choice. The research questions are:

- Is there a change in students' criteria for strategy choice towards a stronger emphasis of tasks characteristics after the intervention?
- Is there a rise in students' adaptive use of the indirect addition strategy?

The sample comprises 54 third graders with good mathematics achievement and taught a variety of strategies. The intervention took place once a week in groups of 7-10 students. In five sessions, different strategies (including indirect addition) and the analysis of task characteristics were addressed. Data for the pre-post-test comparison was collected by identical tests with 10 items on adaptive strategy use and short interviews asking for the criteria of strategy choice for specific subtraction items.

From pre- to post-test there was a substantial increase in the reference to task specific criteria for strategy choice (from 11.0% of the solutions to 35.0%). This indicates that the students more frequently took into account task characteristics when solving subtraction items. As expected, in the pre-test, students showed a low adaptivity in their strategy choice for test items suggesting indirect addition as efficient strategy (overall, only 1.8% of the solutions were efficient). In the post-test, adaptive strategy use for indirect addition items increased (17.4% of the items). Additional, in the post-test data a significant correlation could be observed between the frequency of reference to task specific criteria and adaptive strategy choice ( $r = .51, p < .001$ ).

Based on these results, we hypothesize that the analysis of task characteristics is an important element in teaching students an adaptive use of the indirect addition strategy. However, the post-test results for adaptivity are still on a moderate level.

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# INVESTIGATING RELATIONSHIP BETWEEN READINESS AND SELF-EFFICACY OF PRESERVICE TEACHERS TOWARDS PROFESSION

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Teacher education programs have a great value in preparing quality preservice teachers for the 21<sup>st</sup> century. It has recently observed that one of the most important problems related to the education system in Turkey is preparing preservice teachers and the quality of the prepared teachers. The quality of preservice teachers depends on to what extent the preservice teachers have self-efficacy beliefs for the teaching profession (Çapri & Çelikkaleli, 2008). In addition, to what extent the preservice mathematics teachers feel that they are ready for the teaching profession when they graduate is not much investigated (Mehmetlioğlu, 2010). Hence, it is important to examine both readiness and self efficacy of preservice mathematics teachers to determine the existence of the relationship between them.

The data were collected from 145 third and 4<sup>th</sup> year preservice mathematics teachers. Readiness to Teaching Scale developed by the researcher and Mathematics Teaching Efficacy Belief Instrument (MTEBI; Enochs et al., 2000) were administrated. Cronbach's Alpha value for readiness in teaching scale was found .91. The MTEBI comprised of two subscales, personal mathematics teaching efficacy (PMTE) and mathematics teaching outcome expectancy (MTOE). Cronbach's alpha for the PMTE calculated as .85 and .80 for the MTOE.

The results indicated that there was a statistically significant relationship between readiness and self-efficacy subscales. There was a positive medium relationship between readiness and PMTE ( $r = .47$ ,  $n=145$ ,  $p<.005$ ). The relationship between readiness and MTOE was a positive small relationship between the two variables ( $r = .29$ ,  $n=145$ ,  $p<.005$ ). Outcomes of the study provide valuable information in the process of preparing teachers. In order to improve the efficacy and readiness of preservice teachers, program developers pay attention this relationship and provide courses related to these concepts.

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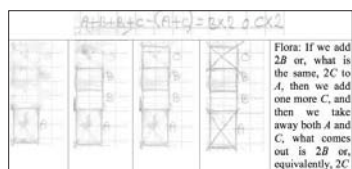
# EXPLOITING CHILDREN'S SPONTANEOUS ALGEBRA SKILLS

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The evidence that pupils own some rough forms of algebraic thinking and exhibit spontaneous algebraic skills seems to be increasingly spreading (see for example Radford, 2010). In this work we propose a possible way to use this spontaneous competence in order to favour its evolution, in a Vygotskian perspective.

In our view, algebra encompasses a wide portion of mathematical discourse, well beyond, and perhaps before, its role of generalizing arithmetic facts. According with (Mason *et al.*, 2009), we'd better think of the algebraic language as related to a natural attitude to observe and detect relations among varying quantities. Therefore, forms of algebraic thinking begin when one starts scrutinizing numerical relations in



searching for generalization or when one is attempting to represent and to solve word problems, but also when one is involved in the observation and study of real-world or physical phenomena, where different magnitudes are linked together. Here we can identify one of the roots of (a mature

meaning/role of) algebra as a sub-category of mathematical discourse deputed to formally treat relations among variables within various scientific contexts.

To develop this attitude means to recognize “structures” even independently from numbers. In this sense, the intermediate systems of representation (Davydov, 1982) appear to be useful tools to see structures, and steps toward further formalizations. In fact, spatial visualizations, due to their perceptive and holistic features, can precede and favour the development of spoken and written language. We will present some excerpts of a long-term work (the above figure is a sample), where the traditional approach to algebra is reversed and algebra is not seen only as a “formal” language into which to translate the arithmetical or the natural language, but it is a semantically meaningful structure rooted in the observation of relations among quantities. Finally, we will stress the importance of adopting this approach as early as with first graders.

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# EVOLUTION OF PUPILS' MATHEMATICAL PROCEDURES TO SOLVE MULTIPLICATION TASKS

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Research with a focus on multiplication is quite extensive. We can identify an emphasis on prominent lines: semantic types of situations, intuitive models and computational strategies. The last one includes empirical studies that relate computation strategies with the type of problems proposed and other that characterize the strategies invented by students (Baek, 2006). Several authors argue that pupils' understanding of multiplication evolves when they face situations that make emerge key aspects, such as, the ideas, the strategies and the models (Treffers & Buys, 2008).

This paper draws on the analysis of procedures used by third grade pupils who participated in a teaching experiment on multiplication, during eight months. The class is the direct source of the data and the main collecting tool is the researcher (the report's first author) through a detailed observation of the events in the classroom. Data includes field notes, transcript of videotaped classroom episodes and pupils' written work.

The analysis of classroom episodes and pupils' written work allows us to characterize the various procedures used by them to solve multiplication problems and its evolution. Three main aspects are revealed. The first one is related with the diversity of procedures which are used by the pupils to solve a problem. The second is related with a pattern of evolution in the use of procedures which are increasingly formal and effective. This progress is not standard and changes with the introduction of new numbers and less familiar relations. A third aspect emphasises the relationship between the proposed contexts and the numbers included, and the pupils' procedures, suggesting the adequacy of the given contexts in the learning of multiplication.

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# A META-ANALYSIS OF MATHEMATICS TEACHERS IN THE INDUSTRIAL INTERNSHIP GIFT PROGRAM

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Research suggests that the quality of the teaching workforce is the single most important factor in predicting student achievement. Effective teachers must have a solid knowledge of academic content, a high mastery of different pedagogical techniques, an understanding of student developmental issues and different ways of learning, and a strong sense of professionalism. Industrial workplace environments are in the unique position of being able to help teachers develop their strengths in most of these categories through summer internships. When teamed with facilitators, industry mentors can provide motivated teachers summer experiences that show the uses of math skills in industry, that increase the teacher's content knowledge, and that provide new teaching strategies; in particular, by bringing suitably adopted industrial applications to the classroom. The Georgia Intern-Fellowships for Teachers (GIFT) program provides teachers an opportunity to connect classroom activities with real world applications and vice versa. The purpose of this evaluation research was to ascertain, using Success Case Methodology (Brinkerhoof & Dressler, 2002), whether the goals of the program that we see informally being achieved are, in fact, supported by data. We identified 23 mathematics teachers who participated in the GIFT program in order to construct individual cases. This approach helped us to uncover patterns and develop themes across cases (Yin, 1994). The case studies document the prior experiences, knowledge, and beliefs these mathematics teachers brought to GIFT, as well as how those factors interacted with their learning from GIFT and from their own teaching experiences. The meta-analysis of the success cases showed that while different in their scope, teachers gained an interest and enthusiasm for their subjects and skills, as well as resources to share with their students. The goals of the GIFT program were achieved by providing teachers with summer intern experiences that increased their content knowledge, challenged them to explore new teaching strategies, showed them the practical uses of science and mathematical understanding and skills, and thereby allowed teachers to give their students an opportunity to see industrial applications of mathematics.

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# THE BARTER MARKET: THE MEASUREMENT CONCEPT IN KINDERGARTENS

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The purpose of the research described below was to develop a learning process relative to measurement for five-year-old children. Kindergarten in Italy encourages the development of independence, skills and good citizenship in children. All this is reflected in daily experiences when a child recognizes and communicates an understanding of fundamental activities and manages transactions with others. Moreover, the child learns to appreciate other points of view and to recognize rights and duties. The cognitive motivation was to teach the mathematical concepts related to measurement. The experience of preparing food and beverages for this project taught them the concepts of weight, volume and length (preparing pasta, juices, blended drinks, pastry cream and chocolate rolls). Finally, in assigning a value to these products in order to exchange them, the children learned about numbers in relation to pricing. The National Curriculum Guidelines aim at indicating the development of specific skills including, among others, communicating your own reasoning and thoughts to others through verbal language. Therefore, the problem of mathematical communication “depends at least as much on what we see as well as on other types of abstract speech”. The question then concerns the “effectiveness of communication” and its semiotic, artefact and visual mediators [Sfard, 2009]. We can see, therefore, that “measurement can constitute an area of near development in which experiences, although not completely understood by a child, can successively be integrated into a network of conceptualization” [Bartolini Bussi, 2008]. The discovery of the importance and use of measurement in a child’s daily life and the invention and implementation of the “barter market” encourage a way of performing an action. By observing nature, things, materials, social and cultural environment, children are able to plan and invent, make projects and shapes taken from real life or create new ones [Clements, 2004].

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# DUALITY AMONG TEACHING AND LEARNING: KEY TEACHING AND LEARNING MOMENTS

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Learning and teaching are complementary processes in mathematics classes, so analyzing the mutual influences among students and teacher's behaviours is necessary to understand those processes. The construct of Hypothetical Learning Trajectory (HLT) (Simon, 1995) has proved to be useful in describing teachers' planning of lessons and expected mathematics students' learning processes. In the same way, we conceive the dual construct of *Hypothetical Teaching Trajectory* (HTT) as a description of expected teachers' teaching processes.

This research<sup>1</sup> aims to analyze characteristics of and relationships among HLT and HTT by identifying *key learning moments* and *key teaching moments*. A key learning moment appears when students move from a state where they do not understand a certain mathematical content to a state where they do understand it. A key teaching moment appears when external agents –the teacher, other students, or a manipulative or technological tool– try actively to induce a key learning moment. In a classroom, such moments usually happen while students are being influenced by external agents.

The definitions of a HLT and its dual HTT can be seen as the design of a teaching unit where key learning moments are expected and corresponding key teaching moments are planned. One of our research objectives is to design and put into practice a teaching unit for a 9th grade class on plane transformations, based on problem solving and including both ordinary and DGS class sessions. We have observed three pairs of students, paying attention to the influence on students' key learning moments of their interactions with teacher, software, and other students. The analysis of fragments of transcripts is a useful methodological tool to identify key learning moments on observed students' outcomes, and precursor key teaching moments on external agents' interactions with those students. Expected results of this ongoing research are identification of real key teaching and learning moments, to compare them with those predicted in the HLT and planned in the HTT, and to characterize the influence of each agent on students' learning.

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# A STRATEGY TO IMPROVE NOVICE STUDENTS' PROOF COMPETENCE

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This paper presents results from an empirical research conducted in 112 Greek grade-9 students. The purpose of this research was to investigate the effects of a coloring strategy on novice students' proof competence, its impact on their meta-cognitive skills, and its effect on students' attitudes. Main focus is put on how to help novice grade-9 geometry proof writers to externalize their thoughts, so as to construct the intermediary steps required to write proofs. Due to the fact that (a) a formal proof consists of and is analyzed into simple justifications, and that (b) novice students often do not know how to begin to write proofs, a reusable matrix pattern named "Reasoning Control Matrix for the Proving Process" (RECOMPP), that helps students to produce reasoning, was used (Dimakos, et al., 2007). Also, a collection of prevailing characteristics of novice geometry students, who fail to write a proof, is presented here. These characteristics have been detected due to the researchers' long instructional experience. Furthermore, the authors introduce the terms "simple justification" and "non-simple justification" in order to construct a model to analyze proofs.

A 6-week course that included lectures was designed. Students were encouraged to create groups of 3 individuals, so as to collaborate with their group-mates, articulate their ideas in public, and compare them with those of their classmates. After having enrolled in the course, students were asked to follow a proving method that required from them to write down their thoughts using two different color pens; a blue one for the given of the geometrical propositions, and a red one for the inferences derived from the given. Following, they were given several examples to practice themselves in proofs. The statistical analysis used descriptive and inferential statistical tests, examining for possible differences between students' pre- and post-test scores in proof writing. Results showed that the proposed coloring strategy significantly improved the ability of novice geometry proof writers to externalize their thoughts, and therefore to construct the necessary intermediary steps of the proving process. Thus, the externalization of their thoughts significantly improved their competence in writing proofs. Moreover, students who used the proposed strategy appeared to have higher metacognitive skills. Finally, their attitudes towards proof were more positive. Major limitations of the research and implications for future research are mentioned.

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# DIFFICULTIES IN TEACHING STATISTICS TO UNDERGRADUATE STUDENS OF PSYCHOLOGY

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The study reported here sought to contribute to a deeper understanding of the difficulties faced by Brazilian Psychology students to learn Mathematics content as required by mandatory curriculum, namely descriptive and inferential statistical analysis. The theoretical notion of didactic contract (Brousseau, 1986) was proposed as a frame of reference that allowed to better understand the fundamental elements of the difficulties related above. These difficulties are frequently associated to the fact that such mathematical content seems outside the typical Psychology program and is offered to students with a previous history of Math phobia, the latter having motivated many students to choose courses in the Humanities (Da Rocha Falcão, 2007). A group of students from a regular Psychology class in a public Brazilian university participated in 4 classes videographed by the researcher with the previous consent of the class teacher and her students. Moreover, this teacher was a member of the staff of the Department of Statistics and, therefore, non-psychologist; she was interviewed by the researcher regarding her representations, expectations and attitudes related to the experience of teaching statistics to these students. The interpretative-clinical analysis performed covered videographic records of the class activities and interviews with the teacher; these data showed strong emphasis on the manipulation of algorithms supported by the use of a calculator. By establishing such emphasis, the instrumental character of statistics in psychological domain is lost to these students. Another important aspect of the analysis to be shown refers to the fact that the discipline is taught too early in the course. In the words of the teacher, *“the students are not yet mature enough, do not feel that they need to study statistical content”*. The data collected in this study calls once more attention to aspects related to the organization of the math classroom as a social-cultural context with participants whose expectations and goals do not necessarily converge. This aspect deserves attention, along with issues of math conceptualization.

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# A FRAMEWORK FOR STUDYING DIFFERENCES BETWEEN PROCESS- AND OBJECT-ORIENTED DISCOURSES

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Learning, or at least one aspect of learning, can be seen as enculturation into a specific type of discourse. The property of mathematical discourse of interest here is an introduced distinction between process- and object-oriented discourses, seen as a generalised idea based on aspects of nominalisations. I describe the first version of a framework to be used when studying process- and object-oriented discourses.

The differences between the types of discourses are here shortly characterised through an imaginary change from process- to object-oriented discourse. In this change descriptions are made to focus on objects and properties of such objects instead of processes, activities or events of some kind, in particular through the following types of changes into more object-oriented discourse:

- *Direct nominalisation*: Using nouns instead of verbs, e.g. “the usage of symbols is the same” instead of “we use the same symbols”.
- *Indirect nominalisation*: Using adjectives instead of verbs, e.g. “the series is convergent” instead of “the series converges”.
- *Structuralisation*: Using structural types of verbs instead of process types of verbs, e.g. “it is the same” instead of “we do the same”.
- *Change of voice*: Using passive verbs instead of active verbs, e.g. “this can be calculated” instead of “we can calculate this”.

In addition, there is a difference between oral and written communication regarding the use of nominalisations; that writing is usually more nominalised than speech (Einarsson, 1978). This result highlights a potential need to include format (oral and written) in the framework for studying process- and object-oriented discourses.

I have done a first pilot study where a small sample of university students were exposed to different types of mathematical presentations regarding variations of discourse and format. An analysis of their evaluations of these different types of presentations showed that there is a general tendency for students to evaluate process-oriented discourse as better than object-oriented discourse. However, while process-oriented discourse was preferred for oral presentations, object-oriented discourse was preferred for written presentations. These results show the relevance of a continued study of process- and object-oriented discourses in mathematics teaching and learning, in particular regarding the interaction between type of discourse and format.

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# EXPLORING MATHEMATICS TEACHERS' USE OF TEXTBOOKS: A MIXED METHODS STUDY

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Mathematics teachers generally select or design the exercises and the mathematical task with respect to available resources and their students (Guedet & Trouche, 2009). Specifically, textbooks are among the mostly used resources by teachers for instructional decisions such as what to teach, how to teach it, what kinds of tasks and exercises to assign to their students. Therefore, textbooks can provide a good model for planning and enacting instruction for mathematics teachers.

This study aims to create, validate, and use an instrument regarding the use of mathematics textbooks by mathematics teachers in Turkey. 6th-8th grade mathematics teachers participated in two phases of research. In the first phase, mathematics teachers' perceptions about the use of textbooks were determined through interviews. This phase concluded with the development of a questionnaire. The second phase consisted of efforts to validate the questionnaire in which 531 mathematics teachers participated. The reason for developing the questionnaire was that there has been no existing instrument to assess the uses of mathematics textbooks by teachers.

The results of the study indicated that the most widely used teaching resources were mathematics textbooks for most of the mathematics teachers. The instrument had satisfactory degrees of validity and internal reliability. It could be used to assess the use of mathematics textbooks by mathematics teachers. The questionnaire involved three dimensions: (1) the use of textbook as a guide in pedagogical decisions (e.g. organizing content, selecting teaching approaches or activities, etc.), (2) direct use of content (e.g. examples, tables, etc.) from textbooks, and (3) the use of problems. Mathematics teachers mostly used the textbooks to select or design multiple choice questions. Data indicated that mathematics teachers gave greater emphasis to solving this type of questions, because of their intentions to prepare students to national high stake exams. Moreover, they derived greater benefit from the sequence of what is taught, as well as how mathematics concepts are taught. On the other hand, they rarely used the definitions and examples in textbooks for instructional decisions.

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# AN ANALYSIS OF TEACHERS' CONCEPTION OF THE PURPOSE OF TEACHING MATHEMATICS

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Teachers' understanding of the goals of mathematics education is important because it affects both their classroom teaching and students' perspective on the purpose of learning mathematics. Building on the previous study of students' conception (Kim & Pang, 2007), this study examined Korean teachers' conception of the purpose of teaching mathematics and compared it with students' awareness.

The survey was conducted with 330 elementary school teachers. The questionnaire consisted of two parts: (a) 10 essay items dealing with teachers' overall understanding of the purpose of teaching mathematics, and (b) 32 items on a scale of seven choices ranging from 'extremely positive' to 'extremely negative' targeted to teachers' perspective of diverse goals of teaching mathematics. Building on the previous study, the goals were categorized as practicality, preparation for the future, understanding of world culture, development of mathematical ideas, improvement of sociability and communicative skills, aesthetic appreciation, tool subject, and academic values.

Teachers showed high positive responses to the items related to the development of mathematical ideas (90.5%), practicality (86.9%), academic values (86.8%), tool subject (86.6%), and preparation for the future (83.8%). The positive percentages decreased in order with regard to aesthetic appreciation (57.6%), understanding of world culture (55.4%), and improvement of sociability and communicative skills (50.6%). In addition, negative responses for these three items were more than 20%. As for how to explain to students why they need to learn mathematics, the common responses from teachers were limited to the usefulness of mathematics in everyday life and to the development of logical thinking. Although teachers agreed more positively than students did in general, the tendency of recognizing the goals of mathematics education was remarkably similar between teachers and students. This result suggests that teachers' conception of the purpose of mathematics education really matters to students' perspective. This study also urges us to help teachers broaden their conception of the purpose of teaching mathematics so that students may be aware of diverse goals of learning mathematics.

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# AN ANALYSIS OF RELEVANT HINTS IN PROBLEM SOLVING

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The last decade witnessed a growing interest for the analysis of teachers' practices in classroom interactions and their impact on students' mathematical learning. A first line of research in this category is related to the construction of mathematical knowledge and understanding through teacher's interventions (e.g. Steinbring, 2005). A second line of research focuses on teacher's interventions to support students work for example, by analyzing hints used by the teacher in problem solving activities (e.g. Brodie, 2001). In the same time, there are just a few, close to none, studies concerning the use of hints in text-or-problem books where the focus has to be on bootstrapping the students' problem solving process and the support of autonomous work. In a context where problem solving occurs in an "outside of class" setting, void of a teacher's presence and collaborative climate, the way these hints are conceived and formulated can have an even more profound impact since there is no possibility for an "on-going" monitoring and on-task adjustment from a teacher's part.

In this study we present an analysis of the hints formulated by 150 in-service teachers with the goal to be included in a textbook. They proposed problems, relevant hints to start problem solving from the student's side, and keywords for the problem classification. We identified 5 categories of hints, differentiated between them by their focus (procedural/knowledge) and their completeness from the perspective of the solving process (partial/complete): giving the answer with no further specification; hints that are meant to avoid a typical error; procedural hints that specify the complete algorithm; hints that are just an intermediary step in the solution algorithm; and hints that refer to the mathematical knowledge (factual/procedural) at the basis of the problem. A first conclusion is that teachers tend to formulate partial and procedural hints which are directive by their overall nature. A second conclusion is that didactic experience doesn't seem to be a factor in the quality of hints. By taking into consideration these results, it seems that teachers do not adapt their strategy of formulating hints to the off-line setting and act as if in classroom.

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# AN ANALYSIS OF PRIMARY TO HIGH SCHOOL STUDENTS' STRATEGIES IN COMBINATORIAL PROBLEM SOLVING

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According to Inhelder and Piaget (1955) combinatorial structures are mathematical logical models that characterize the stage of formal operations. Fischbein (1975) asserts that the ability to solve combinatorial problems can not be achieved without formal education. In the present study we defend that students are able to develop interesting and important strategies for solving combinatorial problems, even before they are formally introduced to Combinatorics at school. Types of responses given by 568 students (from Primary to High School) and their strategies for solving combinatorial problems were analysed, focusing on the dimensions indicated by Vergnaud (1990): meanings, invariants and symbolic representations. The pupils solved eight problems (arrangements, combinations, permutations and Cartesian products), two of each type. The analysis shows that students from different school years are able to understand problems that involve combinatorial reasoning. Although some participants did not finalize their resolutions and failed to find the final correct answer, they developed valid strategies and correctly used valid symbolic representations which demonstrate understanding of the meanings and invariants implicit in the situations. As they advanced in schooling, the students used more formal strategies, however, for small numbers they tended to use less formal strategies, such as drawings and lists of possibilities. Attention should be paid concerning the knowledge already possessed by students – developed since the early years of schooling – and on ways to expand their combinatorial reasoning – a manner of thinking that is useful in the understanding of diverse knowledge – Mathematical and of other areas.

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# DESIGNING SETTINGS FOR DISTANCE LEARNING OF MATHEMATICS

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The design of materials for distance education is challenging mathematics teachers at the universities in our country. As Villareal and Borba (2010) have emphasized, it is important to avoid the mistake of simply transferring to virtual courses the same procedures used in the face-to-face classroom. Although there are many studies focusing on the multiple representations now available due to the contemporary technology, there is hardly any research exploring the textual organization known as hypertext which became possible with the advent of the internet. Such a notion contemplates aspects of the development and of the navigation. Regarding the first, any learning tool may be designed with hyper-textual features; and could be called hypertext. For the second, the environment design is not enough for any reader to behave like a hyper-reader, and to accept the invitation of reading proposed by the designer. In this paper, we report on ideas developed during a previous experiment to prepare an online setting. A workshop using computers was conceived and designed to avoid the exercise paradigm. A version of Graphic Calculus (Blokland, Giessen & Tall, 2006) was introduced to twenty five university students retaking an online first course on Calculus, with a handout, consisting of activities exploring the concepts of functions, limits, and derivatives. The focus was not only on students' knowledge of representations and the use of the mathematical concepts, but also on examining the flow of connections between representations the students make. From the analysis of the written reports on the workshop, one typical student's responses are presented as a case study once revealing characteristics of a hyper-reader in her relationship with the task. Reflecting on the scenario created, we suggest that a combination of hypertext (Levy, 1990) and generic organizer's features (Tall, 1989) may allow the constitution of constructive hypertexts, as alternative settings for online mathematics education.

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# PROSPECTIVE MATHEMATICS TEACHERS' SELF-EFFICACY BELIEFS REGARDING THE USE OF CONCRETE MODELS

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Dealing with concrete representations of abstract mathematical concepts is helpful for children if they are to develop an understanding of those concepts. Many mathematics educators argue that concrete models are not always necessarily more effective than traditional methods and do not guarantee meaningful learning. We can easily infer that teachers have an important role in effectiveness of instruction with concrete models. In the Turkish context, the prior mathematics learning experiences of prospective mathematics teachers are based mostly on traditional instruction, which do not involve the use of concrete models. As they meet with concrete models and develop experience with these materials, they also develop judgements about themselves to be effective in the use of concrete models as teacher. These types of beliefs are called self-efficacy judgements. In this sense, this study was grounded in the work of Bandura (1997) on self-efficacy within social cognitive theory. He suggests that teachers' beliefs on their instructional efficacy partly determine how they structure academic activities in their classrooms. Therefore, the purpose of this study was to investigate prospective middle school mathematics teachers' self-efficacy beliefs on teaching mathematics with concrete models, after they learn about these materials.

Data were collected from third year prospective middle school mathematics teachers attending a university in Turkey during the spring semester of 2008-2009. As part of a mathematics teaching methods course, they were given an instruction on the use of concrete models in teaching mathematics during a three week period. After the instruction, semi-structured interviews were conducted with thirteen prospective teachers.

At the end of the instruction, it was seen that prospective mathematics teachers developed self-efficacy judgments in different aspects. For instance, the participants pointed out that, by using concrete models, they can enable students to better understand the mathematical concepts, increase their achievement and motivation, as well as develop their thinking skills. On the other hand, they also maintained some doubts about using the models and had relatively low self-efficacy judgements about classroom management, when the concrete models were involved in the instruction. In addition, they believed that concrete models have limitations such as preventing abstraction, leading to memorization, and even to misconceptions.

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# BILINGUAL STUDENTS WRITING ABOUT LANGUAGE IDENTITY AND MATHEMATICAL PARTICIPATION

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I explore processes of negotiating language identity and mathematical participation as seen by Spanish and Catalan bilingual students in mathematics classrooms with Catalan being the LoLT –Language of Learning and Teaching (Planas, 2011). I assume that the analysis of the students’ perceptions gives clarity to how negotiation of language identity and mathematical participation works. The research by Setati and her colleagues (e.g., Setati, Chitera, & Essien, 2009) with multilingual students in South Africa is the more direct reference to my current research. My discussion here is limited to the collection of individual narratives written by ten Latin American Spanish speakers who produced extensive responses when addressed the following task:

*Please tell me a little about what language you use during group work in your mathematics classroom and why. I am very interested in who you are and any explanation that you may give me. Write as formally or informally as you like, either in Catalan or Spanish, up to one page. You have around twenty minutes, but you can ask for more.*

Despite the political experience of “monolingual bias”, what I found in many texts is that students are highly engaged in the representation of hybrid language identities during group work mathematical interactions. Drawing on the idea of hybrid language identities, it can be argued that the narrative discourses constructed by the Spanish-dominant speakers do not qualitatively differ much from those constructed by the Catalan-dominant speakers. In their writings students address two forms of resistance that work to maintain the use of their two languages. These forms of resistance are primarily linked to the ideas of: (1) not challenging ‘too much’ some of the imagined expectations, and (2) not damaging opportunities of mathematical learning. The students consider both individual interests –their mathematical learning–, and collective interests –the expectations attributed to others in the class.

## NOTES

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# IS VALID ASSESSMENT OF MATHEMATICS POSSIBLE IN ENGLAND'S CENTRALISED, REGULATED REGIME?

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Since the introduction of the National Curriculum in 1988 English education has been driven by a 'raising standards' agenda in which schools are held to account on the summative assessment outcomes of children (Ball, 2003). Typically, mathematics assessments comprise highly structured items that focus on standard techniques and procedures. They are easy to mark consistently and rarely require students to make decisions about what mathematics to use or how to approach a task (eg Noyes et al, 2008). In other words, they do not assess the entire National Curriculum for mathematics. School inspectors find that 'teaching to the test' dominates much classroom practice (Ofsted, 2009) so many students have an impoverished experience of mathematics (WYTIWYG (*what you test is what you get*), Ruthven (1994)).

The highly centralised and prescriptive system for developing assessments and qualifications is described by Isaacs (2010). The system is regulated by the Office of Qualifications and Examinations Regulation (Ofqual). The purpose of regulation is to ensure that assessments are valid, reliable, comparable, manageable and minimise bias. The regulators are responsible for maintaining standards and public confidence.

However, the rigorous processes in place to ensure consistency of standards actually conspire to stifle innovation (Wolf, 2009). The 'market place' for qualifications at age 16 means that the organisations responsible for their development and delivery will not want to do anything that might jeopardise market share. For National Curriculum tests at age 11 the system of pre-testing with children who are being prepared for the current tests means that innovative items rarely perform sufficiently consistently and reliably, and consequently do not make it through to the final tests.

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# THE ROLE OF GENERALIZATION IN MODELING CYCLE

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In the original study about content analysis of Iran and Australia mathematics textbooks upon composite theoretical framework regard to mathematics literacy, there was a component about generalisation in modelling cycle. In briefest outline, modelling processes start with a problem situated in the extra-mathematical world (EMW). The modeling process continues with formulating the EMW problem in mathematical terms. When this process is complete, the mathematical problem can be solved by the application of mathematical concepts and solution processes. Finally the mathematical solution must be interpreted to provide an answer to the EMW problem, and checked for its adequacy in answering the original question.

Now, we just concentrate on mathematical generalisation that was a part of mathematical problem solving section in modelling cycle. Generalisation importance in the process of mathematical problem solving is such that Mason (1996) has called it the heartbeat of mathematics. Although generalisation is not a feature of mathematical literacy, we decided to investigate the place of generalisation in the mathematics textbooks because almost textbooks pay attention to generalisation at different level and with different breadth and depth. Some factors make a problem as generalisation. For instance, whenever a parameter may replace a number, a wider condition may be considered, or a pattern that works everywhere may be sought, generalisation is in place. Upon content analysis results, none of the textbooks included many problems requiring generalisation, and this is one area of mathematical thinking - not only related to mathematical literacy - that needs more considerations in mathematics textbooks in both countries. After confronting with this results and reflection on them, arise another question that lead us to new direction of research about the role of generalisation on mathematical modelling cycle. What is the place of generalisation on the modelling cycle? Maybe generalisation exists in all part of mathematical modelling cycle! Now how we can produce more support for generalisation in mathematical modelling cycle?

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# ELEMENTARY STUDENTS' ABILITY TO MODIFY AND QUALIFY MATHEMATICAL CLAIMS

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At its core, mathematics involves more than computational procedures, yet researchers have found that mathematics instruction at the elementary school level is mainly answer-driven (Kieran, 2004). Although justifying and arguing are seen by many researchers in the field to be vital aspects of elementary school curriculum, "little research has focused on the issue of understanding and characterizing the notion of proof at the elementary level" (Stylianides, 2007, p. 1). This study sought to explore students' claims and to investigate to what extent elementary school students are able to modify claims (both created by themselves and peers) and qualify claims in order to take into account the probability of the conjecture's truth.

The participants of the study were 22 students in a fourth-grade classroom at a small, suburban elementary school in the Midwestern United States where the teacher did not emphasize mathematical processes such as justification and argumentation. Data were collected during the eight-lesson whole-class teaching experiment in which the researcher taught lessons regarding number properties. The lessons were transcribed, arguments were identified, and the arguments were placed into a model informed by Toulmin's (1958) model of argumentation. Each argument was also coded, reflecting the elements of the model and other key words that emerged from the process of open-coding.

In the initial analysis, there were 89 student arguments in the lessons and the majority of them fell into one of two categories, which emerged in the analysis of the claims: a property of an action (PA) or a property of an object (PO). The preliminary findings suggest that students modified or qualified PAs more often than POs, (about 31% compared to about 17%). At the beginning of the lessons, the students needed to be prompted to provide qualifiers, but did so on their own more often without prompting as the lessons progressed. During this presentation, the researcher will elaborate on these and other findings related to the students' claims and describe possible implications for including mathematical argumentation in lessons at the elementary school level.

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# LOGICAL FALLACIES IN REASONS FOR A CORRECT CHOICE

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In this research, the analysis of a subset of 127 pre-service teachers' responses to a unique probability task is discussed. The task asks the participants to select between two given explanations to the following question: "What is the probability that a three-child family is comprised of two daughters (D) and one son (S)". The first explanation (Jane's) presents four possible outcomes of which one is favourable (i.e., 3S, 2S&1D, **1S&2D**, 3D). The second explanation (Dianne's) presents eight possible outcomes of which three are favourable (i.e., SSS, SSD, SDS, DSS, **DDS, DSD, SDD**, DDD). In this report we focus on the subset of respondents who correctly chose Dianne's response, but got the question right for wrong reasons, and to determine if logical fallacies could be used as a lens for analysing those responses.

Of the participants whose choices were correct but reasoning was wrong, four different logical fallacies were discovered: *begging the question*, *false cause*, *equivocation*, and *division*. Of these, begging the question and equivocation were the most prevalent and are now explained within the context of the research task.

The first logical fallacy, begging the question, occurs in an argument when the truth of what is to be proven is assumed in order to prove it. Participants fell prey to this fallacy when they justified their choice of Dianne's explanation based upon her provision of a greater quantity of possible outcomes. These respondents assumed that a greater quantity of possible outcomes *proves* which explanation is correct without reflecting upon the relevance or accuracy of the outcomes given. In essence, these participants were arguing that 'more' equates to 'correct' regardless of the context.

The logical fallacy of equivocation, on the other hand, is a fallacy of contradiction within the meanings of words used in the argument. In the research task responses, in particular, this contradiction emerged over the respondents' explicit use of the word 'order' and their selection of Dianne's explanation, which is dependent upon order of the genders in the family mattering. In these cases, despite having chosen the explanation that is dependent upon the order mattering, the participants' justifications explain that order does not matter, contradicting the choice made. The analysis of the response justifications through the lens of logical fallacies indicates that defining and understanding of 'order' should be of concern within instruction.

Based upon this preliminary research, we contend that logical fallacies should continue to be examined for their applicability to analysing and understanding justifications given in probabilistic decision-making.

# AN ANALYSIS OF LESSON STUDY IN MATHEMATICS BASED ON THE CULTURAL-HISTORICAL ACTIVITY THEORY: A CASE OF A PROJECT SCHOOL IN THAILAND

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Japanese Lesson Study is a model for “teacher professional development process that allows teachers to work with each other collaboratively as reflective practitioners” (Yoshida, 1999, p.1) which is attracted attention especially mathematics education community in worldwide. Then, theorizing Lesson Study is needed for understanding and communicating among the community. Othani (2009) argued that “Cultural-Historical Activity Theory” (CHAT) offers a powerful theoretical lens for describing and analyzing school communal practice of Lesson Study especially country that adopted it in the beginning stage.

The purpose of this study is to analyse Lesson Study in mathematics in Thailand based on the Cultural-Historical Activity Theory. Thailand has adopted Japanese Lesson Study into Thai mathematics classroom practices as a teacher professional development system since 2002. It gives many challenges for our country in this first decade. Thus, it is suitable for using the theory to analyze the components in activity system of lesson study because it helps us to formulate issues for the development in the future. Ethnographic methods were employed for collecting and analyzing data through participatory observation in Lesson Study Cycle of one project school for two years.

The results showed that Lesson Study in mathematics in the Thai project school has followings the related components of the activity system: subject (teachers), object (students, problem-solving teaching approach), tools (Open Approach, mathematics contents and Japanese mathematics textbooks), rules (rules for practices in Lesson Study processes and Open Class), community (teacher network, supervisors, experts from university) and division of labor (small sub-groups for making lesson plans, school coordinators, internship program, school board and local educational service office’s supporting). The findings suggested the activity system aims to improve professional learning and skills and student’s achievement as outcome.

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# TEACHER QUESTIONING AS AN INDICATOR OF CHANGE

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The improvement of teaching is a critical issue in current educational reforms within the United States. Many educators pursue Master's degrees as a means of improving their teaching; however, little research exists investigating the influence of graduate coursework on practice (Hill, 2007). In seeking to address this weakness in the literature, our research investigated how participation in a Master's degree program influenced mathematics and science teachers' instructional practice. We have focused on teacher questioning as one indicator of change.

Our research is situated within the context of a mathematics-science partnership project, *Institutes for Integrating Content-Knowledge with Classroom-Instruction*. The institutes are conducted as coursework for a 3-year Master's degree program in mathematics and science. Twenty-two teachers from a public school district in a mid-sized city in the United States participated in our program. We collected a set of three video-recorded lessons as part of a three-phase video reflection cycle that spanned the length of the program. During year one, each teacher planned, taught, and recorded a lesson. During year two, teachers reviewed their initial lesson plan, viewed the recorded lesson, and reflected (in writing) on their teaching in light of coursework, readings, and program activities. They modified the lesson plan, wrote a rationale for the modifications, taught and recorded the modified lesson, and reflected on the differences between the two lessons. This process was repeated during year three.

We reviewed the videos and created transcriptions that included the questions teachers posed and the type of student response (e.g., short answer, extended response). Our focus on *questioning* led us to conclude that individual questions were best interpreted within the context of a sequence of questions, what we refer to as a cluster. Transcripts were clustered and analyzed using Chin's (2007) framework for teacher questioning approaches that stimulate productive thinking. This framework is comprised of four overarching categories and subcategories with more specific descriptive features. Data analysis will be completed by the end of May. However, our preliminary findings are: (a) the changes in the type of questions that each teacher asked reflected a shift from teacher-centered to student-centered questions, (b) subtle changes in teachers' questioning strategies impacted the flow of teacher questions. These results, while preliminary, indicate that questioning can serve as a useful lens for conceptualizing changes within teachers' instructional practice.

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# A TEACHING EXPERIMENT TO ORIENT VISUAL REASONING

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The reasons of difficulties faced during visualization have been gathered under various titles in previous studies. According to Presmeg (1986,1992), if present, these difficulties result from the following reasons such as: (1) The fact that a figure does not carry the features of its prototype, (2) The fact that an image, a diagram, or one case concreteness causes the idea to relate to irrelevant details, (3) The existence of certain secondary features but the absence of these features in the general situation, (4) The appearance of uncontrollable visual images, and the fact that these images prevent the richer pieces of the ideas from coming into being. In addition to this, as Presmeg and Balderas- Cañas (2001) suggest, affective factors are also influential over the process of the use of visualization.

The aim of this study is to orient students' visualization process via a teaching experiment, and to enable them to use this process effectively during the problem solving process within the framework of integral. To this end, a four-week teaching experiment was applied. Participants of the study who have non visual preferences were six pre-service mathematics teachers in their second years in the program. Clinical interviews were done with the participants before and after the teaching experiment in order to determine the changes occurring in students' visual preferences.

The results of the study indicated that the use of visual reasoning and the level of change in participants' visual preferences are different for all of them because of the various reasons. In this context, making students be more aware of difficulties that students generally have in visualization process would enable them to be more successful.

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# COGNITIVE CULTURAL ANALYSIS OF LOW ACHIEVEMENT IN TIMSS: EVALUATING WRONG ANSWERS IN 8TH GRADE

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The outcomes herein presented emerge from the analysis of explanations obtained during interviews applied to ten 8<sup>th</sup> grade students, immediately after they have solved a test with eleven problems of the released mathematics items from TIMSS.

The essential element of the theoretical framework for the present research is the cognitive theory of Bruner (1996) that considers the psychological processes as well as the cultural resources and settings involved in cognitive development. Thus, children's rationales were initially classified in base to the origin of their claims as "knowledge acquired at school" or "common sense gained by life experience and cultural influences". Further analyses of children's arguments lead to the identification of subtle situations and difficulties engendering wrong answers, from which three examples are presented. First, a lack of basic information, as when a boy explains his answer to: "A family uses about 6000 L of water per week. Approximately how many litres of water do they use per year?"

Miguel: Oh. It happens that, well, I took... first I put a given number of weeks. It is not the same and some month has four, but I know that... Based on three I was getting the weeks and months they were. A year has twelve months and so.

Though Miguel overcomes the problem, there are children that present a second difficulty: a lack of ability to bring out-of-school knowledge to the classroom.

Edgar (who later showed he could correctly enumerate the months in a year): [A year] has 132 [weeks...]. I've multiplied the months [22] by the weeks [6]...

Finally we detected the senseless application of rules. This is a well documented fact; the particularity here is that the declared rule disappeared from the curriculum long time ago. An explanation of a response to: "If the price of a can of beans is raised from 60 to 75 cents, what is the percent increase in the price?" is shown.

Atilio: Well, supposedly I made a rule of three. I was going to make an equation but I said, no, it will become too hard for me and so, uh, I sort of like went the easy way.

These examples intend to show that low achievement or a slow learning process cannot be solely attributed to the lack of students' mathematical knowledge.

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# PRIMARY TEACHERS ASSESSMENT OF STUDENTS' MATHEMATICAL TEXTS: AN EXPLORATORY STUDY

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Current research recognizes that the assessment of the students' achievements constitutes a predominately social process. To this direction, the present study adopts Bernstein's theoretical framework to look at teachers' pedagogical discourse on assessment, considering that this discourse depicts the pedagogical field that frames their assessment 'identity' and action (Bernstein, 2000; Morgan, 2007). In particular, the paper focuses on the resources teachers draw on in order to form pedagogical discourses of assessment of their students' mathematical texts: 'their personal knowledge of mathematics', 'their beliefs about the nature of the subject matter', 'their expectations of how mathematical knowledge can be communicated', 'their experience and expectations of students and classrooms in general and of individual students in particular', 'their linguistic skills and cultural background'(Morgan et al, 2002).

Eight well experienced primary teachers' answers to a semi-structured interview offered the data for the present study, collected in the context of a project focusing on teachers' assessment practices in mathematics. The teachers were asked to explain their way of dealing with assessment, based on a selection of their students' mathematical texts. A combination of content analysis and grounded theory techniques was used for the analysis of the data. The results showed that the teachers drew on a variety of resources in their assessment discourse. These resources do not differ very much from teacher to teacher, are often incompatible, determine the idiosyncratic interpretations made by the teachers of the students' texts and are barely related to students' mathematical achievements thus leading to divergent assessments.

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# PROVING STYLES AND AFFECTIVE PATHWAYS: A CASE STUDY

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It is widely accepted by mathematics education community and emphasized in the studies that affective factors are very important in mathematics teaching and learning. There has been many researches that investigate the relationship between students' achievement in mathematics and affective factors. Affect should not be considered as separate from cognition and there are many connections between affect and learning and doing mathematics. It is suggested that negative or positive emotions may direct students' reasoning styles and behaviours in engaging with mathematical tasks. Proof construction as an important mathematical task may also be directed by prover's affective state.

This qualitative study investigates the proving process from affective and cognitive aspects. The study shows the interaction between undergraduate students emotional states and the ways they use in constructing proofs. The data of the study were collected by individual interviews, written questions and classroom video recordings. The participants are fourth grade secondary mathematics education students at a public university. This paper is a part of a wide research and aims to explain the interaction of affect and cognition in the proving process through case studies.

The emotional state of the prover is analyzed through Goldin's (2000) model that describes the pathways in local affect. The proving styles of students were described through Weber's (2004) framework.

This case study helps researchers and mathematics educators to gain useful insights on how students' affective states may evoke different strategies and approaches in proving process. This study contributes to explain the proving process from both affective and cognitive perspectives and it is important for making instructional suggestions about enhancing students' proof performances.

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# INTUITIVE AREA ESTIMATION AND ITS RELATIONSHIP TO NUMERIC MULTIPLICATION SKILLS

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This study focuses on the relationship between children's estimations of area and their numeric multiplication abilities.

The prime example concerning area estimation (Wilkening, 1979) pointed out that while young children do attend simultaneously to the relevant stimulus dimensions of height and width, they fail in the correct combination rule for these dimensions. Similar results have been revealed in several developmental studies of cognitive algebra. Anderson (1996) explained this phenomenon in his Information Integration Theory which entails the assumption that the combination of information is getting more complex during the development. The normative integration rule is often developmentally preceded by a systematic employment of a simpler algebraic rule, for example, the adding rule in situations requiring a multiplicative rule.

In the present study, 91 second-grade children (mean age 8;4 years) were asked to estimate the size of rectangles before and after the school-based treatment of multiplication. Furthermore, children's numerical multiplicative abilities were assessed in a paper-and-pencil test subdivided into various types of tasks.

Over 60 percent of the children estimated the size of the area following an intuitive additive combination rule of height + width instead of the normative multiplicative one. The school-based learning of numeric multiplication did not seem to facilitate a shift from an intuitive additive integration rule into a multiplicative rule. Wrong intuitive algebraic rules persisted or the normative rule even shifted to a simplified rule after formal instruction. A further result indicates that scheme of correspondence tasks were solved significantly better by children using the intuitive multiplicative rule. This result supports the hypothesis of Park & Nunes (2001) that the root of understanding multiplication is grounded in the schema of correspondences. Further work on this issue focuses on the relation between the shift from an adding to a multiplicative integration rule and the scheme of correspondence.

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# APPROPRIATION OF ASSESSMENT CRITERIA BY THE STUDENTS: PRACTICE AND REFLECTION OF A MATH TEACHER IN A CONTEXT OF COLLABORATIVE WORK

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Research recognizes self-assessment as a process that can help improve students' learning (Boekaerts, Pintrich & Zeidner, 2000). The appropriation by the students of assessment criteria are essential in this process (Black & Wiliam, 1998). However, it demands a committed investment of the teacher in promoting proper conditions to such development (Wiliam, 2007). Teacher has to reflect on his/her practice (Schön, 1983), if possible, in a the collaborative work (Sowder, 2007). We present part of a study whose main objective is to understand how are designed, implemented and assessed, in a context of collaborative work between teachers, strategies aimed at the appropriation of the assessment criteria by students of 13 years old. We followed an interpretive case study methodology, analysing the practices and reflection developed by a math teacher, Joana, concerning a set of observed lessons. The results support the complexity of both the appropriation of the assessment criteria by the students and the teacher's role in that process (Nunziatti, 1990). The importance of teacher's reflection, enhanced by the collaborative work, contributes to changes in her practices. This work should continue to be refined through a cycle of practice and reflection.

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# UNDERSTANDING PRE-SERVICE MATHEMATICS TEACHERS' DIFFICULTIES IN CONSTRUCTING A MATHEMATICAL MODEL

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The study reported here is part of a broader study that aimed at investigating the development of pre-service teachers' modeling competencies in the context of a semester-long mathematical modeling course. This paper presents the results related to the difficulties pre-service mathematics teachers had in working on a mathematical modeling problem about a company owner who seeks help in renting a storage space. For that, the pre-service teachers were expected to find out the maximum number of cylindrical containers that fit into three different rectangular prism-shaped storages each of which has different rental fee. Nineteen pre-service elementary mathematics teachers participated in the study. Working in groups of three or four, the participants engaged in the modeling activity for about two hours and tried to create a model and write a report to the company owner.

Data were collected through audio and video recordings of the participants while they were working on the modeling activity, audio-recorded interviews conducted after the class, and reflection papers they wrote about their experiences. The analysis of the data revealed that the participants faced with various difficulties in constructing and developing their mathematical models such as struggling to express the verbal information given in the problem into mathematical terms, being unsuccessful in arranging the containers to find a better storage, and having difficulties in revising and refining early mathematical models. The data indicated that their difficulties were partly due to inexperience in using mathematical knowledge productively to solve real-life problems, deficiencies in their understanding of some mathematical concepts, and unfamiliarity with problems requiring them to think beyond the givens, make new assumptions, and attempt to follow different solution paths. These findings implies that teacher candidates should be given opportunities

- to be able to create and strengthen the links between abstract mathematical concepts and real-world situations
- to experience mathematical modeling processes in their formal education,
- to assist them in developing self-confidence and mathematical power in their struggles with modeling processes originated from the nature of unconventional problem.

# TIMSS FROM TURKISH TEACHERS' PERSPECTIVE

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The Trends in International Mathematics and Science Study (TIMSS) assesses the mathematics and science knowledge of fourth and eight graders around the world. More than 60 countries participate in this study. TIMSS was administered every four years, since 1995. Turkey will be participating in TIMSS for the first time, at the 4th grade level in 2011. The schools and which section(s) of these schools will be representing Turkey was determined through random sampling. There are about 200 sections involved. The classroom teachers of these sections were surveyed for this study.

The survey dealt with such questions as demographics, teacher's knowledge and consideration of TIMSS in their lesson preparation, the teaching strategies they have been using, percentage of time they spent in various stages of mathematical learning, textbook use and satisfaction, participating students' mathematical readiness and inclinations, the value their parents' place on learning. The research also aims to evaluate the 4th grade mathematics program and textbook, published by the Ministry of Education, through teachers' level of satisfaction. The survey asks teachers to predict Turkey's success in TIMSS 2011, based on their students' level of competence in regards to questions dealing with various mathematical topics and cognitive levels. Further more, provisions needed in order to enhance Turkey's success in the future, in regards to the mathematics curriculum, quality of mathematics textbooks and in-service teacher training are discussed.

One interesting finding of this research is that while on the one hand most teachers say that they use a constructivist approach to teaching mathematics, on the other hand, few actually use the activities in the textbooks. This indicates a drastic misconception in teachers' understanding and perception of what constitutes constructivism. Most significant factors, contributing towards the poor performance of students, are quoted as their families' socio-economic level and the lack of interest in education per se, both on the part of the students and their parents. Possible solutions suggested, for the improvement of 4th graders' performance in TIMSS, are students' encountering more open ended questions and questions requiring reasoning skills.

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# TWO ROUTES TO THE REIFICATION OF VARIABLE AS A KEY TO CONCEPTUAL CHANGE IN MATHEMATICS LEARNING

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The theory of reification by Anna Sfard is well known as a model of mathematical concept formation. In terms of this theory the same mathematical notion can be conceived in two fundamentally different ways: *operationally* as processes, and *structurally* as objects. The reification undergoes an ontological shift or a qualitative jump from operation to structure. In this presentation about conceptual change, we discuss two routes to the reification of “variable” with the help of the preliminary analysis at first. The reification process of variable will be considered as a key to conceptual change in mathematics learning at second.

In the transition from primary to secondary school mathematics, the teaching and learning of variable can be a crucial didactic situation. Based on the epistemological discussions by Katz (2007) and Sfard & Linchevski (1994), there are two interrelated routes in the teaching and learning of variable: 1) *from arithmetic expression to algebraic expression*; 2) *from proportional relation to functional relation*. To put them briefly, the former corresponds to the syntactic aspect of variable, and the latter corresponds to the semantic aspect of the same notion.

- 1) As far as the arithmetic and/or algebraic expressions are regarded as computational procedures, student’s conception of symbolic notation of variable could be *operational*. If the algebraic expression or equation itself, however, is applied to problem-solving of real situation, the conception of symbolic notation of variable could be *structural*.
- 2) Through the relation between two quantities which vary linearly, student learns proportionality. Such a *dynamic* attitude is based on the concrete (scalar) quantities. If their awareness of linearity in proportional relation can be integrated to the notion of variable in functional relation, where the systematic rule of correspondence will be considered as a unified entity (such as a set of ordered pair), the attitude towards the quantitative relation should be more *static*.

As a result, two routes to the reification of variable imply that the teaching and learning of algebraic expression and functional relation should be integrated from conceptual change perspective. Based on the theoretical discussion, we need to design the relevant teaching situation for student’s conceptual change.

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# MODELLING AS A BIG IDEA IN MATHEMATICS – KNOWLEDGE AND VIEWS OF PRE-SERVICE AND IN-SERVICE TEACHERS

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Modelling is agreed to be a big idea for mathematics as a scientific discipline with high relevance for mathematical literacy. Consequently, teachers should be aware of this big idea and know how modelling relates to a variety of curricular contents. However, especially quantitative empirical research into knowledge and views of pre-service and in-service teachers related to modelling is scarce. Hence, this study concentrates on professional knowledge and views of Austrian pre-service teachers about modelling as a big idea and contains first exploratory comparisons with in-service teachers. The results suggest that especially for a sub-group of the participants there is a need of professional development related to modelling.

In particular, the study focuses on the teachers' knowledge and perceptions about modelling as a big idea and on views about the significance of modelling. The research questions are as follows:

- (1) Are pre-service and in-service teachers able to connect contents through the big idea of modelling and do they have meta-knowledge about the modelling process?
- (2) Which significance do pre-service and in-service teachers assign to the big idea of modelling and how do they see this idea related to specific content areas?

Beyond the evidence, the findings of this study also call for empirical research into the role of the views examined here for the teachers' choice of specific learning opportunities in the classroom and into the interdependency structure of professional teacher knowledge related to modelling. Such deepened analyses could open up ways of effective professional development approaches.

## Acknowledgements

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# LEARNING IN GEOMETRY CLASSES FOR CHILDREN

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One group of teachers planned to build a circuit of obstacles, which would be used in geometry classes for students from 3 to 5 years old. While discussing the proposal in a meeting with a consultant (the author), the teachers came to the conclusion that it would be interesting if they allowed the students to somehow participate in the planning of the circuit, even being conscious that the age of the students would make the activity very challenging. It happened in a public school located in a slum in Belo Horizonte, a southeastern Brazilian city.

The circuit consisted of several obstacles, and the idea was that each child had the challenge to go through each obstacle and finish the circuit reaching a finish line. In the beginning, the teachers explained that the activity would be related to geometry learning because the children would be in contact with geometric shapes throughout the whole activity; hence they would learn some geometric names. However, when the construction of the circuit was critically discussed, it became clear that it is not enough to say that geometry teaching is included in the process only because there are several geometric shapes in the circuit; if so, then in addition to the learning of the names used for geometric shapes, what would be the role of geometry teaching? The question was not clear at all. Besides the children would be treated as alienated learners (Lave and Mcdermott, 2002) if they would only wait for the construction. The questioning led to a radical change in the plans. Now, the activity as a whole would be designed to stimulate and guide the students to play with all materials and, after that, to imagine the position of each material in the circuit. They would express their ideas in groups by making maquettes; then, they would choose the best one and the teachers would finally construct the circuit according to the chosen maquette.

The *Group of Cultural-Historical Studies and Research in Mathematics and Science Education*, headquartered at Universidade Federal de Minas Gerais, was invited to provide help in that exploratory activity. Several classes, planned according the new motive, were videotaped for further analyses, which are in progress. One main concern is about developing a proper approach for a better comprehension of the learning processes, considering that what students and all the adults involved learned was not defined beforehand. The expansive learning approach (Engeström & Sanino, 2010) seems to fit in a configuration like that in which people are open for experiencing situations that are not predictable.

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# PRESERVICE TEACHERS' RESPONSES TO STUDENTS' STRATEGIES IN WHOLE NUMBER SUBTRACTION

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Recognizing meaning in students' mathematical ideas is challenging, especially when such ideas are different from standard mathematics. This study examined, through a teaching scenario task, the reasoning and responses of preservice elementary and secondary teachers to a student's invented strategy for whole number subtraction along with the standard algorithm. In this study, we used a student-invented strategy reported by Carroll and Porter (1997) in which the difference is recorded as a deficit or a negative number (e.g.,  $40-70 = -30$ ) so that no "borrowing" or regrouping is necessary where the subtrahend is larger. Two hypothetical students come up with different solution methods to a two-digit whole number subtraction problem. The first student, Tommy, used a student-invented strategy aforementioned and produced the correct answer. Dan, the second student, found the correct solution using the traditional algorithm for subtraction but failed to understand Tommy's method. Preservice teachers were asked to determine whether each method worked for all whole numbers and then were asked to explain their reasoning. Preservice teachers were also then asked to respond to Tommy and Dan. Analysis results revealed that while all preservice secondary teachers in this study determined the validity and the generalizability of the student-invented method correctly, a large portion of preservice elementary teachers did not initially recognize Tommy's strategy as a legitimate method for subtracting. However, such differences became less distinct when it came to justifying the reasons behind the procedure and providing good intervention. For example, both preservice secondary and elementary gave mostly procedural explanations for Tommy's and Dan's strategies. They also showed a tendency to demonstrate these strategies by 'show and tell' rather than trying to help students construct their own knowledge. In particular, procedural-oriented responses were more evident when student methods were assumed to be incorrect and when asked to connect the student strategy to the traditional algorithm. This result is consistent with the findings from the Son and Crespo' study (2009). This study has implications for teacher educators and future studies. For example, teacher educators must provide opportunities for preservice teachers to explore student-invented strategies and be able to clearly explain the mathematics behind it.

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# MATHEMATICS ANXIETY: A CONSTRUCT MODEL

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Recently, there has been the development of a number of instruments to help researchers investigate the phenomenon of mathematics anxiety — Richardson and Suinn (1972) with the *Mathematics Anxiety Rating Scale* (MARS), Fennema and Sherman's (1976) *Mathematics Anxiety Scale*, and later a *Revised Mathematical Anxiety Rating Scale* (RMARS) noted by Baloglu and Kocak (2006). Mathematics anxiety has been a difficult construct to define and measure. Tests for mathematics anxiety often measure other features, for example, test anxiety, and numerical anxiety, and in these cases mathematics anxiety may be seen as multi-dimensional (Kazelskis, 1998). Many researchers have used mathematics anxiety in association with other factors, such as mathematical performance or attitude to mathematics. These uncertainties and contradictions on how mathematics anxiety is conceptualised has resulted in no commonly accepted construct model and the need to refine measures (Kazelskis .....et al 2000; Ma & Kishor, 1997).

The proposed model applies Modern Measurement Theory namely, the Rasch Rating Scale Model to refine a tentative construct model. The final uni-dimensional model includes attitudinal, cognitive, and somatic indicators and is applicable to in-class instruction, in-class assessment, and out-of-class application. In each of these situations, a person's anxiety will depend on attributes of the person as well as the demands of the situation. Indicators are cumulative, that is a person experiencing high anxiety will also show lower level indicators.

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# PROMOTING STUDENT'S GEOMETRICAL THINKING: A CASE STUDY OF THE INTERNSHIP MATHEMATICS STUDENT TEACHER'S TEACHING PRACTICES

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The van Hiele theory mentions that student's progress from one level to next is more dependent upon instruction than on age or biological maturation (Fuy, Geddes & Tischler, 1995). Consequently, in the field of research in school geometry, one of the current major concerns is about how to improve instructional strategies in order to help students to successfully progress from practical geometry to deductive geometry (Royal Society, 2001). This study focuses on internship mathematics student teacher's emerging teaching practices during the training of professional experience.

The purpose of this study was to characterize teaching practice of the internship mathematics student teacher in promoting student's geometrical thinking. The data was collected by classroom observation and interviewing teacher. The teaching episodes were documented through audiotape and videotape interactive between teacher and student, student and student. Data were analyzed based on the Open Approach as a teaching approach that incorporated into Lesson Study processes (Inprasitha, 2010).

The result showed that the internship mathematics student teacher's teaching practices can promote geometrical thinking by initiating problems and question, making conjectures and presenting student's solutions, exploring examples and student's different/rich ideas. Moreover, internship mathematics student teacher could develop instructional strategy by using whole class discussion and comparison of students' ideas on the blackboard that encourages students to talk about geometric concepts, and to develop expressive language. The findings suggest that students' geometrical thinking levels were passed from level 0 (visualization) to level 1 (analysis) and also passed from level 1 (analysis) to level 2 (informal deduction).

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# THE TRANSITION FROM ARITHMETIC TO ALGEBRAIC FRACTIONS

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Fractions have been identified as a topic of transition from particular mathematical procedures and concepts addressed at the elementary level to the more advanced manner in which operations are performed in algebra (Wu, 2001). At the elementary level, students should learn to work with fractions conceptually (e.g. translate between different representations), and procedurally (e.g. carry out operations). Kilpatrick (2001) suggests that conceptual understanding and procedural fluency are interwoven. Both are needed for students to make further steps in their mathematical growth, such as in the arithmetic-algebra transition (Bruin-Muurling, 2010), for which the concept of a variable is a critical point. At the secondary level students learn to work with algebraic fractions extending their knowledge of arithmetic fractions. We studied students' conceptual understanding and procedural fluency in the transition from arithmetic to algebraic fractions.

The participants were 61 Dutch students (age 13-15) at grade 10 level. The instrument consisted of a test with 9 tasks containing 2 open-ended conceptual tasks and 7 tasks that could be solved procedurally. The first four tasks involved only arithmetic representations (numbers) and the remaining ones, algebraic representations (variables). Data were analyzed qualitatively, focusing on both conceptual understanding and procedural fluency which students revealed.

It was observed that students had a basic conceptual understanding of arithmetic fractions (e.g., representing them on a number line, finding the least common multiple). However, they encountered problems with adding and subtracting rational expressions in one unknown (e.g., bringing terms under a common denominator). It was also noticed that subtraction was more problematic than addition of fractions, as they had problems in applying a minus sign to the whole numerator of an algebraic fraction. In this context, we observed that students' difficulties in the transition from arithmetic to algebra were linked with lack of procedural fluency at arithmetic level.

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# CONNECTING TO THE FOLK MATHEMATICAL CULTURE THROUGH PUZZLES

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A central recommendation of recent revisions of the Indian school curriculum is the forging of connections between the school curriculum and the culture that children are immersed in outside school. One of the ways in which this is implemented in the mathematics curriculum is through the inclusion of puzzles and riddles taken from the folk mathematical culture of India in textbooks, especially at the primary level. These puzzles are interesting not only to children, but also to teachers and form a vehicle for the deepening of mathematical knowledge in in-service professional development programmes for mathematics teachers. The presentation highlights the variety of such puzzles, and discusses the underlying mathematics of some puzzles that have been useful in ongoing teacher professional development workshops.

The following puzzle is an example from the grade 5 mathematics textbook (Math-magic: Book 5, 2008). 'An old woman dies leaving instructions on how to divide her seven horses among her three daughters. The first is to receive one half of all the horses, the second, one half of what the first receives and the third, one half of what the second receives.' This puzzle leads to interesting discussions in teacher workshops of several mathematical aspects: fractions whose sum is close to one, the properties of binary fractions, the sums of the successive powers of two, and the binary representation of numbers.

Another puzzle found in the folklore is the following puzzle related to traditional weight measures. A measure of one *viss* (equal to 40 *palams*) falls down and breaks into four pieces. If the four pieces can be used to measure any (integral) weight from one to 40 *palams*, what is the weight of each of the four pieces? (Rampal, et al. 1998) The solution (1, 3, 9 and 27 *palams*) reveals the underlying base 3 representation of numbers, in which the digits are 0, 1 and  $-1$ . Thus, 26 in base 3 becomes 1001, that is,  $1 \times 3^3 + 0 \times 3^2 + 0 \times 3^1 + (-1) \times 3^0$ . It is interesting to note that this is a fully general ternary representation for positive integers.

The presentation will also connect these puzzles with an enduring trend in Indian mathematics of the representation of positive integers in a variety of ways.

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# EXPLORING STUDENTS' METACOGNITIVE STRATEGIES DURING PROBLEM SOLVING IN A MATHEMATICS CLASSROOM USING THE OPEN APPROACH

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The aim of this study was to explore students' metacognitive strategies while solving problems in the math class, using the open approach. The researcher continuously observed problem solving behaviour of four 1<sup>st</sup> grade students and classroom activities. Qualitative data were used to investigate the triangulation among data sources: video recording, teacher and student interview, and students' written works in the unit on "Addition (2)" (12 periods-60 mins/period). Thereby "metacognitive strategies" refers to thinking ability causing behaviour that a problem solver can control, monitor, and reflect his own thinking process, based on an idea or a way which he valued from existent resources-accumulative recording of previous learning experiences to use as a tool for controlling ideas. Furthermore, metacognitive strategies are used for examining his own thinking and ensuring that he has already achieved his goal.

The findings illustrated that students could show behaviour and abilities reflecting metacognitive strategies related to the open approach: the following 4 steps (Inprasitha, 2010): 1) posing open-ended problem showing students' attempts of understanding problem situations and the thought of proving or solving a problem (students' problematic); 2) students' self learning: (1) learning how to learn and creating problem solving ways by giving value of the ideas and methods learned before and using them as a problem solving tool: making ten, causing continuity, advance and success in problem solving; (2) monitoring and reflecting the problem solving process through descriptive writing of their own thinking process; and (3) attempting to make various problem solving ways beside only finding an answer, leading to investigating, and monitoring their own thinking and reorganizing to create new ways; 3) whole class discussion and comparison related to checking and reflecting the problem solving process together with friends and the teacher, and accepting and correcting mistakes; and 4) summarization through connecting students' mathematical ideas emerged in the classroom related to evaluation and valuation of an efficient idea or way helping solve problems easily and taking less time. The aforementioned decision led to a review of their processes from created choices in the previous steps. The selected ideas or ways were then collected as an essential resource meaningful as a thinking tool to solve other problems. This finding yields mathematical values aspect of metacognitive strategies.

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# **‘HOW TO’ IN THE STUDENTS’ ABSTRACTION PROCESS THROUGH COMPRESSION TO THINKABLE CONCEPT**

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The abstraction process through compression to thinkable concepts is the key to developing increasingly powerful thinking. This point of view focused that instructional must be framed with an awareness of students’ abstraction process to produce thinkable concept (Gray&Tall, 2007). Tall (2007) argued that Lesson Study provided area of the students’ compression to thinkable concept. Moreover, Tall (2008) suggested that Lesson Study is to be the major idea to support children have ‘how to’ in solving problems for compression to thinkable concept. The purpose using Lesson Study In Thai classroom is to be producing ‘how to’ as a tool in thinking to build students’ concept, which is interesting for study that how can it be conducted to thinkable concept. The purpose of this study is to describe ‘how to’ in the students’ abstraction process through compression to thinkable concept under classroom using lesson study and open approach. In this study based on the framework that proposed by Tall&Isoda (2007) was employed to analyze the data collected from classroom observations. It is considered 4 steps of compression operation arithmetic using procedures in problem solving. Data for this study were collected by using teaching experiment, with the four of first graders as targeted group at Kookam Pittayasan School, a project school supported by CRME. The research results revealed that in the students’ abstraction process, they compressed computable symbols and conducted 10 as ‘how to’ in their thinking and thinkable concept at the same time. It is shift steadily from performing sequence of compression in students’ thinking from actions being linked together in increasingly sophisticated ways: accumulation students’ way thinking in 1-3 step to refine important ideas in step 4, and it is realized to extend useable mathematical structure in step 5 also. It happened clearly by compression of knowledge from step-by-step procedure, to the possible choice of several different procedures, to seen the overall effect as a general process that can be carried out in various ways, to compressing it as a thinkable concept. This study predicted that ‘how to’ play a key role to provide students’ a thinkable concept.

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# CHANGING IN THAI SCHOOL: FROM TRADITIONAL TO CREATIVE CLASSROOM

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Mathematical creativity is difficult to develop if one is limited to rule-based applications without recognizing the essence of the problem to be solved (Mann, 2006). When Center for Research in Mathematics Education (CRME) implemented Lesson Study and Open Approach in Thailand, open-ended problems were designed to encourage students to think in several ways. The purpose of this paper is to describe changing mathematics classroom involved teachers' change and consequential students' creativity in Thailand. The data were gathered during four years in research project of CRME. The target group were two pilot schools in this project. Teachers' changing were analysed from their discussion and reflection among Lesson Study teams. Consequently, classroom practice, students' creative thinking, and attitude were analysed from classroom video recordings.

The findings illustrated that three steps of Lesson Study (Inprasitha, 2010) had included planning the lesson, observing the classroom, and reflecting classroom practice, in every week, led change in both of teachers and students. Furthermore, the teacher's roles can be changed from lecturer who use to confided in own teaching style to be patient teacher toward listening to others and their students. For students, they can be concentrated on problem solving for a long time. Especially, students who were sat back in the classroom are confidence to present and share own ideas to their friends. In this long period, they can solved problem diversely and elaborately. These are new roles which contrast both of traditional practices of teachers and students and then led generated creative classroom in Thailand.

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# THE HIGH SCHOOL ANALOGUE OF MATHEMATICS TEACHERS' HORIZON CONTENT KNOWLEDGE: THREE CASE STUDIES

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The purpose of this study was to elicit critical incidents that illustrate how knowledge at the mathematical horizon would guide high school mathematics teachers' judgements about mathematical importance. Three experienced mathematics teachers of two Taiwan high schools were purposefully selected. Data were collected from September 22 to November 23, 2009. The working definition of the term "critical incident" (CI) we use in this paper is based on Flanagan's and Lerman's (1994) definitions. We based this study within the "Mathematical Knowledge for Teaching" (MKT) framework of Ball, Bass and their colleagues(2009) especially their "horizon content knowledge". The results show that lack of horizon content knowledge has caused teachers' misjudgement of the values of a kind of problems in instruction. In addition, the horizon could extend far away from elementary and secondary mathematics curriculum in different directions.

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# EXAMINING MODEL ELICITING ACTIVITIES DEVELOPED BY PROSPECTIVE MATHEMATICS TEACHERS

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Model eliciting activities (MEA) are problem solving activities by using some special principles that students can infer from meaningful real life situations, discover, extend and revise their mathematical structures (Lesh & Doer 2003). The purpose of this study is to examine the process of constructing MEA developed by the prospective mathematics teachers in the context of the suitability of MEA in the MEA principles framework (Lesh et. al, 2000). The participants of the study were selected from mathematics education department of the state university and took a course about mathematical modelling. In this course, students were not informed about the MEA principles. The participants worked collaboratively to form the MEA. These collaborative groups of 2–4 individuals were formed according to the participants' willingness. The data obtained from the MEA generated and solved by the participants as written documents and semi-structured interviews with the participants about the process of generating MEA. The results showed that all of the participants could consider the model construction principle, the reality principle and the construct documentation principle when they were generating their MEA. Some of the participants were partially successful at the model generalizability principle. All of the MEA were not adequate in terms of framework's two principles named effective prototype and self assessment. It is important for prospective teachers to use MEA in their future lessons. In order to support this usage prospective teachers are educated as a person who is not only a mathematical modelling problem solver but also a problem designer.

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# VERIFICATION OF TRIAD FEEDBACK- THE UNIT OF ANALYSIS OF SMALL-GROUP MATHEMATICAL COMMUNICATION

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Small-group mathematical communication (SMC) is an important mathematical learning process. It was defined as the communication which includes characteristics (i.e. rigorousness, economy and freedom) of mathematical communication in small-group working in which students have shared goal and shared meaning. Students' thinking and feeling are more important in the scene of SMC. Thinwiangthong et al. (2010) proposed the "triad feedback" as the unit of analysis of SMC. It modified from dyad feedback (Emori, 2005) and integrated with the unit of analysis of emotional experience (Inprasitha, 2001) for analyzing the SMC to reveal cognitive and emotional aspects.

The purpose of this study was to verify the "triad feedback" hypothetical model in the mathematics classroom using open approach and lesson study. To reach the purpose, one year ethnographic study was used to collect the data from the mathematics classroom. Video of grade 7 students' small-group working, protocol of clinical interviews and students' written works were analyzed by using the "triad feedback".

The result of this study showed that the "triad feedback" was verified by the empirical evidences of SMC in the mathematics classroom using lesson study and open approach. Characteristics of SMC as a cognitive aspect and emotional experience as an emotional aspect were illustrated in analyzing the SMC by the "triad feedback". So, we assert that the "triad feedback" appropriate to be used for analyzing the SMC in order to grasp the students' thinking and feeling.

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# THE PLAN OF MATHEMATICS AND THE NEW MATHEMATICS PROGRAM: TWO PROJECTS OF GIVEN HANDS

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The Plan of Mathematics (PM) is a nationwide project designed to provide the sustained development of school projects aimed at improving students' mathematical learning. The PM embraces students of basic education: from 1<sup>st</sup> to 9<sup>th</sup> grade levels (6 to 14 years-old). 94% of the Portuguese public schools with basic education have elaborated their project, based on their own educational contexts, needs and resources. In 2009/10, the New Mathematics Program for Basic Education (NPMBE) initiated its process of generalization (with 38% of the public schools voluntarily involved), being currently taught at 1<sup>st</sup>, 3<sup>rd</sup>, 5<sup>th</sup>, and 7<sup>th</sup> grade levels. This program emphasizes reasoning and sense making (NCTM, 2009) and reinforces classroom practices such as group work and class discussions, pushing for students' greater involvement in their own learning. The development of both projects is scientifically and pedagogically supported by a national committee and a group of 80 accompanying teachers who work directly with the schools. Data about the projects' development are collected through different types of written reports. Results have indicated greater attention being paid to curricular integration across grade levels, a continuing tendency to use classroom partnerships and teachers' collaborative work as significant strategies to foster students' learning and teachers' professional development, and a profitable association between the projects: e.g., the extra class time allowed by the PM has helped teachers in implementing the NPMBE, especially regarding the establishment of classroom environments aligned with the program recommendations. Schools have reported an increase in students' learning, as also evidenced by the PISA report (OECD, 2010); yet, much attention is still needed to problem-solving, reasoning, and communication. Constraints include difficulties in scheduling time for collaborative work amongst teachers of different grade levels and lack of exemplary and timely materials covering all topics of the NPMBE.

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# MATHEMATICS LECTURING IN THE DIGITAL AGE

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In this study we consider the transformation of tertiary mathematics lecture practice. We undertake a focused examination of the related research with two goals in mind. First, we document this research, reviewing the findings of key studies and noting that reflective pieces on individual practice as well as surveys are more prevalent than empirical studies. Second, we investigate issues related to the transformation of lecture practice by the emergence of e-lectures. We discuss the latter in terms of claims about the efficiencies offered by new technologies and contrast these with possible disadvantages in terms of student engagement in a community of practice.

For this work we sought English-language articles that focused directly on tertiary mathematics lecturing practice with a particular interest in e-lectures. While cognizant of broader relevant terminology (e.g. ‘teaching’), we narrowly focused on variations on the word “lecture” in *titles* primarily from relevant and recognized scholarly sources in the fields of mathematics education, instructional technology and distance learning. The search yielded a total of 43 articles of which 20 were reflective papers and 23 were empirical studies (with 13 using self-report data only and 10 using some form of behavioural data).

In this short oral we detail the nature of methodological approaches employed in the current empirical research and present the major themes that emerge particularly in relation to communities of practice in mathematics (Wenger, 2000). Specifically, we comment on methodological issues surrounding current research and contrast reflective and self-report results with emerging empirical findings.

Overall, while student and lecturer satisfaction is clearly documented, the limited empirical evidence shows a negative correlation between e-lecture use and achievement. Given causality is not claimed, we hope to engage participants in a discussion of questions raised concerning the impact of e-lectures on community and its relationship to learning. We present future directions for research as well as some research-based caution and guidance for implementation.

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# ELEMENTARY SCHOOL CHILDREN'S USE OF GRAPHICAL REPRESENTATIONS AND FRACTIONAL KNOWLEDGE

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Fractions are difficult concepts for children to learn. Graphical representations are ubiquitous in fractions instruction, but how they influence student thinking and learning is not fully understood. Moss and Case (1999) found that teaching decimals and fractions with linear representations resulted in deeper understanding, compared to traditional instruction. On the other hand, Cramer et al. (2008) claim that the circle representation is the most effective representation when comparing fractions. Mathematics education research has not systematically investigated student performance and reasoning on the same fractions task, but performed with different representations. In this paper, we address this concern.

We developed a test with fraction items that involved the circle, rectangle, and number line representations. The problem types included: naming fractions, making fractions, reconstructing the unit from unit fractions and from proper fractions. Each problem type was combined with each representation. We administered the test to 656 US students (141 in 4<sup>th</sup> grade and 515 in 5<sup>th</sup> grade). Using Chi-square statistics, we found that students' performance across representations (within the same problem type) differed significantly. Performance with the number line was lower than that with the other representations, and performance with the circle and rectangle did not differ. We also found differences across fraction problem types (within the same representation): reconstructing the unit items were more difficult than the others. To supplement the quantitative analysis, we conducted three interviews (each) with two 4<sup>th</sup> grade students and a follow-up interview when they were in 5<sup>th</sup> grade. The purpose of the interviews was to conduct a more detailed analysis of students thinking related to representations and fraction concepts. The interviews reveal why the number line is harder, such as decimal bias,  $n/n$  and "1" being different. Thus, systematic comparison of reasoning with different representations clearly shows that students view the number line as a more complex representation compared to circle and rectangle when solving problems with fractions.

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# HIGH EXPECTATIONS, LOW CONFIDENCE – DISCREPANCY IN SELF-IMAGE AS A REASON FOR DISPLEASURE IN MATHEMATICS

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Research in mathematics-related attitudes has almost exclusively considered linear relationships between different dimensions of affect and achievement. We report here some theoretical elaboration and empirical results about a more complex model. The empirical study examines the connection between Finnish grade 5 and grade 8 students' enjoyment in mathematics and their self discrepancies (how good they feel about their skills vs. how well they wish to perform). The idea of discrepancy has been used, for example, to explain mathematics anxiety as a consequence of a difference between outcome expectancy and outcome value (Kyttälä & Björn, 2010).

In our study, we measured the discrepancy of perceived self and mastery goal orientations (MGO). The relationship between this discrepancy and enjoyment of mathematics was of interest. Perceived self was operationalized as a sum variable of students' perceptions of their *self competence*, *self confidence* and *self regulation*. The reliability was high for the indicators for all the variables. We classified the discrepancy and the enjoyment into low, middle and high; cross tabulated them and tested their independency by  $\chi^2$ -statistics. The results show that low discrepancy related most likely with enjoyment of mathematics, whereas large discrepancy related most likely with poor enjoyment. Further, large discrepancy related with poor enjoyment most likely if either the perceived self was low or MGO was high. If MGO was way above perceived skills, the enjoyment of mathematics was lost. Grade 8 girls were over-represented in this group, which may be because they also suffered poor perceived self more likely. This was not visible regarding boys or grade 4 girls.

Our study points out that the relationship between attitudes and self-image may be more complex than previously expected. Positive self-image and positive aspirations are not automatically related to positive emotions: poor self-perception is much more harmless when connected to non-achievable aspirations, and even students with good self-image may lose their enjoyment if the expectations are needlessly high.

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# SEVENTH GRADE STUDENTS' MATHEMATICAL THINKING AND REPRESENTATIONS IN MODEL-ELICITING ACTIVITIES

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Model-eliciting activities (MEAs) include conceptual tools for describing a significant system mathematically (Lesh & Doerr, 2003). In each MEA, students are expected to describe the problem situation and establish a bridge from real world to mathematical world, manipulate the model to make predictions related to the problem situation, translate the results gathered in mathematical world to real life, and verify the model (Lesh & Doerr, 2003). During this process, students share their mathematical thoughts by using and creating external representations such as graphs, tables, and symbols. While MEAs are widely used in mathematics education research, we know little about how these activities could be used to examine students' mathematical thinking in our country. In this study we aimed to explore how seventh grade students develop mathematical thinking while they are engaging in MEAs, and what kind of representations they use. For this reason, researchers reviewed and adapted MEAs in related literature for the study group and designed warm-up and follow up activities. Then students from one seventh grade classroom are presented with four MEAs and required to work as a group of three. Two groups are selected for in-depth analysis. Focal groups' discussions before, during, and after MEAs are observed and detailed field notes are taken by the researchers. Data collected through videotaped observations, field notes, and students' written works are analyzed considering the modeling cycle steps (description, manipulation, transition/prediction, and verification). The preliminary findings suggest that using MEAs are effective in revealing students' mathematical thinking and representations.

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# STRATEGIES IN EARLY SPATIAL REASONING

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The research is focused on the strategies of young children in spatial thinking. It attempts to examine if and how children can improve their initial strategies in spatial activities after a teaching intervention that encouraged them to deal with and develop their own approaches. There many opinions concerning strategy development (Heinze, Star, & Verschaffel, L., 2009; Linchevski, & Schwarz, 2001). In early studies Owens (1999) developed a model of five groupings of strategies especially for aspects of spatial thinking. The adoption of this model and the necessary reconsiderations formed a basis for the analysis of the development of students' strategies related to this study.

32 children, aged 5-6 years, participated in a "pretest-posttest control group design" teaching experiment. Six spatial tasks (from which two were chosen to be presented in this paper) were proposed to the children in a playful mode, before and after the teaching experiment in order to record their achievement, their strategies and possible improvements.

The results indicate that the teaching intervention with carefully designed and challenging spatial activities helped children of lower (or no) strategies to improve their approach to perspective and rotation tasks, while the children with advanced empirical strategies remained mainly to the strategies they had. In general, the development of strategies is not similar to all children, nor linear: it is related to the tasks, the previous experience and the width of available repertoire of strategies. Children adapt relatively fast their strategies in cases they have initial, even not systematic experience, but they need active involvement in relevant tasks within a rich and challenging environment.

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# PROBLEM SOLVING IN GEOMETRY - COMPETENCIES IN COMPLEX CALCULATION AND PROOF

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Mathematics competence as described in educational standards worldwide usually encompasses abilities to perform mathematical reasoning and proof, and to apply mathematical rules and procedures in calculation problems. Students' difficulties in both types of problems have been well documented for decades. The aim of the present study is to clarify commonalities and differences between the two problem types in view of necessary content knowledge components.

Following ideas from psychological problem solving research, a task is considered as a problem for a person, if it cannot be solved straight by readily available solution schemata, procedural knowledge, or algorithms. Geometric calculation problems, accordingly, comprise problems that require multiple rule-based applications of geometric theorems and properties to derive the size of an angle or a length from given information. Explicit reasons are not considered a necessary part of the solution. For geometric proof problems, the presence of all ideas of the correct proof in adequate sequence, and not a formulation in formal symbols, is the relevant criterion for the acceptance of an argumentation (Heinze, Reiss, & Rudolph, 2005).

We conducted a correlation study with  $N = 230$  German 10<sup>th</sup> grade students (high attaining school track) and surveyed their abilities to solve complex calculation tasks (8 open items, not asking for supportive reasoning) and to construct geometry proofs (7 open items). As predictors, we used a geometric knowledge test consisting of 35 mathematical propositions from elementary geometry (grouped into 5 multiple-choice items) which had to be judged as true or false. As an additional control variable, we used a scale of simple single-step geometric calculation problems (4 open items) to survey procedural knowledge (see also Ufer, Heinze, & Reiss, 2008).

The measure of procedural knowledge did not show a significant relation to performance in proof and complex calculation problems. The ability to judge true propositions showed a significant relationship with both task types. The ability to judge false propositions showed a significant relation with proof performance, but only a small, weakly significant relation with complex calculation performance.

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# PRE-SERVICE TEACHERS' QUESTIONING AND THEIR ALGEBRAIC THINKING ABILITY

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The ability to ask questions to uncover student thinking is an important aspect of teachers' knowledge (Sahin & Kulm, 2008). At the same time, research shows that formulating these types of questions is difficult for both practicing and pre-service teachers (Moyer & Milewicz, 2002). Driven by the need to provide directions for teacher preparation programs, the study presented in this report examined the relationship between pre-service teachers' knowledge of algebraic thinking (AT) and the questions they posed to uncover students' algebraic thinking.

Based on their written solutions to 125 AT tasks, the 18 pre-service teachers in this study were divided into high and low AT ability groups. They were asked to probe middle school students' algebraic thinking during problem-based one-on-one interviews. Using an *a priori* coding scheme, we categorized the level of questions that the pre-service teachers posed during the interviews as (1) Checklisting; (2) Instructing; and, (3) Probing. We conducted quantitative and qualitative analyses to determine differences between the questions of the pre-service teachers in high and low AT groups.

Pre-service teachers' own algebraic thinking ability was strongly associated with their ability to probe students' thinking via questioning. Pre-service teachers in the high AT group consistently pressed their interviewees to explain their reasoning about the problem. Pre-service teachers in the low AT group primarily formulated questions to get their students to provide an answer without probing student thinking. Our results demonstrate a close relationship between the pre-service teachers' content (algebraic thinking) and pedagogical (questioning) knowledge. The results imply that teacher preparation programs may benefit from activities that develop content and pedagogical knowledge in ways that mutually inform and strengthen one another.

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# EXPLORING THE MATHEMATIC TEACHING CONCEPTION DEVELOPMENT OF PRACTICE TEACHERS

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This paper mainly explores the developmental processes about mathematics practice teachers' teaching conceptions. In this study, we adopt the case study method, including classroom observations and pre and post-lesson interviews, as the major approach of inquiry to investigate the developmental processes and the values of mathematics practice teachers about teaching conceptions. Six cases of practice teachers in different secondary schools, their mentors and students were involved in the final year study of a three-year longitudinal research project on the beliefs and values of pre-service teachers for secondary mathematics.

The selected teaching *critical incidents of practice* (Skott, 2001) which are collected through the classroom observations of practice teachers' teaching offer a kind of shock or surprise to us, and eventually, they may invoke practice teachers to make decisions for teaching. Then, their implicit principles of and reasons for such decision-making during teaching might become explicit.

Based on our data collected through classroom observations and interviews, we preliminarily addressed some reasons that led the teaching conceptions of practice teachers to change, adjust or maintain. Those include practice teachers' learning willingness, mentors' mentoring strategies and attitudes, teaching circumstances practice teachers engaged in and teaching resources they could utilize, and their professional competencies. The reasons about transformation or maintaining of practice teachers' conceptions in teaching were varied depending on the objective contextual conditions and subjective value judgments. Practice teachers' teaching conceptions were changing or maintaining at different periods reflecting their values upheld for teaching. We thought that there appeared to be some implications about activating the developments of practice teachers' teaching conceptions then to develop their professional competencies. First, the educative programs in universities should be appropriately revised to cultivate practice teachers' learning abilities, and bring out their correct attitudes and learning willingness in teaching. Next, mentors should understand the attributes of practice teachers fully, and then give them proper teaching suggestions. Finally, the teaching resources of schools should be matched up with the needs of practice teachers possibly so that they could display their intended teaching adequately.

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# PROCEDURE AND EXPLANATION IN MATHEMATICAL PROBLEM-SOLVING

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*The purpose of this study is to make clear the relationship between mathematical procedures that students tackle systematically in mathematical problem-solving and the explanation for saying mathematical procedures they tackle are all right by observing a mathematical classroom.*

In this study, “mathematical procedure” is considered as a variety of (physical) activities which students tackle systematically in mathematical problem-solving. What students learn symmetry of square through folding along the diagonal and bring the corners together is a notable example. And the term “explanation” will be used as expression to show us why the mathematical procedures are true. Moreover, “mathematical problem-solving” is considered as an activity in the following way:

Mathematical problem solving is a thinking process in which a solver tries to make sense of a problem situation using mathematical knowledge that he/she has, and attempts to obtain new information about that situation till he/she can resolve the tension or ambiguity about it (Nunokawa, 2010).

Over a period of many years, several articles have been devoted to the study of mathematical explanation (Ex. Steiner, 1978; Yackel, 2001). What seems to be lacking, however, is to make clear why students’ explanation do not easily combine with their mathematical procedure. To find a solution of the problem, I divide students’ explanations in two categories. One is *the explanation of procedures* by which a student reports exactly his/her (physical) activities. Another is *the explanation of reasons* by which a student tries to make his/her reasonable way of thinking. In geometry lesson I observed where students (grade 5 and 8) treated Origami, the teachers demanded their students to show them the two types of the explanation mentioned above. The most remarkable result is that being higher grade students does not necessarily link the two types of the explanation.

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# AN ANALYSIS OF SYMBOLIC REPRESENTATIONS EXPRESSING SITUATIONS OF MULTIPLICATION AND DIVISION IN ELEMENTARY SCHOOL MATHEMATICS

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The purpose of this study is to assess the effectiveness of symbolic representations expressing situations of multiplication and division in elementary school mathematics for acquiring multiplicative properties.

In order to accomplish the above purpose, this study analyzed the assumed student's procedure for problem-solving based on symbolic representations in situations of multiplication and division by applying the framework "The Theory of Conceptual Fields" (G.Vergnaud, 1982, 1988), as follows. First, this study expressed multiplicative situations with symbolic representations. Multiplicative situations are classified into six kinds; (i) multiplication of whole numbers, (ii) partition of whole numbers, (iii) quotient of whole numbers, (iv) rule-of-three problems that the scalar ratio is a whole number, (v) rule-of-three problems that the function ratio is a whole number, and (vi) rule-of-three problems that neither the scalar ratio nor the function ratio are whole numbers. As symbolic representations for expressing situations, two kinds of symbolic representations were selected; an array figure that is a discrete figure and a proportional number line figure that is a continuous figure. Second, this study clarified theorems-in-action for writing expressions of multiplication and division, which are mathematical properties used by students when writing those expressions, in the assumed student's procedure for problem-solving based on symbolic representations.

The results of analysis reveal that (1) only additive properties are used in the procedure for problem-solving based on an array figure. That is, multiplicative properties are not used spontaneously. In contrast, (2) multiplicative properties are used spontaneously, although it may be implicit, by focusing on the relationship of two measures in a measure-space in the procedure for problem-solving based on a proportional number line figure.

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# DESIGNING PROFESSIONAL DEVELOPMENT FOR MATHEMATICS TEACHERS: A CASE STUDY

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This paper describes the design and impact of a modest professional development (PD) programme for secondary mathematics teachers in an English local district. The aim of the PD was to support teachers incorporate constructivist approaches in their teaching. Broadly, constructivist pedagogy features dialogue and collaboration between students working on tasks and the construction of mathematical knowledge through social activity and engagement with the task.

The theoretical basis for the PD is the relationship between teachers' beliefs and practices (Ernest, 1991) and bringing about changes in those beliefs and practices. The PD programme is based on the principle that teacher development is achieved by offering opportunities for individual teachers 'to doubt, reflect and reconstruct' their own classroom experiences in unhurried, 'safe' environments (Wilson & Cooney, 2002, p. 132).

In response to our invitation to participate, eighteen schools (one or two teachers per school) applied to join the programme. In total 30 teachers attended the 6-day PD programme which spread over 10 months.

The evaluation of the PD was carried out using teacher questionnaires administered before and after the programme; student pre and post tests and learning attitude surveys (for a description of the instruments see Swan, 2006) and in-depth interviews and lesson observations with three of the teachers. Data were analysed using a thematic approach using sets of beliefs and practices as a guiding framework; differences and changes in practices were identified. The analysis suggests a change in teachers' beliefs and practices as a result of the PD. Interviews and observations of teachers suggest that the relationship between changes in beliefs and changes in practices is not straightforward, but is influenced by local contexts as well as the teacher's background, experience and disposition.

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# METHODOLOGICAL ISSUES IN RESEARCHING BIG IDEAS IN SMK & PCK

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Within our EU-funded project ABCMaths<sup>1</sup> we are probing for ways in which ‘Big Ideas’ might have been acting as a mediating artefact in our professional development work with teachers of A-level mathematics. We discuss elsewhere both the theoretical framing of our research (PME Short Oral submitted) and some of our initial research findings (PME Research Reports submitted). In this short oral we will present the methodology that lies at the heart of our study. We will invite discussion using Dowling and Brown’s language of operationalisation (2010), and argue the validity of research that seeks to operationalise the theoretical constructs associated with Big Ideas in terms of empirical indicators to be found principally in the developing discourse of our activity, but also in the shifting objects both of this activity and the classroom activity of course participants.

Researching the impact of professional development activity is widely regarded as challenging. Our focus on developing discourse seeks to make allowances for the way that the deliberate introduction of the language of Big Ideas will have constrained teachers’ activity at the same time as it has afforded access to new ways of teaching and thinking about mathematics; this language is introduced explicitly to participants both as mediating artefact and, in itself, as an object of reflection. Tutor-researchers and participants alike ask: How has this language affected my practice? How does(has) its use position(ed) my students? Through a process of elaborated description, to be exemplified in this presentation, we argue the validity of inferring the impact on practice (mathematical knowledge (SMK) and pedagogic content knowledge (PCK)) of the language of Big Ideas.

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# THREE TYPES OF TRANSFORMATION PATTERNS OF SOLVER'S PROBLEM REPRESENTATION LEADING TO PROGRESS OF PROBLEM SOLVING

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The phases of problem-solving process and its progress have been one of the focuses in the research domain of problem solving. From a cognitive and representational perspective, however, problem-solving process is often considered as constructional/transformational process of solver's mental and/or external problem representation.

The purpose of this study based on the above perspective is to relate the progress of problem-solving process with some constructional/transformational processes of solver's problem representation and to pick up three types of construction/transformation patterns of solver's problem representation, which could characterize the progress of problem-solving process. In order to describe and interpret solver's problem representation, this study use a model for a structure of problem-solving competence based on five types of mature internal representational systems (Goldin, 1998). Using those five categories of representational systems in this model, we could interpret solver's (especially mental) problem representation in detail and find its construction/transformation patterns that characterize the progress of problem solving. The patterns picked up in this paper are as follows.

*Construction/Reconstruction:* In solving word problem, for example, solver constructs initial parts of representation from problem statements, and then integrates them into one problem representation. Both processes are typical type of Construction. During problem solving, however, solver sometimes reconstructs her/his problem representation involved in her/his understanding or interpretation of the problem (statements). This process is a Reconstruction.

*Abstraction/Concretization:* Solver sometimes transforms a part of problem representation into more abstract/concrete representation. For example, transformation from concrete operative imagery to abstract pattern imagery in imagistic parts of problem representation might change mutually her/his problem-solving activity from concrete material operation to pattern exploration. This process is a typical type of Abstraction, and the opposite is Concretization.

*Shift:* Based on Goldin's model, we could assume that solver's mental problem representation is a linked complex of five types of representation. And, solver sometimes shifts the main activated representational system among five categories of them (e.g., from imagistic representation to formal notational one) to make progress.

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# RELATIONSHIP BETWEEN USAGE LEVELS OF “LOOKING FOR A PATTERN” AND “SIMPLIFYING THE PROBLEM” STRATEGIES

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Although there are many studies about non-routine problem solving strategies, studies which elaborate on the correlations between usages of strategies are very rare. As expressed by Hershkowitz, Schwarz and Dreyfus (2001), if the students solve a non-routine problem, they might be constructing a new (to them) phenomenon and reflecting on its internal structure, and on its external relationship to things they know already. Although routine problems can be used to fulfill particular didactical functions of teaching students to apply a certain procedure or a definition correctly; only through the careful use of non routine problems can students develop their problem solving ability (Stanic and Kilpatrick, 1988). Therefore, this study was aimed to investigate whether there is any link between elementary school students' implementation levels of “looking for a pattern” and “simplifying the problem” strategies. To this aim, one open-ended non-routine problem for each of these strategies were asked to a sample of Primary 5 (n = 60) and Primary 8 (n = 60) graders. All student scripts were coded as 0 (completely wrong), 1 (partially correct) and 2 (correct). Correlation and regression analysis results that were computed on the basis of this coding showed that a linear positive relationship does exist at fifth grade level. Celebioglu and Yazgan (2009) also found the same positive relationship between usage levels of “looking for a pattern” and “making a systematic list” strategies. However, their study had included second, third, fourth, and fifth graders. Differently, eighth graders were involved in our study and there was not any linear relationship at this grade level. But this does not mean that there is no relationship at all. There may be any other kind of relationship (quadratic, cubic, exponential, and etc) and this can be analysed in depth as a subject of future research in which more pupil participate.

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# WHAT DO ACTIVITIES DESIGNED BY PRE-SERVICE TEACHERS TELL US ABOUT THEIR COMPETENCY IN TEACHING MIDDLE SCHOOL MATHEMATICS?

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The mathematics education program designed and implemented in 2005 in Turkey (MEB, 2007), entailed revisions for the mathematics teacher education programs. As a result, pre-service teachers are required to take several courses including “Micro Teaching in Mathematics Education.” This course provides opportunities for pre-service teachers to develop and imply what they have learned from pedagogical courses. In this course, pre-service teachers could use their “pedagogical content knowledge” (PCK), which includes using the most powerful ways (e.g., representations, analogies, examples, etc.) so that students could comprehend the subject (Shulman, 1986). In this study, we aimed to explore classroom activities designed by pre-service teachers according to PCK elements. In particular, we examined (a) to what extend pre-service teachers can integrate “big ideas” related to the mathematical content into their activities, (b) to what extend they can design their activities in order to improve mathematical processes (i.e., problem-solving, reasoning, communication, connection), and (c) how they can select and apply manipulatives effectively in their activities. Forty five pre-service teachers participated in this study. They were asked to design and implement a micro teaching activity related to a mathematics concept. Data were collected through their written reports of activities and video-recorded observations of their implications. Preliminary findings suggest that pre-service teachers usually did not consider the big ideas related to the mathematical content when designing their micro-teaching activities. Even though most of them tend to use manipulatives, they were not competent in using them effectively. The findings also suggest that while most pre-service teachers were able to design their activities to improve students’ problem-solving skills, they did not put much emphasis on mathematical reasoning, connection, and communication in their activities.

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# EQUITY IN ACCESS TO QUALIFIED MATHEMATICS TEACHERS

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Achieving equity in mathematics education is a challenging though a paramount goal. National Council of Teachers of Mathematics [NCTM] perceives equity in terms of student achievement outcomes, treatment of students, and students' access to educational resources (NCTM, 2008). Equitable access and treatment imply comparable representation of students with various demographics at different levels of education and equitable access to quality curriculum, advanced courses, well-qualified teachers, or favorable school resources.

Mathematics education research in Turkey lacked studies that examined disparities in educational opportunities. Among school-related factors, research has provided evidence that teacher quality (e.g., experience and subject-matter preparation) is an important predictor of learning outcomes (Darling-Hammond, 2000). The present study examined opportunity gaps in access to quality teacher in Turkey using TIMSS 2007 data. This study's research questions were (a) If and to what extent is there differential access to qualified mathematics teachers based on students' SES in middle school? and (b) What is the relationship between mathematics teacher quality and student achievement?

Results showed SES-related inequities in access to qualified mathematics teachers among eighth-grade students. Low-SES students were more likely to be taught by mathematics teachers with less than 3 years of experience or without a degree in mathematics or mathematics education. On the other hand, years of experience and a degree in mathematics or mathematics education were found to be substantially related to student achievement. Low-SES students' teachers were also more likely to report lack of confidence in their preparation to teach various mathematics contents.

The current study illuminates the extent of inequities between low- and high-SES students in their access to qualified mathematics teachers. Moreover, by showing the relationship between teacher qualifications and student achievement, the present study shows Turkish policy-makers the importance of equitable distribution of qualified teachers in closing the achievement gap in middle schools.

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# HOW DO MATHEMATICS TEACHERS UNDERSTAND THE CONCEPT OF VECTOR?

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The development process of vector between linear algebra and secondary education differs. The concept of vector in linear algebra is based on axiomatic and abstract structure as an element of vector space. On the other hand, vector in secondary education is introduced as a directed line segment, algebraic operation using position vector. This research aims to find pre and in-service's Mathematical Knowledge for Teaching(MKT) about the concept of vector.

The purpose of the study was to investigate how pre-service and in-service teachers understand the concept of vector with eighty pre-service and one hundred twenty four in-service teachers performing five questions based on MKT's subdomain and nine participants for the semi-structured interviews.

The result shows that pre and in-service teachers tend to regard vector as not vector itself but an expression of vector. Also, the results show that pre-service teachers have stronger common content knowledge(CCK). On the other hand, in-service teachers have stronger specialized content knowledge(SCK), knowledge of content and teaching(KCT) compared to those of pre-service teachers.

Furthermore, the study proposes CCK, SCK and KCT about the concept of vector. CCK is the knowledge which understands vector as an element of vector space without defining vector as expression means (e.g. ray, line, and arrow). SCK is the knowledge which understands the meaningful essence of vector mathematically and knows the instruction order about various concepts of vector in teaching. KCT is the knowledge which knows how to answer with familiar examples and metaphors when students have difficulties in vector, zero vector and negative vector.

In sum, this research investigated the relationships between subdomains of MKT in that SCK and KCT were connected. Further, the finding of this study sheds light on the relationships between subdomains of MKT from data collection.

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# TEACHERS' ATTITUDES TOWARD STATISTICS<sup>1</sup>

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*This paper investigated the attitudes towards statistics of a sample of 115 statistics in-service teachers. The study explored the influence of certain variables on the attitudes towards statistics and its teaching. A survey and a Likert scale were used to collect information from participants. Analysis of variance was done to analyze the data. The results show that a degree in education and teaching experience are factors associated with positive attitudes of statistics teachers.*

Teachers' attitudes toward statistics might represent either an advantage or an obstacle in teaching (Estrada, Batanero, & Fortuny, 2004). To study teachers' attitudes we gathered information from 115 elementary and secondary statistics in-service teachers from two major cities in Colombia. A 56-item Likert scale was designed for this purpose. The first items correspond to the four dimensions suggested by Schau et al. (1995) in the SATS scale: Affect, Cognitive Competence, Value and Difficulty. The rest of the items explored two dimensions related to attitudes toward teaching (1) Pedagogical Content Knowledge and (2) Perceived utility of teacher preparation. We also collected information about demographic variables that might be related to teachers' attitudes toward statistics such as: city, educational level, gender, teaching experience, professional degree, and number of statistics classes taken.

Results revealed that, in general, teachers have positive attitudes toward statistics and its teaching. The items that received a better punctuation are those related to the dimension that Schau et al. (1995) called *Value* (example: statistics is useful for the typical professional). In contrast, the items that received the poorer punctuation were those associated with the dimension of *Difficulty* (example: I feel insecure doing statistics problems). Other items poorly assessed were associated with the dimension of *perceive utility of teacher preparation* (example: I have the academic training required to teach statistics). This means that although teachers have positive attitudes toward statistics and its teaching, they recognize that their training is weak.

Results also revealed that the variables *teaching experience* and *being professional in education* positively contribute to the teachers' attitudes toward statistics.

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# TEACHERS' SUBJECT MATTER KNOWLEDGE: UAE CONTEXT

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Teachers need to have a solid understanding of the subject they teach as well as knowledge of students and pedagogy in order “to be effective in choosing worthwhile tasks, orchestrating discourse, creating an environment for learning, and analyzing their teaching and student learning” (Lappan & Theule-Lubienski, 1994, p. 253). Subject matter knowledge is only part of the equation but it is positively correlated with student understanding and success as highlighted in the literature. In order to judge whether teachers have profound understanding of mathematics (Ma, 1999) the mathematics education field need studies revealing a profile of teacher knowledge. Considering the fact that United Arab Emirates (UAE) does not have such profile, it is crucial to characterize the quality of mathematics teachers in UAE context. The purpose in so doing is to investigate, understand and characterize the nature of the gap between mathematics teachers' current level of understanding of mathematical ideas and the ideal level of understanding. This will also help us quantify the gap and generate solutions to close the gap, which will inform mathematics teacher education programs, mathematics education researchers and mathematics teachers, both locally and globally. Having a database illustrating this gap will help these parties to work toward creating solutions for closing the gap between where teachers are and at what level they should be. Therefore, this ongoing research project (funded by UAE University, #31D000) aims to develop a profile of mathematics teacher knowledge in UAE context through an investigation of the following research questions: How do (primary and secondary) in-service mathematics teachers understand and think about some of the fundamental mathematical ideas in major mathematics strands (geometry, algebra, measurement, probability and statistics, and numbers)? How do their current understanding compare to expected mathematics teacher knowledge as identified in the relevant literature? What is the nature of the gap between ‘what teachers already know’ and ‘what they should know?’ What are the main components and characteristics of this gap? The data collection, ongoing analysis as well as some preliminary findings will be presented in this session.

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# POSTER PRESENTATIONS







# THE PRE-SERVICE TEACHERS' UNDERSTANDING OF TAXICAB GEOMETRY

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Geometry is one of the subjects that is developed intuitively before the school and plays important roles with these intuitions in arts, physical, biology and other disciplines as well as in entire mathematics beginning from the elementary school. In the Geometry course, the theoretical knowledge covered by Euclidean geometry alone cannot provide students with an opportunity to understand the real world (Baki, 2001; Ada&Kocayusufoglu, 2006). National Council of Teachers of Mathematics emphasized that it was indispensable that students study different geometries, so, axiomatic systems are understood. Taxicab geometry is very close to Euclidean geometry. In this geometry, the calculations of points, straight lines and angle measurements are the same as in Euclidean geometry (Krause, 1975). On the other hand, in Taxicab geometry, the distance between two points is defined.

In present study, it was aimed to determine how to facilitate explorations in Taxicab geometry the primary school pre-service mathematics teachers' when making a deeper understanding of concepts in Euclidean geometry. In line with this aim, it was focused on how the students made sense of the basic concepts of Taxicab geometry and how they related these concepts with Euclidean geometry. Additionally, it was examined where the pre-service teachers made mistakes at both geometries during this relating process. Adopting qualitative research methods, this study included 78 pre-service teachers attending to first grade at the primary school mathematics teaching program. The data of the study was collected through open-ended tasks. Consequently, it was found that the students' views related to Taxicab and geometric structures and relations developed, however, they were not successful at geometric interpretations of their mathematical knowledges. Moreover, it was seen that they had difficulty with reflecting the basic subjects such as distance between two points and absolute value to graphics.

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# THE EFFECT OF TEACHING MATHEMATICS BY USING ACTIVITIES ACCORDING TO PURDUE MODEL ON THE ATTITUDES OF NON-GIFTED STUDENTS

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Purdue model (PM) aims to improve the main thinking skills, individual understanding and mental and creative skills of gifted students (Feldhusen, Kolloff, 1986). If negative attitude of the students to the problem solving doesn't change, we can't expect them to be good problem solvers. (Çanakçı, 2008, quotation Conlrey, 1984). The purpose of this study was to design example activity about "Arithmetic of Conscious Consumption" (ACC) at 7 th grade non –gifted students (NGS) in our country and to investigate the effects of this activity on the mathematical problem solving attitudes (MPSA) of these students. Problem sentence can be expressed as: 'whether the activity which is developed according to the PM, in teaching ACC, have an influence to the MPSA of NGS or not? Sub-problems are as follows :

"Is there any significant difference between control (CG) and experimental group (EG) students' MPSA before and after the application?" and "Is there any significant difference between CG and EG students' MPSA according to their gender differences?"

In this study, we used pre post test model with a control group. While the unit ACC was taught by using activities developed according to the model to 12 EG students, the same subject was taught by using the activities in the National Education Curriculum (NEC) to 10 CG students. It is concluded that the lesson in which an activity designed according to PM was used, is more effective than the lesson in which the activities from the NEC was used, over NGS' MPSA. There is not a significant difference between CG and EG according to their gender.

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# CHANGE OF SOCIOMATHEMATICAL NORMS IN A CLASSROOM: LEARNING MATHEMATICS IN REAL-WORLD

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This study analyzes a change of sociomathematical norms in a classroom where students learn mathematics in real-world.

There are several norms in classroom where students interact with each other. In particular, Yackel & Cobb (1996) identify *sociomathematical norms* which are specific to mathematical activity and mathematical discussion. The norms in mathematics classroom form students' learning. It is therefore necessary for students to change and develop desirable norms so that they can learn mathematics in classroom more productively. As for sociomathematical norms, confliction or harmony can arise from differences between members who obey the previous norms and members who try to create new norms or to change norms. Through the process sociomathematical norms have been developing and changing.

Most of the previous studies illustrate sociomathematical norms in the classroom where students learn mathematics itself (e.g., McClain & Cobb, 2001). It is necessary to investigate sociomathematical norms and their changes in classroom where students learn mathematics in real-world. This investigation would contribute to improve the teaching of mathematics, and would also lead students to understand a role of mathematics in real-world.

This study observes an elementary school classroom for a certain period, and identifies sociomathematical norms and their changes. This study will present the identified norms and their changes peculiar to the classroom where students learn mathematics in real-world.

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# MULTIPLE REPRESENTATIONS OF FRACTIONS: PERFORMANCE AND BELIEFS OF ELEMENTARY STUDENTS

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As conceptual learning comes into prominence in mathematics education, then emphasis on multiple representations is increased. Dufour-Janvier, Bednarz, and Belanger states that

...multiple representations are an inherent part of mathematics; can give multiple concretizations of a concept; can help to decrease certain difficulties students have when trying to complete tasks; and are intended to make mathematics more attractive and interesting (as cited in Patterson & Norwood, 2004, p. 6).

Cognitive and affective developments of any mathematical concept are intertwined firmly. The casual relationship between achievement in mathematics and affective characteristics is important, for this reason the relationship is focused by mathematics educators all along. In this study, our aim was to investigate the 6, 7 and 8<sup>th</sup> grade students' performances of and beliefs about multiple representations of fractions with respect to their grade levels.

The sample was randomly selected from three elementary public schools in Erzurum, Rize and Trabzon. A total of 249 students were included in the study. The data was gathered through administering "Multiple Representations of Fractions Flexibility Test" developed by the researchers and "Beliefs about the Use of Representations Questionnaire" developed by Panaoura, Gagatsis, Deliyianni, and Elia (2009).

The results of the analyses revealed that there was a significant difference between the mean scores of the students in flexibility test ( $F_{2, 246} = 17.006, p < 0.01$ ), and these differences were found to be between 6<sup>th</sup> grade students with 7<sup>th</sup> and 8<sup>th</sup> graders through Scheffe test. There found to be no statistically significant mean difference with respect to beliefs questionnaire ( $F_{2, 245} = 1.362, p > 0.01$ ).

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# THE TEACHER'S ROLES IN PLANNING THE LESSONS PROMOTED ANALYTICAL THINKING

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Higher order thinking is a complex thinking including: analytical thinking, evaluation, and creative thinking (Resnick, 1987; Pogrow, 2005). Sternberg (1998) stated that the analytical thinking was a kind of problem solving ability in which the students would analyze situation, evaluate, make decision, compare, and find the difference in that situation.

The purpose of this study was to the teacher's roles in planning the lessons promoted analytical thinking. The participants in this research were teacher, participant teacher, school coordinator and 1<sup>st</sup> grade students from Kookham Pittayasan School in four-year professional development project implementing Lesson Study and Open Approach conducted by Center for Research in Mathematics Education, Khon Kaen University. Data were analyzed from collaboratively design research lesson, collaboratively observing the research lesson, collaboratively reflection on teaching practice.

The research findings revealed that: Teachers share the goals of lesson using Japanese mathematics textbook in order to planning the lessons promoted analytical thinking as following 1) designing instruction for students to the analyze situation by their own interpretation. An opportunity to comparison of similarities or differences of the ideas by themselves to decision and selected efficiency method which was an important characteristic of analytical thinking 2) designing the materials of everyday life for students as an important tool for supporting ideas and arguments in class discussions.

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# MULTIMEDIA MATERIAL IN CLASSROOM: MATHEMATICS TEACHERS' PERCEPTIONS

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The scenario of the present research is the “Workshop for the use of multimedia educational resources in Mathematics teaching”, which was offered to High School mathematics teachers. This poster presents some of the teachers’ perceptions about the use of multimedia resources in digital format, including videos, computer animations, interactive software, and audios. I emphasize how the nature of each resource can shape the production of knowledge. Teachers’ perceptions have been a topic of interest in last PME conferences (e.g. Chua, Hoyles & Loh, 2010; Beswick, 2009).

The Workshop consisted of two face-to-face meetings in which teachers investigated diverse digital resources and discussed their use for mathematics classroom. Teachers were required to use digital multimedia resources with their students after the first meeting so that they could (a) experiment with the possibilities offered by the resource they chose and (b) share these experiences during the second meeting.

A qualitative research approach was used (Denzin & Lincoln, 2000). The study focused on the teachers’ perceptions regarding the multimedia material and how they used it. Multiple data resources were used, such as field notes, video recording of the sessions, and e-mail surveys submitted to the teachers. By analyzing these resources together, through a process of triangulation, I hope to have minimized possible research biases.

The results of the research show that the nature of the different digital resources shaped knowledge production. The digital resources analyzed can be used within different perspectives and with different focuses. I hope this material may be integrated into class work so that the students effectively engage in experimentation and raise conjectures.

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# **A PROSPECTIVE MATHEMATICS TEACHER'S STRUGGLE WITH THE DISCONNECT BETWEEN THEORY AND PRACTICE**

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Most teachers view their student teaching internship as the most valuable part of their preparation, claiming that most of what they know comes from first-hand teaching experience (Feiman-Nemser & Buchmann, 1985). The purpose of this presentation is to share the story of Julie as she navigates this experience.

Julie is a middle level prospective mathematics teacher enrolled in an undergraduate teacher education program at a large urban state university in the South-Eastern region of the United States. In Fall 2010, she participated in a 2-day per week field-based practicum experience and a university-based mathematics methods course. The course and field experiences in the fall focused primarily on theoretical readings, field-based observations guided by those readings, and lesson plan development. In Spring 2011, Julie completed her full-time student teaching in the same 6<sup>th</sup> grade mathematics classroom where she completed her practicum hours in the fall.

Data for this paper consist of Julie's written assignments from the Fall 2010 mathematics methods course, observation field notes from student teaching, and transcribed interviews from both fall and spring semester. To analyse the assignments and interviews, the researchers read and re-read all narratives separately and collectively to identify general themes. Common themes from Julie's case include the following: philosophical differences between mentor and student teacher, the disconnect between theory and practice, and the question of whether this is the correct career choice.

The quotes from Julie's written work and interviews will be shared during the presentation to explore issues of disconnect. In particular, we focus on the lack of autonomy and flexibility experienced in this setting and possible implications for teacher education.

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# AN EXAM OF ERRORS MADE BY FUTURE SECONDARY MATHEMATICS TEACHERS IN A COMBINATORY QUESTION

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This report comes from a broader study that investigates undergraduate students' errors when solving items dealing with concepts of algebra, geometry and combinatory. The larger ongoing research project involves a total of 417 students from four Brazilian state universities in Bahia. The study incorporates error analysis as both an inquiry and instructional approach. It is an investigation approach when designing the larger research project, planning and analyzing results of the first and second diagnostic test (Cury, 2007, Bortoloti, Santos-Wagner & Bortoloti, 2010). It also incorporates a teaching approach when using ideas from Borasi (1996) and Pinto (2000) in the instruction intervention stages. We use Borasi's principle that error could become a springboard for learning and for using mathematics investigation in the instruction process. And we incorporate Pinto's idea that the error should become an observable one for students. In this work, we explore students' solutions and errors to the following combinatory question: *Three students arrive together at a city to participate in a congress and without having previous hotel reservation. They realize that in each hotel could stay up to two students. Knowing that there are only four hotels at the city, find the maximum number of possibilities of students' accommodation.* In order to solve this item the student should know and apply knowledge concerning counting, permutation, combination and array (Pessoa and Borba, 2010). Here we examine and discuss data from 50 students from 1<sup>st</sup> and 6<sup>th</sup> semester in this question when resolving it in the first diagnostic test.

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# EXPLORING SCHOOL CHILDREN'S 'EVERYDAY' MATHEMATICAL KNOWLEDGE

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It is argued that 'Everyday mathematics' (out-of-school mathematics) represents the functional aspect of mathematical knowledge that is available to all and not hidden. Some studies in the past have shown that children going to schools too gather everyday mathematical knowledge from their surroundings and also from their involvement in income generating activities (for example, Khan, 2004). These studies revealed how mathematics arises spontaneously in everyday activities and that situational variables often influence school students' spontaneous solution procedures. In this report we give some examples of the nature and extent of the knowledge of everyday mathematics present among school going middle-grade children living in a large low-income area of Mumbai city that has a vibrant household based economy. Most children living here are involved in the economic activities from an early age mostly in the family-run small scale manufacturing units like embroidery and zari work, garment stitching, leather work, bag-wallet-purse making, food delivery, etc. Some of the manufactured goods are not only sold in the local markets but also exported to other countries, especially in the middle east.

This report from the preliminary part of an ongoing research study presents two case-studies of children studying in grades V and VII from two municipal corporation-run schools located in the area and involved in zari work and garment making. Data was collected through semi-structured interviews and discussions. Zari work involves skills with needles and symmetrical use of expensive decorative raw materials (small in size, light in weight). The presentation will focus on children's knowledge of subitizing, symmetry, currency and use of variety of units mostly based on convenience and syntactic support from the prevalent practices in their work-domain. Interestingly, many such working students could handle multi-digit operations using oral mode but not in the written form (for example, flawed multi-digit representations: 'two hundred ten' written as '20010', etc.). Many of them did the calculations by considering the numbers as money. It is hypothesised that bringing together such everyday mathematical knowledge and school mathematics can pave way for skill development and effective mathematics learning.

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# PROSPECTIVE TEACHER WORKING WITH SEQUENCES

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In line with the recommendations of authors such as Kaput (2008), the mathematics curriculum in Portugal indicates that elementary school mathematics teaching must support the development of algebraic thinking. This work on early algebraic thinking requires that teachers have a broad knowledge of algebra and how to teach it in order to promote the development of students' mathematics reasoning and understanding. Pictorial sequences provide an opportunity to promote the ability to generalize and to use symbolic representations that may become progressively more formal through the use of algebraic language. Teacher education programs must provide prospective teachers with opportunities to reflect on the work that they must do in the classroom and to understand the approaches that support elementary school students in establishing generalizations based on their analysis of sequences expressed by justification schemes (Rivera & Becker, 2008). This study comes in the context of an Algebra course taught by the author, aiming to promote learners' algebraic knowledge and knowledge about teaching and learning algebra in the elementary school. The course involves 20 prospective teachers who responded to two questionnaires, one before and another after the course, involving multiple mathematical tasks, including tasks with pictorial sequences (Tasks 1 and 2, presented in the poster).

The poster includes prospective teachers' responses to these tasks, examining the generalization of relations that they identify and the representations that they do. It presents data from the whole class as well as the interpretation of three prospective teachers (object of case studies). In addition, the poster presents and discusses their work in solving a task that involves (1) the generalization of pictorial sequences, (2) the construction of pictorial sequences and their generalization, given only one term and (3) the analysis of an episode of a class with grade 2 students from which they explore a pictorial sequence and identify regularities. The results show that after the course, a larger number of prospective teachers generalizes pictorial sequences and uses the algebraic language to express these generalizations, which may assist the development of generalizations in elementary school.

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# WORKING WITH FUNCTIONS – DEVELOPMENT AND EMPIRICAL EXAMINATION OF A COMPETENCE STRUCTURE MODEL (HEUREKO)

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Successful understanding of and working with mathematical functions requires an ability to switch between different mathematical representations. During the first phase of our project a theoretical competence structure model concerning students' competence in switching between different representations of functions was proposed and empirically examined (Leuders et al, 2009). We extend this model to contain the algebraic representation. All in all, the following five representations are included in our current competence structure model: numerical tables (N), situational descriptions (S), graphs (G), and algebraic equation (A). As we examine the competence of students to switch between these representations, the propose dimensions include the following changes from one representation into the other and back: GA – graphic-algebraic, GN – graphic-numerical, GS – graphic-situational, NA – numerical-algebraic, and SA – situational-algebraic. To ensure a broad diversion of difficulties, the model was further extended by including different cognitive action demands in the students' tasks.

The main goal of the study is the empirical examination of the theoretical model, specifically how the proposed dimensions are represented in the collected data.

For the study 120 items were created. These consist of tasks, which each represent one change between representations. They were divided into smaller tests papers with 30 tasks each. Additionally, we employed a questionnaire for students, which measured several variables concerning their learning habits, opinion of math classes, etc. Furthermore, we measured the students' basic competencies concerning the work with functions. Teachers also received a questionnaire, containing questions about the class and a request to rate task examples by difficulty respective to their classes' abilities. 645 students of the 9<sup>th</sup> or 10<sup>th</sup> grade from 8 schools participated in the main part of our study.

The poster will contain graphic representation of the structure model, examples of tasks, a description of used methods as well as participants and first results.

Leuders, T., Bruder, R., Wirtz, M., & Bayrhuber, M. (2009). A competence structure model for solving problems by using mathematical representations. In Tzekaki, M., Kaldrimidou, M., & Sakonidis, C. (Eds.). *Proc. 33<sup>rd</sup> Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 416). Thessaloniki, Greece: PME.

# DEVELOPING STATISTICAL LITERACY: A DESIGN EXPERIMENT APPROACH IN MIDDLE SCHOOL

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The main objective of this poster is to present an ongoing research to enhance statistical education in middle school for further development of statistical literacy. Statistics plays a key role preparing young people to interpret and make critical evaluations about media information and to develop students' capacities for reasoning and communication, providing them with techniques and procedures. Different views and goals for Statistics teaching, the latest ideas about statistical literacy (Batanero, 2002; Watson, 2006), and the new Portuguese Mathematics Curriculum (Ponte et al., 2007) were considered to develop a teaching experiment within a three years intervention project implemented in two classes, and the follow up by the teacher/researcher for three academic years.

In the first phase (7th grade) tasks were implemented to detect potential problems in the organization and interpretation of dummy data, plan a statistical study involving real data and analyze the ability of application of statistics to everyday situations. The second phase (8th grade) included tasks aiming at studying the advancements made and examining how the statistical expertise is mobilized to other areas of knowledge and situations. The research will continue in 9th grade, implementing tasks of communication skills in probabilistic context and characterizing how students apply Stochastic reasoning to genetics and gambling situations.

Preliminary results show improvements in students' critical sense, however, statistical thinking, especially when the data relates to everyday situations, continues to have shortcomings. Statistical work with real data, proved to be fruitful taking into consideration the improvements identified in the written and oral performance of students, which meets the ideas of other researchers mentioned in the theoretical context. Results obtained in the first two years will be presented in a poster with pictures of the project phases and extracts of the implemented tasks.

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# DEVELOPING PROBLEM SOLVING: THE POTENTIALITIES OF DIAGRAMS USED BY YOUNG STUDENTS

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Elisa Pinto

EBI da Malagueira, Évora, Portugal

*This presentation refers to a one year research project aiming to analyse the potentialities of representations, namely diagrams, to promote the development of problem solving of 1<sup>st</sup> grade students. Four case studies of 6 years old students were elaborated based on data obtained by the analysis of their written productions and informal interviews. The students were regularly asked by their teacher to solve problems by themselves. They all were able to draw personal diagrams to solve the problems, and they used them as a tool for reasoning and for communication.*

The idiosyncratic and non conventional representations that students generate in presence of a mathematical problem, namely the diagrams, are essential tools to develop their thinking (Diezman & English, 2001). The use and combination of diverse representations can foster the development of students' capacity to solve complex problems (Goldin, 2002).

This poster refers to a study developed with 1<sup>st</sup> grade students who were asked to solve twelve different problems in the context of a flexible and stimulating classroom. The students' case studies revealed that the children used diagrams combining iconic and symbolic elements. They frequently used the iconic elements to interpret the problem, to give personal meaning to the situation and to unpack the mathematical structure of the problem. The symbolic elements were used to register the data of the problem and their relations (eg., arithmetic operations) and to express the final response and to communicate the solution of the problem to the colleagues. The poster illustrates these conclusions by exhibiting and analysing four diagrams produced by the students in the context of a problem solving situation.

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# MENTAL COMPUTATION WITH RATIONAL NUMBERS: AN EXPERIENCE WITH GRADE 6 PUPILS

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This study aims to analyze mental computation strategies, with rational numbers, used by grade 6 pupils, and understand how short lessons with mental computation activities and the discussion of these lessons and strategies used by pupils can be useful to develop mental computation abilities.

Several studies show that pupils have great difficulty in working with rational number concepts, representations, and operations. Some authors (Callingham, 2004; Caney & Watson, 2003) stress the importance of mental computation in the development of the number sense, of critical thinking and the capacity to make estimates. Recent research McIntosh (2007) underlines the importance of understanding the strategies pupils use in mental and written computations, and teachers can use these results to teach efficient calculation strategies.

This study is based on a teaching unit involving mental computation tasks. These tasks will be used in short mental computation lessons. After doing the tasks, the students will have the opportunity to present and discuss their strategies. Two grade 6 classes from two different schools will participate in this study. Data will be drawn from four cases studies of students, two from each class, collected through individual interviews, video and audio recording of the lessons and researcher's observation notes.

The poster presents in a suggestive way how the research will be done. The information is organized in text boxes and diagrams that show the research methodology and specific techniques used, and some of the mental computation strategies that pupils may use. Some preliminary data from pilot trials will be presented.

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# STUDENTS' DIFFICULTIES IN DIRECT AND INVERSE INFERENCES INVOLVED IN GLOBAL INTERPRETATION OF GRAPHS

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It is found in the literature that global interpretation of graphs are the most difficult to be solved by students (e.g., Janvier, 1978). Carvalho (2008) confirmed these findings in the literature and presented further evidence that students from English schools found easier to solve direct inferences problems than inverse inferences ones. This study replicated the Carvalho's experiment among 30 Brazilian students. The aim of this study was to investigate whether the difficulties observed among students from English schools to interpret graphs involving inverse inferences would also be observed among Brazilian students solving the same type of problems. The participants were 7<sup>th</sup>, 8<sup>th</sup> and 9<sup>th</sup> grade from a private school located in Recife. They were asked to solve problems on points or lines graphs. The research tasks were managed to include some types of problems discussed in the literature. Different difficulty levels of questions were posed: five questions required a local interpretation; four questions demanded students to carry out interpolations; nine required a global interpretation of the graph; and one involved the calculation of average. For the four global questions were reported two types of inferences: direct and inverse. The findings indicated that the students' difficulties about the global interpretations of graphs were related to those questions which involve inverse inferences. The results obtained showed some similarities and contrasts with respect to findings obtained in England.

## Acknowledgment

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# A LATENT GROWTH MODEL: LONGITUDINAL INVESTIGATION OF STUDENT ACHIEVEMENT IN MATHEMATICS AND SCIENCE

Sevket Ceyhun Cetin; Mehmet Sencer Corlu; Mary Margaret Capraro; Robert M.  
Capraro

Texas A & M University

*The relationship between mathematics and science is generally validated by common sense. There is a need to empirically show how student growth is affected as students' transition to high school when the role of mathematics in science courses increases considerably. The purpose of this study is to investigate the causal relationship between mathematics and science objectives that are tested on state-wide student achievement exams in Texas, USA and how this relationship changes over the high school years. Preliminary results over four time points in mathematics and two time points in science, spanning 8<sup>th</sup> to 11<sup>th</sup> grades, showed a greater growth in mathematics than science and a weaker correlation between the two content areas for under-represented ethnicities.*

## INTRODUCTION

American decision makers are investing a significant amount of resources to increase the quality of education in science and mathematics education to compete in tomorrow's world of innovation. An integrated curriculum emerges as an important aspect for reform in mathematics and science education (Czerniak, Weber, Sandman, & Ahern, 1999). There is a need to understand which objectives in school science and mathematics are related and how student achievement is affected by this relationship.

## METHODOLOGY

Data for the longitudinal study came from the Texas Assessment of Knowledge and Skills (TAKS), the state-wide standardized test. The participants were selected from a highly diverse independent school district in a central Texas town with a population of over 100,000 people. A sample of  $N=116$  students ( $n_{\text{Female}}=58$ ,  $n_{\text{Male}}=58$ ) were followed longitudinally from 2006 – 2010. Students' scale scores in mathematics and science were analysed by using descriptive, inferential and causal model methods. Students' demographical information (gender, ethnicity, special education, ESL, etc.), as well as their school grades were included in the SEM model as time (-invariant and -varying) covariates to determine the best model fit.

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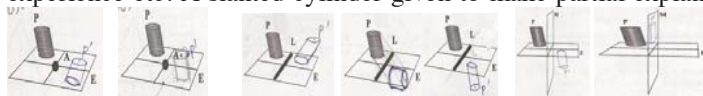
Czerniak, M. C., Weber, W. B. Jr., Sandman, A., & Ahern, J. (1999). A literature review of science and mathematics integration. *School Science and Mathematics*, 99, 421-430.



# A STUDY ON THE PERFORMANCE AND STRATEGIES OF PROBLEM SOLVING IN SPATIAL SYMMETRY FOR PRIMARY SCHOOLS TEACHERS

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Zi Qiang                      National Taiwan      National Taipei University of Education  
Elementary School,      University,

The purpose of this research was to explore elementary schools teachers' performance and the strategies (consisting of correct and error thinking) of problem solving in point, line, and plane symmetries. Owing to the content knowledge of teacher could affect the students' learning, the findings will be as the curriculum design of teacher professional development. A total of 60 subjects were from primary schools in Taoyuan County, they were conducted a written test of spatial symmetry first, then 10 subjects of them were selected from two kinds of backgrounds (science vs. non-science) to be as interviewees. The result was showed as follows: (1) There was no significant difference in performance for gender, but significant difference in performance for background. (2) The comparison of three symmetries of object in 2 dimensional space, a) the correct response rate of the line symmetry(87.78%) was significantly higher than that of the plane symmetry(78.17%) and that of point symmetry (76.25%), (3) The comparison of three symmetries of object in 3 dimensional space, the correct response rate of the line symmetry (28.64%) was significantly lower than that of plane symmetry (54.39%), and the correct response rate of the plane symmetry was significantly lower than that of point symmetry (71.43%). (4) In whole, a) the correct response rate of the line symmetry (62.76%) was significantly lower than that of the plane symmetry (65.71%) and that of point symmetry (74%), b) the correct response rate of symmetry concept for objects in 2D space (82.07%) is significantly higher than that of symmetry concept for objects in 3D space (48.74%). (5) The strategies of problem solving for symmetry: a) by definition, b) by intuition, c) by folding, d) rotation method, e) orthogonal method, f) split-half method, g) parallel method, h) prolonging method and so on. (6) Error types from some of above strategies: a) confusion on line symmetry and plane symmetry, b) confusion on point symmetry and line symmetry, c) confusion on plane symmetry and point symmetry, d) all symmetric concepts in 2D were over extension to 3D, e) any two positioned congruent objects can be overlapped on rotation, folding, or reflection. f) intuition based on life experience etc. A slanted cylinder given to make partial explanations for error types:

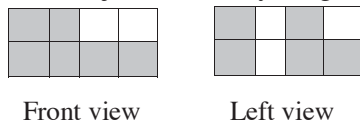


The error rate 23% (error type 6d) and 7% (error type 5g) for first two point symmetry, 58% (error type 6a), 8% (error type 6d), and 8% (error type 6b) for next three line symmetry, 7% (error type 6a) and 5% (error type 6f) for last two plane symmetry.

# A STUDY ON THREE DIMENSIONAL VIEWS OF SPATIAL ORIENTATION FOR PRIMARY SCHOOLS TEACHERS

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Primary School

The purpose of this research was to investigate the spatial ability in three dimensional views of spatial orientation for primary schools teachers. The methodology of this study was a questionnaire survey consisting of three subscales: 3 dimension to 2 dimension organization, 2 dimension to 2 dimension organization, and 2 dimension to 3 dimension organization. 56 teachers from Tao-Yuan county were conducted a written test (Cronbach's  $\alpha$  coefficient was 0.79) first, then 10 teachers from them were selected by their performances as interviewees to find out their problem-solving strategies and error patterns.. The main findings of this research were as follows: 1) there was no significant interaction in spatial orientation ability for fuzzy clusters and teachers' educational background, but there were significant differences in spatial orientation ability for both of fuzzy clusters and teachers' educational background. 2) There was no significant interaction in spatial orientation ability for fuzzy clusters and teachers' gender, and there was no significant difference in spatial orientation ability for gender. 3) There was no significant interaction in spatial orientation ability for teachers' gender and teachers' educational background. Correct problem-solving strategies in each of three subscales were as follows: 1). For 3 dimension to 2 dimension organization: a) direct contrast, b) viewing method of top point, c) counting the blocks. 2). For 2 dimension to 2 dimension organization: a) order method, b) characteristic method, c) counting method. 3). For 2 dimension to 3 dimension organization: a) cross-reference method, b) arranging method by numeric order. Error patterns could be classified into 5 types: a) cross-reference in brain to make confusion, b) response by intuition and without checking, c) without considering the effect in which the former blocks hid the later blocks, d) only considering some views of the whole objects. e) no order to process row by row or column by column etc. Using an example given two views of an object to show some correct strategies of cross-reference to get possible maximum blocks of this object. Similarity to get possible minimum blocks, but not presented here.



(1)Using front view first, then left view

(2) Using left view first, then front view

(1) (2)

2	2	1	1	→	2	2	1	1	2	2	2	2	→	2	2	1	1
2	2	1	1		0	0	0	0	0	0	0	0		0	0	0	0
2	2	1	1		2	2	1	1	2	2	2	2		2	2	1	1
2	2	1	1		1	1	1	1	1	1	1	1		1	1	1	1

# CREATING A CONDITIONAL PROPOSITION ON CO-VARYING GEOMETRY SETTING FOR YOUNG STUDENTS

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University

Jian-Cheng Chen

MingChi University of  
Techonology

We take a closer look at how a theorem be retrieved to solve a geometry question. A geometric theorem is usually introduced in a simplified and specified context in school lessons. It makes our students fall into a “knowing in simplified and applying in complex” situation. Thus many students fail in retrieving a learnt theorem to solve geometry problem. In this study, we want to explore wheher or not and how young students creating a conditional proposition in a complex geometry setting.

There are 15 grade 7 students joint our experiment. We develop 3 exploring tasks on dynamic geometry software. Every task provides a false proposition with one supportive example. The tasks ask students to make as many as examples by dragging, to observe the diagram and record the measures of angles and sides shown on the screen, and to revise the given proposition to be a true one. In these tasks, the students have to modify the ‘premise’ to be the sufficient condition of the given conclusion. In order to provoke students' effort in exploring revised proposition, we design the tasks in unfamiliar geometric context. All the propositions are unfamiliar too.

The result show that 11/15 students succeed in their tasks. The students’ performance shows that there are two kind of successful approaches. One is named *measure relation validating* approach. Firstly they drag and record 3 or 4 random trials. They find out some co-varying relations between the measurements on the recorded table. They then create an inner conjecture and validate it by generating more examples systematically. Most important is that when they make systematic examples, both supportive and counter examples are considered. Hence their final finding is in the format of “if P then Q, if not P then not Q”. That is, their revised proposition is essentially a sufficient and necessary proposition. The other approach is named *figural condition validating* approach. At first they drag the point continuously and make the diagram changed dynamically 2 or 3 times. Some co-varying relations are recognized and one inner conjecture is produced. They then move the point ‘slowly’ and near to the ‘critical position’ of goal condition. In this process, the students may move the point pass through the critical position and return back in order to validate their own conjecture. As the students in co-varying measure validating approach, students in dynamic figural validating approach also create a sufficient and necessary proposition.

In summary, our experiment shows that grade 7 students are able to recognise and graspe the co-varying relation in a complex geometry setting and creating a conditional proposition. This result may applied in redesigning our geometry learning activities.

# EXPLORING STUDENTS' MATHEMATICAL PROFICIENCY IN THE CONTEXT OF INQUIRY-BASED TEACHING

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**Introduction-** Mathematical proficiency composes of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (NRC, 2001). Mathematical representation, attentive use of mathematical language and definitions, articulated and reasoned claims, rationally negotiated disagreement, generalizing ideas, and recognizing patterns— are practices of mathematical proficiency (RAND Mathematics Study Panel, 2003). Inquiry is also central to mathematics. It is a diverse discipline that include of data, measurements, and recognition of patterns. It involves observation of patterns, testing of conjectures, and estimation of results (Jarrett, 1997). Consolidation those reference. Practice of mathematical inquiry can help students to develop mathematical proficiency. It is easier that mathematical proficiency of students emerged from mathematical inquiry context than traditional context.

**Method-** Researchers considered the samples who have 21 boys and 14 girls taught in mathematical inquiry teaching more than two years. We use case study to understand their unique and complex in depth. Our data include classroom video, interview of students and learning recording. Classroom video is video of class , the video of group in mathematics problem-solving, video of the group interactions, and other classroom activities. Interview of students is semi-structured interviews. If researchers interest someone or some questions, we would interview over class. Learning recording are the practices of mathematical inquiry that teacher gave. Researchers would take picture after students work finally.

**Result-** Students' mathematical proficiency were better than better in mathematical inquiry teaching. Students can conceptual understanding through different strategies, and productive disposition can infected through interaction of students. Finally, mathematical inquiry should be used long-term worthily.

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# TEACHER SUPPORT FOR COLLECTIVE ARGUMENTATION

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*Our analysis of a secondary student teacher's practice suggests that teachers use multiple practices when supporting collective argumentation. Furthermore, some of these practices are more effective than others in eliciting particular parts of arguments.*

In 2002, Yackel called for increased attention to the role of the teacher in collective argumentation. Since then, researchers have examined the role of the teacher in classroom communities (e.g., Hufferd-Ackles, Fuson, & Sherin, 2004) and in facilitating discussions about students' work on appropriately high-level tasks (e.g., Stein, Engle, Smith, & Hughes, 2008), but research involving collective argumentation has maintained a focus on student learning, including documenting how ideas come to be taken as shared in classes (e.g., Rasmussen & Stephan, 2008).

As part of a larger project, we observed secondary student teachers in to investigate how they supported collective argumentation. Bridgett, one of the student teachers, supported collective argumentation by engaging in several practices, some of which align with those described by Hufferd-Ackles, et. al (2004) and Stein et. al (2008). Bridgett's support for argumentation included asking questions to prompt parts of arguments, drawing diagrams to prompt or respond to student's contributions, recording student statements in writing on the board, and contributing parts of arguments herself. Our initial analysis suggests that Bridgett asked several different kinds of questions, leading to students contributing different parts of arguments. (Our analysis of arguments follows Toulmin's 1958/2003 model.)

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# INTEGRATION OF GRAPHIC CALCULATOR ON STUDENTS' MATHEMATICAL ACTIVITY

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*The poster aims to present a study concerning the integration of the graphic calculator on secondary students' mathematical activity while they work with functions.*

The graphic calculator is compulsory in secondary mathematics education in Portugal, however we don't know much about its use by students and teachers. The main objective of this research is to understand how students integrate the graphing calculator in their mathematical activity and its role in the learning of functions.

The theoretical framework involves three domains: the instrumental approach, namely the process of instrumental genesis, with respect to the graphic calculator (Trouche, 2004); the learning of functions and the role of multiple representations in conceptual understanding.

The research methodology is within the interpretive paradigm, following a qualitative approach, with a case study design. Data collection started in the previous academic year, with the students on grade 10<sup>th</sup>, and extends through this year, with the students on grade 11<sup>th</sup>, during the study of the topic Functions. Data are collected through classroom observations, clinical interviews with four students, who are the cases, and some written documents produced in the classroom by these students.

The preliminary results show that the instruments produced by the students from the graphic calculator depend on several factors, of which we stress the instrumented schemes that are socially encourage in the classroom, the students' level in the process of instrumental genesis, and their mathematical knowledge. The results also show a tendency for students to use the graphic calculator when are unable to solve a problem analytically. A graphical display will describe this study in terms of the main goals, theoretical background, methodology and examples of students' work with the graphic calculator in some tasks involving functions.

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# THE FUNCTIONAL GRAMMAR OF MATHEMATICS CLASSROOM DISCOURSE

Elizabeth de Freitas & Betina Zolkower

Adelphi University & Brooklyn College

This poster reports on findings from a 3-year, exploratory, qualitative case study of a mathematics lesson study group for beginning mathematics teachers working in under-resourced 6th through 9th grade classrooms in New York City. In this paper we focus on some of the linguistic and semiotic constructs that have proven effective in analysing the unique grammatical and diagrammatic challenges of mathematics teaching and learning. Drawing on transcript and artefact data from our lesson study groups, we show how teachers developed insight into specific linguistic facets of mathematics discourse. We draw on systemic functional linguistics and modal reasoning to study how these constructs function in mathematics classroom discourse.

We focus on two fundamental facets of the functional grammar of mathematics discourse: **(1) Tense/temporality:** One of the most persistent disconnects between everyday language and the mathematics register is the role and representation of temporality and tense. We show how we explored this issue with teachers – using stills and transcripts from video data. We also use transcript data to show the complex role of conditionals, subjunctives, and past, present and future tenses in classroom discourse. Our findings focus on diagramming tools for decoding tense and temporality. We also show how teachers struggled to decode word problems in terms of the relationships between the time of the utterance, the time of the event, and the time of the verbs. **(2) Negation:** Negation operates differently in everyday and mathematics registers. Research participants explored the many different semiotic devices for signifying negation in different mathematics content, and discussed negation of quantifiers and implication. We found that negation functions in complex and diverse ways in mathematics classroom discourse. Negation can be found in (1) exclusion (the use of “not” in terms of set membership - “not a polygon”), (2) inverse operations (multiplying “cancels” division), (3) indirect reasoning (assuming the negation of what one wishes to assert and relying on the law of the excluded middle), (4) opposition (negative numbers as “opposites” of positive numbers), (5) duality (operators that generate complements), (6) appraisal (assertions of wrong, false, no). Our findings also point to how teachers struggle with the negation of quantifiers in mathematics classroom discourse, and we show how this issue maps onto particular discursive habits in the classroom. In this poster we itemize and elaborate on these different forms of negation, and present evidence from the lesson study sessions to show how it manifests in mathematics discourse.



# THE RELATIONSHIP BETWEEN THE BELIEFS ABOUT THE NATURE AND TEACHING OF MATHEMATICS OF THE PROSPECTIVE ELEMENTARY TEACHERS

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The beliefs of the mathematics teachers have a strong effect on the mathematics teaching (Ernest 1989). Furthermore, to determine how the students learn mathematics at both the east culture and western culture, the belief of the teachers about learning and teaching mathematics have a predominant role (Perry, Wong, &Howard 2006). For this reason, the recent studies about mathematical beliefs concentrated on the determination of mathematical beliefs both of the students and the teachers and the effect of these beliefs on their instructional practice (Boz 2008). In the present study, determination of the prospective elementary teachers' beliefs about how the teaching of mathematics should be and the nature of mathematics and to point out the relationship in between these two issues are aimed. The research was conducted in 2010-2011 spring term with 96 students of Department Primary Education in a state university in Central Anatolia. In this study, the description of the Ernest (1989) and Gorman (1991) about the mathematical beliefs are used. For data collection a scale for the "Beliefs about the teaching of mathematics" and "Beliefs about the nature of the mathematics" which was developed by Baydar (2000) were used. Data analysis involved descriptive and inferential statistics. A significance level of 0.05 was set for all inferential tests. The analysis of the data is still going on.

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# TEACHERS' PRATICES AND NUMBER SENSE DEVELOPMENT IN ELEMENTARY SCHOOL

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*In the context of a collaborative setting it was carried out a study on primary teachers' practices when teaching the topic Number and Operations. This Poster presents preliminary results of two case studies, underling teachers' decisions and the challenges they face in the classroom.*

In Portugal, the official guidelines for the teaching and learning Numbers and Operations are going through significant changes. Number sense development has been emphasized, as well as new perspectives about algorithms. These ideas challenge primary teachers' practices, quite often, since they have to develop different approaches to number and operations. To achieve these curricular goals it is important that teachers reflect on the nature of the tasks they propose, and on their potentialities to develop student's understanding, and how to orchestrate whole-class discussions (Stein, Smith, Henningsen & Silver, 2009).

The first author of this proposal conducted a study in the context of collaborative setting that involved two primary teachers. This study, adopts a case study design, and focus on how teachers plan and conduct lessons, and how they construct and implement numerical tasks having as background the idea of hypothetical learning trajectories (Simon, 1995).

In this Poster we will present preliminary results from the two case studies, focusing on: (1) teachers' decisions when they prepare hypothetical learning trajectories, and (2) the challenges they face in the classroom when they propose the numerical tasks to students. The information will be presented in a graphical format including schemas to summarize the learning trajectories and the words that teachers used to explain their decisions and the challenges they faced.

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# **PRESERVICE ELEMENTARY TEACHERS' PERCEPTIONS ABOUT PARALLELOGRAM**

Esra İymen

Gülsinem Pakmak  
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Asuman Duatepe Paksu

*This study is part of an ongoing study that attempts to identify geometry content knowledge of elementary preservice teachers. 45 elementary preservice teachers were interviewed on a parallelogram task. Findings have revealed that these elementary teachers do not possess sound understanding of parallelogram and class inclusion between parallelogram and trapezoid.*

Research studies have emphasized the essential role of teachers' content knowledge in mathematics learning. With this motivation, this study investigated the preservice elementary teachers' geometry content knowledge with focus on parallelogram.

45 elementary preservice teachers who had completed all mathematics and mathematics education courses in their teacher training programme have involved in the study. They were interviewed on a 5<sup>th</sup> grade level task including the knowledge that preservice elementary teachers were expected to possess. The task included a multiple choice question that involved selecting parallelogram between alternatives. Their responses were prompted by further questions about the causes of their answers.

Although the level of interview task was low, the findings revealed that only 82 % of the preservice elementary teachers answered it correctly. Furthermore 21 % of them tried to justify their correct responses with some incorrect explanations. For example some of them had an idea that having only one pair of parallel sides is enough to claim a quadrilateral as a parallelogram. With this perception, they thought that trapezoid is a parallelogram. In addition, some preservice elementary teachers did not identify some quadrilaterals and interpreted relations between the quadrilaterals incorrectly.

Detailed results with suggestions will be displayed in the poster.

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# EXCURSIONS TO AND FROM SEMANTIC OBLIVION

David Easdown

University of Sydney

*This poster highlights the fragility of language and mathematical formalism in communicating ideas. **Syntactic reasoning** lies at the superficial end of the spectrum, involving formal manipulation of symbols, simple rules and substitutions. **Semantic reasoning** is deeper, drawing conclusions from underlying meanings and heuristics. Illuminating the tension and interplay between these complementary modes of reasoning, and heightened awareness, may enhance approaches to successful learning, improve morale and attitudes, and lead to more robust outcomes.*

## Introduction and Background

Apart from Easdown (2009), there appears to be no educational literature exploring the mysterious interplay between syntax and semantics in the way students respond to mathematical ideas, perform calculations and construct arguments. On the other hand, there is a long and rich tradition in mathematics stemming from Euclid's axiomatic method, which developed into a paradigm for formal mathematical argument. This led to Hilbert's questions about decidability by essentially syntactic means (formal manipulations of axioms and substitutions) and Gödel's famous (negative) answer that the gulf between syntax and semantics cannot be bridged in sophisticated mathematics such as number theory or geometry. The modern theory of formal languages, sketched briefly, motivates the terminology of this poster.

## Examples and Discussion

Nuanced examples have been chosen carefully to develop the theme that students use both types of reasoning. The contrast is most stark when they fall into error. Counting legs of an elephant (a variation on the famous trident illusion) demonstrates how almost imperceptible shifts in syntax can lead to incomprehension or confusion. The Rule of Seventy provides a narrative and a variety of possible twists and turns and pitfalls. This leads to other examples concerning fractions and a conjecture relating to threshold concepts, in the sense of Meyer and Land. Examples involving proof and higher algebra are offered, suggesting that the phenomenon under discussion is universal and applies equally to gifted students and experienced mathematicians.

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# CHILDREN'S PERCEPTIONS OF SYMMETRY THROUGH THE WINDOW OF A DYNAMIC GEOMETRY ENVIRONMENT

Susan K Forsythe

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This study was designed to investigate how 13 year old students reason about the properties of 2 dimensional shapes while working in a dynamic geometry environment. The students generated a number of different shapes by dragging two rigid bars which formed the diagonals of a quadrilateral.

A number of dragging strategies have been previously described; for example students use a random dragging strategy in order to investigate which aspects of a figure change and which stay the same (Arzarello et al, 2002). When students are able to predict the result of their actions on the screen they use a proactive dragging strategy (Hollebrands, 2007).

In this study, students used random dragging and proactive dragging to generate shapes such as a kite or a rhombus. Next, they used the Measure menu to check the properties of equal sides or equal angles. Usually the measurements were not exactly equal and then the students were observed to use fine movements of the dragging tool to adjust the position of the bars to make sides and angles equal. I refer to this dragging strategy as 'refinement'.

When the students used proactive dragging and when they used the refinement strategy they appeared to use the symmetry of the shape to help them. They were also observed to drag one bar along the length of the other while maintaining symmetry. A preliminary conclusion from this study is that it may be more intuitive for children to focus on symmetry first and to develop their conceptions of the properties of the shape from this.

The poster will use a pictorial format which will include some screen shots of the computer activity, in a series of positions, to demonstrate the dragging strategies the students used. Text will be placed next to the screen shots to describe the activity and to link the observations with the theoretical background to this study.

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# HOW MATRICES ARE TREATED IN THE NEW GRADE 10 MATHEMATICS TEXTBOOK IN IRAN

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In 2009, the grade 10 mathematics textbook in Iran was changed drastically following the change of grade 9 textbook of 2008. The authors of grade 10 textbook have stated that the main focus of change has been on making “connections between mathematics and science and between mathematics and real world” (in the “Forward” of the book) This statement is especially important since most of the grade10 students- with the exceptions of humanity strand of theoretical branch and vocational branch- math 10 is obligatory course with a national textbook and with a bit of tolerance, the text could be considered as a mean towards mathematical literacy.

In a study, we did a content analysis of grade 10 math textbook taking the above assertion into consideration and in this paper; we only report our findings regarding “Matrices and its application in the real world”. For the analysis of this part, we used Stillman’s (2001) problem classification as **injudicious** problems, **context-separable** problems, standard **applications**, and **modeling** problems.

The content analysis of the matrices showed that only 1% of the problems fell into the “injudicious” category. However, 95% of the matrices problems were in the “context- separated” category. Further, no more than 4% of the problems belonged to the “standard application” category and finally, there was not even one single problem that we could identify it as a “modeling” problem. The findings have raised a major concern about the assertion that made by the authors/ curriculum producers about the main reason of change as to make “connections between mathematics and science and between mathematics and real world”.

Key Words: Grade 10 mathematics textbook; matrices; mathematical connections; problem classification.

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# HOW STUDENTS CONCEIVE THE DIFFERENCE BETWEEN VARIABLE AND PARAMETER; THE CASE OF ALI

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Before we started tutoring a third year university student majoring computer science, we never believed that understanding the difference between **variable** and **parameter** would cause difficulty for even university students! This experience was unique for us in many different ways and this uniqueness encouraged us to do a case study about this young learner's mathematical learning. We like to call our case Ali; a third year computer science student. After consulting with last author (our supervisor), she suggested to design our research as a case study and pilot it with Ali's case. Further, to have more fruitful discussions and reflections during and after the tutorial sessions, we chose to do group tutorial with Ali's permission. And so, our expedition started! In this paper, we share the development of our research question and hopefully in the following papers, we contribute our findings to the community.

To better picture our case, it is helpful to indicate that Ali had a very sophisticated memory. He could look at long formulas and memorize them all, and use them in new situations. However, to our surprise, he had severe difficulties with derivative of functions when variables and parameters were involved. For instance, Ali knew that the formula for the derivative of  $e^{nx}$  was  $ne^{nx}$  but in his view, the derivative of  $e^x$  was  $xe^x$  as well! Further, we noticed that every time we change  $x$  to any other letter such as  $s$  in any differentiable function, Ali was unable to take the derivative. Because for him, variable was  $x$  and nothing but  $x$ . Ali did not realize that every other letter could act as a variable depending on its status. To give another example, he knew that the derivative of  $x^2 + 2x$  is  $2x + 2$ , but for Ali, the derivative of  $s^2 + 2s$  was 0 because there was no  $x$  in it and  $s$  was only like a number- a parameter and thus, its derivative had to be 0.

With these data, we shaped our research question as "how students conceive the difference between variable and parameter" and we like to continue working with Ali as our case.

**Key Words:** Variable; parameter; derivative.

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# THROUGH THE LENS OF IMAGINATIVE EDUCATION – AN AFFECTIVE VIEW OF STUDENT ENGAGEMENT

Pamela Hagen

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This poster presentation will present in a graphical manner of text and photographs, results from a study which examined the issue of student disengagement from learning mathematics. The poster has seven sections which detail the research question, research context, theoretical framework, research methods and data collection, results, themes, implications and recommendations and references.

The research question at the centre of the study was “What does the use of the theory of Imaginative Education (IE) mean to children and for their engagement in elementary mathematics? With the use of the theory of Imaginative Education (Egan, 1997) education is reconceived to be about developing kinds of understanding’ enabling people to make sense of the world in different ways, rather than through the development of decontextualised knowledge.

Qualitative case study (Creswell, 1998) and socio-cultural methodology (Vygotsky 1998) which utilised the notion of student voice (Fielding, 2008), was used to examine six student’s perspective of learning during a geometry unit.

Listening to the students, the ones most intrinsic to education, provided a wealth of understanding about how they experienced the learning of mathematics. The results suggest that the purposeful inclusion of imagination and affective responses, were valuable tools of learning that fostered the cognitive development of mathematics for the students.

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# **TURKISH AND PORTUGUESE VOCATIONAL HIGH SCHOOL STUDENTS' PERCEPTIONS ON ALGEBRA**

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Algebraic concepts, principles, and methods support students' intellectual skills for conceptualizing quantitative information and then reasoning about that information (Christmas & Fey, 1990). However, including vocational high school students, many secondary level students ask themselves or their teachers "Why I am learning equations, functions, permutations etc?" and they think that they will almost never need algebra. On the other hand, algebra is essential to all secondary level students for learning advanced mathematics (Christmas & Fey, 1990); and since students who are training in vocational high schools receive technical education, learning algebra is also important for them. Then, one way or another, we need to find ways to make algebra accessible to those students. Doing this, we first need to study to what extent students in these institutions conceive algebra; because such knowledge would have valuable insights for teaching.

The purpose of the study was to investigate Turkish and Portuguese vocational high school students' perceptions on algebra and then compare it. The participants of the study were 52 (45 males and 7 females) Turkish and 54 (46 males and 8 females) Portuguese tenth and eleventh grade students around 17-18 years old. They had similar academic backgrounds such as mathematics exam results, mathematics interest etc. The data were obtained using a questionnaire which was designed to assess students' perceptions on generally mathematics and particularly algebra. It took students around 30 minutes to answer it. Students' responses were analysed through content analysis.

The results indicated that for both Turkish and Portuguese students algebra is necessary for the daily life. They mostly thought that they learn algebra to understand mathematics better. Concerning the perception of the needs of generally mathematics and more specifically algebra for daily life, both samples related it mostly to functions or jobs requiring the handling of money, rather than to scientific or technological development. Other significant result was that most representations of mathematics revealed by the students were not object-related, but focused on the social and affective value for the subject. At the social dimension we often found the dichotomy "necessary/unnecessary", whilst at the affective dimension the most frequent dichotomy of those representations were "interesting/boring". This may be a result not only of the non-tangible nature of mathematics, but mainly of the way this science is very usually taught, starting from theory to applications and not the reverse. Here is a point for discussion.

Christmas P. T. & Fey, J. T. (1990). Communicating the Importance of Algebra to Students, *In Edgar L. Edwards (Ed), Algebra for Everyone*, Reston, Virginia: NCTM



# THE ANALYSIS OF THE DESIGNED THE EŞME CARPET PROBLEM'S SOLUTIONS THROUGH 7-STAGE MATHEMATICAL MODELLING PROCESS

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The purpose of this study is to analyse the prospective mathematics teachers' approaches and thought processes on a mathematical modelling problem named the Eşme Carpet Problem (ECP) designed by the researchers while working individually. The research was carried out with twenty one secondary prospective mathematics teachers who took a course about mathematical modelling. The data were collected from the written answers of the ECP. To analyse the mathematical modelling process, the researchers compiled 7-stage mathematical modelling process by considering different studies (Borromeo-Ferri, 2007; Blum, 2002; Niss, Blum & Galbraith, 2007). These 7-stage consist of understanding the problem; identifying variables and correlating them; mathematising the problem; constructing mathematical models and correlating them; solving the problem mathematically; deducing results from the solutions, interpreting and adapting to the real world; checking the model's accuracy. In the analysis of the problem solutions, the rubric which was intended for 7-stage mathematical modelling process was used. While examining the solutions of the participants who were informed about this 7-stage mathematical modelling process, it was generally observed that their achievements in the ECP gradually decreased in modelling stages. Almost all of the participants could understand the problem, identify variables and correlate them. Some of the participants directly mathematized the problem while the others preferred using verbal statements before mathematizing the problem. They solved the problem mathematically using their assumptions. Only two of the participants desirably deduced results from the solutions, interpreted these results and adapted them to the real world.

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# MODELLING FOR CONCEPT FORMATION OF PROPORTION: A PERSPECTIVE OF REALISTIC MATHEMATICS EDUCATION

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This study examines a process of concept formation about proportion by focusing on modelling.

Modelling can be classified into the following two types: ‘modelling at functional level’ and ‘modelling at the level of concept formation’ (Andresen, 2007, p. 2042). The former aims at problem solving and involves certain applications of mathematical concepts, methods etc.. The latter corresponds to four levels of learning in Realistic Mathematics Education (RME). The modelling at the level of concept formation becomes a forerunner for the modelling at functional level, though modelling at functional level may be the mainstream in some studies and lessons (ibid, p. 2047). It is therefore desirable for children to experience modelling at the level of concept formation for modelling at functional level.

Gravemeijer (1997) set up four levels of learning in RME: situational, referential, general, and formal levels (p. 340). Those correspond to four levels of self-developed models. That is, ‘model-of’ and ‘model-for’ are placed at the referential and the general levels. Gravemeijer (1997) showed a clear distinction between the two levels, in the process where children constructed formal knowledge from informal knowledge about long division. On the contrary, this study addresses concept formation of proportion, because most of Japanese students could not understand concepts of proportion in realistic situations (NIEPR, 2008).

This study observed the activity where 6<sup>th</sup> graders found the number of many nails and coins in a classroom, and analysed the activity according to four levels of learning in RME. As a result, this study identified the referential and the general levels in concept formation of ‘quantity per unit’, and found the necessity of more detailed levels.

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# INTERACTION BETWEEN GEOMETRIC CONSTRUCTION ACTIVITY AND CONCEPTION OF SPATIAL FIGURE IN 3D DGE

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This study focuses on interactions between geometric construction activity and conception of spatial figure in 3 Dimensional Dynamic Geometry Environments (3D DGE).

Geometric construction activity in DGE has a central role in the learning of spatial figure. Concerning to the construction in 2D DGE, Finzer and Bennett (1995) classified the constructions into the following three levels: underconstraints, appropriate constraints, overconstraints (Finzer and Bennett, 1995). On the contrary there is little research on the geometric construction activity in 3D DGE though the effect of 3D DGE on problem solving has been identified (Bülent & Temel, 2008).

Geometric construction activity and conception of spatial figure in 3D DGE interact each other. When a student constructs spatial figures by geometric construction activity in 3D DGE, the constructed figures do not necessarily correspond to the student's conception. Thus, there is a gap between the constructed figure and the student's conception. For example, when the student constructs a cube not according to a student's conception, he/she will find that the cube can be changed into a rectangular prism by dragging one of the vertexes. In order to decrease this gap, interactions between the constructed cube and the student's conception of it would have emerged.

This study makes a framework to capture conception of spatial figure, and uses Finzer and Bennett (1995)'s framework to capture geometric construction activity. By using these frameworks, interactions between geometric construction activity and conception of spatial figure in 3D DGE would have been identified.

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# LINKING MIDDLE GRADES TEACHERS' UNDERSTANDINGS OF FRACTIONS TO STUDENTS' ACHIEVEMENT

Andrew Izsák, Aijun Wang,

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The University of Georgia

Chandra Hawley Orrill

University of Massachusetts

Dartmouth

Finding empirical evidence connecting what teachers know to what students learn has been a central challenge for mathematics education for nearly a century. Researchers (Hill, Rowan, & Ball, 2005) have recently connected mathematical knowledge for teaching (MKT) and student achievement at the elementary grades in the United States. Their measures of MKT relied on Item Response Theory (IRT). When Hill (2007) examined the MKT of middle grades teachers, however, she found two distinct groups, which violates a core assumption of IRT.

We developed a measure of middle grades MKT and used it to study relationships between teacher learning in professional development and student achievement. Our measure (adapted from Hill, 2007) focused on fractions and proportions and on drawn models. We used the mixture Rasch model, which is designed to detect subgroups (henceforth referred to as Classes) of examinees based on patterns of correct and incorrect responses. Using the mixture Rasch model, we also found two groups in a convenience sample of 201 middle grades teachers.

We recruited 23 middle grades teachers and used our measure of MKT as pretest. We used Class and other variables to construct treatment and control groups. We administered a pretest to the students of these teachers. After the professional development (42 hours/14 weeks)—which focused on fractions, proportions and drawn models—we administered posttests to the teachers and their students.

The poster will display sample test items and ANOVA tests showing significant between-subjects and within-subjects effects on students' gain scores for teachers' Class and Group (treatment or control).

The opinions expressed are the authors', not those of NSF (DRL-0633975).

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# STUDENT'S MATHEMATICAL CREATIVITY IN OPEN APPROACH

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Creativity is an integral part of mathematics (Brunkalla, 2009). But too infrequently is it the case that mathematics pupils are given even the opportunity to display such creative thinking in the subject, particularly in their assessments and assessed in schools which encourages children to think in narrow domains (Haylock, 1987). The open approach class may share the common interest with the class that emphasizes mathematical discussion and communication. The evaluation in the open approach method, where the emphasis is laid on students' ways of mathematical thinking and their creativity rather than correct answers (Nohda, 1993). The purpose of this study was to analyze student's mathematical creativity in open approach in Thai classroom context by using the way of evaluation of the creativity ability test of Akita's idea as follows : divergence, logicity, fluency, flexibility and originality (Akita, 2004). The targeted group was one teacher and 1<sup>st</sup> grade students at Kookham Pittayasan School, Khon Kaen Province, and they used to apply open approach in their teaching. The data were collected through six classroom activities on number and operation. Data analysis were student's written works and field note.

The findings showed that mathematical creativity occurring in classroom showed high level scores in logicity and fluency affecting high level scores of divergence. But, the scores of flexibility and originality were low. Originality score value was found in every group and problem, and found that originality was likely to be advanced from lesson plans 1 to 6

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# DEVELOPING OPEN APPROACH-BASED LESSON PLANS EMPHASIZING USE OF GEOMETER'S SKETCHPAD

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The purpose of the research was to develop open approach-based lesson plans emphasizing use of Geometer's Sketchpad. The study emphasized protocol analysis and analytic description in the context of the 3-year profession development project based on Lesson Study and Open Approach conducted by Center for Research in Mathematics Education, Khon Kaen University together with the Office of Basic Education Commission and the Office of Knowledge Management and Development. The targeted group consisted of 4<sup>th</sup> grade students in Kukam Pittayasan School. The students were given six lesson plan integrating Geometer's Sketchpad and assigned to solve open-ended problems during each period of instruction. Data analysis was the protocol data gathered from the transcription of the audio-video tape recordings of Lesson Study steps: 1) collaboratively design research lesson among the researcher, school coordinator, co-researchers, the participant teachers 2) collaboratively observing the research lesson and 3) collaboratively reflection on teaching practice (Inprasitha, 2010). The research findings revealed that: the development of open approach-based lesson plans emphasizing the use of Geometer's Sketchpad resulted in lesson plans which had passed through various steps of Lesson Study. There are several significant features about the lesson plans as follows: 1) The lesson plans enable the teachers to conjecture their students' mathematical thinking process during classroom activities, 2) The lesson plans enable the teachers to change their role from giving a lecture to exploring or encouraging their students to think during their instruction and 3) The lesson plans enable the teachers to use appropriate technology.

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# DEVELOPMENT OF UNDERSTANDING THE CONCEPTS OF AREA AND PERIMETER

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*Area and perimeter belong to the fundamental concepts of primary school geometry. The poster is devoted to tasks and task cascades aimed at the development of pupils' experience with pre-concepts of area and perimeter both for pupils and their teachers. The tasks come from three different manipulative learning environments: wooden sticks, tiling and paper folding. It presumes that the tasks are posed to a class in schema-oriented educational approach. This approach is briefly described.*

The educational strategy, which we call the schema-oriented strategy, is grounded in our belief that "Learning mathematics requires construction, not passive reception, and to know mathematics requires constructive work with mathematical objects in a mathematical community." (Davis, Maher, Noddings, 1990, p. 2). Scholarly literature gives various connotations of the term 'schema'. The following quote by R. J. Gerrig (1991, p. 244-245) provides a rather loose definition that serves our purposes: "Theorists have coined the term schema to refer to the memory structure that incorporate clusters of information relevant to comprehension ... A primary insight to schema theories is that we do not simply have isolated facts in memory. Information is gathered together in meaningful functional units." Gerrig defines schema within a wide context of a person's life experience. Our definition is narrower (for more details see Hejný, 2008).

An important part of the schema-oriented educational style is the teacher's readiness for the sessions. A teacher should have a deep insight into the matter. We follow this idea and the series of tasks elaborated from the didactical point of view starts with one or two demanding problems for teachers. The pupils' solutions to some tasks will illustrate the work in the environment of wooden sticks in the poster.

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# TENDENCIES TOWARDS DEDUCTIVE REASONING IN SECONDARY STUDENTS' PRE-PROOF IDEAS: A GREEK CASE

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Secondary students' first encounters with proof and proving constitutes a significant, and often challenging part of their mathematical experience (Mariotti, 2006). In the Greek educational context, where the study we report in this paper is being carried out, students' encounter with proof, particularly in the context of Geometry, marks an important moment in students' education, not only in terms of their *mathematical* learning but also as an encounter with a significant element of cultural and national heritage. According to the national curriculum students are expected to encounter proof for the first time in Year 9, for example in the context of proving congruency of triangles or proving algebraic identities. One of the aims of our ongoing study is to investigate Year 9 students' *pre*-proof ideas, namely their perceptions of what proof is, just as they are about to be formally introduced to it, and as evident in their written responses to Geometry problems.

We captured these pre-proof ideas in the early weeks of Year 9. Here we report preliminary analyses of 90 students' responses to a Geometry Test. Specifically we focus on one item of the Test which involved using properties of the perpendicular bisector of a line segment in order to deduce that a given triangle is isosceles. In resonance with previous research on secondary students' geometrical reasoning (e.g. Hoyles & Healy, 2007), a substantial number of student responses – about a half – contained evidence of empirical proof schemes (Harel & Sowder, 2007), particularly perceptual ones. Less expectedly, a significant minority of the responses – about 20 – contained evidence of deductive proof schemes, sometimes in fusion with evidence of schemes of other types. We see this tendency as an apt pedagogical opportunity for facilitating students' often difficult transition to proof.

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# AN INVESTIGATION OF 6<sup>TH</sup> GRADERS' THINKING AND REASONING ABOUT VARIABILITY IN A CHANCE CONTEXT

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*The aim of this study is to explore grade six students' conceptualization of variability in a chance setting. This poster will display the task that students engaged and describe how students' appreciation of theoretical expectation and chance variability plays a role in their thinking.*

"Data analysis and probability" has become one of the main strands of school mathematics curriculum from prekindergarten through grade 12 in many countries. Accordingly there has been a growing body of research on how students think and reason about data and probability (i.e. Shaughnessy, 2007). More recently data and probability topics became part of the national mathematics curriculum in Turkey starting from grade one (MEB, 2005). However, it is noted that there is a lack of research on students' thinking relevant to this content strand, in particular reasoning with data, in our country.

This exploratory study focuses on the idea of variability, which is at the heart of statistical thinking. 19 grade six students were given a survey task focusing on variability in a repeated sampling context with a chance device (adapted from Watson & Kelly, 2004). Students were asked to make predictions for a sampling problem involving a spinner with two equal parts (shaded-white) and to distinguish between reasonable and non-reasonable variability in the given stacked dot plots of sampling outcomes. Based on these responses students were selected for one-on-one interviews where they were given an opportunity to conduct actual trials using simulation features of *TinkerPlots* (Konold & Miller, 2004) software in a slightly different task (with a -shaded and -white spinner) to delve into the vague reasoning in the initial responses. The poster presents examples from student responses to demonstrate some of the student conceptions of variability and some of the difficulties encountered.

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# A TEACHING EXPERIENCE WITH A BLIND STUDENT'S FINGERS: THE AREA OF A CIRCLE

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Well known Russian blind mathematicians, Lev Semenovich Pontryagin, had an accident at the age of fourteen and his mother helped him during his education (Jackson, 2002). Unfortunately most of the blind students are not lucky as Pontryagin. Blind students, their parents and most teachers may think that mathematics is more difficult for blind students, but there are many blind mathematicians such as Lawrence Baggett, Zachary J. Battles and Bernard Morin (Jackson, 2002). Understanding most of the mathematical concepts, paper and pencil may be enough for sighted students. It is same for blind students with a little difference; they need to use tangible and concrete materials. Today a blind student may convert all physics and mathematical articles tangible by the help of LaTeX2Tri tool (Thompson, 2005).

This study aims to investigate how a blind student understands the area of circle by using a tangible material. For this purpose, a method which had mentioned by Greenslade (2011) about the area of a circle was chosen. In Greenslade's study a circle was cut small parts like slices of cake. Circumference of the circle has been taped up before activity. The next step was opening the circle to get a rectangle. These steps were carried out by the participant. She explain that getting a new geometrical shape for a circle was nice to understand. She could not remember the formula of area of a circle but basic rectangle formula helped her to find out the area of circle. The last part of our activity was about calculation. We preferred to use pencil, bant and rubber to symbolise mathematical equations. To get  $\pi$  value it was necessary. While interview she obeyed that there is no way to get the formula of a circle except this kind of approach. Geometrical discovery process took little time than reaching the formula of circle from the formula of rectangle.

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# EXPLORING TEACHERS' UNDERSTANDING ON LESSON PLAN

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Teachers' knowledge and teaching were a foundation of the new reform which teaching necessarily begins with a teacher's understanding of what is to be learned and how it is to be taught (Shulman, 1987). Lesson plan as a window into teacher's conception of lessons which the role of lesson plans as vehicles for examining and improving lessons in lesson study (Shimizu, 2008).

The purpose of this study was to explore teachers' understanding on lesson plan using open approach in the context of lesson study. The research participants included the teachers from four project schools in research project of Center for Research in Mathematics Education, Khon Kaen University, in 2009 academic year. Collecting data was based on cycle of lesson study including: collaboratively plan, collaboratively do, and collaboratively see. Research data collected from the implemented lesson plans, records form after lesson plan implementation, video recordings and teacher interviews. The data was analyzed based on 3 levels of school mathematics curriculum including: intended curriculum, implemented curriculum, and attained or realized curriculum (Kilpatrick, 2009) and based on a perspective on teachers' knowledge about curricular knowledge (Shulman, 1986).

The results showed that teachers' understanding on lesson plan are as follow: 1) related to intended curriculum, indicators for yearly evaluation and learning standards appeared on curriculum documents were used to design lesson plan, 2) related to implemented curriculum, in teaching approach teachers followed the structure of lesson plan in the process of lesson study and 3) related to attained or realized curriculum, the classroom contexts has been changed in the ways that students had their opportunity to think about the activities individually.

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# A FRAMEWORK OF AFFECTIVE ASPECTS ON PROVING

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This study constructs a framework of affective aspects on proving. Proving is important in mathematical education, but its affective aspect seems not to be desirable. Though the previous researches have captured affective aspects without specifying mathematical contents and process, it is necessary to construct a framework specialised in proving in order to improve instruction according to students' affective aspects.

PISA2006 has already assessed affective aspects on science by questionnaires in the assessment of student performance (OECD, 2006). This theoretical basis is the framework for the affective domain in science education (Klopfer, 1976). This framework consists of two dimensions: behavior and phenomenon. The dimension of behavior is based on five levels of internalization (Krathwohl, 1964). The dimension of phenomenon represents important phenomena concerned to the affective domain in science education. This study focuses on the division 'inquiry' at the dimension of phenomenon. The division has three categories: processes of scientific inquiry, scientific inquiry as a way of thought, and inquiry as a way of thought.

This study constructs a framework of affective aspects on proving by adopting the dimension of behavior and the division of inquiry at the dimension of phenomenon. In particular, this study focuses on a process of proofs and refutations (Lakatos, 1976) as inquiry. According to this framework questionnaires will be designed, and the framework will be evaluated and improved through implementation of questionnaires and analysis of its results.

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# A STUDENT TEACHER'S CHOICE OF EXAMPLES IN TEACHING PROBABILITY

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The purpose of this case study is to examine a mathematics student teacher's two lessons using Knowledge Quartet (KQ)'s transformation unit in the context of "choice of examples" for the probability concept (Rowland, Huckstep, & Thwaites, 2005). For this purpose, a student teacher's (Deniz) two lessons regarding probability concept in the 2009-2010 spring term were observed and video-taped, and the semi-structured interviews were realized and audio-taped to discuss some of the episodes of their lessons. In the 2009-2010 fall term, she prepared and conducted six-hour instruction related to the limit concept, they were evaluated in the context of KQ, and she were had reflective interviews about her lessons. In the result of identifying Deniz's two-hour probability instruction in the context of "choice of examples", it was generally seen that she preferred giving examples from real world as part of "teaching concepts and procedures" (Rowland, Turner, Thwaites, & Huckstep, 2009, p.70) aiming at concept formation. On the other hand, she gave examples in the context of "provision of exercises" (Rowland et al., 2009, p.71) aiming at practising those concepts. Deniz wanted their students to remind the concepts (experiment, output, sample space, sample point, space, certain event, discrete event) known from lower secondary and to give examples related to those concepts. After that, she used exercises to make sense of those concepts. She prepared "Facebook Activity" about real world to form the probability function concept. In the interview with the participant, she stated that her aim in preparing this activity was to relate the real world and the function concept learned in the previous years and the probability concept. In the second lesson, she gave examples from real world to introduce isoprobable sample space and she presented three exercises aiming at familiarisation and practice. In the interview concerning her probability instruction, in the process of preparing her lesson, she emphasized that she watched out for the factors that she did not consider in the previous lessons (about the limit concept) in the choice of examples. In this context, evaluating instruction by using KQ is thought as making a contribution to student teachers in the choice of examples.

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# VIEWS OF MATHEMATICS TEACHERS ABOUT MODELLING TASKS

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Professional knowledge and beliefs of mathematics teachers concerning the role of modelling (Blum et al., 2007) for the mathematics classroom might have impacts on the way teachers conceive learning opportunities for their students. In particular, views about tasks (cf. Biza et al., 2007) can give specific insight. In the case of modelling, views about tasks requiring modelling steps might be relevant for instruction-related decisions of teachers when choosing tasks for their classrooms. However, there is still a need for quantitative empirical research about such views and corresponding professional knowledge. Accordingly, this study concentrates on this area. Different levels of globality of teachers' views (Törner, 2002; Kuntze & Zöttl, 2008) were addressed. In particular, on a content-specific level, 230 prospective teachers without long-term classroom experience and more than 75 in-service secondary teachers were asked about their views on characteristics of different tasks.

The results indicate that the prospective teachers preferred tasks with rather low modelling relevance to those requiring more intensive modelling activities. These views might be linked to the fear of these prospective teachers that the goal of mathematical exactness might be underemphasised in modelling tasks. However, the in-service teachers rated the tasks requiring more intensive modelling activities more positively than did the prospective teachers. These results suggest that the in-service teachers might have gained an increased awareness or openness related to learning opportunities contained in modelling tasks on the base of their classroom experience.

The poster presents in more detail the theoretical background, the research questions, design and samples as well as selected results of this study.

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# IN-SERVICE AND PRE-SERVICE TEACHERS' VIEWS ABOUT PROFESSIONAL KNOWLEDGE RELATED TO MODELLING

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For facing the challenges of creating rich learning opportunities related to modelling (Blum et al., 2007) in the mathematics classroom, teachers need specific professional knowledge (cf. Kuntze, in press). This knowledge does not only include content knowledge components like meta-knowledge about the modelling process (e.g. Maaß & Gurlith, 2009), but it encompasses also pedagogical content knowledge components, e.g. about a focused scaffolding of students struggling with the complexity of modelling requirements. For the ongoing professional development of mathematics teachers in this domain, their self-perception of their professional knowledge should be considered to play a filtering role.

As quantitative empirical results concerning such specific aspects of professional knowledge is still relatively scarce, this study examines the views of more than 20 in-service and more than 40 pre-service mathematics teachers working at or preparing for Austrian academic-track secondary schools. The data was gathered in a paper-and-pencil questionnaire using both multiple-choice and open formats.

The results suggest that both pre-service and in-service teachers perceived rather a non-optimal support offered to them during their university professional development phase, related to the area of modelling in the mathematics classroom. Even if the in-service teachers showed a tendency of a less pessimistic view of their PCK related to modelling, the evidence highlights a need of focused professional development.

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# **CHILDREN'S UNDERSTANDING OF INVERSE RELATIONS: MANIPULATING THE DIVIDEND AND THE DIVISOR IN WORD DIVISION PROBLEMS**

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Many of the difficulties in the solving of division problems stem from not comprehending the inverse co-variance between the division terms (Correa, Nunes, & Bryant, 1998; Skoumpourdi & Sofikiti, 2009). Thus it seems relevant to explore how children deal with these relations when solving problems in which the divisor and the dividend are manipulated. Forty low-income children attending the 2nd and 5th grades of elementary school were asked to make judgments about situations involving division (partitive and quotitive) on non-computational tasks. In Situation 1, the problems involved the division of the same quantity to be divided in different number of parts ("Victoria and Robert have bought 40 sweets each to give to their students as a present. Victoria has 5 students and Robert has 8. Who is going to get more sweets: Victoria's or Robert's students?"). In Situation 2, the problem involved the division of different quantities, but this time to be divided in the same number of parts ("Jane has bought 16 sweets and Mark has bought 12 sweets. Jane and Mark have 4 cousins each and they want to give some sweets to each one of their cousins. Who is going to get more sweets: Jane's or Mark's cousins?"). The participants were asked to justify their answers. In both grades, children were more successful in Situation 2 than in Situation 1, showing a better understanding of the inverse relations between the terms of division. This was most evident among the 5th graders. It was concluded that certain contexts make the understanding of these relations easier and that for this could be explored in the classroom.

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# WHY CAN'T AN ELEMENTARY SCHOOL TEACHER TEACH MATHEMATICS SMOOTHLY?

Yuan-Shun Lee, Ying-Mei Chen, Ru-Yi Cao

Taipei Municipal University of Education This research utilized activity theory as the theoretical framework to analyze the factors that influenced the teachers' teaching and the contradictions that appeared in the mathematical teaching activity system. The research methodology adopted an interpretive research method, and the research subjects were 2 elementary school teachers (T1&T2). During the study period, we held a teaching observation and discussion every two weeks. The related data we collected includes teaching videos, teaching discussions and interviews.

According to the elements of activity system, the factors that influence the achievement of teaching goals include instruments, rules, and division of labor in the teaching activities. After analysing T1&T2's teaching activities, we found the difference as follows.

The instruments T1 adopted were various. For example, T1 adopted cooperative learning, class discussion, integration of games into teaching, Information and Communication Technologies (ICT), and reward and punishment system. Relative to T1's teaching, the instruments T2 adopted were not various. T2 adopted didactic instruction in most of class time and he seldom used ICT. About The rules, There was a reward and punishment system in T1's class. Students would get some points when they had good manner. T1 usually encouraged the outstanding groups and individuals. T2 didn't use the reward and punishment system. When students spoke and laughed loudly, T2 would stop them and even punish by forcing them to stand, but it didn't work well. For example, T2 corrected or put a stop to students' inappropriate behaviour 13, 18, 23, and 25 times respectively in math classes of 10/29/2008, 12/24/2008, 4/22/2009, and 5/20/2009. About the division of labor, T1 tried to use many methods to assign tasks when students learned cooperatively in groups. Everyone of each group had his/her own task. T1's students could cooperate and support mutually to complete the tasks assigned by teachers. T2 adopted didactic instruction in most of class time, and the time of interaction with the students was limited. T2 once asked some students to share the solutions of problems with classmates, but T2 didn't assign tasks explicitly, and many students just chatted.

In this research, we also found many contradictions appeared in T2's mathematical teaching activity system.

# A STUDY OF PROBLEM-SOLVING STRATEGIES IN EQUATIONS WITH ONE UNKNOWN FOR JUNIOR HIGH SCHOOL STUDENTS UNDER DIFFERENT UNDERSTANDINGS OF EQUAL SIGN

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*The purpose of this study is to investigate students' understanding of the equal sign (McNeil, Grandau, Knuth, Alibali, Stephens, Hattikudur, & Krill, 2006). Also to investigate students' problem-solving strategies to equations and related these strategies to different understanding types of the equal sign by McNeil et al. The equations were linear equations (7th grade) and quadratic equations (8th grade) with one unknown. The investigators did surveys and development instruments accordingly; for 203 seventh-grade and 215 eight-grade students in a convenient sample. Descriptive statistics were used to analyze data in frequency and percentages. Findings for linear equations included relational definition of the equal sign (close to 50%), and an operational definition was approximately 1/4 of sample. Among various strategies, the primary strategy of operations on the left-hand side of equal sign is the four arithmetic operations; the strategy of an operational definition participant in five equal sign topics is similar to the strategy of one with a relational definition. However, those with a relational definition apply multiple strategies and exhibited varying particular and algebraic property while participants with an operational definition used arithmetic strategies more frequently than participants with a relational definition. For quadratic equations, there are three results. First, participants with a relational definition of the equal sign added to about 80% of the sample. Second, the performance of students with relational definitions is higher than those with operational definitions. Third, participants with an operational definition of the equal sign tended to guess randomly or leave a blank; whereas those relational definitions of the equal sign involved multiple strategies. In all, the researchers suggested that teachers should strengthen students' understanding of equal sign and related students' to prior algebraic knowledge.*

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# THE INVESTIGATION OF THE SENIOR HIGH SCHOOL STUDENTS' STATISTICAL LITERACY

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Quantitative information exists everywhere and statistics are increasingly presented as a way to add credibility to arguments, advertisements, etc. Being able to properly evaluate evidence, reports and claims based on data is an important skill that all students should learn as part of their educational programs. Yet many research studies indicate that adults in mainstream society cannot think statistically about important issues that affect their lives (Ben-Zvi & Garfield, 2005). One of the main arguments presented is that traditional approaches to teaching statistics focus on skills, procedures, and computations, which do not lead students to reason or think statistically. The mathematical literacy of the Programme for International Student Assessment (PISA) focuses on real-world problems, moving beyond the kinds of situations and problems typically encountered in school classrooms. One of the contents of PISA mathematical literacy is uncertainty. This content category includes recognising the place of variation in processes, having a sense of the quantification and explanation of that variation, acknowledging uncertainty and error in measurement, and knowing about chance. It also includes, forming, interpreting, and evaluating conclusions drawn in situations where uncertainty is central. The main purpose of this study is to investigate the senior high school students' performance on the PISA-based statistical literacy assessment and propose workable learning goals for students by focusing on current research study that examines the nature and development of statistical literacy.

Around 1134 10<sup>th</sup> to 12<sup>th</sup> grade students participated in this study. The results showed that there was lots of difference of statistical literacy between the students who received and did not receive the statistics courses ( $F_{(1, 1132)} = 60.12, p < .05$ ), the variances accounted by study experience is 5.5%. The analysis of the pattern of students' responses, only 16% of the students could answer correctly and offer the sufficient explanation to support the answer, and there were about 30% of students can't answer correctly and support their answer appropriately. Some students responded based on the reality but did not consider the information that showed on the charts, graphs, and statistical claims. The implications of the present study, including the plausible approach in meaningful statistics learning and suggestions were made for further research on related issues.

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# THE DESIGN OF LEARNING ENVIRONMENTS : DYNAMICALLY LINKED MULTIPLE REPRESENTATIONS OF COMPLEX MULTIPLICATION

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The purpose of the study is to construct a dynamic linked multiple representations learning environment and to exam the learning effect of complex multiplication. There are three types of complex multiplication : standard form, polar form, and geometry form. Duval (2006) suggested that both treatment and conversion are the sources of students' incomprehension in the learning of mathematics. A learning environment should be provided for students to deeply understand the meaning of complex multiplication by mastered those transformations of representations.

We use GeoGebra and JavaScript to design six dynamically linked learning webpages. Students can be easily interacted with those instructional webpages. Forty-one students who are in grade 11 are chosen to be the subjects for this experiment. All the subjects have learned the concept about complex multiplication. The result from the pretest shows that the students can be divided into three levels of understanding the concept of complex multiplication : (1) Single Representation where the students only construct the standard form; (2) Transferred Representation where students construct both the standard and polar form; (3) Integrated Representation where students integrate the three types of complex multiplication. The students' percentages of these three levels before and after the empirical instruction are listed as below:

	Level(1)	Level(2)	Level(3)
Pretest	59%	34%	7%
Posttest	12%	20%	68%

It presents that dynamically linked learning environment is helpful for students to integrate representations and master the structure of the knowledge. Advance explored for any further possible factors that would affect the teaching phenomenon is needed; besides, applying dynamic visualization to other units for research is suggested.

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# A STUDY OF THE VAN HIELE MODEL OF GEOMETRIC THINKING TO IN-SERVICE KINDERGARTEN TEACHERS IN TAIWAN

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This study was conducted to investigate the van Hiele model of geometric thinking (van Hiele, 1986) to in-service kindergarten teachers.

Using the van Hiele Geometry Test (VHGT), developed by the Usiskin (1982), total of 166 kindergarten teachers were tested. The reliability and validity of the Chinese version, were translated and tested by Wu's Doctoral dissertation (Wu, 1994). The reliability of Chinese version of VHGT of each level were .5549 ( $P = .000$ ), .8153 ( $P = .000$ ), .6492 ( $P = .000$ ), .4505 ( $P = .000$ ), and .7902 ( $P = .000$ ), respectively (Wu, 1994).

After data processing, the following conclusions were drawn from this study: (a) The distributions of the van Hiele levels of geometric thinking of the prospective and in-service kindergarten teachers, from level 1 to level 5, were: 21.08%、30.72%、20.48%、1.81%、0.00%, respective. (b) The distributions of the van Hiele levels of geometric thinking of the prospective kindergarten teachers, through level 1 to level 5, were: 15.38%、29.67%、29.67%、3.30%、0.00%, respective. (c) The distributions of the van Hiele levels of geometric thinking of the in-service kindergarten teachers, through level 1 to level 5, were: 28.00%、32.00%、9.33%、0.00%、0.00%, respective. (d) There was significant difference between the prospective and in-service kindergarten teachers at the distributions of the van Hiele levels of the geometric thinking. **KEYWORDS:** geometric, in-service, kindergarten teacher, thinking, van Hiele

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# TEACHERS' COMMON KNOWLEDGE PRODUCTION

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This poster presents the design and some tentative results of an on-going research project called Teachers' Common Knowledge Production. The aim is to study what kind of professional knowledge teachers develop when they are participating in three Learning Studies (Marton & Tsui, 2004) during one and a half year. Learning Study has shown to be a promising approach to improve students' as well as teachers' learning. Teachers participate in an iterative process to gain knowledge about how their enactments in the classroom affect students' learning. The focus is on the object of learning and particularly what is critical for students to learn a specific subject matter. In a Learning Study variation theory (Marton & Booth, 1997) is used as a framework to understand the relation between what is taught and what is learned. More specifically it is a theory used in practice when analysing and designing lessons and students' learning outcomes. In this study three questions are in focus:

- What kind of knowledge do teachers develop about conditions necessary for student learning?
- How do teachers ability to identify and visualize critical aspects change when they, during one and a half year, systematically in their practice and with an explicit theoretical framework are planning, teaching and evaluating teaching in relation to students' learning?
- How are insights gained through this process reflected in students' learning outcomes?

Tentative results from the first Learning Study show that teachers could learn about what is critical for student learning. They also develop their own understandings of the subject matter as well as students' difficulties and strategies used in mathematics.

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# THE DEVELOPMENT OF THE UNIVERSITY MATHEMATICS THINKING IN BRAZIL

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We understand that a historical approach of mathematics teaching may contribute to comprehend the development of mathematical thinking, since it can provide an unusual overview on “development of mathematical thinking” in the PME. We intend to make use of a historical approach to study the Brazil implementation of the university system, in particular university system for mathematics teaching in Brazil.

The history of the internationalization of knowledge in the ex-European colonies in Latin America, as it is the case of Brazil, shows the introducing in this country of a mathematical movement predominant in Europe in the early 20th century, namely, the formalism. Such a movement came with the Italian mathematicians in 1934 who were brought to compound a European team that would be responsible for creating a university spirit in Brazil.

The archives of the University of São Paulo shows a dispute concerning the chair in the Polytechnics School is an example of resistance to loosing administrative (political) autonomy, in this case, supposedly justified by an epistemological and a statutory argument. Such a lost meant a change of epistemological direction of Calculus teaching. The presence of Fantappiè in the Polytechnic School occurred in the context of a power struggle between the two councils of the Polytechnic School and the newly created University. The latter won the dispute, so that Fantappiè's teaching became established in the Polytechnics School at least until 1935. Nevertheless, in 1938 the chair was reassumed by a traditional engineer, who brought back the old Calculus teaching approach to the Polytechnics School, namely, the Infinitesimal Calculus approach.

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# STUDENTS' PROCEDURES ON MULTIPLICATION TASKS

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*In the context of a teaching experiment it was carried out a research in a third grade class on students' understanding of multiplication. This poster presents the procedures students used to solve a set of tasks specially designed to foster multiplication in the context of number sense development.*

A key aspect of learning multiplication is the transition from the use of additive procedures to multiplicative ones. We can identify research evidence on key conditions that support this transition: a certain conception of 'number', 'sum', 'product' and properties of operations on the one hand, and the mastering of specifics skills and basic facts on the other (Verschaffel, Greer, & De Corte, 2007). In addition curricular guidelines underline that learning multiplication should be anchored in a perspective of number sense development (Treffers & Buys, 2008).

The first author of this proposal conducted a research in the context of a teaching experiment developed in collaboration with the primary teacher of a third grade class. Having as background the idea of hypothetical learning trajectory (Simon, 1995) the team developed a set of multiplication tasks that were implemented by the classroom teacher, in some mathematics lessons, along eight months.

This Poster will present a sequence of tasks and the procedures that students used to solve them. It will also present a discussion that relates the students' procedures with the contexts and numbers present in the tasks. The information will be presented in a graphical format that includes figures with the tasks and the corresponding students' procedures.

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# **“TO GENERALIZE AND SYMBOLIZE USING RELATIONAL AND FUNCTIONAL THINKING IN GRADE 4”**

Célia Mestre

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The new Portuguese mathematics curriculum considers algebraic thinking as one of the four fundamental themes and following recent international orientations, that document recommends that it should begin to be developed in elementary school (Ministério da Educação, 2007). According to the document Principles and Standards (NCTM, 2000), the teaching of Algebra can be conceived as a unifying bond for the mathematic curriculum. This perspective informed a teaching experiment in grade four (students from 9 to 10 years old), focused on two main ideas: i) algebra can provide a solid base for working with numbers, operations and their properties, supporting the latter exploration of algebraic expressions and symbolic appropriation, and ii) the systematic experience with pictorial and numerical patterns can support the emergence of function's concept.

This poster will presents the preliminary results of study carried out from this teaching experiment, which main goal is to understand the development of student's generalization and symbolization capacities when they work on tasks that promote relational and functional thinking. The research follows an interpretative perspective, with data were collection from observation (with video record), students' written records and some clinical interviews. A graphical display will describe this study in terms of the main goals, theoretical background, methodology and examples of students' work

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# INTERPRETATION OF GRAPHS IN A COMPUTING ENVIRONMENT FOR STUDENTS FROM A RURAL SCHOOL

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This study aimed to investigate the process of interpretation of graphs from the use of software TinkerPlots by elementary rural school students of year 5. The research interconnected three main themes: Rural Education; the use of new educational technologies; and Statistics Education (Alves, 2011). Research data was collected with ten students from a rural school in the Agreste of Pernambuco, Brazil. The students never had contact with a computer. Data collection was performed in four research sessions. In the first session was held a collective activity for data collection with all students in the classroom. The second session was comprised of an individual interview with each participant. In the third session the students had their first contact with the computer and the software TinkerPlots and participated in a process of familiarization with the software. In the fourth session the students answered tasks involving interpretation of graphs on TinkerPlots. Data was analysed from a qualitative perspective, which was considered the performance of students in the interpretation of graphs as well as their speech during the research sessions. The results indicated that participants did not have great difficulties on manipulate the TinkerPlots, indicating that they understood the software functions. The high number of correct answers on questions suggested that most students had no difficulty responding to the activities involving the interpretations of graphs. The participants did not consider explicitly the themes of the activities for the interpretation of graphs. Therefore, data from this research suggest us that students with none experience with computer from rural areas are able to interpret graphs in a computer environment for data analysis. These results can be related to easy use of TinkerPlots computing environment which allowed different strategies and construction of multiple representations of a data base.

## Acknowledgment

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# INVESTIGATING 5<sup>th</sup> GRADE STUDENTS' SOLVING NUMBER SENTENCES: EVIDENCE OF RELATIONAL THINKING

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“Relational Thinking” is one form of mathematical approach; it is believed that this thinking is an essential approach for preparing students for algebra (Stephens, 2004). This study investigates student’s way of thinking in written work while they were solving number sentences by focusing on evidence of relational thinking. The targeted group was fifth grade students from Sanambin School, participated in “project for professional development of mathematics teacher through lesson study and open approach” were conducted by Center for Research in Mathematics Education (CRME), Khon Kaen University. Twenty-six fifth grade students participated in this study dealing with two groups of pencil-and-paper number tasks. These two groups of questions include addition questions and subtraction questions. The data was collected from student’s way of thinking written in their tasks. The findings according to Stephens (2004) showed that 1) students had correct understanding about equal sign as the relational sign and they could find the missing numbers of any number sentences by using relational thinking and 2) students compared numbers as being bigger or smaller than corresponding numbers, but they do not represent arrow or diagram for connecting or comparing numbers according to their position. However, most of students had misconception about direction of compensation; they treated the direction of compensation for subtraction same as for addition, so they could not find the correct missing numbers in subtraction numbers task.

Acknowledgement: This work is partially supported by Center for Research in Mathematics Education (CRME), Faculty of Education, Khon Kaen University, Thailand

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# THE RELATION BETWEEN MATHEMATICAL ABILITY AND THE METACOGNITION

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*In this study, we examined the relation between college student's mathematical ability and their "meta-cognition". In order to measure their mathematical ability, we conducted a math test which consisted of some types of problems (e.g. computational problems, simple story problem, complex story problem, logical problem and practical problem) on 58 college students. They assessed whether they could solve these problems before they solved them. As the result, the college students who could assess their ability accurately had more correct answers on the math test. This result suggested that mathematical ability of the college students may relate to "meta-cognition" that they judge their ability properly.*

## METHOD AND RESULTS

In this section, we explain method and result of our research compactly.

### Math test

The math test which we used had 26 problems. Six of them were simple calculation. Eight of them were story problems which should be answered after students read and understood problem sentences. Two of them were logical problems. Four of them were deduction problem that students drew deduction from given conditional statements. Three of them were the problem on sequences of numbers. Three of them were all-round problems required comprehension of given figures and tables.

### The gap of meta-cognition

This index measured to quantify the differences between prediction and result to the problems in math test.

### Results

We find a negative correlation between the gap of meta-cognition and the number of correct answers on the math test( $r=.68$ ,  $p<.01$ ).

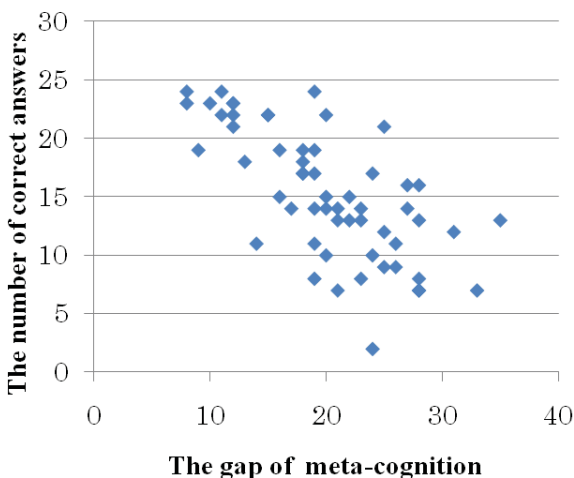


Figure 1 The relation between the number of correct answers and the gap of meta-cognition

# **COPING WITH MULTISTEP ARITHMETIC WORD PROBLEMS: THE CASE OF BELOW-AVERAGE NUMERACY STUDENTS**

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This poster reports on a mixed methods research project wherein the full process of comprehending and solving multistep arithmetic word problems was examined through two linked approaches; Study 1) analysis of students' thinking aloud and scaffolded dialogues from task-based interviews (N=19), and Study 2) correlation analysis of national tests in reading and numeracy (N=1264). Students across the range of ability were selected for both samples 1 and 2.

The research questions regarded: 1) the relationships between reading comprehension and success in solving multistep arithmetic word problems, 2) the cause of errors and difficulties, as well as coping strategies, and 3) the relationships between scaffolding, comprehension, and success in solving word problems (for more detail, see Nortvedt, 2010). The poster will present the results of students in both samples that achieved below-average scores on the national numeracy test.

Overall, reading comprehension was found to have a significant correlation to solving multistep arithmetic word problems (Nortvedt, 2009). Analysis in both samples suggests that below-average numeracy students experienced difficulties in comprehending word problems. When students did not successfully comprehend a problem, they often reduced it to a more simplistic problem within the reach of their mathematical proficiency. Consequently, students did not detect the need to be scaffolded during comprehension. The analysis suggests that students viewed performing calculations as the main activity when solving word problems; little effort in comparison was made toward comprehending. It might be that students did not see interpreting and modelling word problems as a complex task. In addition, below-average numeracy students in Study 1 largely relied upon informal methods when performing multidigit calculations (Nortvedt, 2010), and often required scaffolding in order to perform them.

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# TEACHERS MANAGING THE MATHEMATICS' CURRICULUM

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A key element of teachers' professional practice is the way he/she interprets and manages the curriculum, taking into account the students' characteristics and the conditions and resources of the school. This study draws on several fields of knowledge: teachers' professional knowledge and identity, curriculum management in mathematics, and collaboration and leadership in school context. We strive to understand the practice of collaborative curriculum management in the context of a school mathematics department. Particularly, we address two questions: (i) How teachers conduct curriculum management, in this context, as they attempt to diversify students' learning experiences? (ii) What is the potential of collaborative work around curriculum management in the development of a professional culture at the school?

The methodology is qualitative and interpretive, with four case studies: one the group of teachers involved in innovative collaborative work in a middle and secondary school; and three individual teachers belonging to such group. The data collection includes (i) field observations with reports from working sessions and classroom work taken in a researcher diary, (ii) two interviews with each teacher, and (iii) analysis of documents.

The purpose of this poster is to document results of this study show that the curriculum management made in the context of a collaborative group and the various initiatives of the group in developing innovative practices that involve the development of exploratory tasks are significant changes in educational practice and enable the sustainability of a collaboration culture (Nunes & Ponte, 2010). The poster begins with a presentation of the study including the aims, context and methodology. The focus is them on the teachers' curriculum management in the context of the mathematics department. Some data and results will be presented.

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Note: This work was realized in the context of the project PPPM – *Professional Practices of Mathematics Teachers* supported by FCT – Foundation for Science and Technology (PTDC/CPE-CED/098931/2008).

# IDENTIFYING FACTORS IN THE DIFFICULTIES OF EXPLANATION OF FALSE STATEMENTS

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The purpose of this study is to identify factors in the difficulties in explanation of false statements focusing on learner's interpretations of propositions.

In explanation of false statements, some students cannot generate any counter-examples, others cannot identify an example as a counter-example even if showing the counter-example (Balacheff, 1991; Hoyles & Küchemann, 2003; Zaslavsky & Ron, 1998). The previous researches have focused on learners' knowledge or understanding about counter-examples, but not learners interpretations of the statements.

Our theoretical framework bases on “logic” for “interpretations of propositions” constructed by N. Motohashi (2002). This framework presents three contexts in which each learner is “understanding”, “proving” and “utilizing”.

We report results an analysis of responses to a question in which three hundred students in Japanese junior high schools, were asked to assess the statement about elementary number theory, to evaluate students' explanation of false statement. This question is comprised in “National Assessment on Specific Issues in Mathematics”, and named “a task of pyramids” (National Institute for Educational Policy Research, 2006).

From this analysis, we showed following factors in the difficulties in explanation of false statements. Answers for the question change how the proposition interpreted in the task. Because of unsuitable universe of perspective, an interpretation of proposition in “understanding” might affects an interpretation of proposition in “utilizing”. Furthermore, even if a proposition is interpret in suitable universe of perspective, a way of selecting subject might affect decision about the truth value of statements. Finally, we clarified the important interpretations for counter examples, focusing on relation in “proving”.

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# INVESTIGATING STUDENTS' CHARACTERISTICS OF INTUITIONS IN DECOMPOSING AND COMPOSING ACTIVITY

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Most of the first grade students have experience in 'ordinal number' outside the school. They can count each by each before entering the school. However, it is difficult for them to conceptualize the number 5 as the combination of 2 and 3, 1 and 4, etc (Inprasitha, 2010). Decomposing and composing activity in Japanese textbooks are a key activity to develop the children to operate numbers without counting each by each (Hattori, 2010). Decomposing and composing are importance activity for providing students' how to operate to addition and subtraction on going, and the students need to construct this how to by themselves. The importance of students' doing mathematics related to the construction of mathematical knowledge base on students' intuitions. Many educators considered important of the construction of mathematical concept on intuitions (e.g., Pitta-Pantazi and Tsamir, 2005). The purpose of this study was to investigate student's characteristics of intuitions in decomposing and composing activity. In this study, intuition was defined according to Fischbein (1987). For research design by qualitative research, the data were collected by classroom observations, video recordings, students' tasks and interviews. The target group was four first grade students from one project school, participated in the project for a professional development using innovation of lesson study and open approach. Open approach as a teaching approach emphasized problem solving in mathematics classroom (Inprasitha, 2010). The students engaged in decomposing and composing activity designed for understanding 'decomposing and composing', a concept necessary for later addition and subtraction operation. The findings revealed that students had two characteristics of intuitions as self-evidence and globality; when students observed cards in activity, the students recognized that the whole was bigger than each of its parts, a number could decompose in two numbers or two sets and two numbers or two sets could compose a number. When students were asked to make 10, the students expressed instantaneously that 10 as composition 1 and 9, 5 and 5, 8 and 2 etc. This findings suggested that globality depended on self-evidence.

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# MATHEMATICAL CREATIVITY IN ELEMENTARY CLASSROOM

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The new Portuguese Mathematics Curriculum proposes a different perspective about learning and teaching of school mathematics, with great challenges for both teachers and students. In this setting of innovation creativity plays an important role; it is a dynamic characteristic that students can develop if teachers provide them appropriate learning opportunities (Sheffield, 2009; Leikin, 2009). Leikin (2009) considers 3 components of creativity: fluency, flexibility and originality. Several studies (Torrance, 1974; Levenson, in press) suggest that developing fluency and flexibility in themselves is of great importance to the development of creativity in students. The tasks that can promote these dimensions must be open-ended and ill structured, assuming the form of problem solving, problem posing (including elaboration and generalization) and mathematical explorations and investigations.

We are involved in a project that focuses in the relations among challenging mathematical tasks, manipulative materials, creativity and mathematical ability. We adopted a qualitative exploratory approach study with elementary teachers and students to understand what instructional or manipulative materials are promoters of mathematical creativity and meaningful learning and what characteristics tasks need to foster students' and teachers' creative potential.

The poster presents part of the research showing some open-ended tasks, the way students solved them and their teachers explored them, and reflect on the role of this experience in a curricular context.

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# NUMBER SENSE OF PRESERVICE ELEMENTARY TEACHERS IN PORTUGAL

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*Having a good number sense makes it easier to an elementary teacher to understand and value the informal strategies used by students when solving numerical problems. Training programs for elementary teachers should include opportunities to work, discuss and analyze issues related to the development of number sense, contributing to the reconstruction of concepts and beliefs as well as to develop their mathematical knowledge. In this study we evaluate futures teachers' number sense during their final teaching practice.*

## POSTER CONTENT SUMMARY

**Purpose.** In a shortly section we'll present Background and study purpose.

**Method.** We applied to a 15 students group a questionnaire with nine open-ended questions involving aspects related to various components of number sense, mainly based on Kaminski's (2002) and Tao's (2005) work.

**Results.** Students' performance will be presented using 7 tables, similar to Table 1.

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Calculate  $26 + 9 + 24 + 18 + 41$  and explain the procedures used.

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80% used appropriate and efficient mental strategies; 7% used inappropriate mental strategies; 13% used traditional algorithm.

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Table 1: Students' performance in Question 1.

Discussion. Although the use in few cases of some rigid strategies (use of traditional algorithms) that lead to more lengthy and complex calculations, students revealed a secure and flexible knowledge about natural numbers and their properties. However, students revealed difficulties in working with rational numbers and easily transferred for this set numerical properties of natural numbers without reflecting on the potential wrong consequences.

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# USING ORIGAMI TO SUPPORT THE LEARNING OF MATHEMATICS

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## CONTEXT

My experience of using origami in mathematics classrooms with learners at every stage of education resonates with the considerable evidence from around the world that origami provides a valuable context for learning mathematical concepts in an engaging, accessible and practical way. The proceedings of the international conferences on origami in science, mathematics and education (OSME) contain many articles on the use and benefits of origami in education (Hull, 2002; Lang 2009). In this poster I draw on the OSME contributors and others (eg Wollring) to articulate principles for the use of origami in education and criteria for the selection of models. The poster will be illustrated with examples of models and learner activity.

## PRINCIPLES

The purpose, both mathematical and educational, needs to be clearly articulated. The potential of the resource needs to be understood so that opportunities to extend beyond the intended purpose can be readily exploited. Adequate attention needs to be given to the choice of paper and the technical skills required (eg locating folds accurately and creasing firmly) without distracting from the intended purpose. Careful consideration is needed of the way in which the resource will be engaged with (eg demonstration, modelling, collaborative problem-solving, following written instructions and how these have been produced (commercially or by other learners).

## CRITERIA

The selection of origami models needs to be informed by their potential to illustrate mathematical big ideas and support the development and accurate use of mathematical vocabulary and articulation of mathematical understanding. Identifying and justifying the mathematical properties of folded artefacts leads naturally to proof.

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# EXPLORING STUDENTS' LANGUAGE FEATURES EXPRESSED MATHEMATICAL IDEAS IN OPEN APPROACH

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Socio-cultural theories place language at the heart of learning, as language is the main mediator of social interaction (Lee, 2006). Mathematics has a particular way of using language, its own particular way of expressing ideas, which is termed the mathematics register (Pimm, 1987). The mathematics register is a way of using symbols, specialist vocabulary, precision in expression, grammatical structures, formality and impersonality that results in ways of expression that are recognisably mathematical. But in the traditional teaching, students have not the opportunity to express mathematical ideas. Open Approach as a teaching approach, four phases of the just mentioned teaching approach are: 1) posing the open-ended problem 2) student' self learning 3) whole class discussion and comparison, and 4) summarization though connecting students' mathematical ideas emerged in the classroom (Inprasitha, 2010). Open Approach provide opportunities for students to use their own language to express mathematical ideas.

The purpose of this study was to explore students' language features expressed mathematical ideas in Open Approach. The target group was first-grade students from six classes in three schools in "The project for professional development of mathematics teacher through Lesson Study and Open Approach", Conducted by Center for Research in Mathematics Education. Data were collected by field notes, audio-video recording. Data were analyzed according able to the conceptual framework of mathematics register Pimm (1987).

The results found that the first phase, students use a word that has the same meaning in everyday language. The second phase, students used language to express mathematical ideas to students are the words related to the problem situations and designed materials in problem situations. The third phase, students used language to express mathematical ideas to students and teachers are words that are similar to the mathematical language. The last phase students used language to express mathematical ideas to students and teacher as the mathematical language guided by teacher.

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# COMPUTATIONAL ESTIMATION IN MATHEMATICS CLASSROOM IN CONTEXT OF LESSON STUDY AND OPEN APPROACH

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The purpose of this study was to investigate the computational estimation strategies in context of 5<sup>th</sup> grade students in the context of lesson study and open approach. Lesson study is a system for school-based professional development of teachers. Lesson study cycle is consisted of collaboratively plan lesson, collaboratively do teaching the lesson and collaboratively reflect on teaching approach (Inprasitha & Loipha, 2004). Open approach is strategy of instruction that focus on open-ended problem that is fit to the problem, a situation was created to encourage students to create mathematical problems of the situation on their own problems following four phases : 1) posing open-ended problem to students ; 2) students' self learning ; 3) whole class discussion and comparison and 4) summarization through connecting students' mathematical ideas emerged in the classroom (Inprasitha, 2010). Computational estimation defined as finding an approximate answer to arithmetic problems without actually (or before) computing the exact answer (Lemaire & Lecacheur, 2002), a way of approaching number sense (Wagner. 1993, P.43). Number sense is people's common understanding about number, operations and ability to deal with situations in daily life with number (Yang, Hsu, & Huang, 2004). The data were collected from activity of computational estimation in mathematics classroom of 5<sup>th</sup> grade student at School in Khon Kaen province, Thailand. The data were analysed based on Sowder (1992) of good computational estimation. The results showed that students used the processes of reformulation, translation, compensation and they could use many strategies. The rounding strategy was used the most frequently, the other strategies: front-end strategy, adjusting with front-end, compatible number strategy, special numbers strategy, rounding two numbers, and rounding one number.

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# LEARNING RATIONAL NUMBERS USING DIFFERENT REPRESENTATIONS: AN EXPLORATORY APPROACH

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This study aims to ascertain how a teaching unit that promotes pupils' work in exploratory tasks, using a variety of representations of rational numbers, in different subconstructs and with different kinds of magnitudes may contribute towards grade 5 pupils' understanding of rational numbers and comparing and sorting rational numbers.

Research on rational numbers underlines the role of subconstructs (part-whole, ratio, operator, quotient, measure) and representations (fractions, decimals, percent, number line, natural and pictorial language). Based in previous research, the study assumes that understanding rational numbers is related to flexibility in converting different representations and doing transformations within a representation as well as to increasing independence from pictorial representations and manipulative materials.

The teaching unit had 12 classes (90 minutes each), with additional moments for diagnostic and final assessment. The tasks proposed included explorations, to appropriate representations and develop new concepts using informal and intuitive understandings, and also problems, to relate concepts and develop strategies and exercises to consolidate routines and procedures. Classroom activity involves pupils' autonomous work (small group, pairs and individual) as well as collective discussions for sharing results and discussing strategies and concepts.

This study is a teaching experiment, including an in-depth case study of a pupil (Leonor). Quantitative data was collected (pre- and post-test) as well as qualitative data (video record of classroom discourse, pupils written work, interviews with the case study pupil).

The poster presents the teaching unit and shows the results achieved by all pupils and, particularly, Leonor. The information is organized in text boxes and includes images relating the several aspects of the study. The poster also includes digital images of the work of pupils and graphs representing pre and posttest results.

# EARLY YEARS' TRAINEE TEACHERS MATHEMATICAL KNOWLEDGE FOR TEACHING: THE CASE OF BAR GRAPHS

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Teachers' knowledge is one core dimension that allows achieving the goal of teaching for understanding. For that to be accomplished, teachers must be in possession of a sound knowledge of the mathematics to be taught. Such knowledge is supposed to be acquired during teacher's initial training (amplified than just learning the topics by themselves) and evolve during such training. Such knowledge is conceived here as *Mathematical Knowledge for Teaching* (MKT) (Ball, Thames & Phelps, 2008).

The present research is focused on early years' trainee teachers MKT at the beginning of their training, namely: how they acquire it, how it evolves and what factors influence such evolution. It is grounded on a qualitative perspective under an interpretative approach and a case study design. We elaborated and applied a questionnaire to 31 trainee teachers aiming at accessing and obtaining a deeper understanding on their MKT at the beginning of their training – concerning mainly their contents knowledge. Such focus considers that, in order to be able to seek improvement in teacher training (and likewise in practice), it is essential to know the areas of mathematical knowledge where teachers (qualified ones and trainees as well) find themselves most deficient (Ribeiro & Carrillo, 2011).

These preliminary results allow identifying some mathematical critical features – aiming to discuss some possible reasons underneath such lacunas. The next step will be to research the factors that promote early years' trainee teachers' MKT and how such knowledge evolves (in order to be able to design “teaching activities” that may promote it). In this poster we will present some of the identified mathematical critical situations concerning bar graphs and their relationships with possible misunderstandings linked both to the role/meaning of variables and the increasing complexity topics do gain along schooling.

**Acknowledgements:** The work for this poster has been partially supported by the Portuguese Foundation for Science and Technology (FCT).

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# MODIFYING TOULMIN'S MODEL OF ARGUMENTATION TO REPRESENT ELEMENTARY STUDENTS' ARGUMENTS

Chepina Witkowski Rumsey

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Thurston (1995) and Herbst (2002) posited that students should be given the opportunity to see mathematics as a social process that involves conjecturing, and that students can be partners in the construction of knowledge. If our goal is for students to make and critique mathematical arguments beginning in elementary school, we need a powerful way to understand and study their arguments.

Toulmin's model of argumentation has been used to construct claims and examine the claims of others. His model has been applied across disciplines in order to identify, create, and evaluate arguments in science education, mathematics education, and language arts. I sought to use a modified, field-specific version of Toulmin's model of argumentation to study fourth-grade students' arguments regarding the arithmetic properties during instruction that promoted mathematical argumentation. I modified the model because I found that some aspects of children's arguments were overlooked. I needed a model that would enable me to represent students' arguments authentically without missing relevant components. I modified the model by incorporating components from other mathematics education researchers and made a modified-Toulmin model that is more suited towards studying elementary school students.

In this poster presentation, I will highlight key points and hurdles in developing a modified model of argumentation specific to the elementary level and present the original Toulmin model of argumentation and a revised model for participants to critique. In addition, I will present transcripts from the study and corresponding models on the poster as examples of the situations when the modified model could be appropriate.

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# OCCURRENCES OF POLYSEMES IN WESTERN SCHOOL MATHEMATICS: DIFFERENCES BETWEEN WESTERN AND FIRST NATIONS CULTURES

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This poster introduces upcoming research into the identification of polysemes (words which have multiple meanings) found within Saskatchewan's (a mid-western Canadian province) mathematics curricula. Existing research into language and mathematics include the study of the register of mathematics (e.g., Pimm, 1987), incidences of polysemy involving terms used in Western School Mathematics that have alternate meanings within the Western culture outside of mathematics (e.g., Zazkis, 1998), and the translation of mathematical terminology into culturally relevant and meaningful Indigenous words and expressions (e.g., Barton, 2009). This poster considers a variation of the polysemy research - that of alternate meanings for terms from Western School Mathematics found within the First Nations cultures of Saskatchewan.

Aboriginal scholars (e.g., Absolon & Willett, 2005) note that, despite the processes of colonization that have and continue to work towards the elimination of First Nations languages in Canada, the cultural understandings of those languages are carried forward in the English language used within First Nations communities and homes. This research aims to determine examples of mathematics terms for which this polysemy is a reality, and in doing so, provide a foundation for further research into the implications for the teaching and learning of mathematics for Saskatchewan's First Nations students, while also supporting Saskatchewan's First Nations peoples as they work towards the recapturing and preservation of their cultures and languages. This research can also be used as the groundwork for similar research around the world that seek understanding of the polysemy of mathematical terms between cultures.

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# CONNAISSANCES AND SAVOIRS IN THE FRAMEWORK OF MATHEMATICS RALLYE

Lucie Ruzickova, Jarmila Novotna

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The poster presents some results of research addressing the question whether the framework of mathematics rallye (Brousseau, 2001) allows to observe and to distinguish the situations in which pupils display their *connaissances* and *savoirs* respectively (Brousseau, 1997; Chopin & Novotna, 2011).

In mathematics rallye, the pupils in small groups proceed from one “point” of a labyrinth to another. At each point they get the information and questions which direct them to the next point. Finally, all the pupils try to reconstruct and organize the information gathered by individual groups on their way through different points. Research data has been collected during three consecutive 8 grade lessons related to geometric congruencies and organized according to the principles of mathematics rallye. Pupils worked in groups to gradually carry out specific tasks which required them to apply various solving strategies. The strategies used by the pupils to complete the group work tasks enabled them to formulate certain characteristic properties of congruent transformations. Then, the properties were presented in the follow-up whole class discussion directed by the teacher to institutionalize the newly acquired pieces of knowledge. Classroom field notes and audio and video recordings were examined through qualitative analysis in terms of the form and mathematical content. Each group’s worksheets were analysed to identify the solving strategies used by the pupils and the mathematical knowledge underlying them.

The results of the research suggest that mathematics rallye provides suitable framework for identifying the exhibits of pupils’ *connaissances* and *savoirs*. The poster will include an outline of the principles of mathematics rallye, detailed description of individual group work tasks, and authentic examples of pupils’ solving strategies analysed in terms of *connaissances* and *savoirs*.

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# THE ANALYSIS OF THE EFFECT OF INTERVENTION ON COMBINATORIAL PROBLEM

Takako Sakawaki

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The purpose of this study was to analyse whether intervention on some kinds of combinatorial problems (permutations or “arrangements without replacement”) could improve the understanding of a different kinds of combinatorial problems (combinations). Fischbein & Gazit (1988) showed that tree diagram instruction could improve the understanding of most kinds of combinatorial problems. However, it was considered to be difficult to apply this instruction to combination problems directly. It is necessary to find more effective way of instruction. In this study, the effect of intervention focused on the more conceptual side of these kinds of tasks was analysed.

28 children (5<sup>th</sup> graders) in a peer interaction condition (15 dyads. 2 children dropped out of the study before it finished) and 11 children in an individual condition participated in an intervention session and pre- post1-, and post2- tests. The intervention session was aimed at fostering the understanding of the rules or structures of permutations or “arrangements without replacement”. Pre- and post- tests included different kinds of combinatorial problems. The combination problem in paper and pencil format was chosen for analysis. 2 children who answered inadequately were omitted from the analysis. Sakawaki (2008a) has already shown the effect of this kind of training on the performance of making “arrangements without replacement”.

Subjects’ strategies of making combinations were classified into 4 different types. The result showed the difference between pre- and post test1 was significant in peer condition as a whole by Friedman test and post hoc sign test with Bonferroni correction. The result suggests that this kind of intervention could improve the strategy of making combinations as well. However, Sakawaki (2008a, b) also showed that the understanding of more abstract task was difficult to improve after this kind of intervention. Further research will be required.

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# DEFINING FROM SOCIOCULTURAL APPROACH

Gloria Sánchez-Matamoros; Isabel Escudero; José M. Gavilán

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This poster is situated in the context of a more wide research, which has as aim the study of students' mathematical learning of no-compulsory levels (Sánchez, García, Escudero, Gavilán y Sánchez-Matamoros, 2008). From a sociocultural approach (Sfard, 2008), we assume that mathematics is a special type of discourse and learning mathematics means a change in that discourse. In our research, we analyse it on the basis of the four properties identified by Sfard, (2008): Mathematical words, Visual mediators, Endorsed narratives and Routines.

The context on which we focus on our research is the mathematical process of defining, and the result of it, the definition. The importance of this process and its product has been emphasised in numerous studies (Harel, Selden & Selden, 2006). In this part of the project we try to characterize the changes in the mathematical discourse. The data for our study consists mainly of the transcriptions of the verbal dialogues the small group of students maintained when they solved a mathematical task in the classroom. Here we show the scheme of analysis developed to identify changes in the discourse, and some first results. We will illustrate these aspects with some examples related to how the use of the routines influences the changes that take place in the narratives.

## Additional information

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# MATHEMATICAL CREATIVITY IN PROBLEM SOLVING: DEFINITION

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The aim of this paper was to examine the research line related to mathematical creativity in problem solving. The methodology was to review and synthesis the literatures related to creativity in problem solving.

Creativity is a word which has a wide range of meanings as it is used in daily life. Therefore, the interpretation of the word "creativity" may vary with different people. Creativity is almost infinite. It involves every sense – sight, smell, hearing, feeling, taste, and even perhaps the extrasensory. Much of it is unseen, nonverbal, and unconscious. However, if we are to study it scientifically, we must have some approximate definition. There have been many attempts to define creativity. They all seem to have something in common, and yet each is slightly different.

Guilford (1956) has conceptualized creativity in terms of the mental abilities involved in creative achievement. The divergent thinking category is the factors of fluency, flexibility, originality, and elaboration. He believes that sensitivity to problems and redefinition abilities are also important in creativity. The redefinition abilities involve transformations of thought, reinterpretations, and freedom from functional fixedness in deriving unique solutions.

Saito (1998) defined based on the school context, "Creativity is the ability for making a new worthwhile and valuable thing which is evaluated by the members of the group when students solve a problem and the personal character". However, there is the common definition of creativity, it is "a new worthwhile and valuable" and "make a new ideas".

The definition of creativity in this research is based on Saito (1998), in school context; creativity is the ability for making a new worthwhile and valuable thing which is evaluated by the members of the group when students solve a problem and the personal character. Moreover, the types of problems are important to study mathematical creativity.

The findings were research trends to be 1) Assessment mathematical creativity, 2) Mathematical creativity and problem posing, and 3) collaborative problem solving and creativity.

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# FEEDBACK IN MATHEMATICS LEARNING

Leonor Santos, Institute of Education, University of Lisbon

Jorge Pinto, School of Education of Setúbal

Feedback is a key element for an assessment practice guided in a deliberate way by an assessment for learning (Black & Wiliam, 1998). The research has identified characteristics of feedback to promote learning, such as a descriptive approach focus on improvement (Tunstall & Gipps, 1996), and about the processing of the task or self-regulation (Hattie & Timperley, 2007). Feedback should never come up before the student has had the opportunity to work on a particular task (Wiliam, 2007), why it is desirable to choose tasks developed in two stages, without grading the first one. Several studies, along three years, has been carry on in Portugal, on the scope of the Project AREA. A meta analysis of some of them, that followed an interpretive methodology, and analysed the practices of feedback of three mathematics teachers with students from 8 to 14 years old, pointed out that reflected experience, allows the evolution of the feedback writing, considering the space it gives to the students, as a reviewer of their own mistakes and therefore author of their own learning. Mathematical open tasks tend to originate a feedback to improve justifications and mathematical close tasks a feedback directed to the self correction of specific mathematical aspects. However, the high level challenge tasks revealed the need to the teacher to make longer comments. From the point of view of the students, short feedback is easier to understand and helps them to focus on certain specific aspects of the task. Certain feedback has been ineffective not due to its form as such but due to the students' perception of the task itself. Furthermore, feedback generally helps students with a good performance in mathematics, but it is not always effective for the ones for whom feedback could be most vital. This is a great question for which we still have not found an answer. The poster will use pictorial format.

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# HOW CZECH MATH TEXTBOOKS ADDRESS CRITICAL MATHEMATICAL CONTENT

David Stein

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*The poster compares 4 Czech middle school math textbook series in the way they address critical mathematical content that the authors identify within the text as likely to cause difficulties to the students while being essential. This indication of difficulty can be communicated directly to the student by the textbook or to the teacher by supplementary notes published as didactical helpbook.*

*The poster displays a picture of a representative textbook for each series together with color-coded summaries of how often authors identify critical mathematical content, how (if at all) do they explain their belief the content is critical, and how (if at all) do they provide didactical instruction for students or teachers intended to help them to deal successfully with such critical mathematical content.*

# EXPLORING THE TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING IN TRADITIONAL CLASSROOM

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Most of elementary mathematics teachers in Thailand did not earn mathematics education degree, and model of professional development is a sort of short course training focused on contents (Office of the Education Council, 2010). The objective of this study was to explore mathematical knowledge that teachers need to know for teaching effectively. The targeted group was 100 sixth grade teachers teaching mathematics in southern of Thailand. The theoretical framework conducted in the study is based on “a practice- based theory of mathematical knowledge for teaching” (Ball et al., 2009). The test on mathematical knowledge for teaching focused on number and operation was developed by Hill et al. (2004). The results showed score as the followings:

- 1) The content knowledge (47.43%) indicated that teachers need to know the definitions, concepts of operation, and the common content on higher grade.
- 2) The knowledge of content and student (42.70%) indicated that teachers need to know the students strategies using to find the answer, the students misconception and error, and the student difficulties for solving problem situations.
- 3) The knowledge of content and teaching (25.34%) indicated that teachers need to know the samples for helping students learn to develop several different ideas, using questions ask students in order to help clarify issue, and connecting students ideas.

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# THE STUDENTS' PROCESS OF ABSTRACTION BASED ON ACTION IN COMPRESSION TO THINKABLE CONCEPT OF BLENDING EMBODIMENT AND SYMBOLISM UNDER CONTEXT USING LESSON STUDY AND OPEN APPROACH

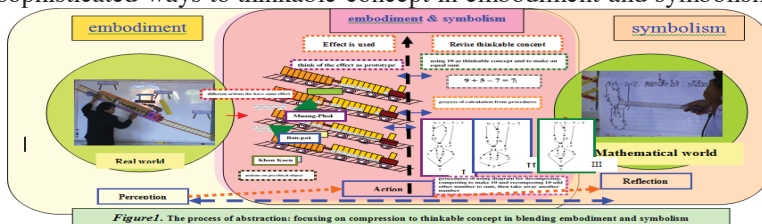
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Gray&Tall (2007) observed that the abstraction process through compression to thinkable concept is the key to developing increasingly powerful thinking. Tall (2007a) noted that to a parallel construction of compression in the symbolism and embodiment to thinkable concept. In addition to this, Tall (2007e) suggested that Lesson Study provided area of the students' compression to thinkable concept. This poster aims to present the first grade students' process of abstraction based on action, it can be seen through compression to thinkable concept in blending embodiment and symbolism from empirical evidence. In Thai classroom using Lesson Study and Open Approach produced the students' mathematical thinking by focusing on open-ended problems with designed material for supporting interaction of students and their problem solving arithmetic. The data collected were used ethnographic study and teaching experiment, with the four of first grader as targeted group at Kookham Pittayasan School, a project school with supported by CRME. The research result revealed that students manipulated with designed materials for supporting and checking their thinking various symbols before into same effect. They gave meaning symbolism enables compression to an efficient symbolic. Especially, in the students' abstraction process based on action, the student's thinking shift steadily from performing sequence of compression from actions being linked together in increasingly sophisticated ways to thinkable concept in embodiment and symbolism.



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# A DIFFERENT APPROACH TO DEVELOPING MATHEMATICAL THINKING: 4MAT SYSTEM

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In literature, definitions of mathematical thinking (MT) indicate that MT is a process, an advanced thinking approach or a cycle. For example according to Tall (1995) MT is an important concept that provides individuals to make sense of the events in their lives. MT begins with perceptions of the objects (input), continues with internal processing (thought) and results in an output (action). According to Woodham (2008), internal factors as individual differences, pre-knowledge, experience, perceptions, belief, and expectation; external factors as family, class environment, teacher, teaching curriculum are effective in the development of MT. Therefore learning style which is one of the individual differences can be a factor during the development of MT. This assumption lead us to think that MT can be developed through an application of 4 MAT (McCarthy, 1990). Because in the 4MAT systems' first quadrant learners exactly understand events, facts and problems. In the second quadrant at the stages of image and inform, learners determine which method/strategy to use and they practice it. In the third quadrant at the stages of practice and extend, learners practice thoughts, they abstract, model, generalise, reason and relate, and in the fourth quadrant at the stages of refine and perform, it is expected from learners to enhance their skills by giving them opportunities to think creative and different and to develop a product. Accordingly, concrete connections exist between 4MAT and MT. In this study we considered these common points and tried to make different connections between 4MAT system and MT development. MT that was assumed as a six-stage process and 4MAT system which is a learning cycle with four quadrants and eight stages were connected to each other. Therefore, a model related with this connection was suggested.

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# PERCEPTIONS OF TEACHERS' USE OF MATHEMATICS CURRICULUM MATERIALS

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After 2003, Ministry of National Education gradually introduced the reform mathematics curriculum for elementary education in Turkey. Since Turkey has a nationwide curriculum, teachers in general are expected to implement curriculum in similar ways. However, considering teaching as a social practice, teachers' use of curriculum will inevitably be affected by the nature of their institutional contexts as well as limitations and affordances they are faced with in their environments as they interact with curriculum (McClain et al., 2009), besides their individual characteristics. Despite overall positive orientations towards reform mathematics curriculum materials, Erbas and Ulubay (2008) report that these materials are only "occasionally" implemented in Turkish elementary classrooms. Therefore, it is important to investigate how teachers make important decisions when they implement mathematics curriculum.

Employing a case-study approach, the researcher has conducted semi-structured one-hour length interviews with four participants, who are at different stages of their careers and work in different settings. Data have been analyzed based on an orientative framework developed by the efforts of design researchers (McClain, et al., 2009). This orientative framework is composed of three constructs: *instructional reality*, *agency* and *professional status*, which will be illustrated in the poster.

The preliminary analysis suggests that teachers' decisions of curriculum use in this study are mediated by their experience, educational level and perceived mathematics knowledge in addition to the three constructs mentioned above. The initial results support the viability of an orientative framework in the context of Turkey. Implications for further research regarding mathematics reform curriculum materials use and professional development aimed towards helping teachers will be provided.

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# LEVELS OF INTERVENTION IN THE CURRICULUM

## A CASE STUDY IN PORTUGUESE SCHOOLS

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*On the poster I intend to present a summary of data collection for an investigation into levels of intervention in the curriculum (Gimeno, 1998) developed in Portuguese schools. The textbooks of mathematics 10 Grade adopted in 2002 came with CD. That includes dynamic geometry applets, programs to teach functions and multiple choice questions.*

### RESEARCH QUESTIONS, DATA COLLECTION AND SOME RESULTS

How are teachers to integrate technological resources in support of school textbooks in mathematics classes in high school? What factors influence mathematics teachers in how to use and integrate technological resources? Data collection was done through workshops of teachers between September and December 2010, in which teachers discussed, planned and implemented math lessons using the technological resources of the textbooks adopted in their schools. The instruments for gathering documents and data were obtained from the analysis of the materials that were produced by teachers who performed the analysis on the technological resources, lesson plans and who prepared and presented their reflections on the lessons. The teachers kept on paper for students to resources that can be divided into the following categories: script support with the instructions that allowed the operation of the proposed tasks in the CD. Worksheets with tasks to accomplish with the help of application on CD and record the findings. Sheets summative or formative evaluation to assess the learning of the tasks performed from a CD. Teachers demonstrate knowledge of resource recovery technologies is proposed that the level of the formal curriculum (Gimeno, 1998), refer to the positive reactions from students about the classes that work with computers and regret for not having better conditions in schools to conduct lessons using technology.

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# DETERMINING VIEWS OF PROSPECTIVE MATHEMATICS TEACHERS RELATED TO MATHEMATICAL MODELLING

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This qualitative research examines views of prospective secondary mathematics teachers regarding mathematical modelling. The participants of the study contained 21 prospective teachers chosen from the Mathematical Modelling Course, an elective course in our teacher education program. At the beginning of the course, it was try to determine what the participants knew about modelling. For this purpose, three open-ended questions presented to the participants and they responded them in a written form. These questions were about their definition of mathematical modelling, the reason why they chose this course and their expectations related to this course. The data obtained from responses of these questions were examined and categorized by using content analysis. Data analysis showed that the participants generally defined mathematical modelling as developing material to concrete mathematics, to relate real world and to comprehend mathematical content. Three participants stated mathematical modelling as solving real problems by transferring mathematical problems. The participants' definitions were quite limited. The reason why they presented limited definitions were that some of them expressed they inferred from word meaning of modelling, some of them used what they listened to prospective teachers who took modelling course in previous year, and some of them responded the questions from their knowledge remained in examining the secondary mathematics curriculum. One participant was determined that she researched about mathematical modelling for preventing her wonder after she selected the course. When considering their definitions, it was observed that they could not accurately define mathematical modelling. Among participants' reasons why they chose this course, it was pointed out their ideas that this course would contribute to their teaching life, discharging Secondary Mathematics Curriculum's requires and developing their mathematical thinking. The participants thought that mathematical modelling would contribute students to behave more actively in mathematics lessons, mathematical subject to be more understandable, permanent, funny and mathematics to be concrete. On the other hand, as seen in Miva (1986; cited in Bukova-Güzel, 2010)'s study, the participants emphasized that mathematical modelling provided a useful occasion to associate mathematics with the real world in this study.

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# CHALLENGES AND CONSTRAINS IN MOVING BEYOND AN EVALUATIVE TEACHING MODE

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The *teaching modes* is a theoretical notion comprised of teachers' inter-related classroom questioning, listening, and responding approaches, teachers' dominant patterns of classroom interaction, their key beliefs about mathematics and its teaching and learning, and their levels of reflective thinking (Tomás Ferreira, 2005). Three teaching modes – evaluative, interpretative, and generative – provided the developmental path for a teacher development experiment (TDE) (Simon, 2000) conducted to provoke, describe, and interpret aspects of teacher development closely related to classroom practice. The TDE was framed in four phases throughout the year-long student teaching practicum of a group of four Portuguese prospective teachers. The generation of accounts of practice for each participant allowed the identification of some factors that constrained the envisioned developmental trajectory, from an anticipated evaluative teaching mode to the increasingly generative one. In this poster, I will elaborate on the case of Diana. Despite holding a perspective of mathematics teaching and learning aligned with current recommendations for school mathematics, Diana experienced many tensions in her quest to teach in a generative teaching mode, creating a great sense of frustration. One significant factor that contributed to such a feeling was related to her sharing of a classroom with a peer who did not (want to) know how to work together in that context. Fighting against external pressures to cover the curriculum and a very strong passiveness and lack of motivation of her students also caused Diana a great sense of helplessness. The lack of sound mathematical knowledge in some specific content topics prevented Diana from teaching how she wanted to teach. Nonetheless, whenever Diana found favourable conditions, she resolved some conflicts between her espoused and enacted beliefs, evidencing many instances of an interpretative teaching mode, and never losing her goal of reaching a generative teaching mode.

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# CREATIVITY, PROBLEM SOLVING AND PROBLEM POSING

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In all areas of knowledge the need to have more creative people, capable of providing innovative solutions to problems has been emphasized. Mathematics is one of the best school subjects for promoting the development of creative thinking. Creativity begins with curiosity and involves students in exploration and experimentation drawing upon their imagination and originality (DFES, 2000). Creativity is typically used to refer to the ability to produce new ideas, approaches or actions. Students' mathematical creativity is characterized by three components: fluency (the production of multiple ideas), flexibility (shown by the students varying the approach or suggesting a variety of different methods) and originality (refers to the novel and unusual ideas generated by the students) (e.g. Leikin, 2009; Silver, 1997). So our purpose as mathematics educators is to provide all students (including future teachers) creative approaches for solving any problems and to think independently and critically.

Many research findings show that mathematical problem solving and problem posing are closely related to creativity (e.g. Silver, 1997). Problem posing can be a powerful strategy to develop problem solving skills and to have good problem solvers; on the other hand, to formulate meaningful mathematical problems, it is necessary to be a good problem solver. So, learning environments with problem solving/posing activity should be used in our classes in order to develop students' creativity. Challenging tasks usually require creative thinking and our recent work with pattern problems showed that this can be one possible way to promote creativity in students.

We adopted a qualitative exploratory approach with elementary teachers to study in what way a didactical experience grounded on problem solving and problem posing is a suitable context to foster mathematical creativity.

We will present some preliminary results through the work of pre-service teachers in the exploration of some of the used tasks.

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# ELEMENTARY SCHOOL TEACHERS PRACTICES: WORKING STUDENT'S MATHEMATICAL REPRESENTATIONS

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This poster presents a study in progress with a double objective: (i) to identify elementary school teachers conceptions and practices related to pupils' mathematical representations; (ii) to understand how these conceptions and practices may change through their participation in a collaborative working group that promotes reflection and sharing of teachers' teaching practices. The theoretical framework assumes that a representation is something that represents something else. However, each representation may have several meanings, and a given concept may have several representations (Goldin, 2008). Teachers need to understand that learning representations is a slow and difficult process for students. In addition, they have to diversify the types of representation that they use and encourage students to connect the meanings of new representations to those that they already know (NCTM, 2000).

This investigation involves a group of five teachers of grade 4 pupils, of the same school ("agrupamento"). First, each teacher is interviewed in order to understand what their conceptions and their understanding about mathematical representations. Then, during one year, the group assisted by a facilitator meets every other week, for one hour and a half. In these meetings, teachers reflect about their work with students, analyze the emerging difficulties and work on tasks that may be used later in their classrooms. The facilitator observes the classes, providing support to the teachers, and making records for future analyses by the collaborative group.

Through diagrams and charts, in a schematically and suggestive way, the poster presents the structure of the research process. Using text boxes it also presents the main elements of the theoretical framework, referring key authors that address representations and teachers' practices. The poster will also present in clearly labelled text boxes the results of preliminary interviews with teachers and explain their views with regard to the use of mathematical representations in the classroom. A box with conclusions will address implications drawn for the continuation of the study.

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# RECONSIDERING THE CATEGORY FRAMEWORK FOR DESCRIBING MATHEMATICS TEACHERS' VALUES

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This paper proposes a modified category framework derived from the 'Values and Mathematics Project' (VAMP) and The 'Values in Mathematics Teaching' projects (VIMTs) for describing teachers' mathematical and pedagogical values, and examines the dialectical relations between values awareness/willingness and teaching, based on case studies of student teachers of secondary mathematics from a follow-up project of VIMTs. VAMP based in Australia was a 3-year research study focusing on exploring teachers' intended and implemented values in the mathematics classroom (Bishop, Seah & Chin, 2003). And VIMTs initiated in Taiwan covered different contexts of schooling focusing on exploring the contents and relationships of in-service mathematics teachers about intended and implemented values, and examined the extent and deepness to which the teachers could clarify or change their own values (Chin & Lin, 2000a; Leu & Wu, 2000).

The case study method, including questionnaire survey, interviews and classroom observations, was used as the major approach of inquiry to investigate the pedagogical values of a group of 6 student teachers. The systematic induction process and the constant comparisons method based on the grounded theory were used to process data and confirm evidence characterized the method of our study. The preliminary results show that students would teach certain values depending on the awareness of values priority, willingness to teach, their teaching capabilities and classroom conditions. So mathematics teacher educators should provide relevant courses to facilitate student teachers to be aware of their implicit values and be willing to enact these values, and to empower student teachers with the knowledge and experiences to teach the values.

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# TEACHER'S LISTENING TO STUDENT VOICES IN MATHEMATICS CLASSROOM

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Many simply may not realize that children have their own mathematical ideas and strategies – which can differ from teachers' own thinking about mathematics – and so they do not expect to hear these ideas and strategies. Even when teachers do ask children to express their thinking, listening effectively is surprisingly hard work (Susan, Empson & Victoria, 2008). Moreover, many teachers who are used to talking and telling find it hard just to listen and try to interpret what is said without offering their own views or “the right answer”. Thus, it is important that teachers become aware that listening is one of the tools they can use to inform themselves about what learners are doing and thinking (Zaslavsky, 2010). This study focus on teacher's listening to student voices as responsive listening refers to listening in which the teacher not only intends to hear the student's thinking but also actively works to elicit, make sense of, and respond to that thinking (Schneier, 2001). The purpose of this study was to searching for guidelines that incorporate listening to student voices in mathematics classroom. The targeted group was 120 teachers who participate in professional development project. Data collection was done by teachers evaluate their own listening behavior of Bommelje and Steil (2004), interviews of the teacher concerning the results of the evaluation of there listening behavior, and data collected from observing classroom activities. These data were used for analyzing the teachers' listening to student voice according to Schneier framework (2001).

The result showed that the mathematics teachers have organized learning experiences for teachers to listening and responding to student voices both in: (a) discussions of student's written work, (b) opportunities for teachers to interact with student and then to reflect on those experiences with their teaching.

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# US NATIONAL MATH ADVISORY PANEL'S SOCIAL NETWORK

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This poster presents analysis of the social context surrounding the US National Math Advisory Panel (NMAP), a group of persons appointed in 2006 by Secretary of Education Spellings to produce a federal policy statement on math education. The work expands previous NMAP analyses, e.g. Gutstein (2008) and Greer (2008).

Two units of analysis focus this study of NMAP's context: the professional biographies, writings and statements of panelist members and the writings and statements of the organizations to which each panelist was affiliated leading up to and during NMAP. This choice reflects a perspective on policy-making whereby private organizations influence the process via the affiliations that individual policy-

makers have with private interests (Wedel, 2009).

The analysis begins with a sociogram that visually expresses the types of individuals and organizations contained within NMAP's context (Figure 1), follows through with a coding process of both panelists' and affiliated organizations' statements regarding math education,

and concludes with a table summarizing the salient themes contained in NMAP's context. Samples from among this table include human capital as a primary motivation for US math education and the deficit of teachers' content knowledge as a primary problem with US math education.

and concludes with a table summarizing the salient themes contained in NMAP's context. Samples from among this table include human capital as a primary motivation for US math education and the deficit of teachers' content knowledge as a primary problem with US math education.

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# EYE MOVEMENT IN READING GEOMETRY: DISTRIBUTION OF TEXT AND FIGURE

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Many studies using eye movement to investigate reading process of multimedia (eg. scientific text, news, or advertising). Some studies indicated that participants had more fixations in picture than text (eg. Rayner, Rotello, Stewart, Keir, & Duffy, 2001), the others claimed the process of reading is text directed (eg. Hegarty, 1992). This study discussed the priority of text and picture in reading geometry.

The study collected eye movement data of 65 college students were not major in mathematics by EyeLink 1000. Materials included 12 geometric descriptions (Figure 1) and two proof items in Chinese. “Square item” used sides and angles of two squares to prove two triangles are congruent. “Circle item” aimed to prove power of a point. The fixation point of drift correction was in the central of screen because text in left part and figure in right part. Results showed: (1) Most of first fixations were in text, instead of figure. In 723 valid data of geometric descriptions, less than 10% first fixations were in figure, and only 17% of these data still stayed in figure as second fixation. In 100 valid data of proofs, 32% first fixations were in figure, and only 9% of these data stayed in figure. (2) In geometric descriptions, the average of TVD (total viewing duration) in text (4080ms) was longer than figure (2400ms),  $F(1, 57) = 362.91, p < .001$ . In two way ANOVA analysis of proofs, there was a significant interaction between item and text-figure,  $F(1, 57) = 33.96, p < .001$ . In Square item, post-hoc data showed TVD of figure (25538ms) was longer than text (21145ms),  $F(1, 57) = 14.22, p < .001$ . In Circle item, TVD of text (22484ms) and figure (20116ms) were similar.

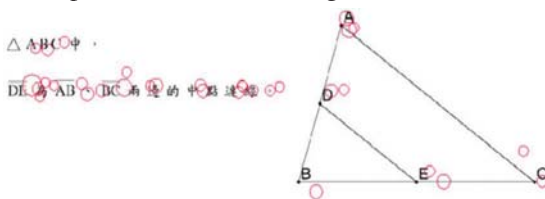


Figure 1: The distribution of fixations in a geometric description

Most of participants were text directed in reading geometry. Text can help them to form primary representation and the function of figure is meaning elaboration. Participants realized the construction of geometric figure and the spatial relationships of elements through descriptions reading. Their eye movement data showed that the duration of text was 1.7 times as long as figure. The argumentation is the key different between proofs and geometric descriptions. In the process of argumentation, figure can provide cue of hypothetical bridging which might explain the increasing ratio of duration in figure.

# INVESTIGATION ON MATHEMATICS CURRICULUM TRANSITION: FROM SCHOOL TO UNIVERSITY

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## AIM AND THE MAIN IDEA OF THE STUDY

It's no doubt that the transition from secondary school to university in mathematics is a very important issue (Clark, 2009), especially after the carrying out of reformation of secondary school courses. Perhaps the most notable feature of existing body of research on linking is the absence of investigation, as much part of research is on the comparison of teaching materials used in university with those in secondary school. Our research is intended to find out the real difficulties and the most important issues in curriculum transition via investigation on freshmen in university and teachers in secondary schools.

## METHODOLOGIES

We have employed an integrated tool: two semi-structured questionnaires and two respective interviews with the freshmen and teachers. Based on our textbooks research and preparatory investigation, we investigated 269 freshmen and 15 teachers in high schools respectively. Furthermore, we interviewed 24 students and 10 teachers to get more feedback.

## CONCLUSIONS

Results of questionnaires show that the difficulties in mathematics curriculum transition are (numbers in braces are the proportions of choosers to all): properties of inverse trigonometric functions(65%), definition of differential(53%), application of polar coordinates (50%), calculation of integral(50%) ,etc. Some content that had been taught in high school needs to be taught again in university. Take integral for example: 52% freshmen think that the definition and calculation of integral should be taught again in detail at university. We have got other useful feedback from 28 questions in the questionnaires and interviews.

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# THE DEVELOPMENT AND APPLICATION OF NUMBER SENSE THREE-TIER TEST FOR FOURTH GRADERS

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The major purpose of the study was to develop a “Three-tier Number Sense Test for 4<sup>th</sup> graders and assess their number sense development. The three-tier number sense test implies that the first tier assesses the students’ knowledge about particular number sense problems, the second tier explores the students’ reasons for the choice they made in the first tier, while the third tier examines the students’ confidence index when they respond the above problems. The three-tier number sense test was designed by the researchers based on the earlier studies (Haki, Ali, 2010; Yang, Li, & Chiang, 2007). It includes 4 number sense components with each component has 8 items. Therefore, the three-tier test includes 32 number sense items totally. 166 fourth graders from a public primary school in southern Taiwan were selected to join the test. The results of factor analysis show that the Cronbach's  $\alpha$  coefficients for the four number sense components of the three-tier test are 0.703, 0.777, 0.785, and 0.734, respectively. The Cronbach's  $\alpha$  coefficients for the whole three-tier test is 0.907. The validity of the three tier-test covered the content validity, specialist validity, and construct validity. To ensure the designed items were representative and not beyond the curriculum scope usually taught to 4<sup>th</sup>-graders in Taiwan, two mathematics educators and five elementary school teachers were invited to review the items. All agreed that the 32 test items were appropriate in terms of item content for 4<sup>th</sup>-graders involved in this study. Therefore, the Three-tier Number Sense Test for 4<sup>th</sup> graders has acceptable reliability and validity. Data also shows that the 4th graders’ NS development between the 4 components of NS appears significantly different. In addition, there is a gap between actually-used problem solving methods and subjectively-perceived problem solving methods.

Keywords : *Fourth Graders, Number Sense, Three-Tier Test*

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# TRIBAL CLASSROOM: CREATING MATH ENVIRONMENTS FOR INDIGENOUS CHILDREN

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*“Mathematics for all” has been internationally considered to be a key issue of mathematics education (NCTM, 2000). And how to improve indigenous mathematics education through culture and local characteristics is already the primary question among education scholars in many developed countries. The Ministry of Education in Taiwan also highlights the importance that school education should take care of all children in mathematics learning, especially for indigenous students who need special attentions (Ministry of Education, 2003). This case study is a part of a four-year project supported by the National Science Council of Taiwan from 2009 to 2013. The main purpose of this project is to improve the environment of mathematics learning and teaching for an indigenous elementary school in Nan-tou County of Taiwan. The school students mainly come from three different tribe, with a majority being the Sediq people. The focus of this presentation would be on the creating of math environments for indigenous children through the Math Weekend Camp. The Camp was accomplished with the help of math volunteers, eleven university students and six in-service teachers. Two sessions have been scheduled for the Math Weekend Camp. Both were completed at the case school. The first session included three mathematical activities, such as “Journey over the Clock” for Grade 1, “It’s Fun TIME!” for Grades 2-3, and “Now on TV” for Grades 4-6. The subject of second session was “Knowing and Constructing Your Home.” For the junior grade students, the activity titled “Beautiful Home” covered how to develop mathematical understanding of 2D graphs and 3D graphs, and related styling/stacking activities. For the middle grade students, the activity titled “I’m a Little Architect” covered how to develop mathematical understanding of and how to calculate the cubic volume. For the senior grade students, through a game-passing activity titled “Knowing A-Z” in campus, it covered how to develop mathematical understanding of the measurement and calculation of length, area, and volume as well as related problems about proportion. Each activity of the Math Weekend Camp was videoed and elaborated to be shared. Through observation and analysis of related documents, the researcher gained an insight into the mathematics learning of indigenous students in the case school, the researcher then can try to develop applicable mathematics learning support and teacher education programs based on these understanding and findings. Hopefully, once the project has been completed after four years, the place-based mathematics learning environments created for indigenous children will continue to be used and be helpful to enhance the tribal children’s mathematics learning.*





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