PROCEEDINGS

of the

36th Conference of the International Group for the Psychology of Mathematics Education

Opportunities to Learn in Mathematics Education

Editor: Tai-Yih Tso

Volume 1

Plenary Addresses, Plenary Panel, Research Forums, Working Sessions, Discussion Groups, National Presentations, Poster Presentations

PME36, Taipei – Taiwan

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PREFACE

It is a great pleasure to welcome you to the 36th Annual Conference of the International Group for the Psychology of Mathematics Education, which is held in Taipei, at the National Chiang Kai-Shek (C.K.S.) Memorial Hall and Wesley Girls High School.

Taipei City is the capital of Taiwan. It was founded in the early 18th century and soon became the political, economic, and cultural center of Taiwan. Taipei is a city where the traditional culture gracefully meets the rapidly changing new developments. The National Palace Museum, holding one of the largest collections of Chinese artifacts and artworks in the world traces it steps back to the dated Chinese culture. The renowned night markets also give abundant taste of our history. The new and modern is well represented by Taipei 101, a 101-floor landmark skyscraper that rose as the world's tallest building when it opened in 2004. The building incorporates many engineering innovations to withstand typhoon winds and earthquake tremors, which are not unfamiliar to the island.

The National Chiang Kai-Shek (C.K.S.) Memorial Hall, the venue for conference registration and opening is another illustration of the traditional culture. Its classic Chinese style architecture is a major tourist attraction, which stands in memory of a former President of the Republic of China, Chiang Kai-Shek. The other venue site, the Wesley Girls High School, is located adjacent to the National Palace Museum. Founded in 1960, Wesley Girls High School is noted for its devoted teachers by whom students are inspired and given the opportunity to pursue life to its full potential.

Hosting PME36 in Taipei is a meaningful challenge and a memorable experience for the local organizing committee composed of members of the Taiwan Association for Mathematics Education (TAME). The theme for PME36 is "Opportunities to Learn in Mathematics Education," which is also what the mathematics education community in Taiwan has been aiming for during the past years. We hope to meet the prospect that education should be developed and promoted in more diversified dimensions by creating opportunities to learn, enhancing proper resources, and providing contemporary learning materials.

The written contributions to our conference are organized into four volumes. The first volume includes the plenary lectures, plenary panel, research forums, discussion groups, working sessions, national presentations and poster presentations. The second volume consists of forty-three research reports. Following that, another forty-four research reports are in the third volume. The fourth and last volume contains the remaining thirty research reports and ninety-seven short orals.

I am grateful to the local organizing committee and all contributors who have dedicated to the success of this conference. I appreciate Dr. Laurie Edwards for setting up ConfTool, Dr, Bettina Roesken for helping me understand all administrative operations of PME, Dr. Joao Filipe Matos for assisting me to manage conference affairs, Dr. Fou-Lai Lin for providing experiences and suggestions about the organization, and the principal of Wesley Girl's high school, Jya-Yi Wu, for providing the venue to hold the conference. I am also grateful to International Program Committee members, Local Program Committee members, and all staffs assisting the conference preparations. I deeply believe that our devoted efforts will lead to a successful experience.

Finally, we hope you enjoy your stay in Taipei and find your participation at the conference fruitful and unforgettable.

Tai-Yih Tso
Chair of PME36

SPONSORS

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National Science Council, Republic of China

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THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION (PME)

History and Aims of PME

The International Group for the Psychology of Mathematics Education (PME) is an autonomous body, governed as provided for in the constitution. It is an official subgroup of the International Commission for Mathematical Instruction (ICMI) and came into existence at the Third International Congress on Mathematics Education (ICME3) held in Karlsruhe, Germany in 1976.

Its former presidents have been:

Efraim Fischbein, Israel	Carolyn Kieran, Canada
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The present president is João Filipe Matos, Portugal.

The Constitution of PME

The constitution of PME was adopted by the Annual General Meeting on the 17th of August, 1980 and changed by the Annual General Meetings on the 24th of July, 1987, on the 10th of August, 1992, on the 2nd of August, 1994, on the 18th of July, 1997 and on the 14th of July, 2005. We print here only two parts of the constitution. As members it is important that you are aware of your rights. The group has the name “International Group for the Psychology of Mathematics Education”, abbreviated to PME. The major goals of the Group are:

- to promote international contact and exchange of scientific information in the field of mathematical education;
- to promote and stimulate interdisciplinary research in the aforesaid area; and
- to further a deeper and more correct understanding of the psychological and other aspects of teaching and learning mathematics and the implications thereof.

PME Membership and Other Information

Membership is open to people involved in active research consistent with aims of PME, or professionally interested in the results of such research. Membership is on an annual basis and depends on payment of the membership fees. PME has between700 and 800 members from about 60 countries all over the world.

The main activity of PME is its yearly conference of about 5 days, during which members have the opportunity to communicate personally with each other about their working groups, poster sessions and many other activities. Every year the conference is held in a different country.

There is limited financial assistance for attending conferences available through the Richard Skemp Memorial Support Fund.

A PME Newsletter is issued twice a year, and can be found on the IGPME website. Occasionally PME issues a scientific publication, for example the result of research done in group activities.

Website of PME

All information concerning PME and its constitution can be found at the PME Website: http://www.igpme.org/

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Hans Freudenthal (Deceased)
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36th CONFERENCE OF THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION(PME36)

Two committees are responsible for the organization of the PME36 Conference.

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PME36 has a website at http://tame.tw/pme36/

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PME International

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1	1977	Utrecht, The Netherlands	Not available in ERIC
2	1978	Osnabrück, Germany	ED226945 3-922211-00-3
3	1979	Warwick, United Kingdom	ED226956
4	1980	Berkeley, USA	ED250186
5	1981	Grenoble, France	ED225809
6	1982	Antwerp, Belgium	ED226943 2-87092-000-8
7	1983	Shoresh, Israel	ED241295 965-281-000-2
8	1984	Sydney, Australia	ED306127
9	1985	Noordwijkerhout, Netherlands	ED411130 (vol.1), ED411131 (vol.2)
10	1986	London, United Kingdom	ED287715
11	1987	Montréal, Canada	ED383532 0771-100X
12	1988	Veszprém, Hungary	ED411128 (vol.1), ED411129 (vol.2)
13	1989	Paris, France	ED411140 (vol.1), ED411141(vol.2), ED411142 (vol.3)
14	1990	Oaxtepex, Mexico	ED411137 (vol.1), ED411138 (vol.2), ED411139 (vol.3)
15	1991	Assisi, Italy	ED413162 (vol.1), ED413l63 (vol.2), ED413164 (vol.3)
16	1992	Durham, USA	ED383538
17	1993	Tsukuba, Japan	ED383536
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21	1997	Lahti, Finland	ED416082 (vol.1), ED416083 (vol.2), ED4l6084 (vol.3), ED416085 (vol.4)
22	1998	Stellenbosch, South Africa	ED427969 (vol.1), ED427970 (vol.2), ED427971 (vol.3), ED427972 (vol.4)

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			0771-100X
23	1999	Haifa, Israel	ED436403 0771-100X
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25	2001	Utrecht, The Netherlands	ED466950 90-74684-16-5
26	2002	Norwich, United Kingdom	ED476065 0-9539983-6-3
27	2003	Honolulu, Hawai'i, USA	ED500857 (vol.1), ED500859 (vol.2), ED500858 (vol.3), ED500860 (vol.4) 0771-100X http://www.hawaii.edu/pme27/
28	2004	Bergen, Norway	ED489178 (vol.1), ED489632 (vol.2), ED489538 (vol.3), ED489597 (vol.4) 0771-100X http://www.emis.de/proceedings/PME28/
29	2005	Melbourne, Australia	ED496845 (vol.1), ED496859 (vol.2), ED496848 (vol.3),ED496851 (vol.4) 0771-100X http://staff.edfac.unimelb.edu.au/~chick/PME29/
30	2006	Prague, Czech Republic	ED496931 (vol.1), ED496932 (vol.2), ED496933 (vol.3), ED496934 (vol.4), ED496939 (vol.5) 0771-100X http://class.pedf.cuni.cz/pme30
31	2007	Seoul, Korea	ED499419 (vol.1), ED499417 (vol.2), ED499416 (vol.3), ED499418 (vol.4) 0771-100X
32	2008	Morelia, Mexico	978-968-9020-06-6 0771-100X http://www.pme32-na30.org.mx/
33	2009	Thessaloniki, Greece	978-960-243-652-3 0771-100X http://www.pme33.eu/pme33/index.php?page=home
34	2010	Belo Horizonte, Brazil	0771-100X http://pme34.lcc.ufmg.br/
35	2011	Ankara, Turkey	978-975-429-262-6 0771-100X http://www.arber.com.tr/pme35.org/

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1	1979	Evanston, Illinois	
2	1980	Berkeley, California (with PME2)	ED250186
3	1981	Minnesota	ED223449
4	1982	Georgia	ED226957
5	1983	Montreal, Canada	ED289688
6	1984	Wisconsin	ED253432
7	1985	Ohio	ED411127
8	1986	Michigan	ED301443
9	1987	Montreal, Canada (with PME11)	ED383532
10	1988	Illinois	ED411126
11	1989	New Jersey	ED411132 (vol.1), ED411133 (vol.2)
12	1990	Oaxtepec, Morelos, México (with PME14)	ED411137 (vol.1), ED411138(vol.2), ED411139 (vol.3)
13	1991	Virginia	ED352274
14	1992	Durham, New Hampshire (with PME16)	ED383538
15	1993	California	ED372917
16	1994	Louisiana	ED383533 (vol.1), ED383534 (vol.2)
17	1995	Ohio	ED389534
18	1996	Panama City, Florida	ED400178
19	1997	Illinois	ED420494 (vol.1), ED420495 (vol.2)
20	1998	Raleigh, North Carolina	ED430775 (vol.1), ED430776 (vol.2)
21	1999	Cuernavaca, Morelos, México	ED433998
22	2000	Tucson, Arizona	ED446945
23	2001	Snowbird, Utah	ED476613
24	2002	Athens, Georgia	ED471747
25	2003	Hawai'i (with PME27)	ED500857 (vol.1), ED500859(vol.2), ED500858 (vol.3),ED500860 (vol.4) http://www.hawaii.edu/pme27/

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26	2004	Toronto, Ontario	http://www.pmena.org/2004/PMENA2004_1.pdf http://www.pmena.org/2004/PMENA2004_2.pdf http://www.pmena.org/2004/PMENA2004_3.pdf http://www.pmena.org/2004/
27	2005	Roanoke, Virginia	http://www.pmena.org/2005/PME-NA_2005_Proceedings.pdf http://www.pmena.org/2005/
28	2006	Merida, Yucatan, México	http://www.pmena.org/2006/cd/book.pdf http://www.pmena.org/2006/
29	2007	Lake Tahoe, Nevada	http://www.pmena.org/2007/PME-NA_2007_Proceedings.pdf http://www.pmena.org/2007/
30	2008	Morelia, Michoacán, México (with PME32)	http://www.pmena.org/2008/Proceedings2008.zip http://www.pme32-na30.org.mx/
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33	2011	Reno, Nevada	http://www.pmena.org/2011/PMENA_Proc_2011.pdf http://www.pmena.org/2011/

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REVIEW PROCESS OF PME36

Research Forums. The international Programme Committee accepted the 1 submitted RF proposals. The proposed structure, the contents, the contributors, and the role were reviewed and agreed by the members of International Program Committee (IPC).

Working Sessions and Discussion Groups. There were 3 Working Session (WS) and 6 Discussion Group (DG) Submissions. The abstracts were all read and commented by the International Program Committee, and 5 DG and 3 WS were accepted. 1 Discussion Groups was recommended to re-submit as Working Sessions by IPC. The nine themes of the group activities planned for the conference covers a wide range of research areas that are relevant for mathematics education.

Research Reports (RR). The IPC received 224 RR proposals. Each paper was blind-reviewed by three peer reviewers. The experienced reviewers contacted for this purpose were not, however, enough. Thus, more reviews were asked from all the reviewers. The majority of the connected PME members responded to the request and contributed decisively to the successful completion of this crucial task.

Reviewers received proposals for review according to the research categories indicated in their Reviewer Information Form. The proposals were sent to reviewers according to the research categories marked by the author(s). All papers with two or three acceptances were accepted. The members of the IPC reexamined all the proposals with one acceptance and two rejections. For the proposals that were finally accepted, the fourth review was added to the existing three ones. For the remaining papers, the IPC offered a Short Oral Communication (SO) or a Poster Presentation (PP) or agreed that the paper should be rejected. Finally, 118 proposals were accepted, 65 were recommended as SOs, 35 as PPs and the remaining ones were rejected.

Short Oral Communications (SO) and Poster Presentations (PP). The IPC initially received and reviewed 96 SOs and 45 PPs proposals, 53 and 35 of which were accepted respectively. In addition, 49 SOs proposals were re-submitted from RRs and 21 PPs were re-submitted from RRs.

The reviewing process was completed during the 2nd Meeting of the International Programme Committee around the end of March 2012.

LIST OF PME36 REVIEWERS

The PME36 Program Committee thanks the following people for their help in the review process:

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Osterholm, Magnus (Sweden)
Ouvrier-Buffet, Cecile (France)
Owens, Kay (Australia)
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Pang, Jeongsuk (South Korea)
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Narrative, Discourse and Mathematics Education: An Historian's Perspective

NARRATIVE, DISCOURSE AND MATHEMATICS EDUCATION: AN HISTORIAN'S PERSPECTIVE

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In this talk I am going to tell a story about my own teaching experience in a liberal study course I gave for undergraduate students at National Taiwan University in the spring semester of 2012. For the goal of the course, "Mathematics and Culture", I adopted an approach of reading mathematical fictions in order to help students making sense of school mathematics they had been familiar with in high school years . By means of questionnaires, I collected how they were inspired by novelist's bring mathematical activities into plots of fiction. I will analyze the feedbacks in terms of narrative and discourse which are regarded by mathematics educators to play significant roles in learning of mathematics.

I. Introduction

"Professor, do you really believe that 0.999…=1?" A voice came from back at a classroom where I had just finished my talk on issues of mathematics and society for an undergraduate math major course given by my colleague Prof. Yiwen Su at her campus. This episode happened just because I had explained to the class how audiences attending my lectures on popular mathematics and mathematical fiction reflected on the equality could be used to demonstrate the tension or conflict between logic vs. meaning.

This episode also reminds of my teaching experience as I explained the same equality in my own class on the theme of mathematics and fiction which is a liberal study course for undergraduate students at National Taiwan University (NTU). My students and I were discussing on one of the plots of the *A Certain Ambiguity*, a mathematical novel among the list of reading assignment. The plot is about how Nico, a Stanford mathematics professor who was teaching a liberal study course, "Thinking about Infinity", in which Nico was addressing a problem concerning infinite sum: 2+1+1/2+1/4+1/8+1/16+…. And one of the students who attended the class, Clair, came up with an elegant argument as follows:

$$\text{Sum} = 2+1+1/2+1/4+1/8+1/16+1/32+\ldots$$

$$(1/2)*\text{Sum} = 1+1/2+1/4+1/8+1/16+1/32$$

By cancellation of the second from the first, Clair gets the result:

$$(1/2)*\text{Sum} = 2+0+0+0+0+0+0++\ldots \Rightarrow (1/2)*\text{Sum} = 2 \Rightarrow \text{Sum} = 4.$$

2012. In Tso, T. Y. (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 5-21. Taipei, Taiwan: PME.

And to which Nico made his comment: "Clair's the proof is clever but it is not correct."

In order to evoke the controversy I drew upon the reason why the equality 0.999…=1 holds with which my students got familiar in their high school classroom. They could easily justify the equality by the following argument:

> Let S=0.999…. 10S=9.999….Cancel S=0.999… from 10S=9.999…, one gets S=1 as desired.

Then why could it happen that Clair's proof is not correct?

The liberal study course mentioned above is called "Mathematics and Culture: An Approach of Reading Math Fiction" which is open for all undergraduate students of NTU, the most prestigious campus in Taiwan. Yet, most of the students in the class are basically from three colleges: College of Liberal Arts, Social Sciences and Law. Although my class is among the required subject on "Quantitative Analysis and Mathematical Competency", one of the eight curricula on liberal study, almost of them have unpleasant experience of learning mathematics in their high-school years. Therefore, if we would like to take into account of sharing mathematics learning opportunity with these students who are nevertheless intellectually advantageous, then the mathematics course in the general education framework should be designed in an accessible and friendly manner. This may well explain the reason why I adopt the approach of reading mathematical fiction. By following characters who are engaged in mathematics in plots, the course is devoted basically to emphasizing how to make sense of mathematical activities which they used to do boring practice or solve hard problem without attainment.

In this talk I am going to reflect on my own teaching experience with these students in the spring semester of 2012. I will analyze feedbacks of my students in terms of narrative and discourse, which are regarded by mathematics educators to play significant roles in learning mathematics. The students' endeavours to make sense of subjects of mathematics they were familiar with are indeed inspired by novelists' bringing mathematics in plots of fiction. Thus, in order to enhance students' mathematical competency, an approach of reading mathematical fiction deserves proper recommendation.

II. Framework and Research Tools

As I was preparing this talk, I have been very alert of Anna Sfard's comments on mathematics education reform in terms of *Principles and Standards for Mathematics Education* (hereafter abbreviated the *Standards*). She says:

> Some … mathematics educators build on an analogy with poetry or music and propose that, beginning with a certain level, we teach students about mathematics rather than engage them in doing mathematics. After all, exactly like poetry and music, mathematical techniques do not have to be fully mastered to be appreciated as part of our culture. It is far from obvious, however, that this is a workable

> proposal: Although one can certainly appreciate and enjoy poetry and music even without being able to produce any, it is probably not the case with mathematics. Another radical solution would be to turn high school mathematics into an elective subject. [Sfard 2000, p. 184]

Above all, it deserves to make it clear that I completely agree with Sfard that acquisition metaphor is as important as participation metaphor. [Sfard 1998] In fact, when I urged the students to engage in doing mathematics, some of them would respond with a little complaint. However, they had already been trained to do mathematics fluently since almost of them come from prestigious high school. And during their school days they were requested to spend a lot of time in mathematics learning. Given the situation that they could do mathematics in a lucid way, at least at senior high school level, we now can urge them moving on to understand how to make sense of mathematical activities.

Since Sfard's critical comments are addressed to the issue of discourse, it deserves here also to cite a definition of discourse by the *Standards*:

> Discourse refers to the ways of representing, thinking, talking, agreeing, and disagreeing that teachers and students use to engage…. The discourse embeds fundamental values about knowledge and authority. Its nature is reflected in what makes an answer right and what counts as legitimate mathematical activity, argument, and thinking. Teachers, through the ways they orchestrate, convey messages about whose knowledge and ways of thinking and knowing are values, who is considered able to contribute, and who has status in the group.

In fact, Sfard's dialogue with the *Standard* apparently has in mind her emphasis on the meta-rules of discourse:

> In mathematics, discourse-specific meta-rules manifest their presence in our instinctive choice to attend to particular aspects of symbolic displays (e.g., the degree of a variable in algebraic expressions) and ignore others (e.g., the shape of the letters in which the expressions are written) and in our ability to decide whether a given description can count as a proper mathematical definition, whether the given argument can count as a final and definite confirmation of what is being claimed. [Sfard 2000, p. 167]

By narrative in this talk I mean what Jerome Bruner discourses in his *The Culture of Education*. According to him, narrative is one of the two fundamental styles of thinking enabling human beings to make their way in the world – the other style being the "paradigmatic" or logic/classificatory one that has typically been associated with mathematics. He maintains that through narrative, we both organize and constitute our experience of the world; we tell stories, make up excuses and impose plots that have a beginning, middle and end.

Bruner describes narratives as particular types of discourses: "Narrative is a discourse, and the prime reason for a discourse is that there is a reason for it that distinguishes it from silence."[Bruner 1996, p. 121] "A story then has two sides to it: a sequence of events, and an implied evaluation of the events recounted."[Bruner 1996, p. 121] Thus, he stresses how, in recounting a series of events, the story-teller presents his or her interpretations of them.

"By using narrative … as our organizing principles, we attempt to show how learners, as well as mathematicians, make claim for mathematical territories by populating the landscape with fictional beings engaged in purposeful activity." [Healy *et al*., 2007, p. 17] Using narrative as an analytical tool, Mor and Noss interpret learner's expressions as mathematical narratives, i.e., "narratives which are intended to communicate or construct mathematical meanings". [Mor and Noss 2008]

More to the point, Wake and Pepin builds on a framework in order to "conceptualize mathematics teachers as 'narrator' revealing a mathematical plot whilst drawing on a range of pedagogic practices in an attempt to engage his or her audience in different ways." In this connection, they also conceptualize the classroom interactions as nested within an evolving systems network, in which teacher and students are mutually constituted through the course of their interactions.[Wake and Pepin 2010, p. 224] However, one should be cautious that there is "a certain ambiguity" in the concept of "narrative" in this talk. For on one hand, narrative is central to fiction in the sense of literary science in which narrative elements were occasionally raised in order to discuss the beauty of mathematical fiction. But on the other hand, narrative is used as a tool here in order to analyze data about students' activities in my classroom.

As for the research tools for this talk, I adopted two questionnaires to collect students' feedback immediately after I explained to them two of Jun-Hong Su's Award Winning Teaching Projects, namely topic on cosine formula and irrational numbers. Jun-Hong Su is an experienced high school mathematics teacher who is now preparing his doctoral dissertation thesis on the history of Chinese mathematics in the period of 1600-1900. He designed several teaching projects in terms of the HPM,[1] and attended the contest, sponsored by The SpringSoft Education Foundation, of teaching projects on high school science. It asked participants to use the PowerPoint software to design and to present their projects. Su combined his experience of teaching mathematics and knowledge of mathematics history to win the first prizes in 3 consecutive years, 2006, 2007, and 2008. And his topic on cosine formula and irrational numbers are the first two we mentioned above.

According to Tsang-Yi Lin's observation, the first teaching project "not only shows the connection between the cosine formula and the Pythagorean Theorem, but also illustrates that we can modify Euclid's proof of the Pythagorean Theorem in the *Elements* to prove the cosine formula". Apparently, familiarity with HPM inspires

[1] HPM refers first to a study group on the relations between history and pedagogy of mathematics, the first affiliated organization of ICMI. Yet, for now it also refers to a discipline devoted to issues of how history of mathematics can be integrated into teaching and learning of mathematics.

Jun-Hong Su to make use of Euclid's proof of Proposition I. 47 of the *Elements* in a profound way. (Cf. Figure 1)

As for the second, Lin Tsang-Yi notes that Jun-Hong Su's aim is to make students truly perceive irrational numbers. For this purpose, "Students are told that irrational numbers are numbers that are not rational numbers. However, this kind of definition or explanation gives students little substance of irrational numbers. To improve it, Su introduces the concepts of 'commensurable' and 'incommensurable' of the *Elements*. First, he shows the connection between rational numbers and commensurable magnitudes, and uses the Euclid algorithm to find the greatest common measure of two commensurable magnitudes. Second, he demonstrates the diagonal and the side of a square are incommensurable to explain the square root of 2 is irrational. (Cf. Figure 2.) Finally, he concludes that irrational numbers are those numbers that cannot be written as fractional numbers, ratios of two integers." [Lin 2012]

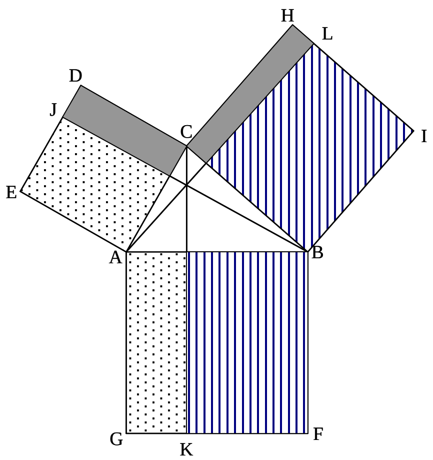

Figure 1

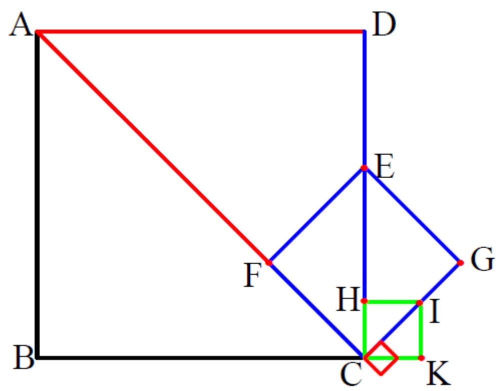

Figure 2

It seems that the term "connection" referred to by Tsang-Yi Lin in the last two paragraphs has different connotation. Since he serves also as a senior high school math teacher, his comments on Jun-Hong Su's teaching projects can be regarded as peer review.[2] I gather it might be relevant to issues of mathematical narrative and discourse. Here let me cite the questions I put in my questionnaires (on April 11, 2012) after Jun-Hong Su's teaching project of cosine formula was introduced to the class:

* What are the major concepts raised in Jun-Hong Su's teaching project? (To name at least three)
* What do you think of his rationale in designing this project?
* Compared with proof of the formula given in high school mathematics textbook, in what aspects you think of this project could do help to your understanding? Explain what you think about.

[2] In this regard, I also urge my graduate students to make their comments which are quite similar to Tsang-Yi Lin's. Note that all but two or three are teachers. Moreover, they are familiar with issues of HPM.

* In his project on “irrational numbers”, Jun-Hong Su primarily uses the concept of commensurability to connect rational numbers and $\sqrt{2}$. In this project, he also establishes a connection between Pythagorean Theorem and cosine formula. Is there any difference in the meaning of the term “connection” related to these two projects? Explain what you think about.

One should be noted that I had already introduced Jun-Hong Su’s project on “irrational number” to the class (on March 7, 2012). And similar questions as the above three had been requested to answer in a questionnaire. As I begin to analyze the data of questionnaires on cosine formula, I will also refer to students’ reflection on various questions I raised in the classroom. Meanwhile, their study background and mathematics learning experience (good and bad) will also be taken into account in due process.

III. Teaching in Context

The course “Mathematics and Culture: An Approach of Reading Math Fiction” (two hours per week) taken by 76 students in the spring semester 2012 is devoted to leading students by means of mathematics vs. narrative to

* understand essence of mathematical thinking and ways of its accessibility.
* investigate how mathematical concepts and methods are represented in the narrative of mathematical fictions/movies.
* explore interaction between mathematics and culture given that mathematical concepts are regarded to be cultural/literary metaphor.

However, since the above activities could not happen without mathematics, the course is also devoted to dealing with related topics or subjects of mathematics such as number system (including real and complex number system), elementary number theory, geometry (both Euclidean and analytic), infinite set theory, calculus, as well as methodology. Needless to say, the goal of the course should meet the requirement of the curriculum, “Quantitative Analysis and Mathematical Competency”.

However, since as I mentioned, NYU is the most prestigious campus in this country, my students are among the most brilliant in their generation. That means they were well trained in linguistic and mathematical comprehension in their high school years. This may well explain why at one occasion I was beginning to discuss how the “analytic” approach of Vieta’s algebraic symbolism makes sense, I asked the students to solve both arithmetically and algebraically the problem of “Chicken and Rabbits in the Same Cage” which comes from *Sunzi suanjing* (Master Sun’s Mathematical Manual, Chinese mathematical classical text, no later than 5^{th} century AD). The problem is read as follows: “Given that chicken and rabbits are put in the cage. Suppose the number of their heads is 35, and the number of their feet is 94. How many chicken and rabbits each respectively?” It was a great pleasure to find that students solved the problem in a confident way. This especially the case when they explained how each arithmetical step meant in their solving the problems. Given my students' skilful expertise of school mathematics, I came to believe once again that I would have emphasized the teaching in sense-making aspects.

The mathematical fictions (all with Chinese version) assigned for reading is as follows:

(1). Apostolos Doxiadis *et al*, *Logicomix: An epic Search for Truth* (graphic novel). Greek authors yet produced in English.

(2). Hiroko Endo, *Sanpou Shoujyo* (Arithmetic Girl). Japanese writer.

(3). Paolo Giordano, *La Solitudine Del Numeri Primi* (The Solitude of Primes Numbers). Italian writer. Originally Italian, then Chinese version.

(4). Denis Guedj, *The Parrot's Theorem: A Novel*. French writer. Originally written in French, then English and finally Chinese version.

(5). Mark Haddon, *The Curious Incident of the Dog in the Night-Time*. English writer.

(6). Catherine Hall, *The Proof of Love*. English writer.

(7). Keigo Higashino, *Yogisha X No Kenshin* (Commitment of the Suspect X). Japanese writer.

(8). Fang-Mei Lin, *Da Vincci's Messy Code* (Da Wenxi Luanma)(in Chinese). Taiwanese author.

(9). Yoko Ogawa, *The Housekeeper and the Professor* (*Hakase no aishita sushiki*). Japanese writer. Originally written in Japanese, then Chinese and English version.

(10). Gaurav Suri & Hartosh Bal, *A Certain Ambiguity*. Indian writers. Originally written in English.

(11). Hiroshi Yuki, *Sugaku Girl: Fermat no Saishu Teiri* (Math Girl: Fermat Last Theorem). Japanese writer.

Among the eleven fictions, my students should pick at least three out of them to write their preliminary reviews.[3] At the end of the semester, they were requested to give a comprehensive report based on what they had already studied through the whole semester. In addition to comparative study on fictions, the students could focus on just one novel. Not surprisingly, Ogawa's *The Housekeeper and the Professor* stood out to be the most favorite text. Apparently one of the reasons for their preference is this novel also has a movie's version (of the same title). Yet, it is also due to the fact that this novel is not merely a literary masterpiece. Still, in its plots Ogawa integrates

[3] They can also pick two movies for the subjects of their reports, namely *Proof* (written by David Auburn, directed by John Madden) and *Agora* (written by Alejandro Amenábar and Mateo Gil, directed by Alejandro Amenábar). In 2012, *Proof* was also performed as a stage play by Greenray Theatre Company, Taiwan.

mathematical concepts/formulas in a just perfect and amusing way. Even so, the writer does not treat mathematics in a shallow sense. I will explain some of her narrative characteristic in the section that follows.

Along with narratives discussed, I introduced mathematical concepts/subjects to the classroom which are somewhat parallel to those appeared in the fictions assigned for reading. For example, since the second week teaching hours were basically devoted to *A Certain Ambiguity*, I would then spend a lot of time explaining how the concept of infinity created by Georg Cantor becomes an issue and help students better understand the mathematical narratives in the fiction. In addition, I engaged the students to read Book I of Euclid's *Elements* (a website managed by David Joyce) in order to make it clear how certainty is assured through a logical chain of propositions in an axiomatic system in ancient Greek context. This episode is significant in the plots devoted to mathematical truth. Since historical and cultural context of mathematics is needed in reading some other fictions like *The Parrot's Theorem* and *Sanpou Shoujyo*,[4] I led my students discussing history of Greek, Chinese, Japanese, Hindu and Arabic mathematics. Still, I drew upon due attention to Platonism and its relevance to Renaissance art and a comparison of the *Elements* and Chinese mathematics classic *Jiuzhang suanshu* (Nine Chapters on the Art of Mathematics, not later than the first century AD.).

As a supplement to mathematical knowledge proper, I also asked my students to refer to Jerry King's *Mathematics in 10 Lessons: The Grand Tour* (2009). In this popular mathematics book, King suggests his readers "How to Read This Book" by drawing upon an inspiring parallelism of mathematics and poetry in terms of content, i.e., *the things said* and form, i.e., *the way of saying*. This may well explain why I assigned the book for study or reference. Yet, since the reading list of fictions were quite much for the students, reading of King's was no requirement so that few students paid due attention to it.

IV. Modelling the Characters

According to the bibliography of Japanese edition, *Hakase no aishita sushiki* (2003),[5] Ogawa refers to a biography of Paul Erdos, which is a Japanese translation of Paul Hoffman's *The Man Who Loved Only Numbers*. [Ogawa 2003, p. 283] In addition to examples of number theory such as Ruth-Aaron pair of 714 and 715 Ogawa takes from the biography, the most illuminating lines to her seems to be Hoffman's argument on insights and connections: "Mathematics is about finding connections, between specific problems and more general results, and between one concepts and another seemingly

[4] *The Parrot's Theorem* is basically devoted to history of mathematics, especially that in Islamic world. On the other hand, in her *Sanpou Shoujyo* (Arithmetical Girl), Hiroko Endo uses 18th century Edo period as the historical context. Readers can thereby get to understand some aspects of history of Japanese mathematics (*wasan*) in 18th century.

[5] It is not included in English edition (2009).

unrelated concept that really is related. No mathematical concept worth its salt stands in isolation.” [Hoffman 1998, p. 208]

Before we learn to appreciate how Ogawa models the character of the Professor, it seems desirable to have a brief summary of the fiction, *The Housekeeper and the Professor*.

The Housekeeper and the Professor has eleven chapters. Its narrator is the Housekeeper who is a young single mother who is hired to take care of the mathematics professor, a number theorist, handicapped by brain injury.[6] The story also involves an old widow, sister in law of the Professor, who is responsible for his living cost, staying in the big house next to that of the Professor. There are four characters in the novel, namely the Housekeeper, the Professor, the old widow and the Housekeeper’s eleven-year son who is called as “Root” by the Professor.

Due to the brain injury, the Professor’s memory can last only 80 minutes. That means his most recent 80 minutes’ memory would be erased automatically so that each new piece of experience including recognition of the persons ever met is as anew as their first meeting. Thus, as he answers the door to the Housekeeper’s call every day he can only “recognize” her by means of the tape slip sticking to his coat. Peculiarly however he preserves the memory prior to 1975, the year of the traffic accident. This may explain why he is still capable of solving problems posed in mathematics journal. And equally interesting is his “hello” with the Housekeeper as she came to his house in the morning: “What is the size of your shoes?” “24.” “What a noble number is! It is the factorial of 4.” “What is your telephone number?” “5761455? How incredible! It is the total amount of the prime numbers up to one billion.”

Apparently it is due the neutrality of numbers that they could protect the Professor himself and insult from intervention of other people. Despite that he was not able to continue his academic career, his mathematical experiences (related with his past memory) was not impeded. The traffic accident only retarded his capability of interaction with other people. Even so, he seems to have been fond of the kid “Root” with a flat head, which looks just like the symbol denoting square root $\sqrt{\ }$. As he first met the kid he explained why the nickname was came across his mind: “You are a Root. You accept any number that comes your way, rejecting none. A truly generous symbol, Root.”

Number indeed has such a magical power. It even can be used to represent some eternal commitment to love. Despite that the Professor is not able to remember what happens this moment he should have still preserved the beloved memory about himself and his sister in law. They could have taken vow on their relationship by means of Euler’s formulae $e^{i\pi} + 1 = 0$, which apparently represents purity, love and trust. Once he

[6] It should be noted that in the movie *The Professor and His Beloved Equation* (2006) which is based on the novel has Root as the narrator instead. Serving as a math teacher he recalls the memory of his mother and the Professor in his first class of high school.

has to make special request from the widow he would show the formula. This happened as the widow prohibited Root's revisit after his mother had already been fired from the caring job. The reason why she got dismissal is due to her bringing Root's company during her duty despite that it was at the Professor's request. One day the Housekeeper was called upon to the Professor's house and saw Root was there. The widow was very angry to claim that she would never allow the reunion of the three, the Professor, the Housekeeper and Root. The Professor was offended to see that the kid was blamed for his innocence. He wrote down the formula and showed it to the old widow. Eventually, she could only concede probably because they share a commitment to the eternality of the beautiful formula. And the Housekeeper was back there for service until the Professor was physically weak and sent to the long-term care facility. Root became a school mathematics teacher. The last sentence of the novel is: "The perfect number 28." Apparently everything with mathematics is just perfect!

Let us come back to Hoffman's exposition on Euler's formula which is the most significant episode Ogawa referred to. We cite Paul Hoffman's relevant passage which will then be made a comparison with what Ogawa has poetically modified into the plots in her novel. Here comes Hoffman exposition first:

> Now what is this constant e? The number e, like π, is a nonrepeating, nonterminating decimal, which Euler calculated to twenty-three places:
>
> 2.71828182845904523536028…
>
> It is a number generated by an infinite series:
>
> $e = 1 + 1/1 + 1(1\times 2) + 1/(1\times 2\times 3) + 1/(1\times 2\times 3\times 4) + 1/(1\times 2\times 3\times 4\times 5) + ...$
>
> The number *e* may not all that "natural", but it is described as such because it comes up often in the mathematical modeling of such basic processes of life as growth and decay. [Hoffman 1998, p. 210]
>
> …
>
> If mathematical success is measured by revealing deep connections among that on the surface don't seem to be related, Euler gets the prize. He is responsible for perhaps the most concentrated and famous formula in all of mathematics, which in one bold stroke ties together π, e and i (the imaginary number, the square root of -1) as well as the most basic whole numbers 0 and 1. For Euler recognized that if you raised the number e to the power π times i and added 1, you'd get 0. Behold the sheer elegance, hieroglyphic beauty, and austere consciousness of Euler's formula $e^{i\pi} + 1 = 0$, which has as much appeal for mystics as it has for mathematicians.
>
> Lost in the beauty and compactness of the formula $e^{i\pi} + 1 = 0$ is a long history, because the acceptance and understanding of numbers like π, *e*, and *i* did not come easily to mathematicians. Nor did the acceptance of much simpler numerical concepts like

zero, negative numbers, and nonrepeating, nonterminating decimals like the square root of 2. [Hoffman 1998, pp. 211-212]

Now it is time to see how Ogawa transforms the above sentences into the following narrative form:

According to Euler's calculations: *e*=2.71828182845904523536028…and so on forever. The calculation itself, compared to the difficulty of the explanation, was quite simple:

$$e = 1 + 1/1 + 1(1\times 2) + 1/(1\times 2\times 3) + 1/(1\times 2\times 3\times 4) + 1/(1\times 2\times 3\times 4\times 5) + ...$$

But the simplicity of the calculation only reinforces the enigma of *e*.

To begin with, what was "natural" about this "natural logarithm"? Wasn't it utterly unnatural to take such a number as your base – a number that could only be expressed by a sigh: this tiny e seemed to extend to infinity, falling off even the largest sheet of paper. …I wondered about Leonhard Euler, who was probably the greatest mathematician of the eighteenth century. All I knew about him was this formula, but reading it made me feel as though I were standing in his presence. Using a profoundly unnatural concept, he had discovered the natural connection between that seemed completely unrelated.

If you added 1 to e elevated to the power ofπtimes *i*, you get 0: $e^{i\pi} + 1 = 0$.

I looked at the Professor's note again. A number that cycled on forever and another vague figure that never revealed its true nature now traced a short and elegant trajectory to a single point. Though there is no circle in evidence, πhad descended from somewhere to join hands with *e*. There they rested, slumped against each other, and it only remained for a human being to add 1, and the world suddenly changed. Everything resolved into nothing, zero.

Euler's formula shone like a shooting star in the night sky, or like a line of poetry carved on the wall of a dark cave. I slipped the Professor's note in my wallet, strangely moved by the beauty of those few symbols. As I headed down the library stairs, I turned back to look. The mathematics stacks were as silent and empty as ever – apparently no one suspected the riches hidden there. [Ogawa 2009, pp. 126-127]

By comparison, one can easily identify how narrative in a biography of mathematician in a form of popular mathematics is transformed into narrative in a literary masterpiece. Such a comparison also well explains Ogawa's claim that mathematics is of supreme beauty is not due to her poetic imagination. Rather, it is all because a mathematician like Euler, by "using a profoundly unnatural concept", "had discovered the natural connection between that seemed completely unrelated"

The above mentioned unexpected aspect of Euler's discovery is regarded by G. H. Hardy as one of the aesthetic qualities of mathematical proof. The other aspect concerns inevitability that lies in the very essence of mathematics as an example of certainty. [Cain 2010, p. 7] Yet, for Ogawa and perhaps anyone else logical inevitability seems to be self-evident. Thus, it is no wonder why she feels no need to go into any further speculation. Instead, as Stephen Snyder, the English translator of the novel, remarks, "Ogawa chooses to write about actual math problem, rather than to write about math in the abstract. In a sense, she invites the reader to learn math along with the characters." [Ogawa 2009, Discussion Questions]

Such an approach apparently echoes Hoffman's reference to Gauss's calculation of the sum of 1+2+3+…+100, by which he uses to argue how insights and connections are crucial to mathematical achievements. [Hoffman 1998, pp. 206-208] In the novel, the Professor asked the Housekeeper and Root to find the sum of the numbers from 1 to 10 without directly adding them. A common method from school would be the one discovered by Gauss : (1+10) = 11, 11×5=55. The Housekeeper spent several days to figure out how. Root shared his experience of doing gymnastics in school. The teacher gave order as follows: "Double lines, face center." The guy in the middle held up his arms and the rest of kids aligned facing him. Once one got nine kids in one line the fifth one would be the center. And this reminded the Housekeeper to think about the concept of "average" (middle). As Root was solving the problem, "we decided to think about 1 to 9 first, and forgot about 10 for right now. The number 5 is in the middle, so it's the uh…" "Average," a hint came from the Housekeeper. Thus, Root was applauded to conclude that "If you add up 1 through 9 and divide by 9 you get 5… so 5×9=45, that's the sum of the numbers 1 to 9. And now it is time to bring back the 10." Eventually the right answer was attained. [Ogawa 2009, p. 57]

Presenting her heuristic for solving mathematical problem aside,[7] Ogawa also uses this episode to share "the path to enlightenment" in ordinary life. Here is the passage about the Housekeeper's description of her own strategy for solving the problem:

> So now I tried leaving 10 aside and lining up the rest of the numbers. I circled five in the center, with four numbers before it and four after. The 5 stood, arms proudly extended, enjoying the attention of all the others. [Ogawa 2009, p. 54]

In other words, the Housekeeper left out the number 10, which is double digits, different from the other nine numerals. After holding the middle average number 5 she got the 45 and then added the 10 to obtain the final answer 55. Apparently this is analogous to how she treated her own life: she put aside the discernible and extreme parts and searched for the average, namely, to converge to the middle. She never complained of her single motherhood. Instead, she gave commitment to universal maternal love.

[7] In his Discussion Question 7 Stephen Snyder asks readers by demonstrating the case "Is there a thematic importance to their method of solving the problem? Generally, how does Ogawa use math to illustrate a whole worldview?" Cf. Ogawa (2009). In fact, here Ogawa also points to the very cognitive aspect of mathematical learning that would interest not only educators but mathematicians as well. This may well explain why she has engaged dialogue with mathematicians and educators on education issues due to the big success of the novel.

V. Connection in terms of Narrative and Discourse

Sixty-two out of the seventy-six students had sent back questionnaires of April 11, 2012. In what follows I will try to summarize and explain the points made in their feedbacks for Question 4, as mentioned above, "In his project on 'irrational numbers', Jun-Hong Su primarily uses the concept of commensurability to "connect" rational numbers and $\sqrt{2}$. In this project, he also establishes a connection between Pythagorean Theorem and cosine formula. Is there any difference in the connotation of the term "connection" related to these two projects? Explain what you think about."

Among the 62 pieces of questionnaires, we leave out 26 of them which apparently did not answer the questions properly due to stress of the class time. The data we are going to analyze is thus 36 questionnaires. Of course, I would agree with Nemirovsky that "[m]athematical narratives are not the exclusive domain of 'right' or 'wrong' ideas. They express the many ways in which people deal with mathematical problems and, for research purposes, they can be more or less illuminating regardless of their 'correctness'." [Nemirovsky 1996, p. 201]

Concerning the connection between rational number and $\sqrt{2}$, some students strove to tell a story of two events/proofs, namely that by applying the concept of the commensurable to side and diagonal of a unit square as well as that by using the method of *reductio ad absurdum*. In this regard, they thought the concept of the commensurable is to make a distinction of rational and irrational number. By contrast, they argued the using Euclid's proof to connect Pythagorean Theorem and cosine formula was for the purpose of generalization. And the connection of the latter was made possible by means of purely logical inference. It is the very reason that they thought the connection touches the essence of mathematical knowledge, namely to generalize, to modify, to expand and to change the scope.

Some other students who were familiar with the method of *reductio ad absurdum* to prove $\sqrt{2}$ is irrational, which is basically algebraic, argued that proof by means of the concept of the commensurable is to bridge geometry and algebra while using Euclid's proof to connect Pythagorean Theorem and cosine formula who are in the same domain, namely geometry. Still, there are students whose observations of the diagrams for proof (Cf. Figure 1 & 2) led them to comment that connection between rational number and $\sqrt{2}$ appealed to the concept of one dimensional line segment while the connection between Pythagorean Theorem and cosine formula appealed to the concept of area (of 2-dimensional region).

If we take methodology into account, both connections involve manipulation of diagrams which were regarded by some students to be concrete and easy to follow. There is one student who pointed out the difference between the manipulations of diagrams. She observed that for the proof of the irrationality of $\sqrt{2}$, "[it] doesn't have a limit, you can continue, create a square within a square while [for the proof on the relation between Pythagorean Theorem and cosine formula] [the manipulation] is not unlimited."

In summary, it is very likely that the students under study regarded the connection between rational number and $\sqrt{2}$ is "horizontal" while the connection between Pythagorean Theorem and cosine formula is "vertical". This may also well explain why they thought the concept of the commensurable was to make a distinction (in definition) between rational and irrational number while Euclid's proof helped to establish implicative relation between two propositions, namely Pythagorean Theorem and cosine formula. In fact, asked to reflect on what they had learned from the course my students seemed to be satisfactory in the sense that in contrast with high school mathematics learning they had widen the horizon and deepen the content of mathematical knowledge as a whole.

On the other hand, we should remember that the students' feedbacks could be better explained in the context of reading mathematical fiction. In terms of situated learning, mathematical activities they engaged in are parallel to those of plots in mathematical fiction. Given the mathematical fiction as it is, my students were quite easy to access some of the mathematics content in the narrative. However, mathematics in the fictions is not merely treated as "facts" but more significantly used as a metaphor and becomes part of the narrative. Therefore, as the literary side is drawn upon, imagination needed for good reading of a mathematical fiction is closely related with not only the writing expertise of the author but cultural literacy of the society to comprehend mathematical narrative as well.[8] After all, as literature scholars put, the nature of the relationship between the author's fictional world and his real world, "is a creative problem for the author, and it is a critical problem for the reader." [Scholes *et al*., 2006, p. 83] No wonder this is also true for author and reader of mathematical fiction.

VI. Epilogue

In their *A Certain Ambiguity*, Gaurav Suri and Hartosh Singh Bal claim their principal purpose in writing the novel is "to show that mathematics is beautiful". Besides, they also "seek to show that mathematics has profound things to say about what it means for humans to truly know something." And then they "believe that both these objectives are best achieved in the medium of a novel." Therefore, narrative for the writers can play a role in the construction of mathematical understandings.

Similar situation occurs as the readers become the narrators. Towards the end of the semester, two students in my class presented their oral report on the play, *Proof*, which is regarded by Robert Thomas as a literary works with "*appearances* of mathematics in literature" [italic original, Thomas, 2002, p. 43], they explained the last screen that the actress was pointing the lines of her mathematics notebook to the actor by saying that "she is making the connection."

[8] In this regard, the English translator Stephen Snyder, as a college professor teaching Japanese , has shared his illuminating readership in his "Discussion Question".

Connection is indeed the central theme of my teaching. I had always emphasized the connection of mathematical concepts, formulas and propositions in due context. My initial intention was to engage the students in mathematical activities so that at least they could revive their prior mathematical experiences. In so doing I could also help them developing a comprehensive viewpoint of mathematics (whose scope is up to calculus) by emphasizing the connections. What a coincidence is at the end of the semester C. P. Snow's issue of the two cultures crossed to my mind. Now I came to realize that the issue is still lively in the sense that the majority of humanity major students hated and even accused mathematics. I hope such an approach of reading mathematical fictions to enhance students' comprehension of mathematics could help to "shorten the distance of mathematics and students", as commented one student.

In any case, this talk is a story about the stories I told in my class. My narrative is as critical as serious. When Paul Feyerabend, a radical philosopher of science, was hospitalized in a critical condition, his doctor made a comment to his visitors by saying that "he is critical but not serious." In response, Feyerabend said: "I am serious but not critical." I hope my talk has been serious in some respects but not critical to some others. Whatever logic or meaning might be considered, 0.999…is equal to 1 truly and forever!

I responded to the student who raised the question in a loud voice: "Yes, I do believe it!"

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PLENARY ADDRESS 2

ICTs as Opportunities for Teaching-learning in a Mathematics Classroom: The Semiotic Potential of Artefcts

ICT AS OPPORTUNITIES FOR TEACHING-LEARNING IN A MATHEMATICS CLASSROOM: THE SEMIOTIC POTENTIAL OF ARTEFACTS

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My contribution intends to propose some elements of discussion in the debate regarding the use of new technologies in mathematics education and intends to do that from a perspective large enough to take into consideration the idea of artefact without circumscribing it to the case of new technologies. I will present a specific theoretical framework, the Theory of Semiotic Mediation, focussing on the key notion of semiotic potential that will be illustrated by examples drawn form different teaching experiments and involving ancient and modern artefacts.

INTRODUCTION

The world in which we live has seen the fast development and spread of all kinds of new tools, in particular digital tools such as personal computers or graphing calculators, but also iPad or iPhone. The use of artefacts [1] and technology development are certainly not a phenomenon exclusive to our times, perhaps only the incredible speed of change, and in same cases the ease of access, can be considered peculiar to our time. In fact, the construction and use of artefacts, for the most varied activities, seems to be one of the salient features of the human species.

Since the very beginning of their appearing new technologies, especially digital technologies, have raised expectations in respect to their educational potential (Howson and Kahane, 1986), and consequently has raised the issue of their integration in school practice. In the last twenty /twenty five years, a great amount of research energy has been devoted to this field of research that still remains very active and see a flourishing of research studies and education projects. ICMI study 17 that took place in 2006 in Hanoi and , and the following book (Hoyles and Lagrange, 2010) witness of such a living field of investigation.

My paper aims to contribute to this research field, starting form the consideration that reflecting on how to enhance mathematical education through the use of artefacts in the classroom, may offers the opportunity to reconsider, perhaps with new eyes, but certainly from a new point of view also the use of ancient tools, sometimes forgotten and neglected. My contribution intends to propose some elements of discussion in the

[1] The word *artefact* is generally used in a very general way and encompasses oral and written forms of language, texts, physical tools used in the history of arithmetic (e.g., abaci and mechanical calculators) and geometry (e.g., straightedge and compasses), tools from ICT, manipulatives, and so on. The way I employ this term is consistent with this use and also with the definition given by Rabardel (1995) that will be discussed in the following.

debate regarding the use of new technologies in mathematics education and intends to do that from a perspective large enough to take into consideration the idea of artefact without circumscribing it to the case of new technologies.

ARTIFACTS AND MATHEMATICAL KNOWLEDGE

An analysis of historical and epistemological development of mathematical ideas shows a complex but very productive interplay between theory and practice. Already in Greek mathematics and particularly in the construction of the geometric theory one can observe the role played by artefacts and specifically by what one can do with them. This makes it very reasonable to agree with the following statements

> "[…] mathematical objects do not come from abstraction from real objects, which describe the characteristic features, but by a process of objectification of procedures." (Giusti, 1999, p. 26, translated by the author)

According to the author, the gesture of tracing has to be considered at the origin of the idea of line (both a straight line and a circle), but mainly what seems to us more interesting is the fact that this gesture is to be related to the use of a particular artefacts: for instance, a rope either stretched between to nails or turned around a pivot, or a pair of compasses.

On the one hand a historic and epistemological study centred on the development of mathematical theories in relations to the use of particular artefacts and their subsequent design development may outline significant paths to be reinvested in mathematics education (Bartolini Bussi and Maschietto, 2006; see also the web site: Mathematical Machine Lab). On the other hand, the cognitive analysis of such key elements (artefacts & modes of use) in relation to the new offer coming form new technologies (ICT) may highlight significant and unpredictable potentialities for introducing students to mathematical knowledge. Reflection on history of Mathematics, and in particular on the evolution from problems solutions through tools' use to the construction of theories, can help to overcome a common misunderstanding and open new opportunities.

Artefacts are generally interpreted as incorporating a pre-existing knowledge, this same knowledge that makes them able to accomplish tasks or solve problems for which they are design. This interpretation is strictly related to the way of thinking of the relationship between practice and theory mentioned before, that is knowledge and in particular theoretical knowledge is often conceived as something pre-existing the solution envisaged with it. More complex artefacts and certainly new computational devices as computers, with all the applications available, or even nets of computers, may be interpreted as more and more sophisticated artefacts incorporating different kinds of knowledge. Take for instance a pocket calculator, it may execute a calculation for us, as far as it *knows* (it has imbedded) the algorithms of arithmetic operations. The fact that arithmetic is a well recognizable part of theoretical mathematical knowledge and a school content to be taught, rises the issue of accessibility of such knowledge through the artefact, or that of its *transparency* (Meira, 1998; Borba Villareal, 2005).

But there is a different and complementary point of view form which artefacts and their mode of use can be analysed. It is the point of view suggested by historic analysis and it is the point of view that see artefacts and their uses as generating new knowledge, new knowledge that can rise at consciousness, that can be shared and formalized becoming theoretical knowledge shared with the community of mathematicians. Thus, the artefact and the modes of its use may appear as key elements in the emergence of mathematical knowledge in the school context. They become a unit of analysis that can guide the design of teaching and learning activities. An historical /epistemological and cognitive study retracing some stages of the evolution of ideas and theories may suggest alternative approaches to teaching and learning Mathematics (Bartolini Bussi & Mariotti, 1999; Bartolini Bussi 2001; Bartolini Bussi and Machietto, 2008).

Traditionally at school, the link between practices and theories that have originated from such practices is broken: a de-contextualized approach to mathematics (Chevallard, 1985) is presented that usually foresees a return to practice only after the introduction of autonomous theories and ways to mange their 'application' to real situations. In fact, the potential of the profound link between practice and theory is ignored, missing the opportunity of exploiting its potential in fostering the construction and the formalization of mathematical ideas. My objective is that of showing how such link may be preserved, or better exploited through a particular teaching and learning model. I will start presenting a specific theoretical framework aimed at describing and modelling the teaching-learning process based on the use of a specific artefacts. Through this frame I will present some examples, drawn from our teaching experiments.

THE THEORY OF SEMIOTIC MEDIATION

The Theory of Semiotic Mediation (TSM) has been elaborated by Bartolini Bussi and Mariotti (2008) on the base of an extensive corpus of data collected in long terms teaching experiments, carried out at all school level (from pre-primary to secondary) and with artefacts of different types, from concrete manipulatives (e.g. abacus, compasses) to virtual manipulative (e. g. softwares like Cabri), from artefact drawn from history to new technological devices.

The TSM is centred around the seminal idea of semiotic mediation introduced by Vygotsky (1978) and it aims to describe and explain the process that starts with the student's use of an artefact and leads to the student's appropriation of a particular mathematical content. The TSM addresses this issue combining a semiotic and an educational perspective, and elaborating on the notion of mediation considers the crucial role of *human mediation* (Kozulin, 2003, p.19; Hasan, 2002) in the teaching-learning process. Taking a semiotic perspective means to acknowledge the central role of signs in the teaching-learning activity. The use of the term 'sign' is inspired by Pierce. We assume an indissoluble relationship between signified and signifier. In the stream of other researchers (Radford, 2003; Arzarello, 2006; see also Saez-Ludlow and Presmeg, 2006) we developed the idea of meaning that originates in the intricate interplay of signs and that teaching and learning activities analysed through a semiotic lens, may be described through the development of what we call a semiotic mediation process[2]. Fostering and guiding this process is a crucial issue and a demanding task for the teacher. In the following sections we outline how it is possible to organize a teaching-learning sequence by integrating the use of an artefact, based on the notion of semiotic mediation. Such description is developed around the key notions of *semiotic potential of an artefact* and of *didactic cycle*.

The semiotic potential of an artefact

According to the previous discussion, the relationship between a specific mathematical knowledge and the use of a given artefact to accomplish a task can be considered from two perspectives: on the one hand, the perspective of the students that accomplishing a task through the use of an artefact construct personal and situated (Lave and Wenger, 1991) knowledge; on the other hand, the perspective of an expert that recognizes in the use of the artefact for solving of the task a specific mathematical knowledge: following Hoyles (1993), one can say that for the expert the artefact evokes such a specific mathematical knowledge. The recognition of the mathematical knowledge may occurs for the expert automatically and unconsciously, on the contrary such a recognition is out of the possibilities of the student who on the contrary has to learn it. Nevertheless it may constitute the motive of a teaching and learning activity and it may inspire the design of a teaching sequence. Thus we will say that the positional notation of numbers may be evoked by an abacus and its use in counting or adding; similarly, we can say that the classic ‘rule and compasses’ Geometry may be evoked by a construction task solved in a Dynamic Geometry System. However, there is the need to distinguish between meanings emerging from the practice based on the use of the artefact and the mathematics knowledge evoked in the expert’s mind. In this respect the case of the abacus is paradigmatic: centuries of practice of computation with the abacus were not sufficient to trigger the move towards the positional notation system for numbers (Menninger, 1958, p. 223).

We think that the distinction between the plane of the individual and the plane of the culture is to be taken in consideration when teaching and learning is concerned. School education emerges from the interlacement of personal endeavour and cultural endeavour, teaching and learning can be interpreted as an activity defined by the shared goal of making students appropriate cultural products (Leont’ev, 1976/64), where the teacher assume the role of a cultural mediator. The notion of *semiotic potential of an artefact* is meant to capture such a distinction, to make it explicit, but at the same time to stress the potential relationship between these two planes - the individual’s plane and the culture’s plane -, relationship that is hinged on the use of an artefact. This leads us to the following definition.

[2] Our perspective is highly consistent with those elaborate by Anna Sfard in her recent thoughtful book (Sfard, 2010; Shard, 2000, p. 42 and following).

By *semiotic potential of an artefact* we mean the double semiotic link which may occur between an artefact, and the personal meanings emerging from its use to accomplish a task, and at the same time the mathematical meanings evoked by its use and recognizable as mathematics by an expert (Bartolini Bussi and Mariotti, 2008, p. 754).

Teaching and learning from a semiotic perspective: the didactic cycle

In a mathematics class context, when using an artefact to accomplish a task and in relation to that task, meanings emerge and students can be led to produce personal signs which can be put in relation with mathematical signs. However becoming conscious of such a relationship is not a spontaneous process for students, on the contrary it can be assumed as an explicit educational goal by the teacher, who can orient her own action towards promoting the production, the evolution and the webbing of signs: from signs that express the relationship between artefact and tasks, to signs that express the relationship between artefact and mathematics knowledge. The process of semiotic mediation consists of such an evolution process that starts with the emergence of personal signs, related to meanings emerging in the accomplishment of a task and the use of a specific artefact, and develops in the collective construction of shared signs related to both the use of the artefact and to the mathematics to be learnt.

Such evolution can be promoted through the iteration of *didactic cycles* (Fig. 3) where different categories of activities take place, each of them contributing differently but complementarily to develop the complex process of semiotic mediation: (a) <u>activities with the artefact</u> based on tasks purposefully designed with the aim of promoting the emergence of meanings referred to the use of the artefact; (b) activities asking <u>individual production of signs</u>, for instance involving students in semiotic activities concerning written productions. Students might be asked to write individual reports on the previous activity with the artefact, reflecting on their own experience, and raising possible doubts or questions; (c) collective activities concerning <u>collective production of signs</u>. Through the collective discussions, and specifically *Mathematical discussion* (Bartolini Bussi, 1998), involving the whole class the teacher's action will be aimed at fostering the move towards mathematical meanings, taking into account individual contributions and exploiting the semiotic potentialities coming from the artefact-use.

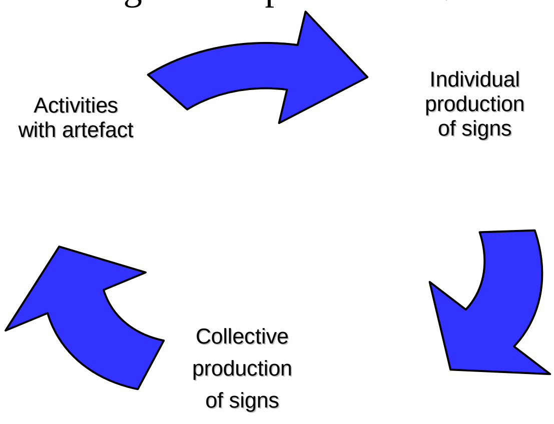

Figure 1 The didactic cycle

The first phase of the semiotic mediation process envisages the emergence of students' personal signs related to the use of the artefact, what we call *unfolding of the semiotic potential*. In the second phase the teacher is expected to foster the social evolution of the emergent personal signs into shared mathematical signs. At each phase of the teaching process, the action of the teacher is required, but her role becomes crucial in the orchestration of class discussions. I use the term orchestration introduced by Bartolini Bussi (1998, p. 68) to describe the teacher's management of the class during a Mathematical discussion, which is described as "a polyphony of voices articulated on a mathematical object", which is one of the goal of the activity of teaching and learning. This metaphor of orchestration is often borrowed, in various meanings, to discuss the integration of tools in the classroom. Trouche (2005, p. 126), for example, defines an instrumental orchestration as an intentional organization of artefacts and actors in a learning environment to assist students' instrumental genesis. This definition is completed by Drijvers et al. (2009), who identify several components in the process of orchestration. As far as a Mathematical discussion is concerned, it can be related to a particular component of an instrumental orchestration, the one Drijvers et al. call didactical performance. Actually, following our model and taking a semiotic perspective, the goal of orchestrating a Mathematical discussion is that of fostering the development of shared meanings recognizable and acceptable by the mathematical community. This goal is not in contrast with the objective of promoting and supporting students' instrumental genesis, but the two objectives remain separate though complementary.

Coming back to our model, interpreting the teaching learning process, and specifically the didactical use of an artefact, through the lens of semiotics we say that the teacher is exploiting the artefact as a tool of semiotic mediation. According to the frame given by Hasan (2002) we can synthesise the process of semiotic mediation as follows.

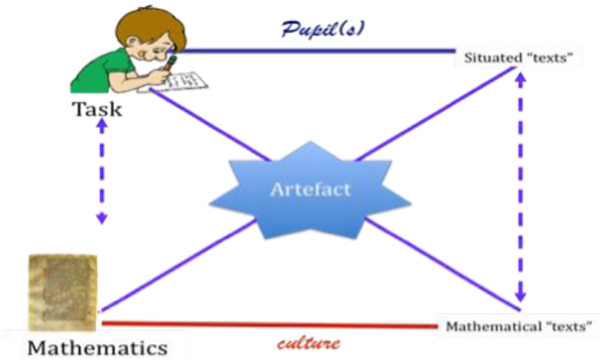

Figure 2 The key elements of the TSM

Mediation occurs because there is a specific mathematical knowledge (content) that is the object of mediation there is the teacher (mediator) who mediates and the student(s) (mediatee) for whom mediation as an effect, there are specific didactic interventions, that we called didactic cycles, creating means and circumstances (modalities) that makes the process of mediation occur. A summary of the key elements of MST is also given in figure 2.

In the following sections, I would like to give two examples illustrating the notion of semiotic potential; the first example concerns concrete manipulative artefacts, the second example concerns a digital artefact, specifically a Dynamic Geometry System, Cabri (Laborde and Bellemain, 1995)

EXAMPLES OF SEMIOTIC POTENTIAL ANALYSIS

Outlining the semiotic potential of an artefact requires an fine grain analysis of the artefact and its use, analysis that encompasses a multiple perspective, including the epistemological and the cognitive perspective, but also a historic and a didactic perspective. The richer is the description of the semiotic potential the richer and more powerful is the base for the design of an instruction sequence centred on exploiting such an artefact as a tool of semiotic mediation.

As far as a cognitive analysis is concerned, the approach developed by Rabardel (1995) reveals its effectiveness. Developed in the research field of ergonomics Rabardel's approach is now widely used in the field of math education (Rabardel and Samurçay, 2001; Lagrange, 1999; Artigue, 2002; Guin, Ruthven & Trouche, 2005), where it is currently referred as the instrumental approach (Trouche, 2000, 2005). The main aspect is that such an approach allows one to distinguish and articulate the use of the artefact with the tasks to be accomplished, and it does it through the distinction between *artefact* and *instrument*: the artefact is an object in se, material or symbolic, designed for answering a specific need, the instrument refers to mix (hybrid) entity with an artefact type component and a cognitive component, called utilisation schemes. This hybrid entity is the product at the same time of the subject and of the object, « le produit à la fois du sujet et de l'objet » (Rabardel & Samurçay 2001). Such an approach, with the notion of utilization scheme, suggests very effective tools of analysis for outlining the semiotic potential of an artefact.

Semiotic potential of counting sticks

Counting sticks (dating back to ancient China, but no only) are thin bamboo or plastic sticks that can be used in different types of counting tasks. The basic schemes of use that may be identified are the following (Bartolini Bussi and Maschietto, 2008):

- sticks are counted one by one,
- sticks are grouped and bundled (and tied with ribbons or rubber bands) into tens,
- ten-bundles are counted up to hundred;
- ten-bundles are grouped and bundled (and tied) into hundreds and so on ..

As far as the grouping by ten is iterated on the bundles already obtained, counting according to the base 10 is clearly evoked. Thus the positional representation of numbers can be evoked too: the necessity of recording either the different phases of counting and/or the final result of counting may originate the production of signs based on digits (0-9) of different value.

Figure 3 working with the counting sticks

In this respect the semiotic potential of the counting sticks appear very similar to that of an abacus. However, the comparison between the schemes of use of an abacus and of that of the counting stick yet gives us some insight of the specificity of one artefact in respect to the other. As far as counting sticks are concerned, the basic schemes are those of bundling and tying (sticks) and their reverse schemes those of untying and displaying sticks; such schemes evokes the key mathematical notions of *carrying* and *borrowing* that are involved in the algorithm of addition and subtraction.

As far as the abacus is concerned, the basic idea is still that of grouping the collection of unity-pebbles down into groups and to handle groups instead of individuals. However the main point consists in the fact that the abacus is a representing device based on the convention that any single pebble may represent either a unity or groups of unities or groups of groups of unities, its referent is given by the position of the pebble on a board or on a tablet divided into a sequence of strips or columns (for details of analysis see Bartolini Bussi and Mariotti, 2008 p. 758). Once a ten group, or a hundred group, is *represented* by a unity-pebble, such a sign has no the direct reference to the quantity that it represents, is has an indirect reference to it *through the convention* of its position on the board. On the contrary, any bundle of sticks (or bundle of bundles of sticks) may afford the direct reference to its quantity (ten, hundred, …), any bundle is, and represents, at the same time a unit and a multiplicity. The move from one reference to the other can be physically performed through the actions of tying and un-tying: actually, any bundle can be un-tied displaying all its units and then tied again to become a unit again. The comparison between counting sticks and abacus, shows how a fine grain analysis may reveal an unexpected complexity in respect to the mathematics meanings evoked by the use of an artefact, at the same time it may highlight possible synergies between the use of different artefacts as tools of semiotic mediation (for a discussion on this issue and its relevance for teacher formation see Bartolini Bussi and Maschietto, 2008).

According to the TSM, the unfolding of the semiotic potential of counting sticks will see the production of different personal signs related to the schemes of utilization developed in the activity, such signs, because of their origin, have the potential to evolve into mathematical signs under the intentional orchestration of the teacher.

Semiotic potential of Cabri in relation to the notion of function

A full analysis of the semiotic potential of artefact Cabri[3] would be too complex and certainly beyond the scope of this paper, I limit myself to explain how some key elements of Cabri can be linked to meanings emerging from their use and how such meanings may evoke specific mathematical meanings referring to the mathematical notion of function. Generally speaking, functional dependency may be seen as an intrinsic feature of a DGS so that working in such an environment means thinking in terms of functional dependency, however, in general, functional dependency remains implicit, i. e. 'in action' (Vergnaud, 1990), but once made explicit, it provides a rich semantic context to make the idea of function emerge.

In a DGS, as Cabri, basic functionalities concern the domain of graphics: they are *construction* and *dragging*: the former allows to generate drawings (graphic traces on the screen) and the latter allows to make these drawings move. The fundamental

[3] For more details with respect to the notion function see (Falcade , 2006; Falcade, Laborde and Mariotti, 2007) and with respect of other mathematical notions, see for example (Mariotti 2000, 2001, 2007, 2012) in the case of the notion of theorem, conjecture and proof.

characteristic of a DGE concerns the stability of a construction under the effect of the dragging tool; that means that whatever property is defined by a command used in the construction procedure, it will be maintained by dragging. Any Cabri object (the drawing, product of a construction procedure) on the screen can be moved using the *dragging tool*, activated through the mouse. In fact, Cabri objects (points, lines, circles, and any other constructed object) move according to two main kinds of motions: direct and indirect motion.

- *direct motion* occurs when a basic element (for instance a point generated by the point tool) can be dragged on the screen by *acting directly* on it: direct motion of a basic element represents the *variation* of this element in the plane; this correspondence between motion and variation constitutes the main characteristic of any DGE, and of Cabri in particular. Thus the functioning of the tool *basic point*, that is the combination of the construction of a point on the screen and its possibility of being freely moved on the screen, evokes the mathematical notion of *variable*. Consistently, a *point on an object* may evoke the variation of a point within a specific geometrical domain, a line, a segment, a circle, and the like; in other words it may be referred to the geometric notion of *point belonging to a figure*, i.e. a sub-set of the plane, or to the notion of a *variable* belonging to (or varying on) a specific Domain.

- *indirect motion* occurs when a *construction* is accomplished; in this case, dragging the basic points from which the construction originates, will determine the motion of the new elements obtained through it; this motion is a constrained motion, because according to the Cabri logic, the geometrical properties defined by the construction must be preserved. In this sense the *indirect motion* evokes the notion of *dependent variable*.

In summary, a construction given, using the dragging tool the user may experience the combination of two interrelated motions, the free motion of the basic points and the dependent motion of the constructed points; in other words, the different utilization schemes of dragging (Mariotti 2011) allows one to feel functional dependency, as the dependency between direct and indirect motion. In this respect, the use of the dragging tool on a specific construction may be considered as referring to the meaning of function as co-variation between dependent and independent variables; while a construction procedure itself may evoke the meaning of function as input/output machine realizing the relationship of co-variation between independent and dependent variables.

Elaborating further on this last example I am going to present an episode illustrating the unfolding of the semiotic potential in the case of the artefact dragging and the mathematical notion of co-variation between independent and dependent variables.

EXAMPLE OF SEMIOTIC POTENTIAL UNFOLDING

According to the TSM, the teaching-learning process starts with the emergence of students' personal meanings in relation to the use of the artefact. The emergence is witnessed by the appearance of specific personal signs that may be related to the use of the artefact but also have clear reference for the expert to specific mathematical notions. The following illustrative example is drawn from a teaching experiment aimed at introducing students to the notions of function and graph using Cabri as a tool of semiotic mediation.

According to the model defined by the TMS a sequence was designed consisting in a sequence of educational cycles based on the analysis of the semiotic potential of dragging presented above. The implementation of the sequence involved four classes of 10th grade students (in France and Italy), students were aged 15-16. (For details see Falcade 2006; Falcade et al. 2007; Mariotti and Maracci, 2010).

We will focus on the very first session to give an example of the semiotic processes which the unfolding of the semiotic potential may consist of.

Semiotic games in the solution of a task

Let us describe the scenario of the first teaching session which was conducted in the computer lab, consisting in the exploration of basic movements that may occur in Cabri. Students are asked to apply an unknown macro (Effect1) to three given points A, B and P, and then to explore systematically the effect of dragging on the different points. In this case, the macro-construction provides the point H as the orthogonal projection of point P on the line AB. In respect to the semiotic mediation process, the intention is that of fostering students' production of personal signs related to the use of dragging after a construction, signs which could subsequently evolve towards the desired mathematical signs of independent and dependent variable, and function . The task proposed is the following.

Task. Displace all the points you can. Observe what moves and what does not. Explore systematically, that is displace a point at time and note which points move and which do not. Summarize the results of your exploration in the table below.

Point which can be dragged	Points which move	Points which do not move

Specific attention is put in the formulation of the task: the expressions "displace" (ita. "spostare"), "move" (ita. "muovere") and "drag" (ita. "trascinare") are present, and are used with different meanings: "displace" and "drag" are used as nearly synonymous to refer to the direct action made by the user upon the points, with the slight difference of being the first a word of 'natural language' and the second a word of 'Cabri language'; while "move" is used to refer to the movement of a point as a result of direct or indirect action upon it. This difference is not made explicit. It is on the pupils to make sense of this difference through their exploration with Cabri. The following excerpt is drawn from the transcript of the exchanges between two students (M and E) working together on the task. The excerpt shows the emergence of students' personal signs strongly related to the actual use of the artefact but pertinent with the target mathematical signs.

Excerpt 1 (transcript of the exchanges between two students facing the task 1 of first session with Cabri)

11) M: Thus [looks at the] … <u>displace</u> (ita. <u>spostate)</u> all the points that it is possible to displace … let's see H, I start immediately with H that was created, let's see, let's see if it can be <u>displaced</u> (ita. si <u>sposta</u>)
12) E: No … start with the [point] A and follow the order A,B,H,P.
13) M: Point H cannot be displaced (ita.<u> non si sposta)</u>.
14) E: Then points that do not move (ita. non si <u>muovono)</u> … H.
15) M: If I <u>move </u> (ita. <u>muovo)</u> A…
16) E: H <u>moves</u> (ita.<u>muove)</u> too.
17) M: Ah … these three points move … point H cannot be displaced. Then A, B e P are in relation to H, thus if I displace A, H can be displaced, if I displace B, H can be displaced
18) E: <u>At the end H can always be displaced </u>…
19) M: if I displace P...
20) E: <u>H can be always displaced</u>! No… we must say which are the points that do not move
21) M: and I know also how it can be displaced … wait, if I displace P, H moves in a circular movement. If I displace B, H moves parallel to point B and , if I displace A on the contrary H … If I displace A then [H] moves in a circular movement. Which ever point I put ... it is sufficient that H turns around H and it makes a circular movement.
22) E: Then … points that can be displaed …
23) M: A , B e P
24)E: Points that do not move … H … points that move …
25) M: then … point that can be displaced …
26)E: A,B e P can be displaced
27) M: <u>Ah, even H can be displaced, but one cannot move it</u>
28) M&E: points that move ... A,B ,P
29)M: then, next question.
30)E: I wanted to ask... points that can be displaced, in what sense … that everytimes [it] moves (ita. si sposta)
31) M: can be displaced … I told you its hard … all of them move but you can displace only three of them. H moves under the action of A, B, and P.

M and E realize that there is a difference between the behaviours of A, B and P, and H with respect to dragging: H cannot be displaced (item13). However verbally articulating this difference (as required) seems a more demanding task, as witnessed by the “conflicting” conclusions drawn by M and E: H cannot be displaced (item13); At the end H can always be displaced (item18).

Throughout the first part of the excerpt the pupils use the expressions “move” and “displace” interchangeably, it is M who first realizes and expresses this difference: “Ah, even H can be displaced, but one cannot move it” (item 27). Actually M mistakes “displace” and “move” with respect to the use of these terms in the formulation of the task, never the less the distinction between the two different meanings is maintained: as clearly expressed in (27) the students agree on the distinction between what moves and what can be displaced. As effect of the their work on the task, the students produced two distinct signs, “displace” and “move” (ita. “spostare” / “muovere”). These two signs directly refer to the activity carried out with the artefact, but they have the potential of being related to the mathematical signs of independent variable (point that can be moved) and dependent variable (points that move but cannot be moved).

According to our definition, the unfolding of the semiotic potential occurred in the emergence of personal signs - point that can be moved / point that move but cannot be moved - related to the meanings of variation, co-variation and dependency. This is an example of what the teacher can expect after the activity proposed by the first task, it is also a good example of how the production of certain signs can be considered the effect of a semiotic process triggered by the formulation of a task, that is by the intentional use of specific signs. A semiotic perspective introduces a specific dimension in the design of the task, besides that concerning the characteristics of the situation envisaged with the aim of developing a specific meanings, a specific dimension concerns the possibility of triggering semiotic games such as that between the signs displace, move and drag in the example above.

CONCLUSIONS

The theoretical model of TSM offers a powerful frame for describing the use of an artefact in a teaching-learning context. Within this model the use of an artefact has a twofold nature: on the one hand it is directly used by the students as a means to accomplish a task; on the other hand it is indirectly used by the teacher as a means to achieve specific educational goals. In this respect, the model offered by the TSM overcomes the problem posed by the conceptual/technical dichotomy recognizing that artefacts and techniques have both an epistemic and a pragmatic value (se also the discussion in Artigue, 2010). The TSM focuses on the epistemic value and does not take explicitly into account the pragmatic dimension, though not disregarding it. Specifically, using the terminology of an instrumental approach, the development of an instrumental genesis (Rabardel, 1995; Trouche, 2005) does not represent one of the educational goals under explicit consideration. In this respect it is important to note that not all the possible artefacts has the same technical impact leading to stable and effective mathematical practices. Compare the case of a CAS and that of the counting sticks, while an instrumental genesis of the former, leading to an expert use of a CAS to solve mathematical problems, may be considered an educational objective in se, this may not be the case of the latter, counting stick have the destiny of obsolescence, as soon as internalized (Vygotsky, 1978) counting stick will be abandoned.

The TSM elaborate the didactical perspective, and in this sense, a specific artefact is to be considered a resource for the teacher, that is "a means to support" teacher's didactical action (Mariotti and Maracci, 2010). The teacher intentionally pilots the process that she/is trying to promote. Regarding this, a meaningful issue concerns the relevance of teacher's consciousness about her/him own role and specifically about the choices that she/he has to undertake. This brings forth the need of clarifying teachers' direct involvement in the integration of ICT. The analysis and the description of teachers' actions is the base on which math educators may contribute to foster teachers' professional development along this dimension.

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PLENARY ADDRESS 3

Opportunities to Learn in Mathematics Education: Insights from Research with "Non-dominant" Communities

OPPORTUNITIES TO LEARN IN MATHEMATICS EDUCATION: INSIGHTS FROM RESEARCH WITH "NON-DOMINANT" COMMUNITIES

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In this paper I frame the discussion of opportunity to learn around the issue of participation and address three questions: whose mathematics problem is it? Whose language gets privileged? And whose knowledge gets valued? I draw on data from several research projects to illustrate some of the dilemmas associated with these questions. I argue that addressing these dilemmas can help us gain a better understanding of issues around the participation of minoritized students in the mathematics classroom.

What comes to my mind when I think of "opportunity to learn"? I think of access to an engaging and relevant schooling experience that is an example of what Ladson-Billings (1995) describes as culturally relevant pedagogy. That is, in this experience, students' social, cultural, and linguistic backgrounds are valued, reflected, and key to their academic advancement. In this schooling experience, students learn how to use mathematics to understand the world around them from a critical stance (Alrø, Ravn, & Valero, 2010; Gutstein, 2006). This schooling experience should capture the four dimensions that Gutiérrez (2009) describes in her framing of equity, namely, access, achievement, identity, and power. For me, a key construct in a discussion of opportunity to learn is that of participation. In Civil and Planas (2004) we argue for a sociocultural view on participation as we write:

> In the psychological or individual approach, the notion of participation is centered on the learner and pays little attention to the characteristics of the learning context. In the social approach, the key notion of participation is viewed as a kind of socialization into the mathematical practices. The participation model, as understood in the sociocultural approach, focuses on the use of discourse and some of its contents (norms, values, valorizations) as crucial mediating tools in order to interpret the mathematical learner in context. The acquisition of concepts and skills is not enough in the process of becoming a mathematical learner. There also needs to be an active participation in the reconstruction of a specific kind of discourse. (p. 8)

In this paper I draw on over 20 years of work in low-income, minoritized communities to share my insights on challenges and affordances for the mathematics education of non-dominant students. I have traced elsewhere (Civil, 2006; 2011a) my entry into the world of research in mathematics education as being originally grounded in preservice teacher education, and more specifically in my interest in elementary teachers' understanding of mathematics and their beliefs about its teaching and learning. In Civil (2006), I argue for the need to combine cognitive and sociocultural approaches. This need came out of my ethnographic work in low-income Latino communities in the

2012. In Tso,T. Y. (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 43-59. Taipei, Taiwan: PME.

Southwest of the U.S. That work highlighted the contrast between community knowledge and school knowledge, as well as between perceptions of success among children in the community versus in the school. As I immersed myself in the Funds of Knowledge for Teaching project (González, Moll, & Amanti, 2005), I became increasingly aware of the richness of knowledge and experiences in the communities where our work was located and the keen interest and concern that parents had for their children's schooling, an interest and concern shared by many of the teachers in the schools I visited. Yet at the same time, teachers and parents often had different perceptions of each other's roles and expectations for the children's education. These differences in perceptions and the presence of a deficit discourse in schools (though certainly not shared by all) made me lean more towards a sociocultural approach.

Yet, as I persevered on applying the principles behind the Funds of Knowledge project, namely contextualizing the mathematics teaching and learning in the richness of children's and their families' everyday experiences, I was often faced with the dilemma of preserving the purity of the funds of knowledge versus foregrounding the mathematics (Civil, 2007). Questions such as "Where it the math?" and "Are we helping students 'advance' in their learning of mathematics"? kept me going back and forth between a cognitive approach and a sociocultural one. Even more recently, as part of the Center for the Mathematics Education of Latinos/as (CEMELA)[1], I would find myself often bringing up the question of "where is the math?" in some of our work. So, for example, in some of projects that we did as part of the After-school Math Club, I sometimes wondered, "is the social justice component taking over the math?" Yes, this question is going to read odd to many of you, while others will probably be nodding in agreement. Those who are nodding in agreement may share similar reservations to others in the field who have raised the question of "where is the mathematics in some of the mathematics education research work?" Martin, Gholson, and Leonard (2010) provide a compelling response to those who raise this question. In particular, I highlight the following from their response:

> Rather than generating concern about studies that do not give priority to mathematics content, it may be more informative to understand why studies that have continued to do so have offered so little in the way of progress for students who remain the most underserved. Minimal progress for these students would seem to demand that we pursue *all* promising areas of inquiry informing us about how to help them experience mathematics in ways that allow them to change the conditions of their lives. (pp. 16-17)

My need to combine cognitive and sociocultural approaches is further affirmed by this call "that we pursue *all* promising areas of inquiry…" Certainly this is not new. Brenner (1998) describes what this may look like in a program of research that addresses the social component, by which "the classroom community needs to facilitate the comfortable and productive participation of all students" (p. 215); the cultural component, which "examines how teachers can respect and incorporate the cultural traditions of the children in the class" (p. 215); and the cognitive component, which calls for the instruction to “enable children to build from their existing cultural knowledge base in mathematics” (p. 215).

The more I worked with students from non-dominant communities, the more I realized that “just” focusing on the mathematics was not enough. I was intrigued by the literature around the role of the context of the problem and how that may affect students’ work on the problems (Cooper & Dunne, 2000; Lubienski, 2002). More recently, largely due to the restrictive language policy affecting the schools where my work was located, I became interested in the interplay between language and mathematics, particularly for students whose home language is different from the language of schooling. And throughout all my work in non-dominant communities, the issue of valorization of knowledge (Abreu, 1995) has been present. The tension between in-school and out-of-school mathematics often goes hand in hand with what forms of mathematics are more valued. Hence, I see three elements at play as I reflect on opportunity to learn in the context of non-dominant communities: the nature of the mathematics problem; the language(s) involved; and the valorization of knowledge. Let me illustrate this situation with the case of Alberto. In Mexico, as a nine-year-old, Alberto contributed to the family business (a bakery) by having his own set of customers; he handled the orders, the money transactions, and the delivery. The problems in his everyday business transactions were relevant to him. He did all of this in his home language, Spanish. His mother was clearly proud of him when she was telling us the story. His knowledge was valued. The year after, in the U.S., as a student in fifth grade, Alberto was disengaged and trailing behind. He was fortunate to be in a bilingual classroom with a teacher who valued his home language and who knew there was more to him than his underperformance in school. His teacher was in the Funds of Knowledge for Teaching project and she chose him as her focal student for the home visit because she wanted to learn about him and his family. Other teachers may have attributed his low performance to his limited knowledge of English, or his prior schooling experiences in Mexico, or to his home environment, using general stereotypical images of low income, immigrant students without seeking to have first hand knowledge of this student and his family.

This brief vignette captures the three dilemmas that are the focus of this paper. The first dilemma is “whose mathematics problem is it?” The second one is “whose language gets privileged? And the third dilemma is, “whose knowledge gets valued?” I argue that understanding these dilemmas can help shed light onto issues of students’ participation in the classroom, which as I said earlier, I view as a key construct in a discussion of opportunity to learn.

WHOSE MATHEMATICS PROBLEM IS IT?

Let me use the bus pass problem, a classic problem in the U.S. literature (Silver, Smith, & Stein; 1995; Tate, 2005) to illustrate what I want to address in this section. A version of this problem goes as follows:

> It costs \$1.50 each way to ride the bus between home and work. A weekly pass is \$16. Which is the better deal, paying the daily fare or buying the weekly pass? (Tate, 2005, p. 36)

The “expected” answer was the single tickets because one would need 2 tickets per day for 5 days in the work week, so a total of \$15. Yet students at an urban middle school chose the weekly pass because in their experience this would be better option: it can be used by more than one person in the family; it can be used on weekends (e.g., for possible job also on weekend); it can be used for more than 2 trips per day (e.g., for more than one job).

I am not arguing that the problems we give need to be necessarily “relevant” to the experiences of the students, though this may be a point worth discussing further. What I am arguing is that whatever problems we use, we need to understand that the answers students give reflect their experiences. As Silver et al. (1995) write,

> Increasing the relevance of school mathematics to the lives of children involves more than merely providing “real world” contexts for mathematics problems; real world solutions for those problems must also be considered. Until the forms of reasoning and problem solving that are developed and used in out-of-school settings are brought into close contact with the forms of reasoning and problem solving being developed in school mathematics, attempts at increased relevance are doomed to failure. (p. 41)

Cooper and Dunne (2000) and Lubienski (2002) write about how working class students seemed to be more likely to bring in real world considerations into the mathematics problems, which sometimes led away from the intended mathematics. And I have to ask, who is to blame, here? Not the students, in my opinion. As we look at textbooks, assessments, new sets of standards, and listen to teachers reporting on their students’ approaches to problems, there is a gap between those who know how to play by the school rules and “just do the math” and those who willingly or not resist this and want to make sense out of the problems. And it seems to me that many of us, mathematics educators, teachers, policy makers, tend to favor those who focus on the mathematics. Maybe we need to reexamine that?

The work of Planas with teachers of immigrant students in Barcelona is a step in that direction. For example in Planas and Civil (2009), we document how through a teacher study group that focused on the development on critical mathematics education tasks, teachers acknowledged their limited knowledge of their students’ realities and used the mathematical tasks as a way to open up a dialogue and in this way learn from their students. The task we described in that article centers on students designing their ideal flat, based on a given flat and some task cards that represent advantages and disadvantages of the flat (e.g., an example of a disadvantage was that the total area of the flat was 65 m^2). The students got engaged in the task and decided to change several of the cards to better reflect their reality. As we write, “by providing a task such as the one of the ideal flat where students can challenge each other and the teacher, we open the channels of participation in the mathematics classroom” (p. 403).

In Planas and Civil (2002) we discuss the students' approach to a task in which the area and population of two neighborhoods in Barcelona were given and the students were to determine in which neighborhood people lived more spaciously. We describe the tensions among the students as they worked in the problem. While some of them interpreted it essentially as a mathematical problem of division of the population by the area, others got more involved in the different types of houses that may be in each neighborhood (e.g., houses with gardens and swimming pools vs. small flats in skyscrapers) and the kinds of families that may live in each (smaller vs. large families) bringing in their view of the world around them. The teacher favored the mathematical approaches and dismissed those that brought in these external considerations. This affected students' participation. In Planas and Civil (2009) we discuss the same problem with the added question of "how many people should move from one neighborhood to the other so that people live in both of them spaciously?" Here is an excerpt between Emilio (an immigrant student) and the teacher:

> E: The second question is wrong.
>
> T: Why?
>
> E: I wouldn't move alone. I'd take all my family.
>
> T: What do you mean?
>
> E: I would change the second question.
>
> T: Don't start again, Emilio! You know the problems are like they are. (Planas & Civil, 2009, p. 150)

In this case, the teacher rejects Emilio's attempt to bring in his personal interpretation about the problem and basically positions the given mathematics problem as external and unchangeable. Van Oers (1996) writes about getting students involved in mathematical activities that are both "real" for the mathematical community and for the students. He writes, "all mathematical learning should take place in the context of a sociocultural activity in which the pupils want to participate and in which they are able to participate given their actual abilities" (p. 104). I take this concept of sociocultural activity in a broad sense and focus on this idea of "pupils want to participate." With this I mean that the question "whose mathematics problem is it" does not imply for me that the problem has to be necessarily grounded in students' lived experiences. I have seen multiple examples of students engaging in mathematical problems that we could characterize as typical problem-solving tasks, but where students are encouraged to bring their own "forms of reasoning and problem solving that are developed and used in out-of-school settings" (Silver et al., 1995, p. 41).

In what follows I illustrate students' engagement in a problem based on a distance-time graph of a bike trip[2]. One of the questions the students had to answer was:

> When is the biker making the most progress or covering the most distance? How do you know?
>
> *¿Cuándo es que el ciclista avanza más o cubre la mayor distancia? ¿Cómo lo sabes?*

The questions were presented in English and Spanish since this was a class composed of only 8 students, all classified as English Language Learners (ELLs), most of them recent immigrants (within the previous two years).

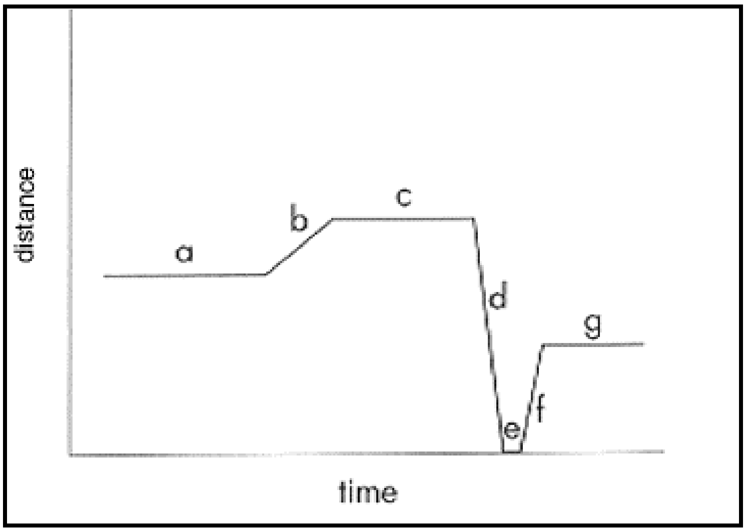

Figure 1: Bike trip graph

Students were in disagreement as to whether the answer was segment b, which had positive slope (hence, "advancing") but less distance in more time or segment d that had negative slope (hence "going back") but greater distance in less time (see Figure 1). Segment f also played a role in the discussion. Here are some excerpts for the discussion.

1 Larissa: It's B

2 Carlos: It's B because it says "the most distance;" it doesn't say… In F he's going fast but is not gaining distance.

3 Ernesto: It's D because it's the longest one, although he's not moving forward, but he's moving somewhere else.

4 Larissa: But it's asking if you're making progress.

5 Ernesto: But it says "or covers the most distance," something like that.

6 Octavio: Distance [points at D]

7 Larissa: But "progress!" You're going down! You're losing distance!

8 Octavio: But it's still distance!

9 Carlos: When does he gain the most distance, dude? And it's in B when he gains the most distance and it's starting from B.

[Discussion goes on for a while; Ernesto and Octavio are confident the answer is D, while Carlos and Larissa think that the answer is B; at my suggestion, they put numbers on the axes]

10 **Marta**: How many kilometers does he travel in D?

11 Carlos: He's not moving forward. He's moving backwards

12 Larissa: He's losing distance.

13 Ernesto: But he's moving.

14 Larissa: But it’s asking you about “progress”!

15 Ernesto Well, he’s progressing backwards.

16 Larissa Oh, that’s right; that’s right, OK.

17 **Marta**: what are you thinking about with the word “progress”?

18 Larissa: That he’s moving forward. That’s what it makes me think about.

19 Ernesto: When you go down you’re going backwards.

20 Larissa: Yes, that’s right. It’s D. It’s D.

21 **Marta**: Let’s see, Larissa. Explain why you think it’s D now.

22 Larissa: Because, because when you read it in Spanish it says “*¿Cuándo es que el ciclista avanza más o cubre la mayor distancia*?” So D is where there’s more distance.

23 Carlos: Why is it more distance?

24 Larissa: (To Carlos) Look. Measure this (points at F). Measure this (points at B). In distance, it’s very little. So this (points at D) is thirty meters, thirty meters?

25 Carlos: But it’s going down.

26 Octavio: But it’s still distance!

27 Larissa: It’s thirty meters in distance (pointing at D) and these are five (pointing at B) and these are fifteen (pointing at F). And it’s asking you, it’s asking you in which letter there is, where it is that he covers the most distance.

What this transcript does not capture is that this was a lively discussion with students at times talking over each other, using nicknames (e.g., students often called Octavio “Costeño” because he came from a region on the coast of Mexico, as opposed to most of the other students who came from a different region), and using casual talk such as “dude” in line 9. O’Connor (1998) discusses transfer issues between home / community discourse practices and school mathematics discourse practices. She writes,

> This leads us to the question of how the classroom must be structured to enculturate students into the habits of mind that let them sustain productive argument that does not disintegrate into personal animosities that overtake the intellectual content. This is pedagogically and theoretically an unsolved problem. (p. 42)

I would add that this question needs to be explored in different contexts. My context in the example above was one in which students felt free to bring in their familial ways of interacting into the classroom, something that may not have been well received in a different classroom setting. So, the question is what are the characteristics of a classroom environment that encourages students, and in the examples discussed, non-dominant students, to make these mathematics problems their own, even when the problems may be more along the lines of “typical” mathematics tasks? The teacher in the “ideal flat problem” (Planas & Civil, 2009) and the teacher in the classroom of the bike trip task gave their students freedom to work on the problem in any way they wanted. They seemed genuinely interested in how the students may approach the problems and were open to “non-mathematical” talk and to the idea of letting their students modify the task (as in the ideal flat problem where students changed some of the task cards to better reflect their reality). The teachers took a risk by letting go of their mathematical agenda. Students, in turn, became engaged in the problems and used mathematics to solve them.

In the bike trip task there is a key element that allowed for the richness of the discussion: students talked in Spanish throughout this whole group discussion. I argue that this was instrumental to the students’ participation in this task, and I expand on this in the next section. Here I limit myself to illustrate one aspect of the role of language in this task, as we see it in Larissa’s interpretation of the English vs. the Spanish version of the question. The word “progress” in the English version of the question corresponds to “progreso” and so, from that point of view, Larissa may have seen segment d as going back (with respect to where the person starts in a), thus not making progress (lines 4, 7, 12, 14, 18). But the Spanish translation of the question does not have that connotation, as it talks about “advancing more or covers the most distance.” Thus the idea of “advancing more” does not have the same connotation as “making the most progress” for Larissa, who then sees “D” as an answer that makes sense (lines 22, 24, 27).

WHOSE LANGUAGE GETS PRIVILEGED?

There is a growing body of research around the teaching and learning of mathematics in multilingual settings (e.g., some recent publications include Barwell, 2009; Moschkovich, 2010; Setati, Nkambule, & Goosen, 2011). In this section I limit myself to some considerations from my work in two different settings. The first setting is in the context of restrictive language policies and the implications of those for the learning of mathematics of immigrant students whose home language is not the language of instruction (ELLs in my context). The second setting is a new one for me. I will give a glimpse of my most recent work in dual language schools where Spanish is actually promoted, and offer some preliminary observations in relation to the participation patterns in the mathematics classroom.

Segregation of English Language Learners

In 2000, bilingual education became severely restricted in Arizona. In 2006, the Arizona state legislature further restricted its language policy by tying increased funding for the education of ELLs to their being in a segregated four-hour daily block, focusing on the learning of the English language. During the 2007-08 school year, the middle school with the classroom of the bike trip task I described earlier implemented a version of this four-hour model. This resulted in the ELL students having five to six of their seven daily classes only with other ELLs in a section of the school that I have labeled Section A in my prior writings (Civil, 2011b; Civil & Menéndez, 2011).

As I describe in Civil (2011b), the contrast between students’ participation in English versus Spanish was striking. When presenting to the whole class in English, their communication was tentative and stilted. The other students did not seem particularly engaged and the whole presentation was more like an exercise they had to go through. When presenting in Spanish or talking in their small groups (where students turned automatically to Spanish), it was a completely different story. Students engaged in lively mathematical discussions (as the excerpt on the bike trip task shows), often bringing in cultural resources such as humor and metaphors to the task. I argue that we would have missed much of the richness of these students' thinking in mathematics if we had limited their communication to English only. So, in a sense, being in this segregated environment allowed us to increase their opportunity to learn by developing an environment in which we encouraged them to talk and communicate about mathematics in either language. The teacher used primarily English due to the language policy in place (though she was herself a native speaker of Spanish, and an English language learner). Most likely, the expectation was that the students would be using mostly English and that was indeed the case in their written work. But as we became more engaged in the research aspect, where our focus was on students working in groups and talking about mathematics, the oral communication was often in Spanish. In a sense we were confronted with the dilemma of code-switching, as described by Adler (2001), as none of us had English as our first language, and in fact in our case, different from the South African context, we all shared the same first language, Spanish. For me, at some level, this was not a dilemma. I was interested in learning about the students' understanding of mathematics and in promoting their learning further by engaging them in mathematically rich tasks, that were also language demanding. Thus, their being able to use both languages gave them more resources. However, as I realized through interviews with the students, being in Section A of the school where ELLs tended to use more Spanish than if they had been in the other sections of the school, was not unproblematic. As I write in Civil (2011b):

> Most of them expressed a desire to move out of Section A, and some believed that they were not learning as much English as they would if they were with the non-ELL students. Thus, in retrospect, it is not entirely clear that these students were necessarily comfortable with the idea of using Spanish in the mathematics classroom, since that may have contributed to their perception that they were not advancing enough in their English. (p. 88)

Not only were students in Section A concerned that they may not be learning as much English as they would have liked to, but also their perception was that the academic level in general was lower. I only worked intensively with a very small fraction of students in Section A, mostly the eight students I have been referring to. I do believe that we pushed these students in mathematics and the students themselves acknowledged that in their conversations with me. But I do not know what the experience was like with the other ELL students or the other subjects. Our interviews with the parents of some of these students showed a clear awareness on their part of the situation. Roxanna, Ernesto's mother, shared the following:

> He [Ernesto] does say he wants to go higher…. He says that he's not very convinced of being there [in Section A]. He wants more. He says, "Mom, just imagine that we are back in Mexico, with the teachers from Mexico because now I even get mixed up because they explain more in Spanish than English. And I am with the expectation that they are going to talk to me in English and I am thinking in English…. I get mixed up, because I want them to talk to me in English and the teacher can't because there are quite a few children who don't understand English well. And the teacher opts to speak Spanish first and when [she] starts talking in English, I am already all tangled up in knots. I am already confused, and I can't get untangled." And that is why he wants to go where "the class in general, from start to finish, [is] in English."

Personally I think that a model in which ELLs are segregated is educationally irresponsible and inequitable. We did the best we could with this situation and we created an environment where students were able to use their home language as a resource for doing mathematics. This resulted in a rich data set where we see students who are often not talked about as examples of "good at mathematics" as indeed very capable of engaging in sound mathematical arguments. As I observed some of them the following year in non-section A mathematics classes, I did not see them as willing to participate. In follow-up interviews they expressed being uncomfortable speaking up because their English was not good enough:

> Larissa: I'm still not learning it [English] well, that's how I see it… So, there are times that I stay quiet because I feel embarrassed if I don't pronounce something well.

The Section A mathematics teacher with whom I collaborated had this insight to offer about the students:

> Matilde: I work only with ELL students. Our kids feel afraid to be in the regular classroom because they feel the other kids have the power. So, even if I have a very brilliant kid… he is not going to be that brilliant because they are going to ask them questions in English so they don't know how to explain themselves and they're going to be quiet. So, they're going to be relegated to the back of the class.

This teacher brings up the notion of power in her reflection. What is the best setting for ELLs? The "regular" classroom? The segregated classroom? In the next section, I turn to a very different setting, with a different kind of dilemma for me, one closely related to the power issue.

English Language Learners in a Dual Language Classroom

The language policy in my current context is quite different from the one in Arizona. Most recently I have started some work in two dual-language (English/Spanish) elementary schools. I conduct mathematics workshops for Latina mothers in both of these schools and in one of them, we have a Teacher Study group where we have been using Complex Instruction to explore status issues in the mathematics classroom (Boaler & Staples, 2008; Cohen & Lotan, 1997; Featherstone et al., 2011). As part of our work with parents and building on my prior work (Civil & Quintos, 2009; Civil & Menéndez, 2012) a group of Mexican, Spanish speaking parents (3 mothers and 1 father) visited a 5th grade mathematics class (ten-year-olds), which was conducted in

Spanish by a native speaker of Spanish. After the classroom visit, we had a debriefing session with the parents. One of them, Graciela, right away commented that she had noticed that the White, non-Latino children participated more than the Latino children. By participation she meant that when the teacher asked for students to share ideas with the whole class it was mostly the "American" children (her words) who answered. This study is very preliminary and thus, there are many elements about the context that we do not know yet. Based on observation only, the Latino/a children tend to be in a minority in the classrooms. Also, most of them come from low-income families with limited formal education; in contrast many of the White, non Latino children come from middle to upper class families, with high levels of formal education.

What I want to discuss here are these parents' perceptions of why they think that the Latino/a children participate less and also bring in some of the teachers' comments on this observation. I draw on data from the classroom visit debriefing and from a parents' group discussion where we asked them to comment on the issue of participation. Graciela's comment below captures what several of the mothers said.

> Graciela: If you notice, Americans have a high level of communication with their children, they let them do things that we, Hispanic, don't…. Our children are very inhibited, it's like they don't have this experience, they haven't done much…. There are many Hispanic children that are low [meaning in performance in school] and why? Because we don't look for people to help them with homework, we don't help them, from very young, to read a book, to learn the colors, we leave it all to the teachers and the Americans, from two years of age [the parents teach them], the colors, the numbers.

In general their comments pointed to differences in approaches to child rearing. These comments are marked by a deficit discourse towards their own approaches and an adoption of the mainstream rhetoric about good parental involvement, as reflected in the activities that "American" parents do with their children. As I pushed them to share what different activities they (or other Latino parents) may engage in with their children, I did not succeed in moving them away from what they thought they were not doing, that is, not teaching them the colors, the numbers, or reading to them.

Going back to the participation question, a couple of the mothers brought up the issue of fear and not feeling comfortable speaking up. They shared that themselves they often did not speak up at parents' meetings for fear to be scolded for not doing the right thing, or for not wanting to ask something that maybe had already been explained but they had missed it or not understood it. They wonder if this fear was transmitted to their children.

> Sandra: They [their children] are scared. We as Hispanics are scared that our answers may be incorrect or that when we answer incorrectly, everyone will make fun of us and for that reason we prefer to remain silent.

When we brought up the issue of differences in participation patterns to the teachers in the Teacher Study Group, here is an excerpt from the conversation:

1 Catherine (teaches mathematics in English): more participation of Anglo kids, probably due to language and the fact that many of them come from highly educated families, but also, from a white kid perspective you are encouraged to ask questions, be cute and obnoxious. And I don't know if this is at true in Latino families.

2 Agena (teaches mathematics in Spanish; and is Latina): They know we are the minority, even here, not comfortable, more shy.

3 Catherine: I would even say it's intimidation; sometimes white kids can be very intimidating, just with their eagerness.

Although these data are very preliminary, they bring up some dilemmas around the dual language program and power issues of a different nature than in the Arizona context. In Arizona, the schools where my research was located were de facto segregated by ethnicity and social class. Hence most of my work there was in low-income, primarily Latino communities. In my current context the schools are very diverse not only in terms of race and ethnicity but also in terms of social class. And this is where the power issues come in, that are hinted at in the parents' and in the teachers' comments.

Valdés (1997) cautions us about the potential for dual-language programs to actually exacerbate the power differential. In her thought-provoking essay she talks about a Latina educator who expressed her concern about dual-language programs by saying "It they take advantage of us in English, they will take advantage of us in Spanish as well." (p. 393) The issue being that dual-language programs often serve two kinds of children, as is the case in my current context, the low-income Latino children with Spanish speaking immigrant parents and the middle class White children, whose parents view the learning of another language as an educational and economic asset. The question is who benefits the most from these programs? As Valdés writes, "Bilingualism can be both an advantage and a disadvantage, depending on the student's position in the hierarchy of power" (p. 420).

Current practices in the teaching and learning of mathematics call for students to engage in argumentation, to communicate their ideas, essentially they assume a higher emphasis on discourse. Discussions about the mathematics education of students whose home language is not the language of instruction cannot ignore the power issues associated with language choice. Segregating ELLs, while it allowed us to engage in rich mathematical discussions (in that particular case in which we all shared a same common first language), is not the answer. Having ELLs with non-ELLs, in English only environments, calls for careful attention on how to promote a meaningful participation of ELLs. ELLs and non-ELLs learning in the home language of the ELLs (such as the dual language program) also raises participation issues. In sum, as Setati (2005) writes, "If we are to explain language practices in a coherent and comprehensive way, we must go beyond the cognitive and pedagogic aspects and consider the political aspects of language use in multilingual mathematics classrooms" (p. 451). Implicit throughout what I have written so far in this paper, is the notion of valorization of knowledge. When we pose a problem and students approach it in a way that is not what we were "expecting" from a mathematics point of view, how do we value that contribution (particularly if they are non-dominant students)? Which language gets valued and when, is of course directly related to whose knowledge we value. I turn to this third dilemma in the closing section.

WHOSE KNOWLEDGE GETS VALUED?

To me it is hard to discuss opportunity to learn and not bring in the concept of valorization of knowledge. As I alluded to in the beginning of this paper, a turning point for me was the contrast between in-school and out-of-school mathematical practices and the notion of success, particularly for non-dominant students. The work of Abreu (1995) around the notion of valorization was instrumental to my thinking: "if we want to understand why a child successful in out of school mathematics does not use this knowledge to inform the solution of school problems, we might want to ask about the valorization of the two practices" (p. 122).

For several years now, my focus has been on looking at this idea of valorization of knowledge from the point of view of immigrant Latino parents who learned different approaches to mathematics, particularly in arithmetic, but also pedagogical approaches (e.g., group work; use of calculator), as well as from the point of view of teachers who work with non-dominant students, mostly children of immigrant origin (Civil, 2011a; Civil & Planas, 2010). My concern is that often, both groups (teachers and parents) bring different valorizations of knowledge and those caught in the middle are the children who have to navigate between home and school practices. That parents, teachers, and students may have different views on what mathematics is and how it should be taught is not surprising. Comments along the lines of "this is not how my teacher wants me to do it" are often shared by parents talking about some of the struggles they meet when trying to help their children with homework. I argue, however, that these comments have different implications when those involved are parents and children from non-dominant communities. As Gorgorió and Abreu (2009) write, "the important issue … is not whether there are or are not differences in the way the division algorithms look, but the reaction of the teacher to this difference" (p. 72). I have documented elsewhere (Civil, 2011a; Civil & Planas, 2010) some of the tensions around parents' and teachers' different valorizations of knowledge. These tensions have to be understood through the power differential that often affects the knowledge of non-dominant communities. Valorizations of knowledge cannot be separated from whose knowledge it is. My sharing the way I learned how to do subtraction with preservice teachers often elicits reactions of surprise, confusion as to how and why it works, and yes, to a certain extent comments along the lines that their way seems "easier." But, my sharing with them algorithms that children from Latin America may bring in, prompts reactions such as "this is nice, but they need to learn to do things the U.S. way."

Let's compare the following comments from two different teachers:

> Caroline: Part of the problem I think that the students are facing is parents didn't learn that way…. The Latino children, if their parents come from Mexico, then they probably did it a different way… and even the algorithms maybe look a little different. If you're looking at algorithms, they're going to be like "my dad does it this way" or "my mom does it this way." And so then you're bringing in another way so that they're seeing maybe even a third or a fourth or a fifth way to attack a problem.

> Dalia: Every Wednesday we are teaching division and multiplication, and the children are doing it the way we ask. This Wednesday when we did it, Eliseo (a student) said "oh no, my mama did it different." And he went to the board and did it that way, and I say, "yes, but that's in mama's home. Let's do it the way that we do it in the school". And it was very close, but it was an approximation. It was an approximation.

Caroline embraced the different approaches that the children brought from home and saw them as opportunities to learn (for herself and for her students). Dalia missed an opportunity to learn.

Our work with teachers and with parents should confront the issue of valorization of knowledge head on. Immigrant parents share their feelings of sadness when they see their children devaluing their knowledge, "last night my son told me that school from Mexico was not valued the same as school here, that is doesn't count. What I studied there doesn't count here."

As Knijnik (2004) writes,

> Our role in these processes of inclusion or exclusion of knowledge in the school curriculum is, above all, political. Such processes, defining which groups will be represented and which will be absent in school are, at the same time, a product of power relations and producers of these relations. A product of power relations, since it is the dominant groups that have the cultural capital to define which knowledge should legitimately be part of the school curriculum. (p. 137)

These dominant groups are the ones who decide whose mathematics, whose language and whose knowledge get valued, and ultimately who has the opportunity to learn. What role do we each play in this process?

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THE INTERDEPENDENCE OF POWER AND MATHEMATICS IN OPPORTUNITIES TO LEARN: A RESPONSE TO MARTA CIVIL

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That teaching is dilemma filled is well known. Dilemmas suggest uncertainty and choice and so judgment in action. Marta Civil's work on opportunity to learn in "non-dominant" communities, brings into focus three tensions or dilemmas that she poses as questions: Whose problem? Whose language? Whose knowledge? These questions are critical, and illuminated through her examples and experience. In response, and with the intention of furthering discussion, I will argue that judgments in relation to each require a discourse of 'who' ***and*** *'what' - an acute awareness of their interdependence. Considerations of the interdependence of power* ***and*** *of mathematical access, in particular, are necessary.*

INTRODUCTION

Through her extensive work and experience in mathematics education in 'non-dominant'/ 'minoritized' communities, Martha Civil confronts what is increasingly acknowledged but challenging to address: " 'Just' focusing on the mathematics was not enough". She brings into focus, three tensions that have emerged over time as dilemmas in her practice. She poses these as the questions: Whose question is it? Whose language gets privileged? and Whose knowledge gets valued? These questions are a function of the key construct of 'participation' from a socio-cultural perspective, and thus all central to 'opportunity to learn'. In her discussion of participation, she draws on her work with Planas (Civil & Planas, 2004) to argue that participation is more than the acquisition of concepts and skills, but participation in 'the reconstruction of a specific kind of discourse', that includes 'norms, values and valorisations'. Embedded in this starting point are Sfard's two metaphors for learning: acquisition and participation (Sfard, 2000), and her warning of the 'danger of choosing just one'. However, what distances these two apparently similar appeals to the substance of our work in mathematics education, is that the 'norms, values and valorisations' refer in Civil's work to 'whose', and in Sfard's to discourse. This tension between 'who' and 'what' continues to separate, rather than mutually engage participation in our community of mathematics education research, where, as Civil highlights, the questions "where/what is the maths?" on the one hand, and "who has access?" on the other struggle to come together.

This tension between this 'who' and 'what' filters through each of the examples Civil provides, as she assists our appreciation and understanding of them. What then to do? In the remainder of this short written response, I pass through a range of literature related to dilemmas of mathematics teaching and learning, including my own. I then

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turn to work in critical literacy for a language with potential to move us forward not with prescriptions of 'what to do', but with 'how' to take on the predicaments and complexities of equity/access work in 'transformative' ways.

Dilemmas of teaching mathematics

In Adler (2001), I argued:

> That teaching in multilingual mathematics classrooms is dilemma-filled is not surprising. Classrooms, after all, are complex sites of practice. The value in identifying key teaching dilemmas and naming them is that they can then become objects of reflection and action. Familiar, taken-for-granted practices can be made strange. … [T]he notion of a 'teaching dilemma' became the key with which to prise open teachers' knowledge of their complex practices. … in a range of multilingual mathematics classrooms in South Africa. (p. 49)

I built on the literature on dilemmas of teaching, drawing specifically from Lampert (1985) and others in mathematics education on the one hand, and Berlak & Berlak (1981) in sociology of schooling on the other. Lampert emphasised practice-based dilemmas, posing questions about the tensions between theory and practice in her work. The Berlaks in contrast, posited the tensions between structure and agency in schooling practices, including curriculum, and teaching. I will not rehearse the story here, only to say that the three dilemmas of teaching mathematics in multilingual classrooms described (code-switching, mediation, transparency) were argued as at once personal, practical, and contextual, and each has some resonance with those in Civil's paper: the dilemma of code-switching is about managing which language; the dilemma of mediation is about managing the boundary between the local/everyday and the scientific and so what knowledge; and the dilemma of transparency is about implicit and explicit practices, and so whose access is enabled. Asking 'which language' and 'what knowledge', however, is not synonymous with asking whose language and whose knowledge gets valued.

Mathematics and power

A critique of my dilemma language provided in Setati (e.g., 2005), is that the relationship between language and power was not sufficiently theorised or engaged, a criticism with which I agree, and one that foregrounds the who. I would argue further now, that it is not only power that was under theorised and engaged. Despite what I thought was attention to the mathematics in play, this too did not receive sufficient attention. Ball's (1993) paper which escaped my attention then, describes, in detail, dilemmas of 'representing content'. She couples the uncertainties she confronted when choosing and using representations for negative numbers, with dilemmas that arose in relation to respecting children as mathematical thinkers (what knowledge gets valued), but without consideration of power.

How mathematics and power inter-relate in moments of practice is central to research and practice in general, and to equity and access concerns in particular. Gutiérrez' (2009) brings power and mathematics together in her framing of equity in mathematics education across four dimensions, where achievement is joined to access, identity and power. The title of her book is an 'and' discourse: learners need to learn to play the game ***and*** change the game. Janks (2010), in a similar way, argues for the inter-dependence of power, access, diversity and (re)design, in her text on critical literacy. She provokes us through questions like: what happens when there is power without access? And, equally, what happens when there is access without power? (p.178). Bringing these questions into the mathematics classroom, let's consider, albeit too briefly within the constraints of this response, Civil's illustration over the tension of 'whose problem is it?'. Civil concludes that the teachers in each of the "ideal flat problem" and the "bike trip task":

> … gave their students freedom to work on the problem in any way they wanted. They seemed genuinely interested in how students may approach the problems and were open to "non-mathematical talk … (taking) a risk by letting go of their mathematical agenda. Students in turn became engaged in the problems and used mathematics to solve them.

The students' participation was enabled. Yet, it is not clear to me from the transcript we have, what mathematics the students 'used', whether they participated in mathematical discourse in extended ways. For if they did not, then we must conclude this might be participation without access, to changing the game without being able to play the game in Gutierrez's terms, or to the power of mathematics in Janks' terms.

Janks discusses the profound complexity of our work as she moves 'beyond reason' to 'desire'. Desire for what one is excluded from, particularly mathematics and language, is not simply of symbolic value – it has material consequences. Both mathematics and English (certainly in the South African context) open and close doors to further study and employment. Desire is a thus double-edged sword for ourselves as teachers with a concern for the other. Janks argues:

> Becoming what we lack, changes who we are. Something is always lost in the process. As educators, changing people is our work – work that should not be done without a profound respect for the otherness of our students. Desiring what one is not should not entail giving up what one is. (p.153)

Marta Civil's illumination of the questions: Whose problem is it? Whose language gets valued? And Whose knowledge gets valued? reflects her profound respect for the otherness minoritized mathematics learners. In my discussion I will use the discourse of dilemmas and a version of Jank's interdependence model to discuss and hopefully take a little further, the dilemmas illustrated in Civil's paper.

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PLENARY ADDRESS 4

Creating Opportunities to Learn in Mathematics Education: A Sociocultural Journey

CREATING OPPORTUNITIES TO LEARN IN MATHEMATICS EDUCATION: A SOCIOCULTURAL JOURNEY

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In this paper I conceptualise opportunities to learn from a sociocultural perspective. Beginning with my own research on the learning of students and teachers of mathematics, I sketch out two theoretical frameworks for understanding this learning. One framework extends Valsiner's zone theory of child development, and the other draws on Wenger's ideas about communities of practice. My aim is then to suggest how these two frameworks might help us understand the learning of others who have an interest in mathematics education, such as teacher educators and mathematicians. In doing so, I hope to move towards a synthesis of ideas to inform mathematics education research and development.

BACKGROUND

The conference theme, "Opportunities to learn in mathematics education", was my point of departure in choosing the focus for this talk, and the first part of my title signals that I'm interested in how such opportunities are *created*. I also want to pose a second question: *Who* has opportunities to learn? Many mathematics education researchers are interested in *students'* mathematics learning or in the professional learning of *teachers* of mathematics. This is where my own research started: investigating how opportunities to learn can be created for, and by, students and teachers in secondary school mathematics classrooms. But there are others who might "learn in mathematics education". Here I'm referring to *mathematics teacher educators*, *mathematics education researchers*, and *mathematicians*, and it is their (our?) opportunities to learn that intrigue me now. So the journey I trace in this paper is both past and future oriented, in the hope that reflection on my past experiences might lead to discussion of new research questions and challenges.

The second part of my title points to the theoretical perspective that has steered this journey. Lerman (1996) defined sociocultural approaches to mathematics teaching and learning as involving "frameworks which build on the notion that the individual's cognition originates in social interactions…and therefore the role of culture, motives, values, and social and discursive practices are central, not secondary" (p. 4). Sociocultural perspectives on learning grew from the work of Vygotsky in the early 20th century. One of the key claims of Vygotsky's (1978) theoretical approach concerns the social origins of higher mental functions, and he introduced the concept of the zone of proximal development (ZPD) to explain how a child's interaction with an adult or more capable peer awakens mental functions that are yet to mature. Early studies that applied Vygotsky's ideas in educational settings tended towards a literal view of learning as internalisation of this interchange between child and adult, but

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more sophisticated interpretations began to emerge in later research that attended to cultural practices and institutional contexts, and the role of personal histories, beliefs and values in shaping teaching and learning interactions. One example of how later researchers extended Vygotsky's original conceptualisation of the ZPD is provided by Valsiner's (1997) zone theory of child development, which introduced two additional "zones" to incorporate the social setting and the goals and actions of participants. Valsiner's work has been used in mathematics education to study opportunities to learn experienced by school students and teachers (e.g., Blanton, Westbrook, & Carter, 2005; Hussain, Monaghan, & Threlfall, 2009).

Vygotsky was also one of several theorists who influenced the development of a practice perspective within sociocultural research, such as the concept of situated learning in a community of practice (Lave & Wenger, 1991). Although this concept arose from studying informal learning in apprenticeship and other out-of-school contexts, community of practice models have been fruitfully applied in mathematics education research focused on school classrooms and teacher professional learning (e.g., Graven, 2004).

The purpose of this paper is to consider how the two lines of sociocultural inquiry identified above, one based on Valsiner's (1997) zone theory and the other informed by Wenger's (1998) ideas about communities of practice, can help build more integrated theories for understanding and creating opportunities to learn in mathematics education. The first part of the paper outlines some research results from studies that applied each of these perspectives to interpret students' and teachers' learning. The second part extends each perspective to new research domains and other learners. Zone theory is proposed as a framework for studying the learning and development of mathematics teacher educator-researchers, and a community of practice perspective is suggested as a means of examining learning through "boundary encounters" between communities of mathematics educators and mathematicians. The paper concludes with some reflections on this proposed research agenda.

A COMMUNITY OF PRACTICE INTERPRETATION OF STUDENTS' AND TEACHERS' LEARNING IN MATHEMATICS EDUCATION

Creating Learning Opportunities for Students

My doctoral research, carried out in the mid-1990s, was motivated by questions about what specific actions a teacher might take to create a culture of mathematical inquiry in a secondary school mathematics classroom (Goos, 2004). This seemed to me to be an important question at a time when curriculum reforms in my country and elsewhere were placing increased emphasis on mathematical reasoning, problem solving, and communication (e.g., Australian Education Council, 1991; National Council of Teachers of Mathematics, 1989). I was attracted to sociocultural themes, evident in research that demonstrated a clear shift away from viewing mathematics learning as acquisition towards understanding learning as participation in the discursive and cultural practices of a community (Sfard, 1998). I used the concept of a community of inquiry to help me understand how one particular teacher structured learning activities and social interactions to develop his students' mathematical thinking. This investigation, carried out over two school years, focused on the detailed practices through which so-called reform approaches were enacted in classrooms.

From analysis of my classroom observation field notes and video-recordings as well as interviews with the teacher and students, I developed a set of five statements that reflected the teacher's assumptions about mathematics teaching and learning:

1. Mathematical thinking is an act of sense-making, and rests on the processes of specialising, generalising, conjecturing and convincing;
2. The processes of mathematical inquiry are accompanied by habits of individual reflection and self-monitoring;
3. Mathematical thinking develops through teacher scaffolding of the processes of inquiry;
4. Mathematical thinking can be generated and tested by students through participation in equal-status peer partnerships;
5. Interweaving of familiar and formal knowledge helps students to adopt the conventions of mathematical communication.

I justified each of these statements with evidence from the data corpus, comprising teacher actions underpinned by each assumption and student actions in appropriating the teacher's mathematical attitudes and pedagogical expectations. Together, the assumptions and actions represented a synthesis of evidence from the study as a whole to show how the teacher created a culture of mathematical inquiry. Although it is difficult to illustrate this evidence in the space available, two brief examples might give some of the flavour of how the teacher's assumptions about sense-making were enacted.

The first example comes from a sequence of two lessons early in the first year of the study when the teacher placed explicit emphasis on the processes of mathematical inquiry. The aim of the lessons was to have the students discover for themselves the algorithm for finding the inverse of a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. The teacher first chose a matrix A with a determinant of 1 and asked the students to find the inverse A^{-1} by using their existing knowledge of simultaneous equations to solve the matrix equation $AA^{-1} = I$. He then elicited students' conjectures about the general form of the inverse matrix, based on the specific case they had examined. Since the nature of the example ensured that students would offer $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ as the inverse, the teacher was able to provide a realistic context for students to test this initial conjecture. A counter-example, whose inverse was found to have the form $n\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, allowed the students to find a formula for n, which only then was labelled by the teacher as the determinant. In this example, the teacher modelled mathematical thinking by presenting a specific problem for students to work on, and eliciting a series of conjectures that had to be tested by students in order to arrive at a valid generalisation.

One of the most convincing signs that students had appropriated the teacher's view of mathematics as sense-making came in a lesson recorded towards the end of the first year. Dylan, a student who had previously been satisfied with knowing rules without reasons, was struggling with a task that asked students to prove that there is a limit to the area of a Koch snowflake curve. The following dialogue occurred after Dylan had spent several minutes with his hand raised hoping to seek the teacher's assistance.

Dylan: (Plaintively) I can't keep going! I want to know why!

Alex: (Looks up, both laugh) Have you got my disease?

Dylan: Yeah!

Alex: (Sounding surprised) Dylan, wanting to know why!

Dylan: Me wanting to know why is a first, but I just want to. It's a proof – you need to understand it. (Alex resumes work, Dylan still has hand raised.)

Creating Learning Opportunities for Teachers

As satisfying as this study proved to be in identifying how the teacher created learning opportunities for his students, it still left me feeling unconvinced that I understood what made this classroom a *community of practice*. For that, I had to turn to Wenger's (1998) social theory of learning. My research interests had shifted towards teacher education, and researchers were starting to invoke the notion of community as a context for teachers' learning (e.g., Graven, 2004). Wenger used community of practice as a point of entry into a broader conceptual framework in which learning was conceived as participating "in the *practices* of social communities and constructing *identities* in relation to these communities" (p. 4, original emphasis). Anne Bennison and I found these ideas useful in our research on the professional socialisation of beginning teachers. One of our research questions asked how communities of practice are formed in a pre-service teacher education program and sustained after graduation and entry into the profession (Goos & Bennison, 2008). This research was prompted, in part, by the unanticipated ways in which our own pre-service students used the course bulletin board as an online space for professional discussions during and after their university program.

We established a Yahoo Groups bulletin board as an alternative to the "official" university learning management system (WebCT), with the aim of creating a mathematics-specific space, one that was not open to other prospective teachers in the secondary pre-service program, and where contributions were voluntary and not graded for assessment purposes as was the case in other pre-service courses. We decided not to moderate these discussions: our participation was limited to modelling and encouraging professional dialogue about issues raised by pre-service teachers during face-to-face classes and practicum sessions. Our approach was consistent with Wenger's (1998) argument that a community cannot be fully designed, even though, as teacher educators in a university setting, we had explicit educational goals and were held accountable for achieving them. Features of the Yahoo Groups bulletin board and the pre-service program afforded unexpected interactions between pre-service and beginning teachers. First, unlike WebCT, the Yahoo Groups site remained accessible to participants after they finished the course. Second, the pre-service program was a four-semester postgraduate degree offered in intensive mode over eighteen months, with a six-month overlap between successive cohorts. We observed that graduates from our mathematics methods course voluntarily continued their professional discussions on the course bulletin board for more than a year after they left the university. Throughout this time they used the bulletin board to engage with the next cohort of pre-service teachers enrolled in the course, by responding to questions and offering advice on teaching strategies for different mathematical topics. They additionally established a separate Yahoo Groups bulletin board for their exclusive use to carry on their own discussions in a different space, and they invited us, the teacher educators, to become members of this community. We were surprised by these developments and curious about how our students seemed to be using both bulletin boards to develop a communal identity as beginning teachers of mathematics.

A significant aspect of the study was our examination of the assumption that a "virtual" community of practice will create opportunities for teachers to learn. In teacher education research, this is a premise that is not always tested to discover whether such a community really exists or what it is actually achieving. Using Wenger's three dimensions of practice – mutual engagement, joint enterprise, and shared repertoire – we analysed almost two years of messages posted to the Yahoo Groups bulletin boards to characterise the activities of the community and trace its emergent structure. Analysis of the frequency and content of messages found evidence that pre-service and beginning teachers increasingly took the initiative in engaging with each other and expanding the community through generational encounters between "old-timers" and "newcomers", in defining what was important to them in the joint enterprise of becoming a teacher of secondary school mathematics, and in constructing a repertoire of participation structures and routines for making sense of their experiences. In this sense, then, their learning was characterised by increasing participation in the practice of "becoming a teacher" (see Goos & Bennison, 2008, for a full analysis of these interactions.)

It is possible to claim that through establishing the Yahoo Groups bulletin board we created emergent, rather than pre-determined, opportunities for these pre-service teachers to learn in mathematics education, in keeping with Wenger's perspective on learning as an informal and tacit process. However, community of practice models are perhaps not well suited to analysing the role of a teacher educator who deliberately sets out to ensure that certain types of learning occur. Encouraged by an earlier experiment with using Valsiner's zone theory in teacher education (Goos, Evans, & Galbraith, 1994), I began to apply this theory more systematically to understand relationships between learning, teaching, and the contexts in which teachers develop their pedagogical identities.

A ZONE THEORY INTERPRETATION OF STUDENTS' AND TEACHERS' LEARNING IN MATHEMATICS EDUCATION

Valsiner viewed the zone of proximal development (ZPD) as a set of possibilities for development that are in the process of becoming realised as individuals negotiate their relationship with the learning environment and the people in it. He extended Vygotsky's original conceptualisation of the ZPD by proposing the existence of two additional zones, the zone of free movement (ZFM) and zone of promoted action (ZPA). The ZFM structures an individual's access to different areas of the environment, the availability of different objects within an accessible area, and the ways the individual is permitted or enabled to act with accessible objects in accessible areas. The ZPA comprises activities, objects, or areas in the environment in respect of which the individual's actions are promoted. The ZPA can include areas that are currently outside the ZFM as well as those that are inside; thus the actions being suggested, while possible, may seem "forbidden" at the present time. The ZFM and ZPA are dynamic and inter-related, forming a ZFM/ZPA complex that is constantly being re-organised by adults in interactions with children. However, a key claim of Valsiner's theory is that children are active participants in their own development: they can change the environment in order to achieve their emerging goals. Thus the process of development is neither completely random nor fully determined; instead, it is directed, or "canalised", along a set of possible pathways jointly negotiated by the child in interaction with the environment and other more mature people.

Although Valsiner's (1997) theory is intended to explain child development, he noted that the ZFM/ZPA complex is also observable in the context of education, both formal and informal. He provided classroom examples to show how teachers can set up narrow or expansive ZFM/ZPA systems, with different implications for the choices allowed to students in completing set tasks. Two different approaches to zone theory are evident in the mathematics education research literature, one of which defines the zones from the perspective of the teacher-as-teacher and the other from the perspective of the teacher-as-learner.

Zone Theory Approach #1: Focus on Teacher-as-Teacher

A teacher's instructional choices about what to promote and what to allow in the classroom establish a ZFM/ZPA complex that characterises the learning opportunities experienced by students. This approach was taken by Blanton, Westbrook, and Carter (2005), who compared the ZFM/ZPA complexes organised by three mathematics and science teachers in their respective classrooms as a means of revealing these teachers' understanding of student-centred inquiry. They found that two of the teachers created the appearance of promoting discussion and reasoning when their teaching actions did not allow students these experiences. Approach #1 is thus useful for explaining apparent contradictions between the types of learning that teachers claim to promote and the learning environment they actually allow students to experience.

Zone Theory Approach #2: Focus on Teacher-as-Learner

Valsiner (1997) argued that zone theory is applicable to any human developmental phenomena where the environment is structurally organised, and thus it seems reasonable to extend the theory to the study of *teacher* learning and development in structured educational environments. Hussain, Monaghan, and Threlfall (2009) proposed a partially developed extension of Valsiner' theory in a study of teachers who participated in a professional development program that introduced them to collaborative learning approaches in primary school mathematics. The analysis initially focused on how the teachers created new ZFM/ZPAs for their students (teacher-as-teacher), but the intervention process led to a parallel transformation in the teachers' ZFMs as they restructured their relationships with students and other mathematical objects in the classroom (possible extension to teacher-as-learner).

My own approach to the use of zone theory goes even further in its explicit focus on the teacher-as-learner. Re-interpreting the zones from this perspective, the teacher's zone of proximal development becomes a set of possibilities for development of new knowledge, beliefs, goals and practices created by the teacher's interaction with the environment, the people in it, and the resources it offers. The zone of free movement structures the teacher's environment, or professional context; so that elements of the ZFM could include perceptions of students (behaviour, motivation, abilities, socio-economic background), access to resources and teaching materials, curriculum and assessment requirements, and organisational structures and cultures of the school. While the zone of free movement suggests which teaching actions are *permitted*, the zone of promoted action can be interpreted as activities offered via teacher education programs, formal professional development, or informal interaction with colleagues that *promote* certain teaching approaches. It is worth noting here that pre-service teachers develop under the influence of two distinct ZFM/ZPAs that do not necessarily coincide – one provided by their university program, and the other by their supervising teacher during the practicum.

In previous studies I have found Approach #2 helpful for analysing alignments and tensions between teachers' knowledge and beliefs, their professional contexts, and the professional learning opportunities available to them in order to understand why they might embrace or reject teaching approaches promoted by teacher educators (Goos, 2005, 2009). One part of this research program has been investigating factors that influence how beginning teachers who have graduated from a technology-rich pre-service program integrate digital technologies into their practice. Table 1 maps onto each of Valsiner's zones a range of factors known to influence teachers' use of technology in mathematics classrooms. Note that this mapping is not intended to define the zones with such precision as to contradict Valsiner's view of "bounded indeterminacy" in relation to developmental trajectories, since pathways of development are *constrained* rather than *determined*. Instead, the adaptation of zone theory provides a way of studying the formation of ZFM/ZPA complexes that support or hinder teachers' learning.

Valsiner's Zones	Factors influencing teachers' use of digital technologies
Zone of proximal development (Possibilities for developing new teacher knowledge, beliefs, goals, practices)	Mathematical knowledge Pedagogical content knowledge Skill/experience in working with technology General pedagogical beliefs
Zone of free movement (Structures teachers' access to different areas of the environment, availability of different objects within an accessible area, ways the teacher is permitted or enabled to act with accessible objects in accessible areas)	Perceptions of students (e.g., motivation, behaviour, socio-economic status, abilities) Access to resources (time, hardware, software, teaching materials) Technical support Curriculum & assessment requirements Organisational structures & cultures
Zone of promoted action (Activities, objects, or areas in the environment in respect of which the teacher's actions are promoted)	Pre-service teacher education Professional development Informal interaction with teaching colleagues

Table 1: Factors affecting teachers' use of technology.

A Case Study of Teacher-as-Learner

Consider the case of Adam, a beginning teacher who participated in the research referred to above (more fully discussed in Goos, 2005). Adam completed his practice teaching sessions at a school that had been designated a Centre of Excellence in mathematics and technology, with government funding to resource all classrooms in the mathematics building with computers, Internet access, data projectors, graphics calculators and data loggers. New mathematics syllabuses additionally mandated the use of computers or graphics calculators in teaching and assessment programs. We could say that this environment offered a zone of free movement enabling integration of digital technologies into mathematics teaching. Adam's supervising teacher, who was the Director of the Centre of Excellence, also encouraged him to use any form of technology that was available for promoting students' mathematics learning. The practicum environment therefore organised a ZFM/ZPA complex that directed Adam's development along a pathway towards technology integration that aligned with his experience as a student in my pre-service course.

After graduation, Adam was employed in the same school but experienced a different set of constraints. Because not all classes could be scheduled in the well-equipped mathematics building, Adam had to teach some of his lessons in other classrooms without computers, data projectors, or Internet access. Now that he was a full-time staff member of the school he discovered that many of the other mathematics teachers were sceptical about using technology. Some of these teachers accused Adam of not teaching in the "right" way. He, in turn, disagreed with their teaching approaches, which in his view betrayed negative perceptions about students:

> You do an example from a textbook, start at Question 1(a) and then off you go. And if you didn't get it – it's because you're dumb, it's not because I didn't explain it in a way that reached you.

Adam was now in a difficult situation that required him to defend his instructional decisions while negotiating professional relationships with other teachers, some of whom did not share his beliefs about teaching and learning. In these circumstances, technology-rich teaching seemed to be neither universally permitted (ZFM) nor consistently promoted (ZPA). Nevertheless, in his first year of full-time teaching Adam continued to expand his teaching repertoire with digital technologies, often preferring to work with graphics calculators as portable tools that could be used in any classroom. He said that he saw technology as a means of giving students access to tasks that build mathematical understanding, and in this he claimed to have been influenced by the university pre-service course and the teacher who had been his practicum supervisor.

It is not possible to explain Adam's appropriation of technology over this period of time by just "adding up" the positive and negative influences listed in Table 1. A zone theory analysis would argue that Adam was an active agent in his own development in two distinctive ways. First, he interpreted his technology-rich ZFM as affording his preferred teaching approach, despite subtle hindrances in the distribution of technology resources throughout the school. He also decided to pay attention only to those aspects of the mathematics department's ZPA that were consistent with teaching approaches promoted by the university pre-service course.

In his second year of teaching Adam was transferred to a school where there was even more limited access to computer laboratories and only one class set of graphics calculators. None of the mathematics teachers were interested in using technology, and they preferred the same kind of teacher-centred, textbook-oriented teaching approaches as some of his colleagues in his previous school. My role now, as a teacher educator-researcher, was to influence Adam's interpretation of the ZFM/ZPA complex to maintain his sense of personal agency. I encouraged him to view the single class set of graphics calculators as an opportunity he could exploit, simply because he was the only teacher who wanted to use them. I also supported him in increasing his involvement in the local mathematics teacher professional association where I hoped he would find a ZPA external to the school that would nurture his potential for further development.

Vince Geiger and I used this zone framework to analyse research on teacher change conducted by other mathematics educators (Goos & Geiger, 2010), thus illustrating its broad applicability across research contexts. We also found ourselves asking what is learned by the teacher educator-researchers who work with mathematics teachers. Working with Laurinda Brown, Olive Chapman, Jarmila Novotna, and colleagues in recent PME Discussion Groups and Working Sessions (2010, 2011, 2012), I have begun to explore this question of how mathematics teacher educators learn and develop.

This is the point where I want to move from past experience to propose new challenges for researching opportunities to learn in mathematics education.

THE LEARNING AND DEVELOPMENT OF MATHEMATICS TEACHER EDUCATOR-RESEARCHERS

There are few published studies of the development of mathematics teacher educators (see Clark, Kotsopoulos, & Morselli, 2009, for one example). Even (2008) noted that neglect of the education of mathematics teacher educators, by comparison to that of mathematics teachers, mirrors earlier research in mathematics education that focused more on students' learning than on teachers' learning.

Theoretical approaches found in existing studies of teacher educator development largely draw on the notion of reflective practice. In mathematics education, Tzur (2001) and Krainer (2008) provided reflective self-studies of their own developmental trajectories, tracing their experiences as mathematics learners, teachers, teacher educators, and mentors of fellow mathematics teacher educators to identify critical events and experiences that advanced their professional knowledge and practice. Reflective practice is claimed to lead to greater awareness of the personal theories motivating one's practice. However, because sociocultural theories take into account the settings in which practice develops, this perspective may have more to offer to those who wish to study the complexity of social practices and situations that engender learning in teacher educators.

Zone Theory Approach #3: Focus on Teacher-Educator-as-Learner

The theoretical approach I propose for studying opportunities to learn in mathematics teacher education extends the zone framework outlined in the previous section. There, I showed how it could be applied in two connected layers: (1) the teacher-as-teacher (TasT in Figure 2) creating classroom ZFM/ZPAs that structure student learning; and (2) the teacher-as-learner (TasL in Figure 2) negotiating the ZFM/ZPAs that structure their own professional learning. At the latter layer the teacher-educator-as-teacher comes into the picture, organising aspects of teachers' ZPAs. *What if we imagined a third layer, with teacher-educator-as-learner?* (TEasL in Figure 2). Now a new set of questions arises. How do our professional contexts as teacher educators structure our interactions with prospective and practising teachers? What activities and areas of the professional environment do we access that promote certain approaches to educating teachers? How do the ZFM/ZPA complexes thereby created canalise our learning and development as mathematics teacher educator-researchers, and how do we negotiate these pathways for development throughout our careers?

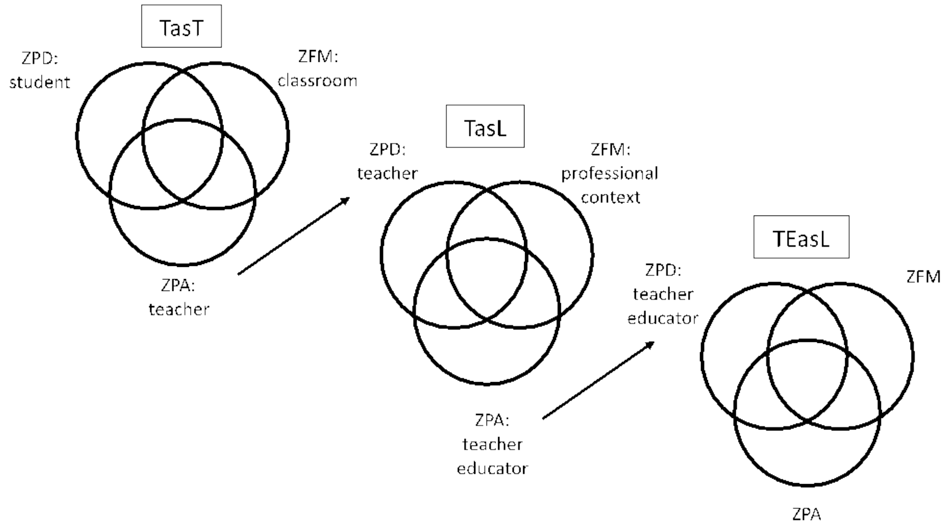

Figure 2. Three layers of application of zone theory.

Analysis of a zone of free movement for mathematics teacher educator-researchers might consider:

- characteristics of our teacher education students, such as their mathematical knowledge and their beliefs about mathematics teaching and learning;
- structural characteristics of teacher education programs, such as class sizes, modes of delivery, and the balance between courses focusing on general pedagogy, mathematics content, and mathematics teaching methods;
- the extent to which curriculum and assessment requirements are influenced by professional accreditation authorities;
- organisational structures that limit the time available for teaching of methods courses;
- challenges in finding suitable practicum placements for prospective teachers;
- university cultures that values research above teaching.

Regarding the zone of promoted action, particular approaches to teacher education are promoted through both our research and our practice. Some researchers have represented mathematics teacher educators' learning as a lifelong process of growth through *practice*. For example, Zaslavsky and Leikin (2004) presented a three-layered hierarchical model of learning, where each successive layer contains the knowledge of mathematics learners, mathematics teachers, and mathematics teacher educators respectively. A recursive relationship exists between the layers as each form of knowledge operates and reflects on knowledge in the layer beneath. There is also space for a fourth layer representing the knowledge of educators of mathematics teacher educators. Tzur's (2001) self-analysis of his own growth as a teacher educators is an example of how an individual moves through these four layers of learning mathematics, learning to teach mathematics, learning to teach mathematics teachers, and learning to mentor fellow mathematics teacher educators.

Mathematics teacher educators are also well positioned to learn from their *research* with teachers, even though this learning is often left unacknowledged and unarticulated (Jaworski, 2001). I recently experienced a striking example of the potential for this type of research to stimulate my own learning. With colleagues from six other Australian universities I am working on a project that aims to provide an evidence base for improving university-based mathematics teacher education. One of our assumptions is that developing pedagogical content knowledge (PCK) is central to teacher education courses, even though we accept that this concept is not so easy to define and even harder to measure. Notwithstanding our reservations about using surveys to investigate pre-service teachers' PCK, we set about designing items that for pragmatic reasons could be administered online and scored automatically. As a project team we had lengthy debates and sometimes heated arguments about what aspects of PCK to incorporate into survey items, what kind of choices to include as possible answers, and which answers were "better" than others. These discussions not only advanced our own understanding of PCK, but they also caused us to question the different emphases we gave to aspects of PCK in our respective teacher education courses (Chick, 2011). The nature of "PCK for mathematics teacher educators" is something that we intend to investigate further.

OPPORTUNITIES TO LEARN ACROSS DISCIPLINARY BOUNDARIES IN MATHEMATICS EDUCATION

It is generally accepted that the preparation of prospective teachers of mathematics needs to include development of mathematics content knowledge as well as pedagogical content knowledge. The Teacher Education and Development Study in Mathematics (TEDS-M), an international comparative study of the competencies of mathematics teachers in sixteen countries at the end of their training, collected outcome measures for mathematics content knowledge and pedagogical content knowledge for pre-service primary teachers as well as information about opportunities to learn – defined in terms of the content and teaching methods experienced during teacher education. Knowledge outcomes differed significantly between participating countries and between teacher education programs within countries, with opportunities to learn within these programs found to be highly relevant to development of these two types of knowledge for teaching (Blömeke, Suhl, Kaiser, & Döhrmann, 2012).

One question that arises from such studies is – who are the teacher educators? Some answers were provided by a survey of a sample of participants in the 15th conference of the International Commission on Mathematical Instruction (ICMI-15), which focused on the professional education and development of teachers of mathematics. Amongst the 21 countries and country-regions included in the sample, mathematicians tended to teach mathematics content courses while mathematics educators taught the mathematics pedagogy courses (Tatto, Lerman, & Novotna, 2010). While there may be questions about who is better placed to help prospective teachers acquire the knowledge they need for teaching mathematics, it has been argued that both mathematicians and mathematics educators have an important role to play (Hodgson, 2001). In my country, however, there have been few instances of productive collaboration between mathematics educators and university mathematicians in the design and delivery of pre-service mathematics teacher education programs. I suspect that Australia is not unique in this regard. These observations, coupled with recent experience of working with mathematicians on teaching-related projects, lead me to wonder about opportunities to learn across disciplinary boundaries in mathematics education. How might such opportunities be recognised or created, theorised, and studied? To sketch out a possible answer, I return to Wenger's (1998) work on communities of practice.

Boundary Encounters Between Communities of Practice

Wenger (1998) describes the three defining characteristics of communities of practice as mutual engagement of participants, negotiation of a joint enterprise, and development of a shared repertoire of resources for creating meaning. Because communities of practice evolve over time they also have mechanisms for maintenance and inclusion of new members. Based on this description, one can accept that mathematicians, mathematics teachers, and mathematics teacher educator-researchers would claim membership of distinct, but related, communities of professional practice. Although communities of practice have "insiders" and "outsiders", they are not completely isolated from other practices or from the rest of the world. There are various ways in which communities may be connected across the boundaries that define them.

Wenger (1998) writes of *boundary encounters* as discrete events that give people a sense of how meaning is negotiated within another practice. The most fleeting of these is the one-on-one conversation between individuals from two communities to help advance the boundary relationship. For example, a mathematics teacher educator might telephone a mathematics teacher who is supervising the practicum experience of one of her pre-service students to discuss problems that the student is encountering at school. A more enriching instance of the boundary encounter involves immersion in another practice through a site visit. For example, a mathematician might visit a school to speak to students and teachers about careers in mathematics. However, both of these cases involve only one-way connections between different practices. A two-way connection can be established when delegations comprising several participants from each community are involved in an encounter. Wenger suggests that if "a boundary encounter – especially of the delegation variety – becomes established and provides an ongoing forum for mutual engagement, then a practice is likely to start emerging" (p. 114). Such *boundary practices* then become a longer term way of connecting communities in order to coordinate perspectives and resolve problems. While boundary practices might evolve spontaneously, they can also be facilitated by *brokering*. Wenger (1998) notes that the job of brokering is complex because it involves translating, coordinating, and aligning the perspectives of different communities of practice. Most importantly, it requires the ability "to cause learning by introducing into a practice elements of another" (p. 109).

Several sites offer potential for productive boundary practices involving two-way connections between communities of mathematics educators, mathematicians, and mathematics teachers. These include the pre-service preparation of teachers, the transition from school to university mathematics, and the development of school mathematics curricula. A research agenda informed by a community of practice perspective might aim to develop a theory of boundary relations between these communities and to examine the processes of learning through exchange of expertise across disciplinary boundaries. If the research seeks to create, rather than only to understand, such opportunities to learn, then additional aims might involve designing, enacting, and analysing different types of boundary practices and investigating the role of a broker in connecting communities.

CONCLUDING COMMENTS

This conference's theme, "Opportunities to learn in mathematics education", is suitably open to different interpretations. I have reframed this theme as two questions: *Who* has opportunities to learn? How are these opportunities *created*? By following two lines of sociocultural inquiry, drawing respectively on zone theory and community of practice concepts, I traced out a past and possible future research trajectory that considers these questions. Two things are important to me in this future research agenda. First, we need to know more about the professional formation of mathematics teacher educator-researchers. Calls for improvements to mathematics education are implicitly based on the assumption that well prepared mathematics teacher educators are available who can foster change in teachers' practices (Zaslavsky & Leikin, 2004). The ethical, social, political and intellectual challenges inherent in bringing about this type of change are well known. However, much less is known about the professional preparation of the mathematics educators who undertake these tasks, or about how they continue to learn throughout their careers. I would also argue that improvements to mathematics education – involving, for example, curriculum development, teacher preparation, and supporting student learning of mathematics as they transition from school to university – would benefit from productive collaboration between the professional communities that have an interest in such issues. Creating opportunities to learn across interdisciplinary boundaries may lead to new understanding of how to integrate the mathematical and pedagogical expertise of community members to enrich mathematics education.

A second notable aspect of my proposed research agenda is a desire to synthesise ideas to create integrated theories about mathematics learning and teaching. Bishop (2010) observed that, as a research community, we are strong on analysis but weak on synthesis, and he called for more integrated research development. The application of zone theory and community of practice concepts to learners other than students and teachers is a small step in this direction.

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PLENARY PANEL

- Lulu Healy, Converor
- Richard Barwell
- Karin Brodie
- K. Subramaniam
- Jean-Baptiste Lagrange

INTRODUCTION TO THE PME PLENARY PANEL 'OPPORTUNITIES TO LEARN IN MATHEMATICS'

Richard Barwell, Karin Brodie, Lulu Healy, Jean-baptiste Lagrange, K. Subramaniam

'Opportunities to Learn in Mathematics Education' has been chosen as the theme of PME 36. In the research panel, this theme will be addressed by five researchers, each of whom brings his or her own view on the factors which shape learning opportunities. In this introduction, the theme, the position papers that the panellists have prepared and the proposed format of the panel are briefly presented.

THE THEME

The plenary panel of this conference will address the theme 'Opportunities to Learn in Mathematics Education'. According to the PME 36 conference organizers, this theme was chosen to meet the prospect that education should be developed and promoted in a more diversified dimension. This alignment of diversity with opportunity signals an extremely broad range of issues that can be viewed through a variety of lenses. Amongst these concerns are questions of equity and access and, in particular, how to avoid the marginalisation of students whose physical, racial, ethnic, linguistic and social identities have resulted in them experiencing what Bishop and Forgasz (2007) have termed "conflicts with mainstream mathematics" (p.1146). Another set of questions that might be raised relates to the roles of those charged with "delivering" opportunities for learning and, indeed, the opportunities they have to develop and transform their practices, so that they are appropriate given the changing profiles of the contexts in which they occur. Contexts themselves become central, with diversity manifested in various forms, including not only differences between curriculum and assessment regimes and the didactic and pedagogic norms within the educational institutions in which teaching occurs, but also in the research cultures through which these might be investigated, as well as in the out-of-school practices within the multiple sites in which mathematics education takes place. Furthermore, since learning is shaped by the language and technologies through which mathematics is expressed, the tensions involved in coordinating the multiple semiotic resources which compose these different learning contexts represent yet another vital area for exploration.

PREPARING FOR THE PANEL

The panel members were invited to structure their contributions around three questions:

- Research has shown that many of the conditions that characterize the context in which learning occurs contribute to mediating the opportunities that different groups of learners have to engage with mathematics. How we look at these conditions deeply influences the ways we think of learning opportunities. Which of these conditions do you privilege in your work?

- What tools (theoretical frameworks and research methods) do you adopt to explore the role of these conditions in mediating access to mathematical knowledge and to focus on the questions of empowerment or disempowerment?
- Can widening our views on mediation help to identify previously hidden opportunities for learning?

The five contributions which resulted from each panel member's considerations on the theme, broadly structured by these questions, are presented in sequence. These papers reflect how the contexts in which we work affect the ways in which we attempt to negotiate between political forces and issues of power, cultural practices and social norms, as well as the individual differences that mediate opportunities to learn mathematics. The contexts of the papers are varied: they involve learning sites in Brazil, South Africa, India, Canada, Greece and France; they focus on a diversity of learners, including students with disabilities, students who learn mathematics in a second language and students who participate in work environments alongside schooling, with attention given to the learning opportunities for teachers and researchers as well as to different student groups; and the contexts also vary in relation to the resources incorporated within them, with a diversity in terms of the bodily resources, the languages and the material and digital artefacts through which mathematics may be accessed and expressed.

As a second step in preparing for the panel, we began to explore the relationships between the positions expressed in the papers. One commonality is that all five of them are concerned with the learning of school mathematics. Another is that, while each panel member adopts a different theoretical framework, they all share in common a notion of learning as a social practice. A third issue which receives some attention relates to how tensions between what might be termed as the dominant voice(s) and the voices of the more marginalised affect opportunities for learning. However, while certain considerations emerged in common, the approaches to them was, not surprisingly, far from uniform. In relation to school mathematics, the papers bring alternative views on the question as to what mathematics is privileged in different school curricula and on how questions related to the epistemology of the content included impact upon different learners' participation in the resulting practices. We might therefore ask "opportunities to learn what mathematics?" or, "what mathematics is worth learning?" In terms of the tension which characterises the third common issue, a second new question can be posed, in which we ask "how can we negotiate the dual aim of empowering learners to participate in the practices of dominant cultures, whilst valuing forms of practice which at times appear to be counter to mainstream trends?"

THE STRUCTURE OF THE PANEL SESSION

The questions mentioned above will inform the next step in preparing for the panel and will also have a role in determining the structure of the panel session. Our aim is that the session will permit a real exchange in views and perspectives both amongst the panel members and with the rest of the PME conference participants. We want the panel to be an opportunity for our own learning. To this end, each panel member will make a short presentation aimed at highlighting their views on the questions posed thus far. We will not attempt to summarise our papers – they are already available here as written texts. Rather, we will attempt to synthesis our individual views on opportunities to learn mathematics in the light of the collective process of preparing for the panel. These brief presentations will be interspersed by reactions and comments from other panel members. The panel will then open for questions from the conference participants and we hope that the dialogue established in this discussion will provide yet another opportunity for learning.

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MATHEMATICAL OPPORTUNITIES FOR STUDENTS WITH DISABILITIES

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As in many other countries throughout the world, the trend to include learners with special educational needs within the mainstream education system began in earnest in Brazil during the latter part of the 1990s. As attention to the rights of this group of learners has increased, so too has their participation in the Brazilian educational system. In 1998, there were a total of 337,326 students with special educational needs enrolled in Brazilian schools. According to data from the 2010 school census, by 2010, this number had more than doubled to 702,605. Even more strikingly, whereas, in 1998, only 48,923 of these students were included in mainstream classes in regular schools, in 2010, this had grown almost 10-fold to 484,332.

In the light of this context, how to ensure that these learners have opportunities to a mathematics education with respects the particular ways in which they experience and participate in the world has become a present and pressing question. Our research [1], began as a response to the pragmatics demands of mathematics teachers who wanted to better understand how to include blinds students, deaf students and students with physical and/or cognitive disabilities in their classrooms. These teachers expressed many concerns. Working with students with disabilities was not something that had been addressed in either initial or in-service education courses, leaving them feeling ill-prepared and uninformed. They reported having little, or no, access to specifically adapted pedagogical resources and pointed to difficulties in locating textbooks in Braille or materials directed at students who use the Brazilian sign language, LIBRAS, Another difficulty that the teachers experienced was in finding literature from which they might learn of successful strategies of inclusion in mathematics classrooms in Brazil. On top of all this, there were also deep concerns about what inclusion actually means in the context of mathematics education. One teacher put his problem thus "*When I was faced for the first time with a blind student in my classroom, I thought I am not a good enough teacher to deal with this situation. I already have problems with students who see. How can I teach anything to someone who cannot.*"

As our work has progressed, so too has our contact with teachers of mathematics within specialized as well as mainstream schools. They also express similar worries about the lack of materials and research specifically addressing learning processes of the students with whom they work. Irrespective of the kind of schools in which mathematics is being studied, it seems that the development of a more inclusive school

[1] Throughout the paper, I will draw on research carried out by myself and other members of the research programme *Rumo à Educação Matemática Inclusive*, PROESP-CAPES, No.). In particular, I would like to acknowledge the contributions of Solange Hassan Ahmad Ali Fernandes in all aspects the empirical work described in this paper.

mathematics depends on deepening our understandings of how mathematics practices and knowledge are mediated through different sensory channels. Perhaps by doing so, we might not only become better able to create learning opportunities for students with disabilities, we might also build more robust appreciations of the relationships between experience and mathematical cognition more generally. The rest of this paper outlines our attempts to contribute to these objectives.

AVOIDING DEFICIT MODELS OF DIFFERENCE

One of our first tasks was to seek a theoretical basis for our research. We were drawn to the socio-cultural perspective of Vygotsky and his colleagues for a number of inter-related reasons. First, in his work in the area of what at the time was called *Defectology,* Vygotsky (1997) warned against focusing on quantitative differences in achievements between those with and without particular physical or cognitive disabilities. Instead, he proposed a more qualitative approach which involves considering how and when the substitution of one tool by another may empower different mediational forms and hence engender different practices (Healy & Fernandes, 2011; Healy & Powell, in press). His approach stressed the potential for development of learners with disabilities, rather than positioning them as deficit in relation to some supposed "norm". In relation to empowering those without access to one or other sensory field to participate in social (cultural) activities, for Vygotsky the solution lies in seeking ways to substitute the traditional means of interacting with information and knowledge with another. For example, he suggested that the eye and speech are "instruments" to see and to think respectively, and that other instruments might be sought to substitute the function of sensory organs (Vygotsky, 1997). Vygotsky's writings suggest that he was attributing, at least implicitly, to organs of the body—more specifically, to the eye, to the ear and to the skin—the role of what he latter denominated psychological tools. And in this sense, as well as empowering their users to participate in otherwise inaccessible activities, substitute tools they can also be expected to restructure the activity in question:

> "...by being included in the process of behavior, the psychological tool alters the entire flow and structure of the mental functions. It does this by determining the structure of a new instrumental act, just as a technical tool alters the process of natural adaptation by determining the forms of labor operation". (Vygotsky 1981, p.137).

Vygostsky's stance suggests that to understand the mathematics learning of those with disabilities, we need to understand how the particular set of material, semiotic and sensory tools through which they attribute meaning to their activities motivate different forms of participation in mathematics. On its own, though, this will give us only one part of the picture. As the above quote stresses, tools do not only shape the meanings that become associated with particular activities, they also shape the activities themselves: the relationships between tools, activity and thinking are reciprocal. This would suggest that as well as recognising that there are different ways by which mathematical knowledge might be appropriated that depend, for example, on whether or not we have access to the visual feel, or on whether we speak with our mouths or with our hands, it is also necessary to recognise the mathematical knowledge itself might undergo some transformation. Moreover, accepting sensory apparatuses mediate mathematical activity adds force to the argument that cognition is embodied, that the way we think cannot be separated from the way we act and that both have their bases in our body, its physical capacities and its location in space and time (Barsalou, 2008; Gallese and Lakoff, 2005). Hence our theoretical framework combines socio-cultural concerns which initiated in the first half of the last century with more contemporary approaches from embodied. To illustrate very briefly how this developing framework is informing our search for a more inclusive school mathematics, the rest of the paper focuses upon the mathematical practices of one blind student as he used his hands rather than his eyes to see.

FEELING MATHEMATICS

In contrast to vision which is synthetic and global, touch permits a gradual analysis, from parts to the whole (Ochaita & Rosa, 1995). Our work is suggesting that this makes the activity of seeing with one's hands a different cognitive practice from seeing with one's eyes and that the properties of the 'seen' object that are privileged may not necessarily be the same in both cases. Lucas's investigations of activities related to the transformation reflection provide a case in point (more details can be found in Fernandes & Healy, 2007). Lucas became completely blind at the age of two. He had already completed High School within the mainstream school system when he participated in the research and he was familiar with a variety of geometrical objects and relations, although he told us he had never studied symmetry or geometrical transformations.

Figure 1: Lucas constructs an axis

He worked on a series of activities involving the transformation reflection during three sessions, each of about an hour and a half. This example is drawn from the second session, during which he was asked to construct the axis of reflection of a number of pairs of symmetrical segments (an example is shown in Figure 1).

After a number of such tasks, Lucas indicated that he had invented a general method by which he might construct an axis of reflection of any figures with axial symmetry. The researcher asked him to explain to her this method, in a way that she might enact the method on an imaginary geoboard displaying two symmetrical segments.

Lucas: Take as your base, one of the extremities of each of the segments.

Res: Any one?

Lucas: The two extremities on the same…sides

Lucas seemed to be aware that this description was not very precise and, as he explained which points he was referring to, he placed his hands as if they too were symmetrical around an imaginary vertical line in the middle of the board. He then traced the imagined symmetrical segments, stressing, with an extra pointing gesture at the end of the movement, the two the extremities, symmetrical in relation to the imaginary vertical axis as shown in Figures 2 and 3. His gestures suggest that as he mentally reenacts his previous activity, he re-evoked the same cognitive resources as in the initial the concrete doings. He then continued onto the next step.

Lucas: You centralize the axis of symmetry on the midpoint between one extremity and the other of each segment.

Res: And how do I find the midpoint?

Lucas: You could use a ruler, but it's simpler to count the pins between one of the segments and the other and localize the pin that will be the midpoint with the same distance between one segment and the other. Then fix the elastic and trace out a line always obeying the distance, for the two extremities, but also so that the other points on the segments keep the same distance from the axis.

Figure 2: Lucas places his hands ready to trace two segments

Figure 3: Lucas taps his two fingers to indicate symmetrical end points

A striking feature of Lucas's method is its dynamic nature. This dynamism is evident in the way he represents the imagined symmetrical segments as he tries to respond to the researcher's first question and in how he treats the axis of reflection as a dynamic trajectory, constrained so that the distance between it and any two symmetrical points belonging to the segments always have the same distance from the line being traced. Dynamic gestures in which blind students re-enact previous tactile explorations as they abstract mathematics relationships appear seem to be rather characteristic of their interactions with geometrical objects (other examples are available in Healy & Fernandes, 2011). Perhaps it is because of the way Lucas had moved his hands over the materials that he talks of line segments as both collections of points and as trajectories – a view which contrasts those usually expressed by sighted students, who tend, at least initially, to treat segments as whole objects (Laborde & Grenier, 1988; Healy, 2002). Moreover, whereas sighted students tend first to focus on the properties within particular objects, Lucas, in common with other blind students we have worked with, begins by looking for the relative positions of the geometrical points that constitute the objects in a mathematically structurable space. Piaget and Garcia (1989) define the first perspective as *intrafigural* and the second as *interfigural*, arguing that they represent the first two of three hierarchically organised epistemological phases through which mathematical ideas develop. That is, their view is that all learners necessarily pass through a stage of intrafigural analyses before reaching the interfigural stage. Our work with blind students suggests, however, that the perspective that comes to be adopted depends on the available means for mediating the ideas and that it would be a mistake to expect those who do not see with their eyes to necessarily follow the same learning trajectories as those who do.

There is one more point to be made before finishing this brief account. As we watched Lucas's explorations, we noticed that symmetrical hand movements were extremely frequent. Again, this form of exploring figures turned out not to be limited to Lucas, but characteristic of the other blind students who participated in the study. This was not something that had been anticipated in the design of the tasks—since they had been developed on the basis of research into sighted learners' understandings of symmetry and reflection (Kuchemann, 1988; Grenier & Laborde, 1988; Healy, 2002). This stresses how a tendency to design learning scenarios for the blind relying exclusively on what we know about the learning trajectories of sighted might not offer the best opportunities for mathematics learning. Moreover, by concentrating more specifically on the how they use their hands to conceive mathematical objects, we are beginning to recognize how very intimate the relationship between bodily groundings and mathematical abstractions is.

ENCULTURATION AND EMPOWERMENT

To end, it seems appropriate to return to our socio-cultural beginning. According to this perspective, mathematics learning can be defined as appropriating the artefacts and practices that historically and culturally represent the body of knowledge associated with mathematics. A danger with this definition is that it might be taken to imply that learning involves an exclusively one way-process of enculturation into the dominant culture (Gutiérrez, 2010). At its most extreme, this might imply that learning to succeed means learning to be like those idealized in the dominant culture. For the learners with whom we work, this would involve a denial of their very identity. Rather than ensuring opportunities for mathematics learning, if enculturation becomes imposition, then those already marginalised can be expected to be ever-increasingly so. The search for a more inclusive mathematics education hence requires that appropriation is not viewed as a one-way process. Rather, it can be seen as a kind of entanglement of perspectives on an activity, out of which emerge new forms of thinking about the objects in question for, at least, some of those involved (Healy & Powell, in press). The social and the individual are both fully present in this entanglement: the activities undertaken and the expressions associated with them being essentially social acts, mediated by *all* the means available to those interacting within the setting in question – not only the material resources and semiotic presentations, but also the bodily resources and ways of being associated with the multiple identities which the learners bring to the setting. Ensuring opportunities hence involves respecting and encouraging diversity in mathematical practices and avoiding the assumption that everyone will, or should, appropriate mathematics in the same way.

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HOW LANGUAGE SHAPES LEARNERS' OPPORTUNITIES TO LEARN

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Learning and teaching mathematics is necessarily a discursive process: it depends on human interaction using language and other forms of meaning making. All forms of mathematics education occur through discursive interaction of some form, whether in the standard classroom model of education, or in alternative forms, such as distance learning or online learning. The role of language in mathematics learning has, of course, been increasingly theorised in recent years, generally drawing on socio-cultural perspectives (e.g. Kieren et al., 2001). The *nature* of language has, however, not been problematized as much as it might. What *is* language? What is it like? And how does the nature of language shape the learning of mathematics?

An examination of these questions highlights, among other things, the way that language is filled with the tensions that arise between the diversity of human society and the uniformity that education so often attempts to impose. These tensions involve issues of social stratification, marginalisation, equity and the nature of mathematics itself. There is much evidence that such issues are widespread and unresolved in mathematics education. A variety of social factors, including social class, race and language background have been shown to influence mathematics learning, from broad measures of attainment, to the ways in which students respond to particular mathematics tasks (e.g. Cooper & Dunne, 2000; Secada, 1992; Zevenbergen, 2000). In my contribution to the panel, then, I discuss how the nature of language plays a role in students' opportunities to learn, referring, in particular, to Bakhtin's (1981) work. I illustrate these ideas, drawing on research in multilingual classrooms. In multilingual classrooms, the role of language becomes at once more problematic and more visible, but the issues that arise are of wider relevance.

LANGUAGE

While there are many theories of language, most 'common sense' understanding is closest to the perspective set out by Saussure (1974). From this perspective, a distinction can be made between the system of any given natural language (*langue*, in Saussure's terms) and the application of this system in actual moments of use (*parole*). This perspective is apparent in the dominant view of languages as discrete, distinct, describable and attached to a political entity, such as a nation state. Much formal linguistic theory (including Saussure's work), however, is based on analysis of dominant Western languages. In recent years, this perspective has been critiqued, particularly by post-colonialist linguists, who argue that languages are hybrid, continuous and difficult to describe in precise terms (Canagarajah, 2009; Makoni & Meinhof, 2004). In mathematics education, a broadly Saussurian view of language also prevails, reflected, for example, in the ideas that language is a tool used for thinking or that language is a resource, as well as the idea that language is the means by which students 'access' mathematics. Even where language is implicated in issues of equity, such as, for example, Zevenbergen's (2000) sense of learners needing to 'crack the code' of mathematics, language is seen in fairly neutral terms, as a kind of capital that can be acquired (or not) and so as a rather static *langue*.

An alternative perspective can be found in the work of Bakhtin, the Russian literary theorist. While his work is largely concerned with the nature of literary language, and in particular, the nature of novelistic discourse, it contains within it a valuable theory of language more generally. Bakhtin does not entirely reject Saussure's distinction between *langue* and *parole*, but his approach is much less structuralist and blurs the kind of clear categories discussed above. His perspective is complex and includes the ideas that the reality of language is in its use, rather than in its structure; and that any utterance is shaped by past experience, current context and future interpretation. In particular, for Bakhtin, there is a constant interplay between the possible meanings of any utterance and other utterances; meaning and structure are situated and relational.

Bakhtin's theory of language has an explicitly social, even political, dimension, encapsulated in his notion of *heteroglossia*, also translated as the social variety of speech types. Heteroglossia refers to the tremendous variety of language-in-use and highlights how this variety is related to social differences: "languages of social groups, 'professional' and 'generic' languages, languages of generations and so forth (Bakhtin, 1981, p. 272). Heteroglossia, therefore, includes the languages of mathematics, as well as the languages of social class, race, region and so on. For Bakhtin, the tremendous social diversity of heteroglossia is in opposition to what he calls 'unitary language,' by which he means the idealised view of languages as pure, correct and systematic: unitary language "gives expression to forces working toward concrete verbal and ideological unification and centralization, which develop in vital connection with the processes of socio-political and cultural centralization" (p. 271). This conceptualisation of language as comprising these two opposing tendencies gives Bakhtin's theory an explicitly political dimension. For Bakhtin, these two ideas of language – unitary language and heteroglossia – are in constant struggle, for which he uses the metaphor of centripetal and centrifugal forces:

> The centripetal forces include the political and institutional forces that try to impose one variety of code over others […] These are centripetal because they try to force speakers toward adopting a unified linguistic identity. The centrifugal forces instead push speakers away from a common center and toward differentiation. These are the forces that tend to be represented by the people (geographically, numerically, economically, and metaphorically) at the periphery of the social system. (Duranti, 1998, p. 76)

There is a sense, then, in which broader social tensions are played out in language, with 'standard' forms of language being less peripheral than 'non-standard' forms associated with more marginalised people. These tensions are apparent on a global scale with the widespread (though not universal) position of English as an elite language used in education, government, (PME conferences) and so on. A key aspect of Bakhtin's theory, however, is the idea that these forces play out in *every* utterance: everything we say is shaped by this tension, so that wider social forces are deeply connected to every moment of interaction. If this is the case, then the inherent tensions of language are present in interaction in mathematics classrooms or other sites of mathematics learning, and so shape students' opportunities to learn.

HETEROGLOSSIA IN MULTILINGUAL MATHEMATICS CLASSROOMS

The literature on multilingual mathematics classrooms (including work on bilingual and second language classrooms) provides many examples of the tensions that Bakhtin describes (see, for example, Adler, 2001; Setati, 2008; see Barwell, 2012, for a review). My current research into second language mathematics classrooms in Canada is revealing similar tensions. The aim of the project is to compare different second language settings. Canada has schooling in its two official languages, English and French. The project compares elementary school mathematics classes in four different settings: a mainstream Anglophone class with ESL learners; a sheltered class for aboriginal learners for whom English is a second language; a sheltered class for new immigrant learners of French as a second language; and a French immersion mathematics class. The social tensions of language described above are readily apparent in all the classes in the study. There is space for one brief example.

In the province of Québec, new immigrant children must attend school in French. If they do not speak French, they attend a *classe d'acceuil* for up to a year to learn enough French to join mainstream classes. In the Grade 5-6 class (10-12 years) that I visited, the students did some mathematics, although less than mainstream classes. I visited the class towards the end of the school year, by which time the students had acquired a degree of basic French. The teacher reported that the main aim of the class was to prepare the students for school life in Québec and to learn to speak and think in French. In mathematics, she focused on vocabulary. All mathematics texts used in class were in French and the teacher insisted on the use of French at all times. French, then, represents a unitary language in this class; the purpose of the *classe d'acceuil* derives from a centripetal force. The following extract comes from a geometry lesson. The students have worked in small groups to sort a set of different geometric forms into two groups and explain their reasoning. One group shares what they have done and the teacher calls on a student to explain why they think certain shapes have been put together (E17 is the student EN is the teacher; the transcript has been simplified; the translation, on the right, is my own):

E17	elle a mis le-la boule comme ça il a mis aussi les choses comme ça et les et les choses comme ça	*she put the-the ball like that he also put the things like that and the and the things like that*
EN	(…) tu parles	*(…) are you talking about*
E17	le rectangle (.) c’est c’est comme la même la même (.) la boule elle a comme la même (la même comme ça) (.) (la même comme ça)	*the rectangle (.) it’s it’s like the same the same (.) the ball has got the same (the same like that) (.) (the same like that)*
EN	la même grosseur?	*the same size?*
E17	oui non pas la grosseur la même (.) le c’est comme (.) c’est comme (…) c’est comme je sais pas comment le dire en français (.) c’est comme la même chose	*yes no not the size the same (.) (…) it’s like I don’t know how to say it in french (.) it’s like the same thing*
EN	le cercle et le rectangle?	*the circle and the rectangle*
E17	non les autres	*no the others*
EN	le cercle et [l’ovale	*the circle and the* [*oval*
E17	[oui	[*yes*
EN	ok et le rectangle lui	*ok and the rectangle*
E17	il est comme	*it’s like*
EN	pourquoi il est avec eux le rectangle?	*why is the rectangle with them?*
E17	parce que il y a des (.) il y a la chose (.) comme ça	*because there are the (.) there is a thing (.) like that*

It is apparent that E17 is a learner of French. He speaks with a Spanish accent, uses non-standard words (*ball* for *circle*) and relies heavily on deictic words (*this*, *that*, *like*) and non-specific nouns (*thing*), accompanied by various gestures. He states explicitly that he cannot find the words he needs in French. The other students in the class also speak with different accents, non-standard words, deictic words and gestures, each in their own way and some of the Spanish-speaking students converse in Spanish from time to time, though not in public. This is heteroglossia.

The tension between centripetal and centrifugal language forces is also apparent: E17 struggles to find the words to express his mathematical thinking in French. The option of expressing himself in Spanish is not made available. For Bakhtin, these various ways of talking are always in dialogue – with each other, with previous utterances and with future possible utterances. This can be seen in the way that E17’s explanation (which continues after this extract) is jointly produced with the teacher. The utterance “I don’t know how to say it in French” is in dialogue with the French-only policy and with other ways of talking that are available to E17. And the mathematical meaning that emerges through his exchange with teacher arises from and is shaped by the dialogue between these various ways of talking, such as the interaction between the words *ball* and *circle.*

CONCLUDING REMARKS

How do the forces of language shape opportunities to learn mathematics? First, in most classes there is a relative positioning of mathematics learning and language learning. In the example, above, mathematics is secondary to French: less time is devoted to mathematics. Second, in most mathematics classrooms, students must work with mathematical texts in one (unitary) language. Third, the particular heteroglossia of any given mathematics class influences the process of mathematical meaning-making. Bakhtin's theory of language highlights a couple of important aspects of this situation. It highlights how students' language is not a distinct variable (a problem to be fixed) and does not exist in isolation; rather, it is in dialogue with other languages in the class, including the teacher's and the language policy of the class. This dialogue is tension-filled. Bakhtin's theory highlights how these tensions are inherent in language. There is no way to eliminate them. It is, therefore, insufficient simply to teach students the language of mathematics, to help them break the code of school mathematics: this idea is based on a unitary language perspective. Moreover, mathematics is itself filled with the tensions of which Bakhtin writes: the heteroglossia of mathematics means that there is no single code to teach. This analysis creates a challenge for equity. If the tensions cannot be resolved, what can we do? I do not have a neat response and I am not convinced that there can be one. Any response, however, will involve finding ways to mediate the tensions that shape students' opportunities to learn mathematics. This, in turn, will entail working with heteroglossia and, in particular, the dialogicality that heteroglossia engenders, a dialogue of differences, to involve all students in creating mathematical meaning.

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OPPORTUNITIES FOR MATHEMATICS TEACHER LEARNING

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Among the many influences on opportunities to learn mathematics, teachers are undoubtedly one of the most important, if not *the* most important. Teachers mediate a range of other influences on learning and can enhance positive influences and shift negative ones. Teacher development programmes usually work with teachers' content knowledge and pedagogical content knowledge (Shulman, 1986) to support stronger knowledge and practice in classrooms. However teacher development programmes which work outside schools and are implemented through workshops or short courses, have generally been found to have little impact on helping teachers provide better opportunities for mathematics learning (Borko, 2004; Stein, Smith, & Silver, 1999).

In this panel presentation, I draw on data from a teacher development programme that works with professional learning communities in and across [1] schools in South Africa, where teachers focus on learner errors as opportunities for learning, both for themselves and for learners (Brodie, 2011; Brodie & Shalem, 2011)[2]. Learner errors are seen as reasoned and reasonable (Ball & Bass, 2003), and as entry points into both teachers' and learners' mathematical knowledge and practice. In this paper I sketch some of the issues that I will raise more fully in the presentation: how a professional learning community focused on learner errors can support teachers from diverse contexts to engage with each other, with their practice, with mathematical knowledge in the curriculum and with their learners.

PROFESSIONAL LEARNING COMMUNITIES

There are a number of definitions of professional learning communities, all of which emphasise two key aspects: professional and collective learning. Professional learning implies learning based on knowledge from practice together with knowledge from research. In learning from practice in our project, teachers analyse their learners' errors in tests and lessons and then decide what they as teachers need to learn in order to engage with the errors. So learner needs inform teachers' learning needs (Katz, Earl, & Ben Jaafar, 2009), or, in the language of this panel, opportunities for learners' learning and opportunities for teachers' learning are strongly intertwined. Research articles written about the particular errors chosen by the teachers show that many errors are common across contexts and help teachers to think about how to deal with the errors. Using classroom data keeps the professional development focused on learners' and teachers' experiences, while bringing in research knowledge helps to develop more extensive content and pedagogical content knowledge.

[1] Teachers form professional learning communities in their schools, which are then networked across schools.
[2] The Data Informed Practice Improvement Project (DIPIP).

The collective nature of the learning provides support for teachers trying new ideas and shifting long-held practices and knowledge, as well as providing for more comprehensive and coherent experiences for learners. In conventional teacher development programmes, where only some teachers participate, there may be resistance from other teachers and learners when they try to develop new ideas. School-based professional learning communities allow teachers to "coalesce around a shared vision of what counts for high-quality teaching and learning and begin to take collective responsibility for the students they teach" (Louis & Marks, 1998, p.535).

LEARNING IN AND THROUGH PRACTICE

Professional learning communities are premised on a notion of learning as social practice (Lave, 1993, 1996). Learning is defined as becoming a better participant in a practice, or, in Sfard's terms (2008) developing the discourse of mathematics, where discourse applies to the language, symbol systems, representations and meanings of mathematics. Social practice theory starts from the notion of practices, which are constituted in communities rather than from conceptual structures that are constructed in the mind. In social practice theory, learning is strongly related to the conditions that constitute learning and the communities in which it takes place. A key question for social practice theory is whether and how learning is transformed across contexts, and how do group and individual learning co-evolve within and across contexts (Borko et al., 2000).

In our project, we work with this view of learning in two ways. First, we discuss teachers' own practices in the community, using classroom artefacts and videotapes of lessons, noting that learning in the community and the classroom co-evolve (Kazemi & Hubbard, 2008). Second, we think about learners coming to know mathematics differently as a social practice, which co-evolves with teachers' coming to know mathematics differently. In particular, if teachers begin to work with errors as opportunities for learning, rather than problems to be avoided, then learners might come to believe in themselves as competent mathematics learners because they make errors and can overcome them.

THE PROJECT CONTEXT

Our argument is strongly informed by our context – the mathematical experiences and achievements of South African learners. As with all aspects of life in South Africa, the education system is characterized by large disparities between rich and poor, and most of our schools and learners are of very low socio-economic status. Most teachers in South Africa teach big classes in very poorly resourced schools. Disaffection and alienation are rife (Motala & Dieltiens, 2008) and failure rates are high, particularly in mathematics, where failure begins as early as grade 3. Learner failure and alienation are compounded through the years of schooling. Teachers experience failure in helping learners to achieve, and much of the lay and professional discourse blames teachers for this failure - they do not spend enough time in classrooms, do not have enough knowledge, or use traditional practices that don't encourage learner thinking.

Any teacher education programme working in South Africa needs to take seriously the mathematical empowerment of both teachers and learners. This raises the question of what mathematics might be empowering, given the alienating nature of the standard curriculum. We choose to work with the standard mathematics curriculum, arguing, together with Young (2008), that traditional subjects such as mathematics do represent powerful knowledge that all learners can learn and deserve access to. Moreover, traditional subjects can be taught in empowering ways, by developing a curriculum of engagement, rather than a curriculum of compliance[3] (Young, 2010). Working across diverse contexts allows teachers from both better and worse resourced schools to work together to construct curricula of engagement.

A focus on errors affords teachers several opportunities to construct a curriculum of engagement, for themselves and learners. In analysing learner errors, they engage with an issue of immediate importance to them and their learners. Teachers from different schools, ranging from poorly-resourced to well-resourced see that their learners make very similar errors, so even "high-achieving" learners are seen to make errors, and teachers from "good" schools are seen to have the same errors occurring in their classrooms. This helps to establish a key principle: that errors are a normal part of the learning process (Smith, DiSessa, & Roschelle, 1993), and that neither teachers nor learners are to blame for errors, although both have the responsibility to engage with them. Research articles from other countries show that learners from well-resourced countries make similar errors. A focus on errors draws on commonalities across very different schools and in doing so, supports teachers in diverse contexts to come to see similarities in their own and their learners' knowledge and practices, and to generate collective responses, where teachers across the spectrum support each other. Focusing on errors can help teachers to mediate a range of influences that deny learning for many learners.

AN EXAMPLE

A teacher presented the following "error"[4] from her class to her professional learning community. The task was for Grade 9 learners to write an equation with variables in it. Each group of learners wrote their equation on the board and the teacher discussed them with the class. One group wrote: $4x \text{ x } 5x = 20x$ and the teacher asked the class: what is wrong with this equation. In the community, two teachers, Shoriwa and Chomane, questioned whether the statement $4x \text{ x } 5x = 20x$ is incorrect and the teacher, Sebolai, insisted that it is. A number of the teachers agreed with her, while a few agreed

[3] For Young, both a curriculum of engagement and a curriculum of compliance are subject-based. However, the first supports learners to access powerful knowledge through subjects while the second retains traditional narrow forms of pedagogy and learning.

[4] In this case the "error" as presented by the teacher turned out not to be an error, but this only became evident to the teachers through the conversation.

with Shoriwa and Chomane. Shoriwa and Lorraine explained how the statement can be seen as correct:

> Shoriwa: Now what you are looking now, you are giving a limited view of that, they are looking at it as an identity, but as an equation it's correct
>
> Lorraine: It's a quadratic equation

This explanation convinced some teachers but not others. For example, Eunice argued:

> that four x times five x, I think it was to be corrected immediately, I'm supporting Linda that now it should be twenty x squared, they must know … because the wrong thing sticks to the mind of the child

While Chomane reflected:

> That one was very interesting, in that when I saw it first, I was also rigid and I only thought of exponentials and forgot that that can be a quadratic, which can be solved

This example brings up a subtle distinction within mathematics, the difference between an equation and an identity, which pushed a number of teachers to the limits of their mathematical knowledge. Until Shoriwa pointed out the difference, many teachers agreed with Sebolai, that the statement was incorrect, thus indicating that they did not usually make this distinction. Once pointed out, some teachers, including Sebolai, still thought that it was incorrect, although others could see the quadratic. Chomane suggested that many teachers (as well as learners) may compartmentalise their knowledge - had they seen the equation in a section on quadratic equations, it is highly likely that they would have solved it, but in this case, the laws of exponents seemed to over-determine their responses. Linda explained this in relation to her teaching:

> if you were solving for x to make this true, in grade nine we're not at that level, we're doing exponents, so if you are saying that is correct you are throwing out your exponential rules, you are just creating one huge misconception, they're not at that level.

Linda was one of the teachers who initially agreed with Sebolai. She was convinced by Shoriwa and Lorraine that in fact there are two ways to see the statement. However, she also argued that in the context of a Grade 9 classroom, it was important to focus on one aspect, the identity and the laws of exponents. She was not yet ready or willing to admit that learners might benefit from her new insight. Chomane was able to take his learning one step further. Having acknowledged above that he had learned to see the statement in a new way, he argued:

> That was a good example, that the teacher could have taken advantage of the misconceptions, extending it to say that equations can have more than one solution and again put an example of that identity, where this one is a special equation with many solutions …

He argued that the difficult mathematics presented an opportunity to teachers, once they had realised their own limited view of the equation, to work in more mathematically sound ways with learners and to bring two usually disparate elements of the curriculum into connection with each other, thus creating opportunities for learning.

The example shows how the professional learning community supports teachers' learning about mathematics and teaching. Through a discussion about an ostensible learner error, a number of teachers came to deepen their own knowledge and re-think their practices. The "error" gave the teachers access to powerful concepts in the curriculum and how they might think about supporting learners' access to these concepts. Some teachers also saw how in keeping their own focus limited, they might deny learners access to deeper mathematics.

CONCLUSIONS

In the conversation, the teachers were willing to challenge each other and themselves in constructive ways in order to deepen their understanding of their own knowledge and develop the discourse of mathematics further for themselves. Their challenge supported enquiry into a difficult area of mathematics – the relationship between equations and identities. In this session a number of teachers shifted their mathematical discourse and deepened their understandings of mathematics, and we see at least one teacher deepening his pedagogical understanding based on his new insights.

Through engaging in analyses and conversations about learner errors, teachers support each other to see their learners and themselves as mathematical reasoners, and errors as reasonable. In this case, the fact that the error turned out to be a teacher error rather than a learner error, gave teachers further opportunities to think about the important of engaging rather than dismissing learner errors and how to make links across the curriculum. Conversations similar to this occurred weekly in this community. On-going sustained investigation of learner errors and the reasons for them support teachers to develop the practice of engaging with their learners' errors providing them better opportunities to participate in mathematics.

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DOES PARTICIPATION IN HOUSEHOLD BASED WORK CREATE OPPORTUNITIES FOR LEARNING MATHEMATICS?

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The National Curriculum Framework for school education in India (NCF, 2005) states as one of its fundamental principles, the building of connections between the school curriculum and the child's life outside school (NCERT, 2006). It recommends that the knowledge gained from outside the school be seen as a resource for learning in the classroom. This is an acknowledgement of research findings in India (Khan, 2004) and elsewhere that children from economically disadvantaged backgrounds acquire significant knowledge outside the school. In India, as in other developing regions in the world, many children participate in the income earning activities of the household, which may enable them to possess impressive levels of knowledge and awareness. Educators have seen in this a potential to not only offset the educational disadvantage stemming from low socio-economic background, but also a possible way of countering the culture of rote learning that is pervasive in Indian education.

Optimism about knowledge acquired by children outside school, especially mathematical knowledge, being a potential springboard for learning school mathematics is evident even in the early writings on 'out-of-school' mathematical knowledge (Nunes, Carraher & Schliemann, 1985). However, despite many studies exploring the contours of such knowledge and its settings, its integration with the school mathematics curriculum remains limited.

Our research has involved a study of 12-13 year olds from low income urban households studying in government schools, who participate in or are exposed to income earning household based work[1]. The objective of the study is to explore the mathematical knowledge that students can access in relation to such contexts, and its potential in learning core areas in the middle school mathematics curriculum such as proportionality, measurement and algebra. The issues surrounding the relation of school learning to knowledge accessed outside the school setting are complex, and several perspectives and approaches have been taken by researchers (Nasir, Hand & Taylor, 2008). Our approach has been to probe the diversity of settings that children within a single classroom have access to, and to situate these in a larger dynamic that shapes both mathematics education and the work settings. The ethnographic study is followed up with instructional sessions with the students aimed at exploring possible connections between the school curriculum and knowledge gained outside school. In the sections below, I'll briefly present a historical background, a theoretical perspective and attempt to situate our study within this context.

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[1]This research is done in collaboration with my colleague, Arindam Bose. I thank him for permission to use data collected from interviews of students.

A HISTORICAL PERSPECTIVE

A look at the elementary school mathematics textbooks from Western India from the late 19th and early 20th Century reveal that the bulk of the curriculum consisted of topics related to everyday commerce (Gokhale, 1921; Potdar, 1922). The several chapters on the four arithmetic operations are separated into two sections dealing respectively with simple and compound operations. Compound operations involve computation with quantities expressed in multiple sub-units. For example, the sub-units for weight had the relation 1 *ser* = 24 *tola* = 24 × 12 *maasa* = 24 × 12 × 8 *gunja*. A weight measure notated as "2 14 5 0" would mean 2 sers, 14 tolas and 5 maasas. Compound operations involved computation with such mixed or "compound" numbers.

A complex and extensive system of measures expressed in the form of conversion tables was an important part of the elementary mathematics textbooks. Conversions between British, Indian and local units were presented in detail. Students were presumably expected to be familiar with these units, and to be able to carry out computation with them. The textbooks also contain an extensive chapter on computing with fractions. Somewhat unexpectedly, the fractions dealt with are base four fractions expressed using an alternating vertical-horizontal "rod" notation. This may have been because the sub-units of money (and some other measures) were based on division by four. The textbooks also contain chapters with problems on simple and multiple proportion and interest calculation.

The extensive treatment of arithmetic and the detailed exercises with a variety of units suggest that such skills were needed and valued in the everyday world of commerce around the time when the textbooks were written. In the "new arithmetic" textbooks from the 1930s, compound operations and base four fractions are completely omitted and the curriculum begins to take a recognizably modern shape (Deshmukh, 1935). In the light of the recommendation of NCF 2005, it is striking that textbooks from a hundred years ago show a strong connection with life outside school, while educators worry about the lack of such connections in modern textbooks.

MATHEMATIZATION AND DEMATHEMATIZATION

Several researchers have identified demathematization as a pervasive trend in the circulation of mathematical knowledge in the culture. "[Demathematization] also refers to the trivialisation and devaluation which accompany the development of materialized mathematics; mathematical skills and knowledge acquired in schools and which in former time served as a prerequisite of vocation and daily life lose their importance." (Keitel, Kotzmann & Skovsmose, 1993, quoted in Jablonka & Gellert, 2007, p. 8) Demathematization with respect to explicit knowledge and skill accompanies the process of the mathematization of society, i.e. the incorporation of implicit mathematical knowledge in artifacts, instruments and practices. "The greatest achievement of mathematics... can paradoxically be seen in the never-ending, two-fold process of (explicit) *demathematizing* of social *practices* and (implicit) *mathematizing* of socially produced objects and techniques." (Chevellard, 2007, p. 60, emphasis original)

The arithmetic of the compound operations in the older Indian textbooks was needed because decimal numbers were used to compute with systems of units that were not decimal. The skills of computing with a variety of compound units and with base four fractions became redundant upon the adoption of a standardization system of units and measures at the national and the international level. Standardization is one of the means by which demathematization takes place. Other ways are the incorporation of arithmetic in artefacts and devices: calculators make paper-pencil calculation redundant; comparative EMI tables make it unnecessary to calculate interests. Demathematization is also devaluation and hence impacts learning opportunities which are framed by what the culture values and perceives as useful.

MATHEMATICAL KNOWLEDGE IN THE HOUSE-HOLD ECONOMY

An explicit objective of out study is to explore connections between such learning and (a possibly redesigned) school mathematics curriculum. The location of the study is a large urban low income locality which had a vibrant house-hold based economy. Roughly one third of the students chosen randomly from a whole class studying in the Urdu medium and a whole class studying in the English medium formed the sample. Extensive interviews of these students have led to a profile of the kinds of income generating work that they participate in or are exposed to, and their basic arithmetic abilities and skills. More detailed ethnographic data is being obtained through interviews done with a selected sub-sample of the students.

Some houses in the locality have small workshops or factory units adjoining or in them, while in many houses activities are done within the house, which include embroidery, *zari* (stitching sequins onto cloth), stitching, garment-making, making plastic bags, leather goods (bags, wallets, purses, shoes) and decorative items, repairing, catering, vending, etc. A large number of children in our study participated in one or more of such kinds of work, often inside the house, and in some cases in a shop or workshop. In some houses, children were discouraged from participation in work and were encouraged to focus on studies. But even children from such households develop a fair knowledge and reality perspective about the activities around them. Nearly all children regularly buy groceries or provisions for daily house-hold needs from neighbourhood shops.

Students in the study generally showed flexible competence in arithmetic in contexts dealing with money (Bose & Subramaniam, 2011). Many could compute mentally and arrive at quick decisions when the situation required addition or subtraction, or multiplication and division by small numbers. Some students struggled with reading and writing numbers larger than 3 digits, although they could deal with such numbers as amounts of money. Detailed interviews with students revealed more about the nature and diversity of work that students participated in. The perspective framed by accounts of mathematization/demathematization helps illuminate many aspects of the settings in which work is carried out. However, the interviews also reveal a situation that is fluid and dynamic in many respects.

The general trend of a shift from craft based industry to large scale factory based manufacture is resisted by small-scale house-hold based industry, which is an outcome of the initiative and enterprise of economically disadvantaged people struggling to make a living. This represents a counter-trend against deskilling and the depletion of craft-based knowledge, as existing knowledge is adapted, modified or new kinds of skills and knowledge arise. Such resistance to deskilling can be read as a counter trend to demathematization, since the need for mathematical skills arises in the course of the bargaining, negotiation and decision making concerning wages, costs, commissions, interest, in dealing with a variety of goods and quantities measured in diverse units. It is emblematic of this counter-trend that old British units such as inch and foot, or even Indian units such as the *gaj* (equivalent of “yard””), that were sought to be exiled through standardization continue to be used in such occupations. A variety of formal and informal units are used to indicate the quantity of raw materials in different sectors of small-scale manufacturing such as tailoring, sequin-stitching, leather work, catering, etc. While formal units belong to a system of units (international or indigenous), informal units are units of convenience, may not be defined precisely quantitatively, and may be partly embodied. An example of an informal unit is a '*mutthi*' or 'fistful' of raw materials, used in *zari* (sequin) work (Subramaniam & Bose, 2012).

The extent of knowledge of measurement among students varies. One student could draw a line one inch long with accuracy, while another student participating in tailoring work confused inches and centimeters (both marked on the tape he was familiar with). Students encounter numbers and measures of different kinds, but the mode of quantification remains obscure. Shirt sizes, for example, were seen as mere numbers bereft of units and without an idea of how the numbers were obtained.

Knowledge related to measurement among the students has aspects of familiarity through participation in work, but is also partial and fragmented. This is reflected in the division of labour and compartmentalization of groups involved in making small articles. Repetitive processes with a stress on the quantity of production characterise not only factory based production but also house-hold based industry. Goods are often delivered as nearly finished goods, with only a small part of the manufacture to be completed in the house-hold. For example, some house-holds are involved for a few months in a year in making '*rakhis*', decorative strips of cloth and thread tied around the wrist for the Hindu festival of '*Rakhsa bandhan*'. Colourful flowers already cut into shape from plastic or paper and decorative threads are collected by a family member from a middleman, and the work to finish the *rakhi* involves only glueing or threading. In a tailoring shop, pieces of cloth that make up a shirt already cut into shape are delivered and the work that remains is only of stitching the pieces together and sewing buttons. A former master tailor who runs the shop now only needs to attend to managing the production of shirts in large numbers.

CONCLUDING REMARKS

An exploration of the mathematical knowledge gained by students through participation in work reveals familiarity with "materialized" mathematics embedded in the culture. Knowledge of mathematics in such contexts may show flexibile competence in some domains such as arithmetic computation, but may be partial and fragmented in domains such as measurement. The settings in which such knowledge is acquired do not foster the gaining of "mastery knowledge", nor is such knowledge readily available in the environment in which work is done. Further, the frequently compartmentalized nature of work, its routinization and repetitive nature are consistent with the limited and fragmentary knowledge gained through them. It is possible that compartmentalization and simplification are the very factors that allow children to participate in the work, but this also means that they and the house-holds have little control, negotiate fluid identities and experience a variety of injustices in the course of such work. School education, perceived as a means to a better future, is seen as distinct from this mileu, and efforts to build bridges between the culture and school education need to take cognizance of these aspects.

The goal of a mathematics curriculum sensitive to the interaction of mathematics in the domains of work and play, must take account of the distinction between mathematics *in* work related activity and mathematics *as* activity, between mathematics *in* the culture and mathematics *as* culture. Connecting the learning of mathematics in school with culture can take the form not only of guided re-invention as in the Realistic Mathematics Education approach (Treffers, 1993), but also of a history or archeology of "materialized" mathematics embedded in the artifacts or practices of a culture. The context of measurement is an instance of such possibilities that need exploration. In instructional sessions with students in the study, the familiar "inch-tape" was a central artefact around which, exploration of the following concepts of measurement was structured: the concept of a unit length, the sub-division of a unit, use of fractional and decimal notation, and the activity of measurement. At the end of two weeks of instruction, in their responses during focus group discussions, students appreciated the fact that they had gained a deeper understanding of something as familiar as an inch-tape. We believe that such "archeology" may have an important place in providing opportunities to learn mathematics to students who obtain a fragmented and partially obscured form of mathematical knowledge from informal work environments.

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OPPORTUNITIES FOR LEARNING WITH DIGITAL TECHNOLOGIES: A QUESTION OF RECONTEXTUALISATION

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Many visions of opportunities for learning provided by digital technologies have been advocated by researchers, using a large diversity of frameworks. Here I consider only a small set of these:

Theory of Didactical Situations (TSD): learning occurs via the interaction with an antagonist milieu, and the institutionalisation of knowing into official knowledge. Technology can be constituent of a suitable milieu (Laborde & Caponni 1994).

Anthropological Theory of Didactics (ATD): learning is developing an adequate relationship to “objects of knowledge” within an institution (Chevallard 1992).

Constructionism: learning is abstracting in situation and constructing a web of meanings. Digital technologies help to provide rich environments where abstracting and webbing meanings are possible (Harel Papert 1991).

Instrumental approach (IA): Learning mathematics with a digital media is through a process of instrumental genesis involving both knowledge of the tool and of mathematics (Verillon et Rabardel, 1995).

The contrast between these visions and the limited impact they have on school practises, as well as the fragmented multiplicity of frameworks in which they are expressed, suggest that visions are tied not simply to the frameworks that researchers privilege but rather to a whole context in which their activity takes place. My assumption is subsequently that opportunities for learning have to be thought of as strongly depending on context especially when using technology. The question is then how to recontextualize opportunities for learning from one context to another.

1. Context and digital media for learning mathematics

Research in the uses of digital media is generally characterized by interventionist agendas and design research methods: conception of innovative educational environments based on particular use of digital technologies, study of what happens as they intervene in school settings. Designers of digital technologies shape their tools from their perceptions of learning mathematics and their epistemologies of mathematics and mathematical activity as well as on the rules and constraints of software production and dissemination, while users shape their conceptualizations of the nature of these media in a process of instrumental genesis also influenced by perceptions and epistemology together with rules and constraints of schools and classrooms. From this relationship of research on the use of digital media to context, particular objects emerge to analyse opportunities for learning. (1) *Participants' behavior* in empirical research is most often a basis for proof of enhanced learning. (2) *Tools and scenarios* are offered by research especially in technology to support evidence of opportunities for learning. Created in a given context, the use of these resources can be problematic in other contexts. (3) *Theoretical frameworks* are finally necessary to communicate about opportunities for learning. Recently in the stream of research about frameworks in math education, interest grew upon the role of communities and cultures which are aspects of the context.

Contextual characteristics also emerge to describe how research activity affects the above objects. Characteristics of *empirical settings* are teachers, students and other actors taking part in the empirical research, their relationship with the artifact(s), and between them, as well as policies and pedagogic norms at several levels from school, to curriculum and assessment regimes, and to national education systems and, etc. Characteristics in *academic settings* include the institutional and cultural environment within which researchers work, particularly the relationships to and expectations of funding agencies, their positioning in relation to colleagues locally, nationally and internationally as well as their relationships with teachers and school for empirical research. In order to give an evidence of the entangled web of relationship between visions of opportunities for learning and contextual characteristics, and to initiate a reflection on recontextualisation, I take the work of the research teams from across Europe brought together through the ReMath project.

2. ReMath: Tools to address the issue of context

The ReMath project carried out ‘cross-experimentation’, i.e. design and analyses of uses of a series of ‘Didactic Digital Artefacts’ (DDA) by different teams in different contexts (Lagrange et al 2010). The project went further to engage in developing cross-case analyses i.e. a unified associative/comparative account of two studies of the same DDA. Below I develop what can be learnt about contextual issues from one cross-case study involving the DDA Cruislet, carried out by a Greek team in charge of the design of this DDA, and Didirem, a French team developing another DDA, Casyopée. Cruislet is a navigation microworld in which a user flights aeroplanes across the Greek geography by issuing navigation instructions in either graphical/Cartesian or spherical/polar coordinate systems, in direct stepwise mode or by way of LOGO programming. Aeroplanes’ movements are defined as vectors, and must take into account not only location, but also the elevation of the landscape they are navigating. Among the six ReMath DDAs, Cruislet is extreme in terms of distance of the tool from usual curriculum. The knowledge at stake with this DDA is at the interface between geography, mathematics and programming. In Greece, a rather long experimentation was organized in grade 10 classes (20 hours) without apparent difficulty while in France, making the use of Cruislet compatible with institutional constraints, resulted in a shorter experiment (6 hours) in the specific settings of project sessions. Moreover in France the negotiation of the scenario with the teachers in charge of the experimentation was a rather difficult process. The influence of the *empirical settings* is then immediately visible in the two scenarios.

With regard to *academic settings*, references to constructionism led the Greek team to consider this experimentation as the study of students' gradual mathematizations in an environment where constructions are journeys using the varied systems of reference and the varied mode of operating. This led the Greek team to especially build on the potential offered by the complex linkage of representations offered by Cruislet for investigating the mathematical meanings that students construct regarding the notion of function as co-variation while navigating in 3D large scale space. The situation is radically different for the French team whose global references are IA, ATD and TDS (above). IA led to pay particular attention to the instrumentalization needs of such a DDA, so complex and so far from algebraic tools or dynamic geometry environments, and to try to find ways of limiting these needs. ATD made French team especially sensitive to the distance with the French curriculum and the attention to be paid to the possible ecology of Cruislet in the French educational system. For French researchers, epistemology is a top concern but they could not rely for supporting their scenario on a stabilized didactic knowledge because the literature regarding the objects implemented in Cruislet is not enough developed. In such conditions, controlled design consistent with TSD became impossible.

3. Cross-analysing a Cruislet experiment

In the cross-analysis the French team did a close study of the Greek experiment and the Greek team provided more data about the epistemological bases of the scenario and a precise account of students' behaviour. For instance, the Greek team reported on a situation based upon the use by students of a procedure that made one aeroplane perform a flight to an arbitrary position while another reached a dependent position, each of its coordinate being a linear function of the coordinates of the first one. Using this procedure first as a black box and then decoding the procedure, students could make sense of the situation by investigating the co-variation of the planes and conceiving the first plane's position as an independent variable and the second plane's position as a dependent variable. The cross-analysis made clear that Cruislet could provide opportunities for constructing the notion of function as co-variation while navigating in a realistic 3D large scale space. Functions as model of co-variation was also a domain of interest for the French team especially for the design of its own DDA, Casyopée. However, the domain of co-variation at stake in Casyopée is 2D geometry and the functions are one variable real functions. In addition, Casyopée is designed to provide opportunities for learning about polynomial, rational or transcendental functions rather than linear functions. Thus, when designing the scenario, French researchers, although informed by the Greek team of the tasks prepared by this latter team, saw these tasks as very far from what they used to propose to the students. It is worth to note that, in the constructivist tradition, no a priori analysis of students' behaviour was made by the Greek team for the cross-experiment, which did not help the French team to recognize the potential of these tasks for their scenarios. On the contrary, the cross-analysis of the Greek Cruislet cross-experiment, focused on the students' behaviour and a better appreciation of constructionism, pointed out an approach to functions that could be very complementary to the approach with Casyopée.

In the cross-experiment, the French team was attracted by features of Cruislet offering opportunities to link mathematics and geography in a multidisciplinary approach also favoured by the realistic context of Greece, and to introduce students to programming. To take advantage of these features, the French team conceived objectives for the teaching experiment far from the curriculum and this is the reason why the experiment was short and in the frame of a special project. In the a posteriori analysis of the first experiment, it appeared that the scenario had a certain potential to promote new teaching/learning situations but also that students' poor instrumental genesis of Cruislet limited opportunities for learning. This misappreciation by the French team can be put into relation with several contextual characteristics.

<u>Contextual characteristics in the empirical settings</u> : Experimenting Cruislet, the French team did not consider the qualitative approach of functional dependencies proposed by the Greek team because of the distance from the curriculum and usual practices and because of the constructionist framework in they were formulated. They rather imagined other opportunities for implementing Cruislet consistent with new trends of the curriculum towards seeing mathematics in coordination with other disciplines and towards the development of an algorithmic approach. The French team identified these trends as an opportunity to implement an experiment with Cruislet, breaking with current practices, but nevertheless providing a response to specific institutional demands.

<u>Contextual characteristics in the academic settings</u> : Casyopée was conceived in close relationship with a group of teachers that were chosen not because they were specially "innovation oriented", but rather because of their ability to create, experiment and disseminate situations that could be acceptable for other teachers. The accompanying epistemology of functions was developed with the idea of real functions of one variable in mind, which seemed to be the easiest notion to implement in the "ecology" of French upper secondary classes. Researchers had some notion of constructionism, but did not consider sharing this with teachers. The epistemology of functions was questioned when researchers and teachers had to look closely at the Greek experiment in the cross-analysis. They recognized that the tasks designed and implemented by the Greek team in a different ecology had the potential to introduce students to a wider understanding of functions that could be useful to consider before or in parallel with the development of competencies in the domain of real functions of one variable favoured by Casyopée use. Looking closely at constructionism in the cross-analysis of the Greek team experiment was also an opportunity to reconsider this approach, and to discuss with the teachers, in the light of a similar field experiment.

4. *Contextual characteristics and visions*

The context of a team working in the field of mathematics education and technology like the French team can be described as an entanglement of contextual characteristics both in the empirical and academic settings. Cross-analyses like the Cruislet case pointed out how this context was supportive for the team's research activity, but also oriented its vision of technology and learning. They also brought support for opening the view: the team could enlarge its epistemological view of the notion of function, and of approaches to this notion using technology and consider the value of a framework like constructionism. This work carried out in common by researchers and teachers was the basis of a publication accessible to French teachers (Lagrange, Le Feuvre, Meyrier 2010) that proposed uses of Cruislet inspired by the cross-experimentation and adapted to the French context, as well as of a reorientation of the Casyopée project towards clarifying its potentialities. This highlights the need to go beyond broad decontextualized visions, to identify contextual characteristics and their influence, and also to use special methods for recontextualizing, as means to respect and encourage diversity in classroom use of technology.

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RESEARCH FORUMS

PME 36

TAIWAN

2012

CONCEPTUALIZING AND DEVELOPING EXPERTISE IN MATHEMATICS INSTRUCTION

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As teachers and their teaching are commonly recognized as key to improving students' mathematics learning, understanding and improving teacher expertise in mathematics instruction are inevitably important. Building upon recent studies on teacher expertise and its improvement, this Forum aims not only toshare and discuss relevant perspectives contributed from five selected education systems, but also to promote further research efforts on this topic. With a focus on "conceptualizing and developing teacher expertise in mathematics instruction", this forum is organized to bring interested PME members to the research frontier and to develop possible collaborative efforts.

THE GOALS, KEY QUESTIONS, AND FOCUS OF THE RESEARCH FORUM

This Research Forum aims to examine the concept and nature of teacher expertise in mathematics instruction valued in selected education systems, and different approaches and practices that are used to develop teacher expertise in mathematics instruction in those education systems. Collectively, this research forum is not only to report specific research findings, but also to provide a platform for understanding and cross-examination of the similarities and differences in conceptualizing teacher expertise in mathematics instruction and the ways that are used to develop teacher expertise in diverse system contexts.

Through this Research Forum, we ask: (1) what aspects of teacher expertise are emphasized/valued in mathematics instruction, what cultural values may be placed behind what can be counted as expertise in mathematics instruction, and (2) what approaches and cultural resources are utilized for developing teacher expertise in mathematics instruction in different education systems?

The focus of this Research Forum is two-folded: one refers to the conception and eminent features utilized in characterizing expert teachers and their expertise valued in Israel, Portugal, Singapore, South Korea and Taiwan. And the other concentrates on specific approaches and/or cultural resources utilized for improving expertise in mathematics instruction in different system contexts.

RESEARCH BACKGROUND

Research on the conception and nature of teacher expertise in mathematics instruction: What do we know?

It is now common knowledge that teacherexpertise in mathematics instruction varies individually and affects teaching performance. However, understanding of the conception and nature of teacherexpertise in mathematics instruction is still very limited. As teachers and teaching have been recognized as vital parts of enhancing students' academic achievement, understanding the nature of teachers' expertise has become an unavoidable issue. In fact, with ever-increasing emphasis on improving students' mathematics learning in current worldwide educational efforts, those who care about finding ways of improving mathematics classroom instruction and teacher education have stressed the importance of knowing and understanding what is needed for making and developing expert mathematics instruction.

Understanding and evaluating teacher expertise has been a perplexing issue in many education systems for years. Taking an international perspective to examine teacher expertise valued in different education systems should help advance our understanding of the issue. For example, existing cross-national studies such as the IEA-study TEDS-M have revealed remarkable differences in the mathematical and mathematics pedagogical content knowledge of future teachers from various countries (see Blömeke, Kaiser, & Lehmann, 2010a, b).The study by Ma (1999) showed remarkable differences between Mainland China and the United States in teachers' knowledge of mathematics for teaching. In particular, Ma revealed the dramatic differences in elementary teachers' knowledge of mathematics between China and the United States, which led to further questions about the nature of expertise that may help connect or distinguish teachers' instructional performance between the East and West (e.g., Li & Kaiser, 2011).

The recent book on expertise in mathematics instruction (Li & Kaiser, 2011) presents a new scholarship in studying teachers' expertise in mathematics instruction. Specifically, the book takes a unique approach to present new research from multiple Western and Eastern countries, and is organized to probe three universal themes: identifying expert teachers, specifying and analysing teacher expertise in mathematics instruction, and understanding expertise in mathematics instruction as it is perceived and valued in different cultures. The book is the first step of undertaking a systematic examination of teacher expertise in mathematics instruction from an international perspective. Many similarities and differences can be identified and cross-examined from the book. For example, Kaiser and Li (2011) highlighted cross-cultural similarities and differences in two aspects: (1) conceptualizing and specifying teacher expertise, and (2) viewing expert teachers and their teaching performance. In particular, cross-cultural differences in teaching practice and people's views of teaching practices suggest an important dimension when examining and understanding teacher expertise in different cultural contexts. Further efforts are needed to study expertise in mathematics instruction.

Building upon the recent studies on expertise in mathematics instruction, this Research Forum is organized with a focus on the nature and development of teacher expertise in mathematics instruction. In particular, the similarities and differences in the conceptualization of teacher expertise between the East and West reported in the book (Li & Kaiser, 2011) suggest some important aspects that need further examination. Such examination can cover multiple aspects, including teachers' knowledge, beliefs, and teaching performance in classrooms. For example, although sound subject knowledge is commonly regarded as being an important part of teacher expertise, it remains unclear what exactly expert teachers know about mathematics. Further research is needed to examine the expert teachers' level of knowledge about school mathematics and to find out if it is important for them to also know advanced mathematics. Moreover, it is unclear how the conception and nature of expertise in mathematics instruction may help differentiate a real expert teacher from a routine expert or an experienced teacher in different cultures. A better understanding of the conception and nature of expertise in mathematics instructionthrough this Research Forum is important, especially when we consider the possibility of improving the quality of mathematics classroom instruction and teacher education in different education systems.

Research on the development of teacher expertise in mathematics instruction: What approaches and practices are developed and used in different education systems?

Educational research has dramatically increased its emphasis on teachers and teaching practices over the past decades (e.g., Sikula, 1996; Townsend & Bates, 2007). The need for improving teachers' expertise has emerged ever-increasingly in various ways, including the expectation for practicing teachers' continuous knowledge development and practice improvement in mathematics instruction, teachers' training for undertaking and implementing changes in curriculum and instruction, and teachers' professional promotion. It is now well recognized that lesson study is an important approach utilized in Japan to facilitate teachers' collaborations and professional development. In fact, there are various approaches developed and used in the pursuit of teacher expertise improvement in different education systems. However, much remains unknown to outsiders about other approaches used in many education systems. For example, online study collaborations are used to improve teachers' expertise in China (e.g., Li & Qi, 2011). Master teachers are an important part of teaching culture in some education systems in East Asia, and master teachers' work stations have been commonly used in China for improving teachers' expertise in mathematics instruction (Li, Tang, & Gong, 2011). Understandably, the improvement of teachers' expertise can be made possible via different approaches in different education systems, including workshops and summer institutes to focus on teachers' knowledge development, working closely with master teachers through mentoring, and peer teacher discussions either in groups or online. Thus, this Research Forum goes beyond the examination of the conception of teacher expertise in mathematics instruction to discuss ways employed to improve teacher expertise.

As mathematics teaching is a cultural activity (Stigler & Hiebert, 1999), ways of improving teacher expertise in mathematics instruction can also be culturally valued practices. Different approaches employed to improve teacher expertise in mathematics instruction can mirror the cultural niche that supports the creation of excellent teachers and high-quality mathematics instruction in that context. An exploration of approaches and associated cultural resources can not only provide others a better understanding of the mechanism, as existed in a system and culture context, that supports the generation and valuation of expert teachers and their expertise in mathematics instruction, but also highlight possible restrictions to simply adapting certain approaches and practices from one context to another. Thus, this Research Forum tends to serve as a window through which mathematics educators can gain a glimpse of various approaches and possible cultural support needed for improving teachers' expertise across several selected education systems.

THE STRUCTURE OF THE RESEARCH FORUM

The Research Forum is to be organized with a format that integrates the use of multiple activities, including formal presentations, small group discussions, pre-prepared commentary, and coordinated Q&A sections. In particular, this format is designed to take advantages of both formal presentations and small group discussions in its two 1.5 hour sessions. The forum will start with formal presentations that aim to share research on mathematics teachers' expertise in selected education systems. The participants will then be invited to join small group discussions to have better opportunity to ask questions and learn further about teacher expertise in mathematics instruction and its development. During the small group discussions, participants may also be invited to share what they know about teacher expertise and/or approaches used to improve teacher expertise in their own education systems. The small group discussions should provide the presenters a good opportunity to prepare a summary of information shared and further explanation as needed, which will be used to kick off the second 1.5 hour session for the whole forum. The presenters will then also present specific approaches utilized for developing and/or evaluating teacher expertise in selected education systems. These presentations will be followed by a commentary provided by the discussant. The session will then be ended with final Q&A between all the audience and presenters.

WHAT IS AN EXPERT MATHEMATICS TEACHER?

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In Portugal, the introduction of new mathematics curriculum for basic education [1] *(ME, 2007) generated new images about the activity of the mathematics classroom and about the role of the teacher, based on the notions of "explorations" and "discussion". This had strong implications to the perspective on teacher expertise accepted by teachers and also by researchers in this country. This paper provides a brief overview of the national context, in terms of curriculum and teacher education, and describes this perspective of mathematics teachers' expertise.*

THE CURRICULUM AND TEACHER EDUCATION CONTEXT

In Portugal, in 2007, the mathematics curriculum for basic education (grades 1-9) was approved, replacing the 1991 curriculum and reflecting current curriculum guidelines, emphasizing ideas such as development of number sense, the development of algebraic thinking since primary school, the development of spatial sense and statistical literacy. It also emphasizes three main "transversal capacities", problem solving, mathematical reasoning and communication, the need of diversifying tasks and representations and of making appropriate use of technology.

The Portuguese educational system establishes the general profile of the teacher, for all subjects and school levels that includes four dimensions (Decree-Law no. 240/2001of 30 August): (i) developing teaching and learning, (ii) participating in school activities and relating to the community, (iii) lifelong professional development, and (iii) handling professional, social, and ethical issues. In this country, since the adoption of the Bologna process (in 2006), the preparation of prospective mathematics teachers involves two stages: first, a 3-years degree provides training in mathematics; second, a 2-years master degree provides professional preparation to teach mathematics in grades 7-12.

PERSPECTIVES ON EXPERT TEACHING

There is no systematic research on expert teaching in the frame of the former basic education curriculum. However, an important document from the Association of Teachers of Mathematics (APM, 1997) stresses that teachers' practice should include elements of "diversification": in the nature of tasks, in the kinds of classroom interaction, in the use of supporting materials, and in the forms of assessment. This emphasis in diversification was well attuned with the general perspective that to

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[1] "Basic education" spans for grades 1-9, that is students who are 6 to 14 years old. In some countries the equivalent expression would be "primary and lower secondary school".

address students with different cultural origins and learning needs, teachers had to introduce many elements of differentiation in their teaching.

Two methodological ideas stand outin the new curriculum for basic education (ME, 2007): The importance of the mathematical tasks that constitute the point of departure for the activity of the students and the communication processes take place in the classroom. The curriculum suggests that the teacher must use a variety of tasks, including problems, exercises, explorations and investigations. Since exercises are used since a long time in Portuguese schools and problems were emphasized in the 1991 curriculum, the novelty here are explorations (open tasks, quite accessible to most students) and investigations (also open tasks but more demanding, see Ponte, 2011). Regarding communication, the curriculum values the development of the students'capacity for oral and written communication, and emphasized the value of moments of collective discussion, creating opportunities for mathematical argumentation, in which teachers asked students to present and explain their solutions to the tasks undertaken, giving the opportunity to the other students to accept or disagree and to present their own claims and justifications.

The supporting documents of this new curriculum indicate that such guidelines may be put into practice using a classroom organization in four main segments: (i) presentation of the task by the teacher, collective interpretation and appropriation of the task by the students; (ii) autonomous work of the students on the tasks, usually in pairs or small groups, with the teacher monitoring the work and providing some support in a careful way, that is, without solving the task for the students; (iii) collective discussion, in which some students presenting their work and all the class discussing it; and (iv) a final synthesis, summarizing the main points of the lesson, that could be done ideally with the participation from the students. It must be noted that, at the time, classes in Portugal lasted for 90 minutes, providing an extended time both for the students' autonomous work, as well as for the collective discussion.

The three key words of this approach are: (i) task, (ii) collective discussion, and (iii) exploratory work. For teachers, before the new curriculum, "task" was not a term much used in daily practice. Whereas technically this term refers to a wide range of situations (including exercises, problems, explorations, investigations, projects, mathematical games, etc.), most teachers tend it to mean some extended piece of work that is more complex than just routine exercises. "Collective discussions" point to classroom interactions in which there is room for students' participation supporting different points of view. And "exploratory work" became the most encompassing designation for this approach, given the prominence of exploratory tasks.

This curriculum change and the extensive production of supporting documents and provision of teacher education and other support processes created a new perspective about what is an expert teacher in Portugal, at least in basic education. It is a teacher who (i) is able to select and perhaps adjust suitable tasks, especially exploratory tasks, involving students actively in mathematical work, stimulating them to develop their own strategies, concepts, and representations and (ii) to conduct classroom discussions that create opportunities for negotiation of meaning, development of mathematical reasoning, and institutionalization of new knowledge.

DISCUSSION

This perspective on expert teaching has been (partially) validated in two ways. First, an in depth independent evaluation of the process of experimentation of the new basic education curriculum, was very supportive of classes that were observed in all different grade levels, from 1 to 9, using exploratory tasks and including highly productive moments of classroom discussion (Fernandes et al., 2011). A second element of validation has been the research studies undertaken at master degree and doctoral level based on teaching experiments that follow this perspective on expert teaching and that have been widely reported in research meetings and professional meetings (e.g., Branco, 2008; Henriques, 2011; Quaresma, 2011; Silvestre, 2006).

This perspective on expert teaching is a variety of deliberate practice (Li & Kaiser, 2012) and is aligned with international views. For example, presentingthe essential features of mathematicsteaching, the NCTM (1991), indicates the key role of worthwhile mathematical tasks (Standard 1) andclassroom communication (Standards 2-3-4, the teacher's and students' role in discourse and tools for enhancing discourse).In their study of the practice of mathematics teachers of the early years, McDonough and Clarke (2003) indicate 25 "practices" that they organize in ten major themes. The first three are strongly related to mathematics and tasks, including mathematical focus,features of tasks and materials, tools and representations. The next four themes include several aspects related to classroom communication and discourse: adaptions/connections/links, organizational style, teaching approaches, learning community and classroom interaction, and expectations. The importance of tasks with a high level of cognitive demand is underlined by Stein, Remillard, and Smith (2007). The value of situations involvingnegotiation of meanings is referred to by Bishop and Goffree (1986). The handling of classroom discussions involving the ability of the teacher in conducting classroom discussions, using a variety of questioning styles (with emphasis in inquiry questions), is currently an active field of research (Ruthven, Hofmann, & Mercer, 2011, Stein et al., 2007).

CONCLUSION

Curriculum documents provide statements about the mathematics to teach and learn and how to conduct and evaluate such teaching and learning. Such documents become important elements in framing new visions of what is expert teaching in teachers and also in researchers. Research and evaluation studies, such as those undertaken in our country provide additional strength to such visions and show that they are viable in practice, at least in small scale. However, a different thing is what happens in large scale. Visions of "expert practice" supported by these documents are supported by researchers and of teachers highly involved in curriculum reform processes. In fact, many of them already suppor5ted such view before the curriculum was approved. But these visions are quite distant from the visions of most practicing mathematics teachers. The fact that within one country different visions coexist at the same time creates an interesting agenda for mathematics education researchers.

CREATIVITY IN TEACHING MATHEMATICS AS AN INDICATION OF TEACHERS' EXPERTISE

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Following the observation that "teaching has often been thought as a creative performance" (Sawyer 2004, p. 12), this paper argues that creativity is an integral component of mathematics teachers' expertise.

CREATIVITY AND EXPERTISE

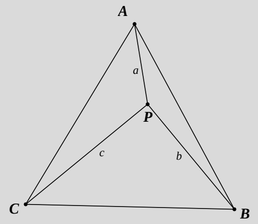
A basic operational definition of creativity widely used nowadays, as suggested by Torrance (1974), is based on four main components: fluency, flexibility, originality and elaboration. *Fluency*relates to the continuity of ideas, flow of associations, and use of basic and universal knowledge. *Flexibility* is associated with changing ideas, approaching a problem in various ways and producing a variety of solutions. *Originality* is characterized by a unique way of thinking and unique products of a mental or an artistic activity. *Elaboration* relates to the ability to describe, illuminate and generalize the ideas. The four components are mutually interrelated, however not all of them are present at the same time. The four components naturally characterize activity of expert teachers: Teachers' expertise is evaluated in terms of fluency in lesson management including fluency in explanation of mathematical ideas that they provide to their students (e.g., Leinhardt 1993). Expert teachers are flexible when reacting to students' unpredicted responses (Simon1997; Leikin and Dinur 2007). Teachers' expertise is associated with their mathematical or pedagogical originality, insofar as it tends to surprise students and, consequently, to raise their motivation. Elaboration of students' mathematical ideas is the main mechanism of moving with students "to a new mathematical territory" (Lampert, 2001). At the same time expert teachers aim to develop students' flexible mathematical reasoning, knowledge and skills that promote fluent problem solving, raising their own novel ideas and elaborating other students' mathematical thoughts (Polya 1963, Even, Karasenty and Friedlander 2009).

I will discuss and illustrate these ideas with Problem 1, which Tami presented to her 11th graders who study mathematics at a high level.

Problem 1:

Distances from point P to the vertexes of the equilateral triangle ABC equal a, b and c and fulfill the equality $a^2 + b^2 = c^2$. Find angle APB.

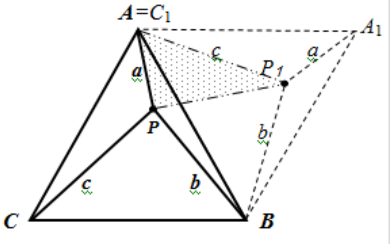
Prove your finding in at least 2 different ways.



FLEXIBILITY AND INQUIRY-BASED INSTRUCTION

Flexibility is a basic component of teacher expertise. In any mathematics classroom, the teachers' flexibility is manifested in the adaptation of teachers' initial agenda for addressing their students' needs and learning processes. Teachers' flexibility has especial importance in an inquiry-based learning environment, which is seen as promoting active learning on the part of students (Da Ponte, 2007). It is characterized by progressive discussion based on the students' own ideas and their need for shared meaning (Yackel & Cobb, 1996). In an inquiry-based mathematics classroom teachers often need to change their lesson plans spontaneously and follow unexpected learning trajectories initiated by the students. The teachers encourage explanation of the students' ideas and conjectures, identify the potential of these ideas, and chart the direction of their further development. In other words, they need to be flexible. In such a classroom, to be effective, a mathematics discussion must both focus on important mathematical ideas and enable discourse that promotes the learners' personal active construction of mathematical meaning through a communicative process (Cobb & Bauersfeld, 1995). The teacher's role in such a discussion is central in promoting students' mathematical understanding by choosing appropriate tasks and providing them with expert assistance. The following description exemplifies how the choice of task performed by a teacher (Tami) and her flexibility in managing the lesson opened opportunities for her students' own creativity.

Tami presented Problem 1 to her students. She planned to guide her students towards two different solutions to the problem. However, since "students in the class were strong" she asked them to think how they could find angle APB. Students found the task to be quite difficult, and Tami suggested they think about possible solutions at home. This was a real inquiry task by definition: an intellectually challenging task that requires independent work by the students.



At the next lesson, Tim suggested a construction in DGE that demonstrated that angle APB equals150^o.His solution then followed the construction. Another student, Sonya, suggested a slightly different solution. Then Tami, the teacher, presented her own solution which was based on rotation (Figure 2). Unpredictably, a third student named Haimclaimed that another solution exists for the points "outside the triangle". The angle then equals 30^{o}. Tami asked Haim to demonstrate his solution to the class.

Tami reported that her decision to change her initial plan was based on her familiarity with the class and her confidence that some of the students would succeed in solving this problem. She reported being surprised by the "outside the triangle" solution and that it had opened up another mathematical question that she "wanted to think about".

THE MODEL OF TEACHERS' CONCEPTION OF CREATIVITY

Expert teachers are usually fluent and flexible in classroom management. Less often do we observe teachers' originality. Teachers' flexibility appears to be strongly connected to their knowledge and confidence. Not less important are teachers' conceptions about creativity and its role in teaching mathematics. Lev-Zamir& Leikin (2011) introduced a new model of creativity in mathematics teaching that is useful for the analysis of teachers' conceptions and their practice. The model suggests distinguishing between (a) mathematical and pedagogical conceptions; (b) teacher-directed and student-directed conceptionsof creativity, and (c) declarative conceptions and conceptions-in-action.

Teacher-directed creativity is expressed in links between creativity in teaching mathematics and teaching-related actions that make teachers themselves creative. For example, Tami's ability to produce two different solutions to Problem 1 demonstrates her mathematical flexibility. Her ability to change the lesson plan demonstrates her pedagogical flexibility. In general, teacher-directed creativity is described by the teachers' in different ways, the most frequent of which are changing mathematical content of the tasks or changing the design of a didactical situation associated with mathematical tasks.

Student-directed creativity is expressed in teachers' actions aimed at fostering students' creativity, including their mathematical flexibility, mathematical originality (imagination), and mathematical elaboration. Tami's decision to allow students to cope with Problem 1 alone, to inquire and produce their own solutions led to original solutions produced by the students. Tim's solution using a mathematical construction in DGE and Haim's discovery of the solution "outside the triangle" is perfect evidence of Tami's students'-directed creativity.

Teachers' declarative conceptions are those expressed by the teachers' in interviews, conversations and their own descriptions of what does it mean to be creative in mathematics teaching. Teachers' conceptions-in action can be observed in teachers' practice. Teachers' expertise is also expressed in compatibility between their declarative conceptions and their conceptions-in-action (Lev-Zamir& Leikin, in preparation). Tami's case exemplifies her conceptions-in-action regarding creativity in mathematics teaching. I argue that the student-directed nature of declarative conceptions of creativity is a fine predictor for teacher's creative expertise.

FROM CREATIVITY IN TEACHING TO TEACHERS' CREATIVITY

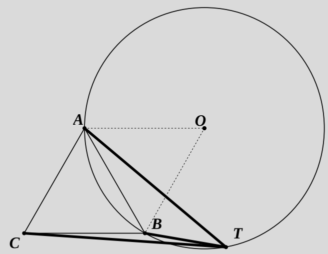
To conclude, this paper suggests that teachers' creativity is an integral component of their expertise and that student-directed mathematical creativity and teachers' creativity-in-action are indicators of their expertise. On the other hand, teachers' student-directed creativity-in-action is one of the main sources of teachers' learning-through-teaching.

When all the solutions were presented in the classroom Tami noticed that the sum of angles "outside the triangle" and "inside the triangle" is 180^o. This observation led to the discovery made by Tami and the new problem that she devolved to her students:

Problem 2:

If ABC is an equilateral triangle and point O satisfies the equality $AO = OB = AB$, then each point T on the circle with center O and radius AB satisfies the equality

$TA^2 + TB^2 = TC^2$.



THE APPROACHES OF DEVELOPING TEACHERS' EXPERTISE IN MATHEMATICS INSTRUCTION IN TAIWAN

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The paper describes ten elements of expertise in mathematics instruction that are highly regarded in Taiwan and groups them around various categories. Furthermore approaches how to foster teacher's expertise are described.

The motivation for studyingTaiwanese teachers'expertise of mathematics instruction derives in part from the findings emerging from TIMSS, PISA, and TEDS-M which show that school students or future teachers in Taiwan surpassed the score of international average. The curriculum that prescribes the mathematics content to be learned is an important factor affecting student achievement. It may be argued that a more important factor is the teachers who deliver the content. Students' mathematics achievement is an indicator of effective instruction and an effect of teachers' mathematics knowledge (Hill, Rowan, & Ball, 2005). The growing interest in examining school mathematics curriculum, high quality classroom instruction, and teacher preparation in an international context is closely related to students' mathematics achievement between Asian countries and their counterparts (Leung & Li, 2010; Li & Kaiser, 2011; Tatto, Schwille, Senk, Ingvarson, Peck, & Rowley, 2008). For example, Leung and Li's book stresses on the policies and practices of curriculum and teachers education of the countries in East Asia, while Li and Kaiser's book focuses on the identification and examination of teacher expertise in mathematics instruction in selectededucational systems between the East and West. TEDS-M study uses the concept of opportunity to learn as central to explain the impact of teacher preparation programs on teacher learning in international contexts (Tatto, et al., 2008). These books provide a platform for sharing and understanding the reform efforts in teacher education from selected high-achieving countries in East Asia. It is important for the rest of world to learn what is happening while improving students' learning. Much remains to be understood about the ways and what culturalresources are utilized to shape the quality of mathematics instruction for improving students' mathematics achievement. The Research Forum is an opportunity for Taiwan to share the approaches and practices that are employed to develop teacher expertise in mathematicsinstruction.

The paper does not focus on the nature of teacher expertise of mathematics instruction valued in different educationsystems; instead, it concentrates on the approaches of developing teacher expertise in mathematics instruction valued in Taiwan. Prior to the development of teacher expertise, what constitutes teacher expertise valued in Taiwan needs to be briefly discussed. The paper begins with a brief description on the nature of teacher expertise in mathematics instruction in Taiwan. It is followed by the development of teacher expertise in mathematics instruction.

NATURE OF TEACHER EXPERTISE IN MATHEMATICS INSTRUCTION

There is no universal agreement on what counts as an expert teacher or teacher expertise in mathematics instruction, while there are similarities and differences on the contents of teachers'expertise between Eastern and Western. Possible differences across countries from the East can be even found in classroom contexts. For instance, expert teacher in Korea has the characteristics of mathematics instruction such as posing questions that further challenge and extend students' activity or mathematical thinking; re-stating in detail or insisted on clear explanation to the student such that the whole class examined the crucial contents; encouraging students to present their methods; and leading students to be actively participated in activities (Pang, 2008). The expertise displayed in mathematics instruction in Taiwan contains at least ten features (Lin & Li, 2011). The expert teachers are skilled in: (1) creating and using tasks with high-level cognitive demands and realistic context for evoking multiple solutions and eliciting the anticipated solutions; (2) sequencing the problems to be posed based on students' learning; (3) predicting students' anticipated solutions; (4) sequencing students' multiple solutions for class discussion based on conceptual development; (5) asking various questions for different purposes; (6) asking key questions in time and asking follow-up questions; (7) interpreting students' productions; (8) highlighting and summarizing the main point at the end of the discussion; (9) transiting from one activity to another corresponding to students' learning; (10) creating specific problems for assessingstudents understanding and as a part of preparation for the next lesson (Lin & Li, 2011). These ten elements of teacher expertise are further clustered into three categories by phases of instruction: (1) prior to teaching, expert teachers master in designing and using tasks that support rich mathematics thinking; (2) during teaching, they purposely selecting and sequencing students' solutions for whole class discussion; critically questioning and using students' errors or misconceptions for discussion; responding to students'questionsadequately, and summing up main points at the end of a lesson; and (3) after teaching, expert teachers skill in creating creative assignments for assessing what and how students learned in the lessons.

APPROACHES OF DEVELOPING TEACHER EXPERTISE

An attempt to helping a novice teacher becoming an expert teacher is commonly enterprise for teacher educators in different educationalsystems across countries. Various approaches have been developed and used in pursing high-quality mathematics instruction in differenteducation systems in East Asia, such as lesson study as an approach for the teachers in Japan (Yoshida, 2008); exemplary lesson for China (Huang &Bao, 2006), and instructional contests for Korea (Pang, 2008); master teachers for Taiwan (Lin, 2008). There is a common feature among these countries that research on teacher development is based on a view of the teacher as an adult learner whose development results from changes in cognitivestructure. The research on teacher development indicates that teachers in teacher professional programs may be different development stages and have very different needs for assistance. Berliner (1989) theorizes that teachers progress through five stages in the journey toward expertise: novice, advanced beginner, competent, proficient, and expert.It implies that: (1) a teacher progressing to be an expert teacher is a long process; (2) a teacher in different development stages needs different assistance. Thus, to be an expert, a teacher needs assistances from various approaches.

The various approaches and cultural resources utilized in Taiwan for assisting teachers developing their expertise include: (1) using textbook accompanying with teacher guides is an approach of helping teachers toward reform-centeredinstruction; (2) taking the instructional contests at the local or national level; (3) lesson study weekly on Wednesday afternoon system-wide; (4) master teachers of mathematicsinstruction in each local area; (5) upgrading academic degree of teachers; and (6) teachers' participation of teacher professional development programs. Due to the limitation of the paper' page space, I only discuss teachers' participation in a longitudinal study of teacher development program as an approach for developing teacher expertise.

This program has been continually funded by the National Science Council since 1997 and the majority of the participating teachers have been accredited as master teachers in mathematics instruction by outside evaluators. There are six in-service teachers that are recruited from the same grade level if at all possible to participate in the program each year. The number of participating teachers in each year is not allowed to be more than eight, in order to maintain each participant for adequate time discussion in professional meetings. They are recruited from the same grade if possible, since the same grade level lends similar contents readily as a base of discussion.

Upon completing the recruitment, several meetings about lesson study are conducted. The lesson study meetings are structured as: before teaching, the instructor to be observed requires accurate identification of teaching objectives,deep analysis of student difficulties in understanding the concepts, read critically the materials in textbook on the basis of students'learning, and it is followed by conducting pre-tests for understanding prior knowledge and what difficulties students have before teaching the lesson. Afterwards, they require restructure and redesign the activities of the lesson and write a brief lesson plan in accordance with students' hypothetical learning trajectory. All participating teachers' lessons were scheduled for observationin turn. In particular, these teachers were scheduled to sit altogether in a classroom to observe a lesson and immediately have a follow-up intensive meeting.

During teaching, the students were divided into groups of 5 to 6. Each participant teacher sits next to students in a group, as the purpose of each student to be observed deeply. The observation is not intended to demonstrate excellent teaching to others rather readily make a focus for the follow-up professional discussion. We do not regularly encourage the instructor to be observed to treat this observation as a demonstration of an exemplary lesson; instead, the observation is expected to be a normal teaching. The classroom observation is the distinguished feature of the teacher professional program from the demonstration of an exemplary lesson engaging in other teacher development in nation-wide. The observers helping the instructor glean the multiplesolutions students coming up the lesson. The instructor' selection and sequence of multiple solutions presented in the lesson are likely to be brought up to the follow-up discussion.

After teaching, the instructorare asked to synthesize and reflect on his/her own teaching and the rest of the participants are invited to articulate what they observed in the lesson with respect to the tasks, students'responses, teacher questioning. Teacher educators in the program in play different roles: facilitators, supporters, and coordinators. The discussion is regularly wrapped withthe framework of the mathematics topic in the lesson, the ideas to be used in the following lesson, the issues to be put in the written cases of teaching, and the items of an assignment for assessing what students learned in the lesson. These activities are used to support participating teachers to improve the quality of mathematics instruction.

In summary, teachers shape and refine their expertise in mathematics instruction through their participation in a longitudinal professional development program, because they have the opportunity to reflect for lesson prior teaching, reflect in teaching, and reflection on lesson after teaching. These reflections are sourced for/from a classroom observation instead of a classroom demonstration, which is the purpose for polishing teaching competences and skills. Thus, their expertise of mathematics instruction is improved gradually.

DEVELOPING KOREAN TEACHER EXPERTISE IN MATHEMATICS INSTRUCTION BY CASE-BASED PEDAGOGY

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This paper describes teacher expertise emphasized in the Korean context and a key issue with regard to increasing elementary teacher expertise in mathematics instruction. It then introduces how a specific case-based pedagogy has beendeveloped and implemented to promote teacher expertise by mathematics-specific analysis ability. This paper is expected to provoke discussion on the nature of teacher classroom expertise and the use of cases to improve such expertise.

TEACHER EXPERTISE IN THE KOREAN CONTEXT

Teacher expertise has received increased concern to improve the quality of mathematics instruction and ultimately to induce students' meaningful learning. The teacher is expected to have solid knowledge of not only mathematics and pedagogy but also student mathematical learning, to provide students with worthwhile mathematical tasks that stimulate their intellect, to orchestrate productive mathematical discourse on the basis of students' contributions in the supportive learning environment, and to analyse her teaching practice against student learning (NCTM, 2007).

Similarly, the Korea Institute of Curriculum and Evaluation (KICE) announced the criteria of assessing mathematics instruction in terms of teacher professional knowledge, planning, implementation, and professionalism (Im&Choe, 2006). Such criteria are closely related to teacher classroom expertise. This expertise covers all sorts of aspects related to mathematics lessons from a teacher's knowledge to her reflection on teaching practice and professional development. As such, the conception of teacher expertise is complex and comprehensive in nature.

Moreover, new aspects have been added to teacher expertise through the recent revisions of national mathematics curriculum. For instance, it has been traditionally valued in Korea that the teacher emphasizes mathematical concepts, principles, and laws on the basis of meaningful questions, while fostering students' problem-solving ability (MOE, 1997). The recent curricular revisions call for additional teacher expertise in enhancing mathematical communication ability, mathematical reasoning ability, and mathematical creativity as well as building students' character as mathematics learner (MEHRD, 2007; MEST, 2011).

Teacher expertise is ultimately related to carrying out effective mathematics instruction. For this reason, teacher expertise has been studied mostly with regard to the process of mathematics teaching and learning. Various approaches to enhance teacher classroom expertise have been implemented in Korea, noticeably through instruction-research contests for teachers as well as voluntary activities by groups of

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teachers to improve their own instruction. Such contests are organized by the educational offices of each province in Korea. In order to identify high-quality instruction in a practical manner, articulate and fragmentary elements are used with the concentration on the teacher's noticeable behaviour during her classroom teaching. Such elements include lesson design, learning environment, learning objective, research topic, student activities, teaching techniques, summary of learning, attainment of a goal, questioning, and instructional materials (Pang, 2009).

The lessons awarded a prize in instruction-research contests are available to the public in video clips via specific websites. They may convey the overall characteristics of effective instruction valued in Korea. However, such lessons do not necessarily reflect on effective *mathematics* instruction, especially at elementary school level, because the criteria of instruction-research contests are not specific to subject matter. In fact, Jang and Pang (2011) report that only 43% of such lessons authorized by the educational offices were analysed either as good or outstanding level in terms of the manual of evaluating mathematics instruction developed by KICE. Similarly, Pang (2011a) examined subtle but important differences in terms of two elementary teachers' expertise in sustaining mathematically significant discourse, even though they established similar social participation patterns. This leads us to explore teacher expertise in terms of paying attention to the mathematics-specific features of a lesson beyond general features which can be common across multiple subject matters.

DEVELOPMENT AND USE OF CASE-BASED PEDAGOGY TO IMPROVE TEACHER EXPERTISE BY MATHEMATICS-SPECIFIC ANALYSIS ABILITY

Building on the increased use of cases to improve mathematics teacher expertise, this paper introduces a specific case-based pedagogy used in Korea with videotaped mathematics lessons and their analytic narrative. The term case-based pedagogy is used to underline a series of pedagogical flow by which teachers not only analyse others' teaching practice but also design, implement, and reflect on their own instruction both individually and collectively. Such pedagogy aims at providing teachers with knowledge and skills to analyse and reflect on a lesson by mathematics-specific ways beyond superficial features.

Teacher expertise by mathematics-specific analysis ability is demanding more for elementary school teachers than for secondary counterparts because they are educated to teach all subjects, which may hinder them from understanding the substantive characteristics of a *mathematics* lesson. This challenge may not easily be solved through their long-term teaching career, partly because common criteria across different subject matters have been used to analyse all elementary instruction. This is the main reason why teacher expertise in terms of mathematics-specific analysis ability is emphasized in this paper.

In order to develop a case-based pedagogy, the videotaped mathematics lessons were first collected from various resources such as public lessons recognized as effective instruction from teaching contests and mathematics teaching demonstrations by expert teachers. In addition, some lessons were purposefully planned and implemented to address key ideas of mathematical teaching and learning which might be difficult to observe in ordinary classrooms. These collected lessons were analysed in terms of productivity of the lesson to raise important issues of mathematics teaching and learning, the specificity of the lesson to understand what happens in the classroom, and the ability of the lesson to represent big mathematical ideas taught across grade levels.

As for the selected lessons, comprehensive narrative cases were developed with two purposes. One was intended to help teachers contextualize the videotaped lesson. For this purpose, the written case included an overview of the case, a detailed description of the lesson, and supplementary materials. More importantly, the written case use was intended to foster teacher expertise by supporting watching, analysing, and reflecting on the specific lesson. For this purpose, the written case included theoretical background, focused analysis, and additional analysis. Specifically, focused analysis is intended to build appreciation in teachers of a mathematics-specific analysis closely related to each case and to demonstrate how to make reasoned judgments of main events. As such, each written case is comprehensive and lengthy.

IMPLEMENTATION OF CASE-BASED PEDAGOGY AND ITS IMPACT ON TEACHER EXPERTISE

The case-based pedagogy has been employedfor both prospective and practicing teachers either by a regular university-based course or by an intensive workshop. The general procedure was similar to different groups of teachers. In the first phase, the main focus was given to discuss the developed cases. The participant teachers were asked to read a part of a given written case, specifically from ‘overview of the case’ to ‘detailed description of the lesson’ in advance. They were then asked to write down whatever stood out while watching the videotaped lesson together. The lesson was extensively discussed on the basis of their comments. Only after this discussion were the teachers encouraged to read the rest of the each written case, specifically from ‘theoretical background’ to ‘focused and additional analysis of the lesson.’ This was intended to help them summarize what they had discussed and to improve their analytic ability.

In the second phase of implementing a case-based pedagogy, the participant teachers were asked to videotape their mathematics lessons and to write a report on their lesson design, implementation, and reflection. For this, prospective teachers used their practicum period, while practicing teachers used their regular mathematics classrooms. Each teacher had an opportunity to present her report with video clips on her lesson and received various feedbacks from peers as well as the instructor.

The impact of case-based pedagogy on teacher expertise appeared in several ways. Most of all, participant teachers’ analytic focus from the early to late comments shifted to attend more to the mathematics-specific features of a lesson. In the early comments, teachers focused more on the classroom atmosphere and general teaching strategies, but in the late comments, these foci decreased substantively. Instead they focused more on mathematical tasks, teaching strategies specifically related to the content to be taught or students' characteristics, mathematical communication, and students' mathematical thinking (see Pang, 2011b for a detailed analysis with regard to prospective teachers). The impact of case-based pedagogy was not limited to the participant teachers'skilled discussion of cases. More importantly, they were able to apply such analytic focus to their own teaching practice and other contexts without specific prompts.

Given the contextual background, this paper illustrates how a specific case-based pedagogy was developed and implemented in Korea to increase elementary teacher expertise in terms of mathematics-specific analysis abilities. As there is little research that confirms cases as pedagogical tools to improve teacher expertise in Asian contexts, this paper is expected to provoke discussions on case use to enhance teacher expertise by mathematics-specific analysis ability across different education systems.

NURTURING EXCELLENCE IN MATHEMATICS INSTRUCTION: SINGAPORE'S PERSPECTIVE

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In Singapore, two initiatives in particular have been launched by the Ministry of Education since 1997 in support of enhancing the practices of teachers to advance excellence in classroom instruction. Along with these initiatives, systemic infrastructure has also been provided to support teachers in their development and learning journeys. The systemic infrastructure exists both at the school and national level. Specific practices exist for mathematics teachers, through which they develop their expertise in exemplary mathematics teaching. Some of these practices are Lesson study, Action research, Research project partnerships, and Professional development activities organised by university scholars as well as master and senior teachers.

BACKGROUND

In Singapore a number of initiatives have been launched by the Ministry of Education since 1997 to enhance the practices of teachers to advance excellence in classroom instruction. The first of the initiatives was the Thinking Schools, Learning Nation (TSLN) vision launched in 1997 (Goh, 1997). This vision places emphasis on the need for teachers to be lifelong learners so that schools keep abreast of advances in knowledge and learning both at the national and international fronts.

The second initiative, following TSLN in 2005, was the Teach Less, Learn More (TLLM) initiative (Shanmugaratnam, 2005). TLLM builds on the groundwork laid in place by the systemic and structural improvements under TSLN, and the mindset changes encouraged in Singapore schools. It continues the TSLN journey to improve the quality of interaction between teachers and learners, so that learners are more engaged in learning and better achieve the desired outcomes of education. TLLM aims to touch the hearts and engage the minds of learners, to prepare them for life. It reaches into the core of education - why we teach, what we teach and how we teach. It is about shifting the focus from “quantity” to “quality” in Singapore’s education. It emphasizes “more quality” in terms of classroom interaction, opportunities for expression, the learning of life-long skills and the building of character through innovative and effective teaching approaches and strategies. It also emphasizes “less quantity” in terms of rote-learning, repetitive tests, and following prescribed answers and set formulae.

Systemic infrastructure has been put in place to support the TSLN and TLLM initiatives. Arising from these initiatives, several specific approaches have also been adopted by teachers to embark on their journeys toward excellence in instructional practices. In the following sections, the systemic infrastructure that is prevailing for teachers in Singapore will be described. Next, specific practices adopted by mathematics teachers in particular will be detailed.

SYSTEMIC INFRASTRUCTURE

In support of TSLN vision, as of 1998 all teachers in Singapore are entitled to 100 hours of training and core-upgrading courses each year to keep abreast with current knowledge and skills. The Professional Development (PD) is funded by the Ministry of Education. To support teachers in mapping their learning trajectories, in 2005 the MOE implemented an Enhanced Performance Management System (EPMS) (MOE, undated). The EPMS is an appraisal system that contains rubrics pertaining to fields of excellence in the education system be it teaching, leadership or senior specialist. These rubrics delineate very clearly the competencies deemed necessary at each level and hence teachers are entrusted with responsibility of their own PD. The entitlement of 100 hours of PD and EPMS as an appraisal system for teachers has created a significant buzz amongst them for learning opportunities.

For teachers to work collaboratively at the school level, in September 2005, in support of the TLLM initiative “white space” was introduced. This is time-tabled time for teachers during curriculum hours to meet, plan and deliberate on their instructional practices. To provide structure for teachers’ collaborative work at the school level, in 2010, the Ministry of Education, unveiled the Professional Learning Communities (PLCs) framework (TDD, 2010). This framework encourages the formation of Learning Teams in schools. These teams have the choice of adopting a range of collaborative methods/tools, such as Learning circles, Action research and Lesson study, to improve instructional practice through development in subject content knowledge and pedagogy.

In 2009, the Academy of Singapore Teachers (AST) was formed. The subject chapters at the academy are led by master teachers. For mathematics there are three subject teachers and the key objectives of the chapter are to i) raise the professional standard in the learning and teaching of Mathematics, ii) serve as a focal point for teacher collaboration and networking, and iii) build a culture of professionalism and pride within the fraternity of Mathematics teachers.

SPECIFIC PRACTICES

In this section, three practices that are commonly used for improving teachers' expertise in mathematics instruction are detailed. The three practices are Lesson study, research projects and professional activities to suit individual needs.

Lesson study

Lesson study, an important tool utilised in Japan to facilitate teachers' collaborations and professional development is also presently commonly used in Singapore for the same purpose. The adoption of lesson study from Japan by educators in Singapore began around the year 2005 (Fan, Lee and SharifahThalha, 2009). In a research project, believed to the first on Lesson study in Singapore, conducted by Fan, Lee and SharifahThalha (2009) from 2006-2007, it was found that through the actions of planning, teaching, reflecting, and revising, teacher participants deepened their knowledge and skills which resulted from the diverse community that worked together in the study. It was also found to be a good means of mentoring the beginning teachers by senior teachers in a school.

Dr Yeap Ban Har, principal of the Marshall Cavendish Institute in Singapore is presently leading numerous Lesson study groups of mathematics teachers. One can follow the chronological development of the groups on

http://singaporelessonstudy.blogspot.com

Research projects

With a quest for professional learning and development that reflects a gradual shift in the centre of gravity away from the University-based, "supply-side", "off-line" forms of knowledge production conducted by university scholars for teachers towards an emergent school-based, demand-side, on-line, in situ forms of knowledge production conducted by teachers with support from university scholars, schools are now welcoming researchers from the university into their schools to work with teachers.Such work is made possible by the Centre for Research in Pedagogy and Practice (CRPP), which was set up in 2002 and funded by the Ministry of Education. CRPP encourages and supports academics from the National Institute of Education to situate their research projects in schools and work with school communities.

Examples of two such projects are the *Enhancing the pedagogy of mathematics teacher (EPMT) project* (Kaur, 2011; 2009) and the *Think-Things-Through (T^3) project* (Yeap&Ho, 2009). The aims of the EPMT project were three fold: to provide teachers with training, to facilitate teachers' work (practice and feedback) at the school level and to enthuse and support teachers to contribute towards the development of fellow teachers. The deliverables, namely resources crafted by teachers (Kaur&Yeap, 2009a; 2009b, Yeap&Kaur, 2010) of the project have contributed to several school-based professional development activities that have had positive impact on classroom practice of many teachers in Singapore.

The T^3 project investigated the effects of the use of word problems that require the consideration of context on students, teachers and classroom environment over a period of three years. The secondary goal of the project was to study teacher change in an informal professional development programme.

Professional activities to suit individual needs

The EPMS entrusts teachers with the responsibility of developing in their fields of work, specifically teaching in this case. Teachers are guided by their mentors in school and self to pursue professional activities that address their needs. For teachers who wish to pursue further professional qualifications, they may enrol for higher degree courses at the universities. Others may choose to enrol for relevant short in-service courses, workshops, seminars and institutes. These professional learning activities are conducted by university academics, master teachers, and senior teachers.

Professional bodies such as the Association of Mathematics Educators (AME), Singapore Mathematical Society (SMS) and the Academy of Singapore Teachers (AST) are active in providing professional development and learning activities for mathematics teachers on a regular basis.

CHALLENGES ASSOCIATED WITH CONCEPTUALIZING AND DEVELOPING TEACHER EXPERTISE IN MATHEMATICS INSTRUCTION

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Researchers from five education systems – Portugal, Israel, Taiwan, South Korea and Singapore – address the theme of this Research Forum: conceptualizing and developing teacher expertise in mathematics instruction. The five contributions are different from each other in what and how they address the Research Forum theme, allowing a glimpse into various perspectives on what an expert teacher is, and on ways of developing expertise in mathematics instruction in different education systems.

Two researchers – João Pedro da Ponte from Portugal and Pi-Jen Lin from Taiwan – describe how expertise in mathematics instruction is viewed in their education systems. They situate this perspective in their national context.

João Pedro da Ponte portrays a new perspective in Portugal about what an expert teacher is,

> It is a teacher who (i) is able to select and perhaps adjust suitable tasks, especially exploratory tasks, involving students actively in mathematical work, stimulating them to develop their own strategies, concepts, and representations and (ii) to conduct classroom discussions that create opportunities for negotiation of meaning, development of mathematical reasoning, and institutionalization of new knowledge.

This perspective of teacher expertise in mathematics instruction is associated in Portugal with the introduction of a new mathematics curriculum that emphasizes the importance of using in mathematics instruction a variety of mathematics tasks, including exploration and investigation tasks, as well as the value of diverse communication processes, including collective discussions.

Pi-Jen Lin from Taiwan outlines 10 elements of expertise in mathematics instruction that are highly regarded in Taiwan. These 10 elements are grouped into three categories by phases of instruction (before, during, and after classroom teaching). In contrast with the description portrayed by João Pedro da Ponte, Pi-Jen Lin explicitly refers to the after-class phase, including assessment as a key aspect of expertise in mathematics instruction in Taiwan:

> (1) prior to teaching, expert teachers master in designing and using tasks that support rich mathematics thinking; (2) during teaching, they purposely selecting and sequencing students' solutions for whole class discussion; critically questioning and using students' errors or misconceptions for discussion; responding to students'questionsadequately, and summing up main points at the end of a lesson; and (3) after teaching, expert teachers skill in creating creative assignments for assessing what and how students learned in the lessons.

Pi-Jen Lin describesvarious approaches for developing teacher expertise in mathematics instruction that are valued in Taiwan, focusing on a longitudinal professional development program that encourages teacher reflection. She argues that by reflecting before, during, and after the lesson, teachers polish teaching competences and skills, and gradually improve their expertise of mathematics instruction.

The other three contributors do not explicitly attend to how expertise in mathematics instruction is viewed in their own country. For example, RozaLeikin from Israel examines what is entailed by expertise in mathematics instruction, not overtly connecting it to the Israeli context. She argues that creativity is an integral component of expertise in mathematics instruction. Leikin adopts a definition of creativity that is commonly used is relation to ideas to teacher classroom behavior, claiming that expertise in mathematics instruction is characterized by fluency, flexibility, originality and elaboration.

JeongSuk Pang from South Korea attends to a particular difficulty to improving expertise in mathematics instruction that is rooted in the way elementary school teaching is structured in many countries, including South Korea,

> Teacher expertise by mathematics-specific analysis ability is demanding more for elementary school teachers than for secondary counterparts because they are educated to teach all subjects, which may hinder them from understanding the substantive characteristics of a *mathematics* lesson.

JeongSuk Pangreports on interesting research findings, which reveal that often mathematics lessons that are identified as good or even excellent bynon-specialized educators in Korea are not evaluated as such by mathematics-specialized educators. To shift teachers' attention more to mathematics-specific features of a lesson, a specific case-based pedagogy was developed and is used in Korea.

BerinderjeetKaur reports on several recent nation-wide attempts developed and implemented in Singapore to nurture excellence in mathematics instruction. She describes a systematic infrastructure provided in her country to support the professional development of practicing teachers. Mathematics teachers in Singapore are offered a variety of possibilities to support the development of expertise, such as, lesson study, research projects, and professional activities to suit individual needs.

This collection of five contributions raises interesting questions. I suggest a few below that could serve as a basis for discussion of challenges that are associated with conceptualizing and developing teacher expertise in mathematics instruction.

Do the above contributions reflect different or similar views on what expertise is? Do they reflect different or similar views on ways to develop teacher expertise in mathematics instruction in different education systems? From what is presented in the five contributions it is not easy to answer these questions. There is a need for more information and specific details in order to examine similarities and differences in conceptualizing teacher expertise in mathematics instruction and its development in different education systems.

Another significant question that emerges from this collection of five contributors is how developing teacher expertise and teacher professional development are related to each other? None of the contributors to this Research Forum proposes a distinction between these terms nor approached the development of teacher expertise as different from teacher professional development in general. This lack of clear distinction between the two terms might be related to viewing the development of teacher expertise as a synonym term to teacher professional development. In this case, this needs to be explicitly stated. Yet, this lack of clear distinction between the two terms might be related to the fact, stated by the organizers of this Research Forum, that "understanding of the conception and nature of teacher expertise in mathematics instruction is still very limited." This further leads to another crucial question: If we do not know what expertise in mathematics instruction is, how can we develop it?

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DISCUSSION GROUPS

RESEARCHERS' AND TEACHERS' KNOWLEDE IN MATHEMATICS PROFESSIONAL DEVELOPMENT

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DISCUSSION GROUP GOALS

In his PME-35 address, Krainer (2011) noted: "One important issue concerning researchers' and teachers' *production of knowledge* is the question of how researchers' and teachers' knowledge is interrelated and exchanged" (p. 50, emphasis in the original). He criticized the research-development-dissemination model of innovation, which embraced a technical rationality to support the unidirectional approach in which knowledge resides with researchers and is passed on to teachers. However, Krainer suggested that from a reflective rationality stand, there was potential for examining teachers' production of knowledge in environments in which external interventions from researchers were present. He suggested that intervention research could apply researchers' knowledge while also generating local knowledge.

The goal of this discussion group, then, is to engage participants in reflections around the ways in which researchers and teachers foster knowledge production in mathematics professional development settings. Participants will examine different ways in which researchers and practicing teachers interact, while considering how such interactions support local and academic knowledge. In particular, participants will consider the role of researcher-produced knowledge in professional development settings and how teachers appropriate and transform such knowledge in practice.

PLANNED ACTIVITIES FOR THE GROUP

The two 90-minute meeting will focus on the sharing of participants' experiences, successes, and challenges in working with teachers and considering the role of research and knowledge production in mathematics professional development. In the first meeting, this sharing will be facilitated by each of the three organizers of the group leading a 25-minute discussion based on her or his own work with practicing mathematics teachers and her or his reflections on the relation between researchers' and teachers' knowledge. At the end of the meeting, participants will spend about 15 minutes creating their own reflection statements about the interrelation and exchange of researchers' and teachers' knowledge within their own work. The first hour of the second meeting will be used to share and discuss participants' generated reflection statements. In the last 30 minutes, the group will summarize major ideas discussed, generating a set of questions and issues that require further examination by the field.

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VISUALIZATION IN MATHEMATICS EDUCATION: TOWARDS A FUTURE RESEARCH AGENDA

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This discussion group provides participants with the opportunity to consider how ideas of visualization have been used in mathematics education research to date and to discuss possible future research directions.

Session 1

A comprehensive survey of PME research on visualization in mathematics education is provided by Presmeg (2006). In this first session, key aspects of this survey are considered alongside a summary of definitions of visualization from the wider research community contained in Phillips, Norris and Macnab (2010). The distinctions that they make between *visualization objects*, *introspective visualization*, and *interpretive visualization* can be discussed in light of Gutiérrez' (1996) use of the terms *external representation*, *mental image*, and *process* (*of visualization*).

Session 2

In this session we consider how, or even if, the ways that visualization is conceptualized and studied changes when different theoretical framings are considered. We will consider visualization in light of the following, as well as being open to include theoretical perspectives offered by participants: Sfard's (2008) argument for a move from "learning-as-acquisition" to "learning-as-participation"; Lakoff & Johnson's (1999) embodied cognition that suggests that individuals' perceptions and interactions guide their conceptual and communication structures.

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EMBODIED COGNITION AND NEUROSCIENCE IN MATHEMATICS EDUCATION RESEARCH

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RATIONALE

Over the past quarter decade or so, it has become widely recognized in the mathematics education research community that human cognition, and mathematical cognition in particular, is embodied cognition. Over this time, scholars and researchers in mathematics education have been theorizing about presuppositions and implications of such a view. Indeed, much has been done in constructing theoretical frameworks for embodied cognition, e.g., from metaphors to gestures to neural manifestations thereof. Concomitantly, a substantial amount of empirical work in mathematical cognition has been coming from the neurosciences (e.g., Dehaene, Butterworth), which aims to ground cognition and learning in brain activity. Relations between mathematics education research and the neurosciences, especially with regard to embodied cognition, and particularly with respect to brain lesion and imaging studies, provide fecund and salient topics for group discussion at PME.

EMERGENT TOPICS

Questions such as the following will provide focal points for this Discussion Group: What is the nature of embodied cognition with regard to neuroscience? What is known and what can be learned from neuroscientific results and approaches to mathematical thinking? What new methods are available to us as we design experiments to test understandings of mathematical cognition as embodied cognition? What is/are the relation/s between subjectively experienced mental actions and objectively observable behaviour with respect to brain activity? How and in what ways are brains engaged in mathematical cognition and learning, and why should it matter to mathematics educators and researchers in mathematics education? How do we ascertain the theoretical and practical relevance of neuroscientific studies? Put differently, what is the ecological validity and educational relevance of such studies?

GOALS AND ACTIVITIES

Our goal is to facilitate discussion in this area by providing brief synopses of previous work in embodied cognition and recent initiatives in mathematics education research and neuroscience. We will use “break out” and “plenary” group discussions as warranted and agreed upon by those attending and participating. We anticipate that discussion and further elaboration of these issues will eventuate in more focused and clearly delineated working group initiatives in subsequent PME conferences.

2012. In Tso,T. Y. (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 153. Taipei, Taiwan: PME.

TEACHING-RESEARCH IN 21ST CENTURY

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With the introduction of the Common Core standards in Mathematics in US in 2014 a new pedagogical framework will be required to assure standards‘ fulfilment, called adaptive instruction that is instruction which adapts itself to the nature of mathematical thinking of students. The process of adaptation requires clear identification of students‘ strength and weakness through ongoing assessment, understanding the role of the challenging concept in the schema of relevant mathematics, and design special strategies or teaching sequences addressing the challenge. This series of activities is a significant component of Teaching-Research activity. In fact the brief produced by the Consortium of Public Research in Education (Daro et al. 2011) asserts explictly: "*Teachers must receive extensive training in mathematics education research on the mathematics concepts that they teach so that they can better understand the evidence in student work (from OGAP-like probes or their mathematics program) and its implications for instruction. They need training and ongoing support to help capitalize on their mathematics program's materials, or supplement them as evidence suggests and help make research based instructional decisions*."

The goal of the Discussion Group **Teaching-Research in 21 Century** is to reflect upon the question To what degree are the practitioners of teaching – research ready to help teachers to transform their teaching into the Teaching-Research approach?

The first session activities will be the guided discussion on the state of Mathematics Teaching-Research at present, its variety of methods, successes and challenges, approaches to Professional Development of Teacher-Researchers and to the development of TR methodology. The essential role of community college mathematics faculty in this process will be explored. The aim of this session is to prepare the second session when the Plan of content and of writing process of the Handbook of Teaching-Research as an Adaptive Instruction will be discussed, agreed upon and designed as the Working Group for the next year annual meeting.

2012. In Tso,T. Y. (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 154. Taipei, Taiwan: PME.

MATHEMATICAL BEAUTY

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Mathematical aesthetics has been cited as one of the most underresearched areas in mathematics education (ESM 2002). The lack of research on this topic is particularly striking compared to its importance in the every day working practices of mathematicians. Dreyfus and Eisenberg attempted to jump-start the topic during PME9 with modest success; some twenty years later, Sinclair began a serious research program which has contributed to our understandings of the role of aesthetics in mathematical practice (2004) and school classrooms (2001). The goal of this discussion group is to build on the momentum of Sinclair and other researchers to build a community of researchers doing serious work in this area.

The aims of the discussion group are to:

- Provide an overview of research in this area, from the fields of mathematics education, history and philosophy of mathematics, and cognitive science
- Discuss examples of proofs, theorems, ideas, etc commonly held to be beautiful.
- Discuss the relevance of research in this area to teaching, including but not limited to curriculum documents, textbooks, teacher practices, and student experiences.
- Discuss different practices/attitudes towards beauty in the teaching of mathematics in different countries.

This is a rather broad agenda for a discussion group, in part because the goal at this stage is to be inclusive and bring together people with diverse expertise. Session 1 will include a short overview of the literature related to mathematical beauty and an open discussion about members' interests related to the above themes. Session 2 will focus on developing specific research questions that could be pursued in the coming year. All are welcome!

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WORKING SESSIONS

PME 36

TAIWAN

2012

LEARNERS' VALUES: THEIR ANALYSIS AND DEVELOPMENT

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VALUES

The topic of values has been in the research media for some years now. It has a history of exploration, with speculative articles, theoretical developments, and exploratory projects being the norm. In some cases the research has focused on values as they are carried by the history of mathematics and the content of mathematical curricula. Other research has explored teachers' values and whether and how they can be developed. Rarer is research about students' values, and this area has proven to be the most intransigent – although arguably the most important – of the three foci.

Presumably the main reason for the existence of this research is to improve the possibilities and potential of mathematics learners. However, the difficulties of doing this research mean that in the main it has been undertaken on a small scale, within a small learner population, with a few sympathetic teachers, and each study conducted within one cultural and societal tradition. This Discussion Group intends to focus on the potential of carrying out more extensive research in a cross-cultural, cross-societal, and cross-lingual manner, using the PME context as the vehicle.

GOALS

Thus the aims of this Discussion Group are:

- To contrast and if possible synthesise ideas from colleagues' diverse research on students' values
- To hear about, and discuss the different methodologies being used by colleagues in researching this area, but chiefly
- To develop some international collaborative research projects for the next years

PLANNED ACTIVITIES

Initial readings and discussions would have begun through a dedicated email list and an online discussion forum before the PME 12 conference. Main themes arising from these will feed into the discussions at the Conference. Anyone present at PME 12 is welcomed to attend our discussion sessions of this Discussion Group.

2012. In Tso,T. Y. (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 159. Taipei, Taiwan: PME.

THE LEARNING AND DEVELOPMENT OF MATHEMATICS TEACHER EDUCATOR-RESEARCHERS

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This Working Session [WS] is a follow up of WS5 on the same topic at the PME 35 conference (Goos, Chapman, Brown, & Novotna, 2011). WS5 built on Discussion Group 4 of PME 34, facilitated by the same team. The Discussion Group explored a range of theoretical perspectives on the learning and development of university-based mathematics teacher educators, a new field of study in which there had been little research to date (Llinares & Krainer, 2006). WS5 focused on research proposals and projects emerging from the Discussion Group. It concluded with participants outlining their plans for collaborating with each other on newly identified research questions dealing with their own learning as mathematics teacher educators working with prospective and practising teachers or the learning of other mathematics teacher educators who are their research participants. Thus, the aim of this follow up WS is to provide opportunities for feedback on research that has been completed or is in progress and for participants to continue working together towards producing a publication. We expect to have sufficient material to propose a journal special issue or edited book based on the work of this group of researchers in order to make a meaningful contribution to the field.

In the first session, participants' completed and in-progress projects will be presented and discussed in order to provide feedback on theoretical perspectives, methodology, and initial or final results.

In the second session, outcomes of the first session will serve as the basis for the type of assistance to offer participants in terms of their research or producing a research manuscript. The WS coordinators will engage with participants in small groups, assisting them to sketch out manuscripts and will discuss plans for a proposal for a WS product.

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2012. In Tso,T. Y. (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 160. Taipei, Taiwan: PME.

EMBODIMENT, GESTURE AND MULTIMODALITY IN MATHEMATICS

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The central purpose of the Working Session is to examine mathematical thinking, learning and communication from the perspective of embodied cognition. From this perspective, mathematics is not an abstract “field”, but a set of practices and knowings derived from physical and intellectual experiences. One tenet of embodied cognition is that mathematics, like other forms of knowledge, is created and communicated via multiple material modalities. These modalities include gesture, speech, written inscriptions, and physical and electronic artefacts. Each modality may contribute in complementary ways to the construction of knowledge, and each can be utilized as a data source in research into mathematical thinking, teaching and learning

The frameworks of cognitive linguistics and semiotics are utilized as analytic tools (Fauconnier & Turner, 2002; Lakoff & Núñez, 2000) in seeking the foundational conceptual structures underlying mathematics as well as the role it plays within language and specific cultures.

The session will be participatory, and attendees will be asked to share videotaped data, interpretations, theories and ideas. Themes and topics addressed in previous years include:

- Gesture and semiotics
- Conceptual integration and conceptual metaphor
- Gesture and embodiment in young children and blind students
- Dynamic geometry and other computer-based tools
- Graphing and other visual modalities
- Language, culture and the body in mathematics

The session will not be limited to these topics, but will be based on the interests of the participants.

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FACTORS THAT FOSTER OR HINDER MATHEMATICAL THINKING

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Stephen Lerman — London South Bank University

This working group builds on the mathematical thinking theme of the PME35 and the previous studies conducted at PME's on that topic. Mathematical thinking is considered to be central to doing mathematics (Burton, 1984; Mason, Burton, & Stacey, 1982) and, in general, is defined as the aspect of thinking processes used in doing mathematics (Chapman, 2011). Important aspects of mathematical thinking are: representing, defining, visualizing, generalizing, classifying, conjecturing, inducing, analyzing, synthesizing, abstracting, proving, formalizing, or modeling (Ubuz, 2011). Empirical findings based on particularly task-based interviews provide a sizable body of evidence that there are unique and joint effects of mathematical thinking aspects on one another. The development of mathematical thinking can be fostered or hindered by various factors such as classroom atmosphere, social and cultural context, assessment, and use of technology (Ubuz, 2011). Considering all these points in mind, the group will focus on particular questions such as:

(a) What is the nature and structure of mathematical thinking?

(b) How are the different aspects of mathematical thinking related to each other?

(c) What are the factors that affect mathematical thinking processes?

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2012. In Tso,T. Y. (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 162. Taipei, Taiwan: PME.

NATIONAL PRESENTATIONS

THE CURRENT EDUCATIONAL SYSTEM AND HISTORICAL DEVELOPMENT OF MATHEMATICS CURRICULUM

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This paper reports on Taiwan current educational system, historical development of mathematics curriculum, and also, research studies on mathematics curriculum in the country. The paper begins with an introduction on Taiwan, its location, population, education in history, and the existing education system with official figures on the number of schools, teachers, and students. The second section is on the historical development on the mathematics curriculum for elementary, middle, and high schools. During each announcement or revision, the documentation included content coverage and major changes in instruction and so each reform also called for teachers' change. Finally, there is a review on local research studies on mathematics curriculum, its comparison to other countries, textbooks and influences on students' learning.

This section reports on the current educational system and historical development of mathematics curriculum. It also includes studies on mathematics curriculum. The three parts are as follows:

PART A. THE CURRENT EDUCATIONAL SYSTEM

About Taiwan

Geographically, it is an island located in the SE of Asia which is convenient to multiple pathways by sea or by air. The population is 23, 000, 000 and its size is 36000 square kilometers.

Education in History

In the year 2011, which is the 100th year of Taiwan, a book on one hundred years in education was published by National Academy for Educational Research. Among the 18 chapters, two of them were on curriculum: curriculum history; and, revisions on standards (http://data.nioerar.edu.tw/public/Data/191518565171.pdf#page=259). The curriculum history chapter divides a hundred year's of education history into four periods. The other chapter reported on the revisions of elementary curriculum standards since 1949 (Ou, 2011). The chapter presents a comprehensive historical account on each revision in elementary curriculum standards as a whole, its meaning and discussed the background for each change. The effort on re-structuring was on the way since 1990 and the establishment of National Academy for Educational Research was finalized in 2010.

2012. In Tso,T. Y. (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 165-177. Taipei, Taiwan: PME.

The current education system

I. The current education system

It supports 22 years of academic study. The education process includes 2 years of preschool education, 6 years of elementary school, 3 years of junior high school, 3 years of senior high school or vocational education, 4-7 years of college or university, 1-4 years for a master's degree program and 2-7 years for a doctoral degree program. (Appendix I: The current education system)

II. Number of schools, teachers, students

Ministry of Education (2011 data), Directorate General of Budget, Accounting and Statistics, Executive Yuan (http://140.111.34.54/statistics/content.aspx?site_content_sn=8869)

TAIWAN		Taipei		Kaohsiung	
21 States		1 State		1 State	
2,659	primary	153	primary	241	primary
742	junior high	62	junior high	79	junior high
3,401	schools	215	schools	320	Schools
99,528	primary	10,871	primary	10,307	primary
51,188	junior high	5,538	junior high	5,852	junior high
149,716	teachers	16,409	teachers	16,159	teachers
1,457,004	primary	139,259	primary	164,392	primary
873,220	junior high	90,149	junior high	99,403	junior high
2,330,224	students	229,408	students	263,795	students

Table 1: Number & schools, teachers and students in Taiwan (2011 data)

III. Compulsory Education

Introduced in 1968 with a 9-year program which included 6 years of elementary education and 3 years of junior high. The MOE is preparing to implement a 12-year compulsory education program that will integrate primary education, junior high and senior high or vocational education in School Year (SY) 2014.

IV. Higher Studies

Among the various disciplines, graduate institutes in Mathematics Education or Science Education that include Math were founded since 1960. The programs allow students to pursue studies (full-time or part time in-service) for master degrees (NTNU: 1960; NKNU: 1980; NCUE: 1994); and, also for doctoral degrees (NTNU: 1987; NKNU: 1999; NCUE: 1999).

PART B. HISTORICAL DEVELOPMENT OF MATHEMATICS CURRICULUM

Taiwan has undergone many revisions of its mathematics curricular in the past: 1968, 1975, 1993, 2000, 2003, 2008, 2011; and each of these revisions was marked by its own philosophy and was a product of the socio-cultural environment of that era (Tam, 2010). Appendix II shows when there is a revision in standards for elementary, middle school, and high schools (Chen, 2007).

A chapter on a brief introduction to the curricular reform efforts pursued in Taiwan and a presentation of some selected features of the recent editions of Taiwan's mathematics curriculum is given in Tam (2010). It also includes a study on 4 curricular in history (1962, 1968, 1975, and 1973) as indicated by textbook coverage.

As reported in this chapter, for 1962 and the 1968 editions, topics like integer, fraction, percentage, and currency were covered relatively more than in the other curricular. In the 1975 edition, topics like calculator, abacus, plane figures, and coordinate were covered relatively more than in the other curricular. In the 1993 curriculum, topics like area, weight, capacity, angle, time and statistical diagram were covered more than the other editions; more emphasis on measurement and statistics.

Research on curriculum reform

From Appendix II, we see that the reform over the past twenty years is frequent. Starting from the year 2000, the curriculum standards combined elementary and middle into one document, called Nine Year Alignment in Curriculum. The combined effort also indicated more communication of elementary and middle educators. The change in curriculum is exhibiting not only contents switch but also changes in teachers' roles. For instance, in the reform in Nine Year Alignment in curriculum, teachers are no longer users of curriculum. They are designers of curricular materials. This change exerted a pressure on teachers and created a tension in implementation. Tam (2010) remarked that this frequent switching of curricular during the recent rounds of reform effort has created a unique problem that deserves careful attention. This is because when new curriculum replaced the old one on a grade by grade status, there is a need to bridge over this discrepancy and to ensure continuity, which is most unfortunate for students and teachers. To close, whenever there is a change in curriculum, there is also a call for teachers' change. For example, in the Nine Year Alignment in Curriculum, a main emphasis is on encouraging mathematics teachers to develop school-based curricular that could suit better to the needs in their own classrooms (Chung, 2005). Also, Leung (2011) reported on the various problems appearing in the implementation and finally gave suggestion on also documenting assessment and professional teaching standards to go hand in hand with a document on curriculum standards, similar to NCTM standards in US.

The development of 2000, 2003, and 2008 Grade 1-9 Mathematics Curriculum Standards

The announcement of 2000 Grade 1-9 Mathematics Curriculum Temporary Guideline (Ministry of Education in Taiwan, 2000) was due to the teaching experiment which highly focused on constructivism was not accepted by parents, teachers, and educators in Taiwan. The 1993 mathematics curriculum in Taiwan put more emphasis on constructive teaching method for elementary mathematics classes. The teaching emphasizes on constructivism began on 1993 and was executed for several years. However, the Ministry of Education in Taiwan decided to draw back the teaching in elementary schools focused on constructive method due to several reasons (e.g., hinder the development of students' computational ability, interfere with the advanced mathematics learning, teachers do not know how to teach, and so on). Therefore, the term "constructivism" was deleted from the 2000 Grade 1-9 Mathematics Curriculum Temporary Guideline (Ministry of Education in Taiwan, 2000). The situation is similar to the change from the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM, 1989) to Principles and Standards for School Mathematics (NCTM, 2000). The previous standards decreased the training of computational ability; however, the later standards draw back the development of conceptual understanding and computational fluency.

The 2000 Grade 1-9 Mathematics Curriculum Temporary Guideline for mathematics curriculum of grade 1 to grade 9 includes six key goals and five major subjects. The six key goals are as follows:

Understanding the concept and relation of number, quantity and shape
Cultivating daily mathematical literacy
Developing ability of forming and resolving mathematics problems
Developing ability of expressing clearly and communicating rationally based on mathematics
Cultivating ability of mathematics critical and analyzing
Cultivating ability of admiring mathematics (Ministry of Education in Taiwan, 2000). "Developing ability of expressing clearly and communicating rationally based on mathematics", "Cultivating ability of mathematics critical and analyzing" and "Cultivating ability of admiring mathematics" are not included in the 82th year of the mathematics curriculum standard.

The five major subjects are Number and Quantity, Algebra, Graph and Space (Geometry), Statistics and probability, and Connection (Ministry of Education in Taiwan, 2000). At the same time, the subject of Connection is not discussed in the 82th year of the mathematics curriculum standard. It highlights that students need to use inner connection to grasp mathematical method for mathematics learning (Ministry of Education in Taiwan, 1993, 2000).

It is still be concerned that following 82th mathematics curricular, the calculation ability is behind the standard presented at the textbook, and meanwhile, 2000 Grade

1-9 Mathematics Curriculum Temporary Guideline does not do much change on the section of number and operation. Besides, 91 school year seventh-grade students do not perform well on first mathematics examination, so some scholars, especially the mathematicians appear to Ministry of Education for revising mathematics temporary curriculum standard because of students' low performance on computational ability. Therefore, under the Ministry of Education's assistance, mathematicians gather some scholars as a revise group to work on this project.

Ministry of Education in Taiwan (2003) announces 2003 Grade 1-9 Mathematics Curriculum Formal Guidelines. On the general teaching goals, they lay emphasis on the importance of calculation. The general goals are as follows:

Cultivating students' calculation, abstract, and inference ability
Learning problem-solving method of application problems
Establishing mathematics foundation for next phase
Cultivating attitude and ability of admiring mathematics
The 2003 Grade 1-9 Mathematics Curriculum Formal Guidelines (Ministry of Education in Taiwan, 2003) also includes five major subjects which are Number and Quantity, Algebra, Geometry, Statistics and Probability, and Connection (Ministry of Education in Taiwan, 2000). The difference on the subjects is to use the Geometry to substitute the subject of Graph and Space.

The major differences between 2000 Grade 1-9 Mathematics Curriculum Temporary Guideline 2003 Grade 1-9 Mathematics Curriculum Formal Guidelines are as follows:

From "four learning stages" to "grade 1-9": on temporary guideline, learning period of nine-year compulsory education was separated into four stages; however, on the formal Guideline, it already has weaken the importance of four stages but now change to grade1-9 learning stages.
From "Competence Indicators" to "year-detailed items": on temporary guideline, curriculum and teaching materials are separated into four stages, and competence indicators substitute for teaching goals in every stage. Formal guideline no longer focuses on competence indicators, but establishes grade 1-9 year-detailed items, which also means every grade has its own teaching goal.
From "subdivision of competence indicators" to "interpretation of year-detailed items": the four learning stages in temporary guideline are included in grade 2-3; therefore, competence indicators can also be used in grade 2-3. That's why it will stress the subdivision of competence indicators to separate teaching goal of every grade. However, formal guideline has grade 1-9 year-detailed items, and has interpretation in every year-detailed items, which can let teacher easier to realize.
No longer emphasizing on "80% of students can study", so teaching materials become "difficult": on temporary guideline, we want 80% students can study curriculum content so math textbook is very easy. Now, the formal guideline curriculum content is more difficult than temporary guideline because we want maintain certain difficulty in math textbook. As for the mission that 80% of students can study, it turns out to be the responsibility of teacher, student and parents.

After the implementation of the 2003 Grade 1-9 Mathematics Curriculum Formal Guidelines and based on the opinions from in-service teacher, textbook publisher, editing group and review committee, the Ministry of Education in Taiwan decided to make some revisions which are based on 92 curriculum outline. Therefore, committees are mainly curriculum outline group, editing group, and review committee of 2003 Grade 1-9 Mathematics Curriculum Guidelines. The 2008 Grade 1-9 Mathematics Curriculum Guidelines (Ministry of Education, in Taiwan, 2008) made a little adjustments and did not change the fundamental idea, but after 92th curriculum outline announced, they will collect opinions of the 2003 Grade 1-9 Mathematics Curriculum Formal Guidelines.

The little adjustments are described as follows:

Mathematical education should have some adjustment, regarded as the core of scientific education and look forward to the complete of mathematics course from elementary to senior high school
We should make some certain regulations about problems caused by textbook editing and reviewers' suggestions.
We should make some revisions regarding editing group's practical question (such as order, isolated materials, scattered materials, and preliminary work)
We should revise or supply the some scant part, and reedit appropriate topic explanation, especially for junior high school
We should adjust original learning stages (1-3, 4-5, 6-7, 8-9) to fit in with other subjects (1-2,3-4,5-6,7-9), and reedit Stages Competence Indicators
There are no adjustment between junior high school and elementary school
Revising part of wording problem
The 2008 Grade 1-9 Mathematics Curriculum Guidelines (Ministry of Education in Taiwan, 2008) also includes five major subjects which are Number and Quantity, Algebra, Geometry, Statistics and Probability, and Connection (Ministry of Education in Taiwan, 2008).

In sum, the major goals of the revised curriculum guideline are: a) to cultivate students' computational ability, the abstract ability, deduction ability, and communication ability; b) to learn to solve applied problem; c) to establish the high school stage mathematic foundation, and hope that can train the students' attitude and ability to appreciate mathematics.

PART C. STUDIES ON MATHEMATICS CURRICULUM IN TAIWAN

For the last decade in Taiwan, the research on mathematics curriculum is mostly done by masters' theses, not much done by the doctoral students or professors. Since, generally speaking, master students' research ability is not as strong and if the research on mathematics curriculum could enhance students' "opportunity to learn (OTL)", then more doctoral students and/or professors should devote into this research topic.

From the perspective of OTL, we divide the Taiwanese mathematic curriculum research into three aspects: (1) the guidelines of mathematics curriculum in Taiwan and other countries; (2) the mathematics textbooks in Taiwan and other countries; (3) the influence of mathematics curriculum reform on students' learning in Taiwan.

The Comparison on the Guidelines of Mathematics Curriculum between Taiwan and Other Countries

From the Taiwanese mathematics curriculum research, the Taiwanese comparative research on the guidelines of mathematics curriculum between Taiwan and other countries include Singapore (e.g. Weng & Chien, 2010), China (e.g. Chiang, 2006), Korea (e.g. Huang, 2006) and Japan (e.g. Hong, 2005). All these studies are between Taiwan and other Asian countries. And the issues include the curriculum system (e.g. Weng & Chien, 2010), curriculum goals (e.g. Huang, 2007) and curriculum design (e.g. Huang, 2006) etc. But there is no study that compares Taiwan with other Western countries (such America and European countries).

Using the curriculum system as an example, there is a permanent department for curriculum development in Singapore and it is responsible for revising the curriculum syllabus and textbooks every ten years. But there is no permanent department or committee in Taiwan that takes charge of revising curriculum in a long-term. It is recommended that we can set up a department for the revision of curriculum guidelines and syllabus. It can also manage the monitor of the curriculum implantation and teaching practice. By doing so, we can have a more consistent, systematic and long-term planning on the mathematics curriculum in Taiwan (Weng & Chien, 2010).

The Comparison on the Mathematics Textbooks between Taiwan and Other Countries

From the Taiwanese mathematic curriculum research, the Taiwanese comparative research on the mathematics textbooks between Taiwan and other countries include China (e.g. Hsu & Hsu，2009), Singapore (e.g. Weng & Chien, 2010), Finland (e.g. Tung, 2011), Spain (e.g. Chiu, 2006) and America (e.g. Yang, Shih, Hsu, & Yu, 2011), etc. The Taiwanese research on mathematics-textbook comparison is quite diverse, ranging from Asian to European and American countries. The issues covering in the research are also quite multiple. From the perspective of OTL, the Taiwanese research includes the levels of mathematics questions (e.g. Weng & Chien, 2010), history of mathematics (e.g. Hsu & Hsu, 2009), connection (e.g. Weng, 2011) and technology (e.g. Chen & Yang, 2010) etc.

Using the level of mathematics questions in the textbook as an example, there are three levels of mathematics questions in the mathematic curriculum syllabus in Singapore: basic/routine questions, non-basic/routine questions and open questions. The "non-basic/routine questions" represent questions that have different questioning format and the "open questions" represent questions that are unfamiliar, requiring further discussion/investigation and sometimes without a single correct answer. In Singapore's textbooks, there are one or two questions that are interesting and challenging in each unit. Contrastingly, most questions in Taiwan's math textbooks are basic/routine questions, with a few non-basic/routine questions (Weng & Chien, 2010). We recommend that we can include some open questions in the textbooks to provide the gifted students the opportunity to have more challenging practice.

Using the issue of technology as another example, other countries, such as America and Singapore, emphasize the integration of technology in mathematics teaching and/or the utilization of technology in calculation. Some topics they incorporation the issue of technology include first grade math (Yang, Shih, Hsu, & Yu, 2011), the fraction in fifth and sixth grade (Wu & Yang, 2007) and the algebra in seventh grade (Chen & Yang, 2010). Contrastingly, even though it is stated in the Mathematics Curriculum Guideline of Grade 1-9 (Ministry of Education, 2003) to connect mathematics with "application of technology and information", it is not realized in the textbooks.

The Influence of Mathematics Curriculum Reform on Students' Learning in Taiwan

From the Taiwanese mathematic curriculum research, there are some studies on the influence of the mathematics curriculum reform on students' learning. They include studies comparing the curriculum of Year 1975 and Year 1993 (e.g. Wang, 2002), studies comparing the curriculum of Year 1993 and Year 2000 (e.g. Liao, 2007) and studies comparing the provisional guidelines of Grade 1-9 Curriculum and the finalized guidelines of Grade 1-9 Curriculum (e.g. Chuang, 2011).

Chuang (2011) conducted a study to evaluate junior high students' mathematics ability in the year of 2002, 2004, 2006 and 2008. He selected 5,000 students each year and analyzed their performance on mathematics from their first "Basic Competence Test for Junior High School Students". The results reported that the students in 2008 performed better than 2006 and the students in 2006 performed better than 2004 and 2002. The students that took the competence test in year 2008 are students under the finalized guidelines of Grade 1-9 Curriculum. The students who took the competence test in year 2006 and 2004 are students under the provisional guidelines of Grade 1-9 Curriculum. And the students who took the competence test in year 2002 are students under the curriculum of year 1994. The study indicated that students have better mathematics ability under the finalized guidelines of Grade 1-9 Curriculum.

A Final Note

As given above, Taiwan mathematics curriculum has undergone various historical developments at different periods of the past 100 years. The reform in curriculum simultaneously exerted pressure and called for teachers' change. Finally, research studies on mathematics curriculum for the country are greatly in need; in order to document its state-of-the-art, to specify deserved themes (such as textbooks development, fidelity of implementation, change in students' learning outcomes,…etc.) that warrant attention from all in the country.

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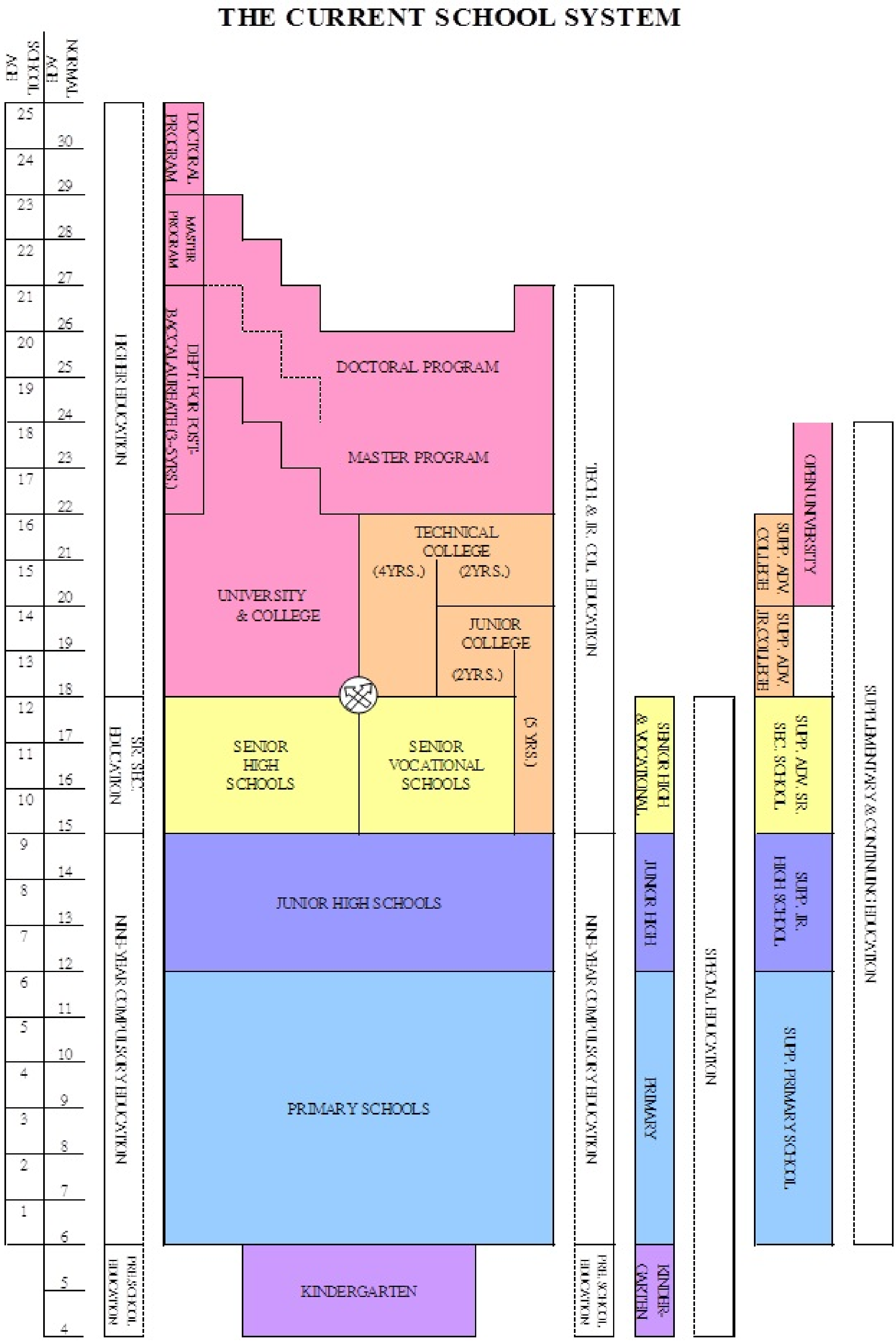
Appendix I: The Current Education System



Appendix II: Year of Revision for Mathematics Curriculum Standards (elementary, middle, and high schools)

Chen (2007)

1930s	1940s	1950s	1960s	1970s	1980s	1990s	2001 ~
1932 Announcement of Standards (Elementary) 1932 Announcement of Standards (Middle) 1932 Announcement of Standards (High) 1936 Announcement of Standards (Elementary) 1936 Revision of Standards (Middle) 1936 Revision of Standards (High) 1940 Revision of Standards (High)							
	1942 Announcement of Standards (Elementary) 1948 Announcement of Standards (Elementary) 1948 Revision of Standards (Middle) 1948 Revision of Standards (High)						
		1952 Announcement of Standards (Elementary) 1952 Revision of Standards (Middle) 1952 Revision of Standards (High) 1955 Revision of Standards (High) 1957 Revision of Standards (Elementary)					
			1962 Announcement of Standards (Elementary) 1962 Announcement of Standards (Middle) 1964 Announcement of Standards (High) 1968 Announcement of Standards (Elementary) 1968 Announcement of Standards (Middle)				
1930s	1940s	1950s	1960s	1970s	1980s	1990s	2001 ~
				1971 Revision of Standards (High) 1972 Announcement of Standards (Middle) 1975 Announcement of Standards (Elementary)			
					1982 Revision of Standards (High) 1983 Announcement of Standards (High) 1983 Announcement of Standards (Middle) 1985 Announcement of Standards (Middle)		
						1993 Announcement of Standards (Elementary) 1994 Announcement of Standards (Middle) 1995 Revision of Standards (High)	
							2002 Announcement of Standards (Elementary) 2002 Announcement of

							Standards (Middle) 2003 Announcement of Standards (Elementary) 2003 Announcement of Standards (Elementary) 2003 Announcement of Standards (High) 2008 Announcement of Standards (Elementary) 2008 Announcement of Standards (Middle) 2008 Announcement of Standards (High)
1930s	1940s	1950s	1960s	1970s	1980s	1990s	2001 ~

OPPORTUNITIES TO LEARN: REFLECTION ON TAIWANESE STUDENTS' RESULTS OF INTERNATIONAL ASSESSMENT

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This paper reported some concerns regarding learning gaps and some research actions based on the results of both TIMSS and PISA studies. The main focus of this paper was to introduced one particular program "After School Alternative Program", directed by Ministry of Education and National Science Council in Taiwan. Preliminary results were also included.

RESULTS OF INTERNATIONAL COMPARISION

Recently, international comparison has been trend in many subject areas. Students in Taiwan have participated in TIMSS since 1999 and in PISA since 2006. The overall performance results of Taiwanese students were above the scale average. At the first glance, the results seemed fine. However, after reviewing details, some problems were observed.

According to both TIMSS-2003 and TIMSS-2007 results, there are 8% of 4th graders and 14% of 8th graders, who did not reach TIMSS intermediate benchmark (i.e., students scored below 475) (Mullis, Martin, Foy, 2008; Mullis, Martin, Gonzalez, & Chrostowski, 2004). These 14% of 8th grade participants in Taiwan were identified as low-achievers, which was the highest proportion among all five leading nations. The percentage of low achievers showed a "quantum jump" from grade 4 to 8 in Taiwan (Lin, 2008). Results from both TIMSS-2003 and TIMSS-2007 reports showed that the variances of the 8th grade results were greater than the variances of many other countries. It indicated that achievement gaps between high- and low-achievers were larger compared with many other countries. In addition, scores of 8th graders were more spread out than that of 4th grades (figure 1 & 2). It seemed that achievement gaps grew larger over the years. Similar results were found in PISA 2006 and PISA 2009.

Country	Average Scale Score	2003 to 2007 Difference	1995 to 2007 Difference	Mathematics Achievement Distribution
Chinese Taipei				
2007	576 (1.7)			
2003	564 (1.8)	12 (2.5)		

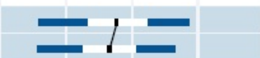
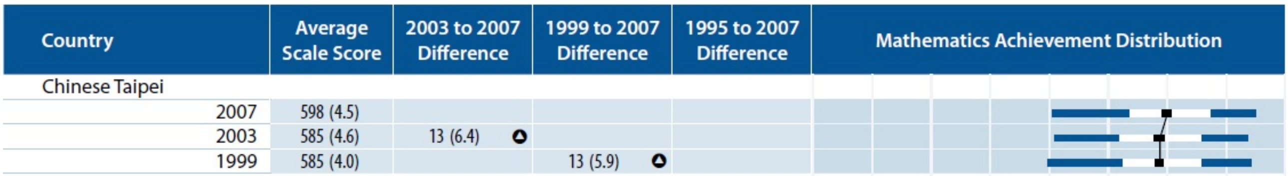


Figure 1. Fourth graders' mathematics achievement – 2003 through 2007 (Mullis, et al., 2008), p. 44

2012. In Tso,T. Y. (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 179-186. Taipei, Taiwan: PME.



Country	Average Scale Score	2003 to 2007 Difference	1999 to 2007 Difference	1995 to 2007 Difference	Mathematics Achievement Distribution
Chinese Taipei					
2007	598 (4.5)				
2003	585 (4.6)	13 (6.4)			
1999	585 (4.0)		13 (5.9)		

Figure 2. Eighth graders' mathematics achievement – 1999 through 2007 (Mullis, et al., 2008), p. 46

The other phenomena found were the results of students' interests and self-confidences in mathematics and science learning in both TIMSS-2003 and 2007. The results showed in TIMSS-2007, for example, only 36% of 4th graders considered themselves at high level of self-confidence in their mathematics abilities, which was second to last among all participating countries; twenty-seven percent of 4th graders self-reported at low level of this index. Only 27% of 8th graders were at high level of this index, and 46% were not confidence in their mathematics abilities at all. The results of this index indicated that students became less confidence in their mathematics abilities as they grew.

RESEARCH ACTIONS BASED ON THE RESULTS OF INTERNATIONAL ASSESSMENT

Besides the official results provided from IEA and/or OECD, there are lots of un-investigated information embedded in the data released. National Science Council (NSC) in Taiwan proposed a special call for proposals in 2004. Researchers who were interested in these international studies were encouraged to perform secondary analyses, and/or to replicate these studies at the local level. In addition, NSC has announced regular call for proposals since 2005, in order to investigate phenomena regarding educational policies, curriculum development, teaching approaches, learning environment, students' mathematics competence, basic abilities, and beliefs toward mathematics, etc. Table 1 is the list of researches funded by NSC since 2005.

Year	Researcher	Research Interest
2005	Chin-Chien Yang	Mathematics and reading progress of Taiwan in TIMSS & PIRLS
	Chia-Cheng Chen	A comparative study of the factors influencing mathematics achievement: The comparison among TIMSS 1995, 1999, 2003
	Fang-Ying Yang	Exploring the epistemic, metacognitive and the affective levels of teacher/student cognition and the styles of teaching and learning in the social and cultural context

Year	Researcher	Research Interest
	Min-Ning Yu	Study of the international comparison of factors affecting mathematics achievement - The comparison of TIMSS 2003, PISA 2003, and TEPS databases used as an example
	Mei-Shui Chiu	Students' TIMSS achievements on different problem types and their interactions with gender, affective responses and learning contexts
	Shin-Feng Chen	An international comparative study on the affective factors of science learning achievement: A longitudinal case study on the databank of TIMSS 1995, 1999, and 2003
	Han-Ping Tam	Data analysis of the TIMSS 2003 study
	Fang-Chung Chang	Research on academic achievement impacted by economic and educational factors: The TIMSS data bank in 1995, 1999, and 2003 years study
2006	Min Ning Yu	The way toward a female scientist---An exploratory study from the TIMSS data analysis
	Chien-Shu Chang	The research for mathematics and reading performance of fourth graders in TIMSS2003 and PIRLS2006 field test
	Min-Hsiung Huang	International comparisons of the variability of student performance within and between classroom: Fourth and eighth grade student in the TIMSS2003
2007	Mei-Yu Chang	The relationship between science curriculum and self-confidence, interest, achievement of students in elementary science learning
	Fang-Chung Chang	Testing the correlation between student's achievement and their mathematical belief: Using the TIMSS2003 data to explore fourth and eighth graders
	Miao-Hsiang Lin	Statistical methods for analyzing contextual perspectives for the TIMSS 2003 International Database
2008	Mei-Yu Chang	The relationship between understanding of TIMSS science items, processes of problem-solving, and achievement of students in elementary science learning
2009	Fang-Chung Chang	Testing the affecting factors of urban and rural student's achievement by HLM: Using the TIMSS2003 data to explore eighth graders

Year	Researcher	Research Interest
2011	Mei-Chung Wang	Using TIMSS 2011 cognitive framework to investigate the comprehension of statistical graph of Taiwanese elementary students
	Mei-Shui Chiu	Development of cultural artifacts, beliefs, knowledge, strategies for teaching and learning mathematics and science in Taiwan: Relationship with PISA and TIMSS outcomes

Table 1. Proposals related to international comparison studies funded by NSC since 2005

PRACTICAL ACTIONS BASED ON THE RESULTS OF INTERNATIONAL ASSESSMENT

Researchers in Taiwan have been interested in identifying reasons that might be the reasons for the existing achievement gaps. In addition, many researchers and educators have been trying to find solutions that might lower these gaps. Given that socioeconomic status [SES] has been proven to be one of the factors affecting students' achievements (Baker, Goesling, & Letendre, 2002; Lubienski & Crane, 2010; McConney & Perry, 2010), providing extra resources for low-SES students might be one way to overcome students' background disadvantage. During 2006, Ministry of Education (MOE) and NSC examined the results of TIMSS 2003 and the observed phenomena cautiously. Based on the results, MOE Department of Elementary Education and NSC Department of Science Education proposed the After School Alternative Program [ASAP]. The idea of this ASAP program is that by providing additional academic supports for disadvantaged students, it might improve students' achievement. Students in this program receive financial support from the government, regardless of the school level (i.e., elementary schools or middle schools) and location (i.e., urban, suburban or rural).

Supported by the MOE, the ASAP program has been implemented in Taiwan since 2007. The main focused group of the ASAP program has been the elementary and middle school students. Figure 3 illustrated the process of the ASAP program. Eligible students, which include minorities, students with low SES status, grand-parenting students, and/or students with learning disabilities, were identified first. After parental consent and achievement screening test, those who scored at the 35 percentile and below will be enrolled into the system. Funds have been provided to classes with seven or more qualified students. These students have two additional afterschool classes twice a week, with adult supervision. Retired teachers, qualified college students, and in-service teachers are recruited to serve as the afterschool teachers. Students' progresses are also monitored regularly. Due to the scale of the ASAP program, an efficient testing system was required in order to evaluate students' progresses and the effectiveness of the program. During 2008, a technology-based testing system (ASAP-tbt) was developed. This testing system also serves as an independent assessment to validate the ASAP remedial effects.

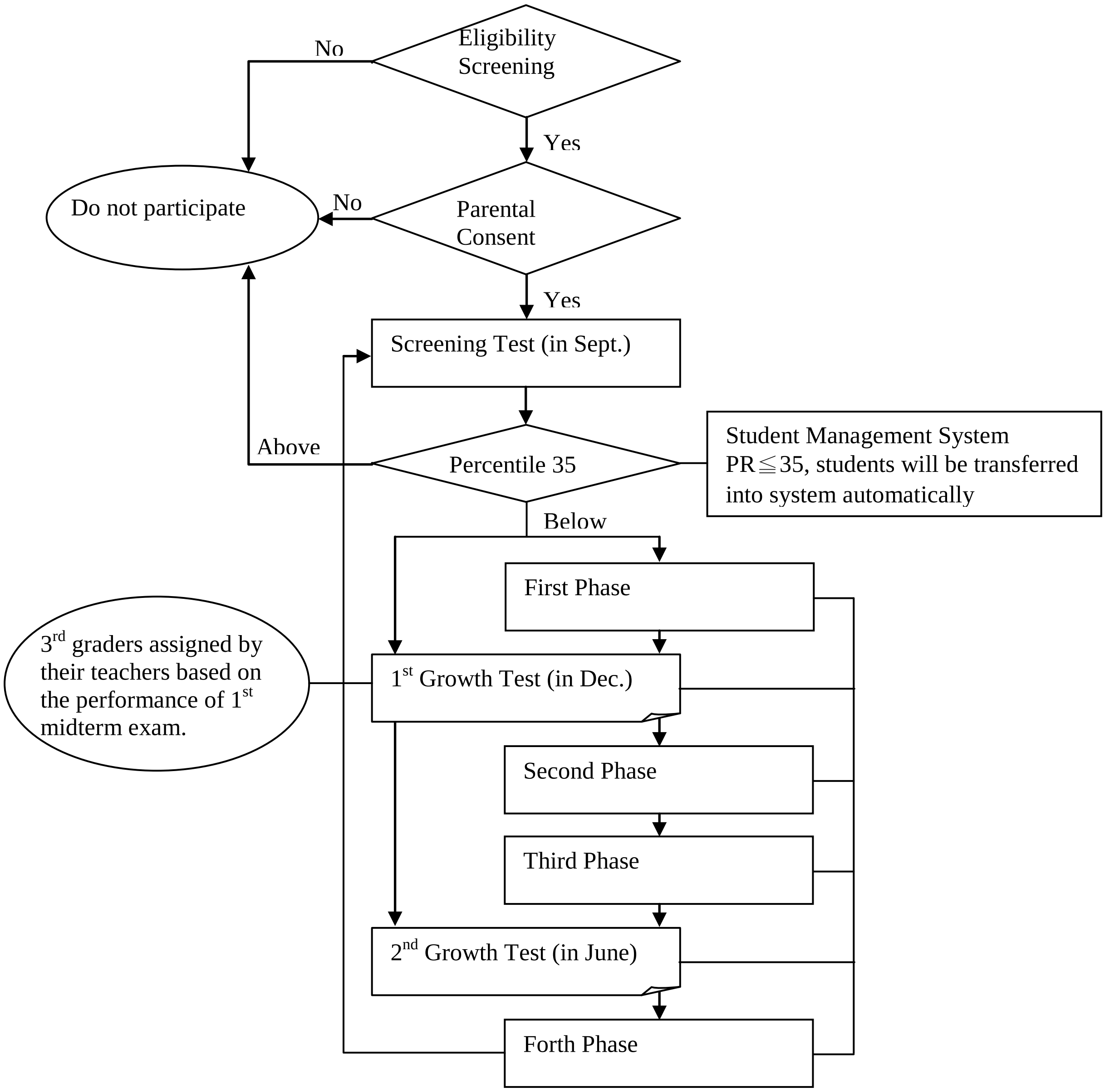


Figure 3. Implantation procedure of ASAP

Based on the schools' reports in the first year, the results seemed promising. Ninety percent of participants reported that they could not finish their homework before attending this program, but they could now after one year (with supervisions). Eighty percent of participants had developed more positive learning attitudes. In addition, 60% of participants had improved on their school test scores (Lin, 2008).

Table 2 is the results of mathematics achievement differences among ASAP students and two control groups (norm and slow learner who are not enrolled in the ASAP program) during 2009-2010. The results indicated that the scores of ASAP students had increased more rapidly, compared to the scores of the two control groups (See Figure 5 & 6).

Grade	group	Screen Test	Growth Test I	Growth Test II
4	ASAP	42.82	45.81	49.42
	Norm	52.28	-	54.32
	Slow Learners in Norm	37.17	-	37.02
8	ASAP	42.16	41.12	49.01
	Norm	50.08	-	53.35
	Slow Learners in Norm	39.55	-	38.14

Table 2. The average scores of the ASAP program and control groups

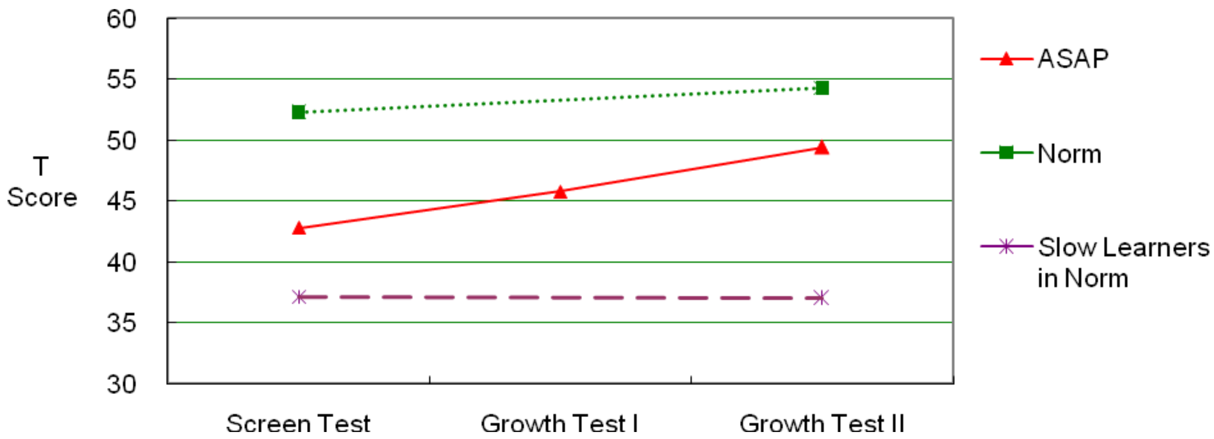


Figure 4. Results of changes on 4th graders mathematics achievement

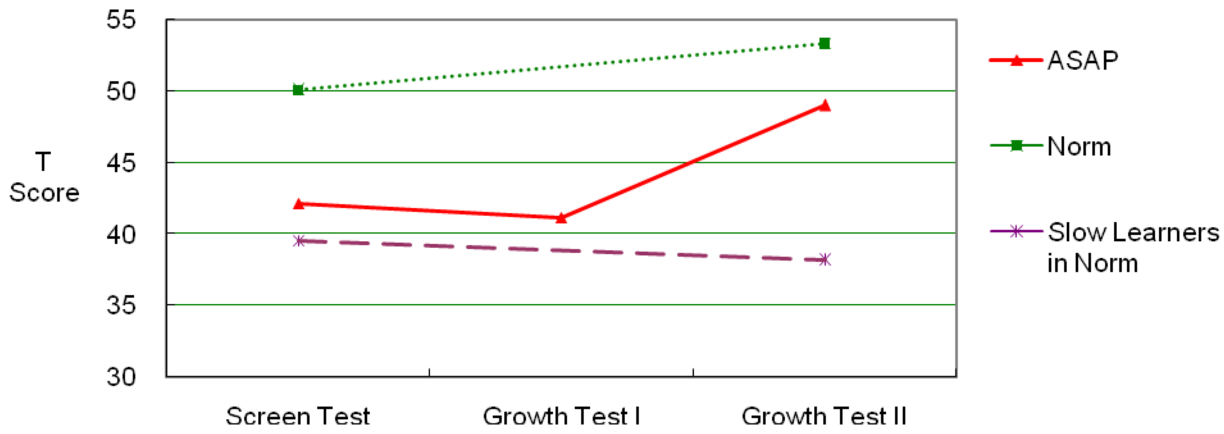


Figure 5. Results of changes on 8th graders mathematics achievement

Another concern regarding ASAP program is on the changes of participants' goal orientations. Utman (1997) argues that positive learning goals would lead to better task performance. In September 2010, there were 708 (20.57%) 4th grade participants categorized as in the avoidance goal, and 788 (22.90%) in the mastery level. Two semesters later, only 446 students (12.96%) were reported in avoidance category, and

1,335 (38.80%) students were in mastery goal level (See Table 3). This finding showed that 4th graders' self-confidence changed from negative to positive. However, the impact on the 8th grade participants was unclear (See Table 4).

June 2011 / Sept. 2010	Avoidance Goals	Performance Goals	Moderately Goals	Mastery Goals	Total (%)
Avoidance Goals	220	299	109	80	708 (20.57%)
Performance Goals	190	478	363	190	1221 (35.48%)
Moderately Goals	26	121	208	369	724 (21.04%)
Mastery Goals	10	16	66	696	788 (22.90%)
Total (%)	446 (12.96%)	914 (26.56%)	746 (21.68%)	1335 (38.80%)	3441

Table 3. The change of goal orientation for 4th graders

June 2011 / Sept. 2010	Avoidance Goals	Performance Goals	Moderately Goals	Mastery Goals	Total (%)
Avoidance Goals	58	48	13	25	144 (21.98%)
Performance Goals	50	110	50	39	249 (38.02%)
Moderately Goals	1	62	22	63	148 (22.60%)
Mastery Goals	0	0	52	62	114 (17.40%)
Total (%)	109 (16.64%)	220 (33.59%)	137 (20.92%)	189 (28.85%)	3441

Table 4. The change of goal orientation for 8th graders

The motto of the ASAP program is: "taking care of every student so that all children get progress academically". The goals of this program are: (1) to reduce low-achievers from 15% to 10% in the future TIMSS survey, wishing for even lower percentage in the future, and (2) to increase students' interests in learning mathematics. ASAP currently is under evaluation. Hopefully, students will continue to benefit from the ASAP.

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MATHEMATICS TEACHER EDUCATION IN TAIWAN

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This paper includes three main parts. The first introduces the historical and cultural background that shapes the current teacher education system in Taiwan, which includes a description of the current mathematics pre-service teacher education system and practice. The second discusses the opportunity to learn (OTL) and its relationship to the knowledge of Taiwan pre-service mathematics teachers at the secondary and primary levels in an international context. The third describes the academic activities and research in mathematics teacher education and the OTL they offer in Taiwan.

INTRODUCTION

Teachers enjoy a relatively high reputation in Taiwan because of the high prestige of teaching jobs and the significant regard for education in traditional Chinese culture. The historical background of political, economic, and social contexts has resulted in a generous and ensured salary and other benefits for current teachers. These incentives make a teaching career extremely attractive for people seeking stable lives. Therefore, becoming a teacher is a competitive task that requires rigorous evaluation and screening.

Mathematics is one of the core academic subjects and is required throughout grades 1-12 in Taiwan. Mathematics teacher competency is therefore one of the most important parameters of school quality. Teacher opportunity to learn becomes essential for producing high-quality teachers.

Teacher opportunity to learn is derived from pre-service academic work, practicum, and in-service professional development. Special in-service opportunities are offered to teachers because the Taiwanese government encourages teachers to pursue higher academic degrees, such as a master’s degree in teaching, through in-service study (MOE, 2006a).

The first section of this paper briefly introduces the historical and cultural background that shapes the current teacher education system, which includes a description of the current mathematics pre-service teacher education system and practice. In the second section, we discuss the opportunity to learn (OTL) and its relationship to the knowledge of pre-service mathematics teachers at the secondary and primary levels in an international context. The third section describes the academic activities and research in mathematics teacher education and the OTL they offer. The concluding section offers a summary of the practice and challenges Taiwan faces and reflects on and envisions mathematics teacher education in Taiwan.

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Section 1: Historical Development and Current System of Teacher education

1.1 Development and Transformation of Teacher Education

Teacher education in Taiwan is dramatically influenced by political, economic, and social contexts. The historical development and transformation of teacher education can be divided into three major periods (Hsieh, Lin, Chao, & Wang, 2009).

1.1.1 Initiation of Teacher Education

The first formal teacher-training program was offered in 1896 at the Kokugo Gakkou (Japanese Language School) during the Japanese colonial period. This program was to prepare Japanese people to become teacher educators, school principals, and teachers (Wu, 1983) to teach the Japanese language among others. In 1899, three "normal schools" were established to mark the first time Taiwanese people had the chance to be educated as primary level teachers to teach primary school arithmetic (Wu, 1983; p. 18). For most of the colonial period, Japanese and Taiwanese pre-service teachers were trained separately. The Japanese government also enacted the first official regulation of teacher education, the "Official Regulation of Taiwan Governor-General Normal School" in 1899 (Lee, 1995; Wu, 1983).

1.1.2 Rise and Decline of the Protectionist Teacher Education System

In 1946, the second year after the Japanese colonial period, the first institution to educate Taiwanese high school teachers– Provincial Taiwan Normal College (predecessor of National Taiwan Normal University)– was established by the Nationalist (KMT) government from mainland China. This marked the beginning of continuous efforts to teach Taiwanese to speak Mandarin and to regenerate Taiwanese culture by pushing primary graduates, who were taught Japanese and in whom the idea of "Japanization" was instilled, to go to junior high schools where the Chinese language and anti-communist ideas were exposed (Cheng, 1998). During this period, the government believed that teacher quality could influence the thinking and inner quality of people, which in turn could influence the development of politics, economy, and national defense (Cao & Liang, 2002). In 1955, President Chiang Kai-Shek used the motto, "Teachers First, Normal Education Foremost," to greatly improve teacher quality (Ministry of Education [MOE], 1976, p. 565).

The KMT government believed that after screening pre-service teachers for preparation in institutions, their preparation and benefits should be covered by the government to attract talented students to a teaching career and to avoid teacher shortage in schools to protect the stability of the teacher education system. Therefore, teachers were educated at the expense of the government and guaranteed job assignments, similar to civil servants. Their education was executed by normal institutions and dominated by the government. A student who could not enter normal institutions had hardly any chance of becoming a teacher. The primary features of the teacher education system in Taiwan during this period were protective, uniform, and closed.

From the 1960s to 1980s, the Taiwan economy improved rapidly, along with living standards. The late 1980s witnessed the shaping of multi-party politics, liberated thinking, and a stronger legislative system. The protectionist teacher-education system did not match the prevalent ideas of a free society and a free economy. Scholars, educational communities, and the opposition party voiced the necessity for more open access to teacher education. This tide finally crushed the decades-old protectionist teacher education system.

1.1.3 Rise of the Competitive Teacher Education System

In 1994, the government enacted the Teacher Education Act (TEA), which opened multiple means toward teacher education in that all four-year universities or colleges were allowed to run teacher education for grades k-12 teachers if they met the requirements for applying as a teacher education institution. The government was no longer responsible for free tuition and job assignments.

The retention policy for teachers remained unchanged; teachers still enjoyed favorable remuneration and benefits. For instance, although teachers were given a two-month summer vacation and a 21-day winter vacation, they were still paid a salary for the entire year and given an additional 1.5 month new-year-bonus and a one-month “merit-of-professional-performance” bonus each year. This new reform paved the way for teacher education but retained the liberal salary and benefits that made the teaching profession more accessible and attractive. Along with a lower demand for teachers resulting from fewer births, the competition to receive teacher education and obtain teaching jobs has been extremely high.

1.2 Reformation of Mathematics Teacher Education

Traditional approaches to teaching mathematics in schools have had a profound effect on mathematics teacher education in Taiwan. School education in Taiwan is focused heavily on helping students achieve high rankings in entrance examinations. Traditional mathematics teaching has been dominated by formal mathematical content and narrative teaching. Junior high and primary school mathematics textbooks prior to 2001 and 1996 were standardized in Taiwan.

In 1997, a new national standardized junior high school mathematics textbook was implemented. Textbook authors initiated open views, such as infusing cartoons and investigations into mathematics textbooks, from which the entrance examination questions formulated. These changes centered on students, the links between mathematics and life, cultivation of student creativity, thinking, and reasoning abilities, and on an active attitude toward learning and appreciating mathematics (Hsieh, 1997). The authors also raised the notion that textbook reform could create widespread teacher education amongst in-service teachers who were using the textbooks and pre-service teachers whose college instructors typically included these textbooks as their teaching materials.

Since 1996, primary mathematics textbooks were edited by private publishers and reviewed by the government. This was a time of constructivist thinking in Taiwan, and textbook writers, affected by such thinking, included many student methods in the textbooks. Although this constructivism movement was later criticized by society, in-service and pre-service mathematics teachers began to consider deeply how students think, shifting toward teacher-centered to student-oriented teaching. Beginning in the 1980s, Dr. Fou-Lai Lin initiated and promoted studies in mathematics education. These researchers – the educators of teachers in Taiwan – began to educate mathematics teachers by combining their practical experience with the results of mathematics education studies, thus moving Taiwan mathematics teacher education toward a new realm.

1.3 Current Teacher Education System

1.3.1 Acts and Programs of Teacher Education

Taiwan teacher education is a strong, national policy-driven system. The current pre-service teacher education system is regulated mainly by the national Teacher Education Act (TEA) and the Teacher Education Act Enforcement Rules (TEAER), enacted in 1994, 1995, and last amended in 2005, 2011, respectively. These regulations established the targets, institutions, recruitment, curricula, and accreditation of the teacher education system. The teacher education institutes include (1) normal universities/universities of education, (2) universities with TE affiliated departments (majors), and (3) universities with teacher education centers. Teacher education programs for grades 1-12 are separated into two levels, primary teachers who teach grades 1-6 for various subjects and secondary teachers who teach grades 7th-9th or 10th-12th for a single subject.

The number of teacher education institutions at the primary and secondary level changed rapidly after the 1994 reformation. In 1995, a total of 29 institutions expanded to 67 in 2004, and then gradually reduced to 49 in 2010 (MOE, 2011a). The number of pre-service mathematics teachers differs among different teacher education universities. In 2007, there were 46 mathematics teacher-education institutions; the number of pre-service mathematics intern teachers, per institution, ranged from 1 to and 90 at the secondary level and 2 to 443 at the primary level.

The teacher-education program is an additional program that includes pedagogical and professional studies and is taken while pursuing academic degrees. Regulated by the TEA, the teacher education curricula (TEC) comprises three parts: General Curriculum, Subject Matter Curriculum (SMC), and Education Professional Curriculum (EPC). Universities or colleges establish the TEC under the guidance and approval of the MOE. After pre-service teachers complete academic degrees and TEC, they spend another half-year completing the Educational Practicum (EP) in primary or secondary schools. The MOE (2005a) also created an EP guideline to ensure its quality in different schools. To become qualified for applying for a teaching job, pre-service teachers must further pass the national common Teacher Qualification Assessment.

The Taiwan government decreed the types of learning experiences and opportunities teacher education programs must provide, the qualifications and process of becoming a teacher, what levels of students teacher education institutions can enroll, and what types of accreditation is required of these institutions.

1.3.2 Screening and selection of mathematics teachers

The government requires public schools and most Taiwanese schools to hold public screenings and selections when employing tenure teachers (MOE, 2005b). The screening and selection for tenure teachers are conducted through written tests, oral tests, teaching demonstrations, and on-site performance tests. Applicants are assessed through a combination of more than two of these methods. Two methods are used to screen and select tenure teachers. One is joint screening and selection held by the department of education of each city/county government and entrusted by schools. Other schools hold screenings and selections by themselves.

Generally, the screening and selection of tenure teachers occurs in two rounds. The first is through written tests to assess the applicant education professional knowledge and subject matter knowledge. Typically, the examination questions are compiled by university professors or senior teachers. Certain applicants, two to five times the quota of tenure teachers, are allowed to move to the second round, which assesses applicants through a 20- to 25-minute teaching demonstration and a short personal interview. Judges for the teaching demonstration are mainly school teachers; occasionally, a university faculty member may be included as an expert from the outside system. The teaching demonstration is a high-pressure one. The teaching topics are drawn by the applicants 20 minutes before the time of the actual demonstration and the applicants can use these 20 minutes to prepare. Those judging the personal interviews are mainly administrative staff, such as school principals. They check applicant educational background, experience, ideas of education, classroom management, willingness to participate in school administration, and so on.

The screening for tenure teaching positions is highly competitive, and is not only held for pre-service teachers, but for all in-service teachers who want to change schools. The average passing rates for screenings and selections across the country during 2007-2010 at the primary, lower secondary, and upper secondary levels are 3.5%, 11.9%, and 6.5%[1] (MOE, 2008, 2009, 2010a, 2011a). Regarding to the pre-service teachers, the average employment rates for tenure teaching positions of pre-service teachers for 2007-2010 are lower than 3.4% for the primary level and 20.2% for the secondary level.

[1] People may attend many screenings; thus, the actual rates of people who pass the screenings should be higher than these data.

1.3.3 Structure of Pre-service Teacher Education Curriculum

1.3.3.1 General Curriculum

The government does not clearly regulate General Curriculum content. Therefore, most teacher education universities accept the completion of a bachelor's degree as completing this curriculum, which requires 128 credits (semester units) and meets the requirements of a specific major.

1.3.3.2 Subject Matter Curriculum (SMC)

SMC is defined as a specific curriculum aimed to improve the strengths of pre-service teachers in subjects they will teach in the future. An SMC for secondary level pre-service mathematics teachers consists of their university mathematics courses. The upper and lower credits of mathematics courses are 30 and 48, respectively, regulated by MOE (2002). For the primary level, there is no regulated SMC because of the nature of interdisciplinary education.

1.3.3.3 Education Professional Curriculum (EPC)

The EPC aims at improving the educational competencies of pre-service teachers. The MOE (2003) has provided a pool of various courses in different areas for the teacher education university to select. The areas of EPC for secondary mathematics pre-service teachers include: Foundation of Education Curriculum, General Pedagogy Curriculum, Materials and Methods of Teaching for Mathematics, and Teaching Practice for Mathematics. A total of 26 credits are required.

The EPC for the primary level differs from the secondary level to include a course in Basic Subject Matter Curriculum in Teaching and increases the Materials and Methods of Teaching course to comprise three to four fields, and the Teaching Practice course is not restricted to mathematics. A total of 40 credits are required.

1.3.4 Educational Practicum (EP)

EP is designed to train pre-service teachers in actual teaching. According to TEAER and other related regulations (MOE, 2005a), intern teachers need to be in schools on a full-time basis for half a year to learn the following content: actual teaching internship (40%), "homeroom" teaching (general class affairs) supervision (30%), administrative work practice (20%), and study and training activities (10%). Each EP school has a team to supervise intern teachers under a systematic plan. The teacher education universities are obligated to visit and counsel the EP schools and intern teachers, handle "back to university training activities" for intern teachers, edit EP counseling literatures, and so on. Fifty percent of intern teacher evaluations are scored by internship supervisors, principals, or directors of EP schools; the other 50% are scored by internship supervisors from universities who typically visit an intern one to three times during his/her EP period.

1.3.5 Quality Assurance of Teacher Education and Teachers

Every phase of the process from entering TE to becoming a certified teacher in Taiwan involves clear requirements. At the entry point to the TE program, the MOE decides the number of admissions for each university, and TE universities have the autonomy to employ their own screening and selection content, criteria, and processes to select qualified entrants from applicants who are eligible after completing their first academic year in university. Many TE universities base their selection on applicant grade in the first academic year, and may support it with tests such as general educational knowledge tests, language tests, attitude tests, or personality inventories. Certain universities also consider student character, moral conduct, and extracurricular activities.

At the exit point, pre-service teachers must take the annual paper-and-pencil Teacher Qualification Assessment. The average passing rates for the 2007-2010 period was 67.4%.

To ensure the quality of teacher education programs, the MOE has conducted periodical evaluations of TE universities. Institutions receiving a third level rating have to stop admitting students. Those that receive a second level rating must decrease student admissions by 20%, and those that receive first level rating can retain the same admission quota (MOE, 2006b, p. 204). In 2007-2009, six TE universities received third level ratings and were disqualified to provide TE programs.

SECTION 2: OTL IN THE CONTEXT OF INTERNATIONAL COMPARISON

The information in this section is based on the results of the international comparison study, the Teacher Education and Development Study in Mathematics (TEDS-M). For details of TEDS-M, please see the series of international reports authored by Teresa M. Tatto and her research colleagues (Tatto et al., 2012) and reports prepared by the authors of this paper (Hsieh et al., 2010).[2] The data were collected from the end of 2007 to mid- 2008. The opportunity to learn various topics in teacher education programs and the preparation outcomes of mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) for lower secondary pre-service mathematics (SPM) teachers and primary pre-service mathematics (PPM) teachers were studied.

2.1 Structure and Substance of Teacher Education Curricula

2.1.1 Topics in Teacher Education Programs

Pre-service mathematics teachers in TEDS-M responded to numerous items that explored whether they had studied various topics in tertiary and school level mathematics, mathematics pedagogy, and general pedagogy as part of their teacher education programs. All topics were classified into areas by TEDS-M according to the results of exploratory and confirmatory factor analyses (Tatto et al., 2012).

[2] The analysis prepared for this report and the views expressed are those of the authors and do not necessarily reflect the views of the International Association for the Evaluation of Educational Achievement or the International Study Center of TEDS-M.

2.1.1.1 Topics in Tertiary and School Level Mathematics

Taiwanese SPM teachers reported studying 16.56 tertiary level mathematics topics of the 19 topics listed in the questionnaire. This amount ranked third among 15 participating countries, which is 1.28 less than leading Russia and 6.75 more than East Asian Singapore. Ten topics were studied by more than 90% of Taiwanese SPM teachers. Most SPM teachers studied topics in the area of Continuity and Functions and Probability and Statistics. The studied rates of all four areas (see Table 1) are significantly and sizably higher than the mean rates of all participating countries or higher-achieving countries.[3]

In the seven listed school-level mathematics topics, Taiwanese SPM teachers studied 6.25 topics and four were studied by more than 90% of SPM teachers. Although Taiwanese SPM teachers had explored the two areas of school level mathematics (see Table 1) extensively when they were at primary or secondary schools, significantly more SPM teachers studied topics in either area than those in other countries.

At the primary level, Taiwanese PPM teachers reported studying 8.64 tertiary level mathematics topics. This magnitude is slightly lower than the international mean of 9.50, but substantially lower than the leading country of Thailand at 15.44. Probability was the only topic taken by more than 90% of teachers. In contrast to the secondary level, the study rates of different areas of tertiary level mathematics are either the same or significantly lower than the mean rates of all participating countries or higher-achieving countries, except for the area of Probability and Statistics (see Table 1).

Levels	Tertiary Level Math Area				School Level Math Area	
	Geometry	Discrete Structures & Logic	Continuity & Functions	Probability & Statistics	Numbers Measurement Geometry	Functions Probability Calculus
Taiwan-Sec	0.81^{**}	0.87^{**}	0.97^{**}	0.97^{**}	0.93^{*}	0.86^{**}
M-Sec	0.66	0.72	0.71	0.77	0.91	0.73
MH-Sec	0.68	0.75	0.72	0.78	0.89	0.75
Taiwan-Pri	0.51	0.57	0.25^{**}	0.86^{**}	0.85^{**}	0.49
M-Pri	0.51	0.58	0.40	0.70	0.87	0.50
MH-Pri	0.49	0.57	0.39	0.67	0.88	0.50

Note. *M-Sec*=international mean of all participating countries at secondary level. *MH-Sec* =mean of Higher-achieving countries at secondary level. *M-Pri*=international mean of all participating countries at primary level. *MH-Pri* =mean of Higher-achieving countries at primary level.
$^{*}p < .05$; $^{**}p < .01$

Table 1: Studied rates of each area of tertiary level and school level mathematics

In the seven school-level mathematics topics, Taiwanese PPM teachers studied 4.51 topics, with a middle ranking. Two of them were studied by more than 90% of PPM teachers. The two areas of school level mathematics show a significantly lower percent

[3] Countries that achieved MCK and MPCK levels beyond the international mean of 500.

of PPM teachers studying topics in the area of Numbers/Measurement/ Geometry than teachers in other countries.

2.1.1.2 Topics Taken in Mathematics Pedagogy

Of the two areas, composed of eight mathematics pedagogy topics, Taiwanese SPM teachers studied fewer courses in the area of Foundations than Instruction (see Table 2). Within each area, the Taiwan pattern of the magnitudes of studied rates for different topics differs from the average approach for all participating countries or higher-achieving countries.

The situation of less OTL than other countries for mathematics topics appeared again for the study of mathematics pedagogy topics, specifically in the area of Foundations, at the primary level in Taiwan (see Table 2).

Taiwanese teachers studied significantly fewer topics in the Context of Mathematics Education, and Affective Issues in Mathematics than did other countries. Further, Taiwanese primary level TE did not emphasize Foundation of Mathematics as much as did other countries.

Levels	Foundations				Instruction					
	FM	CME	DMAT	*Mean*	MI	DTP	MT	MSC	AIM	*Mean*
Taiwan-Sec	0.74**	0.13**	0.76	0.54	0.95**	0.83**	0.88*	0.79	0.39**	0.77
M-Sec	0.69	0.47	0.80	0.65	0.91	0.76	0.83	0.78	0.53	0.76
MH-Sec	0.65	0.37	0.79	0.60	0.93	0.77	0.86	0.86	0.48	0.78
Taiwan-Pri	0.32**	0.11**	0.56**	0.33	0.91	0.76	0.74	0.80**	0.35**	0.71
M-Pri	0.58	0.48	0.76	0.61	0.90	0.74	0.77	0.74	0.52	0.73
MH-Pri	0.50	0.38	0.76	0.55	0.90	0.73	0.79	0.84	0.55	0.76

Note. FM = Foundations of Mathematics; CME = Context of Mathematics Education; DMAT = Development of Mathematics Ability and Thinking; MI = Mathematics Instruction; DTP = Developing Teaching Plans; MT = Mathematics Teaching: observation, analysis and reflection; MSC = Mathematics Standards and Curriculum; AIM = Affective Issues in Mathematics.
$^{*}p < .05$; $^{**}p < .01$

Table 2: Studied rates for mathematics pedagogy topics in two areas

2.1.1.3 Topics Taken in General Pedagogy

In the two areas, composed of eight general pedagogy topics, both Taiwanese SPM and PPM teachers studied fewer topics than did other countries (see Table 3). Within each area, Taiwan patterns of the magnitudes of studied rates for different topics differ from the approaches of international patterns or higher-achieving country patterns. Taiwan underemphasized the philosophy and sociology of education compared to international trends. The Method of Educational Research did not gain equal value compared to other countries. However, the topics of Educational Psychology and Knowledge of Teaching, which are more practical in terms of highly relating to how to teach, gained considerable emphasis in Taiwan.

Country	Social Science				Application					
	HEES	PE	SE	*Mean*	EP	TS	MER	AM	KT	*Mean*
Taiwan-Sec	0.56**	0.63**	0.67**	0.62	0.97	0.73**	0.34**	0.80	0.93**	0.75
M-Sec	0.63	0.74	0.77	0.71	0.96	0.87	0.61	0.82	0.83	0.81
MH-Sec	0.62	0.66	0.73	0.67	0.96	0.82	0.52	0.74	0.85	0.78
Taiwan-Pri	0.61**	0.54**	0.63**	0.59	0.97	0.79**	0.50**	0.72**	0.92**	0.78
M-Pri	0.70	0.77	0.80	0.76	0.97	0.90	0.70	0.81	0.88	0.85
MH-Pri	0.68	0.71	0.78	0.72	0.98	0.89	0.63	0.82	0.91	0.85

Note. HEES = History of Education and Educational Systems; PE = Philosophy of Education; SE = Sociology of Education; EP = Educational Psychology; TS = Theories of Schooling; MER = Methods of Educational Research; AM = Assessment and Measurement; KT = Knowledge of Teaching.
$^{*}p < .05$; $^{**}p < .01$

Table 3: Studied rates for general pedagogy topics in two areas

2.2 Relationship of OTL and Teaching Knowledge

This subsection reports the relationships of OTL of taking different levels of mathematics (school level and tertiary level) and pre-service teacher MCK and MPCK outcomes in Taiwan.

2.2.1 Types of OTL Contributing to MCK and MPCK Outcomes

Few would argue against the influence of learner background on potential to learn. This study formulated two-level hierarchical models relating both selection and OTL variables to MCK and MPCK outcomes. Specific OTL and selection variables were included in the regression analyses as predictors at two levels, the individual level and the mean level, for each participating institution.

Secondary level	MCK			MCK-Tertiary			MCK-Sec.			MPCK		
	Est	(se)	*p*	Est	se	*p*	Est	se	*p*	Est	se	*p*
Intercept	-95.5	(143.2)	0.505	184.8	(130.1)	0.155	-117.1	(124.0)	0.345	-32.0	(105.4)	0.761
Future teacher level												
University level math OTL	**5.6**	**(1.8)**	**0.002**	**3.9**	**(1.9)**	**0.042**	**5.6**	**(1.9)**	**0.003**	**6.5**	**(2.4)**	**0.007**
School level math OTL	-4.0	(4.5)	0.372	-1.3	(3.9)	0.739	-3.2	(3.4)	0.354			
Math Education OTL										-5.2	(3.2)	0.103
General Education OTL												
Marks/grades level received in sec.	-3.9	(3.2)	0.228	0.9	(2.8)	0.741	5.1	(3.3)	0.124	0.4	(3.3)	0.902
Highest math level in sec.												
Institution level												
University level math OTL	**24.2**	**(5.8)**	**0.000**	**18.6**	**(5.8)**	**0.001**	**24.8**	**(5.1)**	**0.000**	**24.8**	**(3.9)**	**0.000**
School level math OTL	**22.1**	**(9.3)**	**0.018**	**18.3**	**(6.7)**	**0.007**	**22.4**	**(10.4)**	**0.032**			
Math Education OTL										8.7	(8.9)	0.333
General Education OTL												
Marks/grades level received in sec.	**65.2**	**(15.2)**	**0.000**	**46.6**	**(15.9)**	**0.003**	**67.7**	**(10.1)**	**0.000**	**62.3**	**(19.9)**	**0.002**
Highest math level in sec.												

Note. Blank spaces indicate that the independent variables in the corresponding row were tested but excluded in the final models for the dependent variables defining the corresponding column. None of the negative coefficients are statistically significantly different from zero at the .05 level.

Table 4: Multi-level analyses relating MCK and MPCK to OTL at the secondary level

Results for the secondary level show that at the individual level only the number of topics taken in tertiary level mathematics had a significant relationship to SPM teacher MCK outcomes, but the estimated effect was small (see Table 4). At the institution level, three variables contributed to MCK achievement with sizable estimated effects; they were the OTL of tertiary level, school level mathematics, and marks/grades level received in secondary school, a selection variable. These OTL and selection variables also contributed to tertiary level MCK and secondary school MCK outcomes. The relationships of OTL and MPCK show that the OTL of school level mathematics was no longer a variable contributing to MPCK.

Results for the primary level show that both tertiary level and school level mathematics OTL at the individual level contributed to the MCK and secondary school MCK outcomes; however, the estimated effects were small (see Table 5). Unlike the secondary level case, no significant effects generated from the institution level at the primary level study. For the primary school MCK, the school level mathematics OTL was replaced by the selection variable as a variable related to this level of MCK outcomes. School level mathematics OTL was also not a variable for MPCK; however, tertiary level mathematics OTL at the institution level joined to contribute to MPCK.

Primary level	MCK			MCK-Sec.			MCK-Pri.			MPCK		
	Est	(se)	*p*	Est	se	*p*	Est	se	*p*	Est	se	*p*
Intercept	478.7	(68.6)	0.000	448.3	(54.9)	0.000	453.5	(204.1)	0.026	458.0	(48.8)	0.000
Future teacher level												
University level math OTL	**4.2**	**(0.9)**	**0.000**	**3.7**	**(0.7)**	**0.000**	**3.3**	**(0.7)**	**0.000**	**2.5**	**(0.4)**	**0.000**
School level math OTL	**4.0**	**(1.6)**	**0.012**	**3.8**	**(0.9)**	**0.000**	2.4	(2.0)	0.239	3.5	(2.0)	0.081
Math Education OTL	1.8	(1.8)	0.321				1.1	(1.6)	0.484	0.9	(1.5)	0.570
General Education OTL										-1.1	(1.0)	0.264
Marks/grades level received in sec.							**9.2**	**(2.6)**	**0.000**			
Highest math level in sec.	-5.0	(8.9)	0.576	-9.9	(8.6)	0.248				-9.0	(7.7)	0.240
Institution level												
University level math OTL	20.2	(10.7)	0.059	20.1	(11.8)	0.088	-0.2	(29.2)	0.995	**18.8**	**(5.4)**	**0.000**
School level math OTL	27.5	(17.2)	0.110	14.9	(17.7)	0.399	21.9	(39.7)	0.581	1.9	(9.4)	0.836
Math Education OTL	-12.0	(7.9)	0.131				-5.1	(7.4)	0.491	-4.0	(4.0)	0.310
General Education OTL										3.5	(4.0)	0.376
Marks/grades level received in sec.							23.3	(35.9)	0.516			
Highest math level in sec.	-25.4	(15.1)	0.094	-18.7	(9.8)	0.057				-9.7	(7.5)	0.192

Note. Blank spaces indicate that the independent variables in the corresponding row were tested but excluded in the final models for the dependent variables defining the corresponding column. None of the negative coefficients are statistically significantly different from zero at the .05 level.

Table 5: Multi-level analysis results relating MCK to OTL in the primary level

2.2.2 Relationship of Individual Topic OTL and Teacher knowledge

To study the relationship between individual topics and pre-service teacher knowledge, a *t* test was used to test whether the means of knowledge scores for studied and non-studied participants differed. Whether the difference between the two means was not only significant, but practically relevant, was evaluated with Cohen's parameter of effect size *d* (the difference between independent means expressed in units of the within-population standard deviation). According to Cohen, the cut points for small, medium, and large effect size based on means were 0.2, 0.5, and 0.8. Differences were regarded as practically relevant if they exceeded 0.2 of a standard deviation.

2.2.2.1 Relationship of Individual Topic OTL and MCK

In the 11 tertiary level mathematics topics studied by less than 92%[4] of Taiwanese SPM teachers, seven topics significantly contributed to SPM teacher MCK outcomes. Six of these topics fall into the areas of Geometry and Discrete Structures and Logic (see Table 1 for the four areas). The topic of Abstract Algebra has a large effect size of 0.89 and the Theory of Real/Complex Functions/Functional Analysis has a medium effect size of 0.73.

In the four school level mathematics topics studied by less than 92% of Taiwanese SPM teachers (see Table 2 for all seven listed topics), only measurement significantly influenced SPM teacher MCK, with a small effect size of 0.38.

For the primary level, in the 19 listed tertiary level mathematics topics, 11 topics significantly related to PPM teacher MCK. These topics fall into the four areas of mathematics. The area of Continuity and Functions included the most, all five topics in this area; the OTL of Advanced Calculus/Real Analysis/Measure Theory enjoyed the largest effect size of 0.96 and Differential Equations were the second largest at 0.72. The remaining tertiary level mathematics topics had small effect sizes.

All school level mathematics topics, except the topic of Functions/Relations/ Equations, had significantly small effects on PPM teacher MCK.

2.2.2.2 Relationship of Individual Topic OTL and MPCK

The OTL of studying six tertiary level mathematics topics influenced the SPM teacher MPCK with all small effect sizes. Four topics fall in the area of Discrete Structures and Logic. Axiomatic Geometry (d=0.44) and Number Theory (d=0.43) had the largest effect sizes.

In school level mathematics, Validation/Structuring/Abstracting is the only topic influencing SPM teacher MPCK with a small effect size of 0.31.

None of the OTL of mathematics education topics related to SPM teacher MPCK. The studies of general pedagogical topics, Philosophy of Education (d=0.23) and Sociology of Education (d=0.24), had even smaller negative effects on SPM teacher MPCK.

For the PPM teachers, the studies of nine tertiary level mathematics topics influenced the MPCK outcomes of PPM teachers. These topics were similar to the topics influencing MCK, but the effect sizes were smaller and no larger than 0.55.

Two of the six school level mathematics topics influencing MCK for PPM teachers were no longer topics influencing MPCK outcomes, but included Measurement and Geometry. The remaining four influenced MPCK.

In contrast to the SPM teacher case, the OTL to study three mathematics pedagogy topics had a positive influence (with small effect sizes) on PPM teacher MPCK

[4] This percentage was chosen to ensure that the participants in the non-studied group met a large sample requirement (30 entries of data) for the *t* test.

outcomes; these topics were Mathematics Instruction, Developing Teaching Plans, and Mathematics Teaching Observation/Analysis/Reflection. Studies of the general pedagogical topic did not significantly influence PPM teacher MPCK outcomes.

SECTION III: MATHEMATICS TEACHER EDUCATION RESEARCH AND ACTIVITIES

3.1 General Description of Mathematics Teacher Education Research

From 1996 to the present, Taiwan has undergone a curricular reformation and has continued with a major reform that links primary and lower secondary curricula. The dramatic change in the structure of mathematics curricula and teaching strategy influenced by constructivism has received intensive debate because of unfamiliarity with the theory and practice of teaching by most in-service teachers. As a result, the policy, content, and practice of pre-service and in-service teacher education have transformed and research activity has thrived accordingly. According to the search data from the National Science Council (NSC) Web site, 810 mathematics education projects were granted from 1998 to 2010. Among them, 82 projects were teacher education and professional development-related (see Figure 1).

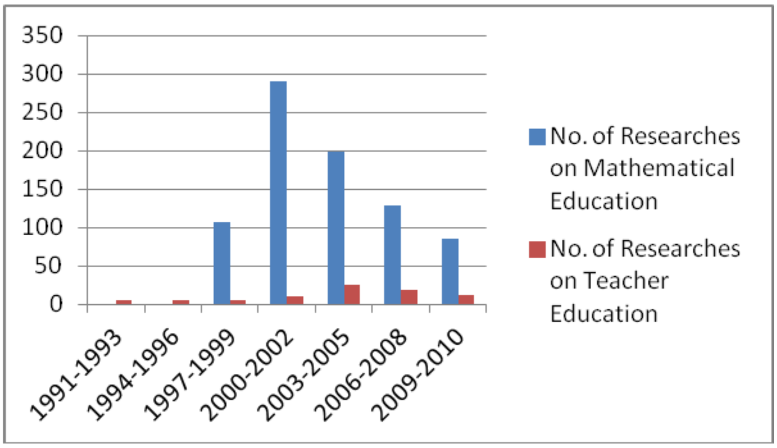

Figure 1: Biannual statistic of research projects granted by NSC from 1991 to 2010. Note that the final column includes only one year of data.

Driven by government policy, many in-service teachers since 1994 have returned to university to obtain credits, master's degrees, PhDs, or EdDs to promote their professional knowledge. The effect of this trend has enlarged academic activity more than ever. In-service teacher engagement in research, supervised by teacher education scholars, has fueled research on valuable authentic teaching problems, and enabled in-service teachers to resolve their own problems more scientifically, such as when using action research. The backbone of research activity in Taiwan consists of scholars in 40 secondary teacher education institutions. Among them, the most active institutions are eight traditional normal or education universities (MOE, 2011b). The major platforms for sharing research findings are journals (SSRCNSC, 2011), such as the *Chinese Journal of Science Education* and the *Contemporary Educational Research Quarterly*. More than 15 international conferences on education have been sponsored by NSC annually and a range of 19 to 41 conferences have been sponsored by MOE in teacher education universities (MOE, 2011c) in the past three years. Research activity has been flourishing in the past two decades. Teachers holding graduate degrees are increasing rapidly and the research network is expected to evolve into a prosperous organism. Various academic societies also hold academic education sessions during their annual meeting or conference.

3.2 Academic Activities from the Perspective of Professional Development

Academic activities in Taiwan are boosted by laws, a regulation system, and an operating system (see Figure 2), which consists of a teaching advisory group and a professional learning community supported by schools and professional scholars.

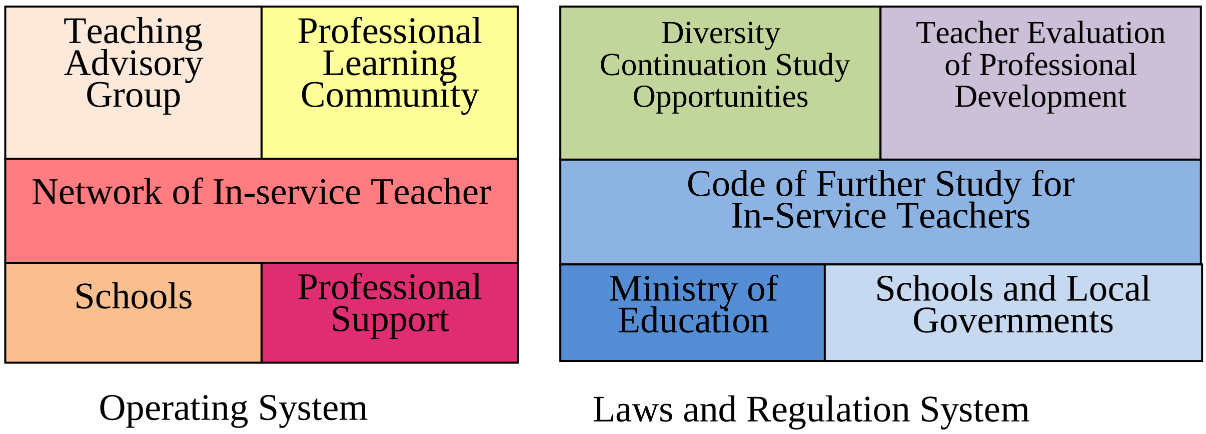

Figure 2: Systems and structures of in-service teacher professional development.

Professional development in Taiwan is threefold according to the type of organization conducting academic activities: university-based, government-based, and Internet community-based. University-based activities offer diverse opportunities for continuation study. Government-based academic activities for professional development of mathematics teachers can be classified into three levels:

(1) Intra-school: In each school, mathematics teachers form a professional group. The class schedule at school is arranged for all mathematics teachers to share a free teaching morning or afternoon per week. Mathematics teachers hold a monthly meeting to resolve problems in their teaching. Scholars and experienced teachers from other schools are invited to give special interest talks.

(2) Countywide: In each county, the county department of education supports all local education programs. The County Compulsory Education Advisory Group (CCEAG), consisting of selected experienced mathematics teachers across the county, conducts classes, workshops, or teaching demonstrations.

(3) Nationwide: The Department of Secondary Education of the MOE supports all national education programs and projects. The National Compulsory Education Advisory Group (NCEAG), consisting of invited expert teachers and professors from universities in different national regions, is an official organization that coordinates all CCEAG activities and offers training courses and workshops for CCEAG members.

Academic activity through the Internet is growing at an amazing speed. Some scholars in Taiwan, such as Lee (2003), and Lee and Wang (2005) have pioneered this topic for years and have yielded fruitful results using information and communication technology in promoting in-service teacher professions. In-service teachers also frequently visit Web sites to share their teaching experiences both in MCK and MPCK. Internet-based professional development is expected to become increasingly important in the future. Table 6 shows the total hours teachers spent attending e-learning in-service education activities by school levels.

Teachers at each level	Total no. of teachers	No. of teachers attended	Total no. of hours spent	Average of hours spent
Upper Secondary	33,699	2,161	22,009	0.65
Lower Secondary	14,849	596	5,952	0.40
Primary	47,286	2,757	39,161	0.83

Table 6: Total hours teachers spent attending e-learning in-service education activities by school level in 2010

Table 7 shows data collected in 2010 of academic activities for in-service teacher professional development (MOE, 2010b).

Teachers at each level	The average attendance hours of each teacher per year	
	2008	2009
Upper Secondary	24.85	34.85
Vocational Secondary	32.52	39.38
Lower Secondary	37.21	47.50
Primary	71.34	88.42

Table 7: Teacher attendance hours of assorted professional development programs

Other than these academic activities, the MOE authorizes teacher education institutions to offer continuation programs for in-service teachers. Table 8 shows the number of teachers attending the continuation programs from 2003 to 2010.

Year	2003	2004	2005	2006	2007	2008	2009	2010
No. of teachers	2150	2823	6690	5435	6427	6475	6390	6360

Table 8: Number of teachers attending continuation programs

3.3 Involvement Opportunities in Mathematics Education Research

According to MOE statistics (2012), the percentage of teachers with master's or doctoral degrees has consistently increased, and reached an all-time high in 2010 (see Figure 3). The percentage is expected to climb to exceed 60% in ten years. A master's degree in Taiwan requires writing a thesis. Thus, all graduate students must perform research to write their thesis. As a result, most in-service teachers are capable of conducting research and are connected well with scholars in institutions.

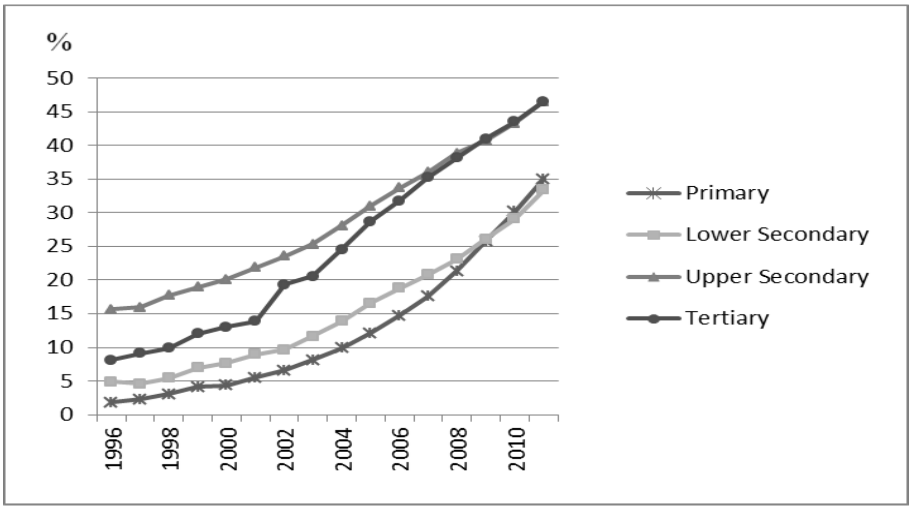

Figure 3: Percentage of teachers with graduate diploma by levels.

Because Taiwan government intends to withdraw the upper secondary school entrance examination by 2013, students must demonstrate their potential and ability by various evidences such as medals won in different types of competitions. National and international science fairs are becoming more important than ever. The National Primary and High School Science Fair is a competition on many levels, with projects winning local exhibitions before being selected to participate in national competitions. According to the 51st science fair bulletin, more than 20,000 projects have been presented annually by teams from primary and secondary schools nationwide in recent years, and over 5,000 projects won the first round in the countywide competition (NTSEC, 2009).

Math project at each level	No. of projects	No. of students participated	No. of teachers participated
Upper Secondary	22	49	32
Lower Secondary	19	47	31
Primary	15	51	27

Table 9: Mathematics projects of different levels in the final list in 2011

Table 9 (NTSEC, 2011) shows the mathematics projects that entered the final list of different levels in 2011. Teachers instructing students to compete in the science fair are intensively engaged in academic activity where they promote their research proficiencies and simultaneously lead their students through projects that are demanding in both mathematics knowledge and creativity. As a prominent competition, the science fair provides a great opportunity for students and teachers to engage in academic activity.

SECTION VI: CONCLUSION

Teaching in Taiwan provides an attractive career in terms of income, working hours, career development opportunities, and job security. The recent teacher education movement started in 1994 as the Teacher Education Act has opened multiple means to teacher education. Becoming a teacher has become an extremely competitive task. A positive side of this situation is that serious competition and a rigorous evaluation and selection process may raise teacher quality. However, the Taiwan teacher education system faces many challenges.

Rapid changes in the numbers of teacher education universities and pre-service teachers have created an unstable teacher education system. Low birth rates in recent years have reduced the quota of new teachers and dramatically increased the number of reserve teachers. Taiwan faces the pressure of reducing the number of reserve teachers; however, it must consider the free market in the teacher profession.

The extremely low passing rates of on-site screening and selection for tenure teaching positions has led many teacher education programs to focus on how to prepare for the 20-25 minute teaching demonstration rather than how to become a good teacher in a real classroom.

The international comparison results have released many figures of Taiwan teacher education. In Taiwan, both individual opportunities to study tertiary level or school level mathematics, within or between universities, influence their mathematics content knowledge achievement. Although Taiwanese secondary level teachers have many opportunities to learn both levels of mathematics, they have fewer opportunities to learn mathematics pedagogy. Teacher education programs at the primary level do not require pre-service teachers to study both levels of mathematics. This is a disadvantage to the mathematics content knowledge of teachers because the TEDS-M findings show that the study of many tertiary and school-level mathematics topics influences teacher performance in mathematics content knowledge.

The lack of opportunity to learn the topic of Affective Issues in Mathematics does not shed light on resolving the problem of low motivation of Taiwanese school students to study mathematics, revealed by TIMSS results.

The Multi-level analyses also show that the academic performance before the entrants enter teacher education programs is a significant variable that influences their preparation outcomes. Taiwan though ranks at the top in the TEDS-M knowledge achievement tests; an attempt to elevate teacher quality continuously is evidenced by the government encouraging universities to increase the ratio of master's to non-master's students in both secondary and primary teacher education programs.

Because the number of teachers earning graduate degrees is increasing rapidly, the research network is expected to evolve into a prosperous organism. As in-service teachers have formed Internet communities, Internet-based professional development is expected to become increasingly more important in the future. In addition to these approaches, the strategic actions for improving teacher education in response to the results of TEDS-M announced in 2010 (MOE, 2010c) indicate enhanced teacher education in Taiwan.

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POSTER PRESENTATIONS

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STUDY ON IMPROVING CLASS PRACTICE POWER OF MATHEMATICS IN TEACHER TRAINING: RELATIONSHIP AMONG THREE POWERS WHICH CONSTITUTE CLASS PRACTICE POWER

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It is very important to improve the crass practice power of mathematics teachers, because it has a strong relation with the mathematical ability of students. There are few studies on improvement of crass practice power of university students during coursework of mathematics education in a university (Akita & Saito, 2009).

The purpose of this study is the elucidation of the actual condition about the class practice power of university students. Especially, we focus on relationship among three powers, the power of a study of teaching, the power of making a teaching plan, and the power of execution a mathematics teaching, which constitute class practice power.

We made three models about the good relationship among the three powers as an assumption. We compared the relationship among the three powers of university students with those models.

Data for this study were collected at a university of education in Japan. The enforcement time was in May and June, 2009. We measured the power of a study of teaching by using the test on teaching objectives, contents and so on. We measured the power of making a teaching plan by using the teaching plan made by student. We measured the power of execution a mathematics teaching by observing a trial lesson practiced by student.

The results of analysis were as follows:

- The correlation between the score of test and the score of a trial lesson is 0.83; it is strong positive correlation. The students' power of execution a mathematics teaching has strong relationship with the power of a study of teaching.

- The correlation between the score of test and the score of the teaching plan is 0.29 and the correlation between the score of the teaching plan and the score of a trial lesson is 0.32; those are weak positive correlation. The students' power of execution a mathematics teaching has no relationship with the power of making a teaching plan.

- Three powers which constitute class practice power of a college student are not connected, and it differs from the models severely.

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THE MATHEMATICAL SELF-CONCEPT OF TALENTED STUDENTS PARTICIPATING IN A MATH CLUB

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Learning is primary a cognitive activity, but is influenced by affective factors such as self concept. It is the image an individual holds of him/her self. Many previous studies have confirmed the influence of self concept on achievements and on the will to further study (e.g. Ireson & Hallam, 2009). Academic self-concept is formed on two simultaneous sets of comparisons: External comparisons, in which students compare their self-perceived performance in a particular school subject with the perceived performance of other students and internal comparisons in which students compare their self-perceived performance in a particular school subject with their performance in other school subjects. Participation in programs for high level students often leads to a decline in self-concept, when students compare their own performance with their peers and realize there are students as good as they are, or better than themselves. This effect has been termed the "Big-Fish-Little-Pond Effect" (Marsh et al., 1995).

This study focused on the mathematical self-concept of mathematically talented students participating in the "Kidumatica" after school math club (Amit, 2009). Mathematical self concept was assessed in the beginning of the school year (pre test) and in the end of the school year (post test). No statistically significant decline of self concept measures was found. This is in contrast to several previous studies that concluded that participation programs for high level students has negative effects on the participants (e.g. Marsh et al., 1995). The characteristics of "Kidumatica" contributing to these findings, such as meaningful mathematical tasks, teacher- student interactions and no grades or report cards will be discussed and elaborated.

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A STUDY OF MATHEMATICAL TASKS IN ELEMENTARY MATHEMATICS TEXTBOOK

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In terms of instruction, Kilpatrick et al (2001) argue that the quality of instruction depends, for example, on the tasks selected for instruction and their cognitive demand. The tasks teachers assign to students influence to a large extent how students come to understand the curriculum domain. Moreover, tasks serve as a context for student thinking not only during, but also after instruction. This premises that tasks, most likely chosen from textbooks, influence to a large extent how students think about mathematics and come to understand its meaning (Doyle, 1988) .

The aim of the study of the nature of the mathematical tasks and the level of the cognitive demands of the mathematical tasks in elementary textbook.

Selected textbook series from the 1th mathematics textbooks currently used by the Basic Education Curriculum B.E.2544 (A.D.2001) by the Institute for the Promotion of Teaching Science and Technology, Ministry of Education. The data source consists of the tasks in the 1th mathematics textbook in Thailand elementary school. Data were analyzed by using the Mathematical Tasks Framework (Stein & Smith, 1998)

The research findings revealed that: more than 85% of mathematical tasks required low levels of cognitive demand revealed that nature of the mathematical tasks focused on producing correct answers and typical textbook word problems.

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CREATIVE USE OF PATTERNS IN PRE-SCHOOL TO ENHANCE LEARNING IN OTHER AREAS

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Pre-school and elementary school curriculum (NCTM, 2000) recommend that teachers provide opportunities for students to engage in activities that connect mathematics to other areas, generating significant learning experiences. It is fundamental that teachers use the natural curiosity of students in mathematics, to make sense of the world around them and understand the utility of mathematical tools. Tasks related to pattern exploration promote the establishment of several types of connections, with real life, between different topics of mathematics and even with other curricular areas (e.g. Orton, 1999; Vale & Pimentel, 2011), because of their transversal nature. Creativity is emerging as a fundamental aspect in different domains of education, including mathematics, and it is undeniably linked with curiosity and challenge. Exploratory tasks tend to implicate the use of processes like experimenting, conjecturing, investigating, communicating and creating, enhancing the use of creative approaches (Vale & Pimentel, 2011). Children's experiences with art, music, literature, science, and other areas, linked to the exploration of mathematical patterns can translate into noteworthy and creative approaches, creating opportunities for them to solve real-life problems making sense of the processes involved.

With this study we tried to analyse the impact of patterns in pre-school children learning in different areas of the curriculum. The research was based on a case study of a class with 25 five years old children. In each of the four tasks children were engaged in understanding phenomena related to certain areas, like a repetitive structure in a story, how to build wrapping paper, reading and playing a musical score and exploring the effects of light reflection. The results showed that, because of the existence of patterns, students were able to learn, in an insightful way, facts related to other areas, predicting what came next in a story, building wrapping paper having recognized its structure, quickly reproducing the rhythm of a certain music and perceiving what happens when they explore reflection in mirrors. Students found these tasks motivating and challenging, indicating an effective and creative learning environment.

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PHENOMENA OF DEVELOPING MATHEMATICAL PEDAGOGICAL CONTENT KNOWLEDGE – A LONGITUDINAL REPERTORY GRID

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Mathematics teachers' constructs of mathematics lessons will affect their action in class. To explore prospective mathematics teachers' constructs in different semesters, we asked them (N=424) to compare and analyse different mathematics lesson plans using an adapted Repertory Grid questionnaire. Within a longitudinal cohort (N=42), we found a development in their constructs. These results are used to develop a partly automated feedback, which supports the learning process of prospective teachers.

Teachers' *mathematical pedagogical content knowledge* (MPCK) (Blömeke, 2011) is a necessary but not sufficient condition for a good and successful mathematics lesson. Thus, it is important to support teachers' development of MPCK from the very beginning of their teacher education course. To explore the development of MPCK in teacher education, an adapted *Repertory Grid Method* (Kelly, 1955) questionnaire was designed (Bausch, Bruder & Prescott, 2011).

The survey described is a project between the University of Technology Sydney (UTS) and the Technische Universität Darmstadt (TUD). It is designed as a cross-sectional study with longitudinal components. Based on the data of 424 prospective teachers, a quantitative evaluation system was developed (Bausch, Bruder & Prescott, 2011). The poster will present the results of the 42 TUD students who were surveyed in their first and third semesters. To explore their MPCK development during their teacher education, we sought themes in their survey responses and found four different phenomena of MPCK development. Students' perspectives on the lesson plans changed in different ways: Some are more detailed in their lesson plan analysis, some change the focus of their analysis, some lose facets or foci, and some get more multifarious in their lesson plan comparison. These results are used to create an individual partly automated feedback, which is furthering participants' individual development of MPCK.

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ASSESSING PROBLEM SOLVING – A RATING PROCEDURE FOR EXPLORATIVE PROCESSES IN WRITTEN DOCUMENTS

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Mathematics teachers who are expected to regard 'doing mathematics' as a key feature of mathematics learning should experience mathematics as a problem solving activity already in pre-service teacher education. To achieve this goal pre-service teachers at University of Education Freiburg regularly take part in a problem solving course working on open-ended problems, keeping records of their work and reflecting their thoughts, emotions and preliminary ideas in a written journal. In the research project FORMAT "Mathematics Teachers as Researchers" we intend to measure (among other variables) the increase of problem solving competences. Due to the characteristics of the problems we mainly focus on assessing the quality of explorative mathematical processes as described in the framework of mathematical discovery as 'quasi-experiment' by Polya (1954). For that purpose Leuders, Naccarella & Philipp (2011) developed a set of more than 20 categories describing how students experiment mathematically.

Our aim was to develop a rating procedure drawing on these categories in order to detect the quality of the problem solving processes and to measure the progress of the participants. For that reason we reduced the set of categories to four meta-categories (*e*)*xample*, (*d*)*escription*, (*c*)*onjecture* and (*m*)*etacognition*. First the raters define homogeneous units of meaning within the writings and assign one of the categories to each unit. The ensuing assessment within these meta-categories comprises the aspects: number of systematically created examples (e), tables in use (e), number of example-based hypotheses (c), making a plan (m), and additionally influence of missing knowledge, achievement of solution. Two independent raters showed satisfactory to very good ICC after training. Thus the rating procedure appears to be useful in rating extensive explorative problem solving processes in a quantitative and reliable way.

The poster presents the categories with descriptions from the rating manual, examples of the rating process and some preliminary results from applying the rating procedure to measure the impact of the intervention described above.

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RISK ZONES: ZONES OF POSSIBILITIES?

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A landscape of investigation is defined by Skovsmose (2001) as a learning environment characterized by invitation from the teacher to develop an investigative activity and its acceptance by students. Since there is no guarantee that students will accept the invitation of the teacher, a landscape of investigation has two aspects: unpredictability and possibilities. Penteado (2001) defines a situation with those properties as a risk zone. We have endeavoured to encourage teachers to enter a risk zone. The reason for this is to provide the possibility to work in a landscape of investigation where knowledge is constructed collectively by students and teacher, because there is possibility in the classroom to doubt and argue. In the presentation we will show pictures related to some researches in a landscape of investigation. Thus we can refer to: Biotto Filho (2008) working with students of a Brazilian public school to understand how a landscape of investigation can contribute to growth of students' social awareness; Biotto Filho (2011) conducting a work in a orphanage to investigate how a landscape of investigation can motivation in learning mathematics; and Milani (2011) working with dialogue between prospective teachers and their students in a landscape of investigation. Based on this researches, we argue that a movement of towards a landscape of investigation, is a possibility for the teacher: to deal with the students' different intentions in mathematics classes, to contribute to growth of students' social awareness, to provide motivation in learning mathematics, and promote dialogue as a form of communication between teacher and students.

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DEVELOPING THE USE OF DIAGRAMMATIC REPRESENTATIONS IN THE PRIMARY CLASSROOM

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Leinhardt et al. (1991) emphasised the important role that representations play in explanations of mathematics; external representations can provide a link between the concrete experiences of students and the more abstract world of mathematics (Bruner and Kenney 1965). Pape and Tchoshanov (2001) provided examples of types of external representations, including numerals, algebraic equations, graphs, tables, diagrams, and charts. Larkin and Simon (1987) use the term *diagrammatic* representation to describe “a data structure in which information is indexed by two-dimensional location” (p. 68).

The present study was a funded project aimed at developing primary children’s understanding of mathematics, through developing teachers’ use of diagrammatic representations in the classroom. The study involved 8 maths coordinators from primary schools taking part in professional development sessions looking at teachers’ use of diagrammatic representations for mathematics. The sessions drew on research on how diagrammatic representations can be used, specifically looking at representations of multiplication and fractions. Coordinators taking part in the project attended three one-day sessions and were then asked to work with teachers in their schools in order to put these ideas into practice. The impact of the project was be assessed through pre- and post-tests (with comparison control schools) for pupils on multiplication and fractions. Lesson observations and interviews with teachers were also conducted. However, this poster presentation will outline the quantitative findings of this project, highlighting some of the representations used and the impact of the project on pupils and teachers. It will also put forward implications for future research in this area.

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INTERNSHIP STUDENT TEACHERS' TEACHING PRACTICE IN THE CONTEXT OF LESSON STUDY

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Prospective teachers learn from their activity and their reflection on their activity, and such learning takes place in variety of places, as they interact with others, notably their university teachers, colleagues, school mentors, school students, and other members of the community (Lin & Ponte, 2008).

The objective of this research was to study internship student teachers' teaching practice in the context of lesson study. The target group included 4 fifth year undergraduate students in Mathematics Education, faculty of Education, Khon Kaen University, 2010 school year. The data were collected through classroom observation, classroom reflection, interviewing internship student teachers and questionnaire based on Inprasitha's conceptual framework of lesson study and open approach (Inprasitha, 2010).

The research findings found that, in lesson study that include 3 steps, collaboratively design research lesson, collaboratively observing research lesson and collaboratively reflection on teaching practice. The internship student teachers' behaviours were

1) In collaboratively design research lesson, internship student teachers, in-service teachers and school coordinator collaborated in creating the open-ended problem situation as well as teaching steps, questions, conjecturing students' approach to be occurred and making major material and supplementary material, following the Japanese mathematics textbook and the classroom's data. They were collaboratively design research lesson one time per week.

2) In collaboratively observing research lesson, had 2 cases. In case of teaching, internship student teachers taught following the steps of open approach. And in case of observing, they observed the students' thinking process, students expressed in class and took note what they observed in their notebooks.

3) In collaboratively reflection on teaching practice, they talked about students' approach, the student's real existing approach which wasn't relevant to the conjectured one and guidelines for instructional management in next period. They were collaboratively reflection on teaching practice one time per week.

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FACILITATING WORLD-WIDE COMMUNICATION AND COLLABORATION IN MATHEMATICS EDUCATION RESEARCH USING A VIRTUAL WORLD

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Virtual environments such as “Second Life” <www.secondlife.com> are emerging as major cultural influences with significant opportunities and possibilities for mathematics education. Second Life (SL) is a massively multi-user on-line social interaction virtual environment where individuals design and inhabit their own "avatars" or virtual bodies. In SL, individuals can socially and collaboratively interact in real time through their avatars with the avatars of others, via gestures and actions, and communicate through text messaging and voice over internet. In this session, we (the David Wheeler Institute for Research in Mathematics Education at SFU <http://blogs.sfu.ca/research/davidwheeler>, in collaboration with the ENL Group <www.engrammetron.net>) have researched, developed, and implemented an initiative to do just that. In this short oral, we introduce a virtual David Wheeler Institute in SL to foster creativity, and facilitate communication and collaboration amongst mathematics education researchers and graduate students world-wide.

Thus far, we have held conference and lecture sessions simultaneously, both in the real world (e.g., middle top video inset), and in the virtual conference facility “Wheeler Island” in SL (right video inset). Real world attendees were logged in as virtual world attendees. One attendee (middle lower video inset) attended the virtual session remotely using an ENL-based computer with eye-tracking monitor. Attendees interacted using voice over internet. Power point slides for our conference presentations are controlled from a virtual podium. Both real and virtual sessions were recorded and the acquired data were subsequently integrated, coded and analysed as per the figure above. We discuss results and demonstrate this virtual facility and its potential for facilitating communication and collaboration in mathematics education research, as well as its potential for designing and conduction experiments online, along with some clips from our new virtual lecture series.

2012. In Tso, T. Y. (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 218. Taipei, Taiwan: PME.

ANATOMY OF AN "AHA" MOMENT

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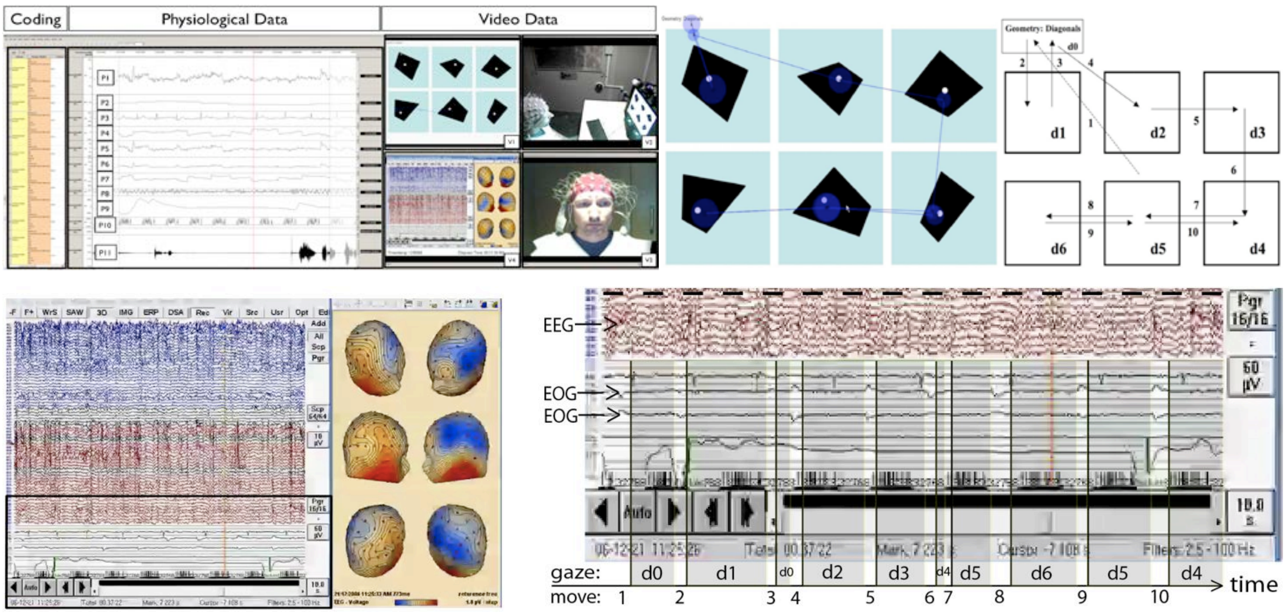
In this session we present the results of a detailed observation of an "aha" moment. Using a full suite of observational techniques, including audiovisual recordings, screen and keyboard capture, eye-tracking, electrooculography, electrocardiography, electroencephalography, and respiration, we illustrate our acquisition, capture, and analysis of 10 seconds of data, revealing extremely rich and extremely fast reasoning associated with a moment of insight. We demonstrate how further analysis consistent with results from the cognitive neurosciences manifests this moment in brain activity.



The above figures illustrate the data we acquired and analysed that we will present and discuss. The panel on the upper left consists of a single movie frame from the integrated and time synchronized data acquired over the 10s time period in question. The panel on the upper right illustrates in detail our analysis of the participants' eye-movements during this period of time. The panel on the lower right illustrates the psychophysiological data over the same time period. Finally, the panel to the lower right maps results from the eye-tracking analysis onto the psychophysiological data, enabling more detailed analysis of the electroencephalographic data (EEG). In that regard, we used independent component analysis (ICA) to separate brain signals from coherent noise sources, thereby isolating and identifying a burst of energy in the gamma range (~28-40Hz) in the superior anterior temporal cortex known from studies in cognitive neuroscience to be associated with moments of insight. Combining these results with our behavioural data has led us to a rich interpretation of the relationship in this case between insight and reasoning from this 10s data set, which we anticipate and are hopeful will evoke some interesting discussions.

MULTIPLE REPRESENTATIONS OF FUNCTIONS: HOW ARE THEY USED BY STUDENTS WORKING WITH TECHNOLOGY?[23]

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Ana Patrícia Gafanhoto

Escola Secundária Mouzinho da Silveira, Portalegre

This poster refers to a one year research project aiming to analyse how students use the different representations of functions when they work with dynamic software that provides algebraic, graphical and tabular representations of functions. We concluded that the students chose the representations to respond to the tasks accordingly to its mathematical demands, revealing a good sense of the advantages of each representation.

The use of multiple representations has the potential of making the process of learning functions more meaningful and effective (Friendland & Tabach, 2001). Technology can be exploited to privilege certain types of representation over others, focusing attention in specific aspects of function (Ferrara, Pratt & Robutti, 2006).

Five case studies of small groups of students of a 9th grade class were elaborated based on data obtained by the analysis of the written responses and digital files that they produced when solving diverse mathematical tasks, including modelling tasks. Students were able to use the different representations but they used the tabular one as a way to obtain particular numerical values. They revealed a tendency to use the numerical and algebraic representations when they had to find the image of an object (or vice versa). They used the graphical representation when they had to describe the behaviour a function or to compare functions. The poster illustrates these conclusions by exhibiting and analysing the different representations used by the groups of students when solving two tasks with different mathematical demands.

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MATHEMATICAL REPRESENTATIONS OF FRACTIONS: COMPARISON BETWEEN THAI AND JAPANESE MATHEMATICS TEXTBOOK

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Mathematics should be presented in instruction by taking complex subject matter and translating it into representations that can be understood by student (Fennema & Franke, 1992). Goldin (2003) defined representation as a configuration of signs, characters, icons, or objects that stand for, or "represent" something else.

Traditional approach of teaching in Thai context, Inprasitha (1997) stated that there are more than 90% of Thai teachers used mathematics textbooks as an instructional media. Furthermore, in teaching fractions teacher usually used bar or pie chart like approach from textbook for described concept of fractions to their students. Inprasitha, Isoda & Ohara (2004) noted from the International Cooperation Project Towards the Endogenous Development of Mathematics Education that teachers expressed the idea that fractions was one of the easiest topics to teach but the student's scores was not reflect their confidence. The purpose of this study is to address different approach of teaching fractions in context of Thai and Japanese. Document analysis is tool for getting research result. The results showed that mathematical representation of fractions in Japanese mathematics textbook are more meaningful to student understanding than Thai approach.

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AMBIGUITY ALGORITHM IN ANALOGICAL REASONING AND PROBLEM SOLVING

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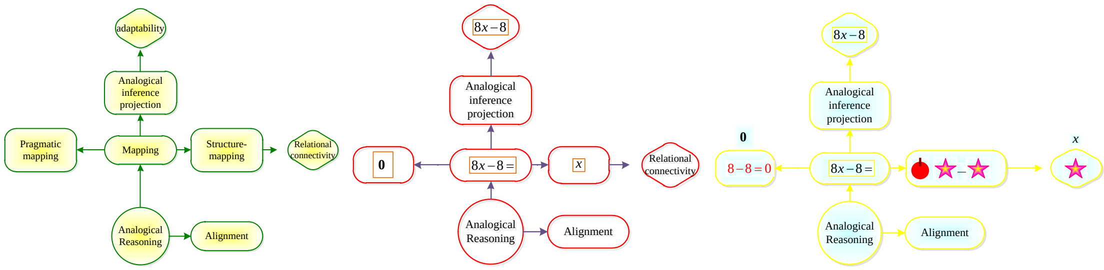
Most of learners based on their prior experiences to think, generate, and process their learning behaviors individually. Learners often use analogies from known domains to fill gaps in mapping and inferring in related domains. Consequently, the individual’s schema abstraction may adopt to infer, solve, promote, and transfer to new problems. However, the ambiguity algorithm in analogical reasoning and problem solving may possibly guide to misuse the algorithm to make analogical reasoning in the specific domain unsuitably. Furthermore, the algorithm of reasoning and inferring processes may psychological logic base rather than mathematical logic. The mapping processes of analogical reasoning (Gentner,1983, Markman, & Gentner,2000,Gentner,2002) are structure and pragmatic processes and analogical inference projection. This research is to detect the unsuitable analogical reasoning and to find the possible analogical mapping processes of ambiguity algorithm. The figure 1 illustrates the unsuitable analogical reasoning in question, $8x-8=$, ①8 (9.52%) ②0 (4.76%) ③ x (19.05%) ④ $8x-8$ (66.67%) and the possible analogical mapping processes of ambiguity algorithm.

Obviously, the ambiguity algorithm is reasonable for the individual in analogical reasoning and problem solving but not justifiable in mathematical reasoning and problem solving.

Figure 1: The mapping processes of analogical reasoning



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COOPERATIVE PEER INTERACTION AND INDIVIDUAL COGNITION WITHIN COGNITIVE, INTERACTIVE, AND TRANSPARENT TEACHING INTERFACE

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This paper presents an teaching interface, Cognitive, Interactive, and Transparent Teaching Interface (CITTI), to support the perceptible and distinguishable information for Cooperative Peer Interaction and Individual Cognition learning and teaching to insight, monitor and communicate the sensory information within teaching and learning coordinate plane in junior high school. Cooperative learning and teaching interactions will communicate the competition, motivation and situation which are based on dynamical sensory information and strategy modification to let individual to detect individual's cognition and metacognition. The active and constructive processes are the essential of self-regulation for learners to plan, monitor, and control their own learning process (Winne & Hadwin, 1998; Zimmerman, 1990; Winne, 2001; Zimmerman & Schunk, 2001). Owing to learning is a complex task, several different aspects and characteristics of learners, instructors, and materials must be taken into consideration. Furthermore, in self-regulation, not only learners/instructors need to stay on learning/teaching for fulfillment, but learners/instructors also need to deploy their comprehension, evaluation and modification for acknowledgment. In CITTI, individuals' emotions and motivations could be inspired and influenced via supportive thinking, cooperative acting, and directive operating on individual's cognition and metacognition compatibly. Meanwhile, individual's misconception and miscomprehension could be exposed and detected by active comparing, judging, and evaluating individuals' manipulations, actions and operations with others incompatibly.

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LEARNING OPPORTUNITIES FOR MATHEMATICAL PROOF: THE PRESENTATION OF GEOMETRY PROBLEMS IN GERMAN AND TAIWANESE TEXTBOOKS

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The basic features of the curriculum, as content, organization, and sequencing, have an impact on students' conception of proof, and the activities of problem solving might be an easy way for students to experience the process of proof. We chose mathematical proof as the topic to be discussed, as proof is a prototypic mathematical activity but difficult to master for students. The purpose of texts from mathematics textbooks is quite broad, but generally the mathematical goals can be briefly summarized as the acquisition of concepts, principles, skills, and problem-solving strategies (Shuard & Rothery, 1984). We present that the curricular materials (textbooks) for the beginning of learning mathematical proof differ significantly in Germany and Taiwan. We focused on comparing *the structure of content* of mathematics textbooks, including (1) the textual structure of content and (2) the structure continuity (flow) of knowledge.

We chose two general but important topics in introducing as mathematical theorems in German and Taiwanese textbooks. They are *the sum of interior angles of a triangle* and *the Pythagorean theorem*. We examined how these statements are presented in Germany and Taiwan by inspecting six different textbook series, three from each country. The details of analytical framework in analysing the structure of content will be introduced in our poster presentation.

By analysing two topics from different textbooks in Germany and Taiwan, we found that the types of text in Germany are diverse within textbook series while in Taiwan are static. German textbooks provide logical reasoning with hierarchical statements to a proof (theoretical way), e.g. from parallel postulates to the sum of interior angles of a triangle; from properties of similarity of triangles to Pythagorean theorem group. Taiwanese textbooks start with authorized knowledge and then sets varied worked examples or immediate practices in order to make students familiarize themselves with a learned statement (practical way). However, both educational societies try to enhance students' development of reasoning skills, which is important for proof competence. We found important differences in the presentation of paths to proof in German and Taiwanese textbooks, which reflect our view on two different mathematical philosophies. Proof in German textbooks emphasizes argumentation as a mode of validation whereas in Taiwan it is introduced as a mode of generalization and application with facts starting from the activity of conjecture.

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DEFINING FROM RELATIONSHIP BETWEEN SCHOOL COORDINATOR AND TEACHER

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The key aspect of collaborative work between two bureau government departments is to bridge the gap. Especially educational section, university and school had difference sociocultural. Center for Research in Mathematics Education (CRME), Khon Kaen University had conducted the project for professional development through lesson study and open approach, he concerned of dissimilarity context, CRME (Inprasitha, N., *et* al, 2008) originated new function, since 2006, for mathematics education graduate students to be connector between CRME and project schools in term of School Coordinator. In addition, he set initial lesson study team for driving the project.

In this paper, I analyses preliminary study of defining from relation of school coordinator and teacher in the project school. Which one of the purposes of my ongoing study is how to investigate relationship of school coordinator in fostering teacher to do research on their practice. The data were collected by videotaping through academic year 2554 on lesson study team's activities, were situated in the context of lesson study process and interviewing five school coordinators and five teachers, for basic information of my research.

From the long-run relationship of school coordinator and teacher, the context on which I focus in 3rd phase: collaboratively see, teacher and school coordinator were reflected truthfully and sincerely. The findings revealed, teachers reflected about their teaching concentrate on representing the experience - problems, confusion, nervousness – of practice to themselves, and to school coordinator. The findings suggested that school coordinator and teacher should do collaborative work on practice. They had massive of information that insight from insider and outsider (Ball, 2000).

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DESIGNED PROGRAM FOR PRESERVICE KINDERGARTEN TEACHERS TO IMPROVE MATHEMATICAL KNOWLEDGE AND TEACHING THROUGH PEER CO-LEARNING

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The goal of research is to explore a professional program can lead pre-service kindergarten teachers to learn mathematical knowledge and teaching mathematics for young children by peer co-learning. In the class, pre-service kindergarten teachers engage in productive unguided peer discourse to gain a shit which here based on how to learn mathematical knowledge, teach mathematics and confirm the value of mathematics. We show how the designed program contributes to this shift, and more than that: we aimed at identifying the kinds of discourse process are interwoven in peer collaboration. And additional goal is the development of a methodology for investigating, and presenting knowledge and social shifts that evidence productivity in peer discourse and learning a pair of pre-service teachers adopted.

The experiment was carried out during the academic year 2010-2011 with a regular group of pre-service kindergarten teachers in teacher–education program at the University, Taiwan. The 36-h module was conducted in 2-h sessions during the pre-service teachers' first year. The objective was to develop capabilities related to specific competencies in the training of professionals to teach mathematics. At this time, the Taiwan curriculum for kindergarten-school teachers was based on two years of the study. Of the 40 students enrolled, completed all the activities of the module. Their average age was 21. Data collection includes the subjects of learning and practice teaching in the class were observed, interview the subjects' responses after each class, videotaped, transcripts were analysed as well.

The result showed the designed program was effectiveness in learning and teaching mathematics. The participants' post-test scores (11.32) in mathematical knowledge higher than pre-test score (9.25) had significant difference ($p<.000$) in their performance. Referring PC of scaffolding, it provided them to know how to teach young children with various strategies in the class. However, more than CK to teach in the preschool, they needed further enhancing PCK. Except that, they confirmed their mathematical interests in the program. Form survey of their mathematical attitudes, the data indicated mathematical attitude scores higher than whole scales means (38.59>30). Hence, they were strong identities about mathematics and were willing to teach mathematics for young children in their carriers during program attending.

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A STUDY OF MATHEMATICS TEACHERS' INTERACTIVE PATTERNS ON THE ASYNCHRONOUS DISCUSSION NET ENVIRONMENT

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This study analyzes seven elementary school teachers' communication processes on the asynchronous discussion net environment. Content analysis was used to determine what communication patterns appeared and what teachers discussed. The results of this study showed that seven teachers showed the "asynchronous interaction" communication pattern. "Unsocial interaction" and "dominant leader interaction," as described by scholars (Milson, 1973; Roth, 1995) did not occur. This study speculates that the communication pattern displayed by these teachers is related to the background of them. The participants of this study are volunteer on-site teachers who are accustomed to expressing their ideas; thus, communicating and interacting with others is not difficult for them. Therefore, the communication pattern featuring a single leading speaker is unlikely to occur.

Because only seven teachers participated in this asynchronous Web-based teaching case discussion, despite their communication pattern gradually adopting a "symmetric interaction" pattern, the ideal condition of "every teacher communicating with every other teacher" did not occurred. Previous studies(Lee, 2006) have suggested that the "no participation" situation of Web-based communication primarily occurs when the number of participants is small or the participants' lack enthusiasm. Therefore, Web-based discussions with more participants are recommended in the future.

Overall, the asynchronous teaching case discussions focus on mathematical teaching cases, allow the participating teachers from different regions to gradually form better communication patterns and enhance their discussion content. The result of this study agrees with that of previous studies (Merseth, 1996), that is, teachers can understand the significance of teaching through case discussions, which also benefit teachers in developing practical responses and problem-solving abilities. The result of this study validates the perspectivethat Internet network connections enable learning to overcome the constraints of time and space and can increase the interactivities of learning (He,& Guo, 1996).

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A CONTENT ANALYSIS ON MIDDLE SCHOOL MATHEMATICS TEXTBOOKS WITH ALIGNMENT TO TAIWAN MATHEMATICS CURRICULUM STANDARDS

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Taiwan mathematics curriculum standards had undergone various revisions at different times of the century; and there were 4 major strands in the last version (MOE of Taiwan, 2003). Research on textbooks is important in that they determined how teachers teach, to a great extent (Lloyd, 2008). In this study we followed Tam (2010) and analyzed textbook by coverage, using pages as unit of analyzes. The textbook we used is the one published by government printer in 2010. Results are three. First, the total coverage (grade 7, 8, 9), in descending order is: Algebra, Geometry, Number, and Statistics. Second, among the three grades, grade 7 emphasized mainly on Numbers and Algebra; grade 8 and 9 mainly on Algebra and Geometry. The coverage of Statistics is minimal and found in grade 9. Finally, there are few chapters using the same naming as given in grade 6 elementary textbooks (mostly on Numbers), indicating a deeper presentation of contents and a vertical integration of contents in Number Strands. Results on coverage by percentages gave implications for the amount of instructional time during implementation.

Keyword:

Mathematics Textbooks, Middle School, Content Analysis

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EFFECTS OF CULTURAL ARTEFACT USE ON STUDENT MATHEMATICS MOTIVATIONS, EFFORT, AND ACHIEVEMENT

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AIM AND BACKGROUND

The aim of this study is to identify effective cultural artefact use that may have short-term and long-term effects on student motivations (confidence, interest, and value), effort use, and achievement in mathematics. Artefact use in mathematics learning reflects the constraints and affordances of a culture in relation to student learning processes and outcomes. The identification of effective artefacts can not only improve student mathematics learning outcomes but also provide valuable knowledge for supporting student mathematics learning.

METHOD

The research participants were 193 Grade-6 students from five classes of a primary school in Taiwan. At Time 1 and Time 2, the participants indicated their self perceptions of mathematics confidence, interest, value, and effort on 18 items in a five-point Likert scale; they also indicated their use of 8 artefacts (parents, older siblings, peers, cram schools, tutors, mathematics assessment books, other mathematics books, and computers) for learning mathematics (21 items in total). Their Time-1 and Time-2 school mathematics achievement data were also collected.

RESULTS AND DISCUSSION

The Likert-scale items show acceptable internal consistency reliability and construct validity, with the values of Cronbach's α being all above .60 and factor loading all above .40. The percentages of the participants who used the artefacts for supporting their mathematics learning were 66% (parents), 31% (older siblings), 55% (peers), 36% (cram schools), (4%) tutors, 68% (mathematics assessment books), 32% (other mathematics books), and 13% (computers). The results of regression analysis show that the use of other mathematics books has a short-term effect on student mathematics confidence, interest, value, and effort, and a long-term effect on confidence and value. Tutor use has a short-term negative effect, but a long-term positive effect on confidence, interest, and effort. Mathematics assessment books have a short-term effect on confidence and achievement. The results indicate proper ways to include cultural artefacts in supporting student mathematics learning.

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PROSPECTIVE TEACHERS' DIFFICULTIES IN SOLVING BAYES PROBLEMS

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In this research we assess the competence of 196 prospective secondary teachers (95 in the last year of the Batchelor in Mathematics; 101 in the Master of Secondary Education, specialty in Mathematics) in solving a typical Bayes problem, and compare our results with those by Díaz and Batanero (2009) with a sample of 414 psychology students. Bayes problems are included in the curriculum for high school level and reasoning about them appears in diagnosis, evaluation, decision making and applications of statistical inference. Moreover research related to Bayesian reasoning suggests that this ability does not develop without a specific instruction (Koehler, 1996) and the robustness and spread of the base-rate fallacy (Tversky & Kahneman, 1982). A qualitative analysis of the different steps needed to solve the problem was carried out and a semiotic analysis of a typical correct and incorrect response in each category was performed. Only 37.7% participants finished the complete solution of the problem and 15.8% were unable to identify the data or build an adequate representation. Only 65.8% participants identified the problem as a problem of inverse probability and only 52.5% computed the total probability in the denominator of the Bayes' formula. Results were very worse than those by Psychology students. To conclude, these results suggest the need to reform and improve the probability education these prospective teachers are receiving during their training.

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ENCOURAGING LEARNING WITH MULTIPLE REPRESENTATIONS IN THE MATHEMATICS CLASSROOM

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The ability of dealing flexibly with distinct representations of a mathematical concept has been shown to be a key factor for successful mathematical thinking and problem solving (e.g. Lesh, Post & Behr, 1987; Panaoura et. al., 2009); thus, many national standards emphasize the usage of multiple representations. However, empirical evidence concerning specific professional knowledge of mathematics teachers and possible effects on students' competencies in using multiple representations is rare. Our project "La viDa–M" ("encouraging learning with multiple representations in the mathematics classroom" – in German: "**L**ernen **a**nregen mit **vi**elfältigen **Da**rstellungen im **M**athematikunterricht") aims therefore not only at investigating students' competencies and beliefs regarding multiple representations, but also at correlating them with the specific professional knowledge and views of their teachers by using a multi-level approach. La viDa–M is a project carried out at Ludwigsburg University of Education.

Central to the first project phase is the investigation of learners' competencies in dealing with multiple representations and the question of how they might be influenced by their teachers' specific professional knowledge. A core questionnaire unit on teachers' knowledge and views concerning multiple representations has already been developed and used in a prior study. It will be enlarged and complemented by questionnaires and tests for students. Combining the empirical results of the first project phase with further theoretical conceptualizations, the second phase concentrates on developing and evaluating learning environments, tailored specifically to the identified learning needs of students and teachers. The designed course material will be used for an intervention study based on a treatment-control design. In addition it can be part of teacher professional development activities focusing on dealing with multiple representations in the mathematics classroom.

The poster presents and visualizes in more detail the theoretical background, the emerging research questions and the design of both phases of our project.

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TEACHING COMPLEX ANALYSIS: DESIGN OF VISUALIZATION MATERIALS FOR DISTANCE TEACHING

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This poster presents part of the results obtained within a project for teaching innovation in the Universidad Nacional de Educación a Distancia (UNED) called 'Visualization and teaching complex variable functions: design and use of materials'. The UNED is one of the biggest distance universities in the world, with more than 200,000 students. Distance teaching at UNED has traditionally been focused on a textbook, and the support of the teachers either by telephone or e-mail and rarely a few classes. This is changing thanks to the development of ICT. UNED is currently adapting to it thanks to the new opportunities for distance teaching brought by technological developments. In particular, the use of dynamic geometric software (GeoGebra), graphical representation software (SciLab) and other tools offer the possibility to exploit the strong geometrical component Complex Analysis possesses to enhance students' comprehension of the concepts. With this aim the project has been created. Taking into account the scarcity of previous research in visualization on Complex Analysis on the one hand and our aim of designing new materials on the other, Design Based Research methodology has been chosen. Two cycles of research are planned, each of them consisting of four steps: 1. Exploration and design of materials. 2. Use of the materials. 3. Evaluation of the design and use. 4. Revision of materials.

In the poster, the second cycle of research will be referred. Some of the materials designed will be shown and analyzed and the following question will be explored: Is it possible to establish a classification of the designed materials according to the topic and the representations chosen and the kind of task posed? On the other hand, the team is composed by the teachers of five subjects with contents on Complex Analysis. This variety of teachers will allow us to study differences among them: How different teachers interpret visualization in complex variable functions? Which problems do they focus on? How do they choose to create materials for students? The comparison between the different ways different teachers create materials and the feedback and evaluation of the materials may shed new insights on the role visualization plays in teaching and learning Complex Analysis.

Additional Information

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INVESTIGATING TEACHERS' ASSESSMENT AND TEACHING PRACTICES: WHAT CAN WE LEARN FROM EXTENSIVE CLASSROOM NARRATIVES

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Recently, a new Mathematics curriculum for basic education (Grades 1-9) has been introduced in the Portuguese educational system. The new curriculum implementation was strongly supported by a special designed program aimed at providing scientific and pedagogical support to all teachers of Grades 1-9. Before its generalization to all basic education schools of the country, 40 teachers of the different grade levels had put the new curriculum into practice for a three-year period. These teachers had the opportunity to attend a number of workshops and to share their experiences in a variety of contexts. At the same time the Ministry of Education appointed a team of three mathematics educators to conduct a three-year evaluation study of the whole process of implementation of the new curriculum. As a whole, the study aimed at: a) understanding the implementation process of the new curriculum (e.g. teacher training programs, curriculum materials distributed, teaching planning); b) understanding teachers' classroom assessment and teaching practices as well as students' participation (e.g. teaching planning and organization, classroom dynamics, teachers' and students' main roles in the teaching and assessment processes, assessment uses, predominant assessment tasks, assessment dynamics; nature and frequency of feedback); and c) understanding students' learning (e.g. students' performance in a variety of tasks, students' results in a test).

In order to reach those goals, researchers observed 94 hours of classroom work within the six classes (two per grade) involved in the study (Grades 4, 6, and 9). Based upon observations, interviews, and document and artifacts analysis, three extensive narratives (one per grade level) were produced illustrating assessment and teaching practices of the participant teachers.

These narratives described and analysed a diversity of pedagogical episodes that took place within the classes of the participant teachers, enabling one to highlight a number of results and conclusions. This presentation will share them as part of what the authors have learned in this three-year evaluation study on teachers' assessment and teaching practices in the context of the launching of a new Mathematics curriculum for Grades 1-9.

TEACHERS REFLECTIONS ON TEACHING DEVELOPMENT

Anne Berit Fuglestad

University of Agder

In a three year long developmental research project teachers were challenged to use an inquiry approach to their teaching. Often observed in mathematics classroom is teaching from the blackboard and pupils solving tasks. The recent TIMSS advanced report for Norway (Grønmo, Onstad, & Friestad Pedersen, 2010) confirmed that the dominating activity in classrooms is to solve tasks similar to the ones in the textbook and with little reasoning and discussion of strategies or little with pupils choosing their own ways of solving complex problems.

The project, named Teaching Better Mathematics, aimed to initiate and support development of teaching through the close collaboration of teachers and didacticians (i.e. university researchers) in learning communities. Furthermore, inquiry in the meaning to wonder, ask questions, seek information and to work deeply into problems rather than solve a lot of similar tasks, was promoted as a teaching approach and as stance or a way of being (Jaworski, 2006). Inquiry was seen as providing for and stimulating pupil's work as well as stimulating the discussions and preparations of teachers and didacticians in the project work.

The project activities encompassed regular workshops for the teachers and didacticians with plenary presentations by didacticians on mathematical or didactical topics and by teachers on experiences with an inquiry approach in classrooms. An important part and highly valued in workshops were the group discussions to practice inquiry, discussing mathematical and didactical issues. School teams met locally to plan, share information and discuss their own activities.

This poster reports evidence from focus group interviews with teachers in secondary schools. The teachers expressed satisfaction, engagement and enjoyment over the project and an inquiry approach. Teachers claim they did not change very much, but gradually transformed and developed awareness of their teaching approach and moved towards using more open tasks intending to provide for wondering and investigations. In particular they promoted inquiry based tasks as a starting point to stimulate students' attention and learning. The teachers valued highly the workshops including the discussion and collaboration with colleagues from other schools and their own.

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WHY MATHEMATICS EDUCATORS *SHOULD* BE BOTHERED ABOUT POVERTY

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One thing is clear; success at mathematics is not evenly distributed across sections of society. Studies have shown that poverty has a stronger influence on achievement than instructional quality, leading to a policy imperative that if we want all pupils to do well a key element of education policy must be to reduce social inequity. I do *not* want to argue everyone should be bothered about the effect of poverty; it depends on your politics. Whilst some researchers are concerned about this, many are not and the literature is full of examples of studies that are class-blind, based upon the assumption that pupil backgrounds are irrelevant to learning. I see three stances taken in our discipline which mirror, albeit a bit crudely, the political spectrum.

- **A radical stance** - Taking social class seriously, challenging the status quo;
- **A moderate stance** - Taking the status quo seriously, desiring equity;
- **A conservative stance** - Rendering class invisible and unproblematic.

Kitchen suggests three challenging policy changes.

> Transforming the mathematics education culture to value the mathematical preparation of the majority over the achievements of a select few... Acknowledging that mathematical education is a political endeavour. .. The need to question the role of an education in mathematics, particularly at schools that serve high-poverty communities. (Kitchen 2003)

Sarah Lubienski studied mathematical experiences of pupils with an eye to looking at pupils' backgrounds (Lubienski, 2007). Whilst she naturally expected to find SES differences what she actual found were very *specific* differences in two main areas – *whole class discussion* and *open-ended problem solving*.

So what we might do about this? A challenge for all of us is to fight the demons that cause us to expect little from learners from less affluent backgrounds and to recognize the influence that poverty has on all aspect of teaching and learning mathematics. Engaging explicitly with class and social differences in learning has been shown to have the potential to open up greater opportunities for higher order thinking (Jorgensen et al. 2011), and for raising the intellectual quality of pupil cognition. Class is always a latent variable whose invisibility obscures possibilities for action.

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HOW IS ARGUMENTATION USED BY STUDENTS IN THE SECONDARY MATHEMATICS CLASSROOM?

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Several authors have developed research around practices of argumentation in the mathematics classroom. In the context of our research agenda, the first author has initiated a PhD work to look for evidence in relation to discursive exchanges among secondary school students discussing mathematical ideas. The basic question aims at examining justification processes interactively carried out by students together with the teacher in mathematics lessons. Our approach to the question is organized by means of three goals: 1) to identify critical classroom episodes in which certain uses of justification are mathematically inconsistent; 2) to explore some of the reasons given by students to support/reject these uses; and 3) to determine relationships between the students' reasons and their diverse interpretations of mathematical knowledge. For the initial stage of the research, the instructional design has been agreed with the teacher of the classroom. Main data has been collected in a sequence of problem solving-based lessons with thirty students aged 15 and 16, first involved in pair work and then in whole-group discussion. Three pairs have been audio and video-recorded. At this moment, with an eye on Steinbring's epistemology-oriented interaction analysis (Steinbring, 2005), aspects of data reduction are being decided.

In this Poster we describe the design experiment, show transcripts of selected critical episodes to illustrate data and propose some questions that emerge from preliminary analyses. These questions are expected to guide the future steps in the research. We are aware of the difficulty of grasping private meanings concerning mathematical knowledge out of students' public talk. Hence, we are in the process of integrating the communicative and the epistemological perspectives in the structural analysis of students' argumentations. This integrated perspective needs to be interpreted in terms of a useful practical framework for the analysis of the classroom environment.

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LEARNING DIARIES AND SELF-REGULATION IN MATHEMATICS

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Many students face serious problems when starting a university course in science, technology, engineering, or mathematics. At Ruhr-Universität Bochum, the project MP² (Math/Plus/Practice) aims at supporting engineering students in mathematics. Various interventions are being tested in order to find out which can help students to develop learning strategies that can assist them to successfully complete their studies.

MP² has so far covered a period of two years, starting in summer 2010, preceded by planning and accompanied by evaluation. It is divided into two parts, Math/Plus and Math/Practice. The first aims at improving learning strategies and motivation, the latter (which is the centre of a different paper) at providing examples of practical engineering applications of mathematics in connection with motivating project work. By summer 2012, more than 300 students will have been involved in our project, divulging multitudinous data. There are numerous interventions in Math/Plus (e.g. tutorials, an e-learning course, a helpdesk, a revision course, questionnaires and, evidently, learning diaries) which were applied to different groups of students.

At the beginning of their university studies, as well as at the end of their first semester, students were asked to fill in the LIST questionnaire on learning strategies (Wild & Schiefele, 1994). Thus, developments and differences between the project groups could be found, interpreted and (in some cases) attributed to the project interventions. The learning diaries' (Schmitz & Wiese, 2006) specific aim was to encourage the students to modify their learning behaviour. Over a period of ten weeks in their first crucial year at university students recorded not only their learning times, frequencies and strategies but also their mental state and motivation. Therefore the learning diaries enable us to compare these to the learning strategies students claimed to use in the pre and post LIST questionnaires. Hence they allow us to get a more detailed view of the regulation of learning behaviour in mathematics.

The poster presents excerpts from the LIST questionnaire and the learning diary, details of the project work as well as selected results.

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CONTINUOUS MATHEMATICAL LEARNING BIOGRAPHY FROM KINDERGARTEN TO ELEMENTARY SCHOOL

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Patterns and structures are the basics of Mathematics. When children have their first experience with patterns and structures the topic is always accuracy, the arrangement and the repeatability. If they understand the structure or the phenomena in an ornament for example, they can explain, continue, generalize or draw the ornament, and this kind of action is the first step for concept formation. It also seems that children with less knowledge of mathematics do not perceive or use mathematical structures. Children with good knowledge of mathematics are mostly successful, because they know how to use patterns and structures (Wittmann & Müller, 2007).

This poster presentation shows how mathematics-based material can serve as a bridge between kindergarten and elementary school. The way it can be used enables kindergarten and elementary schools to combine the two institutions to be responsible for a continuous education biography. This project, which is called MATHElino, is based on an idea of Kerensa Lee (2010) and works with the principle “equal material in a large quantity”. For this, children from elementary school and kindergarten meet once a week for two hours in one of the institutions to work together with this mathematics-based material (pattern blocks, wooden cubes, dice, mugglestones, flaggings and distance puzzles). The main idea for such a workshop is that children from both institutions work cooperatively with the same material to develop their mathematical thinking and mathematical language. If this process is successful, they talk and discuss a lot about structures within the material and sometimes they even like to put down their solution on paper.

Based on this situation, the poster presentation shows the results of the cooperative work of four institutional partnerships (one kindergarten always cooperates with one elementary school) where the children work together (two children from kindergarten cooperate with two children from elementary school). The results of this experimental research show that children from kindergarten and elementary school can work cooperatively with the same material. This research also reveals what kind of position the children will engage by working together and how the elementary school teachers and kindergarten teachers will handle this situation.

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A STUDY ON THE AMBIGUITY OF 'ABSTRACTION' IN THE PROCESS OF GENERALIZATION

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Generalization (that is, reasoning from particular(s) to general) is known as one of most important process in mathematics education. Nevertheless, previous studies have focused on the process of generalization itself, and not explicitly paid attention to its justification. Someone may think that generalization is justified by proving. But when one intends to justify a generalization, proving is insufficient reasoning for justification. For instance, according to Lakatos (1976), even if Euler's *polyhedron* theorem is proved, no one answer what particular polyhedrons are involved into *(general) polyhedron*.

According to Kant, this kind of problem results from the ambiguity of 'abstraction'; it has dual meanings, namely *aliquid absrahere* [abstract from some things] and *ab aliquibus abstrahere* [abstract something] (Kant, 1770). "The former denotes that in a concept we give on attention to other matters in whatsoever way they may be connected with it; but the latter, that is not given but in the concrete and so as to be separated from what it is conjoined with (*ibid*)." Kant critiqued the former; because, essentially, the former intends to leave from one's perception, therefore the generality will be self-existent and limitless generality. In contrast, the latter is based on one's perception and not intends to leave from it. Therefore, Kant was seeing *generality as not concepts but how to use concepts*; that is, generality is only derived from what concepts are used in a proof.

In mathematics education, the former is more emphasized because it is reasonable. In fact, commonly, while one intends to develop mathematics, only abstracted from some things are used, and no one has to use abstracted something. Therefore, many students deal (or teachers and researchers may think) particulars as no more than a starting point of generalization, so when particulars are rather generalized, they're not very necessary resources for generalization. On the contrary, according to Kant's viewing of abstraction, one's choice of concepts in a proof is often very intuitive so empirical reasoning (i.e. to check what concepts are used in a proof) is necessary resource for generalization. In this sense, we intend to give more important state to particulars, and require students to sensitive justification through referring to particulars from a general.

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HIGH-SCHOOL STUDENTS' USE OF GRAPHICAL REPRESENTATION IN SOLVING CONDITIONAL EQUATIONS: FACTORS THAT HINDERED STUDENTS' LEARNING OF MATHEMATICS

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Student performance in national examinations is often used as a gauge of the students' ability to do well in the subject, or at least in answering structured exam-type questions. Institutionally, national examinations were used and believed to measure students' knowledge and skills in subject matter. Some would even use it as a predictor on future student performance, or use it as an indicator of the students' preparedness for higher levels of study. Nevertheless, there were trends of exceptions, where previously high achieving students perform poorly in high school with respect to the learning of mathematics. Apart from possible isolated incidents, the study unveils the factors that could have contributed to this phenomenon.

A purposefully drawn sample of four students who had scored distinctions in their previous national examinations and failed their end-of-year high-school summative examination was selected to participate in the study. The four students were individually asked to answer a questionnaire comprising of three exam-typed questions, with different levels of difficulty, to find solutions to different types of conditional equations using the graphical approach. Upon their completion of the un-timed assignment, they were then interviewed to review problems that they had encountered, to explain reasons for their approaches in solving the questions, as well as on other researcher's observations of them working through the assignment.

It was found that fundamental gaps existed in students' understanding of mathematical concepts which were taught and assessed during their earlier school days. They were still able to do well in the national examinations because they had mastered the skills to answer exam-typed questions instead of having acquired the holistic conceptual knowledge that is needed for further construction of knowledge at higher levels. Other factors include the lack of mathematical habits of thinking which includes formulating and verifying conjectures, varied use of mathematical representations, understanding procedures, and formulating connections across mathematical concepts. Students also lacked confidence in pursuing their hypotheses which hindered their experimental willingness to attempt questions more thoroughly. Many of these could be attributed to their previous experiences in traditional approaches of assessment.

Findings from the study surfaced essential elements which are lacking in the current form of assessments at the middle school levels, which prove to be pertinent and indispensable in the progressive construction of knowledge at higher levels.

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MATHEMATICS INTERN TEACHERS' CONCEPT IMAGES FOR MATHEMATICS TEACHING: THE ASPECT OF STUDENTS' MATHEMATICAL THINKING IN CLASSROOM

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This study proposed the idea "*concept image for mathematics teaching* (CIMT)" by analogizing to the idea "*concept image*" proposed by Tall & Vinner (1981). When the mathematics teachers make their decisions, their CIMT should not be ignored. Among the various mathematics teaching concepts, this paper focuses on teachers' concept of student's mathematics thinking in the mathematics classroom.

Qualitative research method was designed for this study. Most of data was collected by a questionnaire and the rest came through interview. The results in this paper mainly come from two open-ended questions in the questionnaire: (1) what is your view of mathematics teaching? and (2) what is your view about the role which students' mathematics thinking plays in the mathematics classroom and when should a teacher let students think? With discover-oriented method, data was analysed by content analysis and inductive analysis method.

We found that more than 70% of the intern teachers approved the roles that students' thinking plays in mathematics class. However, only around one fourth (15/62) of them spontaneously exhibited some aspects related to students' mathematical thinking when describing their views about mathematics teaching. Similarly, we found that when facing practical teaching scenarios, they do not usually evoke the image of "let students think" as their bases of teaching strategy. In other words, even though these intern teachers know how important students' thinking is for mathematics learning, they barely give students opportunities or time to think in mathematics class.

Another important discovery was that most intern teachers who approved the roles of students' thinking would evoke the image of "students' cognitive aspect" at the same time. In short, for these intern teachers, the main function of the students' thinking is in the aspect of students' cognition. Three main sub-categories were found in this situation:

- Students' thinking is an important factor to students' concept development.
- Students' thinking is an effective method to help them retain knowledge.
- Students' thinking helps the outputs and applications of knowledge.

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THE EFFECTS OF TEACHING TRIGONOMETRY BY USING DGS

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In current high school mathematics, trigonometry deals with real-world problem by applying the properties of triangles. Solving a trigonometric problem usually involves transforming situations to diagrams, formulating equations through perceived relational structures, and solving equations to obtain the answers. Low-achieving students often have problems in the first stage of transformation. To help the low-achieving students, we designed realistic situations with DGS. And the aim of this study was to find out the effects of their learning with DGS.

Tso (2001) suggests that to design a digital learning environment we need to consider three aspects: the nature of the subject content, theories of learning, and features of the technologies. We based our literature review on these aspects, and integrated them to form our theoretical framework. We investigated their change of problem-solving skills in trigonometry after the teaching activities with DGS. This study used a method of experiment on a single group with pre-test and post-test for them. Before and after the teaching activity, questionnaires of trigonometry were administered to the students. The subjects were 40 low-achieving 11th-grade students.

Our analysis shows that students had significant improvement in handling trigonometry problem after learning activities. Further analysis suggests that their improvements rested mainly on “figure-constructing” and “expression-formulating”, but there was no significant change in “solution-finding”.

The focus of this study was visualization, hoping to provide students with meaningful visualization by virtual DGS, so they could understand situational problems more effectively. The outcome of the teaching showed that it was helpful in students’ figure-constructing when they were solving trigonometry questions, but their performances in expression-formulating and solution-finding were still relatively low. In other words, the major difficulty for students might not be in figure-constructing, but in expression-formulating. In the future, we believe the teaching of trigonometry should focus on strengthening students’ ability to find relational equations from diagrams, that is, on guiding students to observe diagram structures and transforming them into equations. It takes more research to find good strategies for this aim.

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THE IMAGE OF <,>,= BY PRE-SCHOOL TEACHERS

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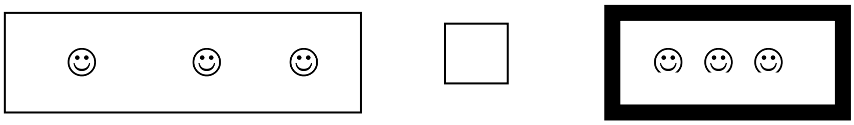
Many activities for young children require them to place mathematical signs between nonmathematical objects. Later in school, it sometimes causes them to use the signs incorrectly between numbers. For example, a child in grade 1 wrote: 6< **4** because "The size of four is bigger and thicker than the size of six". Such cases led to conducting research on pre-school teachers in order to see how they use the mathematical signs themselves and how they teach them to the children. The research has been ongoing for 10 years with 15 different groups of 298 pre-school teachers. The participants had to write the correct sign or to write that it's not possible to put any sign between two objects. For example: o □ **O**

Part (35%) of the students put = because "there is 1 ball on each side", and the others (65%) put < because "the right ball is bigger". No one gave the answer - that it's not possible to put any sign. Another case:



In this case most of the students (94%) thought that every sign can be possible: " = because we have a rectangle on each side or the same amount of faces", "> because the left rectangle is longer than the other" and "< because the left rectangle is thinner than the other", 6% didn't know which sign to choose. No one answered that it's not possible to put any sign. Another example: **4** □ 6 . Most (75%) of the students wrote **4**>6 because "The size of four is bigger than the size of six" and at the same time they also wrote **4**<6 because "It's correct mathematically". For them it's correct to use two different signs at the same time. The image of the signs **<,>,=** for the pre-school teachers, can be used not only in the mathematical sense. For them the signs can be used in many ways and they don't see a problem if the child writes: **5**>5. They say: "We teach the child to use the sign > between two objects, in this case the size is important, in another case the length is important. It depends on the context." In this case, only cognitive conflict brings the pre-school teachers to understand that it is not possible to use two different signs between two numbers, at the same time. The use of the same words in everyday life and in mathematics (Ilany and Margolin, 2008), leads to misconceptions in the meaning of the mathematical signs.

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COMPARING TURKISH AND AMERICAN MIDDLE SCHOOL MATHEMATICS TEXTBOOKS: A CONTENT ANALYSIS

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This comparative study examined mathematics problems found in Turkish and American textbooks at the middle school level. In particular, it identified characteristics of mathematics problems by employing a problem content analysis methodology. Previous studies have explored particular characteristics such as mathematics features, contextual features, and performance requirements. In addition to these three characteristics, the current study also took into account the use of technology.

Analysis of textbook problems provides information on curricular expectations of developing students' mathematical competence not immediately evident through textbook content analysis. Few existing studies have shown that textbooks can be analyzed to understand their potential impact and that instructional approaches embedded in textbooks can be explored to uncover how textbooks differ in the teaching and learning of mathematics problem solving. Because textbooks organize their instructional content into different textbook units such as lesson units, textbook units can be examined to show variations in textbook organization. Although the question of how Turkish reform curriculum teaches mathematics in general has been addressed in the literature, no emphasis was given to explore the question of how mathematics problems are presented in textbooks. It is, thus, of interest to explore textbooks' inclusion and use of several content presentation features.

The findings in this study revealed that compared with American textbooks, Turkish textbooks contained: 1) more pure mathematics problems but fewer real-life-application problems, 2) more mathematics problems in the cognitive domains of applying and reasoning but fewer in the cognitive domain of knowing, and 3) more emphasis on explanations and solution processes in their problems but no problems involving the use of technology. In general, American textbooks included fewer multiple step problems and were dominated with problems of low mathematical and cognitive requirements.

Based on these findings, educational policy recommendations can be put forth to highlight the need for Turkish government to reform its mathematics curriculum by considering a careful integration of the use of technology as well as a practical adoption of the applied mathematics problems of which middle school students can make the connection in their everyday life. Furthermore, both countries can benefit from promoting more mathematically challenging problems pertaining to joint efforts to facilitate students' development of mathematics competence.

A DEVELOPMENT OF MATHEMATICAL MENTAL REPRESENTATION TEST FOR UNIVERSITY STUDENTS

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Some kinds of placement tests for mathematics have been developed in the world. But these tests ask to solve a lot of problems and a large amount of time is needed to complete it. Dehaene (1997) used the magnitude decision task to assess the mental representation for natural numbers. And Okamoto & Wakano (2010) reported that the children's mental representation assessed with the magnitude decision task correlates the performance of simple addition problem in first graders. These results suggested that the mathematical mental representation is an index as mathematical achievements. We developed a mathematical mental representation test (MMRT) covering high school mathematics contents. The purpose of this study is to examine that it is useful to a readiness test for university students.

Method Participants were 40 university students who had the course of mathematics for social sciences. Mathematical mental representation test includes 12 problems concerning a sequence of numbers, vectors, matrix operations, and functions. We will figure out all problems in the poster. Each problem was presented by a PC, and a response time was recorded via wireless response devices.

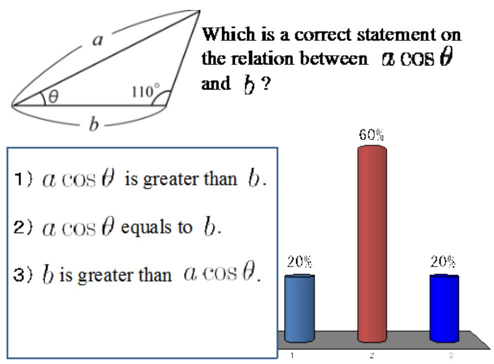

Figure 1. An example of problems in MMRT

Results and Discussion An analysis for the responses in MMRT showed that the problem of trigonometric functions was most difficult and students could not use the representation of trigonometric functions (see Figure 1). Table 1 shows that there is a significant correlation between MMRT and the score of the National Center Test for University Admissions, but not for the semester exam. It suggests that MMRT is useful to assess student's mathematics achievements, but it could not predict their achievements after the 1st semester.

Table 1. A relationship between MMRT and mathematics achievements

	Score of Center Test	Score of 1st semester exam
Score of MMRT	0.406*	0.142
RT of MMRT	−0.364*	−0.071

* < .05

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INTERACTION BETWEEN BELIEF AND PEDAGOGICAL CONTENT KNOWLEDGE OF TEACHERS WHILE DISCUSSING USE OF ALGORITHMS

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Elementary education in India has long held teaching of algorithms as the prime focus of teaching mathematics at this level. Likewise there is co-occurence of widespread belief among teachers that there is just *one best algorithm* for each operation that should be focused while teaching in the classroom. This has been challenged in the new curriculum framework (NCERT, 2006, p.19) by providing space for alternative methods that students come up and engaging students in understanding why algorithms work.

In this poster we will graphically display the results of thematic analysis of a session conducted in a workshop as part of a 2 year long professional development program involving 4 primary and 8 middle school mathematics teachers from a public school system. The session involved discussion about subtraction algorithm followed by multiplication where in teachers engagement with belief about teaching algorithm was witnessed. For some teachers the engagement was in form of resistance to engage with alternative methods and questions about how they work, as they perceived it to cause confusion among students since they would not be able to understand the concepts. Resistance was also on account of the rules related to algorithm which teachers felt cannot be broken like “borrowing from left from the same number”. Teachers also engaged by sharing the explanation of algorithms which ranged from procedures involving numbers to use of concepts like place value and distributivity for understanding the algorithm. Teachers voiced their challenges to these explanations using students' thinking and understanding as proxy. In comparison to subtraction, discussion of multiplication involved sharing of alternative methods, but teachers stressed the importance of students getting correct answers and speed or ease of calculation rather than conceptual clarity. These forms of engagement resulted in interaction between beliefs held by participating teachers and the pedagogical content knowledge related to algorithm leading towards engagement of teachers in understanding how algorithms work by teacher educators and why different algorithms give correct answers. These engagements might be the first steps towards teachers engaging with alternative ways to find solutions and evaluating generality of the alternative methods.

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GROWTH STAGES FOR INSTRUCTION-DESIGN TEACHERS

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We discuss three issues on professional growth for instruction-design teachers: (1) In what situations and to which extent is a teacher characterized as an instruction-design teacher? (2) What growth stages of the teachers can be identified through their participation in design-based professional development? (3) Is the identified growth stages generalizable to other mathematics teachers and other research settings?

Accordingly, we define teachers as instructional designers when they participate in creating any kinds of resources (e.g., task, manipulatives, tests) for a particular teaching purpose. By designing instructional materials, teachers have opportunities to enrich their pedagogical power through actively transforming and coordinating different sources of information into the creation of the work. We further used a selected mathematics teacher, Hao, to illustrate two distinguished growth stages: stage for understanding school mathematics and that for experiencing fundamental ideas in mathematics. Hao at the understanding-school-mathematics stage aimed at creating tasks that allow students to learn well-defined mathematics knowledge in textbooks and to apply the learned knowledge to solve a variety of problems. But when he headed to experiencing-fundamental-ideas stage, he changed his design intention and was able to create tasks in correspondence with the intention. He thought the importance of tasks is the opportunities for students to experience the fundamental ideas from which mathematical knowledge are developed and to engage in learning mathematics through discussions between students and teachers. In line with the design intention, he created a task titled as “centers in a triangle” by coordinating his understanding of curriculum materials, the aims set up in professional development, and peer teachers’ design experiences. The task entails students to activate their common senses and explore fundamental ideas for the follow-up learning of mathematics. Hao’s growth in profession was recognized by other teachers in the professional development but they expressed the difficulties in creating such work. Teacher Li said

Li: All of my designs look similar…no matter how hard I have tried…I feel the constraints of my designs as I could not go further…I would like to know and analyze Hao’s brain to see how he can create such good tasks.

Li’s reflection pointed out the significant growth difference in the two stages. In view of that, the follow-up research question is the extent to which the classification of the stages can be used to explain different teachers’ professional growth and be generalized to other research settings?

THE EFFECTS OF THE AFTER SCHOOL ALTERNATIVE PROGRAM ON TAIWANESE FOURTH AND EIGHTH GRADERS' MATHEMATICS ACHIEVEMENT AND GOAL ORIENTATION

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Achievement gaps constitute important barometers in educational and social progress. The TIMSS provides information on the achievement gaps among different countries in mathematics and science. For Taiwan, the result of TIMSS 2003 showed that there was an excellent average performance. However, the proportion of low achievers was high and the overall students' learning interest and self-efficacy were quite low. In 2006, in order to narrow the achievement gaps in Taiwan, the policy of After School Alternative Program (ASAP), proposed by Ministry of Education and National Science Council, aims at ensuring both academic excellence and equity by providing new opportunities and challenges for Taiwan to advance the goal of closing the achievement gap. Assessing ASAP's impact requires more rigorous scrutiny of new evidence from assessment results. The After School Alternative Program technology-based testing project (ASAP-tbt) was proposed to play a confirmatory role as an independent assessment to validate the ASAP remedial effects, by examining whether and how recent math assessment trends in average achievement as well as achievement gaps are systematically related to Taiwan accountability policies under ASAP. The purpose of this study is to offer precise analyses of ASAP fourth and eighth graders' math achievement results during 2009-2010. We adopt HLM (Hierarchical Linear Model) to obtain the slop of the learning growth by compare 3 times profile in math achievement of ASAP fourth and eighth graders. We also concern the change of the proportion of negative mathematics learning affect for these students. In order to have a precise understanding about ASAP effect on low achievers' learning. There are two reference groups, norm and low achievers that do not participate in ASAP, are involved in this study. By comparing ASAP students' math achievement and affect profiles of 3 time points, the results show that the effects of ASAP not only on improving academic performance, but also on the reducing the negative goal orientation about mathematics learning.

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HIGH SCHOOL TEACHERS' PERFORMANCE IN MATHEMATICAL INQUIRY ACTIVITIES AND THEIR BELIEF CHANGES

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Nowadays, mathematical inquiry is considered as a framework for teaching and learning mathematics (Siegel, Borasi & Fonzi, 1998). Moreover, inquiry strategy is adopted for teachers' professional development as early as in Dewey's time, and it could help teachers reveal the dilemma between belief and daily practice (Corckett, 2002). We define inquiry as settling doubt and fixing belief within community, which is followed the perceptions of Peirce and Dewey. Additionally, teachers' beliefs have been recognised as crucial determinants for their teaching and influenced in significant way by their experiences (Wilson & Cooney, 2002). Due to there are few issues related to how teachers perform and how they change their beliefs in inquiry activities, thus this study focuses on the explorations of such viewpoints. Consequently, this study is organised to help teacher revisit mathematics inquiry in order to see how they do mathematics inquiry and how they change belief about inquiry. Twenty high school teachers are orchestrated to participate in a professional development programme and all of them are invited as learners to engage in activities which are purposefully designed with four key chronological phases of mathematics inquiry cycles (setting the stage and focusing the inquiry, carrying out the inquiry, synthesising/communicating results from the inquiry, and taking stock and looking ahead) (Siegel, Borasi & Fonzi, 1998), in particular, which are focused on generalising mathematics patterns. The whole process of activities is videotaped, participants are interviewed and an open-ended questionnaire for examining teachers' belief change is conducted both before and after the study. The research results was derived from the analysis within all of the qualitative data and interpreted with the paradigm of phenomenology. It shows that teachers could apply cognitive strategies flexibly and generalise specific mathematics patterns for interpreting the phenomenon of problems; further, most of their beliefs changing trajectory about inquiry teaching and learning could be traced during the implementation of study.

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PARTICIPATION IN A SCHOOL MATHEMATICS PRACTICE WITH ROBOTS: RACING WITH ROBOTS[24]

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With this Poster presentation we pretend to describe and analyze students' participation (Wenger, 1998) in a school mathematics practice in which robots play a central role. This study has the purpose to understand how the use of robots, as artifacts mediators' of the learning of mathematics contributes to: students' mathematical communication, students' mathematical reasoning and to their ability to problems solving. We assume learning as participation in the sense of the situated perspective of learning (Lave and Wenger, 1991 and Wenger, 1998).

Within a qualitative methodology we have been collecting data in a 8th grade class with alternative curriculum paths in a secondary school during six sessions, 180 minutes each. We recorded all the sessions with two video cameras. The work had a project methodology in which we propose students to build in groups (of four students), a race car (NXT robot) out of Lego bricks. Students had to program the car to race independently, keeping in mind that: must start the race upon the starting signal (using the sensor sound); the route has to be made so that the cars do not clash with each other, that is, they should not leave their black line (using the color sensor). Each group of students created a prototype of a race route, with provided parts, so that two robots could race at the same time. The race route had to be fair, that is, a route that would provide the two cars (robots) with the same probability of winning. Students chose the race route to be used and then they built it with real dimensions. In this process many mathematical concepts were displayed. After all groups had managed to program the cars, the races were held: each car ran six laps. The laps were timed and recorded. With the data collected from the races, each group made a statistical study. Additionally to statistical data analysis, conclusions were provided and generalizations were also established.

In spite of we are now in an initial phase of data analysis we can already foresee some findings that show themselves as promising concerning to mathematical learning.

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A STUDY OF CONCEPT OF PRIME NUMBERS TO TEACHERS AND STUDENTS IN THE ELEMENTARY SCHOOL

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This study was undertaken to explore the variety in the performance of the teachers and the students in the elementary school from different grades and different genders in the test of concept of prime numbers.

Most of the in-service teachers have a through understanding of the definitions of the "prime number", "composite number", "greatest common factor (GCF)", and "least common multiple (LCM)". Although most of the students have a through understanding of the definitions of the "prime number" and "composite number", they confused the definitions of the "greatest common factor (GCF)" and "least common multiple (LCM)". A lot of teachers, but only a few students, think that one is NOT a "prime number". Some of the students can not identify the odd numbers and prime numbers (Wu, Ma, & Horng, 1998).

The test was designed by Wu, Ma, & Horng (1998) and included 20 multiple choice questions. The reliability of Cronbach's α is 0.840 ($p < .01$).

The participants were 134 elementary school teachers and 223 elementary school students among 5th grade to 6th grade. The size of teachers from lower grade to higher grade is 57, 47 and 69, respectively. The size of students from 5th grade to 6th grade is 77 and 146, respectively.

The researcher used independent-sample t-test, three-way ANOVA to analysis the difference of the participants of different grades and different genders in the test.

After data processing, the following conclusions were drawn from this study: (a) The average of the passing rate of the students was 55.7%, that of the teachers was 85.1%. (b) There was significant difference between the male and female teachers but no significant difference among 1st and 2nd grades' teachers, 3rd and 4th grades' teachers, and 5th and 6th grades' teachers at the concept of prime numbers. (c) There was no significant difference among 1st to 6th grades students and different genders students at the concept of prime numbers.

KEYWORDS: concept, elementary school, prime numbers, students, teachers

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THE PASSINGS RATES OF THE MA-WU'S TEST OF PRACTICAL REASONING ABILITIES

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This research was undertaken to develop the test of mathematically problem-solving abilities in living situation for elementary school students. The researchers picked 179 typical problems from 1,622 daily life problems. These problems were posed by 98 elementary school students in Taiwan when they made authentic mathematical activities during two semesters. At last the researchers chose 20 problems according to object, source material, structure, and verbal (linguistic, semantic) to develop and complete the Ma-Wu test of mathematically problem-solving abilities in living situation.

The instrument used in this study, the Ma-Wu's Test of Practical Reasoning Abilities (MWTPRA), was specifically designed for this project due to there were no suitable Chinese instruments available (Wu & Ma, 2011).

The participants were 494 elementary school students who were randomly selected from 8 elementary schools in 6 counties/cities in Taiwan. There were 236 girls and 258 boys. The numbers of participants, from northern, central, to southern area, were 165, 204, 125 students, respectively.

Mayer (1992) divided five types of knowledge for problem solving: linguistic, semantic, schematic, strategic, and procedural knowledge.

Based on the questions of Test I of Ma-Wu's Test of Practical Reasoning Abilities, the passing rate of practical reasoning abilities of the elementary school students were 60.42%.

The passing rate of *problem translation* sub-processe was 71.40%, 61.07% at *problem integration* sub-processe, 53.79% at the last two sub-processes (*solution planning and monitoring* as well as *solution* execution). It seemed that the problem translation sub-processe is the easiest one for students, followed by problem integration sub-processe. The last two sub-processes (solution planning and monitoring as well as solution execution) is the most difficulty one for students.

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ON THE ANALYSIS OF CLASSROOM INTERACTION IN COMMUNITY COLLEGE TRIGONOMETRY CLASSES

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We report analyses of classroom interaction in trigonometry classes taught at a community college attending to two dimensions: the novelty of mathematical questions that instructors ask and the interactional moves that the instructors use to encourage student involvement in the lesson. Community college is a U.S. tertiary institution that provides the first two years of baccalaureate degrees, and vocational, technical, and enrichment education. These institutions are typically open access, non-residential, and low-cost relative to other types of tertiary institutions. They are an attractive option for students because their classes are usually small, thereby allowing more opportunities for instructor-student interaction. The classes followed a lecture approach, which deemed current frameworks to analyze classroom interaction insufficient. We analyzed 21 trigonometry lessons taught by five instructors using a mathematical questions novelty framework (Mesa, Celis, & Lande, 2011) and the teacher moves framework (Scherrer & Stein, 2012). The analyses indicate that in these highly interactive classrooms most of the interaction centered on providing information and asking questions students knew how to answer; teacher moves primarily guided students on a particular solution path. However there are important variations across teachers that suggest different opportunities to learn afforded by these lectures. We illustrate the two analyses with examples of the findings and identify their strengths and weaknesses in representing the interaction.

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STUDY OF A TRANSLATION BETWEEN GRAPHICAL AND SYMBOLIC LANGUAGES: AN EXAMPLE WITH THE COEFFICIENT OF CORRELATION

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In this presentation, the focus is on translation (according to Duval, 1995) between two mathematical languages, the graphic and the symbolic. Specifically, 21 university students enrolled in a statistics course as part of their undergraduate program in communications were asked to transpose a graph illustrating the coefficient of correlation (see figure 1) into symbolic language $r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1) s_x \cdot s_y}$

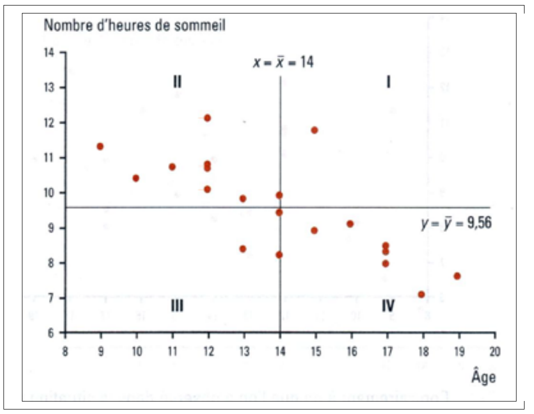

Figure 1: Example of a graphic representation in the form of a "cluster of points" and of the distribution of population items among the four quadrants

Our interest was focused on undergraduate students because we had the assurance that all the significant units necessary for the requested conversions had been addressed previously in the progression. Our choice was therefore focused on this cohort of 21 students (STT1995 course – Université de Montréal), because they were repeaters who had already taken the course the previous year (and perform the level 2 of processing complexity, according to Pressley, 2006). This study revealed that, despite these methodological precautions, only two students were able to perform the task assigned to them (only one student made a real attempt at writing and conversion - the other one had made, by his own admission, a simple effort at memorization). Almost all the students were unable to determine the acceptability with respect to mathematical writing standards of the equation they attempted to produce. 13 among the 19 students who failed were unable to initiate a written process, despite the semi-directed character of the interview. This finding shows a very low mastery of mathematical languages which corresponds to a situation analogous to illiteracy.

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WHAT HONG KONG STUDENTS SAW AS IMPORTANT IN THEIR MATHEMATICS LESSONS : A CASE STUDY

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Abstract: *The paper reports the analysis of the data of one Hong Kong teacher taken from the Learner's Perspective Study with a focus on the students' perspectives. A sequence of 18 consecutive lessons has been recorded. Analysis of the post-lesson students' interviews was carried out to find what the students saw as important.*

The learning outcomes achieved by a student are based on their own actions and behaviors. These behaviors in turn, are affected by the students' beliefs about their capacity as learners and mathematics; and the teachers' philosophy and behaviors in the classrooms (Koehler and Grouws, 1992). A simple framework is constructed to show the relationship between the teacher and the student within the classroom process. Within the framework, the teacher and the students are the major actors. The teacher beliefs, teacher practice, student beliefs and attitudes, student behaviour and outcomes are inter-related.

The Learner's Perspective Study (LPS) an international video study of "well-taught" Grade 8 mathematics lessons (Clarke, et al. 2006) applied (i) three-camera set-up capturing both teacher and student actions, and (ii) the technique of video-stimulated recall in post-lesson interviews to obtain participants' reconstructions of the lesson and the meanings that particular events held for them personally.

I carried out a case analysis for a sequence of 18 consecutive lessons by a teacher recognized as a very good and competent teacher locally form the LPS Hong Kong data set with a focus on the students' perspectives on their mathematics lessons, with an attempt to make a connection between the teacher's and the students' perspectives. Analysis was carried out to seek answers for the following research questions:

1. How were the students' attitudes towards their mathematics lessons?
2. How did the students see their teacher's instructional practices?

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ALGEBRA PROBLEM TYPES IN HONG KONG SENIOR SECONDARY MATHEMATICS TEXTBOOKS: CHANGES IN THE NEW SENIOR SECONDARY CURRICULUM

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Textbooks play a significant influence in the classroom teaching in Hong Kong. Teachers are used intensively for teaching of secondary mathematics. Thus the problems in the textbooks are extremely important for quality teaching and learning. With the launching of the New Senior Secondary Curriculum (NSS) by the Education Bureau (EDB), all the senior secondary schools will use new sets of mathematics textbooks. The new curriculum aims at helping students to "develop in students the generic skills, and in particular, the capability to use mathematics to solve problems, reason and communicate" (EDB 2007, p.10).

The problem types presented in two sets of mathematics textbook series (one before the implementation of NSS syllabus and the other after) and the similarities and differences between these the textbooks series published in the different time period will be studied. A conceptual framework developed by Zhu (2006) is applied. The problems are classified as seven types: routine problems vs. non-routine problems, traditional problems vs. non-traditional problems, open-ended problems vs. closed-ended problems, application problems vs. non-application problems, single-step problems vs. multiple-step problems, sufficient data problems, extraneous data problems and insufficient data problems, problems in purely mathematics form, problems in verbal form, problems in visual form and problems in a combined form. Interim results show that the problems in the NSS Form Four textbook are solely in routine problems (100%) and in sufficient data problems (100%); were dominantly in traditional problems (99.3%, 1862 out of 1875), open-ended problem (99.0%, 1856 out of 1875), non-application problem (88.1%, 1651 out of 1875), multiple-step problem (84.1%, 1577 out of 1875); and were both mostly in purely mathematical form (47.1%, 883 out of 1875) or in combined form (46.1%, 865 out of 1875).

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A PROCESS FOR ENHANCE INTERNSHIP REFLECTIVE PRACTICE THROUGH COLLABORATIVELY REFLECTION

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A process in which teachers by working with other teachers to examine and critique one another's teacher techniques of Japanese Lesson Study functions as a means of enabling teacher to develop and study their own teaching practices (Baba, 2007). According to the phases of Lesson Study Cycle, Collaboratively reflection was a phase for work practitioners reflecting the studied things from Mathematics Classroom Activities (Inprasitha, M., Pattanajak & Inprasitha, N., 2010). Approaches to in-service teacher education have also emerged from the perspective of communities of learning, in which researchers are collaboratively involved the process of teachers' professional development (Hino, 2010). The purpose of this study is to describe the roles of graduate students in lesson study process during reflecting seminar after worked within internship practicum. The target group as graduate student lesson study team supervisors in Mathematics Education Program of 2010-2011 academic years. Data was collected by IC recording, video recording then analyse scenario and statement of reflection seminar of internship practicum. The result showed that the roles of graduate students reflection in deeply more over time and support internship thinking with divergent thinking, reasoning and self learning with the others.

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TEACHER EDUCATORS' PERSPECTIVES ON WORKING WITH LEARNER MATHEMATICAL THINKING: A ZAMBIAN STUDY

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This poster presents part of a larger study on what and how learner mathematical thinking is recognised and focused on in teacher education in Zambia. We present some Zambian teacher educators' orientations towards the discourse of engaging with learner mathematical thinking. Our focus here is on mathematical reasoning, one of three notions identified by Even and Tirosh (2002) as important for teachers' work with leaner mathematical thinking.

In their interviews teacher educators themselves talked about the programme and their teaching in it. We will present their talk and show that in this particular programme, following Bernstein (1996), learner mathematical thinking is not a strongly classified part of the curriculum. Nevertheless, it is part of what the teacher educators deemed important. We then show here what they said pertaining to mathematical reasoning as part of the discourse of learner mathematical thinking and 'how' they do so. Even & Tirosh (2002) point to the need for teachers to develop in learners both instrumental and relational understanding in Skemp (1976) terms, or both procedural and conceptual understanding in Kilpatrick et al. (2001) terms. Of the four teacher educators interviewed, three referred in either *instrumental* or *relational* ways to mathematical reasoning, or to both. We present these different ways and how they were analytically recognised. We then argue that these differing ways make available different opportunities to learn, and that this has particular significance in a weakly classified curriculum. A critical question is raised about a programme that is preparing teachers for teaching; specifically whether this could or should be a more explicit part of the programme.

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ATTEMPTING EQUAL OPPORTUNITIES TO LEARN – NORWEGIAN EXPERIDENCES FROM USING NATIONAL MAPPING TESTS IN PRIMARY SCHOOL

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Norwegian schools are inclusive schools and are expected to provide equal opportunities to all students to learn mathematics. Students with identified special needs in mathematics should receive extra attention and, if necessary, be offered special needs education. However, primary school teachers tend to wait and see if students can overcome their difficulties with the assistance provided in the general teaching activities (Nordahl & Hausstätter, 2009). Thus, targeted intervention for at-risk children is often delayed until they have developed a real problem in learning mathematics. This is recognized at a national level and recently early intervention became a national policy (MER, 2006). In 2008 a mandatory national mapping test to help Grade 2 teachers screen their students in mathematics was implemented. Optional mapping tests became available for Grade 3 in 2009 and in 2011 for Grade 1 (NDET, 2011). All tests were targeted at low achievement levels resulting in a ceiling effect by design. In 2011, the top 50% of the Grade 2 students solved an average of at least 80% of the test items correctly (Nortvedt & Throndsen, 2011). The poster will present analysis of 2011 test results. Analysis indicates that schools that use the mandatory Grade 2 test only and schools that, in addition, use optional tests had similar results with comparable number of students identified as being at risk. While at-risk children at all grade levels were confident in comparing quantities and counting by ones, still these students demonstrated less knowledge of numbers and counting than their peers. Group counting and sorting numbers were identified as critical aspects in numeracy as displayed by the tests. In addition, small differences were observed regarding the mathematical knowledge displayed by at-risk students in Grades 1, 2, and 3.

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FROM SAVOIR OF FRACTIONS THROUGH CONNAISSANCE OF PERCENTS TO SAVOIR OF PERCENTS

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The poster presents some results of a teaching experiment introducing the concept of *percent* to 6th grade pupils. The experiment is a part of a longitudinal research project (Ruzickova, Novotna, 2011) identifying teaching frameworks which allow observing and distinguishing the situations in which pupils display their *connaissances* and *savoirs* respectively (Brousseau, 1997).

Research data has been collected during two consecutive 6 grade lessons with certain characteristics of *mathematics rallye* (Brousseau, 2001). First, pupils worked in groups to solve five real-world problems concerning proportional reasoning. Although no fractional notation was used in the problem statements, the tasks required the pupils to compare parts of different wholes or to express the same parts of different wholes. Therefore, the pupils referred to their *savoirs* of the concept of *fraction* to describe the situations.

Then, in the follow-up whole class discussion directed by the teacher, the solving strategies were presented. Each task required a solving strategy based on finding a common denominator or any "common unit" of different wholes, thus representing a piece of *connaissance* of the notion of *percent*. Finally, the teacher institutionalized the newly acquired pieces of *connaissances* by suggesting one hundredth as a suitable "common unit", thus introducing the savoir of *percent*.

Classroom field notes and audio and video recordings were examined through qualitative analysis in terms of the form and mathematical content. Each group's worksheets were analysed to identify the solving strategies used by the pupils and the mathematical knowledge underlying them. The poster will include detailed description of individual group work tasks and authentic examples of pupils' solving strategies analysed in terms of connaissances and savoirs.

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DO YOUNG CHILDREN NOTICE WHAT COMBINATORIAL SITUATIONS REQUIRE?

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Inhelder and Piaget (1955) claim that combinatorial reasoning is a characteristic of the stage of formal operations, in which hypothetical-deductive thinking is required. However, Pessoa and Borba (2009), based on empirical evidence, argue that combinatorial reasoning begins to develop very early. Matias, Santos and Pessoa (2011) observed that 70% of the 5 and 6 year-old children interviewed noticed conceptual invariants (properties and relations) of *arrangement* problems. In the present research, a pilot study, six five-year-old kindergarten children answered, by manipulation of figures, four combinatorial problems, one of each type: *arrangment*, *permutation*, *combination* and *Cartesian product*. Vergnaud (1990) points out invariants (properties and relations) as important components of concepts. In the case of Combinatorics, invariants, of each kind of problem (*arrangment*, *permutation*, *combination* and *Cartesian product*), are related to element *selection*, i.e., whether or not to use all elements of the problem situation, to element *order*, i.e., whether or not the order of elements designate different possibilities, and to the *exhaustion* of all possibilities. The children generally found it easier to *select* the necessary elements, however, had greater difficulty in understanding how to deal with the invariant of *ordination* and a even greater difficulty to *exhaust* all possibilities. It was, thus, observed that young children intuitively notice some invariants of Combinatorics and it can be concluded that simple combinatorial situations can be concretely worked already with kindergarten children.

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EXPLORING HOW TO THAI TEACHER USE JAPANESE MATHEMATICS TEXTBOOKS

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Nicol, C & Crespo, S. (2006) stated that mathematics textbooks provide framework for what is taught, how it might be taught, and the sequence for how it could be taught. It is difficult to find out how teachers use textbooks because the availability of textbooks does not assure their use, and because their use varies considerably from teacher to teacher (Moulton, 1997). Teachers and educators recognize that There are different ways to use textbooks are significant for student learning (Takahashi, 2010). Japanese mathematics textbooks have been accepted were a problem-solving structure such as Takahashi (2006) stated that Japanese mathematics lessons, especially for elementary grades, include a significant amount of problem solving was designed to create interest in mathematics and stimulate creative mathematical activity in the classroom through students' collaborative work. A most teachers use textbooks as their primary instructional materials (Shimahara & Sakai, 1995; Sugiyama, 2008; Takahashi, 2010).

This study was to explored fifty-eight primary school teachers on "Project for Professional Development of Mathematics' teacher through Lesson Study and Open Approach" about the approaches to using Japanese mathematics textbooks while planning the lesson. The method was questionnaires-check lists, open ended questionnaires and interviewed. The data were analysed by approaches to using textbooks ranking from; adherence, elaboration and creation (Nicol & Crespo, 2006) for classification the teachers' approaches to using mathematics textbooks.

The results were as follows: 1) Before teachers attend the project, teachers had approaches to using mathematics textbooks ranking from adherence (25.77%), creation (27.84%) and elaboration (46.39%) and 2) During teachers attend the project, teachers had approaches to using mathematics textbooks ranking from adherence (21.71%), creation (37.14%) and elaboration (41.14%). The percentage of approaches about using mathematics textbooks ranking indicated that: percentage of adherence and elaboration were decrease and creation were increase after teachers attend the project.

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AN INVESTIGATE OF STUDENTS' LANGUAGE IS USED TO EXPRESS MATHEMATICAL IDEAS

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Language is a tool through which pupils build their knowledge of mathematics, and knowledge is built in social settings (Vygotsky 1978; Bruner 1996). The important goal of education is not to get students to take part in the conventional exchanges of educational discourse, even if this is required of them along the way. It is to get students to develop new ways of using language to think and communicate, 'ways with words' which will enable them to become active members of wider communities of educated discourse (Mercer, 1995). According to Open Approach, teaching was aimed to every student for being able to study mathematics in serving one's competency aligned with decision making level by oneself during study. In addition, the student had opportunity in bargaining the meaning with other students (Nohda, 2000). Those techniques allowed the students show their ideas and problem solving technique through their own language meaningfully. Language in its broadest sense is the mechanism by which teachers and pupils alike attempt to express their mathematical understanding to each other. The means of mathematical communication can be classified under 6 types: 1)"Ordinary" language 2) Mathematical verbal language 3) Symbolic language 4) Visual representation 5) Unspoken but shared assumptions 6) Quasi-mathematical language (Pirie, 1998).

The objective of this research was to investigate of students' language is used to express mathematical ideas using a qualitative research methodology which the researcher participated in the lesson study by collaboration in planning the lesson, teaching observation, and reflection with team members. The data collected from various sources including participatory observation, interviewing, and artifacts from classroom activities. Three phase of lesson study were videotaped and then transcribed, and analyzed based on Pirie's (1998) approach regarding to mathematical communication technique. The findings found that student communicate mathematical idea by language in 6 type and the most type that student used to communicate was ordinary language.

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PROSPECTIVE TEACHERS' MKT WHEN INTERPRETING THE PART-WHOLE REPRESENTATION: THE ROLE OF THE WHOLE

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Fractions are among the most complex mathematical concepts that children encounter in primary education (Behr, Harel, Post & Lesh, 1993). Students' limited understanding might be related to how their teachers understand and interpret fractions. In order to improve students' understanding on fractions it is of fundamental importance that teacher education focuses more on teachers' knowledge. This knowledge is conceived here as the *Mathematical Knowledge for Teaching* (MKT) (Ball, Thames & Phelps, 2008).

By combining a qualitative methodology and an instrumental case study, we focus on early years' prospective teachers MKT on fractions, and on their revealed understanding about the role of the whole. Data is from a sequence of tasks assigned to these prospective teachers in the context of a course focusing on the subject matter knowledge sub-domains of MKT. In the analysis we focus on prospective teachers' mathematical critical situations: their revealed gaps in knowledge, their different interpretations of fractions, and on the role of the whole. Our aim is to obtain a deeper understanding of the mathematical reasons why such gaps occur, in order to be able to design materials to improve teachers' training and the ways in which we, as teacher educators, approach such training. In this poster we will present the preliminary results obtained.

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A SEQUENCE OF ONLINE DIDACTIC ACTIVITIES FOR TRIGONOMETRIC FUNCTIONS

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A GOVERNMENT PROJECT

By 2009 the government of the Mexican state of Veracruz launched a program to develop didactic materials that could be used by students from 12 to 18 years old that were attending any school in Veracruz. Many projects were presented, one of them is named “Design, development and programming of online didactic activities for teaching mathematics in Veracruz” and its focus is to design four sequences of activities to let students to generate mathematical concepts. The work team is form with researchers and professors of the National Polytechnic Institute, the University of Veracruz and the Ministry of Education of Veracruz, all them in Mexico.

THE SEQUENCE

In Mexico, High school curricula include definitions, properties and applications of trigonometric functions. Our sequence has 20 activities and they are based on specific characteristics of sine, cosine and tangent functions. The first activity starts drawing a random graphic of f(x)= A sin(x) where “A” is an integer from -10 to 10. Then the user (a student) tries to draw an identical graphic by entering a value for A. The second activity involves the graphic of the function f(x)= sin(A x) where “A” is also an integer from -10 to 10. Also the user has to draw an identical graphic of the random graphic.

The third activity involves the function f(x)=sin(x+ A), and so on. The sequence continues mixing functions sine and cosine with expressions like:

f(x)= A sin(x) + B cos(x) where A & B are integers from -10 to 10, and so on until we get the expression f(x)= A sin (B x) + C cos(D x) where A, B, C & D are rational numbers from -10 to 10.

SOME RESULTS

The sequence has already been applied to three groups of high school students and we are doing the analysis of the answers. In this poster we will show some of the results we have got from our students and a brief resume of the analysis. The poster will include images of two activities and photos of students solving it. Images of students’ remarks about how they felt solving the activity will be added and finally we will include a few teachers remarks on the usefulness (or not) of the activities.

UNDERSTANDING THE WAY BASE AND POSITION IN THE DECIMAL NUMBER SYSTEM ARE PRESENTED IN TEXTBOOKS

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Gilda Guimarães - Universidade Federal de Pernambuco – Brazil

The Indo-Arabic number system is historically one of the most difficult achievements of mankind. This system, efficient and economic, is based on position and basis strategies whose sophistication is a cause of much of the learning problems that emphasizes the investigation. Textbooks are the most universal and popular educational instruments in classrooms and their influence on learning are of greatest importance.

The aim of this paper is to analyze the treatment of base and position of the decimal number system in textbooks.

The eighteen books analyzed dedicate more that 70% of their activities to this subject matter. However, these activities take place in the context of algorithmic calculations machined in the base and position strategies are hidden. Only 14% of the activities that has as its aim the learning of the DNS explicit the strategy of base and slightly more than 40% the positional. The strategies of base and position are complex, however the textbooks don't giving the student the opportunity to reflect on the same. Emphasis is laid upon the value of a number in a position, but there is no argument about what a base system, and specifically, a base-ten system, may be, nor do they propose that students reflect on the role of position. It is also important to reflect on the absolute and relative value of numbers. It should be taken into consideration that the use of didactic materials is not always sufficient for understanding the system (for example, multibase blocks can help understand the concept of base, but not position). Grouping and the opposite relationship, break down in various bases is fundamental to understanding what a base system is; however, what is set in textbooks is often the assertion that "10 units are equal to 1 ten" without any other emphasis or reflection on the procedure.

The role of zero is not analyzed. There are activities in which the symbol of zero is shown or it is said that "0" is registered in the absence of units, but there is no reflection on zero in decimal writing, as a means to maintain grouping. This is key since it is evidence of the importance of position.

We consider that researchers and teachers need to observe how a mathematical concept has been covered in textbooks to contribute to the overcoming of the difficulties found in the researches and in large-scale studies of the area.

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REASONING IN PRIMARY SCHOOL? AN ANALYSIS OF 3RD GRADE GERMAN TEXTBOOKS

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Although 'argumentation and reasoning' is one of the five procedural standards in the curriculum, primary teachers use reasoning very little in their lessons. Since mathematics educators and the ministry of education demand for more reasoning even in primary schools the question raised, how it may be strengthened in the lessons. In German mathematics lessons the textbook plays a dominant role. So, textbooks may be useful tools to develop teachers practice in reasoning.

Since no standard textbook exists in the German curriculum, we focused on two major questions first: Do German mathematics textbooks support mathematical reasoning through specific demands? Do the textbooks differ in supporting reasoning?

For our analysis we differentiated two different forms of reasoning (Adam 2011). If reasoning is seen as a pre-form of proof, the communicative structure is often a monological process, which a student shall fulfil by his or her own. If reasoning is much more focused as a part of argumentation the communicative structure is a dialogical one with at least two persons.

We hypothesized that books which contain more text ask for more mathematical reasoning than textbooks which contain only little text. The amount of words and its relation to the pages of the book were used as indicators to differentiate between these two groups of books.

Special linguistic elements as e.g. the interrogative sentences starting with "why", can be seen as explicit indicators for the demand for reasoning. In addition, there can be found implicit demand for reasoning as well, e.g. "Can this be true?" or "Who is right?" Both categories were defined in detail and used as the basis for the content and document analysis of the textbooks.

The analysis shows that over all less than 10% of the tasks ask for reasoning; the amount of explicit and implicit reasoning do not differ very much over all books, although some seem to favour explicit demand whereas others favour implicit demand. Whereas the amount does not differ significantly between books with much text and those with little amount of text, interesting qualitative differences can be seen.

The poster will present translated parts of textbooks as examples, a description of the methods used as well as graphic representations of the results.

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VARIOUS WAYS OF UNDERSTANDING IN MATHEMATICS TEACHER EDUCATION

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In recent years, most of the research on teacher education has been carried out with primary and secondary teachers up to the 10th class. There is less research into the field of knowledge by upper secondary level teachers, both prospective and in-service teachers. In this study content knowledge is not separated from pedagogical content knowledge as is done in most studies; the attention has been shifted to a different way of dealing with mathematical understanding. The initial point is the assumption that teacher training at most universities worldwide focus on procedural skills rather than on conceptual and relational understanding; the same applies to the research when it comes to the implementation of subject-matter content knowledge.

Therefore, the study distinguishes – based on Skemp (1979) and Herscovis and Bergeron (1983) – three main kinds of understanding:

- *Basal understanding*
- *Instrumental understanding*
- *Advanced understanding*

The study is designed as an international comparative study and will revise whether the superiority of East Asian Countries in comparison with Western Countries, as international comparative studies such as the IEA study TEDS-M (Blömeke, Kaiser & Lehmann, 2010) show, can be maintained, even if the focus is on prospective upper secondary level teachers taking into account the above described multi-faceted model of understanding. Furthermore, it should be clarified how the different kinds of understanding are related to each other and whether the shortcomings and deficits even in elementary routine procedures – as acknowledged in national and international studies – are due to a lack of basal understanding.

The presentation will describe the design of the planned study and exemplify the distinction between the various kinds of understanding using innovative test items.

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INTERDISCIPLINARY ALGEBRA WITH IPADS

Susan Staats, Alison Link, Alfonso Sintjago, Douglas Robertson

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Can iPad apps support students' contextual understanding of a mathematics application? This poster reports on an assignment in which students conducted either app-based or internet research to write a story about social relationships during an HIV epidemic. The University of Minnesota College of Education and Human Development provides all entering students with an iPad to use in learning communities and interdisciplinary seminars that are the foundation of the program. Students keep the iPad until they graduate or transfer from the college. The students described here were enrolled in an interdisciplinary learning community linking college algebra and world literature.

The students completed a modelling assignment in which they traced the logistic trajectory of an HIV epidemic. Because the logistic model can be understood as a calculation of disease prevalence based on potential contact between infected and uninfected people, it both assumes and obscures an underlying social back-story. Students wrote from the standpoint of a journalist describing the epidemic at early, middle and late stages in the epidemic describing how people might relate to each other and issues that might structure these relationships. Several internet sites and free apps were recommended to help students build this contextual understanding of epidemics.

Overall, students cited internet resources more frequently than apps in their writing. A frequently cited app, World Bank Datafinder, had been used extensively in the class. Some students also used the World Factbook app. Overall, though, a great deal of contextual knowledge was derived from internet web searches. Students used the mode of accessing information that was most familiar to them. Adoption limitations have been noted in other studies of hand held devices (Oliver & Goerke, 2008; Valstad. 2011). By contrast, iPads were used more extensively in a classroom assignment on making microloans through the website Kiva.org. In this activity, the experiential quality of the assignment and the utility of loan tracking apps may have contributed to greater usage. In general, students were not overwhelmingly "early adopters" of the use of apps to develop contextual knowledge. The poster will offer writing samples of students who used apps and those who used internet searches in contextual writing.

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INTERDISCIPLINARY ALGEBRA CURRICULUM MODEL

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Interdisciplinary education is a high-impact practice for early undergraduates, but implementing it can daunting. Interdisciplinary education often requires complex and costly organizational changes such as learning communities, team teaching or longitudinal programs that allow for strong disciplinary grounding and gradual development of integrative skills (Boix Mansilla, Miller & Gardner, 2000).

This poster outlines a model for interdisciplinary curriculum design that reduces organizational complexity and cost by enabling a single mathematics instructor to deliver authentic interdisciplinary curriculum. The model encompasses five components: (a) an introduction covering content from a partner discipline, (b) an essay, fictional story, memoir, or poem— co-written by a creative writer in any genre and by a specialist in a partner discipline, (c) explicit learning goals for algebra and for content in the partner discipline, (d) discussion and homework questions, and (e) a bibliography of readings in the partner discipline.

This model is theoretically grounded in aspects of interdisciplinary pedagogy that echo mathematics modeling pedagogy– a process orientation to learning and critical reflection (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003; Maaß, 2006). The model is also inspired by the need to extend the principles of Universal Instructional Design from the learning needs of students to include the learning needs of instructors (McGuire, 2011). A sample module uses short fiction to connect the mathematics of finance to psychologist Erik Erikson's theory of psychosocial development.

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UNDERSTANDING DIFFICULTIES IN SOLVING EXERCISES: A NEW POINT OF VIEW

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Many students face serious problems when solving mathematics tasks. For a long time teachers have wondered about the reasons and origins of these difficulties. In 2010 the author started collecting data at different schools and universities in order to analyse the solutions of several tasks in advanced calculus and linear algebra. Since that time more than 150 students have been involved in the collection of data about the solution of different types of tasks.

Students' solutions show that their way to the solution is often very laborious and seldom planned or structured from the beginning. Investigating the solutions step by step it is often difficult to identify those steps where the difficulties occurred. The reason might be that there is no distinction between the concept for the solution and its application. When building a concept it seems to be difficult for students to find an appropriate *method* and its position inside the solution with the aim of solving the exercise and applying it to the problem at hand.

To examine and understand difficulties during the solution by focusing on the methods one needs to distinguish between building a concept by *conceptualization*, an *operation* and an *application*. These procedures cannot be separated from each other. To understand the connections between them in consideration of the applied methods the procedures have been examined with reference to *thinking* and *skills* (e.g. Kaenders & Kvasz, 2011).

A lot of solutions of different mathematic exercises of many students from school and university are analyzed and examples are presented. They show that mistakes and their presumptive reasons concerning different processes and methods might be located. With these results it is possible to formulate an assumption about thinking and skills, which might help to reduce related problems in the future.

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A HISTORICAL RESEARCH ABOUT SECONDARY MATHEMATICS TEACHER EDUCATION IN JAPAN

Yuki Suginomoto and Hideki Iwasaki

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The main objective of this poster is to present a research on history about secondary mathematics teacher education in Japan. In concrete terms, this poster makes a comparative study of secondary mathematics teacher education curriculum.

Japanese lesson study, called jugyokenkyuu is a professional development process that Japanese teachers engage in to systematically examine their practice, it is well described in "the teaching gap" (Stigler & Hiebert, 1999). Since the Meiji era, Japanese lesson study has worked. The lesson study has origins in Japanese elementary teacher education school (Lewis, 2002). Because the lesson study carried a big role for teacher education in Japan, Japanese researchers have not been interested in research for mathematics teacher education. But, the lesson study has not worked in the secondary school level. Additionally, because there is a national curriculum for children, and this curriculum has the force of law, Japanese lesson study tends to focus on only teaching method.

In the early days, most of mathematics teachers for secondary school learned as simply function as rite of passage and only teach mathematics "and" pedagogy in a pre-service level. For that reason, pre-service teacher education wasn't based on practical aspects, and in-service teacher education was paid little attention in teacher education research domains. In the periods of new university, teacher educators began to treat teacher education as a discipline like mathematics education in modern day.

In recent years, qualities that are required of teachers are distributed in two major compartments. One is personal abilities for living through that period of time. The other is abilities for getting along with people. The lesson study will contribute to develop abilities for getting along with people, but it seems that the lesson study will not contribute personal abilities for living through that period of time. For this reason, we should construct a new lesson study in japan. For putting this into practice, we need to motivate teachers. In the current national curriculum in japan, there are content-free subjects of mathematics. Mathematics teacher should make a curriculum for a lesson.

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EXPLORING SENIOR HIGH SCHOOL MATHEMATICS TEACHERS' HORIZON CONTENT KNOWLEDGE

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The research presented here is part of an ongoing study of Taiwanese senior high school teachers' Mathematical Knowledge for Teaching (MKT) (Ball, Thames & Phelps, 2008). At the high school level this issue has yet to be explored. We focus in this report in teachers' Horizon Content Knowledge (HCK), a sub-domain of MKT, in the teaching of combinatorics. HCK is defined as an awareness of how mathematical topics are related over the span of mathematics included in the curriculum, an awareness of the large mathematical landscape in which the present experience and instruction is situated and an understanding of the broader set of mathematical ideas to which a particular idea connects (Ball, Thames, & Phelps, 2008; Ball & Bass, 2009). It has four constituent elements: a sense of the mathematical environment surrounding the current "location" in instruction, major disciplinary ideas and structures, key mathematical practices, and core mathematical values and sensibilities. Data were collected from three senior high school teachers, T1, T2 and T3. We use a mixed-methods approach (Smith, 2006). Our findings suggest first, one of the participants might provide a concrete example of a teacher with the HCK; second, the HCK produces improvements in teachers' mathematical quality of instruction; third, mathematical knowledge for teaching seems different from collegiate mathematics, fourthly, the HCK may be acquired and developed through teacher-student dialogues, and finally, there is a further relationship between the HCK and the KCS.

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EVOLUTION OF PROPORTIONAL REASONING PROBLEMS

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Research in proportional reasoning (PR) describes students' ability to think proportionally as a gradual increase in local competence: it progresses from cultivating learning adaptations of multiplicative inquiries to extending transferrable mastery to PR contexts. As such, students' PR problem-solving strategies develop little by little as students gain more and more exposure to solving formal mathematical expressions of a proportion. Young children begin approaching PR problems with integer multiples using such elementary strategies as the building-up/down strategy. After realizing that these strategies may not be effective for solving PR problems with non-integer multiples, children may look for an algorithm that is straightforward and applicable to more general PR problems. Successful problem-solvers will eventually conclude that the cross-multiplication strategy provides an answer to this goal. However, many researchers in the mathematics education community maintain that mastery of the cross-multiplication strategy does not necessarily equate with the ability to think proportionally. The practice of teaching to the test particularly exacerbates learning blind applications of the cross-multiplication strategy, which in turn leads to excessive emphasis on procedural algorithms without adequate conceptual foundations of PR.

Assessments based on traditional psychometric frameworks (e.g., item response theory) are generally geared toward measuring overall student performance and providing summative scores. By design, they are primarily useful in determining students' relative rankings along a continuum, and not in identifying their specific strengths and weaknesses. In contrast, cognitive diagnosis model (CDM) is a newly developed psychometric framework that can provide profiles of mastery and non-mastery of specific fine-grained attributes and can be used toward formative assessment. By developing attributes that are both descriptive and prescriptive, CDMs can provide instruction-relevant feedback that can encourage teachers to integrate carefully all necessary mathematics concepts in their lesson plans, including the conceptual and procedural aspects of PR problem-solving.

The current study describes the evolutionary process of developing proportional reasoning (PR) items based on the CDM framework. It is part of a larger research project on cognitive diagnosis assessment in the subject area of PR. It presents changes in the structural and contextual components of PR items during three stages of item development, particularly by taking into account both anticipated and observed students' mathematical thinking and problem-solving behavior in PR. Given a set of defensible fine-grained attributes instilled in appropriate assessments or lesson plans, cognitively-based items can serve as effective measures not only to provide informative and prescriptive assessment, but also to facilitate productive and creative mathematical problem-solving experiences.

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DO REPRESENTATIONAL MODI AFFECT STUDENTS' LINKING OF REAL-LIFE SITUATIONS TO MATHEMATICAL MODELS?

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The translation of a problem situation into a mathematical model constitutes a key – but far from obvious – step in the modelling process. We focus on two elements that can hinder that translation process by relating it to the phenomenon of students' overreliance on the linear model on the one hand (De Bock, Van Dooren, Janssens, & Verschaffel, 2007), and their (lack of) representational fluency on the other hand (Verschaffel, De Corte, de Jong, & Elen, 2010). More concretely, we investigated: (1) How accurate are students in connecting descriptions of realistic situations to "almost" linear models, and (2) Does accuracy and model confusion depend on the representational mode in which a model is given?

To answer these questions, sixty-four students in the first year of Educational Sciences were confronted with a written multiple-choice test consisting of twelve verbal descriptions of realistic situations they had to connect to an appropriate mathematical model. For each situation the appropriate model was either linear (i.e. of the form $y = ax$) or "almost" linear: inverse linear ($y = a/x$), affine with positive slope ($y = ax + b$ with $a > 0$), or affine with negative slope ($y = ax + b$ with $a < 0$). These models were given either in graphical, tabular or formula form (each representation was provided in one third of the cases). Data were analyzed by means of a repeated measures logistic regression analysis followed by multiple pairwise comparisons.

Results are in line with findings from several other studies showing the "default" role of the linear model, this time in situations in which an "almost" linear model is (more) appropriate. More importantly, this study highlights the impact of representational modi on students' (in)appropriate model assignments. A particular representation may facilitate correct reasoning about a situation with one mathematical model, but be misleading when representing a situation with another model.

An implication for mathematics education is the need for explicitly discussing differences between linear and various types of "almost" linear functions as well as the affordances and constraints of various representations of these functions.

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PRODUCTIVE FAILURE IN LEARNING: DO STUDENTS REALLY HAVE TO FAIL THEMSELVES?

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When students solve problems to a yet unknown concept prior to instruction they externalize their (mostly erroneous) pre-concepts. The teacher then can build on these erroneous pre-concepts in the following instruction. Our findings suggest that building on erroneous pre-concepts can foster learning. This is true for both – pre-concepts that have been externalized by the learners themselves as well prototypical pre-concepts.

PROBLEM-SOLVING PRIOR TO INSTRUCTION ALLOWS BUILDING ON STUDENTS' PRE-CONCEPTS

Productive Failure (PF, Kapur, 2009) suggests that students can benefit from first solving mathematical problems to a yet unknown concept, followed by instruction that builds on student-generated solutions, although the solutions most likely display erroneous pre-concepts. Kapur could show that students in a PF condition learn more than students who receive Direct Instruction (DI) right away. Upon closer inspection, however, students in the DI condition in fact receive a different form of instruction than students in the PF condition: The teacher directly presents the canonical solution, rather than building on typical student-generated solutions and pre-concepts. Thus, when comparing the two conditions, the timing of the instruction and the form of instruction has been confounded. The DI setting may also benefit from a classroom discussion about typical pre-concepts (Hammann, 2003). In a quasi-experimental study we varied the form of instruction in two DI conditions (a regular DI condition and a DI-S condition where instruction built on typical pre-concepts) and compared these conditions to PF conditions. The DI-S and the PF conditions did not differ regarding their learning outcomes, but outperformed the DI condition. Thus, it seems important to include students-generated solutions and pre-concepts in the instruction, but students do not have to first grapple with problem-solving themselves. However, problem-solving prior to instruction enables teachers to diagnose typical pre-concepts.

In addition to the description of our study, the poster will display the learning task together with student-generated solutions to prompt a discussion about which pre-concepts are particularly suitable to build on during instruction.

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OBSERVATION STUDENTS' IDEA AS LEARNING TO LISTENING

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The importance of listening as the foundation from which teacher education should be conducted. Listening to students promoted reflection which can lead to a conception of teaching grounded in adaptation (Cooney & Krainer, 1996, p. 1181). Listening to students as giving careful attention to hearing what students say (and to see what they do), trying to understand it and its possible sources and entailments (Arcavi & Krainer, 2007, p.112). Teachers have share reflections about a study lesson they have observed, for group member to be assigned to take detailed minutes. This way can have available for future references a good record of all the ideas that were generated during their work together (Fernandez & Yoshida, 2004, p.9).

This study focus on investigate how teachers observing students' idea to improve the way of teaching. Data collected from teachers' reflection of observation students' idea in classroom activity and interviews of the teacher concerning the results of the reflection. The result showed that how to observed students' ideas are present on teachers' reflection from hearing of teachers that;

1. Purpose of the activities that need to happen in the classroom.
2. Students' ideas expectation in the classroom.
3. Problem situation in the classroom and use of appropriate teaching materials.

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PARTICIPANT PERSPECTIVES OF 'ENGAGED TO LEARN PEDAGOGY': DOES THEORY MATCH PRACTICE?

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The fit between theory underpinning 'Engaged to Learn Pedagogy' (Williams, 2009), and participating student and teacher perceptions of its features is examined. Theoretically, this pedagogy engineers conditions for 'flow' (Csikszentmihalyi, 1997), a state of high positive affect during creative activity. Flow conditions specific to mathematical problem solving include a) *spontaneously* created intellectual challenges associated with 'discovered complexities'; overcome by b) constructing new knowledge whilst unravelling these complexities (Williams, 2002). Cognitive and social elements associated with this creative activity are considered using Williams' Spontaneous Learning Model (2007), which integrates theory from Krutetskii (1976 in Williams, 2007), Dreyfus, Hershkowitz, and Schwarz (2001), and Vygotsky (1933/1966). Video-stimulated post-lesson interviews, with an upper elementary school student and teacher, undertaken after two years of participation in the research were selected to illustrate perceptions of enacted Engaged to Learn Pedagogy. Subjects participated in six problemsolving activities (twelve 80-minute sessions). The student identified "freedom to think" as different, and important, and the teacher focused intensely on group development of new knowledge without 'expert other' input. Consistencies between theory and enactment of Engaged to Learn Pedagogy are evident between their perspectives and flow conditions.

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COMPREHENSION TESTS AND EYE MOVEMENTS IN READING MATHEMATICS: VERBAL COMPARING WITH EQUATION

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Mathematical symbol, equation, and figure are used in some newspapers, magazines and professional documents. Österholm (2006) found high school and university students who read group theory text written without symbols had better comprehension than those readers who read the similar content written with symbols. It meant mathematical texts using symbols demand a special skill for reading comprehension. The purpose of this study was to investigate the effect of different representation of verbal and equation on reading comprehension and eye movements.

This study collected eye movement data of 42 university students were not major in mathematics by Eye-Link 1000. The experimental materials were two texts in Chinese rewritten from popular science books, and each text had a graph. The topics were "how to figure out Earth radius" and "how to estimate the height of an island". Both texts had two versions written with verbal (eg. the arc length is equal to the central angle multiply by radius) or equation (eg. $d = \theta \times r$) in the key areas, but other parts were the same. There were 5 and 7 key areas in the two texts. Participants were randomly assigned to one version. After reading each text, participants were asked to evaluate the degree of difficulty and complete a true/false test, their response time was recorded simultaneously. There were three findings in the result: (1) there were no significant differences between two versions on reading comprehension, response time, and difficult rating. (2) Readers who read the version of equation represent- tation had more reading time than whom read the version of verbal representation, and the tendency was similar with the key areas showing readers spending more total viewing durations on the equation version. (3) Readers who read the equation version had higher ratio of reading time on graphic section than those readers read the version of verbal representation, approximately 40% versus 30%. This study showed university students had similar comprehension whether read the version of verbal or equation. However, eye movements showed readers consumed more time on processing equation representation, and need to refer to graphs more often.

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IS EXPLORATION REDUNDANT?

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This study, adopting the framework of Hsieh, Horng and Shy (in press) about integrating hands-on exploration in the production of proof, probes into whether and how hands-on exploration influences students' construction of proofs requiring auxiliary. In this study, paper- folding is used in the exploration.

RESEARCH DESIGN

The content to teach to students in our teaching experiment was the *property*: an angle (named *chord-tangent-angle* and abbreviates as *CT-angle* in this paper) formed by a chord and a tangent is equal to half its intercepted arc. The teaching activities of the experiment included two stages. At the first stage, named *real-value exploration-inferring stage*, students had to construct necessary steps to figure out the size of a *CT-angle* through folding papers. They were not restricted to any particular method of folding or figuring out the size of the angle. Differing from the first stage in which numerical information was provided, students in the second stage, named *symbol exploration-inferring stage,* worked with a *CT-angle* of which the measure of the intercepted arc was x degree. Further, they were asked to write down in detail the steps of deriving the measure of the angle, and then finally re-write this in the form of mathematical proof with the use of as many symbols as possible.

RESEARCH RESULTS

The result shows that, by hands-on exploration, students can actively construct proofs that are otherwise difficult when simply taught by a teacher. One of the reasons for the success is due to the event that, by (partially) randomly folding papers, students create an entry point to a solution, which is impossible to achieve in a regular paper-and-pencil task, in which students have no sense or idea of, or no pathway to prove. The folding action transforms the figures and creates many auxiliary objects that either uncover or create more information about the figures; this generates new stimulation to students' mind. On the other hand, the students' mental operation of analyzing or selecting evokes successive actions. A need for starting with hands-on exploration and progressing towards sieving out mathematical objects and steps for proving cannot be omitted for certain students to produce proof.

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A STUDY OF INTERACTION EFFECTS BETWEEN LEVELS OF MATHEMAITCS PROFICIENCY AND READING ENGAGEMENT FOR TAIWANESE ADOLESCENTS' MATHEMATICAL LITERACY: A CASE OF TAIWAN IN PISA 2009

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Introduction

The purpose of the study was to investigate the interaction effects between levels of mathematics proficiency and reading engagement for Taiwanese adolescents' mathematical literacy by analyzing the data sets from PISA 2009 (Programme for International Student Assessment 2009, PISA 2009).

Methodology

The subjects were 5,831 Taiwanese students of 15-year-old in PISA 2009. A full model of 2-way ANOVA was used to detect the interaction effects between levels of mathematics proficiency and reading engagement for students' mathematical literacy. In the model, the two factors were levels of mathematics proficiency and reading engagement and the dependent variable was mathematical literacy. The factor "levels of mathematics proficiency" was defined by two groups as high-performing and low-performing groups. The factor "reading engagement" was categorized by reading attitude, reading strategy, diversity of reading material and reading habit. The dependent variable "mathematical literacy" was calculated by averages of 5 plausible values in math.

Conclusions

Several evidences of interaction effects between levels of mathematics proficiency and reading engagement were found in the study. There were statistically significant differences of reading engagement in the two performing groups. The results will provide substantive guidelines for educators, practitioners, and researchers in related communities.

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THE DIFFERENCE ON ESTIMATION PERFORMANCE OF 8TH-GRADERS BETWEEN CONTEXTUAL AND NUMERICAL PROBLEMS

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To compare the differences of estimating strategies used by 8th-graders when solving contextual and numerical problems, the estimation instruments were designed based on the studies of Reys et al., (1991) and Sowder (1992). Both instruments are parallel which implies that the numbers used in both instruments are same, however, the presented types are different. Each instrument includes three different components which are reformulation, transformation, and compensation. Each instrument includes 12 questions which consist of each component include 3 questions, respectively. The Cronbach α for contextual and numerical problems are 0.75, respectively. 198 8th-graders from junior high schools in south Taiwan were selected to participate into this study. The t-test result shows that there is a significant difference on the estimation performance between contextual and numerical problems at α=0.05. The correct percentage for the numerical problems and contextual problems are 45% and 33%, respectively. This implies that these 8th-graders perform better on numerical problems than contextual problems. In addition, the t-tests results also show that there are significant differences on the performance of reformulation, transformation, and compensation between numerical and contextual problems at α=0.05, respectively. The correct percentages for numerical and contextual problems on reformulation, transformation, and compensation are 27.4% vs 48.7%, 37.3% vs 46.2%, and 33.3% vs 40.7%, respectively. The results show that sample students perform better on numerical problems than contextual problems for each component. In addition, data also show that sample students tended to use the written method to solve numerical and contextual problems. Implications will be discussed.

Keywords*: 8th-grade, Contextual problems, Numerical problems*

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