



Proceedings

OF THE 37TH CONFERENCE OF THE
INTERNATIONAL GROUP FOR THE PSYCHOLOGY
OF MATHEMATICS EDUCATION

» Mathematics learning across the life span «

Volume 5

PME 37 / KIEL / GERMANY
July 28 – August 02, 2013

Editors

Anke M. Lindmeier
Aiso Heinze



IPN

Leibniz Institute for Science
and Mathematics Education



Proceedings

OF THE 37TH CONFERENCE OF THE
INTERNATIONAL GROUP FOR THE PSYCHOLOGY
OF MATHEMATICS EDUCATION

» Mathematics learning across the life span «

Volume 5

PME 37 / KIEL / GERMANY
July 28 – August 02, 2013

Editors

Anke M. Lindmeier
Aiso Heinze



July 28 – August 2, 2013

Cite as:

Lindmeier, A. M. & Heinze, A. (Eds.). Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education, Vol. 5. Kiel, Germany: PME.

Website: <http://www.ipn.uni-kiel.de/pme37>

The proceedings are also available via <http://www.igpme.org>

Copyrights © 2013 left to the authors

All rights reserved

ISSN 0771-100X

ISBN 978-3-89088-291-8

Volume 1 ISBN 978-3-89088-287-1

Cover Design: Sonja Dierk

Logo: Verena Hane

Composition of Proceedings: Maria Schulte-Ostermann, Johannes Effland

Production: Breitschuh & Kock GmbH, Kiel

TABLE OF CONTENTS

VOLUME 5

SHORT ORAL COMMUNICATIONS

A DESIGN AND STUDY OF A MATHEMATICAL REPRESENTATION TO ADVANCE STUDENTS' KNOWLEDGE OF THE ADDITION OF FRACTIONS	1
Yasaman Abtahi	
METACOGNITIVE ABILITIES OF YOUNG CHILDREN IN MATHEMATICS ACTIVITIES	2
Fahretdin H. Adagideli, Engin Ader	
CONSTRUCTION OF CRITICAL THINKING SKILLS BY THE INFUSION APPROACH IN "PROBABILITY AND STATISTICS IN DAILY LIFE"	3
Einav Aizikovitsh-Udi, Miriam Amit	
LETTER AS OBJECT MISCONCEPTION IN JUNIOR SECONDARY SCHOOL ALGEBRA	4
Zarina Akhtar, Vicki Steinle	
STUDY ON CULTIVATION OF CREATIVITY IN MATHEMATICS EDUCATION: OVERCOMING OF INHIBITORY FACTOR OF FLEXIBLE IDEA	5
Miyo Akita, Noboru Saito	
STRUCTURAL AND OPERATIONAL MATHEMATICS: AN INTEGRATED APPROACH IN E-LEARNING	6
Giovannina Albano	
PARENT-CHILD INTERACTION IN MATHEMATICS LEARNING – INSIGHTS INTO MATH-EXPERIENCE-DAYS	7
Natascha Albersmann	
LEARNING OPPORTUNITIES IN MULTILINGUAL MATHEMATICS CLASSROOMS ACROSS TWO COUNTRIES	8
Annica Andersson, Eva Norén, Núria Planas	
A STUDY ON CONCEPT FORMATION OF GEOMETRIC FIGURES AT PRIMARY LEVEL IN METRO MANILA: FOCUSING ON COMPONENTS OF FIGURES	9
Mitsue Arai	
LINEARITY PREPONDERANCE ON 7 TH GRADE STUDENTS' SOLUTION STRATEGIES IN LENGTH-AREA PROBLEMS	10
Rukiye Ayan, Mine Işiksal-Bostan	
PROSPECTIVE MATHEMATICS TEACHERS INTERACTING IN A CHAT CONCERNING THE DEFINITION OF POLYHEDRON	11
Marcelo A. Bairral	

STUDENTS' CONCEPT IMAGES ABOUT LIMIT OF A FUNCTION AT INFINITY.....	12
Başak Barak, Tangül Uygur Kabael	
CONDITIONS FOR A SUCCESSFUL USE OF COMPUTERALGEBRA IN MATHEMATICS TEACHING.....	13
Bärbel Barzel, Raja Herold	
INVESTIGATING PROBLEM POSING PROCESSES OF PRESERVICE PRIMARY MATHEMATICS TEACHERS	14
Burcu Nur Baştürk, Ayşe Simge Ergin, Elif Türnüklü	
THE DYNAMICS ASSOCIATED WITH CLASSROOM IMPLEMENTATION OF MATHEMATICAL TASKS	15
Burcu Nur Baştürk, Sibel Yeşildere İmre	
TELPs – ANALYSING AND SUPPORTING MATHEMATICS TEACHERS' PCK	16
Isabell Bausch, Regina Bruder, Anne Prescott	
TEACHERS' REFLECTIONS UPON CONSTRUCTION OF THEIR OWN MADE-TESTS	17
Semir Becevic	
IMPLICATIONS OF THE DIDACTIC CONTRACT FOR TERTIARY MATHEMATICS TEACHING PRACTICES.....	18
Mary Beisiegel	
INDIVIDUAL DIFFERENCES IN FRACTION KNOWLEDGE AND THEIR RELATION TO THE INDIVIDUAL'S LEARNING APPROACH TO MATHEMATICS	19
Maria Bempeni, Xenia Vamvakoussi	
MATHEMATICS STUDENTS USING RESOURCES	20
Margot Berger	
WHAT MATHEMATICAL COMPETENCES CAN BE LEARNED FROM WEB-BASED LEARNING RESOURCES?	21
Tomas Bergqvist	
A TEACHER-TRAINING-STUDY FOR IN-SERVICE TEACHERS: FORMATIVE ASSESSMENT IN COMPETENCY-ORIENTED MATHEMATICS	22
Michael Besser, Dominik Leiss	
SITUATIONAL FIXATIONS IN THE USE OF DECIMAL FRACTIONS	23
Angelika Bikner-Ahsbahr	
VIDEO RESOURCES FOR MATHEMATICS TEACHER - LEARNING DURING PEER ASSESSMENT TASKS	24
Yaniv Biton	

A QUESTIONNAIRE TO SURVEY OPPORTUNITIES TO LEARN IN PRIMARY SCHOOL MATHEMATICS CLASSROOMS	25
Katrin Bochnik, Stefan Ufer	
SITUATED LEARNING TRANSFER OF ALGEBRAIC SYNTAX: LINEAR EQUATIONS AND VIRTUAL BALANCE	26
Maricela Bonilla, Teresa Rojano	
FLEXIBLE CALCULATION: KEY IDEAS FROM STUDENTS' SOLUTIONS	27
Joana Brocardo, Fátima Mendes, Catarina Delgado	
MATHEMATICAL CONTENT KNOWLEDGE OF ELEMENTARY PRESERVICE TEACHERS: GEOMETRY AND MEASUREMENT	28
Christine Browning, Alden Edson, Patrick Kimani, Fatma Aslan-Tutak	
7 TH GRADE STUDENTS' REASONING AND PROOF SKILLS IN ALGEBRAIC STATEMENTS	29
Ayşegül Çabuk, Mine İşiksal-Bostan	
UNDERSTANDING MATHEMATICS INQUIRY-BASED CLASSROOM PRACTICE: TEACHER'S ACTIONS AND INTENTIONS	30
Ana Paula Canavarro, Hélia Oliveira, Luís Menezes	
MENTAL COMPUTATION WITH RATIONAL NUMBERS: STUDENTS' STRATEGIES WITH PERCENT	31
Renata Carvalho Carrapiço, João Pedro da Ponte	
PRIMARY SCHOOL TEACHERS' PRACTICES IN STATISTICAL INVESTIGATION TASKS	32
Ana Caseiro, João Pedro da Ponte, Cecília Monteiro	
NEEDS-BASED MATHEMATICS TEACHERS' CONTINUOUS PROFESSIONAL DEVELOPMENT	33
Michael Casper, Bettina Roesken-Winter	
AN ANALYSIS OF TURKISH STUDENTS' SELF-CONFIDENCE AND VALUE BELIEFS IN MATHEMATICS AND PREDICTION OF MATHEMATICS ACHIEVEMENT IN TIMSS 2007	34
Kübra Çelikdemir	
CULTIVATING LEARNING THE CONCEPT OF NEGATIVE NUMBERS IN THE CONTEXT OF ALGEBRA AND VICE VERSA.....	35
Sepideh Chamanara, Amir H. Asghari	
RELATIONSHIP BETWEEN RELIGIOUS BELIEFS AND VIEWS ON EFFECTIVE MATHEMATICS TEACHING AND LEARNING FROM THE PERSPECTIVES OF MATHEMATICS TEACHERS	36
Yip-Cheung Chan, Ngai-Ying Wong	

A STUDY OF TENTH GRADERS' MATHEMATICAL MODELLING PROCESS IN TAIWAN	37
Ching-Kuch Chang, Ming-Feng Chen, Shu-Chuan Huang, Yi-Chen Yu	
A CASE STUDY OF ADJUSTMENT ON MATHEMATICS TEXTBOOK IN TAIWAN: USE POLYHEDRON AS AN EXAMPLE	38
Shu-I Chang, Jing Chung	
A STUDY ON RELATIONSHIPS BETWEEN ARITHMETIC OPERATING FLEXIBILITY, ACADEMIC ACHIEVEMENTS AND REASONING ABILITIES OF PRIMARY SCHOOL STUDENTS	39
Chia-Huang Chen, Hsiu-fei Lee	
TEACHER GROWTH THROUGH DESIGNING $0.\bar{9}=1$ TASK	40
Jian-Cheng Chen, Fou-Lai Lin, Hui-Yu Hsu, Hui-I Liao, Kai-Lin Yang	
EXPLORING THE GAP BETWEEN VERBAL DESCRIPTION AND FIGURAL PATTERN OF GEOMETRIC THEOREM ON SECONDARY STUDENTS	41
Ying-Hao Cheng, Hui-Yu Hsu, Jian-Cheng Chen	
ISSUES IN IMPLEMENTING AN INCOHERENT CURRICULUM BETWEEN MATHEMATICS AND PHYSICS	42
Mei-Shiu Chiu	
A THIRD-SPACE OF MATHEMATICAL PRACTICE: IMPLICATIONS FOR TEACHER EDUCATION	43
Marta Civil, Bryan Fede, Marcy Wood	
TEACHERS AND OBSERVERS LEARNING FROM DEMONSTRATION LESSONS IN MATHEMATICS EDUCATION	44
Doug Clarke, Anne Roche, Rose Knight	
PARENTAL BELIEFS ABOUT MATHEMATICS AND MUSIC LEARNING.....	45
Csaba Csíkos, Gabriella Dohány	
COGNITIVE SCIENCE AND MATHEMATICS EDUCATION: INVESTIGATING THE YOUNG MIND AND FRACTIONS.....	46
Julie Cwikla, Jennifer Vonk	
FRAMING ELEMENTS OF MATHEMATICS TEACHERS' PROFESSIONAL PRACTICE	47
João Pedro da Ponte, Marisa Quaresma	
THE LEARNING APPROACHES OF UNIVERSITY STUDENTS IN A REQUIRED MATHEMATICS COURSE	48
Bettina Dahl Søndergaard	

PRE-SERVICE SECONDARY MATHEMATICS TEACHERS' LEARNING ABOUT STUDENTS' THINKING	49
M. Gozde Didis, A. Kursat Erbas, Bulent Cetinkaya, Erdinc Cakiroglu	
CAN IN-SERVICE TEACHER TRAINING HELP IMPROVING STUDENTS' PROBLEM SOLVING COMPETENCES?.....	50
Ulrike Dreher, Lars Holzäpfel, Timo Leuders	
CHANGING MENTAL MODELS WHILE ACTING WITHIN STATISTICAL SITUATIONS	51
Andreas Eichler, Markus Vogel	
EXAMINING CONSTRUCTS OF STATISTICAL VARIABILITY THROUGH A SEMIOTIC MEDIATION LENS.....	52
George Ekol	
DEVELOPMENTAL TREND OF STUDENT LEVEL OF REASONING ASSOCIATED WITH STRATEGY USE IN PATTERN GENERALIZATION	53
Rabih El Mouhayar, Murad Jurdak	
HOW LEARNING COMMUNITIES IMPROVE MATHEMATICS PERFORMANCE ON A RURAL BAHAMIAN ISLAND	54
Marcella D. Elliott-Ferguson, Yolanda A. Rolle	
THE IMPACT OF PRIMARY SCHOOL ON SECONDARY SCHOOL - THE EXAMPLE OF DIVISION BY ZERO	55
Christian Fahse	
MEANINGS OF SPECIFIC TERMS RELATED TO THE CONCEPT OF FINITE LIMIT OF A FUNCTION AT ONE POINT	56
José Antonio Fernández-Plaza, Luis Rico, Juan Francisco Ruiz-Hidalgo	
DESIGNING VIDEO INSTRUCTION FOR MATHEMATICS	57
Mikaël Flens, Pauline Vos	
LINKING DRAGGING STRATEGIES TO VAN HIELE LEVELS	58
Susan K. Forsythe	
IDENTIFYING CHANGES IN DISCURSIVE MATHEMATICAL PRACTICES	59
Cristina Frade, Steve Lerman, Luciano Meira	
TEACHERS DISCUSSING INQUIRY BASED TEACHING WITH DIGITAL TOOLS	60
Anne Berit Fuglestad	
LEARNING TO LEARN TOGETHER IN A COLLABORATIVE ENVIRONMENT AND AN ALGEBRAIC MICROWORLD.....	61
Eirini Geraniou, Manolis Mavrikis	

EMPOWERING CONTROL BEHAVIORS OF STUDENTS IN CONSTRUCTING GEOMETRICAL PROOF	62
Hosein Ghaffari, Amir H. Asghari	
NURSES' CALCULATION OF MEDICINE DOSES: FORMULA VERSUS PROPORTIONAL REASONING	63
Roslyn Gillies	
CONSTRUCTION AND IMPLEMENTATION OF VIDEO-BASED INTERVIEWS WITH STUDENTS FOR THE ANALYSIS OF THEIR ARGUMENTATION PRACTICES	64
Manuel Goizueta, Núria Planas	
REDESIGNING INTERVENTIONS FOR ENGINEERING STUDENTS: LEARNING FROM PRACTICE	65
Birgit Griesse, Bettina Roesken-Winter, Michael Kallweit, Eva Glasmachers	
DIFFERENT APPROACHES OF FOSTERING STUDENTS' MODELLING COMPETENCIES – AN EMPIRICAL STUDY	66
Susanne Grünewald	
REFLECTION OF THE MATHEMATICS TEACHER ON HIS RESOURCES.....	67
José Guzmán, David A. Páez	
TEACHER'S WAYS OF GUIDING STUDENTS IN OPEN PROBLEM SOLVING	68
Markus Häikiöniemi, Henry Leppäaho, John Francisco	
A CODING ON 5 TH AND 6 TH GRADERS' PICTURES AND TEXTS ON WHAT MATHEMATICS MEANS TO THEM	69
Stefan Halverscheid, Annika Nieger, Sarah-Elena Wege, Kirsten Werner	
“SIMPLE” & “NOT SIMPLE” PARAMETRIC EQUATIONS	70
Dina Hassidov, Bat-Sheva Ilany	
AN INNOVATIVE APPROACH TO NEGATIVE NUMBERS	71
Mathias Hattermann, Rudolf vom Hofe, Harry Kullmann	
'STRATEGY KEYS' - AN ESSENTIAL TOOL FOR (LOW-ACHIEVING) MATHS STUDENTS	72
Raja Herold, Bärbel Barzel, Markus Ehret	
THE BENEFITS OF PROBLEM-SOLVING WHEN FOLLOWED BY INSTRUCTION THAT BUILDS ON STUDENT SOLUTIONS.....	73
Lars Holzäpfel, Katharina Loibl, Nikol Rummel	
CHILDREN'S CONCEPTIONS OF MEASUREMENT OF AREA AND VOLUME	74
Hsin-Mei E. Huang	

INTRODUCTORY SETTING DESIGN WITH THE PURPOSE OF CREATING STUDENTS' MATHEMATICAL ACTIVITY	75
Kazuhito Imai, Tomomi Takada	
AN ASSESSMENT OF STUDENTS' MATHEMATICAL LEARNING PROCESS USING TEACHING AND LEARNING BASED ON OPEN APPROACH	76
Maitree Inprasitha, Preecha Kreaowan, Somkuan Srichompoo, Narumon Changsri	
PRESERVICE TEACHERS' CONCEPTIONS OF ALGEBRA AND KNOWLEDGE OF STUDENT THINKING	77
Isil Isler, Eric Knuth	
IDENTIFICATION OF COGNITIVE CHARACTERISTICS OF MATHEMATICALLY GIFTED PRIMARY SCHOOL STUDENTS	78
Adela Jaime, Angel Gutiérrez	
VALIDATING AN ASSESSMENT OF PRE-SCHOOL TEACHERS' MATHEMATICAL KNOWLEDGE	79
Lars Jenßen, Simone Dunekacke, Wibke Baack, Martina Tengler, Hartmut Wedekind, Marianne Grassmann, Sigrid Blömeke	
RELATION BETWEEN IMITATIVE AND CREATIVE MATHEMATICAL REASONING WHEN SOLVING PHYSICS TASKS	80
Helena Johansson	
CONSTRUCTING A SITUATION MODEL OF A TRIGONOMETRY PROBLEM: AN EXPLORATORY STUDY	81
José Antonio Juárez López, Josip Slisko Ignjatov	
MATHEMATICS LANGUAGE AND MATHEMATIZING CAPABILITIES OF PRE-SERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS	82
Tangul Kabel, Draga Vidakovic	
EXPERT RATINGS AS AN INSTRUMENT FOR VALIDATING RESULTS OF VIDEO-BASED TESTING	83
Gabriele Kaiser, Jessica Benthien, Martina Döhrmann, Johannes König, Sigrid Blömeke	
THE INTERPLAY BETWEEN FLUENCY AND APPRECIATION IN SECONDARY STUDENTS' FIRST ENCOUNTER WITH PROOF	84
Ioannis Kanellos, Elena Nardi, Irene Biza	
STUDENTS' PERCEPTIONS OF BRACKETS	85
Annasara Karlsson, Robert Gunnarsson	
ANALYSIS OF TAIWANESE STUDENTS' CONCEPTUAL STRUCTURE OF QUADRATIC FUNCTIONS	86
Ching-An Ko, Feng-Lin Lu, Tai-Yih Tso	

ASSESSING PRE-SERVICE TEACHERS' UNDERSTANDING OF MULTIPLICATION AND DIVISION	87
Ho Kyoung Ko, Nan Huh, Jihwa Noh	
IDENTIFYING MULTIPLE FUNCTIONS OF TEACHERS' QUESTIONING IN GERMAN AND JAPANESE MATHEMATICS CLASSROOMS	88
Yuka Koizumi, Yoshinori Shimizu	
SCHOOL MATHEMATICS AND BUREAUCRACY	89
David Kollosche	
STUDENTS' LEVEL OF 3D GEOMETRICAL THINKING: THE INFLUENCE OF 3D REPRESENTATIONS	90
Yutaka Kondo, Taro Fujita, Susumu Kunimune, Keith Jones	
CHANGING TEACHERS' BELIEFS IN A PROCESS OF COLLECTIVE PRODUCTION OF MULTIPLE PROOFS	91
Margo Kondratieva	
FACTORS THAT CAUSE CONTEXT DEPENDENCY IN FUNCTION BETWEEN MATHEMATICS AND SCIENCE	92
Masato Kosaka	
CHANGES IN TEACHING EQUATIONS WITH ONE UNKNOWN AFTER PARTICIPATING IN LEARNING STUDIES	93
Angelika Kullberg, Ulla Runesson, Pernilla Mårtensson	
OPERATIONALISING THE ZPD AS A 'SEMANTIC SPACE'	94
John Kuznirczuk	
USING MATHEMATICAL MODELLING TO ENGAGE STUDENTS IN REAL WORLD PROBLEM SOLVING	95
Meek Lin Kwek	
A FOUR-FIELD MODEL FOR ANALYSING DIALOGUES IN MATHEMATICS CLASSROOMS	96
Niclas Larson	
EFFECTS OF FUTURE MATHEMATICS TEACHERS' PRECONDITIONS ON THEIR KNOWLEDGE AT THE END OF THE TEACHER EDUCATION IN GERMANY AND TAIWAN	97
Christin Laschke, Gabriele Kaiser, Sigrid Blömeke	
MODELING AND ALGORITHMIC APPROACH TO A RANDOM SITUATION: A HELP FOR STUDENTS' UNDERSTANDING?	98
Dominique Laval	
HOW TEACHERS NOTICE AND ANALYSE MATHEMATICS LESSONS	99
Claudia Lazarevic	

AUSTRALIAN INDIGENOUS STUDENTS AND NAPLAN MATHEMATICS ACHIEVEMENT	100
Gilah C. Leder, Helen J. Forgasz, Jennifer Halliday	
EQUATIONS REDUCIBLE TO QUADRATIC FORM: A STUDY OF STUDENTS' ERRORS	101
Chee Peng Lee, Jaguthsing Dindyal	
THREE LEVELS OF UNITS AND MULTIPLICATIVE STRUCTURE IN FRACTIONAL CONTEXTS	102
Hyung Sook Lee	
STUDENTS' UNDERSTANDING ON UNIT AS A FOUNDATION OF MULTIPLE INTERPRETATIONS OF FRACTIONS	103
Ji Young Lee, Jeong Suk Pang	
MATH/PLUS - SUPPORTING FUTURE MATHEMATICS TEACHERS IN THEIR FIRST YEARS' STUDIES	104
Malte Lehmann, Bettina Roesken-Winter	
DEVELOPMENT OF THE CREATIVE DISPOSITION TOWARD MATHEMATICS SCALE	105
Yuh-Chyn Leu, Mei-Shiu Chiu	
IMPROVING DIAGNOSTIC JUDGEMENT OF PRESERVICE TEACHERS BY REFLECTIVE TASK SOLUTION	106
Juliane Leuders, Timo Leuders	
TASKS THAT MAY OCCASION MATHEMATICAL CREATIVITY: AVA'S EMERGING PERSPECTIVES	107
Esther Levenson	
A PRELIMINARY EXPLORATION OF LEVELS OF UNDERSTANDING DURING AN INTERACTION EFFECT	108
Tsai-Wen Lin, Chao-Jung Wu, Scott Sommers	
NUMERACY EXPERIENCES AND MODELLING BEHAVIOURS	109
Minnie Liu	
MATHPEN: EXPLORING HANDWRITING RECOGNITION TECHNOLOGY FOR ONLINE MATHEMATICS EDUCATION	110
Mandy Lo, Julie-Ann Edwards, Christian Bokhove, Hugh Davis	
USING DIFFERENT LEARNING STRATEGIES TO PROMOTE STUDENTS' COMPREHENSION OF GEOMETRY PROOF	111
Feng-Lin Lu, Tai-Yih Tso	

STUDENT-DRIVEN PROPORTIONAL REASONING APPROACHES TO AN EARLY ALGEBRA TASK	112
Anna L. V. Lundberg, Thomas Hillman	
MATHEMATICAL COMMUNICATION IN PRESCHOOL	113
Marita Lundström	
INSTRUMENTAL GENESIS: THE CONCEPT OF OBJECT	114
Kate Mackrell	
MULTIPLICATIVE STRUCTURES AND PROPORTIONAL REASONING: TEACHERS' EXPLORING & STUDENTS' LEARNING	115
Joakim Magnusson	
ELEMENTARY STUDENTS' GENERALIZATIONS AND THE GROWTH OF THEIR MATHEMATICAL ACHIEVEMENT	116
James A. Maher, Sarah B. Berenson	
LEARNING MATH THROUGH SOCIAL JUSTICE ISSUES	117
Ami Mamolo	
PCK AND TPCK: THE CASE OF THE AREA OF A TRAPEZOID.....	118
Agida G. Manizade, Dragana Martinovic	
AN ADIDACTICAL MILIEU THAT FAILS TO PREPARE FOR AN INTENDED STATEMENT OF EQUIVALENCE	119
Heidi Strømskag Måsøval	
INVESTIGATING FRESHMEN'S UNDERSTANDING OF SCHOOL MATHEMATICS: FOCUS ON THE GRAPH OF THE QUADRATIC FUNCTIONS.....	120
Yuki Masuda	
MATHEMATICAL REASONING PROCESSES: GENERALIZING AND JUSTIFYING IN THE STUDY OF REAL NUMBERS	121
Joana Mata-Pereira, João Pedro da Ponte	
PRESCHOOL MATHEMATICS EDUCATION WHICH COOPERATED WITH ELEMENTARY SCHOOL EDUCATION	122
Nanae Matsuo	
THE CO-CONSTITUTED SPACE OF LEARNING IN LESSONS INTRODUCING THE EQUATION OF A STRAIGHT LINE	123
Tuula Maunula	
QUANTITATIVE REASONING IN ENVIRONMENTAL SCIENCE: LEARNING PROGRESSION FOR 6 TH TO 12 TH GRADES	124
Robert Mayes, Franziska Peterson	

PRIMARY SCHOOL STUDENT’S MATHEMATICAL MODELLING COMPETENCY IN BUILDING HOUSE ACTIVITY	125
Samira Mehraein, Abolfazl Rafiepour Gatabi	
JUDGING “AS MUCH AS” BEFORE FORMAL INTRODUCTION TO MEASURE	126
Maria Mellone, Anna Baccaglini-Frank, Francesca Martignone	
GENERALISING AND SYMBOLISING IN COLLECTIVE DISCUSSIONS WITH GRADE 4 STUDENTS	127
Célia Mestre, Hélia Oliveira	
RAQUEL AND ISOLDA IN A MUTUAL INTERVIEW ABOUT DIALOGUE IN MATHEMATICS EDUCATION	128
Raquel Milani	
TEACHER LEARNING IN OBJECT FOCUSED PROFESSIONAL DEVELOPMENT	129
Shadrack Moalosi	
MEASURING FLOW IN MATHEMATICAL TASKS	130
Ana Belén Montoro Medina, Francisco Gil Cuadra	
EXPLORING PRE-SERVICE TEACHERS’ KNOWLEDGE OF INSTRUCTIONAL STRATEGIES FOR MATHEMATICS	131
Jayaluxmi Naidoo, Vimolan Mudaly	
WHAT SUCCESS AND CHALLENGE DID TWO ZAMBIAN MATHEMATICS TEACHERS SHOW THROUGH LESSONS?	132
Nagisa Nakawa	
TEACHERS’ QUESTIONS IN GUIDING FOURTH GRADERS IN OPEN-ENDED PROBLEM SOLVING	133
Liisa Näveri, Anu Laine, Anu Kankaanpää, Maija Ahtee, Erkki Pehkonen	
SYMMETRY IN SPATIAL PERSPECTIVE TASKS - AN INTERVIEW STUDY AT THE BEGINNING OF FIRST GRADE	134
Inga Niedermeyer, Silke Ruwisch	
DIFFERENT TRANSLATION ACTIONS IN THE FIELD OF FUNCTIONAL RELATIONSHIPS	135
Renate Nitsch, Regina Bruder	
INCREASED EFFICIENCY WHEN ENGAGING IN CREATIVE MATHEMATICAL FOUNDED REASONING	136
Mathias Norqvist, Yvonne Liljekvist, Bert Jonsson, Johan Lithner	
THE MATHEMATICAL TASKS FOR REFLECTIVE THINKING IN POLAR COORDINATES	137
Hye Mi Oh, Oh Nam Kwon, Jee Hyun Park, Ji Eun Lee	

PROSPECTIVE MATHEMATICS TEACHERS' CRITICAL THINKING PROCESSES REGARDING CONDITIONAL PROBABILITY IN THE CONTEXT OF POPULAR MEDIA TEXTS	138
Mehtap Ozen, Erdinc Cakiroglu	
HOW EXPERIENCES WITH LINEAR GROWTH INFLUENCE STUDENTS' UNDERSTANDING OF EXPONENTIAL GROWTH?	139
Zekiye Ozgur, Amy Ellis, Torrey Kulow, Muhammed F. Dogan, Caroline Williams, Joel Amidon	
PRE-SERVICE TEACHERS' CONCEPTIONS ON MATHEMATICAL CREATIVITY	140
Areti Panaoura, Georgia Panaoura-Maki	
SECONDARY PRE-SERVICE TEACHERS' SUBJECT MATTER KNOWLEDGE AND KNOWLEDGE ABOUT STUDENTS: MEANING OF EQUIVALENT FORMS IN ALGEBRA.....	141
Joo Young Park	
10 YEARS OF DYNAMIC GEOMETRY IN PME.....	142
Gilmer Jacinto Peres, Rúbia Barcelos Amaral	
LATE-ARRIVED IMMIGRANTS IN SCHOOL AND PERFORMANCE IN ALGEBRA	143
Jöran Petersson	
MATHEMATICAL CHALLENGE AND SENSITIVITY TO STUDENTS IN UNIVERSITY LECTURING: AN UNEASY BALANCE.....	144
Georgia Petropoulou, Barbara Jaworski, Despina Potari, Theodossios Zachariades	
DIAGNOSTIC COMPETENCES OF MATHEMATICS TEACHERS – WHAT KIND OF KNOWLEDGE DO TEACHERS USE?	145
Kathleen Philipp, Timo Leuders	
EXPLORING THE EPISTEMIC FACET OF THE DIDACTIC-MATHEMATICAL KNOWLEDGE REQUIRED TO TEACH THE DERIVATIVE.....	146
Luis R. Pino-Fan, Vicenç Font, Juan D. Godino	
ASSESSMENT CRITERIA AS A TOOL FOR MATHEMATICS LEARNING.....	147
Jorge Pinto, Leonor Santos	
MODALITY OF TASK PRESENTATION AND MATHEMATICAL ABILITY IN A STUDY ABOUT SPATIAL ABILITY	148
Meike Plath	
THE USE OF LEARNING JOURNALS AS ASSESSMENT.....	149
Hilary Povey	

EARLY DEVELOPMENT OF MATHEMATICS COMPETENCES - AN INTERVENTION IN KINDERGARTENS IN A LOW INCOME COMMUNITY	150
Glória Ramalho, Mafalda Magalhães, Susana Batista, Sofia Ferreira, Inês Elias	
PROSPECTIVE PRIMARY MATHEMATICS TEACHERS' DIAGNOSTIC COMPETENCE: MODELLING THE PROCESS	151
Simone Reinhold	
ACCELERATING EARLY CAREER TEACHERS' KNOWLEDGE OF STUDENTS AND CONTENT IN ALGEBRA	152
Steve Rhine, Mike Charles, Colin Starr	
IN-SERVICE TEACHER EDUCATION AS A FACTOR TO THE INSTITUTIONAL DEVELOPMENT	153
Margarida Rodrigues, Cristina Loureiro	
TEACHERS' BELIEFS REFERRING TO TEACHING WITH TECHNOLOGY	154
Paul Rögler, Bärbel Barzel, Andreas Eichler	
DEALING WITH COVARIATION: MISCONCEPTIONS AND THE EFFECT OF REPRESENTATION FORMS	155
Tobias Rolfes, Jürgen Roth, Wolfgang Schnotz	
LONG-TERM DEVELOPMENT OF STUDENTS' REPERTOIRE OF RATE OF CHANGE PROCEDURES	156
Gerrit Roorda, Pauline Vos, Martin Goedhart	
THE RELATIONSHIP BETWEEN A GRAPH AND ITS DERIVATIVE GRAPH: IMPROVING STUDENTS' UNDERSTANDING	157
Ulf Ryberg	
FROM MATHEMATICS TEACHERS TO TEACHER EDUCATORS: EMERGENCE OF A COMMUNITY OF PRACTICE	158
Laura Sabater, M. Carmen Penalva-Martínez, Germán Torregrosa-Gironés	
RESEARCH TRAINING NEEDS OF PRACTICING MATHEMATICS TEACHERS	159
Brigitte Sánchez Robayo, José Torres Duarte, Jaime Fonseca Gonzáles, Jorge Rodríguez Bejarano	
STUDENT ENGAGEMENT AND MATHEMATICAL LEARNING: LOOKING SMART VS. STAYING OUT OF TROUBLE.....	160
Lina Sanchez Leal, Roberta Y. Schorr, Gerald A. Goldin	
PROSPECTIVE KINDERGARTEN AND PRIMARY SCHOOL TEACHERS' UNDERSTANDING OF THE MEAN	161
Raquel Santos	

INSTRUMENTAL ORCHESTRATION TYPES PLANNED BY PRE-SERVICE MATHEMATICS TEACHERS	162
Rüya Şay, Tuğçe Kozaklı, Hatice Akkoç	
TEACHERS' BELIEFS REGARDING THE BENEFIT OF GRAPHICAL REPRESENTATIONS	163
Angela Schmitz, Andreas Eichler	
INVESTIGATING BELIEFS OF KINDERGARTEN AND PRIMARY SCHOOL TEACHERS TOWARDS MATHEMATICS LEARNING.....	164
Stephanie Schuler, Gerald Wittmann	
THE CO-EVOLUTION OF PROBLEM POSING AND PROBLEM SOLVING IN THE COURSE OF SARAH'S ON-GOING SOLUTION ACTIVITY	165
Volkan Sevim, Victor V. Cifarelli	
A STUDY ON THE PROCESS OF SOLVING CONTEXT PROBLEMS BY PROSPECTIVE TEACHERS	166
BoMi Shin	
PERCEPTION OF VISUAL MODELS IN LEARNING OF BINARY RELATION THEORY	167
Anna Shvarts	
LESSON STUDY IN TEACHER EDUCATION	168
Charlotte Krog Skott, Camilla Hellsten Østergård	
METAPHORS AND DIDACTICAL SITUATIONS	169
Jorge Soto-Andrade	
GENERAL COGNITIVE AND READING ABILITIES AS IMPACT FACTORS ON STATISTICAL LITERACY	170
Ute Sproesser, Sebastian Kuntze, Joachim Engel	
THE ALGEBRAIC THINKING PROJECT: FUNCTIONS	171
Colin Starr	
DISCUSSIONS THAT BUILD UPON CHILDREN'S THINKING	172
Ruti Steinberg	
AN EMPIRICAL STUDY OF TREATMENTS AND CONVERSIONS OF REPRESENTATIONS	173
Anna Stenkvist	
PRIMARY TEACHERS IN ENGLAND AND HONG KONG – LEARNING FROM EACH OTHER?	174
Susan Steward, Linda Hargreaves	

READING COMPREHENSION AND CALCULATION AS PREDICTORS OF MATH WORD PROBLEM SOLVING.....	175
Marina Stibric	
STUDENTS' BELIEFS WHILE SOLVING AN EXERCISE.....	176
Hannes Stoppel	
A STUDENT'S IDENTITY FORMATION IN ELEMENTARY MATHEMATICS CLASSROOMS.....	177
Hitoshi Takahashi	
COLLABORATION BETWEEN IN- AND PRE-SERVICE TEACHERS FOR CAPTURING THE STUDENTS' MATHEMATICAL IDEAS	178
Sampan Thinwiangthong, Wipaporn Suttiamporn, Maitree Inprasitha	
VALID STRATEGIES, INCORRECT ANSWERS: STUDENTS WITH LEARNING DISABILITIES SOLVING WORD PROBLEMS.....	179
Helen Thouless	
FOSTERING PROBABILISTIC INTUITIONS OF RISK AND DECISION MAKING UNDER UNCERTAINTY.....	180
Christoph Till	
ENCULTURATION INTO MATHEMATICAL PRACTICE: AN EXPLORATIVE STUDY INTO THE STUDENT PERSPECTIVE	181
Stephanie Treffert-Thomas	
CHARACTERISTICS OF ADAPTIVE TEACHER BEHAVIOR IN MATHEMATICAL MODELLING: A CASE STUDY	182
Natalie Tropper	
FRACTIONAL KNOWLEDGE AND GRAPHICAL REPRESENTATIONS.....	183
Zelha Tunc Pekkan, Martina Rau, Vincent Aleven, Nikol Rummel	
REFLECTIVE JOURNALS FOR TEACHER CHANGE: IMPLEMENTATION OF MATHEMATICS REASONING APPROACH IN 6 TH GRADE.....	184
Ece Ulucan, Umut Girgin, Recai Akkus	
WHAT DO COLLEGE CALCULUS STUDENTS BELIEVE ABOUT MATHEMATICS?.....	185
Figen Uysal, Jessica Ellis, Chris Rasmussen	
SEEKING INTERDISCIPLINARITY AND NEW WAYS OF DOING SCIENCE: A HISTORICAL AND CHALLENGING APPROACH.....	186
Andrea V. Rohrer	
GRAPHICAL REPRESENTATIONS AS CATALYTIC TOOLS FOR SHARPENING TEACHING-LEARNING DISCOURSES.....	187
Pieter van Jaarsveld	

CONSIDERING EXTRINSIC ASPECTS IN THE ANALYSIS OF MATHEMATICS TEACHER-TRAINING PROGRAMS	188
Yuly Vanegas, Javier Diez-Palomar, Vicenç Font, Joaquim Giménez	
MATHEMATICAL REPRESENTATIONS USED BY GRADE 3 TEACHERS	189
Isabel Velez	
TRANSFORMING THE LEARNING ATTITUDE THROUGH SCIENTIFIC-POPULAR BOOKS ENGAGED IN MATHEMATICAL INSTRUCTION.....	190
Chih-Yeuan Wang	
THE USE OF DISCURSIVE ANALYSIS FOR UNDERSTANDING PROSPECTIVE TEACHERS' GEOMETRIC THINKING.....	191
Sasha Wang	
THE ASPECTS OF STUDENTS' EXPLANATIONS IN AN ERROR ANALYSIS SITUATION.....	192
Keiko Watanabe-Mukai	
THE RELATIONSHIP BETWEEN MEDIUM OF INSTRUCTION AND MATHEMATICS ACHIEVEMENT IN SOUTHERN AND EASTERN AFRICAN COUNTRIES.....	193
Koji Watanabe	
STUDENTS' GOAL DIRECTED BEHAVIOR: IMPLICATIONS FOR MODELING STUDENT UNDERSTANDING	194
Eric Weber	
COMPARING THE EFFECTS OF TWO DYNAMIC VISUALIZATION INTERVENTION PROGRAMS ON STUDENTS' TRANSFORMATIONAL GEOMETRY ABILITIES	195
Xenia Xistouri	
DISCOURSE PATTERNS EMPLOYING CHORAL RESPONSE IN MATHEMATICS CLASSROOMS IN SEVEN COUNTRIES	196
Lihua Xu, May Ee Vivien Wan, David Clarke	
A COMPARATIVE LOOK AT ENGLAND AND TAIWAN TEXTBOOKS BASED ON THE NOTION OF ABSTRACTION.....	197
Kai-Lin Yang	
A CASE STUDY OF EDUCATIVE POWER	198
Kai-Lin Yang, Fou-Lai Lin, Hui-Yu Hsu, Jian-Cheng Chen, Ying-Hao Cheng	
RELATIONSHIP BETWEEN HIGH SCHOOL STUDENTS' UNIVERSITY ENTRANCE EXAM SCORES AND THEIR NON-ROUTINE PROBLEM SOLVING SKILLS.....	199
Yeliz Yazgan, Huseyin Ozan Gavaz, Bahar Kirmizi	

A CASE STUDY ON PRE-SERVICE MATHEMATICS TEACHERS' COVARIATIONAL REASONING IN PEER LEARNING	200
Seçil Yemen-Karpuzcu, Fadime Ulusoy, Mine Işıksal-Bostan	

AN ANALYSIS OF STUDENTS' BASES FOR WRITING AN EXPRESSION OF MULTIPLICATION BY A DECIMAL IN RULE-OF-THREE PROBLEMS.....	201
Watarai Yohei	

MATHEMATICAL RICHNESS OF TALKS IN SIX CHINESE MATHEMATICS CLASSROOMS: THE AMOUNT OF ACADEMIC WORDS AS AN INDICATOR	202
Jun Zou, Yujing Ni, Xiaoqing Li	

POSTER PRESENTATIONS

INSTRUMENTAL GENESIS OF GRADE 10 STUDENTS LEARNING TO USE DYNAMIC 3D GEOMETRY SOFTWARE	207
Francisco Javier Alba, Angel Gutiérrez	

VALUES IN THE SWEDISH MATHEMATICS CURRICULUM	208
Annica Andersson, Anette de Ron	

AN INVESTIGATION OF NOVICE MATHEMATICS TEACHER EDUCATORS' EXPERIENCES IN TRANSITION FROM DOCTORAL STUDENT TO FACULTY MEMBER	209
Mary Beisiegel, Jennifer A. Eli	

ODE COURSE AND PRE-SERVICE TEACHERS CONCEPTION ABOUT LINEAR ALGEBRA OBJECTS	210
Barbara Bianchini, Silvia Machado	

A LANGUAGE OF DESCRIPTION FOR ANALYSING MATHEMATICS CLASSROOM COMMUNICATION	211
Lisa Björklund Boistrup, Niclas Larson	

COMPARTMENTALIZATION IN DUTCH TEXTBOOKS ON THE MULTIPLICATION OF FRACTIONS	212
Geeke Bruin-Muurling, Koeno Gravemeijer, Maaïke Koopman, Perry den Brok	

USING MULTIMEDIA CASES WITH PROSPECTIVE TEACHERS	213
Ana Paula Canavarro, Luís Menezes	

COMPARISON OF FRACTION DIVISION CONTEXT BETWEEN MATHEMATICS TEXTBOOKS IN TAIWAN AND USA	214
Shu-I Chang	

TAIWAN'S ELEMENTARY STUDENTS' BELIEFS ABOUT MATHEMATICS TEACHERS	215
Chia-Huang Chen, Hsiu-fei Lee	

TEACHER' UNERSTANDING OF PROOF BY CONTRADICTION	216
Hyungmi Cho, Oh Nam Kwon, Hye Mi Oh	
EXPLORING HIGH-SCHOOL MATHEMATICS TEACHERS' EXTEMPORANEOUS AND IMPROMPTU SPECIALIZED CONTENT KNOWLEDGE: TWO CASES STUDIES	217
Yi-An Cho, Chien Chin, Pai-Tang Lin, Ting-Wei Chen	
ITERATION AS THE COMMON METHODOLOGICAL TOOL FOR ACADEMIC AND CLASSROOM RESEARCH.....	218
Bronislaw Czarnocha	
CONSTRUCTING THE PROBLEM SOLVING CITIZEN	219
Jonas Dahl	
SELECTING AND ADAPTING MATHEMATICAL TASKS THAT FOSTER NUMBER SENSE DEVELOPMENT: THE PRACTICE OF ONE PRIMARY TEACHER	220
Catarina Delgado, Joana Brocardo, Hélia Oliveira	
FRACTIONS: AN EPISTEMOGRAPHICAL VIEW	221
Jean-Philippe Drouhard	
STRATEGIES OF PROBLEM SOLVING	222
Maria Madalena Dullius, Virginia Furlaneto	
CULTURE OF SOLVING PROBLEMS – A TOOL FOR EVALUATION OF CULTURE OF PUPIL SOLVING OF MATHEMATICAL PROBLEMS	223
Petr Eisenmann, Jarmila Novotná, Jiří Příbyl	
GRADE 3 AND 4 STUDENTS' DIFFERENT WAYS OF DISCERNING MATHEMATICAL PATTERNS	224
Anna-Lena Ekdahl	
WHAT IS CRITICAL WHEN EXPERIENCE RATIONAL NUMBERS WITH MEASURING TASKS	225
Helena Eriksson	
UNFOLDING THE PROPERTIES OF LOGARITHMS	226
Roger Fermsjö	
MATHEMATICIANS' DISPOSITIONS ON TEACHING UNDERGRADUATE MATHEMATICS USING SOFTWARE	227
Gregory D. Foley, Heba Bakr Khoshaim	
PERCEPTIONS ABOUT MATHEMATICS IN SCHOOL SUBJECTS	228
Andualem Tamiru Gebremichael	

THE EFFECT OF NUMERICAL DISTANCE IN COMPARING FRACTIONS GOES BEYOND ARITHMETIC PROFICIENCY	229
David M. Gómez, Gonzalo Mena-Carrasco, Pablo Dartnell	
ATTITUDES AND EXPERIENCES OF TEACHERS LEARNING WITH DYNAMIC GEOMETRY IN A COLLABORATIVE, DISCURSIVE, AND ONLINE ENVIRONMENT	230
Loretta Grisi-Dicker, Muteb Alqahtani, Arthur B. Powell	
CONSTRUCTING OBJECTS FOR TEACHER REFLECTION ON LEARNING MATHEMATICS	231
Cindy Hmelo-Silver, Carolyn A. Maher, Marjory F. Palius, Robert Sigley, Alice Alston	
BELIEFS OF EARLY CHILDHOOD TEACHERS IN MATHEMATICS	232
C. Hüttel, H. Wadepohl, E. Rathgeb-Schnierer	
DESIGNING A DANISH ‘MATH COUNSELOR’ PROGRAM	233
Uffe Thomas Jankvist	
STRUCTURE, LEVEL AND DEVELOPMENT OF PROFESSIONAL COMPETENCIES OF PRE-SCHOOL (KINDERGARTEN) TEACHERS IN THE FIELD OF MATHEMATICS	234
Lars Jenßen, Simone Dunekacke, Wibke Baack, Martina Tengler, Hartmut Wedekind, Marianne Grassmann, Sigrid Blömeke	
STRATEGIES FOR EVALUATING CLAIMS – AN ASPECT THAT LINKS CRITICAL THINKING AND STATISTICAL THINKING	235
Sebastian Kuntze, Einav Aizikovitsh-Udi, David Clarke	
MATHEMATICAL IDEAS ON CONSTRUCTING 3D DIGITAL FIGURES SHARED ON-LINE WITHIN A GROUP	236
Chronis Kynigos, Foteini Moustaki	
MULTIPLICATIVE THINKING IN RELATION TO MULTIPLICATION OF TWO TWO-DIGIT NUMBERS	237
Kerstin Larsson	
RELATIONSHIPS IN MATHEMATICS TEACHER EDUCATION IN GERMANY AND TAIWAN – THE ROLE OF INDIVIDUAL AND INSTITUTIONAL CHARACTERISTICS	238
Christin Laschke, Gabriele Kaiser	
THE INFLUENCE OF MULTIPLE SOLUTIONS ON MATHEMATICS TEACHING ...	239
Shin-Yi Lee	
INTEREST AND SELF-DETERMINATION IN THE TRANSITION TO STUDYING MATHEMATICS	240
Michael Liebendörfer, Reinhard Hochmuth	

THE LATENT CLASS ANALYSIS OF SENIOR HIGH SCHOOL STUDENTS' STATISTICAL INTERESTS AND VALUES	241
Su-Wei Lin, Wen-Chun Tai	
OPPORTUNITY TO LEARN: A STUDY FROM TAIWAN TIMSS 2007 SURVEY OF MATHEMATICS LEARNING ACTIVITIES	242
Su-Wei Lin, Jeng-Shin Wu	
ACCESS TO PROFESSIONAL DEVELOPMENT OPPORTUNITIES FOR MATHEMATICS TEACHERS IN RURAL USA.....	243
Agida G. Manizade, Laura Jacobsen	
PROGRAM FOR CONTINUOUS TRAINING IN MATHMATICS: JUSTIFYING ITS EFFICACY	244
Cristina Martins, Leonor Santos	
EVOLUTION OF STUDENTS' MATHEMATICAL PROCEDURES IN DIVISION TASKS.....	245
Fátima Mendes, Joana Brocardo	
A TRAJECTORY TO DEVELOP GRADE 4 STUDENTS' ALGEBRAIC THINKING ...	246
Célia Mestre	
TEACHER EDUCATION: SUPPORTING STUDENTS IN THEIR FIRST YEAR OF STUDIES IN MATHEMATICS	247
Kathrin M. Nagel, Florian Quiring, Jana T. Beitlich, Kristina Reiss	
ASSESSING GRADE 2 STUDENTS' UNDERSTANDING OF THE NUMBERLINE: THE ISSUE OF THE ITEM FORMAT.....	248
Guri A. Nortvedt, Andreas Pettersen	
ELEMENTARY AND SECONDARY SCHOOL TEACHERS' PERSPECTIVES OF EFFECTIVE MATHEMATICS TEACHING	249
Jeongsuk Pang, Misun Kwon	
KINDERGARTEN TEACHERS' BELIEFS ABOUT MATHEMATICS EDUCATION IN PRESCHOOL.....	250
Rubina Pereira, Gloria Ramalho	
ITERATIVE RESEARCH: DEVELOPING A LEARNING PROGRESSION FOR QUANTITATIVE REASONING.....	251
Franziska Peterson, Robert Mayes	
LATE-ARRIVED IMMIGRANTS IN SCHOOL AND PERFORMANCE IN ARITHMETIC FOR NEGATIVE NUMBERS	252
Jöran Petersson	

DEVELOPING REASONING ABOUT SIMPLE ADDITIVE STRUCTURES: ONE TASK FOR ELEMENTARY STUDENTS	253
Elena Polotskaia, Victor Freiman, Annie Savard	
STUDENTS' EXPECTATIONS ABOUT MATHEMATICS AT UNIVERSITY	254
Stefanie Rach, Aiso Heinze	
GEOMETRY THINKING LEVELS AMONG STUDENTS AND TEACHERS	255
Shaker Rasslan, Amal Rasslan-Sharif, Manal Yosif	
LEARNING FROM TRAINING TO IMPROVE KINDERGARTEN PROSPECTIVE TEACHERS' KNOWLEDGE ON POLYGONS	256
C. Miguel Ribeiro	
FROM TASKS FOR STUDENTS TILL TASKS FOR TEACHERS: IMPROVING KNOWLEDGE	257
C. Miguel Ribeiro, Alexandra Gomes	
TEACHER KNOWLEDGE AND THE IMPLEMENTATION OF INVESTIGATION TASKS	258
Helena Rocha	
PROOF SCHEMES: A STUDY WITH 9 TH GRADE PUPILS	259
Margarida Rodrigues, João Filipe Matos	
DEVELOPMENT OF CHILEAN AND FINNISH TEACHERS' CONCEPTIONS ON MATHEMATICS TEACHING WHEN USING OPEN-ENDED PROBLEMS	260
Laia Saló i Nevado, Päivi Portaankorva-Koivisto, Erkki Pehkonen, Leonor Varas, Markku Hannula, Liisa Näveri	
DEVELOPMENTAL INDICATORS OF PROSPECTIVE MATHEMATIC TEACHERS' NOTICING OF STUDENTS' UNDERSTANDING OF THE DERIVATIVE	261
Gloria Sánchez-Matamoros, Ceneida Fernández, Salvador Llinares, Julia Valls	
OUT-OF-FIELD MATHEMATICS TEACHING: PROGRESS IN INDIGENOUS SECONDARY SCHOOL CLASSROOMS	262
Satwant Sandhu, Gillian Kidman, Tom J. Cooper	
IDEAS TOWARDS THE EXTENSION OF THE THEORETICAL CONSTRUCT OF MATHEMATICS TEACHERS' KNOWLEDGE	263
Thorsten Scheiner	
UNDERSTANDING TRIADIC STRUCTURAL RELATIONS BETWEEN PART, WHOLE, AND FRACTION	264
Andrea Schink	

TEACHERS THINK ABOUT HOW TO STRUCTURE MATHEMATICS LESSONS TO DEVELOP STUDENTS' PROBLEM-SOLVING COMPETENCE	265
Stephanie Schlump, Michael Neubrand	
SELF GENERATED EXTERNAL REPRESENTATIONS IN THE CASE OF FRACTIONS	266
Stefan Schumacher, Jürgen Roth	
TEACHING PRACTICE WITH THE INQUIRYING QUESTIONS TO FOSTER DESIRABLE ATTITUDE OF MIND	267
Yusuke Shogimen	
EYE TRACKING RESEARCH OF NOVICE EXPERT DIFFERENCE IN MUPLTI-REPRESENTATIONAL LEARNING	268
Anna Shvarts, Dmitry Cumachenko	
TEACHING QUALITY OF MATHEMATICS UNIVERSITY COURSES	269
Ulrike Siebert, Stefanie Rach, Aiso Heinze	
THERE ARE SEVERAL DIFFERENT APPROACHES TO FINDING SOLUTIONS FOR PROBLEMS: WHICH IS HELPFUL FOR ME?	270
Nina Sturm, Renate Rasch, Wolfgang Schnotz	
TEACHERS' PROFESSIONAL DEVELOPMENT REGARDING THE DIAGNOSIS OF FUNCTIONS AND GRAPHS	271
Julia Teller, Bärbel Barzel	
INVESTIGATING PRESERVICE ELEMENTARY TEACHERS MATHEMATICAL BELIEFS AND LEARNING PREFERENCES	272
Figen Uysal, Yüksel Dede	
SENSE MAKING OF TRIGONOMETRIC RELATIONSHIPS IN THE CONTEXT OF LESSON STUDY	273
Nellie C. Verhoef	
HOW TO CRACK THE CODE	274
Ragnhild Vikström	
FACILITATING STUDENTS' MATHEMATICAL REASONING.....	275
Hannatjie Vorster, Trudie Benadé	
THE ROLE OF GENERALIZATION IN STUDENT THINKING ABOUT AVERAGE RATE OF CHANGE.....	276
Eric Weber, Allison Dorko	
LESSON STUDY AND LEARNING STUDY. DOES IT MATTER?	277
Anna Wernberg	

STUDENTS RESISTING STUDENT-FOCUSED TEACHING PRACTICES	278
Richard Wester	
UTILIZATION OF FIGURES DURING PROOF READING AMONG LOW AND HIGH-PRIOR KNOWLEDGE READERS	279
Chao-Jung Wu	
THE STUDY OF GEOMETRIC CONTENTS IN THE MIDDLE SCHOOL MATHEMATICS TEXTBOOKS IN TAIWAN, U.S.A AND SINGAPORE	280
Der-Ching Yang, Ting-Yun Cheng	
THE STUDY OF INTUITIONS IN ONE PROSPECTIVE TEACHER'S CONSTRUCTIONS OF MATHEMATICAL OBJECTS	281
Andonis Zagorianakos	
INDEX OF AUTHORS AND CO-AUTHORS VOLUME 5	285



SHORT ORAL COMMUNICATIONS

A DESIGN AND STUDY OF A MATHEMATICAL REPRESENTATION TO ADVANCE STUDENTS' KNOWLEDGE OF THE ADDITION OF FRACTIONS

Yasaman Abtahi
University of Ottawa, Canada

Operations with fractions are among the most challenging concepts in elementary school mathematics (Lamon, 2007). This research project aimed to examine 5 fifth grade students' reasoning as they attempted to construct meaning for the concept of addition of fractions using Cuisenaire rods, fraction circles, and a newly designed learning environment called the fraction board. To analyse participants' interaction with these representations, task based semi-structured interviews were conducted, during which participants solved addition of fractions problems. A framework

<u>Ideas</u>	Induction	Physically Distributed Learning (PDL)
	Off-loading	Repurposing
Adaptable		
Stable		

Figure 1 Stable Adaptable
 Environment

developed by Martin and Schwartz (2005) was used to study how physical action (adaptation of the environment) can support thinking and learning (interpretation), figure 1. The findings indicated that for previously unseen representations, participants' interaction with the environment created a recursive feedback. Participants adapted the environment to construct new interpretations on mathematical ideas and in return, these new interpretations fed the new adaptations of the environment (PDL). And for the known representations, participants relied on the environment to reduce the cognitive burden of the task (off-loading). Data analysis also showed the importance of feedback integrated within these learning environments, e.g. sizes of fraction circles, to help participants uncover the structural regularities of the environments and hence solve the problems. Further studies are needed to understand how various forms of feedback within a learning environment can help students better understand the concept of the addition of fractions.

References:

- Lamon, S. (2007). Rational numbers and proportional reasoning: Towards a theoretical framework for Research. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 629–667). Reston, VA: NCTM.
- Martin, T., & Schwartz, D. (2005). Physically distributed learning: Adapting and reinterpreting physical environments in the development of fraction concepts. *Cognitive Science* 29, 587–625

METACOGNITIVE ABILITIES OF YOUNG CHILDREN IN MATHEMATICS ACTIVITIES

Fahretdin H. Adagideli*, Engin Ader**

*Istanbul University, **Boğaziçi University

Veenman, Van Hout-Wolters, Afflerbach (2006); in one of the recent major reviews of metacognition and self-regulation; concluded that it is a relatively late emergent ability at the age of 8-10. Aim of this present study is to investigate metacognitive and self-regulatory abilities of young children at the age of 4-6. We use the coding scheme, Cambridge Independent Learning (C. Ind. Le) which is developed specifically to code and analyze metacognitive and self-regulatory abilities of young children, and adopted the underpinning theoretical framework (Whitebread et al., 2009). Moreover, we focus on exhibition of these abilities in mathematical activities. Therefore, main ground of the study is framed:

- What type of metacognitive and self-regulatory abilities do young children exhibit during mathematics activities?

This study has an observational methodology since other methodologies such as think aloud protocols and self-report methodologies have inadequacies. Participants in the study are 33 children at the age of 4-6. After three problem solving activities are designed to foster children's metacognitive and self-regulatory abilities, they were allowed to collaborate in groups of two or three during these activities. Children are video-recorded during each activity. Video episodes of activities are transcribed and particular episodes are coded according to metacognitive elements they represented.

As far as the initial findings are concerned, metacognitive and self-regulatory skills are displayed in several ranges of areas by children during children's activities. It is a valuable finding so far that metacognitive regulation abilities (planning, monitoring, control and evaluation) are more frequently identified than metacognitive knowledge (about person, task and strategy). Further analyses will be discussed through extracts from collected data.

References

- Veenman, M. V. J., Van Hout-Wolters, B. H. A. M., & Afflerbach, P. (2006). Metacognition and learning: conceptual and methodological considerations. *Metacognition and Learning*, 1, 3–14.
- Whitebread, D., Coltman, P., Pino Pasternak, D., Sangster, C., Grau, V., Bingham, S., et al. (2009). The development of two observational tools for assessing metacognition and self-regulated learning in young children. *Metacognition and Learning*, 4, 63–85.

CONSTRUCTION OF CRITICAL THINKING SKILLS BY THE INFUSION APPROACH IN “PROBABILITY AND STATISTICS IN DAILY LIFE”

Einav Aizikovitsh-Udi, Miriam Amit

Beit Berl Academic College, Israel Ben Gurion University, Israel

This paper examines the feasibility of utilising the mathematics classroom for the development of higher order-thinking skills, where “critical thinking skills” are used as a surrogate for higher order thinking skills in general, and the paper reports research into the development of these critical thinking skills in a mathematics classroom in Israel through the teaching of probability.

To create the learning unit investigated in this research, the mathematical content of an existing learning unit called "Probability and Statistics in Daily Life" (Lieberman & Tversky, 2001) was "infused" with a hierarchical progression of critical thinking skills according to Ennis' taxonomy (Ennis, 1991). 18 students were taught by the researcher in the *Kidumatica* program at Ben-Gurion University and 20 students were taught by the researcher in a high school in central Israel. Our research examined the processes of construction of critical thinking skills during the study of "Probability in Daily Life" learning unit in the infusion approach. The skills were: (a) identifying variables; (b) referring to sources; (c) identifying assumptions; (d) evaluation of statements; (e) suspending judgment; (f) offering alternatives. Tests were administered at the end of each stage and it was possible to calculate the mean score for each distinct skill at each stage. This allowed comparison of the retention of each skill, once taught, as increasingly sophisticated skills were introduced.

The results indicate that the instructional program produced measurable gains in student achievement and retention of the targeted skills. This research suggests that critical thinking skills do not develop spontaneously and even good students require explicit instruction. It was clear that the construction and teaching of critical thinking skills are critically determined by the specific content and tasks the teacher uses. From this research, we can conclude that the use of the infusion approach to teach critical thinking skills while also teaching conventional probability and statistics content was effective with students acquiring critical thinking skills they also valued.

References

- Ennis, R. H. (1991). Critical thinking: A streamlined conception. *Teaching Philosophy*, 14(1), 5-25.
- Lieberman, V., & Tversky, A. (2001). *Critical thinking: Statistical considerations and intuitive judgments*. Tel Aviv: Open University. (In Hebrew)

LETTER AS OBJECT MISCONCEPTION IN JUNIOR SECONDARY SCHOOL ALGEBRA

Zarina Akhtar, Vicki Steinle

International Islamic University, Pakistan University of Melbourne, Australia

The *letter as object* misconception in algebra has been reported in the literature for about 30 years; for example, Rosnik (1981) and Clement (1982). MacGregor & Stacey (1997) and Chick (2009) noted the use of “fruit salad algebra” (that is, the letter *a* stands for the object apple and *b* for bananas) in several Australian textbooks. MacGregor and Stacey found that the students at one school were adversely affected by the use of a textbook which taught that letters can be used as abbreviated words and labels. Despite known problems, this discredited teaching strategy persists.

In this study we analyse students’ responses to a particular online assessment within the *smart test system*. This assessment system (www.smartvic.com/smart/index.htm) is designed to provide teachers with diagnostic information about their students’ understanding, as well as the presence of any misconceptions, so that they can prepare teaching to address student needs. The data for this study came from 850 Australian students from early secondary school, Years 7 to 9. These students completed three items which were designed to detect the *letter as object* misconception.

Not surprisingly, performance improved from Year 7 to Year 9 on each of the three test items. However, on average, only 43% of Year 9 students were correct on each item. We conclude that between 50% and 70% of Year 7 students bring the *letter as object* misconception to their learning of algebra and that about half of Year 8 and one quarter of Year 9 students in this sample have this misconception. The smart test system was designed to make the results of mathematics education research readily available to teachers. We expect that the information provided to teachers about their own students will increase teachers’ pedagogical content knowledge in the particular topic, so that they can provide teaching to remove such known misconceptions.

References

- Chick, H. (2009). Teaching the distributive law: Is fruit salad still on the menu? In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia*, 1, (121-128). Palmerston North, NZ: MERGA.
- Clement, J. (1982). Algebra word problem solution thought process underlying a common misconception. *Journal for Research in Mathematics Education*, 3(1), 16-30.
- MacGregor, M. & Stacey, K. (1997) Students’ understanding of algebraic notation: 11-16 *Educational Studies in Mathematics*, 33(1), 1-19.
- Rosnik, P. (1981). Some misconceptions concerning the concept of variable. Are you careful about defining your variables? *Mathematics Teacher*, 74(6), 418-420.

STUDY ON CULTIVATION OF CREATIVITY IN MATHEMATICS EDUCATION: OVERCOMING OF INHIBITORY FACTOR OF FLEXIBLE IDEA

Miyo Akita, Noboru Saito

Naruto University of Education, Japan; Rissho University of Education, Japan

The purpose of this study is to show how students can enhance mathematical creativity. Flexibility is an important factor for creative problem solving. Many researchers suggest that flexibility is an important factor for creative problem solving (Torrance, 1976; Haylock, 1987; Saito & Akita, 2000). However, in the lesson of the present school mathematics, students' flexible thinking is hardly used.

We focused on the relationship between the intelligibility of learning contents and exertion of mathematical creativity. We analysed students' tendency to act according to problem solving. The results revealed that many students memorize a solution without understanding the mathematical background (Akita & Saito, 2013). We found that the inhibitory factor of students' flexible idea in mathematical problem solving. We called it "Temporary plateau of thinking in problem solving".

In this study, We analysed the influence of fixation of a solution by using the problem of constructing a perpendicular bisector and proposed the problems which break out of the "Temporary plateau of thinking in problem solving". When students used that problem, they were able to consider various solutions than the usual problem. These results show that the teachers can make student overcome the temporary plateau of thinking in problem solving, if the teacher makes the problem for developing the flexible thinking and gives it to students. And students can understand mathematics more deeply by using that.

References

- Akita, M., Saito, N. (2013). *Teaching for Creative Thinking: Try to Improve the Level of Understanding about Mathematics*. The 6th East Asia Regional Conference on Mathematics Education (EARCOME6) Proceedings, Vol.3, 329-338.
- Haylock, D. W. (1987). Framework for assessing mathematical creativity in school children. *Educational Studies in Mathematics*, 18(1), 59-74.
- Saito, N., Akita, M. (2000). The Relation between the Creativity Test and the Creativity Attitude Test Scores in Mathematics Learning. *Journal of Japan Society of Mathematics Education*, Vol.6, 35-48.
- Torrance, E.P. (1976). *Creativity in the Classroom*, National Education Association Publication.

STRUCTURAL AND OPERATIONAL MATHEMATICS: AN INTEGRATED APPROACH IN E-LEARNING

Giovannina Albano
University of Salerno

In this paper we try to combine research results in Mathematics Education with the opportunities provided by technology in order to design and implement an e-learning course. The aim of the work is to investigate how to apply theoretical constructs, such as the dual nature of conceptions and the process of the knowledge construction (Sfard, 1991), into new teaching environment such as e-learning platforms.

The definition of procept has been exploited in order to represent the operational knowledge in a cognitive structure, based on a web metaphor and a hierarchical structure, where the web nodes (parent nodes) are the procepts, and the hierarchical nodes (child nodes) are the various procedures associated to each procept, connected by edges (Crowley & Tall, 1999). Such cognitive structure has guided the design of some WebMathematica learning activities (WM-LA), each of them consisting in more interactive and dynamic problems (PWM), that is step-by-step algorithm. According to Sfard's hierarchical three-phases path for knowledge acquisition, moving along the cognitive structure, the interaction of a student with a WM-LA and PWMs refer on one hand to the *interiorization* and on the other hand to the *condensation*. In fact interactions with PWMs allow the student to become skilled in performing procedures (e.g. Cramer, Gauss), until he is able to think at one idea with various aspects (e.g. solving linear system). Then the possibility of packing long sequences of operations into a block (procept), allows to define the granularity of a "step" of a PWM. The underpinning teaching idea refers to the procept's cognitive power to think about operational conceptions, fostering actually the students to starting off structural thinking.

As future work, we plan to include the WM-LA in the ACE Teaching Cycle, constituted by the three components: activities, class discussion and exercises (Asiala et al., 1996).

References

- Asiala, M., Brown, A., DeVries, D., Dubinsky, Ed, Mathews, D., Thomas, K. (1996). A Framework for Research and Curriculum Development in Undergraduate Mathematics Education. *Research In Collegiate Mathematics Education II, Issues in Mathematics Education* (CBMS), American Mathematical Society.
- Crowley, L. & Tall, D. (1999). The roles of cognitive units, connections and procedures in achieving goals in college algebra. *Proc. Of the 23rd Conference of PME*, 2, 225-232.
- Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics* 22, 1-36.

PARENT-CHILD INTERACTION IN MATHEMATICS LEARNING – INSIGHTS INTO MATH-EXPERIENCE-DAYS

Natascha Albersmann

University of Wuppertal, Germany

Parents assume an important role in their children's educational development and likewise in their mathematical development. The kind of influence parents can exert depends on the way parents attempt to support their children, as well as on the circumstances and manner of parent-child interactions in a mathematical situation (Hoover-Dempsey & Sandler, 1995). In the following a family math project is presented, which aims at integrating parents more actively in their children's mathematical education. In this paper the focus lies on the question, how parent-child interactions can be fostered by the participation in such a family-math-project.

The presented findings refer to the first of three workshops of the so called math-experience-days, in which a total number of 22 parent-child teams from a 5th grade of a German Gymnasium participated on a voluntary basis. Guiding for all workshops is the pedagogical principle of problem-based discovery learning (Bruner, 1961). Within the meaning of this approach, parents and children embark on a discovery journey and slip into the role of investigators, ready to explore unknown mathematical problems together as a team. Reflective journals are used by parents and children at the end of each workshop to report on their experiences. In order to encourage reflection, guiding questions are displayed in the journals, which at the same time constitute an a priori designed system of categories. Following the principles of qualitative content analysis (Mayring, 2005) the journal entries, especially those that contain hints on parent-child interaction, were examined, significant themes identified and categories were elaborated.

The findings show that such a family math project offers parents and their children the possibility to experience mathematics in a neutral, assessment-free setting. Away from achievement orientation or school pressure they can investigate mathematical problems at eye level and gain new feelings of competence. These results already give an insight in the potential of such a project, which will be investigated more closely during the upcoming workshops.

References

- Bruner, J. S. (1961). The act of discovery. *Harvard Educational Review*, 31 (1), 21–32.
- Hoover-Dempsey, K., & Sandler, H.M. (1995). Parental involvement in children's education: Why does it make a difference? *Teachers College Record*, 95, 310–331.
- Mayring, P. (2005). *Qualitative Inhaltsanalyse. Grundlagen und Techniken*. Weinheim: Deutscher Studien Verlag.

LEARNING OPPORTUNITIES IN MULTILINGUAL MATHEMATICS CLASSROOMS ACROSS TWO COUNTRIES

Annica Andersson, Eva Norén; Núria Planas
Stockholm University; Universitat Autònoma de Barcelona

There is an existing literature on mathematics education and language diversity based on the examination of data and findings across countries (e.g., Barwell & Setati, 2005). However, little data has paid attention to how learning opportunities for bilingual students are differently created in different political and pedagogical classroom contexts. We attempt to explore the uses of the students' languages in the learning of mathematics, especially for cases in which the home language is different from the language of instruction –Catalan in Catalonia, and Swedish in Sweden. Our common methods are associated with the qualitative and interpretive approach to research, mainly lesson observation and ethnographic interviewing.

Drawing on large studies that we have conducted with our teams in several multilingual classrooms of Catalonia and Sweden, we will offer examples in which bilingual students take up learning opportunities during the involvement in mathematical activities, sometimes despite actions by other students and/or the teacher, and even though the language of instruction is not their own. A key assumption is that all students may become agentive in either adopting or rejecting discursive positions on the role and use of languages and cultures in the mathematics classroom. Students whose language is not the language of instruction may come to privilege their own home culture and language at concrete moments of their learning.

From the analysis of several lessons, we conclude that through discourses that allow for negotiation of language and cultural practices, the multilingual mathematics classroom becomes permissive for students to experience more and different learning opportunities than usual. From the perspective of the research agenda in mathematics education, it can be inferred the importance of developing studies on culturally responsive pedagogies together with studies on language classroom practices.

Acknowledgements

Project EDU2012-31464, Spanish Ministry of Economy and Competitiveness, in collaboration with Stockholm University, Aalborg & Malmö University.

References

Barwell, R., & Setati, M. (2005). Multilingualism in mathematics education: A conversation between the north and the south. *For the Learning of Mathematics*, 25(1), 20-23.

A STUDY ON CONCEPT FORMATION OF GEOMETRIC FIGURES AT PRIMARY LEVEL IN METRO MANILA: FOCUSING ON COMPONENTS OF FIGURES

Mitsue Arai

Graduate School for International Development and Cooperation,
Hiroshima University, Japan

Components are considered as important teaching contents in school education. Especially in the process of verbalization in Level 2 (Students recognize shapes by their properties) of Van Hiele's theory. The aim of this study is to clarify how students modify their experience based concepts and obtain scientific concepts of geometric figures focusing on components from the cognitive aspects. In this context, Nunokawa (1992) stated that ways of recognition of figures changed from recognizing a figure as an indifferentiated image to recognizing by a set of characteristics. In this paper, ways of students' recognition of figures are described, such as how they use components, why they did not use them.

A survey questionnaire composed of classification and identification with their reasons was administered to students in grade 3 (N=81) and 4 (N=109) in two schools in Metro Manila, Republic of the Philippines. The researcher classified the descriptions of reasons in several categories: identifying by name, by components, by impression, which were based on Clement's ideas (1999). Then in the case of identification, a group of the students using "components" in the case of classification was analysed by comparing with other groups.

It was revealed that the students using "name" in classification had a poor extension, for example in the case of choosing triangles most of them recognized a sector as a triangle. Meanwhile the students using "components" had a view point of identification such as number of sides and vertices, however, they could not utilize the components such as straight or curved lines in unfamiliar shape's case. Thus their concepts of geometric figures were limited yet, just simple knowledge such as name and number of sides, which exist separately without systemic organization. These kinds of knowledge disturb students to think flexibly. In order to examine the status of this, intensional understanding and extensional understanding were considered.

References

- Clement, D. H. et al. (1999). Young Children's Concepts of Shape. *Journal for Research in Mathematics Education*, 21(2), 132-144.
- Nunokawa, K. (2010). A Consideration on van Hiele Levels from the perspective of Recognizing Figures. *Bulletin of Institute of Education in Tsukuba University*, 16(2), 139-152. (In Japanese)

LINEARITY PREPONDERANCE ON 7TH GRADE STUDENTS' SOLUTION STRATEGIES IN LENGTH-AREA PROBLEMS

Rukiye Ayan, Mine İşiksal-Bostan
Middle East Technical University, Turkey

A significant number of relationships in daily life involve non-linear relationships in nature. Yet, experiencing mathematical problems related to linear relationships in school curriculum, students might develop a severe habit of looking for a linear relationship between any two quantities (Freudenthal, 1983). In the literature, this situation is referred as illusion of linearity (De Bock et. al., 1998). The problems requiring examination of the relationship among the length, perimeter, area and volume of reduced or enlarged figures might give clues about students' solution strategies for linear and non-linear processes. Hence, the purpose of this research study is to examine 7th grade students' solution strategies for problems including non-linear relationships in the case of length and area of geometrical shapes. The sample of the study consisted of fifteen 7th grade students selected from a private middle school in the capital city of Turkey. A problem solving achievement test including 11 essay type problems adapted from the literature was implemented. Yet, within the scope of this study the answers on the two items related to the relationship between the length and area of geometrical shapes were analyzed qualitatively. Results showed that most of the students applied linear solution strategies for both problems. However, there was some remarkable number of students who gave correct answers for the problem requiring participants to find the area of a rectangular region after being doubled. Nevertheless, students who gave a correct answer for this problem failed to establish the quadratic relation between the length and area of the geometrical shape. The solution strategy used by these students was to find the side length of the doubled region and then to calculate the area of the enlarged figure. On the other hand, when the figure was not rectangular but irregular none of the students was able to find the area of the doubled region. It is suggested that problems including non-linear relationships must be given place in the mathematics curriculum beginning from the early years of the elementary school.

References

- De Bock D., Verschaffel L. & Janssens D. (1998). The Predominance of the Linear Model in Secondary School Students' Solutions of Word Problems Involving Length and Area of Similar Plane Figures. *Educational Studies in Mathematics*, 35, 65–83.
- Freudenthal, H. (1983). Didactical phenomenology of mathematical structures. Dordrecht: Reidel.

PROSPECTIVE MATHEMATICS TEACHERS INTERACTING IN A CHAT CONCERNING THE DEFINITION OF POLYHEDRON

Marcelo A. Bairral

Federal Rural University of Rio de Janeiro, Brazil

In mathematics education, research studies that analyze the construction of geometric concepts through interactions on chat are scarce. This study focuses on prospective mathematics teachers discussing about the definition of polyhedrons. The report is part of an ongoing research project¹ that analyses interactions in virtual learning environments (VLE).

VLE are mediated by different technologies and artifacts. In VLE individuals can exchange ideas and develop their mathematics concepts, without hierarchy or domination from one participant on another. In our VLE one way to exchange geometrical concepts is through the use of writing. Writing about mathematical ideas allows individuals to review, at different moments, their understanding concerning some concepts. In this study we focus on written online interaction regarding the definition of polyhedrons (Tanguay & Grenier, 2010).

Our VLE (<http://www.gepeticem.ufrj.br/cursos.php>) is structured around a vision of work that breaks with the axiomatic approach and the memorization of formulae in geometry classes. In this report the analytical process was focused on interactions in chat. The transcription of writing chat analyzed here comprised 343 lines. We used the following procedures for data reduction: chat transcription (a file provided by the platform itself), numbering (in lines) of interactions, removal of lines which contained no ideas related to the concepts we were intent on focusing, re-reading interactions and organizations in turns.

Participants showed they deepened conceptual aspects in three scopes: one associated with geometric solids in general; another one with aspects focused on the elements (faces, vertices and edges), and a third one, focusing on 3D. These approaches are not sequential, hierarchical, nor individual. They arose from the discussion and they developed and deepened with the constituted online group. In order to identify those scopes, our analysis switched from a global look on the interactions (on the motivations of participants, the expression of their curiosity, their doubts, etc., to a focus where we tried to highlight the mathematical ideas that were most explicit in their interactions.

Reference

Tanguay, D., Grenier, D. (2010). Experimentation and proof in a solid geometry teaching situation. *For the Learning of Mathematics*, 30(3), 36-42.

¹ Research supported by CNPq and Faperj.

STUDENTS' CONCEPT IMAGES ABOUT LIMIT OF A FUNCTION AT INFINITY

Başak Barak, Tangül Uygur Kabael
Anadolu University

Limit is one of the fundamental concepts that a lot of concepts like continuity, derivative, integration etc. are firmly connected about it (Cornu, 1991). In the literature, many sources of difficulties about the concept of limit emerged and one of them is about concept of infinity. For instance, even if it is taught that infinity is not a real number, students may consider it such as a real number (Tall, 1992). This study is a part of a larger study aimed to determine the students' concept images and concept definitions of limits of functions. Focus of this study was students' concept images about limit of a function at infinity. Data was obtained from clinical interviews with eleven pre-service middle school mathematics teachers taking Analysis I course and analyzed qualitatively by using content analysis technique in terms of concept images (Tall and Vinner, 1981). Findings of the study revealed that powerful sources of students' difficulty with limit of functions at infinity were students' perception way of infinity and their concept images based on right and left hand limit. Most of the students considered the concept of infinity as a real number. Since these students' concept images about limits of functions included taking right and left hand limit, they tended to approach infinity from right and left hand. They approached to positive infinity in order to find right hand limit and they approached to negative infinity in order to find left hand limit. On the other hand, some students tried to find limit by taking image of infinity under the function while some of others found the limit of a function at infinity by approaching the points on graph (Przenioslo, 2004).

References

- Cornu, B. (1991). Limits. In D. Tall (Ed.) *Advanced Mathematical Thinking*. Dordrecht, The Netherlands: Kluwer Academic Publishers, 153-166.
- Przenioslo, M. (2004). Images of the limit of function formed in the course of mathematical studies at the university. *Educational Studies in Mathematics*, 55, 103-132.
- Tall, D. O. & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12(2), 151-169.
- Tall, D. O. (1992). The Transition to Advanced Mathematical Thinking: Functions, Limits, Infinity, and Proof, in Grouws D.A. (ed.) *Handbook of Research on Mathematics Teaching and Learning*, Macmillan, New York, 495– 511.

CONDITIONS FOR A SUCCESSFUL USE OF COMPUTERALGEBRA IN MATHEMATICS TEACHING

Bärbel Barzel, Raja Herold
University of Education Freiburg

A lot of studies show the potential of integrating Computeralgebrasystems (CAS) in teaching and learning mathematics (Heid & Blume 2008; Zbiek et al. 2007). Nevertheless, schools and school administration have an ongoing debate about chances and burdens about implementing CAS in the classroom. A lot of countries still do not involve CAS in their examinations and mathematics classrooms.

CASE-X was a meta-study with the aim of investigating conditions that ensure a successful use of CAS in mathematics classrooms. For this reason, we have reviewed the whole research field that dealt with the integration of CAS in learning, teaching and assessing. Due to the focus of the project, criteria for selection were elaborated. Only current results (last 10 years) concerning the implementation of CAS and conditions to ensure this implementation were considered. Eventually, about 275 publications have been included in the review. The results of the meta-study show that CAS has enormous advantages and benefits. CAS can have an effect as catalyst towards a student-centred and understanding-oriented teaching. Moreover, students' competences can be promoted, such as:

- The acquisition of conceptual knowledge especially in the field of algebra can be fostered via CAS.
- Procedural knowledge and skills can be acquired in a CAS environment and do not necessarily need to loose importance.
- Mathematical language use in written communication can be stimulated.

To improve these competences and to ensure a positive development, certain institutional conditions are essential, for instance: CAS should be obligatory in curricula and assessment; final examination should be divided into two parts – with and without technology ; a network of information for parents and students as well as a good system of ongoing teachers professional development.

CASE-X is initiated and funded by the ministry of education, Thuringia.

References

- Heid, M. K.; Blume, G. W. (2008): *Research on Technology and the Teaching and Learning of Mathematics: Syntheses, Cases and Perspectives. Research Syntheses*. Charlotte, North Carolina: National Council of Teachers of Mathematics (NCTM).
- Zbiek, R. et al. (2007): Research on Technology in Mathematics Education – A Perspective of Constructs. In: F. Lester (ed.), *Second handbook of research on mathematics teaching and learning*, 1169-1207. Charlotte, NC: Information Age.

INVESTIGATING PROBLEM POSING PROCESSES OF PRESERVICE PRIMARY MATHEMATICS TEACHERS

Burcu Nur Baştürk, Ayşe Simge Ergin, Elif Türnüklü
Dokuz Eylül University, Türkiye

Mathematical problem posing is one of the important abilities of teaching mathematics. Many researchers have approved that the ability to pose meaningful problems had an equally important role as the ability to solve them. Mathematical problem posing is defined as the process by which, on the basis of mathematical experience, students construct meaningful mathematical problems (Ellerton and Stoyanova, 1996).

While dealing with the studies of problem posing, generally young children and university students are observed. Although a few studies examined university students, there is little research studying with preservice teachers. This study is aimed to report how the preservice teachers pose mathematical problems and their thoughts about their own problem posing process. So, following Koichu and Kontorovich (2012) the Billiard Task (adapted from Silver et al., 1996) and a semi-structured task from Leung's research are used in order to collect data. The tasks are given to 53 pre-service primary mathematics teachers who are studying in an education faculty in Türkiye. Additionally the interview is also used with the chosen four of subjects in order for deeper understanding of their thoughts. All the interviews are taped. All the data are analysed by using Leung's (2012) classifying strategy.

As a result, pre-service primary mathematics teachers' thoughts about problem posing and their problem posing processes are classified and some challenges about problem posing are found. One of interesting results is the differences between their thoughts about the aim of the problem and what they pose to achieve that aim.

References

- Kontorovich, I. & Koichu, B. (2012). Dissecting success stories on mathematical problem posing: a case of the Billiard Task. *Educational Studies in Mathematics*. doi: 10.1007/s10649-012-9431-9
- Leung, S. S. (2012). Teachers implementing mathematical problem posing in the classroom: challenges and strategies. *Educational Studies in Mathematics*. doi: 10.1007/s10649-012-9436-4
- Silver, E.A. & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education*, 27(5), 521-539.
- Stoyanova, E. & Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. In P. Clarkson (Ed.), *Technology in Mathematics Education* (pp. 518-525). Melbourne: Mathematics Education Research Group of Australasia.

THE DYNAMICS ASSOCIATED WITH CLASSROOM IMPLEMENTATION OF MATHEMATICAL TASKS

Burcu Nur Baştürk, Sibel Yeşildere İmre
Dokuz Eylül University, Türkiye

Task design has received increased attention in recent years (Swan, 2007; Breen & O'shea, 2010). However teachers face the challenge of bridging the gap between designing the task and classroom implementation of the task. The ability to implement mathematical tasks had an equally important role as the ability to design them (Stylianides & Stylianides, 2008, Henningsen & Stein, 1997).

The present study reports on classroom implementation of mathematical tasks. It is aimed to determine the factors which affect the fidelity of task implementation and to reveal the indispensable features of an implementation process should include. The research is conducted with two primary mathematics teachers. Teachers' lessons are videotaped, and interviews were conducted before and after their lessons. This study is a qualitative research that used video analysis from transcribing video and audio tape of two primary mathematics teachers. Data were analysed based on Stylianides & Stylianides (2008)'s analytical framework and Henningsen & Stein (1997)'s conceptual framework.

The results indicated that an understanding of pedagogical value of a task and familiarity with the content knowledge are necessary elements of implementation process. It is also found that the ability to shift the students' attention regarding to the purpose of a task is an essential part of classroom implementation.

References

- Breen, S. ve O'shea, A. (2010). Mathematical Thinking and Task Design. *Irish Mathematics Society Bulletin*.66: 39-49
- Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28, 524-549.
- Stylianides, G. J., & Stylianides, A. J. (2008). Studying the classroom implementation of tasks: High-level mathematical tasks embedded in 'real life' contexts. *Teaching and Teacher Education* 24 (2008) 859-875.
- Swan, M. (2007). The impact of the task-based professional development on teachers' practices and beliefs: A design research study. *J Math Teacher Educ*.10: 217-237.

TELPS – ANALYSING AND SUPPORTING MATHEMATICS TEACHERS' PCK

Isabell Bausch, Regina Bruder, Anne Prescott
TU Darmstadt and University of Technology Sydney

Mathematics Teachers' Pedagogical Content Knowledge (PCK) is necessary to plan and design a mathematics lesson. The Teacher Education Lesson Plan Survey (TELPS) aims to explore this knowledge during the teacher education program in order to develop feedback that supports mathematics teachers' development of PCK.

TELPS is a Repertory-Grid-Survey (Kelly, 1955) aiming to explore mathematics teachers' development of PCK (Shulman, 1986) during their teacher education program. TELPS is a project of the University of Technology Sydney (UTS) and the Technische Universität Darmstadt (TUD), designed as a panel-study and implemented in the teacher education program of both universities.

The core of TELPS is the comparison of two mathematics lesson plans. The results of this comparison are documented in a grid forming the database for the analysis of the PCK used by the teachers. Based on the grids a quantitative evaluation system was developed to code the data (Bausch, Bruder & Prescott, 2011).

We analysed 526 pre-service teachers' grids. At both universities, the beginners focus on aspects of motivation in their comparison of the lesson plans. The final semester pre-service teachers have a different view of the lesson plans. At TUD, the last semester pre-service teachers name aspects of initial situation, didactic content analysis, goals, internal differentiation, and tasks. In the last semester at UTS, aspects of the lesson's goals are named more often.

The presentation will show how the results of the participants are used in a web tool to give automatic feedback that supports mathematics teachers' development of PCK. This feedback includes the results of all participants and compares the individual results with the results of the other participants, supporting the self-reflection of pre-service teachers' learning process. This feedback could also be a part of a learning portfolio.

References

- Bausch, I., Bruder, R., & Prescott, A. (2011). Personal constructs of planning mathematics lessons. In Ubuz, B. (Ed.) *Proc. of the 35th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 2, pp.113–120). Ankara, Turkey: PME.
- Kelly, G. A. (1955). *The psychology of personal constructs*. New York: Norton.
- Shulman, L.S. (1986). Those Who Understand: Knowledge Growth in Teaching. In: *Educational Researcher*, 15, S. 4-14.

TEACHERS' REFLECTIONS UPON CONSTRUCTION OF THEIR OWN MADE-TESTS

Semir Becevic

Mathematical Sciences, Chalmers and University of Gothenburg

The case of the new syllabus at upper secondary schools in Sweden calls for a deeper consideration directed to teacher made tests. Besides of great influence that teacher made tests might generally have on educational and assessment practices, the new syllabus brings a strong and obvious focus on couple of mathematical abilities. With the roots in the internationally based movement towards developing higher-order of mathematical thinking (Kilpatrick, Swafford, & Findell, 2001) an interesting attempt to connection of the mathematical abilities and the new marks is made by marking instructions. With the emphasis on sporadic research insight in teacher made tests, especially regarding test production and evaluation from the constructors' points of view, the aim of this study becomes illuminating teacher motives and reflections upon test constructions and scoring practices. The research question is:

- How do teachers reflect upon and reason about their own test constructions and marking practices in alignment with the new ability request?

Seven upper secondary school teachers have in individual, semi-structured interviews reflected upon their own test construction practices. Some of the teachers' previously conducted tests had in advance been collected, investigated and used as foundation to the interviews. The methodological approach of the Grounded theory (Glaser & Straus, 1967) is applied. By three types of coding, this inductive method has subsequently contributed to generating a substantial theory about teachers' reasons upon test constructions and evaluations. Within the theory two phase and three labels are detected. There are constructional and marking phase. The first label considers only mathematics at different levels. The second label regards also present of abilities besides of the mathematics while the third one takes into account both mathematics and abilities at different qualitative levels. The result shows a movement through the space influenced both by the mathematics and by the abilities at different qualitative levels. This challenging interplay in test constructions calls for farther research.

References

- Glaser, G. B., & Straus, L. A. (1967). *The discovery of grounded theory: strategies for qualitative research* New York: Aldine de Gruyter, cop.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding it up: Helping children learn mathematics Retrieved 12.11.2012., from <http://www.nap.edu/catalog/9822.html>

IMPLICATIONS OF THE DIDACTIC CONTRACT FOR TERTIARY MATHEMATICS TEACHING PRACTICES

Mary Beisiegel
Oregon State University

The body of research concerning how future mathematicians are prepared for teaching has grown significantly in the past decade. This research has stemmed from concerns about tertiary mathematics teaching practices and their impact on undergraduate learners (Seymour & Hewitt, 1997). However, despite new preparation programs for tertiary mathematics teachers, many future mathematicians have maintained problematic teaching practices. In response, the study described here was an investigation of mathematics graduate students' (MGSs; future mathematicians) experiences and the implications for tertiary teaching practices. Interviews with MGSs from a large, doctorate-granting university were conducted over the course of an academic year. The interviews were transcribed verbatim and analysed through a hermeneutic lens coupled with thematic analysis (Braun & Clarke, 2006). Some findings are presented here.

The didactic contract has been described as the implicit set of rules that establish how teachers and students interact (Brousseau, 1997), where often students listen while the teacher presents mathematics. Encountering the contract as learners, the MGSs struggled with the teaching they encountered, describing their roles in classrooms as passive and disconnected from mathematics. Despite this struggle, however, the MGSs spoke ardently of how they would adhere to this same form of teaching once they became professors. The cognitive dissonance experienced by means of the didactic contract had implications for their practices in the following ways. First, they began to see teaching as a process of replication – of text and of practice, of presenting material with little, if any, interaction. Beyond the sense of replication came feelings of resignation. While each of the MGSs stated their aspirations to provide a different experience to students, they also felt that they must resign their aspirations for their teaching practices. Finally, in attending to the didactic contract, there was a cost. Specifically, the MGSs developed a sense of despondence, of hopelessness about what they could do as tertiary teachers of mathematics.

References

- Braun, V., & Clarke, V. (2006). Thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77-101.
- Brousseau, G. (1997). *Theory of didactical situations in mathematics* (edited and translated by N. Balacheff, M. Cooper, R. Sutherland & V. Warfield). Dordrecht, Netherlands: Kluwer.
- Seymour, E., & Hewitt, N. (1997). *Talking about leaving: Why undergraduates leave the sciences*. Boulder, CO: Westview Press.

INDIVIDUAL DIFFERENCES IN FRACTION KNOWLEDGE AND THEIR RELATION TO THE INDIVIDUAL'S LEARNING APPROACH TO MATHEMATICS

Maria Bempeni, Xenia Vamvakoussi
University of Athens, University of Ioannina

This study aims at contributing to the discussion on individual differences in students' conceptual and procedural fraction knowledge and on the possible reasons underlying such differences (e.g., Hallett et al., 2012). Seven 9th graders were asked to solve 19 fraction tasks during an individual interview. We found individual differences in the way students combined conceptual and procedural knowledge to deal with the tasks. Specifically, three students displayed procedural fluency but hardly any conceptual understanding (Procedural profile); one student displayed strong conceptual understanding, but practically no procedural fluency (Conceptual Profile); and three students appeared to combine conceptual understanding with procedural fluency (Conceptual-Procedural Profile). Similar to Stathopoulou & Vosniadou (2007), we hypothesized that the students with strong conceptual understanding adopted a deep approach to mathematics learning (associated with the intention to understand), whereas the students with poor conceptual understanding adopted a superficial approach (associated with the intention to reproduce). We selected three representatives of the distinct profiles and interviewed them in depth about their learning approaches to mathematics. The transcripts were initially coded along three categories used by Stathopoulou and Vosniadou (i.e., Goals, Study Strategy Use, and Awareness of Understanding), while two new categories emerged, namely Preferred Tasks and Motivation. Consistent with our hypothesis, the representatives of the Conceptual and the Conceptual-Procedural Profiles were very similar across all categories, whereas they differed from the representative of the Procedural Profile across all categories. These findings indicate that individual differences in conceptual and procedural fraction knowledge may be associated with the individual's approach (deep/surface) to mathematics learning.

References

- Hallett, D., Nunes, T., Bryant, P., & Thorpe, C. M. (2012). Individual differences in conceptual and procedural fraction understanding: The role of abilities and school experience. *Journal of Experimental Child Psychology*, 113, 469-486.
- Stathopoulou, C., & Vosniadou, S. (2007). Conceptual change in physics and physics-related epistemological beliefs: A relationship under scrutiny. In S. Vosniadou, A. Baltas, & X. Vamvakoussi (Eds.), *Reframing the conceptual change approach in learning and instruction* (pp. 145-164). Oxford, UK: Elsevier.

MATHEMATICS STUDENTS USING RESOURCES

Margot Berger

University of Witwatersrand

I examine the ways in which in-service high school mathematics teachers shape and are shaped by resources (textbooks, class discussions, tasks, GeoGebra) as they participate as learners in pre-calculus and calculus courses. These courses are part of a post-graduate education programme which is geared towards extending and deepening mathematics content knowledge of teachers (the students). The courses are designed around modified principles of the flipped classroom (Bergmann & Sams, 2012). Students (that is, the teachers) are expected to self-study specific sections of work from a prescribed mathematics **textbook** prior to class time. Class time is devoted to a **discussion** of the self-study material followed by **GeoGebra** and/or pencil-and-paper **tasks**. Each course consists of eleven weekly three-hour sessions. There were sixteen students in the courses.

The framework for this research is based on the theory of instrumental genesis (Drijvers & Trouche, 2008). Its questions are: How do the participating teachers use the resources for their own learning? Which resources? What informs this usage?

Data were collected from structured ‘reflection sheets’ wherein students reflected on how they used the different resources for learning. These sheets were handed in weekly. To gain further insight into students’ use of the multiple resources, six students were interviewed at the end of the courses. Classroom discussions were audio-taped.

I draw on data from the interviews and reflection sheets to show how two students use the resources. Data was analysed using grounded theory and content analysis. Thabo (with strong mathematical background) focussed his textbook reading on definitions and graphical representations; he used class discussions to clarify and consolidate his mathematical ideas and he used various tasks to make connections between mathematical ideas. In contrast, Jane (for whom many mathematical concepts were new) relied heavily on worked examples in the text. She avoided definitions and was unable to see links between concepts in the text unless they were made explicit. During discussions her focus was largely on improving her pedagogical content knowledge. She used GeoGebra primarily as a tool for verification. I hypothesize that students’ use of resources relates to specific aspects of their different mathematical backgrounds.

References

- Bergmann, J. & Sams, A. (2012). *Flip your classroom*. USA: ISTE; ASCD.
- Drijvers, P. & Trouchè, L. (2008). From artifacts to instruments: A theoretical framework behind the orchestra metaphor. In G. W. Blume & M. K. Heid (Eds.), *Research on Technology and the Teaching and Learning of Mathematics*, Vol. 1. 363-391. Charlotte, NC: Information Age.

WHAT MATHEMATICAL COMPETENCES CAN BE LEARNED FROM WEB-BASED LEARNING RESOURCES?

Tomas Bergqvist

Umeå Mathematics Education Research Centre, Umeå University.

In a large study (Bergqvist et al., 2010), the teaching in 200 Swedish lessons was analysed regarding students' opportunities to develop mathematical competencies. The results show an emphasis on rote learning and procedural handling. This study presents a comparison between the classrooms in the 2010 study and what is offered by two commonly used web-based learning resources. The research questions are:

- What competencies can students develop using the web-based learning resources?
- Are there any differences from what they are meeting in the classroom?

The two resources analysed are *Khan Academy*, a well-known international resource, and *Matteboken.se*, a resource for students where videos of solutions to Swedish National test tasks are presented. 15 task presentations from the Swedish resource and 15 presentations from *Khan Academy* was analysed using MCRF (Lithner et al., 2010), a framework for analysis of empirical data concerning mathematical competencies.

The results indicate that there is a small difference between the two chosen web-based resources, where *Matteboken.se* to a large extent focus on procedures in the same way as in Swedish classrooms. In the videos at *Matteboken.se*, reasons or arguments for how a task should be solved were totally absent. All presentations concerned how to carry out a solution, without discussions of possible options. Videos at *Khan Academy* offer more information concerning how things are done, with explanations of what different expressions mean and examples of multiple solutions. However, mathematical competencies, for example mathematical reasoning and problem solving are almost as rare as in the Swedish resource. The main conclusion from the study is that the possibilities to develop mathematical competencies using web-based learning resources are as limited as in Swedish classrooms.

References

- Lithner, J., Bergqvist, E., Bergqvist, T., Boesen, J., Palm, T., Palmberg, B. (2010): Mathematical Competencies – a research framework. In Bergsten Jablonka & Wedege (Eds.), *Mathematics and mathematics education: Cultural and social dimensions*. Proceedings of MADIF 7, the Seventh Mathematics Education Research Seminar, Stockholm, January 26-27, 2010. (pp 157-167). Linköping, Sweden: SMDF.
- Bergqvist, E., Bergqvist, T., Boesen, J., Helenius, O., Lithner, J., Palm, T., Palmberg, B. (2010): *Matematikutbildningens mål och undervisningens ändamålsenlighet – Gymnasieskolan hösten 2009*. En rapport från Skolinspektionen. Publicerad av NCM (Nationellt Centrum för Matematikutbildning).

A TEACHER-TRAINING-STUDY FOR IN-SERVICE TEACHERS: FORMATIVE ASSESSMENT IN COMPETENCY-ORIENTED MATHEMATICS

Michael Besser, Dominik Leiss

University of Kassel and University of Lueneburg, University of Lueneburg

Assessing and reporting students' performance regularly without giving marks is a central part of formative assessment and is expected to influence students' learning in a positive way (Hattie & Timperley, 2007). Furthermore it is generally supposed that teachers' knowledge influences the quality of teaching and students' learning (Baumert et al., 2010). Based on these ideas a teacher-training-study (as part of the research project Co CA) has been conducted which aims at answering the following questions: (1) Is it possible to conduct and evaluate a teacher training that fosters teachers' pedagogical content knowledge (PCK) and general pedagogical knowledge (PK) about formative assessment in competency-oriented mathematics? (2) Does a successful teacher training have any influences on the teachers' way of teaching respectively the quality of teaching?

Trying to answer these questions, from February 2013 to May 2013 25 mathematics teachers have been trained all in all 6 days to implement central ideas of formative assessment in every-day teaching of competency-oriented mathematics (experimental group A). In contrast, another 25 mathematics teachers have been trained in general ideas about competency-oriented mathematics (experimental group B). For being able to compare these two groups – that is for being able to analyse the impact of the teacher training on teachers' knowledge and on the quality of teaching – , pretest and posttest on teachers' PCK and PK have been developed and used to evaluate the teacher trainings. Furthermore questionnaires asking for the quality of teaching have been given to the teachers' students. First results of the pretest hint at a lack of knowledge of nearly all participating teachers concerning formative assessment in competency oriented mathematics. Therefore it is expected that experimental group A outperforms experimental group B in the posttest as well as in the quality of teaching. Overall it is intended to point out the importance of teacher training for in-service teachers on teachers' knowledge and the teaching of mathematics.

References

- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y.-M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180.
- Hattie, J. & Timperley, H. (2007). The power of feedback. *Review of Educational Research*, 77(1), 81-112.

SITUATIONAL FIXATIONS IN THE USE OF DECIMAL FRACTIONS

Angelika Bikner-Ahsbahs
Bremen University

The development of the concept of decimal fractions often is accompanied by constraints such as misconceptions, epistemological obstacles, and intuitive rules which may cause mistakes in a consistent way. However, new intervention studies show that the students' success of overcoming mistakes often is not consistent nor are the kinds of mistakes which we encounter. These kinds of mistakes depend on situational conditions on which students are fixed. "Zero means nothing" is such a situational fixation to which occasionally learners are tied leading to false interpretations of decimal fractions if zero is a digit number. Situational fixations describe situational constraints which may explain why actions with decimal fractions cannot be expanded and mistakes cannot be overcome. In this short oral, an empirical case study is presented that aims at answering two questions: On which typical situational conditions are the students fixed when building the concept of decimal fractions? How do these situational fixations constrain the use of decimal fractions?

The investigation of the two questions is conducted within a frame of networking (Bikner-Ahsbahs & Prediger, 2010) two theories, the theory of Abstraction in Context (Dreyfus, Hershkowitz, & Schwarz 2009) and the theory of object relations (Oerter, 1982). *Situational fixations* are object relations indicated by mistakes or by recognizing, building-with and other actions that are tied to the contextual conditions in the situation. Four lessons of support in the use of decimal fractions are conducted, videotaped, transcribed and analyzed. Based on the first three lessons, the methodology of ideal type construction is used to answer the first question developing types of situational fixations on decimal fractions. The results are installed to answer the second question by analysing the fourth lesson: Situational fixations act as filters letting pass specific actions; if they overlap, the students' potential to act decreases.

References

- Bikner-Ahsbahs, A., & Prediger, S. (2010). Networking of Theories—An Approach for Exploiting the Diversity of Theoretical Approaches. In B. Sriraman, & L. English (Eds.), *Theories of mathematics education: seeking new frontiers, Advances in Mathematics Education series, Vol. 1* (pp. 479-512). New York: Springer
- Dreyfus, T., Hershkowitz, R., & Schwarz, B. (2001). Abstraction in Context II: The case of peer interaction. *Cognitive Science Quarterly*, 1(3/4), 307-368.
- Oerter, R. (1982). Interaktion als Individuum-Umwelt-Bezug. In E. D. Lantermann (Ed.), *Wechselwirkung. Psychologische Analysen der Mensch-Umwelt-Beziehung* (pp. 101-127). Göttingen: Verlag für Psychologie.

VIDEO RESOURCES FOR MATHEMATICS TEACHER - LEARNING DURING PEER ASSESSMENT TASKS

Yaniv Biton
Shaanan College

The idea of students' involvement in the assessment process has recently taken a prominent place in education. Research suggests that peer assessment, which exposes students to deciding on the assessment criteria and grading the other students' works, is an effective teaching and learning strategy (Kollar & Fischer, 2010). The goal of our research was to follow the students' process of learning as emanating from peer-assessment, but it was difficult to ignore the process of learning undergone by the teacher during the peer-assessment relating to our research. The present research focuses on opportunities for learning by the teacher, following the viewing of the mathematical discourse carried out by the students during their search for criteria for assessing the task.

Six 90-minute peer-assessment activities in two classes were conducted (i.e., each class worked on three mathematical problems). The students' collaborative work was recorded using three video cameras and four audio-taping devices placed at different tables. The video recording was necessary because it enabled us to record the conversations and simultaneously observe the students' writings. All the written materials (the solution of the task and the evaluation pages) were collected at the end of the activity.

It was found that the peer-assessment activities instigated meaningful interactions, both among the evaluators and among the evaluators and evaluatees. In particular, the students negotiated the evaluation criteria, the mathematical validity of the solutions under evaluation, and details of the grading process.

The present research indicates that not every suggestion ends in mutual agreement. These situations, in which the students disagree, partly or in whole, with the suggested criterion, have created situations of imbalance-balance during the formulation of the criterion. The main characteristic of these situations was that of uncertainty regarding the need for a certain criterion, thus impeding the continuation of the process. This was an additional challenge with which the evaluators had to cope. These situations also afforded the teacher an opportunity to evaluate the students' difficulties and derive conclusions about what was missing in her teaching, thus enabling her to learn from the peer-assessment activity and improving her future teaching.

Reference

Kollar, I., & Fischer, F. (2010). Commentary: Peer assessment as collaborative learning: a cognitive perspective. *Learning and Instruction*, 20, 344-348.

A QUESTIONNAIRE TO SURVEY OPPORTUNITIES TO LEARN IN PRIMARY SCHOOL MATHEMATICS CLASSROOMS

Katrin Bochnik, Stefan Ufer
University of Munich, Germany

National survey studies repeatedly report significant differences in mathematical achievement between students from German and non-German speaking homes already at the beginning of primary school. Thereby, disadvantages concerning knowledge of mathematical concepts can be explained by instructional language proficiency (Heinze et al., 2009). Adequate instructional language skills are required as a basis for the construction of mathematical knowledge during instruction (Gorgorió & Planas, 2001), and represent a prerequisite for sustainable opportunities to learn (OTL). Here, we consider in particular the opportunity to learn by following and participating in mathematical instruction and engaging in mathematical practices.

Since data on the role of OTL in the development of mathematics skills in children from non-German speaking homes is rare, the aim of the present pilot study was to develop an instrument to identify language-related differences in elementary school students' reported OTL. Each item of the questionnaire represents one mathematical practice or communication situation. Students report if the described situation usually occurs in their classroom (perception), and if they participate in it (participation). While the perception items were included mainly to acquaint students with the description of the situations, our main focus was on the participation items. We developed three scales of items, corresponding to communicative (e.g. explaining solutions), cognitive (e.g. finding a different solution), and receptive participation (e.g. understanding an explanation) in OTL. First results of an associated pilot study with $N = 95$ third graders showed medium reliabilities for the theoretical scales (Cronbach's α : 0.48-0.59), indicating a more complex empirical structure than the initial theoretical constructs. While t-tests did not yield significant group differences of reported OTL, response frequencies suggest first tendencies, in particular towards a lower cognitive participation of children from non-German speaking homes.

A deeper analysis of the factorial structure of the questionnaire and its optimization will be the next step before using the instrument in a larger cross-sectional study.

References

- Gorgorió, N. & Planas, N. (2001). Teaching mathematics in multilingual classrooms. *Educational Studies in Mathematics*, 47, 7–33.
- Heinze, A., Reiss, K., Rudolph-Albert, F., Herwartz-Emden, L., & Braun, C. (2009). The development of mathematical competence of migrant children in german primary schools. In M. Tzekaki (Ed.), *Proc. 33rd Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 3, pp. 145-152). Thessaloniki, Greece: PME.

SITUATED LEARNING TRANSFER OF ALGEBRAIC SYNTAX: LINEAR EQUATIONS AND VIRTUAL BALANCE

Maricela Bonilla, Teresa Rojano
Cinvestav-IPN, México

We analyze the situated learning transfer processes in the case of teaching algebraic syntax for resolution of linear equations using a concrete, virtual and dynamic balance model¹ (see previous outcomes in Rojano & Martínez, 2009). We resort to the theoretical elements proposed by Greeno, Smith & Moore (1993) who emphasize the importance of affordances (qualities of the objects or settings that enable an individual to carry out an action) and suggest that transfer depends on a capacity to perceive the *affordances* for practice that are present in a new situation. From this perspective we analyze how the *affordances* that are present in the balance model are recognized and transferred by the subjects to algebraic syntax.

The participant (pre-algebraic) students engaged in a four sessions work in which they passed from manipulating objects in the virtual model to choose the operation that had to be performed in the equation in order to solve it, to actually operate on the algebraic representation of the equation. At the end of the study the students showed a significant progress in equation resolution and it can be asserted that the majority of them were able to transfer the actions undertaken with the concrete model sign system (virtual balance) to actions executed with the algebraic sign system. In other words, they transferred the *affordances* that they perceived from removing objects in the balance to the algebraic syntax domain. We moreover observed that the transfer processes go through different stages depending on the sign system to which the actions were transferred.

References

- Greeno, J. G., Smith, D. R. & Moore, J. L. (1993). Transfer of situated learning. In D. K. Detterman & R. J. Sternberg (Eds.), *Transfer on trial: Intelligence, cognition and instruction* (pp. 99-167). Norwood, NJ: Ablex Publishing Corporation.
- Rojano, T. & Martínez, M. (2009). From Concrete Modeling to Algebraic Syntax: Learning to solve linear equations with a virtual balance. In S. L. Swars, D. W. Stinson, & S. Lemons-Smith (Eds.), *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 235-243). Atlanta, GA: Georgia State University.

¹ This reserach was partially funded by Conacyt – Mexico (Grant No. 168620).

FLEXIBLE CALCULATION: KEY IDEAS FROM STUDENTS' SOLUTIONS

Joana Brocardo^{1,2}, Fátima Mendes^{1,2}, Catarina Delgado^{1,2}

¹Escola Superior de Educação, Instituto Politécnico de Setúbal

²Unidade de Investigação do Instituto de Educação, Universidade de Lisboa

To promote adaptive thinking and flexible computation is a strong curricular goal for compulsory education all over the world. Recent studies document the difficulty to achieve it, highlighting the tensions that occur during the process of learning of numbers and operations (Thompson & Saldanha, 2003; Threlfall, 2009). In fact, students show difficulties in using adequate and flexible ways of thinking, revealing a lack of conceptual understanding to make decisions on which computation process is more appropriate to solve a particular situation.

This paper is a report from one pilot study of a project aiming to analyze the critical issues related with the development of adaptive thinking and flexible computation. The project plan is based on design research and assumes a three-tiered teaching experiment design (Lesh, Kelly & Yoon, 2008).

The sample comprises the analysis of 50 clinic interviews with students from 6 to 15 years old focused on the resolution of 5 different numerical tasks.

Findings indicate that there is no relation between the use of flexible approaches, reflecting the use of a method that is efficient for calculating in each particular problem and students' age. They also indicate that students that use flexible approaches have a good knowledge of number facts, confidence in the methods they use and well developed numerical skills. The solutions of the students enable us to conjecture on a network of numerical facts, numerical relations and arithmetic properties that will support the development of several teaching experiences, to be developed in the context of the project.

References

- Lesh, R., Kelly, A. E., & Yoon, C. (2008). Multi-tier design experiments in mathematics, science and technology education. In A. E. Kelly, R. Lesh, and J. Baek (Eds.), *Handbook of design research in education: Innovations in science, technology, mathematics and engineering*. New York: Routledge.
- Thompson, P. W., & Saldanha, L. (2003). Fractions and multiplicative reasoning. In J. Kilpatrick, G. Martin & D. Schifter (Eds.), *Research companion to the Principles and Standards for School Mathematics* (pp. 95-114). Reston, VA: National Council of Teachers of Mathematics.
- Threlfall, J. (2009). Strategies and flexibility in mental calculation. *ZDM Mathematics Education*, 41(5), 541-555.

MATHEMATICAL CONTENT KNOWLEDGE OF ELEMENTARY PRESERVICE TEACHERS: GEOMETRY AND MEASUREMENT

Christine Browning⁽¹⁾, Alden Edson⁽¹⁾, Patrick Kimani⁽²⁾, Fatma Aslan-Tutak⁽³⁾

⁽¹⁾Western Michigan University, ⁽²⁾ California State at Fullerton, ⁽³⁾Boğaziçi University

There has been a recent emphasis in the mathematics education community in describing the needed and desired mathematical content knowledge for teaching, with various descriptions emerging from research. Understanding this knowledge is vital for mathematics educators as we develop courses to meaningfully prepare preservice teachers as well as knowing how to help preservice teachers understand the needed content knowledge for themselves. Hill, Ball, & Shilling (2008) distinguish between different types of knowledge in a mathematical-knowledge-for-teaching framework. A PME-NA working group designed a study to focus on one aspect of this framework, namely, subject matter (mathematical content) knowledge. Several subgroups of the working group were organized around different mathematical content areas with the following research questions: *What research has been conducted on elementary/middle school preservice teachers' content knowledge?* and *What is known from this research about preservice teachers' content knowledge?* We will report our findings on the area of geometry and measurement.

The methods of the study included an ERIC database search of peer-reviewed journals that focus on elementary or middle school (age 3-14) preservice teachers' geometry and measurement content knowledge. Each study went through an independent review that detailed the research questions, study type and research design, lens and/or approach used, selection and description of participants, conditions of and procedures for data collection, data analysis, findings, and conclusions/implications. Based upon an analysis of the compiled studies, findings show that preservice teachers have weak conceptions in geometry and measurement content knowledge. However, instructional strategies that incorporate technology (e.g., dynamic geometry software) or analyzing children's work, have been shown to strengthen preservice teachers' understanding. In order to work from what preservice teachers do know (CBMS, 2012), it appears that a greater focus on developing strong foundational knowledge is needed in geometry and measurement courses.

References

- Conference Board of the Mathematical Sciences. (2012). *The mathematical education of teachers II* (Issues in Mathematics Education, Vol. 17). Providence, RI: American Mathematical Society & Mathematical Association of America.
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400.

7TH GRADE STUDENTS' REASONING AND PROOF SKILLS IN ALGEBRAIC STATEMENTS

Ayşegül Çabuk, Mine İşiksal-Bostan

Başkent University, Türkiye, Middle East Technical University, Türkiye

The activity of mathematical reasoning including development of proofs provides looking into the meanings of mathematical ideas, statements or facts which promotes proficiency in learning mathematics. Stylianides and Ball (2008) explained this educational role of proof as "proving can be a vehicle for deep learning in all content areas (algebra, geometry, etc.), so students' engagement in this activity can have significant influence on their mathematical education more broadly" (p. 309). However, even if NCTM (2000) reflects an important issue that reasoning and proof should be included from prekindergarten to grade 12, studies in literature generally include secondary school students and pre-service mathematics teachers. With the light of the idea that the amalgam of reasoning and proof has significant effect on middle school students' mathematical learning, this study aims to investigate 7th grades students' reasoning and proof abilities. The sample will include 20 students of grade 7 from a private middle school in the capital city of Turkey. Data will be gathered during second week of March. Students will answer an achievement test including 2 essay type of problems about algebra adapted from literature (Lappan, et al., 1998/2004). The answers of three items will be analyzed qualitatively in terms of students' reasoning and proof processes. Based on the findings in literature, it is expected that even students in earlier ages have proving and reasoning abilities (Lester, 1975), they will generally use some numerical examples and drawings, specifically empirical evidences to verify and accept proofs (Healy & Hoyles, 2000). It is suggested that in order to develop reasoning and proof abilities, tasks which require reasoning and proving should be included not only in secondary school both also in elementary school mathematics curriculum.

References

- Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Philips, E. D. (1998/2004). *Prime time*. Connected Mathematics Project. Menlo Park, CA: Dale Seymour Publications.
- Lester, F. K. (1975). Developmental aspects of children's ability to understand mathematical proof. *Journal for Research in Mathematics Education*, 6 (1), 14-25.
- National Council of Teachers of Mathematics, (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Stylianides, A.J., & Ball, D.L. (2008). Understanding and describing mathematical knowledge for teaching: knowledge about proof for engaging students in the activity of proving. *Journal of Mathematics Teacher Education*, 11 (4), 307-332.
- Healy, L., & Hoyles, C. (2000). A study of proof conceptions in algebra. *Journal for Research in Mathematics Education*, 31 (4), 396-428.

UNDERSTANDING MATHEMATICS INQUIRY-BASED CLASSROOM PRACTICE: TEACHER'S ACTIONS AND INTENTIONS

Ana Paula Canavarro*, Hélia Oliveira+, Luís Menezes~

*Univ. of Évora & Research Unity of Institute of Education, Univ. of Lisbon, Portugal,

+Institute of Education of University of Lisbon, Portugal

~Higher School of Education of Viseu and CI&DETS, Portugal

This paper focuses on the inquiry-based teaching of mathematics (Stein, Engle, Smith & Hughes, 2008). It is our purpose to deepen our understanding of this complex practice of the teachers, considering both the actions and the intentions behind the actions they perform (Ponte & Chapman, 2006). We analyse the case of one of the teachers we work with in a broader Design Research project where the research on classroom practice and the planning of teacher training develop in articulation (Canavarro, Oliveira, & Menezes, 2012). We provide a description of the actions the teacher performed when students work autonomously on a mathematical task, as well as her reasons to justify her actions. We conclude that the practice of the teacher is oriented by two different but interrelated main purposes: to promote the mathematical learning of the students and to manage the classroom.

References

- Canavarro, A. P., Oliveira, H., & Menezes, L. (2012). A framework for mathematics inquiry-based classroom practice: the case of Célia. Paper presented at the 12th International Congress on Mathematical Education, Seoul, Korea.
- Ponte, J. P. & Chapman, O. (2006). Mathematics teacher's knowledge and practices. In A. Gutiérrez & P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education: past, present and future* (pp. 461-464). Rotterdam: Sense.
- Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking. *Mathematical Thinking and Learning*, 10(4), 313-340.

Acknowledgements

This study is supported by national funds by FCT – Fundação para a Ciência e Tecnologia through the Project Professional Practices of Mathematics Teachers (contract PTDC/CPE-CED/098931/2008).

MENTAL COMPUTATION WITH RATIONAL NUMBERS: STUDENTS' STRATEGIES WITH PERCENT

Renata Carvalho Carrapiço, João Pedro da Ponte
Unidade de Investigação do Instituto de Educação
Universidade de Lisboa

This study aims to analyse grade 6 students' mental computation strategies with rational numbers represented as percent in the context of a teaching experiment. We want to know the strategies that students use in percent mental computation tasks and the aspects of number sense that may be identified in such strategies. Following Reys, Reys, Nohda and Emori (1995), we regard mental computation as the process of computing an exact arithmetic result without external support. When computing mentally with rational numbers students may use known numerical facts and memorized knowledge about numbers and operations, as well as relationships such as change of representations and equivalence. We assume that computing mentally with rational numbers represented as percent develop important skills for students' daily life and also that a systematic work in mental computation, including collective discussions, may contribute to develop students' fluency in using different strategies (such as equivalence between representations, benchmarks and operations), as well as to the development of number sense (Barnett-Clarke, Fisher, Marks, & Ross, 2010).

This study is qualitative, with a teaching experiment design, involving 20 grade 6 students and their mathematics teacher. Data were collected using video recording of classroom episodes and data analysis focuses on episodes of whole class discussions. The teaching experiment consists of 10 mental computation tasks carried out weekly for about 15-20 minutes at the beginning of the class followed by whole class discussions. The study shows that, in mental computation with percent, students use strategies based on the change of representation (percent to fraction or decimal), halving, equivalence and numerical relationships involving part-whole relationships, showing number sense.

Acknowledgement

Study supported by national funds by FCT – Fundação para a Ciência e Tecnologia, Project *Professional Practices of Mathematics Teachers* (contract PTDC/CPE-CED/098931/2008). The first author was supported by FCT grant SFRH/BD/69413/2010.

References

- Barnett-Clarke, C., Fisher, W., Marks, R., & Ross, S. (2010). Developing essential understanding of rational numbers: Grades 3-5. Reston, VA: NCTM.
- Reys, R. E., Reys, B. J., Nohda, N., & Emori, H. (1995). Mental computation performance and strategy use of Japanese students in grades 2, 4, 6, and 8. *Journal for Research in Mathematics Education*, 26(4), 304-326.

PRIMARY SCHOOL TEACHERS' PRACTICES IN STATISTICAL INVESTIGATION TASKS¹

Ana Caseiro; João Pedro da Ponte; Cecília Monteiro

Escola Superior de Educação de Lisboa; Instituto de Educação da Universidade de Lisboa; Escola Superior de Educação de Lisboa

Statistics is an essential field in our society, and students must get a sound preparation on this topic. To be able to interpret and critically evaluate statistical studies, students must conduct statistical investigations in school (Groth, 2006). To provide them productive learning experiences in statistics, teachers need to know what kind of tasks they may propose. They also need to know how they must lead, plan, conduct and reflect about the lessons in which they propose their students to undertake investigations. Therefore, the study of teachers' practices is necessary to support the quality of the teaching and learning process.

Assuming that collaboration is an important strategy for conducting investigations on teachers' practices and a fundamental means for their development (Hargreaves, 1998), this study aims to understand 1st cycle (grades 1-4) teachers' practice and its development. In this paper, our focus is in the statistics investigations that teachers propose to their students, sustained in a collaborative work context. To achieve this goal, this research follows a qualitative and interpretive methodology, using a multiple case study design. Participants are three 1st cycle teachers that teach grades 3-4. Data are collected by means of participant observation in sessions of the collaborative working group and of the classes taught by the teachers, as well as through the documents produced by them. Data analysis is supported in the literature review and new categories are established as necessary.

Preliminary data shows that, during statistical investigations, teachers' questioning of the students and the examples that they chose was quite different among the participants. Teachers' reflections are focused on different aspects of the task, according to their sensitivity and understanding of issues that arise from this type of statistical work.

References

- Groth, R. E. (2006). An exploration of students' statistical thinking. *Teaching Statistics*, 28(1), 17-21.
- Hargreaves, A. (1998). *Os professores em tempos de mudança. O trabalho e a cultura dos professores na idade Pós-Moderna*. Lisboa: Mc Graw-Hill.

¹ Work funded by FCT – Fundação para a Ciência e a Tecnologia, PORTUGAL, under the scope of the project Developing statistical literacy: Student learning and teacher education (PTDC/CPE-CED/117933/2010).

NEEDS-BASED MATHEMATICS TEACHERS' CONTINUOUS PROFESSIONAL DEVELOPMENT

Michael Casper, Bettina Roesken-Winter
Ruhr-Universität Bochum

Continuous professional development (CPD) has been assigned a key role for improving mathematics teaching and learning. One crucial approach lies in providing long-term CPD offers that balance theoretical input and implementation in practice. In accordance, teachers' professional growth can be captured in terms of four analytic domains that are connected by the mediating processes of *reflection* and *enactment* (Clarke & Hollingsworth, 2002): first, the *personal domain*, comprising teacher knowledge, beliefs and attitudes; second, the *domain of practice*, covering professional experimentation; third, the *domain of consequence*, meaning salient outcomes; and fourth, the *external domain*, containing sources of information, stimulus or support. Based on this model, a CPD course provided by the German Center for Mathematics Teacher Education (DZLM), in which experienced teachers are engaged for one year, is evaluated. Data was collected for all four domains beginning with the personal domain, in order to answer the questions what knowledge and experiences teachers possess for the specific field of problem solving and how such information can be best integrated in offers from the external domain.

The sample comprises 14 experienced and influential teachers engaged in a course organized in three layers of theoretical input on problem solving and its teaching, exploring problem solving issues in practice, and reflecting these experiences. In relation to the personal domain, the teachers were asked to explain their experiences with and understanding of teaching problems from both perspectives as teachers and teacher educators in an open questionnaire. The data analysis was based on content analysis and, among others, yielded the following crucial points. Teachers often see no need to know about problem solving theory. They are well aware of restraints in time and resources that prohibit establishing a problem solving culture. All information was used to continuously customize the course to the exact needs of the group of teachers involved. In the future we plan to do the same in reference to the domains of practice and consequence and hence expect further fascinating insights.

References

Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education*, 18, 947–967.

AN ANALYSIS OF TURKISH STUDENTS' SELF-CONFIDENCE AND VALUE BELIEFS IN MATHEMATICS AND PREDICTION OF MATHEMATICS ACHIEVEMENT IN TIMSS 2007

Kübra Çelikdemir

Middle East Technical University, Turkey

The purpose of the present study is to investigate Turkish eighth-grade students' self-confidence in mathematics (SCM) and values on mathematics (SVM) and to determine how these constructs predict students' mathematics achievement in TIMSS 2007. Expectancy-value theory (Wigfield & Eccles, 2000) forms the theoretical framework of the study. SCM reflects students' perceptions about their ability in mathematics and perceptions of task difficulty and SVM refers to whether students perceive mathematics achievement as advantageous to their future education (Martin, Mullis & Foy, 2008) which represent the expectancy aspect and utility value aspect respectively in expectancy-value theory. 4498 eighth-grade Turkish students in 146 elementary schools participated to TIMSS 2007 study. There were 2093 girls (46.5%) and 2405 boys (53.5%). Students' self-confidence and values were measured through self-report student questionnaire. Therefore, 8th grade mathematics achievement test scores and data from student questionnaire were used for analysis.

To investigate self-confidence in and value on mathematics variable, descriptive analysis was conducted. Results showed that the majority of the students are at the high level for valuing mathematics (85.9%) index variables whereas the frequencies of the levels of students for the self-confidence are close to each other. In other words, while most of the Turkish students like mathematics and perceive mathematics as important for their lives, they are not so confident in their mathematical ability. On the other hand, to determine how these constructs predict students' mathematics achievement, multiple linear regression analysis was conducted. Results showed that the students' self confidence in mathematics and value of mathematics significantly accounted for 27% of variance in mathematics achievement, ($R=.52$, $F=743.94$, $p<.05$). The relative strength of the individual predictors indicate that only self-confidence in mathematic was significant, $p<.05$. In other words, students who were confident in their ability demonstrated higher achievement in mathematics. In summary, it was found that while most of the Turkish students put value on mathematics, relatively smaller number of students were confident in their ability in mathematics which significantly predicted their mathematics achievement in TIMSS 2007. These results may be come from developmental reasons like the age group of the students (Wigfield & Eccles, 2000).

References

- Martin, M. O., Mullis, I.V.S., & Foy, P. (2008). *TIMSS 2007 international mathematics report*: Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.
- Wigfield, A. & Eccles, J. S. (2000). Expectancy-value theory of achievement motivation. *Contemporary Educational Psychology*, 25, 68-81.

CULTIVATING LEARNING THE CONCEPT OF NEGATIVE NUMBERS IN THE CONTEXT OF ALGEBRA AND VICE VERSA

Sepideh Chamanara, Amir H. Asghari
Shahid Beheshti University, Tehran – Iran

The goal of this study is to propose a learning environment that cultivates the learning of the concept of negative numbers while introducing and extending the notion of variable in a way that it contains both negative and positive numbers. We hope that the proposed learning environment let the learners perceive a symbolic representation of a variable in such a general sense. For this aim, we use the design experiment (a kind of design-based research) which have both a pragmatic bent –“engineering” particular forms of learning- and a theoretical orientation –developing domain specific theories by systematically studying those forms of learning and the means of supporting them (Cobb, Confry, diSessa, Lehrer, & Schuable, 2003).

Regarding the notion of negative number, although we accept the dual nature of the concept (operational & structural; Sfard, (1991)), we do not adhere to the “subtraction” as the process that produces negative numbers (as Sfard (1991) suggests). We introduce and support a rather radical approach in which the object perception of negative numbers precedes the process perception of those numbers. Accordingly, we will design a situation in which the negative numbers are introduced at first by geometric existing objects – it means points - and then, the “addition” would be developed on this new set of objects in such a way that the addition properties would be preserved. This would be done in an algebraic context by which the new objects – negative numbers- would be introduced in relationship with other existing objects. Also we use the history of negative numbers that supports our approach (Heefffer, 2008).

We hope the results from this design research shed light on the knowledge about learning negative numbers and the “operational, structural, *inter-structural*” conception of this mathematical concept. The “inter-structural” conception of a concept is our extension of Sfards framework which fails to describe the relationship between the new object and the older existing objects and our design will support it. In this short oral, we also show some results obtained from working with algebraically naive students who have not seen negative numbers in formal education.

References

- Cobb, P., Confry, J., diSessa, A., Lehrer, R., & Schuable, L. (2003). Design Experiments in Educational Research. *Educational Researcher* , 32 (1), 9-13.
- Heefffer, A. (2008). Negative Numbers as an Epistemic Difficult Concept: Some Lessons From History.
- Sfard, A. (1991). On The Dual Nature of Mathematical Conceptions: Reflections on Process and Object as Different Sides of the same Coin. *Educational Studies in Mathematics* (22), 1-36.

RELATIONSHIP BETWEEN RELIGIOUS BELIEFS AND VIEWS ON EFFECTIVE MATHEMATICS TEACHING AND LEARNING FROM THE PERSPECTIVES OF MATHEMATICS TEACHERS

Yip-Cheung Chan, Ngai-Ying Wong
The Chinese University of Hong Kong

Mathematics teachers' beliefs and conceptions about mathematics and mathematics education is a research focus in recent decades (Philipp, 2007). Despite the study of numerous factors influencing teachers' beliefs and conceptions, to date, there are no systematic empirical ones on teachers' religious beliefs and worldviews. Our previous study (Chan, Wong & Leu, 2012) is the first step to fill this research gap, yet it based on questionnaires. The current study extended to semi-structured interviews (which are more informative) with fifteen mathematics teachers. Among them, 5 were Buddhists, 5 were Christians and 5 claimed not subscribing to any religion. The interview questions were about their views on the nature of mathematics, effective mathematics lesson, and possible impact of religious beliefs on mathematics teaching. A questionnaire on how teachers' religious beliefs might influence their teaching was used as triangulation. Narrative analysis was used for the analysis of the data. There is evidence that teachers' beliefs about mathematics education align with their religious worldview. A Buddhist teacher who had a wide perspective in seeing Buddhist doctrines emphasised on "seeing wider" in order to look for connections between different mathematics topics. A Christian teacher thought that mathematical understanding can be enhanced bit-by-bit by means of applying formulae to do mathematics questions even if these formulae are not completely understood. She pointed out that this view is similar to how she developed her Christian faiths. She only understood the Biblical teaching partially when she decided to become a Christian. Her understanding on Christian doctrines increased gradually as she put her faith into practice. More cases will be discussed on the spot.

The authors acknowledge the financial support by The Chinese University of Hong Kong Research Committee Funding (Direct Grants) (Project Code: ED11490).

References

- Chan, Y. C., Wong, N. Y., & Leu, Y. C. (2012). Do teachers with different religious beliefs hold different values in math and math teaching? In T. Y. Tso (Ed.). *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, p. 254). Taipei, Taiwan: PME.
- Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Charlotte, U.S.A.: Information Age.

A STUDY OF TENTH GRADERS' MATHEMATICAL MODELLING PROCESS IN TAIWAN

Ching-Kuch Chang, Ming-Feng Chen, Shu-Chuan Huang, Yi-Chen Yu

Graduate Institute of Science Education, National Changhua University of Education,
Taiwan

We paid more and more attention to enhance and develop students' mathematical competency through school education (e.g., Kaiser et al., 2011) in the modern world. Research has shown that students' mathematical competency is promoted by engaging in model-eliciting activity (MEA) (Lesh & Doerr, 2003) collaboratively in small groups. In Taiwan, we trained in-service teachers to implement MEA and also get students' responses about modelling learning. The study reported the modelling process of two groups of tenth graders. Teacher C's class (16 students) and Teacher H's class (19 students) were in different but similar suburban high schools in Taiwan. These students' mathematical performances were at average level. We used the modelling cycle (constructing, simplifying/structuring, mathematising, working mathematically, interpreting, validating, exposing) of Blum (2011) as an analytical framework to analyse the transcripts of students' group discussion and their learning sheets. Teacher C founded that students' modelling processes were closely related to the modelling task and showed different key points according to the task. Second, not all seven-step of the modelling cycle would happen in order in every modelling task. Third, students' modelling processes mainly circulated on mathematising, working mathematically and interpreting. Teacher H founded that modelling process was not a linear process and students had a long mess which was related to the complicity of the task and students' modelling experience from the beginning of solving the task. If students got more modelling experience, there would be less teacher's intervention. The seven-step schema was helpful to teachers to enhance students' modelling process better.

References

- Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser et al. (eds.), *Trends in teaching and learning of mathematical modelling, international perspectives on the teaching and learning of mathematical modelling*. (pp. 15-30). New York: Springer.
- Kaiser, G., Blum, W., Borromeo Ferri, R. & Stillman, G. (2011). Trends in teaching and learning of mathematical modelling-Perface. In G. Kaiser et al. (eds.), *Trends in teaching and learning of mathematical modelling, international perspectives on the teaching and learning of mathematical modelling*. (pp. 1-5). New York: Springer.
- Lesh, R., & Doerr, H. M. (2003). *Beyond constructivism: models and modeling perspectives on mathematics problem solving, learning, and teaching*. American, Lawrence Erlbaum Associates, Inc.

A CASE STUDY OF ADJUSTMENT ON MATHEMATICS TEXTBOOK IN TAIWAN: USE POLYHEDRON AS AN EXAMPLE

Shu-I Chang, Jing Chung

National Taipei University of Education, Taiwan

Over the past ten years, a phenomenon has been noticed in Taiwan, that is, most of the teachers who teach mathematics in primary five and six are junior or substitute teachers who usually have trouble to interpret the textbook properly and correctly and have less idea on materials adjustment. Sherin and Drake (2009) present the methods how teachers adjust material with continuous spectrum of creation, replacement and omission. Generally speaking, the factors which may affect the way a teacher adopt and adjust the material are teacher's professional knowledge, content of material, and any restrictions caused by time and space. Therefore, the purpose of this study is to explore how a primary five teacher adjusts the textbook in the lessons of polyhedron as well as the factors affect the adjustment.

This study invites an expert teacher, Mrs. Fang, who has twenty years experience, as an object and collects data through observation in class and interview after class. Also, the textbook used in class, homework, and teacher's manual are studied to be reference.

It is found that Mrs. Fang adopts four methods in activity adjustment- rearrangement, addition, omission and replacement, as well as revision of question. Through her long-termed teaching experience, she finds her students often have difficulties in the perspective view of cuboid, so she throws in lots of activities to elaborate it in her lesson plan. Moreover, the factors which affect the adjustment are found to be contents of textbook, teacher's ample knowledge in instruction, teacher's concerns on students' learning difficulty, and piles of interesting activity in database. Scheduling before important examination and quantity of teaching aid may be influential, too.

This study finds the expert teacher adopts various methods when she makes adjustment in the unit of polyhedron and her instruction is different from the way other teachers utilize the textbook normally. It is expected that, from this example, teachers can be enlightened on the utilization of textbook, that is, teachers do not have to follow the book hundred percent, but try the possibility of making adjustment on textbook. Furthermore, this case study may inspire the planning of teacher development program. It is suggested a session for expert teachers to share their experience with others in the program so that pre- and at-service teachers have channel to learn from expert teachers and further, make improvement in their own instruction.

References

Sherin, M. G., & Drake, C. (2009). Curriculum strategy framework: Investigating patterns in teachers' use of a reform-based elementary mathematics curriculum. *Journal of Curriculum Studies*, 41(4), 467-500.

A STUDY ON RELATIONSHIPS BETWEEN ARITHMETIC OPERATING FLEXIBILITY, ACADEMIC ACHIEVEMENTS AND REASONING ABILITIES OF PRIMARY SCHOOL STUDENTS

Chia-Huang Chen, Hsiu-fei Lee

Department of Mathematics Education National Taichung University of Education,
Department of Special Education National Taitung University

The study examined the performance of fourth and fifth graders in a multiplication and division operating flexibility test, and explored the relationships between arithmetic operating flexibility, academic performance in other courses and reasoning abilities. We designed a multiplication and division operating flexibility test, according to the types of acquisition and progress levels proposed by Lampert (1986). Our research sample consisted of 181 fourth graders and 169 fifth graders students in Southern Taiwan. We performed both quantitative and qualitative analysis. A t-test and regression was used to examine the divergences and relationships between variables. We then selected two fourth graders and two fifth graders who had performed well in the flexibility test with whom structured questionnaire-type interviews were conducted. The results were as the followings: 1. The fourth and fifth graders did not show significant differences in the flexibility test as a whole, although fifth graders outperformed fourth graders in implementation. 2. Students with better operating flexibility also demonstrated better academic performance in Chinese and mathematics, they also scored remarkably higher in the reasoning tests related to language and patterns and space. 3. Students' academic performance in Chinese was an important factor in overall operating flexibility (15.3%). The results of the interviews also indicated that the student's proficiency in Chinese and language reasoning positively affected his multiplication and division operating flexibility.

References

Lampert, M. (1986). Knowing, Doing, and Teaching Multiplication. *Cognition and Instruction*, 3(4), 305-342.

TEACHER GROWTH THROUGH DESIGNING $0.\bar{9}=1$ TASK

Jian-Cheng Chen**; Fou-Lai Lin*; Hui-Yu Hsu*; Hui-I Liao***; Kai-Lin Yang*

* National Taiwan Normal University ** Ming Chi University of Technology

***Taren Junior High School

This paper presents professional growth of an experienced mathematics teacher (to whom referring pseudonym Flora) as she challenged a pedagogical problem through a process of designing $0.\bar{9}=1$ task and enacting the task with students. Perspectives including essence of mathematics learning, mathematics history, epistemology, and pedagogy were used to investigate Flora's growth evolution. Problem-Based Learning (PBL) (Hmelo-Silver, 2004) was adopted for analyzing the growth as PBL highlights the importance of recognizing deficiency of self-knowledge and developing the competence in solving realistic problems. Similarly, professional growth of teachers means that they learn how to solve pedagogical problems in realistic teaching contexts and to facilitate students to experience the essence of mathematics learning. The reasons for Flora to challenge the problem is that students often think $0.\bar{9}<1$ intuitively and may use different viewpoints to interpret the relation between $0.\bar{9}$ and 1. For example, Flora had interviews with three adults who have learned the infinite geometric series and have majors in physics, social science, and engineering respectively. Her interview showed that the one who majors in physics used the definition of atom, the minimal unit, to argue that $0.\bar{9}$ is smaller than 1 as subtracting $0.\bar{9}$ from 1 obtains a positive number but not 0.

To help students understand the meaning of the recurring number $0.\bar{9}$ and its relation to 1, Flora designed a task which aims to create cognitive conflicts for students so that they can reflect and explore the mathematics. The task includes activities: (1) exploring the essence of the recurring number $0.\bar{9}$ on a number line and denseness of real numbers; (2) conjecturing the answer for $9/9$ based on the observation of $1/9=0.\bar{1}$, $2/9=0.\bar{2}$, and so on; and (3) using an algebraic approach to prove that $0.\bar{9}=1$ ($x=0.\bar{9}$; $10x=9.\bar{9}$; $9x=9$ so that $x=1$). Before enacting the task with students, Flora anticipated that the first two activities were more likely to help students understand the meaning of the recurring number $0.\bar{9}$ and accept $0.\bar{9}=1$, whereas the third one only introduces the formal reasoning. However, the experimentation of the task was not consistent with her anticipation as a relatively high portion of students accepted $0.\bar{9}=1$ because of the third activity but not the first two. The inconsistency created opportunities for Flora to explore the reasons accounting for students' responses to the task, the relation between mathematics and student learning, and pedagogy. Of importance is that she recognized she should not subjectively decide how students learn. Instead, she should listen to students before determining the teaching sequences.

References

Hmelo-Silver, C. E. (2004). Problem-based learning: What and how do students learn? *Educational psychology review*, 16(3), 235.

EXPLORING THE GAP BETWEEN VERBAL DESCRIPTION AND FIGURAL PATTERN OF GEOMETRIC THEOREM ON SECONDARY STUDENTS

Ying-Hao Cheng, Hui-Yu Hsu, Jian-Cheng Chen

Taipei Municipal University of Education, National Taiwan Normal University,
MingChi University of Technology

Duval(2002) proposed that the crucial process of constructing a proof is to recognize all the necessary information and apply appropriate geometric theorem. The results of students' performance in different countries showed that many students had heavy difficulties in applying theorems. A geometric theorem is learnt in both verbal and figural format. It was always introduced with theorem name, *verbal description*, and a *figural pattern*. The verbal description explained the premise and conclusion. The figural pattern established the prototype figural image. Fischbein(1993) showed that there are different meanings of a geometric figure depend on the learners' cognitive processing. In this study, we introduce one survey of how grade 7 students connected the verbal description and figural patter of a geometric theorem. There were 314 grade 7 students participated. The task provided 5 theorems they had learnt. Every one contained their verbal description, a complex graph with labels, and 4 expressions. Fig 1. is one of the items. We asked them to choose the correct expression associated to the theorem.

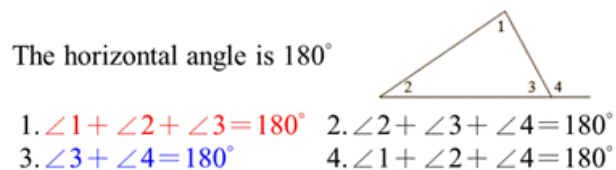


Fig 1: The example of item

The data showed that (as item in Fig 1) there were 23% of grade 7 students chose correctly, 30% chose both option 3 and 1, and 36% chose option 1 only. Option 1 (the sum of interior angles of a triangle) seems to be a more attractive expression in a graph with triangle. In concluding, the specific and simplified figural pattern in textbook for learning may narrow down students' visualization process and make them unable to recognize an appropriate theorem for reasoning.

References

- Duval, R. (2002). Proof understanding in mathematics: what ways for students? Plenary papers in the 2002 International Conference on Mathematics: Understanding Proving and proving to understand. Department of Mathematics. National Taiwan Normal University. 2002.11.16-19.
- Fischbein, E. (1993). 'The theory of figural concepts'. Educational Studies in Mathematics 24(2), 139-162.

ISSUES IN IMPLEMENTING AN INCOHERENT CURRICULUM BETWEEN MATHEMATICS AND PHYSICS

Mei-Shiu Chiu

National Chengchi University, Taiwan

THEORIES, CONTEXTS, AND METHODS

Coherent curriculum design is required between mathematics and physics given that the two domains are closely interwoven and when mathematics curriculum calls for external connections with life as mediating means (Askew, Venkat, & Mathews, 2012; Lonning & DeFranco, 2010). Incoherence between mathematics and physics saliently occurs in the high-school curriculum of 2010 in Taiwan. Grade-11 science students study motion and dynamics in physics without any prior learning experiences of trigonometry and trigonometric function in mathematics. Qualitative research methods were used to investigate the perspectives and actions of 51 science students, 22 mathematics and physics teachers, and 3 curriculum developers/professors, with an aim to identify the issues in the implementation of the curriculum.

RESULTS AND DISCUSSION

In the national intended curriculum level, the issue is domain boundaries: Mathematics emphasizes abstraction, procedures, and theorems, while physics emphasizes scientific advances, concepts, and unified truth. In the teacher implemented curriculum level, the issue is fix curriculum: Mathematics teachers feel relaxation and independence given fewer, easier, and self-contained contents, while physics teachers feel anxiety and helplessness given wider contents and insufficient mathematics ‘tools’ for physics. In the student received curriculum level, the issue is diverse cognitive developments: Students learned the quickly-taught new mathematics by physics teachers, with a negative impression of physics teaching in school. The findings suggest that the curriculum development process based on ‘hierarchical democracy’ needs to be transformed into a renewed framework, with equitable expert and teacher curricula at the bottom to support excellent student curriculum based on ‘rational democracy’.

References

- Askew, M., Venkat, H., & Mathews, C. (2012). Coherence and consistency in South African primary mathematics lessons. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 27-34). Taipei, Taiwan: PME.
- Lonning, R. A., & DeFranco, T. C. (2010). Integration of science and mathematics: A theoretical model. *School Science and Mathematics*, 97(4), 212-215.

A THIRD-SPACE OF MATHEMATICAL PRACTICE: IMPLICATIONS FOR TEACHER EDUCATION

Marta Civil⁽¹⁾, Bryan Fede⁽¹⁾, Marcy Wood⁽²⁾

⁽¹⁾ University of North Carolina, Chapel Hill, USA ⁽²⁾ University of Arizona, USA

The work presented here is part of a project grounded on a co-educational model where preservice teachers (PTs), mentor teachers (MTs), and university faculty (UF) participate in several joint events. These events are aimed at creating a third space (Martin, Snow, & Torrez, 2011) where participants develop new meanings and practices, moving away from the disconnect that often exists between university-based teacher preparation programs and teaching practices in schools.

We focus on the summer institute in which MTs and PTs engaged in the practice of mathematics by exploring patterns and justifying algebraic rules. The overarching research question was, “What is the nature of the interactions among MTs, PTs, and UFs in this joint event?” We build on research on mathematical interactions (e.g., Mueller, Yankelewitz, & Maher, 2012) and expand upon it through a third-space lens. The analysis of the video-taped problem-solving sessions of 2 MTs and 2 PTs reveal the interplay of mathematical authority and pedagogical authority. One MT alternated between engaging as a teacher and as a problem-solver; the other MT engaged only as a teacher, making connections to the classroom. The PTs were seen by the MTs as the mathematical experts, yet their approach reflected a procedural view of school mathematics, which was challenged through the UFs’ questions.

This research revealed the challenges of supporting PTs and MTs in crafting a third-space understanding of mathematical practices which honors their pedagogical and prior experiences with mathematics but also invites an exploration of the mathematical practices advocated by the UFs. Our findings suggest design principles for mathematical tasks, including structures for highlighting participants’ differences in mathematical practices, with the goal of developing third-space understandings.

Acknowledgment

This material is based upon work supported by the National Science Foundation under Grant No. DRL-1019860.

References

- Martin, S., Snow, J., & Torrez, C. F. (2011). Navigating the terrain of third space: Tensions with/in relationships in school-university partnerships. *Journal of Teacher Education*, 62, 299-311.
- Mueller, M. Yankelewitz, D & Maher, C. (2012). A framework for analyzing the collaborative construction of arguments and its interplay with agency. *Educational Studies in Mathematics*, 80, 369-387.

TEACHERS AND OBSERVERS LEARNING FROM DEMONSTRATION LESSONS IN MATHEMATICS EDUCATION

Doug Clarke, Anne Roche, Rose Knight
Australian Catholic University

Demonstration lessons have the potential to promote teacher change and raise the quality of the teaching and learning in a classroom (Grierson & Gallagher, 2009). The purpose of demonstration lessons is to use a “prelesson, classroom demonstration lesson observation, and postlesson debrief” cycle as a catalyst for in-depth reflection on mathematics teaching and learning” (Loucks-Horsley et al., 2003, p. 212).

As part of a professional learning project, university mathematics educators taught demonstration lessons during primary school visits, with grade levels and content negotiated with teachers. Observing teachers (typically 15-20 observers over three lessons) completed questionnaires and participated in discussion before and after the lessons. Our main research question was the following: What do teachers attend to when observing a demonstration lesson in mathematics and what change(s) in practice do teachers report might occur as a result of this experience?

Questionnaires items included “What are you planning to focus on in your observations with regard to teaching? ... student learning? Is there anything that occurred today that you believe might contribute to a change in your teaching?”

The research team analysed the written responses of a random sample of 200 teachers to these questions and others. These data sets were analysed using an iterative process of inductive coding and collaborative revision by teams of coders.

Teachers’ intended observation foci included strategies for building engagement, questioning, and catering to the needs of all students. In relation to teachers’ intended changes in practice (445 comments), teachers were looking to raise the level of engagement of their students with a more active approach, differentiate according to student needs, place a greater emphasis on small group work, plan their questioning more carefully, focus on the use of appropriate materials and representations, and increase their level of use of mathematical language.

Demonstration lessons in mathematics are under-researched and this study has the potential to make an important contribution to the literature.

References

- Grierson, A. L., Gallagher, T. L. (2009). Seeing is believing: Creating a catalyst for teacher change through a demonstration classroom professional development initiative. *Professional Development in Education*, 35(4), 567-584.
- Loucks-Horsley, S., Love, N., Stiles, K. E., Mundry, S., & Hewson, P. W. (2003). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, CA: Sage Publications.

PARENTAL BELIEFS ABOUT MATHEMATICS AND MUSIC LEARNING

Csaba Csíkos, Gabriella Dohány
Institute of Education, University of Szeged

Theoretical background The role of parental beliefs has been investigated in different domains such as Physics (e.g., Yeung, Kuppan, Foong et al., 2010), Music (e.g., Koh, 2011) and Mathematics. In Mathematics, students' parents' beliefs about causal attributions for success in mathematics (Eccles & Davis-Kean, 2005), and the accuracy of parents' information (Pezdek, Berry & Renno, 2002) have been studied.

Research questions The current research aims at studying parental beliefs in mathematics and music in parallel. (1) How do secondary school students' parents see the connections between math and music learning? (2) What is the most sensitive age for talent recognition and development? (3) How do they perceive the role of music and mathematics in a life-span perspective?

Methodology A self-administered questionnaire was sent out to parents in four classes (specialized in music, environmental studies, humanities, and one without subject-specialization; N=117) of an upper secondary school of a county seat town. The questionnaire covered statements in five-point Likert-type scale about (1) scientifically approved phenomena (e.g., Mozart-effect) and naïve beliefs about mathematics and music learning, (2) talent development and its sensitive age, and about (3) "the role of music and mathematics in getting along in life".

Results (1) Parents see the importance of mathematics in music learning in solfège and in keyboard playing. (2) The optimal age for talent recognition (7.38 years for mathematics and 5.85 for music) and for talent development (6.94 and 5.94, respectively) seem to be controversial. The role of music and mathematics from different aspects of getting along in life proved to in part stereotypical, but in many cases surprising and alerting (e.g., relative unimportance of mathematics in private life and in creativity, as compared to music.)

This research was supported by the Hungarian Scientific Research Fund (OTKA #81538) and by the PRIMAS project (European Commission FP7, GA #244380)

Eccles, J. S., & Davis-Kean, P. E. (2005). Influence of parents' education on their children's educational attainments: the role of parent and child perceptions. *London Review of Education*, 3(3), 193-204.

Koh, C. K. (2011). *Adolescents' motivation to study Music as compared to other school subjects: a Singaporean perspective* (Doctoral dissertation). Retrieved from https://www.ideals.illinois.edu/bitstream/handle/2142/24262/Koh_CheeKang.pdf?sequence=1.

Pezdek, K. Berry, T., & Renno, P. A. (2002). Children's mathematics achievement: The role of parents perceptions and their involvement in homework. *Journal of Educational Psychology*, 94(4), 771-777.



Yeung, S. A., Kuppan, L., Foong, S. K., Wong, D. J. S., Kadir, M. S., Lee, P. C. K., & Yau, C. M. (2010). Domain-specificity of self-concept and parent expectations influences on short-term and long-term learning of Physics. *New Horizons in Education*, 58(2), 54-72.

COGNITIVE SCIENCE AND MATHEMATICS EDUCATION:
INVESTIGATING THE YOUNG MIND AND FRACTIONS.

Julie Cwikla, Jennifer Vonk
University of South Alabama; Oakland University

Little is known about young children’s knowledge of fractions prior to formal teaching (e.g. Empson, 2003; Hunting & Davis, 1991). Both children and teachers experience difficulty with these concepts. Therefore, it is critical to discover whether young children are capable of acquiring such concepts before the whole number bias is deeply ensconced, and to understand what strategies they use when solving fraction and fair sharing problems prior to formal curricular introduction.

We presented 36 three to six-year-olds with seven orally presented fair sharing problems ascending in difficulty. For example, “Jade wanted to share six carrot sticks with her four friends. How can she do this fairly?” If children had difficulty expressing their solution either verbally or pictorially, we provided visual prompts in the form of pre-fabricated two-dimensional sketches. They were then encouraged to indicate how the items could be distributed fairly.

Pre-School 4.58 Years	Kindergarten 5.67 Years
	
Table 1: Examples of student work	

Children’s solutions were scored on the basis of both verbal and pictorial responses. We documented key differences in overall conceptual understanding between the youngest children and the older children. Scores decreased for all children as the problems increased in difficulty. However, the older children performed better overall. Kruskal-Wallis tests revealed significant differences in total scores between the youngest children (< 4.5 y.o.) and two older groups (4.6 - 5.5 y.o. and > 5.5 y.o.), $X^2=14.94$, $p<.001$.

Table 1 illustrates the differences in written response sophistication as children mature solving the above carrot sharing example. Our larger data set indicates students can successfully attempt and solve fractional tasks in the preschool years.

References

Empson, S. B. (2003). Low-performing students and teaching fractions for understanding: An interactional analysis. *Journal for Research in Mathematics Education*, 34, 305-343.

Hunting, R. P. (Ed.); Davis, G. E. (Ed.) (1991). *Early Fraction Learning*. New York, NY, US: Springer-Verlag Publishing.

FRAMING ELEMENTS OF MATHEMATICS TEACHERS' PROFESSIONAL PRACTICE

João Pedro da Ponte, Marisa Quaresma

Instituto de Educação Universidade de Lisboa; Unidade de Investigação do Instituto de Educação, Universidade de Lisboa

The study of teachers' professional practice is an important topic in mathematics education. Our goal is to analyse how this practice is framed by the tasks proposed to students and classroom communication. Teaching practice is regarded as an activity, identifying the teacher's overall goals and motives as well as the actions used to achieve them. We look at the teacher's decisions in face of foreseen and unforeseen events. Attention is also given to contextual elements (as curriculum, students, and classroom) and the teacher's orientations, dispositions, and competencies. During a lesson, the students may work as whole class, small group, pairs, and solo, which one with specific communication patterns. The combination of such forms of work may sustain the development of students' mathematical concepts, representations, procedures, strategies, generalizations, justifications, and appreciation. The study methodology is qualitative, based on observations of twelve 90-minutes classes of a grade 5 teacher, video recorded and transcribed. The topics included the notion and representation of rational numbers as fractions, decimals and percent. Data analysis of lessons and segments was made with the participation of the teacher.

The results show that this teachers' professional practice is framed by the goal of engaging students in mathematical activity, searching strategies to solve the tasks proposed. Such teaching develops in a context of curriculum change, based in two elements: (i) the teacher values this exploratory approach, and (ii) the students are eager to participate in such activity. In this teaching practice the selection of appropriate tasks is critical – leading to students' learning of important ideas, fostering their involvement, and including elements of challenge. How classroom communication is handled also plays a key role. In this case, dialogic communication, with frequent inquiry questions, revoicing the students' discourse and providing negotiation of meaning, provided opportunities for students' explanations and justifications. The teacher's decisions concerned moments of transition from a lesson segment to another and driving communication. We conclude that in studying teacher practice it is useful to pay attention both to the nature of the activity in its context and to the agency of the teacher.

Acknowledgement

This study is supported by national funds by FCT – Fundação para a Ciência e Tecnologia through the Project *Professional Practices of Mathematics Teachers* (contract PTDC/CPE-CED/098931/2008).

THE LEARNING APPROACHES OF UNIVERSITY STUDENTS IN A REQUIRED MATHEMATICS COURSE

Bettina Dahl (Søndergaard)
Aalborg University, Denmark

The study focuses on students who are not in a science, engineering, or mathematics study programme but who are nevertheless required to take a mathematics course to fulfil the university's breadth requirement. More research is needed on such students. The research questions are: What are learning approaches of these students and how are these approaches related? Do their approaches relate to their learning outcome?

A student's learning approach is crucial for the quality of learning (Biggs et al., 2001). In a *surface* approach, the students use low cognitive level processes such as rote learning to avoid failure with minimal effort. Students with a *deep* approach focus on the underlying meaning. The *strategic* approach is a well-organised surface approach focused on what is required in the exam. The learning approach is a function of the student's characteristics and the teaching style. ASSIST (Approaches and Study Skills Inventory for Students) (Tait et al., 1998) was used to measure the students' learning approaches in 2011 at a US university. 87 of 191 students answered. The response rate is thus 46%, which is satisfactory, but may still be a limitation for extrapolation.

The students mainly adopt a strategic approach. Since the course is a requirement, one might have anticipated a surface approach. There is a strong positive correlation between the deep and strategic approaches ($r = .52, p < .00$), a negative correlation between the surface and strategic approaches ($r = -.26, p = .03$), but no correlation between the surface and deep approaches ($p = .94$). In other studies, the deep approach correlates negatively with a surface approach. The difference might be related to the nature of the course. In terms of learning outcome, the surface approach has a strong negative correlation ($r = -.58, p < .00$), the strategic approach a positive correlation ($r = .29, p = .01$), but there is no correlation to the deep approach ($p = .52$). In other studies, the deep approach correlates positively with learning outcome. Correlations do not establish causation, but the results can form the hypothesis that the students experience the strategic approach to be useful and they feel no need for a deep approach.

References

- Biggs, J. B., Kember, D., & Leung, D. Y. P. (2001). The Revised Two Factor Study Process Questionnaire: R-SPQ-2F. *British Journal of Educational Psychology*, 71, 133-149.
- Tait, H., Entwistle, N. J., & McCune, V. (1998). ASSIST: a reconceptualisation of the Approaches to Studying Inventory. In C. Rust (Ed.), *Improving students as learners*. Oxford: Oxford Brookes University, The Oxford Centre for Staff and Learning Development. See also: <http://www.etl.tla.ed.ac.uk/questionnaires/ASSIST.pdf>.

PRE-SERVICE SECONDARY MATHEMATICS TEACHERS' LEARNING ABOUT STUDENTS' THINKING

M. Gozde Didis, A. Kursat Erbas, Bulent Cetinkaya, Erdinc Cakiroglu
Middle East Technical University

Teachers' understanding of students' conceptions, errors and difficulties is one of the important components of knowledge for teaching mathematics (Shulman, 1986). Various studies indicate gaps between how students think about particular topics in mathematics and what pre-service teachers know about those. Thus, it is often suggested that teacher education programs should help (pre-service) teachers' to increase their awareness across students' various and sometimes incorrect common cognitive processes and thinking (e.g., Tirosch, 2000). The aim of this study was to investigate pre-service teachers' learning about students' mathematical thinking within an undergraduate course context where they first worked on thought-revealing non-routine tasks and then examined actual solutions made by real students. The research question guided the study was: "What do (or might) pre-service secondary mathematics teachers learn about students' thinking by examining works they produced?" Data were collected in an undergraduate course designed to improve prospective mathematics teachers' knowledge in and about mathematical modeling in teaching (and learning) mathematics. The participants were 25 pre-service secondary mathematics teachers enrolled in the course. The data collection consisted of four two-week long cycles. In each cycle, the pre-service teachers worked on a non-routine task in groups of 3-4 to produce their own solutions to the task. The next week they were provided with high school students' written works on the same task and classroom videos of students' discussions about the task while they were solving it in groups. The pre-service teachers were asked to collaboratively analyze students' thinking manifested in their written works and the videos and to write about students' solution strategies, strengths and challenges of their solutions, mathematical concepts used in their works and other things they noticed. The data obtained from students' individual reflection papers and focus group interviews were analyzed. The results indicated that while the pre-service teachers had limited knowledge about students' thinking and prominently used their own experiences as students while they were required to think about students' thinking in particular situations, the examination of students' written works and classroom videos enabled them to identify and recognize that students can produce various kinds of valid (and invalid) solution strategies that might be different than the ones they can produce. Moreover, the pre-service teachers also learned about students' common weakness and strengths in particular mathematical topics.

References

- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.
- Tirosch, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *JRME*, 31(1), 5-25.

CAN IN-SERVICE TEACHER TRAINING HELP IMPROVING STUDENTS' PROBLEM SOLVING COMPETENCES?

Ulrike Dreher, Lars Holzäpfel, Timo Leuders
University of Education Freiburg, Germany

Research on effectiveness of in-service teacher training programs focusses on different aspects. One topic currently being studied is the impact of the training on participants. Several impact factors at the teacher level have been identified: E.g. the extension of pedagogical and diagnostic knowledge, a focus on student learning processes, and the link between input, application and reflection (Lipowsky, 2010). Effects of professional development programs can be measured on different levels: (1) Teachers' self assessment, (2) Changing teachers' cognitions, (3) Teaching practice and (4) Changing student achievement (ibid.). There is a scarcity of results on the effects on level (4). The main focus of most studies is the achievement with respect to content knowledge. This study focuses on the attitude change of teachers and students as an impact factor on student achievement. Because the training content involves "problem solving strategies" we can assume that conceptual change processes of teachers occur and that these changes play an important role in attitude change at multiple levels (Posner et al, 1982). The training is based on Bruder and Collet's (2008) extensive program on problem solving implementation which focused on the improvement of students' problem solving competences and instructional implementation. In an on-going research project we aim at supporting teachers in fostering students' problem solving competences. In this paper we report on results with respect to the following research questions:

- What is the impact of the training on the students' achievements?
- What is the role of teachers' beliefs?

In a quasi-experimental study (n= 995 secondary school students in 54 classes) we implemented an in-service teacher training. We show that the training significantly improved student achievement. In a regression analysis we find a correlation between the number of problems and the increase in post-test. Subsequently additional testing instruments will be used to measure the changing attitudes of teachers and students. Preliminary results and individual testing instruments will be presented and discussed.

References

- Collet, C. & Bruder, R. (2008): Longterm-study of an intervention in the learning of problem-solving in connection with self-regulation. In: Proceedings 32th PME
- Lipowsky, F. (2010). Lernen im Beruf. Empirische Befunde zur Wirksamkeit von Lehrerfortbildungen. In: F. Müller & A. Eichenberger u.a. (Eds.): Lehrerinnen und Lehrer lernen. Konzepte und Befunde zur Lehrerfortbildung. (pp. 51-70) Münster: Waxmann.
- Posner, G., Strike, K., Hewson, P., & Gertzog, W. (1982): Accommodation of a scientific conception: Toward a theory of conceptual change. *Science Education*, 66(2), 211-227.

CHANGING MENTAL MODELS WHILE ACTING WITHIN STATISTICAL SITUATIONS

Andreas Eichler, Markus Vogel

University of Education Freiburg, University of Education Heidelberg

Most of research in statistics education follows the approach of intervention studies (Jones & Thornton, 2005). Thus, there is a lack of research that according to Piaget investigate the statistical thinking of young students, who did not gain schooling in statistics (ibid.). For this reason, our research concern unschooled young students' thinking when working within elementary statistical situations.

The part of our research we will present in our talk concern interviews with 15 pairs of students (grade 2 and grade 4, age 8 to 10). None of these students have gained schooling in statistics before. The interview study consists of different parts. Firstly, the students work on tasks representing simple statistical situations for their own. The statistical situations involve for example jumps of paper frogs, the filling process of candy bags or throws of different dice. Afterwards, the students have to discuss their considerations. Finally, the students were asked to re-enact the statistical situation given in the task.

We analyse the students' considerations according to the construct of mental models (Johnson-Laird, 1983). Further, we use the construct of conceptual change (Vosniadou, 2002) to analyse changes of the students' mental models that occur when the students re-enact the statistical situations. Empirical results of our data analyses yield evidence that these theoretical approaches are suitable for finding deep insights into young students thinking processes when coping with demands of elementary statistical situations. In our talk, we present the theoretical framework in more detail, the task, and, finally findings concerning the students' change of their mental models.

References

- Johnson-Laird, P. N. (1983). *Mental models*. Cambridge: University Press.
- Jones, G.A., & Thornton, C.A. (2005). An overview of research into the teaching and learning of probability. In G.A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 65–92). New York: Springer.
- Vosniadou, S. (2002). Mental Models in Conceptual Development. In L. Magnani & N. Nersessian (Eds.) *Model-Based Reasoning: Science, Technology, Values*. New York: Kluwer Academic Press.

EXAMINING CONSTRUCTS OF STATISTICAL VARIABILITY THROUGH A SEMIOTIC MEDIATION LENS

George Ekol

Simon Fraser University

Statistical variability is considered one of the key elements in developing students' statistical thinking (Wild & Pfannkuch, 1999). Studies, however, show that while students can compute formal measures of variability—range, interquartile range, 5-number summary, standard deviation etc.,—they are challenged by the meanings of these measures and their connections to other concepts (Garfield & Ben-Zvi, 2008). In this study, I assumed that part of students' challenges is imposed by the predominantly static environment in which they learn the notions of variability. I thus designed two sketches using dynamic geometry software (*Sketchpad*) to explore how students reason with the notions of variability in a dynamic computer-based environment. Participants first made predictions and then tested them by dragging statistical data points on the horizontal axis using the dragging tool. I interviewed five undergraduate students registered in a one-semester university level introductory statistics course. Data collection followed one-on-one task-based clinical interviewing and all interview proceedings were videotaped and transcribed. Qualitative analysis of video transcripts (speech, drawings, gesture) relied on two theoretical perspectives: awareness of variability (Wild & Pfannkuch, 1999) and use of signs as means of solving mathematical problems (Bartolini Bussi & Mariotti, 2008). Results show that in the static environment, measures of variability (e.g., standard deviation, mean, and distribution) were more likely to be considered as individual mathematical objects with no links among them. However, after interacting with the sketches, my analysis suggests that students were more likely to link the concepts and clearly state the functional relationships among them.

References

- Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics: Artifacts and signs after a Vygotskian perspective. In L. English, M. Bartolini, G. Jones, R. Lesh, & D. Tirosh (Eds.), *Handbook of international research in mathematics education, second revised edition*. Mahwah: Lawrence Erlbaum.
- Garfield, J., & Ben-Zvi, D. (2008). *Developing students' statistical reasoning: Connecting research and teaching practice*, New York: Springer.
- Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review*, 67(3), 223–265.

DEVELOPMENTAL TREND OF STUDENT LEVEL OF REASONING ASSOCIATED WITH STRATEGY USE IN PATTERN GENERALIZATION

Rabih El Mouhayar, Murad Jurdak
American University of Beirut

This paper explored how student level of reasoning, associated with strategy use in pattern generalization, developed across grade level. For this purpose, a test was designed by the researchers that included four pattern problems. The test was given to 1232 students from grades 4 to 11 and they were asked to predict figural steps. The Structure Of the Learned Outcomes (SOLO) taxonomy, developed by Biggs and Collis (1982), was used to classify students' responses into five hierarchical increasingly complex levels of reasoning: from inability to engage in the task (prestructural); to using one element (unistructural); to several elements in the consecutive terms of the pattern (multistructural); to several elements integrated together by relating the step number with the growth of the pattern (relational); to a generic case (extended-abstract).

Analysis of data showed that: (1) in each grade level, the students used several strategies; (2) the developmental stages were across clusters of grade levels and not from grade level to the next; and (3) the level of reasoning associated with some of the strategies (in particular those that related the figural step number with the growth of the pattern) increased by more than one SOLO level while with the other strategies (those that focused on counting and/or adding consecutive terms of the pattern) it remained within the same SOLO level. The findings of the present study indicate that the developmental trend of student level of reasoning associated with strategy use fits the "overlapping waves' theory" (Siegler, 1996), one of the neo-Piagetian developmental theories, that focuses on the gradual learning process.

The findings have relevance for teaching. Being aware of students' strategies and of the level of reasoning associated with each strategy may assist the teacher in planning classroom activities that foster the development of student level of reasoning in pattern generalization.

References

- Biggs, J., & Collis, K. (1982). *Evaluating the Quality of Learning: The SOLO taxonomy*. New York: Academic Press.
- Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*. New York: Oxford University Press.

HOW LEARNING COMMUNITIES IMPROVE MATHEMATICS PERFORMANCE ON A RURAL BAHAMIAN ISLAND

Marcella D. Elliott-Ferguson, Yolanda A. Rolle
The College of The Bahamas

Learning communities are known to foster a culture where there is collective effort toward a common purpose or goal (Bielaczyc & Collins, 1999). Despite the fact that the archipelago of The Bahamas has more than 20 inhabited islands, one island continues each year to outperform the others on the standardize mathematics national exams. (Hufferd-Ackles, Fuson & Sherin, 2004).

While many schools in the Bahamas are in that rural island environment, the question of why the schools on this island continue to outperform the others has been raised. Our research investigated:

What factors make these schools on this island consistently outstanding in mathematics.

We analysed students' school grades (GPA) and their performance on the standardize national exam, Bahamas General Certificate of Education (BGCSE), comparing them to schools on other rural islands with similar demographics. We also interviewed 49 students, 4 teachers and 4 administrators at these schools.

The results indicate that this close-knit island has created a learning community that has benefited them in and beyond the mathematics classroom. In addition, through discourse students have been able to raise their mathematics proficiency levels to the level of productive disposition. A significant factor to their performance was the facilitation by the community to make mathematics real to the students in their everyday lives. It should be noted that many of the teachers are not native to that island and hence it is the elders in the community that facilitate the success of the students. Leadership by example was clearly the order of the day.

References

- Bielaczyc, K., & Collins, A. (1999). Learning communities in classrooms: A reconceptualization of educational practice. *Instructional design theories and models*, 2, 269-291.
- Hufferd-Ackles, K., Fuson, K. & Sherin, M. (2004). Describing level and components of Math-Talk learning community. *Journal for Research in Mathematics Education*. 35 (2), 81-116.

THE IMPACT OF PRIMARY SCHOOL ON SECONDARY SCHOOL - THE EXAMPLE OF DIVISION BY ZERO

Christian Fahse
Universität Koblenz-Landau

There still seems to be a need for empirical evidence on how concepts established in primary school have an impact on topics of the secondary school level. In the present study we concentrated on students' concepts of division by zero. The answers of 311 students in a German secondary school were analysed on the basis of a filled in questionnaire asking the students to explain their results of $7:0$. Studies concerning division by zero are reported in the educational literature (e.g. Ruwisch, 2008, Tsamir & Sheffer, 2000). Our quasi-longitudinal study examined four different class levels (7, 9, 11, 13) and focusses on the primary school's influence. The research questions were:

- How do the given answers develop over time, i. e. by grade levels?
- Is there an influence of primary education in regard to this specific topic?

The percentage of right answers to $7:0$ starts at about 0% in grade 4 (according to Ruwisch, 2008) and then increases to 27%, 49%, 58% in grade 7, 9, 11 resp. until 75% in graduating classes. Being asked where they learned the answer, about 40 % of all students referred to primary school. Conclusions need to be drawn with caution: Since the official curriculum of primary school does not treat the subject some students may simply associate the subject to primary school. This hypothesis is supported by the fact that in all grades those students who refer to primary school had a higher percentage of wrong answers (e.g. in grade 7: 85% versus 56%, $p < .05$) and were more likely to use concrete arguments instead of formal ones.

Six groups of four or more students, each now in grade 7, visited six different forms in primary school. These six groups were analysed by a chi-squared test of homogeneity using bootstrap due to the small size of the sample. The present data suggest that there is no influence of primary school on the given results but on the way of argumentation (use of concrete examples) and the underlying notion of zero (both weakly significant, $p = .088$ and $p = .084$). In summary there seems to be an influence of primary school education but in a different way than assumed by the students themselves. The important role of coding (i.e. non-cardinal) views of zero (25% in grade 7) and their transition from primary to secondary school are left to further investigation.

References

- Ruwisch, S. (2008). Vorstellungen über null und Null. In: E. Vasarhélyi (Hrsg.): *Beiträge zum Mathematikunterricht*. Vorträge auf der 42. Tagung für Didaktik der Mathematik. Münster: WTM-Verlag
- Tsamir, P., Sheffer, R. (2000). Concrete and Formal Arguments: The Case of Division by Zero. *Mathematics Education Research Journal*, 12(2), 92–106.

MEANINGS OF SPECIFIC TERMS RELATED TO THE CONCEPT OF FINITE LIMIT OF A FUNCTION AT ONE POINT

José Antonio Fernández-Plaza, Luis Rico, Juan Francisco Ruiz-Hidalgo
Universidad de Granada (Spain)

The influence of colloquial meanings and everyday uses of some specific terms from the language of Calculus, namely, “to approach,” “to tend,” “to reach,” “to exceed,” “to converge” and “to limit,” has been reported in several studies, such as Cornu (1991) and Monaghan (1991). We performed an empirical study that focused on the personal meanings that students in upper secondary education (17-18 years old) have of the above mentioned specific terms (we exclude “to limit,” whose meaning is given by the context) related to the concept of finite limit of a function at a point. We use an interpretive framework based on conceptual analysis to establish both the mathematical and colloquial meanings of these terms. We aim to contrast these meanings with students’ definitions; we also consider the effective terms that students use in fact to explain their definitions.

We conducted a semistructured interview for this purpose. The analysis revealed that students showed colloquial and mathematical interpretations, as well as other different interpretations. We stress some distinctions between “to approach” and “to tend” by students, such as “to approach” is more general; “to tend” implies reachability; the action of “to tend” is finite; “to tend” is a technical term. “To reach the limit” is mostly interpreted as “to arrive at or to touch the limit” in a colloquial sense, although some students consider “to reach” as “to know the exact value of the limit or the value of $f(x)$ (continuity).” “To exceed the limit” has a particular interpretation as “to be both above and below the limit”. Finally, some students provided an original definition of “to converge” (this term was unfamiliar to students in the context of finite limit of a function at a point), “The right and left-sided limits are the same number”. This way, our prior analysis of meaning can be extended with other different meanings previously unexpected.

Acknowledgements

This study was performed with aid and financing from Fellowship FPU AP2010-0906 (MEC-FEDER), projects EDU2009-11337 and EDU2012-33030 of the National Plan for R&D&R (MICIN), Subprogram EDUC, and group FQM-193 of the 3rd Andalusia Research Plan (PAIDI).

References

- Cornu, B. (1991). Limits. In D. Tall (Ed.) *Advanced Mathematical Thinking*. (pp.153-166). Netherlands: Kluwer Academic Publishers.
- Monaghan, J. (1991). Problems with the Language of Limits. *For the Learning of Mathematics*, 11(3), 20-24.

DESIGNING VIDEO INSTRUCTION FOR MATHEMATICS

Mikaël Flens, Pauline Vos

Zaanlands Lyceum (The Netherlands), University of Agder (Norway)

Video instruction is emerging in mathematics education; on Youtube a growing number of teachers offer explanations, as for example from www.khanacademy.com. Video instruction supports self-study, both at school and at home. Videos offer students a teacher who can be ‘rewound’ or ‘fast-forwarded’, yet who cannot react to questions. We studied design principles for videos that assist in learning mathematics. The first study was an expert appraisal, in which exemplary videos were shown to mathematics teachers, students (grades 10-12) and video designers. The videos differed in length, tone, format (instructor unseen or visible), use of worked examples (Hilbert et al., 2007) and a procedural/conceptual approach (Kilpatrick et al., 2001).



Figure 1: Mathematics video with instructor unseen (left) or instructor visible (right).

The first study yielded characteristics that the participants preferred. In the second study the found characteristics were applied to the production of 13 videos that were used in an intervention on the topics of *Equations & Inequalities* with 55 students (grade 10). The evaluation of the intervention confirmed that students preferred a clear lay-out, avoidance of unnecessary information and colouring, a serious but friendly voice, and more procedural mathematics. The students confirmed unanimously that the videos were useful for improving their learning. Thus, well-designed videos can supplement instruction, but it is open to argument whether they can replace face-to-face interaction.

References

- Hilbert, T. S., Renkl, A., Schworm, S., Kessler, S., & Reiss, K. (2007). Learning to teach with worked-out examples: a computer-based learning environment for teachers. *Journal of Computer Assisted Learning*, 24(4), 316–332.
- Kilpatrick, J., Swafford, J. & Findell, B. (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

LINKING DRAGGING STRATEGIES TO VAN HIELE LEVELS

Susan K. Forsythe
University of Leicester

Students working in Dynamic Geometry environments interact with geometric figures by dragging objects on the computer screen. Arzarello et al, (2002) provide a cognitive analysis of dragging strategies. For example *wandering dragging* is used to investigate properties of a figure and *guided dragging* is used purposefully to give a figure a basic shape. *Dummy locus dragging* entails dragging an object along an invisible path in order to maintain a certain property. Additionally, Baccaglini-Frank and Mariotti (2010) describe *maintaining dragging*; a form of dummy locus dragging where the desired intention is to keep a certain property constant.

This study investigates whether students' dragging strategies can provide insight into their geometric reasoning using the levels devised by Van Hiele (1986) as a framework for analysis. Briefly the first three levels are: VH1 - visual holistic; recognising and naming shapes, VH2 - descriptive; understanding shape properties, VH3 - structural; e.g. making connections between shape properties and hierarchical classification.

Ten pairs of students (12-13 years of age) participated in the study, working with a dynamic figure with fixed length perpendicular diagonals. They were asked to investigate which shapes they could make by dragging the figure. The dialogue and on-screen activity were recorded and analysed, focusing on the dragging strategies used and the geometrical reasoning employed by students.

Alongside *wandering dragging* and *guided dragging*, (using holistic reasoning, VH1) students were observed to use two new dragging styles. *Refinement dragging* was used to make small adjustments to the figure if displayed measurements of sides and angles indicated it were not exact (using VH2 reasoning). In *dragging maintaining symmetry* (a special case of maintaining dragging), students dragged in a purposeful manner thus generating a family of shapes with symmetry as a common property which suggests development towards VH3.

In conclusion, students used holistic, descriptive and structural reasoning when working with the dynamic figure. Their dragging styles could be linked to the Van Hiele levels giving insight into students' geometrical reasoning while they used dragging and supported their development towards VH3.

References

- Arzarello, F., F. Olivero, D. Paola and O. Robutti (2002). A cognitive analysis of dragging practices in Cabri environments, *ZDM*, Vol. 34(3), pp. 66-72.
- Baccaglini-Frank, A and Mariotti, M. A. (2010). Generating conjectures in Dynamic geometry: The maintaining dragging model. *International Journal of Computing in Mathematics Learning* 15: 225-253.
- Van Hiele, P. (1986). *Structure and Insight*. New York: Academic press.

IDENTIFYING CHANGES IN DISCURSIVE MATHEMATICAL PRACTICES

Cristina Frade, Steve Lerman, Luciano Meira

Federal University of Minas Gerais; London South Bank University; Federal University of Pernambuco

In this paper we propose four domains to observe students and student-teachers' changes in discursive mathematical practices. *The vocabulary domain* is the set of words/symbolism with which one narrates the story of (past, present and prospective) knowing (reading, writing, speaking, doing) about specific mathematical content or aspects of the mathematical practice. Changes in this domain are identified if one is able to recreate or enlarge vocabulary. *The communicational domain* carries a sense of agency and reciprocity. In a previous work, the first author demonstrated that the management of specific vocabulary for the purpose of conversation, especially when one is first faced with a certain mathematical idea or situation, has to some extent a tacit phase and this can become gradually explicit by exercising communicative acts mediated by the teacher, peers or other mediational means. Changes are identified if one is able to (i) gradually improve tacit vocabulary towards the explicit, and/or (ii) communicate relevant aspects of vocabulary that had not been communicated before. *The domain of norms* refers to the regulation of behavior regarding both the mathematical vocabulary and its communication. This subsumes a non-static process in which the application of norms takes into account the contingencies and circumstances to which the learning is subject. Changes are associated with the individual's ability to recreate/redescribe positioning towards those norms or to create new ones in conformity with the meanings produced in these practices. The last domain involves *affective positionings*. Changes here are identified if one is able to develop increased satisfaction with a certain mathematical practice. Using Meira & Lerman's (2010) notion of ZPD as a symbolic space, semiotically mediated, that emerges from a past, through a present, to future relationships, we suggest that these four domains operate as an analytical tool to observe and identify when learning occurs, and in which domain it occurs. Further discussion will be provided.

References

Meira, L., & Lerman, S. (2010). Zones of Proximal Development as fields for communication and dialogue. In C. Lightfoot & M. C. D. P. Lyra (Eds.), *Challenges and strategies for studying human development in cultural contexts* (pp. 199-219). Rome, Italy: Information Age.

TEACHERS DISCUSSING INQUIRY BASED TEACHING WITH DIGITAL TOOLS

Anne Berit Fuglestad
University of Agder

In this paper I report from workshop activities in a developmental research project in which teachers discuss inquiry based activities with digital tools, here computer software, to support pupils' mathematical activities and stimulate learning. The theoretical framework is based on sociocultural theory and project activities funded on key ideas of inquiry and learning communities (Jaworski, 2007). Inquiry is related both to teachers' investigations, wondering and asking questions for their own learning and in planning stimulating activities for their pupils. Theories of instrumental genesis (Trouche, 2005) and teachers' documentation work (Gueudet and Trouche, 2009) is used to seek insight into the teachers' challenges and development arising during the genesis of their own personal instrument and the development of their own documentation systems for teaching. Instrumental genesis is the two-way process by which the user develops an artefact (e.g. computer software) into as a personal instrument. In a similar way the teacher develops a personal documentation system.

The developmental part of the project aimed to support development of teachers' of competence and inquire into how digital tools like spreadsheets, dynamic geometry and a graph plotter can be used in inquiry based teaching. The research was conducted in close collaboration with the developmental work, collecting data using audio and video recordings and written notes from workshops and other project activities. The aim of the paper is to explore and expose key elements in teachers' discussions, in order to understand the developmental process by researching: What are the key issues in teachers' engagement and reflections regarding use of ICT for mathematics teaching in their classes during the project workshops? Findings include: reflections on constraints e.g. curriculum guidelines, the teachers own knowledge and how it develops and inquiry into mathematical tasks and software and use.

References

- Gueudet, G. & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, 71, 199-218.
- Jaworski, B. (2007). Theoretical perspectives as a basis for research in LCM and ICTML. In B.Jaworski, A. B. Fuglestad, T. Breiteig, R. Bjuland, S. Goodchild, & B. Grevholm (Eds.), *Læringsfellesskap i matematikk - Learning Communities in Mathematics* (pp. 121-137).
- Trouche, L. (2005). Instrumental genesis, individual and social aspects. In D.Guin, K. Ruthven, & L. Trouche (Eds.), *The Didactical Challenge of Symbolic calculators* (pp. 197-230). Springer.

LEARNING TO LEARN TOGETHER IN A COLLABORATIVE ENVIRONMENT AND AN ALGEBRAIC MICROWORLD

Eirini Geraniou, Manolis Mavrikis
Institute of Education, University of London

This short oral presents work undertaken in the context of an EU project (<http://www.metafora-project.org>) that designed a web-based platform where groups of students are challenged to solve a problem together. Among other tools, the platform includes appropriately designed microworlds, a web-based argumentation tool and a planning tool, which are used by students to share ideas, organise their thoughts, discuss and argue. The overarching aim behind the system and pedagogy is to promote ‘learning to learn together’ (L2L2) (Wegerif, 2013). Our specific objective is to encourage students to actively participate in their own development of mathematical reasoning and sense-making processes (c.f. Yackel and Cobb, 1996).

We will present data from 12 students split in 4 groups whose task (presented as a challenge) was to construct figural patterns in a microworld designed to promote algebraic generalisation and to discuss the equivalence of the algebraic rules they derived. We follow a group discourse analysis triangulating with the data recorded from the interviews and the various questions during the sessions, but also comparing students’ justification strategies to a framework (presented in a previous PME conference) derived from earlier studies with the microworld when students were working face-to-face on a paper-based task. In this work we are interested in investigating several groups of students tackling the same challenge in a blended learning scenario supported from the design of the collaborative tools.

Key findings include an increased awareness on behalf of the students on the process and outcomes of L2L2 and an appreciation of the collaborative tools’ value with respect to visualizing and structuring their arguments. Our previous findings with respect to the possible justification strategies were replicated with the added value that the persistent nature of the students’ plan and argumentation in the collaborative tools supported a reflection phase that made the L2L2 agenda explicit with evident seeds of sociomathematical norms (ibid). This reflection phase and a subsequent classroom plenary raised discussions of what counts as mathematically different as opposed to just offering “solutions different from those already contributed” i.e. a social norm (ibid, p.461). Next steps involve investigating how transferable these L2L2 skills are in other situations with or without the support of this platform.

References

- Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458-477.
- Wegerif, R. (2013). *Dialogic: Education for the Internet Age*. London: Routledge.

EMPOWERING CONTROL BEHAVIORS OF STUDENTS IN CONSTRUCTING GEOMETRICAL PROOF

Hosein Ghaffari, Amir H. Asghari
Shahid Beheshti University

Geometry has a longstanding reputation as a building block for teaching deductive reasoning. Nevertheless, previous researches show that teachers are faced with many challenges in this direction. The main important ones are helping students to understand the structure of a geometric proof, to construct such a proof, and to write it properly.

During the last century, many attempts have been made to help students to understand, to construct, and to write geometric proofs. Probably, the most famous attempt was introducing two-column proofs (Herbst, 2002). It is now commonly believed that failed since it had strong focus on form rather than content. However, seeing proof constructing as a problem-solving task, two-column proofs can be regarded as a tool to enhance students' behavior at the control level (In Schoenfeld's sense, 1985). This is also the aim of the present study, though with a different tool.

The present study used a revised version of "Reasoning Control Matrix for the Proving Process" (Dimakos et al, 2007). It was a written form with 7 different sections that should be filled by students while solving a geometric problem. Different sections of this tool, asks students to write down their hypothesis, figures, partial proofs, auxiliary elements, scaffoldings and overall thinking process. Unlike two-column proofs, this is designed to encourage students to adopt a backward strategy and somehow plan the chain of reasoning needed before writing the complete proof.

The tool was administrated to a class of thirty-four second grade high school students and it was used time and again throughout the school year in their geometry class. Students' written responses to a range of theorems were collected and analyzed as problem solving protocols.

The results show an improvement in students' understanding of different aspects of the problems, notably, the distinction between hypothesis and conclusion, the role of figures and auxiliary elements, and the ways they are all related to each other.

References

- Dimakos, G., Nikoloudakis, E., Ferentions, S., & Choustoulakis, E. (2007). Developing a proof-writing tool for novice lyceum geometry students. *The Teaching of Mathematics*, X, 2, 87-106.
- Herbst, P. (2002). Establishing a custom of proving in American school geometry: Evolution of the two-column proof in the early twentieth century. *Educational Studies in Mathematics*, 49, 283 -312.
- Schoenfeld, A. H. (1985). *Mathematical problem solving*. New York: Academic Press.

NURSES' CALCULATION OF MEDICINE DOSES: FORMULA VERSUS PROPORTIONAL REASONING

Roslyn Gillies

Deakin University, Australia

Accurate calculation and measurement of medicine doses is critical, with little or no room for error. Nurses begin to develop their skills for calculating doses during their pre-registration education, and continue to adapt and extend these skills throughout their professional lives. Wright (2008) questioned the effectiveness of the commonly taught “nursing formula” because it removes the problem from its context; relies on rote procedures and memory; and limits development of conceptual understanding, which is important for promoting recall and skill transfer. Few researchers have investigated the calculation strategies nurses use in clinical practice or how they relate to the strategies learnt as students. In one of the few such studies, Hoyles, Noss and Pozzi (2001) examined the mathematical practices of 12 paediatric nurses and found they made little use of the “nursing rule”, preferring to use proportional reasoning strategies that retain the situational meaning of the problem.

This paper presents some of the findings from a study that aimed to identify the medicine dose calculation strategies nurses use in clinical practice and compare these with the strategies students are taught in pre-registration programs. The first aim was addressed through naturalistic observation, interviews, questionnaires and focus groups, and the second through a survey of university nurse educators. During 100 hours of observation, 75 registered nurses in three Australian hospitals were observed as they administered over 1500 medicine doses to more than 440 patients. Findings from the observations showed that less than one third of the medicines administered required a calculation, with nurses demonstrating a strong preference for simple mathematical techniques such as fraction operations, multiplicative reasoning and repeated addition when calculations were needed. Furthermore, nurses only reported using the “nursing formula” for approximately 3% of the dose calculations. While a “one-size-fits-all” formula will always deliver the correct dose if correctly applied, preliminary findings suggest that more effective student learning might result if curriculum developers take account of the many other successful “intuitive” dose calculation strategies that experienced nurses demonstrate in their clinical practice.

References

- Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education*, 32 (1), 4-27.
- Wright, K., (2008). Drug calculations part 1: A critique of the formula used by nurses. *Nursing Standard*, 22 (36), 40-42.

CONSTRUCTION AND IMPLEMENTATION OF VIDEO-BASED INTERVIEWS WITH STUDENTS FOR THE ANALYSIS OF THEIR ARGUMENTATION PRACTICES

Manuel Goizueta, Núria Planas
Universidad Autónoma de Barcelona

Our study contributes to the research field on the development of argumentation practices in the mathematics classroom. We argue the importance of exploring meta-mathematical knowledge in relation to what is acceptable as a mathematical argumentation and what is acceptable as a contribution to validate it. The underlying epistemological issues of the classroom discursive practices, along with the access to certain socio-mathematical norms, become then relevant.

Inspired by Sfard and Kieran (2001), we search for mathematical, epistemological and normative aspects of the interaction in a mathematics classroom with students aged 14-15. We first collected data (videos, protocols, and field notes) from a total of 22 students solving probability problems, in small groups and through whole group discussions. The initial experiment served to create the main experimental design, which included a refined version of the problems, and of the problem solving dynamics. At that stage eight focal students were interviewed. The structure and contents for such interviews were decided on the basis of selected ‘critical’ video-clip episodes from the problem-solving lessons that would lead students to reflect more deeply on their argumentations, as well as their individual and collaborative ways of arguing. All video-based interviews were videotaped and transcribed.

In our presentation we will summarize methodological findings around the design, implementation and evaluation of the main experiment, with particular attention to the use and potentiality of ‘critical’ video-clip episodes and video-based interviews. The purpose of making the students visualize and discuss certain episodes was to investigate the criticality of classroom practices with regards to the diverse competing interpretations of what counts as a mathematical argumentation in a particular problem-solving environment. We will also comment on the implications, for future research in mathematics education and classroom discursive practices, of having documented competing interpretations of argumentation.

Acknowledgements

Project EDU2012-31464, Ministry of Economy and Competitivity, Spain.

References

Sfard, A., & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students’ mathematical interactions. *Mind, Culture and Activity*, 8(1), 42-76.

REDESIGNING INTERVENTIONS FOR ENGINEERING STUDENTS: LEARNING FROM PRACTICE

Birgit Griese, Bettina Roesken-Winter, Michael Kallweit, Eva Glasmachers
Ruhr-Universität Bochum

Engineering students' difficulties with mathematics in their first years' studies have been reported in several studies. The project MP -Math/Plus aims at advancing engineering students' performance in mathematics by focusing on learning strategies and self-regulation. Its concept involves continuous critical reflection and redesign of all project elements, forming cyclic processes of thought and instruction experiments typical of design-based research (Burkhardt & Schoenfeld, 2003; Gravemeijer & Cobb, 2006).

Thus, after the first round with only a moderate pass rate, the MP -Math/Plus interventions were modified true to the idea of demanding more commitment from the participating students, providing them with coordinated and linked interventions (including workbooks, learning logs, group meetings and a helpdesk, all focusing on a weekly topic and promoting each other), and planning the withdrawal of our personal support in three phases until the end of the project timeframe. Alternatively nurturing peer support paid respect to students' affective needs.

Evidence of the impact of these changes was given by empirical data, including, among others, demographic variables and exam performance. Out of the 783 (145 female, 19%) engineering students who attended the mathematics exam in March 2012, 58 (22 female, 38%) of these had been able to profit from all of the project interventions. A pass rate of 70.69% as compared to 53.33% in a reference group of comparable abilities suggested the desired effect. Closer inspection, however, revealed no significant differences when comparing exam scores, only the comparison of passes and fails proved almost significant ($p=.052$). Comparing exam scores with reference to gender (male $t(43.487)=1.32$, $p>.05$, $r=.197$; female $t(27.949)=2.91$, $p=.007^*$, $r=0.482$) supplied the insight that women profited more strongly from the redesigned project interventions than men.

References

- Burkhardt, H., & Schoenfeld, A. (2003). Improving Educational Research: Toward a More Useful, More Influential, and Better-Funded Enterprise. *Educational Researcher*, 32(9), 3–14.
- Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational Design Research* (pp. 45–85). London, New York: Routledge.

DIFFERENT APPROACHES OF FOSTERING STUDENTS' MODELLING COMPETENCIES – AN EMPIRICAL STUDY

Susanne Grünewald
University of Hamburg

The project deals with the question of the practicability and the effectiveness of different approaches to foster students' mathematical modelling competencies, which include abilities and a willingness to solve real-world problems by using mathematical modelling (Maaß 2006). Projects to foster students' modelling competencies can mainly be assigned to one of two approaches: either a holistic (tackling whole modelling problems with an increasing complexity) or an atomistic (tackling sub-processes of modelling processes separately) approach (Blomhøj & Jensen 2003). Within the modelling project ERMO (Acquirement of modelling competencies) a holistic and an atomistic approach of mathematical modelling were compared.

During the project 17 classes of 9th grade integrated into their mathematics lessons six modelling activities that either belonged to the holistic or the atomistic approach. To reconstruct the development of various aspects of the students' modelling competencies a written test in a pre-, post- and follow-up-design was implemented. In addition, guided interviews with 8 from 14 teachers were conducted.

The interviews point out that both modelling approaches can be implemented in everyday mathematics lessons. In addition the teachers reported significant progress especially concerning the competence of simplifying the given problems and making assumptions. Teachers of the atomistic group commented that tackling only sub-tasks is less motivating for students and weaker students may experience especially problems in interpreting given models not developed by them. By contrast, teachers of the holistic group stated that the students were able to solve complete problems from the very beginning, although some teachers missed the possibility to foster single sub-competencies separately when necessary. To conclude the most suitable approach to foster students' modelling competencies in everyday mathematics lessons might be an approach with mostly holistic and some atomistic characteristics.

In summer 2013 first results of the evaluation of the modelling tests will be available and presented at the conference. It is expected that sub-competencies of mathematical modelling can be fostered more effectively by the atomistic approach and the competence of performing complete modelling tasks by the holistic approach.

References

- Blomhøj, M., & Jensen, T. H. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning, *Teaching Mathematics and its applications*, 22(3), 123-139.
- Maaß, K. (2006). What are modelling competencies? *ZDM - Zentralblatt für Didaktik der Mathematik* 38(2), 113-142.

REFLECTION OF THE MATHEMATICS TEACHER ON HIS RESOURCES

José Guzmán, David A. Páez

Center of Research and Advanced Studies IPN, México

Several researchers consider that the resources, used by the mathematics teacher, are essential in his teaching practice. This article aims at answering the question: What does the teacher learn from reflecting upon the resources used when teaching algebraic concepts? Gueudet and Trouche (2010) propose the *Documental Approach in Didactics* to analyse the teacher's *resources*. According to this theory, resources are artefacts related to teaching practice and it refers to everything that is used by the teacher in order to solve a given problem; in addition, the teacher's interaction with available resources is known as *documental work*, and a *documental genesis* is produced. This study is of a qualitative kind, data were collected by means of non-participative observation. Four secondary teachers of nine degree algebra participated. For the purpose of this paper, we only focus on one teacher: Peter (pseudonym). He was video-recorded when working on diverse lessons related to algebra, of their textbook; afterward, he was interviewed about used resources, for example, when solving a problem regarding the concept of the angle of inclination of the straight line and its relation to the slope. In class, although one of her students solved the problem, Peter did it differently; however, the problem and the textbook do not discuss the meaning of this angle of inclination and its relationship with the slope of the straight line. Besides, during class, Peter does not reflect on these concepts. As a resource, Peter identified the angles of inclination of two straight lines, whose functions are $d = 100t$ and $d = 80t$, elongating them to the origin of the plane; thus justifying that in this way is possible to "measure" and compare both angles. And he used the opening in both angles of inclination as a mathematical resource (Gueudet & Trouche, 2010). During the interview Peter considered other resources: to move the vertex of the angles of inclination to other coordinates. Also he assures to notice the need of having reflected and thus having given an argument of why the straight lines are elongated to the origin of the coordinates. According to the initial question, we conclude that Peter carried out a reflection of his teaching practice which was carried out in his class. Mainly, that reflection emerged upon analysing the resources he used in the solution of the problem, and, with that, a modification in its documental genesis (Gueudet & Trouche, 2010). However, the results of the analysis from both his class and the interview show Peter's lack of mathematical resources to give a solution to the problem.

References.

Gueudet, G. & Trouche, L. (2010). Des ressources aux documents travail du professeur et genèses documentaires. In G. Gueudet & L. Trouche (Eds.), *Ressources vives*. (pp. 57-74). Rennes, France: Universitaires de Rennes/ INRP.

TEACHER'S WAYS OF GUIDING STUDENTS IN OPEN PROBLEM SOLVING

Markus Häikiöniemi¹, Henry Leppäaho¹, John Francisco²

¹University of Jyväskylä, ²University of Massachusetts

Previous research suggests that the teacher plays a crucial role in guiding students' problem solving (e.g., Anghileri, 2006). However, less is known about the particularities of teacher guidance when dealing with open problems, which include multiple possible solution pathways (Nohda, 2000). The aim of this study is to understand how teachers can support students' open problem-solving processes.

We examined one Grade 9 mathematics lesson in which 14 students worked in pairs on an open problem using GeoGebra. Data was collected by videotaping and capturing the computer screens.

Figure 1 presents a sample of three of the seven student pairs' problem-solving processes. The teacher-initiated transitions are indicated by the dashed arrows.

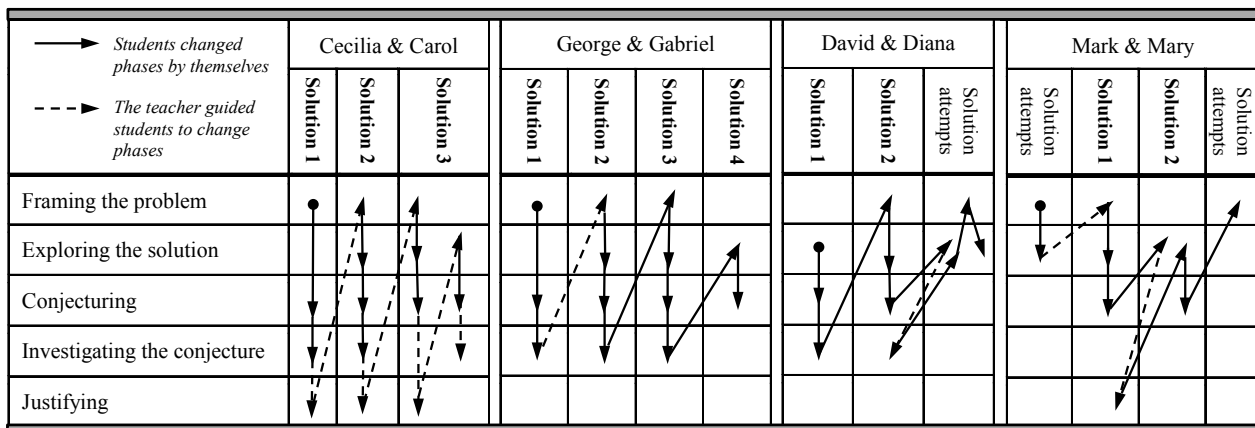


Figure 1. Mappings of three student pairs' open problem-solving processes.

By examining the teacher-initiated transitions, we found nine ways in which the teacher guided the students to change phases: 1) narrow the starting situation, 2) widen the starting situation, 3) use a reasoning-based strategy, 4) explain the solution, 5) reflect critically on a conjecture; 6) test a conjecture in different situations, 7) build more precise solution, 8) return to justification, and 9) listening and trying to understand students' explanation.

References

- Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education*, 9, 33–52.
- Nohda, N. (2000). Teaching by open-approach method in Japanese mathematics classroom. In Nakahara, T. & Koyama, M. (Eds.), *Proc. of the 24th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 39–53). Hiroshima, Japan: PME.

A CODING ON 5TH AND 6TH GRADERS' PICTURES AND TEXTS ON WHAT MATHEMATICS MEANS TO THEM

Stefan Halverscheid, Annika Nieger, Sarah-Elena Wege, Kirsten Werner
Georg-August-University Göttingen

Using pictures and texts for investigating mathematical beliefs has shown that a lot of children's works show an instrumentalist view according to Ernest. This suggests an open coding to finer categories within the category of the instrumentalist view. This was carried out in several steps on content related and process oriented criteria. An inter-rater analysis is carried out and discussed.

Using pictures and texts to investigate mathematical beliefs. Since questionnaires on beliefs have not been implemented successfully yet, an alternative to the use of questionnaires is desirable. Rolka & Bulmer (2005) started to use pictures for investigating mathematical beliefs on statistics. This approach has been further developed (Rolka & Halverscheid, 2011) to a two step design. First, the children are asked: "Imagine you are an artist or a writer and you are asked to show on this sheet of paper what mathematics is for you." Second, guiding questions are used to ask them to write on their beliefs expressed in their work.

Research design. The two steps were given to 157 5th graders and to 188 6th graders. The first third was used for an open coding in which all features were collected, coded and categorized. An essential, here omitted, part of the work is the description of their criteria. An inter-rater analysis on them was carried out with three raters.

Results. The resulting categories are listed with inter-rater results.¹ The latter are good for the facts and quite satisfactory concerning the contexts.

Facts: Numbers (0,65), arithmetic signs (0,88), routine tasks (0,83), specialist's terminology (0,77), 2D geometry (0,82), 3D geometry (0,91), angles (1), units (0,83), Roman and binary numerals (0,87), computations in geometry (0,87), fractions (0,79), coordinates and graphs (0,77).

Context: loose collection of objects (0,68), school (0,84), learning (0,47), applications (1), creative presentation (0,68), presentation of a math. area (0,57), math is hard (0,65), math is fun (0,84), independent math. activity (0,83), math is merely computing (0,65), math is logical (0,60).

References

- Bulmer, M., & Rolka, K. (2005). The "A4-Project" – Statistical world views expressed through pictures. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 193-200). Melbourne, Australia: PME.
- Rolka, K. & Halverscheid, S. (2011). Researching young students' mathematical world views. *ZDM* vol. 43 (4), 521-533.

¹ Median of Cohen's kappa measuring the mutual inter-rater reliabilities.

“SIMPLE” & “NOT SIMPLE” PARAMETRIC EQUATIONS

Dina Hassidov, Bat-Sheva Ilany

Western Galilee College, Israel; Beit-Berl College, Israel

"Problems with parameters test the solver's knowledge of mathematics deeply and detect its weak point" (Sedivy, 1976). He also claims that students encounter more difficulties with parametric equations than with numerical coefficient equations.

This study focused on how 133 advanced mathematics high school students, 115 student teachers and 6 teachers solve parametric equations. The **parametric equations** were divided into two types (Ilany, 1997): **“Simple” parametric equations** - those in which variable in question is eliminated and the solution process requires standard steps of solving first degree or second degree equations. We relate to the knowledge required to solve these equations as procedural knowledge (For example: Find x in: $x^2 + 8ax + 4a^2 = 0$). **“Not Simple” parametric equations** - in which the solution is not achieved by elimination but needs to be performed in several stages. Moreover, judgment and thinking must be exercised before commencing their solution, such as finding the link between the parameters. We relate to the knowledge required to solve the “not simple” equations as conceptual knowledge (For example: find values of m for them the 2 solutions of the equation are the same: $x^2 + 4mx + 3m - 2(m + 3) = 0$).

The research tools included a questionnaire of 5 “Simple” and 5 “Not Simple” parametric equations, interviews and observations.

Generally it was found that the participants encountered difficulties in solving parametric equations, stemming from the very fact that the equations contained parameters. Two principal factors affected the solution of the simple parametric equations: **letter type** and **order of the equation terms**. The “not simple” parametric equations posed an additional difficulty, which was to understand the problem. Therefore the participants had less success with the “not simple” parametric equations than with the simple parametric equations.

In the conference we will present examples of specific difficulties in solving "simple" and "not simple" parametric equations as well as the main results of our study.

References

- Sedivy, J. (1976). A Note on the Role of Parameters in Mathematics Teaching. *Educational Studies in Mathematics*. 7,. 121-126.
- Ilany, B. (1997). *The Concepts of Variable and Parameter of Pre-Service Teachers and High School Students*. Doctoral dissertation (in Hebrew), Tel Aviv University, Tel Aviv, Israel.

AN INNOVATIVE APPROACH TO NEGATIVE NUMBERS

Mathias Hattermann, Rudolf vom Hofe, Harry Kullmann
University of Bielefeld

This paper reports on a pilot study for the investigation of the learning and teaching of negative numbers in the laboratory school of Bielefeld, Germany. The theoretical framework of our study consists of the theory of *basic ideas* or *Grundvorstellungen* (GVs, vom Hofe 1998). The theory of GV's has its correlate in English speaking communities in the term of *concept image* (Tall and Vinner). The ideas of GV's are in line with current research for the teaching of negative numbers using *metaphorical reasoning* (Chiu 2001). The most important research question is the following:

- What mental representations (GVs) can be identified in students' explanations dealing with different didactic models after having finished the course of instruction to negative numbers?

On the methodological basis of lessons studies 3 researchers and 3 teachers worked together to plan a course of instruction for negative numbers that lasted for 12 weeks in one class (grade 7), in which GV's and metaphors played a fundamental role. A card game (for + and -) and a vector-model (for \cdot and $:$) were used to explain the arithmetic operations. Every lesson was audio- and videotaped and the researchers' meetings were audiotaped. In two different evaluations students were invited to write a letter to a classmate to explain him how to solve different tasks. Each time the results were validated by interviews. In the analysis (e.g. task: $-2 \cdot (-3)$) we distinguished between pure syntactic explanations in the letter/the interview (is 6, because minus \cdot minus = plus) and model based explanations (is 6, because the arrow -2 is reflected at 0 and stretched by 3). As a result we can state that 10 out of 17 students used the card-game to explain + and -, whereas only 2 out of 20 students from the same class used the vector model to explain \cdot and $:$ operations. In the presentation further results and identified GV's will be given. In a second step the results will be used to revise the course of instruction. After that the study will be continued with two more classes in grade 7 and a course on a propaedeutic level for grades 3 and 4 will be conceptualized.

References

- Chiu, M. (2001). Using metaphors to understand and solve arithmetic problems: Novices and experts working with negative numbers. *Mathematical thinking and learning*, 3(2-3), 93–124.
- Hofe, R. vom (1998). On the Generation of Basic Ideas and Individual Images: Normative, Descriptive and Constructive Aspects. In J. Kilpatrick (Ed.), *Mathematics Education as a Research Domain: A Search for Identity* (pp. 317–331). Kluwer Academic Publishers.

‘STRATEGY KEYS’ - AN ESSENTIAL TOOL FOR (LOW-ACHIEVING) MATHS STUDENTS

Raja Herold, Bärbel Barzel, Markus Ehret
University of Education Freiburg

The project “Mathe sicher können” (“Fit for maths”) aims at the development and research of structures, concepts and materials for maths lessons, facing especially low-achieving students. Four universities are involved with the Technical University Dortmund as leading partner. At the University of Education Freiburg, we focus on methods in the classroom concerning especially small group interaction. This is to foster students’ mathematical learning and to establish a communicative, interactive and meaningful learning environment.

Current research figured the importance of metacognitive strategies (Montague 2011) and of teaching students to use heuristics (Gersten et al. 2009). Dealing with problems one key obstacle is to understand a problem first (Mason, Burton & Stacey 1985). Only if students understand what to do, they can go further in means of solving a problem. With the method “strategy keys” as a clear organisational structure we particularly guide pupils towards a reflective use of metacognitive and heuristic strategies. The central research question is to investigate the way students use these keys. One aspect of interest is whether or not different categories of keys support an effective use and reflection of strategies. For the students we offer two bunches of keys: 1) Getting stuck before getting started, and 2) Getting stuck while solving a problem. For both, we integrate general hints (metacognitive), general mathematical heuristics and topic-specific strategies.

These questions were investigated in a qualitative study with video sequences in different classrooms and accompanied by pupils’ interviews. Preliminary results show the value of the strategy keys to foster independent learning and especially a reduction of frustration for low-achieving students when getting started to solve a problem.

“Mathe sicher können” is initiated and funded by Deutsche Telekom Stiftung.

References

- Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, O., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, 79(3), 1202-1242.
- Mason, Burton, Stacey (1985). *Thinking Mathematically*. London: Addison Wesley
- Montague, M., Enders, C., & Dietz, S. (2011). The effects of cognitive strategy instruction on math problem solving of middle school students with learning disabilities. *Learning Disability Quarterly*, 34, 262-272.

THE BENEFITS OF PROBLEM-SOLVING WHEN FOLLOWED BY INSTRUCTION THAT BUILDS ON STUDENT SOLUTIONS

Lars Holzäpfel², Katharina Loibl¹, Nikol Rummel¹

¹Ruhr-Universität Bochum, ²Pädagogische Hochschule Freiburg

Multiple studies have shown benefits of problem-solving prior to instruction for conceptual knowledge (e.g. Kapur, 2012). The solutions that students generate during problem-solving can be compared to the canonical solution during subsequent instruction in a classroom discussion. This comparison may guide students' attention to the relevant aspects of the canonical solution (Durkin & Rittle-Johnson, 2012). So far, it is unclear whether the benefits of problem-solving prior to instruction for conceptual knowledge are based on the cognitive processes related to the problem-solving activity or originate from comparing students' solutions to the canonical one during instruction. To investigate these effects separately, we conducted a quasi-experimental study with 247 10th graders varying the two factors *timing of instruction* (problem-solving prior to instruction versus instruction prior to problem-solving) and *form of instruction* (standard instruction focusing on the canonical solution versus instruction that builds on typical student solutions). Instruction and problem-solving each took place during a regular math lesson of 45 minutes on two consecutive days. Students learnt the concept of variance. Learning outcomes were assessed by a posttest with items testing for procedural skills and for conceptual knowledge. We calculated a two-factorial MANOVA with the factors timing of instruction and form of instruction and the outcome variables procedural skills and conceptual knowledge. For procedural skills, we found a small effect for timing of instruction favouring instruction prior to problem-solving ($F[1,243]=4.04$, $p=.05$, $\eta_p^2=.02$). Neither the form of instruction ($F[1,243]=0.03$, $p=.87$) nor the interaction of timing and form ($F[1,243]=0.25$, $p=.62$) was significant. For conceptual knowledge we found a small effect for timing of instruction favouring problem-solving prior to instruction ($F[1,243]=7.16$, $p=.01$, $\eta_p^2=.03$) and a large effect for form of instruction favouring instruction based on typical student solutions ($F[1,243]=32.60$, $p<.01$, $\eta_p^2=.12$). We further found a significant interaction ($F[1,243]=5.30$, $p=.02$, $\eta_p^2=.02$) indicating that the timing of instruction has no effect when combined with standard instruction. In summary, our findings support the notion that problem-solving prior to instruction can prepare students for the acquisition of conceptual knowledge from subsequent instruction. However, in our study this benefit only came to bear when the subsequent instruction built on typical student solutions.

References

- Durkin, K., & Rittle-Johnson, B. (2012). The effectiveness of using incorrect examples to support learning about decimal magnitude. *Learning and Instruction*, 22(3), 206-214.
- Kapur, M. (2012). Productive failure in learning the concept of variance. *Instructional Science*, 40(4), 651-672.

CHILDREN'S CONCEPTIONS OF MEASUREMENT OF AREA AND VOLUME

Hsin-Mei E. Huang

Taipei Municipal University of Education, Taipei, Taiwan, R. O. C.

Findings from interviews investigating fifth-grade children's conceptions of measurements of area and volume indicate a relationship between the quality of children's understanding of the two spatial measures and their reasoning for solving problems involving area and volume measurement.

Introduction. Mathematics researchers proclaim the importance of understanding the meaning of what-it-is-to-measure and measurement vocabulary through activities for assisting children to meaningfully construct measurement concepts (Van de Walle, Karp, & Bay-Williams, 2013). This study examines fifth-grade children's conceptions of measurements of area and volume and how the quality of their understanding of the two measures affects their reasoning about area and volume measures.

Theoretical Framework. Concepts of measurement of area and volume are closely related because the two spatial measures result from multiplicative relationships involving the lengths of the sides. Moreover, acquisition of the attribute of units of measures supports the comprehension of multiplicative relationships, which represents composites of rows and columns that fill in 2- or 3-D space (Lehrer, 2003).

Method. Forty-six fifth-grade children (20 boys and 26 girls) participated in interviews. They were recruited from two public elementary schools in Taipei, Taiwan.

Findings & Implications. The results show that all of the children demonstrated good memory of the formulas and metric units for measuring area and volume. Nevertheless, more than 50% of the children confused "measuring area" with "area" and were unclear about the difference between "measuring volume" and "volume." About 63% of the children knew how to distinguish the differences between area measurement and volume measurement, whereas approximately 37% of the children held misconceptions in this regard. For solving problems involving reasoning of area and volume, children who were in the correct-conception group outperformed the children who were in the misconception group, $t(44) = -2.15, p < .05$. Learning concepts about volume measurement should involve transfer of knowledge taken from measuring area. Instructors need to direct students' attention to attributes of measurement and help them to distinguish the characteristics of various measurements, such as length, area, and volume.

References

- Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics. Teaching developmentally*. (8th ed.). New York: Allyn & Bacon.
- Lehrer, R. (2003). Developing understanding of measurement. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 179-192). Reston, VA: NCTM.

INTRODUCTORY SETTING DESIGN WITH THE PURPOSE OF CREATING STUDENTS' MATHEMATICAL ACTIVITY

Kazuhito Imai, Tomomi Takada

Fukuoka University of Education, Shirouzu Elementary School, Japan

Raising children's awareness of values and the significance of learning mathematics is one of the aims of mathematics education in Japan. The theory of situated learning (Lave & Wenger, 1991), a new viewpoint on learning which emphasizes paying attention to 'activity', is considered to have potential to achieve this aim. Transforming mathematical content into a form of activity should make it easier for students to become aware of the values and significance of learning mathematics. The purpose of this research is to design a mathematical learning environment (lesson) based on the theory and to form specific principles of the introductory setting design with the purpose of creating students' mathematical activity in the classroom. In this paper, the creation of students' mathematical activity is interpreted as evidence of their awareness of values and significance of learning mathematics. Imai (2010) has formed general principles of lesson design as follows: (GP1) Choose a mathematical activity from daily life in order to stage it in the classroom. (GP2) Make students involved in the actual mathematical activity staged by the teacher. It has also formed specific principles of the introductory setting design with the purpose of creating students' mathematical activity of dividing in the classroom as follows: (SPd1) Preparing genuine articles. (SPd2) Considering in advance and preserving the balance between the number of students and things to be divided/shared. (SPd3) Establishing actual human relations among students. Based on these principles, a case-study lesson on the concept of division for a class of 29 second graders who hadn't studied the concept before was conducted by one of the authors of this paper in 2013. At the introductory stage of the lesson, the students received 6 boxes, each containing a different design of a dozen pencils (they didn't know how many pencils there were in each box) as a present. They were then asked to decide what to do with them. In the consideration process, it was the students who asked about the total number of pencils they received and decided to divide them equally among one another. Examining the students' response revealed the effectiveness of the principles for creating students' mathematical activity not only for dividing but also for counting.

References

- Imai, K. (2010). Forming the principles of the introductory setting design of creating students' mathematical activity in the classroom. In Y. Shimizu, Y. Sekiguchi, & K. Hino (Eds.), *Proc.5th East Asia Regional Conference on Mathematics Education* (Vol. 2, pp. 63-70). Tokyo, Japan: Japan Society of Mathematics Education.
- Lave, J. & Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*, Cambridge: Cambridge University Press.

AN ASSESSMENT OF STUDENTS' MATHEMATICAL LEARNING PROCESS USING TEACHING AND LEARNING BASED ON OPEN APPROACH

Maitree Inprasitha, Preecha Kreaowan, Somkuan Srichompoo, Narumon Changsri
Center for Research in Mathematics Education, Faculty of Education, Khon Kaen
University

The purposes of this research were to: 1) describe the “how to” assess student’s mathematics learning process using teaching and learning focusing on open approach, 2) develop a model to be integrated between an instructional activity based on open approach and Thai traditional classroom. This research used a qualitative methodology focus on mathematics learning process in grade 8. In this research, it used the teaching and learning based on open approach as a context to investigate student’s mathematics learning process. Five lesson plans had used for classroom instruction which were developed by the researchers from the Center for Research in Mathematics Education (CRME) and mathematics teachers who participated in this research. There were three phases in this research. In the first phase, the researchers from CRME and mathematics teachers constructed 5 lesson plans together. In the second phase, it used the open approach lesson plans in real mathematics classroom, the researchers and mathematics teacher observed classroom together to assess student’s mathematics learning process. In the third phase, the case study teacher has collaboratively worked with researcher team doing post-discussion or reflection on teaching practice.

The results were the following; 1) An instructional activity based on Open Approach illustrated “how to” has assessed students’ mathematical learning processes as follows: (1) observing (2) journals (3) gathering and arranging information and valuing students’ ideas, student’s solution and also prioritize students’ presentations (4) questioning (5) setting tasks (6) their learning through drawings, action, role play, concept mapping, as well as writing. According to these ways of assessment as an authentic assessment, students’ learning processes can be assessed as follows: (1) problem solving (2) proof and reasoning (3) representation (4) communication.

2) To use protocol analysis and theoretical framework to evaluate mathematical thinking processes had suitability enable to evaluate students mathematical learning processes, mathematics teacher enable to understand and analyze student’s learning processes from student’s verbal that they represented. The results conduct a model that described the relationship between learning processes assessment and teacher professional development using Lesson Study which is consisted of 3 phases (1) collaboratively plan (2) collaboratively do (3) collaboratively reflect.

Acknowledgement: This research was supported by National Research Council of Thailand (NRCT) and Center for Research in Mathematics Education, Khon Kaen University.

PRESERVICE TEACHERS' CONCEPTIONS OF ALGEBRA AND KNOWLEDGE OF STUDENT THINKING

Isil Isler, Eric Knuth
University of Wisconsin-Madison

For many students, algebra continues to be a gatekeeper to future academic and employment opportunities. As a result, reform efforts in recent years have sought to integrate aspects of algebra into elementary and middle school mathematics curricula. In the U.S., for example, the influential Common Core State Standards for Mathematics (Common Core State Standards Initiative, 2010) has called for the inclusion of algebraic thinking as early as kindergarten. Not surprisingly, Blanton (2010) contended that envisioning algebra as a K-12 strand places serious demands with regard to the preparation of teachers, especially at the elementary and middle school levels, in terms of gaining the knowledge and tools necessary to develop and foster their students' algebraic thinking.

This study investigates preservice teachers' conceptions of algebra and the extent to which they recognize student thinking in early algebra tasks. Semi-structured individual interviews were conducted with ten participants. The interview protocol consisted of seven tasks designed to explore elementary and middle grade students' understanding of core aspects of early algebra (including generalized arithmetic, functional thinking, equality and the equal sign). Prior to presenting the tasks and posing questions about them, preservice teachers were asked to define algebra and algebraic thinking, and to illustrate the definitions by giving an example.

The preservice teachers' conceptions of algebra and algebraic thinking as well as their justifications for the categorizations of the tasks as algebra or not related primarily to a symbolic view of algebra and the manipulation of equations. In addition, the preservice teachers were relatively successful in recognizing student thinking, however, they were not as successful anticipating student misconceptions (e.g., student demonstrating an operational view of the equal sign). These results as well as other findings will be discussed in greater detail during the presentation.

References

- Blanton, M. (2010). Early Algebra. In Z. Usiskin, K. Andersen & N. Zotto (Eds.), *Future curricular trends in school algebra and geometry* (pp. 45-61). Charlotte, NC: Information Age Publishing, Inc.
- Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association and the Council of Chief State School Officers.

IDENTIFICATION OF COGNITIVE CHARACTERISTICS OF MATHEMATICALLY GIFTED PRIMARY SCHOOL STUDENTS

Adela Jaime, Angel Gutiérrez

Departamento de Didáctica de la Matemática - University of Valencia (Spain)

The need to pay due attention to mathematically gifted (m-gifted) students is being recognized by most current curricula. To do it adequately, it is necessary to be aware of the specific characteristics of these students' ways of mathematical reasoning. Some researchers have described abilities integrating the mathematical talent for Secondary school students (Krutetskii, 1976; Ramírez, 2012), but this information is very limited for Primary school students (Freiman, 2006). The aim of this research is to identify styles of reasoning characteristic of m-gifted Primary school students when solving problems. Our hypothesis is that these students tend to solve problems in quite different ways, some times in very surprising ways, including originality, intuition, efficiency, visualization and other mental abilities.

By applying a case study methodology, we observed four m-gifted children, aged 7 to 9 years and studying mathematics in grades 2 to 4. They were asked to solve this problem: *The colour of a traffic light changes in this way: Green, yellow, red, green, yellow, red, etc. Which colour will the 26th light be? Which colour will the 330th light be?* Multilink cubes were available. Data were video recording of students actions and dialogues, and their written calculations.

We present and compare the ways those students solved the problem, to evidence that they put to work diverse mental abilities and mathematical contents to solve the problem in different ways. These results confirm our hypothesis stated above, and allow us to identify characteristics of m-gifted students and degrees and styles of m-giftedness.

The results reported are part of the research project *Analysis of Learning Processes by Primary and Middle School Mathematically Gifted Students in Contexts of Rich Mathematical Activities*, funded by the Spanish Ministry of Economy and Competitiveness as part of the Non-oriented Fundamental Research Sub-program of the R&D&I National Program (EDU2012-37259).

References

- Freiman, V. (2006). Problems to discover and to boost mathematical talent in early grades: A challenging situations approach. *The Montana Mathematics Enthusiast*, 3(1), 51-75.
- Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. Chicago, USA: The University of Chicago Press.
- Ramírez, R. (2012). *Habilidades de visualización de los alumnos con talento matemático* [Abilities of visualization in mathematically talented students] (unpublished PhD dissertation). Granada, Spain: University of Granada. Available in <http://fqm193.ugr.es/produccion-cientifica/tesis/ver_detalle/7461/descargar>.

VALIDATING AN ASSESSMENT OF PRE-SCHOOL TEACHERS' MATHEMATICAL KNOWLEDGE

Lars Jenßen¹, Simone Dunekacke¹, Wibke Baack¹, Martina Tengler², Hartmut Wedekind², Marianne Grassmann¹, Sigrid Blömeke¹

¹ Humboldt-Universität zu Berlin, ² Alice Salomon University of Applied Sciences

This paper examines the professional competence of future pre-school teachers. Current research reveals that mathematical literacy acquired in the early years is a strong predictor for later school achievement (Krajewski & Schneider, 2009). The development of mathematical literacy in early education depends on a stimulating environment by pre-school teachers (Reynolds, 1995).

Referring to teacher studies like TEDS-M, a model of pre-school teachers' professional competence was developed. Based on the concept of Shulman (1986), mathematical content knowledge (MCK) can be regarded as one important facet for creating learning environments. We developed a structural model of MCK dimensions and a model of MCK levels as well as an assessment that accurately and economically measures MCK.

The presentation provides information on the following research questions:

- Are we able to validate empirically the underlying dimensional structure of MCK?
- Does the assessment represent MCK adequately (content validity)?

With respect to the first research question, we hypothesize a two-dimensional MCK structure according to which mathematical domains (like number, data) are linked to mathematical processes (like argumentation, problem solving). Data were collected during a pilot study and their results will be presented.

With respect to the second research question, results of a panel of expert analysts which legitimate the assumption of content validity will be presented and discussed. Selected Items of the test will be presented and will be put up for discussion.

References

- Krajewski, K. & Schneider, W. (2009). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, 19, 513-526.
- Reynolds, A. (1995). One Year of Preschool Intervention or Two: Does it Matter? *Early Childhood Research Quarterly*, 10, 1-31.
- Shulman, L.S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, 15(2), 4-14.

RELATION BETWEEN IMITATIVE AND CREATIVE MATHEMATICAL REASONING WHEN SOLVING PHYSICS TASKS

Helena Johansson

Mathematical Sciences, University of Gothenburg

This on-going pilot study is based on a framework that distinguishes between *creative mathematical reasoning* (CR) and *imitative reasoning* (IR) (Lithner, 2008). The former one refers to a reasoning that is anchored in intrinsic mathematical properties and that includes some novelty to the reasoner. If instead the anchoring is in surface properties and the reasoning consists of remembering an answer or following a process step by step, it is IR. The authors overall hypothesis is “how students reason mathematically when solving tasks in physics might have an impact on *their learning of physics* (as the reasoning has on their mathematical learning (Lithner, 2008))”. As an approach to this hypothesis, the framework was used in a previous study by the author to categorise tasks from ten Swedish national physic tests with respect to the kind of reasoning required for solutions.

In this pilot study the above hypothesis is examined further. A quantitative analysis using the Mantel-Haenszel (MH) procedure (Mantel & Haenszel, 1959) is conducted on the categorised physics tasks. The addressed question is “Does students’ success on CR tasks depend on their success on IR tasks?” Success is referred to as when a task is completely solved.

The sample used so far comprises 2612 upper secondary students’ results on tasks from one of the ten categorised physics tests, as well as a teacher indicator for each student. In the MH-procedure one IR-task is compared to one CR-task while possible influence from the teacher is controlled for. The obtained chi-2 statistics (one d.f.) is $17.4 > 3.84$, which is the chi-2 (one d.f.) limit for a 95% confidence interval. This indicates that it is more likely to succeed on a CR-task if the student has succeeded on an IR-task. More studies have to be done in order to generalise the result and to decide if the MH-procedure is an appropriate method to use for analysing dependent between different kinds of reasoning. In the presentation I hope to discuss this further.

References

- Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational studies in Mathematics*, 67(3), 255- 276.
- Mantel, N. and Haenszel, W. (1959). Statistical aspects of the analysis of data from retrospective studies of disease. *Journal of the National Cancer Institute*, 22(4), 719-748.

CONSTRUCTING A SITUATION MODEL OF A TRIGONOMETRY PROBLEM: AN EXPLORATORY STUDY

José Antonio Juárez López, Josip Slisko Ignjatov

Facultad de Ciencias Físico Matemáticas, Benemérita Universidad Autónoma de Puebla

In Mathematics Education the resolution of verbal mathematical problems has been amply researched. The students' performance in mathematical problem solving determines the success or failure of each teaching strategy. For this reason, since long ago, we try to have an insight about the causes that affect such performance improvement. The difficulties for students increase enough when the problems involve a process of mathematical modelling. It is well established, as much theoretically as experimentally, that the modelling process consists of several phases (Borromeo, 2006). The first stage is the construction of a mental model of the situation, which is a construct in the episodic memory representing the event or situation on which the text problem is related with and is necessary for the understanding of a mathematical problem and its subsequent resolution (Nesher et al, 2003). In this sense, van Dijk and Kintsch (1983) point out the need for such models, arguing that these are not merely justifiable constructions, also essential to make understand the comprehension discourse and memory phenomena.

The aim of this research was to verify the role of situational model construction during the textual understanding of a trigonometric problem known as "the fallen tree". The present type of study we introduce is an exploratory nature comparative by applying comparative worksheets with four different versions of this problem: "The broken tree forming a triangle", "The broken tree", "The broken bamboo forming a triangle" and "Bamboo broken". Our hypothesis was that formulating the problem with the bamboo would be easier for students to construct the appropriate situation model. In these activities, students were asked to read the problem and draw the situation described in each version of the problem. The versions of the problem were applied to 193 high school students.

The results show the importance of the proper construction of situation model for textual understanding of the problem and its possible implications for the subsequent construction of the mathematical model and solving process.

References

- Borromeo, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *The International Journal on Mathematics Education*, 38(2), 1–8.
- Nesher, P., HersHKovitz, S., & Novotna, J. (2003). Situation model, text base and what else? Factors affecting problem solving. *Educational Studies in Mathematics*, 52(2), 151-176.
- van Dijk, T. A. & Kintsch, W. (1983). The Cognitive Model. In van Dijk, T. A. y Kintsch, W. (Eds.), *Strategies of discourse comprehension*. (pp.333-385). New York: Academic Press.

MATHEMATICS LANGUAGE AND MATHEMATIZING CAPABILITIES OF PRE-SERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS

Tangul Kabael, Draga Vidakovic
Anadolu University, Georgia State University

Aim of the mathematics learning is shifted from gaining isolated mathematical concepts and skills from daily life to absorbing mathematical knowledge and skills to reality (De Corte, 2004). Mathematizing one of the seven capabilities of the mathematical literacy in PISA framework is described as transforming a real life problem to a mathematical form or interpreting or evaluating the mathematical outcome in relation to the problem situation (OECD, 2010). Another capability of PISA framework is using symbolic, formal and technical language and operations (OECD, 2010). Thompson and Chappell (2007) indicated the importance of using the language of mathematics as an element of mathematical literacy. The goal of this study was to investigate the capabilities of mathematizing and using symbolic, formal and technical language and operations of pre-service middle school mathematics teachers in USA and Turkey. We also intended to evaluate pre-service mathematics teachers' capability in using representations in mathematizing process. This is a qualitatively designed study in which the data was collected through open-ended tests and clinical interviews with sixteen pre-service middle school mathematics teachers, eight of them from the USA and the eight from Turkey. Collected data was analyzed qualitatively by using content analysis technique. The results of the analysis indicated that mathematizing and using graph representation capabilities of the participants from USA were more powerful than that of their Turkish counterparts. On the other hand, Turkish participants were more successful in using the language of mathematics than the participants from USA but were unable to use their conceptual knowledge in mathematizing process.

References

- De Corte, E. (2004) Mainstreams and Perspectives in Research on Learning Mathematics From Instruction, *Applied Psychology*, Vol.53, pp, 279–310.
- OECD (2010). PISA 2012 Mathematics Framework. Retrieved December 8, 2012 from <http://www.oecd.org/pisa/pisaproducts/46961598.pdf>
- Thompson, D.R., & Chappell, M.F. (2007). Communication and representation as elements in mathematical literacy. *Reading and Writing Quarterly*, 23, p. 1-18.

EXPERT RATINGS AS AN INSTRUMENT FOR VALIDATING RESULTS OF VIDEO-BASED TESTING

Gabriele Kaiser, Jessica Benthien, Martina Döhrmann, Johannes König, Sigrid Blömeke

Hamburg University, Vechta University, Köln University, Humboldt University

The TEDS-FU-project, a follow up study of the Teacher Education and Development Study in Mathematics (TEDS-M), aims to analyse the competence development of mathematics teachers at the end of their professional education and the first years of work experience (for details on TEDS-M see Blömeke et al., in press). As part of teachers' competencies, observational, analytical and alternative procedural skills are assessed by video analyses in this longitudinal study. Expertise research shows that there are systematic differences between experts' and novices' perception and analysis of classroom situations, amongst others experts identify key instructional incidents more precisely and also offer more profound alternative approaches for reactions (cf. Carter et al, 1987). These differences in the perception and analysis of classroom situations are in part assessed by four-point rating scales.

The rating scales of TEDS-FU were generated by adapting parts of established instruments from other video studies; they are connected strongly to the video vignettes that were developed for the study and which consider relevant perceptual and analytical elements of mathematics teachers' expertise. Due to the novelty of the approach and the instruments used for assessing teachers' expertise an essential and inevitable research goal is the extensive analysis of reliability and validity of the evaluation instruments through expert ratings.

Altogether, the majority of rating scale items that were developed for the TEDS-FU study achieved highly satisfying results in the expert ratings. However, a few distracting elements clearly emerged in the rating processes and needed strong attention in trying to optimise the rating scale items for video analysis. One main result is that expert ratings on items referring to psychological constructs often do not reach sufficient consensus, which point to principle difficulties in assessing controversially described and understood educational and psychological constructs.

References

- Blömeke, S., Hsieh, F.-J., Kaiser, G., & Schmidt, W. (Eds.) (in press). International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn – IEA TEDS-M Results. New York: Springer.
- Carter, K., Sabers, D., Cushing, K., Pinnegar, S., & Berliner, D. (1987). Processing and using information about students: A study of expert, novice, and postulant teachers. In: Teaching and Teacher Education, 3, 147-157.

THE INTERPLAY BETWEEN FLUENCY AND APPRECIATION IN SECONDARY STUDENTS' FIRST ENCOUNTER WITH PROOF

Ioannis Kanellos*, Elena Nardi*, Irene Biza**

*University of East Anglia, **Loughborough University

Proof is a key interest for mathematics education research as its teaching is tightly connected with students' appreciation of and fluency in it. Our study aims to take a snapshot of how this fluency emerges in the early stages of proof teaching and to explore the relation between fluency and appreciation. Our data analysis tool is the Harel & Sowder (2007) *Proof Scheme* taxonomy. We present here the responses to three Algebra items (A1, A2, A3) of 85 Year 9 Greek students (14-15 years of age). A1 and A2 (a and b) measure primarily proof fluency in employing identities concerning the sum and difference of squares in algebraic proofs. A3 (a and b), measures primarily proof appreciation in asking students to evaluate whether they/their teacher would accept an empirical-inductive argument.

We see proof fluency as evidenced by the presence of D.T. (*deductive transformational*) proof schemes in the scripts; and proof appreciation as reflected in the ability to recognise and value a D.T. proof. We noted high percentages of no solution for items A2a and A2b (58%, 52%; low fluency) and low D.T. percentages for A1 and A2a (14%, 8%; low fluency). However, we noted high percentages of D.T. responses for A2b, A3a and A3b (35%, 45%, 42%; high appreciation) and low percentages of no solution in A3a (27%). These scores suggest that, although proof fluency seems relatively low, proof appreciation seems rather high. Across the scripts fluency and appreciation are related in a range of ways: high fluency but less confident appreciation; limited fluency but solid appreciation; and, high fluency and high appreciation. The first and third are the least surprising – and are often reported in the literature. However the second is remarkable: our scrutiny of the broader set of data (classroom observations, teacher interviews) revealed trace in the student scripts of the teacher's explicit and regular statements concerning what constitutes acceptable mathematical proof, and more widely, fostering high proof appreciation. It seems that fostering an appreciation of proof can start from the students' early encounter it – and even as their technical facility with proof is still emerging.

References

Harel, G., & Sowder, L. (2007). Toward a comprehensive perspective on proof. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*, National Council of Teachers of Mathematics, (pp. 805-842).

STUDENTS' PERCEPTIONS OF BRACKETS

Annasara Karlsson, Robert Gunnarsson

School of Education and Communication, Jönköping University, Sweden

Brackets are essential elements to define a structure in mathematical expressions. Moreover, it is well documented that students' problems with arithmetic calculations are mainly connected to the structure of expressions (Blando, Kelly, Schneideer & Sleeman, 1989). Students tend to focus on the numbers, detaching them from the operations (Linchevski & Livneh, 1999). But, brackets in themselves are not easily interpreted (Kieran, 1979), and the use of superfluous brackets may impede the development of a structure sense (Gunnarsson, Hernell & Sönnnerhed, 2012). However, in previous research the full spectrum of students' problems with brackets has not been described in detail.

Our research aim is therefore to analyse the perceptions of a large number of students of the bracket as a symbol and as a structure element. In particular, we would like to answer the questions "how do students perceive the bracket as a symbol in mathematical expressions?" and "how do students structure their calculations using brackets?". The sample was 84 eighth grade students (age 14-15) in eight classes in four different schools. Data was collected by a questionnaire containing 10 questions/problems asking the student to describe or handle, in total, 35 different arithmetic expressions. The findings suggest that the students have qualitatively different perceptions of what a bracket is. In short, the word 'bracket' was interpreted as either a single arch or as a pair with or without content. But in mathematical expressions the pair could be formed in new ways embracing e.g. another single bracket-arch or an equal sign. Moreover, the students could spontaneously use superfluous (useless) brackets and insert incorrect brackets to detach numbers from operations. In the presentation, the different perceptions observed in the sample will be described in detail and by examples from the students.

References

- Blando, J.A., Kelly, A.E., Schneider, B.R., & Sleeman, D. (1989). Analyzing and modeling arithmetic errors. *Journal of Research in Mathematics Education*, 20(3), 301-308
- Gunnarsson, R., Hernell, B., & Sönnnerhed, W.W. (2012). Useless brackets in arithmetic expressions with mixed operations. In T.Y. Tso (Ed.), *Proc. 36th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 2, pp. 275-282). Taipei, Taiwan: PME.
- Kieran, C. (1979). Children's operational thinking within the context of bracketing and the order of operations. In D. Tall (Ed.), *Proc. 3rd Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 2, pp. 275-282). Taipei, Taiwan: PME.
- Linchevski, L., & Livneh, D. (1999). Structure sense: The relationship between algebraic and numerical contexts. *Educational studies in Mathematics*, 40(2), 173-176.

ANALYSIS OF TAIWANESE STUDENTS' CONCEPTUAL STRUCTURE OF QUADRATIC FUNCTIONS

Ching-An Ko, Feng-Lin Lu, Tai-Yih Tso

Department of Mathematics, National Taiwan Normal University

Students' exploration for mathematical structures and properties of the quadratic function as well as manipulation of conversion between representations which is learning obstacle of students will help them learn more abstract concepts of the function. The purpose of this study aims to explore Taiwanese students' conceptual structure and comprehension on the representations of the quadratic function. We have developed a diagnostic questionnaire based on A cognitive analysis of problems of comprehension in a learning of mathematics (Duval, 2006) and Concept definition, concept image and the notion of function (Vinner, 1983). Therefore, there are two dimensions which involve the mathematical structure and representations of quadratic functions, in the questionnaire. Through quantitative and qualitative analysis of 30 ninth-graders which are middle school students in Taiwan, we can understand students' comprehension differences regarding different representations and mathematical structures and investigate similarity and variation between authentic cognitive construction and experts' intended cognitive construction. The results show that: (1) Only 26.7% students are able to exactly grasp definition of the quadratic function because they consider the concept image to be the concept definition of the quadratic function; (2) 70% students can effectively grasp graphics properties and its structure of quadratic functions; (3) Half of students can't grasp the concept of the graphics motion in the quadratic function since it is hard to determine relation between graphical properties and structures of the quadratic function if the coefficient of the quadratic function is presented by symbols for students; (4) Only about one-third of students can grasp the conversion between the algebraic representation and graphical representation of the quadratic function because of their past learning experiences of quadratic functions, tendency to be operated by vertex-form of quadratic function and considering maximum and minimum value never co-occur under any domain. Therefore, learning of quadratic function should focus on the graphic motion of the quadratic function as well as the conversion between algebraic structures and graphical representations. We suggest the development of digital learning environment in the future study since digital learning environment can characterized by changing the coefficient of quadratic functions and its graph with dynamic linked multiple representations.

References

- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61(1), 103-131.
- Vinner, S. (1983). Concept definition, concept image and the notion of function. *International Journal of Mathematical Education in Science and Technology*, 14, 293-305.

ASSESSING PRE-SERVICE TEACHERS' UNDERSTANDING OF MULTIPLICATION AND DIVISION

Ho Kyoung Ko, Nan Huh, Jihwa Noh

Ajou University, Korea, Kyonggi University, Korea, University of Northern Iowa, USA

Hill et al. (2008) described specialized content knowledge (SCK). SCK is the mathematical knowledge on which teachers draw to select appropriate tasks, explain whether a nonstandard method would work in general, and analyse what students know and need to know. SCK is an important concept in that it helps to give credit to the teaching field as a profession in signifying that the knowledge needed for teaching is specialized. Being able to write relevant story problems is an important task of teaching, which requires a solid understanding of the mathematical idea being illustrated and an anticipation of possible strategies students might use along with pedagogical technique that can be used to teach it. Therefore, tasks in which pre-service teachers are asked to create relevant story problems can be a meaningful context for learning SCK and assessing their learned SCK.

In our study, 135 Korean pre-service elementary teachers and 99 US pre-service elementary teachers were asked to write story problems that could be answered by solving $\frac{2}{3} \times \frac{5}{7}$ and $6 \div \frac{3}{4}$. Their written responses were analysed into four categories: 1) the story illustrates the given operation correctly and meaningfully, 2) the story involves the type of the given operation but shows minor error, 3) the story illustrates a different mathematical operation, and 4) unanalysable response. In order from Category 1 to Category 4, the Korean pre-service teachers' response percentages regarding the multiplication problem were 30%, 14%, 1% and 56%, while the distribution of their responses on the division problem appeared to be 30%, 8%, 9% and 53%. The percentages of the US pre-service teachers' responses on the multiplication problem were 17%, 17%, 43% and 22%. Many of the participating pre-service teachers from both nations were not adept at creating a meaningful context that illustrates a mathematical operation, possibly due to their lack of understanding of the meanings of multiplication and division with fractions and/or their rare experience with this type of mathematical tasks. Tasks of writing story problems can be a pedagogical tool to help pre-service teachers connect to the mathematics they need to teach.

References

Hill, H. C., Ball, D. & Schilling, S. (2008). Unpacking Pedagogical Content Knowledge: Conceptualizing and Measuring Teachers' Topic-Specific Knowledge of Students. *Journal for Research in Mathematics Education* 39(4), 372-400.

IDENTIFYING MULTIPLE FUNCTIONS OF TEACHERS' QUESTIONING IN GERMAN AND JAPANESE MATHEMATICS CLASSROOMS

Yuka Koizumi, Yoshinori Shimizu
University of Tsukuba, University of Tsukuba

It is widely acknowledged that “teaching is a cultural activity” (Stigler and Hiebert 1999), which takes place in particular cultural and social environments. Cross-national research therefore provides an opportunity to better understand our implicit assumptions about teaching and learning processes.

The TIMSS 1995 Videotape Study (Stigler & Hiebert, 1999), which analysed eight-grade mathematics classes in Germany, Japan and the United States, revealed that there are interesting similarities between German and Japanese classrooms. For example, mathematical concepts were likely to be “Developed” in German and Japanese lessons, specifically focusing on the stage of introducing new mathematical concepts. Based on the result, this study aims to reconsider the similarities and differences identified in the TIMSS Videotape Classroom Studies from the participants’ perspectives. For this aim, subsets of the data on German and Japanese lessons in the Learner’s Perspective Study (Clarke, Keitel & Shimizu, 2006) are analyzed. The data analysis is conducted both quantitatively and qualitatively that intended to capture the outline of appearance of teacher’s questioning and to describe the existing patterns in which the German and Japanese teachers used questioning in their mathematics classrooms.

The results of analysis reveal that there are multiple functions of teachers’ questioning within the sequence of mathematics lessons in Germany and Japan. Both of the teachers’ questioning functioned as a guide for students to solve a task in the whole class discussions. On the other hand, there are different functions of connecting mathematical ideas and building understanding of mathematical concepts both within and across the lessons. Those differences illustrate the cultural values behind what can be considered as quality mathematics teaching in different contexts.

References

- Clarke, D., Keitel, C., & Shimizu, Y. (eds.) (2006). *Mathematics Classrooms in Twelve Countries: The Insider’s Perspective*. Rotterdam: Sense Publishers.
- Stigler, J. W. & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World’s Teachers for Improving Education in the Classroom*. New York: NY, The Free Press.
- Koizumi, Y. (in press). Similarities and differences in teachers' questioning in German and Japanese mathematics classrooms, *ZDM - The International Journal on Mathematics Education*.

SCHOOL MATHEMATICS AND BUREAUCRACY

David Kollosche

Universität Potsdam, Germany

With regard to the dominance of traditional calculation tasks, school mathematics has often been called ‘bureaucratic’. However, this attribution has received hardly any attention in research. Skovsmose’s (2005, pp. 11–12) suggestion that the dominance of calculation tasks serves as a preparation for jobs in administration gives rise to the research questions of this study: In how far is calculation connected to bureaucracy and what does that tell about the social functions of school mathematics?

My theoretical analysis compares Max Weber’s (1947) theory of bureaucratic administration with Sybille Krämer’s (1988) history of formalisation, looking for developmental analogies and conceptual similarities. The latter are more deeply understood under the light of the epistemological shift around 1600 which was first analysed by Michel Foucault (1970). Finally, my research comes back to mathematics education, drawing some conclusions concerning the social functions of school mathematics.

From the comparison of bureaucracy and calculation, it can firstly be concluded that most advancements in the formalisation of calculation temporally and locally collided with the existence of strong bureaucratic administrations. It can secondly be found that both calculation and bureaucracy share a common style of thought which is based on the reduction of situations to cases and the impersonal treatment of those cases along pre-given rules. Finally, it can be shown that this style of thought follows an epistemological trend which was fully cultivated around 1600 and is typical for modern thinking – a trend which establishes a man-made and autonomous symbolic realm unconnected with ‘reality’. On this basis, it can not only be argued that calculation tasks – unlike any other tasks in school – require, train, cultivate and test a bureaucratic style of thought; it can also be argued that mathematics education introduces the student to a style of thought typical for modernity. Although hardly recognised in research, the cultivation and testing of a bureaucratic style of thought can be understood as a central social function of school mathematics. Furthermore, mathematics education can be understood as an institution which implicitly creates acceptance for bureaucratic practices. Taking this serious can inspire further research and allow a deeper understanding of problems in the practice of mathematics education.

References

- Foucault, M. (1970). *The Order of Things*. New York: Pantheon.
- Krämer, S. (1988). *Symbolische Maschinen [Symbolic Machines]*. Darmstadt: WBG.
- Skovsmose, O. (2005). *Travelling Through Education*. Rotterdam: Sense.
- Weber, M. (1947). *Theory of Social and Economic Organization*. New York: Oxford Univ. Press.

STUDENTS' LEVEL OF 3D GEOMETRICAL THINKING: THE INFLUENCE OF 3D REPRESENTATIONS

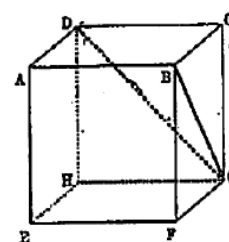
Yutaka Kondo¹, Taro Fujita², Susumu Kunimune³, Keith Jones⁴

¹Nara University of Education, Japan; ²University of Exeter, UK; ³Shizuoka University, Japan; ⁴University of Southampton, UK

While representations of 3D shapes are in common use in the teaching of geometry in lower secondary school, it is known that the ways that 3D shapes are represented in 2D can provide challenges for students (Parzysz, 1988). In this paper, we use the levels of 3D geometrical thinking proposed by Gutiérrez (1992) to analyse students' reasoning about a 2D representation of a 3D shape.

Gutiérrez's level descriptors synthesise the two aspects of a) recognising the properties of 3D shapes and comparing 3D objects, and b) manipulating different representational models of 3D objects and obtaining correct answers. The levels can be described as follows: Level 1 No manipulations of representations; Level 2 Some manipulations; Level 3 More advanced manipulations; Level 4 Effective manipulation.

As a part of a larger study, in the research reported in this paper we analysed classroom episodes from two experimental lessons in which 28 Grade 7 students (aged 12-13) tackled a challenging problem in 3D geometry. The problem was to find the size of angle BGD in the accompanying diagram of a cube.



Through analysis of the students' responses during the lessons, we found that students at lower levels of 3D thinking could not manipulate representations effectively, e.g. they drew a net of the cube and by referring to the net concluded that angle BGD was 90 degrees. In contrast, students operating at higher levels of 3D thinking could reason correctly; for example, they could change the oblique parallel projection of the cube to an orthogonal projection in order to demonstrate that angle BGD was 60 degrees. Nevertheless, we found that even these students sometimes were unable to reason correctly for the problem. Based on our analysis, we propose that it is necessary to modify the level descriptors suggested by Gutiérrez in order to better capture students' understanding of representations of 3D shapes. Our proposal is to sub-divide levels 2 and 3 with two sub-levels each.

References

- Gutiérrez, A. (1992). Exploring the links between van Hiele levels and 3-dimensional geometry. *Structural Topology*, 18, 31–48.
- Parzysz, B. (1988). 'Knowing' vs. 'seeing': problems of the plane representation of space geometry figures, *Educational Studies in Mathematics*, 19(1), 79–92.

CHANGING TEACHERS' BELIEFS IN A PROCESS OF COLLECTIVE PRODUCTION OF MULTIPLE PROOFS

Margo Kondratieva
Memorial University, Canada

Teachers' beliefs about types of acceptable proofs may vary significantly from reliance on verbal, visual, empirical, generic, or inductive reasoning to strict requirements of following formal steps, symbolic and deductive derivations (Hanna & De Villiers, p. 332). Teachers' beliefs may become an obstacle for students' learning if, for example, a teacher on a purely formal basis rejects an insightful student's solution. Thus, it is important for teachers to (1) recognize many aspects of proofs such as explanation, exploration (ibid, p. 3), (2) value multiple proof tasks (ibid, p. 198), and (3) consider ability of students to prove as a developing skill (ibid, p. 45). But how may teachers acquire such a perspective? Here I focus on item (3).

This project aims to observe instances of *productive* collaboration (when participants reveal and build on mathematical connections between individual ideas) in a group of math teachers working on a proof. The research question is: "What kinds of changes do instances of productive collaboration produce in teachers' beliefs about proofs?"

The following statement allows both elementary and advanced proofs: "In a square ABCD with E the mid-point of CD, join B to E and drop a perpendicular from A to BE at F. Then DF and AB are of equal length." Forty primary and secondary school teachers were asked to verify it by using reasoning available at their grade level, to negotiate their solutions, and to report reflecting on the process of their collaboration.

Instances of productive collaboration were evident in many cases. For example, in order to show that segments DA, DF, DC are equal, a primary teacher draws a circle for which the segments appear to be its radii. A secondary teacher confirms this fact using Cartesian equations of a circle and lines. Another teacher shows that ADF is an isosceles triangle by means of a synthetic proof that its altitude DK is also a median. Her group mate illustrates this idea by folding (along DK) the figure drawn on paper, and indicating that points A and F will coincide. Both examples reveal a cognitive unity (ibid, p. 118) of an empirical argument and rigorous proof construction.

Teachers agreed that instances of productive collaboration initiate a paradigm shift: By recognizing how the mechanisms of secondary school approaches are rooted in primary level activities, teachers start to view learning to prove deductively as a gradual and multimodal process originated from reflections on empirical actions.

References

Hanna, G., De Villiers, M. (Eds.) (2012). *Proof and Proving in Mathematics Education: The 19th ICMI Study*. New York: Springer.

FACTORS THAT CAUSE CONTEXT DEPENDENCY IN FUNCTION BETWEEN MATHEMATICS AND SCIENCE

Masato Kosaka

Graduate School for International Development and Cooperation,
Hiroshima University, Japan.

Connecting mathematics and science is supported by a number of countries and educational associations. However, students often fail to connect both subjects by themselves (e.g., Kosaka, 2012). This phenomenon is known as context dependency, meaning that learning is context dependent. Although context dependency between mathematics and science has been recognized, little is known about factors that cause context dependency between mathematics and science. The objective of this study is to examine the factors that cause context dependency in the case of the topic “Function” between mathematics and science at high school level in Zambia.

One hundred and sixty-five students in Grade 12 at a high school in Zambia were chosen for this research. Their academic performance was average among high schools in Zambia. The research was conducted by using the same two types of tests about function providing different context between mathematics and science. The instruments were developed based on previous studies (e.g. Ishii et al., 1996). The two tests were conducted on the different dates in order to avoid the influence of the first test. Context dependency was studied by using chi-square test. Factors that cause context dependency were examined through students’ used solving methods and interview answers.

The results showed that the five questions out of twelve caused context dependency. The analysis concluded that there are two factors that cause context dependency. One is the difference in formulae the students learn in each subject. When the interviewer told them to use mathematics formula in science test, they were able to solve it by using the mathematics formula, and vice versa. The other is the difference in characteristics of the subject. When the interviewer explained the situation by using the science context in mathematics test, they were able to solve it, and vice versa. In order for students to be able to apply the concept of function in both mathematics and science, teachers need to understand that the same concept is taught in a different way in mathematics and science.

References

- Ishii, T., Minowa, A. & Hashimoto, Y. (1996). A study on a Cooperative Instruction between Mathematics and Science: Suggestions to overcome context dependency. *Journal of Science Education in Japan*, 20(4), 113-117 (In Japanese).
- Kosaka, M. (2012). Context dependency in function between mathematics and physics: focusing on high school pupils in Zambia. *Proceeding of 12th International Congress on Mathematics Education* (p. 7606). Seoul, Korea: ICME.

CHANGES IN TEACHING EQUATIONS WITH ONE UNKNOWN AFTER PARTICIPATING IN LEARNING STUDIES

Angelika Kullberg, Ulla Runesson, Pernilla Mårtensson

University of Gothenburg, University of Jönköping, University of Jönköping

This paper reports on findings from a study about teachers' change of teaching practice after participation Learning studies (Marton & Tsui, 2004). Before they participated, we recorded each teacher's reflections on one particular lesson plan before it was taught, and video recorded the enacted lesson. The same thing was done when the teachers had taken part in three Learning studies (LS) two years later. We report on the analysis of data from one of the teachers in the project teaching equations with one unknown. Students' learning of algebra has been studied extensively (cf., Kieran, 2007). This study contributes to previous research by exploring differences in how the same content is taught differently by the same teacher. The research question elaborated concerns; in what ways did the teacher change his teaching after participating in three consecutive Learning studies?

The data was analysed with a variation theory framework (Marton & Tsui, 2004). The analysis demonstrates that the teacher's intentions with the lesson, how it should be taught, as well as how it was taught in class were different before and after the LS. In the "before lesson" (lesson 1) the teacher taught the topic by using two word problems that were set up as equations with one unknown and another task solved with visual representation. The method of solving was taught in terms of a procedure of a technique. In the "after lesson" (lesson 2), however, the teacher taught the topic without word problems, only by numeric representation. The answers to the equations (the x-value) were already given, and the emphasis was not on how to solve the equation. Instead, aspects of the solving of equations with one unknown were highlighted; that is, how different operations (addition, subtraction, multiplication and division) work in an equation became the focus. In this way, the method of solving was taught with emphasis on the conceptual understanding of using the operations as well as on the procedure. Consequently, lesson 1 and 2 made it possible for the learners to discern different features of the topic taught.

References

- Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels. Building meaning for symbols and their manipulation. In F. K. Lester (Ed.), *Second handbook of research on Mathematics Teaching and Learning*. Charlotte, N.C: Information Age.
- Marton, F., & Tsui, A. B. (2004). *Classroom discourse and the space of learning*. Mahwah, N.J.: Erlbaum.

OPERATIONALISING THE ZPD AS A ‘SEMANTIC SPACE’

John Kuszniarczyk

Melbourne Graduate School of Education, University of Melbourne

Vygotskii’s zone of proximal development (ZPD) is an object of pedagogical knowledge that perhaps suffers from being more ‘known of’ than known. Its underlying principle is easily stated as the ‘distance’ between a subject’s present developmental state and a potential level of development (determined by guided problem solving activity) (Vygotskii, 1978, p. 86) but its explanatory power risks being lost if it is not understood in relation to the rest of Vygotskii’s work. Meira and Lerman (2001) begin to address this risk with their conception of the ZPD as a ‘symbolic space’ that emerges (or not) in interaction rather than the simplistic notion of an individual’s readiness to learn. Using Meira and Lerman’s report (op. cit.) as its point of departure, this research ‘unpacks’ the ZPD as a ‘semantic space’ and compiles a toolset for the analysis of spoken interaction to arrive at examples illustrating the possibility of its operation, the aim being to establish a systematic and empirical basis for understanding the ZPD and its function in the maths classroom.

This research’s interdisciplinary approach synthesises complementary accounts of semiosis taken from linguistics, sociology, psychology and literary criticism to arrive at a coherent ‘organising principle’ for the objects used to describe the maths ‘lesson’ as a site of meaning making.

Transcripts of natural episodes of maths classroom interaction are analysed and presented as examples of generic interactional ‘tropes’ along with descriptions of the semantic features that indicate the possibility of the ZPD’s operation in the ‘semantic space’ between individuals at any given moment. These examples and the organising principle used to describe them are intended for the benefit of classroom teachers, providing a critical vocabulary and practical examples to support their understanding of a complex and powerful principle. The value of this approach is not in the certitude of descriptions of educational objects like the ZPD but in the disciplined and systematic critique of such descriptions based on empirical evidence.

References

- Meira, Luciano, & Lerman, Stephen. (2001). The zone of proximal development as a symbolic space *Social Science Research Papers No. 13*. London: South Bank University, Faculty of Humanities and Social Science.
- Vygotskii, L. S. (1978). *Mind in society: the development of higher psychological processes*. Cambridge: Harvard University Press.

USING MATHEMATICAL MODELLING TO ENGAGE STUDENTS IN REAL WORLD PROBLEM SOLVING

Meek Lin Kwek

Raffles Girls' School (Secondary), Singapore

What can Mathematics teachers do to prepare students for handling the complexities in the 21st century workplace? This scenario is an impetus for learning and applying mathematical content as a basic tool - for thinking critically within and across disciplines, processing information, working collaboratively with others in a technologically driven environment. The relevance of being equipped with these skills in the twenty-first century implies the need for a future-oriented perspective on the teaching and learning of problem-solving.

Teaching through problem-solving is an approach that treats problem-solving as integral to the development of an understanding of any given mathematical concept or process (Lesh and Zawojewski, 2007). Mathematical modelling is one such approach.

In this study, 120 grade 9 high-ability students participated in a modelling activity in which they were facilitated by teachers in using a four-step problem-solving model (Polya, 1945) to elicit their own mathematics as they work the problem. The students were first presented with some statistical data from which they applied their mathematical knowledge to construct and analyse the models, used them to make prediction about future trends and evaluated their usefulness. In this process, the teachers structured the discussions by documenting students' (1) understanding of the problem task, (2) plan of action, (3) carrying out the mathematical procedures, and (4) examining the solution with the assumptions made.

The students' task report, discussion document and perception survey results were analysed for consistency and quality of performance. The findings present preliminary evidence of the opportunities provided for the learning environment, teachers' instruction and facilitation, as well as students' performance as they engage themselves in explicitly using the real world context to draw upon several disciplines such as social studies and economics.

References

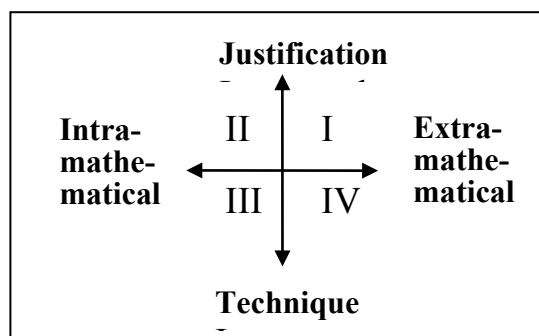
- Lesh, R. & Zawojewski, J. (2007). Problem-solving and modeling. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*, (pp.763-804). Reston, VA: NCTM.
- Polya, G. (1945). *How to solve it*. Ed. Tecnos. Madrid. Espana.

A FOUR-FIELD MODEL FOR ANALYSING DIALOGUES IN MATHEMATICS CLASSROOMS

Niclas Larson
Linköping University

The transition from lower to upper secondary school in mathematics is problematic for many Swedish students, indicated by the achievement levels on the national tests. In order to understand the character of those problems, mathematics classrooms from the last year in lower secondary school and the first year of upper secondary school were studied, where a sample of students were followed during their transition. Classrooms at both school levels were video and audio taped with a focus on whole class teaching as well as teacher-student dialogues. The analysis of the data aims to identify differences in the character of the knowledge emphasised in the two school sectors, which may contribute to the observed difficulties during the transition. Some ideas of how to develop theoretical tools flexible enough to analyse and describe the classroom practices in both lower and upper secondary school are presented. The appropriateness of the theoretical approach chosen for studying this transition problem has been further discussed in Larson and Bergsten (in press).

In this paper a four-field model that draws on Bernstein's concept *classification* (Bernstein, 2000) and the notion *praxeology* from the anthropological theory of didactics (e.g. Chevallard, 2002) is presented (see figure). It can be used for analysing the extent to which the tasks discussed in the dialogues are 'pure' or 'applied' mathematics and the knowledge emphasised is focussed on techniques for solving the tasks or on the justification of the techniques. A first application of the model for analysing the teacher-student dialogues supports its usefulness for identifying such differences between the mathematical work at the two school levels. That the work was placed mainly in field III at the lower level may be critical for the transition.



References

- Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research and critique* (Rev. ed.). Lanham: Rowman and Littlefield.
- Chevallard, Y. (2002). Organiser l'étude 1. Structures & fonctions. In J.-L. Dorier, M. Artaud, M. Artigue, R. Berthelot, & R. Floris (Eds.), *Actes de la 11e école d'été de didactique des mathématiques* (pp. 3-32). Grenoble: la Pensée sauvage.
- Larson, N., & Bergsten, C. (in press). Comparing mathematical work at lower and upper secondary school from the students' perspective. To appear in the proceedings of CERME8, Antalya, Turkey, 6-10 February 2013.

EFFECTS OF FUTURE MATHEMATICS TEACHERS' PRECONDITIONS ON THEIR KNOWLEDGE AT THE END OF THE TEACHER EDUCATION IN GERMANY AND TAIWAN

Christin Laschke, Gabriele Kaiser, Sigrid Blömeke

Humboldt University of Berlin, University of Hamburg, Humboldt University of Berlin

The results of the “Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)” replicated for future mathematics teachers on the country level the findings of large-scale assessments of K-12 student achievement, i.e. Taiwanese teachers outperformed their German counterparts (Blömeke & Delaney, 2012). The question is which characteristics prior to teacher education affected the future teachers’ knowledge at the end of their training, whether differences in the relationship between preconditions and outcomes existed and – if so – whether these differences point to cultural differences. Cultural theories with respect to the Chinese Confucian heritage and the classical Latin/Greek/Christian tradition including their respective educational traditions serve as a frame of reference to explore the latter research question.

Two representative samples of 771 future lower-secondary mathematics teachers from 13 federal states in Germany and 365 future teachers from 19 training units in Taiwan in their final year of teacher education were used to examine the effects of preconditions on mathematical content knowledge, mathematics pedagogical content knowledge and general pedagogical knowledge. The teachers’ demographic background (gender, parent education, first language), their prior knowledge (high-school achievement, level of mathematics lessons), affective characteristics (intrinsic, pedagogical, extrinsic motives) as well as family and money related constraints during teacher training were used as predictors while controlling for differences in opportunities to learn. Because of the nested data structure, hierarchical linear models were estimated.

Whereas the Taiwanese teacher-education system is apparently able to develop high achievers widely independent from their preconditions, the knowledge of German teachers is significantly affected by these (e.g. by prior knowledge). This result may imply that efforts to improve teacher education should be directed towards recruiting school graduates with more favourite preconditions. Hence, the attractiveness of the teaching profession and teacher education in Germany will be discussed.

References

Blömeke, S. & Delaney, S. (2012). Assessment of teacher knowledge across countries: A review of the state of research. *ZDM – The International Journal on Mathematics Education*, 44 (3), 223-247.

MODELING AND ALGORITHMIC APPROACH TO A RANDOM SITUATION: A HELP FOR STUDENTS' UNDERSTANDING?

Dominique Laval

L.D.A.R. – Diderot University (Paris 7)

Introduction and Theoretical Framework

The new French curriculum for upper secondary mathematics introduces building algorithms as a task for every topic. The underlying assumption is that an algorithmic approach can help students access to mathematical notions. Considering the case of probability our hypothesis is that building algorithms to simulate random situations can support students' understanding of the underlying probability law: **1.** Identifying random variables in a situation, simulating by way of a random generator and observing frequencies and means can help to approach the underlying law **2.** Various environments (by hand, spreadsheet, algorithmic languages...) offer specific opportunities and constraints for simulating the same random situation.

Our theoretical framework is based on a transposition of "Paradigms and Geometrical Workspaces" (Kuzniak, 2003) and on Duval's (2006) semiotic approach in order to consider activities and registers associated to specific simulation environments. We consider also Keune & Henning's, (2007) levels of modelling skills in order to evaluate students' activity when building algorithms for simulating.

Methodology and First Research Findings

The method is to implement and evaluate teaching units at various grades based on random situations chosen in order that the underlying laws are not directly accessible to students. Consistent with hypothesis 2, we choose to ask students to build a simulation in varied environments. Our results confirm the specific opportunities and constraints of the environments: by hand helps an entry into the situation, but simulation is limited, the spreadsheet allows an easy implementation but does not really contribute to understanding the law; algorithmic languages allows students to a better identification of different steps in constructing the law, while the necessity of mastering formalized languages creates new challenges. Our research work continues, investigating the contribution of algorithmic activities to students' understanding of other mathematical ideas like proof.

References

- Duval, R. (2006). Quelle sémiotique pour l'analyse de l'activité et des productions mathématiques ? *Relime, Numero Especial*, pp. 45-81.
- Henning, H., & Keune, M. (2007). Levels of Modelling Competencies. *Modelling and Application in Mathematical Education-The 14th ICMI Study*, pp. 225-232. Berlin: Springer.
- Kuzniak, A. (2003). *Paradigmes et espaces de travail géométriques, Notes d'habilitation*. Paris : IREM Université de Paris VII.

HOW TEACHERS NOTICE AND ANALYSE MATHEMATICS LESSONS

Claudia Lazarevic
University of Hamburg

Mathematics teachers' noticing as a central facette of professional competence currently is a highly discussed topic in mathematics educational research (Sherin et al. 2011). This issue is also focussed in studies like COACTIV and TEDS-FU. The connections between professional competence and teachers' noticing and analysing are the starting point of qualitatively oriented study. My study aims at developing a theoretical model of this relation.

14 novice teachers for mathematics in primary school, who have worked as teachers for one or two years, took part in the study. In order to obtain insight into teachers' noticing and analysing, all participants had to analyse a video vignette during a guided focused interview. Within these interviews the teachers were asked about their own teaching, about the shown video vignette and about their knowledge of selected concepts of mathematics education.

The interviews are evaluated based on methods of grounded theory. As theoretical framework Schoenfeld's model of teaching as a decision making process (Schoenfeld 2011a) and his assumption that "teachers' noticing is intimately tied to their orientations", (Schoenfeld 2011b) are guiding the data analysis.

The data analysis shows, that the teachers analyse the video vignette by using different orientations, which leads to two different types of how teachers analyse lessons. On the one hand, normative schemata like knowledge from didactics of mathematics or general pedagogy can be identified as guiding orientations; these orientations can be described as the normatively structured. On the other hand, teachers evaluate the lessons according to the reaction of the shown students and the students' working discipline, to their experience or to their feelings; these orientations can be described as intuitively structured. The study will lead to various types of noticing and analysing mathematics lesson by teachers. This can lead on the long run to models of the continuous professional of teachers.

References

- Schoenfeld, A. (2011a). How we think. A theory of goal-oriented decision making and its educational applications. New York: Routledge.
- Schoenfeld, A H. (2011b). Noticing matters. A lot. Now what? In M. G. Sherin, V. R. Jacobs & R. A. Philipp (Eds.), Mathematics teacher noticing. Seeing through teachers' eyes. (pp. 223-238). New York: Routledge.
- Sherin, M. G., Jacobs, V. R., Philipp, R. A. (Eds.) (2011): Mathematics teacher noticing. Seeing through teachers' eyes. New York: Routledge.

AUSTRALIAN INDIGENOUS STUDENTS AND NAPLAN MATHEMATICS ACHIEVEMENT

Gilah C. Leder, Helen J. Forgasz, Jennifer Halliday
Monash University

The *National Assessment Program – Literacy and Numeracy* [NAPLAN] was introduced in Australia in 2008. Each year since then, NAPLAN tests have been administered nationally to students in Years 3, 5, 7, and 9. Results from these tests in different years are reported on the same achievement scale so that a student's performance on successive testings can be compared.

Indigenous status, based on students' self-identification as being of Australian Aboriginal or Torres Strait Islander descent, is among the student background information gathered. Indigenous students, on average, perform well below their non-Indigenous peers on traditional measures of achievement. For example, national NAPLAN data indicate that Indigenous students in Year 5 perform at approximately the same level as non-Indigenous students in Year 3. A similar two year gap in performance is found on the NAPLAN tests administered at the other year levels. Little prominence is generally given to Indigenous students who perform well on the NAPLAN numeracy tests. This tendency can lead to essentialising, inappropriately, all Indigenous students as weak mathematically. Whether examples of high achieving Indigenous students can be found was explored in this study.

We examined data gathered from Indigenous students attending schools involved in a large national project, the *Make it Count* [MiC] project, which was established to improve the learning outcomes of Aboriginal and Torres Strait Islander students in mathematics. Some 1450 useable numeracy NAPLAN results were available. This comprised both one-off data from some schools, and repeated information from other schools. The latter enabled longitudinal performance patterns to be traced for individual students.

In each year (2008-2012) that the NAPLAN tests have been administered to date, some MiC Indigenous students scored in the highest NAPLAN band for their year level: 15 students over the five year period. Six of these were in Year 3 in 2011 or 2012 so that no later test results are yet available. Longitudinal data were available for five of the other students. None of these scored in the highest band two years later and for four students the NAPLAN score had decreased over the two year period.

On the one hand, our data revealed that there are high achieving Indigenous students. On the other, sadly, initial high achievement was not sustained two years later. Whether stereotyped expectations, or other factors contribute to this decline is worthy of further exploration.

EQUATIONS REDUCIBLE TO QUADRATIC FORM: A STUDY OF STUDENTS' ERRORS

Chee Peng Lee, Jaguthsing Dindyal

Fairfield Methodist School, Nanyang Technological University

The study of algebra at the secondary level invariably involves the solution of quadratic equations as one of the main topics. While the standard quadratic equation $ax^2 + bx + c = 0, a \neq 0$ is identified fairly easily by students and is solved by some standard methods such as factorization or by the use of the standard formula, the same cannot be said about equations such as $e^{2x} - 3e^x - 4 = 0$; $2\sin^2 z + 7\cos z = 4$; and $3\log_y 5 + \log_5 y - 4 = 0$. In the last three equations, the quadratic nature becomes obvious only after some manipulations and after using an appropriate substitution. The difficulty of students in solving these types of equations is compounded by the fact that these equations cut across several content domains such as trigonometry, exponential and logarithmic functions. In this study we explored the types of errors made by a group of 26 secondary school students when solving these types of equations. The research questions were: (1) What kinds of errors do students make when solving equations that are reducible to quadratic form? (2) Why do the students make these errors? A 7-item test was constructed and administered to all of the students. The test was scored out of 30 marks ($M=20.8$, $SD=5.48$) and specific errors made by the students in the various stages of the solution process were noted and categorised. Nine students with different performance levels were each interviewed for about 30 minutes after the test. The results show that several students made errors in identifying the structure of the equations, particularly those involving a complex trigonometrical function and exponential function. For the solving part, equations involving the logarithm function was the most problematic for the students. Teaching students to identify structure requires relational understanding and is important to incorporate instrumental teaching to ensure proper application of the rules pertaining to the function. Errors can be used as springboard of mathematical inquiries in teaching or remediation (Borasi, 1994).

References

Borasi, R. (1994). Capitalizing on Errors as "Springboards for Inquiry": A Teaching Experiment. *Journal for Research in Mathematics Education*, 25(2), 166-208.

THREE LEVELS OF UNITS AND MULTIPLICATIVE STRUCTURE IN FRACTIONAL CONTEXTS

Hyung Sook Lee

Eastern Washington University, USA

Three levels of units are considered as critical in the construction of multiplicative structure (Olive & Steffe, 2002). This study investigated in what sense coordinating three levels of units is needed in creating a multiplicative structure in fractional contexts and in what ways the coordination of three levels of units enables establishing a relationship between two quantities using a fraction.

When children coordinate two composite units, three levels of units are created. While distributive partitioning is an operation that requires coordinating two levels of units in fractional contexts, the splitting operation is required, but does not suffice, for coordinating three levels of units (Hackenberg, 2007). In this study, multiplicative structure in fractional contexts is conceived as a scheme that requires simultaneously performing a splitting operation at two levels, one on a posited quantity and the other on an imaginary unit quantity.

A 15-week teaching experiment was conducted with an eighth grade mathematics teacher. The teacher engaged in fractional reasoning two ways: watching selected video clips of a middle school student solving fraction problems and solving fraction problems of the same general type that the student solved. Ashley showed progress in her multiplicative fractional reasoning in terms of an ability to coordinate three levels of units and an awareness to differentiate a fractional operation from its result.

During the last six weeks of video watching sessions, she watched Mike responding to “If you share seven candy bars among 9 people, how much of all the candy will one person have?” As she commented on Mike’s construction of one share consisting of seven pieces, Ashley was explicit in the notion of three levels of units involved saying, “He wasn’t making the connection that I have one-ninth or I divided each individual bar into nine pieces and I have seven bars.” She also said, “Repeating operation (an operation provided by JAVA: Bars, a computer software program) calls Mike’s attention to connecting the parts based on one bar with the entire bar.” It is evident that she conceived one bar as a whole unit on its own as well as a unit being used to comprise 7 bars. She was in the coordination of three levels of units. However, it is unclear whether she could establish a multiplicative relationship, $1/9 \times 7 = 7 \times 1/9$, between one share and one bar as she differentiated $1/9$ as an operation performed on 7 bars from $1/9$ as a resultant amount of an operation on one bar.

References

- Hackenberg, A. J. (2007). Units coordination and the construction of improper fractions: A revision of the splitting hypothesis. *Journal of Mathematical Behavior*, 26, 27-47.
- Olive, J., & Steffe, L. P. (2002). The construction of an iterative fractional scheme: The case of Joe. *Journal of Mathematical Behavior*, 20, 413-437.

STUDENTS' UNDERSTANDING ON UNIT AS A FOUNDATION OF MULTIPLE INTERPRETATIONS OF FRACTIONS

Ji Young Lee, Jeong Suk Pang

Paldal Elem. School, Korea National Univ. of Ed.

The flexible reasoning about unit is fundamental to multiplicative reasoning, proportional reasoning, and even algebraic thinking. In particular, students need to have a firm grasp of unit in learning fraction because even basic addition of fractions requires the same unit (Barnett-Clarke et al., 2010; Lamon, 2012). However, the unit itself in fraction has not been fully emphasized in a mathematics curriculum. This hinders students' conceptual understanding of fractions. Against this background, this study explores how students identify units embedded in multiple interpretations of fractions. It then examines students' understanding on unit to analyze their levels of units suggested by Steffe (2010).

A total of 150 sixth graders who learned multiple interpretations of fraction from 6 typical elementary schools in Korea were surveyed by a questionnaire consisting of 40 tasks with regard to identifying units in multiple interpretations of fractions: (a) part-whole relationships, (b) measures, (c) quotients, (d) ratios, and (e) operators. Each meaning of fractions was given in 4 representations (i.e. number line, area model, a set of discrete objects, and number). For an in-depth interview, both 3 students who showed a flexible use of unit and 3 counterparts who experienced difficulties in using a unit were selected to explain their strengths or difficulties.

Students' overall performance varied according to interpretations and representations of fractions. However, many students had difficulties in identifying and using the unit in a given context. In case of the task (i.e. how many pencils are in 1 bag if 12 pencils represent $\frac{3}{4}$ of a bag?), most of the students regarded 12 pencils as 1 unit and then found the wrong answer by multiplying 12 by $\frac{3}{4}$ without understanding the context. Only some students answered correctly as follows: since $\frac{3}{4}$ unit is the same as three $\frac{1}{4}$ unit and there are 4 pencils in $\frac{1}{4}$ unit, 1 unit represents 16 pencils. The episodes of interview showed that these students can use three levels of units. The focus on unit can prepare students for proficiency in learning fraction. In this respect, this study is expected to suggest instructional implications on what to focus on in teaching fraction.

References

- Barnett-Clarke, C., Fisher, W., Marks, R., & Ross, S. (2010). *Developing essential understanding of rational numbers for teaching mathematics in grades 3-5*. Reston, VA: NCTM.
- Lamon, S. J. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. New York: Routledge.
- Steffe, L. P., & Olive, J. (2010). *Children's fractional knowledge*. New York: Springer.

MATH/PLUS - SUPPORTING FUTURE MATHEMATICS TEACHERS IN THEIR FIRST YEARS' STUDIES

Malte Lehmann, Bettina Roesken-Winter
Ruhr-Universität Bochum

The transition from school to university is a challenge for many students. Future mathematics teachers especially need support in three domains. First, they are challenged to adapt their learning strategies (Griese, Glasmachers, Härterich, Kallweit & Roesken, 2011). Second, the complexity and formal rigour of mathematics at university level demand new ways of thinking and diversified approaches (Rach & Heinze, 2011). Third, future teachers particularly need help to see the connection between mathematics in school and at university (cf. Gueudet, 2008).

At Ruhr-Universität Bochum, the project “Math/Plus” aims at supporting future mathematics teachers to attain the skills described above. Our manifold interventions are based on experiences in a successful project with engineering students (Griese et al. 2011). The focus here is on learning strategies, and how students’ involvement in “Math/Plus” increases their development.

In a pre/post-design the students were asked to fill in a modified LIST questionnaire (Griese et al., 2011) at the beginning and at the end of the course. On average the students improved their skills in *organisation* (pre: $M=2.72$, $SD=0.75$; post: $M=3.13$, $SD=0.80$), $t(24)=-2.0142$, $p<.05$). Furthermore *metacognitive strategies* were enhanced (pre: $M=2.94$, $SD=0.58$; post: $M=3.38$, $SD=0.43$), $t(24)=-2.887$, $p<.01$. *Repeating* is the third factor with an increase (pre: $M=3.02$, $SD=0.54$, post: $M=3.60$, $SD=0.69$), $t(24)=-2.8254$, $p<.05$. Other factors showed no statistically significant differences.

Based on these results we can assume that “Math/Plus” has positive effects on some relevant learning strategies ranging from simple cramming techniques (*repeating*) to more sophisticated structuring (*organisation*), and to *metacognitive strategies* such as planning, monitoring and self-regulation.

References

- Griese, B. Glasmachers, E., Härterich, J., Kallweit, M. & Roesken, B. (2011). Engineering students and the learning of mathematics. In B. Roesken & M. Casper (Eds.), *Proceedings of the 17 MAVI Conference* (pp. 85-96). Bochum: PSE.
- Gueudet, G. (2008): Investigating the secondary-tertiary transition. *Educational Studies in Mathematics* 67, 237-254.
- Rach, S. & Heinze, A. (2011). Studying Mathematics at the University: The Influence of Learning Strategies. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol 4, pp. 9-16). PME: Ankara.

DEVELOPMENT OF THE CREATIVE DISPOSITION TOWARD MATHEMATICS SCALE

Yuh-Chyn Leu; Mei-Shiu Chiu

National Taipei University of Education; National Chengchi University, Taiwan

AIM AND THEORETICAL BACKGROUND

The aim of this study is to develop a psychological scale that can be used to investigate creative dispositions towards mathematics for primary-school students. Characteristics of ‘big’ creators in mathematics may inform those of ‘mini’ creators in mathematics classrooms (Beghetto & Kaufman, 2009, p. 42).

METHOD

The research participants were 372 Taiwanese Grade-4 students (123 girls and 249 boys) with high mathematics achievements in school. They filled in the Creative Disposition toward Mathematics Scale (CDMS), adapted from the 21 characteristics of potentially creative thinkers in mathematics identified by Carlton (1959, pp. 414-417) and Mathematics Affect and Gifted Behaviour Questionnaires. They also took the Mathematics Achievement Test, Mathematics Reasoning Test, and Mathematics Creativity Test.

RESULTS AND DISCUSSION

The results of exploratory and confirmatory factor analyses resulted in a 10-item CDMS, with a clear structure of 2 factors (intuitive association and deep innovation). The coefficients of Cronbach’s alpha were .815 and .790. The results of correlation analyses showed that the 2 factors had positive relations with self-reported measures on mathematics confidence, mathematics interest, passion, creativity, and intelligence ($r = .284 \sim .552$), and a negative relation with mathematics anxiety ($r = -.290$ and $-.208$). The relations between the 2 factors and their mathematics achievement, Reasoning, and creativity scores were positive and generally low, though most were significant ($r = .051 \sim .259$). The above results show that the CDMS has acceptable internal reliability, and construct, concurrent, and criterion validity.

References

- Beghetto, R. A., & Kaufman, J. C. (2009). Do we all have multicreative potential? *ZDM Mathematics Education*, 41(1), 39-44.
- Carlton, L. V. (1959). *An analysis of the educational concepts of fourteen outstanding mathematicians, 1790-1940, in the areas of mental growth and development, creative thinking and symbolism and meaning*. Unpublished doctoral dissertation, Northwestern University, IL.

IMPROVING DIAGNOSTIC JUDGEMENT OF PRESERVICE TEACHERS BY REFLECTIVE TASK SOLUTION

Juliane Leuders, Timo Leuders

University of Education (Pädagogische Hochschule) Freiburg, Germany

Knowledge about content and students is considered a major dimension of knowledge for teaching (Hill, Schilling & Ball 2004). A typical situation for teachers is the diagnostic analysis (assessment) of students' solutions and the identification of strengths and misconceptions, potentially drawing on (i) scientific knowledge on students' concepts, (ii) experience from classroom practice or (iii) task-specific knowledge derived from own reflective solution attempts. The extent to which i to iii are being used depends on the teacher knowledge and the task at hand. In this study we investigate the conditions which support preservice teachers (who are not yet able to use (i) or (ii)) to construct task-specific knowledge for appropriately analysing students' solutions.

Pre-service teacher students for mathematics at primary level (N=126) are asked to analyse students' solutions of explorative problems in arithmetics (Wittmann, 2001). We assess their analyses with respect to breadth and depth, thus constructing a measure for their diagnostic competence. Two groups of students are tested: students with and without specialisation on mathematics as a subject (differing in the amount of mathematics and mathematics education). Comparing the diagnostic competence between the two groups we find lower values in the non-specialised group. The opportunity to solve the problems by themselves has a noticeable impact on the students' diagnostic competence. We expect to find further improvement of diagnostic competences when giving them the opportunity to compare different solutions within small groups and thus to enhance their knowledge of the task's solution space.

This research complements the findings of Morris, Hiebert & Spitzer (2009) who show that the diagnostic competence of pre-service teachers can be improved by analysing the sub-skills ("unpacking") in simple tasks of decimal number addition.

References

- Hill, H.C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal*, 105, 11-30.
- Morris, A., Hiebert, J., Spitzer, S. (2009). Mathematical Knowledge for Teaching in Planning and Evaluating Instruction: What Can Preservice Teachers Learn? *Journal for Research in Mathematics Education*, Vol. 40, No. 5, 491–529
- Wittmann, E. Ch. (2001). Drawing on the richness of elementary mathematics in designing substantial learning environments. *Proceedings of the 25th PME*. Vol. 1, 193-197

TASKS THAT MAY OCCASION MATHEMATICAL CREATIVITY: AVA'S EMERGING PERSPECTIVES

Esther Levenson
Tel Aviv University

The teacher plays an important role, not only in promoting mathematical knowledge among students, but also in promoting students' mathematical creativity. One of the important elements of teachers' practice is choosing tasks to be implemented with students. Several studies have shown that different types of tasks may promote different aspects of creativity (e.g., Silver, 1997). Given the plethora of tasks available to teachers, how can we help teachers recognize tasks that have the potential to promote mathematical creativity in their classroom? This paper presents the impact of a graduate course on one teacher's changing perspective regarding tasks that may occasion mathematical creativity. At three points during the course, Ava, an experienced secondary school teacher, was requested to submit mathematical tasks that, in her opinion, had the potential to occasion mathematical creativity and explain her choices. Using Levenson's (2013) framework, the following questions were investigated: (1) Can we see a change in the task features and cognitive processes that Ava associated with tasks that may occasion mathematical creativity? (2) Was there a change in affective issues Ava associated with these tasks?

The findings (see Table 1) indicated that Ava began with a task requiring a moment of creativity. However, her second and third tasks were about developing long-term creative thinking processes. Solving tasks in different ways promotes flexible thinking. Encouraging students to come up with new ideas, may develop their creative disposition. Additional findings will be discussed during the presentation.

	Task one	Task two	Task three
Task features	Source: 12 th grade exam; 1-2 final answers.	Source: 12 th grade text book; 1 final answer; many solution paths.	Source: 7 th grade textbook; one final answer.
Cognitive demands	Requires an unconventional solution path.	Connects different mathematical domains.	Requires generalization of a pattern and coming up with a rule.
Affective issues	-	-	Surprise; value of group work; mathematical disposition.

Table 1: Ava's perspectives of tasks which may occasion mathematical creativity

References

- Levenson, E. (2013). Tasks that may occasion mathematical creativity: Teachers' choices. *Journal of Mathematics Teacher Education*. DOI 10.1007/s10857-012-9229-9
- Silver, E. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM Mathematics Education*, 3, 75-80.

A PRELIMINARY EXPLORATION OF LEVELS OF UNDERSTANDING DURING AN INTERACTION EFFECT

Tsai-Wen Lin¹, Chao-Jung Wu¹, Scott Sommers^{1 2}

¹National Taiwan Normal University; Ming Chuan University²

Statistical literacy - the ability to interpret, evaluate and integrate statistical information- enables people to make decisions critically and informatively (Gal,2002). Interaction is scientific thinking and an important aspect of statistical literacy since individuals need to identify the relationship between variables with prior knowledge of statistics (Zohar, 1995). This includes such examples as considering the interaction between medicine and food before making the decision to take it. This study aims to investigate individual levels of understandings in the concept of interaction. The research questions are:

- How do adults explain interaction effect and represent it in a graphical way?
- Does explicit representative information from text and graphs influence cognitive processes and results when adults are answering?

A semi-structured questionnaire was used to examine different types of study during the interaction of two interaction items and four questions. Participants were 12 volunteers from colleges in Taipei.

Results indicated three ways in which people construct understanding: a lack of understanding of the variables (type 1), a preliminary understanding of the variables (type 2), and an understanding of independent and dependent variables (type 3). Classification of graphs representing interaction was analysed according to these three types. Type 1 understanding involves drawing graphs based on the way interaction effects are described in the items or research results. Type 2 understanding involves drawing an interaction effect with and a procedural picture that presents two independent variables but no correct presentation of the dependent variable. Type 3 understanding involves the use of a line graph, a bar graph and tables to present the interaction effect and independent variable. In addition, different representations of interaction and domain-specific knowledge influence participants' response during interaction. Prior knowledge of statistics was shown to have an effect on type of study during reasoning about interaction.

References

- Gal, I. (2002). Adults' Statistical Literacy: Meanings, Components, Responsibilities. *International Statistical Review*, 71(1), 1-51.
- Zohar, A. (1995). Reasoning about interactions between variables. *Journal of Research in Science Teaching*, 32(10), 1039-1063

NUMERACY EXPERIENCES AND MODELLING BEHAVIOURS

Minnie Liu

Simon Fraser University, Canada

Numeracy has gained importance in the field of mathematics education in recent years. While there exists much discussion about numeracy and the closely related mathematical literacy, these terms lack agreed upon definitions. As such, there is little to help us delineate numeracy, numeracy tasks, and numerate behaviour. On the other hand, modelling, model-eliciting activities, and modelling behaviour are well represented in the research literature (Lesh et al., 2003). This study aims to map the characteristics of model-eliciting activities on to numeracy tasks by comparing students' numeracy task behaviours to established modelling behaviours.

This study involves a group of grade 8 (age 12-13) mathematics students ($n = 28$) and a group of grade 9 (age 13-14) mathematics students ($n = 29$). Working in groups of three to four, students were assigned a numeracy task. The teacher (the author) acts as a facilitator by providing encouragement, prompting discussions, and mobilizing knowledge in the room. Data includes in class observations, field notes, class discussions, and impromptu interviews.

Preliminary results indicate that while both groups of students exhibit modelling behaviours, there are distinctive differences in behaviours and fluency between the two groups. For example, the solutions produced by the grade 8 group are largely based on the data presented in the problem. On the other hand, the grade 9 group is able to extend their thinking and solution smoothly beyond the present scenario, naturally account for various situations, and create suitable solutions and make generalizations. While it is not surprising that the grade 9 group demonstrate more mature modelling behaviours than the grade 8 group, it is interesting to notice these distinguishable differences in behaviours and fluency generated by a small amount of task experiences. Roughly speaking, the grade 9 students have one more year of task experience than the grade 8 students. This translates to approximately eight to ten more numeracy tasks, or 15 to 20 hours of task experiences. Although this could be considered a big difference between novice problem solvers, it seems that these experiences accumulate quickly and allow students to improve at numeracy tasks rapidly.

References

Lesh, R., & Doerr, H. (Eds.). (2003). *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching*. Mahwah: Lawrence Erlbaum Associates.

MATHPEN: EXPLORING HANDWRITING RECOGNITION TECHNOLOGY FOR ONLINE MATHEMATICS EDUCATION

Mandy Lo, Julie-Ann Edwards, Christian Bokhove, Hugh Davis
University of Southampton, UK

Mathematical language is more than a means of communication and pooling of information; it is an active process by which people think, learn and work towards joint understanding (Mercer, 1995). Although collaborative learning is increasingly a research focus in mathematics education, most of these studies are conducted offline (e.g. Swan, 2006). Since the online use of mathematical notations is said to be a barrier to mathematical communication, this study aimed to a) investigate this claim through empirical evidence, b) explore the feasibility of using mathematics handwriting recognition technology to overcome the mathematical challenges, and c) propose a potential solution to the perceived problem.

Of the 4819 mathematical statements studied from the most recent 500 threads of mathematical discussions on an online help forum, 50% were entered using Latex and 44% using plain text, though the choice between the two is topic dependent ($p < 0.001$). Case studies also demonstrated the lack of clarity of plain text and the cumbersome nature of Latex. An expert helper, despite his more than 11,000 posts, stated: “*I wasn’t about to Latex it all*” and communicated through scanned handwritten mathematics work instead. These limitations were further confirmed by 80 participants through an online questionnaire, in which 72% believed that handwriting recognition technology will prove to be useful for communicating mathematics.

MathPen (an online mathematics handwriting recognition system) was conceptualised to address current issues and provide an interface for spontaneous and effective mathematical communication online. A short demonstration video was produced and sent to seven experts who a) have been teaching mathematics for the last 10 years, b) have some experience of handwriting recognition technology and c) were not previously known to the researcher. They unanimously agreed that, if MathPen materialised, it is a strong candidate for overcoming present challenges and opening the way for online collaborative learning in mathematics education. MathPen is currently being developed. This presentation will offer further details of the study from which MathPen was conceptualised and report on early use of MathPen with students.

References

- Mercer, N. (1995). *The guided construction of knowledge: Talk amongst teachers and learners*. Multilingual Matters Limited.
- Swan, M. (2006). *Collaborative Learning in Mathematics: A Challenge to our Beliefs and Practices*. London: NIACE/ NRDC.

USING DIFFERENT LEARNING STRATEGIES TO PROMOTE STUDENTS' COMPREHENSION OF GEOMETRY PROOF

Feng-Lin Lu, Tai-Yih Tso

Department of Mathematics, National Taiwan Normal University

Comprehending geometry proof is a complicated cognitive process for middle school students, but it is also one of the fundamental elements to construct and verify geometry proofs successfully. Although reducing task complexity of geometry proof problems by segmentation can lower students' cognitive load, there are mere or limited effects on promoting students' comprehension (Tso et al., 2011). Lu and Tso (2013) analysed eighth graders' learning efficiency and they found students can efficiently comprehend geometry proof at local level by segmentation. The purpose of this study aims to promote students' learning efficiencies at micro, local and global levels in the meantime. We applied different learning strategies in comprehending geometry proof and investigate students' learning performance as well as their cognitive load. The experiment was adopted 3x3 posttest-only control group design. The participants were 292 eighth graders which are middle school students in Taipei in Taiwan. They were divided into three levels according to their mathematics achievement and then randomly assigned to study a segmented (S), a segmented with a structural overview (SSO), or a segmented with practice (SP) version of a geometry proof problem. The major findings are described as follows. First, comparing with students in SSO and S group, practice can significantly lower students' cognitive load and raise their confidence in comprehending geometry proof. Second, providing a structural overview appears to be beneficial to comprehending geometry proof at global level while giving the learner an opportunity to practice after each segment is beneficial to their understanding of geometry proof at local level. Future research needs to explore the delayed effects of these learning strategies or presentation on other platforms, such as designing heuristic geometry worked-out examples involving various learning strategies in dynamic mathematics environment.

References

- Lu, F. L., & Tso, T. Y. (in press). *Efficiency of reducing task complexity by segmentation on eighth graders' reading comprehension of geometry proof*. Poster session presented at the 6th East Asia Regional Conference on Mathematics Education, Phuket, Thailand.
- Tso, T. Y., Lu, F. L., Tzeng, S. C., Wu, H. M., Chen, M. J., & Tan, N. C. (2011). Impact of reducing task complexity by segmentation on experts' and novices' reading comprehension of geometric proof problems. *Bulletin of Educational Psychology*, 43(Special Issue on Reading), 291-314. (In Chinese)

STUDENT-DRIVEN PROPORTIONAL REASONING APPROACHES TO AN EARLY ALGEBRA TASK

Anna L. V. Lundberg, Thomas Hillman
University of Gothenburg, Sweden

In autumn 2011, Sweden introduced a new curriculum that, among other changes, introduces the concept of proportionality earlier than before. Under the new curriculum, proportionality is introduced during year six, the same year that early algebra and in particular the concept of variable is first included. In this presentation, we will examine the role of proportionality in Swedish early algebra lessons by examining the discursive and representational resources at play. To illustrate our analysis will draw on one lesson from a corpus of over 100 hours of early algebra classroom video recordings collected as part of the VIDEOMAT project. The particular sequence that we will discuss was identified through inductive multimodal analysis (Jewitt, Kress, Ogborn, & Tsatsarelis, 2001) that was conducted during a process of repeated shared data sessions (Jordan & Henderson, 1995). Following identification of this sequence that involves students working in groups on a patterning task, we drew on the Vygotskian tradition to interpret the relationships among discourse, gesture and forms of visual representations. In particular, we followed Kaput's (1998) assertion that basic notations are fundamental and complex cultural achievements and adopted a view of representations as "a relationship of symbolisation between two representational systems" in line with Goldin (1998).

Based on this analysis, we found that despite not having been introduced by the teacher or the textbook as a possible strategy for solving algebraic problems, the students made extensive use of proportional reasoning during the lesson. Given that the new Swedish curriculum introduces proportionality as a concept during the same year that algebra is introduced and the importance of supporting the building of connections between different mathematical concepts (e.g. Goldin, 1988), the student-driven use of proportional reasoning to solve algebraic tasks offers a significant pedagogical opportunity.

References

- Goldin, G. (1998). Representational systems, learning, and problem solving in mathematics. *The Journal of Mathematical Behaviour*, 17(2), 137-165.
- Jewitt, C., Kress, G., Ogborn, J., & Tsatsarelis, C. (2001). Exploring Learning Through Visual, Actional and Linguistic Communication: the multimodal environment of a science classroom. *Educational Review*, 53(1), 5-18.
- Jordan, B., & Henderson, A. (1995). Interaction analysis: foundations and practice. *The Journal of the Learning Sciences*, 4(1), 39-103.
- Kaput, J. J. (1998). Representations, inscriptions, descriptions and learning: a kaleidoscope of windows. *Journal of mathematical behaviour*, 17(2), 265-281.

MATHEMATICAL COMMUNICATION IN PRESCHOOL

Marita Lundström
University of Gothenburg

The study has the overall aim to evaluate and describe how preschoolers communicate mathematics with other children and adults. The question posed in this presentation is: How can children's mathematical communication be described in preschool and how do they use mathematical concepts in interaction with others?

Previous research has shown that preschoolers who spontaneously use and understand numbers, number and quantity in everyday life are better at solving mathematical problems in school (Hannula & Lehtinen, 2005). It has also been shown that young children's mathematical communication can be developed if teachers integrate mathematics in playful activities (Hye Young & Reifel, 2011; Klibanoff et.al, 2006). The study is based on a sociocultural perspective where children's understanding of mathematical concepts can be developed through social interaction (Wertsch, 2007). Thirty-two children between the ages of 3-6 years and three teachers participated and they are observed under the duration of 12 months in daily activities in preschool. Their interaction was documented with video camera, photos` and field notes.

The empirical data shows that children use mathematical concepts when they communicate and they are active to help each other to understand and to be understood in their interaction with others. It also shows that a supportive environment where children can be challenged to use a mathematical language can contribute to their understanding of mathematical concepts.

References

- Hannula, M. & Lehtinen, E. (2005). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction* 15 (2005) 237-256.
- Hye Young Jung & Stuart Reifel (2011): Promoting Children's Communication: A Kindergarten Teacher's Conception and Practice of Effective Mathematics Instruction, *Journal of Research in Childhood Education*, 25:2, 194-210.
- Klibanoff, R., Levine, S.C., Huttenlocher, J., Vasilyeva, M. & Hedges, L. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk." *Developmental Psychology*, 42 (1), 59-69.
- Wertsch, J. V. (2007). Mediation. In H. Daniels, M. Cole & J. Wertsch (Eds.), *The Cambridge companion to Vygotsky* (pp. 178-192). New York, NY: Cambridge University Press.

INSTRUMENTAL GENESIS: THE CONCEPT OF OBJECT

Kate Mackrell

Institute of Education, University of London

The question of the relevance of instrumental genesis (Rabardel, 2002) to task and technology design was raised in an analysis of the design of a Cabri Elem activity book (Mackrell, Maschietto & Soury-Lavergne, 2013). Both instrumentation needs and observed instrumentalization contributed to design. However, a major problem with the scope of instrumental genesis in the Cabri Elem environment was identified: the artifact comprised not only a component of the instruments developed to mediate action, but also the screen objects upon which action took place and the environment within which the action took place, both of concern to the designer. A preliminary attempt to extend instrumental genesis in order to be able to consider the objects and environment provided by an artifact revealed differing interpretations of the key concept of object: object-as-thing (which is defined as that with which the subject interacts and which includes physical, virtual and mathematical objects), or object-as-goal (which is defined as the motive or objective of an activity).

It is argued that object-as-thing is used in Rabardel's conception of instrumented activity situations (IAS), with "object of action" referring to the focus of the action (on an object-as-thing) rather than its goal. Rabardel also assumes that subjects act purposefully: hence goal is not lost even if not explicitly identified. In contrast, all objects-as-things acted upon are lost if object is identified with object-as-goal. The interpretation of "object" as object-as-goal has not prevented "object-as-thing" from being the most common way in which "object" is used in the literature on instrumental genesis, but, together with the limitation of the artifact, has prevented a fine-grained IAS analysis of student action in a technological environment.

There are other reasons for considering object-as-thing; it corresponds to the meaning of "object" in natural language and philosophy. It is fundamental to human cognition, and, in mathematics, is key to discussions of the ontological status of mathematics, the representation of mathematical objects and the reification process by which mathematical processes become perceived as objects. Object-as-thing is both more observable in and less defined by the action than is object-as-goal.

This usage is problematic in activity theory, however; further analysis is needed.

References

- Mackrell, K., Maschietto, M., & Soury-Lavergne, S. (2013). Tasks in technology environments: Theory of didactical situations and instrumental genesis in a Cabri Elem environment. http://cerme8.metu.edu.tr/wgpapers/WG15/WG15_Mackrell.pdf Accessed: 24 February 2013.
- Rabardel, P. (2002). People and technology: a cognitive approach to contemporary instruments. <http://ergoserv.psy.univ-paris8.fr> Accessed: 20 July 2012.

MULTIPLICATIVE STRUCTURES AND PROPORTIONAL REASONING: TEACHERS' EXPLORING & STUDENTS' LEARNING

Joakim Magnusson

University of Gothenburg, Sweden

One of the most complex and mathematically difficult topic to teach, and also regarded as crucial to success in higher mathematics and science, is the area of fractions, ratios and proportions (Lamon, 2007). The aim of this study is to describe in what way students develop their ability to reason proportionally, during a Learning Study. To reason proportionally is being able to identify the structural multiplicative relationship between two quantities and extend this relationship to other pairs of quantities. Learning Study is a methodological approach (Lo et al, 2005), a cyclic process in which the handling of a specific content, is planned, filmed and afterward analysed by teachers and researchers. This process is combined with a theory of learning, Variation Theory (Marton & Tsui, 2004), which is used as a guideline during the study, which strives to challenge the students' conceptions of the content (i.e. proportional relations). It is of key interest to identify and to help the students to discern aspects that makes it possible for them to see the content in a more complex way.

The on going empirical study began in February 2013 and comprises 3 classes with 60 students in the age of 13-15 years. To examine if teaching has any influence on students' learning, pre and post test will be conducted, in combination with an interview of 5 – 6 students per class. The results from these tests, combined with a Variation Theory analysis of filmed sequences from the lesson, could inform the teachers and researchers what the students have to discern. Analysis of the first lesson is used to make a revised lesson plan, which is used in a new class with new students in a cycle including the three classes. Comparisons between how the content is handled in the three different classes and how this effects the students' learning will be discussed during the presentation.

References

- Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the national council of teachers of mathematics* (pp. 629-667). Charlotte, NC: Information Age Pub.
- Lo, M.L., Pong, W. Y. (2005). *Fort each and everyone. Catering for individual differences through Learning Studies*. Hong Kong: Hong Kong University Press.
- Marton, F. & Tsui, A. B. M. (2004). *Classroom discourse and the space of learning*. Mahwah, NJ: Lawrence Erlbaum Associates

ELEMENTARY STUDENTS' GENERALIZATIONS AND THE GROWTH OF THEIR MATHEMATICAL ACHIEVEMENT

James A. Maher, Sarah B. Berenson

Rutgers University, University of North Carolina Greensboro

The Common Core State Standards recommends reasoning as one of eight mathematical practices for inclusion in states' K-12 curriculum (National Governors Center for Best Practice, 2010). Anticipating the national curriculum's emphasis on reasoning we began a two-year after school enrichment program for students to promote algebraic reasoning. A free and voluntary program, ON TRACK students elected to attend up to three, 5-week sessions each year. The research question was: Is there a relationship between 4th and 5th grade students' abilities to generalize explicit rules and their percentile growth on end-of-grade state mathematics achievement tests [EOG]? This mixed methods study used the framework of Lannin, Barker, and Townsend (2006) to code students' generalizations. Students from two urban and four rural schools were encouraged to work together in student-centered contexts, sharing their observations, solutions and justifications with the rest of their peers throughout these one-hour sessions. The pattern finding activities encouraged the use of concrete materials for modeling linear, quadratic and exponential functions. Sources of data included students' written generalizations and justifications of the day's activity, as well as, raw scores on state EOG tests. Analyses included coding students' generalizations as explicit (rule finds the n th term), recursive (rule finds n th from $n-1$ term), and missing/incorrect rules. Multiple regression analyses were used to compare the EOG growth of: (a) 4th and 5th graders who could write at least two explicit rules in 10 activities (ER) and (b) those students who wrote less than two explicit rules, wrote recursive rules or had incorrect or missing generalizations (OR). Results indicate significant EOG growth ($< .0001$) for ER 4th graders with growth highest for those ER students with low 3rd grade scores. The multiple regression analysis of ER 5th grade EOG growth shows a significant relationship ($< .0001$) and ER students with low 4th grade EOG scores showing the greatest growth on their 5th grade EOG scores. We suggest the need to further explore the relationships between student generalizing ability, reasoning, and the growth of mathematical achievement among elementary students.

References

- Lannin, J. K., Barker, D. D., & Townsend, B. E. (2006). Recursive and explicit rules: How can we build student algebraic understanding? *The Journal of Mathematical Behavior*, 25(4), 299–317.
- National Governors Association Center for Best Practices. (2010). *Common core state standards in mathematics*. Washington, DC: Chief State School Officers.

LEARNING MATH THROUGH SOCIAL JUSTICE ISSUES

Ami Mamolo
York University

Mathematics lessons that teach to issues of social justice have been used to make mathematics meaningful in multicultural classrooms (e.g. Bateiha, 2010), and to improve student understanding of world issues (e.g. Bartell 2011). According to Gutstein's (2006) model for Teaching Mathematics for Social Justice (TMSJ), a balance must exist between Social Justice Pedagogical Goals (SJPG) and Mathematics Pedagogical Goals (MPG). Most studies focus on how mathematical knowledge can influence understanding of social issues and teaching for social justice. This research is part of a broader study which explores how engagement with "social justice context problems" can influence understanding of mathematics and mathematics teaching.

The participant in this case study was an experienced secondary mathematics teacher, referred to as Alba. She was invited to answer a social justice context problem and then participate in a follow-up interview. Details of the problem and interview will be provided in session, but are omitted here due to space considerations. Analysis sought instances of change in her orientation toward mathematics teaching.

In her interview, Alba contrasted the task with her prior school experience which "has created a kind of stereotype." She notes "a discomfort" that there was no "neat and final solution" because one cannot separate the data from its social context ("you have to see it all together," and "you need to learn about the social issues... otherwise you get confused with the data"). The task "reinforced how powerful mathematics is" in interpreting non-mathematical situations, and opened for her "a new window... to make teaching relevant, meaningful, helpful..." These comments suggest a shift in orientation toward mathematics in the classroom – part of the MPG of the TMSJ model – while the former comments suggest a shift in how Alba was "reading the world with mathematics" – part of the SJPG of the TMSJ model (Gutstein, 2006).

Results suggest engaging with social justice context problems can create a disturbance that may shift a teacher's view of helpful mathematics learning experiences away from the stereotypical ones. Such experiences may also foster new understandings of specific math concepts embedded in the task; on-going research attends to this issue.

References

- Bartell, T. (2011). Learning to teach mathematics for social justice: Negotiating social justice and mathematical goals. *Journal of Research in Mathematics Education*, 41, 5-35.
- Bateiha, S. (2010). Mathematics for critical numeracy: A case study of social justice mathematics course for preservice elementary teachers. Unpublished doctoral dissertation, University of Oklahoma, USA.
- Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy of social justice*. New York, NY: Routledge.

PCK AND TPCK: THE CASE OF THE AREA OF A TRAPEZOID

Agida G. Manizade, Dragana Martinovic
Radford University, University of Windsor

This paper presents preliminary results of the study that examined the nature of teacher's pedagogical content knowledge (PCK) and technological pedagogical content knowledge (TPCK) (Hill, Ball & Schilling, 2008; Mishra & Koehler, 2006) related to the area of a trapezoid. The following were our research questions: 1) What is the nature of teachers' PCK & TPCK related to the area of a trapezoid?; 2) In what ways, if any, did activities presented in the study affect teachers' professionally situated knowledge (e.g., PCK and TPCK)?

There were 23 secondary mathematics teachers participating in our graduate course on Euclidean & Non-Euclidean Geometry. Four instruments were used: (1) Usiskin's standardized Van Hiele test (VHT); (2) a questionnaire designed to assess teachers' subject matter knowledge of trapezoid and its area, and related TPCK; (3) adapted PCK instrument to measure teachers' professionally situated knowledge; and (4) a reflection to measure change in teachers' self-reported perceptions of the best ways to teach the area of a trapezoid. *All the instruments & completed analysis will be shared during the presentation.* The quantitative analysis was conducted on (1), while instruments (2)-(4) were analyzed qualitatively. First, the teachers spent 30-40 minutes reflecting on questions on their knowledge of trapezoid and its' area, and the ways they typically taught it. Next, through discussion, the whole class developed seven distinct strategies for deriving the area of a trapezoid. The formal proofs, done in small groups or individually, accompanied each strategy. The PCK instrument with a sample of nine student solutions (some correct and some with mathematical limitations) was administered. To assess teachers PCK & TPCK, researchers asked participants to analyze these solutions using a specific set of questions. Finally, teachers were asked to reflect on their experiences & describe in what ways, if any, the activities affected their professionally situated knowledge.

The preliminary results identified components of teachers PCK as well as their misconceptions & preferred strategies. Teachers used technology rarely & there was inadequate sensibility in demarcating special cases of trapezoids from general cases. Results of teachers' PCK items & selected VHT items & had a positive correlation.

References

- Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, 108(6), 1017-1054.
- Hill, H. C., Ball, D. L. & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400.

AN ADIDACTICAL MILIEU THAT FAILS TO PREPARE FOR AN INTENDED STATEMENT OF EQUIVALENCE

Heidi Strømskag Måsøval

Sør-Trøndelag University College, Norway

An *adidactical milieu* is a subset of the students' environment with only those features that are relevant with respect to the knowledge aimed at by the teacher (Brousseau, 1997). The concept of milieu models the elements of the material or intellectual reality on which the students act and which may be an obstacle to their actions and reasoning. The research question addressed in the project is: *What features of the milieu constrain students' establishment of algebraic generality in a given shape pattern?*

Research participants were two groups of students (three in each group) and a teacher of mathematics. The data are transcripts of students' (video-recorded) collaborative engagement with a task on algebraic generalisation of a shape pattern. The students were given the following task (made by the teacher):



- How many cubes will there be in the fourth shape? And in the fifth?
- How many do you think there will be in shape number 10? And in shape number n ?
- What kinds of numbers are present in these shapes? In each row, and totally in the shape?
- Express as a mathematical statement what the shapes seem to show, in words and in symbols.

The teacher's aim with the task was to express in natural language, and transform into algebraic notation, the mathematical statement that the sum of the first n odd numbers is equivalent with the n -th square number (possibly represented by $\sum_{i=1}^n 2i - 1 = n^2$). The students, however, produce a formula for the numerical value of the n -th element of the shape pattern: $F(n) = n^2$ (Group 1) and $a_n = n^2$ (Group 2).

The incomplete achievement in the situation of formulation can be explained by two features of the milieu. First, there is a problem with the design of the task: It does not provide the students with knowledge that enable them to formulate a conjecture about equivalence of two different expressions for the numerical value of the general element of the shape pattern. Second, there is a problem with the concept of a mathematical statement: The teacher's meaning of this concept (a theorem) is different from the students' interpretation (a statement about the numerical value of the n -th element).

References

Brousseau, G. (1997). *The theory of didactical situations in mathematics: Didactique des mathématiques, 1970-1990* (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Eds. & Trans.). Dordrecht, The Netherlands: Kluwer.

INVESTIGATING FRESHMEN'S UNDERSTANDING OF SCHOOL MATHEMATICS: FOCUS ON THE GRAPH OF THE QUADRATIC FUNCTIONS

Yuki Masuda

Department of Children Studies, Tokyo Seitoku University, Japan

Improvement of pre-service mathematics teacher education in a private university is a serious problem in Japan. Many Japanese freshmen in a private university who wanted to be an elementary school teacher in their future had little opportunity for studying mathematics in their high school days because it was not necessary to study mathematics to pass the entrance examination. Previous studies of pre-service mathematics teacher education suggested that a variety of approaches (i.e. using exemplary mathematics activities, solving problems from a wide variety of school mathematics, using investigation, and so on.) could lead to positive learning outcomes for pre-service teachers (Rowland et al., 2001; Ponte & Chapman, 2008).

The purpose of this study is to investigate the private university students' (freshmen's) understanding of school mathematics and to explore teaching approach for them. For this purpose, freshmen majoring education in the private university (N = 139) were given the set of assessment tasks which were developed under four broader categories: (a) Reading and understanding a situation using their logical thinking ability and mathematical knowledge, (b) Their understanding of number and operation, (c) Finding characteristics of the graph of functions and (d) Geometric construction of similar figures. More than 70% of them did not study mathematics to pass the entrance examination. This paper focuses on investigating the results of the category (c), especially their difficulties in understanding the graph of the quadratic functions.

The results showed two major difficulties in finding three characteristics of the graph of the quadratic function ($y = -x^2 + 6x - 8$) as follows; (i) About 4% of them could answer three characteristics and 70% of them could not find any characteristics. (ii) It is difficult for them to answer the characteristics using mathematical terms (i.e. convexity, axis, vertex, and coordinates) they could image correctly. Implications for planning lessons for them are discussed in terms of developing their understanding of the graph of the quadratic functions considering some mathematical terms of the linear or quadratic functions.

References

- Rowland, T., Martyn, S., Barber, P., & Heal, C. (2001) Investigating the mathematics subject matter knowledge of pre-service elementary school teachers. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 121-128. Utrecht, Netherlands: PME.
- Ponte, J. P., & Chapman, O. (2008). Preservice mathematics teachers' knowledge and development. In L. English (Ed.), *Handbook of international research in mathematics education*, pp. 225-263. New York, NY: Routledge.

MATHEMATICAL REASONING PROCESSES: GENERALIZING AND JUSTIFYING IN THE STUDY OF REAL NUMBERS

Joana Mata-Pereira, João Pedro da Ponte

Unidade de Investigação do Instituto de Educação, Universidade de Lisboa; Instituto de Educação Universidade de Lisboa

The ability to reason mathematically is essential to use effectively mathematics in different situations and to develop a deep understanding of this science as logical and coherent knowledge. We view mathematical reasoning as using existing mathematical information to reach new conclusions through deductive, inductive or abductive ways. Accordingly, mathematical reasoning includes using processes such as formulating questions, conjecturing, generalizing, testing, and justifying. So, our aim is to analyse the reasoning processes of grade 9 students while solving tasks involving algebraic properties of real numbers. We focus our attention on generalization and justification as key mathematical reasoning processes. By identifying commonalities among several cases, students may develop generalizations that lead them to use and get a deeper understanding of concepts, symbols and representations. By using mathematical properties, definitions and representations, students argue their solutions providing justifications. We also consider the representations and sense making processes involved, given their close relationship to reasoning.

Data collection was conducted by the first author in a grade 9 class with an overall weak performance, but with a productive working environment. The classroom teacher was in charge of the activities and the researcher accompanied the students in doing the tasks as a “supporting teacher”. As a qualitative research based on participant observation, data collection used direct observation – with audio and video recording of classes –with collection of documents. The results suggest that, in formulating generalizations, an inductive approach is predominant, but there is also a role for deduction, when students generalize based on their knowledge of mathematical properties. Other generalizations that consider a variety of characteristics of the situation, suggest the use of abductive reasoning. Regarding justification, the students do not seem to give prominence to the characteristics necessary for it to be valid, featuring both valid and invalid justifications. However, in the use of counterexamples to refute a statement, the students seem to be concerned especially for the non-random selection of test cases. Overall, some students were able to perform certain generalizations and justifications using inductive, deductive and abductive reasoning and articulating their reasoning processes with sense making and representations.

Acknowledgement

This study is supported by national funds by FCT – Fundação para a Ciência e Tecnologia through the Project Professional Practices of Mathematics Teachers (contract PTDC/CPE-CED/098931/2008).

PRESCHOOL MATHEMATICS EDUCATION WHICH COOPERATED WITH ELEMENTARY SCHOOL EDUCATION

Nanae Matsuo
Chiba University, Japan

There is broad agreement about early mathematics education for young children. In particular, some research shows the necessity of teaching young children mathematics, and its effects (e.g. Clements & Sarama, 2012). However, since there have been few studies conducted about early childhood mathematics which cooperated with elementary school education, we need to tackle the issue of research on elementary school and preschool cooperation in mathematics education.

This research project aims to clarify the difference of views between elementary school teachers and preschool teachers, and propose a cooperative mathematics educational program for bridging that difference. Teachers' views on the relation between the contents of the mathematics taught at elementary schools and preschool children's play was assessed by a questionnaire. The items of this questionnaire are set up based on the contents of textbooks used in Japan (Shimizu et al, 2010) and also the teacher-tested activities founded on NAEYC guideline and NCTM standards (Copley, 2000). In order to show the gap between the views of elementary school teachers and preschool teachers, we need to specify what kind of child's play is accepted by many elementary school teachers but not by preschool teachers as being related to the contents of mathematics education.

The sample consists of 166 elementary school teachers and 108 preschool teachers in Japan. The findings indicate that there is a big difference between the views of elementary school teachers and preschool teachers, especially in the situation of playing shop. Furthermore, the analysis of many preschool teachers' opinions suggests that playing shop is not connected to learning addition and subtraction. Therefore, it seems that children can't learn mathematics in elementary school based on their experience in preschool. In summary, it can be concluded that we have to plan and perform teaching early mathematics in preschool by cooperating with elementary school teachers. Moreover, we propose the activities of playing shop which are carried out in preschool as a trial for the cooperative program.

References

- Clements, D. H. & Sarama, J. (2012). Mathematics Learning, Assessment, and Curriculum. In R. C. Pianta (Ed.), *Handbook of Early Childhood Education* (pp. 217-239). New York, NY: The Guilford Press.
- Copley, J.V. (2000). *The young child and mathematics*. Washington, D.C., NW: NAEYC.
- Shimizu, S. et al (2010). *Waku-Waku-Sansu 1 ~6*. Osaka: Keirinkan. (in Japanese)
- This research was supported by Grant-in-Aid for Scientific Research (C) (No.24501040) in Japan.

THE CO-CONSTITUTED SPACE OF LEARNING IN LESSONS INTRODUCING THE EQUATION OF A STRAIGHT LINE

Tuula Maunula

Department of Pedagogical, Curricular and Professional Studies
University of Gothenburg, Sweden

The overall purpose of this study is to investigate the teachers' and students' co-constitution of the space of learning in mathematics lessons. Students' questions and comments on the content taught are sometimes seen as interruptions of the lesson. It could also be seen as something that informs the teacher about the conceptions of the students or as something guiding the lesson in a certain direction. The theoretical perspective in this study is Variation Theory (e.g. Marton & Booth, 1997, Runesson, 1999 and Lo, 2012) and one of the key concepts used is the space of learning (e.g. Marton & Tsui, 2004, Häggström 2008), which addresses the potential for learning in a lesson.

The main research question is; how will the potential for learning be shaped by the students' and teachers' co-constitution of the content in a lesson? By analysing the patterns of variation and invariance of the content in the lesson, forming different spaces of learning in different lessons, this study will discuss the potential for learning offered in the lessons. The study is conducted in the last year of compulsory school and in the first two years of upper secondary school in Sweden. The students are 16-18 years old, and this sample comprises 12 videotaped lessons of the introduction of the equation of the straight line.

In the presentation of this on-going research the differences in space of learning constituted in the lessons will be discussed. Other questions addressed in the study are: what could be learnt from using students' questions in teaching and how could instruction be informed from comparing different ways of introducing the straight line.

References

- Häggström, J. (2008). Teaching systems of linear equations in Sweden and China: what is made possible to learn? Göteborg: Acta Universitatis Gothoburgensis.
- Lo, M. L. (2012). Variation Theory and the Improvement of Teaching and Learning Göteborg: Acta Universitatis Gothoburgensis.
- Marton, F & Booth, S. (1997). Learning and Awareness. Mahwah, N.J.: Erlbaum.
- Marton, F. & Tsui, A. B. (2004). Classroom Discourse and the Space of Learning. Mahwah, N. J.: Erlbaum.
- Runesson, U. (1999). Variationens pedagogik. Göteborg: Acta Universitatis Gothoburgensis.

QUANTITATIVE REASONING IN ENVIRONMENTAL SCIENCE: LEARNING PROGRESSION FOR 6TH TO 12TH GRADES

Robert Mayes, Franziska Peterson
Georgia Southern University, University of Wyoming

Research recommends that learning and curriculum designs be organized around learning progressions. The Consortium for Policy Research in Education (Corcoran, Mosher, & Rogat, 2009) identified learning progressions as a promising model that can advance effective adaptive instruction teaching techniques and change the norms of practice in schools. We are building a learning progression for quantitative reasoning (QR). Based on extensive literature reviews and pilot studies supported by the NSF Pathways Project, we define QR as mathematics and statistics applied in real-life, authentic situations that impact an individual's life as a constructive, concerned, and reflective citizen.

We hypothesize that QR is essential for data-based and modeling approaches to learning the sciences (Duschl, 2007). We have developed assessments that inform a QR progression and teachers' adaptive instruction strategies. The purpose of this study is to verify a hypothesized learning progression for QR using environmental sciences as a context addressing three different scientific scales (micro/atomic, macro, and landscape). Creating learning progressions is an iterative research process that involves grounding the lower anchor in domains accessible to 6th graders, then identifying intermediate levels of understanding through which they pass on their way to attainment of the upper anchor at 12th grade. Data collection includes student closed-form QR assessments and follow-up QR interviews for a stratified random sample of students completing the closed-form assessments.

The result of this research is a revised learning progression for QR in the context of environmental science. Our prior qualitative research indicates no consistent increase in student attainment of higher levels of QR as they move from grade six to grade twelve; no consistent differences in the QR ability of students at the macro scale, micro/atomic scale, and landscape scale; and students struggled to select the appropriate QR tool (e.g. proportional reasoning) to apply within an environmental sciences context.

References

- Duschl, R.A., Schweingruber, H.A., & Shouse, A.W. (2007). *Taking science to school: Learning and teaching science in grades K-8*. Washington, D.C.: National Academies Press.
- Corcoran, T., Mosher, F. A., & Rogat, A. (2009). *Learning progressions in science: An evidence-based approach to reform*. Philadelphia, PA: Consortium Policy Research in Education.

PRIMARY SCHOOL STUDENT'S MATHEMATICAL MODELLING COMPETENCY IN BUILDING HOUSE ACTIVITY

Samira Mehraein, Abolfazl Rafiepour Gatabi
Faculty of Mathematics and Computer-Shahid Bahonar;
University of Kerman, Iran

Although there are some emphasis on modelling and application in Iranian national curriculum in mathematics section, but there is only a few work on modelling and application activities in research and action domain. Current study tries to introduce the modelling activity for primary level students in Iran. In this study, 340 sixth grade Iranian children from nine classroom participated. All of them were children of 12 years of age at single-sex schools. Before starting to work on Building House activity, the children were presented with a brief introduction of modelling cycle. After that students start to work on this activity in a group of five. Data were collected through video recording and field notes.

Building House Modelling Activity

Ali s' Family want to build a house with three floors, (each floor has 150 square meters area). There are two different ways for building this house: pillars of ferrous and the concrete. Which method Have more advantageous for Ali s' Family? If they sell one of the floors before the finishing the house (pre-sale) and they have to pay fine for each day delay, what is the best way to build the house in your opinion?

Students work on this problem for 2 sessions (each 70 minutes). After completing worksheet by students in their group, teacher (first author of this contribution) ask one student from each group for presenting their solution for whole students in the classroom. In this part, Children freely expressed their thoughts and ideas for best method of building house activity. We code students modelling performance upon Ludwig, Xu (2010) theoretical framework. Analysis of data shown that some groups of students can finalise modelling cycle and give reasonable response to the Building House Modelling Activity. Between whole class discussion students reveal different alternative solutions for this activity. This shown modelling activity potential for fostering student's mathematical communication skills. Finally, we believe like English (2006) that primary school children can participate successfully in modelling activity in meaningful way.

References

- Ludwig, M.; Xu, B. (2010). A Comparative Study of Modelling Competencies among Chinese and German Students. *Journal for didactics of Mathematics*. Vol. 31, pp: 77–97.
- English, L. D. (2006). Mathematical modelling in the primary school: children's construction of a consumer guide. *Educational Studies in Mathematics*, vol. 63, pp: 303–323.

JUDGING “AS MUCH AS” BEFORE FORMAL INTRODUCTION TO MEASURE

Maria Mellone**, Anna Baccaglini-Frank*, Francesca Martignone*

*Università di Modena e Reggio Emilia, **Università di Napoli Federico II – Italy

The common western approaches to number in mathematics education prevalently assign a primitive and a priori role to natural numbers and to the action of counting discrete, countable magnitudes (Iannece, Mellone & Tortora, 2009). A different approach was proposed by Davydov who placed the experience of measuring continuous quantities as preliminary to the introduction of numbers (Davydov, 1982). Drawing from Davydov's assumptions we carried out an explorative study, to analyse the behaviour of children at the beginning of the first grade when working with a particular kind of magnitude, so to say, intermediate between discrete and continuous: rice. We interviewed and videotaped 19 first grade children from an Italian school at the beginning of the school year. The children were interviewed one at a time, outside of the classroom setting, in a social interaction with the interviewing researcher (one of the authors). The interviewer picked up a bag containing about $\frac{1}{2}$ kg of rice and she poured about 200 grains onto a table lifting the bag and slowly letting the rice pour out. Then the interviewer would ask the child: “Now can you please give me as much as you have in front of you?”, and then when the child had finished: “How are you sure they are the same [pointing to one pile of rice and then to the other]?” offering the child a variety of artefacts to choose from if he/she thought they would be helpful. Our observations show that only few children chose to count the grains, managing the substance as discrete, and, not surprisingly these happened to be the children who were particularly confident in handling large numbers. On the other hand, the children who treated the rice as continuous made different attempts to reduce the complexity of comparing two continuous quantities by, for example, flattening the piles and evaluating their surfaces; or comparing the heights of piles or putting them in two equal containers and so on. In this direction we recognized in the children's behaviours the similar remarkable insight to simplify the complexity of the comparison of two quantities by acting on them in order to focus only on one of their properties: in the discrete approach the chosen property is of course the numerosity; while in continuous approaches they chose to focus on the length, surface or volume.

References

- Davydov, V. V. (1982). The psychological characteristics of the formation of elementary mathematical operations in children. In T.P. Carpenter *et al.* (Eds.), *Addition and Subtraction: A cognitive perspective*. Hillsdale, NJ: Lawrence Erlbaum, 224-238.
- Iannece, D., Mellone, M. & Tortora, R. (2009). Counting vs. measuring: reflections on number roots between epistemology and neuroscience. In M. Tzekaki, M. Kaldrimidou & H. Sakonidi (Eds.). *Proceedings of PME33* (Vol. 3, pp. 209-216). Thessaloniki, Greece: PME.

GENERALISING AND SYMBOLISING IN COLLECTIVE DISCUSSIONS WITH GRADE 4 STUDENTS¹

Célia Mestre¹, Hélia Oliveira²

¹ Unidade de Investigação do Instituto de Educação, Universidade de Lisboa

² Instituto de Educação, Universidade de Lisboa

Algebraic thinking can be regarded as “a process in which students generalise mathematical ideas from a set of particular instances, establish those generalisations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways” (Blanton & Kaput, 2005, p. 413). This perspective informed a study with one grade 4 class with the purpose of developing algebraic thinking through the school year, focussed on the processes of generalisation and progressive symbolisation. Taking into account the importance of the social context of the classroom, within a perspective of a dialogic inquiry of mathematical knowledge development, the aim of this study is to analyse how the collective discussions contribute to the development of students’ generalisation and symbolisation processes.

Following an interpretative perspective, the research was conducted through the implementation of a year-long teaching experiment (Gravemeijer & Cobb, 2006) and data were collected by the observation of lessons (with video recording) and from the students’ written work. This presentation will centre in the analysis of segments of collective discussions of the students’ work from one of the mathematical tasks in the teaching unit. The results show that the students assumed an active role, explaining and justifying their ideas and questioning colleagues. This has contributed to a reconstruction of the first generalisations that came out in the class and conducted to more sophisticated forms, where the final version can be seen as a *collective generalisation* (Ellis, 2011). These findings stress the important role of the collective discussion of the mathematical ideas in promoting students’ ability to generalise and to use meaningfully the mathematical symbols.

References

- Blanton, M. & Kaput, J., (2005). Characterizing a classroom practice that promotes algebraic thinking. *Journal for Research in Mathematics Education*, 36(5), 412–446.
- Ellis, A. B. (2011). Generalizing - Promoting actions: how classroom collaborations can support students’ mathematical generalizations. *Journal for Research in Mathematics Education*, 42(4), 308-345.
- Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational Design Research* (pp. 45-85). London: Routledge.

¹ This communication is supported by national funds through FCT - Fundação para a Ciência e Tecnologia - in the frame of the Project Professional Practices of Mathematics Teachers (contract PTDC/CPE-CED/098931/2008).

RAQUEL AND ISOLDA IN A MUTUAL INTERVIEW ABOUT DIALOGUE IN MATHEMATICS EDUCATION

Raquel Milani

State University of Sao Paulo – Unesp, Rio Claro – SP, Brazil

This paper describes an interview between Isolda and myself (Raquel, the author of this paper), two teachers and researchers on mathematics education. The interview is part of the doctoral research that I have been developing whose central question is “how do prospective teachers plan and sustain the dialogue (a special pattern of communication which promotes learning) with their students in the context of supervised teaching practice?” In this context, I acted as a practitioner-researcher and planned some dialogue activities with the supervisor teacher (Isolda). Once the planning and the implementation of dialogue activities would be made by Isolda and myself, it was not conceivable to impose one’s ideas about dialogue over the other. Therefore, Raquel and Isolda arranged a mutual interview about dialogue which had two aims: to know what each teacher understood about dialogue in mathematics education, and delimitate common aspects in these perspectives.

The interview was mutual, open and implemented by email. In three rounds of questions and answers, Raquel and Isolda deepened perspectives about dialogue in mathematics class. For them dialogue involves assuming a pedagogical stance according to which mathematical knowledge should be discovered by students who must be engaged in activities and express their ideas and ways of thinking. The teacher is the one who guides the process of discovery. Listening, talking and asking questions are common actions to the teacher and the students. Dialogue is intentional, i.e., it aims at learning. These common aspects would be emphasized and integrated into Raquel and Isolda’s discourse in order to talk to the prospective teachers about dialogue.

As a result of what has been experienced by Raquel and Isolda and based on the concepts of dialogue by Bohm (1996), and by Alrø and Skovsmose (2002), and the concept of inter-view by Kvale and Brinkmann (2009), I propose a characterization of mutual interview: it is a dialogue between people interested in a subject of mutual interest, who listen actively to each other and assume the roles of both the interviewer and the interviewee, and is characterized by general actions of seeing, thinking and constructing common knowledge together, as well as by specific dialogic acts.

References

- Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education - intention, reflection, critique*. Dordrecht: Kluwer Academic Publishers.
- Bohm, D. (1996). *On Dialogue*. Londres: Routledge.
- Kvale, S., & Brinkmann, S. (2009). *InterViews: Learning the Craft of Qualitative Research Interviewing* (2nd ed.). Thousand Oaks, CA: Sage.

TEACHER LEARNING IN OBJECT FOCUSED PROFESSIONAL DEVELOPMENT

Shadrack Moalosi

University of the Witwatersrand

In this Short Oral I discuss part of the results from an ongoing study which is located in a five year professional development (PD) programme. I am concerned with whether and how teachers' participation in object focused PD mediates their professional knowledge, and the question: What comes to be their lived object of learning? My focus is teachers' lived experiences of PD activities which were aimed at enabling them to deepen and extend their knowledge of functions per se – and so what the project termed 'object focused'. Pedagogy was backgrounded. My research is framed by Variation Theory (Lo, 2012) and the contention that learning is a function of discernment, and so variation with respect to the object of learning. To answer my research question above I draw more generally from phenomenography, as well as from Adler and Davis (2006) who elaborate the duality of the object of learning in mathematics teacher education and offer tools with which this duality can be grasped.

Nine teachers (selected from 25 in the PD) participated in individual two-hour semi-structured interviews focused on learning in the PD. Analysis was both deductive (identifying whether teaching and/or mathematics was the object in focus) and inductive (through the development of themes that captured and described teachers' responses). In their talk, teachers' descriptions shifted between learning about functions per se and about the teaching of functions, with these shifts related to specific foci in the interview. For example, where resources from the PD were in focus – resources that supported teachers' engagement with functions and their representations – the teachers' talk was dominated by how these have informed their teaching of functions. In contrast, when function representations themselves were in focus in the interview, talk shifted to learning of functions. These results indicate that even though functions per se were foregrounded and pedagogy backgrounded, teachers' lived object of learning encompassed both aspects of mathematics and teaching mathematics. The data indicate further that teachers' lived object of learning may be related to the teacher's personal growth or utilitarian needs.

References

- Adler, J., & Davis, Z. (2006). Opening Another Black Box: Researching Mathematics for Teaching in Mathematics Teacher Education *Journal for Research in Mathematics Education*, 37(4), 270-296.
- Lo, M. L. (2012). Variation Theory and the Improvement of Teaching and Learning Available from <https://gupea.ub.gu.se/>

MEASURING FLOW IN MATHEMATICAL TASKS

Ana Belén Montoro Medina, Francisco Gil Cuadra
University of Almeria

When someone experiences flow they are so concentrated on the activity at hand that nothing else seems to matter. They forget everything around them, lose all track of time and enjoy the activity (Nakamura & Csikszentmihalyi, 2002). Flow depends on the task, the person and the environment in which it is performed. Several researches point out that students get higher performance and become committed to an activity when they feel flow doing it, this is why we are interested in knowing what kind of tasks promote flow more frequently. Early researches on flow have been conducted with gifted children (Heine, 1997). Following Schweinle, Turner y Meyer (2008) our work, which analyzes flow experienced with different tasks, is focused on students who are not high in mathematical talent.

The literature review shows lack of unified and clear flow definition and operationalisation. The aim of this study is to test two models proposed in the literature with pre-service teachers when they perform mathematical tasks: the correlated two-factor flow model, including concentration and enjoyment, and the correlated two-factor flow model, in which interest is an indicator of enjoyment. For this purpose, a questionnaire and Confirmatory Factorial Analyses were used.

The questionnaire which includes 8 items measuring concentration (2 items), enjoyment (4 items) and interest (2 items), was administered to 230 pre-service teachers on completing each of six Measurement and Geometry tasks.

Results of the Confirmatory Factorial Analyses support the idea that interest is an external factor closely related to flow experience, rather than an indicator of flow experience itself. That is to say, the two-factor flow model, including concentration and enjoyment, provides better fit to data: loadings above .6, reliability of indicators exceed .5 and acceptable values of absolute and relative goodness of fit indices (GFI=.96, RMR= .39, CFI=.903 y RMSEA=.069). Moreover, the values of the Average Variance Extracted (.61) and Composite Reliability (.87) suggest that this instrument is suitable for identifying flow experiences with mathematical tasks.

References

- Nakamura, J., & Csikszentmihalyi, M. (2002). The concept of flow. En C.R. Snyder y S.J. Lopez (Eds.), *Handbook of Positive Psychology* (pp. 89-105). Oxford: Oxford University Press.
- Heine, C. A. (1997). *Tasks Enjoyment and Mathematical Achievement*. Unpublished Doctoral Dissertation, University of Chicago, Illinois.
- Schweinle, A., Turner, J.C., & Meyer, D.K. (2008). Understanding young adolescents' optimal experiences in academic settings. *The Journal of Experimental Education*. 77 (2), 125-143.

EXPLORING PRE-SERVICE TEACHERS' KNOWLEDGE OF INSTRUCTIONAL STRATEGIES FOR MATHEMATICS

Jayaluxmi Naidoo, Vimolan Mudaly
University of KwaZulu-Natal, South Africa

Pre-service teacher preparation is considered an unquestionable practice to prepare teachers for the classroom situation. Teachers need to be knowledgeable in various areas (Chong, Choy, & Wong, 2008), thus this training includes preparing prospective teachers with respect to aspects of content knowledge, pedagogical content knowledge and general pedagogical knowledge. Additionally being able to interpret concepts as discussed by learners is of importance (Davis & Simmt, 2006). This study was located at one University in KwaZulu-Natal (KZN), South Africa. This qualitative, interpretive study examined Mathematics Postgraduate Certificate in Education (PGCE) students' awareness and perceptions of instructional strategies. These students were registered for a one year full time programme. Once the students completed the yearlong programme, they began their careers as teachers in the General Education and Training phase which incorporates learners in Grades 7 – 9 in South Africa.

This study was framed within Shulman's teacher knowledge model, focusing on teachers' pedagogic content knowledge. Pedagogic content knowledge necessitates the merging of content and pedagogy (Ball, Thames, & Phelps, 2008). Within the ambits of pedagogic content knowledge, this study examined instructional strategies students were aware of, their perception of each of these strategies and their personal preference while teaching. Qualitative data was collected during the 2012 academic year from fifty PGCE mathematics students via three questionnaires. Thematic coding and interpretive techniques were used to analyse the data. An initial analysis of the data exhibited that these students have limited awareness and understanding of instructional strategies required for the effective teaching and learning of mathematics. In the presentation an in-depth discussion of the results, findings and recommendations will be provided. These findings are important for advancing mathematics teacher and curriculum development.

References

- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of teacher education* 59(5), 389 - 407.
- Chong, S., Choy, D., & Wong, A. F. L. (2008). *Pedagogical knowledge and skills of preservice primary school teachers*. Paper presented at the AARE Conference 2008.
- Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational studies in mathematics*, 61(3), 293 - 319.

WHAT SUCCESS AND CHALLENGE DID TWO ZAMBIAN MATHEMATICS TEACHERS SHOW THROUGH LESSONS?

Nagisa Nakawa
Tokyo Future University

This reports what success and challenges Zambian teachers held during action research in collaboration with the author. The qualitative analysis shows that both teachers drastically improved their pedagogical skills in class and developed their reflective views on individual students' learning. On the other hand, there were insufficient mathematical content-based discussions related to their content knowledge and pedagogical content knowledge and lack of self-regulation.

FRAMEWORK OF ANALYSIS

TEDS-M's teachers' competencies were applied as a framework of analysis (Hill et al., 2008). They show that teacher competencies are cognitive abilities and affective-motivational characteristics. This presentation will start from the data obtained and discuss which factors of competencies teachers developed and undeveloped in the action research.

DATA ANALYSIS AND CONCLUSION

Qualitative analysis was conducted utilising the records on lesson observations and sheets, field-notes and recorded data. All the teachers' reflections were transcribed and classified in planning, implementation and evaluation of about 25 lessons conducted in two regions in Zambia. The classifications determined two kinds of changes that were 'drastic change' and 'qualitative gradual change'. We classified them and identified the positive changes on assessment of learning, general pedagogical knowledge, views on learning, and views on mathematics lessons and others.

Consequently, the analysis showed teachers' development on general pedagogical knowledge in the cognitive abilities, deepened beliefs about mathematics and the teaching and learning of mathematics in the affective-motivational characteristics. More importantly, their beliefs were strengthened by students' learning and could be a driving force to change their pedagogical skill. In this sense, cognitive abilities and beliefs were intertwined. They also showed the challenge on content knowledge and pedagogical content knowledge in the cognitive abilities and self-regulation including reflection on mathematical contents in affective-motivational characteristics.

References

Hill, C.H., Ball L. D., & Schilling, G. S. (2008). Unpacking pedagogical content knowledge: conceptualizing and measuring teacher's topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39, 372-400.

TEACHERS' QUESTIONS IN GUIDING FOURTH GRADERS IN OPEN-ENDED PROBLEM SOLVING

Liisa Näveri, Anu Laine, Anu Kankaanpää, Maija Ahtee, Erkki Pehkonen
University of Helsinki, Finland

The problem-solving literature contains plenty of advice how a teacher can guide students' problem-solving activity. For example, the teacher can encourage active involvement by employing a range of teaching approaches (e.g., Anghileri 2006) or the teacher may use careful questioning to promote students' reasoning (e.g., Sahin & Kulm 2008). The purpose of this presentation is to analyze both teachers' and their pupils' questioning during a problem solving lesson when a nonstandard problem is used.

Eight Finnish teachers and their fourth grade pupils took part in this study. The lessons dealing with an open-ended problem "Gary the Snail" was implemented in September 2011. Based on the videotapes the teachers' questions were classified into six categories using inductive content analysis.

The teachers' questions were categorized as follows: Task assignment and marking the solution, Way of working, Progress of working, Asking for justification, Deepening of understanding, and The others i.e. incoherent or unconnected to problem solving. It seems that the teachers asked mostly about the task assignment, because the pupils had difficulties to get started in working with the problem. The teachers asked plenty of good questions with which they prompted the pupils to understand the problem instead of just telling the pupils what they should do. However, only a few of the questions aimed specially to understanding. The portion of these questions varied a lot (0%-47%) according to the teacher. The teachers differed quite clearly both as to the number of questions and as to what kind of questions they asked. During this problem solving lesson the total number of questions per teacher varied from 16 to 90.

We were also interested in studying pupils' questions in order to get a better overview of the whole amount of questioning in different classes. The pupils' questions which they posed to their teachers were classified into the following three categories: Solving the problem, Checking of understanding, and The others, i.e. incoherent or unconnected to problem solving. The average proportion of the pupils' questions from all the questions was about 30%. In most of the classrooms when the number of the teacher's questions increased also the number of the pupils' questions increased. This is an interesting finding that should be studied more closely.

References

- Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education*, 9, 33–52
- Sahin, A., & Kulm, G. (2006). Sixth grade mathematics teachers' intentions and use of probing, quiding, and factual questions. *Journal of mathematics teacher education*, 11 (3), 221–241.

SYMMETRY IN SPATIAL PERSPECTIVE TASKS - AN INTERVIEW STUDY AT THE BEGINNING OF FIRST GRADE

Inga Niedermeyer, Silke Ruwisch
Leuphana University Lüneburg, Germany

Spatial perspective-taking as a component of spatial ability is defined as the ability to imagine how objects appear from another point of view than one's own. Subsequent to Piaget and Inhelder's (1967) famous "three-mountains-task" the effects of many different task characteristics on the ability to coordinate perspectives were examined (see Fehr 1978 and Newcombe 1989 for an overview). However the effect of the use of symmetric and asymmetric objects on this ability has not yet been considered in research.

This research project therefore aims to examine the effect of using symmetric and asymmetric objects in spatial perspective tasks. Since symmetric objects have two side views that are mirror-images of each other and differ only in their left-right-orientation, we suppose that spatial perspective tasks with symmetric objects are solved less often or less well than tasks with asymmetric objects. In symmetric tasks, we also assume that the two side views, which are symmetric to each other, are more often confused with each other.

The study was conducted as an interview-study with 95 children at the beginning of first grade (average age: 6 years 8 months). Besides of symmetry the objects' type (toy animals, cuboid buildings), their orientation and the type of view (side view or front/back view) were varied. Children's solution rates, types of errors and explanations were analyzed. The differences between symmetric and asymmetric tasks are not reflected in the solution rates, but in the distributions of two other factors: the type of mistakes and the kind of explanations children used to justify their solutions. For symmetric tasks, the children more often confused the side-views and had more difficulties to justify their decisions than for asymmetric tasks. The statements for the asymmetric side views show that the differences between the two side views, which lay in the front-back-dimension, helped the children to distinguish the two. In the presentation the design and some results will be discussed.

References

- Fehr, L. A. (1978). Methodological inconsistencies in the measurement of spatial perspective taking ability. *Human Development* 21(5-6), 302-315.
- Newcombe, N. S. (1989). The development of spatial perspective taking. In H. W. Reese (Ed.), *Advances in child development and behavior* (Vol. 22, pp. 203-247). New York, USA: Academic Press.
- Piaget, J., & Inhelder, B. (1967). *The child's conception of space*. London, UK: Routledge & Kegan Paul.

DIFFERENT TRANSLATION ACTIONS IN THE FIELD OF FUNCTIONAL RELATIONSHIPS

Renate Nitsch, Regina Bruder
Technical University of Darmstadt

In order to describe learning processes the social activity theory gives an adequate background. One focus of this theory is a hierarchical structure of learning activities (Kaptelinin, 1994). In our study, embedded in the project HEUREKO (HEUristic discoveries of REpresentations of functional relationships and the diagnosis of mathematical Competencies of students) we differentiated basic translation actions between different forms of representations in the field of functional relationships and analysed their structural relations. According to Leinhardt, Zaslavsky and Stein (1990) translation actions can be differentiated into two main categories: Interpretation and Construction. Based on this general classification we consider four *elements of cognitive actions*: Identification (recognizing the most important aspects of the given form of representation), Construction (building a target representation which is not given), Description (describing the solving process verbally) and Explanation (arguing why a specific solution is correct or incorrect). In the context of the HEUREKO project we analysed their structure resulting in a 3-dimensional competence structure model: Identification and Construction form two separate dimensions, in the third dimension, Description and Explanation are summarized as a combined action, because both demand verbalization.

Based on the results of a pilot project, 120 designed tasks were divided into four different assessment booklets in the form of a multimatrix design. 650 students from grade 9 and 10 of eight high schools (Gymnasium) took part in our main study. A comparison of different alternative models using probabilistic test models verified our postulated 3-dimensional competence structure model. A specific feature of our model is the possibility of multiple item loading, i.e. one item can be related to several elements of cognitive action. The most important finding is the separation of the two elementary actions Identification and Construction. The multidimensional structure shows that Identification is essential for all other actions whereas Construction additionally contains the building of a not given form of representation. These two actions are especially meaningful for the learning process and should both be explicitly involved in exercises. The multidimensional structure suggests several difficulty levels which could lead, including further research, to a hierarchical structure.

References

- Kaptelinin, V. (1994). Activity theory: Implications for Human-Computer Interaction. In M.D. Brouwer-Janse, T.L. Harrington (Eds.): *Human Machine Communication for Educational Systems Design* (pp. 5-14). Berlin: Springer.
- Leinhardt, G., Zaslavsky, O. & Stein, M.K. (1990). Functions, Graphs, and Graphing: Tasks, Learning and Teaching. *Review of Educational Research*, 60(1), pp. 1-64.

INCREASED EFFICIENCY WHEN ENGAGING IN CREATIVE MATHEMATICAL FOUNDED REASONING

Mathias Norqvist, Yvonne Liljekvist, Bert Jonsson, Johan Lithner
Umeå University, Karlstad University, Umeå University, Umeå University

Starting from research insights related to inefficient rote learning, the Learning by Imitative or Creative Reasoning design research project studies an approach to teaching based on student's own creation of knowledge and compares this to the common imitative model of teaching. Mathematical tasks were designed based upon a mathematical reasoning framework (Lithner, 2008) utilizing a specific didactical practice situation (Brousseau, 1997). Thus, the participating students were engaged either in algorithmic reasoning (AR), or in creative mathematical founded reasoning (CMR), during practice. The research questions were: What are the outcomes when comparing imitative and creative designs? How can these be understood?

The sample consisted of 102 students at the Natural science program in Swedish upper secondary school (16-17 year olds). The participants were matched into two groups (CMR & AR) based on a cognitive composite score (Raven matrices & operation span), gender, and mathematics grade. The intervention comprises training tasks related to the target knowledge, i.e. 14 formulas. The training session was computer based and took about half an hour to complete. Except for the presented algorithmic representation in the AR-tasks the training tasks were identical for the two groups. The retention interval was 6-8 days, and the same test was administered to all students. The test tasks were also computer based, the first task evaluated knowledge of a specific formula and then, if needed, a possibility to reconstruct the formula was given in two consecutive tasks. The CMR group significantly outperformed the AR group on all three test tasks. The cognitive composite score was found to be a significant predictor for performance in the AR group but not in the CMR group. The result indicates that CMR performances as a function of CMR practice is unrelated to individual's cognitive resources. Instead, successful training was found to be the predictive factor for performances in the CMR group. The opposite pattern was seen for the AR group. In addition when removing participants with the highest 33% cognitive composite score the pattern remained. Hence, the differential effects between CMR and AR were not driven by the cognitively stronger students. Our suggestion is that CMR model of teaching is more effective and neutral in terms of basic cognitive resources.

References

- Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Dordrecht; Boston: Kluwer Academic Publishers.
- Lithner, J. (2008). A Research Framework for Creative and Imitative Reasoning. *Educational Studies in Mathematics*, 67(3), 255-276.

THE MATHEMATICAL TASKS FOR REFLECTIVE THINKING IN POLAR COORDINATES

Hye Mi Oh, Oh Nam Kwon, Jee Hyun Park, Ji Eun Lee
Seoul National University

Advanced mathematics is an elective subject introduced to the 2009 revised high school curriculum for the first time in Korea. Since there is no research about polar coordinates in pedagogical perspectives, the purpose of this research is to analyse pre-service secondary teachers' reflective thinking for tasks on polar coordinates.

Four interview tasks which were adapted from Kwon & Lee (2013)'s research and modified, consisted of two features of polar coordinates: multi-valued expressions and symmetry. We analyzed each semi-structured interview case by using the main stages of the process of reflective thinking as a framework (Gagatsis & Patronis, 1990).

Participant WJ initially tried to solve the tasks about multi-valued expressions by using the features of Cartesian coordinates. He did not think of the relationship between the solution of the equation and the intersection of the graphs. Reflecting on his thoughts, he was aware that one graph could represent two polar equations. Finally, both WJ and HJ failed to see their own process of solving the tasks according to r and θ synthetically.

WJ tried to find the symmetry of the polar equation graph in task 3 using algebraic conditions about symmetry, $f(-\theta) = -f(\theta)$ that he had used to find the symmetry of equations in the Cartesian coordinates. After sketching the graph of the polar equation partially and substituting some particular points for two variables r and θ in the polar equation, he noticed that the polar equation did not satisfy the two algebraic conditions about symmetry. WJ was provided the opportunity to develop his understanding of polar coordinates through tasks that cause the reflective thinking.

This study is intended to show that the tasks arousing cognitive disequilibrium provide pre-service secondary teachers with the opportunity to develop their incomplete understanding of polar coordinates through reflective thinking. Teachers should experience the enhanced process of reflective thinking through questioning to improve recognition of disequilibrium.

References

- Gagatsis, A., & Patronis, T. (1990). Using geometrical models in a process of reflective thinking in learning and teaching mathematics. *Educational studies in mathematics*, 21(1), 29-54.
- Kwon, O.N., & Lee, J. E. (2013). Mathematical Tasks in the Advanced Mathematics Class for Nurturing Reflection, In Beinderjeet Kaur & Toh Tin Lam (Eds,) *Nurturing Reflective Learners, Yearbook 2013*. Sinagpore: Worldcientific.

PROSPECTIVE MATHEMATICS TEACHERS' CRITICAL THINKING PROCESSES REGARDING CONDITIONAL PROBABILITY IN THE CONTEXT OF POPULAR MEDIA TEXTS

Mehtap Ozen, Erdinc Cakiroglu
Middle East Technical University, Turkey

Critical thinking is regarded as an educational goal to nurture citizens actively participating in the society. In order to contribute to this goal in schools, mathematics teachers should have the ability of critical thinking with mathematical concepts in various contexts. In this sense, popular media texts are interesting contexts to explore which may include ambiguous or misleading language, particularly regarding concepts of probability. Conditional probability is one of the essential concepts in the media texts to make sense of uncertainty in real world, which often misused or confused by consumers of the statistical and probabilistic information (Gal, 2005). In this sense, this study aimed to investigate the extent to which prospective middle school mathematics teachers make use of critical thinking skills in the newspaper articles including conditional probability statements. The participants of the study consisted of four prospective middle school mathematics teachers. The data were collected through in-depth interviews. The participants were asked to think about the newspaper article regarding the blood test asserted to detect a pregnancy with Down syndrome. The data analysis was conducted on the basis of six critical thinking skills (interpretation, analysis, evaluation, inference, explanation, and self-regulation) conceptualized with a consensus among experts (Facione, 1990). The findings of the study highlights (a) participants' tendency to interpret the statements by clarifying their meanings (b) mostly inappropriate inferences regarding accuracy rate of the test (c) insufficient evaluation to judge the credibility of the statements in the newspaper article (d) mostly insufficient analysis of closely related statements, which requires understanding the difference between $P(A|B)$ and $P(B|A)$ (e) intertwined relationship among the critical thinking skills. The reasons underlying the findings could derive from their language skills, knowledge of health context, difficulty in noticing the conditional event or misuse of conditional probability statements. In this sense, the findings of this study emphasize the necessity of critical thinking skills in the contexts of media, particularly in conditional probability statements, which sometimes include vague or ambiguous meanings.

References

- Gal, I. (2005). Towards "probability literacy" for all citizens: building blocks and instructional dilemmas. In G. Jones (Ed.), *Exploring probability in schools: challenges for teaching and learning* (pp. 39-63). New York: Springer.
- Facione, P. A. (1990). *Critical thinking: A statement of expert consensus for purposes of educational assessment and instruction. Research findings and recommendations*. ERIC, ED 315423.

HOW EXPERIENCES WITH LINEAR GROWTH INFLUENCE STUDENTS' UNDERSTANDING OF EXPONENTIAL GROWTH?

Zekiye Ozgur, Amy Ellis, Torrey Kulow, Muhammed F. Dogan, Caroline Williams;
Joel Amidon

University of Wisconsin-Madison; University of Mississippi

Exponential growth is a critical topic in school algebra and in higher mathematics, but research on students' thinking suggests understanding exponential growth remains an instructional challenge. These documented challenges call for investigations into students' learning about exponential growth. Building on the body of research suggesting reasoning with quantities support students' understanding of functional relationships (Smith & Thompson, 2007), this research project aims to investigate students' learning about exponential growth in the context of two continuously covarying quantities, height and time. Specifically, we examine how students' experience with linear growth affect their learning of exponential growth.

This study was situated at a public middle school and consisted of a 12-day teaching experiment with three eighth-grade students (ages 13-14) in which the second author was the teacher-researcher. All sessions were videotaped and transcribed. Data analysis relied on retrospective analysis techniques (Simon et al, 2010) to characterize students' changing conceptions throughout the teaching experiment.

Two conceptual foci occurred in the development of students' understanding of exponential growth: *correspondence reasoning* and *covariational reasoning* (Ellis et al, 2012). Our most recent findings suggest that students' experience with linear growth influenced their development of correspondence rules. The influences of students' linear models in the formation of students' correspondence reasoning for exponential growth, as well as the implications of these findings, will be further discussed in the presentation.

References

- Ellis, A. B., Ozgur, Z., Kulow, T., Williams, C., & Amidon, J. (2012). Quantifying exponential growth: The case of the Jactus. In R. Mayers and L. Hatfield (Eds.), *Quantitative Reasoning and Mathematical Modelling: A Driver for STEM Integrated Education and Teaching in Context*. WISDOMe Monographs (Vol. 2). Laramie, WY: University of Wyoming.
- Simon, M., Saldanha, L., McClintock, E., Akar, G., Watanabe, T., & Zembat, I. (2010). A developing approach to studying students' learning through their mathematical activity. *Cognition and Instruction*, 28(1), 70-112.
- Smith, J. P., & Thompson, P. W. (2007). Quantitative reasoning and the development of algebraic reasoning. In J. J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the Early Grades* (pp. 95-132). Mahwah, NJ: Lawrence Erlbaum Associates.

PRE-SERVICE TEACHERS' CONCEPTIONS ON MATHEMATICAL CREATIVITY

Areti Panaoura*, Georgia Panaoura-Maki**

*Frederick University **Pedagogical Institute Cyprus

Contemporary curricula emphasize the development of students' creative thinking, in all aspects of learning. Creative ability in mathematics refers to the ability to perceive patterns and relationships using complex and non-algorithmic thinking and being capable of original thinking (Livne & Milgram, 2006). Even when the teachers acknowledge the importance of their role in fostering mathematical creativity, they report several factors that inhibit the manifestation of mathematical creativity (Shriki, 2008). In order to be able to foster their students' mathematical creativity they should acquire suitable pedagogical knowledge during their training. The aims of the present study were to investigate: (1) pre-service teachers' conceptions on mathematical creativity and (2) their ability to transfer in their lesson plans the pedagogical knowledge they acquired about mathematical creativity after the completion of a training course on creativity. The research was carried out at a sample consisted of ten students, during their studies at a pedagogical department. They were asked to define creativity, expressed their opinions about the relation of creativity and mathematics education and mainly propose creative teaching mathematical activities. Qualitative analysis concentrated on their ability to transfer their ideas into teaching plans.

Results revealed the initial conceptions of some pre-service teachers that mathematics is not a creative subject and that it is difficult to develop creativity in mathematics. They included on their initial definitions of creativity only the concept of "originality" – referring to the final product – and the use of different processes in order to investigate and encounter a new situation. Qualitative analysis of their manuscripts suggested that their conceptions on mathematical creativity were still limited, as they preferred to use typical and routine mathematical activities. Programs of teacher education have to be much more explicit in discussing with the prospective teachers what it means to foster students' creative thinking in primary mathematics and in what ways this can be done.

References

- Livne, N. & Milgram, R. (2006). Academic versus creative abilities in mathematics: two components of the same construct? *Creativity Research Journal*, 18 (2), 199-212.
- Shriki, A. (2008). Towards promoting creativity in mathematics of pre-service teachers: The case of creating a definition. In R. Leikin (Ed.), *Proceedings of the 5th International Conference on Creativity in Mathematics and the Education of Gifted Students* (p.p. 201-210). Haifa.

SECONDARY PRE-SERVICE TEACHERS' SUBJECT MATTER KNOWLEDGE AND KNOWLEDGE ABOUT STUDENTS: MEANING OF EQUIVALENT FORMS IN ALGEBRA

Joo Young Park

Teachers College, Columbia University

Research on teaching has started to include investigations of teachers knowledge and understanding of students' way of thinking related to specific topics, as well as the issue of the nature and quality of teachers' responses to students' questions, remarks and ideas (McDiarmid & Ball 1988); however, there is little research on teachers' subject matter knowledge and knowledge about students' understanding in a specific topic at secondary level. Algebraic equations and inequalities play an important role in various mathematical topics including algebra, trigonometry, linear programming and calculus. All students in Grade 9-12 should understand the meaning of equivalent forms of expressions, equations, and inequalities and solve them fluently (NCTM, 2000). Accordingly, this study examined pre-service teachers' content knowledge and 'knowledge about students' in the context of written equivalent expression, equations and transformation of inequalities.

Two pre-service secondary mathematics teachers completed non-standard mathematics tasks followed by semi-structured interview. The pre-service teachers were asked to justify their answers and explain their way of dealing with students' solutions. Data was analyzed by a content analysis and Shulman (1986)'s two sources of pedagogical content knowledge: knowledge of subject matter and knowledge about students: knowing "that" and knowing "why". The result showed that two prospective teachers performed meaningless procedures not having a deep understanding of mathematical structures. Two teachers' solutions were not generalizable, and further probing revealed that "knowing why" influences the prospective teacher's pedagogical content specific decision. Even though this study provided with illustrations of the prospective secondary mathematics teachers' knowledge on the specific content, clarification of the complex ideas surrounding knowledge for teaching algebra entails.

References

- McDiarmid, G. W., and Ball, D. L. (1988). "Many Moons": Understanding teacher knowledge from a teacher education perspective. *The Journal*, 2(2), 23-37
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- Shulman, L. (1986). "Those who understand: Knowledge growth in teaching." *Educational Researcher* 15(2).

10 YEARS OF DYNAMIC GEOMETRY IN PME

Gilmer Jacinto Peres, Rúbia Barcelos Amaral
CEFET-MG/UNICAMP-SP, UNESP-RIO CLARO

The Psychology Mathematics of Education (PME) gathers researchers from all over the world, but what are the characteristics of their researches? This study analyses papers on Dynamic Geometry (DG) in last ten years from PME-27 to PME-36 focus on Research Reports, identifying categories. If we consider *that purposes of research in Mathematics Education are manifold* (Kilpatrick, 1992, p.3), we understand the ways by which these researches show the DG view in the world. In this article we are inspired on Drijvers, Kieran & Mariotti (2010) research for identifying which categories are present in the last ten years of PME research. We look for specific words used in articles: *Geometry; Computer; Student; Software; Teaching and Learning*. The small amount was found in papers of PME-27 (3) and the bigger in PME-36 (13).

Thus we could identify and group the papers published by PME edition which related to DG on similar topics. There are some papers that can be included in more than one category, but in this paper we just took in account the category of the greatest relevance. We have found eleven categories which the respective quantity of articles follows: 1st - *State of the art in DG* [2]; 2nd - *How the teachers use DG* [3]; 3rd - *Dynamic Geometry Environment (DGE) to study specifics geometry content* [3]; 4th - *Making Conjectures and Proofs with DGE* [5]; 5th - *Material production in DG by teachers* [2]; 6th - *DG and Pedagogical Theories* [11]; 7th - *Training and use of DGE* [3]; 8th - *Student performance with DGE* [16]; 9th - *The DGE for other Mathematics content study* [6]; 10th - *DG and Teachers training* [8]; 11th - *Differences between DGE and paper and pencil use* [4]. These eleven categories show the overall vision about DG researches developed and publicized in the last ten PME editions.

The study of the researches found in the categories with the largest amount of works indicate that pedagogical theories, like Van Hiele levels for example, can leverage activities that involve DG (6th category); and how students deal with these features helps their activities doing (8th category). In the 9th category, for other example, we realize that DG can be used to increase the understanding of others mathematical content beyond geometry, which shows new potential for the DGE use. In this event we want to share the works classified in these categories, promoting discussion and further study on this subject.

References

- Drijvers, P., Kieran, C., & Mariotti, M. A. (2010) Integrating technology into mathematics education: theoretical perspectives. In C. Hoyles & J.-B. Lagrange (Eds.), *Mathematics Education and Technology-Rethinking the Terrain: The 17th ICMI Study*. 13. (pp. 89-132). Springer US.
- Kilpatrick, J. (1992) A history of research in mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. (pp.3-35). New York: Macmillan.

LATE-ARRIVED IMMIGRANTS IN SCHOOL AND PERFORMANCE IN ALGEBRA

Jöran Petersson
Stockholm University

This paper reports from a study on immigrant and native students tested in algebra as one of several topics in school mathematics. In general immigrant students perform lower the later they have immigrated (Böhlmark, 2008). Students often find working with algebra difficult and Kieran (1992) noted that new-beginners in algebra often read algebraic expressions from left to right and might ignore the brackets.

In this study 358 school year 9 students in six Swedish schools, with an over average percentage of immigrants, took a test. Several test problems were formulated so that they were likely to not cause too much of language obstacles for second language learners. In this report the test problem “ $a = 2$, $b = 4$. What is $a(b+2)+b$?” is in focus. This problem was characterized in Duval’s (2006) semiotic registers as mainly “computations” and scarcely dependent on natural language.

An important result is that the students who immigrated during school years 8 – 9 performed better than the native students and much better than students who immigrated during school years 1 – 7. Most wrong solutions among all student categories were due to misuse of the distributive law in line with Kieran (1992). There were few arithmetic errors.

In this on-going research project one conclusion is that there seems to be a need to see early and late immigrants as having different challenges in being second language learners. The former have difficulties in following some advanced topics in mathematics teaching and the latter have difficulties in understanding some test questions. A second conclusion is that there is a need in research to look at specific topics in mathematics, especially advanced compulsory school mathematics such as negative numbers and algebra, for these student categories.

References

- Böhlmark, A. (2008). Age at immigration and school performance: A siblings analysis using Swedish register data. *Labour Economics*, 15(6), 1366-1387. doi: 10.1016/j.labeco.2007.12.004
- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61(1-2), 103-131. doi: 10.1007/s10649-006-0400-z
- Kieran, C. (1992). The learning and teaching of school algebra. In D.A. Grows (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 390-419). Reston, VA: National Council of Teachers of Mathematics.

MATHEMATICAL CHALLENGE AND SENSITIVITY TO STUDENTS IN UNIVERSITY LECTURING: AN UNEASY BALANCE

Georgia Petropoulou*, Barbara Jaworski**, Despina Potari*, Theodossios Zachariades*

*University of Athens, Greece ** Loughborough University

Current research recognizes that teaching a large number of university students in a lecture format has very little been investigated. We study first year's Calculus university teaching in two Greek mathematics departments. Calculus is a compulsory course in both departments and it is taught exclusively in a lecture format in classes of approximately one hundred students. In this paper we focus on the ways that two experienced lecturers try to include students more overtly in the lecture; addressing students' difficulties and at the same time drawing them into mathematical culture. We analyse dialogues from each lecture to characterise teaching using the Teaching Triad (Jaworski, 1994). This suggests how the lecturers employ "management of learning", "sensitivity to students" and "mathematical challenge" to engage their students and enable them to make meaning of the mathematics of the lecture. A balance between sensitivity to students (in both cognitive and affective domains) and mathematical challenge is an indicator of effective mathematics teaching (Potari & Jaworski, 2002).

We find that the lecturers attempt to encourage students to participate in the process of creating mathematics. One of the lecturers offers challenge that a few students can follow and the lesson moves smoothly towards his goals. The other lecturer seems to reduce mathematical challenge to meet the students' needs. Both lecturers build on responses and comments from a small number of students. They attempt to draw on students' inputs and synthesize these inputs to make them meaningful to the rest. These teaching actions seem to indicate a balance between sensitivity to students and mathematical challenge in these cases of university teaching. Further analysis of the data is needed to explore the relation of the elements of the Teaching Triad in the university mathematics teaching and its potential to characterise mathematics teaching at the university lecturing.

References

- Jaworski B. (1994). *Investigating Mathematics Teaching: A Constructivist Enquiry*. London: Falmer Press.
- Jaworski, B. (2002). Sensitivity and Challenge in University Mathematics Teaching. *Educational Studies in Mathematics*, 51, 71-94.
- Potari, D. & Jaworski, B. (2002). Tackling Complexity in Mathematics Teacher Development: Using the teaching triad as a tool for reflection and enquiry. *Journal of Mathematics Teacher Education*, 5, 351-380.

DIAGNOSTIC COMPETENCES OF MATHEMATICS TEACHERS – WHAT KIND OF KNOWLEDGE DO TEACHERS USE?

Kathleen Philipp, Timo Leuders

University of Education (Pädagogische Hochschule) Freiburg, Germany

Diagnostic competences of teachers are important for students' learning success. In addition to formal diagnostic tests, there are also informal diagnostic situations which influence instruction. In mathematics such diagnostic situations are often linked to dealing with tasks, e.g. (i) Teachers analyse and select tasks with respect to their potential diagnostic value and (ii) teachers evaluate students' solutions to a task. While there is plenty of research on the precision of teachers' diagnostic judgements ('veridicality') there many open questions remain with respect to the cognitive processes of teachers during the assessment process and to the domain specificity of their diagnostic competence. The research of Ball et al. (2008) suggests the importance of mathematical knowledge. Morris et al. (2009) show that "unpacking" the sub-goals of a task may be an important facet of diagnostic competence. For an in-depth investigation of teachers' diagnostic competence we pursue the following research questions:

- What kind of processes can be identified in teachers' diagnostic judgements?
- What kind of knowledge do teachers use during these processes?

Six well experienced secondary teachers of German lower secondary school were first asked to judge two given tasks (on fractions, e.g. "Which fraction can be found between $\frac{1}{2}$ and $\frac{1}{3}$?") and to evaluate three students solutions afterwards. Think-aloud-protocols of their reflections of their own assessment-processes offered the data for the present study. Interpretative content analysis techniques were used for the analysis.

The results show that a variety of different diagnostic processes occur, drawing on (i) mathematical knowledge, (ii) knowledge of students' (mis)conceptions and (iii) intuitive judgements based on experience. In the analysis it becomes obvious that the ability to unpack learning goals is inadequate to clarify students' misconceptions completely.

References

- Ball, D. L., Thames, M. H., & Phelbs, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education*, (59), 389–407.
- Morris, A. K., Hiebert, J., & Spitzer, S. M. (2009). Mathematical Knowledge for Teaching in Planning and Evaluating Instruction: What Can Preservice Teachers Learn? *Journal for Research in Mathematics Education*, 40(5), 491–529.

EXPLORING THE EPISTEMIC FACET OF THE DIDACTIC-MATHEMATICAL KNOWLEDGE REQUIRED TO TEACH THE DERIVATIVE

Luis R. Pino-Fan, Vicenç Font, Juan D. Godino

In Pino-Fan et al. (2012) we presented the results obtained in the first part of a study that explored what we call the epistemic facet of prospective secondary teachers' didactic-mathematical knowledge about the derivative. The study as a whole involved three stages: 1) design of a questionnaire using the Onto-Semiotic Approach (OSA) (Godino, et al., 2007) to evaluate teachers' knowledge about the derivative; 2) analysis of the results obtained in a pilot application of this questionnaire; and 3) based on the information gathered in stage 2, development and application of a final version of the questionnaire, and analysis of the results obtained. This third stage also included interviews in order to obtain a more detailed understanding of prospective teachers' knowledge. The results of our research show that the variable 'type of cognitive configuration activated in the prospective teachers' answers are useful for understanding the kind of didactic-mathematical knowledge they possess. This variable was analysed by means of a tool that we refer to as the 'configuration of primary mathematical objects and processes', one which facilitates the analysis and categorization of certain features of the epistemic facet of prospective teachers' didactic-mathematical knowledge. The design of the questionnaire used in this study, as well as the responses of prospective teachers to it, reveal the complex set of mathematical practices, objects and processes that are brought into play when solving tasks related to the derivative. Teachers need to become aware of this complexity during their training so that they will be able to develop and assess the mathematical competence of their future students.

Acknowledgements

This study was conducted within the framework of two teaching training projects, EDU2012-32644 (University of Barcelona) and EDU2012-31869 (University of Granada).

References

- Godino, J. D., Batanero, C. & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics Education*, 39(1), 127-135.
- Pino-Fan, L., Godino, J.D., Font, V. & Castro, W.F. (2012). Key epistemic features of mathematical knowledge for teaching the derivative. In Tso, T.Y. (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 297-304). Taipei, Taiwan: PME.

ASSESSMENT CRITERIA AS A TOOL FOR MATHEMATICS LEARNING

Jorge Pinto, Leonor Santos

Institute of Education of Lisbon University, Polytechnic Institute of Setúbal

According to Vygostksy, learning is performed through a process of mediated activity, where the environment is part of the intention to learn (Bart, 2007). Thus, language as a tool of mediation becomes central. It is through language that the meanings for the different activities are negotiated, activities in which the student gets involved, learns and recognizes herself as a learner. Reducing the gap between what is, and what is not, important - especially in mathematics - is a prerequisite for mathematics learning (Hannula, 2006; Wiliam, 2007). Assessment criteria constitute a mediation tool that gives immediate meaning to tasks and allows the student to perceive the specific assessment criteria in a continuum of more general learning. But, for this to really happen, it is not enough to state the criteria, it is also necessary to negotiate them, allowing their ownership by the students. Based on two studies conducted in two classes (one with six-year-old students working with problem solving, the other with eleven-year-old students in the context of mathematical reasoning), we formulated the following research questions: Will students show evidence that they have appropriated ownership of evaluation criteria? In what way assessment criteria guide students in understanding the mathematical capacities that are being developed? Both studies used an interpretive methodological approach supported by student interviews, participant observation of classes and documental evidence. Based on the progressive construction of assessment criteria that involved both students and teachers, the results show that students used and reshaped the assessment criteria in a progressive fashion. They displayed understanding of the mathematical activities that comprise problem solving and mathematical reasoning. In parallel, the students developed their capacity to self-regulate. These results permit us to conclude that the assessment criteria can be an important resource for mathematics learning, regardless of the student's age.

References

- Bart, B. (2007). L'établissement de l'intersubjectivité comme outil de médiation: participer pour apprendre. In L. Allal & L. Lopez (Orgs.), *Régulation des apprentissages en situation scolaire et formation*. Bruxelles: de Boeck.
- Hannula, M. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics*, 63, 165–178.
- Wiliam, D. (2007). Keeping learning on track. In F. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1053-1098). Charlotte: Information Age Publishing.

MODALITY OF TASK PRESENTATION AND MATHEMATICAL ABILITY IN A STUDY ABOUT SPATIAL ABILITY

Meike Plath

Leuphana University Lüneburg, Germany

On the one hand, psychological research leads to the assumption that different ways to present spatial tasks may influence the solving process. But only few is known about the kind of influence the task presentation has on spatial ability (e.g. Nigl and Fishbein 1974). On the other hand, the relationship between spatial and mathematical abilities was discussed in a variety of studies. However, the participants mostly were older students and the tasks often covered only small parts of spatial ability (e.g. Smith, 1964; Fennema & Tarte, 1985). Therefore, this study focuses on the combination between different ways to present spatial tasks and the correlation between spatial and mathematical abilities. Two research questions are emphasised: 1) *Does the way the tasks are presented to the children influence which strategy they chose and how successful they are?* 2) *Do children's mathematical abilities influence their success in solving spatial tasks?*

27 fourth-graders with high and 30 with low mathematical abilities participated in the study. In face-to-face interviews, they were asked to solve four different types of spatial tasks with 38 items and to explain their solving strategies. These tasks were presented as real cube objects or photographs of these objects.

The analysis of the solution strategies shows that the modality of the task presentation does not have an overall influence on the solution rates or the success of the strategies. Differences only occurred in the mental rotation tasks. The assumption that children solve spatial tasks more easily when faced with real objects in comparison to photographs could not be confirmed.

Although the mathematically high-achievers got overall significant better results. These differences could only be observed in two of the four types of tasks. Interestingly, no differences could be detected in the flexibility of using different kinds of strategies.

References

- Nigl, A. J., & Fishbein, H. D. (1974). Perception and conception in coordination of perspectives. *Developmental Psychology*, 10(6), 858-866.
- Smith, I. M. (1964). *Spatial ability. Its educational and social significance*. San Diego: Robert R. Knapp.
- Fennema, E., & Tarte, L. A. (1985). The use of spatial visualization in mathematics by girls and boys. *Journal for Research in Mathematics Education*, 16(3), 184-206.

THE USE OF LEARNING JOURNALS AS ASSESSMENT

Hilary Povey
Sheffield Hallam University

It is now widely understood that assessment practices have a key influence on learning. Summative assessment tends to have a damaging effect overall and in particular on learners' self-awareness and deep engagement (Harlen & Deakin Crick, 2003). This research concerns the use of learning journals for the assessment *of* learning (summative), *for* learning (formative) and *as* learning, developing further the concept of *educative assessment* (Povey & Angier, 2006).

The project falls within an action research paradigm with elements of an auto-ethnographic approach. The participants were graduates in other disciplines studying mathematics for a year at undergraduate level to prepare for initial teacher education. The project was inspired by an article about using learning journals with secondary pupils (Coles & Banfield, 2012) - 'one of the things we were most struck by was the power of these journals in supporting pupils to revisit and take on their work in a manner that seemed to provoke new learning and awarenesses' (p 11). Students were given a scrapbook at the beginning of a pure mathematics module. The format was chosen to encourage informality, personalisation and freedom in the students' responses. The teaching of the module was interspersed with 'scrapbook sessions' where scissors, glue and other materials were provided and students reviewed and extended their learning; these sessions were supplemented by substantial amounts of independent out-of-class work by the students on the scrapbooks. The scrapbooks were submitted throughout the module whenever an individual chose for formative feedback - "post-its" attached to their work - and finally for summative assessment.

Data for the study comprises the journals themselves and written reflections from and interviews with the participants. Initial analysis indicates that the journals were highly effective in eliciting an awareness of and engagement in learning through the students choosing what to include, articulating what had been achieved and assimilating and responding to the formative feedback.

References

- Coles, A. & Banfield, G. (2012). Creativity and mathematics: using learning journals. *Mathematics Teaching*, 228, 6-11.
- Harlen, W. & Deakin Crick, R. (2003). Testing and motivation for learning. *Assessment in Education*, 10(2), 169-207.
- Povey, H. & Angier, C. (2006). The assessment of undergraduate mathematicians: re-crafting assessment of learning to provide opportunities for assessment as learning. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds), *Proc. 30th Conf. of the Int. Group for the Psychology of Mathematics Education*, 4, 353-360. Prague: PME.

EARLY DEVELOPMENT OF MATHEMATICS COMPETENCES - AN INTERVENTION IN KINDERGARTENS IN A LOW INCOME COMMUNITY

Glória Ramalho, Mafalda Magalhães, Susana Batista, Sofia Ferreira, Inês Elias
ISPA, IU K'CIDADE

This presentation will illustrate a program designed for kindergarten children and display the evidence sustaining its positive effect in their number sense development.

Several authors argue for an early approach to mathematics education from preschool on (e.g. Brissiaud, 1989; Ginsburg & Baroody, 2003; Griffin, 2004). This early approach of mathematical competences is known to foster children's later development and contribute to their more successful school progress. Based on this literature and following learning trajectories present in the literature, we developed a program intended for children 4 to 5 years old attending kindergarten in a community considered to be in need of an education intervention. This program was developed by the education team of K'CIDADE, an initiative of the Aga Khan Portugal Foundation.

This program was created with two objectives: i) to improve children's development of mathematical competences, specially with respect to number sense; ii) to foster a better awareness of kindergarten teachers about the importance of strategies favoring that development.

This program took place during 4 months through 21 sessions which included enjoyable activities which lasted about 45 minutes. Two hundred and four children attended this program. In order to evaluate its potential effect on children's number sense, an experimental pre-test – pos-test design was implemented which included a control group of kindergarten children in the same region. TEMA 3(Ginsburg & Baroody, 2003) was administered in order to assess children competences.

Evidence gathered showed the existence a positive effect of the program with respect to children's number sense development, especially the least competent as assessed in the pre-test. Kindergarten teachers rated the intervention with positive marks and asked for further in-service training.

References

- Brissiaud, R. (1989). *Como as Crianças Aprendem a Calcular*. Lisboa: Instituto Piaget.
- Ginsburg, H., & Baroody, A. (2003). *Test of Early Mathematics Ability, 3rd Edition*. Austin, Tx: Pro-Ed.
- Griffin, S. (2004). Building number sense with Number Worlds: a mathematics program for young children . *Early Childhood Research Quarterly*, 19, 173 – 180.

PROSPECTIVE PRIMARY MATHEMATICS TEACHERS' DIAGNOSTIC COMPETENCE: MODELLING THE PROCESS

Simone Reinhold

Institut für Didaktik der Mathematik und Elementarmathematik, TU Braunschweig

Diagnostic competence is a key aspect of teachers' professionalism. Acting within a diagnostic situation that intends to enlighten students' ways of (mathematical) thinking can be regarded as integral element of a multidimensional circular process (cf. Klug et al., 2013). Taking a qualitative view, this study goes beyond measuring accuracy of teachers' judgements of students' achievements, too, and tries to capture (prospective) primary teachers' way of proceeding in diagnostic mathematics interviews:

- What elements characterize (prospective) mathematics teachers' strategies during a one-on-one diagnostic mathematics interview?
- What types of strategies can be identified and to what extent are they used in a flexible way?

In ongoing data collection, participants of several university courses (prospective primary school mathematics teachers) are asked to prepare and conduct a videotaped one-on-one diagnostic mathematics interview with a child (grade 1). This experience is followed by a retrospective interview. The diagnostic strategies that have been analysed by qualitative coding so far often resemble to basic processes in qualitative data analysis and include acts like collecting, coding, sorting or contrasting. Interpreting relevant facts may also be driven by general dimensions of diagnostic strategies (e.g. topographic vs. symptomatic search; Cegarra & Hoc, 2005).

As the results of the study will offer an empirically grounded theoretical framework for a qualitative view on activities in mathematics interviews, the study contributes to the improvement of (preservice) teacher education: Details or types of diagnostic strategies can be discussed or taught. Learning about facets of such strategies, being aware of them and flexibly considering various strategic elements of diagnostic approaches can raise the benefit we derive from one-on-one diagnostic mathematics interviews in mathematics education.

References

- Cegarra, J. & Hoc, J.-M. (2005). Cognitive styles as an explanation of experts' individual differences: A case study in computer-assisted troubleshooting diagnosis. *International Journal of Human-Computer Studies*, 64, 123-136.
- Klug, J.; Bruder, S.; Kelava, A.; Spiel, Ch. & Schmitz, B. (2013). Diagnostic competence of teachers: A process model that accounts for diagnosing learning behavior tested by means of a case scenario. *Teaching and Teacher Education*, 30, 38-46.

ACCELERATING EARLY CAREER TEACHERS' KNOWLEDGE OF STUDENTS AND CONTENT IN ALGEBRA

Steve Rhine, Mike Charles, Colin Starr
Willamette University, Pacific University, Willamette University

The Algebraic Thinking Project accelerates ECTs' understanding of students' misconceptions and ways of thinking about algebra through modified mathematics methods courses at four universities. The project created an Encyclopedia of Algebraic Thinking, a Formative Assessment Database, and iOS apps for the development of ECTs' knowledge of students and content. A combination of quantitative and qualitative data will determine effectiveness.

Students' have significant mental hurdles in making the transition from arithmetic to algebra (Moseley & Brenner, 2009). Improving the quality of algebra teaching, the most important school-related factor influencing student achievement, will likely improve students' success in algebra. Early career teachers (ECTs) are not as effective as teachers who stay in teaching longer than five years (Henry, Bastian, & Fortner, 2011). Veteran teachers develop extensive knowledge of students' challenges when learning algebra. Ball, Thames, & Phelps (2008) define this type of Math Knowledge for Teaching as *knowledge of students and content* or understanding of students' errors, justifications, misconceptions, developmental sequences, etc. The Algebraic Thinking Project (ATP) accelerates the development of ECTs' *knowledge of students and content* to improve students' success in algebra.

Over 800 articles spanning the last three decades examine why students struggle in algebra. Based on these studies the ATP created an Encyclopedia of Algebraic Thinking, a searchable Formative Assessment Database of problems from research, and iOS apps that address challenging algebra topics. Four northwest universities redesigned their math methods courses for 35 ECTs with those resources. Each ECT's algebraic content knowledge and orientation towards incorporating students' algebraic thinking into their instructional decision making is assessed. The Teacher Disposition towards Students' Thinking (TDST) instrument helps analyze eight videotaped case studies. In June, 2013 we will have final data and analysis completed.

References

- Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.
- Henry, G. T., Bastian, K.B. and Fortner, C. K. Stayers and Leavers: Early-Career Teacher Effectiveness and Attrition. (2011). *Educational Researcher*, 40(6), 271-280.
- Moseley, B. & Brenner, M. E. (2009). A comparison of curricular effects on the integration of arithmetic and algebraic schemata in pre-algebra students. *Instructional Science: An International Journal of the Learning Sciences*, 37(1), 1-20.

IN-SERVICE TEACHER EDUCATION AS A FACTOR TO THE INSTITUTIONAL DEVELOPMENT

Margarida Rodrigues, Cristina Loureiro

Escola Superior de Educação do Instituto Politécnico de Lisboa

The success of curriculum reforms depends on the action of teachers as agents of curriculum decisions. So the in-service teacher education is a priority in order to achieve the goals outlined by these reforms. The teachers as continuous learners must improve the reflection on, in and for their professional practices within the profession. The demands of the professional development conception throughout life are consistent with the establishment of a lasting partnership between higher institutions and the elementary teachers and their institutions (Sowder, 2007). After the collaboration of ESELx in the implementation of the national in-service teacher education program in Mathematics for elementary teachers, we maintain the link with the trainers of this program supervising the training activities that we propose to accreditation and that they conduct within their schools.

The research questions are: 1) what contributions can give formalized training for the teacher professional development, including the professional knowledge?, 2) how can progress a school through supervising and collaboration processes?, 3) what contributions can provide the linkage with the elementary teachers for professional development of mathematics teachers of ESELx, and 4) how evolve a pre-service teacher education school taking into account an institutional project of in-service teacher education? The study will be conducted within an interpretative perspective, involving the mathematics teachers of ESELx, the trainers of elementary teachers and the directors of their schools. The data will be collected by: (a) semi-structured interviews, and (b) documental analysis.

The expected results will be focused on the internal legitimating of in-service teacher education as a line of an institutional strategy and on the potentialities of the collaborative work developed by the teams involving elementary teachers and higher teachers in the interaction between theory and practice and in the experiment teaching.

References

Sowder, J. (2007). The mathematical education and development of teachers. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157-223). Charlotte: Information Age Publishing Inc., & NCTM.

TEACHERS' BELIEFS REFERRING TO TEACHING WITH TECHNOLOGY

Paul Rögler, Bärbel Barzel, Andreas Eichler
University of Education Freiburg

In the mathematics education research, it is widely accepted that mathematics teaching with technology could facilitate students' conceptual knowledge when this teaching complies specific conditions (Zbiek et al., 2007). These conditions comprise for example a student-centred teaching style, a constructivist view on teaching instead of a transmission view on teaching (Staub & Stern, 2002) or teaching goal to facilitate an instrumental genesis of the technology tool (Zbiek et al., 2007). According to these demands concerning a promising teaching of mathematics with technology, the teachers' beliefs towards the teaching and learning of mathematics that determine the teachers' classroom practices and impact on their students' learning (Calderhead, 1996) play an important role. For this reason, the teachers' beliefs referring to mathematics, the teaching and learning of mathematics and particularly the teaching and learning of mathematics with technology is the focus of the project that is addressed in the research reported in this paper.

To investigate teachers' beliefs with respect to the mentioned aspects, we regard two groups of teachers. The first subsample of teachers consists of about 300 teacher trainers that are engaged in the project T³ – Teachers Teaching with Technology – that aims to promote teaching with technology in schools. The second subsample consists of about 300 ordinary mathematics teachers. We collect data using questionnaires and semi-structured interviews according to existing scales referring to different aspects of teachers' beliefs (e.g. Staub and Stern, 2002).

Referring to preliminary results of pilot studies we expect a confirmation of the dependency between regarding teaching as transmission and using technology as „computational tool“ on the one side and teaching as construction and technology as „instructional tool“ on the other side. Impressions and first results will be presented at the PME conference.

References

- Calderhead, J. (1996). Teachers: beliefs and knowledge. In D. C. Berliner (ed.), *Handbook of education* (pp. 709-725). New York: MacMillan.
- Staub, F. & Stern, E. (2002). The nature of teacher's pedagogical content beliefs matters for students' achievement gains: quasi-experimental evidence from elementary mathematics. *Journal of Educational Psychology*, 94 (2), 344-355.
- Zbiek, R.; Heid, K.; Blume, G.; Dick, Th. (2007): Research on Technology in Mathematics Education – A Perspective of Constructs. In F. Lester (ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1169-1207). Charlotte, NC: Information Age.

DEALING WITH COVARIATION: MISCONCEPTIONS AND THE EFFECT OF REPRESENTATION FORMS

Tobias Rolfes; Jürgen Roth, Wolfgang Schnotz
University of Koblenz-Landau; Campus Landau, Germany

When dealing with the concept of function, secondary school teaching mainly focuses on the input-output view. In this view, every value of the domain corresponds to a precise value of the range. Researchers stress that a second aspect – the aspect of covariation – is not sufficiently implemented in mathematical curricula (e.g., Thompson, 1994). Covariational thinking requires taking the variation of input and output into account: In which way will the function value vary if the x-value is varied?

Two main research questions are:

- What kind of misconceptions surface when dealing with covariational tasks?
- Does the representation of the function (table of values vs. graph) have an effect on students' covariational thinking performance?

The sample comprised 27 students of grade 7. Using physical materials to build different models of cubes, the students explored and compared the covariation of linear and quadratic functions with a discrete domain within a 40-minute, guided intervention. For each function they were asked to produce a table of values and a graph as forms of representation. After the intervention, the students' covariational thinking performance was tested with six paper-and-pencil tasks.

In the qualitative analysis of misconceptions we discovered students' confusion between the first and second difference. Many students incorrectly identified a function with constant second differences as a linear function instead of a quadratic function. The quantitative analysis revealed that the form of representation had a large effect on the construction task. The students performed significantly better at value table production than graph construction (sign test: $g = .35$, $p < .001$). In the presentation, these and further findings will be discussed in detail.

These results raise the question of whether the table of values is a neglected form of representation for covariational reasoning since mathematics teaching mainly uses the graphical and symbolic approach. Furthermore, we conjecture that a table of values could be more adequate for quantitative exploration whereas a graph might be more appropriate for qualitatively exploring covariation. A further topic for investigation is the problem of how to avoid confusion between the first and second difference.

References

Thompson, P. W. (1994). Students, Functions, and the Undergraduate Curriculum. In E. Dubinsky, A. H. Schoenfeld, & J. J. Kaput (Eds.), *Research in Collegiate Mathematics Education I* (pp. 21–44). Providence, RI: American Mathematical Society.

LONG-TERM DEVELOPMENT OF STUDENTS' REPERTOIRE OF RATE OF CHANGE PROCEDURES

Gerrit Roorda*, Pauline Vos⁺, Martin Goedhart*

^{*}University of Groningen, ⁺University of Agder

This study aims to contribute to understanding the development of students' use of rate of change procedures. Students learn several rate of change procedures, on one hand in mathematics classes (e.g. the derivative and using graphic calculator options), on the other hand in physics classes (e.g. using formulas such as $v=g \cdot t$). For students it is hard the relations between these procedures. We took students' ability to relate different procedures as indicator of their understanding. Based on the description of procedural fluency and conceptual understanding (Kilpatrick, Swafford & Findell, 2001) we analyzed students' repertoire of procedures in two dimensions: breadth (amount of procedures) and connectedness (relations between procedures).

To gain insight into students' development of their repertoire over a longer period of time, we carried out a longitudinal multiple case study with ten students. While the students (all following a science track) moved from grade 10 to grade 12, four task-based interviews were conducted with half year intervals. The interviews were designed to provide in-depth information on students' repertoire. The tasks asked for the calculation of an instantaneous change, but guiding terms (e.g. 'derivative' or 'slope') were explicitly avoided. Instead, situations were described with variables having a real-life meaning.

Our study shows that after the introduction of calculus in mathematics, and kinematics in physics, most students have a narrow and disconnected repertoire. While a few students can relate procedures, most students are unsure about which procedures can be used and how these procedures are interrelated. In the consecutive interviews most students demonstrated a broader and more connected repertoire. There were major differences in students' development, such as: the preference for procedures, the moment at which the repertoire became connected, and the final breadth and connectedness.

With respect to procedures learnt in different school subjects, we observed that most students eventually relate the tangent method (learnt in physics) and symbolic differentiation (learnt in mathematics). However, we did not observe that students could relate mathematics procedures and physics *formulas*.

References

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

THE RELATIONSHIP BETWEEN A GRAPH AND ITS DERIVATIVE GRAPH: IMPROVING STUDENTS' UNDERSTANDING

Ulf Ryberg

University of Gothenburg, Sweden

Over the last decades, it is reported by several researchers that students show weak understanding of the concept of derivative in the graphical context (e.g. Asiala et al., 1997). Many students facing a graph express the need of an algebraic formula so they, often without really knowing why, can use the differentiation rules. This pattern has been well known for a long time and the need of different representations (e.g. graphical) is proposed (e.g. Hähkiöniemi, 2006).

The aim of this planned study is to describe in what way it is possible to improve students' understanding of the relationship between a graph and its derivative graph. The methodological approach is Learning Study (e.g. Lo, 2012), an iterative and cyclic process where researchers and teachers cooperate in designing a lesson plan and analysing the implemented lesson. The current lesson plan is framed by variation theory (ibid.) and strives to make it possible for the learners to discern the critical aspects of the learning object (i.e. derivative) and thereby enable them to interpret the relationship between graphs and derivative graphs. Critical aspects are a key concept in variation theory and describe what the learner must discern about the object of learning to develop further knowledge. These aspects are found both theoretically, via previous research result, and empirically by studying the students' knowledge before the study start. The study is going to be conducted in March 2013 and comprises 80 students at three different programs (science, engineering, social science) at the Swedish upper secondary school. Pre- and post-tests will take place to measure the development and the results will be complemented by three qualitative interviews (one student in each program) to describe the learning situation from the students' point of views and to make it easier for teachers to facilitate students' learning in this area. The results of the tests and the interviews will be discussed at the presentation.

References

- Asiala, M., Cottrill, J., Dubinsky, E., & Schwingendorf, K. (1997). The development of students' graphical understanding of the derivative. *Journal of Mathematical Behavior*, 16 (4), 399-431.
- Hähkiöniemi, M. (2006). *The role of representations in learning the derivative*. Report 104 Jyväskylä: Department of mathematics and statistics
- Lo, M.L. (2012). *Variation Theory and the Improvement of Teaching and Learning*. Göteborg: Acta Universitatis Gothoburgensis

FROM MATHEMATICS TEACHERS TO TEACHER EDUCATORS: EMERGENCE OF A COMMUNITY OF PRACTICE

Laura Sabater, M. Carmen Penalva-Martínez, Germán Torregrosa-Gironés
University of Alicante (Spain)

Recent surveys have revealed the absence of research on mathematics teacher educators (MTEs) and have suggested directions for future research. Different authors pointed out the existence of few formal education programs or curricula that prepare novice MTEs (Jaworski & Wood, 2008). In this paper we use the notion of community of practice to analyze the emergence of a *community of practice* (Wenger, 1998) of four novice MTEs considered as a context of learning. The majority of MTEs are mathematicians that start to be interested on mathematics education, e.g., they are interested on the question how to promote and understand students' learning. This interest could appear during the initial education or later, after a period of teaching as mathematics teachers. Many of the novices MTEs in our primary and kindergarten mathematics teacher education programs at the University of Alicante are also secondary school mathematics teachers. Most of them have not received a specific education as MTEs and do not have time for formal education. The MTEs group in the mathematics education department can be considered as a professional organization in which they interpret and share their experiences.

Data come from a questionnaire that supports the reflection of MTEs about the coordination meetings of the subject during the four months of the kindergarten teacher education program. During the teacher education program, there were communication and coordination mechanisms that can promote the formation of the community of practice. The analysis of mathematics teachers' reflections and the negotiation of meanings of resources, and of their practice as teacher educators in the coordination meetings show different aspects of the transition from mathematics teachers to teacher educators. The dimensions of a community of practice, mutual engagement, negotiation of a joint enterprise and developing a shared repertoire are used in order to describe and explain this transition.

References

- Jaworski, B. & Wood, T. (2008). *The International Handbook of Mathematics Teacher Education: The mathematics Teacher Educator as a Developing Professional*. Rotterdam: Sense Publishers.
- Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. New York: Cambridge University Press.

RESEARCH TRAINING NEEDS OF PRACTICING MATHEMATICS TEACHERS

Brigitte Sánchez Robayo, José Torres Duarte, Jaime Fonseca Gonzáles, Jorge Rodríguez Bejarano
Universidad Distrital Francisco José de Caldas (Bogotá - Colombia)

As one part of the development of the research project called “Training in and toward investigation of practicing mathematics teachers”, some research training needs were identified from nine case studies and from establishing the dimensions and characteristics which surface around researching practice. First, the notion about need was established from different approaches about human needs, for instance, Maslow (cited Elizalde, A., et al., 2006) defines need as the absence or lack of something and Tejedor (1990, cited González, B., 2011) states that the term need refers to the difference between a common situation and some desired situation. Then need was considered as “the **difference** between and actual status and a desire status or ought, or something that allows the **transit** to one desire status or ought” (Sánchez, et. al., 2012, pg. 2165). Secondly, with respect to dimensions and characteristics identified, the dimensions are social, personal and performative and they are composed by characteristics as interaction in the social dimension, the critique and problematization in the personal dimension and the strategy in the realizative dimension.

Regarding these conceptions, nine case studies were carried out using no participative observation and in-depth semi-structured interviews. The studies were done with practicing teachers from different locations in Bogota and various conclusions refer to the need for teachers to work within a team whose members share the same interests and are accompanied by research professors.

References

- Elizalde, A., et al., (2006). A critical review of the human needs debate from the approach focused on the person. In POLIS, Journal of the Universidad Bolivariana (Chile). Original title: Una revisión crítica del debate sobre las necesidades humanas desde el Enfoque Centrado en la Persona. Vol. 5 n° 15, 2006. Recovered on 28 of february of 2012 from <http://www.istas.ccoo.es/descargas/Revisi%C3%B3n%20cr%C3%ADtica%20de%20las%20necesidades.pdf>,
- González, B., (2011). Professional profile and teacher training needs specialized in learning difficulties. Thesis Universidad Rovira I Virgili. Original title: Perfil profesional y necesidades de formación del docente especialista en dificultades de aprendizaje. Recovered on 28 of february of 2012 from <http://www.tesisenred.net/handle/10803/42932>.
- Sánchez, B., et al. (2012). *Training needs for investigation and research teacher's characteristics*. In III International congress and VIII National congress of research in education, pedagogy and teacher training. Original title: Necesidades de formación en investigación y características del profesor investigador. pp. 2161 – 2174.

STUDENT ENGAGEMENT AND MATHEMATICAL LEARNING: LOOKING SMART VS. STAYING OUT OF TROUBLE

Lina Sanchez Leal, Roberta Y. Schorr, Gerald A. Goldin
Rutgers University, New Jersey, USA

When exploring mathematical ideas with peers, students solving the same problem may experience very different affect (Jansen, 2008). We consider affective, behavioural, and cognitive dimensions of engagement (Fredricks, Blumenfeld & Paris, 2004), using the theoretical construct of engagement structures – idealized, highly affective patterns recurring in individuals, inferred from observed behaviours and interviews. Each includes several components; most importantly, a characteristic in-the-moment *motivating desire*, behaviours including social interactions oriented toward fulfilling the desire, and accompanying emotional states.

Our research goal was to identify patterns of mathematical engagement by 7th-graders in urban, low-income communities, and their impact on learning. We observed 55 students in 3 classes, working in groups during 8 videotaped sessions over 2½ weeks, using SimCalc MathWorlds® software to provide dynamic, linked computer-based representations (graphs, tables, algebraic symbols, geometric figures). We administered math pre- and post-tests and an engagement survey (4 times during the study). We interviewed selected “focus” students: a retrospective, stimulated-recall interview focusing on engagement, and a task-based interview with mathematical problems.

We report and compare results for two “focus” students who worked together. “Jen” refrained from talking to peers throughout the sessions, as she wanted to “stay out of trouble.” “Eric” dominated the resources and discussions, wanting to “look smart” to the teacher and his peers. When his answers were challenged he withdrew, saying he did not “care anymore,” but moments later re-engaged. Despite the vast difference in engagement patterns, both students showed substantial mathematical learning.

We conclude that the nature of the students’ engagement is *highly complex, dynamic, and situation-dependent*, and their dominant, contrasting patterns both had *adaptive value*. We draw some inferences for ways to broaden perspectives on the study of “in the moment” mathematical engagement.

References

- Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research*, 74, 59–109.
- Jansen, A. (2008). An Investigation of Relationships between Seventh-Grade Students' Beliefs and Their Participation during Mathematics Discussions in Two Classrooms. *Mathematical Thinking And Learning*, 10(1), 68.

PROSPECTIVE KINDERGARTEN AND PRIMARY SCHOOL TEACHERS' UNDERSTANDING OF THE MEAN

Raquel Santos

Unidade de Investigação do Instituto de Educação, Universidade de Lisboa

The current Portuguese curricular documents give significant emphasis to statistics. Teachers' knowledge of this topic may influence future generations of students, since a solid knowledge is essential to promote a learning environment where students want and are able to learn mathematics. A central notion in statistics is the mean. According to Jacobbe (2012), over the past 20 years, regarding the understanding of this concept, studies focusing on teachers were fewer than studies conducted on students. Hence, it becomes important to understand prospective teachers knowledge about the mean, in particular if they show computational or conceptual understanding of the mean, as the development of the latter type of understanding generally arises later than a computational understanding (Leavy & O'Loughlin, 2006).

Participants in this study are 18 Portuguese prospective primary (grades 1 to 6) and kindergarten teachers in their 3rd year. Here, we focus on their answers to three questions of a questionnaire assessing prospective teachers' understanding of the mean. Data were analysed from several perspectives: correctness of solution and evidence of conceptual or computational understanding. The results show that very few (one/two) participants demonstrate conceptual understanding of the mean in answering the questions. Furthermore, a computational understanding of the mean is very common in participants in several questions. This study shows that prospective teachers know the statistical content at an insufficient depth to be able to teach it properly. In order to strengthen their preparation, a diversity of statistical problems involving the mean should be addressed during preservice teacher education, since this will certainly make a difference regarding the way in which their future students will understand this topic.

Acknowledgement

This work is financed by national funds through FCT – Fundação para a Ciência e Tecnologia under the project *Developing Statistical Literacy* (contract PTDC/CPE-CED/117933/2010).

References

- Jacobbe, T. (2012). Elementary school teachers' understanding of the mean and median. *International Journal of Science and Mathematics Education*, 10, 1143-1161.
- Leavy, A., & O'Loughlin, N. (2006). Preservice teachers understanding of the mean: moving beyond the arithmetic average. *Journal of Mathematics Teacher Education*, 9(1), 53-90.

INSTRUMENTAL ORCHESTRATION TYPES PLANNED BY PRE-SERVICE MATHEMATICS TEACHERS

Rüya Şay, Tuğçe Kozaklı, Hatice Akkoç
Marmara University, İstanbul, TURKEY

Instrumental orchestration is defined as the arrangement of the artefacts available in a learning environment for external steering of students' instrumental genesis (Trouche, 2004). Drijvers (2012) and Tabach (2013) distinguish eight types of instrumental orchestration: technical-demo, explain-the-screen (including the mathematical content), link-screen-board (linking representations on the central screen to representations on the board or in the textbook), discuss-the-screen, spot-and-show (the teacher brings up previous student work on a central screen), sherpa-at-work (students' ideas shared by whole-class on a central screen), work-and-walk-by (the teacher monitors students' progress who works individually or in pairs with computers), not-use-tech (the teacher chooses not to use technology). The aim of the current study is to determine types of orchestrations planned by pre-service mathematics teachers.

Twenty-four pre-service mathematics teachers participated in the study and prepared lesson plans for teaching various topics in mathematics and geometry using software such as Geogebra, Graphic Calculus and Geometry Sketchpad. The lesson plans were analysed to reveal different types of orchestration planned by pre-service teachers.

The results indicated that “not-use-tech” and “work-and-walk-by” were the most common orchestration types while “serpa-at-work” and “spot-and-show” were not observed at all. Another result is that the participants used more than one orchestration types in their lesson plans. It was also found that most of the participants planned to use technological tools to promote student participation. On the other hand, some of the efforts to encourage student participation could not be categorised by the types of orchestration (e.g. students working on their own computers being instructed by the teacher in a step-by-step manner) as also indicated by the findings from Tabach (2013).

References

- Drijvers, P. (2012). Teachers transforming resources into orchestrations. In G. Gueudet, B. Pepin & L. Trouche (Eds.), *From text to 'lived' resources; mathematics curriculum materials and teacher development* (pp. 265-281). New York/Berlin: Springer.
- Tabach, M. (2013). *Developing a general framework for instrumental orchestration*. Paper presented in WG15, Cerme8 conference, 6-10 February 2013, Antalya, Turkey.
- Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning*, 9, 281-307.

TEACHERS' BELIEFS REGARDING THE BENEFIT OF GRAPHICAL REPRESENTATIONS

Angela Schmitz, Andreas Eichler
University of Education Freiburg

Particularly when secondary students are regarded, it is widely accepted that graphical representations of mathematical objects can promote students' mathematical understanding (e.g. Presmeg, 2006). Further, students' understanding could be defined as students' ability to switch among different representations of mathematical objects, for example between a symbolic representation and a graphical representation or also between two graphical representations. However, research also shows limitations of learning with multiple external representations (Ainsworth, 2006). Although research yields some evidence referring to the functions, benefits and limitations of multiple external representations for students' learning, we know little about mathematics teachers' knowledge and beliefs about multiple external representations and, in particular, graphical representations of mathematical objects that impact on their classroom practices (Calderhead, 1996).

For this reason, the research approach presented here focuses on secondary mathematics teachers' beliefs regarding the benefit of graphical representations. We use a qualitative approach based on semi-structured interviews involving 12 secondary teachers. The interview addresses the teachers' beliefs regarding a variety of graphical representations as well as their beliefs concerning mathematics and mathematics teaching. We analyse the interview transcripts according to grounded theory using deductive codes as well as inductive codes (Glaser & Strauss, 1967).

Results of the analysis of one interview with a secondary teacher that are to be presented at PME show on the one side the benefit of existing categories concerning multiple external representations (e.g. Ainsworth, 2006), but also give evidence towards further data driven categories that underline a specific view on these teachers' beliefs from a mathematics education perspective.

References

- Ainsworth, S. E. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction*, 16, 183–198.
- Calderhead, J. (1996). Teachers: beliefs and knowledge. In D. C. Berliner (Eds.), *Handbook of education* (pp. 709-725). New York: MacMillan.
- Glaser, B., Strauss, A. (1967). *The discovery of grounded theory*. Chicago: Aldine.
- Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. In A. Gutierrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education* (pp. 205-235). Rotterdam: Sense Publishers.

INVESTIGATING BELIEFS OF KINDERGARTEN AND PRIMARY SCHOOL TEACHERS TOWARDS MATHEMATICS LEARNING

Stephanie Schuler, Gerald Wittmann
University of Education Freiburg, Germany

Mathematics learning across the lifespan should be – for each individual person – a continuous mathematical learning biography. But learning biographies are characterised by transitions between educational systems. Within every system educational and instructional practices are highly influenced by teachers' beliefs (Staub & Stern 2002). Due to different curricula, traditions and teacher trainings the transition from kindergarten to primary school is discussed in the context of continuity and discontinuity. Two ideal-typical positions exist (Roßbach 2006): (1) Differences should be reduced to increase continuity and to allow a smooth transition. (2) Discontinuities are challenges beneficial to (personal) development.

The research goal of the presented qualitative survey is to describe the beliefs of kindergarten and primary school teachers towards mathematics teaching and learning with regard to continuity and discontinuity. The underlying data corpus consists of video observations in kindergarten and primary school (n=12), as well as guided interviews (n=12) and two focus groups with altogether 35 experienced kindergarten and primary school teachers. The data was analysed by coding according to the qualitative content analysis (Mayring, 2000).

On the surface we can note continuity concerning the content area: in both institutions teachers' focus on number and operations. But the analysis of the reasoning reveals also differences of the beliefs on mathematical learning:

- View 1: Mathematics is omnipresent in every day life. Mathematical learning happens naturally with every day materials.
- View 2: Every day materials in kindergarten are sufficient but special training programmes ensure the preparation for school. Mathematical learning is twofold: individual learning according to individual interests as well as guided learning with minimal goals for every child.
- View 3: In kindergarten children should learn mathematics like counting but no special school topics. Mathematical learning aims at the same suppositions of all children in order to facilitate school learning (homogeneity).

References

- Mayring, P. (2000). Qualitative Content Analysis. *Forum Qualitative Social Research* (FQS), Vol. 1, No. 2. <http://www.qualitative-research.net/index.php/fqs/article/view/1089/2385>
- Roßbach, H.-G. (2009). Institutionelle Übergänge in der Frühpädagogik. In L. Fried & S. Roux (Eds.). *Pädagogik der frühen Kindheit* (pp. 280-291). Berlin: Cornelsen-Scriptor.
- Staub, F. C.; Stern, E. (2002): The Nature of Teachers' Pedagogical Content Beliefs Matters for Students' Achievement Gains: Quasi-Experimental Evidence from Elementary Mathematic. *Journal of Educational Psychology*, 94 (2), 344-355.

THE CO-EVOLUTION OF PROBLEM POSING AND PROBLEM SOLVING IN THE COURSE OF SARAH'S ON-GOING SOLUTION ACTIVITY

Volkan Sevim, Victor V. Cifarelli

School of Education, Virginia Commonwealth University,
Department of Mathematics & Statistics, University of North Carolina at Charlotte

Silver (1994) defined within-solution problem posing as “problem formulation or re-formulation [that] occurs within the process of problem solving” (Silver, 1994, p. 19). The current study viewed problem posing as a form of sense-making through formulation and re-formulation, which aids the solver’s on-going development of goals and purposes throughout problem solving. In this report, our examination of within-solution problem posing focuses on an episode of a Mathematics Education graduate student solving a number array task. Drawing from this episode, we seek answers to the following research questions: 1) How does problem posing evolve from the solver’s ongoing interpretations of the problem situation, 2) How do these posed problems contribute to the solver’s problem solving? Our view is that problem posing needs to be considered as occurring throughout problem solving. As students act to solve problems, we believe that they use results to monitor the usefulness of current goals and revise or reorganize their goals and purposes as needed to solve the problem. Problem posing is then a series of transformation of the original problem, with each successive problem posed indicating both progress towards a solution as well as providing possibilities for action to further expand the scope of the original problem. The case illustrated in this report involves one participant from a graduate course in Mathematics Education. The data consisted of the videotaped protocols, the experimenter’s field notes, and the participant’s written work. Transcripts of the student’s verbal responses were generated and protocol analytic techniques were used. Our results indicate that the co-evolving processes of problem solving and posing enabled Sarah to move from a lower level of generalization, which is a typical inductive generalization of a sequential pattern based on the correspondence between the sums of entries in $N \times N$ blocks and the square number sequence, to a higher level of generalization, which is a more scientific generalization that captures the mathematical properties of the array. We conclude that problem posing performed in the solution of a problem helps to both broaden the solver’s perspective of the original problem as well as expand its scope.

References

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics*, 14 (1), 19-28.

A STUDY ON THE PROCESS OF SOLVING CONTEXT PROBLEMS BY PROSPECTIVE TEACHERS

BoMi Shin

Chonnam National University

Context problems defined as problems of which the situation is experientially real to students offer opportunities for them to develop meaningful mathematical tools and understanding (Van den Heuvel-Panhuizen, 2005). Teachers need to possess ability to properly solve problems which can arouse students thought process (Pelczer, Singer, & Voica, 2011) because their sense of problem-solving have profound influence on how to effectively guide learners' problem-solving process (Liu & Thompson, 2004). Therefore, this study aims to analyze characteristics appearing from the process of context problem-solving by 44 prospective mathematics teachers. Analysis data were collected through a 1-hour paper and pencil test with 3 linear programming problems.

The analysis results are as follows: Firstly, 19 subjects inappropriately set up equations for solving the problems by only considering superficial descriptions of the conditions instead of underlying signification of real context in the problems. This phenomenon is apparently known as "suspension of sense-making" (Schoenfeld, 1991). Secondly, 20 subjects had difficulties with recognizing how changes of the conditions affect the solving process and did not reflectively investigate the context of the problems to measure validity of their answers. Thirdly, 13 subjects employed a trial-check strategy of substituting specific values one-by-one and 10 subjects used expository writing rather than algebraic inequalities or the coordinate plane in presenting their arguments, reasons and solutions of the problems. In the presentation, further results and suggestions on the design of the teacher education program will be discussed in detail.

References

- Liu, Y., & Thompson, P. (2004). Teachers' personal and pedagogical understanding of probability and statistical inference. In D. McDougal (Eds.), *Proc. 26th Annual Meeting of the Int. Group for the Psychology of Mathematics Education* (pp.1-7). Toronto: PME-NA.
- Pelczer, I., Singer, M. F., & Voica, C. (2011), In-service and pre-service teachers' strategies of task adaptation. In Ubuz, B. (Eds.). *Proc. 35th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 3, pp. 369-376). Ankara. Turkey: PME.
- Schoenfeld, A. (1991), "On Mathematics as Sense-Making: An Informal Attack on the Unfortunate Divorce of Formal and Informal Mathematics". In J. Voss, D. Perkins, & J. Segal (Eds.), *Informal reasoning and education* (pp. 311-344). Lawrence Erlbaum Associates: Hillsdale, NJ.
- Van den Heuvel-Panhuizen (2005). The role of contexts in assessment problems in mathematics. *For the Learning of Mathematics*, 25(2), 2-9.

PERCEPTION OF VISUAL MODELS IN LEARNING OF BINARY RELATION THEORY

Anna Shvarts

Lomonosov Moscow State University

There are three types of signs in C. Pierce's classic papers: index, iconic and symbol. The problem is that mathematical visual models don't produce an appropriate image immediately and automatically. M. Roth (2008) writes about the special work that should be done by an interpreter to see the graph as a sign of a phenomenon. Moreover, perception transforms under changing theoretical knowledge and the genesis of specific actions (Davydov, 1990; Radford, 2010).

The goal of our experiments is to explore conditions when visual models are perceived correctly and pass into internal visual representation. The research material was binary relations which could be represented by formulas and graphs. At first we gave two lectures, one with graphs and another without them ($n=79$) and checked understanding by tasks. Results evidenced that visual models alone don't help students to understand new mathematical material.

At the second stage we gave a prefatory training for half of the students. The training tasks were focused on translation of binary relations from verbal descriptions to graphs and vice versa. They had to construct specific actions with graphs. According to the results, two independent variables (the presence of prefatory training and the presence of graphs) interact significantly (ANOVA, $F=5,1074$, $p=0,030$, $n=40$): visual models will help students to solve problems only if material was preceded by special training.

To explore how the perception of graphs changed after participating in prefatory training, we recoded students' eye movement during their learning. We found out that untrained students more often coordinated their listening with appropriate perceptions of graphs ($p=0,015$, $n=20$). Trained student spend more time looking at graphs in comparison with formulas ($t=-3,6$, $p=0,005$) while untrained students gaze at formulas and graphs for the same duration during problem solving. Ultimately, to represent material visually is more common for students who were educated to perceive graphs. The work was funded RHF, grant number 12-36-01408.

References

- Roth, W.-M. (2008). The dawning of signs in graph interpretation. L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in mathematics education: epistemology, historicity, classroom, and culture*, Rotterdam: Sense, pp. 83-102
- Radford, L. (2010). The eye as a theoretician: Seeing structures in generalizing activities, *For the Learning of Mathematics*, 30(2), 2-7.
- Davydov, V. V. (1990). *Types of generalisation in instruction: Logical and psychological problems in the structuring of school curricula*. Reston: National Council of Teachers of Mathematics.

LESSON STUDY IN TEACHER EDUCATION

Charlotte Krog Skott, Camilla Hellsten Østergård
University College Metropol, Denmark

Generally, research on teachers holds acquisitionist perspectives on human functioning and sees knowledge as abstract and static. Recent critique claims that such perspectives downplay the importance of the context, and that knowledge is not located in the “head” of the individual teacher, but attached to and situated in the complex and social contexts of mathematics classrooms (Hodgen, 2011). Adopting this more contextualized and dynamic perspective, the emphasis shifts towards communities of practice (Wenger, 1998) and the concept of knowledge is replaced by knowing.

In an ongoing study we adopt a similar perspective on the development of knowing mathematics in teaching and use lesson study with prospective teachers for the lower secondary level to do so (Corcoran, 2012). The participants work with teacher educators to set up lesson study groups for their practicum. We adopt a community of practice perspective to address the questions of what the potentials and constraints are of these groups for the participants’ development of mathematical knowing in teaching. We use a qualitative approach inspired by grounded theory. The data include classroom observations of 6 groups of 4 prospective teachers, joint reflections on their research lessons, supervision, semi-structured interviews, and written reports.

One potential of lesson study is the development of a common language on teaching, originating in joint planning, observations and reflections on research lessons. The discussions and observations may also facilitate increased awareness of the complexity and multiple forms of knowing needed in teaching. Interview statements from prospective teachers on “...how much you really need to consider” to teach well are indicative. A third potential is that their thorough and collaborative planning support changes in their focus from “what to do” to “what to learn” and towards stronger connections between activities and learning objectives. One constraint is that the prospective teachers are often challenged by their school-based mentors, who question lesson study, because the collaboration takes too much time. Also, the current political discourse on education suggests that teachers should teach more and prepare less.

References

- Corcoran, D., & Pepperell, S. (2011). Learning to teach mathematics using lesson study. In T. Rowland & K. Ruthven (Eds.), *Mathematical knowledge in teaching* (pp.213-230). Dordrecht: Springer.
- Hodgen, J. (2011). Knowing and identity: A situated theory of mathematics knowledge in teaching. In T. Rowland & K. Ruthven (Eds.), *Mathematical knowledge in teaching* (pp. 27-42). Dordrecht: Springer.
- Wenger, E. (1998). *Communities of practice*. Cambridge, MA: Cambridge University Press.

METAPHORS AND DIDACTICAL SITUATIONS

Jorge Soto-Andrade

CIAE and Mathematics Department, University of Chile

The relation and interplay between metaphors (Lakoff & Núñez, 2000) and didactical situations (Brousseau, 1998) in mathematics education has practically not been explored yet. We intend to do so using the metaphorical approach itself as a meta-theory. We propose a working metaphor, the *voltaic metaphor* to describe one possible relation: when “didactical tension” becomes high enough, in an ongoing didactical situation, metaphors spark spontaneously in the classroom, triggering an unlocking cognitive dynamics. These emerging metaphors are not necessarily conceptual metaphors, they might also refer to the previous life history or circumstances of the learners.

To explore the role of metaphors and metaphorizing in didactical situations, since 2006 we have carried out didactical engineering and collective case studies involving a broad spectrum of learners in our country: in service primary and secondary school teachers, prospective pre-school and secondary school teachers, first year university students aiming at majoring in humanities and social sciences, undergraduates majoring in mathematics and natural sciences besides juvenile offenders enrolled in a social re-insertion program. As a result, recurrent phenomena were observed: In all classes idiosyncratic metaphors emerged when didactical tension increased sufficiently (provided that the learners felt that the usual ban on metaphorizing was lifted). For instance, in a didactical situation where students or teachers without previous knowledge of probability approached concrete random walks, a “solomonic” or “hydraulic” metaphor emerged: the walker splits into pieces instead of walking randomly. This enabled them to solve in friendly and enactive ways the random walk for a small number of steps, constructing the concept of probability on the way. We also observed the emergence of life-metaphors in juvenile offenders (in semi-freedom): they saw an “escape point” or they “deviated from the straight way” to construct creative solutions to an open ended geometric problem. Further research seems necessary, also on affective factors and resilience contributing to sustain and endure the didactical tension suggested by our voltaic metaphor.

References

- Brousseau, G. (1998). *Théorie des situations didactiques*. Grenoble: La Pensée Sauvage.
- Lakoff, G., & Núñez, R. (2000). *Where Mathematics comes from*. New York: Basic Books.

GENERAL COGNITIVE AND READING ABILITIES AS IMPACT FACTORS ON STATISTICAL LITERACY¹

Ute Sproesser, Sebastian Kuntze, Joachim Engel
Ludwigsburg University of Education, Germany

This study investigates the relationship between the competency of using representations and models in statistical contexts on the one hand and potential impact factors in the area of general cognitive abilities and reading skills on the other.

Existing definitions and competency models in the domain of statistical literacy (SL) have hardly taken into account the role of potential impact factors such as (non-)verbal cognitive abilities or reading comprehension. For the case of the competency “using representations and models in statistical contexts” (Kuntze, Lindmeier & Reiss, 2008), this study focuses on the following potential influencing variables:

Cognitive abilities: For any specific competency construct it should be verified how closely it is linked to general cognitive abilities. For estimating the interrelatedness with such variables, verbal and non-verbal cognitive abilities can be considered.

Reading abilities: This is particularly interesting as SL can be described by the metaphor of data-related reading (Curcio, 1987; Kuntze et al., 2008). Conversely, reading data from representations such as bar diagrams is addressed in the PISA reading competency tests. In order to avoid this intersection domain, we chose indicators of reading comprehension and reading speed.

564 German students (260 female, 304 male) of grade 8 aged between 12 and 16 years (mean 13.54, SD 0.66) took part in this study. For measurement, we used the competency test by Kuntze et al. (2008) and well-established psychometric test instruments addressing general cognitive and reading abilities. Correlation and regression analysis were used to examine the underlying interdependencies in the first analysis steps. The preliminary results suggest that verbal cognitive abilities appear to be more closely connected to the competency score than general reading abilities.

A deeper comprehension of statistical literacy and its impact factors could not only fill gaps in basic research, but also help to develop specific learning material in this area.

References

- Curcio, F. R. (1987). Comprehension of mathematical Relationships Expressed in Graphs. *Journal for Research in Mathematics Education*, 18(5), 382-393.
- Kuntze, S., Lindmeier, A. & Reiss, K. (2008). „Using models and representations in statistical contexts“ as a sub-competency of statistical literacy – Results from three empirical studies. *Proceedings of the 11th Int. Congress on Math. Education (ICME 11)*.

¹ This study is supported by research funds of Ludwigsburg University of Education.

THE ALGEBRAIC THINKING PROJECT: FUNCTIONS

Colin Starr
Willamette University

Students almost universally struggle with the concept of a function. Yet, most have already developed an accurate idea of what a function is without realizing it (e.g., in the USA, each student has a unique Social Security number). Functions are at times concrete and at times abstract; geometrical and algebraic; algorithmic and conceptual. For most high school students, a thorough understanding of functions takes years of repeated exposure to all facets of functions. In a systematic review of the literature, the Algebraic Thinking Project's Functions team read 146 research articles from the last thirty years about students' struggles with functions. We distilled the most useful parts for teachers into a series of entries in an Encyclopedia of Algebraic Thinking based on a few themes that consistently appeared in the literature.

First, students need to see functions presented in a variety of formats (e.g., Panasuk & Beyranevand, 2010). Students who only see functions represented as algebraic expressions come to believe that functions *are* algebraic expressions and no more. Second, students need to see a wide variety of functions. Students who only see continuous functions come to believe that functions *must* be continuous, and students who see only smooth functions (e.g., no cusps) come to believe that functions *must* be smooth (e.g., Vinner & Dreyfus, 1989). Finally, function notation is a barrier for many students (e.g., Coady & Pegg, 1994); for example, if f is a non-linear function, then $f(x+y)$ does not equal $f(x)+f(y)$ in general, but if a is a number, then $a(x+y) = ax+ay$, which leads many students to believe that all functions are linear.

Across three decades of research, authors consistently concluded that variety is critical for students to come to a solid understanding of functions; articles in the Encyclopedia reflect this.

References

- Coady, C., & Pegg, J. (1994). Tertiary students' understanding of second order relationships in function notation.
- Panasuk, R., & Beyranevand, M. (2010). Algebra students' ability to recognize multiple representations and achievement. *International Journal for Mathematics Teaching and Learning*. Paper presented at the 17th Annual Mathematics Education Research Group of Australasia Conference.
- Vinner, S., & Dreyfus, T. (1989). Images and Definitions for the Concept of Function. *Journal for Research in Mathematics Education*, 20(4), 356-366.

DISCUSSIONS THAT BUILD UPON CHILDREN'S THINKING

Ruti Steinberg

Kibbutzim College of Education, Tel Aviv, Israel

Children learn mathematics in a more meaningful way when they solve challenging problems using and discussing their own solution strategies (Carpenter, Fennema, Franke, Levi, & Empson, 1999). Good discussions promote learning by allowing children to explain and defend their mathematical ideas (Sfard, 2008). Pierson (2008) examined classroom discourse and found higher achievement when teachers were more responsive to students' ideas in the discussions. Learning was also enhanced when dialogue was intellectually demanding, such as asking for justifications and examples. Pierson (2008) found that high-level mathematical discussions are not the norm. This paper identifies features that are characteristic of good discussions in constructivist classes. I will present several examples of meaningful discussions and analyze how teachers facilitate them, with reference to the kind of questions they use, their actions and their decisions. Good discussions are rare, so it is important to document examples that can be used to support teacher training and development.

Methods: Discussions were observed and documented by field notes and videos in a large project in which elementary school teachers learn to conduct constructivist problem solving classes. The discussions were transcribed.

Results and discussion: The discussions were analyzed to find out what make them effective. The teacher built the discussion on the students' thinking so we can see a high level of responsiveness to the students and an aim at understanding. This made the discussions very rich and gave the teachers ideas to create new tasks and questions. The teachers asked the children to explain, justify, and participate in discussions. They suggested new problems for the class to solve that were related to the discussion and brought the children to develop new ideas. How did the teacher use her questions and remarks? Examples: She solicited ideas from the students and encouraged them to participate in the discussion (*Do you understand why he did that? What problem is she doing?*), clarified solutions, asked for more information (*What did you draw here?*), and asked the students to give justifications (*Who can convince us that what Lior did is right? What is your opinion? Is he correct?*).

References

- Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L. & Empson, S.B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.
- Pierson, J. (2008). *The Relationship Between Patterns of Classroom Discourse and Mathematics Learning*. Doctoral dissertation, The University of Texas at Austin. 3324548
- Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourse, and mathematizing*. New York: Cambridge University Press.

AN EMPIRICAL STUDY OF TREATMENTS AND CONVERSIONS OF REPRESENTATIONS

Anna Stenkvist

Dept. of Philosophy and History, Royal Institute of Technology, Stockholm, Sweden

Almost all theoretical knowledge is represented by signs. In mathematics signs, or representations, are especially important since the objects of learning are abstract. The presentation will be about the operational handling of representations - an ability which generally is regarded as necessary to comprehend mathematics. Previous studies in the field have classified the signs into representation systems or semiotic registers (Duval, 2004, 2006; Goldin, 1996). The presented study considers two semiotic registers used in mathematics consisting of geometric figures and symbolic representations. It examines the statement that to treat a semiotic register, here the symbolic one is easier for students than to convert between registers, here from a geometric to a symbolic register (Duval, 2004, 2006).

A database is used, which contains upper secondary students' results of a common national test. The study uses approximately 2000 works of two years divided per grade, in a four level grading system. Only problems requiring no more than a final answer are considered. The proportions of the students' correct answers, received by treatment or conversion respectively, are compared.

The results show a tendency which confirms that it is easier for students to treat representations within a semiotic register than it is to convert between registers. The results also show that this tendency is a bit different depending on the students' marks. A remaining work is to examine the generality of this phenomenon and the possible causal connections.

References

- Duval, R. (2004). A crucial issue in mathematics education: the ability to change representation register, *ICMI (International Commission on Mathematical Instruction) the 10th International Congress on Mathematical Education, ICME-10*. Copenhagen, Denmark.
- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics, *Educational Studies in Mathematics*, 61, 103–131.
- Goldin, G., Kaput, J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. Steffe, L. et al. (Eds.), *Theories of Mathematical Learning*, Erlbaum, Hillsdale, NJ, 397–430.

PRIMARY TEACHERS IN ENGLAND AND HONG KONG – LEARNING FROM EACH OTHER?

Susan Steward, Linda Hargreaves
Faculty of Education, University of Cambridge, UK

The SPeCTRM project (Social Pedagogical Contexts for Teaching and Research in Mathematics) is currently being carried out in primary schools in England and in Hong Kong. The project is exploring the potential of group work to transform mathematics teaching in the two countries through teachers learning from each other.

BACKGROUND TO THE RESEARCH

England's policy makers, aiming to improve England's position in international league tables, have looked to mathematics teaching practices in high scoring countries such as Hong Kong; conversely, policy makers in Hong Kong, aiming to modernise their primary classroom practices, have worked with academics from countries such as England to promote more innovative and creative teaching approaches, including collaborative group work.

Research suggests, however, that high attainment in mathematics may be more closely linked to differences in culture and philosophy than curriculum and pedagogy (Askew et al., 2010). Hong Kong mathematics classrooms compared with those in England can be characterised as opposite ends of a spectrum: teacher rather than pupil-centred, focused on the collective rather than the individual, and attributing success to effort rather than to innate ability (Leung 2001). A greater use of group work in mathematics, therefore, may act as a bridge between these two cultures and facilitate dialogue between teachers through shared experiences.

PARTICIPATING TEACHERS AND SCHOOLS

Upper primary phase (Grades 3-5) teachers in 10 schools in England have been paired with Grade 4 teachers in 10 schools in Hong Kong. Comparing these two samples, we have found a greater diversity in England than in Hong Kong on school and teacher characteristics. Initial data from English teachers (self-efficacy questionnaires and interviews) underline the particular importance that England's teachers place on *differentiation* based on individual children's national curriculum levels, whereas those in Hong Kong focus on the class's chronological age. After participation in the project we aim to re-interview teachers to investigate whether these characterisations of pupils and their learning in mathematics may have changed.

References

- Askew, M., Hodgen, J., Hossain, S., and Bretscher, N., (2010) Values and variables - Mathematics education in high-performing countries. London: Nuffield Foundation
- Leung, F.K.S. (2001). In search of an East Asian identity in mathematics education. *Educational Studies in Mathematics*, 47, 35-51.

READING COMPREHENSION AND CALCULATION AS PREDICTORS OF MATH WORD PROBLEM SOLVING

Marina Stibric

Department of Education, Faculty of Psychology, University of Bergen, Norway
Department of Psychology, University of Zagreb, Croatia

Solving mathematical word problems includes comprehending the text, constructing a mental model of a problem situation, choosing appropriate problem solving strategy, performing the calculation and interpreting the solution (Reusser, 1990). Dependence on reading comprehension is what differentiates word problems from computational tasks expressed with mathematical symbols, and what can mediate their difficulty compared to computational tasks at different educational stages.

Since mathematical word problem solving includes reading comprehension and calculation, it is important to determine what the relationship between these skills is.

This correlational study included 185 pupils ($n=97$ female; age: $M=125.3$, $SD=3.85$ months) attending fourth grade of primary school. Pupils solved mathematical word problems, computational and reading comprehension tasks. Measure of reading comprehension included reading a paragraph from a children's novel followed by open-ended questions about the text. Computational tasks were constructed in line with the fourth grade curriculum and comprised four arithmetical operations. Mathematical word problems included three types of word problems (i.e., combine, change and compare problems), according to Riley and Greeno (1988).

Mathematical word problem solving was correlated to calculation ($r=.39$; $p<.001$) and reading comprehension ($r=.43$; $p<.001$), while the correlation between calculation and reading comprehension was $r=.35$ ($p<.001$). Regression analysis revealed that these calculation and reading comprehension explained 25% of the variance of word problem solving with reading comprehension being the strongest predictor. Results showed that reading comprehension can be more important for word problem solving than computational skill. This goes in line with theories of mathematical word problem solving that emphasize understanding the relations in the text of the word problem. These findings can be used in developing teaching practices that enhance word problem solving. A follow up study that is currently being conducted is exploring do these relations change depending on participants' age, culture and area of mathematics.

References

- Riley, M. S., & Greeno, J. G. (1988). Developmental analysis of understanding language about quantities and of solving problems. *Cognition and Instruction*, 5, 49-101.
- Reusser, K. (1990). *Understanding word arithmetic problems. Linguistic and situational factors*. Paper presented at the Annual Meeting of the American Educational Research Association (Boston, MA, April 16-20, 1990).

STUDENTS' BELIEFS WHILE SOLVING AN EXERCISE

Hannes Stoppel

Westfälische Wilhelms-Universität Münster

In contrast to students' beliefs concerning mathematical problem solving, relationships between changes of their beliefs when solving an exercise of familiar type and the structure of their solution stood neglected. During the study 60 students of grade 13 of senior high schools had to solve an exercise on analytic geometry. Right before and straight after solving the exercise the students had to fill in a questionnaire on mathematical problem-solving beliefs (Kloosterman & Stage, 1992) and epistemological beliefs (Hofer, 2000). The study took place when the students had already started revising before their final exams in mathematics.

Analysis of the solutions enabled identification and description of the relationships between various categories of solutions (Stoppel, 2012), several types of reflective thinking, and execution (cf. Zehavi & Mann, 2005). Furthermore, evaluation of the questionnaires connected these characteristics to the methods and their application to the objects (Leontiev, 1978) which the students had used. While investigating the questionnaires and the solutions connections were found between the development of beliefs from before to after solving the exercise dependent on the activities, the categories of solutions as well as the thoughts and skills. Interestingly, students often drew wrong conclusions about the difficulty of an exercise and their individual ability in solving it. This corresponded to choosing a suitable method and applying it to an unsuitable object. These insights facilitate not only a deeper understanding of students' thoughts, difficulties and mistakes when solving exercises, but also imply new perspectives for instructors when teaching problem solving.

References

- Hofer, B. K. (2002). Epistemological world views of teachers: From beliefs to practice. *Issues in Education*, 8(2), 167–173.
- Kloosterman, P., & Stage, F. (1992). Measuring beliefs about mathematical problem solving. *School science and mathematics*, 92(3), 109–115.
- Leontiev, A. (1978). *Activity, Consciousness, and Personality*. Englewood Cliffs, N.Y.: Prentice-Hall.
- Stoppel, H. (2012). Understanding Difficulties in Solving Exercises: Phasing Solutions.
- Zehavi, N., & Mann G. (2005). Instrumented Techniques and Reflective Thinking in Analytic Geometry. In B. Sriraman (Ed.), *The Montana Mathematics Enthusiast*, Vol. 2, no. 2 (pp. 83–92).

A STUDENT'S IDENTITY FORMATION IN ELEMENTARY MATHEMATICS CLASSROOMS

Hitoshi Takahashi

Joetsu University of Education, Japan

This study investigates what student identity is formed and how this identity is formed in elementary classrooms. This study further looks at how the identity of the student influence the acquisition of mathematical knowledge.

In mathematics classrooms, students' humanity should be cultivated while the acquisition of mathematical knowledge should be encouraged. Identity is part of humanity. Approximately, there are two kinds of identities. One is personal identity or ego identity, and the other is group identity (Erikson, 1959). In mathematical education, Cobb, Gresalfi & Hodge (2009) investigated on the identity of eighth grade students in algebra classrooms. In the classrooms, students have two kinds of identities: normative identity and personal identity.

Twenty-four lessons of third graders were observed for five months at an elementary school that is attached to a university. We selected one third grader, whom we called Taro, to interpret his identity on mathematical learning. Eight out of 24 lessons were videotaped. These eight lessons are on division with remainder.

Data indicate that having a role in those classrooms formed Taro's identity on mathematical learning, and that harmony between his mathematical group identity and personal identity leads to acquiring knowledge on division. At first, Taro's participation in elementary mathematics lessons was not enough. In the later lessons, the teacher encouraged Taro to participate in the lessons by calling on him, making him express his ideas, and making him approve his classmates' ideas. These were part of the norms of his classroom's group identity. When the teacher induced Taro to form a group identity, Taro's personal identity was formed concomitantly. Because the teacher gave Taro roles in the elementary mathematics lessons, Taro felt a sense of belonging in the classroom and began to be conscious of his mathematical ability. This conscientization allowed Taro to perceive his selfsameness and continuity. Furthermore, this conscientization allowed for the perception of the fact that others recognize Taro's sameness and continuity. In conclusion, mathematical identity, which was formed through the students' mathematical activities, was very important.

References

- Erikson, E. H. (1959). *Identity and the life cycle*. New York: Norton.
- Cobb, P., Gresalfi, M. & Hodge, L. L. (2009). *An interpretive scheme for analyzing the identities that students develop in mathematics classrooms*. Journal for Research in Mathematics Education, 40, 1, 40-68.

COLLABORATION BETWEEN IN- AND PRE-SERVICE TEACHERS FOR CAPTURING THE STUDENTS' MATHEMATICAL IDEAS

Sampan Thinwiangthong, Wipaporn Suttiamporn, Maitree Inprasitha
Center for Research in Mathematics Education, Faculty of Education, Khon Kaen
University, Thailand

Most teachers in Thailand still use a traditional teaching style focusing on the coverage of contents (Inprasitha, 2010). They cannot capture the students' mathematical ideas in the classroom. Teachers' ability in capturing the students' mathematical ideas was needed in the classrooms focusing on new teaching approach. In order to enhance the teachers' ability to do that, Center for Research in Mathematics Education had establishing the context of collaboration between in-service teachers and pre-service teachers based on the innovations of Lesson Study and Open Approach and theory of community of practice (Lave and Wenger, 1991). This paper aimed to describe the collaboration between in-service teachers and pre-service teachers which offer the teachers can capture the students' mathematical ideas in the pilot schools. 44 internship students in mathematics education program who had experiencing teaching practice and 132 in-service teachers in the pilot schools were asked by questionnaire. They had analyzed students' mathematical ideas in the classroom with students' artifact and interviewing.

The results showed that pre-service and in-service teachers gradually built the process of collaboration in order to capture the students' mathematical ideas, follow by 3 steps of Lesson Study. They had working in *collaboratively plan* by creating problem situation, anticipating students' respond and mathematical ideas and preparing teaching materials; *collaboratively do* by one teacher take role as the classroom teacher and another one take role as the observer; *collaboratively see* by reflecting discussion about students' respond and mathematical ideas, group the ideas and note for revising that lesson plan. They can capture several of students' mathematical ideas and use it for improving of collaboration to enhance Open Approach as a teaching approach in problem solving classroom.

References

- Inprasitha, M. (2010). One Feature of Adaptive Lesson Study in Thailand: Designing Learning Unit. *Proceedings of the 45th Korean National Meeting of Mathematics Education*. Gyeongju, Korea.
- Lave, J. & Wenger, E. (1991). *Situated Learning*, Cambridge University Press, Cambridge.

VALID STRATEGIES, INCORRECT ANSWERS: STUDENTS WITH LEARNING DISABILITIES SOLVING WORD PROBLEMS

Helen Thouless
University of Washington

There has been considerable research using the framework of Cognitively Guided Instruction (CGI) to examine the strategy use of typically developing children as they solve word problems (Carpenter et al, 1999). This framework has helped mathematics educators understand the typical developmental trajectory of these skills. However, there has been limited research using the CGI framework with students with LD.

This current study does use the CGI framework to answer the following research questions: (1) Do students with LD use valid strategies when solving word problems? (2) How accurate are students with LD when solving word problems? (3) What are the barriers to accuracy for students with LD?

In this study, fifteen fourth grade students with LD participated in individual clinical interviews. The interviews used the CGI framework to examine the students' responses to six word problems.

The results of this study show that although these students usually had valid strategies for solving the problems, they solved less than half of the problems correctly.

With further analysis, it was found that 39% of the errors when the students had a valid strategy were due to counting errors. These counting errors occurred during direct modelling when the students counted blocks twice, skipped blocks, allowed blocks from different piles to mix, or were unsure of the counting sequence beyond one hundred. These counting errors also occurred during skip counting when the students missed a decade or counted the wrong number of times.

In this study students with LD usually understood the context of the problem but there were other barriers to their solving the problems accurately. One barrier to their solving problems accurately was their difficulties with counting accurately, either from inaccurate one to one counting, errors in the counting sequence, or inattention. These findings suggest that teachers need to continue focusing on counting for students with LD into the upper elementary grades.

References

Carpenter, T., Fennema, E., Franke, M., Levi, L., Empson, S. (1999) *Children's mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

FOSTERING PROBABILISTIC INTUITIONS OF RISK AND DECISION MAKING UNDER UNCERTAINTY

Christoph Till

Ludwigsburg University of Education

Since risk communication in environmental, economic and medical issues is based on probabilities and statistics, it is important that young students develop a good understanding of their basic elements for coherent decision making in times of uncertainty. Risk is communicated and modelled by a combination of different mathematical concepts for example proportions, expected values and conditional probability. That is why basic elements and intuitions of probabilistic and statistical thinking can and should already be fostered in primary school. Prior research shows that the use of natural frequencies – instead of fractions - is an accessible and adequate tool for communicating risks (Gigerenzer, 2002; Martignon & Krauss, 2009) not just to adults but also to young children. Natural frequencies can be applied in classrooms using concrete, enactive tinker-cubes and iconic representations. In this study basic mathematical concepts of risk and intuitions of chance are fostered through an intervention focusing on natural frequencies. The research questions are:

- Which basic intuitions do children aged 8 to 10 have about pre-concepts of probability and decision making under uncertainty?
- How can an intervention affect these intuitions and pre-concepts and foster them?
- We created a short unit (4 lessons) including basic concepts like comparison of proportions, sample distributions, validities of features in a playful experimental learning environment. The sample comprises 260 fourth graders from 12 classes. In this experiment 200 school students were part of the treatment group; 60 students only were tested (control group). Data for the pre-post-follow-up-test comparison was collected by tests with 29 items on stochastic tasks in order to measure student's competence development. We expect the intervention to have an impact on the way students perceive, communicate and reckon with risk. Our experience in the classrooms appears to confirm that primary school students are able to reckon with risk and improve their intuitions of basic stochastic concepts, when quantities and probabilities are communicated in 'natural' representation formats. In the presentation, results will be reported and discussed in more detail.

References

- Gigerenzer, G. (2002). *Reckoning with risk*. London, England: Penguin Books Ltd.
- Martignon, L & Krauss, S. (2009). Hands-On Activities for Fourth Graders: A Tool Box for Decision-Making and Reckoning with Risk. *International Electronic Journal of Mathematics Education*, 4(3), 227-258.

ENCULTURATION INTO MATHEMATICAL PRACTICE: AN EXPLORATIVE STUDY INTO THE STUDENT PERSPECTIVE

Stephanie Treffert-Thomas
Loughborough University

I am reporting from my research into undergraduate students' views and perceptions of mathematics and mathematical learning. The purpose is to find out how and to what degree students' view of mathematics has changed during the first year of a university linear algebra course. This research aims to extend the focus of a previous study of university teaching and the lecturer's perspective to learning and the student perspective. Using activity theory as my theoretical framework I employed both quantitative and qualitative methods of data collection and analysis to gather the student viewpoint.

In past research I focussed on the teaching of linear algebra and the lecturer's motive and goals for student learning (Thomas, 2012). The lecturer stated explicitly (in research meetings with the researchers as well as in lectures directly to students) that he wanted to enculturate students into the way that research mathematicians think and work when doing mathematics. The purpose of my current work is to find out if, and to what degree students' views have changed as a result of the teaching approach taken, and how this contributed to students developing their understanding of key concepts in linear algebra.

I administered a questionnaire (adapted from Perrenet & Taconis, 2009) as a preliminary step before more focussed interviews with students. The questionnaire contained twenty statements that describe an aspect of mathematics or mathematical learning. Students responded by circling one of five points on a Likert scale. The same questionnaire was administered twice, at the beginning and at the end of the course. Analysis of the questionnaires showed that students' responses remained unaltered in 50% of cases. I will follow-up the analysis with interviews specific to linear algebra. I present my findings from the questionnaires, and from the interviews to date.

References

- Perrenet, J. & Taconis, R. (2009). Mathematical enculturation from the students' perspective: shifts in problem-solving beliefs and behaviour during the bachelor programme. *Educational Studies in Mathematics*, 71, 181-198.
- Thomas, S. (2012). An activity theory analysis of linear algebra teaching within university mathematics. Unpublished doctoral thesis, Loughborough University, UK.

CHARACTERISTICS OF ADAPTIVE TEACHER BEHAVIOR IN MATHEMATICAL MODELLING: A CASE STUDY

Natalie Tropper

Leuphana University, Lüneburg, Germany

In pedagogy and mathematics education there was and still is a controversy about what kind of teacher behavior (extreme restraint vs. active support) is appropriate for the promotion of specific competences. However, especially in autonomy-oriented learning environments there are a lot of indicators for the importance of teachers' individual student support (e.g., Pauli & Reusser, 2000). Therefore teachers are required to act *adaptively*, i.e. to optimally fit their actions to the learners' individual, social and cognitive conditions. Although the term *adaptivity* is currently used in numerous pedagogical contexts, it has not been sufficiently operationalized until now and effective components of an adaptive teaching practice have to be worked out.

The present study's research question directly arose from this situation: 'Are there characteristics of teacher interventions that indicate adaptive behavior?' A laboratory study (part of DISUM) was conducted with 1 teacher and 2 ninth grade classes participating. In each class the teacher acted in a classroom-like setting with 16 students in groups of four. The students had to work on a mathematical modelling task. The teacher's only instruction was to intervene as autonomy-supportive as possible during the task processing. To allow a microanalysis of the interventional behavior all task processing situations were videotaped and transcribed. Afterwards, several cases were selected where students had problems in different steps of the mathematical modelling cycle and the teacher tried to help them overcome their problems. These cases were analyzed in detail by a qualitative category system (see Leiss, 2010). Finally the adaptivity of the teacher's behavior was rated in each case (e.g. depending on peculiarities in the category system or the fit of the teacher's reactions to the underlying student problem). A remarkable result is that although the analyses revealed several aspects that were obviously not adaptive (e.g., formal expectations as a frequent intervention cause) no characteristics could be found that clearly indicated adaptivity. In the end, individual student support is a highly situational activity, so simple rules for adaptive intervening are not likely to be found. Nevertheless, further research in this area can be worthwhile in order to be in the long run able to prepare teachers for their intervention practice.

References

- Leiss, D. (2010). Adaptive Lehrerinterventionen beim mathematischen Modellieren. *Journal für Mathematik-Didaktik*, 31 (2), 197-226.
- Pauli, C. & Reusser, K. (2000). Zur Rolle der Lehrperson beim kooperativen Lernen. *Schweizerische Zeitschrift für Erziehungswissenschaft*, 22 (3), 421-441.

FRACTIONAL KNOWLEDGE AND GRAPHICAL REPRESENTATIONS

Zelha Tunc Pekkan¹, Martina Rau², Vincent Alevan², Nikol Rummel³

¹ Yeditepe University, Istanbul, Turkey

² HCI Institute, Carnegie Mellon University, Pittsburgh, USA

³ Institute of Education, Ruhr-Universität, Bochum, Germany

External graphical representations are mainly used as teaching tools for fraction instruction. In this presentation we discuss how 656 fourth and fifth grade students perform with same fraction test items with different representations and we present the error analysis of 120 test papers in relation to conceptions of specific graphical representations. We will discuss the results in the presentation.

Learning fractions is difficult for children even though they receive many years of instruction (NAEP 2007). Moss and Case (1999) found that teaching decimals and fractions with linear representations resulted in deeper understanding, compared to traditional instruction. On the other hand, Cramer et al. (2008) claim that the circle representation is the most effective representation when comparing fractions. Mathematics education research has not systematically investigated students' performance and categorization of errors on the same fractions task, but performed with different external graphical representations. In this presentation, we address this concern. We developed a test with fraction items that involved the circle, rectangle, and number line representations. We administered the test to 656 US students. Using Chi-square statistics, we found that students' performance across representations (within the same problem type) differed significantly. Performance with the number line was lower than that with the other representations, and performance with the circle and rectangle did not differ. We also found differences across fraction problem types (within the same representation). In addition to the analysis, we analysed 120 of the test papers and categorized the most common errors in relation to the use of the representations. Systematic comparison of error analysis of the fraction test with different representations clearly shows that students view the number line as a more complex representation compared to circle and rectangle.

References

- Cramer, K., Wyberg, T., & Leavitt, S. (2008). The role of representations in fraction addition and subtraction. *Mathematics Teaching in the Middle School*, 13(8), 490-497.
- Moss, J., & Case, R. (1999). Developing Children's Understanding of the Rational Numbers: A New Model and an Experimental Curriculum. *Journal for Research in Mathematics Education*, 30(2), 122-147.
- National Assessment of Educational Progress (2007). Retrieved from <http://nces.ed.gov/nationsreportcard/pubs/main2007/2007496.asp>

REFLECTIVE JOURNALS FOR TEACHER CHANGE: IMPLEMENTATION OF MATHEMATICS REASONING APPROACH IN 6TH GRADE

Ece Ulucan¹; Umut Girgin²; Recai Akkus¹

¹ Abant Izzet Baysal University, Bolu; ² THK Gazi Elementary School, Istanbul

Teacher change needs more attention in terms of conceptual knowledge and beliefs about students' learning. Teacher knowledge and beliefs about how to organize learning environment and how the students think and learn are effected by the knowledge and beliefs about how teacher sees learning and about how to promote learning (Borko & Putnam, 1996). Teachers need to reflect on their experiences of teaching so that they can increase their awareness of teaching and learning occurring in the classroom.

The purpose of this action research study was, through the reflective journals kept before and after the lessons and at the end of the unit, to analyze the process of implementation of Mathematics Reasoning Approach whose aim is to change teacher pedagogy for scaffolding students' problem solving skills and mathematical thinking. Two teachers, at two different schools, implemented the Natural Numbers unit with grade 6 students, which lasted three weeks. There were 4 journals before the lesson, 6 after the lesson and 1 at the end of the unit. The journals were analyzed by three independent coders and common themes were constructed.

Based on the data analyses, four themes came out. First, the teachers realized that they lacked of conceptual understanding of natural numbers during lesson planning (e.g., what are the properties of the greatest common factor?). Second, direction of communication in the classroom changed from teacher-student to student-student-teacher. Third, creating dialogical interaction (argumentation) needed time and using concept mapping helped the teachers direct the discussions to the main ideas. Fourth, the teachers realized that questioning is a key ingredient for creating mathematical argument. More results with examples will be presented at the conference.

References

Borko, H. & Putnam, R. (1996). *Learning to Teach. Handbook Of Educational Psychology*.

WHAT DO COLLEGE CALCULUS STUDENTS BELIEVE ABOUT MATHEMATICS?

Figen Uysal¹, Jessica Ellis², Chris Rasmussen²

¹Bilecik Şeyh Edebali University ² San Diego State University

Teachers' and students' mathematical beliefs have been studied from different perspectives. Students' mathematics-related beliefs can have a substantial impact on their interest in mathematics, their enjoyment of mathematics, and their motivation in mathematics classes (Kloosterman, 2002, p.247). In our study, we aimed to investigate college students' mathematics-related beliefs regarding some affect-related concepts, such as confidence, enjoyment and interest, and specifically how the beliefs expressed by Calculus students relate to their persistence in Calculus, as well as other variables.

Data for this study come from a large-scale national survey of mainstream Calculus I instruction that was conducted across a stratified random sample of two- and four-year undergraduate colleges and universities in the U.S. during the Fall term of 2010. For the purpose of this analysis, we focus only on Science, Technology, Engineering, and Mathematics (STEM) intending students who responded to both pre and post term surveys and whose instructors did as well, resulting in a data set of 5345 students from 421 instructors from 145 institutions. The 19 items of the student surveys related to beliefs were subjected to principal components analysis (PCA). This revealed three factors: (a) affect; (b) beliefs about nature of mathematics; and (c) beliefs about teaching and learning of mathematics. Initial analyses reveal that students who choose to persist in Calculus express higher affect about math, and more novice beliefs about the nature of mathematics and the teaching and learning of math, when compared to students who choose to switch out of Calculus. Interestingly, over the course of Calculus I these differences in beliefs became more extreme, suggesting that persisters' beliefs are more flexible toward the novice direction than switchers'.

The literature tells us that students' beliefs about mathematics are related to their success in mathematics. Thus, it is important for us to first identify the differences in beliefs between students who continue studying Calculus and those who don't.

References

Kloosterman, P. (2002). Beliefs about mathematics and mathematics learning in the secondary school: Measurement and implications for motivation. In G.C Leder E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?*, (pp.247-269), The Netherlands: Kluwer.

SEEKING INTERDISCIPLINARITY AND NEW WAYS OF DOING SCIENCE: A HISTORICAL AND CHALLENGING APPROACH

Andrea V. Rohrer

Universidade Estadual Paulista (Unesp), Guaratinguetá, Brazil

The theory of embodied cognition had its beginning in the 1980s, and had emerged as a consequence of the several theoretical and empirical limitations observed in cognitivism. This new theory posited the observer as an active constructor of her/his reality, "based on non-arbitrary culturally determined forms of sense-making which are ultimately grounded in bodily experiences." (Nuñez et al., 1999) The theory of embodied mathematical cognition represents the idea of considering a new interdisciplinary research: adding mathematics to psychology, mathematics education and cognitive science, in order to attempt to explain how, from a non-objectivist perspective, mathematical concepts are constructed. However, Anderson (2003) has suggested that there has been little discussion on what the meaning of embodiment is, as if it would be of no relevance to embodied cognition. On the contrary, it plays a quite important role on how are we to interpret the embodiment mathematical, concepts, ideas, abstractions, imaginations.

Based on an historical review, this paper aims to presenting some misconceptions regarding how the notion of embodiment has led misconceptions within the theory of embodied mathematics and to comparing at least two other research approaches (radical constructivism and ethnomathematics) to this theory.

It will be shown that in fact the notion of embodiment has been in fact interpreted in at least two different manners: one supporting proto-arithmetic abilities (Edwards, 2011), and another rejecting this latter (Nuñez, 2011); and also that, rather than a non-objectivist science, it is possible to speak of relative objectivity.

References

- Anderson, M. L. (2003). Embodied Cognition: A field guide. *Artificial Intelligence*, 149, 91-130.
- Edwards, L. (2011). Embodied cognitive science and mathematics. In M. Pytlak, T. Rowland, E. Swoboda (Eds.), *Proc. 7th Cong. of the European Society for Research in Mathematics Education* (http://www.cerme7.univ.rzeszow.pl/WG/16/CERME7_WG16_Edwards.pdf). Rzeszów, Poland: CERME.
- Nuñez, R., & Edwards, L., & Matos, J. F. (1999). Embodied cognition as grounding for situatedness and context in mathematics education. *Educational Studies in Mathematics*, 39, 45-65.
- Nuñez, R., (2011). No innate number line in the human brain. *Journal of Cross-Cultural Psychology*, 42(4), 651-668.

GRAPHICAL REPRESENTATIONS AS CATALYTIC TOOLS FOR SHARPENING TEACHING-LEARNING DISCOURSES

Pieter van Jaarsveld

Wits School of Education, University of the Witwatersrand, Johannesburg

In a national context of continuing poor performance at the national exit level examination in South Africa, and recurring annual reports of the Department of Basic Education that detail the shortcomings of examination candidates, this research has as its focus the preparation of teachers for secondary school mathematics teaching. As part of a practice based research programme my third year group of 56 prospective secondary teachers was required to present procedural solutions and written discourses that would emulate an episode of teaching under test conditions, in line with what was being emphasised in lectures. From a phenomenographic analysis in a qualitative approach, the data revealed that student teachers shared the same shortcomings as the national cohort of school leavers, at procedural competence and particularly at being able to articulate the nature of mathematical objects. Students therefore lacked the essential teaching skill of being able to communicate the essence of the mathematics with meaningful exactitude. It appears that students are algebraically bound and cannot metacognitively evaluate mathematical objects or interrogate their own understanding of them. Ball et al (2004) poses the question about what it is that teachers need to know to teach mathematics effectively. This research attempts to answer that question. It uses the notion of representation as posited by Kaput (1985) who suggests that learners' understanding of mathematical concepts is reflected in their ability to represent the concept. This research emphasises the need to represent abstract algebraic forms in a concrete graphical way. The concreteness of the representation was then used as a catalyst for students to produce an exact and meaningful description of the object that facilitated understanding in an episode of teaching and learning.

References

- Ball, D.L., Bass, H., Hill, H. C. (2004) Knowing and Using Mathematical Knowledge in Teaching: Learning what matters. In Buffler, A., Laugksch, R. (Eds.) *Proceedings of the 12th Annual Conference of the South African Association for Research in Mathematics, Science and Technology Education*. Durban, SAARMSTE.
- Kaput, J.J. (1995) Representation and problem solving: Methodological issues related to modelling. In Silver, E. A. (Ed.) *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, NJ: Erlbaum.

CONSIDERING EXTRINSIC ASPECTS IN THE ANALYSIS OF MATHEMATICS TEACHER-TRAINING PROGRAMS

Yuly Vanegas, Javier Díez-Palomar, Vicenç Font, Joaquim Giménez
University of Barcelona, Spain

Previous researches on parent involvement in teaching and learning mathematics (Díez-Palomar, Gatt, & Racionero, 2011; Civil, & Bernier, 2006) suggest that interactions among family or community members have strong impact on students' mathematics performance. We draw on the Onto-Semiotic Approach (OSA) (Godino, Batanero & Font, 2007) to analyze whether training teaching programs include or not these “external” components. OSA proposes a didactic analysis of teaching processes that considers five levels of analysis: 1) identifying mathematical practices; 2) developing configurations of objects and mathematical processes; 3) analyzing didactic trajectories and interactions; 4) identifying the system of rules and meta-rules; and 5) assessing the educational competences of the teaching process.

The Criteria of Teaching Suitability (CTS) emerge as a methodological tool to measure how teacher-training programs met those 5 levels of analysis. CTS assess how mathematical practices, objects, processes, etc., included within teacher-training programs, led future teachers to master on teaching competences.

We argue that teacher-training programs use to focus on mathematics contents and processes; however there is a lack of attention to “extrinsic” components of learning domain, such as family and community engagement. Evidence suggests that “extrinsic” components appear in a succinct manner in teacher-training programs. Hence, we conclude that there is a need to find ways including such elements to improve mathematics teachers' training.

Note: Data comes out from projects I+D+I EDU2009-08120 and EDU2012-32644 funded by the Ministry of Science and Innovation of Spain.

References

- Civil, M., & Bernier, E. (2006). Exploring Images of Parental Participation in Mathematics Education: Challenges and Possibilities. *Mathematical Thinking and Learning*, 8(3), 309-330.
- Díez-Palomar, J., Gatt, S., & Racionero, S. (2011). Placing Immigrant and Minority Family and Community Members at the School's Centre: The Role of Community Participation. *European Journal of Education*, 46(2), 184-196.
- Godino, J.D., Batanero, C., & Font, V. (2007). The Onto-Semiotic Approach to Research in Mathematics Education. *ZDM. The International Journal on Mathematics Education*, 39(1-2), 127-135.

MATHEMATICAL REPRESENTATIONS USED BY GRADE 3 TEACHERS

Isabel Velez

Unidade de Investigação do Instituto de Educação, Universidade de Lisboa

The aim of this paper is to study the representations used by grade 3 teachers in introducing and supporting students' work on tasks and in discussing results and solutions, and to understand the reasons why they choose a given representation. The way teachers use representations in their teaching practice has a fundamental impact in students' learning (Stylianou, 2010). Bishop and Goffree (1986) point that teachers must provide students with opportunities to learn and understand different types of representations, respecting students' learning pace, and enabling them to understand and apply representations and to establish connections. The work of the teacher is analyzed using Schoenfeld's (2000) analytical model of teaching practice that focuses on action plans and decisions. This model takes into account the sequence of actions that the teacher perform and in combination with the analysis of teacher's reflective discourse after the class leading to a better understanding of their practice. The participants in this study are three grade 3 teachers from two primary schools with the same administration near Lisbon. Data was gathered by video recording during class observations and audio recording in post class interviews to teachers and was analyzed through content analysis. When teachers propose a problem harder than usual they tend to use several representations giving the students different routes to find a solution, but always leading students towards symbolic representations. The present study suggests that teachers encourage students to make connections between informal and symbolic representations. The results underline the importance that, while planning tasks, teachers take into account the characteristics of their students, anticipating their representations and difficulties.

Acknowledgement

This study is supported by national funds by FCT – Fundação para a Ciência e Tecnologia through the Project *Professional Practices of Mathematics Teachers* (contract PTDC/CPE-CED/098931/2008).

References

- Bishop, A., & Goffree, F. (1986). Classroom organization and dynamics. In B. Christiansen et al. (Eds.), *Perspectives on mathematics education* (pp. 309-365). Dordrecht: Reidel.
- Schoenfeld, A. (2000). Models of the teaching process. *Journal of Mathematical Behavior*, 18(3), 243-261.
- Stylianou, D. A. (2010). Teachers' conceptions of representation in middle school mathematics. *Journal of Mathematics Teacher Education*, 13, 325-343.

TRANSFORMING THE LEARNING ATTITUDE THROUGH SCIENTIFIC-POPULAR BOOKS ENGAGED IN MATHEMATICAL INSTRUCTION

Chih-Yeuan Wang

General Education Centre, Lan Yang Institute of Technology

This paper mainly explores what scientific-popular books transform the learning attitude in mathematical instruction. In this study, author adopts the case study method, including classroom observations and pre and post-lesson interviews, as the major approach to investigate the attitude change of students after scientific-popular books engaged in mathematical instruction. Four cases of students in a certain institute of technology, they were involved in summer remedying course of 2011 academic year.

Jenkins (2010) indicated that mathematical scientific-popular books are excellent media to complete mathematics pedagogy through integrating vary courses, because they can develop proper and meaningful discussion, furthermore they are full of interesting. van den Heuvel-Panhuizen, van den Boogaard, & Doig (2009) found that it is popularized to connect children literature and mathematics.

Based on the data collected through classroom observations and interviews, researcher preliminarily addressed some results about the investigation. Those include that it can transform students' learning attitude and stimulate their learning motive through scientific-popular books engaged in mathematical instruction. At the same time, engaging scientific-popular books in mathematics course can also improve their learning achievements. Author thought that there appeared to be some implications about changing learning attitude, stimulating learning motive and improving learning achievements. First, teachers should conduct relevant scientific-popular books into mathematical pedagogical activities to improve learning achievements and motive. Secondly, to carry out the complementarities, replacement, connection and extension between scientific-popular books and mathematics materials should depend on teachers' pedagogical profession, students' cognitive development and the logicity and correlation of mathematical course. Thirdly, to let students comprehend more easily, the contents of scientific-popular books should connect mathematics theory with life experience. Finally, because the research is only in accordance with the remedying course, author suggests that the correlative investigation should be carried out in normal mathematics course classrooms.

References

- Jenkins, K. (2010). Positioning picture books within the mathematics curriculum. *Australasian primary mathematics classroom*, 15(2), 28-32.
- van den Heuvel-Panhuizen, M., van den Boogaard, S., & Doig, B. (2009). Picture books stimulate the learning of mathematics. *Australasian journal of early childhood*, 34(8), 30-39.

THE USE OF DISCURSIVE ANALYSIS FOR UNDERSTANDING PROSPECTIVE TEACHERS' GEOMETRIC THINKING

Sasha Wang

Boise State University, Idaho, U.S.A

The study revisits the van Hiele theory (1985) with a discursive lens to investigate prospective teachers' geometric thinking through their *word use*, *visual mediators*, *routines* and *endorsed narratives* (Sfard, 2008). The van Hiele Geometry test developed by Usiskin (1982) were used as a pre- and post-test to gather initial information on changes of the prospective teachers' geometric thinking. Clinical interviews were conducted to investigate changes in students' geometric discourses. Aligning prospective teachers' test results with analyses of their geometric discourse from the clinical interviews, we found variations of geometric discourses at the same van Hiele level among different prospective teachers, as well as changes in geometric discourses within a van Hiele level for the same prospective teacher (Wang, 2011). For example, Sam's van Hiele geometry tests suggested that there was no change after a semester of instruction in van Hiele levels, but the analyses of Sam's geometric discourses showed changes in her word use, a change in her understanding of related geometric concepts, and changes in her substantiation routines. Sam's routine procedures operated at an object level at the pre-interview, and ten weeks later she was able to use endorsed narratives (i.e., mathematical axioms) to verify her claims at the post-interview. The discursive framework, complimenting with the van Hiele levels, provided tools allowing for the detailed examination of prospective teachers' geometric thinking. It helped to understand learning as change in discourses while students move toward a higher van Hiele level. In particular, a fine-grain analysis of prospective teachers' mathematical word use reveals valuable insight toward their understanding of related concepts. Teachers' routines offer valuable information about what they do as a course of action to endorse narratives, and help differentiate whether a prospective teacher reasons at an object level or a meta-level.

References

- Sfard, A. (2008). *Thinking as communicating: human development, the growth of discourses, and mathematizing*. Cambridge.
- Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry* Chicago: University of Chicago.
- Van Hiele, P.M. (1986). *Structure and insight: A theory of mathematics education*. Orlando, FL: Academic Press.
- Wang, S. (2011). *The van Hiele theory through the discursive lens: Prospective teachers' geometric discourses*. (Doctoral dissertation). Retrieved from ProQuest.

THE ASPECTS OF STUDENTS' EXPLANATIONS IN AN ERROR ANALYSIS SITUATION

Keiko Watanabe-Mukai
Shiga University

The purpose of this study is to make clear the aspects of student's explanation by which he or she could reflect his or her way of thinking in mathematical problem-solving. This paper is focus on the explanations for the errors of 25 students of teacher training course in a probability problem-solving. A great deal of effort has been made on the study of student's explanation in mathematical problem solving. What seems to be lacking, however, is to make clear how we analyse the student's explanation. To find a solution of the problem, I analysed student's explanation in two categories. One is the explanation of procedures by which a student reports exactly his/her activities. Another is the explanation of reasons by which a student tries to make his/her reasonable way of thinking and to show us why his/her procedures are true. And I set the learning situation, the error analysis situation, to observe that how students make the relationship the explanations of procedure and of reason. In this research, I analysed student's explanation about given error of another person. Then, Students were requested to explain why given answer for the probability problem is error. The Followings are the probability problems (Q), an error answer (EA), and steps (Step n) showed students:

- Q: A box contains ten cards which are written the number from 1 to 10 one by one. You pick up one card, confirm the number, and return the card into the box. You repeat this trial three times. What is the probability that the maximum value of the card number is 6?
- Step1: Solve the given problem (Q).
- Step2: Confirm only the correct answer (The answer is 91/1000).
- Step3:(EA-Procedure) $3 \times (1/10 \times 6/10 \times 6/10) = 108/100$ (EA-Reason) In three times trials, certainly we once pick up a card written the number 6 and twice pick up cards written the number 6 or less
- Step 4: Find out and report each of *error point* for the procedure and the reason in the given error answer (EA).

The most remarkable result is that students spontaneously consider the connection EA-Procedure and EA-reason and then investigate whether an EA-procedure is justifiable in the error analysis situation (Cf. Peled and Zaslavsky, 2008).

References

Peled, I. & Zaslavsky, O. (2008). Beyond Local Conceptual Connections: Meta-Knowledge about Procedure. *For the Learning of Mathematics*, 28(3), 28 -35.

Acknowledgment This work was supported by JSPS KAKENHI (22730689)

THE RELATIONSHIP BETWEEN MEDIUM OF INSTRUCTION AND MATHEMATICS ACHIEVEMENT IN SOUTHERN AND EASTERN AFRICAN COUNTRIES

Koji Watanabe

Graduate School for International Development and Cooperation,
Hiroshima University

This study is undertaken to better understand the relationship between medium of instruction and mathematics learning for the students for whom medium of instruction is an additional language, especially focusing on 14 southern and eastern African countries (namely, Botswana, Kenya, Lesotho, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Swaziland, Tanzania, Uganda, Zambia and Zanzibar). Obviously, the role of medium of instruction ability is important and essential for students to learn mathematics in the classroom. There are a lot of researches about the relationship between the medium of instruction and learning of mathematics in Africa (e.g., Barwell et al., 2007; Mamokgethi, 2012).

In this study, the approach of international comparison by using the raw data of international educational survey will be adopted in order to reveal their holistic characteristics. The data which is analysed in this study is collected by the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) in between 2000 and 2002. This data set consists of the data of mathematics and reading achievement test written by their medium of instruction, for instance Mozambique is in Portuguese, Tanzania and Zanzibar are in Kiswahili and other countries are in English.

In this analysis, there are two points of view. One is to focus on test score. By using hierarchical linear model (HLM), the international comparison of mathematics test score will be conducted through controlling the influence of reading ability. Another one is to clarify the answer pattern of each country by focusing on item discrimination.

As a result, the countries have gotten higher level mathematics test score (namely, Uganda, Kenya and Mauritius) in HLM analysis have similar answer pattern, and by contrary, lower level countries (namely, Namibia, Lesotho, Zambia and Malawi) have also similar answer pattern. In addition, the results suggest that the countries which English is not medium of instruction possess different types of answer pattern.

References

- Barwell, R., Barton, B., & Mamokgethi. S. (2007). Multilingual issues in mathematics education: introduction, *Educational Studies in Mathematics*, Vol.64, pp.113-119.
- Mamokgethi, S. (2012). Mathematics Education and Language Diversity: Background, Findings and Future Research Directions, *ICME12 Pre-proceedings*, p.41.

STUDENTS' GOAL DIRECTED BEHAVIOR: IMPLICATIONS FOR MODELING STUDENT UNDERSTANDING

Eric Weber
Oregon State University

A significant challenge in mathematics education is how to create learning environments that support students' development of new ways of thinking. Teaching experiments, hypothetical learning trajectories (e.g. Simon & Tzur, 2004) address this issue by developing models of how students might come to understand an idea in a particular way. These models are based on predicting and analyzing students' actions, characterizing students' progression with specific tasks. However, students' goal directed behavior is not often considered as part of these models even though they directly affect the observable actions from which models of their thinking result. This study explored the role of students' goals in characterizations of their thinking by analyzing the results of a teaching experiment in two phases, the second of which explicitly attended to the students' motivation and goals.

This study took place in two phases, each with a specific research question. First, I made interpretations about students' thinking within a teaching experiment that were not sensitive to the goals the students had within the instruction. The research question driving the first phase of analysis was *what ways of thinking do students develop about functions and their graphs?* In the second phase, I conducted a retrospective analysis in which I accounted for goals the students had in engaging with the instruction and materials. The research question driving this phase was *how did the students' goals in engaging with the tasks affect their observable actions?*

The results of the study suggest that accounting for the students' goals during the teaching experiment revealed that the students were able to think in a myriad of creative ways, but had avoided doing so because of their goal of being correct. Simply put, they were afraid to take chances on conjectures and relied on what they knew would be considered as acceptable.

The results of this study suggest that if we construct inferences based only on behavior without asking why a student behaved in that particular way, we may fail to explain the depth and complexity of a students' understanding. The recent focus on modeling the development of student thinking via learning progressions and trajectories both in research and in national standards makes systematic consideration of students' goal directed behavior an important area of research and practice.

References

- Simon, M., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning*, 6(2), 91-104.

COMPARING THE EFFECTS OF TWO DYNAMIC VISUALIZATION INTERVENTION PROGRAMS ON STUDENTS' TRANSFORMATIONAL GEOMETRY ABILITIES

Xenia Xistouri

Department of Education, University of Cyprus

There seems to be a disagreement in the findings of various studies regarding the potential of technology-based instruction to develop transformational geometry abilities. Given that there are different types of dynamic visualizations, we suggest that it may be too simplistic to generalize results from studies which use only one type of dynamic visualization. Setting off from a discrimination of visualizations proposed by Moreno-Armella, Hegedus, and Kaput (2008), this project aimed to compare the potential of two similar intervention programs, one using a discrete dynamic visualization and one using a continuous dynamic visualization, to develop primary school students' ability in transformational geometry.

After controlling for initial differences, two groups of approximately 40 sixth-grade students (total of 79) received a twelve-session intervention program on transformational geometry concepts by the same teacher, based on the same principles and with the same activities, but each with a different type of dynamic visualization – discrete or continuous. Students' abilities in transformational geometry concepts were measured before and after the intervention program. The results suggest that the continuous dynamic visualization instruction group significantly outperformed the discrete dynamic visualization instruction group ($t_{(77)} = 2.05, p < .05$). Moreover, the results show that only the continuous dynamic visualization group had a significant increase in transformational geometry ability ($t_{(39)} = 3.74, p < .01$). Our findings suggest that it may be more beneficial for students if instruction of transformational geometry concepts in primary school is based on the use of continuous dynamic visualizations, in contrast to discrete. The results also call for a need to develop awareness regarding the different characteristics of instructional visualizations and for evaluating their effects on students' mathematical abilities.

Acknowledgments

This work falls under the Cyprus Research Promotion Foundation's Framework Programme for Research, Technological Development and Innovation 2009 -2010 (DESMI 2009-2010), co-funded by the Republic of Cyprus and the European Regional Development Fund (Grant:PENEK/0609/57).

References

Moreno-Armella, L., Hegedus, S. J., & Kaput, J. J. (2008). From static to dynamic mathematics: historical and representational perspectives. *Educational Studies in Mathematics*, 68(2), 99-111. doi:10.1007/s10649-008-9116-6

DISCOURSE PATTERNS EMPLOYING CHORAL RESPONSE IN MATHEMATICS CLASSROOMS IN SEVEN COUNTRIES

Lihua Xu, May Ee Vivien Wan, David Clarke
University of Melbourne, Australia

Choral Response refers to a classroom event in which all students in a group respond orally in unison to a question or statement from the teacher. Wang (2010) distinguished two functions of choral response: a regulative function and an instructional function. Four variations of choral response were identified: co-reading, co-narrating, simple answering, and tag answering (where the response is ritualized rather than informative, serving purposes such as re-focusing student attention). To date, research has not examined choral response within classroom discourse patterns.

This paper reports the fine-grained analysis of one lesson from each of 22 classrooms in Melbourne, Hong Kong, Shanghai, Berlin, Tokyo, Seoul, Singapore, and San Diego. Three video cameras were used (teacher camera, student camera and whole class camera) and each provided an audio record from which classroom speech could be analysed. We distinguished three types of public utterances: teacher utterance, choral utterance, and (individual) student utterance. Seven types of choral response were identified: Yes/No (Select Choice); Numerical; Mathematical Symbolic Expressions; Mathematical Terms; Mathematical Procedures; Mathematical Propositions; and Non-mathematical Responses. Choral response was located within identifiable patterns of discursive practice in the various mathematics classrooms.

We identified two discourse patterns associated with the use of choral response. The most common pattern featured a teacher question, then a choral answer, followed by teacher feedback/evaluation/follow-up (TQ/CA/TF), and three variations were identified. In the second discourse pattern, an individual student was asked to respond to a teacher question, and a choral evaluation by the class of that student's response was requested by the teacher (TQ/SA/CE). Both echo the familiar IRF structure (teacher initiation, student response, teacher feedback/follow-up), but are based on different authority structures in the classroom and have different affordances for student participation in classroom discourse. The two discourse patterns differ in the sophistication of mathematical content, the rules of participation and the form of participation. In TQ/CA/TF, the content is simple and the discourse establishes the taken-as-shared status of certain knowledge. In TQ/SA/CE, students have the opportunity to exercise judgement. In both cases, we suggest choral response is best thought of as monologic rather than dialogic, but should be considered a legitimate form of verbal participation useful to initiate students into mathematical discourse.

Reference

Wang, T. (2010). Teaching Mathematics through Choral Responses: A Study of Two Sixth Grade Classrooms in China. NY: The Edwin Mellen Press.

A COMPARATIVE LOOK AT ENGLAND AND TAIWAN TEXTBOOKS BASED ON THE NOTION OF ABSTRACTION

Kai-Lin Yang

Department of Mathematics, National Taiwan Normal University

In this study, attention was directed towards the notion of abstraction to analyse how the Pythagoras theorem was handled in England and Taiwan mathematics textbooks. Abstraction is a key adaptive mechanism of human cognition and an essential process in the personal construction of mathematical knowledge. By integrating the meaning of abstraction from a constructive-empirical and a dialectic perspective of abstraction, Yang (2013) defined mathematical abstraction as a mental or social activity through which subjects intentionally identify, reconstruct or apply new mathematical objects represented or mediated by using semiotic tools (models, artefacts or multiple representations). Then, three key components: object, subject and semiotic tool, were focused to produce six attributes. The six attributes included generality of objects, connections of subjects' experience, types of semiotic tools, subjects' need for objects, the changes of semiotic tools relative to the same object, and the assumed purposes for subjects to use semiotic tools in context.

Against the six attributes underlying the three components of abstraction and their relations, this study analyzed and compared how England and Taiwan textbooks arranged the topic of the Pythagorean theorem, focusing on the main research question: What are similarities and differences in the ways of arranging each attribute between the two countries?

We used a content analysis methodology to study on the selected topic. 23 and 44 analysis units were identified respectively in English and Taiwan textbooks according to the categories of text intentions which include (1) to introduce something related to the topic, (2) to explore the idea of the topic before describing it, (3) to describe the idea of or about the topic, (4) to explain or elaborate the topic, (5) to show worked examples, (6) to practice, (7) to adapt or extend the idea of this topic. In addition, we also formulated a workable classification as to each attribute.

As to the similarities, the two textbooks were characterized as the recursive way to arrange generality, plenty connections of students' experience, and no social need for interactions with others. As to the differences, in the England textbook, exercises were arranged collectively, and an explorative activity was provided for extending this theorem. In the Taiwan textbook, one exercise was arranged to follow each worked example, and each situational problem was provided with ritual or geometrical figure.

References

Yang, K.-L. (2013). A Framework for Analysing Textbooks Based on the Notion of Abstraction. *For the learning of mathematics*, 33(1), 31 – 37.

A CASE STUDY OF EDUCATIVE POWER

Kai-Lin Yang^{*1}, Fou-Lai Lin^{*1}, Hui-Yu Hsu^{*1}, Jian-Cheng Chen^{*2}, Ying-Hao Cheng^{*3}

^{*1} Department of Mathematics, National Taiwan Normal University

^{*2} Department of Electrical Engineering, MingChi University of Technology

^{*3} Department of Mathematics and Computer Science Education, Taipei Municipal University of Education

Previous researchers have examined the learning of teacher educators by self-studies on personal teaching practices. Researchers also tried to characterize the learning of teacher educators as a continuing search for appropriate and effective actions. Some researchers also have investigated types of knowledge that teacher educators require and the ways to challenge and facilitate the development of beginning teacher educators as well as how the nature of educators' work shaped their identity (Loughran, 2011). Recognizing the values and contributions of those studies, we aim to amplify the enquiry of educative power through exploring the features of an experienced mathematics teacher educator-researcher (MTE-R), X. In particular, it is interesting that how he initiated at-the-moment actions to facilitate teachers' professional growth based on his knowledge of both research and practice.

To this end, three main types of data were collected for investigating the educative power of the educator X: videotape corpus of the professional development workshops, interview records and participating teachers, and written materials. Based on the conceptual framework of educative power of MTE-Rs (Lin et al., 2011), we focused on X's overall competencies of reasoning, communication, and connection across strands of learners: students, teachers, and educators.

Our analyses showed that features entailed in the X's power can be characterized as communicating with educative phenomenology, reasoning onto emergent models about educative phenomenology, and maintaining a dialectical and coordinated connection between practice and research. The intrinsic components concerned by the X regarding to the three features include essence of mathematics and sense of students' cognition. However, awareness of different teachers' needs is not well taken into account. This work contributes to research on the profession of MTEs by providing a model of educative power.

References

- Loughran, J. J. (2011). On becoming a teacher educator. *Journal of Education for Teaching*, 37(3), 279-291.
- Lin, F. L. et al. (2010). Conceptualizing Educative Power of Researcher-Educator. In In Ubuz, B. (Ed.). *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 351. Ankara, Turkey: PME.

RELATIONSHIP BETWEEN HIGH SCHOOL STUDENTS' UNIVERSITY ENTRANCE EXAM SCORES AND THEIR NON-ROUTINE PROBLEM SOLVING SKILLS

Yeliz Yazgan, Huseyin Ozan Gavaz, Bahar Kirmizi
Uludag University, Bursa/TURKEY

Problem-solving is an active experience involving cognitive and metacognitive strategies (Nik Pa, 1991). Involving problem situations that students are not familiar with and are not expected to have previously solved, or have not encountered regularly in the curriculum, non-routine problem-solving gives students the opportunity to think and reason systematically (Salleh & Zakaria, 2009). Therefore, the ability to solve non-routine problems has been one of the important aims of the many mathematics curriculums for the past fifteen years (Wong & Tiong, 2006).

Purpose of this study is to seek whether there is any relationship between high school students' scores on university entrance exam and their non-routine problem solving skills. To this aim, firstly a paper-pencil test that comprises 9 non-routine open-ended problems was conducted to 85 high school students. Participants were from 5 different high school in Bursa/Turkey and they were 12th grade students. All answers for each problem were coded as 0 (blank or wrong), 1 (incomplete) and 2 (correct). So, every student had a total point between 0 and 18. Three months later, all students took nationwide standard university entrance exam, and their general scores on this exam were obtained from their schools. Lastly, Spearman's rho correlation coefficient was computed to see what the level of relationship between these two kinds of scores is.

Spearman's rho showed that there is a strong relationship between high school students' scores on university entrance exam and their success on non-routine problem solving ($r = .71$ $p < .001$). This result shows that students should be given the opportunity to solve non-routine problems (NCTM, 2000) since this kind of problems demand flexibility in thinking which students can exhibit in the other areas except mathematics.

References

- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author
- Nik Pa, Nik Aziz. (1991). *Program pendidikan pelajar pintar cerdas, teori dan praktik. [Education program for gifted students: Theory and practice]*. Selangor: Jiwamas Printer.
- Salleh F.; Zakaria E. (2009). Non-routine problem-solving and attitudes toward problem-solving among high achievers. *International Journal of Learning*; 16(5):549-560.
- Wong, K. Y., & Tiong, J. (2006, May). Diversity in heuristic use among Singapore pupils. Paper presented at the Research Association of Singapore Conference: Singapore

A CASE STUDY ON PRE-SERVICE MATHEMATICS TEACHERS' COVARIATIONAL REASONING IN PEER LEARNING

Seçil Yemen-Karpuzcu, Fadime Ulusoy, Mine Işıksal-Bostan
Middle East Technical University, Turkey

College mathematics students have difficulties on rate of change and covariant aspect of a function situation (Carlson et al., 2002). Therefore, it is important to identify how prospective mathematics teachers apply covariational reasoning abilities when analysing dynamic events in which independent variable changes continuously. When considered from social perspective, students are able to engage actively in learning process while interpreting the events through interaction and discussion with peers. For this reason, the aim of this research was to investigate prospective middle school mathematics teachers' covariational reasoning abilities for dynamic events with peers. Data was collected from 46 third year prospective teachers through Matching Bottles and Graphs instrument that includes matching 6 volume-height graphs with figures of 6 bottles filled water continuously at a constant rate and writing reasons for selected correspondence. After analysing students' responds to the instrument, semi-structured interviews were conducted with pairs to examine their covariational reasoning about dynamic events in peer learning. In this study a pair, Aysu and Buket, was selected and analysed. Whereas Buket answered all items, Aysu answered only two items correctly. Interview data was analysed through identifying students' expressions using five levels of mental actions described in the covariational reasoning framework (Carlson et al., 2002). In the following, the flow of mental actions and the pattern of interactions between two students were identified in the light of the double coding model (Dreyfus, Hershkowitz, & Schwarz, 2001). Analysis of interview data revealed that, while Aysu generally appeared to exhibit *quantitative coordination level* (Level 3), Buket applied reasoning at *average rate level* (Level 4) by emphasizing the concepts of slope and rate of change. On the other hand, the pattern of interactions between students indicated that Aysu was able to apply Level 4 reasoning while discussing with Buket. Aysu also realized her conflict about switching the role of variables in the graphs throughout interaction process. In conclusion, results revealed that students had a progress on covariational reasoning in dynamic function events in peer interactions. In line with the findings, it is suggested that mathematics teacher programs should include tasks that enhance covariational reasoning abilities for dynamic function events in peer learning environments.

References

- Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modelling dynamic events: A framework and a study. *Journal for Research in Mathematics Education*, 33(5), 352–367.
- Dreyfus, Hershkowitz, R., & Schwarz, B. B. (2001). Abstraction in context: The case of peer interaction. *Cognitive Science Quarterly*, 1(1), 307-358.

AN ANALYSIS OF STUDENTS' BASES FOR WRITING AN EXPRESSION OF MULTIPLICATION BY A DECIMAL IN RULE-OF-THREE PROBLEMS

Watarai Yohei

Graduate School of Comprehensive Human Sciences, University of Tsukuba

In Japan, the canonical meaning of multiplication by a decimal is defined as "multiplication of a reference quantity by a ratio" in elementary school mathematics. However, it is difficult for students to understand the canonical meaning of multiplication by a decimal. Because of that, it is also difficult for students to write an expression of multiplication by a decimal in problem-solving. Therefore, the purpose of this study is to clarify bases for writing an expression of multiplication by a decimal by students who don't understand the canonical meaning of multiplication by a decimal.

In order to accomplish the above purpose, this study analyzes problem-solving procedures used by students (10-11 years old) by applying the framework "The Theory of Conceptual Fields" (Vergnaud, 1988), as follows. First, this study classified bases for writing an expression of multiplication according to *theorems-in-action* based on multiplicative structures, on problem-solving procedures used by students in *rule-of-three problems* (Nakamura's lesson (Nakamura, 2007)). Second, this study described *theorems-in-action* based on both multiplicative structures and additive structures according to students' interpretations of a meaning of multiple number, on problem-solving description expressed by students.

The results of analysis reveal that (i) students who interpret a meaning of multiple number with reference to additive structures and can use a multiplicative proportional relation come to be able to write an expression of multiplication of reference quantity by a ratio through problem-solving of rule-of-three problems that require a multiplication by a decimal. On the other hand, (ii) students who interpret a meaning of multiple number with reference to additive structures and use only an additive proportional relation don't write an expression of multiplication by a decimal even if they use a phrase "If one measure becomes N times, another measure will also become N times".

References

- Nakamura, T. (2007). An analysis of description concerning multiplication by a decimal. *Proceeding of 40th Conference of the Japan Society for Mathematical Education*, 361-366 (In Japanese).
- Vergnaud, G. (1988). Multiplicative structures. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades*, Hillsdale: L. Erlbaum Associates, 141-161.

MATHEMATICAL RICHNESS OF TALKS IN SIX CHINESE MATHEMATICS CLASSROOMS: THE AMOUNT OF ACADEMIC WORDS AS AN INDICATOR

Jun Zou, Yujing Ni, Xiaoqing Li

Department of Education Psychology, Department of Psychology
The Chinese University of Hong Kong, Shenzhen University

Talking or speaking mathematically in mathematics classroom is considered to be important for facilitating children's mathematical. But, how could we judge one classroom to be mathematically richer than the other? Most of those studies were based on the analysis of language used in classroom, at the discourse or lexical level (Clarke & Xu, 2008). Also, the effects of mathematical talks on students learning needed to be examined further. Therefore, the present study was to exam whether the amount of academic words could differentiate different classroom instructions and whether the differences on the amount of words are related to the differences of students' learning outcomes. Six fifth-grade mathematics teachers in China mainland were selected based on their scores of mathematics richness from a previous study (Li, 2011) and one lesson was selected from each teacher. Linguistic features of the six instruction transcriptions were analysed. Also, learning outcomes of students in the six classrooms were assessed for three times. The results showed that the amount of academic words varied among the six lessons. The differences in the amount of academic words among the classrooms were found to be associated with students' gain scores on solving complex problems, but not on computation and solving simple problems. Therefore, the amount of academic words could be used as an indicator of mathematical richness. And the mechanism of its association to students learning outcomes needed to be investigated further.

References

- Clarke, D., & Xu, L. H. (2008). Distinguishing between mathematics classrooms in Australia, China, Japan, Korea and the USA through the lens of the distribution of responsibility for knowledge generation: public oral interactivity and mathematical orality. *ZDM Mathematics Education*, 40, 963-972.
- Li, Xiaoqing. (2011). Quality of instructional explanation and its relation to student learning in primary mathematics [D]. HongKong: The University of HongKong.



POSTER PRESENTATIONS

INSTRUMENTAL GENESIS OF GRADE 10 STUDENTS LEARNING TO USE DYNAMIC 3D GEOMETRY SOFTWARE

Francisco Javier Alba, Angel Gutiérrez¹

Departamento de Didáctica de la Matemática - University of Valencia (Spain)

Even for students who master dynamic plane geometry software, learning to use dynamic 3d geometry software (D3dGS) is not easy (Hugot, 2005). This is a quite recent type of software that has scarcely been approached from the instrumental genesis (Flores, 2009). We present part of the results of a research project aimed to investigate aspects of the learning of D3dGS by students while solving 3d geometry tasks. To analyze students' learning and actions, we adopted an instrumental genesis approach (Rabardel, 1999). Our research objectives are to identify and analyze i) difficulties of interpretation and use students have to manage points, lines, planes, and solids in Cabri 3D, ii) difficulties they have while dragging to manage those objects, and iii) possible schemes developed by students in these instrumented tasks.

The subjects were a classroom group of eleven grade 10 students using Cabri 3D to solve a set of tasks designed to help them learn to use the software. Our analysis of students' interactions with the software is based on the information that students produced while they were solving instrumented activities (audio recording of conversations and screen recording of actions on the computer).

Our main conclusions are that students have difficulties using Cabri 3D mainly when i) objects move out of the screen, ii) points cannot be dragged out of the base plane, iii) students have to change their viewpoint (glass ball), and iv) they do not take profit from on screen feedback (help and message windows), but iv) adequate instrumented tasks may help students to overcome their difficulties by developing schemes of use and inducing instrumentalization processes to take place.

References

- Flores, J. V. (2009). *Gênese instrumental na interação com Cabri 3D: Un estudo de transformações geométricas no espaço*. Sao Paulo, Brazil: Pontificia U. Católica de Sao Paulo. Available in <http://www.pucsp.br/pos/edmat/do/tese/jesus_flores_salazar.pdf>.
- Hugot, F. (2005). *Une étude sur l'utilisabilité de Cabri 3D*. Grenoble, France: Univ. J. Fourier. Available in <<http://www-diam.imag.fr/memoiresDEA/memoireF.Hugot.PDF>>.
- Rabardel, P. (1999). Éléments pour une approche instrumentale en didactique des mathématiques. *Actes de la 10^e Université d'Été de Didactique des Mathématiques*, 203-213. Available in <<http://ergoserv.psy.univ-paris8.fr/Site/Groupes/Modele/Articles/Public/ART372248700765426887.PDF>>

¹ This is part of the research project *Key moments in learning geometry in technologic collaborative environments*, funded by the Spanish Ministry of Science and Innovation as part of the Non-oriented Fundamental Research Sub-program (EDU2011-23240).

VALUES IN THE SWEDISH MATHEMATICS CURRICULUM

Annica Andersson, Anette de Ron
Stockholm University

Values can be seen on a societal level as part of a social process in a society where, for example, political forces and ideologies forms values. One way those values are manifested and affects mathematical education is in the Swedish National Curriculum. This research study is part of a larger international project with the aim to investigate teachers' and students' values when learning mathematics (Seah & Wong, 2012). This poster focuses how the values from the international study are consistent with values expressed in the Swedish national mathematics curriculum.

The methodology mainly based on Faircloughs (1995) model for discourse analysis support us to identify the consequences of how the uses of certain words make values explicit or implicit in the curriculum.

The results indicate that values expressed in the Swedish curriculum align with a number of the students' and teachers' values expressed in the international study, however they might be described and represented in very different ways. To give an example the mathematical values of rationalism and objectivism (Bishop, 1991) has a strong representation in the curriculum. Words consistent with these values are for rationalism e.g. argument/argumentation, reason/reasoning and mathematical proof and for objectivism e.g. application, tools and objects. These words are frequently used in the curriculum. We can also see that the words argument/argumentation and reason/reasoning are used much more frequently than other words consistent with rationalism. These results support our understanding whether the values in the international study are consistent with values regarded as important in a Swedish context and also further support our understanding of the alignment of students', teachers' values and values in the document texts.

References

- Seah, W. T., & Wong, N. Y. (2012). What students value in effective mathematics learning: A „Third wave project“ research study. *ZDM- The International Journal on Mathematics Education, Values in East Asian Mathematics Education - the Third Wave*, 44(1), Retrieved January 7, 2013.
- Bishop, A.J. (1991). *Mathematical enculturation: a cultural perspective on mathematics education*. Dordrecht: Kluwer Academic Publishers.
- Fairclough, N. (1995). *Critical discourse analysis: the critical study of language and social change*. Edinburgh: The Longman Group.

AN INVESTIGATION OF NOVICE MATHEMATICS TEACHER EDUCATORS' EXPERIENCES IN TRANSITION FROM DOCTORAL STUDENT TO FACULTY MEMBER

Mary Beisiegel, Jennifer A. Eli

Oregon State University, The University of Arizona

Understanding the doctoral experiences of future mathematics teacher educators (MTEs) is significant to the field of mathematics education as MTEs will not only become the teachers of pre-service elementary, secondary and post-secondary mathematics teachers, but they might also become researchers in mathematics education. However, Even (2008) recognized “there is almost no research on the education of mathematics teacher educators” (p. 58) and Goos (2009) conceded “almost nothing is known about the [...] development of mathematics teacher educators” (p. 210). To address these concerns, the goal of this study was to investigate MTEs’ doctoral preparation in the U.S. and their transitions from doctoral student to faculty member, with a particular comparison of MTEs who earned their doctorate in colleges of education but found employment in one of two areas: either a college of education or department of mathematics. A survey instrument, which included 27 questions partitioned into demographics and open-response items, was developed and reviewed by a panel of expert MTEs, and 69 out of 81 novice MTEs responded. Participants’ responses to open-response items were analysed with the six-stage process of thematic analysis (Braun & Clark, 2006).

In this poster, the following information and results will be presented: (1) the various paths from doctoral student to MTE faculty member in the U.S.; (2) the categories of MTEs and how many participants fell into each category; (3) disparities in the way different types of departments attend to the development of MTEs; (4) themes that were similar and different among the respondents based on their area of employment; and (5) implications for doctoral programs for MTEs and experienced MTEs in charge of doctoral preparation.

References

- Braun, V., & Clark, V. (2006). Thematic Analysis in Psychology. *Qualitative Research in Psychology*, 3(2), 77-101.
- Even, R. (2008). Facing the challenge of educating educators to work with practicing mathematics teachers. In B. Jaworski & T. Woods (Eds.), *The international handbook on mathematics teacher education: The mathematics teacher educator as a developing professional* (Vol. 4, pp. 57-73). Rotterdam, Netherlands: Sense Publishers.
- Goos, M. (2009). Investigating the professional learning and development of mathematics teacher educators: A theoretical discussion and research agenda. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 209-216). Palmerston North, NZ: MERGA.

ODE COURSE AND PRE-SERVICE TEACHERS CONCEPTION ABOUT LINEAR ALGEBRA OBJECTS

Barbara Bianchini, Silvia Machado
Pontificia Universidade Católica de São Paulo

Researches in Mathematical Education, specifically on the learning and teaching of Linear Algebra, show that students in general, and specially the pre-service ones have difficulty in acquiring the basic knowledge of the topic. However, we believe that Linear Algebra is an essential topic in the mathematics' teacher education, because it is the background for the K-12 Education, e.g. matrix, systems of equation, etc. It is important to highlight that the study of Linear Algebra provides situations for the development of the algebraic thinking, such as demonstration, generalization and deduction, among others, in addition to clarifying the link between algebra and geometry and their notions, which are the basis for a lot of notions which are developed in Computer Science.

We hereby present the results already obtained by an ongoing qualitative research in which the goal is to investigate the conceptions concerning Linear Algebra elementary concepts of ODE pre-service teachers. In order to do so, we have adopted the APOS model (Dubinsky, 1991), which allows us to identify mathematics objects' conception.

The results described refer to concepts of basis and linear transformation of a vector space over R . The data were obtained through the analysis of activities and evaluations done during a Linear Algebra course, and by semi-structured interviews which were conducted with students who had already finished this course.

Two students were interviewed with the objective of identifying their vector space basis' conceptions. One of them showed to have *object concept* of basis and the other one, *process concept* only for linear combination; however, the data did not allow us to identify his conception of the object. We interviewed two other students aiming at identification of their conceptions about linear transformation. One of them showed to have a slight comprehension of *action concept* of linear transformation. Concerning the other student, not only did he register an incomplete definition of linear transformation, but he also failed to give meaning to this definition, not using it to justify his decisions on the veracity of the examined examples. This led us to consider there were not enough elements to classify him in any type of conception.

Our conclusion is that the course did not enable students to have the same performance in terms of concept building, which depended more on the potential of each one than the proper virtual environment.

References

Dubinsky, E. (1991). Reflective abstraction in advanced mathematical thinking. In: Tall, David. *Advanced Mathematical Thinking*. Holanda: Kluwer Academic Publishers, 1991.

A LANGUAGE OF DESCRIPTION FOR ANALYSING MATHEMATICS CLASSROOM COMMUNICATION

Lisa Björklund Boistrup, Niclas Larson
Stockholm University, Linköping University

The context of this poster is a study investigating to what extent and how the studies of mathematics influence Swedish students when they apply for a certain programme (a pre-set course) in upper secondary school and how they experience and perceive the transition from lower to upper secondary school with respect to the teaching and learning of mathematics. The problem complex will be studied through quantitative methods (survey) combined with qualitative methods (interviews and video).

This poster focuses on one part of the study, where the data consists of videos from two mathematics classes in lower and three mathematics classes in upper secondary school. The videos contain front teaching episodes as well as dialogues between a few students and the teacher. In this poster we address a part of the analytical framework. We will describe initial analyses inspired by three social semiotic *meta-functions* (described in Van Leeuwen, 2005). Our interpretations of these functions are aligned with two concepts developed by Bernstein (e.g. 2000); *framing* and *recognition rules*. This approach provides a *language of description* for performing analyses close to data and also building a basis for further analyses relative to Bernstein's theoretical concepts. In our analyses, the social semiotic meta-functions then become "a device for transforming observed empirical instances of a phenomenon of interest into theoretically relevant data" (Jablonka & Bergsten, 2010, p. 39).

One of the meta-functions is the *ideational*. In short, this refers to our experiences and representations of the world. When this function is interpreted from within the concept of framing, our analysis focuses on the explicitness of the criteria for legitimate knowledge productions within the activity. We look for how clear it is to the students what they are expected to do and to learn. The other two meta-functions are the *interpersonal* and *textual* which will be described in the poster. The three meta-functions will be illustrated with analysis of excerpts from classroom episodes. The findings are discussed in terms of framing and recognition rules.

References

- Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research and critique* (Rev. ed.). Lanham: Rowman and Littlefield.
- Jablonka, E. & Bergsten, C. (2010). Theorising in mathematics education research: differences in modes and quality. *Nordic Studies in Mathematics Education*, 15(1), 25-52.
- Van Leeuwen, T. (2005). *Introducing social semiotics*. London, UK: Routledge.

COMPARTMENTALIZATION IN DUTCH TEXTBOOKS ON THE MULTIPLICATION OF FRACTIONS

Geeke Bruin-Muurling, Koenno Gravemeijer, Maaike Koopman, Perry den Brok
TU Eindhoven

The Netherlands is known for the Realistic Mathematics Education (RME) approach (e.g. Freudenthal, 1968), but less is known about how this mathematics reform is actually implemented in RME based Dutch textbooks. Against this background, an analysis of multiplication of fractions is presented. The domain of fractions is the subject of a considerable amount of educational research. So-called “subconstructs” (e.g. Kieren, 1980, Streefland, 1991) have been widely accepted as fundamental for fraction learning.

For our analysis we selected 444 fragments from 4 major textbooks (grade 6) that involve fractions, of which 82 involved multiplication. We analyzed the use of contexts, models and types of tasks against the background of RME theory.

As expected, we found elements of RME in the textbooks. Multiplying fractions was firmly grounded in contextual problems which were experientially real for the students. Several subconstructs were addressed in the chosen contexts and different models of multiplication were used such as repeated addition, part of and area. However, multiplication of fractions remained strongly connected with informal strategies, contexts and models. More importantly, these contexts and models appear to be compartmentalized in four categories that depend on the type of numbers involved.

This is an important deviation from RME. It becomes very difficult for students to come to grips with formal fraction multiplication when the individual number-specific solution procedures students have to generalize over, are ingrained as independent, unrelated, solution procedures that do not transcend the contexts from which they originate. In 2012 we have started a research project based on these results in which the role of teacher in supporting students learning is further explored (NWO-PROO, project nr: 411-10-703).

References

- Freudenthal, H. (1968). Why to teach mathematics so as to be useful. *Journal of Research in Mathematics Education*, 16(1), 3-17
- Kieren, T.E. (1980). *Recent research on number learning*, chapter The rational number construct – its elements and mechanisms, pp 125-150. ERIC/SMEAC, Columbus.
- Streefland, L. (1991). *Fractions in Realistic Mathematics Education, a paradigm of developmental research*. Dordrecht: Kluwer Academic Publishers.

USING MULTIMEDIA CASES WITH PROSPECTIVE TEACHERS

Ana Paula Canavarro, Luís Menezes

Univ. of Évora and Research Unity/Institute of Education/Univ. of Lisbon, Portugal
Higher School of Education of Viseu and CI&DETS, Portugal

This paper deals with the use of multimedia cases focused on mathematics inquiry-based teaching on teacher initial training. Each case includes video episodes from different phases of the lesson, the lesson's plan, students' solutions of the task, teacher's explanations about her intentions and actions, and questions that promote reflection about key issues of the planning and conducting of the lesson. We took Brunvand's (2010) perspectives for the production of the video based materials by including multiple resources that allow the users to notice relevant content about teaching and to confront and widening their perspectives. As Alsawaie and Alghazo (2010) point, the videos provide a realistic and complete portray of the classroom and foster the analysis of challenging teaching practices. Our aim is to analyse how prospective teachers evaluate the use of the multimedia cases in their training and to identify the professional knowledge they developed due to that formative experience.

A class of 14 female prospective elementary teachers used one 4th grade multimedia case for one month, in 3 hours weekly lessons, exploring and responding in groups to the included questions. After that, each group planned and conducted a lesson following a mathematics inquiry-based approach. Finally, each prospective teacher wrote a reflective report about her learning and responded to an anonymous questionnaire concerning the case. The prospective teachers valued most: learning about a new way of teaching mathematics; knowing students' solutions that allowed them to raise their expectations about students' mathematical reasoning; and listening to the teacher's voice that allowed them to give meaning to her challenging teaching. So, the multimedia cases are powerful resources for initial teacher training.

This paper is supported by FCT – Fundação para a Ciência e Tecnologia - Project P3M - (contrato PTDC/CPE-CED/098931/2008).

References

- Alsawaie, O. N., & Alghazo, I. M. (2010). The effect of videobased approach on prospective teachers' ability to analyze mathematics teaching. *JMTE*, 13(3), 223-241.
- Brunvand, S. (2010). Best practices for producing video content for teacher education. *Contemporary Issues in Technology and Teacher Education*, 10(2), 247-256.

COMPARISON OF FRACTION DIVISION CONTEXT BETWEEN MATHEMATICS TEXTBOOKS IN TAIWAN AND USA

Shu-I Chang

National Taipei University of Education, Taiwan

Fraction division is a difficult concept for most of the students. Children tend to acquire only mechanistic procedures, like the procedure of “invert and multiply” (Okazaki & Koyama, 2005). As distinguished contexts may affect not only how a teacher delivers this topic, but also students' learning outcomes (Tarr et al., 2008), the purpose of this study is to compare the context of fraction division in two versions of textbooks, which are Hanlin (HL), a popular version of textbook in Taiwan, and Everyday Mathematics (EM) developed by Chicago school mathematics project. The textbook, workbook and teacher's handbook of HL as well as Student Reference Book, Student Math Journal and Teacher's Lesson Guide of EM are the study resources. Content analysis is employed as the study method.

Followings are the comparison results. First, fraction division in two versions both correspond to the indicators of sixth grade. It, however, is distributed in EM with the spiral mode at fifth and sixth grade and is with the block structure at sixth grade in HL. Second, the sequences of activities are also different. In HL, the activities are arranged in the following order- 'fraction divided by a whole number', 'a fraction divided by one with the same denominator', 'one whole number divided by a fraction', 'a fraction divided by one with different denominator (remainder is zero)', 'a fraction divided by one with different denominator (remainder isn't zero)'. However, in EM, the main activity at fifth grade is to introduce common denominator method and followed by the introduction of the algorithm, “invert and multiply”, at sixth grade. Third, the introduction methods of the algorithm, “invert and multiply”, are different. The HL version emphasizes the formulating process of the algorithm. In EM, students at sixth grade are presented that the result from common denominator method they learn at fifth grade is the same with it from 'invert and multiply'. Further studies are suggested to find which route is better to learn fraction division.

References

- Okazaki, M., & Koyama, M. (2005). Characteristics of 5th graders' logical development through learning division with decimals. *Educational Studies in Mathematics*, 60, 217–251.
- Tarr, J., Reys, R., Reys, B., Chavez, O., Shih, J., & Osterlind, S. (2008). The impact of middle-grades mathematics curricula and the classroom learning environment on student achievement. *Journal for Research in Mathematics Education*, 39(3), 247-280.

TAIWAN'S ELEMENTARY STUDENTS' BELIEFS ABOUT MATHEMATICS TEACHERS

Chia-Huang Chen, Hsiu-fei Lee

National Taitung University, National Taichung University of Education
Department of Special Education, Department of Mathematics Education

Effective learning in Mathematics is a cognitive, affective, and socio-cultural issue of significance. How do students effectively learn Mathematics and what values do they hold towards the subject? These questions are being actively explored in the Third Wave project. Cross-national researches (Bishop, 1996, 1999; Cai & Wang, 2010; Peng & Nyroos, 2010; Seah, 2010) have adopted socio-cultural perspectives to explore relevant values and further understand the learning of Mathematics. This study examined the differences between Aboriginal students in Eastern Taiwan and Han Chinese students in Western Taiwan in terms of effective learning strategies and values from their teachers. This study comprised two major objectives. First, we explored what values Aboriginal students in Eastern Taiwan and Han Chinese students in Western Taiwan hold toward the learning of Mathematics to understand their relevant emotions and beliefs. Second, we compared the views of effective learning strategies in Mathematics between different ethnic groups. The sample in this study comprised a total of 331 students (171 fifth graders and 160 sixth graders each from Eastern and Western Taiwan). Data was collected from structural questionnaires developed by the authors. Statistical analysis of variance was conducted on the questionnaire responses, which ranged from the functions of mathematics, mathematical methods, mathematical problem solving, how to learn mathematics effectively, and views towards mathematics in conjunction with socio-cultural factors such as the educational background and occupations of parents. From the data, we obtained the values of students towards the learning of mathematics, their affective responses towards mathematics, and the factors that influence the effective learning of mathematics. The results of this study facilitate the understanding of how students in Eastern and Western Taiwan view the learning of mathematics and can function as reference for the provision of appropriate teaching methods and strategies based on the characteristics and needs of students in different areas.

TEACHER' UNERSTANDING OF PROOF BY CONTRADICTION

Hyungmi Cho, Oh Nam Kwon, Hye Mi Oh
Seoul National University

Growing efforts are underway to make proof central to students' mathematical experiences across all grades. Achievement of this goal depends on teachers' knowledge of proof. This paper investigates secondary mathematics teachers' understanding of proof by contradiction into two levels; logical structure and the validity of proof.

If a statement S can be expressed as an implication $p \rightarrow q$ with proposition p and q , a proof by contradiction of S is direct proof of $p \wedge \sim q \rightarrow r \wedge \sim r$ where r is any proposition. The logical structure of proof by contradiction is difficult to understand. It may be challenging to assume that what is to be proved is false, and it is difficult for one's mind to follow the deductive steps when false hypotheses and contradictions are involved. Antonini and Mariotti (2008) analyse proof by contradiction. They call $S(p \rightarrow q)$ that one wants to show truth, principal statement, and another statement $S^*(p \wedge \sim q \rightarrow r \wedge \sim r)$ that one really proves, secondary statement. They indicate the validity of the statement $S^* \rightarrow S$ depends on the logical theory: modus ponens, contraposition, principal of contradiction, and principal of excluded middle.

In this study, we conducted interviews with 23 secondary school teachers. The interviews began with individual reflections on whether two given arguments (purported proofs) are true or not. As for proof in the interview process, this study asked teacher to evaluate two proofs.

We found that teachers had an invalid or limited viewpoint of the logical structure of the proof by contradiction. So they fail to evaluate the validity of the argument. Furthermore we can find seven types of the logical structure that teacher think the hypothesis and thesis about the secondary statement(S^*). This study can inform the knowledge required by mathematics teacher educators in order to effectively teach proof to in-service teachers. In the presentation, further results will be discussed in detail.

References

- Antonini, S., Maritotti, M. A. (2008). Indirect proof: with is specific to this way of proving? *ZDM Mathematics Education*, 40, 401-412.

EXPLORING HIGH-SCHOOL MATHEMATICS TEACHERS' EXTEMPORANEOUS AND IMPROMPTU SPECIALIZED CONTENT KNOWLEDGE: TWO CASES STUDIES

Yi-An Cho¹, Chien Chin², Pai-Tang Lin², Ting-Wei Chen²

¹Graduate Institute of Science Education, National Taiwan Normal University

²Department of Mathematics, National Taiwan Normal University

The relatedness of teachers' mathematical knowledge to high-quality instruction has long been discussed. While some hypothesize that the mathematical knowledge teachers need to know should be 'more' or 'deeper' than that known by students, Ball and their colleagues (e.g., Ball, Thames & Phelps, 2008) suggested that there is a special kind of mathematical work entailed in teaching which is qualitatively different from these hypotheses, and that they coined the term MKT for the mathematical knowledge needed to carry out the work of teaching mathematics, including responding to students' "why" questions, recognizing what is involved in using a particular representation, evaluating the plausibility of students' claims (often quickly) and so on. They identified six sub-domains of MKT, one of which is the specialized content knowledge (SCK), the mathematical knowledge not typically needed for purposes other than teaching. However, Ball and Bass (2000) thought that teaching is like a practice embedded with both regularities and endemic uncertainties, and that the sources of uncertainty in teaching are partially derived from its foundations: the impossibility of knowing definitely what students know or ask, and the necessarily incomplete nature of knowledge of teaching, and even the inherent indeterminacy of mathematical knowledge itself that is germane to a given instructional context. Therefore, this paper aims to concentrate on teachers' performances which are inclined to focus on dealing with students' questions, claims or confusion, and investigate how these questions, claims or confusion help teacher develop their SCK.

We used systematic classroom observations and the follow-up interviews to explore the extemporaneous, impromptu, and implicit aspects of the SCK. The study results properly describe the two teachers' SCK as well as its relationship with their knowledge of content and teaching (KCT) and knowledge of content and student (KCS), while they are facing students' questions, claims, ideas or confusion.

References

- Ball, D. L., & Bass H. (2000). Interweaving content and pedagogy in teaching and learning to teaching: Knowing and using mathematics. *Multiple perspectives on mathematics teaching and learning* (pp. 83-104). London: Ablex Publishing.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.

ITERATION AS THE COMMON METHODOLOGICAL TOOL FOR ACADEMIC AND CLASSROOM RESEARCH.

Bronislaw Czarnocha

Hostos Community College, CUNY, NYC

The poster is the report from the continuous search to identify methodological and theoretical frameworks in Mathematics Education which can be used as the common basis for the integration of educational research and teaching practice. The presented results are the work of teacher-researchers of the Bronx, NY employing the methodology of Teaching-Research NYCity Model (Czarnocha, 2002; Czarnocha and Prabhu 2006). The intensification of that search is dictated by realization on the part of the research community that *“it is [only] the teacher who can affect to the greatest extent the achievement of one of the main purposes of the research enterprise, that is, the improvement of students’ learning of mathematics”* (Kieran et al, 2013), and that *“to implement instruction that genuinely and effectively supports student construction of mathematical meaning and competence teachers must not only understand cognition-based research on students’ learning, they must also be able to use that knowledge to determine and monitor the development of their own students’ reasoning.”* (Batista, 2004). Other reports (e.g. Daro et al, 2011) confirm that to be successful in the process of learning improvement in mathematics classes, the teacher has to be able to integrate both the research result and the research methodology into classroom teaching; the teacher has to become the teacher-researcher.

The report identifies **Iteration** as one of the common methodological tools on the basis of which the required transformation can be accomplished; it discusses the difference Iteration plays in academic and classroom research, and, by restructuring the relationship between theory and practice, makes it possible to accomplish both goals: the introduction of research results and research methodology into classroom teaching.

The poster presents the iteration methodology in the development of the trajectory of linear equations in the class of integrated arithmetic and algebra at a community college in the Bronx, NYC with the help of Teaching-Research cycle (Czarnocha, Prabhu 2006), which in the presented case was composed of two semester long courses of the subject with different cohorts of students and the same instructor.

The proposed classroom teaching-research iteration starts at the identification of a common error and/or misconception in the classroom (here, the task: “Solve for y in terms of x ”), and ends in creation of the learning trajectory prototype after three subsequent iterations, the details of which (together with REFERENCES) will be presented in the poster.

CONSTRUCTING THE PROBLEM SOLVING CITIZEN

Jonas Dahl
Malmö University

Emphasising problem solving in the Swedish mathematics curriculum is (as in many other curricula) because it is seen both as an end in itself, but also as a means to “guarantee that all students could develop the general competences required [...] as citizens in an increasingly complex society” (Skolverket, 2011, p. 11). Therefore, I suggest that the goal of mathematics problem solving in schools is about creating, what I call, *the problem-solving citizen*. From this assumption, the question becomes one of how this goal is achieved. A crucial part of the question is what kinds of problems are featured in this construction. Roughly, problems can be divided into *pure mathematical* and *applied* ones. Drawing on Bernstein (2000) and Dowling (2005), I argue that for students to become problem solving citizens, they need to leave the horizontal (context-bounded, mundane) discourse, used in applied questions, and enter the vertical (context-free, esoteric) discourse of mathematics, used in pure mathematical questions.

Therefore, my research question is: In different upper secondary programs, how is the problem-solving citizen recontextualised into tests and students’ perceptions?

In the poster, I describe how different agents (curriculum makers, national test makers and teachers as test makers) make use of the different kinds of problems (pure/applied) in relation to the goal of producing problem solving citizens. I also report on a pilot study in which students were interviewed on their perceptions about problems and problem solving. My initial analysis shows that discussions of problem solving, as well as descriptions of the kinds of problems that are or should be used in policy documents are ambiguous. Applied problems seem more commonly used for teaching and assessing problem solving. The pilot study suggests that students can see the difference between pure and applied problems and recognise that this difference is important in regard to their future as problem-solving citizens.

References

- Bernstein, B. (2000). *Pedagogy, symbolic control and identity : Theory, research, critique* (Rev. ed ed.). Lanham, Md.: Rowman & Littlefield Publishers.
- Dowling, P. (2005). *The sociology of mathematics education :Mathematical myths pedagogic texts*. London: Routledge Falmer.
- Skolverket. (2011). Upper secondary school 2011. Retrieved 05/30, 2012, from <http://www.skolverket.se/publikationer?id=2801>

SELECTING AND ADAPTING MATHEMATICAL TASKS THAT FOSTER NUMBER SENSE DEVELOPMENT: THE PRACTICE OF ONE PRIMARY TEACHER

Catarina Delgado¹, Joana Brocardo¹, Hélia Oliveira²
Escola Superior de Educação, Instituto Politécnico de Setúbal¹
Instituto de Educação, Universidade de Lisboa²

Number sense development demands approaches to number and operations teaching quite different from the traditional one, based on the use of algorithms (Yang, 2003). Teachers must be able to help student to reason and to use appropriate computation methods (Yang, 2003). To achieve these objectives it is important that they reflect on the nature of the mathematical tasks they propose and on their potentialities to develop student's understanding (Stein et al., 2009), on the different strategies students can use, on the meaning they can give to the contexts and on the way students can mentally manipulate the numbers presented in the task (Fosnot & Dolk, 2001). This study aims to characterize the practices of two primary teachers in the selection and adaptation of tasks focused on developing students' number sense, in the context of one year long collaborative setting with the first author of this poster. Data were collected from four semi-structured interviews, along the 30 collaborative working sessions (teachers and researcher) and the teachers' classrooms (26 observed lessons).

In this poster we present aspects of Manuel's practice, one of the teachers. The findings indicate that he gradually developed a greater sensitivity to analyse students thinking and to relate it to the tasks' characteristics. Manuel's decisions regarding the adaptation and implementation of the tasks were strongly based on the choice of the numbers involved and the characteristics of the contexts that the students have explored. Finally, he explicitly began to prefer more open tasks that provided the possibility for students' use of various strategies.

References

- Fosnot, C., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division*. Portsmouth, NH: Heinemann.
- Stein, M. K., Smith, M., Henningsen, M., & Silver, E. (2009). *Implementing standards-based mathematics instruction*. Reston: NCTM.
- Yang, D. C. (2003). Teaching and learning number sense - an intervention study of fifth grade students in Taiwan. *International Journal of Science and Mathematics Education*, 1(1), 115-134.


FRACTIONS: AN EPISTEMOGRAPHICAL VIEW

Jean-Philippe Drouhard

CEFIEC, Universidad de Buenos Aires (Argentina)

The aim of the poster is to present how a model of knowledge organization called "epistemography" (Drouhard, 2010) can be used to better understand the nature of fractions when learned and taught. The model is based on three concepts : "epistemographical analysis of knowledge", "micro-paradigms" and "didactic bunches".

The **epistemographical analysis** consists in analyzing a given piece of knowledge (let us consider for instance the sum of two fractions) according to five dimensions:

- *Notional*: What is the sum of two fractions? How is it defined? (according to how fractions are defined)? What are its properties? How is it related to other operations (like subtractions) and so on?
- *Semiotic / linguistic*: How to represent the sum of two fraction? With a "+" sign as in $\frac{1}{2} + \frac{1}{4}$ or just by juxtaposition as in:  or in: $2\frac{1}{2}$? According to Duval (2000) conversions between registers of semiotic representations are a key to understanding (to access the notional knowledge, with our words).
- *Instrumental*: How to calculate the sum of two fractions?
- *"Nomological"* which is knowing the rules of the game: In some circumstances $\frac{3}{4}$ is a good answer to "the sum of $\frac{1}{2}$ and $\frac{1}{4}$ is?" while $\frac{6}{8}$ is not (neither $1\frac{5}{2}$).
- *Identification* knowledge, which allows to identify the "game" (characterized by its rules) and to decide for instance if an adequate answer is " $\frac{3}{4}$ " or "0.75".

A **micro-paradigm** is an epistemographically coherent set of objects (which means that they share the same registers of representation, they are defined the same way etc.). It is very important for learning to notice that "fractions" belong to two quite different micro-paradigms: the integer fractions like $\frac{3}{4}$ (the corresponding mathematical objects being rational numbers) and the generalized fractions like $\frac{1-i}{1+i}$.

A **didactic Bunch** is a set of micro-paradigms, belonging to different words, sharing some epistemographical components. Integer or generalized fractions, alongside with time and avoirdupois measures, and parts of a collection of isolated objects etc., are part of the same didactic bunch, which is the "fractions" didactic bunch.

References

- Drouhard, J-Ph. (2010). Epistemography and algebra. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.): *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education*. Lyon: INRP. pp. 479-486.
- Duval, R. (2000). Basic issues for research in mathematics education. In T. Nakahara & M. Koyama (Eds.), *Proceedings of PME 24*. Hiroshima University. pp. 1.55 – 1.69.

STRATEGIES OF PROBLEM SOLVING¹

Maria Madalena Dullius, Virginia Furlaneto
Centro Universitário UNIVATES

Mathematics is usually considered difficult by students and is a concern of teachers regarding to school performance. Our challenge is to improve the quality of education and to do this we must be clear about what we want students to learn and what and how to teach that this learning happens. Problem solving is emphasised by the National Curriculum Parameters as a starting point of mathematical activity, offering to the student the opportunity to mobilise knowledge and develop the ability to manage the information that is at their fingertips. Whereas the resolution of a problem implies in the understanding and in the presentation of answers using appropriate procedures, it should be noted that there are several ways to achieve the same result, ie, there are numerous strategies that the students can use in this process.

In developing this research, we got supported by the use of different strategies for solving problems, based on authors such as Cavalcanti (2001), Krulik and Reys (1997) and Pozo (1998). We highlighted as research issue to investigate "What are the different strategies that students use to solve problems and how these interfere in this process?". This research is predominantly qualitative and is characterised in a case study. Initially, 158 students solved a selection of eight problems. The resolutions were analysed under the perspective of different strategies that could be used in solving mathematical problems. We have detected the predominance of formal calculation with a low rate of success in the resolution of most problems. From this result we are developing actions that aim to improve the teaching and the learning of problem solving, exploring different strategies. We are conducting an ongoing education with teachers and experiencing a pedagogical intervention with students.

References

- Cavalcanti, C. (2001). Diferentes formas de resolver problemas. In K. S. Smole & M. I. Diniz (Eds.). *Ler, escrever e resolver problemas: Habilidades básicas para aprender matemática*. Porto Alegre: Artmed.
- Krulik, S., & Reys, R. E (1997). *A resolução de problemas na matemática escolar*. São Paulo: Atual.
- Pozo, J. I. (1998). *A solução de problemas: aprender a resolver, resolver para aprender*. Porto Alegre: Artmed.

¹ This work integrates the project Observatory of Education, which is being developed at the University Center Univates with the support of CAPES, the Brazilian government entity focused on the formation human resources.

CULTURE OF SOLVING PROBLEMS – A TOOL FOR EVALUATION OF CULTURE OF PUPIL SOLVING OF MATHEMATICAL PROBLEMS

Petr Eisenmann²⁾, Jarmila Novotná¹⁾, Jiří Příbyl²⁾

¹⁾ Charles University in Prague, ²⁾ J. E. Purkyně University in Ústí nad Labem

One of the concepts of the Theory of Didactical Situations is pupils' Culture of Problem Solving (CPS) (Brousseau and Novotná, 2008). In our conception, CPS consists of four components: Intelligence, Creativity, Reading comprehension, and Ability to use the existing knowledge (Břehovský et al., 2003).

The poster presents a tool for assessing CPS. The tool is made of three parts: **A) Psychological screening** (*Váňa's test of intelligence* (VIT), *Christensen-Guilford test of creativity* (CGT), *Test of the ability reading with comprehension* (RC)), **B) Testing the ability to use existing knowledge** (AUK), **C) Evaluation by the mathematics teacher** (was gained in interviews of the authors with them). Piloting of the constructed tool in two 8th grades of two lower secondary schools in the Czech Republic (38 pupils aged 14 – 15) confirmed that it is suitable for assessing individual pupil's CPS.

For each class, mutual dependencies between components were statistically tested with the following results: There is no significant link between VIT and AUK and between CGT and AUK; there is a significant link between RC and AUK: pupils with low RC show worse results in AUK. The fact that there are individual differences in the level achieved in VIT and in other components of CPS in one respondent confirms the hypothesis that children have certain skills and abilities that are not made use by the school. VIT gives results of mechanical work and drill, not of everything the children are able to do. Traditional teaching strategies are not sufficient to fight school stereotypes. We believe children can be "awaken" if teachers work with them in new ways.

Interviews with the teachers confirm that higher (or lower) performance in solving of mathematical problems is significantly correlated to better (or worse) pupil's evaluation in the following three components of CPS: VIT, RC, AUK.

Acknowledgement: This research was supported by the grant GACR P407/12/1939. The psychological survey was carried out by Z. Hadj-Moussová.

References

- Brousseau, G., and Novotná, J. (2008). La culture scolaire des problèmes de mathématiques. In Sarrazy, B. (Ed.), *Les didactiques et leurs rapports à l'enseignement et à la formation. Quel statut épistémologique de leurs modèles et de leurs résultats ?* [CD ROM]
- Břehovský, J., Eisenmann, P., Novotná, J., Ondrušová, J. and Příbyl, J. (2013). Heuristic strategies in problem solving of 11-12-year-old pupils. *Proceedings to SEMT 2013*, in print. Praha: UK-PedF.

GRADE 3 AND 4 STUDENTS' DIFFERENT WAYS OF DISCERNING MATHEMATICAL PATTERNS

Anna-Lena Ekdahl

School of Education and Communication, Jönköping University, Sweden

Students understanding of number sequences and visual patterns have previously been described in different reports. The problems encountered have been characterized as lack of different strategies for finding structures in various patterns (Hargreaves, Shorrocks-Taylor, Threlfall, 1998) and difficulties to describe the visual patterns (Warren & Cooper, 2007). However, there are only a few studies in the early grades which focus on what pattern in number sequences and visual patterns the students really discern. The aim of this study is therefore to describe young students' qualitatively different ways of discerning number sequences, and to identify features critical for them being able to generalize the patterns.

In a set of 50 students (grade 3 and 4) a pen-and-paper test on different number patterns were used to select the informants. Nine students with qualitatively different answers to the test were then selected for semi-structured video-recorded interviews. The interviews were verbatim transcribed and analysed in two stages. The analysis was made within the framework of phenomenography and variation theory (Marton & Booth, 1997). In this process, a variation in what aspects had been discerned and focused on emerged. Based on the results, the qualitative differences between the perceptions helped identify the aspects critical for discerning number patterns.

In brief, six qualitatively different categories could be discerned from the data. These are described in detail on the poster. As an example, one of the categories "Relationship between some parts" shows that one way the students find a pattern in a sequence is to focus on *some* of the numbers, or parts, but not the entire given pattern. In addition, critical feature of discerning patterns could be identified as differences between the different ways of discerning the patterns. These results have implications for what learners in a teaching situation need to be able to discern, in order to develop a more comprehensive and differentiated way to describe number patterns.

References

- Hargreaves, M., Shorrocks-Taylor, D. & Threlfall, J. (1998). Children's strategies with Number Patterns. *Educational Studies*, 24:3, 315-331.
- Marton, F. & Booth, S. (1997). *Learning and awareness*. Mahwah, N.J: Erlbaum.
- Warren, E., & Cooper, T. (2007). Generalizing the Pattern Rule for Visual Growth Patterns: Actions that Support 8 Year Olds' Thinking. *Educational Studies in Mathematics*, 67(2), 171-185.

WHAT IS CRITICAL WHEN EXPERIENCE RATIONAL NUMBERS WITH MEASURING TASKS

Helena Eriksson

Department of Mathematics and Science Education, Stockholm University

When students are going to meet the concept of rational numbers the approach of education is important for the students' possibilities to experience these numbers (Schmittau, 2003). Rational numbers can be seen as multifaceted (Davydov & TSvetkovich, 1991) and in relation to the introduction of these numbers lots of difficulties must be taken into account. The rationals are the first numbers students meet that consist of not just one number but a relation between two numbers (Davydov & TSvetkovich, 1991). In this study the activities used to introduce these numbers consist of measurement tasks inspired by Davydov and Tsvetkovich (1991). Activity theory is used as a lens to make it possible to challenge a usual arithmetic context in Swedish mathematical education. The tasks are tested and developed for the benefit of a more structural approach of education that develops abstract thinking (compare Schmittau, 2003).

The purpose of this study is to explore the learning object "experience rational numbers as numbers". The research question is "what critical aspects of these numbers can be found in a challenged context"?

The learning object is explored in two Learning Studies. The iterative model of Learning Study makes it possible to explore a specific learning object during a process of developing teaching (Marton & Booth, 1997). The study is conducted together with three teachers and ninety-four students in third and fourth grade in a Swedish public school. The students joining the study represent a diversity referencing to cultural origin, socio-economic conditions and special needs. Because of the heterogeneity of the student groups and the measurement tasks in a Swedish educational context, we can find more and new critical aspects for this learning object. The poster will show some of the critical aspects that have been found in the study.

References:

- Davydov, V., & Tsvetkovich, Z. (1991). On the Objective Origin of the Concept of Fractions. *Focus on Learning Problems in Mathematics*, 13 (1) 13-64.
- Marton, F., & Booth, S. (1997). *Learning and awareness*. New Jersey: Lawrence Erlbaum Associates.
- Schmittau, J. (2003). Cultural-Historical Theory and Mathematics Education. i I. Kozulin, B. Gindis, V. Ageyey, & M. Miller, *Vygotsky's educational theory in cultural context* (ss. 225-245). Cambridge: Cambridge University Press.

UNFOLDING THE PROPERTIES OF LOGARITHMS

Roger Fermsjö

Department of Mathematics and Science Education, Stockholm University

Using logarithms as a tool in mathematics is a difficult task for students, and a very common error is that the symbol ' \lg '¹ is discerned and treated as a factor, e.g. $\lg x + \lg 5 = \lg(x+5)$ (Liang & Wood, 2005). If mathematics is seen as a subject of arithmetical operations (see Van Oers, 2001), the cause of these errors and/or misconceptions may be due to the procedure-oriented teaching, and the fact that the special structure and meaning of logarithms is lost.

The aim of this study is to contribute to a deeper understanding of the concept of logarithms in the didactic field of mathematics; its meaning, how it is constructed and its features in relation to the barriers that might arise in the conceptual formation.

Variation Theory (see Marton & Pang, 2006) and Activity Theory (see Davydov, 2008) were used both for planning the intervention and for the data analysis. A so called "germ-cell" model for logarithms and different tasks were constructed, which gave the opportunity to explore the structure of logarithms and discern the original intent – to substitute multiplication with addition.

The samples were collected in one Learning Study² in spring -13, which comprise about 90 students in upper secondary school in Sweden. Collected data from tests, video and interviews can show if typical errors are still made, and if there are some persistent misconceptions. The data set can also reveal the meaning of the students' own actions in the learning activity.

The results from the study provide both a didactic and a methodological contribution, e.g. a greater understanding for critical aspects of the learning object, and further knowledge in the field of research dealing with misconceptions and errors.

References

- Davydov, V. V. (2008). *Problems of Developmental Instruction: A Theoretical and Experimental Psychological Study*. (P. Moxhay, Trans.) New York: Nova Science Publisher, Inc.
- Liang, & Wood. (2005). Working with logarithms: students' misconceptions and errors.
- Marton, F., & Pang, M. F. (2006). On Some Necessary Conditions of Learning. *The Journal Of The Learning Sciences*, 193–220.
- Oers Van, B. (2001). Educational Forms of Initiation in Mathematical Culture. *Educational Studies in Mathematics*, 46, pp. 59-85.

¹ There are different notations of logarithms so the ISO 31-11 standard is used in this text.

² See Marton and Pang (2006) for further explanations of Learning Study.

MATHEMATICIANS' DISPOSITIONS ON TEACHING UNDERGRADUATE MATHEMATICS USING SOFTWARE

Gregory D. Foley, Heba Bakr Khoshaim

Ohio University, Prince Sultan University, College for Women

Academic mathematicians' opinions vary regarding software use in undergraduate mathematics instruction. This study examined these opinions and underlying attitudes via interviews and a subsequent survey involving 50 randomly selected PhD-granting institutions in the United States. Most prior related work had ignored skeptics and critics. This investigation studied the full range of views. The research questions were

- What are academic mathematicians' dispositions toward software use in undergraduate mathematics instruction?
- What are the reasons underlying these dispositions?

An exploratory sequential research design built, expanded, and tested a model to explain mathematicians' dispositions. This model subsumed the Fishbein and Ajzen's (1975) attitude framework. We reviewed anecdotal evidence, published opinions, related theories, and research results to build an initial model and interview protocol. We interviewed 11 mathematicians to expand the model and develop a survey built on Fleener (1995). A survey of 183 mathematicians tested the factors in the expanded model. We triangulated the interview and survey data with the reviewed literature to identify the underlying factors for the use or nonuse of software.

We found that the average mathematician has a moderate, slightly positive attitude toward software use in teaching. Small numbers of mathematicians either strongly oppose (4.9%) or strongly support (17.5%) software use in undergraduate instruction. Most value the benefits of software but are concerned about its potential harm.

Among the 8 factors and 16 subfactors identified initially, the triangulation process suggested that software characteristics, perceived effect on learning, and instructor's personality are the three most influential factors. Among the remaining factors, students' level, students' major, instructor's educational background, and teaching background carry weight. There were inconsistent results with regard to institution and research interest as factors, which may warrant future investigation.

References

- Fleener, M. J. (1995). A survey of mathematics teachers' attitudes about calculators: The impact of philosophical orientation. *Journal of Computers in Mathematics and Science Teaching*, 14, 481–498.
- Fishbein, M., & Ajzen, I. (1975). *Belief, attitude, intention, and behavior: An introduction to theory and research*. Reading, MA: Addison-Wesley. Retrieved from <http://www.people.umass.edu/ajzen/f&a1975.html>

PERCEPTIONS ABOUT MATHEMATICS IN SCHOOL SUBJECTS

Andualem Tamiru Gebremichael

Universitet i Agder

This paper reports about Ethiopian students' perceptions of relevance of mathematics to other subjects. Relating mathematics to other subjects enhances students' interests and motivations (Michelsen & Sriraman, 2009). The research question that I attempt to answer is "how do students perceive the relevance of mathematics to other school subjects?" CHAT is employed as an analytic tool. Particularly, I used Engeström's triangle which models activity (Cole & Engeström, 1993) within which perception is mediated by tools, community, division of labor and rules. The students in this study participate in the activity of schooling with the motive of joining the university. In order to realize their activity, they take various actions such as studying school subjects. These actions are directed towards the goal of learning mathematics and other school subjects. Interviews and classroom observations were employed to collect pilot data, and the results were used as items in the questionnaire. The analytic process resulted in four characterizations of students' perceptions: 'I see mathematics in other subjects'; 'I use mathematics in other subjects'; 'I see preparatory mathematics in other subjects' and 'I use preparatory mathematics in other subjects'. The students strongly held these perceptions. Natural science students held more strongly than social science students. Students also compare the time cost that mathematics incurs at the expense of the other subjects (Cf. Eccles & Wigfield, 2002). This tension between mathematics and other subjects is important in understanding students' perceptions. As a mathematics teacher, I remember that we (teachers) perceived social science students as being disinterested in mathematics. Now, I question if we were inclusive of this student group in our teaching. This needs further investigation.

References

- Cole, M. & Engeström, Y. (1993). Cultural-historical approach to distributed cognition. In G. Salmon (Ed.), *Distributed cognitions: Education and educational considerations* (pp. 1-43) Cambridge, UK: Cambridge University Press.
- Eccles, J. S., & Wigfield, A. (2002). Motivational beliefs, values, and goals. *Annual Review of Psychology*, 53, 109-132.
- Michelsen, C., & Sriraman, B., (2009). Does interdisciplinary instruction raise students' interest in mathematics and the subjects of the natural sciences? *ZDM-Mathematics Education*, 41, 231-244

THE EFFECT OF NUMERICAL DISTANCE IN COMPARING FRACTIONS GOES BEYOND ARITHMETIC PROFICIENCY

David M. Gómez, Gonzalo Mena-Carrasco, Pablo Dartnell
University of Chile

Practitioners and researchers have observed that the usual writing of fractions impairs the learning of fractional concepts and procedures by focusing students on the whole numbers composing fractions rather than the numerical magnitude of the fractions themselves (e.g. Bonato, Fabbri, Umiltà, & Zorzi, 2007). Knowledge of fractions' numerical magnitude is crucial for a deep understanding of fractions (Siegler, Thompson, & Schneider, 2011).

Schneider and Siegler (2010) presented young adults with a fraction comparison test, whose results suggested that participants had access to the numerical magnitude of the presented fractions. These authors, however, did not address the role of arithmetic proficiency. In this work, we use stepwise linear regression to assess whether proficiency in mental arithmetic subsumes the effect of numerical distance between fractions in young adults' responses.

Thirty-three young adults studying non-mathematical disciplines answered a fraction comparison test and the arithmetic section of WAIS-III (Wechsler Adult Intelligence Scale). We considered response times as dependent variable, separating test items into different types. Chi-square tests showed that a model including both WAIS scores and numerical distance between fractions in each item performs significantly better than a model including only WAIS scores for all item types excepting when the fractions to be compared had the same numerator. This shows that numerical distance between fractions provides additional information beyond that of arithmetic proficiency. Items where fractions have the same numerator (e.g. comparing $\frac{1}{2}$ and $\frac{1}{3}$) may be treated differently by just comparing denominators, a strategy that eliminates the need to access numerical magnitude of the intervening fractions. These results stress the relevance of fostering students' knowledge of the numerical magnitude of fractions.

References

- Bonato, M., Fabbri, S., Umiltà, C., & Zorzi, M. (2007). The mental representation of fractions: Real or integer? *Journal of Experimental Psychology: Human Perception and Performance*, 33(6), 1410-1419.
- Schneider, M., & Siegler, R. S. (2010). Representations of the magnitudes of fractions. *Journal of Experimental Psychology: Human Perception and Performance*, 36(5), 1227-1238.
- Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, 62(4), 273-296.

ATTITUDES AND EXPERIENCES OF TEACHERS LEARNING WITH DYNAMIC GEOMETRY IN A COLLABORATIVE, DISCURSIVE, AND ONLINE ENVIRONMENT

Loretta Grisi-Dicker, Muteb Alqahtani, Arthur B. Powell
Rutgers University, Rutgers University, Rutgers University

The rapidity of social, scientific, and technological changes requires that mathematics teachers throughout their professional lives continue to learn new technologies, pedagogies, and content. From a sociocultural perspective, we view mathematics learning as a discursive, participatory process (Sfard, 2008) and believe that teachers gradually develop their Technological Pedagogical Content Knowledge by interacting discursively in small groups (Mishra & Koehler, 2006). In our study, 32 middle and high school teachers participated in an online 10-week course in which they interacted in small teams in an online collaborative environment, known as Virtual Math Teams with GeoGebra (Stahl, 2009), focusing on mathematical practices and discourse. Teachers engaged in asynchronous individual preparation and synchronous collaboration to solve open-ended mathematics problems and then reflected on the logs of their synchronous discursive interactions to identify successful and unsuccessful discourse moves.

Using conventional content analysis (Hsieh & Shannon, 2005) of data from pre- and post-course focus group interviews, we inquire into possible shifts in teachers' attitudes and experiences toward an instructional focus on collaboration, discourse, and dynamic mathematics. Our results show that from course activities teachers' attitudes and experiences shifted importantly. The collaborative structure of the course helped the teachers overcome difficulties learning dynamic geometry, and caused them to shift their attitudes positively towards introducing structured collaboration with their students. The teachers evolved to see discursive interactions as promoting group and whole-class learning. In sum, teachers integrated learning of technology, pedagogy, and mathematics through their active participation in collaborative and discursive problem solving.

Acknowledgements: This work is based upon research supported by the National Science Foundation, DRK-12 program, under award DRL-1118888. The findings and opinions reported are those of the authors and do not necessarily reflect the views of the funding agency.

References

- Hsieh, Hsiu-Fang, & Shannon, Sarah F. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research*, 15(9), 1277-1288.
- Mishra, Punya, & Koehler, Matthew J. (2006). Technological Pedagogical Content Knowledge: A Framework for Teacher Knowledge. *Teachers College Record*, 108(6), 1017-1054.
- Sfard, Anna. (2008). Thinking as communicating: Human development, the growth of discourses, and mathematizing. Cambridge: Cambridge.
- Stahl, Gerry. (Ed.). (2009). *Studying Virtual Math Teams*. New York: Springer.

CONSTRUCTING OBJECTS FOR TEACHER REFLECTION ON LEARNING MATHEMATICS

Cindy Hmelo-Silver, Carolyn A. Maher, Marjory F. Palius, Robert Sigley, Alice
Alston
Rutgers University

Video is an important resource for understanding how students learn and how teachers recognize the details of student learning. In the Video Mosaic Collaborative (VMC), we provide opportunities for teacher education through the use of a digital repository that offers resources and a tool for creating multimedia artefacts, the VMCAnalytics. VMCAnalytics are multimedia narratives that trace student learning of mathematical concepts and ways of reasoning. The VMC (videomosaic.org) builds on extensive prior longitudinal research that followed the same cohort of students doing mathematics from elementary through high school and beyond, video recording as they provided justification for their solutions to strands of problems. (Maher, 2005; Maher, Powell, & Uptegrove, 2010). This poster examines what VMCAnalytics show about teacher knowledge of students' mathematics learning.

Our study examines how the VMCAnalytic has been used to date and how users may have been influenced by courses or personal goals. We describe 27 multimedia artefacts that were created by participants from four courses or published on the VMC by researchers. Our poster shares information collected on the length of the VMCAnalytics, number of events, ratings of coherence, clarity, and depth in both mathematics education content and learning sciences lenses. These data are presented based on an analysis of VMCAnalytics by course context in which they were created.

Results suggest that users incorporated themes clustered around course goals, but also situated within broader cognition and instruction context. We found that teacher-learners in mathematics education courses engaged with issues of the complexity of learning mathematical content to include collaboration, discussion, building representations, and transfer within the broader settings in which learning occurs. Different users brought various lenses that offer the opportunity for creative insights into their observation of student learning. Future research will address how these factors influence the construction of VMCAnalytics as multimedia artefacts.

References

- Maher, C. A. (2005). How students structure their investigations and learn mathematics: Insights from a longitudinal study. *Journal of Mathematical Behavior*, 24(1-14).
- Maher, C. A., Powell, A. B., & Uptegrove. (2010). *Combinatorics and reasoning: Representing, justifying and building isomorphisms*. New York: Springer Publishers.

BELIEFS OF EARLY CHILDHOOD TEACHERS IN MATHEMATICS

C. Hüttel*, H. Wadepohl**, E. Rathgeb-Schnierer*

*Pädagogische Hochschule Weingarten; **Leibniz Universität Hannover

Due to the increasing importance of first learning processes in early childhood (Kucharz, 2012), the PRIMEL study¹ examine the quality of free-play and lessons in four domains (mathematics, natural science, art and physical education). Additionally, the study analyzes how the quality is influenced by teachers' attitudes, self-concepts and knowledge.

This paper is based on the mathematical perspective which investigates teachers' attitudes, self-concepts and knowledge in this domain. Therefore, studies show a positive connection between the way to teach and the teachers' beliefs (Charlesworth et al., 1993). The main research hypothesizes are that teachers with a high level of self-concepts and positive attitudes realize more domain specific interactions as well as teachers with a broad knowledge and a high level of self-concepts show more domain specific actions in free-play and lessons. Therefore, a questionnaire about teachers' attitudes regarding their own relation and the relevance of the domain as well as the self-concepts and knowledge has been developed. The sample consists of $N = 90$ early childhood teachers; $n = 30$ with a vocational training, $n = 30$ with a University degree in pedagogics (both in Germany) and $n = 30$ teachers with an Early Childhood Education University degree (in Switzerland).

Bivariate correlation analyzes of a subsample of $N = 55$ teachers show significant correlations between teachers' attitudes regarding the relevance of the domain and the self-concepts ($r = .537$, $p \leq .01$) as well as between the self-concepts and teachers' knowledge ($r = .570$, $p \leq .01$). Furthermore, the teachers' general high mean in a five-point Likert scale is noticeable ($\bar{x} = 4.10$; $s = .52$). Regarding two of the groups ($n = 31$ non-academic (D); $n = 16$ academic (CH)) no significant differences could be found in the four scales. We assume that further analyses can show differences.

The poster will be illustrating on the right side the theoretical framework and the subject as well as on the left side the methods and further results of the teachers' beliefs.

References

- Charlesworth, R., Hart C. H., Burts, D. C., Thomasson, R. H., Mosley, J. & Fleege, P. O. (1993). Measuring the Developmental Appropriateness of Kindergarten Teachers' Belief and Practices. *Early Childhood Research Quarterly* (8), 255–276.
- Kucharz, D. (Hrsg.). (2012). *Elementarbildung. Bachelor/Master*. Weinsheim und Basel: Beltz.

¹ A joint project with the Goethe University of Frankfurt, Leibniz University of Hanover, the University of Education Weingarten and the University of Koblenz-Landau in cooperation with the University of Teacher Education Schaffhausen (CH) and St. Gallen (CH). The project is funded within the "Ausweitung der Weiterbildungsinitiative Frühpädagogische Fachkräfte" (AWiFF) by the German Federal Ministry of Education and Research. Within AWiFF 16 research projects in the field of professionalization and qualification in preschool are sponsored by the German Federal Ministry of Education and Research.

DESIGNING A DANISH ‘MATH COUNSELOR’ PROGRAM

Uffe Thomas Jankvist
Roskilde University

This poster addresses how decades of mathematics education research results can inform practice by describing an in-progress developmental research project on designing and implementing an in-service upper secondary school teacher education program in Denmark. The program aims to educate a ‘task force’ of so-called math counselors; math teachers whose purpose it is to identify students with learning difficulties in mathematics, investigate the nature of their difficulties, and carry out interventions to assist the students in overcoming them. The program runs over 3 semesters (approx. 400 hours). Each semester comprises both a theory part and a practice part adhering to an overall semester theme: (1) concepts and concept formation in mathematics; (2) reasoning, proofs and proving in mathematics; (3) models and modeling in mathematics.

In each semester the teachers are introduced to a selection of relevant theoretical mathematics education literature focusing on constructs, frameworks and solid findings. In addition to this more general literature, the teachers are also provided with a list of specific literature on relevant issues, studies and findings related to the semester theme. The description of mathematical competencies by Niss & Højgaard (2011) makes up an overall framework for all three semesters. Our criteria for selecting both general and specific literature are that it should motivate to understand the learning problems which arise in the teachers’ everyday practice.

Through a series of research-based specially designed detection tests, the teachers identify students at their own schools, diagnose the student’s learning difficulties through interviews, etc., and design and implement interventions accordingly – which also sums up their future practice as math counselors and illustrates that the education program contains an element of learning by doing. (As for the students to receive math counseling, we aim at those who unsuccessfully tries hard to learn mathematics, and who really want to get help.) The teachers are asked to document and report everything, and at the end of each semester hand in a mini-project report applying theoretical constructs to their collected data and findings. The poster will display a selection of the teachers’ findings of students’ (conceptual) learning difficulties.

The responsible for the design and implementation of the program are Mogens Niss and Uffe Thomas Jankvist. The work is – as part of the STAR-project – supported by the European Social Fund through grant no. ESFK-09-0024.

References

Niss, M., & Højgaard T. (Eds.). (2011). Competencies and Mathematical Learning Ideas and inspiration for the development of mathematics teaching and learning in Denmark. English Edition, October 2011. *IMFUFU tekst* no. 485. Roskilde: Roskilde University.

STRUCTURE, LEVEL AND DEVELOPMENT OF PROFESSIONAL COMPETENCIES OF PRE-SCHOOL (KINDERGARTEN) TEACHERS IN THE FIELD OF MATHEMATICS

Lars Jenßen¹, Simone Dunekacke¹, Wibke Baack¹, Martina Tengler², Hartmut Wedekind², Marianne Grassmann¹, Sigrid Blömeke¹

¹ Humboldt-Universität zu Berlin, ² Alice Salomon University of Applied Sciences

The development of mathematical competencies of a child starts with its birth. As these competencies highly predict later school achievement (Krajewski & Schneider, 2009), its development should be supported by pre-school teachers. Therefore, pre-school teachers need general pedagogical (PCK), mathematical content (MCK) and pedagogical content knowledge (MPCK) with respect to Shulman (1986).

By analyzing formal pre-school curriculums, a requirement based and curricular valid model of competencies was developed, focusing on the three knowledge categories of Shulman and beliefs about mathematics and personal attitudes. In addition, a model of competence levels was developed by considering special literature. Competence levels are a heuristic for test construction and needed for describing different types of competent pre-school teachers.

Measuring competences is based on an achievement test. In addition, two questionnaires are committed. The first one captures opportunities to learn (OTL), the second one beliefs about mathematics and personal attitudes. In a cross-longitudinal-design the achievement test and the questionnaires will be deployed by two cohorts (beginning and end of pre-school teacher-training). The data will be analysed by confirmatory factor analysis with respect to the item response theory and the data's multi-level-structure.

We expect results with high variance of professional competences. Indicators therefore are, graduation needed for starting pre-school teacher-training is different and OTL during pre-school teacher training vary especially in mathematics. With respect to results of primary and secondary school teachers there is likely to be high correlation of MCK and MPCK. The discussion focuses on the question if pre-school teacher training prepares future pre-school teachers adequately to support the mathematical development of young children. Moreover, recommendations for future training of pre-school teachers will be reflected upon.

References

- Krajewski, K. & Schneider, W. (2009). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, 19, 513-526.
- Shulman, L.S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, 15(2), 4-14.

STRATEGIES FOR EVALUATING CLAIMS – AN ASPECT THAT LINKS CRITICAL THINKING AND STATISTICAL THINKING

Sebastian Kuntze*, Einav Aizikovitsh-Udi**, David Clarke***

* Ludwigsburg University of Education, Germany;

** Beit Berl Academic College, Israel; *** University of Melbourne, Australia

Basic skills in statistics are a prerequisite for participation of responsible citizens in the democratic decision-making process. As described by the notion of *statistical literacy* (e.g. Watson & Callingham, 2003), competencies in interpreting data, being able to deal with statistical variation as well as using statistical models in order to evaluate claims plays a core role. Evaluating claims is also crucial in the theoretical framework of critical thinking (CT, e.g. Ennis, 1998). In particular, the process of critically evaluating claims presupposes that claims are questioned actively, challenged by possible counter-evidence, and reviewed by corresponding reasoning and argumentation. This strongly suggests that CT and statistical thinking (ST) elements relevant for statistical literacy have an intersection domain from a theoretical point of view. Astonishingly, connections between CT and ST have hardly been examined empirically and the theoretical frameworks have followed separate paths, which raises the question as to how the connections between CT and ST can be characterised.

On the theoretical level, research on scientific reasoning and dealing with hypotheses (e.g. Kuhn, 1989; Bullock & Ziegler, 1994) can describe an important common aspect of CT and ST, namely the area of strategies for evaluating claims in both domains. The poster gives an overview of this theoretical background and confronts it with empirical findings (cf. Aizikovitsh-Udi et al., 2013), which suggest that this theoretical perspective affords the description of connections between CT and ST in an insightful way. Promoting CT and ST in mathematics classrooms will be greatly assisted by a theoretical framework that identifies and connects the constituent elements of both.

References

- Aizikovitsh-Udi, E., Kuntze, S. & Clarke, D. (2013). Exploring the Relationship between Statistical Thinking and Critical Thinking. [Paper presented at the annual conference of the American Education Research Association, San Francisco, USA, 2013].
- Bullock, M. & Ziegler, A. (1994). Scientific Thinking. In F. Weinert & W. Schneider (Eds.), *The Munich Longitudinal Study on the Genesis of Ind. Competencies*. Munich: MPI.
- Ennis, R. R. (1989). Critical thinking and subject specificity: Clarification and needed research. *Educational Researcher*, 18, 4–10.
- Kuhn, D. (1989). Children and Adults as Intuitive Scientists. *Psych. Review*, 96, 674 -689.
- Watson, J., & Callingham, R. (2003). Statistical literacy: A complex hierarchical construct. *Statistics Education Research Journal*, 2(2), 3-46.

MATHEMATICAL IDEAS ON CONSTRUCTING 3D DIGITAL FIGURES SHARED ON-LINE WITHIN A GROUP

Chronis Kynigos, Foteini Moustaki

Educational Technology Lab, School of Philosophy, Department of Pedagogy

The past years, mathematics educators have come to increasingly appreciate and implement in their classrooms group activities -instead of individual ones- also integrating the use of digital tools, such as microworlds (Kafai and Resnick, 1996). When, however, groups of students working together are not situated in the same classroom, communication becomes problematic. Computer supported collaborative learning (CSCL) environments may provide students an on-line virtual workspace in which they are able to exchange ideas, present the outcomes of their explorations or even share the digital artefacts they create (Mor et al., 2006). In this study, we attempt to highlight how the students' mathematical ideas on constructing 3d figures with a half-baked microworld (Kynigos, 2007) were shaped and influenced by the on-line discussions they had in such a CSCL system with the rest of the group members.

Half-baked microworlds (Kynigos, 2007) hold an interesting idea, but they are incomplete by design, challenging students to deconstruct them and make sense of the buggy behaviour. The "Twisted Rectangle" is a 3d Turtle Geometry microworld in which the 3d shape generated when running a Logo program appears to be a broken one. The shape's side lengths and angles are expressed in terms of variables. However, exploring the shape's geometrical properties and drawing projections on 2d planes, it becomes evident that -through the use of trigonometric functions- one of the side variables can be expressed in terms of the other side and angle variables.

In their joint attempts to repair the figure, four 10th grade students working in dyads, exchanged ideas through a CSCL system. The meaning generation processes in which they engaged with regard to constructing 3d figures seemed to be shaped both by their mathematical activity with the half-baked microworld and by their social activity as they discussed on-line. The mathematical ideas, which were made objects of discussion and reflection, fuelled students' explorations which resulted in extending the original ideas and providing new insights on how to implement them.

Acknowledgements

Metafora: "Learning to learn together: A visual language for social orchestration of educational activities". EC - FP7-ICT-2009-5, Technology-enhanced Learning, Project No. 257872.

References

- Kynigos, C. (2007). Half-Baked Logo microworlds as boundary objects in integrated design. *Informatics in Education*, 6(2), 335–358.
- Mor, Y., Noss, R., Hoyles, C., Kahn, K., & Simpson, G. (2006). Designing to see and share structure in number sequences. *Int. J. Technology in Mathematics Education*, 13(2), 65-78.
- Kafai, Y., & Resnick, M. (1996). *Constructionism in Practice: Designing, Thinking, and Learning in A Digital World*. Mahwah, NJ: Lawrence Erlbaum.

MULTIPLICATIVE THINKING IN RELATION TO MULTIPLICATION OF TWO TWO-DIGIT NUMBERS

Kerstin Larsson
Stockholm University

Multiplicative Thinking (MT) is distinctly different from Additive Thinking (AT) (Clark & Kamii, 1996) and to fully master MT is difficult for many students (Van Dooren, De Bock, & Verschaffel, 2010). To successfully multiply two two-digit numbers one needs to coordinate several aspects of multiplications such as recall of multiplication facts and the use of commutative and distributive properties. This study aims to examine relationships between calculations and MT among 5th grade students. It is a part of the author's PhD project, which is a longitudinal study following 27 students during two years, starting in grade 5. The research question for this part of the study is 'How do students, using AT and MT respectively, perform multiplication of two two-digit numbers?'

The students solved word problems which reflected different multiplicative situations; equal groups, arrays and multiplicative comparison, where especially multiplicative comparison can reveal if the students use AT or MT (e.g. Clark & Kamii, 1996). Based on the students' work on the text problems they were divided into three groups; students using only AT, only MT and students using both AT and MT. In an individual interview they also multiplied two two-digit numbers and explained how they carried out the computation. The calculation methods were compared between the three student groups.

There seem to be a correlation between the use of AT or MT and calculations with two two-digit numbers. The cumbersome method of repeated addition was found only among the students who exclusively used AT and the distributive property was never used by them. Another finding was that the same invalid method was used by students in all three groups. This method might have been influenced by the addition algorithm, since units were multiplied with units, tens with tens and no cross multiplication was done.

Acknowledgement: This research is financially supported by funds from The Royal Swedish Academy of Sciences.

References

- Clark, F. B., & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1-5. *Journal for Research in Mathematics Education*, 27(1), 41-51.
- Van Dooren, W., De Bock, D., & Verschaffel, L. (2010). From addition to multiplication ... and back: The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction*, 28(3), 360-381.

RELATIONSHIPS IN MATHEMATICS TEACHER EDUCATION IN GERMANY AND TAIWAN – THE ROLE OF INDIVIDUAL AND INSTITUTIONAL CHARACTERISTICS

Christin Laschke, Gabriele Kaiser

Humboldt University of Berlin, University of Hamburg

The results of the “Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)” replicated for future mathematics teachers on the country level the findings of large-scale assessments of K-12 student achievement, i.e. Taiwanese teachers outperformed their German counterparts (Blömeke & Delaney, 2012). The question is which individual aspects and program characteristics affected the future teachers’ knowledge at the end of their training, whether differences in the relationship between preconditions and institutional conditions and outcomes existed between countries and – if so – whether these differences point to cultural differences. Cultural theories with respect to the Chinese Confucian heritage and the classical Latin/Greek/Christian tradition including their respective educational traditions serve as a frame of reference to explore the latter research question.

Two representative samples of 771 future lower-secondary mathematics teachers from 13 federal states in Germany and 365 future teachers from 19 training units in Taiwan in their final year of teacher education were used to examine the effects of individual and institutional characteristics on mathematical content knowledge (MCK), mathematics pedagogical content knowledge (MPCK) and general pedagogical knowledge (GPK). The teachers’ demographic background, their prior knowledge, affective-motivational characteristics as well as family and money related constraints during teacher training were used as predictors on the individual level. While controlling for individual aspects the role of opportunities to learn (OTL), teaching methods and entry selection were examined. The analyses were carried out with HLM.

Whereas the Taiwanese teacher-education system is apparently able to develop high achievers widely independent from their preconditions, the knowledge of German teachers is significantly affected by these. On the institutional level especially the OTL had a strong direct influence on future teachers’ knowledge in Germany as well as in Taiwan. In addition to this model the poster will display precise empirical results and their link to cultural context.

References

Blömeke, S. & Delaney, S. (2012). Assessment of teacher knowledge across countries: A review of the state of research. *ZDM – The International Journal on Mathematics Education*, 44 (3), 223-247.

THE INFLUENCE OF MULTIPLE SOLUTIONS ON MATHEMATICS TEACHING

Shin-Yi Lee

Taipei Municipal University of Education, Taiwan

The National Council of Teachers of Mathematics (NCTM, 2000) indicated that students should develop their “flexibility in exploring mathematical ideas and trying alternative solution paths” (p. 21). Literature indicated that generating multiple solutions in mathematical problem solving provides an opportunity for students to stretch their thinking and develop their creativity (Cai & Kenney, 2000; Krulik & Rudnick, 1994). Lee (2011) found that students who performed better in multiple solutions tended to improve more on their mathematical problem solving performance. Very little, however, was known about the influence of multiple solutions on mathematics teaching (Silver, Ghouseini, Gosen, Charalambous, & Strawhun, 2005).

The purpose of this study was to explore how students’ multiple solutions in mathematical problem solving could influence a mathematics teacher’s teaching. In the study, an eighth-grade class were asked to solve a mathematical problem in multiple solutions. Students were instructed to write down all work and not to erase or black out anything they had written. There were no time limits for the students to complete their solutions. Data for this study were comprised of the students’ multiple solutions to the problem and the teacher’s reflection journals on the students’ multiple solutions. To analyze the data, the student’s solutions and the teacher’s reflection journals were analyzed separately and collectively to identify correspondences. The findings indicated that the students often used what the teacher taught in their initial solutions and revealed their deficiency in their alternative solutions. The results of this study suggested that students’ multiple solutions could assist their teacher in identifying their misconceptions or learning difficulties in school mathematics.

References

- Cai, J., & Kenney, P. (2000). Fostering mathematical thinking through multiple solutions. *Mathematics Teaching in the Middle School*, 5(8), 534-539.
- Krulik, S., & Rudnick, J. A. (1994). Reflect ...for problem solving and reasoning. *The Arithmetic Teacher*, 41(6), 334-338.
- Lee, S. Y. (2011). The effect of alternative solutions on problem solving performance. *International Journal for Mathematics Teaching and Learning*, September 21st.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Silver, E. A., Ghouseini, H., Gosen, D., Charalambous, C., & Strawhun, B. T. F. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *Journal of Mathematical Behavior*, 24, 287-301.

INTEREST AND SELF-DETERMINATION IN THE TRANSITION TO STUDYING MATHEMATICS

Michael Liebendörfer, Reinhard Hochmuth
Leuphana Universität Lüneburg

Our study focusses on interest in mathematics and its development in the first academic year. The secondary-tertiary transition has a wide range of problems, see e.g. (Gueudet, 2008). Specifically in Germany, drop-out rates in the first year are around 40 %. We use self-determination theory (SDT) by (Deci & Ryan, 2000) and its connection to interest theory given by (Krapp, 2005). We work out whether and how the basic needs postulated by SDT (competence, autonomy, social relatedness) are satisfied and relate them to students' interest by applying different interest theories. For instance, we investigate relations between interest in school mathematics, university mathematics and studying. For this purpose, we analyse interview data of 14 students from a bachelor and secondary school teacher's course. They volunteered for two interviews, one in week 3/4, the other one in the 2nd half of the semester. The interviews, taped and transcribed, were analysed by qualitative content analysis.

First findings show that the basic needs can be experienced very differently, depending on the individual's beliefs and orientations. We often found negative competence and autonomy experiences, whereas relatedness is less problematic. The former is mainly due to compulsory weekly homework assignments, where students apply their expectations from school homework, although now even understanding the task sheet is challenging for many of them. Not knowing appropriate learning strategies, they can hardly act autonomously and don't feel competent. The latter is additionally affected by an inverse big-fish-little-pond effect. Social relatedness is ensured by forming learning groups. Although still struggling, students feel slightly better after some weeks, but individual interest couldn't emerge so far. However, situational interest sometimes arose and positive experiences of basic needs were happily emphasized in students' reports.

The poster is organised in boxes for the different kinds of interest and a central matrix, where the columns are formed by the basic needs of SDT and the rows are given by theoretical and empirical descriptions.

References

- Deci, E. L., & Ryan, R. M. (2000). The "what" and "why" of goal pursuits: Human needs and the self-determination of behavior. *Psychological inquiry*, 11(4), 227–268.
- Gueudet, G. (2008). Investigating the secondary–tertiary transition. *Educational Studies in Mathematics*, 67(3), 237–254. doi:10.1007/s10649-007-9100-6
- Krapp, A. (2005). Basic needs and the development of interest and intrinsic motivational orientations. *Learning and Instruction*, 15(5), 381–395. doi:10.1016/j.learninstruc.2005.07.007

THE LATENT CLASS ANALYSIS OF SENIOR HIGH SCHOOL STUDENTS' STATISTICAL INTERESTS AND VALUES

Su-Wei Lin, Wen-Chun Tai
National University of Tainan, Taiwan

Statistics is a critical tool for young people as they confront issues and challenges at the personal, occupational, societal, and scientific levels. It shows that knowledge of statistical is quite valuable to serve the needs of citizens and lifelong learning. The ability to interpret and critically evaluate messages that contain statistical elements, termed statistical literacy, is paramount in our information rich society (Gal, 2003). Besides, of all the psychosocial factors, interest in statistics and value of statics are considered to be two of the best predictors of achievement in an educational context. The main purpose of this study is to identify the subgroups of high school students based on their interest in statistics and value of statics, and investigate the students' statistical literacy among these subgroups. Latent class analysis (LCA) identifies unobservable subgroups within a population. We conducted LCA to understand the impact of patterns of multiple affective variables, as well as the antecedents and consequences of complex behaviours, so that interventions can be tailored to target the subgroups that will benefit most. Around 1268 10th to 12th grade students participated in this study. The research tools included the scale of interest in statistics and value of statics. The internal consistency (reliability) of the total scale was extremely high (Cronbach alpha = .89) confirming that the items of the scale are clearly measuring the same construct. The internal consistency coefficients for the subscale scores were also adequate, being .91 for interest in statistics, and .80 for value of statics. Three latent classes were identified: low interest and low value (class 1: 28.4%), low interest and high value (class 2: 34.5%), and high interest and high value (class 3: 37.1%). The students identified as class 2 have the highest statistical literacy, followed by the class 3 subgroup, and the statistical literacy of class 1 subgroup was lowest. The results of this study provide important information for statistics educators about both the achievement-related and attitude-related outcomes of schooling. By combining information from the assessment of statistical literacy and the survey of interests and value that predispose students to use their statistical literacy, a more complete picture emerges. With this information, we can develop the suggestions for interventions that target the neediest individuals.

References

Gal, I. (2003). Teaching for statistical literacy and services of statistics agencies. *The American Statistician*, 57(2), 80-84.

OPPORTUNITY TO LEARN: A STUDY FROM TAIWAN TIMSS 2007 SURVEY OF MATHEMATICS LEARNING ACTIVITIES

Su-Wei Lin, Jeng-Shin Wu
National University of Tainan, Taiwan

The opportunity to learn (OTL) concept was first introduced by International Association for the Evaluation of Educational Achievement (IEA) as a mean to ensure the validity of cross-national comparisons of student mathematics achievement. As the name indicated the construct, OTL was conceptualized to what is taking place in school and classrooms to support student' learning and progress. The well-known definition of OTL is given by Husen (1967) as "whether or not ... students have had the opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test". Opportunity to lean can be used to improve content validity, explain variation of countries and to be the references for education revolution. The measurement of opportunity is mainly based on teacher survey. However, the opportunity to learn should be separated from teacher judgments of student learning because learning something is not the same thing as having an opportunity to learn (Leimu, 1992). Thus, the purpose of this study is focus on constructing opportunity to learn based on mathematics learning activities both from student and teacher questionnaires in TIMSS 2007. Based on Taiwan TIMSS 2007 data, we use exploratory factor analysis to select OTL variables and conduct the correlation between OTL and achievement measures for Grade 8. The results of exploratory factor analyses extract 3 factors from 17 items about OTL in TIMSS 2007 student questionnaire, named "Mathematics content knowledge instruction", "the type of problem solving in usual", and "Using mathematical tools and Cooperative learning". Among these three factors, the first two factors are positive related to students' achievement, but the third factor is negative. The results of this study also show that it is more appropriate to apply mathematics learning activities from student self-report to construct opportunity to learn indices. Moreover, these indices can explain large proportion of between school variations and within schools variations as well as can be very independent variables to predict student achievements.

References

- Husen, T. (Ed.). (1967). *International study of achievement in mathematics: A comparison of twelve countries*. I. Stockholm: Almqvist & Wiksell; New York: John Wiley & Sons.
- Leimu, K. (1992). Explorations in opportunity-to-learn. Finnish national analyses of IEA/SISS data. *International Journal of Educational Research*, 17, 291-317.

ACCESS TO PROFESSIONAL DEVELOPMENT OPPORTUNITIES FOR MATHEMATICS TEACHERS IN RURAL USA

Agida G. Manizade, Laura Jacobsen
Radford University, Radford University

In this paper, we report preliminary results of a professional development program designed to meet the needs of 37 secondary mathematics teachers from 33 school divisions in the southeastern part of the United States. Participating teachers previously had limited access to quality professional development opportunities and were from different parts of the state. They participated in the synchronous distance learning/on-line professional development program of six to ten credit hours of graduate coursework. During this study, researchers measured: (1) gains in teacher content knowledge, (2) impact on student achievement, and (3) progress towards meeting the assessed needs of participating school divisions and private schools. The researchers developed and administered rigorous measures of teachers' knowledge, analyzed assessment data of student achievement, and conducted surveys and classroom observations during this project.

We are reporting on our preliminary data. Pre and post-tests were developed & implemented at the beginning of the course & at the end. Tests were scored based upon percent correct. Results were analyzed via paired sample t-tests to determine if any significant improvements were made in teachers' knowledge after participating in these courses. A 5-item on-line survey was developed for superintendents who had teachers enrolled in the professional development project. Based on the survey results, superintendents were satisfied with the professional development program. An 18-item on-line quantitative survey was developed to get feedback from teachers in the professional development program. Teachers' scores significantly increased between the beginning and the end of the courses demonstrating that the goal of having teachers' knowledge increase was met. In addition, based on the self-reported data, teachers reported positive experiences with the distance education professional development program. The data collection and analysis on this project is on-going and the complete report will be provided at the PME presentation along with copies of all surveys and selected assessment instruments developed in the project.

References

Matsumura, L. C., Slater, S. C., Wolf, M. K., Crosson, A., Levinson, A., Peterson, M., et al. (2006). Using the Instructional Quality Assessment toolkit to investigate the quality of reading comprehension assignments and student work. (CSE Report 669). Los Angeles: National Center for Research on Evaluation, Standards, and Student Testing and Center for the Study of Evaluation.

PROGRAM FOR CONTINUOUS TRAINING IN MATHEMATICS: JUSTIFYING ITS EFFICACY

Cristina Martins¹, Leonor Santos²

¹Escola Superior de Educação, Instituto Politécnico de Bragança; ²Instituto de Educação, Universidade de Lisboa

Mathematics teacher education is an emerging field of study within mathematics education (Sztajn, 2011). It is necessary to prepare Mathematics teachers and to provide them, as they progress in their career, with opportunities to build their knowledge about mathematics and about pedagogy, in an environment which encourages them to take risks and to reflect (Sowder, 2007). This poster is based in a Ph. D. Thesis on Education by the first author (Martins, 2011), which had as main goal to study the Professional development of 1st cycle teachers through their participation in a Program for Continuous Training in Mathematics for 1st Cycle Teachers (PFCM).

Following a methodological approach interpretative in nature, with a case study design, semi-structured interviews were conducted with the three case teachers, classes and work sessions were observed, and documental data was gathered, specifically concerning the portfolios built within the scope of the training program. Categories for analysis were constructed within the theoretical framework and, afterwards adjusted or complemented on the basis of information stemming from the analysis itself.

From Aida's participation we highlight a new vision about Mathematics, from Dora's the improvement of her relationship with Mathematics, and from Sara's the relevance she started bestowing on communication within the Mathematics class. In this poster we wish to discuss these differences. The starting point of these three teachers was different, as were their expectations regarding PFCM. The characteristics of PFCM were, in the teachers' words, capital to their Professional development, namely concerning: (i) Rotation between sessions; (ii) Role of the teacher in the sessions; (iii) Tasks undertaken; and (iv) Reflection allowed. We thus wish to offer some recommendations for the organization of professional development programs for teachers of Mathematics.

References

- Martins, C. (2011). Professional development of 1st cycle of basic education teachers: Contributions from participation in a program for continuous training in Mathematics (Ph. D. Thesis). Lisboa: Instituto de Educação da Universidade de Lisboa.
- Sowder, J. T. (2007). The mathematical education and development of teachers. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 157-223). Charlotte: Information Age Publishing.
- Sztajn, P. (2011). Standards for reporting mathematics professional development in research studies. *JRME*, 42(3), 220-239.

EVOLUTION OF STUDENTS' MATHEMATICAL PROCEDURES IN DIVISION TASKS

Fátima Mendes^{*+}, Joana Brocardo^{*+}

^{*}Escola Superior de Educação, Instituto Politécnico de Setúbal

⁺Unidade de Investigação do Instituto de Educação, Universidade de Lisboa

A key aspect of learning division is working both on multiplication and division facts, helping to develop students' awareness of the relations between these two operations (Anghileri, 2003). Some authors also argue that students do not find division much different from multiplication and that they are naturally able to adapt their multiplication procedures to the division contexts without much difficulty (Ambrose, Baek & Carpenter, 2003).

The aim of this research is to understand how the 23 students of one third grade class learn multiplication (and division), having as background the idea of hypothetical learning trajectories (Simon, 1995). The report's first author conducted the research in the context of a teaching experiment carried out, in the classroom, during eight months. The researcher together with the classroom teacher developed sequences of multiplication and division tasks, which were implemented by the teacher, in the classroom. To highlight the relationship between multiplication and division operations was one of the key ideas that supported the development of the hypothetical learning trajectory.

The data in the study includes field notes, transcript of videotaped classroom episodes and students' written work. A content analysis was used for data analysis.

The results reveal that students' use a diversity of procedures and relate their procedures with the tasks' characteristics. The results also stress the importance of relating multiplication and division to solve with success the division tasks. Finally, it was clear that some everyday life contexts help to develop appropriate division procedures, namely enhancing the use of arrays by the students.

References

- Ambrose, R., Baek, J. M., & Carpenter, T. P. (2003). Children's invention of multidigit multiplication and division algorithms. In A. J. Baroody, & A. Dowker (Eds.), *The development of arithmetic concepts and skills* (pp. 305-336). Mahwah, NJ: Erlbaum.
- Anghileri, J. (2003). Issues in teaching multiplication and division. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools* (pp. 184-194). Buckingham: Open University Press.
- Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145.

A TRAJECTORY TO DEVELOP GRADE 4 STUDENTS' ALGEBRAIC THINKING¹

Célia Mestre

Unidade de Investigação do Instituto de Educação, Universidade de Lisboa

In the recent Portuguese mathematics curriculum, algebraic thinking is considered one fundamental theme in mathematics and it is recommended that it should start to be promoted in elementary school. Assuming generalisation as one central process and considering generalised arithmetic and functional thinking as two key domains of algebraic thinking, a developmental study was conducted in a grade 4 class with the principal goal of fostering students' algebraic thinking. Instead of introducing a new theme, the exploration of algebraic thinking was led through a curriculum integration perspective, favouring the connection between arithmetic and algebra. Thereby, the study was conducted in order to enable "students to work with several layers of awareness of generality in all areas of their mathematics curriculum prior to any formal introduction to algebra" (Britt & Irwin, 2011, p. 153), through the exploration of mathematical tasks which allow the use of arithmetic to develop and to express generalizations and the exploration of numerical and figurative sequences to describe functional relationships.

The study was conducted through the implementation of one year-long teaching experiment, adopting a research design perspective (Gravemeijer & Cobb, 2006). Data were collected from the lessons observation (with video recording), students' written records and some clinical interviews with the students. Analysis of the data reflects some particular characteristics of the trajectory that contributed to students' development of arithmetic and algebraic ways of thinking. Those characteristics were: bringing out the generalised character of arithmetic, exploring relational thinking in different situations, and using the work with regularities and sequences to promote the emergence of students' functional thinking.

References

- Britt, M. S. & Irwin, K. C. (2011). Algebraic thinking with or without algebraic representation: A pathway for learning. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 137-159). New York, NY: Springer.
- Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational Design Research* (pp. 45-85). London: Routledge.

¹ This poster is supported by national funds through FCT - Fundação para a Ciência e Tecnologia - in the frame of the Project Professional Practices of Mathematics Teachers (contract PTDC/CPE-CED/098931/2008).

TEACHER EDUCATION: SUPPORTING STUDENTS IN THEIR FIRST YEAR OF STUDIES IN MATHEMATICS

Kathrin M. Nagel, Florian Quiring, Jana T. Beitlich, Kristina Reiss
Technische Universität München

The transition from school to university mathematics is considered a challenge for students. Failure often causes frustration and dropout. But we need motivated students. Only when students are motivated, they can become excellent in their studies. Excellent graduates will likely also motivate their students' interest. And those interested students will likely become motivated students at university themselves. Achieving such a positive feedback loop is one of the goals of the TUM School of Education. Here, we will present two methods (one of which we present in detail) on how to overcome the gap between school and university by supporting students in their first year of studies in mathematics. This way, there will be a better chance that interested school students will turn into motivated university students.

One method is to offer a two-week transition course ("Brückenkurs") preceding the first semester. The aim of this course is to introduce basic contents, e.g. complex numbers, and fundamental mathematical methods, e.g. proof. Mathematical reasoning is essential for understanding undergraduate courses in mathematics successfully. Because students often do not learn how to do proofs in mathematics at school, we concentrate at this topic (Heinze & Reiss, 2007).

Second, we support our students during the first two semesters through additional recitation sections complementing the basic lectures in Linear Algebra and Calculus. Mathematics in school is normally taught in a rather concrete way. In contrast, students at university have to deal with the generalization and abstraction of mathematical contents. Our goal is to foster students' abilities for mathematical communication and problem solving by presenting and explaining mathematical topics to each other. Furthermore, we point out connections between school and university mathematics and inspire students to adapt their existing concept images to the formal definitions given in the lectures (Vinner, 1991). To evaluate the recitation sections, students ($n = 67$) completed a questionnaire. 85.7% of the students indicated that they were satisfied with the recitation sections. 80.1% believed these sections to be useful for their further studies. Due to these positive ratings we plan to transfer our concept to other mathematics courses.

References

- Heinze, A. & Reiss, K. (2007). Reasoning and proof in the mathematics classroom. *Analysis*, 27, 333-357.
- Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 65-81). Dordrecht: Kluwer.

ASSESSING GRADE 2 STUDENTS' UNDERSTANDING OF THE NUMBERLINE: THE ISSUE OF THE ITEM FORMAT

Guri A. Nortvedt, Andreas Pettersen
University of Oslo

In 2008, Norway introduced mapping tests to help Grade 2 teachers to identify students with weak conceptual understanding of numbers and lack of procedural fluency in counting, addition and subtraction (Throndsen & Turmo, 2012). The test was developed to identify the weakest 20%, including students with mathematics learning difficulties (Nortvedt, 2012). Tests for Grades 1 and 3 with similar aims have subsequently been developed. A key aspect of the test concept is students' understanding and use of the number line, including knowledge of the number sequence, counting of concrete objects or structured groups of objects and placing the correct number on the number line, and using an unlabelled line. At the moment, we are developing the second generation of screening tests. Tests should have a ceiling effect by design, and they should discriminate more efficiently and securely between students just above and below the 20% cut-off score (the two lowest quintiles). Consequently, the item format plays a crucial role; the size of the numbers, the labelling of the number line, and the grouping of the objects influence the difficulty level of the item considerably. The poster will present examples of item formats that discriminate well between students in the two lower quintiles in addition to items that discriminate better between other quintile groups. All items have been piloted and the results from the item analyses (classical test theory and item response theory) will be presented. Preliminary analysis demonstrates that the first quintile group can confidently place small numbers (< 20) on the number line. They need more time than other students to count up and down the number line, especially when only end points are labelled. When attempting to identify the value (structured groups, for instance groups of coins with values of 1, 5 and 10) and tie this amount to the number line, they might struggle both to count and to identify the correct number on the number line. It may be assumed that students start counting from 0 on the number line when solving these items, rather than using the identified numbers. Consequences for item development as well as teaching activities will be discussed.

References

- Nortvedt, G. A. (2012). Attempting equal opportunities to learn - Norwegian experiences from using national mapping tests in primary school. In T. Y. Tso (Ed.), *Opportunities to learn in mathematics education. Proceedings of the 36th Conference of the International Group of the Psychology of Mathematics Education* (Vol. 4, pp. 1-259). Taipei, Taiwan: PME.
- Throndsen, I. S., & Turmo, A. (2012). Primary mathematics teachers' goal orientations and student achievement. *Instructional Science*, 41(2), pp 307-322.

ELEMENTARY AND SECONDARY SCHOOL TEACHERS' PERSPECTIVES OF EFFECTIVE MATHEMATICS TEACHING

Jeongsuk Pang, Misun Kwon

Korea National University of Education, Yangji Elementary School

High-quality mathematics instruction is necessary to enhance students' mathematical learning. The teacher is one of the most important factors in good mathematics instruction. A main teaching method and its concomitant result in students' learning depend on how the teacher thinks of effective mathematics instruction. In this respect, teachers' perspectives on what constitutes good mathematics teaching are important. This paper explores Korean teachers' perspectives of effective mathematics teaching.

This study compares and contrasts the perspectives of effective mathematics teaching by 135 elementary school teachers, 132 middle school ones, and 124 high school ones using a questionnaire. The questionnaire consisted of two parts. Part I asked teachers to describe both aspects they regarded as important to good mathematics lessons and aspects which they thought led to not-good lessons, along with reasons. Part II then asked teachers to check how much they might agree the 48 items related to good mathematics teaching in terms of 5 Likert scales. The items were categorized into 4 main domains and 7 sub-domains. The questionnaire additionally asked teachers to prioritize the domains to examine the importance placed on each domain.

An analysis of Part I showed that all groups of teachers thought in common that enhancing students' self-directed learning is good mathematics teaching. Mainly elementary school teachers thought that using concrete materials and real-life contexts are important, whereas secondary school counterparts gave priority to students' motivation and engagement. An analysis of Part II showed that the domain of teaching and learning was selected as the most important by all groups of teachers. Elementary school teachers tended to agree more upon the 48 items than their counterparts did. Statistically significant differences appeared with regard to 11 items between elementary and middle-school teachers, 39 items between elementary and high-school teachers, and 29 items between middle-school and high-school teachers. For instance, teaching to improve problem-solving ability and using concrete materials were agreed upon more by elementary school teachers than by their secondary school counterparts.

Along with brief background information for this study, the poster will document (a) teachers' choices among domains of good mathematics teaching, (b) average scores of teachers' perspectives of 48 items, and (c) comparative analysis according to the groups of teachers. The similarities and differences among the groups of teachers in this study are expected to stimulate lively discussion of what constitutes high-quality mathematics instruction within and across various educational contexts.

KINDERGARTEN TEACHERS' BELIEFS ABOUT MATHEMATICS EDUCATION IN PRESCHOOL

Rubina Pereira, Gloria Ramalho
ISPA, IU

This research analyses beliefs and attitudes towards mathematics of 70 Portuguese preschool teachers who worked with children 4 to 5 years old in the school year 2011/2012.

Much research has been done on the importance of teachers' beliefs about mathematics education in preschool (e.g. Benz, 2010a, 2010b, Lee & Ginsburg, 2009). Evidence shows that teachers' beliefs about math education are relevant to their teaching practices (Ponte, Matos & Abrantes, 1998).

Seventy preschool teachers were surveyed through a questionnaire that was a translation into Portuguese of Benz (2010a) questionnaire. This instrument covers questions about educators' views of mathematics (affective visions and view of math nature), and how mathematics is learned. Views about math nature are classified as Formalism (exact reasoning and calculations done through rules and procedures); Application (practical application) and Process (problem solving processes through discovery and understanding). Educators' beliefs about how mathematics is learned are organized into Transmission and Construction.

Results are for the most part consistent with Benz results: i) Most teachers classified math as important, useful and challenging; negative adjectives were seldom checked; ii) Teachers view math predominantly as application, followed by formalism, and finally, process; iii) Learning math was taken to happen slightly more as a construction than as a transmission.

A better understanding of the nature of teachers' beliefs about early mathematics education seems to be essential to build further action in professional development, and ultimately contribute to improve children's math understanding and learning.

References

- Benz, C. (2010a). Kindergarten educators maths. In M. F. Pinto, & T. F. Kawaaki (Eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2) (pp.201-207). Belo Horizonte: PME.
- Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood*, 34, 37-45.
- Ponte, J. P., Matos, J. M., & Abrantes, P. (1998). *Investigação em educação matemática*. Lisboa: Instituto de Inovação Educacional.

ITERATIVE RESEARCH: DEVELOPING A LEARNING PROGRESSION FOR QUANTITATIVE REASONING

Franziska Peterson, Robert Mayes

University of Wyoming, Georgia Southern University

Research has suggested that learning progressions can help inform curriculum design and professional development, as well as advance effective adaptive instruction teaching techniques (Duschl, et al., 2007; Corcoran, Mosher, & Rogat, 2009). The central component of this study is to develop a learning progression (LP) on how students' quantitative reasoning abilities in environmental science progress. Our proposed LP consists of three overarching components: quantitative act (QA), quantitative interpretation (QI), and quantitative modeling (QM).

Creating an LP is an iterative research process which – for us – began about two and a half years ago. Initially, a hypothesized LP was constructed based on extensive literature reviews. This hypothesized LP was tested empirically via semi-structured interviews ($N=39$). The qualitative data were analyzed using grounded theory, which led to a revision of the LP as well as the interview assessments. Subsequently, our revised assessments were tested ($N=14$) and that data informed our LP again. During this phase our interview data was used to strengthen our LP through student exemplars giving meaning and examples to our components in each level of our LP. The next step in this iterative cycle was to develop mostly closed-form assessments based on the qualitative data and revised LP to help inform our LP on a larger scale ($N\approx 2,000$). This data collection will take place spring and summer 2013.

The result of this iterative research will be a revised LP for QR in environmental science as well as revised assessments which can provide a potential formative assessment tool for teachers. We will present how our qualitative data informed our LP and assessments as well as our qualitative and quantitative data in synthesis. Our previous results have not indicated a consistent progress in students' QR abilities as they move from 6th to 12th grade.

References

- Duschl, R.A., Schweingruber, H.A., & Shouse, A.W. (2007). *Taking science to school: Learning and teaching science in grades K-8*. Washington, D.C.: National Academies Press.
- Corcoran, T., Mosher, F. A., & Rogat, A. (2009). *Learning progressions in science: An evidence-based approach to reform*. Philadelphia, PA: Consortium Policy Research in Education.

LATE-ARRIVED IMMIGRANTS IN SCHOOL AND PERFORMANCE IN ARITHMETIC FOR NEGATIVE NUMBERS

Jöran Petersson
Stockholm University

This paper reports from a study on immigrant and native students tested in negative number arithmetic as one of several topics in school mathematics. In general immigrant students perform lower the later they have immigrated (Böhlmark, 2008). Students often find working with negative numbers difficult and Vlassis (2004) identified two kinds of conceptual changes needed for productive use of the minus and subtraction sign. First, the arithmetic rules are different for addition and subtraction. For example, subtraction is not commutative. Second, the minus sign has a flexible use exemplified by the expression $(-23) + 9 = 9 - 23 = -(23 - 9)$. The expression uses the minus sign as a signed integer (non-operational), a subtraction (operational) and a reflection (operational) and a flexible change between these.

In this study 356 school year 9 students in six Swedish schools, with an over average percentage of immigrants, took a test. Several test problems were formulated so that they were likely to not cause too much of language obstacles for second language learners. In this report the test problem “Calculate $12 - 23 + 9$ ” is in focus. This problem was characterized in Duval’s (2006) semiotic registers as mainly “computations” and scarcely dependent on natural language. One result is that about half of the solutions contained sign errors. There were few arithmetic errors and few other unclassified errors. A second result is that the students who immigrated during school years 1 – 7 clearly underperformed compared to the other students while those who immigrated during school years 8 – 9 performed slightly better than the native students. In this on-going research project one conclusion is that there seems to be a need to see early and late immigrants as having different challenges in being second language learners. The former have difficulties in following some advanced topics in mathematics teaching and the latter have difficulties in understanding some test questions. A second conclusion is that there is a need in research to look at specific topics in mathematics, especially advanced compulsory school mathematics such as negative numbers and algebra, for these student categories.

References

- Böhlmark, A. (2008). Age at immigration and school performance: A siblings analysis using Swedish register data. *Labour Economics*, 15(6), 1366-1387. doi: 10.1016/j.labeco.2007.12.004
- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61(1-2), 103-131. doi: 10.1007/s10649-006-0400-z
- Vlassis, J. (2004). Making sense of the minus sign or becoming flexible in ‘negativity’. *Learning and Instruction*, 14(5), 469-484. doi: 10.1016/j.learninstruc.2004.06.012

DEVELOPING REASONING ABOUT SIMPLE ADDITIVE STRUCTURES: ONE TASK FOR ELEMENTARY STUDENTS

Elena Polotskaia, Victor Freiman, Annie Savard
McGill University, Université de Moncton, McGill University

The current math curriculum for elementary school in Quebec pays a special attention to the development of students' problem solving skills, and in particular, a full and flexible understanding of additive structures involved in addition and subtraction word problems. According to practitioners in our region, many teachers experience difficulties in attaining this goal. In the scope of a granted collaborative research project, during the design phase, we analysed the traditional task of word problem solving and concluded that the task itself does not directly promote the structural understanding. To design a new task, we applied to the case of solving simple additive word problems the idea of Sfard (1991) about possible cognitive duality of mathematical reasoning. We hypothesised that an operational view - addition and subtraction as operations (Vergnaud, 1982) and systemic view – additive relationship between three quantities (Davydov, 1982) - should complement each other. Instead of “finding the answer”, the new task invites students to analyse the situation where the numerical data is incoherent with the semantic meaning. Then students should try to “correct” the situation using appropriate arithmetic operations.

The new way of work with additive structures is now used by 6 teachers of grade 1. Students' problem-solving activity is videotaped. Using qualitative analysis, we will try to find out how the new practice affects students' behaviour and performance in solving different types of additive problems. We hope that the new approach will help students develop a full and flexible understanding of additive structures involved in addition and subtraction word problems.

Our poster will present design principles, examples of the new task and student's productions.

References

- Davydov, V. V. (1982). Psychological characteristics of the formation of mathematical operations in children. *Addition and subtraction: cognitive perspective* (pp. 225–238). Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, 22, 1-36.
- Vergnaud, G. (1982). A Classification of Cognitive Tasks and Operations of Thought Involved in Addition and Subtraction Problems. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: cognitive perspective* (pp. 39–59). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

STUDENTS' EXPECTATIONS ABOUT MATHEMATICS AT UNIVERSITY

Stefanie Rach, Aiso Heinze

Leibniz Institute for Science and Mathematics Education, Kiel, Germany

The transition from school to university is a difficult passage in students' learning biography, especially in mathematics. At this transition, the character of mathematics changes: In school, computations and modeling of real-world problems play an important role, whereas at university, students with a major in mathematics get acquainted with mathematics as a scientific discipline which consists of formal concept definitions and deductive proofs. Accordingly, the learning activities and the corresponding cognitive demands change considerably when students enter university. Some authors assume that such a discontinuity causes learning problems because students have inadequate expectations concerning scientific mathematics and the corresponding cognitive demands (e.g. Crawford et al., 1998). Against this background, we investigate the following questions:

- What are students' expectations about university mathematics when entering university and how do these expectations develop in the first four weeks?
- Do students' expectations influence their learning success?

Our sample comprises 187 first-year students with a major in mathematics. Data on students' expectations is based on students' ratings of twelve mathematical tasks on the topic "differentiation" at the beginning of their study and four weeks later. Each task corresponds to exactly one of the three mathematical activities "schematic computation", "real-world modeling" and "proof". Students' ratings for each task range from "3 = appears" to "0 = not appears" on a four-point Likert scale depending on whether students expect that the task will occur in their first Analysis course or not.

A confirmatory factor analysis confirmed the three factor model of expectations. When entering university, students showed a tendency of expecting proofs ($M = 2.28$) but little modeling ($M = 0.97$). Expectations about computations were ambivalent ($M = 1.59$). The expectations about computations ($t(186) = 4.78, p < .001, d = 0.36$) and modeling ($t(186) = 6.42, p < .001, d = 0.47$) decreased substantially within the first weeks, proof only slightly ($t(186) = 2.50, p < .05, d = 0.18$). A regression analysis showed no significant impact of the expectations on the score of the Analysis I exam.

Summarizing the findings, it seems that students' expectations are only partially realistic at the beginning but they are adjusted within four weeks. The results do not indicate that there are sustainable negative effects on the learning success.

References

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1998). Qualitatively different experiences of learning mathematics at university. *Learning and Instruction*, 8(5), 455-468.

GEOMETRY THINKING LEVELS AMONG STUDENTS AND TEACHERS

Shaker Rasslan^{1 2}, Amal Rasslan-Sharif³, Manal Yosif¹

⁽¹⁾Al-Qasemi Academic College of Education , Israel; ⁽²⁾Beit Berl- Academic College of Education, Israel; ⁽³⁾The Academic Arab College of Education, Israel.

Not a few studies conducted in recent decades report difficulties that students encountered in learning geometry. Van Hiele levels of thinking in Euclidian geometry examined by two groups: 102 ninth grade students, and 16 in-service mathematics teachers who are studying for M.Ed. program (8 elementary and 8 middle and secondary teachers). All the participants were from the Arab sector.

“Arabic Version” of Usiskin’s Questionnaire was translated and processed. The goal was, to build a tool, fit into teaching geometry for the Arab sector in Israel. The analysis based on the approach used by Usiskin (Usiskin, 1982): “a student who answers correctly by at least 4 from 5 of the questions who represents a specific level, and controls all the previous levels, called “control” this level”. The results show that only 65%, 37% and 26% of the students in our sample control levels 1, 2, and 3 (respectively). Moreover, 81%, 56% and 56% of our teachers' sample control levels 1, 2 and 3 (respectively). Levels 4, 5 do not existed among the students and the teachers of our sample. Which mean that most of our students' sample reveals **illiteracy** in geometry; and most of the teachers in our sample have “**serious difficulties**” in geometry.

References

Usiskin, Z. (1982). *Van Hiele Levels and Achievement in Secondary Geometry* (Final report). Chicago: Department of Education. University of Chicago

LEARNING FROM TRAINING TO IMPROVE KINDERGARTEN PROSPECTIVE TEACHERS' KNOWLEDGE ON POLYGONS

C. Miguel Ribeiro

Research Centre for Spatial and Organizational Dynamics (CIEO), University of Algarve (Portugal)

Kindergarten assumes a fundamental importance in and for developing pupils' mathematical knowledge foundations for future learning. Thus, kindergarten teachers assume a major importance in introducing pupils to elementary, but powerful and core, mathematical ideas that are at the basis of their future formal learning. One of such core ideas concerns definitions and the example and non example space of polygons. In such, teachers' knowledge is a core element in improving pupils' achievement, understanding and reasoning. The particularities and specificities of such knowledge should contribute, amongst others, to approach the topics in a way that would allow pupils to perceive the connections between different mathematical topics – in one same level and between different levels–, leaving the door open to the elaboration, and its understanding, of a dense net of concepts, being polygons and the space of definitions a core element of such network.

The focus of the research is on prospective kindergarten teachers' knowledge on some geometrical topics (here polygons), and combines a qualitative methodology with an instrumental case study. Data concerns prospective teachers' productions while answering a sequence of tasks elaborated to develop their Mathematics Teaching Specialized Knowledge (Carrillo, Climent, Contreras & Muñoz Catalán, 2013), and audio recordings of the subsequent discussions and reflections. The analysis focus on prospective teachers' argumentations and representations as a way of exteriorizing knowledge – main difficulties and content of their comfort zone. This corresponds to the first phase of the project, which will be address on the poster, and it's perceived as the basis for the following phases. The next steps concerns designing tasks specifically for teachers training, aimed at developing their specialized knowledge for teaching, taking into consideration the specificities of such knowledge.

Acknowledgements: The work for this poster has been partially supported by the Portuguese Foundation for Science and Technology (FCT). It forms part of the research project "Mathematical knowledge for teaching with respect to problem solving and reasoning" (EDU2009-09789), General Directorate for Research and Management of National Plan I + D + i. Ministry of Science and Innovation of Spain.

References

Carrillo, J., Climent, N., Contreras, L. C., & Muñoz-Catalán, M. C. (2013). Determining Specialized Knowledge for Mathematics Teaching. *Proceedings of CERME 8* (pp. to appear). Antalya: ERME.

FROM TASKS FOR STUDENTS TILL TASKS FOR TEACHERS: IMPROVING KNOWLEDGE

C. Miguel Ribeiro, Alexandra Gomes

Research Centre for Spatial and Organizational Dynamics, University of Algarve
(Portugal) CIEC/IE – University of Minho (Portugal)

Tasks assume a main importance in the teaching-learning process as the core way of developing learners' abilities and knowledge. Although there is a vast research and investment in (mathematical) tasks, it focuses mainly on students and the possible impact of tasks in their learning/outcomes. They leave aside (not completely, but mostly) teachers' role and knowledge in preparing, modifying and implementing tasks that would allow a high quality of instruction (in the sense of Hill, 2010). Considering that most individuals (both students or teachers) perform better any actions/set of actions if they experience them by themselves, one of the critical aspects in the process of developing such tasks – challenging tasks with a higher cognitive level that would also permit developing teachers' knowledge – concerns the possibilities of living similar kind of experience that they are expected to allow their students (e.g., grounded on problem solving; facing difficulties; trying to overcome difficulties; explain procedures; making sense of others processes).

Aiming at developing teachers knowledge for teaching, taking into consideration the specificities of such knowledge (in the sense of Carrillo, Climent, Contreras & Muñoz Catalán, 2013), we focus on conceptualizing tasks in and for teachers training. The research combines a qualitative methodology with an instrumental case study. Tasks conceptualization is approached from different points of view and contexts in an intertwined manner, and working with different educational agents (from students to educators) having as a starting point the development of mathematical challenging tasks for students and, from those, developing tasks that foster (prospective) teachers' and educators' knowledge and abilities. In this poster we will present the different phases of the considered conceptualization and some preliminary results considering its potentialities for developing (prospective) teachers' specialized knowledge.

References

- Carrillo, J., Climent, N., Contreras, L. C., & Muñoz-Catalán, M. C. (2013). Determining Specialized Knowledge for Mathematics Teaching. Proceedings of CERME 8 (pp. to appear). Antalya: ERME.
- Hill, H. C. (2010). The nature and predictors of Elementary Teachers' Mathematical Knowledge for Teaching. *Journal for Research in Mathematics Education*, 41(5), 513-545.

TEACHER KNOWLEDGE AND THE IMPLEMENTATION OF INVESTIGATION TASKS

Helena Rocha

Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa (Portugal)

The main focus of this poster is a new model of teacher's knowledge: Knowledge for Teaching Mathematics with Technology (KTMT). Central in this model is a set of inter-domains knowledge divided in two main areas: Mathematics and Technology, and Teaching & Learning and Technology. This model is the basis for a study that intends to understand the knowledge held by the teacher and its influence on the implementation of investigation tasks using graphing calculators.

The study presented on this poster intends to understand the knowledge held by the teacher and its influence on the implementation of investigation tasks using graphing calculators. The theoretical framework focuses on the professional knowledge and on the graphing calculator, and emphasises the KTMT model developed during this study (Rocha, 2012). This model has implicit three stages of knowledge development and adopts four basic domains of knowledge (Mathematics, Teaching and Learning, Curriculum, and Technology), two sets of inter-domain knowledge (Mathematics and Technology, and Teaching and Learning and Technology) and an Integrated Knowledge. The study adopts a qualitative and interpretative methodological approach, undertaking two teacher case studies. Data were collected by semi-structured interviews, class observation, and documental data gathering. Data analysis was conducted combining the theoretical framework and the categories that emerged from the interpretation of data collected for each teacher. The results of the study suggest that the two teachers are in different stages of their KTMT development. This difference is clear in the way they emphasise different domains of knowledge in their practice, with the teacher in an intermediate stage of her KTMT development revealing a tendency to privilege one or other of the inter-domains knowledge. The teacher in an advanced stage of her KTMT development proposes in class a diversified set of investigation tasks where the calculator is an important resource. The other teacher proposes a set of tasks less diversified and tends to be more directive when students face difficulties.

The poster adopts a visual metaphor, using the primary colours (blue, red and yellow) to represent the base knowledge and the painter's palette to represent the curriculum, the secondary colours (green and orange) to represent the inter-domains knowledge and a tertiary colour (brown) to the integrated knowledge.

References

Rocha, H. (2012). *A integração da calculadora gráfica no ensino da Matemática: Estudo sobre as práticas curriculares de professores do ensino secundário* (Unpublished doctoral dissertation). Universidade de Lisboa, Lisboa.

PROOF SCHEMES: A STUDY WITH 9TH GRADE PUPILS

Margarida Rodrigues, João Filipe Matos

Escola Superior de Educação do Instituto Politécnico de Lisboa, Instituto de Educação da Universidade de Lisboa

The study is rooted in the theoretical framework of social practice. There is a substantial literature within mathematics education that stresses the prominent role of proof in the curriculum because it gives a more comprehensive vision of the nature of mathematics and promotes mathematical understanding. Although students reveal a great difficulty in constructing proofs, they can develop their proof schemes if taught appropriately (Harel and Sowder, 2007).

The research aims to identify the ways in which students validate their mathematical results, relating them to the social practice developed in the classroom. The questions posed were: 1) what is the nature of proof in a school context?, 2) what is the role of proof in students' mathematical activity?, and 3) how does the construction of proof relate to the social practice developed in the mathematics classroom? The study was conducted within an interpretative perspective, involving a 9th grade class. A group of four students was selected to be videotaped. Data was collected by: (a) participant observation, (b) semi-structured interviews, and (c) documental analysis.

The results show that two factors are relevant in the development of proof schemes: the specific examples and the function of proof. Students are more successful in constructing proofs when they use an informal and narrative form. Proof is seen by students as having multiple functions: discovery, verification, explanation and communication. The teacher has a decisive role in negotiating with students the need of a proof. The communication of proofs, realised by insight, is intrinsically related to explanation and to a more complete understanding of mathematics, which increases students' ownership of meaning. The poster will illustrate the main results by exhibiting some of students' productions and its analysis.

References

Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 805-842). Charlotte: Information Age Publishing Inc., & NCTM.

DEVELOPMENT OF CHILEAN AND FINNISH TEACHERS' CONCEPTIONS ON MATHEMATICS TEACHING WHEN USING OPEN-ENDED PROBLEMS

Laia Saló i Nevado , Päivi Portaankorva-Koivisto , Erkki Pehkonen , Leonor Varas ,
Markku Hannula , Liisa Näveri
University of Helsinki , Universidad de Chile

This poster presents the cases of two Chilean and two Finnish elementary teachers' reflections on their own professional development during a research project where they learn about and used open-ended problems to teach mathematics. The focal point of the project is teachers' professional development along with both teachers and pupils' development in mathematical thinking and understanding when dealing with open-ended problems. This poster is particularly focused in the teachers' own conceptions of their development.

In mathematics education, problem solving is considered as a method to promote pupils' high-order thinking and understanding. The use of open-ended problems challenges teachers to modify their roles in class. A teacher is no more a deliverer or transmitter of information, but a guide and facilitator for learning, and a planner of learning environments. Thus, teachers need to alternate and to improve their own conceptions of teaching and learning (cf. Pehkonen 2007).

The research question was: How do the teachers themselves perceive that their conceptions about mathematics and teaching of mathematics have changed during the project?

The data gathered through teacher interviews, classroom observations, videotaped discussions and the researchers' field notes during March-April 2012 in Finland and during November-December 2012 in Chile.

The data indicates that during the project the teachers increased their pedagogical content knowledge, subject matter knowledge and motivational components. Teachers claim they give more room for pupils' ideas and rely on pupils' learning in pairs or in groups. Furthermore, they allege that also the weakest pupils seem to be involved with problem solving.

References

Pehkonen, E. 2007. Über "teacher change" (Lehrerwandel) in der Mathematik. In: *Mathematische Bildung - mathematische Leistung: Festschrift für Michael Neubrand zum 60. Geburtstag* (Hrsg. A. Peter-Koop & A. Bikner-Ahsbals), 349–360. Hildesheim: Franzbecker.

DEVELOPMENTAL INDICATORS OF PROSPECTIVE MATHEMATIC TEACHERS' NOTICING OF STUDENTS' UNDERSTANDING OF THE DERIVATIVE

Gloria Sánchez-Matamoros¹, Ceneida Fernández², Salvador Llinares², Julia Valls²
¹University of Seville (Spain); ²University of Alicante (Spain)

In the last two decades, research on teacher learning has focused on the skill of noticing considered as a fundamental element of expertise in teaching (Jacobs, Lamb, & Philipp, 2010; Mason, 2002). A particular focus for mathematics teacher's noticing is students' mathematical thinking. The aim of this study is to characterize the development of prospective secondary school mathematics teachers' noticing of student's understanding related to the derivative concept.

Eight PMTs participated in an instructional intervention addressed to develop prospective teachers noticing of students' understanding of the derivative. In this instructional intervention, PMTs answered two questionnaires (after and before the instruction). In both questionnaires, PMTs had to identify characteristics of the students understanding related to the derivative concept. The two questionnaires were designed considering students' learning trajectory (research-based descriptions of how students' understanding evolves over time from initial ideas to increasingly complex understanding) related to the derivative concept because we believe that it is essential for mathematics teachers understand how students' understanding of the derivative concept develop along high school. We analysed PMTs answers to the two questionnaires. The focus was the extent to which PMTs attended to the noteworthy mathematical elements in the student's answers and how they used them to identify the students understanding.

Findings provide indicators of three levels of development of prospective secondary school mathematics teachers' noticing of students' understanding. These levels are related to how PMTs describe the students' answers using noteworthy mathematical elements and how PMTs use the knowledge of high students' learning trajectory.

Acknowledgements

The study reported here has been financed in part by Ministerio de Educación y Ciencia, Dirección General de Investigación, Spain, under Grant no. EDU2011-27288 and in part by the Birth Project GRE10-10 de la Universidad de Alicante, Spain.

References

- Jacobs, V.R., Lamb, L.C., & Philipp, R.A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41, 169-202.
- Mason, J. (2002). Researching your own practice. The discipline of noticing. London: Routledge Falmer.

OUT-OF-FIELD MATHEMATICS TEACHING: PROGRESS IN INDIGENOUS SECONDARY SCHOOL CLASSROOMS

Satwant Sandhu, Gillian Kidman, Tom J. Cooper
Queensland University of Technology

Mathematics in-field teachers are regarded as important for learning mathematics. Darling-Hammond (2000) relates the lower academic results found in rural and remote schools to be related to the lack of in-field teachers. Little research has been carried out studying the teaching practices of mathematics in-field teachers to rural and remote Australian students. Further, even less attention has been paid to the mathematics out-of-field teacher in rural remote settings. This research examines the effectiveness of a non-traditional mathematics teaching approach by an out-of-field teacher, versus the traditional mathematics teaching approach of an in-field mathematics teacher with underperforming Indigenous students.

Teaching styles and student engagement were observed in two classrooms over a period of one year through school visits. Class A is a Year 8 foundation class consisting predominantly of Indigenous students taught by an out-of-field mathematics teacher. Class B is a Year 10 foundation class consisting predominantly of Indigenous students taught by an in-field mathematics teacher. Teacher interviews were conducted with both teachers after each observed lesson to discuss the lesson and strategies used.

Observations show that the out-of-field mathematics teacher's lessons started each topic by relating the topic to students' reality, followed by short activities (group and individual), with students working from a variety of positions (desk and floor). Difficult concepts were explained using a variety of hands-on activities and real-world examples. Students responded enthusiastically, relating concepts to their life experiences and worked well in groups or individually. Observations of the in-field mathematics teacher's lessons showed an opposite engagement response from the students; most concepts were shown through traditional chalk-and-talk pedagogy, concepts presented as formulae, and students working mainly from their desks. Students were seen to have difficulty grasping concepts, often disengaging from the teaching and reluctant to participate.

In summary, it was observed the out-of-field mathematics teacher made pedagogical adjustments that compensated for his lack of mathematics teacher training. He turned his lack of mathematics training into an advantage by using a kinaesthetic teaching approach from his physical education in-field expertise. The in-field mathematics teacher had difficulty in adjusting his pedagogy to cater for his underperforming Indigenous student cohort.

References

Darling-Hammond, L. (2000). New standards and old inequalities: School reform and the education of African American students. *Journal of Negro Education*, 69(4), 263-287.

IDEAS TOWARDS THE EXTENSION OF THE THEORETICAL CONSTRUCT OF MATHEMATICS TEACHERS' KNOWLEDGE

Thorsten Scheiner

University of Hamburg, Germany

The underlying assumption of this paper is that how one frames the theoretical construct of mathematics teachers' knowledge depends a great deal on how one understands the nature of the development of mathematical thinking and mathematical concept construction. Therefore, it is assumed that a theoretical construct of mathematics teachers' knowledge must be *linked* both to a theory of mathematical knowledge that reflects the nature of knowledge structures and to a theory of mathematical concept construction that reflects various ways of mathematical thinking. The focus of this paper is limited to the former aspect taking into account two levels of knowledge structures, namely (1) a fine-grained level including knowledge structures such as conceptions and internal representations and (2) a conceptual level including knowledge structures described in terms of concept image and concept definition. This subdivision is in line with Schoenfeld, Smith, and Arcavi (1993).

The theoretical paper explicitly links conceptualizations of teachers' knowledge with theoretical approaches and research findings within mathematics education research concerning the two mentioned levels of knowledge structures. This is considered as a powerful *tool* both to give deeper insights into a broader image of mathematics teachers' knowledge. Based on theoretical and empirical work regarding knowledge structures, first considerations of various components of mathematics teachers' knowledge are discussed.

In this paper as a part of a broader work grounded in research on both teachers' knowledge and mathematics education, the author does not discuss a new theoretical framework but a first step towards a thorough extension of the theoretical construct of mathematics teachers' knowledge. Although the ideas are inherently incomplete, it is hoped to stimulate a valuable discussion and make significant advances to our understanding of the theoretical construct of mathematics teachers' knowledge.

References

Schoenfeld, A. H., Smith, J. P. III, & Arcavi, A. (1993). Learning: The microgenetic analysis of one student's evolving understanding of a complex subject matter domain. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 4, pp. 55-175). NJ: Lawrence Erlbaum Associates.

UNDERSTANDING TRIADIC STRUCTURAL RELATIONS BETWEEN PART, WHOLE, AND FRACTION

Andrea Schink

IEEM Dortmund, Germany

Many empirical studies focus students' individual use of fractions and analyse typical cognitive obstacles and mistakes in their products and processes. Whereas most of them concentrate on dyadic structural relations between part and whole, the here reported study stresses the importance of triadic structural relations between part, whole, and fraction in different representations and constellations. Those relations constitute the meaning of a fraction: On the one hand, one has to understand that a fraction simultaneously refers to both, a special whole and part; on the other hand, part and whole hold a relation expressed by the fraction. That is why part, whole, and fraction have to be thought of as a networked trinity. Even if these relations are basic, they are not easy to understand: To establish structural relations, students have to (re-)form and use units (cf. Lamon, 2007), which express their interpretations of structural relations. Taking the resources of individual interpretations into account, the main research question was: How do students construct structural relations between the part, the whole, and the fraction? (cf. Schink, 2013).

The study followed a mixed methods design, comprising a paper-pencil test with 153 7th-graders who had nearly finalized their fraction course and 12 interviews with two 6th-graders each, at earlier points in their learning trajectories. For data analysis, the quantitative and the qualitative data was triangulated with respect to the use of structural relations between part, whole, and fraction.

The results offer insights into individual cognitive obstacles but also into resources: They show that students may think of structural relations between part, whole, and fraction in quite different ways than it would be mathematically correct. Successful individual strategies show the power of exploratory operative variations of the given structures, where the formation and use of units seems to be critical.

The study does not only provide a sharpened language for analysing individual understandings, but offers also practical consequences for designing learning arrangements for weaker students.

References

- Lamon, S. J. (2007). Rational Numbers and Proportional Reasoning. Toward a Theoretical Framework for Research. In F. K. Jr. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 629-667). Charlotte, NC: IAP.
- Schink, A. (2013). Flexibler Umgang mit Brüchen – Empirische Erhebung individueller Strukturierungen zu Teil, Anteil und Ganzem. Wiesbaden: Springer Spektrum.

TEACHERS THINK ABOUT HOW TO STRUCTURE MATHEMATICS LESSONS TO DEVELOP STUDENTS' PROBLEM-SOLVING COMPETENCE

Stephanie Schlump, Michael Neubrand
Carl von Ossietzky University Oldenburg, Germany

Despite the fact that problem solving (Pólya 1945, Newell & Simon 1972) is a key competence of mathematical literacy in secondary schools, a considerable lack of information exists on how teachers structure lessons about mathematical non-standard problem solving. Based on this situation and following the model of “Educational Re-Construction for Teacher Education (ERTE)” (van Dijk & Kattmann 2007), three research questions will be addressed:

- How can teachers structure lessons about mathematical non-standard problem solving? (theoretical perspective)
- What do teachers think about the structuring of mathematics lessons to develop students' problem-solving competence? (empirical perspective)
- Are there consequences for teacher education? (design perspective)

To structure mathematics lessons two aspects should be distinguished: the short-term structuring of lessons (e.g. Pólya 1967, Newell & Simon 1972) and the long-term structuring (e.g. according to the model of Bruder & Collet 2011).

To receive information about teachers' cognitions, a complex constructed qualitative interview is used. The interviews take video vignettes from a theoretically constructed teaching unit as starting points and stimuli. The sample consists of twelve experienced secondary schools teachers, each having at least ten years of professional experience. The data are analysed by using the method of Content Analysis. The results show that the teachers are indeed able to describe concrete options for actions towards specific cognitive activities within the problem-solving process.

It is expected, that consequences for teacher education will be formulated based on the theoretical and empirical results, as it is also one of the goals of the ERTE model.

References

- Dijk, E. Van, & Kattmann, U. (2007). A research model for the study of science teachers' PCK and improving teacher education. *Teaching and Teacher Education*, 23, 885–897.
- Bruder, R. & Collet, C. (2011). *Problemlösen lernen im Mathematikunterricht*. Berlin: Cornelson Scriptor
- Newell, A. & Simon, H.A. (1972). *Human problem solving*. Engelwood: Prentice-Hall.
- Pólya, G. (1945). *How to solve it*. Princeton, NJ: Princeton University Press.

SELF GENERATED EXTERNAL REPRESENTATIONS IN THE CASE OF FRACTIONS

Stefan Schumacher, Jürgen Roth

University of Koblenz-Landau, Campus Landau, Germany

In the student laboratory “Math is more” at the University of Landau students work out curricular content independently. In case of the presented study students worked on “basic ideas” of fractions. Besides the learning of fractions a main focus of the study is the students’ ability to externally represent their cognition process and their insights in form of so called “cognition-protocols”. Previous research on self-generated external representations (ER) focuses on their role in problem solving processes (Cox 1999). Generating ER for the use in “cognition protocols” is underrepresented in the research today.

There are two main research questions, namely: (1) Do students who learn in the student laboratory reach at least the same learning success as students who get taught basic ideas of fractions in a teacher based learning environment at school. (2) How does the learning success of students correlate with the ability to generate ER of their insights and do students benefit in this sector from working in the laboratory.

A total of 190 six grade students take part in the study. The students are split in an experimental group (n=148) who visited the student laboratory and a control group (n=42) who got taught in school. The study contains three measurement points, a pre a post and a follow-up test that is running at the moment.

Tasks (so called “video items”) to measure the ability to generate “cognition protocols” and a test on basic ideas of fractions have been developed and will be presented together with first research results.

It is expected, that the students who got taught in school learn more in general, due to the higher cognitive load of the students working in the laboratory because of the unknown learning environment (Schmidt, Di Fuccia, Ralle 2011). But it can be assumed, that students who are capable of representing their insights externally in independent learning processes achieve higher scores in the fraction test than the ones who are not able to do so, because it’s easier for them to retrace what has already been learned in previous stages of the independent learning process.

References

- Cox, R. (1999). Representation construction, externalised cognition and individual differences. *Learning and Instruction*, 9, 343-363.
- Schmidt, I., Di Fuccia, D. S., Ralle, B. (2011). Außerschulische Lernstandorte- Erwartungen, Erfahrungen und Wirksamkeit aus der Sicht von Lehrkräften und Schulleitungen. *MNU* 64(6). S. 362-369.

TEACHING PRACTICE WITH THE INQUIRYING QUESTIONS TO FOSTER DESIRABLE ATTITUDE OF MIND

Yusuke Shogimen

Graduate School of Education, Hiroshima University, Japan

Today, it is said that Japanese students are lacking in the willingness of attempting to solve problems. It has turned out from the results of the annual national mathematical achievement survey for 6th and 9th grade students in Japan. The objective of this study is to provide general suggestions from perspective of action research.

The subjects of the investigation are 40 Japanese students in grade 9. We conducted two preliminary investigations: lesson observation and questionnaire investigation. We found some students gave up solving problems from these investigations; hence our research question is how we can improve these students' attitude toward problems. We assume that factors behind the reluctance of students to solve problems are not only difficulty of problems but also lack of desire to solve problems. According to Lester (1983), "the individual or group confronting it wants or needs to find a solution" (p.232), and "the individual or group must make an attempt to find a solution" (p.232). So, we can suppose that problems which a teacher gives are not a students' own problem. If a student makes questions by himself or herself from the aim of inquiring into new mathematical objects and ideas, we call this type of questions as *the inquiring questions* (for example, after finding a relationship between a homothetic ratio and a ratio of area, the students under our observation made a question about a relationship between a homothetic ratio and a ratio of volume). In order to approach the research question, we hypothesized that when a lesson with the inquiring questions was conducted, students have the desire to solve a problem. The lesson with the inquiring questions consists of five aspects; (i) presenting the inquiring question, (ii) sharing lesson's problem which arises from the inquiring question, (iii) exploring the problem for answering the inquiring question, (iv) solving the problem, (v) setting up opportunity for students to create new inquiring questions. They are used in the next lesson.

After these lessons in the research, we analyzed students' comments about the lesson, the inquiring questions in each lesson, the pre- and post-questionnaire, and videotaped lessons. We found that students themselves felt to investigate mathematical ideas deeply and that the lessons with the inquiring questions fostered students' desirable attitude towards difficult problem for them. As a result, we suggested that the inquiring questions were effective for improving students' willingness.

References

Lester Jr, F. K. (1983). Trends and Issues in Mathematical Problem-Solving Research. In R. Lesh & M. Landau (Eds.), *Acquisition of Mathematics Concepts and Processes* (pp. 229-261). Massachusetts: Academic Press.

EYE TRACKING RESEARCH OF NOVICE EXPERT DIFFERENCE IN MUPLTI-REPRESENTATIONAL LEARNING

Anna Shvarts, Dmitry Cumachenko
Lomonosov Moscow State University

Eye tracking is one of the extensively developing instruments in multimedia learning research (Van Gog, 2010). The coordination of several representations is a current problem for mathematical education as far as the nature of mathematical concepts is multi-representational (Duval, 2006).

We used eye tracking in novice-expert methodology to understand how the perception of a combination of formulas and graphs during listening of a lecture differ between undergraduate students not majoring in mathematics ($n=10$) and graduates majoring in mathematics ($n=5$). The lecture devoted to the introduction to binary relation theory was accompanied by formulas and graphs presented on a monitor. After listening students had to detect which properties are attributed to several binary relations. To check how students represent binary relations we exposed the description of all attributes used in tasks on the monitor in both representations (by formulas and by graphs). By tracking which area was more attended we saw which of the representations was more useful for each student.

Preliminary results showed that novices spend longer looking at graphs, while experts spend more time on formulas during the learning phase. Also, we found out the predominance of saccades between different representations in expert behavior as it was suggested by Andra et al. (2009). The qualitative analysis showed that novices were more precise in following the lecturer by looking at appropriate points while an example was sounded. During the problem-solving experts almost never looked at graphs while novices focused on graphs and ignored formulas ($F=28,630$, $p<0,001$).

In summation, the connection between representations is important during listening while during problem solving students choose the favorable representation and use it to refresh their knowledge. Our next task is to expand the sample.

The research was funded by RFH, grant number is 12-36-01408.

References

- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational studies in mathematics*, 61, 103-131.
- Van Gog, T., & Scheiter, K. (2010). Eye tracking as a tool to study and enhance multimedia learning. *Learning and Instruction*, 20, 95-99.
- Andrà, C., F. Arzarello, F. Ferrara, K. Holmqvist, P. Lindstrom, O. Robutti, and Sabena, C. (2009) How students read mathematical representations: an eye tracking study. *Proc. 33rd Conference of the International Group for the Psychology of Mathematics Education* Eds. Tzekaki, M., Kaldrimidou, M. & Sakonidis, C. (Eds.). (Vol. 2, pp. 123-131). Thessaloniki, Greece PME.

TEACHING QUALITY OF MATHEMATICS UNIVERSITY COURSES

Ulrike Siebert, Stefanie Rach, Aiso Heinze

Leibniz Institute for Science and Mathematics Education, Kiel, Germany

High drop-out rates in mathematics give evidence of students' learning problems during the transition from school to university. These problems might be related to a change of the learning opportunities because mathematics at university is not offered in a didactically prepared learning environment but presented as a pure scientific theory where heuristic processes remain implicit. Since there are only a few empirical case studies examining the teaching quality of mathematics courses at university (e.g., Bergsten, 2007), we started a study with the aim of developing an observational instrument for analyzing aspects of the teaching quality of lectures and tutorials. Here, we particularly consider the introduction of concepts and the proving processes.

We observed for three weeks the lectures and the associated 10 tutorials of the course "Analysis 1". Regarding the introduction of concepts we focussed on the relation between formal definitions and corresponding mental representations. The proving process was rated by the framework of Heinze and Reiss (2004) which focus the explicit discussion of the single proving phases. For both aspects, the standardized observation used a four-point Likert scale (3: well treated to 0: not treated). The interrater reliability was acceptable for all aspects ($ICC > .72$).

The findings indicated that the introduction of concepts in the lectures is characterized by formal definitions ($M = 2.25$, $SD = 0.50$) for which hardly any motivation is given ($M = 0.13$, $SD = 0.25$). Furthermore, (counter-)examples ($M = 1.25$, $SD = 0.96$) and links to mental representations ($M = 0.88$, $SD = 1.03$) were rarely observed. For the proving processes, the phases of exploration were underrepresented: development of the assumption ($M = 1.24$, $SD = 0.88$) and exploration of the assumption ($M = 1.48$, $SD = 0.80$). Similar results were found in the tutorials. The organisation of a formal proof ($M = 2.10$, $SD = 0.50$) seemed to be more important than the generation of a proof idea ($M = 1.48$, $SD = 0.69$). Significant differences can be found for the tutorials regarding the latter two aspects of the proof process. It seems that tutorials put an emphasis either on generating ideas or presenting a formal proof.

This study gives an insight into the teaching quality of mathematics courses at university. Our results - limited to one university and one course - can especially confirm the expected dominance of formal definitions and absence of proof processes.

References

- Bergsten, C. (2007). Investigating Quality of Undergraduate Mathematics Lectures. *Mathematics Education Research Journal*, 19(3), 48-72.
- Heinze, A., & Reiss, K. (2004). The teaching of proof at lower secondary level – a video study. *ZDM - International Journal on Mathematics Education*, 36(3), 98-104.

THERE ARE SEVERAL DIFFERENT APPROACHES TO FINDING SOLUTIONS FOR PROBLEMS: WHICH IS HELPFUL FOR ME?

Nina Sturm, Renate Rasch, Wolfgang Schnotz
Universität Koblenz-Landau, Campus Landau

A current study (Hohn, 2012) shows that word problems in mathematics lessons are not satisfactorily solved across all grades. It is difficult for most students seeing this kind of problem for the first time to convert the words into a mathematical sentence (Charles/Lester, 1982). From a psychological point of view, using external forms of representation can relieve the working memory and generate capacity for individual, creative thinking processes (Schnotz et al., 2010).

Based on this theoretical background, a training program for fourth grade pupils has been developed to foster the use of external forms of representation for solving word problems in regular classroom settings and has been implemented within the framework of a pre-study. It aims to determine whether the training program helps learners to develop their own representations, implement them to find solutions and transfer them to other contexts. In addition, the between-subject design investigates whether communicative settings have positive effects on problem-solving competences and/or finding solutions.

The initial findings indicate that the training program can help learners to develop self-generated forms of representation to solve word problems. Further results will be presented on the poster.

References

- Charles, R. & Lester, F. K. (1982). *Teaching Problem Solving: What, Why and How*, pp. 8- Palo Alto, Calif.: Dale Seymore Publications.
- Hohn, K. (2012). Gegeben, Gesucht, Lösung? Selbstgenerierte Repräsentationen bei der Bearbeitung problemhaltiger Textaufgaben, (Doctoral thesis, University Koblenz-Landau, Germany). Retrieved from <http://d-nb.info/1028021070/34>
- Schnotz, W. Baadte, C. Müller, A., & Rasch, R. (2010). Creative thinking and problem solving with depictive and descriptive representations. In L. Verschaffel, E. De Corte, T. de Jong, & J. Elen (Eds.) *Use of representations in reasoning and problem solving* (pp. 11-35). London: Routledg.

TEACHERS' PROFESSIONAL DEVELOPMENT REGARDING THE DIAGNOSIS OF FUNCTIONS AND GRAPHS

Julia Teller, Bärbel Barzel

Institut für Mathematische Bildung, University of Education Freiburg

Diagnostic competencies represent one prerequisite to effective lesson planning and student achievement (Anders et al., 2010). Large scale studies such as COACTIV found significant deficiencies in diagnostic competencies for mathematics teachers at the secondary level. This study replies to the obvious need of teacher training to foster diagnostic competencies, and investigates whether a lesson-accompanying training can facilitate the development of diagnostic competencies in the school setting.

The training focuses on the concept of function as a central component of the mathematics curriculum. It provides input, practice and reflection opportunities for a scope of diagnostic activities from assessing students' achievement to the diagnostic potential of tasks. Additionally, this study aims to facilitate competence development through comparison of teachers' prognosis of students' achievement with their actual achievement based on the results of a centrally administered test. The successive analysis of discrepancies is known to be a trigger for teachers to adjust their subjective judgments (Helmke et al., 2004).

The sample comprises 26 secondary teachers of three German school types (Werkreal-, Realschule and Gymnasium). The training includes three meetings over the course of six weeks (January/February 2013). The effects of the training on teachers' diagnostic competencies will be determined by quantitative and qualitative data analysis of the pre- and post-test with 8 items (mainly open format), taking into account the variables educational institution (University vs. University of Education), amount of teaching experience in general and concerning the concept of function as well as the school type. A first glance at the test results indicates an increase in the use of pedagogical content knowledge while diagnosing students' achievement and the diagnostic potential of tasks. Further results based on the analysis of qualitative data will be presented and discussed at the PME37-conference.

References

- Anders, Y. et al. (2010). Diagnostische Fähigkeiten von Mathematiklehrkräften und ihre Auswirkungen auf die Leistungen ihrer Schülerinnen und Schüler. *Psychologie in Erziehung und Unterricht*. München, Basel: Ernst Reinhard Verlag. 57:175-193.
- Helmke, A., Hosenfeld, I. & Schrader, F.-W. (2004). Vergleichsarbeiten als Werkzeug für die Verbesserung der diagnostischen Kompetenz von Lehrkräften. In R. Arnold & C. Grieser (Hrsg.), *Schulleitung und Schulentwicklung* (S. 119-144). Hohengehren: Schneider.

INVESTIGATING PRESERVICE ELEMENTARY TEACHERS MATHEMATICAL BELIEFS AND LEARNING PREFERENCES

Figen Uysal¹, Yüksel Dede²

¹Bilecik Şeyh Edebali University ²Istanbul Medeniyet University

It may be many factors that effect students' mathematics learning such as teaching strategies and methods and students' learning preferences. There is also a growing body of research showing the influence of students' beliefs on their mathematical learning. Addressing the beliefs preservice elementary school teachers hold toward mathematics is critical to improving the mathematical performance of students, because these beliefs can have a strong influence on his/her approach to teaching mathematics. Raymond (1997) defined mathematics beliefs as personal judgments about mathematics formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics. In our study, we aimed to investigate the relationship between Turkish preservice elementary teachers' beliefs about teaching mathematics and learning preferences. The research was conducted in 2010-2011 spring term with 96 students of Department Primary Education in a state university in Central Anatolia in Turkey. The sample involves 72 female and 24 male student teachers. For data collection, two instruments were used: a) "Mathematics Learning Preferences [MLP]" scale which was developed by Dede and Yaman (2006), in our study it was found three factors such as teacher-centered learning, working with group and individual learning preference. b) Beliefs about the Teaching of Mathematics [BaToM]" scale which was developed by Baydar (2000). The BaToM was one factor and reflected the child-centredness beliefs. Data analysis reveal that there was a significant correlation between preservice elementary teachers' mathematics teaching beliefs and learning preferences at the working with group factor and teacher-centered factor. In other words student teachers' mathematics teaching beliefs were related to working with group learning preference and teacher-centered learning preference. We are planning to present and discuss more detailed information about our measurement instruments and results using tables and graphics in the poster.

References

- Baydar, B. (2000). Beliefs of preservice mathematics teachers at Middle East Technical University and Gazi University about the nature and teaching of mathematics. Unpublished Master Thesis.
- Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education*, 28(5), 550-576.
- Dede, Y., & Yaman, S. (2006). Science and mathematics learning preference of primary school students. *International Journal of Enviromental and Science Education*, 1(2), 172-180.

SENSE MAKING OF TRIGONOMETRIC RELATIONSHIPS IN THE CONTEXT OF LESSON STUDY

Nellie C. Verhoef
University of Twente

This study focused on sense making of mathematics through perception, operation and reason in the context of trigonometry (Chin & Tall, 2012). Lesson study - in which teachers collaboratively study teaching and learning in daily practice by means of lesson plans, live classroom observations and post-lesson discussions and reflections – was used as a strategy to involve mathematics teachers in the development of sensible mathematics (Fernandez & Yoshida, 2004). The lesson study team developed sense making research lessons with regard to the transition from triangle trigonometry to circle trigonometry using static as well as dynamical icons (Bruner, 1966). The research question was: What icons help students to make sense of this transition?

Six Dutch mathematics teachers from different secondary schools participated in the school year 2011-2012. The research instruments consisted of teachers' lesson plans and video-tapes of (a) the enactment of the research lessons, (b) the post-lesson discussions at school and (c) the reflection meetings at the university. Student activities - classified in personal experiences (perception), procedures (operation) and argumentation with both (reason) – were related to teachers' iconic interventions.

Two teachers used a windmill as an icon. The windmill elicited the use of symmetry and coordinates and helped students to reason about trigonometric characteristics. Three teachers introduced a dynamic cogwheel as an icon to influence the idea of moving around a circle. One of the teachers emphasized the relation between height and number of turns of the radius. He used an applet to show the relation between the arc length and the number of cogs. This teacher was aware of students needs to use symbols in order to argue. The plenary discussion at the university revealed the mathematical weakness of this icon because of the space between the cogs. One teachers' lesson could not be observed because of unexpected practical impediments.

This study shows that the windmill helps students to relate ratios in right-angled triangles to symmetry properties and x - and y - coordinates in a unit circle. The cogwheel helps students - by emphasizing the arc length and the number of cogs - to relate the arc length and the height which directly makes reference to the functional representation of the sine function. The cogwheel failed mathematically because of the space between the cogs. The students had no burden of this failure.

References

- Bruner, J. S. (1966). *Towards a Theory of Instruction*. New York: Norton.
- Chin, K. E., & Tall, D. O. (2012). Making Sense of Mathematics through Perception, Operation & Reason: The Case of Trigonometric Functions. *Proceedings of PME 36*.
- Fernandez, C., & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. Mahwah, NJ: Erlbaum.

HOW TO CRACK THE CODE

Ragnhild Vikström

Department of Pedagogical, Curricular and Professional Studies, University of
Gothenburg

This presentation is about “Why young pupils without special problems cannot crack the code in mathematics in comparison to how they find the code for reading”. Many pupils who use the proceeding of “counting” for solving subtraction and additive tasks often end up with the wrong results, for instance one number back or forward. How come that some pupils easily find the answer to an open statement as $2 + _ = 9$? The purpose is to identify and explore the ability of the spatial sense for instance “to see” or “not to see”, immediate and sequential experiences of numbers and subtract and generalize (Neuman, 1987).

Previous research shows that spatial skills are regarded as important prerequisites for learning mathematics (Nes, 2010; Solène, 2011). However, little work has investigated the various underlying pathways, and particular 3D. This study is based in a qualitative research and with phenomenography as a theoretical approach with variation theory as an analytic tool. The experiment is a pilot study, an exploratory observational study with video recording of nineteen pupils aged 6 - 8 years old. They were individually interviewed and laboratory materials such as Lego, Cuisenaire, 3D blocks and also a computer, were used in order to identify and gather insight into their strategies of spatial structuring - and perception of number (Nes, 2010). The results from this research will be used in a further Design Base Study which is a typical test environment for new ideas.

References

- Nes, F.v.D.v.E.(2010). Spatial structuring and the development of number sense. A case study of young children working with blocks. *Journal of Mathematical Behavior* 29, 145-159.
- Neuman, D. (1987). *The origin of arithmetic skills. A Phenomenographic Approach*. Acta Universitatis Gothoburgensis, Göteborg.
- Solène, K.L.P.a.E.G. (2011). The visual and visuo-haptic exploration of geometric shapes increases their recognition in pre-schoolers. *International Journal of Behavioural Development*, 35 (1).18-21.

FACILITATING STUDENTS' MATHEMATICAL REASONING

Hannatjie Vorster, Trudie Benadé

North-West University, Potchefstroom Campus, South Africa

In a study at a university in RSA on the transition from school to university Benadé (2013) showed that the students were not able to do creative problem solving. Educators at all levels have the responsibility to consciously facilitate students' reasoning skills using the topics they teach. Our view of the process of teaching is based on the reduced reasoning levels of Van Hiele and Tall's three worlds of mathematical thinking (as quoted by Benadé, 2013); different types of mathematical knowledge and the geometrical skills identified by Cangelosi (2003) and Hoffer (1987). We argue that effective reasoning cannot exist without conceptual knowledge, understanding of relationships and problem solving strategies; otherwise it results in imitative reasoning.

Both Van Hiele and Tall argue that a student's understanding develops sequentially and if one of these levels/worlds is skipped, learning is impeded. It follows that at all levels each topic has to be rooted in the visual level/embodied world. It is in this level/world that understanding of concepts develops. Tall's symbolic world can be linked to Van Hiele's descriptive level. It becomes possible for the student to describe concepts by means of words and symbols discovered through inductive reasoning which the student should knowingly practice. Finally the student enters the formal third world of rigid definitions and symbol language where deductive reasoning skills have to be developed. The teaching process becomes a repeated spiral through the three worlds. Initially reasoning even in the third world is simple. Each time a new topic is added knowledge in the embodied world is broadened and reasoning in the symbolic and formal worlds becomes more sophisticated.

The poster will include a diagram organized in four parts. First the problem statement, second the description of the above mentioned theories, third a representation of the merging of the theories to facilitate the progression in the development of reasoning and fourth a representation of how the teaching of different topics could add to the goal of facilitating creative reasoning.

References

- Benadé, C.G. (2013). The transition from secondary to tertiary mathematics: exploring means to assist students and lecturers. (Unpublished PHD). North-West University, Potchefstroom, RSA.
- Cangelosi, J.S. (2003). Teaching mathematics in secondary and middle school: an interactive approach. 3rd ed. Englewood Cliffs, N.J: Prentice Hall.
- Hoffer, A. (1981). Geometry is more than proof. *Mathematics Teacher*, 74, 11-18.

THE ROLE OF GENERALIZATION IN STUDENT THINKING ABOUT AVERAGE RATE OF CHANGE

Eric Weber, Allison Dorko
Oregon State University, Oregon State University

Multivariable calculus requires students to work with systems that are more complex, yet rely on the principles learned in single variable calculus. However, how students use their knowledge of single variable calculus as they negotiate multivariable settings is not well understood. The purpose of this study was to consider how students generalize their conceptions of average rate of change from two- to three-space. Specifically, we investigated if the quantitative and covariational reasoning found to support a mature conception of rate of change in two-space (e.g., Thompson, 1994) also played a role in multivariate calculus students' ways of thinking about average rate of change.

Subjects were 16 multivariable calculus students from a pool of volunteers from six course sections at a large American university. We interviewed them in weeks 2 and 8 of a ten-week term to capture their conceptions about rate of change before and after instruction about rates of change in three-space. We asked students about the process they would follow for finding the average rate of change for given one- and two-variable functions; what average rate of change meant; and how they saw average rate of change as similar and different in different dimensions.

We found that while nearly all students set up their measurement for average rate of change of a one variable function in the same way, only 2 of the 16 students described rate of change as measuring how fast one quantity is changing with respect to another. Those two students were able to determine that in the two-variable case, they needed to find how fast f was changing in a particular direction. They used their understanding of average rate of change in the plane to reason with partial derivatives. The other students were concerned with finding "something on top and something on the bottom" and did not explicitly identify measuring how fast a quantity was changing with respect to another as their goal. Many students' generalization was based on the structure of the calculation, not quantification.

Based on these results, we suggest that multivariable calculus instructors review rate of change in two-space from the perspective of quantities changing, then extend this to rate of change in three space. We emphasize making this connection explicit to students to help them generalize. We believe that the current trend toward learning trajectories could make use of our findings and recommendations.

References

Thompson, P. W. (1994). Images of rate and operational understanding of the fundamental theorem of calculus. *Educational Studies in Mathematics*, 26, 229-274.

LESSON STUDY AND LEARNING STUDY. DOES IT MATTER?

Anna Wernberg
Malmö University

Lesson study is used in Japan by teachers to improve teaching and student learning. This form of collaboration conducted by teachers was put forward as one explanation for excellent student performance on international achievement tests in mathematics (Stigler & Hiebert, 1999). The researchers believe that lesson study develops teachers' teaching skills and thereby provides better possibilities for their students to learn. Another approach, learning study, was developed from ideas about lesson study by Marton and colleagues (e.g. Marton and Tsui, 2004). A learning study, like a lesson study, is a systematic and collaborative way for teachers to plan and revise lessons. One significant feature of a learning study is that the approach is based on an explicit theoretical framework, variation theory, exploring what students need to discern in order to learn a certain capability.

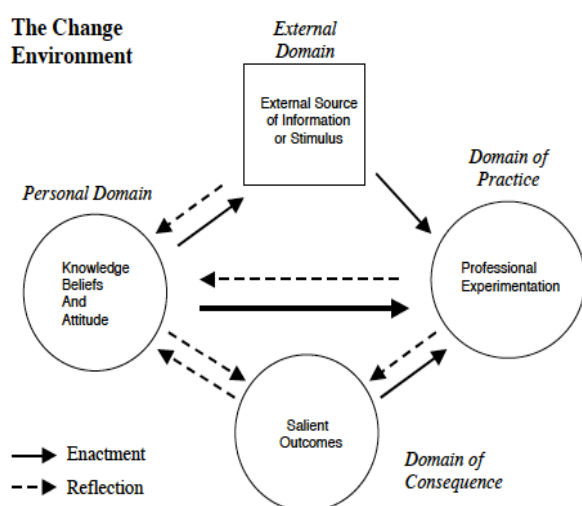


Figure 1. The interconnected model of professional growth. From Clarke and Hollingsworth (2002), p. 951

In the analysis, I use Clark and Hollingsworth's (2002) four components, from their model of teacher growth, to describe differences and similarities in the learning and lesson study approaches. My research data comes from two previously completed studies, one a lesson study and the other a learning study. I analyse the teachers' development in regard to their understanding of how students understand the content they are taught and the implications that this has on the students' possibilities to learn. One result of this study suggests that teachers' involvement in a learning study leads to a deeper understanding of the pupils understanding of the content taught.

References

- Clarke & Hollingsworth (2002). Elaborating a model of teacher professional growth. *Teacher and teacher education* 18 (947-967)
- Marton, F., Tsui, A. B. M. (2004). *Classroom discourse and the space of learning*. Mahwah, NJ: Erlbaum.
- Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: Summit Books.

STUDENTS RESISTING STUDENT-FOCUSED TEACHING PRACTICES

Richard Wester

Malmö Höskola, Sweden

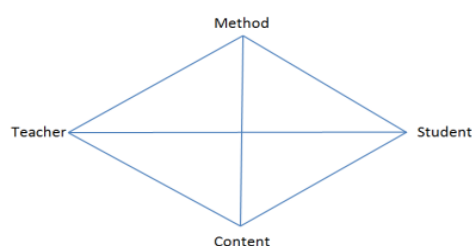


Figure 1: Model of interactions in a teaching episode.

As can be seen in Figure 1 taken from Alexandersson (1994) a teaching episode involves a meeting between the teacher, the students, mathematical content and teaching methods. In my research, I am exploring differences in teacher and student conceptions of mathematics and mathematical teaching practices. Pehkonen (2001) argues that students' beliefs and learning are connected and will influence each other. Students beliefs will be a filter through which the students perceive the teaching method, the content and the teacher.

Kilpatrick, Swafford, and Findell (2001) contend that teaching practices should promote students' development of mathematical proficiency. Efficient practices could include students discussing with each other, justifying their strategies, doing hands on activities with manipulatives, and so on. However, students may not value such teaching practices and thus not always result in improved student learning.

I am working with a Swedish teacher, who has made big efforts to change her teaching to include what Kilpatrick et al. (2001) called efficient practices. Eleven of her students in grade 8 were interviewed in May 2012, in three focus groups about their perception of mathematics teaching and how they learn mathematics. An analysis of the students' discussion was provided to the teacher in January 2013, who reflected in an interview upon the information about how the students viewed her teaching.

As a result of the interviews, obvious differences in beliefs and perceptions between the teacher and students are presented. Students' beliefs and attitudes were considerably resistant to the teachers' introduction of new teaching practices. In my poster, I will show examples of the mismatches.

References

- Alexandersson, M. (1994). *Metod och medvetande*. Göteborg: Acta Universitatis Gothoburgensis.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up : Helping children learn mathematics*. Washington, D.C.: National Academy Press.
- Pehkonen, E. (2001). In Grevholm B. (Ed.), *Matematikdidaktik : Ett nordiskt perspektiv*. Lund: Studentlitteratur.

UTILIZATION OF FIGURES DURING PROOF READING AMONG LOW AND HIGH-PRIOR KNOWLEDGE READERS

Chao-Jung Wu

National Taiwan Normal University

Lin, Wu, and Sommers (2012) reported the average ratio of total fixation duration in reading figures was about 50% while college students reading a geometric proof. However, are there any differences on utilization of figures among low and high-prior knowledge (PK) readers? and any comprehension differences between them?

The participants were 51 undergraduate students who are non-math related background. Reading materials were three geometric proofs were adapted from junior-high- school textbooks. Each of them included text and figure. The geometric ability of participants was evaluated by a PK test (10 scores). The participants' eye movement during reading were collected by Eyelink 1000. Thier comprehension was assessed by comprehensive tests (7 yes/no items each proof) and transformative-recall tests.

The high-PK (scores ≥ 8) consisted of 16 participants and the low-PK consisted of 25 participants (≤ 6). According to the result of *t*-test, comprehensive test had no PK effect. But the PK effect from accuracy of recall test indicated that high PK group had better apprehension than low PK, $t = -3.92$, $df = 40$, $p < .001$, $d = -1.24$. Based on the eye movement result of 3 way ANOVA, there is a marginal interaction of PK and IA (text and figure) on TFD, $F(1, 39) = 3.541$, $p = .067$, $\eta^2 = .083$ (Fig 1). The TFD of figure was longer than text in high PK. On the contrary, TFD of text was longer in low PK. However, the ratio of TFD in reading figures of both PK groups was about 50%.

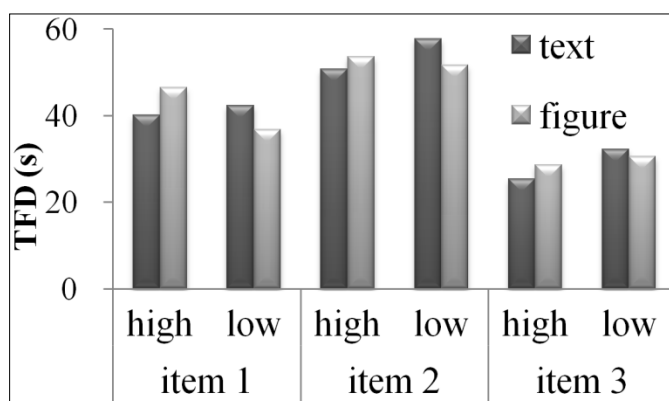


Fig 1 TFD of text and figure in high PK and low PK

The results suggest that high-PK participants were more geometric knowledge and strategic in processing in the sense that they spent relatively more time on segments of figures. On the other hand, low-PK, without the able to capture implicit properties from geometric figure and their reading were more dependent on text.

References

Lin, T. W., Wu, C. J., & Sommers, S. (2012). The influence of reading figures in geometry proof on eye movement. In T. Y. Tso (Ed.), *Proceedings of PME 36*, Vol. 3, 147-152. Taipei, Taiwan: PME.

THE STUDY OF GEOMETRIC CONTENTS IN THE MIDDLE SCHOOL MATHEMATICS TEXTBOOKS IN TAIWAN, U.S.A AND SINGAPORE

Der-Ching Yang, Ting-Yun Cheng

Graduate Institute of Mathematics and Science Education,
National Chiayi University, Chiayi, Taiwan

Mathematics textbooks play a key role in supporting students' learning and helping teachers to teach mathematics (Shield & Dole, 2013; Yang, Reys, & Wu, 2010). The textbook not only influences the teacher's teaching way and content, but also influences knowledge and method of student's study (Reys, B., Reys, R., & Rubenstein, 2010). In addition, geometry is an important topic from grade 1 to 12. Therefore, this study applied the content analysis method to compare the differences of geometric contents in the middle school mathematics textbooks among the KH (Kang Hsuan) in Taiwan, the CMP (Connected Mathematics Program) in U.S.A., and the MSN (New Syllabus Mathematics) in Singapore. Results show that the KH and MSN textbooks included over 90% of closed-ended problems. The CMP included about one-thirds of open-ended problems. The three textbooks all included about 70% to 80% of "Problems in a Combined Form" and "Problems in a Verbal Form". In addition, the CMP included the lowest percentages on the problems in a purely mathematical form. However, KH and NSM include about a half and one-fifth of problems in a purely mathematical form, respectively. The content design of KH and NSM put more emphasis on symbolic representations and provide clear procedures of algorithms and computations. The NSM also emphasizes proficiency on written computation through practice and provides students with diverse problem solving strategies. However, the CMP emphasizes daily-life problems to enhance students' learning motivation and highlights the understanding and application of mathematics concepts. Implications for possible curriculum revision and future research studies are discussed.

Keywords: CMP; Geometry; KH; mathematics textbooks; NSM

References

- Shield, M., & Dole, S. (2013). Assessing the potential of mathematics textbooks to promote deep learning. *Educational Studies in Mathematics*, 82(2), 183-199.
- Stein, M., Remillard, J., & Smith M. (2007). How curriculum influences student learning. In F. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp.319-369). Greenwich, CT: Information Age.
- Yang, D. C., Reys, R. E., & Wu, L. L. (2010). Comparing how fractions were developed in textbooks used by the 5th- and 6th-graders in Singapore, Taiwan, and the U.S.A. *School Science and Mathematics*, 110(3), 118-127

THE STUDY OF INTUITIONS IN ONE PROSPECTIVE TEACHER'S CONSTRUCTIONS OF MATHEMATICAL OBJECTS

Andonis Zagorianakos
Manchester Metropolitan University

The research is a study of the Husserlian approach to intuition, as it is substantiated by Hintikka, in the case of a prospective teacher of mathematics. It is a case study based on data collected from a course where the students were free to choose their own ways of exploring the tasks while working in groups, without the teacher's intervention. A phenomenological approach that takes objects as self-given and focuses on the student's intuitions reveals mathematical objects that surfaced from her investigation and the particular circumstances that led to these objects. The research exemplifies the two intuitive stages introduced by Husserl, while introducing a method of discerning them, and argues for a new approach to intuitions and the essential part that they play in the construction of mathematical objects.

SUMMARY

Intuition is seen as the mediator of the “overlap of my consciousness and reality” and it has two principal features emanating from Husserl's theory and an implied one accompanying the second feature, which allow us to *trace* and *classify* them as such:

- *Intentionality*, namely the directedness of consciousness towards objects, which is for Husserl a fundamental attribute of all conscious acts, especially in their ‘pregnant’ state, as intuition certainly is. Intentionality is the property that makes intuition an objectifying act: “what makes seeing an essence an intuition is not that it is seeing an *essence*, but that it is seeing an *object* which is ‘itself given’” (Hintikka, 2003, p. 181).
- *Immediacy* is the intuitions’ second critical feature, the one that distinguishes them from other concepts such as imagination, and it brings with it the *feeling of certainty*.

In contradistinction to the Kantian approach the current study adopts a Husserlian perspective, where “there is a level of experience that has not yet been subjected to the objective categories, a level of experience that is the ground of the objective categories” (Tito, 1990, p.78).

References

- Hintikka, J. (2003). The notion of intuition in Husserl. *Revue internationale de philosophie*, 2003/2(224), 169-191.
- Tito, J. M. (1990). *Logic in the Husserlian context*. Evanston, Illinois: Northwestern University Press.

INDEX OF AUTHORS AND CO-AUTHORS

Volume 5

A

Abtahi, Yasaman	1
Adagideli, Fahretudin Hasan	2
Ader, Engin	2
Ahtee, Maija	133
Aizikovitsh-Udi, Einav	3, 235
Akhtar, Zarina	4
Akita, Miyo	5
Akkoç, Hatice	162
Akkus, Recai	184
Alba, Francisco Javier	207
Albano, Giovannina	6
Albersmann, Natascha	7
Aleven, Vincent	183
Alqahtani, Muteb	230
Alston, Alice	231
Amaral, Rubia Barcelos	142
Amidon, Joel	139
Amit, Miriam	3
Andersson, Annica	8, 208
Arai, Mitsue	9
Asghari, Amir H.	35, 62
Aslan-Tutak, Fatma	28
Ayan, Rukiye	10

B

Baack, Wibke	79, 234
Baccaglioni-Frank, Anna	126
Bairral, Marcelo Almeida	11
Barak, Basak	12
Barzel, Bärbel	13, 72, 154, 271
Baştürk, Burcu Nur	14, 15
Batista, Susana	150
Bausch, Isabell	16
Becevic, Semir	17
Beisiegel, Mary	18, 209
Beitlich, Jana T.	247
Bempeni, Maria	19
Benadé, Trudie	275
Benthien, Jessica	83
Berenson, Sarah	116

Berger, Margot	20
Bergqvist, Tomas	21
Besser, Michael	22
Bianchini, Barbara Lutaif	210
Bikner-Ahsbahs, Angelika	23
Biton, Yaniv	24
Biza, Irene	84
Björklund Boistrup, Lisa	211
Blömeke, Sigrid	79, 83, 97, 234
Bochnik, Katrin	25
Bokhove, Christian	110
Bonilla González, Maricela	26
Brocardo, Joana	27, 220, 245
Browning, Christine Annette	28
Bruder, Regina	16, 135
Bruin-Muurling, Geeke	212

C

Çabuk, Ayşegül	29
Çakıroğlu, Erdinç	49, 138
Canavarro, Ana Paula	30, 213
Carrapiço, Renata Carvalho	31
Caseiro, Ana	32
Casper, Michael	33
Çelikdemir, Kübra	34
Cetinkaya, Bulent	49
Chamanara, Sepideh	35
Chan, Yip-Cheung	36
Chang, Ching-Kuch	37
Chang, Shu-I	38, 214
Changsri, Narumon	76
Charles, Mike	152
Chen, Chia-Huang	39, 215
Chen, Jain-Cheng	40, 41, 198
Chen, Ming-Feng	37
Chen, Ting-Wei	217
Cheng, Ting-Yun	280
Cheng, Ying-Hao	41, 198
Chin, Chien	217
Chiu, Mei-Shiu	42, 105
Cho, Hyungmi	216
Cho, Yi-An	217

Chumachenko, Dmitry	268
Chung, Jing	38
Cifarelli, Victor	165
Civil, Marta	43
Clarke, David	196, 235
Clarke, Doug	44
Cooper, Thomas	262
Csíkos, Csaba	45
Cwikla, Julie	46
Czarnocha, Bronislaw	218

D

da Ponte, João	
Pedro	31, 32, 47, 121
Dahl, Jonas	219
Dahl Soendergaard, Bettina	48
Dartnell, Pablo	229
Davis, Hugh	110
de Ron, anette	208
Dede, Yüksel	272
Delgado, Catarina	27, 220
Den Brok, Perry	212
Didis, Makbule Gozde	49
Diez-Palomar, Javier	188
Dindyal, Jaguthsing	101
Dogan, Muhammed F.	139
Dohány, Gabriella	45
Döhrmann, Martina	83
Dorko, Allison Jeanette	276
Dreher, Ulrike	50
Drouhard, Jean-Philippe	221
Dullius, Maria Madalena	222
Dunekacke, Simone	79, 234

E

Edson, Alden J.	28
Edwards, Julie-Ann	110
Ehret, Markus	72
Eichler, Andreas	51, 154, 163
Eisenmann, Petr	223
Ekdahl, Anna-Lena	224

Ekol, George	52
EL Mouhayar, Rabih Raif	53
Eli, Jennifer	209
Elias, Inês	150
Elliott-Ferguson, Marcella Diana ...	54
Ellis, Amy	139
Ellis, Jessica	185
Engel, Joachim	170
Erbas, Ayhan Kursat	49
Ergin, Ayşe Simge	14
Eriksson, Anna Helena	225

F

Fahse, Christian	55
Fede, Bryan	43
Fermsjö, Roger Claes	226
Fernández, Ceneida	261
Fernández Plaza, José Antonio	56
Ferreira, Sofia	150
Flens, Mikaël	57
Foley, Gregory D.	227
Fonseca González, Jaime	159
Font, Vicenç	146, 188
Forgasz, Helen	100
Forsythe, Susan Kathleen	58
Frade, Cristina	59
Francisco, John	68
Freiman, Victor	253
Fuglestad, Anne Berit	60
Fujita, Taro	90
Furlanetto, Virginia	222

G

Gavaz, Huseyin Ozan	199
Gebremichael, Andualem	
Tamiru	228
Geraniou, Eirini	61
Ghaffari, Hosein	62
Gil Cuadra, Francisco	130
Gillies, Roslyn Kay	63
Giménez, Joaquim	188

Girgin, Umut.....	184
Glasmachers, Eva	65
Godino, Juan.....	146
Goedhart, Martin	156
Goizueta, Manuel	64
Goldin, Gerald A.	160
Gomes, Alexandra	257
Gomez, David Maximiliano	229
Grassmann, Marianne.....	79, 234
Gravemeijer, Koeno	212
Griese, Birgit	65
Grisi-Dicker, Loretta	230
Grünewald, Susanne	66
Gunnarsson, Robert	85
Gutierrez, Angel	78, 207
Guzman-Hernandez, Jose	67

H

Hahkioniemi, Markus	68
Halliday, Jennifer	100
Halverscheid, Stefan.....	69
Hannula, Markku.....	260
Hargreaves, Linda.....	174
Hassidov, Dina	70
Hattermann, Mathias	71
Heinze, Aiso	254, 269
Herold, Raja.....	13, 72
Hillman, Thomas	112
Hmelo-Silver, Cindy	231
Hochmuth, Reinhard	240
Holzäpfel, Lars	50, 73
Hsu, Hui-Yu	40, 41, 198
Huang, Hsin-Mei E.....	74
Huang, Shu-Chuan	37
Huh, Nan.....	87
Hüttel, Caroline	232

I

Ilany, Bat-Sheva	70
Imai, Kazuhito	75
Inprasitha, Maitree.....	76, 178

Işıksal-Bostan, Mine	10, 29, 200
Isler, Isil.....	77

J

Jacobsen, Laura	243
Jaime, Adela	78
Jankvist, Uffe Thomas	233
Jaworski, Barbara	144
Jenßen, Lars.....	79, 234
Johansson, Helena	80
Jones, Keith	90
Jonsson, Bert	136
Juárez López, José Antonio	81
Jurdak, Murad	53

K

Kabael, Tanguel	12, 82
Kaiser, Gabriele.....	83, 97, 238
Kallweit, Michael	65
Kanellos, Ioannis.....	84
Kankaanpää, Anu	133
Karlsson, Annasara	85
Khoshaim, Heba Bakr	227
Kidman, Gillian.....	262
Kimani, Patrick M.....	28
Kirmizi, Bahar.....	199
Knight, Rose.....	44
Knuth, Eric	77
Ko, Ching-An	86
Ko, Ho Kyoung	87
Koizumi, Yuka	88
Kollosche, David.....	89
Kondo, Yutaka	90
Kondratieva, Margo	91
König, Johannes	83
Koopman, Maaïke	212
Kosaka, Masato	92
Kozakli, Tuğçe	162
Kruwan, Preecha	76
Kullberg, Angelika.....	93
Kullmann, Harry	71

Kulow, Torrey	139
Kunimune, Susumu	90
Kuntze, Sebastian	170, 235
Kusznirczuk, John	94
Kwek, Meek Lin	95
Kwon, Misun	249
Kwon, Oh Nam	137, 216
Kynigos, Chronis	236

L

Laine, Anu	133
Larson, Niclas	96, 211
Larsson, Kerstin	237
Laschke, Christin	97, 238
Laval, Dominique	98
Lazarevic, Claudia	99
Leder, Gilah	100
Lee, Chee Peng	101
Lee, Hsiu-fei	39, 215
Lee, Hyung Sook	102
Lee, Ji Eun	137
Lee, JiYoung	103
Lee, Shin-Yi	239
Lehmann, Malte	104
Leiss, Dominik	22
Leppäaho, Henry	68
Lerman, Stephen	59
Leu, Yuh-Chyn	105
Leuders, Juliane	106
Leuders, Timo	50, 106, 145
Levenson, Esther	107
Li, Xiaoqing	202
Liao, Hui-I	40
Liebendörfer, Michael	240
Liljekvist, Yvonne	136
Lin, Fou-Lai	40, 198
Lin, Pai-Tang	217
Lin, Su-Wei	241, 242
Lin, Tsai-Wen	108
Lithner, Johan	136
Liu, Minnie	109
Llinares, Salvador	261

Lo, Mandy	110
Loibl, Katharina	73
Loureiro, Cristina	153
Lu, Feng-Lin	86, 111
Lundberg, Anna L. V.	112
Lundström, Marita	113

M

Machado, Silvia Dias Alcantara...	210
Mackrell, Kate	114
Magalhães, Mafalda	150
Magnusson, Joakim	115
Maher, Carolyn A.	231
Maher, James	116
Mamolo, Ami	117
Manizade, Agida	118, 243
Mårtensson, Pernilla	93
Martignone, Francesca	126
Martinovic, Dragana	118
Martins, Cristina	244
Måsøval, Heidi Strømskag	119
Masuda, Yuki	120
Mata-Pereira, Joana	121
Matos, João Filipe	259
Matsuo, Nanae	122
Maunula, Tuula	123
Mavrikis, Manolis	61
Mayes, Robert Lee	124, 251
Mehraein, Samira	125
Meira, Luciano	59
Mellone, Maria	126
Mena-Carrasco, Gonzalo	229
Mendes, Maria de Fátima	27, 245
Menezes, Luís	30, 213
Mestre, Célia Maria	127, 246
Milani, Raquel	128
Moalosi, Shadrack	129
Monteiro, Cecília	32
Montoro Medina, Ana Belén	130
Moustaki, Foteini	236
Mudaly, Vimolan	131

N

Nagel, Kathrin M.....	247
Naidoo, Jayaluxmi.....	131
Nakawa, Nagisa.....	132
Nardi, Elena.....	84
Näveri, Liisa.....	133, 260
Neubrand, Michael.....	265
Ni, Yujing.....	202
Niedermeyer, Inga.....	134
Nieger, Annika.....	69
Nitsch, Renate.....	135
Noh, Jihwa.....	87
Norén, Eva.....	8
Norqvist, Mathias.....	136
Nortvedt, Guri.....	248
Novotna, Jarmila.....	223

O

Oh, Hye Mi.....	137, 216
Oliveira, Hélia.....	30, 127, 220
Østergård, Camilla Hellsten.....	168
Özen, Mehtap.....	138
Ozgur, Zekiye.....	139

P

Paez, David Alfonso.....	67
Palus, Marjory F.....	231
Panaoura, Areti.....	140
Panaoura-Maki, Georgia.....	140
Pang, Jeongsuk.....	103, 249
Park, Jee Hyun.....	137
Park, Joo young.....	141
Pehkonen, Erkki.....	133, 260
Penalva-Martínez, M.Carmen.....	158
Pereira, Rubina.....	250
Peres, Gilmer Jacinto.....	142
Peterson, Franziska Isabel....	124, 251
Petersson, Jöran.....	143, 252
Petropoulou, Georgia.....	144
Pettersen, Andreas.....	248

Philipp, Kathleen.....	145
Pino-Fan, Luis Roberto.....	146
Pinto, Jorge.....	147
Planas, Núria.....	8, 64
Plath, Meike.....	148
Polotskaia, Elena.....	253
Portaankorva-Koivisto, Päivi.....	260
Potari, Despina.....	144
Povey, Hilary.....	149
Powell, Arthur B.....	230
Prescott, Anne.....	16
Pribyl, Jiri.....	223

Q

Quaresma, Marisa.....	47
Quiring, Florian.....	247

R

Rach, Stefanie.....	254, 269
Rafiepour Gatabi, Abolfazl.....	125
Ramalho, Maria Gloria.....	150, 250
Rasch, Renate.....	270
Rasmussen, Chris.....	185
Rasslan, Shaker Ahmad.....	255
Rasslan-Sharif, Amal.....	255
Rathgeb-Schnierer, Elisabeth.....	232
Rau, Martina.....	183
Reinhold, Simone.....	151
Reiss, Kristina M.....	247
Rhine, Steve.....	152
Ribeiro, C Miguel.....	256, 257
Rico, Luis.....	56
Rocha, Helena Cristina.....	258
Roche, Anne.....	44
Rodrigues, Margarida Maria.....	153, 259
Rodríguez Bejarano, Jorge.....	159
Roesken-Winter, Bettina.....	33, 65, 104
Rögler, Paul.....	154
Rojano Ceballos, Teresa.....	26

Rolfes, Tobias.....	155
Rolle, Yolanda Antoinette.....	54
Roorda, Gerrit.....	156
Roth, Jürgen.....	155, 266
Ruiz-Hidalgo, Juan Francisco	56
Rummel, Nikol	73, 183
Runesson, Ulla.....	93
Ruwisch, Silke.....	134
Ryberg, Ulf Anders	157

S

Sabater, Laura.....	158
Saito, Noboru.....	5
Saló i Nevado, Laia	260
Sánchez Robayo, Brigitte Johana	159
Sanchez-Leal, Lina.....	160
Sanchez-Matamoros, Gloria.....	261
Sandhu, Satwant	262
Santos, Leonor.....	147, 244
Santos, Raquel	161
Savard, Annie	253
Şay, Rüya.....	162
Scheiner, Thorsten.....	263
Schink, Andrea	264
Schlump, Stephanie	265
Schmitz, Angela	163
Schnotz, Wolfgang	155, 270
Schorr, Roberta Y.....	160
Schuler, Stephanie	164
Schumacher, Stefan	266
Sevim, Volkan	165
Shimizu, Yoshinori.....	88
Shin, BoMi	166
Shogimen, Yusuke.....	267
Shvarts, Anna	167, 268
Siebert, Ulrike.....	269
Sigley, Robert.....	231
Skott, Charlotte Krog	168
Slisko Ignjatov, Josip	81
Sommers, Scott.....	108
Soto-Andrade, Jorge	169

Sproesser, Ute	170
Srichompoo, Somkuan	76
Starr, Colin	152, 171
Steinberg, Ruti.....	172
Steinle, Vicki.....	4
Stenkvisst, Anna	173
Steward, Susan	174
Stibric, Marina.....	175
Stoppel, Hannes.....	176
Sturm, Nina	270
Suttiamporn, Wipaporn.....	178

T

Tai, Wen-Chun	241
Takada, Tomomi	75
Takahashi, Hitoshi.....	177
Teller, Julia.....	271
Tengler, Martina.....	79, 234
Thinwiangthong, Sampan	178
Thouless, Helen Rachel.....	179
Till, Christoph	180
Torregrosa-Gironés, Germán	158
Torres Duarte, José.....	159
Treffert-Thomas, Stephanie	181
Tropper, Natalie	182
Tso, Tai-Yih	86, 111
Tunc Pekkan, Zelha.....	183
Türnüklü, Elif.....	14

U

Ufer, Stefan	25
Ulucan, Ece	184
Ulusoy, Fadime	200
Uysal, Figen	185, 272

V

V. Rohrer, Andrea	186
Valls, Julia.....	261
Vamvakoussi, Xenia	19
van Jaarsveld, Pieter Paul.....	187

Vanegas, Yuly	188
Varas, Leonor	260
Velez, Isabel	189
Verhoef, Nellie C.....	273
Vidakovic, Draga.....	82
Vikström, Ragnhild Maria.....	274
Vogel, Markus	51
vom Hofe, Rudolf.....	71
Vonk, Jennifer	46
Vorster, Hannatjie.....	275
Vos, Pauline.....	57, 156

W

Wadepohl, Heike	232
Wan, May Ee Vivien	196
Wang, Chih-Yeuan.....	190
Wang, Sasha	191
Watanabe, Koji.....	193
Watanabe-Mukai, Keiko	192
Weber, Eric.....	194, 276
Wedekind, Hartmut	79, 234
Wege, Sarah-Elena	69
Wernberg, Anna Madeleine	277
Werner, Kirsten	69
Wester, Richard	278
Williams, Caroline.....	139
Wittmann, Gerald	164

Wong, Ngai-Ying.....	36
Wood, Marcy.....	43
Wu, Chao-Jung.....	108, 279
Wu, Jeng-Shin	242

X

Xistouri, Xenia	195
Xu, Lihua.....	196

Y

Yang, Der-Ching.....	280
Yang, Kai-Lin	40, 197, 198
Yazgan, Yeliz.....	199
Yemen-Karpuzcu, Seçil	200
Yeşildere İmre, Sibel.....	15
Yohei, Watarai	201
Yosif, Manal.....	255
Yu, Yi-Chen	37

Z

Zachariades, Theodossios	144
Zagorianakos, Andonis	281
Zou, Jun.....	202

