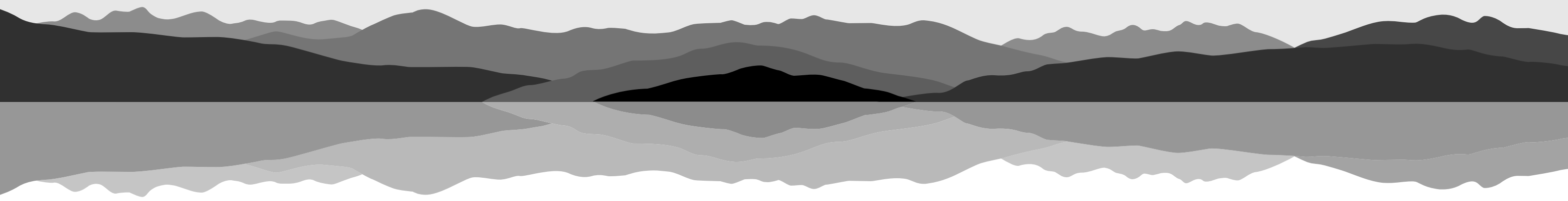
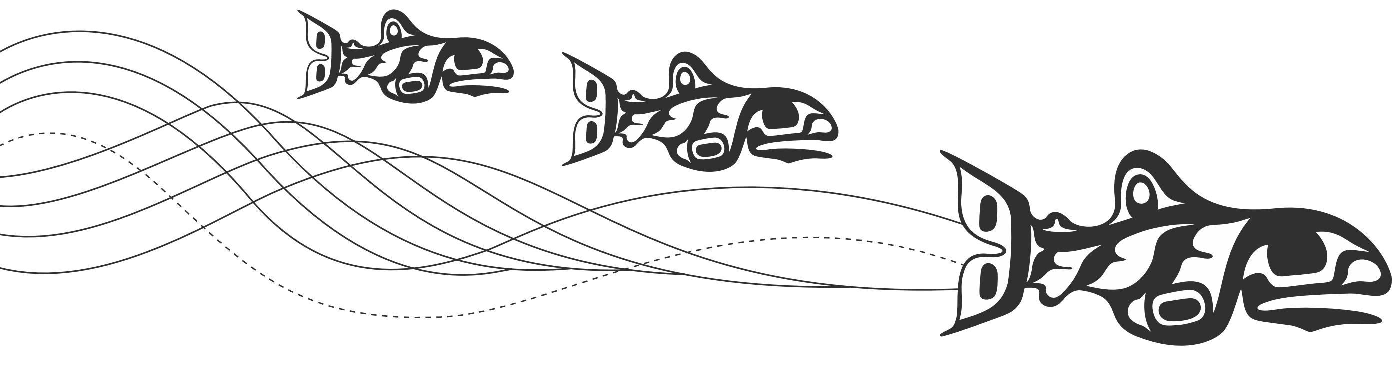


PME 38 / PME-NA 36

Proceedings

Vancouver, Canada | July 15-20, 2014

Of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education



Volume 6

Editors | Susan Oesterle, Cynthia Nicol, Peter Liljedahl, Darien Allan

PME 38 / PME-NA 36

Vancouver, Canada | July 15-20, 2014

Cite as:

Oesterle, S., Nicol, C., Liljedahl, P., & Allan, D. (Eds.). (2014). Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education (Vol. 6). Vancouver, Canada: PME.

or

Oesterle, S., Nicol, C., Liljedahl, P., & Allan, D. (Eds.). (2014). Proceedings of the Joint Meeting of PME 38 and PME-NA 36 (Vol. 6). Vancouver, Canada: PME.

Website: http://www.pme38.com/

The proceedings are also available via http://www.igpme.org

ISSN 0771-100X

Vol. 6 ISBN 978-0-86491-366-1

Set ISBN: 978-0-86491-360-9

Cover Design: Kirsty Robbins, University of British Columbia

Salmon Design: Haida artist, William (Billy) NC Yovanovich Jr. Ts'aahl

Formatting Assistance: Robert Browne, Jacky McGuire

Production: Lebonfon, Québec

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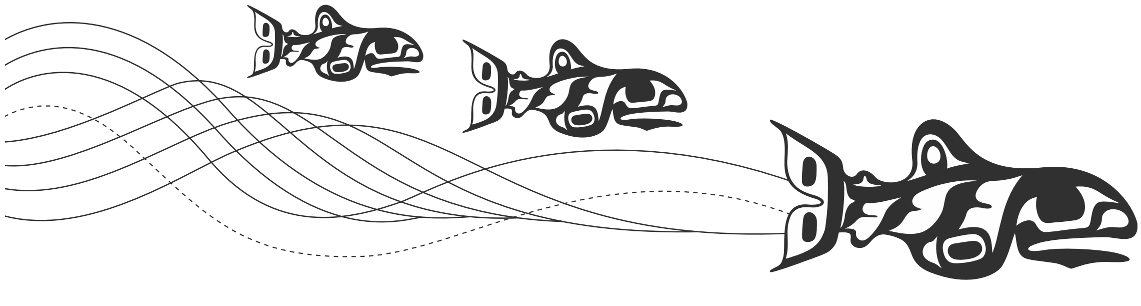

MEASURING CHANGE IN MIDDLE SCHOOL STUDENTS' ATTITUDES TOWARD MATHEMATICS OVER TIME

Aysha Abdul Majeed
The University of Adelaide, Australia

Research recognizes that attitudes toward mathematics have powerful impact on effective engagement and achievement in mathematics (e.g. Reynolds & Walberg, 1992). The present study adopts Neale's (1969) definition which examines students' attitudes toward mathematics, as a multidimensional construct with four dimensions; self-confidence, value, enjoyment and motivation.

The instrument *Attitudes Toward Mathematics Inventory* (ATMI) was considered in relation to Neale's definition of attitudes toward mathematics, so that statistical results can be related to the theoretical concerns that led to the research. Studies reveal that Years 7 and 8 are significant in development of negative disposition toward mathematics (Hembree, 1990), hence the sample of the study. A total of 544 students of Years 7 and 8 in 13 schools in South Australia participated in the three cycles of data collection. The purpose of this paper is to examine change in the students' attitudes toward mathematics over time and to analyse gender and grade related changes. Repeated measures ANOVA was employed to determine whether or not change took place over time. The analysis showed decline of positive attitudes toward mathematics in the mean scaled score for the sub-scales of ATMI, except for the sub-scale of *Value*. The sub-scale of *Self-Confidence* declined over the academic year but did not show statistical significance. The sub-scales, *Enjoyment* and *Motivation* showed decline which reached statistical significance at 95% confidence interval. The analysis revealed that a negative disposition set in the middle school years in just one academic year. The present investigation signifies growth in the Australian research literature by providing empirical evidence about the development of attitudes toward mathematics.

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FEEDBACK FROM MATHEMATICAL ARTEFACTS

Yasmine Abtahi
University of Ottawa, Canada

Among all the topics in elementary mathematics curriculum, fractions are the most mathematically complex concepts. Cramer & Wyberg (2009) argued that it is crucial for students to use different representations, including artefacts, while learning fractions. Artefacts are: "objects that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (Swan & Marshall 2010, p. 14). Cramer & Wyberg (2009) noted that the physical structures (e.g., colours, sizes) of mathematical artefacts have strengths and limitations. I refer to the relations between the physical components of the artefact as "feedback"; for example, the relations between the colours and sizes in fraction circles. I performed a meta-analysis of 52 fraction-related tasks drawn from literature to examine the role of feedback from the artefacts in understanding of fractions. My research question was: How does the feedback from the artefact play a mediating role between the children's interaction with the artefacts and their understanding of fractions? I used a framework developed by Martin and Schwartz (2005) to study how physical action can support understanding of fractions, figure 1. My analysis showed the recursive roles of feedback. Children used the feedback from the artefact to: a) modify (i.e., make changes to) the artefact, b) to construct new interpretation of the task or to solve the problem, figure 2. Furthermore, findings showed that children used the feedback from a mathematical artefact according to their initial idea of the task and of the artefacts. Further studies are required to examine the role of feedback in realising the affordances and constrains of different mathematical artefacts in understanding of different fractions concepts.

Ideas		
Adaptable	Induction	Physically Distributed Learning
Stable	Off-loading	Repurposing

Figure 1: Stable *Environment* Adaptable

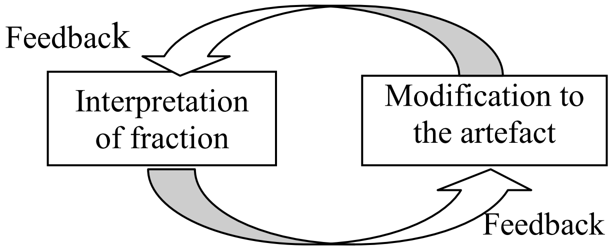

Figure 2: The recursive role of feedback

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ELEMENTARY SCHOOL TEACHERS' PERCEPTIONS OF THE CAUSES AND EFFECTS OF MATHEMATICS ANXIETY: A SOCIOCULTURAL PERSPECTIVE

Atinuke Adeyemi
University of Windsor

Elementary mathematics classrooms appear to be the starting points of mathematics anxiety as elementary teachers experience different levels of mathematics anxiety that could be transferred to their students (Hadley & Dorward, 2011). This could have negative consequences on the students' achievement and attitudes toward mathematics. While most studies on mathematics anxiety have been conducted with pre-service teachers, college students, adolescents, and school children, few studies have involved elementary school teachers; thus, necessitating the need to further explore potential factors, contributing to the development of mathematics anxiety among in-service elementary teachers in Canada.

The study aims to examine the levels, causes, and effects of mathematics anxiety that exist and how it differs by gender and other demographic factors, among in-service elementary school teachers in Southern Ontario. The research questions are: i) What are the levels of mathematics anxiety among in-service elementary school teachers and how do teachers' mathematics anxieties differ in view of varied sociocultural factors? and ii) What are the self-reported causes of mathematics anxiety and the effects on the perceived teaching abilities of in-service elementary school teachers?

This study will draw on sociocultural theory by Vygotsky (1981) to understand the social and cultural factors, such as ethnicity and parental influences, which contribute to teachers' mathematics anxiety. A sample of about 472 in-service elementary teachers in Grade 1 to 8 in a Public School Board is expected to participate in the study. Data will be collected with survey instruments and through individual interviews and analysed using descriptive and inferential statistics as well as thematic analysis.

The findings are expected to provide substantial knowledge of the levels, development, and the causes of mathematics anxiety that would help in creating manuals and planning appropriate interventions programs and strategies that would help alleviate mathematics anxiety among teachers. This study will contribute to the body of research and knowledge on mathematics anxiety among elementary school teachers, particularly regarding various sociocultural sources of mathematics.

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TEACHING THE EQUAL SIGN: WHEN DOES TELLING WORK?

Emmanuelle Adrien, Kim Desmarais, Allyson Cooperman , Helena P. Osana
Concordia University

Children often have difficulty solving non-canonical equivalence problems (e.g., 9 + 1 = 6 + •) because of deep-seated misunderstandings of what the equal sign ("=") means. Our research program addresses the ways in which elementary students respond to different types of instruction about the meaning of the equal sign as a relational symbol. Some evidence indicates that "telling" students what it signifies can be effective, but engaging students actively in discussions that challenge their thinking (called the "conceptual change approach"; Vosniadou & Verschaffel, 2004) can promote conceptual reorganization. The goal of the present study was to explore the conditions under which telling students how to think about the equal sign is beneficial in the context of a lesson based on the conceptual change approach.

Twenty (N = 20) second-grade students met with the third author for a 60-minute individual interview. To obtain baseline data, she first asked the students to solve equivalence problems presented symbolically and non-symbolically (i.e., with blocks). During the second part of the interview, the researcher engaged the students in a discussion during which they were asked to explain why they thought a series of non-canonical equations were true or false. Based on their justifications, the researcher presented subsequent equations to challenge the students' thinking with the objective of modifying, or re-organizing, their conceptions about the equal sign.

At certain points during the lesson, the researcher told the students directly how to think about the equal sign. We focused our analysis on the nature of four students' understanding at these specific points in time. Two students, the *Understanders*, acquired a relational understanding of the equal sign soon after they were told how to think about it; in contrast, the *Non-Understanders* created a rule that they rigidly and inappropriately transferred to other problems. Differences in prior knowledge can account for these results. The Non-Understanders were unable to solve any of the symbolic or non-symbolic problems in the first part of the interview, and struggled with counting and computation throughout. The Understanders had more initial knowledge about mathematical equivalence, more efficient computational strategies, and were more articulate about their own, sometimes even incorrect, understandings of the equal sign. In sum, telling students how to think about the equal sign may be most effective when they have the conceptual prerequisites that will enable them to modify their existing theories into correct relational understandings.

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A STUDY ON ROLE OF RELATIONAL REPRESENTATION ON CREATIVITY IN MATHEMATICS EDUCATION

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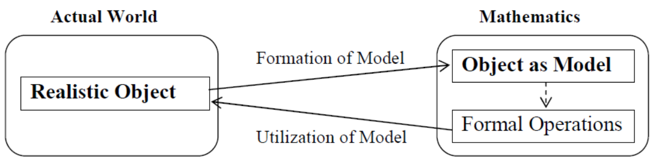
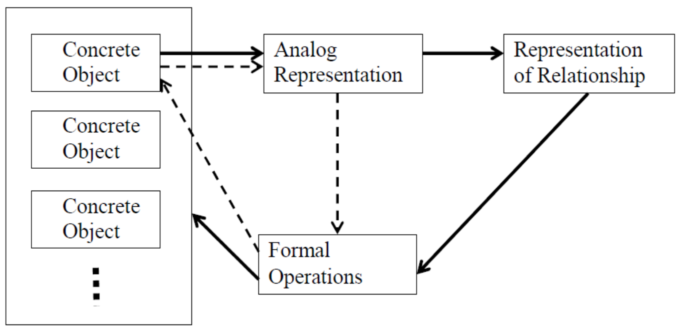
The purpose of this study is to show how students can enhance mathematical creativity. Flexibility is an important factor for creative problem solving. Many researchers suggest that flexibility is an important factor for creative problem solving (Torrance, 1976). However, in the lesson of the present school mathematics, students' flexible thinking is hardly used.



In mathematics, we translate the realistic object into the object as model. A model is a representation of the essential aspects of a real object. This will allow us to use all the results that someone proved. Models can also shed light on the mathematical structures themselves. This will allow us to use the results for other problem solving. It is important that to find out the representation of the essential aspects from the realistic object for addressing the object as model. The representation is classified into two broad categories, the analog representation and the digital representation. In a mathematical class, the representation of relationship must be formed in a student's brain.



We tested the "Representation Model in Mathematics learning" using in real situation of learning the perpendicular bisector. Students had the tendency to memorize a solution without understanding the mathematical background. The dotted line in the "Representation Model in Mathematics learning" shows the situation of the representation of most students. The student was merely solving as the procedure. The student was not able to consider a solution by himself/herself, when he had forgotten the procedure. This fact has given a big warning to teaching mathematics. Even if a student has a correct answer, he/she does not necessarily understand leaning contents.

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THE GEOMETRIC PERFORMANCE OF FIFTH TO EIGHTH GRADERS

Ayman Eleyan Aljarrah
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This paper is a response to the difficulties I faced in my attempts to assess students' van Hiele levels of geometric thought (Aljarrah, 2001). I determined that it may be possible to use the students' performance on a geometric thinking levels test as criteria to make judgements about their geometric thinking levels, and about the characteristics of those levels. My study was driven by the following questions: 1) Does students' performance on a geometric thinking levels test differ with respect to their grade levels? 2) Does students' performance on a geometric thinking levels test differ with respect to different geometric concepts? And 3) Does students' performance on a geometric thinking levels test differ with respect to different parts of the test that measure different levels of geometric thought?

The original descriptive survey research was conducted with a population of 5th to 8th grade public education students in the city of Irbid/Jordan. Two-stage cluster sampling was used in selecting the 600 participants. To build the test, performance tasks were identified based on the description of the first four levels of Van Hiele's model of geometric thought: recognition, analysis, ordering, and deduction (Khasawneh, 1994). The test was built based on three geometric concepts: the angle, the triangle, and the rectangle. The final version of the test included 39 items; apportioned equally across the three concepts. The score on each item was either 0 or 1.

The analysis of variance ANOVA was used to compare the means of the scores of the four grade levels. Analysis indicated significant differences in the students' performance on the geometric thinking levels test due to grade level, $F(3,596) = 3.12$, $p < .05$. A repeated-measures ANOVA was used to compare the means of students' scores on each of the three concepts. It indicated significant differences in students' performance on the geometric thinking levels test due to geometric concept, $F(2,1198) = 230.80$, $p < .05$. Also, a repeated-measures ANOVA test was used to compare the means of students' scores on each of the four parts of the geometric thinking levels test. It indicated significant differences in students' performance on each of these parts due to the geometric thinking level, $F(3,1797) = 802.43$, $p < .05$.

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DESIGN AND USE OF AN INSTRUMENT FOR EVALUATING STUDENTS' MATHEMATICAL CREATIVITY IN THE CREATIVE MODELING PROCESS

Miriam Amit, Talya Gilat
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Rapid progress through scientific discoveries and the proliferation of technological innovations that are globally transforming our lives highlight the crucial need for developing and encouraging students' creative potential.

Model eliciting activities (MEAs) give the student opportunities to deal with non-routine, open-ended “real-life" challenges (Lesh & Doerr, 2003). These authentic problems encourage the student to ask questions and be sensitive to the complexity of structured situations, as part of developing, creating and inventing significant mathematical ideas (Amit & Gilat, 2013). A prerequisite for the development of any skill or ability is an evaluation system (Guilford, 1967). However, despite the importance of evaluating students' mathematical creativity, which manifests itself in the creative mathematical-modeling process, and its original outcomes, such evaluations are limited (Coxbill, Chamberlin, & Weatherford, 2013). Therefore the aim of this paper is to explore the design and use of an instrument for evaluating students' mathematical creativity as manifested in the creative modeling process. The instrument, termed Mathematical Creativity Evaluation Model (MCEM), was developed as part of a more inclusive intervention study aimed at exploring how experience in model-eliciting activities (MEAs) of “real-life” situations affects students' mathematical creativity. The MCEM is based on the researchers' operational definition factors: Appropriateness, Mathematical Resourcefulness and Originality, and utilizes students' qualitative modeling products to evaluate and score their mathematical creativity. This instrument was successfully applied for 157 students, and may be further developed to assess other similar creative mathematical problem-solving processes.

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EIGHTH-GRADE MATHEMATICS TEACHERS' REFLECTIONS ON PRACTICE

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In a review of literature investigating the following teacher question from the National Council of Teachers of Mathematics' (NCTM) *Linking Research & Practice* (Arbaugh, Herbel-Eisenmann, Ramiriz, Knuth, Kranendonk, & Quander, 2009) report: "What is the effective way to break the cycle of teaching based on how the teacher (lecture mode) was taught" (p. 19), we found that reflection is a key ingredient in teacher change. Using Rodgers' (2002) definition of reflection based on Dewey's (1933) work, we analyzed cases from mathematics practitioner journals that illustrated teachers' changed practice to discover how teachers' practice could change and the role of reflection in contributing to their change. For our study, we will use Rodgers' definition to examine the practice of three 8th-grade teachers who have different amounts of experience. Our research question is: How does reflection as defined by Rodgers (2002) manifest in three 8th-grade mathematics teachers' teaching practice?

Using an interpretive case study design, we will observe the three teachers' classroom teaching, interview them about reflection in their practice, and collect artifacts (e.g., lesson plans and handouts) from their teaching. In addition to field notes, researchers will also keep observation journals to further track the impact of teachers' reflection on their teaching practice. Transcription and artifact chunks that focus around the same idea/activity will be coded according to Rodgers' (2002) reflection definition criteria.

Possible themes that are expected to emerge from the data include impacts of reflection on teachers' selection of teaching strategies, class collaborations, task design, and attitudes toward their professional development. We expect mathematics teacher educators may gain insight from this study regarding the role of reflection in teachers' practice, and how it evolves over teachers' careers. This insight may help to craft professional development opportunities that support practicing teachers in developing competencies that more closely reflect reform practices (Arbaugh, et al., 2008).

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PROSPECTIVE MATHEMATICS TEACHERS' ADAPTIVE EXPERTISE

Glenda Anthony, Jodie Hunter, Roberta Hunter
Massey University

Adaptive teachers who are "driven by the moral imperative to promote the engagement, learning, and well-being of each of their students" (Timperley, 2013, p. 5) are crucial in addressing the vision of mathematical proficiency for all students. However, although adaptive expertise is identified as a critical component of quality teaching understanding how best to support prospective teachers (PTs) develop adaptive expertise is emergent. In this paper we use Timperley's (2013) framework of expertise to examine one PT's learning within a classroom inquiry (CI). Making the developmental trajectory of adaptive expertise explicit is an important first step in understanding how we might occasion such learning within initial teacher education.

A part of a design experiment to support PTs learn the work of ambitious mathematics teaching, we implemented a series of practice-based cycles of rehearsal, planning and school-based enactments of high-leverage practices (McDonald et al., 2013), followed by an CI focused on implementing rich group tasks and mathematical discourse. A content analysis of the dataset (PTs' assessment journals and media presentations, field notes of PTs' teaching sessions and feedback sessions, and interviews with 14 of the 23 PTs at the beginning and end of the CI) mapped PTs shift in focus (i) from self to students and (ii) to increasingly complex understanding about teaching and learning.

Selected as a PT who initially lacked self-confidence in mathematics, the 'telling' case illustrates how the practice-based inquiry approach (Zeichner, 2012) supported the PT to be able to use confronting moments as opportunities to reconstruct her conceptual framework—through integration of practice and theoretical evidence. In particular, the PT was able to move away from believing that her own learning experiences need be the norm. Her consideration that learning might be more than an individual experience was accompanied by a move towards a more socially just view of mathematics participation. These findings provide added weight to calls for increased practice-based approaches within teacher education.

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WHAT IS THE ROLE OF MATHEMATICS IN A MODELLING PROJECT? THE EMERGENCE OF CONTRADICTIONS IN AN ACTIVITY SYSTEM

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Developing modelling projects guided by critical mathematics education (Skovsmose, 2005) does not mean simply to represent aspects of reality using mathematical concepts. It means also to address problems arising from students' reality at the same time that students question the extent to which mathematics is determinant in the solution that they have found. In this way, students may reflect upon, and discuss about, mathematics in action (Skovsmose, 2005). A modelling project guided by critical mathematics education was carried out in the context of a research that has the general objective to characterize and analyse expansive learning (Engeström, 2001), according to Cultural Historical Activity Theory (CHAT), lived by students while they develop the modelling project. According to Engeström (2001), contradictions are a driven force to expansive learning and our focus, in this communication, is the possibility of emergence of contradictions in a modelling project. In this study, the objective is to analyse the actions performed and the statements made by one of the students (Rafael), related to what was, for him, the role of mathematics in the project. Regarding methodological aspects, the context of the research was the development of the modelling project by five students (Rafael was one of them) from three different courses of the Exact Science area of the university. The construction of data was made by means of observation, written report by the students and interviews. Our analysis was based on a few episodes chosen from the data. The results point out that Rafael seemed to support some characteristics of ideology of certainty, as proposed by Borba and Skovsmose (1997, p. 18). According to this ideology, "mathematics can be applied everywhere and its results are […] better than the ones achieved without mathematics." Furthermore, we raise the hypothesis that Rafael's actions and statements led to the emergence of contradictions in the activity system of the modelling project, since one of the worries of critical mathematics educations is to question the use of mathematics to sustain certainty when it is applied to solve real problems.

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MULTILINGUAL CHILDREN'S MATHEMATICAL REASONING

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Many educators often assume that children should be competent in the language of instruction before they engage with mathematical content. A review of recent research in this area demonstrates that the knowledge of the language of instruction is only one aspect of becoming competent in mathematics (Moschkovich, 2007). Drawing on a sociocultural understanding of learning and development allows us to move away from what multilingual learners cannot do, and instead focuses our attention on describing the multiple resources multilingual children use to communicate mathematically (Moschkovich, 2007). It also focuses our attention on the role of context in the development of children's expression. The research question for this study was: How do multilingual children express mathematical reasoning during collaborative problem-solving?

Two multilingual families, each with 3 children between the ages of 8 and 12, participated in a mathematical problem-solving activity. The language spoken at home for one family was Farsi, and for the other, Arabic. In my analysis of the children's discursive productions in terms of informal reasoning categories (Voss, Perkins, & Segal, 1991), we see that the children produced more claims than any of the other reasoning categories (e.g., question, analogy, hypothesis). The claims were not produced in isolation, but rather one claim often provided the evidence for another claim. This suggests that the children were developing a form of argument supported by evidence. The source of the evidence came primarily from the children's mathematical knowledge and world knowledge. The findings also show that during the mathematical problem-solving activity, the children in both families spoke predominantly in English, which suggests that the students' experiences and the context in which the mathematical activity took place influenced the use of language (Planas & Setati, 2009).

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EXAMINING THE RELATIONSHIP AMONG TEACHERS' MATHEMATICS BELIEFS, THEIR EFFICACY BELIEFS AND THE QUALITY OF THEIR INSTRUCTIONAL PRACTICES

Fetiye Aydeniz
Indiana University

Recognition of the significance of teachers' beliefs to mathematics education has informed the ways teacher educators design courses and experiences for teachers in training and how professional developers work with teachers in practice. The findings from a substantial body of research suggest a strong relationship between teachers' beliefs about mathematics teaching and learning and the ways they influence their teaching practices (Pajares, 1992). However, questions still remain about the strength of this relationship, whether beliefs are predictive of instruction and what other factors influence teachers' decisions and actions (Philipp, 2007). In this regard, this study is guided by these specific questions:

1. How do inservice teachers articulate their beliefs about mathematics, their knowledge of mathematics, and their ability to influence student learning?
2. How do inservice teachers teach mathematics, and what are the factors that influence their teaching practice?
3. How consistent are inservice teachers' mathematics beliefs and practices, and what factors influence the degree of consistency?

This research will be a collective case study as it involves more than one case at one site. I will analyze data of six teachers working in the same school, which are a part of a larger professional development project. Data will be drawn from three main sources: teacher interviews, teacher efficacy surveys, and video recordings of classroom instruction. Data from inservice teachers' semi-structured interviews will be analyzed using the thematic analysis with open coding technique. MQI (Mathematical Quality of Instruction) will be used to analyze teachers' classroom instruction. Additionally, the data generated by teachers' efficacy surveys will be analyzed using descriptive statistics to measure teachers' beliefs about their mathematics knowledge and their ability to influence their students' learning. Based on preliminary analysis, there appears to be greater alignment than misalignment between teachers' mathematics related beliefs and their instructional practices. However, this alignment does not always bring about high quality instruction. In the presentation, these results will be discussed in detail.

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TEACHERS' CHALLENGES WITH PROPERTIES OF OPERATIONS & CHILDREN'S MULTIPLICATIVE STRATEGIES

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The standards in *Common Core State Standards* (CCSS) (National Governors Association, 2010) emphasize that students in grades 3-5 should be able to develop multiplication and division strategies based on properties of operations, before focusing on the standard algorithms. This means that teachers need to understand mathematics embedded in student strategies, so that they can identify strategies based on properties of operations, and develop instruction to promote them. A number of researchers discussed that teachers need mathematical knowledge for teaching to be able to meet such demands (e.g. Ball, Thames, & Phelps, 2008). This paper aimed to portray teachers' mathematical knowledge that emerged when they analysed students' multiplication and division strategies.

The data of this study are from author's work with 3rd-5th grade teachers in US who participated in a 3-year professional development program, focusing on promoting children's mathematical thinking. This study investigated teachers' challenges when they attempted to analyse student strategies for multiplication and division in written and video samples in terms of expectations in the CCSS after they discussed distributive and associative properties of multiplication.

The analyses reveal that the participating teachers were more successful in identifying student strategies based distributive property of multiplication over addition. For example, most teachers identified the distributive property in a third grader's strategy, 8x100 = 800, 8x25= 200, 800+200 = 1000, for 8x125=☐. However, they had more difficulty when another third grader solved 8x24=☐ based on 8x25=200 and 200-8=192. For division strategies, the teachers were more successful identifying embedded properties when students solved division as multiplication with a missing factor (e.g. 108÷6=☐ as ☐x6=108). However, it was difficult when students decomposed the dividend and summed up partial quotients. For example, for 6104÷4, when a fourth grader decomposed 6104 into 4000, 2000, 100, and 4, divided each by 4, and added the partial quotients, most teachers did not think the strategy was based on any properties of operations. The results indicate unique mathematical challenges that teachers face, and call for more support.

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EFFECT OF MATHEMATICS CONTENT KNOWLEDGE ON MATHEMATICS PEDAGOGICAL CONTENT KNOWLEDGE

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Knowing mathematics is not enough for effective teaching and a teacher should make his own knowledge understandable for students. Shulman (1986) defines Pedagogical Content Knowledge (PCK) as the knowledge which allows a teacher convert his own content knowledge to a form that students can understand. PCK consists of several components (such as Students' Knowledge, Lesson's Organization ...) and therefore it includes, among others, a teacher's understanding of how a student learns a subject, how to design and manage a lesson. Literature emphasizes that teachers need to have deep and wide Mathematics PCK (MPCK). Recently, researchers recommended carrying out qualitative studies investigating field experience of candidate teachers (e.g. Youngs & Qian, 2013). In line with these recommendations, this study aims at examining how pre-service elementary classroom teachers' Mathematics Content Knowledge (MCK) affect the components of MPCK.

The researchers worked with 12 pre-service teachers practicing in 5th grade mathematics classrooms through Teaching Practicum course. Observation, interview and field notes were used as data collecting tools. Each pre-service teacher was observed for 3-4 hours and was interviewed about their instruction following each lesson. Qualitative data gathered were analyzed with induction and deduction methods.

The pre-service teachers' lack of MCK made it difficult for them to use MPCK effectively. Their weaknesses in MCK were apparent in their evaluation and interpretation of the students' responses and explanations, as well as in giving feedback to these comments. Similarly, in cases where pre-service teachers' MCK was good, but their MPCK was insufficient, the lessons were also ineffective. In these instances, they had difficulties in reducing the level of instruction to the level of the students, and they used some knowledge which had not been mastered by the students. Furthermore, their lack of planning knowledge sometimes caused them to embed too many activities in one lesson. All the pre-service teachers had difficulties in using mathematical terminology while they were giving instructional explanations.

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A MATHEMATICAL LITERACY EMPOWERMENT PROJECT

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Quantitative literacy, defined here as numerical, mathematical and financial literacies (NMFLs) are pressing economic, social and cultural challenges. There is a strong link between citizens' basic numerical or mathematical abilities and their financial prosperity and civic engagement. It is becoming evident that quantitative literacy is increasingly necessary for fully contributing citizens of tomorrow (Steen, 2001). This study applied a qualitative research approach to describe personal flourishing (Grant, 2012) which includes participants making meaning and sense of important aspects in their lives, having a positive identity, and being financially stable. Personal flourishing was investigated within the context of numerical, mathematical, and financial literacy (NMFL) education.

Four families participated in a weekly evening program (eight consecutive weeks). Count On Yourself (COY) was designed to inform participants about NMFLs. COY provided a parallel program for adult and child literacy: while parents were involved in a financial literacy course, their children participated in a *Math Camp* led by teacher candidates from a local university. The goal was for both adult and child participants to become more mathematically empowered. Analysis of the interview and focus group qualitative data showed that participants described a sense of personal flourishing, gained confidence and skills, and felt financially empowered enough to teach/transfer that knowledge to their children. The author proposes a conceptual framework linking personal flourishing with quantitative literacy and the participants' voices, and suggests that this framework be used in future studies to investigate and describe participants' personal flourishing with respect to NMFLs. This initiative integrated Freirean research and practice and addressed the practical needs of a community. A robust social justice research vision was applied to offer numerical, mathematical and financial literacies in a community.

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MATHEMATICS GRADUATE STUDENTS ENACTING A NEW PARADIGM OF INSTRUCTION IN REDESIGNED COURSES

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Redesigned courses that call for increased student engagement in mathematical practices have new roles for mathematics graduate teaching assistants (MGTAs) that steer away from lecture-based instruction. Researchers describe instructional practices for such courses as facilitating the development of students' competence and confidence in their problem-solving abilities, creating a classroom atmosphere conducive to exploratory learning, and providing student-centered, activity-based instruction (Ganter & Haver, 2011). However, such practices are difficult to enact as many MGTAs have not experienced this type of instruction, and researchers have suggested offering professional development opportunities to support redesign efforts. Thus, a two-day professional development seminar was created that described the rationale for the redesigned course, illustrated the practices listed above through videos of instruction, and emphasized the National Research Council's (1999) three key findings of how students learn. Additionally, 10 two-hour support meetings with similar emphases were offered over the duration of the term as the MGTAs taught the course. The research questions for this study addressed MGTAs' views of: (1) teaching and learning, (2) their roles and students' roles in the classroom, and (3) the challenges they faced as they taught a redesigned college algebra course.

Four interviews were conducted over the length of the term with MGTAs assigned to the course. Interview transcripts were analysed using thematic analysis (Braun & Clarke, 2006). Findings indicate that MGTAs found the transition to the redesigned course to be challenging, but particularly inspiring in learning about how students learn. They also expressed great enthusiasm for working with students in new ways. However, despite the professional development support, some MTGAs' enactment of the new paradigm of instruction was mitigated by their previous lack of exposure to models of student engagement as well as their own successes in lecture-based courses, which prevented them from understanding the need for the redesign.

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"AT MOST" AND "AT LEAST"

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This paper describes a pilot study whose aim is to examine mathematics teachers' understanding of the quantifying conditions "at most" and "at least". This research was instigated after observing a recurring mistake among teachers attempting a sorting task whose instructions included these conditions. It is accepted that children's understanding of numerically quantified expressions takes many years to develop. Teachers must be able to develop their students' understanding of these quantifying conditions and to do so must themselves have a well-based knowledge, must know meanings for terms and explanations for concepts (Ball, Thames, & Phelps, 2008).

A sorting activity was given to 28 primary school mathematics teachers and 17 student teachers. They were shown a picture of 16 quadrilaterals (adapted from Britton & Stump, 2001) and were requested to sort them into four categories (at least one right angle, no right angle, two pairs of parallel sides, at most one pair of parallel sides) in a 2X2 table.

The participants understood the condition "at least" but not the condition "at most" They understood "at most one" to include "one" but not "none". One of the most frequent occurring mistakes (16 teachers and 10 student teachers) was not including quadrilaterals with no parallel sides in the category "at most one pair of parallel sides". In subsequent discussions with the student teachers it was discovered that this non-understanding of "at most" occurred in a mathematical context but not in a real-life situation.

We strongly recommend a more extensive research with a wider participant base. Since children's understanding develops slowly and since the learner will meet these two quantifying conditions frequently in his school career in different branches of mathematics such as calculus, probability, etc., we recommend developing tasks that aim to deepen the knowledge and understanding of these terms as early as primary school.

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MATHEMATICS VERSUS MISCHIEF IN THE SECONDARY CLASSROOM: A STUDY OF TEACHERS' PRIORITIES

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Our research programme (Nardi, Biza & Zachariades, 2012) examines teachers' priorities when they make decisions in the secondary mathematics classroom. We use tasks in which we invite teachers to consider a mathematical problem and typical student responses to the problem. Tasks are in the format of fictional classroom situations. We collect written responses to the tasks and conduct follow-up interviews.

The task we discuss here aims to elicit perspectives on a key issue (e.g. Jaworski, 1994): classroom management often interferes with commendable learning objectives. The task is as follows. A class is asked to solve the problem: "When p=2.8 and c=1.2, calculate the expression: $3c^2+5p-3c(c-2)-4p$". Two students reach the result (10) in different ways: Student A substitutes the values for p and c and carries out the calculation; Student B simplifies the expression first and then substitutes the values for p and c. When Student A acknowledges her difficulty with simplifying expressions, Student B retorts offensively ("you are thick") and dismissively ("what can I expect from you anyway?"). Both solutions are correct and Student B's approach particularly demonstrates proficiency in important algebraic skills. But Student B's behaviour is questionable. Respondents to the task (21 prospective mathematics teachers) were asked to write, and then discuss, how they would handle this classroom situation.

Responses were analysed in terms of: *balance between mathematical* (arithmetic vs algebraic solution) *and behavioural* (verbal mischief verging on offensive treatment of a peer) *aspects*; and, *social* and *sociomathematical norms* (Cobb & Yackel, 1996) which participants intended to prioritise in their classroom. While most participants discussed *social* (e.g.: peer respect; value of discussion) and *sociomathematical* (e.g., value of different solutions in mathematics) *norms*, 10 of the 21 focused almost exclusively on *behavioural* aspects and made limited or no reference to *mathematical* aspects. We credit this type of tasks with allowing this insight into teachers' intended priorities and we posit that this lack of *balance* between mathematical and behavioural aspects merits further attention both in teacher education and research.

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CRITTER CORRAL: DESIGN AND EVALUATION OF AN IPAD GAME FOSTERING PRE-K NUMBER CONCEPTS

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Critter Corral (CC) is a free iPad app that helps Pre-K children develop number sense through gameplay. It targets important number concepts including cardinality, relative magnitude, numeral identification, estimation, and 1:1 correspondence (Pupura & Lonigan, 2013.) Children fix a Wild West town by helping its 5 businesses (games). In creating CC we applied 3 research-based design principles:

- Games integrate multiple number concepts and ways of representing number (Griffin, Case, & Siegler, 1994.)
- Numerical tasks are integral to the story and help characters achieve meaningful goals, such as cooking the right amount of food.
- Feedback helps children attend to relative magnitude, rather than simply focusing on correct/incorrect (author).

A study this fall involved 209 transitional kindergarteners (age 4-5) in 14 classes in an urban district. After a pre-test, 7 classes played CC for 5 weeks (Cohort A), and 7 classes did not (Cohort B). Five iPads were kept in each Cohort A classroom. Teachers had students play for 15 minutes, 3-4 times/week (when and how at teacher discretion.) Students completed a mid-test. Then they switched - Cohort B played the game for 5 weeks, while Cohort A did not. This was followed by a post-test.

At the 5 week mid-test, students who played CC (Cohort A) showed higher gains than students who did not, $F(1,201)=5.2$, $p=.02$. After the crossover, Cohort B played CC and showed similar gains from mid- to post-test. We are currently examining in-game process data to look for patterns of student actions and learning.

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THE IMPACT OF A NON-GERMAN FAMILY LANGUAGE ON FOUR FACETS OF MATHEMATICAL COMPETENCE

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Differences in mathematical skills between students from German and non-German speaking homes are reported already by the end of grade 1 (Heinze et al., 2009). First exploratory analyses indicate that language-related disparities in arithmetic skills can be explained by basic cognitive abilities, while only proficiency in the instructional language German explains differences in knowledge of mathematical concepts and representations (e.g. assigning numbers to the number line; Heinze et al., 2009). Nevertheless, a systematic analysis how home language and instructional language proficiency affect different facets of mathematical understanding is still not at hand.

The aim of our pilot study was to systematically investigate (a) the influence of a non-German home language on the four different facets of mathematical competence: basic arithmetic skills, knowledge of mathematical concepts, mathematical word problems, and knowledge of manipulatives which are usually used for classroom communication (e.g. base-ten blocks) and (b) if these differences could be explained by instructional language proficiency and basic cognitive abilities.

For our pilot study with $N = 95$ third graders we constructed new instruments to survey the four facets of mathematics skills. We found significant differences in favour of learners from German speaking homes for word problems ($F(1,87)=5.47$; $p<.05$; $\eta^2=.06$), understanding of manipulatives ($F(1,87)=9.01$; $p<.01$; $\eta^2=.09$), and mathematical concepts ($F(1,87)=12.12$; $p<.01$; $\eta^2=.12$), but not for basic arithmetic skills ($F(1,86)=2.86$; $p>.05$; $\eta^2=.03$). Controlling for general cognitive abilities in analyses of covariance, differences remained significant for mathematical concepts ($F(1,86)=7.98$; $p<.01$; $\eta^2=.09$) and manipulatives ($F(1,86)=5.78$; $p<.05$; $\eta^2=.06$). Controlling for instructional language proficiency reduced both effects considerably.

Our results extend Heinze et al.'s (2009) findings and show that instructional language skills play a major role in the acquisition of mathematical concepts, but to a lesser extent for learning standard procedures and tasks, like short word problems. For conceptual understanding, differences could not be explained completely by language proficiency. Moreover, longitudinal studies that study different facets of mathematical understanding are necessary to explain the genesis of these disparities.

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HIGH SCHOOL STUDENTS INTERACTING IN A COLLABORATIVE VIRTUAL ENVIRONMENT: PLANE TRANSFORMATIONS

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Our research aimed to better understanding how high school students develop, in a collaborative manner, ideas about transformations in the plane, while working in a virtual online environment, performing tasks involving matrix notation. In general, school curricula reduce such tasks to a procedure for operating on matrices, calculating determinants, and solving linear systems, in a fragmented way. Throughout our practice, we observed that this conventional approach does not contribute much to students' understanding of planar transformations. This research took place in face-to-face and online modes at a federal school in of Rio de Janeiro with high school students. The analysis relied on discourse and interactions within the virtual online environment—Virtual Math Team (VMT). Students communicated via chat, drawings, or using the software GeoGebra to solve problems

The tasks and the use of technologies were conceived on the basis of our assumptions about the importance of discursive (Sfard, 2008) and embodied experiences, (Nunez, 2000) and Argumentative Strategy Model (Castro & Bolite Frant, 2011). We adopted Design Research methodology, which allows for modifications in the tasks during the development of the research. This methodology is also iterative and, in this research, was organized in three cycles, so it contributed to the theoretical and pragmatic development of this investigation. Data collection included worksheets generated by VMT, videos, transcripts and the researcher's diary. Analysis was based on our theoretical assumptions.

The results showed arguments and cognitive mappings that emerged during students' interactions in VMT, as Transformation Is Change, and the Identity Matrix Is Neutral Element. The VMT was a fertile environment for interaction and cooperation between the participants, their involvement went beyond the allocated time on VMT, they continue interacting through social networks to solve the tasks.

We will present findings about the elaboration of tasks and about the development of student's discourses.

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WHAT CAN INFLUENCE COMBINATORIAL PROBLEM SOLVING

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The present study aimed to identify factors that can influence performance in combinatorial problem solving. Vergnaud (1990), taken as a basic theoretical framework, points out three dimensions of conceptualization: *situations that give meaning* to concepts, concept *invariants* and *representation systems* that can symbolize the concepts. Thus, we sought to observe how factors associated with these dimensions affect combinatorial reasoning.

The study was conducted in two phases. In the first phase, 568 students (1st to 12th grade) completed a test with eight questions (two of each type: *arrangement, combination, permutation* and *Cartesian product*, half of the problems resulting in fewer possibilities and half with higher order of magnitude as solution). In the second phase, 128 students participated (6th grade), divided into various test types, with different types of problems, set from the control of the number of steps of choice. This study aimed to determine whether previous findings could also be explained in terms of the number of steps of choice in combinatorial problems.

In the first phase of the study it was observed that, in general, the problems followed the order, from easiest to most difficult: *Cartesian product, arrangement, permutation and combination – permutation* being the most difficult for children from initial school years. Significant differences ($p<.001$) were found with regard to performance in the different types of problems, as well as a function of years of schooling (F (10, 557) = 37,892; $p<.001$) and the total number of possibilities of the problem (t = -16 325; $p< .001$), with performance also affected by the mode of symbolic representation used. The second phase of the study confirmed greater difficulty with the higher total number of possibilities and showed that the number of steps of choice is also a variable that can affect performance because, unlike the first stage of the study, *permutation* problems become easier compared with *Cartesian products* which have the same number of steps of choice (t (23) = 2.713; $p< .001$).

The results of the study indicate variables to be considered in proposing school combinatorial problems: type of situation (and their invariants and appropriate forms of symbolic representation), total number of possibilities and number of steps of choice involved in the problems.

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CHARACTERISTICS OF LEARNERS' MATHEMATICAL THINKING STYLES IN DIFFERENT CULTURES

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Mathematical Thinking Styles (MTS) are characterized as the way in which an individual prefers to present, to understand and to think through mathematical facts and connections by certain internal imaginations and/or externalized representations (Borromeo Ferri 2010). According to Sternberg (1997) "A style is a way of thinking. It is not an ability, but rather, a *preferred* way of using the abilities one has." However since many studies focus on mathematical thinking of individuals in different contexts, there is still less research about MTS (analytic, visual, integrated) of secondary school students in different cultures. One central research question of the MaTHSCu-project (Mathematical Thinking Styles in School and Across Cultures) was whether in this international comparative study characteristics of three earlier identified Mathematical Thinking Styles (analytic, visual, integrated) can be measured quantitatively with 15 and 16 year old learners.

Earlier qualitative studies gave a lot of insight into MTS of secondary learners and teachers while investigating their cognitive processes. But till 2012 a lack of research was there regarding a construction of appropriate scales for MTS. This was directly connected with the open question whether the construct of MTS can be determined quantitatively which was one of the central goals within the MaTHSCu-project. More interesting research questions than with small case studies were possible, such as: 1) Are there correlations between MTS and beliefs? 2) Are there correlations between preferences for certain MTS and mathematical performance? 3) Are there cultural differences in the characteristics of MTS? The questionnaire included MTS-scales and open problems. After the final pilot study the scales had satisfactory reliabilities (Cronbachs α): visual (.77), analytic (.90). The sample of the studies comprises N=1370 students of grade 9 and 10 of the countries Germany, Japan, South Korea, Turkey. The results show a stronger preference for visual and analytic thinking styles in Japan than in Germany. Similar results can be determined between German and Turkey. An interesting result became evident concerning the correlations between different MTS and the performance of the students. Those students with a preference for analytic representation have the best marks. These correlations are significant in Germany: .418** and Japan: -.164**. In the presentation further results will be discussed in detail.

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VALIDATING THE PSM6

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Problem solving has been a notable theme within mathematics education. The importance is clearly seen in the recently adopted Common Core State Standards for Mathematics (CCSSM) in most of the United States. New standards mean old measures need revisions and revalidation or new measures must be created and validated. The purpose of this study is to examine the psychometric qualities of a new measure addressing the CCSSM and students' problem-solving ability called the Grade 6 Problem Solving Measure (PSM6).

Our framework for problems and problem solving stems from the notion that problems are tasks such that (a) it is unknown whether a solution exists, (b) a solution pathway is not readily determined, and (c) there exists more than one way to answer the task (Schoenfeld, 2011). Problem solving goes beyond the type of thinking needed to solve exercises (Polya 1945/2004).

One hundred thirty-seven grade six students volunteered to complete the PSM6 during one administration. Content validity was assessed qualitatively through an expert review. All other psychometric properties were evaluated using Rasch modeling for dichotomous responses (Rasch, 1960/1980).

The expert review panel had favorable reviews of the items. The resulting statistics suggested that items worked together to form a unidimensional measure capable of assessing a wide range of problem-solving abilities. A variable map agreed with an a priori hypothesis indicating construct validity was high and the PSM6 meaningfully measured the theorized construct of Problem Solving.

The purpose of the PSM6 is to inform students and teachers about an individual's mathematical problem-solving ability and to capture growth in this area. It provides useful information about sixth-grade students' problem-solving ability while also addressing CCSSM found across all sixth-grade domains. This measure will advance the field because there are few, if any, psychometrically-sound instruments that provide the information about students' problem-solving abilities within the context of the CCSSM like the PSM6.

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MIDDLE GRADES MATHEMATICS TEACHERS' PERCEPTIONS OF THE COMMON CORE STATE STANDARDS

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The recent adoption of the Common Core State Standards (CCSS) in the United States represents a significant change to individual state mathematics standards (Porter, McMaken, Hwang, & Yang, R., 2011). Intended changes include shifts in placement and rigor of mathematical content, as well as expectations for student mathematical practices. The realization of these intended educational reforms is largely dependent on teachers who serve as what Schwille et. al. (1983) calls policy brokers. To what extent teachers will enact the CCSS in their classroom is based partially on their own individual capacity, but also on external factors. Spillane (1999) frames these external factors as enactment zones, "spaces in which the reform levers meet the world of practiconers and 'practice', involving the interplay of teachers' personal resources with 'external' incentives and learning opportunities" (p. 171). It is in using this framework of enactment zones that this study examines the following research question.

- In what ways do middle grades mathematics teachers perceive the influence of external factors in the adoption of the CCSS in their classrooms?

The sample for the study consisted of 15 middle grades mathematics teachers. Data was collected through interviews. Participants discussed the way in which they perceived the CCSS have been integrated into their classroom and the impact of this process on their own practice. Interviews were recorded, transcribed, and analysed using a constant comparative approach. Preliminary results indicate that certain school based factors including content and grade specific workshops were related to teachers' optimistic view for long term impact of the CCSS. Implications of the study suggest a need to continue examining ways in which teachers can best be supported in integrating the reforms suggested by the CCSS.

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DESIGNING A LEARNING ENVIRONMENT FOR ELEMENTARY STUDENTS BASED ON A REAL LIFE CONTEXT

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Enabling students to solve real life problems is considered a major aim of school mathematics. In order to successfully deal with mathematical modelling tasks they have to implement different types of strategies. When the context is complex, learners have to be able to detect and search for necessary data which proves to be a difficult task especially for younger students. Effective teaching of modelling competence depends on certain factors for example a "balance between teacher's guidance and students' independence" (Blum & Borromeo Ferri, 2009, 45). However "[t]he literature indicates using contexts to teach mathematics can be difficult and few detailed exemplars exist." (Harvey & Averill, 2012, 41). Using Design Research methods (Bereiter, 2002) the ZooMa project attempts to construct and evaluate a learning environment based on the real life context of animals living in a zoological garden. One of the goals is to support the students' modelling process.

The sample comprises 212 fourth graders in Germany, which had to solve a task about the weight of rhinoceroses with the aid of a specifically designed fact booklet. It includes much more information than necessary. The findings indicate that students with prepared strategical support are more successful than students without (200) = -2.009, $p<.05$, $r = 0.1399$. A closer look shows that a hint to look for the missing data (e. g. in the book) leads to significantly better results $t(200) = -3.005$, $p<.001$, $r = 0.43$. Furthermore over 80 percent of the students judge this specific hint to be helpful. More details (e. g. on the hints) and other results will be given in the presentation.

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MENTOR TEACHERS' SHIFT TO A FOCUS ON STUDENT LEARNING IN URBAN CONTEXTS

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This research reports on experiences between pre-service and mentor teachers with respect to student learning from a co-planning, co-teaching, and cognitive coaching perspective. The results from 28 mentors who participated in two focus group interviews indicated that these structured collaborations helped mentors shift their thinking during planning, teaching, and assessing to focus on student learning.

Developing systems of support to facilitate success for classroom teachers, particularly those teaching in the context of complex, large urban school districts are crucial (Costa & Garmston, 2002). Effective schools promote collaboration amongst teachers who focus on improving student learning by sharing successful practices and supporting one another to overcome daily changes (DuFour & Eaker, 1998). In this work, we studied co-planning, co-teaching (CPCT) and cognitive coaching (CC) between pre-service (PST) and mentor teachers (MT) from one large urban school district, and how these collaborations influenced the MT planning, teaching, and assessing. The aim of this report is to share how experiences between PST and MT within a CPCT and CC context helped MT shift their perspective on their teaching practice to becoming more focused on student learning.

The data used in this study were part of a larger research project in which PST were paired with a MT in a high-needs urban district while they earn a master's degree in math, science, or foreign language in middle childhood or secondary education in exchange for three years of teaching in the same district. Twenty-eight MT participated in two one hour video-recorded focus group interviews where they were split into two groups and given thirty minutes to answer questions related to their experiences with the project. Videos were transcribed and coded for instances in which MT commented on how CPCT and CC with PST helped them shift their focus towards student learning.

Three themes that emerged as a result of the analysis included the MT shift to a focus on student learning, changes to teaching and instruction, and positive effects of collaborations. Vignettes from this research will be shared. These findings suggest that CPCT and CC helped facilitate the mentor teachers' shift to focus on student learning.

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METHODS, CHALLENGES AND AFFORDANCES OF VIDEO DATA USE WITH MATHEMATICS TEACHERS

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Video allows us to view and re-view direct teaching practices and/or student thinking processes, making it a powerful catalyst for reflection. Our collaborative action research with over 50 mathematics teachers and 1000 students (JK-Grade 8), produced approximately 4 terabytes of video data of students, teachers and classrooms over 3 years. Our research program involved developing and testing novel digital video data analysis strategies with teachers to support their professional learning through the close examination of mathematics teaching and related student learning. We used active axial coding (Charmaz, 2003) to describe processes and merge themes in the data sets (including transcripts, focus group interviews, and extensive field notes) to answer the question, *What methods of video data collection and analysis support teachers and researchers who are inquiring collaboratively into mathematics teaching practices and related student learning?*

Raw video data is time-consuming to process and to review. To use video more effectively, researchers and teachers identified: (i) 'critical moments' with evidence of student learning; (ii) spontaneous teacher moves; and (iii) unanticipated student responses. These clips became the focus for teacher-researcher analysis in professional learning sessions, and were analysed using explicit video analysis guides (to examine student learning) and think-aloud protocols (in which a teacher watched video of him or herself teaching). Benefits were: heightened attention on mathematics content (what do the students know and do?) and noticing (Mason, 2002); increased precision of instructional decision-making (what precise next steps will help?); broader video study use (see video studies at www.tmerc.ca). Van Es & Sherin (2010) have called for further exploration of effective norms for facilitation of teacher video analysis to inform "the design of video-based professional development that is both productive and meaningful for teachers" (173). This study contributes new theoretical knowledge of methods for working with digital video alongside practitioners for greater impact on practice.

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ARGUING, REASONING, PROVING – SIMILAR PROCESSES?

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Whereas there is a consensus in mathematics education about the importance of the acquisition of mathematical competences like arguing, reasoning or proving, dissent on terminological issues still prevails. Some competency models for mathematics education, such as the one which forms part of the Swiss Curriculum (D-EDK, 2013), include the competence of "Arguing". This is not specific enough and thus renders an accurate interpretation quite difficult. In the German model (KMK, 2005), they use "mathematical arguing", which is more concrete, while the NCTM's (2000) Standards and Principles list "reasoning and proof". Do they all refer to the same competence?

Alongside this inconsistency as far as terminology is concerned, there is another bone of contention, namely the confusion about the actual relation between the processes of arguing, reasoning and proving (e.g. Reid & Knipping, 2010). The controversy is focused on the relation between arguing on the one hand, and proving and/or reasoning on the other hand, in which the latter concepts are usually taken for granted and not further clarified or even properly defined. The currently ongoing debate is mainly dominated by two different stances: one which claims a close connection between proving and arguing (e.g. Pedemonte, 2007), and one which regards proving and arguing as quite different activities (e.g. Duval, 1991). Until recently, a comprehensive model which tried to relate the three mathematical activities in question on a theoretically substantiated basis was lacking. Now, a promising attempt at developing such a model (cf. Brunner, 2014) is available and shall be put forward in a brief outline. Moreover, the cognitive processes of arguing, reasoning and proving were associated with different types of mathematical proof, which resulted in a process model of proving in school mathematics. This model shall be presented as well.

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A STUDY ON THE LONGITUDINAL EVALUATION OF MATHEMATICS TEACHER EDUCATION IN GERMANY

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Mathematics teacher education for lower and upper secondary schools in Germany is characterized by high drop-out rates up to 40 % during the study entrance phase, caused amongst others by the gap between school mathematics and university mathematics. In the past years several approaches have been developed to overcome this discontinuity by revising the mathematics teacher education curriculum. However, the scientific outcomes of these changes have not yet been assessed satisfactorily. The "Teacher Education and Development Study – Telekom (TEDS-Telekom)" (Buchholtz & Kaiser, 2013) is a longitudinal evaluation study basing on and deepening the international comparative study TEDS-M 2008 (Tatto et al., 2012). Amongst other considerations, the study analyzed the 2-year-development of the professional knowledge of 167 future teachers in the field of academic mathematical content knowledge (MCK) in the area of calculus, linear algebra and elementary mathematics from an advanced standpoint and mathematical pedagogical content knowledge (MPCK) following the main research questions:

- How does the professional knowledge of different types of future teachers develop longitudinally and in comparison to non-student teachers?
- How do the various knowledge facets of professional knowledge correlate with each other and within the sample structurally and longitudinally?

The evaluation of the future teachers took place using three 90-min paper-and-pencil tests. Methods of Multidimensional Item Response Theory were used in order to scale the data longitudinally and compare the achievements of the future teachers. Overall, results point to different developments of the knowledge of future teachers and non-teaching students, however MCK and MPCK develop differently. Furthermore, strong correlations between certain knowledge facets were identified.

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E-BROCK BUGS©: AN EPISTEMIC MATH COMPUTER GAME

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Educational computer games can provide a rich platform for the development of mathematical thinking (Devlin, 2011). Such games fall in the category of "epistemic games" (Shaffer, 2006). Based on principles outlined by Devlin (2011), we designed, implemented, and launched the free online computer game E-Brock Bugs© (2013), which seeks to prompt a player's mathematical thinking about basic probability concepts (Broley, 2013). In this paper, we examine the design of E-Brock Bugs using Annetta's (2010) **six "I's" nested model** of "serious educational game" design, enriched with Devlin's (2011*) key aspects of an epistemic mathematics game*.

After selecting an avatar, the player of E-Brock Bugs may adopt her "**Identity**" as the saviour Bug City has been waiting for and experience "**Immersion**" into a world in which *mathematics arises naturally*, and where she is motivated to defeat the bullies that now terrorize it. Throughout her journey, the player interacts with i) non-player characters: the bullies, who use probabilistic games to control the City, and the loyal friends who cheer her on ("**Interactivity**"), and ii) mathematics, through *exploration*. As the player progresses from one district to the next, the probabilistic concepts involved in the bullies' challenges carefully scaffold in complexity ("**Increased Complexity**"). As such, the player *regularly tests* or self-assesses her understanding of the ideas encountered while saving previous districts ("**Informed Teaching**"). In support of the *learning by doing* approach, the player is encouraged to learn through play and at her own pace (*self-paced learning*). Having challenged a bully at least once, the player may choose to interact with Smarty, who can provide mathematical insight into the challenge to assist the transition from the bully's game, to the mathematics involved, and back to where the mathematical knowledge can be put to *immediate use* ("**Instructional**"). In the case of our epistemic mathematics game, the nested six "I's" components ultimately return to the "Identity" as mathematician, forming a Klein bottle model.

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ARITHMETIC WORD PROBLEM SOLVING: THE MICROANALYSIS OF SUBJECT/MATERIAL DIALECTIC

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Many researchers, such as Radford (2013) argue that students construct their knowledge, and that learning is a social act. This is the era of socioconstructivism. However, how the student performs this construction has not yet been finely studied. For building presupposes above all the use of materials (Poisard, 2005). My doctoral research argues for subject/material dialectic and methodology of microanalysis to understand the effectiveness of the materials students use to construct meaning during the activity of arithmetic problems solving.Indeed, the subject/material dialectic is built upon the basis of tool/object dialectic (Douady, 1986). She argues that the dialectic refers to a change in the use of tools. For Douady, changes in the direction of thought are at the centre of mathematical activity. Her central idea of the tool/object dialectic is to get students to think like a mathematician. However, I strongly argue that it is important to understand how students construct their knowledge to be able to effectively bring them to think like a mathematician.

I conducted a research on six third graders who have solved three arithmetic problems in a familiar context, close to their regular classroom. Data was collected using three instruments: semi- structured interview, direct observation, and worksheet. All sessions were video recorded. I conducted a microanalysis of the data using the following procedure: 1) Carefully viewing the recordings to denote the progress of participants action (e.g., gestures, glances and traces) while solving the problems. Traces include words, drawings, and symbols. 2) Establishing links to denote the relationship between the gestures and the traces. 3) From the participants' responses to the interview questions, I denoted the link between thoughts and traces of the participants. The finding of this microanalysis shows that students who solved an arithmetic problem, build meaning from the use of a variety of materials (e.g, their drawings, fingers and other concrete objects). Moreover, results show that students' development of thoughts toward thinking like a mathematician was apparent in their use of traces (e.g., from more informal drawings to mathematical symbols).

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DESIGNING THE MATHCITYMAP-PROJECT FOR INDONESIA

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The MathCityMap-Project (www.mathcitymap.eu) provides opportunities for students to engage with math in math trails supported by GPS-enabled smartphones (Jesberg & Ludwig, 2012). Students use the app to receive tasks and know coordinates, routes, tools needed, and hints on demand. They solve tasks on site and enter their answers into the app. Thus, teachers get feedback by the system. The goal of this project is to challenge students to solve tasks and to make them enjoy learning. It may manifest motivation. A meta-analysis found that both, outdoor programs and motivation have a strong impact on achievement (Hattie, 2009). This paper is a report from a pilot project study which aims to explore needs and conditions of math education in Indonesia, particularly in Semarang, in order to develop the MathCityMap-Project for Indonesia (the MCM-Indonesia). It was motivated by the low Indonesian students' achievement. The question was 'how can the MCM-Indonesia promote the motivation of student?'

In PISA 2012, 75.7% of Indonesian students did not reach the baseline level 2 in mathematics. In the national exam, the proportion of Semarang secondary school students who were able to solve problems related to area, volume, and angle was below 50%. To get more in depth information, discussion was conducted involving 4 experts and 10 mathematics teachers from Semarang secondary schools. The topic was about designing the MCM-Indonesia by considering teaching experiences cross checked with theoretical reviews, and based on data and previous researches. First results show that students have difficulties in solving real problems, particularly in geometry. They cannot extract relevant information and use basic algorithms to solve tasks. It is known in Indonesia that most of the students have low motivation in learning math. They fear math and assume that math classes are boring. Building upon Hattie's results (Hattie 2009), outdoor tasks may have a positive impact on that issue. Settings need to be varied, such as outdoor learning and using advanced technology. The tasks are designed with focus on the subject geometry for 8th graders during 2nd semester (dry season). The tasks are located around the school district area so they are easily accessed by students. Students work in groups using the MCM-app. Findings indicate that the MathCityMap-Project with special features can be implemented in Indonesia to improve students' motivation. The challenge of solving tasks and the joys of discovering math in real context with the aid of smartphones might raise motivation and the effectiveness of MCM-Indonesia to increase students' achievement can be proved at the next stage. In the presentation, further results will be discussed in detail.

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STUDENT-CENTRED INQUIRY LEARNING: HOW MIGHT MATHEMATICAL THINKING EMERGE

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Learning can be enhanced when students develop a learning trajectory that affords some ownership of the process and meaning. In order to establish such ownership, teachers often attempt to design problem-solving contexts that relate to real-life situations (Lowrie, 2004). Student-centred inquiry learning is a democratic teaching approach, where meaningful contexts are central to individual learning trajectories. Students pursue questions, issues or inquiries that are of genuine interest to them and curriculum is collaboratively co-constructed (Brough, 2012). Some mathematics educators contend that learning should be initiated by contexts that require mathematical organization, contexts that can be mathematised (Freudenthal, 1968).

This paper examines the research question: How might student-centred inquiry facilitate mathematical thinking? Data are drawn from a research project where mathematics was situated within authentic student-posed problems. A contemporary interpretive frame was utilised and mixed methods were used to collect data. The project took place with a class of 13-15 year-olds in a purpose-built secondary school. The findings indicated that mathematics centred on real-life learning was purposeful and engaging, with students frequently employing conceptual content beyond the expected level. Mathematical thinking in measurement, number and statistics was initiated, as well as generalisation. Critical thinking, especially the use of compare and contrast, was prevalent in the analysis as the students sought trends and patterns in the data.

Some students were initially negative about the inquiry process in mathematics but became very positive due to investigating within an area of personal interest. Another key aspect was the effective meshing of student inquiry and teacher or student-led mathematical content workshops. Student inquiry facilitated the identification of mathematical concepts and processes that individual students required.

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STATISTICAL LITERACY AND THE CURRICULAR KNOWLEDGE OF MATHEMATICS TEACHERS

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This paper focuses on the curricular knowledge of mathematics teachers, assuming it as an important component of the professional knowledge of teachers that influences what they teach and how they teach (Ponte & Chapman, 2006). It is our purpose to describe teachers' curricular knowledge about the teaching of statistics and to analyse if it comprises the idea of developing statistical literacy of the students. Statistical literacy is a main issue in the orientations of the current mathematics curricula but it constitutes a main challenge for many teachers (Batanero, Burrill, & Reading, 2011).

We analyse the case studies of three middle school teachers that were investigated for one year, based on interviews and classes' observation. We provide a description of the curricular knowledge of each one, focusing the objectives they state for the teaching of statistics, the statistical contents they most value and how they approach them in their classes, as well as the tasks they propose to their students. We conclude that teachers do not mention explicitly the idea of statistical literacy although they refer to the necessity of developing students' capacities to interpret and understand the reality. The tasks they choose are always related to a context. However, these contexts are not necessarily relevant to students' lives. The tasks explore nothing but the statistical description of the situation and do not ask students to critically evaluate them, or to discuss the eventual manipulations that may occur in a statistical study.

Acknowledgements

This study is supported by national funds by FCT – Fundação para a Ciência e Tecnologia through the *Project Developing Statistical Literacy* (contract PTDC/CPE-CED/117933/2010).

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TEACHERS' TALK AND TAKE UP OF POSITIONING

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In mathematics classrooms, words and actions send students messages about who they are as learners, what they are capable of, what mathematics is, what it means to know and do mathematics, and so on. These messages relate to "positioning," which refers to "the ways in which people use action and speech to arrange social structures" (Wagner & Herbel-Eisenmann, 2009 p.2) We operationalize the idea of positioning in our work with teachers in a study group by considering issues of status and power between/among students, issues of authority and agency between the teachers and students, and what constitutes the practice of 'mathematics' based on the kinds of activities and tasks in which students engage. We see all of these aspects relating to students' developing identities. Here we examine the following question: In what ways did a group of secondary mathematics teachers talk about and take up in their action research the idea of positioning over the course of a two-year study group focused on mathematics classroom discourse?

The study group involved the co-authors of this article and four mathematics teachers who were working in a culturally, linguistically, and racially diverse middle school. Our work is grounded in the sociocultural and sociolinguistic perspectives, and as such, we see learning as related to how one participates in the discourse practices of a community. We use tools described by Herbel-Eisenmann and Otten (2011) to analyse the ways in which teachers' participation shifted over time. The primary data source for our analysis is video-recordings of 18 study group sessions. The study group met seven times for about 36 hours in the 2012-13 AY and continues to meet bi-weekly during the 2013-14 AY in order for the teachers to engage in cycles of action research related to their interests and questions.

Our analyses are underway, so we report tentative findings here and elaborate further in our presentation. Teachers' early focus on positioning centred on students, especially related to students' status and identity. Later, as teachers engaged in action research, their focus turned to the positioning of mathematics itself, with less emphasis on status, power, and authority. In particular, they identified the importance of using rich tasks, but needed prompting to reflect on the relationship between the positioning of mathematics and students' identity. These findings offer insights into how teachers make explicit the ways the positioning of mathematics, students, and the teacher are continuously affected by the discourse in their classroom.

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MATHEMATICS TEACHERS' TRAJECTORIES TO TRANSFORMING THEIR PRACTICE

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This paper is based on the first year of an ongoing five-year nationally funded project aimed at supporting change in the mathematical culture of one school by infusing contemporary research findings into the community. The focus here is on two of the mathematics teachers to understand changes in their thinking and the trajectories to transforming their practice that resulted from the intervention to support their growth. The project is grounded in the literature supporting the need for better developed teachers' knowledge of mathematics and professional learning through collective knowledge building. To enable attendance to change at the individual, collective, and institutional levels, the research is framed theoretically in complexity science that involves a complex, systems approach (Davis & Sumara, 2006). The project is located in a grade 2 to12 school that is noted for innovations in literacy learning and special needs, but had placed little emphasis on mathematics learning and had a culture oriented towards traditional teaching. A core group of 10 teachers participated in the first of a three-stage intervention held at their school. This stage consisted of four courses linked to a year-long graduate certificate. The courses dealt with concept study (Davis & Renert, 2014), mathematical understanding and its implications for task design, and teachers' constructions of their mathematical and pedagogical identities.

The two teachers considered here taught grade 7 and worked as a team. Data sources included semi-structured interviews on the culture of mathematics teaching in the school and changes to practice and written course assignments that included reflective journals and unit plans. Data analysis involved an emergent approach that identified themes of changes in the teachers' thinking and teaching. Findings indicated that the intervention enabled them to set new goals for their practice that included relational understanding, problem solving, and autonomy in learning. These goals triggered their trajectories to transform their practice from a *teacher-controlled approach*. For one teacher, this approach initially changed to an *abandonment approach* and for the other, toa *rescue approach*. Development of a *reflective stance* that focused on understanding what worked and did not work for the students helped them to shift towards a *noticing approach* by the end of the first year. The trajectories, which will be illustrated through the teachers' teaching of fractions prior to, during and after the intervention, provide a basis to understand how teachers could accomplish desirable changes in their practice.

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INTEGRATION OF CONJECTURING AND DIAGNOSTIC TEACHING: USING PROCEDURALIZED REFUTATION MODEL AS INTERMEDIATE FRAMEWORK

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Refutation through generating examples and making conjectures is an efficient and effective activity to scaffold students in developing critical thinking. Lin and Wu Yu (2005) proposed a proceduralized model that can structure teaching for refutation. Recognizing that conjecturing is the backbone of mathematics learning, we attempted to integrate conjecturing and diagnostic teaching, and used proceduralized refutation model as an intermediate framework to coordinate both. The revised teaching sequence based on the proceduralized refutation model includes activities: (1) students observe a mathematics situation that can elicit their misconceptions (e.g., the median of a triangle is also its altitude); (2) students generate both supportive and counter-examples based on the observation of the mathematics situation; (3) students discern common characteristics among the supportive examples (e.g., medians of different isosceles triangles are also their altitude); (4) students revise the mathematics situation and provide correct statements; and (5) students justify the revised mathematics situation (e.g., providing more supportive examples or proving the statements using relevant properties).

We argued that the revised teaching sequence based on the proceduralized refutation model not only facilitates students in developing the competence of argumentation and conjecturing, but also helps students to correct their misconceptions. Activity (1) can be seen as eliciting students' misconceptions, and activities (2) and (3) are adopted for creating cognitive conflict for students. Regarding activities (4) and (5), they are for adjusting students' misconceptions so that students can achieve cognitive equilibrium again. The experiments with 8th grade students who have not learned the mathematics content showed that the revised model can successfully correct their misconceptions and enhance the understanding of mathematics. Taking the mathematics situation "the median of a triangle is also it altitude" as an example again, the model scaffolded students to discern the difference between isosceles triangles and non-isosceles triangles and their relation to altitudes. Students then made conjectures based on the discernment of the examples and justified their conjectures using different approaches (e.g., dividing an isosceles triangle into two congruent right triangles or proving the conjecture by the Pythagorean Theorem).

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LEARNING AND TEACHING AXIAL SYMMETRY AT THE TRANSITION FROM PRIMARY TO SECONDARY SCHOOL IN FRANCE

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The study presented here tries to connect the difficulties that pupils experience at the transition from primary to secondary school in France with the continuity and ruptures identified in teaching practices in the two grades. This case study follows up pupils during 5th grade and 6th grade mathematics sessions about axial symmetry.

The study exposed here is part of a larger project on the subject of transition in mathematics and physics between primary and secondary school in France, especially focusing on populations identified as "socially at risk". Among the multiple factors that are responsible for the frequent decline in achievement following transition pointed out by research (McGee et al., 2004), we try to identify ruptures in teaching practices causing difficulties for the students. We chose to focus our study on axial symmetry because we showed that this subject is representative of both continuity and ruptures identified in the curriculum as well as in textbooks (Chesnais & Munier 2013). Our object here is to further this study by following up some pupils from their fifth grade (last grade in primary school, 9-10 y. o.) to their sixth grade (first grade in secondary school, 10-11 y. o.). The data that we collected is constituted of videos of classroom sessions that they attended on the subject of axial symmetry during 5th grade in two classrooms and during 6th grade. We also ran tests at both levels and interviews with the teachers.

The theoretical framework used to study the teachers' practices is the double approach (Robert et Rogalski 2002, Vandebrouck, 2012). We articulate the analysis of pupils' and teachers' activities in order to identify both the difficulties experienced by pupils and the ways teachers deal with items related to transition issues. We thus try to identify ruptures that may be responsible for pupils' decline in achievement.

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GROUP INTERACTION AND COLLECTIVE ARGUMENTATION

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We view collective argumentation as relevant to all settings of teaching and learning mathematics in that it gives value to processes of collaboration, negotiation and construction of shared knowledge (Krummheuer, 2007). By stressing these processes, we shift the focus from the nature and elements of argumentation to its role, use and effect in group activity aimed at mathematics learning (Rasmussen & Stephan, 2008).

Our data consists of audio and video recordings of five lessons in a teaching experiment that was conducted in a mathematics classroom with students aged 15 to 16. Two types of information were grouped, and a system of dependent codes was produced for each type. The codes of interaction label identifiable responses by students to peer actions aimed at the resolution of the task (e.g., 'initiating', 'rejecting'), while the codes of argumentation label explicit mathematical content changes involved in the construction of collective argumentation by students (e.g., 'from visual to inductive reasoning', 'from procedure to numerical comparison'). A tool was created to search for relationships between codes of the two systems.

The created tool has been proved to represent its intended purpose: the move for group interaction to collective argumentation as the two of them develop from pair work to class discussion. More generally, our results illustrate basic and combined patterns of interaction with an effect on the elaboration and progress of conceptual mathematical learning in a secondary mathematics classroom. It has been found that the initial pair-work structure is mathematically positive in that it contributes to the expansion and problematization of contents of the interaction during class discussion. Moreover, the combined patterns of interaction point to a phenomenon of thematic discontinuity, with various levels of intensity, generated by the oscillation of different mathematical ideas that occur successively or by unpredictable alternate turns.

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SUPPORTIVE AND PROBLEMATIC ASPECTS IN MATHEMATICAL THINKING OVER THE LONGER TERM

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This paper exemplifies a framework for mathematical thinking over the longer term based on an empirical study. A sample of 45 secondary mathematics PGCE student teachers responded to a questionnaire designed to seek the possible mismatch between the conceptions of graduate students destined to become teachers and the ways of thinking of learners that they will teach in school.

The empirical data focuses on three successive levels of learning in trigonometry, namely *triangle trigonometry* involving lengths that are magnitudes without sign, and angles in right-angled triangles strictly between 0 and 90°, *circle trigonometry* involving angles of any size, sides that have signs and functions that vary dynamically, and *analytic trigonometry* that gives new insights unavailable to school children involving infinite power series and complex numbers.

The research focuses on discontinuities involving changes of meaning as mathematics becomes more sophisticated. Aspects that occur in student conceptions at one stage may be *supportive* at a subsequent stage and aid generalisation or *problematic* and impede new learning. The paper reveals how an overall supportive conception may involve problematic aspects and a problematic conception may include supportive aspects. Examples are given of conceptions held by students that involve problematic aspects that will cause difficulties in teaching at various levels. This has high significance in the preparation of teachers who have conceptions in mathematics that are at variance with the level of thinking involved by the range of students at different stages of personal development.

The material in this paper is developed by the author in his PhD thesis (Chin, 2013) and has contributed to the wider framework of the development of mathematical thinking from birth to adult and on to the boundaries of research (Tall, 2013).

It is a significant evolution of ideas developed throughout the history of PME.

You may read the full version from

https://dl.dropboxusercontent.com/u/92731980/KEChin_PME38_2014.pdf

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HOW TEACHERS USE COGNITIVE DIAGNOSIS MODEL DATA FOR INSTRUCTIONAL DECISION MAKING

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This research is to examine how four middle school mathematics teachers use CDM-based (Cognitive Diagnosis Model) formative assessment data for future instruction. The U.S. Department of Education (2011) reported that teachers desired to examine students' performance results of assessment in three ways; classroom level, specific mathematics concepts, and item analysis. One of the ways to provide such information is to use generalized DINA (G-DINA; de la Torre, 2011), which is one of CDMs, because one of features of CDMs is to present the pattern of existence and absence of fine-grained learning goals based on students' responses to each item. With this assessment framework, teachers could understand whether students successfully learned on fine-grained learning goals.

Assessment results that we provided to teachers included latent probability that individual students' mastering each CCSSM-based standard instead of overall score or rank order of students. Teachers received information on each student, class-level data for pre- and post-assessments for one trimester. Teachers' use of the data was primarily explored through multiple unstructured interviews with the teachers as well as field notes and survey. The interviews were conducted when designing assessments, offering assessment analysis data to the teachers, and inquiring how teachers used the assessment data.

The first response of teachers when they received the assessment results was that students' mastery information from the assessments generally concurs teachers' existing understandings of each student's ability. Majority of teachers did not plan on making instructional changes until the following assessment results were reported. Validation of the effectiveness of instruction was the main focus of teachers when data is offered, which is reactive rather than a proactive in using data. Teachers need to understand what assessment data mean and to have proactive attitude toward implementation of information about their students' learning in order to improve instructions. As teachers are ready to evaluate their own teaching from provided data, the next step should be further education of teachers on what assessments inform.

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THE PERCEPTION OF FUNCTIONS: EYE-TRACKING STUDY OF STUDENTS WITH DIFFERENT MATHEMATICAL BACKGROUND

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In 1996 Heider and French proposed the information reduction hypothesis. It claims, that people learn to distinguish task-relevant and task-irrelevant information (stage 1) and to ignore redundant information (stage 2) through practice. Through a set of laboratory experiments the authors show, that task –redundant information is ignored at the perceptual level, and this process is under our voluntary control (Haider & Frensch, 1996, 1999). These results were replicated by Canham and Hegarty (2010) as applied to the weather maps. The authors show that domain knowledge can guide the perception of complex graphics: information selection changed after a short tutorial.

In our current study we investigated how domain knowledge and training change the perception of function graphs in solving math tasks. We compared eye movements of participants of 3 levels of mathematics competence (n = 27). The tasks were: (1)to compare coefficients of the two graphs of line functions, (2)to detect whether this plot is a graph of a function or not, (3) to identify whether the presented function is an increasing or a decreasing one.

The preliminary results of the study are: (1) experts distinguish task-relevant from task-redundant information better than novices and use some heuristics based on their domain knowledge. (2) Heuristics could be used without conscious control. (3) According to eye-tracking data there are some differences in strategies applied by physicists and mathematicians, which could be explained through peculiarity of function concept in these domains of knowledge.

Acknowledgements:

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INVESTIGATIONS INTO LEARNERS' ATTITUDES TOWARDS STUDYING OF A MATHEMATICAL CONTENT

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This research aims at determining some of the affective and cognitive manifestations of learners unveil during a mathematical learning process. Embodied cognition and learning (Campbell, 2010; Varela, Thompson & Rosch, 1991) is the theoretical component of this study. Based on the classifications detailed in Johnson, Onwuegbuzie and Turner (2007), this is a qualitative dominant mixed methods research. The methods used to record, observe and analyze the study consist of behavioral methods that are audio-visuals to analyze the facial expressions and gestures of the participants, as well as eye-tracking; and physiological methods that are eye-blink, heart and respiration rates and signatures. The study material involves the division theorem and some related concepts such as divisibility. The participants were three undergraduate students (Linda, Betty and Susan - names assigned arbitrarily). The results indicate that the methods used to record participant reactions during a study session are effective to spot learners' studying strategies and underlying affective states, both of which grow differently with respect to content type (i.e. numerical, verbal) and previous experiences. Linda often checked the time if her time is limited, especially where she faces with challenging content. Betty, from the moment she was informed she will study mathematics, started showing indications of anxiety that was continuous throughout the study and only diminished during breaks. Susan almost always shifted her attention anywhere else whenever she sees numbers and calculations. She reported this type of content as 'well understood' even if she did not attend to it, indicating her self-report data does not reflect the factual. The small portion of results presented here shows the strong potential of using psychophysiological and behavioral methods for studying learner reactions in a mathematical context. Once applied to studies with larger sample sizes, these methods can be used as a way to generate learner profiles. The results also indicate a growing need for developing custom pedagogies in schools and classrooms for learners with different previous experiences with, and different attitudes towards, mathematics.

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ASSESSMENT MODEL AWARENESS AND PRACTICES OF PRIMARY TEACHERS: AN EXPLORATORY STUDY

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This research examined assessment methods of 15 primary teachers in the mathematics classroom. Data was collected through semi-structured interviews and was analysed for patterns and themes. Three themes were identified.

Assessing student learning has been a natural way to examine best practices for teaching. Testing upon the completion of a learning cycle, summative assessment, was popular during the No Child Left Behind era in the U.S. and it does not necessarily lead to higher student achievement (Nichols, Glass, & Berliner, 2006). Researchers reported some teachers have neglected classroom formative assessment in an obsession with summative standardized testing (Stiggins, 2007). Formative assessment is an essential component of classroom work and can raise achievement (Black & William, 1998). This research hypothesized that the defining terms of summative and formative may not be understood. Before judging the value of summative and formative assessments, it is imperative to answer the research question, “How comprehensive is mathematics teachers’ assessment knowledge?”

The questions posed of 15 experienced primary teachers in a semi-structured interview offer data for this study on familiarity with assessment definitions and options as well as which mathematics assessment models are being practiced. The data was analysed for patterns with a combination of content analysis and grounded theoretical techniques.

The results identified three themes: 75% of the teachers were not familiar with assessment terms, each used at least one formative assessment tool, and a link between teacher training education, and understanding terms and available assessment options was absent and required to gain greater insight into the assessment methods available.

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STUDENTS' UNDERSTANDING OF ECONOMICAL APPLICATIONS IN CALCULUS – A SPECIAL KIND OF KNOWLEDGE TRANSFER?

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Historically the scientific fields of calculus and physics took a rather parallel path as far as the philosophy of sciences is concerned (Sonar, 2011). In the 20th century one can observe an increasing use of methods of calculus within the field of economics (Möller, 2004). The motives behind this procedure are various and aim at the attempt to give the younger field of economics a scientific foundation and acceptance.

Recently these considerations are regarded as a part of mathematical modelling. This framework offers the possibility to investigate teacher students' perceptions of science beliefs and their cognitive understanding of the underlying processes (Jablonka, 1986). Research questions are:

- What strategies do they use solving these problems?
- Do they recognize this as a modelling process?
- Are they aware of the existence of alternatives?
- Do they recognize a knowledge transfer?

At the University of Erfurt a sample of 85 teacher students (secondary I level) problems were given and they had to answer with paper and pencil. The given problems cover not only applied calculus problems but also ask for the understanding of economical applications – that is the relationship between reality and mathematics.

The study shows that teacher students have a naïve realistic view of science in general and specifically in the use of mathematics as a tool.

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UNIQUE COMBINATIONS: HIGH COGNITIVE DEMAND TASKS, MATHEMATICAL PRACTICES, PRE AND INSERVICE TEACHERS

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While types of professional development vary significantly, researchers have come to a general consensus that content focus, active learning, coherence, duration, and collective participation are essential components of effective PD (Coggshall, 2012). Unfortunately, necessary funding for PD addressing all five components tends to exceed availability. This qualitative study examined the impact of a one-day PD on teachers' abilities to develop high cognitive demand (HCD) tasks and understand the Common Core State Standards Mathematical Practices (CCSSM-MP).

Inservice teachers (n=12) were carefully selected and invited to work collaboratively with our preservice teachers (n=11) to learn about and develop activities to then be disseminated for use by other educators in our state. In this way, the group shared coherence through common theoretical and pedagogical perspectives. A framework for analyzing mathematical tasks (Arbaugh & Brown, 2005) along with constant comparative analysis on the written participant responses and open-ended surveys was used to examine the perceived impact of this PD on the participants' ability to develop HCD tasks and make sense of the CCSSM-MP.

Findings indicate that 73% of the tasks developed by the 11 teams were of high-cognitive demand. Further, participants reported increased understanding of both HCD tasks and CCSSM-MP, but were left with questions about how to help transition students from traditional materials and alignment with new testing requirements.

Overall, this study suggests that using HCD tasks is an effective motivator and context for understanding the CCSSM-MP and that inservice teachers and preservice teachers are able to effectively collaborative to design HCD tasks they plan to implement in their classrooms.

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ACCELERATING JUNIOR-SECONDARY MATHEMATICS

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Unfortunately, in Australia there is a prevalence of mathematically underperforming junior-secondary students in low-socioeconomic status schools. This requires targeted intervention to develop the affected students' requisite understanding in preparation for post-compulsory study and employment and, ultimately, to increase their life chances. To address this, the ongoing action research project presented in this paper is developing a curriculum of accelerated learning, informed by a lineage of cognitivist-based structural sequence theory building activity (e.g., Cooper & Warren, 2011). The project's conceptual framework features three pillars: the vertically structured sequencing of concepts; pedagogy grounded in students' reality and culture; and professional learning to support teachers' implementation of the curriculum (Cooper, Nutchey, & Grant, 2013). Quantitative and qualitative data informs the ongoing refinement of the theory, the curriculum, and the teacher support.

In 2013, the curriculum's vertical structure was primarily focussed on number and algebra concepts. Modules explored whole and fractional number, patterning, and whole number and fractional number operations. The relatively narrow focus of the modules facilitated the development of curriculum materials. However, classroom observations and teacher interviews suggested that the narrow focus restricted the use of students' reality and culture as authentic contexts for developing mathematical understanding. In 2014 the curriculum has been refined. The vertical 'trunk' of number and algebra has been augmented with lateral 'branches' of concepts from other strands of mathematics. An ongoing challenge of the project is to clearly articulate the structured sequence and so scaffold teachers' implementation of the curriculum.

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MATHEMATICAL MODELING AND MATHEMATICAL UNDERSTANDING AT THE HIGH SCHOOL LEVEL

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Teaching mathematics can be demanding, especially when teachers are seeking for mathematical understanding. It is not uncommon to have high school students disengaged and facing difficulties within mathematics classes. I argue that if students engage in the modeling process of life-like situations, they can better develop mathematics understanding and acquire mathematics knowledge. Indeed, Lesh (2007) emphasizes that mathematical problems are usually not solved in the simple ways that are shown in mathematics classes and he asserts that "modeling abilities are among the most important proficiencies students need to develop" (p.158).

In this sense, this research investigates the use of modeling approaches within high school mathematics classes, in order to promote changes in students' mathematical reasoning and skills. In other words, students are expected to authentically participate in modeling experiences, in a way that might better prepare them to deal with their daily academic, personal and professional lives.

The research intervention will take place in a spontaneous, self-organized and non-linear environment, allowing and prompting the emergence of long-lasting mathematical understanding. Thus, complexity science is the chosen theoretical framework. Davis and Simmt (2003) five necessary conditions to implement and develop a complex environment (internal diversity, redundancy, decentralized control, organized randomness and neighbour interactions) are required characteristics of the intervention setting. The applied modeling activities will be analysed for mathematical understanding based on the five strands of mathematical proficiency proposed by Kilpatrick et al (2001), namely: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition.

If the expected outcomes prove effective, modeling activities might be considered as a way of enhancing high-school students' mathematical understanding and skills. Hence, this research might be a potential contribution to research in mathematics education.

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INQUIRY SKILLS IN THE FRAMEWORKS OF LARGE-SCALE INTERNATIONAL ASSESSMENT SURVEYS: A COMPARATIVE ANALYSIS OF TIMSS, PISA, AND IBL

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Inquiry skills (i.e., thinking and reasoning skills that are necessary for and may be improved by inquiry-based learning) have long been recognized as essential components in a renewed pedagogical approach of mathematics and science education. The main research question of our study is how inquiry skills are represented in two large-scale studies of mathematical and scientific learning outcomes, in PISA and TIMSS, with an emphasis on the PISA mathematical literacy framework.

In the literature, several different taxonomies of inquiry skills have already been proposed. Both Fradd, Lee, Sutman and Saxton (2001) and Wenning (2007) identified several phases of classroom inquiry that may be matched with inquiry skills, e.g. identifying a problem, conducting an experiment, collecting meaningful data, and reporting the results. Fostering inquiry skills in classroom situations may be facilitated by making obvious that both PISA and TIMSS definitely represent those skills in their frameworks, and consequently, in their tasks. A comparative analysis of the TIMSS and PISA frameworks (Mullis et al., 2009; OECD, 2013) shows the following pattern: both in the fields of mathematics and science, the TIMSS and PISA assessment frameworks address a variety of inquiry skills with the exception of the 'on-site' design and implementation skills. Our research provides evidence on how the 'formulating situations mathematically' processes in the PISA mathematical literacy study can be compared to the inquiry skills taxonomies of science literacy.

This research was supported by the SAILS project (European Commission FP7, GA #289085)

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THE PURPOSE AND IMPLICATIONS OF FIRST LANGUAGE USE IN SECOND LANGUAGE MATHEMATICS TASKS

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This research examines the crucial role of language in French immersion (FI) mathematics. Although first language (L1) use is a practice that has traditionally been strongly discouraged in FI programs, researchers working in other multilingual contexts have underscored the importance of understanding how multilingual students learn mathematics through an L2, and how, at times, they rely upon their L1 to do so (e.g., Barwell, 2009; Moschkovich, 2005).

This study is grounded in Vygotskian (1978) theories of language and learning, and in particular, the roles of language in interaction, and of language interdependence, or the positive relationship and transfer that exists between the L1 and the second language (L2) (Cummins, 2000). It explores FI students' use of their L1, English, as they work collaboratively on mathematics problems. The research questions are: 1) Do the frequency and nature of L1 use change as the nature and difficulty of the mathematical tasks change? 2) What prompts L1 use during the completion of mathematical tasks? and 3) What effect, if any, does L1 use have on students' completion the mathematical tasks?

Pairs of secondary school FI students are recorded while working together in their L2 on a series of mathematical problems. Data analysis is informed by sociocultural theory and discourse analysis approaches. Based on the results of preliminary data analysis, I will describe how students' L1 serves as a cognitive tool as they work through increasingly complex mathematical tasks. I will discuss what these results suggest about if, when, and how immersion students' L1 may be allowed, or encouraged, in the FI mathematics classroom, and thus contribute to an open dialogue about how these students might best learn both mathematics and the French language.

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THE FRAMING OF FRACTION AND FAIR SHARING TASKS

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Little is known about young children's knowledge of fractions prior to formal teaching (e.g. Cwikla, 2014; Empson, 2003; Hunting & Davis, 1991). We presented 180 young children (ages 3-5) with a series of cognitive tasks. There are 60 children in each of three age-groups and we tested each child twice over the course of two years. Children were assessed on the ability to complete fair sharing tasks, pro-social measures, and theory of mind tests. Each child participated in three sessions of 30-60 minutes in length each, in the first year, and two approximately 40 minute sessions in the second year, giving us potentially over 1000 hours of video to code. We also analyzed some of this data with regard to the basic demographic variables such as average family income (SES), number of siblings and birth order and IQ test data has been scored.

Specifically our mathematical tasks were framed either socially or physically. For example a task might ask the child to share a snack with their friends (social) or place items into containers (physical). A repeated measures ANOVA comparing performance on socially and physically framed tasks (context) as a function of age and sex revealed a significant effect of age ($F_{3,\,101}= 21.25, p < .001$) but no interactions and no other effects. Children performed progressively better on both types of tasks with age.

Overall, we have discovered so far that prosocial behavior and theory of mind may not predict performance on fair sharing problems, regardless of context. However, verbal reasoning may differentially impact performance on socially framed fair sharing problems. Variables related to the home environment such as number of siblings and household income did not significantly correlate with our outcome variables. However, we have not yet analyzed changes over time as the second year of data is still being coded. We have also discovered significant age related differences in the types of strategies employed to solve these fair sharing problems with children able to partition accurately at the age of five.

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ASSESSMENT OF UNIVERSITY STUDENTS' TEAMWORK COMPETENCIES IN GROUP PROJECTS

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The theory of constructive alignment (Biggs & Tang, 2011) states that the teaching and assessment should be aligned with the intended learning outcomes (ILO) to help ensure that the students are learning what is intended. At Aalborg University, Faculty of Engineering and Science (FES), students work half the time each semester in groups on problem based learning (PBL) projects where learning is self-directed and some of the ILOs focus on PBL collaborative and process competencies. Previously, the project assessment was a group exam where students presented their project and the examiners asked questions in a group discussion. But in 2006-12 the government banned group exams in Denmark and FES projects then were assessed in individual oral exams. Kolmos and Holgaard (2007) concluded that these exams did not test PBL process competencies such as collaboration and teamwork. In 2013 FES reintroduced the group exam but now it includes an individual part where each student is quizzed alone. This paper studies if the new group exam according to the students is aligned with the ILOs about PBL collaborative and process competencies, particularly for mathematics.

In order to make comparison with Kolmos and Holgaard (2007), some questions were reused and the students were for instance asked to what extent they agreed or disagreed (5-point Likert) that during the exam "one can complement and expand on others' answers" or "show one's ability to participate in group work". The questionnaire was piloted after the January 2013 exams and an extended version was distributed online to 4588 FES students in June. 1136 responded, hence 25% response rate (the same as Kolmos and Holgaard (2007)), which according to Nulty (2008) is acceptable.

The majority of the students found that the group exam was better to assess the teamwork competencies than the individual exam. The difference between the answers of the mathematics students and all the FES students are not statistically significant. However, the group exam is not without problems and the study showed that the group exam needs to be managed well by the examiners to secure this alignment in practice, for instance when groups of students of different levels of ability are assessed.

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TRANSFORMING A CULTURE MATHEMATICS TEACHING BY TRANSLATING CONTRAINTS INTO LICENSES

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We report on an attempt to affect the culture of mathematics teaching in a school. The research is oriented by the conviction that educational change is better studied at the institutional level than the individual level, following Ulm's (2011) assertion, "changes at a school cannot … be brought about by 'lone wolves.' Instead, we need cooperation … involving practically all members of the math teaching staff." (p. 5) We part company with Ulm on the advice to engage all staff, as an attempt to involve all presents risks of coercion and resistance. Thus, our strategy has been to *invite* all teachers but to work intensely only with those who self-select, based on the hypothesis that success is more likely if the project scales up from an enthusiastic and informed core group that might infuse research insights into the system.

The school was chosen for many reasons, including a self-reported need to improve mathematics pedagogy and a teaching staff with little turnover, which facilitates a multi-year study. The five-year, design-based study has multiple data sources, including student understanding, attitude, and achievement, along with discourse patterns among teachers. Activities over the first year of the five-year project were focused entirely on a core group of 11 teachers.

In initial interviews, teachers noted their practice did not align with reform emphases and excused that fact by pointing to administrative pressures, students' need for "structure," parental demands, and colleagues' expectations – along with worries about time, coverage, and disciplinary expertise. Those teachers then completed three courses. The first focused on mathematical concept study, the second on current theories of learning, and the third on task design and implementation. In follow-up interviews, after those courses, the elements that had previously been reported as constraints had been reformatted as licenses. In particular, after experiencing in-class successes with students, the excuse that they needed structure was replaced by an obligation to present them with challenges and to allow them to struggle. Similar shifts were noted in imaginings of parents' and colleagues' reservations. As educational researchers and teacher educators, we conclude that it is likely more important to focus on reformatting *constraints* as *licenses* than to attempt to *remove obstacles* or *overcome barriers*.

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STUDENT INTERPRETATIONS OF AXIOMATIZING

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Students often struggle in collegiate, proof-oriented courses because they interpret and use elements of mathematical theory differently from mathematicians. For instance, students' reasoning about definitions often focuses more upon exemplars or prototypes (*referential*) rather than defining properties (*stipulated*). Even mathematicians' views of geometric axioms shifted over time from being true statements about planes to being useful assumptions for proving (Freudenthal, 1973).

By conducting iterative teaching experiments in guided reinvention (Gravemeijer, 1994) of axioms of neutral, planar geometry, I investigated the following research question, "How do students interpret the nature of axioms and the purpose of axiomatizing both initially and after reinventing a body of axiomatic theory?" The data reflects four semesters of teaching experiments at a large, public university in the USA, alternating between small groups (2-4) and whole class experiments. Students' decisions about defining and axiomatizing were informed by observations of geometric phenomena on familiar (Euclidean, spherical, hyperbolic) and idiosyncratic planes. I identified students' interpretations of axioms through open and axial coding of their explanations and use of both axioms and example planes for making decisions in the reinvention process and for proof construction.

Four categories of student interpretations of axioms emerged. *Exclusive referential* interpretations treated axioms as true statements about prototype planes (Euclidean or Spherical), but treated other planes (e.g. finite planes) as illegitimate. *Inclusive referential* interpretations treated axioms as descriptions of familiar and unfamiliar planes alike and viewed adding axioms as explicating more geometric intuitions into the formal system. *Stipulated* interpretations proposed that different sets of axioms constitute different geometric realities. *Formal* interpretations viewed the axiomatic theory as the objects of study, attending to independence and provability relations among mathematical statements. The final three interpretations were not mutually exclusive in student reasoning, and are each compatible with modern mathematical practice. Students expressing the first interpretation tended to struggle in axiomatizing and proving because they could not curtail their reasoning to reflect the structure provided by the axioms, often relying on spatial intuitions instead. Many students shifted toward latter interpretations, suggesting metamathematical learning and greater focus on mathematical structure as a result of engaging in axiomatizing.

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THE ENTANGLEMENT OF RATIONALITY AND EMOTION IN MATHEMATICS TEACHING

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Habermas (1998)'s idea of *rationality* deals with how a rational being can give account for her orientation toward validity claims in the context of discursive activities. Whereas it recognizes an interrelation of the epistemic, teleological and communicative roots of rationality, it seems to dismiss aspects related to the essence of the human being. One aspect concerns the fact that any decision-making process is not detached from body and *emotion* (Immordino-Yang & Damasio, 2007).

My research aims to study *how* emotional engagement affects the rationality of the mathematics teacher, who makes decision in the social context of the classroom. I am especially interested in the *entanglement* of rationality and emotion for her to respond appropriately in teaching situations. I draw on the notion of the *emotional orientation*—which Brown and Reid (2006) offer as a means to examine the teacher's decision-making—to identify a source for talking about the entanglement.

The participants to the research are three secondary school teachers in the context of introducing algebra, in particularly the topic of linear equations. I focus on their interviews and the activity in their grade 9 classrooms. Through the beliefs that are declared in the interview and the belief-related actions during videotaped activity, I identify emotional orientations. I analyse how the emergence of these orientations in the classroom serves the purpose of rationally reaching goals and understanding.

The results suggest that rationality and emotion together characterize the teacher as the weave and the warp together shape a fabric. The rationality and the emotional orientation of the teacher are intertwined as the weave and warp of the fabric. The warp relates to emotional orientation as well as the weave relates to actions. Looking at the backwards of the fabric, we also discover all prior experience of the teacher, without which she would not be the same teacher. I will give the case of one teacher.

I propose here that we need to recast the being of the teacher in the classroom as an *emotional rational being* with agency. This argument aligns with a new materialist theory of embodiment, which rethinks the relationship between matter and meaning, as well as between body and concept, in mathematics teaching and learning.

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TEACHERS' BELIEF SYSTEMS ABOUT TEACHING NON-ROUTINE MATHEMATICAL PROBLEM SOLVING: STRUCTURE AND CHANGE

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Teachers' "belief system" (Green, 1971) is regarded as essential in shaping their decision making and teaching practice in the classroom. However, the extant studies focused more on *pre-service primary* mathematics teachers' *domain-general* belief system (e.g., beliefs about mathematics teaching), and how it may change after certain *one-off* interventions. Very few studies have examined *in-service secondary* teachers' *domain-specific* belief system and its change. Besides, it remains unclear whether teachers' changed belief system can persist. Therefore, situated in a project that used the "design-experiment" approach, this study aims to answer two research questions: (a) What are the structures of three Singapore teachers' belief systems about teaching non-routine mathematical problem solving (MPS) at the end of the two cycles, and (b) Are there any changes in the teachers' belief systems across the two cycles?

Three secondary teachers participated in this study, consisting of two research cycles. All teachers engaged in a two-day MPS workshop and several on-going professional development sessions before and during their teaching MPS, respectively. During both cycles, all teachers taught a 10-lesson module of non-routine MPS, in which Schoenfeld's (1985) framework and Polya's (1957) model of MPS was embedded. Data sources included interviews, teachers' reflective journals, and the researchers' field notes. Inductive coding and constant comparison methods were used.

As consistently identified at the end of two cycles, the three teachers belief systems about teaching non-routine MPS were generally constituted by five dimensions: *beliefs about the effects of the MPS teaching on students' learning, beliefs about MPS, beliefs about scaffolding for students, beliefs about the capacity to teach MPS*, and *beliefs about practice issues of teaching*. Similar but different changes were recognized for the first four dimensions, while little change for the last dimension (e.g., time for implementation). The results corroborate the coherent nature and the psychological strengths of individual beliefs in the belief system (see Green, 1971). Future research can explore the "mechanism" about the changes in teachers' belief systems in a more fine-grained way. More detailed results will be discussed in the presentation.

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CHARACTERISTICS OF STUDENT WORK SAMPLES RELATED TO STUDENT TEACHER PROFESSIONAL NOTICING

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Jacobs, Lamb, and Philipp (2010) proposed a construct of professional noticing of children's mathematical thinking that can be used to evaluate teachers' decision-making processes when analyzing students' written responses. For preservice student teachers, analysis of written student work is a documented area of weakness due to lack of experience working with actual students (Jacobs et. al., 2010). Yet research has shown many positive affordances associated with teachers analyzing work from their own classrooms (Kazemi & Franke, 2003). The teachers are generally more engaged because the students are familiar and the challenge of assisting them is real. Since student teachers are engaged with their own students, attention to their students' thinking should be viewed as a means of student teacher development.

This design research study analysed student teachers' engagement with professional noticing through a set of researcher-designed tasks focused on analysis of the student teachers' students' multidigit addition and subtraction work. Four student teachers placed in first grade and three student teachers placed in second grade participated in the study. Data was analysed via ongoing and retrospective analysis. Both predetermined and open codes were used. The predetermined codes were based on the aforementioned professional noticing of children's mathematical thinking framework (Jacobs et. al., 2010).

The purpose of this paper is to share some characteristics of the student teachers' student work samples that influenced their engagement with professional noticing. Student work samples that were hard to interpret due to the student teachers' lack of questioning their students often decreased engagement with professional noticing. Easy and expected responses, for simple problems also led to less engagement. There was not much to understand; therefore, the student teachers did not provide evidence or specifics. In contrast, many times the student work samples that were the most difficult to interpret were those that were those that were coded with the highest level of professional noticing because the student teachers were most committed to interpreting the solutions. Further results and examples will be provided during the presentation.

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SOCIOPEDAGOGICAL NORMS ESTABLISHED THROUGH TEACHERS' DISCOURSE ABOUT MATHMATICS INSTRUCTION

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Discourse in mathematics professional development (PD) meetings focuses not only on content but also on instruction. In such settings, beyond social norms for discourse participation and sociomathematical norms for talking about mathematics (Van Zoest et al, 2011; Yackel & Cobb, 1996), there is a need to establish sociopedagogical norms to talk about mathematics instruction. Defining these norms will help PD providers establish what counts as productive discourse about mathematics instruction. For example, what kinds of evidence and justifications are necessary when one is examining mathematics teaching? What types of descriptions of mathematics students and mathematics instruction are most productive? The purpose of this paper is to share our initial investigation into such questions.

Our research was carried out in the context of Project xxx, a 40-hour, yearlong PD program for elementary teachers. The PD focused on promoting mathematics discourse in elementary classrooms as a viable approach to support the development of conceptual understanding. In PD discussions, participants considered their own instruction as well as video footage of classroom episodes. For this study, teacher discourse surrounding mathematics instruction was analysed for a cohort of 17 second-grade teachers. We developed a coding scheme based on existing literature on norms and applied it to our data. We also allowed for alternate codes and norms to emerge and used constant comparison to make sense of the socio pedagogical norms that were in place during Project xxx PD discussions.

Preliminary results highlight the importance of norms relating to the teachers' discussions about mathematics instruction. We conjecture that the teachers' tentative or certain stance (Van Zoest et al., 2011) and whether the teacher was talking to her fellow participants or the facilitator are different based on whose instruction is being discussed (i.e. own instruction or another's instruction). Based on these results, we suggest that PD providers establish sociopedagogical norms for PD discussions that focus on mathematics instruction.

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EXPLORING MATHEMATICIANS' AND MATHEMATICS EDUCATORS' PERSPECTIVES ABOUT MATHEMATICS

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This study originates from the belief that mathematicians think of themselves as people who do 'real math' and they see others, especially math educators, as people who do not do mathematics (Shulman, 1986). Many mathematicians are sceptical of the reform in mathematics education because they believe it will create mathematically incompetent individuals (Boaler, 2008). On the other hand, math educators tend to encourage reform in efforts to enable students to learn mathematics meaningfully (Boaler, 2008). Following are the research questions to look at the disparity in their views more closely;

- How do mathematicians, math educators and individuals who transitioned to math education from the field of mathematics describe their beliefs about mathematics? In what ways are they similar or different?
- In what ways are individuals' backgrounds and the degree of involvement in mathematics and math education influential on their beliefs?

The participants represent three groups; mathematicians, mathematicians who have transitioned to educational studies and math educators. A total of six people participated as two people from each category. Two semi-structured interviews were conducted. The first one elicits on ideas about mathematics, background as a learner and experiences/perspectives about teaching. The second interview asks participants to watch two short videos from two math classes. Then, they are asked to reflect on what they see in the videos. For the data analysis, thematic analyses were conducted. The preliminary findings showed that the variation in individuals' ideas about the nature of mathematics, mathematics teaching and learning largely depends on their experiences as a learner and a teacher. This variation held true even for the same group of participants. Moreover, the findings revealed that based on her heavily traditional math learning background one of the participants believed that being able to do the basic calculations in mind is a necessary part of learning mathematics and she disallowed her students to use calculators during her college level math classes. Further results will be discussed in detail during the presentation.

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EXPLORING CONSTRUCTS OF STATISTICAL VARIABILITY USING DYNAMIC GEOMETRY

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Variability—how spread out the distribution of a data set is from the centre—is a foundation concept in introductory statistics in that it links to many other concepts, such as regression analysis and sampling distributions. Research studies (see Garfield & Ben-Zvi, 2008) however, reveal that while students can calculate measures of variability such as the range, variance and standard deviation in a data set, they rarely can connect them with other concepts that they learn in statistics. The current study assumes that a dynamic geometrical sketch, hereafter called the DMS, offers students better chances of understanding the meaning of variability through process of semiotic mediation. The study draws on two different theoretical perspectives: i) using signs to mediate meanings of mathematical concepts (Falcade, Laborde & Mariotti, 2007); and, ii) aggregate reasoning with statistical data (Wild & Pfannkuch, 1999), focusing on students' understanding of the overall characteristics of a data set, including its shape, and the spread from the centre. Five student participants enrolled in a one semester university introductory statistics course were clinically interviewed about their understanding of the meaning of variability, before interacting with the DMS. The participants were also interviewed while they interacted with the DMS, to enable the author compare expressions in the static, as well as in the dynamic mathematics environments.

Findings suggest that the dynamic mathematics sketch produced signs that mediated the meaning of statistical variability during and after participants interacted with the sketch. The dynamic sketch also seemed to support students' aggregate reasoning with statistical data in that after interacting with the sketch, fewer participants relied on recalling from the textbook, but presented the meaning of variability in their own words. However, it appears that using DMS to learn concepts calls for more time of practice on the part of students given that at the end, some students still relied on the static way the concept of standard deviation is presented in some text books.

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SHORTCUTTING ABDUCTION TO ENABLE JUSTIFICATION

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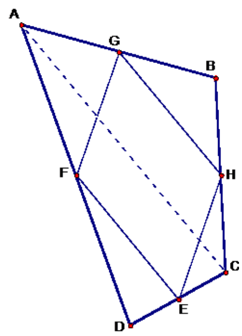
For a task meant to elicit student generalizing and justifying, we have developed a theoretical tool for interpreting how the task's implementation facilitates student justifying. Based on the specific types of reasoning students use, we have observed two general modes of implementation. In an *anticipatory* task implementation, students develop a general claim through reflective abstraction, which allows them to readily justify it. In a *sleuthing* task implementation, the students arrive at a general claim by reasoning empirically about the features of mathematical objects or results. Although at this point they can readily produce an empirical justification, it often requires extensive further abductive reasoning for them to make an analytical justification (Harel & Sowder, 1998). In general, abductive reasoning seeks a rule accounting for an observed or desired result, then infers that the rule's condition is plausibly true in the case at hand. We adapt this to describe the reasoning a student uses to find an analytical justification for a general claim (if P, then Q): they seek a plausible "rule" by which Q logically follows from P. Often this involves finding (i) a rule P'=>Q that accounts for why P implies Q, and/or (ii) a way to represent or transform the objects in the claim's hypothesis P into a form P' the rule applies to.



For example, college geometry students constructed a generic quadrilateral ABCD using Geometer's Sketchpad, connected its midpoints to form a new "midpoint quadrilateral" EFGH. They quickly empirically conjectured that for any quadrilateral (P) its midpoint quadrilateral is a parallelogram (Q), but sleuthed unsuccessfully for a justification for 15-20 minutes. The instructor then drew a diagonal AC of the original quadrilateral, circled triangle ABC, and asked about the relationship between AC and GH. Many students then readily recalled and used the Triangle Midpoint Connector Theorem, which they had learned the previous day: In triangle ADC (P'), GH||AC and FE||AC (Q'), so GH||EF (Q, with similar reasoning to show FG||EH).

Our theoretical tool reveals how the teacher's scaffolding shortcutted the abductive sleuthing by representing the objects in the hypothesis P in a form that allowed the students to invoke their known rule P'=>Q', lowering the cognitive demand in one aspect in order to purposefully enable the production of high-level justification.

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MATHÉMATIQUES EN FRANÇAIS, MATH IN ENGLISH: DISCOURSE IN AN ELEMENTARY SCHOOL FRENCH IMMERSION MATHEMATICS CLASSROOM

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French immersion education is an area of growth in Canada. It has been well established in the literature that curriculum content, including objectives in mathematics, can be mastered in a French immersion setting. Literature also supports the idea that students can transfer mathematical learning from one language to another (Cummins, 2000), yet the characteristics of mathematical discourse during the transition from one language of instruction to another remains largely unexplored.

The goal of this research was to describe the mathematical discourse in an elementary French immersion classroom before and after a transition to mathematics instruction in English. In order to highlight the interplay between communication and cognition, four categories of word use, visual mediators, endorsed narratives, and routines were used as a framework to analyze the mathematical discourse (Sfard, 2007).

The sample was an elementary French immersion class. Data, consisting of digital audio recordings from six classroom visits and interviews with students and teachers, as well as photocopies of student work, were collected during two periods: in the students' grade three year, when mathematics instruction was in French, and one year later, when the students were in grade four and the language of mathematics instruction was English. This data was then analyzed using Sfard's framework.

Analysis has highlighted the role of conventional and unconventional mathematical word use in endorsing narratives. Fluent speakers of the language of instruction used unconventional as well as conventional mathematical words to endorse narratives, while the endorsements of less fluent speakers utilized more conventional expressions and vocabulary.

This work provides insight into the mathematical thinking in the classroom, with a view to improving pedagogy in multilingual mathematics settings. Further investigation is needed to examine the effects of these patterns of communication on student cognition as they use language to construct their mathematical understanding.

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THE CHARACTERISTIC OF STUDENTS' ALGEBRAIC THINKING: FOCUS ON THE LINEAR EQUATION WITH TWO UNKNOWNS AND THE LINEAR FUNCTION

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Sfard and Linchevski (1994, p.191) analyzed the nature and the growth of algebraic thinking from an epistemological perspective supported by historical observations. Eventually, its development was presented as a sequence of even more advanced transitions from operational conception to structural conception. This model was subsequently applied to the individual learning. The focus was on two crucial transitions. One was from the purely operational algebra to the structural algebra of an unknown and the other was from here to the functional algebra of a variable. The difficulties experienced by the learner at both these junctions was illustrated with much empirical data.

However previous studies only pointed out a part of the difficulty of algebraic learning. This study focuses on the scene in which students must transform the view of the linear equation; from "equality" to a "functional relationship". This study is a part of the basic research for identifying the difficulty of students in the scene to transform the view of letters in equation; from the "unknown" to "variable". The purpose of this study is to identify the characteristics of the students' algebraic thinking.

This study was a qualitative research based on data collected through a survey. For This purpose, the 9th grade 154 students were given the set of assessment tasks about the graph of a linear equation with two unknowns and the distinction between linear equation with two unknowns and equation of linear function. In order to analyze data from the viewpoint of functional approach, some categories were made by identifying skills and concepts for functional approach.

The results showed two characteristics as follows; (1) 26% of them recognized that a set of points isn't a line. Therefore the students recognized the graph of linear equation with two unknowns as a discrete graph. (2) 37% of them considered that the graph becomes a straight line if the linear equation with two unknown is solved for y. Namely, the student distinguished the linear equation with two unknowns and the linear function by the form of expression (i.e. explicit function or implicit function). In the other words, the students couldn't distinguished the equation and the function by mathematical definition.

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EFFECT OF PERCEIVED RANK AND MATH ANXIETY ON METACOGNITION IN UNDERGRADUATE MATH EDUCATION

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Metacognition is crucial for academic performance. Inability to assess ability accurately can lead to non-optimal learning. Students have paradoxically displayed high overconfidence in predicting math performance despite being math anxious (Erickson & Heit, 2013). Other work indicates that individuals' beliefs about their relative ability (rank) within a group of peers can affect metacognitive judgments and performance satisfaction (Brown, Gardner, Oswald, & Qian, 2008). If metacognitive calibration is unaffected by math anxiety, is it instead aligned with how individuals rank their own ability when compared a group of peers?

Our sample comprised 53 undergraduate students, who predicted their performance on a math test. After the test, they provided a postdicted score. Participants also reported predicted and postdicted scores for the rest of the participants in the study. They were not told their own scores or others' scores. Then, participants completed a shortened Math Anxiety Ratings Scale (MARS, Alexander & Martray, 1989).

Individuals' perceived rank of their own ability relative to their peers' was highly correlated with metacognitive judgments of their own performance (r=0.60, p<0.001 for predictions; r=0.68, p<0.001 for postdictions). Those who ranked themselves higher among peers tended to be less accurate (over-confident) in their metacognitive calibration, while those who ranked themselves lower among their peers had more accurate self-estimates of performance. MARS scores indicated that students were math anxious. Math anxiety was unrelated to metacognitive calibration but was moderately correlated with math ability (r= –0.27, p<0.05). Metacognitive calibration improved in postdictions of self-ability compared to predictions. However, estimates of peer performance did not change significantly from predictions to postdictions. From these analyses, we hypothesize that individuals' perceived rank of ability among peers, not actual ability nor math anxiety, plays a role in determining metacognition .

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INTEGRATING TECHNOLOGY IN MATH CLASS: HOW, WHEN AND WHY?

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According to findings in the research field on technology in mathematics education, it is extremely important that teachers should understand and become aware of the affordances, constraints, and general pedagogical nature of technology as a new resource (Ruthven & Hennessy, 2002). In this paper we focus on the decision to use technology for teaching and learning maths, attempting to answer the following research question: what are the crucial factors influencing the awkward process of integrating technology in math class? To do this, we report and discuss results from three different studies. The first study concerned teachers' perceptions of technology in math class. Findings from an anonymous questionnaire (submitted to 129 teachers) revealed, in particular, that in their perception of the use of technology in mathematics class, both pre-service and in-service teachers lacked a clearcut awareness of the opportunity it offers to create a new "milieu" and change the "economy" of the solving process. The second study aimed at investigating how teachers orchestrate activities in a technology-rich class. Among the teachers we videotaped and interviewed, we focus on the case of Enza, whose teacher training "in action" experience helped her to verify potentialities and constraints of her orchestrations and to focus on the opportunity to use technology within an integrated learning environment. The aim of the third study was to analyse the relationship between work with manipulatives and technologically instrumented work. The research results of a teaching experiment (conducted with 5th grade pupils) revealed that within an integrated laboratory approach, students may develop not only a change in their geometrical work but also a feeling for the need to obtain proof of any explanation. These studies have strengthened our conviction that crucial factors influencing the awkward process of integrating technology in math class are: an awareness of the potential and limitations of technology in the teaching field and a knowledge of the related pedagogical content; the decisions the teacher takes in determining when to integrate technology in everyday teaching practice and how to structure the learning environment; the choices s/he makes when facing the problems that new environments require a new set of mathematical problems and that the instrumentation process and its variability need to be managed.

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PROSPECTIVE ELEMENTARY TEACHERS' ANALYSIS OF STUDENT THINKING

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Research shows that prospective elementary teachers who study children's mathematical thinking strengthen their content knowledge as well as develop more sophisticated beliefs about mathematics, teaching, and learning (Philipp et al., 2007). Additionally, awareness of children's mathematical thinking can lead teachers to shift their practice from demonstrating procedures to engaging students in problem solving and concept building (Fennema et al., 1996).

The aim of this research project is to identify the extent to which prospective elementary teachers were able to anticipate and make sense of children's mathematical thinking around fraction comparison. As part of a larger study on task design, 25 prospective elementary teachers enrolled in a mathematics content course were given three fraction comparison problems and asked to anticipate the types of incorrect responses a child might give for each comparison. They then viewed a video of a 5th grade student solving the same fraction comparison problems and described the child's misconceptions. Reflective analysis of the prospective teachers' written responses and discussions about the video showed that they were generally able to anticipate children's misconceptions around fraction comparisons, but they initially struggled to understand the 5th grader's misconceptions following the viewing of the video.

The results support the need to expose prospective teachers to children's mathematical thinking as one way to help prospective teachers develop mathematical knowledge for teaching. In the presentation, detailed results will be discussed.

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ASSESSING MATHEMATICAL BASIC KNOWLEDGE BY MEANS OF ADAPTIVE TESTING ELEMENTS

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In answer to the common dilemma of limited testing time and the amount of contents, an assessment tool has been developed which uses adaptive elements for a stepwise break down of the demanded cognitive actions.

Especially when it comes to assessments covering the basic knowledge of several years, bundling the plenitude of relevant contents is a challenge. A common strategy is the use of complex multi-step items demanding to combine several elementary contents and cognitive actions. This way, potential deficits in elementary knowledge are hard to localise. Single-step items, on the other hand, are highly time-consuming and do not demand any combination of contents or actions, although that should as well be part of basic knowledge. The developed tool however presents a pool of complex main items to each student. Only if it comes to failure, the student is led through a loop of elementary (single-step) items. Thus testing time can be used more effectively and possible sources of errors deriving from elementary deficits might be detected. The adaptive breaking down of the items is realized on the level of contents and cognitive actions. On the one hand there is a separation by terms, theorems and procedures and on the other hand by the two elementary cognitive actions Identification and Construction (Nitsch et al., in press). Thus the term adaptivity is used here in the sense *of branched testing* according to Kubinger (1988).

An assessment tool designed as described above is being tested within the field of differential calculus. The test is supposed to reveal whether the constructed loops contain the most important elementary contents and cognitive actions suggested by theoretical analysis. Another focus is set on the evaluation by students and teachers. Since the tool is able to give hints on deficits in elementary knowledge, it is expected to be perceived as a significant device when it comes to the decision what e.g. repetition units should focus on. Another focus is set on the deviations of individual processing time, since students go through different item sequences. Due to the construction of the elementary items as single-step items deviations are expected to be moderate. Further results concerning the implementation of the tool in classroom will be presented.

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A MATHEMATIC LESSON ON MEASUREMENT – ANALYSED THROUGH DIFFERENT LENSES; LEARNING ACTIVITY AND VARIATION THEORY

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Cobb and Yackel (1996) claim that different perspectives could be complementary when analysing empirical data. The aim of this study was to analyse a mathematic lesson on measurement by using two theoretical approaches; Learning Activity theory (Davydov, 2008) and Variation Theory (Marton & Tsui, 2004). Both theories state that the multiplicity of human is determined by social practice, i.e. human being and society cannot be separated, but Learning Activity is related to students' learning actions in a learning task, whereas Variation Theory offers a tool for analysing how the content is handled in lessons.

The empirical data consists of one digitally recorded mathematic lesson in School no. 91, a federal school in Moscow. The lesson was on different measure units in a first grade class. One aim for this lesson was to find a general method for comparing volumes using a unit of measurement, by jointly solving a hands-on activity.

The recording of the lesson was analysed from the different perspectives. When comparing the theoretical analyses, we found that from both perspectives the use of several student conceptions simultaneously could be seen as something that would increase the learning possibilities. The two theories also led to different, but not contradictory, conclusions. In the LA perspective the task can only be solved if the students are able to create a unit of measurement. A new piece of knowledge is needed and also developed by the students. From the VT perspective questions could be raised if some aspects of the content still are taken for granted and therefore not possible to discern. One sequence from the lesson will be shown and discussed during the presentation, highlighting the strength of using the two theories when analysing empirical data.

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COULD NARRATIVES HELP PRE-SERVICE PRIMARY TEACHERS TO DEVELOP THE SKILL OF NOTICING STUDENTS' MATHEMATICAL THINKING?

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Noticing what is happening in a classroom is an important skill for teachers but noticing effectively is complex. Previous research indicates that it is possible to enhance the pre-service teachers' learning of noticing students' mathematical thinking. Our study is embedded in this literature and examines if the task of writing "narratives" could help pre-service teachers to develop the skill of noticing students' mathematical thinking. Narratives are a form of expressing teachers' practical understanding of mathematics teaching (Chapman, 2008). So, the narratives of pre-service teachers describing what they notice about the teaching of other teachers and students' mathematical thinking might be a relevant tool in their learning to be a teacher.

Participants were 41 pre-service primary school teachers that were in the period of teaching practices at primary schools (practicum). Pre-service teachers had to write a narrative related to the students' mathematical competence. We provided them with specific guidelines: describe "in detail" the mathematics teaching-learning situation (the task, what the primary school students did, what the teacher did), interpret the situation (evidence of students' understanding of the mathematical competence) and complete or modify the situation in order to help students to develop other aspects of the mathematical competence identified (Jacobs, Lamb & Phillip, 2010).

The narratives written by prospective teachers in this context focused on identifying some evidence of students' mathematical understanding help prospective teachers to enhance their ability of noticing the students' understanding. Our study provides new results to understand how the skill of noticing students' mathematical thinking can be developed in teachers' education programs.

Acknowledgments

The research reported here has been financed by the Ministerio de Educación y Ciencia, Dirección General de Investigación, Spain, under Grant no. EDU2011-27288.

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HOW TEACHERS UNDERSTAND MATHEMATICAL PRACTICES

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The Common Core State Standards for Mathematics (CCSSM) are being implemented in K-12 schools across the United States. The CCSSM outline eight Standards for Mathematical Practice (SMP) that focus on the development of mathematical disposition among children. We investigated mathematics teachers' understanding of these standards.

With the introduction of the CCSSM, efforts to establish national curricula have been made. The CCSSM consist of the Content Standards, which outline specific mathematics classroom content, and the Standards for Mathematical Practice (SMP), four of which include: reason abstractly and quantitatively (SMP2), model with mathematics (SMP4), look for and make use of structure (SMP7) and look for and express regularity in repeated reasoning (SMP8). Due to their novel nature, we examined prospective and practicing teachers' understanding of these four standards using Mathematical Knowledge for Teaching, as a lens for analysis of data (Ball, Thames, & Phelps, 2008).

We administered surveys to 48 inservice teachers (ISTs) who were mathematics coaches participating in a professional development program. The survey asked the ISTs to explain each SMP. Additionally, semi-structured interviews with 6 preservice teachers (PSTs) were conducted in an effort to obtain in-depth data on their understanding of the demands of these standards.

Analysis of the surveys and interviews revealed varied and inconsistent interpretations of the SMP for both teacher groups. Additionally, the PSTs' already established beliefs and their school-based experiences with the SMP influenced how they interpreted the SMP and judged their utility in the classroom.

These findings have implications for teacher education programs. Results cause concern for the implementation of the SMP in the classroom; if teachers do not have a clear understanding of the SMP, they will likely not be implemented the classroom. Work must be done to improve and solidify teachers' conceptions of the SMP. Further research should include if and how teachers' conceptions of the SMP influence their planning and instructional practice.

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TEACHERS LEARNING TO PROMOTE STUDENTS' MATHEMATICAL REASONING

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INTRODUCTION

Reform movements in mathematics education recommend that mathematical reasoning play a central role in classrooms (see e.g., NCTM, 2000). Yet, teachers still find it challenging to promote thoughtful mathematical activity in classrooms (e.g., Cohen, 2004; Jacobs et al., 2006). Often they often lack the knowledge and skills to do so. This paper reports on a research project that tried to promote such skills among in-service middle-school mathematics teachers. The teachers facilitated research sessions on students' development of mathematical ideas. Videos of their interactions students were analysed for insights on how the tried to influence students' thinking.

THEORETICAL PERSPECTIVE

Telling has been downplayed in teaching due to perceived inconsistencies with constructivism. Yet, Lobato et. al. (2005) argue that it can still be instructionally important if it is redefined as "a set of actions that serve the function of stimulating students' mathematical construction via the introduction of new mathematical ideas into a classroom conversation" (p. 110) and two ways of introducing new information are distinguished. *Initiating* actions seeks to stimulate students' thinking. *Eliciting* actions try to determine students' thinking. Promoting students' reasoning is using appropriate initiating and eliciting actions to induce students restructure their thinking.

RESULTS AND IMPLICATIONS

Overall the results from the research project highlight the importance of teachers' knowledge of how students think and build mathematical ideas. This has implications for discussions on skills required to promote meaningful learning in classrooms.

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PRE-SERVICE TEACHERS’ NOTICING FOCUSED ON THE USE OF REPRESENTATIONS IN CLASSROOM VIDEOS

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With mathematical objects being abstract, teaching and learning mathematics cannot be imagined without the use of representations that stand for those objects. However, the mere existence of multiple representations in the classroom is not sufficient by itself but rather *the way* representations are dealt with is crucial for the students’ development of mathematical understanding (cf. Duval, 2006). Both the professional knowledge and professional vision of teachers play an important role in this context: In the complexity of the classroom, teachers have to notice significant events concerning the use of representations in the sense of identifying and interpreting such relevant classroom interactions based on their corresponding professional knowledge (cf. Sherin & van Es, 2008).

Since empirical studies on teacher noticing under the focus of using representations are scarce, we carried out an exploratory study with 31 pre-service teachers. The aim was to find out what pre-service teachers *notice* concerning the use of representations when they observe classroom situations, in particular how they *evaluate* the observed situations and what *reasons* they give for their evaluations. To stimulate noticing, we showed the pre-service teachers two authentic classroom videos in which the use of representations played an important role and asked them to complete a questionnaire comprising both open-ended and multiple-choice questions. The answers to the open-ended questions were coded according to a top-down coding scheme by two raters ($0.66 < \kappa < 0.87$).

The findings indicate that the answers of the pre-service teachers were substantially influenced by the question formats: Answering the open-ended questions, most pre-service teachers described for instance the use of representations in the classroom videos as supportive for the students’ understanding whereas the multiple-choice question formats strongly increased the number of critical evaluations. In the presentation, further results and sample answers will be shown and discussed in more detail, alongside with implications for further research on teacher noticing under the focus of using representations in the mathematics classroom.

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TEACHER'S MATHEMATICAL INQUIRY IN WORKSHOPS

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In a project combining research and development the aim was to promote inquiry approaches to mathematics teaching in order to build students' understanding and teachers' flexibility in their work with students. Key activities in the project were workshops in which teachers and researchers worked together inquiring into mathematical tasks, discussing both mathematics and teaching approaches.

Recent TIMSS and PISA reports document that the major activity in Norwegian mathematics classrooms are students working on tasks which are similar to those in textbooks (www.pisa.no, www.timss.no). Furthermore, the reports reveal that limited time is used discussing solution strategies and students finding their own methods. This points to fairly teacher centred and traditional teaching, and motivate the project work to promote inquiry activities for students in order to develop deep understanding.

The theoretical framework is a sociocultural view of learning with activities in the project based on inquiry and learning communities (Jaworski, 2009). By collaborating in communities teachers and researchers learn from each other. Inquiry applies to both mathematics and mathematics teaching through activities of wondering, questioning and investigating; thus promoting teachers' learning and the development of teaching approaches. The research is based on qualitative methods, using video and audio recordings, and a grounded approach to the analysis, combining observation data, information from interviews and teachers own presentations in workshops.

Teachers participating in courses or projects often focus on their students' work and discuss teaching approaches and less often inquire into the mathematics per se in order to deepen their understanding. It is expected that engagement in mathematics will feed back and enhance the teaching approaches and collaboration with colleagues.

The research question for this paper deals with teachers' mathematical inquiry in the workshops: What kind of mathematical inquiry was observed in the workshops and how did the experiences impact the teachers further planning and work with students?

Results from research encompass teachers' engagement with mathematics that led to mathematical challenges which the teachers solved through joint activity. In other cases their inquiry challenged their mathematical knowledge. The teachers enjoyed the work on mathematics and this seemed to stimulate more talking about mathematics and closer collaboration between teachers in schools.

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THE VALUE OF GEOMETRICAL CONSTRUCTIONS: DISCOVERING, REASONING AND PROVING IN GEOMETRY

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The pedagogic value of geometrical constructions has been recognised for many decades and beyond (e.g. Polya, 1962) However, Tam and Chen (2012) have also identified that students often find it difficult to make a link between the construction procedures and their underlying geometric properties. Such issues arise whether constructions are performed with physical instruments or with computer software. The aim of this paper is to provide a guiding principle that can be used to inform the use of geometrical constructions in ways that foster students' proving processes (be this with or without technology).

In our research, we approach this through our central research question: how does reasoning and proving emerge during the process of geometrical construction? In doing so, we focus on issues around 'discovering' and proving. We refer to the word 'discovering' in line with De Villiers (1990). When students solve challenging geometrical construction problems such as 'Construct a square which goes through the two given points A and B', it is expected that students might 'discover' facts, reasons and relationship, for example (a) what 'conditions' are necessary to construct a shape; (b) their methods of construction; (c) reasons why their methods are correct; (d) relationship between the conjecture and conclusion.

We report two cases from Grade 7 (12-13 year old) which undertook the construction of a square described above to consider our research question. As we expected the students 'discovered' various methods to construct a square ((a) and (b)), but at the same time, they had difficulties to use properties of shapes to reason why their construction would be correct ((c)and (d)). However, after they experienced more constructions and reasoning, they gradually became aware how they could utilise properties of shapes, and a shift of their reasoning took place, i.e. a shift from relying on visual appearances or measurement to reasoning with properties of shapes. Detailed analysis of the students' construction processes and their reasoning will be presented in the actual presentation.

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USING VIDEO CASES TO LEARN TO PAY ATTENTION TO CHILDREN'S THINKING

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Paying attention to children's conceptions and their reasoning is one way for teachers to develop their pedagogical content knowledge. Mathematics education researchers have theorized that if teachers listen to children, understand their reasoning, and teach in ways that reflect this understanding, not only will teachers provide those children with a better mathematics and science education, but also have a powerful effect on the way teachers view mathematics learning (Jacobs, Lamb, & Philipp, 2010). This study focused on how preservice teachers conceptualize theoretical models of a student's mathematical thinking through the use of a video case. The purpose was to further understand how preservice teachers think about students' mathematical reasoning and how they use evidence to support models of students' thinking.

Participants in the study were nine preservice teachers in a methods course at a large university in the Midwestern portion of the United States. The course was designed to improve content and pedagogical knowledge by focusing on how young students learn and think about mathematics. The preservice teachers were asked to complete a video case focused on how one kindergarten child, Grace, was comparing and representing numbers to ten. In one of the components of the video case PSTs were asked to write a model to describe Grace's thinking. In another component they were asked to provide evidence to support their model. Data were analyzed qualitatively by examining the preservice teachers' models and evidence to determine how they were thinking about Grace's understanding in accordance with a research based rubric, termed Prediction Assessment Rubric (Norton, McCloskey, & Hudson, 2011).

We found that all of the PSTs created a model of how Grace was thinking and used some evidence in their models, however the quality of the models and the way in which they used the evidence varied. Only four of the PSTs produced models that were rated at the highest level in the rubric, these were models that included a reasonable explanation of how Grace was thinking and used evidence to support their explanation. We conclude that although video cases can provide opportunities to engage in model building, and all of the PSTs in our study attended to evidence to create their models, PSTs do not find it easy to create such models.

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EXPERIENCING TECNOLOGY IN MATHEMATICS CLASSROOMS: THE CASE OF SIMCALC

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Considering pre-service mathematics teachers lack discussions regarding teaching with the use of technology in their formation, we developed a project to initiate undergraduate students in research, in an attempt at helping them to elaborate activities to teach mathematics using a software. For that, under our supervision, one student, who was already teaching mathematics in secondary school, engaged on the steps of a documental research study aiming to develop activities regarding the concept of function, using a software of her choice, to be administered to 9th graders. She analysed some Brazilian mathematics textbooks, a few Brazilian research studies regarding the use of technology to teach functions, and the softwares used in these studies: Cabri-Géomètre and Geogebra. She also evaluated two other softwares that she had contact with: Winplot and SimCalc. Analysis were made in the light of the theoretical framework of the Three Worlds of Mathematics (Tall, 2013), the conceptual embodied world, the *operational symbolic* world and the *formal axiomatic* world.

With this study, she realised the analysed textbooks do not provide relations between mathematical worlds, since they rarely show connections between graphs and algebraic laws of functions, for instance. Considering the softwares, the one of her choice was SimCalc, mainly due to the software's "world window", in which an actor moves according to a function, and it is possible for a student to see and analyse its behaviour. She also concluded that SimCalc enables a student to transit between different representations of functions, which would provide a journey through worlds of Mathematics. She designed some activities in which students would analyse the behaviour of the actor in the computer screen and decide what kind of function it could be related to. In the end of the project, she had the opportunity to choose four students of hers to administer those activities, and found out how engaged they were. She also observed that the "world window" helped them to specially connect characteristics from embodied and formal worlds, and they realised some of symbolic characteristics in the activities as well.

We also found that undergraduate mathematics students can design useful activities for the use of technology in their future classrooms if they are engaged in a situation of discussion, exchange of ideas and analysis of what can be done with such tools.

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TRADITIONAL NUMBER-DICE GAMES: AN OPPORTUNITY FOR EARLY MATHEMATICAL LEARNING

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Scientific findings of several intervention studies show that play activities can influence mathematical learning in a positive way (Young-Loveridge, 2004; Ramani, & Siegler, 2008). Many of these studies focus on children with learning difficulties or at school age, and nearly all games used in these studies were especially designed for the intervention. However, it is an open question whether traditional dice games – played "normally" without a special training focus – can foster children's mathematical development at an early age. This question is of particular interest in the lively discussion, whether and to what extent early mathematics education needs direct instruction or whether it can be integrated into play and everyday situations.

In an experimental intervention study we have explored whether children can improve their mathematical knowledge and skills while playing traditional number-dice games in "normal" play situations in kindergarten.

The sample comprises N=95 kindergarteners, who were randomly assigned to intervention (N=48) and control group. Children in the intervention group played traditional dice games (e. g. ludo) with usual number-dice, children in the control group played games using colour/symbol dice. Each child had seven 30 minutes play sessions in a small group with an adult. The adults gave no explicit mathematical instructions. Data for the pre-post-test-comparison was collected by a standardized test with 64 items on counting, subitizing, knowledge of numerals and calculating.

Children who played number-dice games showed significantly higher learning gain from pre- to post-test than children in the control group ($F(1,92)=13.57$, $p<.001$, $\eta 2=.13$). The effects of the intervention were independent of gender, migration background or intelligence.

Detailed analyses show that children in the intervention group improved especially in the aspects of mathematical knowledge which are known as predictive for further mathematical learning. The results indicate that also informal learning situations like number-dice games can be a good opportunity for early mathematical learning.

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NEGOTIATING *NOS/OTR@S* RELATIONSHIPS MATHEMATICALLY: HOW TWO PRE-SERVICE MATHEMATICS TEACHERS ENGAGE MARGINALIZED MIDDLE SCHOOL STUDENTS AT AN AFTER-SCHOOL CLUB

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Most after-school mathematics club research focuses on the youth and not the prospective teachers who volunteer. Yet community-based field experiences are unique opportunities for pre-service mathematics teachers to gain authentic experience working with youth of diverse backgrounds (Mule, 2010). In these out-of-school spaces, pre-service secondary mathematics teachers have the flexibility to develop engaging activities that involve undergraduate-level mathematics.

This study is part of a longitudinal study documenting the development of PSMTs who are enrolled in an equity-based mathematics teacher education program at a major Midwest R1 university. The PSMTs volunteer five times per semester at *I Do Mathematics (IDM)* an after-school mathematics club that takes place at a local public library. We primarily recruit Black and Latin@ middle school students.

My analysis is informed by *political conocimiento* (Gutiérrez, 2012). Teaching mathematics is a political act and seeking solidarity (*nos/otr@s* relationships) with marginalized students may help to engage them to do mathematics. *Nos/otr@s* relationships guides my analysis of how two pre-service secondary mathematics teachers (PSMTs) negotiate relationships with students and engage them in mathematical discussions at *IDM*

This qualitative interview study investigates the approaches of Rafael (a Caucasian male) and David (an Asian male) Scholars. They described how they facilitated the IDM activities they designed (respectively, a combinatorics card trick and a variation of the Bridges of Konigsberg). By facilitating a challenging activity, working with students, and engaging them in mathematical discussions, the PSMTs negotiated *nos/otr@s* relationships that resulted in them exploring rigorous mathematics .

This study has implications for teacher education programs to consider community-based field experiences for PSMTs to develop the stance of negotiating relationships with marginalized students to engage them in mathematics.

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STUDENT ACHIEVEMENTS AND TEACHER PRACTICE IN TWO ENVIRONMENTS

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Integrating ICT-based teaching and learning may improve students' achievements in mathematics (Hinostroza, Guzman & Isaacs, 2002). The mathematics teacher is a key individual in determining what kind of technology will be used in the classroom. Indeed, teaching practices have been found to have an impact on student learning (Ely, 1996). The current study was aimed to examine consequences of integrating ICT into school mathematics on the achievements of fifth-grade students.

The study took place at a unique point in a school's transition towards integrating technology. Several classes were already equipped with data projectors, while others were not, creating a research arena in which two fifth-grade classes at the same school had different learning environments. One class of 35 students studied with Mr Ted in a technological environment, while the other class of 33 students studied with Mr Norris in a non-technological environment.

Three achievement tests were administered for both classes. The first test served to map students' knowledge at the beginning of the school year. No significant difference was found between the two classes on the initial test scores. Second and third tests were administered at the beginning of February and toward the end of April. T-tests conducted for the second and third tests; the differences between the average scores for the two samples were found to be statistically significant ($t(46.65)=3.542$; $p<0.01$ for the second test and $t(61)=2.736$; $p<0.01$ for the third test). That is, while in the preliminary mapping the average score was identical among students in the two classes, the exams at the later stages indicated significant differences in achievements between the two classes. Yet it is not the presence of digital technology alone that makes the difference, because it is the teacher who guides students' actions in class and at home.

In search of potential reasons for the achievement gap between the two classes, several lessons in each class were observed, recorded, and analysed. Four main themes emerged that revealed differences in teachers practice: classroom interaction; motivation for learning; time management; and use of representations. We assume that a combination of these factors influenced the differences in student achievements.

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IDENTIFYING EQUITABLE TEACHING PRACTICES IN MATH

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This paper is a report from a pilot study designed to measure pre-service teachers' abilities to identify equitable mathematics instruction. Leveraging the methods and frameworks of mathematical knowledge for teaching, cultural modeling, and ambitious teaching, our project developed a framework of domains of equitable teaching practices. This framework was used to develop an observational tool entitled Mathematical Quality and Equity (MQE) and is the instrument used in this study. The research questions guiding this study are:

- Can we measure pre-service teachers' abilities to identify equitable teaching practices?
- What are knowledge demands of pre-service teachers to notice and provide evidence of equitable teaching?

The data collected for this project was a part of a pilot study where approximately 70 elementary pre-service teachers completed the above mentioned MQE survey in Fall 2012. From this group, 21 students were randomly selected to participate in cognitive interviews. The 70 completed observational surveys and data from cognitive interviews are the data set for this pilot study.

Each of the students' responses were analyzed and coded to identify the accuracy of their ability to identify equitable teaching practices. Furthermore, grounded theory techniques (Glaser and Strauss, 1967) were used to code students' responses to the open ended responses on the tool and also to code the responses offered in the cognitive interviews.

Preliminary findings indicate that using the MQE tool reveals qualitative evidence about pre-service teachers' ability to identify equitable teaching practices. Results also revealed common themes of students' responses that indicate that some pre-service teachers are able to identify equitable teaching practices and offer evidence to justify their responses while others were not. These results provided data to begin categorizing levels of pre-service teachers' knowledge and skill as they enter our program and will be used in future analyses for comparing their responses at the end of the program (Spring, 2014). Additionally, these categories contribute to an emerging theory about the knowledge demands and skill required for identifying (and later producing) equitable teaching practices in mathematics classrooms.

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UNDER CONSTRUCTION: THE CONCEPTUALIZATIONS OF MATHEMATICAL COMPETENCE OF A GROUP OF AFRICAN AMERICAN ADOLESCENT STUDENTS

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Over the last three decades, there has been a spate of initiatives designed to increase the mathematics proficiency of American students. However, the resulting fixation on test-score gaps of different racial groups, and the relative underperformance of African American students have been influential in establishing, maintaining, and disseminating narratives that position students of color, especially African Americans, as mathematically incompetent—or worse, incapable. The present study adopts Martin's (2000) framework for analysing mathematics identity and mathematic socialization to examine the mathematics competence and self-concept beliefs of a groups of African American students. This study also aims to explore the ways these students construct their beliefs.

The sample comprises six African American adolescent students who participated in a 2011 three-week academic enrichment program at a university in a metropolitan city in the southeastern United States. Participants were in grades seven through nine. This research relies on qualitative, case study methodology. Students participated in three semi-structured interviews. Each of the students' parents also participated in one semi-structured interview. Students composed mathematics autobiographies and mathematics logs to provide insight into their interpretations of their experiences with mathematics. Students also composed graphical social knowledge structures to represent their self-descriptions and self-esteem beliefs. Analyses were conducted, and themes were analysed within and across cases.

The results show that these students' conceptualizations of mathematical competence align fairly consistently with Kilpatrick et al.'s notion of strands of mathematical proficiency. Students' mathematics self-concept beliefs were largely irrespective of their mathematics achievement. Participant parents sought to put their children in settings to reinforce positive, robust mathematics identities and mathematics self-concepts. In addition to family influences, participants' interactions with teachers and peers were also essential in these participants' constructions of their mathematics competence and self-concept beliefs.

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SPACING PATTERNS AFFECT ENGINEERING STUDENTS' APPLICATION OF ARITHMETIC PRECEDENCE

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Arithmetic precedence states that, in mixed arithmetic expressions with no parentheses like 2+3×5, multiplications must be calculated before additions. Although arithmetic precedence may be a conceptually simple topic, its learning is far from successful. Important numbers of university students (e.g. prospective teachers, see Glidden, 2008) fail to apply correctly the rules of precedence. Some researchers have proposed as a solution the addition of extra parentheses during learning, but results are not always encouraging (e.g. see Gunnarsson, Hernell, & Sönnerhed, 2012).

An alternative approach is the search for implicit, perceptual ways of teaching arithmetic precedence. Spacing patterns may be one such way, as Landy and Goldstone (2010) have observed: They asked young adults to compute the value of expressions spaced as 2 + 3×5 or 2+3 × 5, and documented that the latter form elicits more errors and longer response times than the former. However, these authors did not control for the degree of mathematical expertise of their sample and it is therefore possible that their results partly stem from limited practice with arithmetic expressions.

We thus investigated whether the phenomenon reported by Landy and Goldstone (2010) affects a population with strong mathematical expertise, such as engineering students from a highly competitive university (N=24). We used decimals (e.g. 2+3×0.5) to raise task difficulty, and presented these expressions with different spacing patterns. Accuracies were all close to 100%, but response times varied systematically according to the relation between spacing and arithmetic precedence. Our findings confirm the importance of visual display in the calculation of arithmetic expressions by expert young adults. We discuss the potential implications of these results for the development of an implicit method to teach arithmetic precedence.

This research was funded by the programs CONICYT PIA (grant CIE-05) and CONICYT PAI/Academia (grant 79130029).

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TEACHERS' BELIEFS ABOUT MATHEMATICS AND MULTILINGUAL STUDENTS

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Teachers' established beliefs hold that mathematics is the easiest subject for second language learners because there is little language involved. On the other hand teachers' established beliefs maintain that increased English vocabulary is mainly what students need to be successful in mathematics (Moschkovich, 2002). Recent research argues that language is important and that multilingual students have resources such as codeswitching, gestures, and objects they may use in their learning of mathematics (Moschkovich, 2002). What teachers believe matters as beliefs inform teachers' pedagogical practices (Pajares, 1992). I draw on the work of Negueruela-Azarola, (2011) to examine beliefs. From this sociocultural perspective (Vygotsky, 1978), teachers' beliefs are conceptualizing tools for thinking about activity. I understand beliefs as arising from teachers' lived experiences. The research question, then, is:

- What are teachers' experiences teaching mathematics to multilingual students?

The research design involved interviews with five teachers who were asked to describe their experiences teaching mathematics to multilingual students. The data were analysed looking for examples of teachers' beliefs as inferred from their discussions about their practices. Beliefs were categorized as *epistemological* or *efficacy* beliefs.

The findings demonstrated that teachers' epistemological beliefs were consistent with their classroom practises. For instance, how teachers referred to their multilingual students reflected their beliefs. The findings also showed that teachers could hold beliefs that, while consistent within their own belief systems, were inconsistent with their training, or were inconsistent with their school's philosophy. Teachers' efficacy beliefs were found to be related to their epistemological beliefs.

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POTENTIAL EFFECTS OF AN INITIATION TO RESEARCH ON THE PROFESSIONAL DEVELOPMENT OF TEACHERS

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The aim of this study is to evaluate the effects of the introduction of an initiation to research in the training of pre-service primary school teachers. It leads us to build a methodology to analyse both the contents of the training and the practises of the beginner teachers during their first years in a class, when teaching mathematics. We try to understand the potential use of some tools from the field of Mathematics' Education, by the educators as well as the beginner teachers.

A RESEARCH GROUNDED IN A PARTICULAR REFORM CONTEXT

In our university in Créteil, initiation to research now takes a rather large part in the training of pre-service primary school teachers and we would like to understand what kind of leverage it could offer to educators. In order to do that, we analyse the contents and methods of the training sessions offering an initiation to research in Mathematics' Education, and compare them with what is proposed in other sessions, to characterize the specific *transposition* of didactical tools (Chevallard, Y., 1989).

We then analyse and compare the expected and actual practises of beginner teachers when they teach mathematics, during their first years in a class, assessing their level of *Vigilance Didactique* (Charles-Pezard, M., 2010).

We also rely on the hypothesis, from the Double Approach (Robert, A. & Rogalski, J., 2005), that we have to consider not only what happens in the classroom but also the context of teaching as a job, with its particular constraints. It also applies if we want to analyse the choices made by the educators for teacher training.

We follow a group of students, during their two first years in class, through observations in their classroom and questionaries. We will present our multidimensional grid of analysis and the results obtained after the first year.

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EQUAL SIGN UNDERSTANDING IN FOUR COUNTRIES

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In this study the authors aim to expose differences and similarities in the understanding of the equal sign by children from United States, Uruguay, Turkey and Korea. The preliminary findings show similarities between Uruguay and US children in relational thinking items. Further comparative findings from Turkey and Korea will be added in the presentation.

This project is a pilot for a cross-cultural study that explores middle school students' understandings of the equal sign. Studies (e.g. Falkner et al., 1999; Knuth et al., 2006) found that young learners experience a great deal of difficulties with the transition from arithmetic to algebra due much part to ill-conceived understandings of equal sign such as understanding it as computation not as comparing the quantities on both sides. Based on the results of these studies, the authors have design a project to explore differences and similarities in the understanding of the equal sign demonstrated by students who speak different languages from different countries (Uruguay, Turkey and Korea). The research question is: How is the understanding of equal sign similar or different among students speaking different languages?

In this introductory pilot study, the authors conducted interviews with six 12 or 13 old middle school students in Uruguay. Borrowing the framework of the above listed studies, students were asked open number sentences, ($12 + 23 = \square + 20$), true or false number sentences ($4 + 1 = 5$ and $4 + 1 + 35.456{,}987 = 5 + 35.456{,}987$) and relational thinking questions ($35 + a - a = \square$). Students' justification of each answer was encouraged during the interview. The result of Uruguayan students' data showed difficulties similar to the studies done in US mostly with relational thinking items. Both Spanish and English are Indo-European languages, and the comparison with Korean and Turkish (Altaic languages) should provide an interest contrast. Further comparative results will be discussed in detail in the presentation and the findings are expected to add to the literature about the role of language and culture in shaping children's understandings of equal sign in school mathematics.

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DO FUTURE PRIMARY SCHOOL TEACHERS KNOW HOW TO CLASSIFY?

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Today, due to the value research has received, at all levels of teaching, the act of classifying and creating classifications has been greatly emphasized, since it is common knowledge that every activity involving investigation hitches on categorization, that is, data organized into groups so that they can be interpreted in a more objective fashion.

According to Piaget and Inhelder (1980) for a classification to be correct, its categories must present exhaustiveness (represents all facts and occurrences possible) and exclusivity (coherence in order that any result can be represented in one way only). However, Luz, Guimarães and Ruesga (2011) state that teachers and students alike display difficulties creating criteria to classify any given group of elements. To know how to classify is not an ability one learns through life's experience, rather, it is up to the school to make it possible by creating a systematic work.

The purpose of this study is to investigate how 113 Brazilian, Spanish and Canadian future teachers create criteria to classify and use them in a free classification. They were given, individually, a set of 9 cartoon pictures which were to be classified in groups (2 or 3 groups). In the three classes, the majority of the students managed to get one correct classification. However, a highly significant difference ($X^2 = 13{,}717$, gl 1, $p \leq .000$) was observed on account of the amount of groups to be formed (two to three groups), and only those who defined a descriptor were able to perform a correct classification. To classify in three groups the difficulties are evident when they make use of more than one criterion at the same time. The difficulty experienced by teachers and students alike may be explained in part by the lack of a systematized work in school on what classifying is all about.

It was also observed that there was no significant difference between countries. This result reinforces the notion that classifying is not a so simple task as we were led to think. This result brings about questions that will not go away: If future teachers have difficulty classifying, how will they be able to teach future students? The formation process of teachers must lead them to a systematic reflection which will allow them to build criteria to classify a given set and observe exhaustiveness and exclusivity.

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FOSTERING ENGAGEMENT OF ELEMENTARY MATHEMATICS STUDENTS: A CASE STUDY

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The significance of considering affect as important to students' engagement with mathematics is recognised within mathematics education (e.g. DeBellis & Goldin, 2006; McLeod, 1994). However, implementation of this recognition in classroom practice is not widespread.

This qualitative instrumental case study examined the question of whether the use of a particular framework, Imaginative Education (IE) (Egan, 1997, 2005), based in a Vygotskian styled socio-cultural framework, had any meaning to students and their engagement with elementary mathematics. IE is an approach to education that effectively engages both students' emotions and their imaginations in learning.

A focus group of six students were tracked through a six week geometry unit designed with an IE theoretical framework. The researcher acted as Teacher-Researcher collecting data using students' math journals, activity pages, transcripts of audio and video taped semi-structured individual and group interviews, a teacher/researcher diary and a detailed unit overview and lesson plans. The study gathered rich descriptive data focused on bringing out the students' perspective of their experience.

Results indicate the students' demonstrated positive engagement with mathematics and that use of the IE theory utilizing the students' imagination and affective responses allowed multiple points of access. Three conclusions of the study were that students expanded mathematical awareness through making a variety of connections, they developed self-confidence in learning mathematics through using emotions and imagination, and were able to use cognitive tools to engage with mathematics.

This research confirms the importance of utilization of students' affective responses and suggests methods to combine emotive and imaginative responses to assist students' engagement in mathematics. In the presentation, specifics and samples of students work and teachers unit and lesson plans will be presented and discussed.

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FOSTERING STUDENTS' MEASUREMENT ESTIMATION PERFORMANCES IN A SHORT PERIOD OF TIME

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Measurement estimation, that is physical measurement without the aid of tools, is a central aspect of school mathematics and an important skill for daily life. Even though children and adults alike are often poor at estimating measurements, little research has been devoted to teaching and learning of measurement estimation. Especially there is a lack of knowledge about the effectiveness of using strategies if estimating measurements (Joram, 2005). Since a common strategy one uses in every-day life if estimating measurements is comparing an object to be estimated to some standard unit or an object whose measurement is known (so-called benchmark strategy) (Bright, 1976), an interventional study has been conducted focussing on the following research question: Is it possible to foster students' measurement estimation performances in a short period of time by teaching them using benchmark strategies if doing measurement estimation?

Trying to answer this question, in January 2013, 55 students of an experimental group have been trained within two hours in developing accurate benchmarks of linear and area measurement units and on using these benchmarks for measurement estimation successfully. In comparison another 55 students have not been trained in doing so. For being able to compare these two groups a pretest on general intelligence (20 items, α=.74) and a posttest on measurement estimation performances (12 items, α=.72) have been administered. Analyses point out that the estimates of those students who participated in the intervention are more accurate than those students who did not. Or in other words: Students of the experimental group score higher in the posttest than those of the control group ($t(104)=2.83$; $p<.01$; effect size Cohen's $d=0.55$). Furthermore, regression-analyses with pretest performances and treatment-condition as predicting-variables and subjects' posttest performances as depending variables hint at a significant influence of the treatment condition. It can therefore be concluded that it's possible to foster students' measurement estimation performances by benchmark instruction in a short period of time.

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A PRESERVICE TEACHER'S IDEAS ABOUT ENGAGEMENT

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Fredericks, Blumenfield, & Paris, (2004) note that engagement can be considered a meta-construct consisting of three components: "Behavioral engagement encompasses doing the work and following the rules; emotional engagement involves interest, values, and emotions; and cognitive engagement involves motivation, effort, and strategy use" (p. 65). In this study, we were interested in understanding when and how prospective teachers "notice"—describe, attend to, interpret and respond to (see Sherin, Jacob, & Phillip, 2011) student engagement using the components of engagement described above. Our research questions are: How did a prospective teacher describe and interpret the engagement of the students that she observed and/or taught; what particular aspects of engagement did she tend to focus on; and, how did these shift over time.

We use a case study approach to analyze data collected from Ms. C, a 20 year old math major enrolled in a secondary math education program at a large university. Data were collected during a one semester practicum (where she regularly observed math classes and taught three lessons) and student teaching (where she was the instructor for one semester). Ms. C was observed by project researchers and interviewed several times during each semester. She also kept written reflections about her experiences. Data include her reflections, interview transcripts (coded to reflect the types of engagement described above), as well as researcher field notes.

Initially, Ms. C described engagement as occurring when students were "completing the assignment without focusing on other things." She said that she could identify engagement by examining "face expressions." A "blank stare" meant that the student was not engaged. By the end of her student teaching, she appeared to shift from the more behaviourally oriented indicators to those involving cognitive and affective factors. For example, she pointed out that it was important to focus on "what each student was doing each day" because "it changed," and she needed to try "relating" to each student "as much as possible" so that she could see when and how they were engaged. She felt it important to pay attention to "the kids reactions … and seeing what kids are paying attention [to]…" Our overall results indicate that what Ms. C. described, attended to, and interpreted about engagement became more nuanced and refined, and more specific to individual students.

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TRACING STUDENT EXPERIENCE USING MATHEMATICS AUTOBIOGRAPHIES

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In North America, many students develop a fear of and/or dislike for mathematics during their elementary and secondary school years, and these negative feelings tend to persist into adulthood (Boaler, 2008). To better understand Canadian students' experiences, and the ways that these negative feelings may develop as a result of school experiences, we designed a project to explore research questions that address: (1) students' lived experiences of learning mathematics in Canadian schools, (2) the images of mathematics that students are developing in Canadian schools and how they persist over time, (3) the nature of students' mathematical identities, and (4) the role teachers and schools play in shaping these identities. The study draws on enactivism, a theory of embodied cognition that emphasizes the interrelationship of cognition and emotion in learning and that troubles the positioning of self and identity as purely individual (and static) phenomena (Varela, Thompson, & Rosch, 1991).

Data collection for the project has begun, and will continue in Alberta and Ontario for the next two years. Participants include students ranging from Kindergarten to the university level, as well as members of the general public. Drawing on narrative inquiry methodology (Clandinin, 2007), we are collecting mathematics autobiographies from the participants in a variety of formats: written narratives, drawings, and individual interviews. Analysis involves both emergent and thematic coding, and findings will be presented in creative multimedia formats, such as composite cases and *Wordles*.

Preliminary findings are revealing important connections between students' identity formation in relation to mathematics and both the nature of the teaching and learning they experienced in school and the significance of their wider context, including parental influences. These initial findings suggest that this rich source of autobiographical data will assist teachers and policy-makers in understanding their roles in shaping students' experiences and mathematical outcomes.

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THE ROLE OF EXAMPLES & NONEXAMPLES IN DEFINING

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Learning about the definition and properties of a mathematical object from a series of examples is important in mathematical thinking. Educators must decide which examples to select and how to incorporate them into instruction. In this 2x2 experiment, we cross two learning methods with the type of examples given as feedback. Our primary measure (full study includes 5 measures) is how many of the following key features of *polygon* students provide: a polygon is a (1) closed figure made up of (2) straight line segments that (3) do not cross, intersecting only at the vertices. Prior research (Hillen & Malik, 2013) suggests that sorting objects before viewing them in the respective groups helps draw attention to features. We predict that showing polygons and near-miss non-polygons that violate the definition will be more effective than showing only polygons (e.g. Shumway, 1971).

In this study, community college students have been participating in a one-on-one think-aloud protocol. Twelve of 48 total have participated to date. As a pre-test, participants generate a list of polygon properties. Half the participants begin with examples: all polygons or both polygons and non-polygons grouped and labelled. The other half sort the shapes into polygons/non-polygons before seeing one of the two example sets. All then complete a post-test in which they list polygon properties.

	Examples Only	Sort + Examples
Polygons Only	0.33	1.33
Polygons + Non-Polygons	0.67	2.67

Table 1: Number of key properties listed on post-test by condition (maximum = 3).

Although all students had studied polygons, none of the 12 identified any key features at pre-test, instead listing imprecise details (e.g. “shape with many sides”). At post-test, even with a small sample, we find a main effect of sorting ($p < 0.05$) and a trend toward polygons + non-polygons. Similar results were found on other measures. We will share additional results about participants’ construction of definitions in the presentation. Sorting and viewing both positive and negative instances help make explicit rules for classification and generation of mathematical objects. This finding helps inform the selection of examples for instruction.

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ELEMENTARY TEACHERS AND HIGH COGNITIVE DEMAND TASKS

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SUMMARY

There are multiple calls for increasing students' work with mathematical practices and high-cognitive demand tasks. The National Research Council has called for moving toward embedding mathematics in problem solving and sense-making (White-Fredette, 2009). High cognitive mathematics performance tasks provide students with an opportunity to show conceptual and procedural understanding, problem solving ability, and mathematical reasoning skills (Van de Walle, 2004). This study asked how teachers interpreted the cognitive demand of a task in order to provide an *opportunity* for students to engage in sense making and problem solving.

The purpose of this qualitative study was to investigate the factors relevant to teachers' understanding of high cognitive demand tasks and the meaning of the process standards. The data included a pre/post survey and a pre/post task evaluation and observation notes and transcripts of the teachers vetting sessions after they created a mathematical task. The intervention was a weeklong summer professional development with follow-up interviews in the fall. The data was coded using a constant comparative, thematic analysis approach (Glesne, 2011).

The results revealed the teachers had difficulty creating high cognitive demand mathematical tasks because they struggled with the role of the curriculum standards. This was most evident in participants' discussions of tasks. It was also evident in the directions the teachers wrote on the tasks. They needed to see the content stated (in some cases verbatim) using language from the standard in order to "meet the standard". Preliminary results show that when they implemented the tasks they realized the tasks did not elicit mathematical reasoning and thus were not high cognitive demand tasks.

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RESEARCH ON AFFECT OF PRESERVICE ELEMENTARY TEACHERS IN UNIVERSITY MATHEMATICS CONTENT COURSES: 1990-2012

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As advocated by Ma (1999) there is a critical need for *profound understanding of fundamental mathematics* (PUFM) by elementary teachers. A significant opportunity to influence the nature and quality of that understanding rests in the university mathematical content courses taken by prospective elementary teachers (PTs); and while there are several complex factors that impact knowledge development in these courses, the affect (e.g., attitudes, beliefs, and emotions) that PTs bring to and acquire during university mathematics content courses is of salient concern.

Compelling warrants for this project include the significant influence of teachers' *affect* on their learning, coupled with the pervasive tendency of elementary teachers to have negative affect toward and avoidance of mathematics. Accordingly, the goal of this investigation was to describe the state of research on mathematical content knowledge preparation of PTs in university mathematics courses from 1990 to 2012, where all or part of the study looked at affective factors.

Using a combination of key words, three researchers explored electronic databases. In addition, two researchers conducted a targeted search of 11 journals considered to have high scholarly regard in the fields of mathematics education and teacher education. After careful screening and removal of overlapping articles, 15 studies were identified that were published within the time period 1990-2012, had PTs as subjects, were conducted in the context of university mathematics content courses, and investigated affective factors.

Analysis of the studies revealed three broad categories: snapshot research that examined PTs at one point in time (n=1), comparative studies that contrasted experiences of two groups of PTs (n=6), and change studies that followed a group of teachers across a period of time to describe shifts as a result of an intervention (n=8). Methods used during the studies include: four qualitative, five quantitative, and six with some combination of qualitative and quantitative. Example key findings include: differences when data are disaggregated by high and low achievers, an absence of impact of demographic factors on affect, and affect as more malleable than content knowledge.

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MATHELINO: KINDERGARTEN AND ELEMENTARY SCHOOL CHILDREN EXPERIENCE MATHEMATICS TOGETHER

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The transition from kindergarten to elementary school takes place between the poles of continuity and discontinuity. The first step which should be taken in order to improve this transition is to enhance continuity and reduce differences. The second step is to consciously look at discontinuity as a challenge fostering development and therefore supporting the children by addressing these differences. (see Roßbach, 2006). An improvement of transition can be achieved by making some structural changes in kindergartens and elementary schools, coordination of educational curricula (in terms of content and material), training and further education of staff as well as cooperation between kindergarten and elementary school (see Knauf, 2004).

In this study, we analyzed the outcome of the cooperative work with children from kindergarten and elementary school, who met once a week for two hours in one of the institutions. The cooperative meetings were prepared and performed by school teacher and kindergarten educator tandems. We also analyzed the outcome of cooperative teacher-educator-tandems training courses (n=20), which took place every 12 weeks (over a 2-year-period). A group discussion was video-recorded and transcribed. Particular episodes were analyzed by coding to the qualitative content analyses (see Mayring, 2012). A questionnaire was carried out at the end of the training courses to assess the attitude of the kindergarten educator and elementary school teacher in terms of the variables: Cooperation between the institutions, education of the staff and cooperation between the children from kindergarten and elementary school.

First results, taken from a group discussion and also from the use of questionnaires, show that the project has an impact on the cooperation between the children, kindergarten educator and school teacher as well as on the management level of the institutions. Essential for these developments are common training courses in which the further development of professionalism and understanding of each other's institution must be one focal point. During the weekly cooperation meetings it could be insured that the children have effectively learned from and with each other as well as that they had an intense verbal exchange of mathematical terms.

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ESTIMATION STRATEGIES OF YOUNG CHILDREN USING MULTITOUCH TECHNOLOGY

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We investigated potential benefits of integrating the visual affordances of a dynamic geometry software—Sketchpad® Explorer—with the multitouch affordances of the iPad, for facilitating young children's access to advanced mathematical ideas. We present the results of implementing an activity focused on area estimation in three 4th-grade classrooms in the US. Within this representational infrastructure (Noss and Hoyles, 1996) geometric objects can be manipulated to discover its inherent mathematical structure. It can also mediate children's conjecturing and testing processes (Mariotti, 2001). We ask whether, and how, small groups of three students strategize in making sense of the underlying mathematics of an area estimation activity in the dynamic multimodal environment. Fourteen groups were asked to estimate the area of a fixed quadrilateral using dynamic circles. They were able to simultaneously use up to 15 circles, and adjust the diameter of each circle by touching and dragging. A "sum" meter in the top left hand corner offered a readout of the total area of all circles moved onto the quadrilateral, so the sum of circles' area provided an estimation of the quadrilateral's area. Small-group work was video recorded and transcribed and strategies were codified. *Strategies* are the methods that children used to solve the task and represent their ways of thinking. We found six strategies: (1) Taking into account the circles' size to measure the shape's area, (2) Using one circle on the entire shape to measure the shape's area, (3) Taking into account the size of the circles to fill the whole shape, (4) Filling the quadrilateral with circles trying to fit them into the shape, (5) Locating one circle on each corner, (6) Locating circles on the shape's perimeter and multiplying the number of circles on one side by the number of circles on another side. In strategies 1 and 2 children consciously use the area of the circles to measure the quadrilateral's area; due to circles not having corners, they overlapped or left portions of circles outside the shape to compensate uncovered space. In strategies 3 and 4 although children do not measure consciously, they consider the size of the circles as a key resource. Strategy 5 shows the salience of corners as an attribute. In strategy 6 children use the rule of the perimeter of a regular quadrilateral. Findings show that the dynamic multitouch technology was a meditational tool that facilitated children's mathematical representations and collaborative building of area estimation strategies.

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CLASSROOM USE OF REPRESENTATIONS BY NOVICE SECONDARY MATHEMATICS TEACHERS

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What we know is a function of our experiences (Radford, 2014; von Glasersfeld, 1995), and the way that individuals gain access to mathematics is through their experience with representations of that mathematics (Duval, 2006). A major way in which students gain access to mathematics is through their in-class experience with mathematical representations – activities orchestrated by their classroom teachers. The research goal of this study was to characterize novice secondary mathematics teachers' mathematical representing and use of mathematical representation in their *classroom mathematics* (i.e. mathematics in which they and/or their students engaged during classroom instruction). Because a major influence on teachers' use of representations in their classrooms would seem to be their use of representations in their own mathematical problem solving, we studied not only each teacher's classroom mathematics (through several series of weeklong observations) but also his or her personal mathematics (through five 1.5-hour task-based interviews). We studied representing and the use of representation by three teachers during the last two years of their teacher preparation program and the first year or two of their teaching careers. We identified three characteristics that differentiated the representational work of these teachers. First, the level of generality of explanations differed from teacher to teacher. Second, the features of representations emphasized by teachers in the classroom varied in terms of their mathematical relevance. Third, the use of representations for mathematical ideas varied in terms of the interconnectedness of their mathematical features. We found evidence that teachers' use of representation in their classroom mathematics was influenced not only by their personal mathematics but also by factors such as their management of learning and sensitivity to students (Jaworski, 1994).

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MATHEMATICS AT UNIVERSITY: PRACTICES, VALUES AND PARTICIPATION

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In this paper we report on a developmental design research study in an engineering mathematics undergraduate course, where previous attempts to increase participation by introducing a mathematical modelling approach had failed, mainly due to a very strategic-oriented student attitude.

A re-design of the teaching approach was then adopted, using ideas from socio-political and socio-cultural theories (Pais 2013, Williams 2012). Consistent with these perspectives, the new teaching strategy tried to reconcile *in practice* the contradiction between mathematics as exchange value of capital in the market and as use-value in the development of the mind. By incorporating tasks designed for the development of employability skills (e.g. communication skills, problem-solving skills), the aim was for students to engage with mathematics not only as a rather useless but unavoidable and potentially convenient-to-have subject but also as something that could prove intellectually useful and that could appeal to their future aspirations.

We used a socio-cultural theoretical framework – where learning is seen as participation (Lerman 2001) – to evaluate the impact of this strategy. We asked: (1) What were the students' perceptions of the value of the teaching strategy? (2) What was the impact of the strategy on students' participation in mathematically meaningful activity? We collected data from students' written feedback (in weeks 4 and 11 of a 12-weeks course) and used peer observation (during 5 weeks along the course) to reflect on and refine the teaching strategy and to analyse the students' learning.

Results showed a substantial participation of most of the students in the different activities that were designed to enhance their learning, in spite of some of these not counting towards the assessment of the course. The data showed that most students believed that these tasks could be useful to their future and that developing these skills was worth investing time and effort, hence conferring use-value to the mathematics.

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QUANTIFYING MATHEMATICAL DISCOURSE[1]

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Methods for analyzing classroom discourse have proliferated in recent years in mathematics education together with the interest in promoting discourse-rich instruction. Most of these methods were qualitative. We build upon Sfard's (2008) communicational framework and Systemic Functional Linguistics (Halliday & Matthiessen, 2004) to suggest a method for quantitative analysis of mathematical talk.

Sfard conceptualizes mathematical learning as change in participation in a certain type of discourse. She distinguishes between *objectified* talk that speaks about mathematical objects as existing of themselves and *syntactic* talk that merely deals with the manipulation of mathematical signifiers. Most often, such syntactic talk is accompanied by *personified* talk where the participants talk about what they *do* to the mathematical signifiers. This objectified/syntactic distinction partially coincides with the distinction between 'conceptual' and 'procedural/calculational' talk. Our goal in this work was to come up with a coding method that would enable capturing quantitatively the difference between personified and objectified talk so that we can compare what type of mathematical talk students get access to.

The data analysed included full transcriptions of two similar courses about functions for prospective elementary-school teachers, taught by two instructors. The 'objectified' vs. 'personified' distinction was captured by SFL's system of Transitivity. Three linguistic categories were found to be most relevant: (a) Material (b) Existential and (c) Relational processes including (c1) attributive and (c2) identifying processes.

Findings revealed significant differences between the instructors' mathematical talk. While one of the instructor's talk was mainly personified, the other' was more objectified. These quantitative and linguistic differences were validated against former qualitative analysis of several classroom episodes. The significance of the study lies in the presentation of a relatively simple and straightforward method to quantify and compare mathematical talk.

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[1]This study was partially supported by the Israel Science Foundation, no. 446/10.

MATH PATHS FROM SECONDARY TO POST-SECONDARY

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Mathematics education is one area where post-secondary institutions report students are often the most under-prepared (National Governors Association, 2011). In the US, the community college is often a public, two-year institution that may include vocational programs or serve as a transition to a four-year university. While there are multiple math course sequences, many of the remedial course sequences are not effective for students in terms of students' achievement (i.e., students often fail or withdraw from mathematics courses) (Bahr, 2013). The research question for this study asked: what are community college faculty perceptions of math preparation in high school and what are the goals for college-level math courses?

This study examined the results of a survey about the STEM goals and objectives of community college faculty. The survey focused on their interests in high school curriculum, their perception of students' motivation and potential for success in community college, and the types of skills and abilities students should develop in high school to prepare for community college. Nine of the 34 grantees contacted from the government grant program responded to the survey.

The results indicate that community colleges were connecting with employers of their students to design and develop their programs and initiatives. However, they perceived disconnects between high school preparation in mathematics (and STEM broadly) and college-level coursework. The community college population is very diverse and often draws low-income, first-generation, or non-traditional age students. The goal of the institution is to connect the workplace and students' current skills.

There was also interest in developing problem solving, communication and reasoning skills related to STEM at the college level. To both provide remediation for missing skills and prepare students for the workplace, some remediation was situated in applied settings. Traditionally, students needing remediation in mathematics took separate developmental math courses. Some initiatives are integrating context-based remediation into courses that are directly part of the degree program in order to provide the content in a workplace setting (to increase students' learning) and to reduce the time to degree completion for students who have limited resources.

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PRE-SERVICE TEACHERS' EVOLVING CONCEPTIONS OF PARTITIVE DIVISION WITH FRACTIONAL DIVISORS

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Only a few studies have examined pre-service teachers' (PSTs') conceptions of partitive division with proper fraction divisors (e.g., Lo & Luo, 2012). However, prior research has not focused on the evolution of PSTs conceptions of the partitive interpretation of division. Our study fills this void by using a constructivist lens to examine how PSTs' conceptions evolve in an instructional setting.

Seventeen PSTs were interviewed before and after participating in a two-part lesson on proper fraction partitive division in a content course for PSTs at a university in the Mid-Atlantic region of the U.S.A. The two-part lesson emphasized reasoning covariationally with the dividend and divisor for partitive division. One-on-one semi-structured interviews involved five proper fraction partitive division tasks.

A grounded theory analysis of the data revealed four levels of PSTs' conceptions, which represent *pivotal intermediate conceptions* (Lobato et al., 2012) in the evolution of PSTs' thinking about partitive division. At Level 1, the PSTs conceived partitive division as partitioning only the dividend and they had a limited understanding that the goal of partitive division is to find a unit rate. At Level 2, PSTs recognized that partitive division involves partitioning both the dividend and divisor in a coordinated manner but continued with a limited understanding that the goal of partitive division is to find a unit rate. At Level 3, PSTs continued to conceive partitive division as partitioning the dividend and the divisor but had a more robust understanding that the goal of partitive division is to find a unit rate. At Level 4, PSTs conceived partitive division as involving partitioning and/or iterating the dividend and divisor and continued with the robust understanding that the goal of partitive division is to find a unit rate. Fourteen of seventeen PSTs moved from a lower to a higher level of conception by the post-interview; no PSTs regressed.

These four levels could serve as a tool for research and teaching for discriminating between PSTs conceptions of partitive division. The levels also suggest ways in which PSTs could develop explicit understandings of partitive division. In future research, we will design activities to support PSTs in advancing levels.

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UNIVERSITY STUDENTS' USES OF MATHEMATICAL DEFINITIONS THAT ARE NEW TO THEM

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University students often do not know there is a difference between dictionary definitions and mathematical definitions (Edwards & Ward, 2004), yet in order to succeed in their university mathematics courses, they must often construct original (to them) mathematical proofs. Only a little research has been conducted to discover how university students handle definitions new to them (e.g., Dahlberg & Housman, 1997).

Our specific research question was: How do university students use definitions to evaluate and justify examples and non-examples, in proving, and to evaluate and justify true/false statements? Data were collected through individual task-based interviews with volunteers from a transition-to-proof course. Each student was provided with a particular definition and asked to consider examples and non-examples, construct a proof, and consider true/false statements, in that order. Altogether there were five definitions: function, continuity, ideal, isomorphism, and group, but each student was asked to consider only one of the five. We used content analysis and grounded theory for the analysis.

We concentrate here on the definition of function, which was the Bourbaki ordered pair definition. Results suggest significant interference of students' previous knowledge, which was not always appropriate, in students' understandings of this definition. We observed a tendency to ignore the given definition and use only their concept images (Tall & Vinner, 1981). One of our conjectures with regard to the definition of function is that such interference occurred due to the students having worked with functions in several previous courses. Is there a way to prevent students from using their inappropriate previous knowledge? We expect less interference as we analyse the later interviews, when the definitions were not only new to the interviewed students but also more abstract.

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THE RELATIONSHIP BETWEEN UNDERGRADUATE MAJOR AND SPECIALISED CONTENT KNOWLEDGE WHEN ENTERING METHODS COURSES

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This study investigated the influence of undergraduate mathematics background on teachers' conceptual understanding of elementary mathematics as needed for teaching. While teachers with mathematics related backgrounds appeared slightly stronger overall in conceptual knowledge than other teachers, few participants generally appeared well-prepared.

This study investigated the conceptual mathematics knowledge of middle-school preservice teachers (PSTs, N=260) based on university courses prior to the mathematics methods course. Our work is based on current understandings of mathematics content knowledge for teachers in which specialised knowledge is deeply rooted in conceptual understanding of elementary concepts (Silverman & Thompson, 2008).

PSTs' conceptual understanding of selected content areas was explored using a fine-grained lens, with participants grouped into three broad categories related to their previous undergraduate courses, namely mathematics, sciences, and arts. Pencil and paper survey responses formed the data for the current analysis. After examining descriptive statistics, selected item responses were organised into categories based on response-type (see Kajander & Holm, 2013) and scored on a scale ranging from "blank" and "incorrect rule" to "more than one correct model or explanation".

All mean subgroup scores of conceptual knowledge of mathematics were below 30%, *regardless of background*. The majority of all participants were unable to demonstrate even a partial conceptual understanding of integer subtraction (75% of math, 95% of science, and 96% of arts), or fraction division (60% of math, 93% of science, and 94% of arts), with stating a rule usually offered as the "explanation". Given the small sample of participants with mathematics background however, (N=8) further study is needed.

Based on our data, we argue that much more - or other - than standard undergraduate mathematics is needed to support appropriate teacher development in mathematics.

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STUDYING INTERN TEACHERS' CONCEPT IMAGES FOR MATHEMATICS TEACHING THROUGH THE ASPECT OF STUDENTS' MATHEMATICAL THINKING

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Tall & Vinner (1981) described the *concept image* as "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and process (p.152)". Based on this definition, this study proposed the idea "concept image for mathematics teaching (CIMT)" and tried to explore the relationship between teachers' CIMT and their decision making in teaching. Among the various aspects of teaching concepts, this paper focused on the aspect of student's thinking in mathematics teaching.

A qualitative research method was designed for this study. The participants were 62 secondary mathematics intern teachers from National Taiwan Normal University, which is one of the major teacher preparation institutions in Taiwan. The data were collected by a questionnaire and were performed content analysis to identify the primary patterns and themes. The results reported in this paper are mainly inducted from three short essay questions, which are about the views about mathematics teaching, the role of students' thinking and the timing for letting students think.

The result shows that most of the intern teachers approved the roles that students' thinking plays in mathematics class. However, only around one fourth of them exhibited the aspects related to students' thinking without hint. They thought that mathematics teaching should focus on developing students' thinking ability and should provide students the opportunities to think.

We also found that, about the roles of students' thinking in mathematics class, the intern teachers usually evoked three main categories of concept images, including the important factor to develop students' mathematical concept, the effective method to help them retain knowledge, and the key to help them to apply the knowledge. We can know that most of the intern teachers who evoked the image of student thinking would also evoke the image of students' cognition. As to the appropriate timing for students' thinking, there exist three sub-categories of concept images from the teachers:

- When facing the questions that students are capable of figuring out.
- When students make errors, feel difficult and have cognitive conflicts.
- After teaching the main concept, teachers should leave time for students to think and absorb.

Reference:

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*(2), 151-169.

HOW DO ENGINEERING STUDENTS GENERATE COUNTEREXAMPLES OF CALCULUS CONCEPTS?

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Conventional calculus education has emphasized “good” functions and examples; thus, students are accustomed to using familiar skills and manipulating signs rather than focusing on reasoning and verifying mathematical concepts, definitions, and theorem conditions. Generating examples of mathematical objects may be a complicated task for both students and teachers; however such tasks yield substantial educational potential. In mathematics education, requesting that students generate examples is a particularly valuable tool (Watson & Mason, 2005). Zazkis and Chernoff (2008) also suggested that the convincing power of counterexamples depends on the extent to which they are in accord with individuals’ example spaces. This study, a particular focus is on engineering students’ reasons behind their uses of different strategies in generating counterexamples.

The participants comprised 15 first-year engineering students at a university of technology in Taiwan who previously completed courses of derivative and definite integral. The questionnaire contained three false mathematical statements regarding differentiation and integration. The students were asked to determine the validity of the mathematical statements and provide an example or a counterexample backing their decision. The primary data sources were the written responses to the questionnaire and interviews. The analysis focused on identifying the students’ strategies used to generate counterexamples for the false statements and the justifications provided.

The results suggest that the counterexamples that engineering students generating are not well-defined as logical entities. The strategies used by students can be characterised as an unwarranted and far too extensive reduction of complexity of calculus concepts. The most frequent type of reduction of complexity seems to be to focus the counterexamples on algorithmic procedures that used to carry out in textbook exercises. The findings also show that in engineering students’ strategies the reasoning is founded on mathematical experiences that are dependent on their experiences from their learning environment. For example, although using graphical representation is a simple method of generating counterexamples, the students preferred using algebraic representation.

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INTERMEDIATE PARTICIPATORY STAGES OF THE CONCEPT OF UNIT FRACTION: TWO STUDENTS WITH LEARNING DISABILITY

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Special education fraction instruction is limited in its use of Reductionist frameworks to define children as passive responders in instruction. The present study utilizes markedly different, constructivist teaching/learning processes to illustrate evolving conceptions of unit fractions evidenced in the cases of Ana and Lia, two 11-year-old girls labeled as low achieving. In particular, the paper focuses on the *Reflection on Activity-Effect Relationship* (Ref*AER) framework (Simon, Tzur, Heinz, & Kinzel, 2004) to explicate how the girl's conceptions grew from a prompted (participatory) to an unprompted (anticipatory) stage. We hypothesized Type-I and Type-II reflection grounded in partitioning-via-iteration (Steffe & Olive, 2009) would move the girls to coordinate the size of unit fractions relative to the number of same-size iterations needed to make the whole, forming the concept of inverse order relation. Such tasks (e.g., share between four) involve a) constructing a length below a whole the size of the equal share; b) iterating the length a number of times equal to the number of sharers, and; c) adjusting the size of the length depending on if the previous estimate was too long or too short until a length is found such that exactly [four] copies reproduces the whole.

Analyzed data came from six tutoring sessions of a teaching experiment. The girls were cases of the phenomenon under study due to disability labels and reductionist teaching received in school. Researchers used retrospective analysis to understand initial conceptions of unit fractions and constant comparison to document evolving conceptions in relation to the stage distinctions. Analysis revealed each girl did not initially anticipate the nature of adjusting an estimate (making it longer or shorter). They then progressed into the participatory stage of foretelling the nature of adjustment but not the relative size of adjustment (how much longer or shorter). Reflections then progressed to qualitatively assessing the relative size adjustment through a guess and check method (not yet relating adjustment to iterations). Finally, each girl evidenced an anticipated coordination of relative size adjustment with the number of iterations/people sharing and used their anticipations to solve novel problems. Results suggest constructivist-based teaching can enable such children to learn and achieve like their normally achieving peers.

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SUPPORTING SECONDARY MATHEMATICS TEACHERS' UNDERSTANDINGS OF CULTURE IN THE MATH CLASSROOM

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Vignettes reflecting intersections of culture and mathematics teaching supported secondary mathematics teachers' awareness that culture matters, development of broader conceptions of culture, and attendance to relational aspects of teaching.

Supporting the mathematics learning of *every* student, particularly those traditionally marginalized, is a significant challenge for mathematics education. Culturally responsive teaching can address this need. It can be challenging to convince secondary mathematics teachers about the importance of culture in mathematics education (Leonard, 2008), in part because teachers believe math is culture-free and thus the teaching and learning of math exist outside cultural influences (Bishop, 1988). Realistic vignettes contextualize scenarios prompt reflective responses (Schoenberg & Ravdal, 2000).

This research examines how vignettes reflecting intersections of culture and mathematics teaching supported 60 mathematics teachers' developing awareness that culture matters in mathematics education. Qualitative analyses show that this project supported *all* teachers' awareness of the role of culture in mathematics education. For about a quarter of the teachers this was a new realization, generally, or in terms of attending to students' culture (e.g., "I am far less aware of the culture that my students have than I thought."). Further, about half of the teachers developed new conceptions of culture not considered before, now seeing values, beliefs and attitudes as cultural. Finally, this project supported many teachers in attending to the importance of the relational aspects of teaching, such as knowing students across cultural lines. Teachers were ready to learn more about instructional practices that would support their take up of these understandings, but also felt overwhelmed by these ideas, suggesting implications in teacher development. Results of this research contribute to the knowledge base on interventions in mathematics teacher education that can orient secondary mathematics teachers toward culturally responsive teaching.

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THE EFFICACY AND IMPACT OF A HYBRID PROFESSIONAL DEVELOPMENT MODEL ON HANDHELD GRAPHING TECHNOLOGY USE

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Online teacher professional development (oTPD) is becoming more prevalent as the ability to harness technology to bring teachers and resources together becomes easier. However, Dede et. al. (2009) state that most evidence of effectiveness is from participant surveys completed immediately after the professional development ends with no consideration for long term impact. This research project aims to determine how a hybrid, face-to-face and online, professional development model affects the teacher's use of handheld graphing technology in the classroom.

The sample comprises 14 of the 22 teachers who participated in the professional development for two days, approximately 20 hours, of face-to-face meetings followed by 18 ninety-minute online sessions over a 10-month period. Data collected consists of two surveys, administered midway and at the completion of the online sessions, about the participants' perspectives about growth in skill with the technology and providing support to other teachers in using the technology. Surveys included a 1 to 5 point scale ranking and an opportunity to provide written descriptions and examples for the ranking. Since Dede et. al. (2009) call for additional measures of teacher change that are more objective, three participants were selected for classroom observations and interviews during the following school year. Teachers were asked how their use of the graphing technology has changed since the professional development ended.

Survey results showed an increase in ability (mean 3.86 to 4.21) to use the graphing technology, but a decrease in ability (mean 4.36 to 4.21) to mentor other teachers, citing face-to-face sessions would have been more effective. Classroom observations and interviews indicate teachers have integrated graphing technology use throughout lessons rather than the beginning or end, modified their curriculum to be more discovery and discussion based, and worked more effectively with teachers who are reluctant to learn new technology.

Based on these results, this model of professional development appears to have a positive impact on classroom use of graphing technology. Future work will continue to examine the long-term effectiveness and impact of the oTPD.

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A STUDY ON CHANGE OF LESSON PRACTICE IN ZAMBIAN MATHEMATICS TEACHER: FOCUS ON TEACHERS' SUBJECT MATTER KNOWLEDGE ON THE LESSON STUDY

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In the Republic of Zambia, Lesson Study Support Project was started by JICA (Japan International Cooperation Agency) technical cooperation in 2005. The main purpose of this project was to promote implementation of Lesson Study through existing In-service training program. The Lesson Study contributed to disseminate methodology of Lesson Study to stakeholders. However, the Zambian Lesson Study was pointed out several qualitative problems (Ministry of Education, 2009). There were so many teachers who consider simply conducting the group work as the learner-centered lesson. So, Zambian teachers have easily received superficial idea and they have some difficulty to develop those ideas through lesson practice.

Therefore, in this research, author considered the ideal future for the Zambian Lesson Study through grasping and analysing the current situation of Zambian teachers' subject matter knowledge on Mathematics lesson. The research was conducted at 5 schools with focus on groups of Mathematics teachers. The main purpose of this research was to investigate how teachers changed through the Lesson Study cycle. The sources of data were mainly participation observation during the Lesson Study cycle.

Through the result of analysis, it was clarified that Zambian Mathematics lesson have following two problems. One problem is that Zambian teacher aim simply general teaching method through the Lesson Study cycle. Lesson improvement is entirely focused on using of chart, concrete teaching aids and group activity. Therefore the Lesson Study doesn't function to improve the subject matter knowledge of teachers. The other problem is curriculum and textbook of Zambian Mathematics. In the unit of fraction, the contents are biased toward procedural knowledge of the operation. So, students, even teachers are unable to acquire conceptual knowledge of fraction.

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A COMPARISON OF ELEMENTARY AND MIDDLE GRADES STUDENTS' ALGEBRAIC REASONING

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This study compares the functional thinking and equation solving performance of third-grade students who participated in a year-long early algebra intervention with that of sixth- and seventh-grade students who experienced only their regular curriculum. Results of a written assessment show that third-grade students performed similarly to sixth- and seventh-grade students, suggesting that early algebra instruction can have a positive impact on students' achievement in middle school.

Algebra's role as a gatekeeper to higher mathematics has led initiatives in the USA to re-conceptualize algebra as a longitudinal K–12 strand of thinking (e.g., Common Core State Standards Initiative, 2010). However, given the well-documented difficulties students in middle grades have with algebraic concepts, we wondered how the algebraic thinking of young children after an early algebra intervention compares to that of older students. We conducted a one-year early algebra intervention with about 100 third-grade students and administered a pre/post assessment to measure their understanding of core algebraic concepts. Two items from this assessment that addressed functional thinking and equation solving were also given to about 200 sixth- and seventh-grade students from the same school district who experienced a more traditional arithmetic-based mathematics education in elementary school. Results show that while only 4% of third-grade students wrote a function rule in words at pre-test to describe a set of data, 46% of the students did so at post-test. In comparison, 48% of sixth-grade and 26% of seventh-grade students wrote a rule in words. No third-grade student wrote a function rule using variables at pre-test, but 38% did so at post-test. In comparison, 35% of sixth-grade and 48% of seventh-grade students used variables to write the rule. Moreover, only 12% of third-grade students correctly solved an equation at pre-test, while 73% did so at post-test. In comparison, 92% of sixth-grade and 86% of seventh-grade students solved the equation correctly. These results suggest that, with even a limited intervention, third-grade students can perform as well as middle grade students in traditional settings, underscoring the importance of early algebra experiences in elementary grades to prepare students for the formal study of algebra in later grades.

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RELATIONSHIPS BETWEEN TEACHER CHARACTERISTICS AND KNOWLEDGE PROFILES FOR FRACTIONS

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There is enduring international interest in the mathematical knowledge of teachers (e.g., Rowland & Ruthven, 2011). We report on data from a paper-and-pencil instrument that highlights dynamic reasoning necessary for responding to students' thinking in a particular domain—fractions and fraction arithmetic in terms of quantities. The instrument consisted of two parts. The first part assessed four distinct components of reasoning highlighted in past fractions research: (1) Referent Units, (2) Partitioning and Iterating, (3) Appropriateness (recognizing which situations can be modelled by multiplication or by division), and (4) Multiplicative Comparisons. Reasoning about fraction arithmetic in terms of quantities (e.g., length) involves all four components. The second part surveyed teachers' professional experience, preparation, demographics, motivation for using drawings of quantities, and use of such drawings in their instruction. We examined relationships between teachers' facility with the four components for reasoning about fractions, as measured in the first part, with aspects of experience and practice reported in the second part.

The sample included 990 in-service U.S. teachers of students 12 to 14 years of age. We designed the first part of the instrument to be multidimensional and analysed item responses using the log-linear cognitive diagnosis model (Rupp, Templin & Henson, 2010). The analysis placed teachers into one of 16 reasoning profiles based on their mastery or non-mastery of the four components. Roughly 25% were masters of all four components, 20% were masters of no component, and the rest were distributed over other profiles. We used cross-tabulation and mean comparison to identify differences and Chi-square and t-tests to evaluate the statistical significance of differences. We found statistically significant relationships between reasoning profiles and years of teaching experience, mathematical preparation, motivation to use drawings of quantities, and tendency to use such drawings with students before introducing algorithms, among other characteristics. The results suggest that teachers' capacities for reasoning as captured by the profiles are intertwined with teachers' motivation and reform-oriented instructional practices, and they illustrate the value of using cognitive diagnosis models to assess teacher knowledge.

Acknowledgement: The National Science Foundation supported this research (DRL-0822064). The authors' views do not necessarily reflect those of the Foundation.

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PREPARING PROSPECTIVE MATHEMATICS TEACHERS TO WORK WITH STUDENTS WHO STRUGGLE

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The teaching of mathematics to students who are typically developing can be a challenging task, and an even more demanding assignment can be teaching students who struggle (van Garderen, Scheuermann, Jackson, & Hampton, 2009). According to the Teaching Principle from the *Principles and Standards of School Mathematics*, "effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000, p. 17). Traditionally, teacher education programs have placed little emphasis on preparing mathematics teachers to work with struggling learners (Allsopp, Kyger, & Lovin, 2007). The purpose of this study is to examine how a course situated in an informal learning environment affects the preparation of secondary mathematics prospective teachers to work with students who struggle in mathematics.

In this embedded mixed methods research design, data was collected from seventy-five prospective secondary mathematics teachers. The qualitative data (e.g., semi-structured interviews and weekly reflections) were analysed using a constant comparative method. The quantitative data were collected pre and post via two survey instruments and were compared to the results of the emergent patterns and themes from the qualitative data analysis. The findings revealed that the prospective teachers were positively impacted to learn about and implement research-based instructional strategies to work with struggling mathematics learners. More specifically, the informal learning environment simulated a situation where the prospective teachers were able to practice instructional methodologies in real, contextual situations. In the presentation, results will be discussed in detail.

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THE ROLE OF EXPERIENCE IN THE DEVELOPMENT OF MATHEMATICAL KNOWLEDGE FOR TEACHING

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A growing body of international research has documented the importance of teachers' pedagogical content knowledge for students' learning (e.g., Baumert et al, 2010). Much remains to be discovered about how teachers develop mathematical knowledge for teaching (MKT; Ball, Thames, & Phelps, 2008). One empirical question is whether teachers develop MKT on the job. For example, interactions with students may help novice teachers notice common student errors (a component of MKT), and more knowledgeable colleagues may help novices develop pedagogically appropriate responses to those errors. It is also plausible that topic-specific MKT develops independently; the same teacher may have a great deal of knowledge for teaching geometry but relatively little for algebra. Thus, another empirical question is whether the topics that comprise a teacher's curriculum have implications for the kinds of MKT that he or she is able to learn in the context of professional experience.

In this longitudinal study, I used survey data (including an MKT instrument narrowly focused on multiplicative reasoning (MR) topics) and multilevel models (time nested within individuals) to investigate the following research question: *How do changes in MKT MR differ between those who teach MR topics directly and those who do not?* I hypothesized that only those teachers who taught MR topics would develop MKT MR over the course of the study, and furthermore, that this development would be contingent on the quality of teachers' interactions with colleagues and the number of hours they participated in mathematics-focused professional development. The sample included 199 teachers of students 12 to 14 years of age in the US state of Georgia, surveyed 3 times over one semester (5 months).

I found that MKT MR did increase during the study; however, the change in MKT MR was not related to whether or not teachers taught MR topics, nor was it related to the reported quality of teachers' interactions with colleagues. Moreover, change in MKT MR was not related to the number of hours of mathematics-focused professional development. These findings suggest that the mechanisms of MKT growth may be general rather than topic-specific such as the content domain of MR, and may depend on factors that go beyond those of professional development and collegiality. Additional findings and limitations of the study will also be discussed.

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MATHEMATICAL REASONING AND BELIEFS

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We present a research project on students' mathematical reasoning and how beliefs are indicated in their arguments. Preliminary results show that students express beliefs that task solving does not include reflection or much struggle. The results underpin earlier studies stressing expectations as a theme of belief.

It has been shown that students working with textbook tasks of a routine character express beliefs of safety, expectations and motivation used as arguments for central decision making (Sumpter, 2013). These types of tasks are often tackled by the students by using known algorithms (Boesen, Lithner & Palm, 2010). This is one of the expectations students have on textbook tasks. Do arguments and the expressed beliefs change and if so, how when students get the chance to work on tasks of non-routine character? Non-routine tasks are essential in developing the competence of mathematical reasoning. This ongoing study explores how beliefs could work as arguments for the central decision making in students' reasoning while solving non-routine tasks, as a support in their learning process. We apply the framework and the notion of Beliefs Indications used by Sumpter (2013). By categorizing tasks in relation to the reasoning demands, non-routine tasks have been selected (Jäder, Lithner & Sidenvall, 2014). Data is collected by video recording task solving sessions and interviews.

Preliminary results show that students express beliefs not to include reflection or much struggle in task solving. Beliefs of expectations are used as arguments for central decision making.

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EXAMINING EFFECTS OF TEACHER PREPARATION: EFFORTS TO TEACH MATHEMATICS CONCEPTUALLY

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To investigate the effects of mathematics teacher preparation on novice teachers' classroom practices, we analysed teachers' practices in terms of their potential to support students' conceptual understanding, as promoted in the preparation program.

Every year, the United States spends millions of dollars preparing thousands of teachers with no real evidence for the effectiveness of this expenditure (Cochran-Smith & Zeichner, 2005). We are in dire need of empirical data on how teachers learn to teach mathematics effectively to inform the goals and design of teacher preparation.

To address this need, we followed six K-8 teachers as they transitioned from their teacher preparation program into their first three years of classroom teaching. We investigated their teaching of four topics – three "target" topics and one "control" topic – developed in grades K-8. The three target topics (multiplication of multi-digit whole numbers, subtraction of fractions, and division of fractions) were topics that were well developed in the teacher preparation program; the control topic (finding the mean of a data set) was not directly addressed in the preparation program. Two classroom observations from each teacher each year (one lesson on a "target" topic and one on the "control" topic) were analyzed in terms of the degree to which they exhibited two key features of instruction identified as having the most empirical support for developing students' conceptual understanding of mathematics: 1) explicit attention to key mathematical ideas, and 2) opportunities for students to grapple with key mathematical ideas (Hiebert & Grouws, 2007). Post-observation interviews captured novice teachers' perceptions of their efforts to teach for conceptual understanding.

Results indicate that teacher preparation supported these novice teachers in enacting instruction that makes key mathematical ideas explicit, which contrasts with previous findings (e.g., Borko, et al., 1992). These novice teachers were more capable of going beyond procedural instruction with their students during target topic lessons.

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EXAMINING PRE-SERVICE TEACHER EDUCATION: THE EFFECTS OF A METHODS COURSE ON TEACHERS' BELIEFS

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For pre-service teachers (PSTs), particularly those who have only experienced traditional approaches as a student, teacher education courses can be a revelatory experience in their development as educators. Such courses should provide opportunities for PSTs to discuss reformed approaches to teaching math and experience doing math in a reformed way (Battista, 1994). This study explored elementary PSTs' changes in beliefs about math education through two research questions: 1) Have the beliefs of elementary PSTs changed as a result of a math methods course; and 2) What course-related factors made them change their beliefs?

Participants for the study were enrolled in a math methods course in their first year of a two-year teacher training program in a Canadian faculty of education. In the program's first year, PSTs took a required math methods course. Components of the course included: reading research-based articles, class discussions, presentations, and active participation incorporating small and large group learning.

In Phase I of this mixed methods study, data was gathered via a pre- and post-course survey about their beliefs about math. Phase II consisted of a reflective essay, which was the final assignment for the course, and one-on-one interviews that followed-up on themes that arose from reviewing participants' surveys and essays. PSTs taking the course could participate in any or all of the components of their choice.

From a class of 28 PSTs, 19 consented to the use of their surveys and essay for the research study and 12 were interviewed for Phase II. A Wilcoxon Signed Rank Test found that the PST's average responses on the survey increased from 4.22 to 4.51 (Wilcoxon w=28, p=0.013) thus indicating that PSTs shifted their math beliefs towards a more reformed approach.

The PSTs changed their beliefs related to some practices (student-centered learning, non-traditional assessment strategies, classroom discussions and open-ended tasks) and PSTs shared that they enjoyed the opportunity to experience math education in a different way. Other themes (manipulatives) did not yield similar results. While many of the experiences from the methods class were new to the PSTs, the discrepancy of changing beliefs in some areas but not others requires additional investigation.

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PROCESSES OF MATHEMATICAL REASONING: FRAMING FROM MATH EDUCATOR DISCOURSES

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Helping students develop their mathematical reasoning (MR) is a goal of several curricula. But what is meant by mathematical reasoning isn't quite clear and we sometimes assume that everybody knows what it is. Wanting to clarify the meaning of MR, this research project aimed to qualify from a theoretical perspective MR as it might occur in the classroom. To do so, a literature search based on anasynthesis (Legendre, 2005) was undertaken. From the analysis of the mathematics education research literature on MR, two main aspects characterize MR.

A commognitive framework (Sfard, 2007) underpinned the anasynthesis of the literature. First, for a commognitive researcher, research development is equivalent to the development of a research discourse. A researcher has to build on other researchers' works and try to develop a common discourse. This is why the data of this study are the mathematics education literature resources dealing with MR. Second, from a commognitive point of view, MR is a discursive process that derives utterances about objects or mathematical relations by exploring the relations that tie them together. By this, we mean that MR extends an existing discourse about already existing mathematical objects. In contrast, mathematical thinking is a synonym for mathematical discourse. It is composed of particular wordings, mediators, narratives, and routines. It is more than MR. MR is a kind of mathematical thinking process that contributes to its development by broadening it with derived utterances.

The analysis points out that MR can be characterized by two main aspects: the structural and the processual. The structural aspect refers to the different structures that can define discursive units (deductive, inductive, abductive). The processual aspect refers to the different discursive actions that can be undertaken. The structural aspect isn't sufficient to characterize MR, a discursive process consisting of several units. In particular, this analysis allows, among others, differentiating among exemplifying, generalizing, conjecturing, justifying, and proving on the basis of process actions and purposes, and helps in understanding how they are intertwined. Differentiating MR processes can help in designing opportunities to develop MR in the classroom.

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TEMPORAL PATTERNS IN COGNITIVE DEMAND OF TEACHER QUESTIONING

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Students build a powerful understanding of mathematics when they are involved in discourse that challenges them to explore, communicate, and justify their own mathematical ideas (Boerst, Sleep, Ball, & Bass, 2011). Students' exposure to communication through meaningful teacher questioning has correlated with higher student achievement and deeper levels of thinking (Stein, Grover, & Henningsen, 1996). Therefore, creating opportunities for communication, which allows students to think at higher cognitive levels, is important for educators, especially with repeated exposure to mathematical concepts. A lack of research in this area leads to the following research question: What patterns occur in the cognitive demand of teacher prompts at the introductory, developing, and review levels of content development?

This case study comprised of one eighth grade mathematics teacher known to employ practices of student discourse. Semi-structured interviews were conducted and six periods of pre-algebra classes were observed. Observation data were coded, prompt-by-prompt, for the level of cognitive demand using an adapted framework by Stein et al. (1996). The levels are: 1) Memorization/ recall of a fact; 2) Use of procedures and algorithms without attention to concepts or understanding; 3) Use of procedures and algorithms with attention to concepts or understanding; 4) Making relevant connections; 5) Employment of complex thinking and reasoning strategies (conjecturing, justifying, interpreting, etc.).

Results indicated that prompts were most frequent, at all levels of cognitive demand, in introductory lessons and decreased as content became more familiar. One possible explanation could be teachers' belief that students need less guidance when revisiting concepts. Since higher-level prompts were not a focus at later points in development based on the results, researchers and educators need to explore how teachers can focus more time giving prompts that encourage students think more critically. Educators need to understand that students should be exposed to learning situations that require higher level thinking with rigorous mathematics when revisiting a concept. Further results will be discussed in the presentation.

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IT'S ABOUT TIME: HOW INSTRUCTORS AND STUDENTS EXPERIENCE TIME CONSTRAINTS IN CALCULUS I

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This report investigates the relationship between the pace at which Calculus I material is covered and student persistence onto Calculus II. Faculty often cite coverage concerns as a reason to not implement student-centered instructional approaches. Research, however, has found that such instructional approaches support deeper student understanding, longer retention of knowledge, and increased persistence in a STEM major (Larsen, Johnson, & Bartlo, 2013; Rasmussen & Ellis, 2013).

Here we address the question: What is the relationship between student persistence and student and instructor reports of the amount of time spent in class to develop difficult ideas? Theoretically, we see this research question as investigating aspects of the didactical contract between students and faculty (Brousseau, 1997). We draw on data collected during the Characteristics of Successful Programs in College Calculus (CSPCC) project, a large empirical study designed to investigate Calculus I. We analyzed student and instructor survey responses regarding the pace of the course and student reports of changes in intention to take Calculus II.

Our results indicate that we are most likely to lose STEM-intending students in classes in which they do not feel like they have enough time to learn difficult material. This attrition rate was most pronounced when both students and their instructor felt that they did not have enough time. Such classes were characterized as very traditional, with high levels of lectures and low levels of any other instructional practice. These results are contrasted with results from classes in which students and instructors both feel that they have enough time, where there is a variety of traditional and student-centered instruction, and where students are more likely to continue with their intentions of taking further calculus courses.

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SIXTH GRADE STUDENTS' WAYS OF THINKING ASSOCIATED WITH SOLVING ALGEBRAIC VERBAL PROBLEMS

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The mathematics of the middle school includes the transition from arithmetic to algebra. The development of algebraic thinking requires some particular mathematical skills such as quantitative reasoning, generalizing and problem solving (Cai & Knuth, 2011). According to Thompson and Smith (2007), quantitative reasoning provides a smooth transition from arithmetic reasoning to algebraic reasoning as a glue. On the other hand, algebraic verbal problems play a significant role in developing quantitative reasoning in middle grades. As Harel (2008) emphasized, the important thing in problem solving is the ways of thinking rather than answering the problems correctly in school mathematics.

The purpose of this study is to investigate, within the framework of DNR (Harel, 2008), the sixth grade students' ways of thinking and ways of understanding associated with problem solving in the context of an algebraic verbal problem. This is a qualitatively designed study in which the data was collected through clinical interviews with 10 sixth grade middle school students. Data was analyzed qualitatively by using content analysis technique. In this study most of the participants (seven participants out of ten) tended to use the strategy of look for a keyword by skipping the making a plan step of problem solving process. It was seen that these participants preferred to use the strategy of trial and error when they realized the look for a keyword strategy did not work. The participants, who were not able to reason, were thought to have arithmetic way of thinking. Furthermore, these participants could not use any different strategy from problem solving approach which they recalled. Remaining three participants, who were able to reason quantitatively, grasped the relationships in the problem situation and were able to solve the problem by using the strategy of looking for relevant relationships among the given quantities. Thinking way of these participants was thought to be quantitative way of thinking.

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EFFECT OF SELF-EVALUATION ON PRE-SERVICE MATHEMATICS TEACHERS' SELF-EFFICACY WITH RESPECT TO LANGUAGE OF MATHEMATICS

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Mathematics teachers need to know teaching the language of mathematics, universal language with its own language system, by connecting conceptual structure of mathematical knowledge. However, Gray (2004) found that they ignore knowledge and skills of mathematical language in teaching mathematics. Gray examined the reason for such neglect and obtained that teachers are either unaware of how to teach this language or they may not believe that they can implement such language training. According to Gray, this may be explained by Bandura's (1997) self-efficacy theory. With regard the development of teachers' beliefs about ability to teach, the studies verified that the self-evaluation had a positive effect on teachers. In the current study, it is aimed to investigate the effect of self-evaluation of pre-service elementary mathematics teachers on their self-efficacies with regard to using and teaching language of mathematics. In the study sequential explanatory mixed design was used. In the first phase of the study the self-efficacy instrument was developed and administered to 23 elementary pre-service mathematics teachers. Then the participants taught a middle school level mathematics subject in the micro teaching process of the language of mathematics course. Their teaching processes were recorded and they evaluated themselves by using the records. Afterwards, the instrument administered the participants again. In the second phase eight participants were interviewed by clinical interview method in order to elaborate their responses. The data of the first phase was analyzed by using paired samples t-test whereas the data of second phase analyzed qualitatively by using content analysis technique. The results of the paired samples t-test indicated that there was no significant difference between pretest and posttest self-efficacy scores. The reason of no significant difference was revealed by the results of the interviews in the second phase. Interviews indicated that participants perceived the language of mathematics as using native language or using pedagogical approaches and they weren't aware of the responsibility in teaching the language of mathematics as well as mathematical concepts. As a result, it was concluded that the effect of self-evaluation on individuals' self-efficacies may depend on their awareness or knowledge of the context.

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TOWARD THEORIZING THE EMBODIMENT OF MATHEMATICAL COGNITION IN THE CLASSROOM

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Embodiment theory is a theoretical trend in mathematical cognition research in which the objective is to understand students' knowing. It is based on the background that cognition is embodied. If mathematics is considered a product of imagination rooted in human activities (Lakoff & Núñes, 2000), mathematics education research should focus on the role of bodily actions for cognizing and understanding mathematics. This article aims to examine the roles of bodily actions as a diagrammatic incarnation of reasoning (de Freitas & Sinclair, 2012) from this theoretical perspective.

To accomplish aims of this article, qualitative research was conducted. For data collection a series of six mathematical lessons about spatial geometry were implemented and exemplary episodes were analyzed. In particular, during the final lessons (5^{th} and 6^{th}), the students dealt with a given problem by the teacher and were asked "What shape is the resulting section if we cut the cube cleanly by a plane holding certain three points?" They spontaneously inquired into geometrical properties about the position relation of lines and planes through the parallel projection.

From exemplary episodes from the final lessons, a student's time series gesture which was similar to a process of reasoning was observed (Fig.1). This reasoning process could be described as the statement that a section is a quadrilateral and includes two right angles. In this sense the gesture was considered as a type of diagrammatic incarnation. In addition, the gesture might have a function to realize and develop the subject of discussion in the classroom. The inferences regarding the shape of the section based on perception is incarnated by gestures in 3D space as shown in Fig.1. Moreover, inferences with gestures enable students to focus on the position relation of the necessary sides to demonstrate and create a naive theory of space geometry as a basis of demonstration.

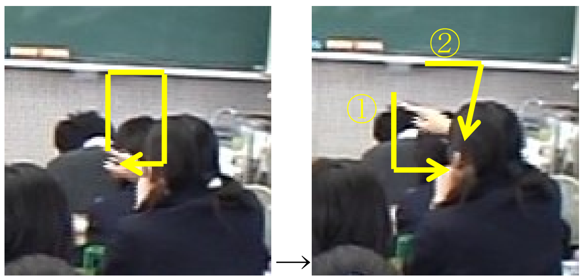

Figure 1: A students' gesture

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ANALYSIS OF PRE-SERVCE MATHEMATICS TEACHERS' DIAGRAMATIC REASONING SKILLS AND THEIR VAN HIELE LEVELS

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Diagrammatic reasoning skills are of great importance to many professions, among them teaching of Mathematics. The principal issue is that of teacher preparedness. In particular, teaching Geometry requires subject-matter content knowledge and diagrammatic reasoning skills. When solving geometric problems we "argue from a diagram". Informal arguments eventually become translated into formal statements we call proofs. "This translation from visual to verbal suggests a possible method of moving from visual mathematics to formal mathematics." (Pinto, Tall, 2002) Successful geometry problem solvers are characterized by their spontaneous abilities to access geometric rules (Chinnappan, 1998). Moreover, the quality of teachers' knowledge for teaching is characterized by the richness of the interconnected schemas of their subject-matter content knowledge (Chinnappan & Lawson, 2005)

Presented study attempted to learn whether there is a relationship between pre-service teachers' levels of geometric thought (van Hiele model) and their diagrammatic reasoning skills. In the course of the study participants were presented with "visual proofs" of certain theorems from high school mathematics curriculum and asked to prove/explain these theorems by reasoning from the diagrams. Participants' clinical interviews were analysed with respect to their van Hiele levels (Usiskin, 1982); and their solutions to visual proofs were compared with solutions given by a group of "experts".

The results of the analysis showed that pre-service mathematics teachers' diagrammatic reasoning skills were much weaker than their attained van Hiele levels would indicate.

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VIDEOTAPED LESSONS AS RESOURCES FOR PROFESSIONAL DEVELOPMENT OF SECONDARY MATHEMATICS TEACHERS

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Video is used as a tool for teachers' professional development for the past 50 years, yet the rapid advancements of digital video has enabled a significant amplification in this field. Video is seen as a window to the authentic practice of teaching, which allows teachers to unpack complex issues through observing, re-observing and reflecting on specific occurrences (Sherin & van Es, 2009). Three main trends of video uses for mathematics teacher development can be detected: Introducing new curricula via video cases that model how teaching these curricula may be enacted; using videos of lessons as a source for feedback and evaluation; and enhancing teachers' proficiency to notice and understand students' mathematical thinking in screened episodes. The VIDEO-LM project suggests a different direction, in which videotaped mathematics lessons are used as "vicarious experiences". Groups of secondary teachers watch whole lessons of other teachers and analyze them together according to a framework we have formed, based on analytical tools offered by Arcavi & Schoenfeld (2008). The framework consists of the following 6 components, with which teachers can reflect on their own practice while watching lessons of others (Karsenty & Arcavi, 2014): mathematical and meta-mathematical ideas; goals; tasks; interactions with students; dilemmas; and beliefs. In a pilot study, two groups of teachers (n=22) participated in 30 hours of video workshops, using this framework. The study explored the following questions: What may be the gains of video-based teacher discussions around the VIDEO-LM framework, in terms of the teachers' mathematical knowledge for teaching (MKT)? Can changes be traced in the teachers' discourses after exposure to the framework?

The data included video-documentations of sessions, the groups' e-mail exchanges, field notes and questionnaires. Data was analyzed using various content analysis methods. The results show that participants have collectively generated new mathematical knowledge for teaching, as shall be demonstrated in the presentation. Also, there is evidence that they have shifted, to some extent, from evaluative to reflective comments. These promising preliminary results will need to be further triangulated in the forthcoming stage of the study, involving more groups of teachers.

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A STUDY OF LESSONS FOR LOW ATTAINERS IN PRIMARY MATHEMATICS

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From the findings of a study on low attainers in primary mathematics in Singapore (Kaur & Ghani, 2012) and past research it is apparent that the challenge for teachers of low attainers in mathematics is to provide instruction comprising a good balance of "how they would like to learn" type of activities and the kind of instruction that is effective for such pupils. Therefore, Kaur (2013) hypothesized that teaching activities comprising group work, where pupils make sense of mathematical concepts using manipulatives, and direct instruction, where pupils hone their mathematical skills, could enhance the learning of mathematics by low attainers. An appropriate amalgam of such activities will facilitate the development of both conceptual and procedural knowledge. Lastly, the mathematical tasks that form the bedrock of the activities must be an appropriate mix of knowledge building (higher order) and performative (lower order) mathematical tasks.

An intervention project, pedagogy for low progress learners of mathematics, is presently underway in some schools in Singapore. In this study we are exploring the themes and respective codes that may be used to analyse the lessons. We are studying the video-records of sequences of lessons conducted by two teachers in the project. We are using the grounded theory approach (Glaser & Strauss, 1967) to identify activity segments that characterise the lesson sequences being studied. As an exploratory study, activity segments – "the major division of the lessons", serve as an appropriate unit of analysis for examining the structural patterns of lessons since it allows us "to describe the classroom activity as a whole" (Stodolsky, 1988, p.11). Our preliminary analysis has shown that the four main classroom activity segments are: reviewing of past knowledge, introduction of new knowledge, guided practice and seatwork. The distribution of time across the four segments, appear to vary from lesson to lesson, depending on the position of the lesson in the sequence. We are also studying the mathematical tasks and instructional strategy used in the respective segments.

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STUDENTS' MENTAL REPRESENTATIONS OF GEOMETRIC VECTORS AND SETS RELATED TO LINEAR ALGEBRA

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Linear algebra is a cognitively and conceptually difficult subject for many students due to its abstract and formal nature. Some researchers pointed out that the use of embodied notions helps students to understand concepts in the formal theory of linear algebra (Stewart & Thomas, 2007). In linear algebra, embodied notions are visualised in the world of geometric vectors where vectors are represented as arrows and the formal theory is described in terms of sets. Hence having mature mental representations of geometric vectors and sets seems to be important for success in learning linear algebra. In this study, focusing on the notions of linear independence and subspace, we examined students' mental representations of geometric vectors and sets related to these notions.

The participants of this study were 107 first-year university engineering students who had a linear algebra course. We implemented a test to assess these students' mental representations of geometric vectors and sets. The test consisted of five problems. In three problems, students were asked to determine linear independence of geometric vectors in space given in pictures. In the other two problems, students were asked to give examples of elements in sets consisting of an infinite number of column vectors.

As for linear independence of geometric vectors, only 15% of the students could not answer correctly in the case of three linearly independent vectors and only 12% of the students could not answer correctly in the case of three linearly dependent vectors. However, in the case of four vectors, 37% of the students could not answer correctly. The result of the qualitative analysis of students' answers suggested that some students cannot imagine the space spanned by three linearly independent geometric vectors and hence they fail to recognize the linear dependence of four vectors in space. As for sets, only 14% of the students made conceptual errors or gave no answer in the case of a set of all solution vectors of a linear equation, but in the case of a set of all linear combinations of three 4-dimensional column vectors, 41% of the students made conceptual errors or gave no answer. These results indicate that some students do not have mature mental representations of geometric vectors and sets. Future research will address whether and how these mental representations influence the learning of linear algebra.

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DISTRICT PROFESSIONAL DEVELOPMENT SUPPORTING STUDENTS' OPPORTUNITIES TO LEARN ALGEBRA 1

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Algebra has received the attention of policymakers and educators, who consider the first high school Algebra course as a gatekeeper to students' advanced mathematics success, college and career readiness, and crucial to preparing a globally competitive workforce. One important way that districts support students' opportunities to learn Algebra is through the professional development (PD) of Algebra teachers. Recommendations for effective PD emphasize the importance of models that are collaborative, centered on teachers inquiring into their practice, and grounded in content (e.g., Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009). Using these recommendations as an analytic framework, we investigate the following research question: How are district decision makers using PD to improve students' opportunities to learn Algebra?

Data was collected as part of a mixed-methods study that included a survey of a nationally-representative sample of 993 district decision-makers and 12 district case studies. The survey collected data at a macro-level on Algebra policies and practices. The case study districts, representing variation in size, region, and achievement, afforded a micro-level illustration of national trends. Preliminary findings suggest that district decision-makers have identified PD as an area in which changes are needed in order to improve Algebra instruction. There was also near-universal (94%) agreement with a statement, "Changing teaching practices through targeted professional development is critical to achieving the goal of early algebra completion." On average, districts are offering 14 hrs/yr of mathematics PD, although 58% offer fewer than 10 hrs/yr, with very little that is Algebra-specific. Case study findings reveal that many districts are not able to offer formal mathematics PD, but that a common approach is creating time for teachers to collaborate in content-groups, focused on the creation of curriculum maps or assessments for alignment purposes. This approach to PD offers a creative solution with collaborative benefits for districts facing budgetary constraints. It is questionable, however, whether this approach fosters communities of practice engaged in inquiry, with the collegial discourse necessary for deepening teachers' knowledge of content and pedagogy. These findings suggest that while PD is important to district decision-makers, they are not able to make the commitment to PD that aligns with research-based recommendations for improving students' opportunities to learn Algebra.

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USING WRITING PROMPTS TO DEVELOP PEDAGOGICAL CONTENT KNOWLEDGE WITH PRESERVICE TEACHERS

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Pedagogical content knowledge (PCK) requires an understanding of conceptions that students bring with them to the classroom (Shulman, 1986). However, preservice mathematics teachers' (PSMTs) development of PCK may be impeded by the *expert blind spot*, where novice teachers with advanced understanding of content may fail to recognize students' misunderstandings of concepts (Nathan & Petrosino, 2003). We contend that engagement in and reflection on *writing to learn mathematics* (WTLM) might be one means of overcoming the expert blind spot and enhancing PCK. WTLM involves using writing prompts that require explanations of mathematical content or processes (e.g., How do you know that 1/4 is greater than 1/5? Explain the difference between the commands solve, simplify, and evaluate). Our research question for this study is: *How does engagement in and reflection on WTLM enhance PSMTs' understanding of student learning of mathematics?*

We used a qualitative case study to examine seven PSMTs in a unique seminar where they received credit to teach a college algebra course and met each day to discuss content and pedagogy. During the seminar, PSMTs engaged in five cycles where they (a) wrote a response to a WTLM prompt, (b) engaged in collaborative reflection via asynchronous web discussions, (c) gave a quiz to their students with the same WTLM prompt, and (d) reflected again on what they learned from students' responses. PSMTs completed a PCK questionnaire at the beginning and end of the seminar that was analyzed using a simple t-test. Responses to writing prompts and reflection posts were analyzed using content analysis to identify changes in the ways PSMTs think about teaching and learning mathematics and other issues related to PCK.

PCK questionnaire results show an increase in PSMTs' beliefs about readiness to teach mathematics. We found no statistical significance in changes related to beliefs about learning mathematics, however qualitative analysis of WTLM data shows indications of PSMTs growing awareness of their expert blind spots. Reflections indicating understandings of learners' needs increased significantly in both quantity and quality. We look forward to sharing examples of PSTs' development of PCK and expert blind spot awareness and to consider implications for student learning.

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IMPACT OF A CREATIVE CHARACTER EDUCATION PROGRAM ON PRE-SERVICE TEACHERS' PERSPECTIVES AND BELIEFS IN LEARNING MATHEMATICS

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The purpose of this study is to investigate how a creative character education program affects participants' perspectives on creativity and character and their beliefs about creativity and character education. The NCTM standards take into account the importance of creativity and character education by referring to mathematical creativity, awareness of mathematical values, and confidence (NCTM, 2000). In this study, creative character education program can be seen as an educational approach for synthesizing creativity and character in learning mathematics (see Gardner, 2009; Sternberg, 2003). Fifty seven pre-service teachers participated in a semester-long program and their responses to the pre- and post- questionnaires were collected. In order to foster creativity, the program design emphasizes three mathematical contents such as storytelling, manipulatives, and representation activities. For instance, our goal in using storytelling is to expand creativity by prompting students' interest, curiosity, and imagination about mathematics through mathematical enrichment activities using the history of mathematics, modeling using a STEAM project, and real-life stories. In order to cultivate character in learning mathematics, we make full use of three mathematical processes interwoven across mathematical contents: classroom norms, cooperative learning, and multiple assessments.

The *t*-test result shows that the mean scores for many questions about participant perspectives on creativity in the post-questionnaire were significantly higher than those in the pre-questionnaire. For instance, in the case of storytelling, participants were significantly more likely to report loving reading about mathematics and exercising mathematical imagination than before. In addition, changes in participant perspectives on character can be described in relation to mathematical processes emphasized throughout the program. Through the use of multiple assessments, for instance, they seemed to cultivate their character components such as responsibility, honesty, and sincerity which reached statistical significance. Note also that the participants were more convinced about that students' creativity can be improved whenever they learn new things. All these findings suggest that merging mathematical processes with mathematical contents holds the promise of making a creative character education program possible in learning mathematics.

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INVESTIGATING SOUTH KOREAN SECONDARY MATHEMATICS TEACHERS' LEARNING WITHIN A SCHOOL-BASED PROFESSIONAL LEARNING COMMUNITY

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The latest 2012 Programme for International Student Assessment (PISA) results from the OECD showed that students in South Korea performed 5th out of 64 countries at mathematics, science, and reading. Moreover, published by the Varkey GEMS Foundations, the 2013 Global Teacher Status Index, which gauged the social standing of teachers in 21 countries revealed that teachers in South Korea have a higher social standing (ranked 4th) than the countries surveyed in Europe and U.S. Therefore social expectations for better education in mathematics in Korea have been very high, which led to increasing attention to the mathematics teacher education. However, little research has investigated how Korean teachers teach mathematics and what types of professional development opportunities are afforded to further enhance their professionalism. The present study reports an analysis of seven Korean high school mathematics teachers learning as they participated in a Professional Learning Community (PLC) created on their own initiative. We will also share some challenges that the teachers had in sustaining the community and as the result how the initial PLC model was evolved to fit the need of the community. The theoretical perspective on teacher learning in PLC was informed by Lave and Wenger's (1991) view of learning as social participation and Wenger's (1998) four categories (meaning, practice, identity, and community) of learning. The data for this research come from audio/video records of 23 PLC meetings collected throughout a year and participating teachers' 24 classroom videos, and participating teachers' survey on their own teaching practices. Initial findings of the study include that the teachers were extremely challenged when opening video-recorded data of their own teaching to the public and discussing it closely. However, the tension was resolved as they elaborated their initial PLC model (classroom observation → collective reflection) by adding more collective practices such as developing collective meaning of good teaching via reading literatures and a program of inviting experts in the field. Moreover, some of the teachers altered their meanings of good teaching and identities as a mathematics teacher, which resulted in the change of their teaching practices.

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MIDDLE SCHOOL STUDENTS' IMAGES OF SPEED IN DYNAMIC FUNCTIONAL SITUATIONS

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Covariational reasoning to recognize varying quantities and construct a relationship among those quantities is fundamental to learning of a function concept (Oehrtman, Carlson, & Thompson, 2008). Perceiving spatiotemporal objects, tracing their change, and establishing a relationship in a dynamic situation inevitably involve physical concepts as time and speed. Therefore students' concept of speed would be closely associated with the students' initial learning of function. In the present study, we report characteristics of students' understanding of speed emerging in their exploring several dynamic situations, informed by Thompson's (1994) four hierarchical levels of images of speed, and provide its potential relevance to their learning of function.

Our qualitative case study was conducted with four Korean 7th grade students who learned a formula for speed (as distance/time) and a formal definition of function in school. The students, with an e-book designed for a tablet PC, freely investigated given dynamic functional situations. They were asked to find varying quantities in the situations and express the relationship among the quantities in their own ways.

Despite their prior knowledge concerning function, results indicate that the students had difficulties in constructing a function when speed was involved as one of the varying quantities. One student understood speed as accrue of discrete distance (Image 1), one recognized speed as a distance moved in one time unit (Image 2), and two conceived speed as simultaneously and continuously accrued value of time and distance (Image 3).

The results also imply that a concept of 'speed' in mathematics education should not be dismissed as a particular type of rate or as just an application of a function concept to a changing situation. Rather a dynamic functional situation entailing time and speed can be one of the most suitable environments to introduce the concept of function, and students' development of covariational reasoning involving a speed concept can play a critical role along the students' learning trajectory with functions.

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THE REMARKABLE PEDAGOGY OF CHRISTOPHER HEALY

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In the late 1980s Christopher Healy began teaching a high school geometry course in which the students, collectively, wrote the textbook, deciding democratically what topics would count as geometric content, and what results are soundly reasoned:

> After each presentation … there is a vote on whether the material presented is true and worthy of entry into the book. This process produces some of the most difficult moments for me, because students have presented and voted down things that I feel are significant parts of geometry. Still, I believe it imperative that I not interfere. (Healy, 1993, p. 87)

Through this process, his working class students were transformed from typical teenagers into scholars who care passionately about mathematics. As one student put it, " 'In this class we make enemies out of friends arguing over things we couldn't have cared less about last summer' "(quoted in Healy, 1993).

This paper analyzes Healy's pedagogy using a sociology-based framework for learning cultural practices that distinguishes unconscious processes of *enculturation* from conscious processes of *acculturation*. For example, through "unconscious introjection" (Parsons, 1951) natives of France come to prefer closer physical proximity for conversation than do Americans, whereas an American emigrating to France may consciously emulate French proxemics in order to fit in.

Educationally, acculturation methods of modeling cultural practices are appropriate only in case students already are identified with the target culture (Kirshner, 2011); otherwise, intrapersonal conflicts may arise (e.g., Clark, Badertscher, & Napp, 2013). Healy's careful portraits of students' evolving identities illustrate the efficacy of enculturational methods in which target practices are tacit, and no coercion for participation exerted. Healy also illustrates the soundness of theorizing—and addressing!—learning goals independently: Cultural practices need not be taught in conjunction with skills or concepts (Kirshner, 2011).

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NOTICING AND WONDERING WITH TECHONOLOGY

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This presentation will discuss the design-based research process to develop a software tool that teachers can use to enhance their noticing and wondering of student work. The presentation will situate that discussion of software development within the larger EnCoMPASS project goal of developing an online community of teachers engaged in the creative assessment of student work to support mathematical thinking.

The Math Forum is an online educational community dedicated to supporting students and teachers to engage in meaningful mathematics. One practice that has emerged as central to disrupting teachers and students traditional answer-focused approaches to mathematics is *Noticing and Wondering* (Fetter, 2008) as a protocol for thinking about mathematics and "reinforc[es] the expectation of problem solving as a process" (Hogan & Alejandre, 2010, p. 33). The process of *Noticing and Wondering* has always been mediated through technology; further enhancing the "slowing down" of the problem solving process, and in doing so, honors student-generated ideas.

Our current NSF-funded project, EnCoMPASS, is designed to develop an online teaching community focused on understanding and improving mathematical thinking through work with formative assessments centered on student thinking. A major component of this work is the iterative design of software that will scaffold productive mathematical noticings and wonderings in order to provide individualized feedback to students and to collaborate with other members of the EnCoMPASS community to discuss student work. In this way the project attempts to create meaningful conversations around student thinking that serves to support strengthened instructional decision-making and to contribute to the ongoing conversation about teachers noticings (e.g. Sherin, Russ & Colestock, 2011). This presentation reports on the development of the software, the associated community and presents examples of teacher engagement with *Noticing and Wondering* in technologically mediated forms.

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IMPROVING TEACHERS' ASSESSMENT LITERACY IN SINGAPORE MATHEMATICS CLASSROOMS

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Singapore, like many countries, has been implementing reforms in education that are aligned with current vision of teaching and learning 21^{st} century competencies (e.g., conceptual understanding, higher-order thinking, and inquiry habits of mind). Teachers' assessment literacy plays an important role in influencing the success of this reform in the classroom. This study investigated an intervention to improve the quality of Singapore teachers' assessment literacy in mathematics teaching and learning.

Based on the Singapore Mathematics Curriculum Framework, teachers in Singapore are urged to shift their assessment practices toward the use of alternative forms that are aligned with the higher-order curricular goals. But there are ongoing concerns about teachers' lack of assessment literacy to achieve this shift (Koh & Luke, 2009). To address this situation, this study focused on designing and testing an intervention framed in a *flexible performance of understanding* perspective (Darling-Hammond & Adamson, 2010). The criteria of *authentic intellectual quality* were used as the guidelines for teachers to design authentic assessment tasks.

Participants of the 2-year study consisted of 18 grade 5 teachers from four schools. Teachers from two of the schools formed an experimental group and those from the other two schools formed a control group. The experimental group received ongoing, sustained experiences in authentic assessment task design and rubric development while the control group received only one workshop in authentic assessment at the end of each year. Data sources included 116 teachers' assessment tasks and 712 related students' work before and after intervention. Data analysis included comparison of mean scores on the criteria of authentic intellectual quality for teachers' assessment tasks and students' work and between the control and experimental groups. Results indicated that the experimental teachers increased their competence to design assessment tasks that were of high authentic intellectual quality while the control teachers' tasks focused less on students' mathematical understanding, thinking, problem solving, and connections. The study suggests that teachers' assessment literacy can be significantly improved through this type of intervention. Details of the intervention, teachers' changes, and samples of the tasks before and after intervention will be presented.

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TRANSPARENT REASONING AND CRYSTALLINE CONCEPTS

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Emergence of a *crystallin*e concept refers to a phenomenon where an object of mathematical study "which originally was a single gestalt with many simultaneous properties, and was then defined using a single specific definition – now matures into a fully unified concept, with many properties linked together by a network of relationships based on deductions" (Tall et al, 2012, p.20). Such concepts result from the process of cognitive development of a learner engaged in educational activities that call for perception, reflection on, formalization and connection of various aspects of the object. Possession of crystalline concepts by a learner is essential for flexible and effective mathematical thinking. The goal of this research is to expose a relation between crystalline concepts and the practice of transparent, insightful reasoning.

The participants were 23 pre-service secondary mathematics teachers. In one hour they were asked to solve six algebraic problems and explain their results in a simple way. Most challenging was the task: "Find all real x for which (a) $||x-1|+2|>1$; (b) $\log_2(x-3)=\log_3(2-x)$". Even though before the test the teachers had revised definitions and properties of the absolute value and logarithm, only five of them gave an acceptable solution in (a) and only three in (b). This outcome was consistent with my informal observations of pre-service teachers in the previous five years. Having at least six courses in undergraduate mathematics makes them familiar with many formal rules and algorithms, but does not always afford a clarity and flexibility of thinking. An interview with unsuccessful solvers of the above task revealed their preference for procedural approaches in teaching and thinking, as well as shortage in experiencing transparent reasoning and simple but astute explanations. During the test they overlooked reasoning such as "since $|x-1|$ is always non-negative, the inequality in (a) is valid for all x", or "comparing the domains of both sides, we see that the equation (b) has no solutions". However, the teachers have arrived at this type of reasoning during the interview. An insight of a simple solution often occurred to them while they, responding to my questions, were listing, linking and explaining properties, and thus moving towards crystalline concepts of the absolute value and logarithm. My data suggest that formation of crystalline concepts by a learner both enhances simplicity and transparency of their reasoning and depends upon learner's exposure to basic but enlightening ideas related to more advanced methods.

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IDENTIFYING A FRAMEWORK FOR GRAPHING FORMULAS FROM EXPERT STRATEGIES

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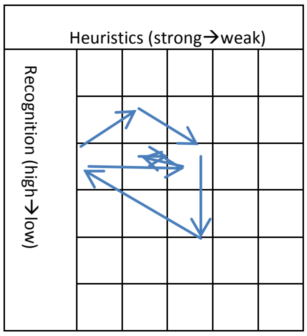
Graphing formulas with paper and pen helps learners to connect the algebraic and graphical representation of a function, which is important in learning to read algebraic formulas. To be able to graph formulas, a repertoire of functions that can be instantly visualized by a graph (formula-graphics) is necessary, as well as heuristics to tackle more complex functions. Which strategies are essential to graph formulas effectively and efficiently, however, is largely unknown. In this research we identify a framework to describe the variety of strategies for graphing formulas, based on studies of chess expertise. These show that experts use structured knowledge for recognition at different levels, which in turn facilitates the use of search heuristics. Similar interdependence of recognition and heuristic search can be applied to graphing formulas. On this basis a two dimensional framework was formulated, with the dimensions recognition and heuristic search. Five experts and three maths school teachers, who graphed the formula $f(x) = 2x\sqrt{8-x} - 2x$, validated the framework. Participants used different combinations of recognition and heuristic search at different levels. For every participant, the strategy was represented as a path in the framework (see figure). Experts' paths are predominately on the upper and/or left side of the framework; some experts focus on their large repertoire of formula-graphics while others focus on strong heuristics, such as qualitative reasoning about domain, infinity behavior, and symmetry. Both strategies can give efficient results. Not all teachers participating in this study turned out to have sufficient expertise in graphing formulas to complete this task. We conclude that the framework appropriately and discriminatively describes the different observed strategies. The framework can be used to compare teachers' and learners' strategies to those of experts and to design learning trajectories for teachers and learners.



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FIRST INSIGHTS IN DEVELOPMENTAL ASPECTS OF MATHEMATICAL THINKING IN THE EARLY YEARS

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For the purpose of analysing the longitudinal development of mathematical thinking, statuses of participation of a child are reconstructed and theoretically described in terms of the concept of the "Interactional Niche in the Development of Mathematical Thinking" (NMT).

Our research interest is the development of an empirically grounded theory concerning the generation of mathematical thinking of children between the ages of 3 and 10. In earlier analyses Krummheuer created a conceptual framework for this theory: the "Interactional Niche in the Development of Mathematical Thinking" (NMT; Krummheuer 2012; 2013). At PME 37 we employed the NMT concept to describe the relationship between mathematical content areas and the early development of mathematical thinking (Schütte & Krummheuer 2013). Here we present our first insights in developmental aspects of mathematical thinking applying the NMT concept to analyse the *personal participation* of a child in different situations of play and exploration, which, in a longitudinal design, are presented over the whole time span of its stay in German Kindergarten (ages 3 to 6).

With respect to NMT we can show that this child participates in different ways in peer situations: its rather active form of participation relates more to non-mathematical themes; when it comes to "mathematics", however, it shifts into the status of a observing participation. First conclusions about how this pattern of participation might impact this child's development of mathematical thinking will be drawn.

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THE ANALYSIS OF DISCURSIVE CONFLICT ASSOCIATED WITH THE STUDY OF EQUIPROBABILITY

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The notion of equiprobability has been an issue in probabilistic teaching and learning. Constructing a complete sample space is one of the key patterns in producing growth in probabilistic thinking (English, 2005).

From a commognitive standpoint (Sfard, 2007), meta-level learning involves changes in the meta-rules of discourse. Discursive conflict, which is the situation that arises when meta-rules of mathematical discourse are different from each other, is practically indispensable for meta-level learning. However, previous research is mainly limited to algebraic discourse. Hence, our study concerns probabilistic discourse, with a particular focus on the notion of equiprobability. The research questions are;

- How are meta-rules of students' discourse on equiprobability?
- If the meta-rules are different, how do they influence the development of probabilistic discourse?

In this research, we used a case study. The participants were 11 high-ability tenth-grade students and one high school teacher. Although they had already learned the classical definition of probability, they were unfamiliar with the notion of equiprobability. Data were collected in the form of video documentation and worksheets.

We observed that students had difficulty in understanding equiprobability. And the meta-rules of their discourse were different. The difference in meta-rules involved the use of the word *outcome*. The meta-rule of S1's discourse regulated outcome as an element of the sample space. On the other hand, outcome in S2's discourse was used as the situation that exists at the end of experiment, like an element of the sample set. The discursive conflict which occurred when students solved same task in a different, unfamiliar way provided opportunities for students to figure out the inner logic of interlocutors' discourse. The results shows that the leading discourse and the students' and teacher's respective role are required for the development of probabilistic discourse. In the presentation, further results will be discussed in detail.

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MATHEMATICS TEACHERS VOICE ON HOMEWORK

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Homework (HW) is an integral part of the learning process. Currently, there is a renewed controversy about its effectiveness (e.g. Cooper, 2007). Limited research on HW in the mathematics education demonstrates general misperception of teachers in the benefit of HW assignments. Since teachers' attitudes may predict their behaviors (Bryan, 2012), examining it as it pertains toward HW is crucial. Hence the following questions guided our research: (1) what are teachers' attitudes? (2) are there gender differences? (3) does experience influence their attitudes? (4) are there differences among the teachers of elementary, middle and high school grades?

This research was divided into two parts. Part I served to build the questionnaire. It was based on interviews with 25 teachers. The interviews dealt with the rationale behind assigning HW, their advantages and disadvantages, and the types of HW given. Through inductive analysis categories were extracted from both the interview data and the literature. Based on this process a 33-items questionnaire was constructed and validated. The items were grouped into the following domains: *cognitive* (HW improves students' achievements, HW establishes good learning habits, exercising the learnt skills, etc.), *pedagogical* (HW contributes to evaluating students' knowledge and the teaching process, etc.) and *affective* (developing students' independence, responsibility, self-discipline and self-confidence, suitability of HW as an education tool for students, etc.). In Part II of the study the questionnaire was distributed to 154 teachers of mathematics, having different professional experiences who teach in elementary, middle and high schools. The questionnaire's Alpha Cronbach was found to be 0.78. In general, teachers' attitudes toward HW were found to be moderate (average – 3.48/5). Teachers held both positive and negative views concerning HW. No significant differences between male and female teachers were found, except for more male teachers considered HW to be unsuitable. The influence of HW on the development of cognitive skills was considered the most meaningful. In addition, experienced teachers found the affective domain more significant than the less experienced teachers ($p<0.05$). This research may serve as a foundation for other studies to explore perspectives in HW suitability, such as intervention programs aimed at enriching math teachers' tool-set.

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THE 'I SPY' TROPE: OPERATIONALISING THE ZPD

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The analysis presented here uses a methodology designed to systematically operationalise objects of pedagogical knowledge by locating them in descriptions of actual pedagogical situations. The *mathematics period* is its object of interest and the *episode, rhetorical unit, message* and *clause* are its units of analysis. Episodes are analysed according to a kind of *poetics*. Rhetorical units, messages and clauses are analysed according to a *functional grammar* and *text semantics*.

Vygotskii's zone of proximal development (ZPD) is used as an example of a *pedagogical knowledge object*. Following Meira and Lerman (2001), the ZPD is treated as a quality of the *symbolic space* between subjects in interaction. Figurative descriptions of the ZPD, taken from the literature, are 'translated' into the more literal semantic qualities of such a symbolic space. In the example presented here, the ZPD is operationalised by identifying these qualities in an episode of natural mathematics classroom interaction that is itself identified with the 'I spy' *trope*.

An episode can be described as being a trope when it is of a recognisably generic kind. The trope is used in this reflexive process as a pedagogical device, as a 'launching place' for a movement from the implicit recognition of a form of action to the explicit description of its structure and function. By describing interaction using explanatory principles that account for the resources that make that interaction possible, and by linking descriptions of the ZPD to those resources, a plausible case can be made for the systematic description of the material conditions that identify a given tropical episode and indicate the possibility of the ZPD's operation in it.

It is not enough that classroom teachers know 'what works', they need to be able to explain why it works, and they need to be able to do so on the basis of the empirical evidence of natural classroom interaction and some or other theory of teaching and learning. The demonstrated feasibility of establishing an evidential base for objects of pedagogical knowledge makes a valuable contribution to our theorising and our practice. It creates the expectation that the use of such knowledge objects be expressed in terms both empirically grounded and theoretically coherent by providing a systematic and auditable means of doing so in a *shared discourse of practice*.

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IT IS WHAT YOU LEARN *AFTER* YOU HAVE SOLVED THE PROBLEM THAT REALLY COUNTS

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Despite a great deal of research on how writing can be used in mathematics classes and its benefits for mathematics teaching and learning (e.g., Cross, 2009), writing plays a minimal role, if any, in secondary and tertiary mathematics education (Bergqvist & Österholm, 2012). Pupils who are not from early on confronted with writing learning environments, tend not be open to the idea of writing in mathematics and very often view writing only as a part of language and social studies classes (Cross, 2009). Consequently, it is important to create a more positive writing climate at school; educating preservice teachers for so doing is a tool to achieving this goal. This research project aimed at exploring the experiences of preservice teachers as they engaged in mathematics problem solving and prepared different types of written reports, namely booklets comprising of a problem solving writing protocol and a post reflection protocol about problem solving, and reflection papers on their learning processes. Moreover, I wanted to see how these experiences shape their attitudes towards incorporating writing into their mathematics lessons.

The sample comprised a cohort of 24 mathematics preservice teachers. The intervention took place throughout the semester in a problem solving course. Every 3-4 weeks the preservice teachers received a set of problems to solve and were asked to keep a writing booklet. In addition, they had to keep a journal designed to encourage them to relate what they were learning in class in general and with respect to writing to their own practice or experience. Data analysis involved identifying pertinent passages from collected data about relevant ideas using techniques from both content analysis and grounded theory through which categories and its subcategories were constructed. The results showed that preservice teachers' responses differed based on their beliefs about mathematics and writing in mathematics itself, type of writing they were engaged in, and their target audience. Moreover, I was able to observe a shift in their beliefs and attitudes towards writing and its incorporation in mathematics lessons as the semester progressed. These and further results will be discussed in the presentation in detail.

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EXTRACTING A DECISION-MAKING FRAMEWORK FROM CONSTRUCTIVIST TEACHING

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A constructivist theory of knowledge (von Glasersfeld, 1995) implies two radical shifts for the mathematics teacher. First, instead of the educational goal to focus on the transmission of knowledge, the teacher must provide opportunity for invention of knowledge that may be deemed mathematical. This process of knowledge invention in the learner is heavily dependent on social interaction. Second, the teacher would not attend to the arrival upon a singular truth nor the accurate or efficient mimicry of one's own knowing. Instead, the teacher must engage in an effort to create (or recognize) models for the mathematical ways of knowing she attributes to her students, and recognize these models are fabrications of her own observation.

The research to be presented emerged through my collaboration with Daniel, a high school mathematics teacher. He and I shared an interest in student's mathematical identity; in particular, an identity in which the student owned the authority for mathematical knowledge (Gutiérrez, 2013; Schoenfeld, 1989). This study utilized a grounded theory approach to characterize the decision-making of the constructivist mathematics teacher. The study was driven by two curiosities: What decisions are constructivist mathematics teachers who attend to identity and authority making when enacting mathematical activities? What *considerations* impact those decisions? The primary sources for data were ongoing conversation via multiple modalities, and observed or co-taught classroom experiences.

Although certainly an incomplete framework, this study yielded several focal points for decision-making. Daniel had great interest to refine the presentation of mathematical problems. Closely related were a concern for the manner in which initial information was provided, and how students were suggested to begin exploration. A second decision type was to what extent the divergence of student-defined mathematical exploration productive in the classroom environment. The last, and highly unresolved concern, was what to do when an effort toward collective resolutions or ways of knowing proved illusive. Examples from Daniel's teaching activity will be used to illustrate these decision-making frameworks.

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AUSTRALIA, CANADA, AND UK: VIEWS ON THE IMPORTANCE AND GENDER STEREOTYPING OF MATHEMATICS

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Data from successive PISA testings (involving 15-year-old students) reveal that the mathematical literacy achievement of males is consistently higher than that of females in Australia, Canada, and the UK – countries in which English is the dominant spoken language. Models put forward to explain gender differences in mathematics learning outcomes commonly include society at large, that is the general public, and significant others, including parents. Our aims were to examine, by country, the views/beliefs of members of the general public and of students about aspects of mathematics.

The adults comprised pedestrians approached in the street and Facebook users who responded to an on-line survey about the gender stereotyping of mathematics (traditionally considered a male domain) and the importance of studying mathematics for future jobs and careers. The Australian, Canadian and UK adult samples comprised respectively 615, 105, and 58 pedestrians and 119, 35, and 58 Facebook respondents. The relevant survey items were: Who are better at mathematics, girls or boys? Is studying mathematics important for getting a job? Is it more important for girls or boys to study mathematics?

The student data came from participants in PISA 2012 in the three countries and their responses to four attitude items: ST29Q05: Learning mathematics is worthwhile for me because it will improve my career prospects/chances; ST29Q08: I will learn many things in mathematics that will help me get a job; ST35Q04: My parents believe it's important for me to study mathematics; ST35Q05: My parents believe that mathematics is important for my career.

The responses of the adult and student samples in the three countries were remarkably consistent and led to a number of robust conclusions. Both students and members of the general public in the three countries deemed mathematics to be an important subject and indicated that they thought studying mathematics to be important for getting a job. Among those who held stereotyped views, studying mathematics was considered to be more important for boys than girls. Many in the different samples considered boys and girls equally capable of doing mathematics and thought studying mathematics to be important for both boys and girls, but among those who held stereotyped views, boys were more frequently considered to be better at mathematics.

Despite long standing concerns about gender differences in mathematics performance in Australia, Canada, and the UK, and the implementation of intervention programs aimed at achieving equity, perceptions of gender role differentiation, we found, persist. Not only did a sizeable group among the general public, but also substantial numbers of students, still subscribe to traditional views about differences in the suitability of mathematics for boys and girls.

A STUDY ON MATHEMATICAL KNOWLEDGE FOR TEACHING COMPLEX NUMBERS

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Students learn only about a formal manipulation of complex numbers, instead of understanding conceptual aspects of complex numbers (Sfard, 1994). Prior study can be classified into two parts: one is to explain students' difficulty due to formal nature of complex numbers; the other is to enhance and complement curriculum of complex numbers. A teacher plays an important role in implementation of curriculum for the students. However, there lacks enough research that focuses on teachers that teach the concept of complex number compared to the number of researches that deals with curriculum and students. This research aims to find mathematical knowledge that is essential for teachers teaching complex numbers, and also elaborate the meaning of applying the findings to teaching complex numbers.

This research started with realizing the important role of teachers in several problems with a teaching and learning complex numbers. Based on Ball et al (2008), the mathematical knowledge a teacher should possess is renamed as a Mathematical Knowledge for Teaching complex numbers (MKTc). In order to study MKTc, a review of researches about general Mathematical Knowledge for Teaching was conducted. It was found that MKTc can be deduced not only from didactical analysis which consists of historico-genetic analysis, mathematical analysis, curricular analysis of the concept of complex number but also from analysis of work of teaching complex numbers. Based on the MKTc, a test to review a prospective teacher's knowledge needed to teaching complex numbers was designed and carried out. As a result, instability of preservice teachers' MKTc was noticed. For example, most pre-service teachers said that square root of i is not a complex number as square root of -1 is not a real number. In the presentation, detailed contents of MKTc will be discussed.

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SUCCESSES AND CHALLENGES OF SECONDARY PRESERVICE MATHEMATICS TEACHERS IMPLEMENTING PROJECT-BASED LEARNING

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One inquiry-based instructional approach that has been increasingly popular is project-based learning (PBL). Unlike units in which a project is used as a culminating experience, project-based learning poses a realistic situation at the beginning of a unit and uses the need to create a deliverable product to drive the content through an extended inquiry process. Research is scant on teacher-designed PBL units, and even more on preservice teachers and the supports they need. Because of the complexities in designing and implementing math PBL units, the question that motivates this study is: What are the successes and challenges preservice teacher (PSTs) encounter as they design and implement math PBL units? A set criteria used to evaluate the rigor and relevance of PBL units is the Six A's: Authenticity, Academic Rigor, Applied Learning, Active Exploration, Adult Connections, and Assessment Practices (Markham, Larmer, & Ravitz, 2003). This framework provided a lens for examining the math units and how they were implemented.

The study involved ten mathematics PSTs who were in an accelerated teacher credentialing program, earning a Masters of Arts in Teaching and a teaching licensure in 12 months. Data sources included pre- and post-written reflections, student-generated artifacts, and unit plans. This study employed grounded theory techniques, developed by Glaser & Strauss (1967), to analyze data. Examining multiple sources helped generate informed questions and contributed to investigate how teachers engaged in the design, implementation, and adaptation of the unit.

There were similarities between the ten PSTs. All had difficulty with having students actively exploring (one of the Six A's). All PSTs found difficulty in supporting students' meaningful engagement with content while balancing the context of the unit (Authenticity). They also struggled to support students in demonstrating their understanding of underlying mathematics with which they were engaging (Academic Rigor). Using the Six A's framework to analyze outcomes, additional findings will be shared.

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NURTURING DEEPER MATHEMATICAL UNDERSTANDING THROUGH VIRTUAL MANIPULATIVES EXPLORATION

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A study of middle school students was conducted while they explored Virtual Manipulatives (VMs), which are expected to present "opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002, p.373). The goal of this study is to compare the differing levels of learning opportunities using VMs in two contexts.

Sixth-grade students (N=152) explored a VM called Peg Puzzle from the National Library of Virtual Manipulatives. In the first context, students explored Peg Puzzle and completed a survey asking about their learning process and satisfaction levels. Two pairs of students were randomly selected and their discussions were observed and recorded during the exploration. In the second context, a brief semi-structured discussion on strategy development occurred prior to other procedures used in the first context.

In the first context, about 44% of the students reported that they practiced or learned mathematics while exploring the VM. Many students perceived that a VM presented as math game, such as the Peg Puzzle, did not increase their learning of mathematics. Observers noted that students mainly relied on a trial-and-error strategy. In the second context, about 77% of students replied that they learned mathematics. The recorded discussion revealed a decreased dependency on the random trial-and-error strategy and an increased level of math-specific content discussion (e.g., predict results, test prediction through pattern recognition, use of algebraic expressions) compared to those in the first context.

These comparative results indicate that simply using VMs does not guarantee meaningful learning. For students to gain the most benefits from using VMs, teachers must design quality tasks that incorporate the demand for mathematical challenge (Jaworski, 1994) and uncertainty (Zaslavsky, 2005). Detailed results of surveys, observations, and recorded pair discussions will be shared and further discussed in the presentation.

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PRE-SERVICE TEACHERS' ABILITIES TO ATTEND TO STUDENTS' MATHEMATICAL THINKING IN THE MOMENT OF TEACHING IN EARLY FIELD EXPERIENCE

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This study investigated how six pre-service teachers attended to students' mathematical thinking while teaching. Findings suggest that pre-service teachers are weak in their abilities to use students' thinking in the moment of teaching and frame questions to probe young children's mathematical reasoning.

When teachers have in-depth understanding of students' mathematical thinking, they can plan lessons that address students' needs (Jacobs, Lamb, & Philipp, 2010). Also, effective interaction between teachers and students boost students' understanding (Chin, 2006). By employing interaction patterns and attention to students' thinking (Chin, 2006; Jacobs et al., 2010) as dual theoretical lenses, this study focused on how six PSTs attended to students' mathematical thinking through their interactions with students in a preK-2 grade mathematics field experience classroom. The research questions guiding this study are: (1) What are the forms of feedback provided by PSTs in the follow-up move of the initiation – response – follow-up format in the moment of teaching? (2) How do elementary PSTs attend to students' mathematical thinking in the act of teaching? To address the research questions, six junior elementary PSTs' math lesson videos were analysed by using a grounded theory methodology.

Findings suggested that the six PSTs demonstrated six types of the feedback move: (1) affirming or directly correcting the student's answer; (2) affirming the answer and asking a follow-up question; (3) attempting to ask a follow-up question but ending up with affirmation; (4) asking a follow-up question but not giving enough time for the student's complete explanation; (5) asking a follow-up question but funnelling rather than focusing on the student's thinking; and (6) asking a follow-up question and reflecting the student's thinking in the flow of the lesson. In addition, the PSTs in this study displayed weakness in using students' thinking in the teaching moment and framing questions to probe young children's mathematical reasoning. Implications are discussed how to develop the expertise of attending to students' mathematical thinking in teacher education program.

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EXPLORING THE DIFFERENCE BETWEEN STUDENT TEACHERS' AND EXPERIENCED TEACHERS' MATHEMATICAL CREATIVITY

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Mathematical creativity contributes to high-level mathematical thinking (Haylock, 1987; Kattou & Christou, 2013; NCTM, 2000). It is important for teachers to enhance their students' mathematical creativity in today's classrooms (Levenson, 2013). Little, however, was done to develop teachers' mathematical creativity, and even less was known about the difference, if any, between student teachers' and experienced teachers' mathematical creativity. Such information is important because it could be of assistance in developing teachers' mathematical creativity.

The purpose of this study was to explore the difference, if any, between elementary student teachers' and experienced teachers' mathematical creativity on a mathematics task. The participants of the study comprised eight elementary student teachers and eight elementary experienced teachers. They were asked to identify mathematical concepts that could be developed using geoboards in the study. Based on Kattou and Christou (2013), fluency, flexibility and originality were used to determine the participants' mathematical creativity on the task, where fluency was determined by the number of appropriate mathematical concepts for geoboards, flexibility was determined by the number of different mathematical concepts, and originality was determined by the number of mathematical concepts that were rarely developed using geoboards. The findings of the study indicated that the student teachers showed more mathematical creativity on the task than the experienced teachers. More research needs to be undertaken that investigates the causes of the difference in mathematical creativity on the task between the student teachers and experienced teachers.

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MATHEMATICAL EDUCATION OF FUTURE ENGINEERS: EXPLORING PROBLEM SOLVING SKILLS

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Engineering students have to master physics tasks that involve many mathematical challenges. Besides declarative and procedural knowledge they need problem solving skills to be successful (SEFI, 2013). For investigating students' problem solving skills it is possible to focus on the *inner structure* considering heuristics and beliefs, or on the *outer structure* considering timing and organization of processes. We focus on heuristics and timing by drawing on Polya's (1945) phases of the problem solving process, and Bruder and Collet's (2011) work on heuristics which places the methods in the center and explains the process of problem solving by heuristic tools and heuristic strategies. The KoM@ING project aims at investigating competencies of engineering students. By quantitative studies, our project partners have gained IRT-scaled test on mathematical and physical-technical skills. In our qualitative study we investigate students' difficulties when working on these tasks. Our research question is: Is it possible to explore and understand the barriers of the mathematics and physics tasks that students encounter in terms of their problem solving skills?

Participants in the video study were 37 engineering students. They worked in small groups on the five easiest and most difficult tasks from the IRT-test. Data was gathered by applying the *thinking aloud* method and analyzed by using the categories derived from the theoretical frame. Results show that Polya's phases can be found in all students' work, but that they are differently relevant, as only 34% of all phases could be observed in the easy tasks. When solving the difficult tasks students usually pass through all phases. All students frequently used heuristics tools, but only up to 22% did so when working on the easy tasks. The same applies for the use of heuristic strategies which occurred in up to 30% when students worked on those tasks. While *working forward* is the main strategy used in in the easy tasks, many different strategies, like *conclusion of analogy or principle of decomposition*, were found when students solved the difficult tasks. While the incomplete use of Polya's phases and the restricted choice of heuristics have no negative effect on success in the easy tasks, the use of different strategies and passing through all phases increases the success in difficult tasks.

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REASONING USING GEOMETRICAL CONSTRUCTION

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A geometrical proposition generally exhibits an inseparable connection between geometrical figures and proving. The main function of those geometrical figures is to determine the key ideas of proofs for solving problems (Duval, 1995). Duval (1998) suggested that geometry learning involves three types of cognitive processes: visualization, construction, and reasoning. These three processes fulfil different functions and support each other in learning geometry. However, visualization may not always facilitate reasoning in geometrical activity. In general, the idea of proving problems mainly focuses on the properties between the elements presented in the geometrical figure rather than the construction of the figure. Hence, some geometry proofs that can be either proved by geometrical construction or geometrical properties were designed in this study. Student learning outcomes after these two proving instructions are mainly analyzed for investigating the relationship between construction and reasoning.

Two 45-min lessons were used for this experimental instruction, one for administering the pretest and another for instruction and a posttest. Students in the experimental group were taught to learn the proving process from the idea of geometrical construction; students in the control group were conducted to use the properties from the given figure. In this pilot study, quasi-experimental research design was used to investigate the learning effects on reasoning of 48 ninth-grade students. The students were divided into low and high previous knowledge groups according to the pretest. The findings indicated that students with low previous knowledge in the experimental group did not benefit from the idea of geometrical construction. A different demonstration from the traditional proving problems created an obstacle for and imposed a heavy cognitive load on them. However, students with high previous knowledge in both groups can understand the proving idea. The process of the construction steps can be considered as an effective tool on reasoning when students understand the figural concepts. However, the mechanism for visualization from geometrical construction that triggers the key elements to facilitate reasoning required further discussion.

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CURRICULUM FOR LOW-ACHIEVING MATHEMATICS STUDENTS IN SINGAPORE

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Curriculum design for low-achieving secondary mathematics students is “hard” because the typical profile of these students reveals multi-faceted areas that need attention: Mathematically, they lack the requisite content resources for a number of secondary mathematics topics (Mercer & Mercer, 2005); affectively, many of them display low levels of interest which affects motivation to learn mathematics (Karsenty, 2004). We present here a crystallisation of our earlier studies (e.g., Leong et al., 2013) with lower-achieving classes in a Singapore secondary school over a 4-year period. In particular, we propose critical elements in curriculum design for these students.

We propose five elements—namely, (1) Motivation for learning; (2) Study Habits; (3) Problem Solving Disposition; (4) Skill fluency and (5) Disciplinarity. Elements (1) and (2) directly address the common deficiencies in the academic portrait of low achievers: Lack of interest in learning, easily distracted, short span of concentration, and poor note-taking skills. In addition, we think that low-achieving students should develop a different disposition when confronting difficulties in mathematics learning: Make good informal attempts at problems through the use of suitable heuristics—such as drawing a diagram or substituting numbers. We define this approach towards mathematics problems as Problem Solving Disposition—listed as (3) above. Element (4) is the most popular conception of doing mathematics—as consisting of remembering rules and applying them correctly to standard questions. We agree that mastering a set of mathematical content and skills is an important part of learning mathematics, but it should be complemented by an emphasis on (5) Disciplinarity. Upholding the disciplinarity of mathematics in teaching involves a careful weaving of activities that will enable the students to experience mathematics as more than arbitrary rules—to appreciate pattern-observation, relating concepts, and sense-making.

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ASSESSING AND SUPPORTING DIAGNOSTIC SKILLS IN PRE-SERVICE MATHEMATICS TEACHER EDUCATION

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Diagnostic activities are considered to be pivotal to effective teaching. The teachers' knowledge, skills and beliefs connected to diagnostic activities can be summarized as *diagnostic competences* and are often defined as the ability of teachers to accurately assess students' performance. They can be measured by evaluating teachers' answers to tasks that resemble diagnostic situations, e.g. the analysis of students' written solutions (Hill, Schilling, & Ball, 2004). In our study we capture a certain component of diagnostic competence by focussing on pre-service teachers' understanding of students' solutions without time pressure. To evaluate the role of content knowledge in this diagnostic process we examine whether preceding own solution efforts have a positive influence on the diagnostic competence.

We presented primary students' solutions for open ended tasks to the teachers and asked them to analyze these solutions by phrasing up to five short written statements. Using a system of inductively developed categories we found two different types of diagnostic judgments: "specific diagnosis" (e.g. "did not calculate +9 but +10-1") and "unspecific diagnosis" (e.g. "worked systematically"). We measured the *richness* of the diagnostic judgments as the sum of statements fitting in the two types.

The teachers' diagnostic judgments could be categorized in 23 categories with an average inter-rater reliability of κ=0.76. Our hypothesis was that the richness of diagnostic judgments would increase after own solution efforts by the teachers. The randomized assignment to experimental and control group (N=53 and N=57) showed no statistically significant differences with respect to distributions of age, semester, mathematical competence, and teacher training program. Our hy-pothesis could be confirmed by an analysis of variance ($F=5,1$; $p<0,001$) with an effect size of $\eta^2=0.12$: Own solution efforts increase the richness of diagnostic judgments. Further analysis showed that the quality of the judgements seemed to rely mostly on noticing *specific* features of students' solutions. This effect of increased specificity of diagnostic judgements also occurred in a similar study of Morris et al. (2009) who elicited the "unpacking of learning goals" from pre-service teachers.

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THE MATHEMATICAL BELIEFS AND INTEREST DEVELOPMENT OF PRE-SERVICE PRIMARY TEACHERS

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Interest in mathematics as a motivational person-object relationship has proven to be a good predictor of intense and effective learning (Köller, Baumert & Schnabel, 2001). We investigate the interest development of pre-service primary teachers in their first academic year. Since the object of interest depends on their personal perception, we sketch the students' views on mathematics using beliefs on the nature of mathematics (cf. Rolka, Rösken & Liljedahl, 2006) which we expect to affect their future interest. We focus on dynamic aspects (process and utility) as well as a system aspect. At school, the first two aspects (but not system) are known to be positively correlated with interest. Our main research question is whether beliefs help predicting future interest in addition to present interest.

The data were collected with paper-and-pencil questionnaires in the first (T1) and last lectures (T2) of a first semester course and at the end of a second semester course (T3) held at Kassel University, Germany. The sample consisted of $N=62$ students who participated in all three surveys. To measure interest and beliefs, we used well-tested Likert scales with four to seven items. Reliabilities (α) ranged from .63 to .81.

There was a considerable decline in interest and process as well as utility beliefs during the first semester, followed by a slight recovery in the second semester. Correlations of dynamic beliefs and interest were around .50 ($p<.001$), as expected. The system aspect neither showed significant changes nor correlations with interest. For the main question, we first took interest at T1 (or T2) as independent variables to predict interest values at T2 (T3) using linear regression. Including beliefs as predictors increased the explained variance (R^2) in interest from .44 to .47 for T2 and from .43 to .50 for T3. Surprisingly, system beliefs for T3 were the only additional significant predictor explaining the whole additional 7% of the variance in future interest.

We conclude that system aspects should be included in future research but also interventions which today mainly focus on dynamic beliefs (e.g. Rolka et al. 2006). More detailed results will be given in the presentation.

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THE PERCEIVED EFFICACY OF THE COURSE FOR ADVANCING PROSPECTIVE TEACHERS' UNDERSTANDING OF STUDENTS' MATHEMATICS UNDERSTANDING

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To configure the knowledge that teachers may need, Darling-Hammond (2006) pointed out that future teachers are required to understand how students learn and what various students need, and to develop their productive disposition towards seeking answers to difficult problems of teaching and learning and their ability to learn *from* practice as well as to learn *for* practice. Therefore, "Psychology of Mathematical Learning" is the first course for our prospective teachers to learn mathematics teaching. When learning to tackle the pedagogical problems and willing to consider the various learning needs of classroom students, prospective teachers are required to incorporate an appreciation for student-centred teaching into their teaching. Without the ability to understand how students learn mathematics, and how different students learn mathematics differently, teachers lack the foundation that can help them figure out how to interact with students when students fail to understand and solve a given mathematics task.

A learning cycle was conceptualized for designing this course emphasizing on prospective teachers' knowing-to. This cycle began with making sense of student understanding. In the second stage, the prospective teachers divide into small groups by themselves; each group needs to design their own research question as well as tool, and then conduct a survey or interview with students. In the final stage, these prospective teachers need to compare their research results with literature for validation and then come up with their claims. It's noticeable that these claims might be somehow biased because the sample data are all in small size.

In this study, we examined these prospective teachers' perceptions of their engagement, their attitudes towards the course, and their perceived impact of the course on their understanding of students' mathematics understanding. Results indicated that most of the prospective teachers perceived that this course can support their learning to understand students' mathematics understanding. Nonetheless, few prospective teachers showed that they did not like learning by doing and preferred listening to teacher educators' lecture. The function of doing in the course is discussed.

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USING THEORETICAL PRINCIPLES IN THE EVALUATION OF TEACHERS' TASK DESIGN FOR CONJECTURING

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Conjecturing is the backbone of mathematics learning (Mason, Burton, & Stacey, 1982). Lin, Yang, Lee, Tabach, and Stylianides (2012) proposed four theoretical principles and suggested they can function as intermediate framework to scaffold teachers in designing conjecturing tasks. According to Lin et al., *observation* principle highlights the opportunity for students to engage in discerning examples purposefully and systematically in order to understand and make a generalization about examples. *Construction* principle concerns the opportunity for students to construct new mathematical knowledge based on their prior knowledge. *Transformation* principle points to the importance of making conjectures by transforming given statements, formulae, algorithms, principles and so on. Regarding *reflection* principle, it emphasizes the chance for students to further explore mathematics problems and improve their conjectures.

In this study we take a further step in examining how the four theoretical principles can be practically used to evaluate conjecturing tasks designed by teachers. The data analysed here was from two semester-long professional development programs with a group of experienced mathematics teachers. In the programs, participating teachers were required to design instructional tasks through which they have opportunities to intensely explore curriculum materials and student learning as well as to incorporate professional development materials into their designs. We particularly selected a sequence of conjecturing tasks designed by a mathematics teacher and examined the extent to which the tasks can afford opportunities for students to experience the conjecturing. The main finding is that good conjecturing tasks should not only facilitate students engaging in observation, construction, transformation, and reflection individually. The most importance is to scaffold students in actively transiting among the four-principle-oriented tasks so that they can experience the essence of conjecturing and develop model for learning mathematics. Additionally, the analysis also reveals the importance of constructing examples by students; which work has potential to lead students actively transit among the four-principle-oriented tasks.

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STUDENTS’ MATHEMATICAL ARGUMENTATIONS IN FOURTH-GRADE CLASSROOM

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Researchers draw attentions on reasoning, proving, and conjecturing. Existing empirical studies suggest students should have early opportunities to incorporate conjecturing and proving into mathematical learning. Thus, this study was designed to identify what mathematical argumentation looks like and how it is developed in a fourth-grade classroom where the teacher designed and implemented a task for conjecturing with respect to quadrilaterals cut diagonally. Argumentation is seen as a social resource (Schwarz, 2009). This study takes Toulmin’s (1958) scheme and adapted from Knipping’s (2008) structure as an approach of analysing argumentation in classroom. The scheme consists of data, warrant, backing, and conclusion.

Wen-Wen, one of the six teachers, participated in the study. Wen- Wen thought a task in a textbook deprived of her 24 students the opportunity with exploring the relationship of two triangles where any quadrilaterals were cut diagonally. Hence, she redesigned and implemented a task into classroom. The mathematical argumentation of fourth-graders was developed through five stages of conjecturing: constructing cases, formulating a conjecture, examining the correctness of the conjecture, validating the truth of the conjecture, generalizing the conjecture, and justifying the generalization. This study contributed to differentiating the meaning of examination, validation, and justification involved in the argumentation. Fourth graders preferred to use empirical operation as a warrant for supporting their conjectures. Deductive reasoning was used by fourth graders, too. For instance, several students stated that “*square can be divided into two congruent isosceles right triangles, because when you cut it diagonally, a right angle and two equal sides of the square were left for the new triangles*”. Verifying a conjecture by deduction is more valid and more powerful than by measurement. However, the issue is how to design a task that are likely to give rise to conjecturing, and then to lead to mathematical argumentation.

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STUDENTS' JUDGMENT OF CONVINCING MATHEMATICAL ARGUMENTS: A BETWEEN-GRADE COMPARISON

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In current curriculum, systematic presentation of mathematical proof is often embedded in a high school geometry course with the intention that such interventions will help learners recognize proof as a fundamental mathematical reasoning method. However, this goal remains unfulfilled (Harel & Sowder, 2007; Hersh, 2009). In order to seek ways to improve students' mathematical reasoning capacities, it is important to first understand their comprehension of mathematical arguments and identify existing misconceptions.

To serve such a purpose, a Survey of Mathematical Reasoning was developed to evaluate students' understanding and judgment of different types of mathematical arguments. The survey contains four mathematical conjectures, each supported by a variety of arguments, including empirical and algebraic justifications (Harel & Sowder, 2007). Participants of the study were asked to determine if each argument successfully proved why the corresponding conjecture is true by providing a "yes," "no" or "not sure" answer to each item. Middle and high school students in a Midwestern state took the survey in the spring of 2013. This report focused on the comparison between 7th graders, who had been just introduced to the use of symbolic notations in mathematics, and 10th graders, who had studied proof in a geometry course. Each of the two groups consisted of over 300 students from multiple schools.

In analyzing the data, particular attention was paid to 1) Whether students could recognize that experimenting with a few examples is insufficient to prove a mathematical statement true for general cases; 2) Whether students could recognize that symbolic representation has the capacity to demonstrate the general validity of a mathematical statement; and 3) Whether 10th graders, compared to 7th graders, demonstrated a more mature understanding of what argument is mathematically valid. Quantitative results of the study suggested insignificant differences between the 7th and 10th graders in their responses to most items in the survey, indicating minimum understanding of proof even after their exposure to proving techniques. Content of the survey instrument and data details will be shared in the presentation.

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TEACHING THROUGH CULTURALLY-BASED MATHEMATICS

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This presentation reports on the learning experiences of teachers who have engaged in culturally-based inquiry units to connect Indigenous knowledge to school mathematics. We will share teachers' responses to the experiences of working with elders in their classrooms to help students learn mathematics through cultural tasks.

The marginalization of Mi'kmaw youth from mathematics has been a long-standing concern in Mi'kmaw communities and this concern extends to many Aboriginal communities across Canada. The Minister's national working group on education (Indian and Northern Affairs Canada, 2002) stated that a key area to be addressed in Aboriginal education in Canada is the development of culturally relevant curricula and resources in areas of mathematics and science.

The purpose of this study is to examine the implementation of culturally-based mathematics inquiry units in Mi'kmaw schools. Mi'kmaw Kina'matnewey (MK) stands out as an example of a Regional Aboriginal Education Authority that is experiencing success. MK schools are striving to meet curriculum expectations while maintaining a strong sense of Mi'kmaw cultural identity. For this study, inquiry units were developed drawing from conversations with elders and from having elders work alongside teachers in classrooms to teach cultural tasks. Thus the children learned mathematics through learning about cultural practices. Such practices have proven to be effective in other Indigenous contexts. (Kisker et al., 2012) Yet such inquiry practices are a shift for teachers in many MK schools. For this part of the study, we interviewed teachers who have participated in implementing these inquiry units.

In our presentation, we will share stories of teacher learning through engagement with elders in the classroom. We will report on how teachers are describing the value of having elders work with them in the classroom and how these experiences shape teacher knowledge with respect to culturally-based mathematics teaching.

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PROBLEM POSING IN PRESERVICE PRIMARY SCHOOL TEACHERS' TRAINING

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We consider that for an adequate teaching practice in primary schools it is crucial that preservice primary school teachers (PPST) experience learning mathematics in active ways, beyond the mechanical and senseless use of algorithms. In this sense, and according to Bonotto (2013) and Tichá & Hošpesová (2013), problem posing is a motivating and participative means to encourage learning, deepen knowledge and developing mathematical thinking.

In this paper, we report about strategies for stimulating posing problem capacity in PPST. The aim was to contribute to develop their competencies, both mathematical and didactic. We refer to experiences with 30 PPST who were freshman at a Peruvian university. This is part of a wider and on-going research in which in-service primary school and high school teachers are included.

The methodology we have developed combines both problem-solving and problem-posing during the learning of a selected mathematical object. First, we design a problem and we ask to the students to solve it, first individually and then by groups. Then, we promote the sharing of the group solutions among the class and we use these experiences to fully develop the mathematical object. Next, in order to expand the learnings around this mathematical object and to encourage didactical reflections, we ask our students to pose a problem individually at first and then in groups. The problem posed by a group should be solved by themselves and then by other group. Finally, all groups' problems and solutions are discussed and examined by the whole class for feedback. One of the research experiences with PPST was related to divisibility: after coming to the result that when adding any two even numbers, an even number is obtained, we asked our students to pose contextualized problems whose solution demands the use of such result. Our observations and qualitative analysis indicate that the methodology described above contributes to PPST's significant learning as well as obtaining our research goals.

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BILINGUAL TEACHER NOTICING: ENGAGING BILINGUAL CHILDREN IN PROBLEM SOLVING

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Given the complexities of the elementary mathematics classroom, with many stimuli competing for a teacher's attention, what is the basis for the in-the-moment decisions a teacher makes while engaging children in problem solving? The construct of *teacher noticing*, defined as what teachers "attend" to in a classroom setting and how they "make sense" of the events that occur (Sherin, Jacobs, & Philipp, 2011), brings to light those in-the-moment decisions upon which teaching moves may be further examined. This study further uses the construct of *professional noticing of children's mathematical thinking* (Jacobs, Lamb & Philipp, 2010) in order to document and understand *bilingual teacher noticing*, the ways in which bilingual teachers identify, reason about and make decisions in the situations that occur when engaging bilingual children in problem solving. This study investigates what three bilingual teachers notice as they participate in a teacher study group to analyse and reflect on their experiences in weekly problem solving small groups.

This qualitative study documented what third grade bilingual teachers noticed when participating in a semester-long teacher study group. Teachers met weekly with their small groups of students to engage in problem solving tasks; small groups were videotaped. Teachers then met in the teacher study group to view videotape, analyse student work and discuss challenges and insights. The teacher study groups were videotaped, and then transcribed and coded for instances of *professional noticing of children's mathematical skills* of attending, interpreting and deciding how to respond.

All three noticing skills were documented from the study group, but there was a larger percentage of interpreting and attending as compared to deciding how to respond. The levels of interpreting and attending were often superficial, for example, solely focusing on incorrect or correct answers versus student reasoning. In addition to the rest of the results of this study, understanding these teachers' noticing can provide us with an idea of how to help improve how bilingual teachers engage bilingual students in problem solving.

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WHAT IS A MODEL IN THE CYBERNETIC WORLD?

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In general, a model is understood as a simplified representation of a real system. In the case of mathematical models, it is assumed that this representation can be done using mathematical concepts and symbols. However, according to the report of the Working Group 6 'Applications and Modelling' of the Eighth Congress of European Research in Mathematics Education (available at http://cerme8.metu.edu.tr/wgpapers/Reports/WG6_Report.pdf), there is a lack of consensus regarding what constitutes a model in Mathematical Modelling (MM).

We believe that in cybernetic world (understood as any environment produced with digital technologies) this issue gain new perspectives. In the reality of the cybernetic world the physical relations established in the mundane can either be experienced or totally ignored. Thus, in the MM of the cybernetic world there is not necessarily a reference to the empirical (as in physics), making room for imagined situations to pass from the virtual to the actual. Actualization is the process by which something moves from a situation of potentiality (virtual) to the actual state. According to Granger (1994), the reference is responsible for the relation between the actual fact and the virtual fact, and in the reality of the cybernetic world, this reference is based on the scientific and technological device being used, which supports the actualizations that occur in it (Bicudo & Rosa 2010). Thus, actualizations in the sphere of the cybernetic world emerge from the relation of the human being with the scientific apparatus, through commands in a computer programming language, for example.

Based on these ideas and guided by qualitative research methodology we run an experiment with undergraduate students in mathematics, in order to explore the relation between MM and the cybernetic world. Those students developed electronic games using the Scratch programming environment. We argue that models created in a computer-programming environment are singular, as the fact actualized by the model in the reality of the cybernetic world is completely determined by the model itself - the model defines the possibilities of the player's action. Also, those models can incorporate sounds and visual-aesthetic aspects into their structure, as well as aspects related to spoken language, making them *technological/mathematical models*. These findings can opens up new theoretical and practical possibilities for studies in MM in mathematics education.

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UNDERSTANDING 4TH GRADERS IDEAS OF THE INVERSE RELATION BETWEEN QUANTITIES

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Children understand the inverse relationship between quantities in division situations (see Correa, Nunes & Bryant, 1998), and understand the logical invariants (ordering and equivalence) of fractions in particular situations (see Nunes & Bryant, 2008). Thus, it is important to know how these aspects are related. This study aims to analyze how the understanding of the inverse relation between size and number of parts in division situations is related to the concept of fraction in quotient and part-whole interpretations. Three questions were addressed: (1) How do children understand the inverse relationship between size and number of parts in partitive and quotitive division? (2) How do children understand this inverse relation in part-whole and quotient interpretations of fractions? and (3) Is there a relationship between this understanding in partitive and quotitive division and fractions presented in part-whole and quotient interpretations?

A survey by questionnaire was applied to 42 fourth-graders, during the mathematics class. The questionnaire comprised 22 problems: 6 division problems (3 partitive division, 3 quotitive division); 8 part-whole fractions problems (4 ordering, 4 equivalence); 8 quotient fractions problems (4 ordering, 4 equivalence). Results indicate that in quotient fractions problems children understand better the inverse relation between quantities; and the ordering problems are more accessible to understand the inverse relation between numerator and denominator. They also suggest that quotitive division is easier than partitive division. A correlational analysis shows that children's performance in ordering and equivalence fraction problems in quotient interpretation are related to each other ($r<.31$, $p<.05$); and ordering problems in quotient and in part-whole interpretations are related ($r<.45$, $p<.05$). The division problems are related to parte-whole interpretation; and the success in partitive division problems is related to the success in quotitive division problems ($r<.53$, $p<.001$). Based on these results, it is hypothesized that the quotient and part-whole interpretation of fractions and of partitive and quotitive division contributes differently for the understanding of inverse relationship between quantities.

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PROFILES OF TEACHER PROFESSIONALLY SITUATED KNOWLEDGE IN GEOMETRY

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This paper presents results of the pilot study that created profiles of teachers' professionally situated knowledge in geometry by examining their pedagogical content knowledge (PCK) and technological pedagogical content knowledge (TPCK) (Hill, Ball, & Schilling, 2008; Mishra & Koehler, 2006) related to the area of polygons. The main research questions were: 1) What is the nature of teachers' profiles of professionally situated knowledge in geometry (e.g., PCK and TPCK); 2) What are the main components of this knowledge; and 3) How to create profiles of teachers' professionally situated knowledge in geometry?

There were 22 secondary mathematics teachers participating in our study. Four instruments were used: (1) Usiskin's standardized Van Hiele test (VHT); (2) adopted Matsumura and others' Instructional Quality Assessment rubrics (IQA); (3) a questionnaire designed to assess teachers' subject matter knowledge and related TPCK; (4) PCK instrument to measure teachers' professionally situated knowledge; and (5) a reflection to measure change in teachers' self-reported perceptions of the best ways to teach the area of polygons. The *instruments & all results will be shared during the presentation*. The quantitative analysis was conducted on (1)-(2), while (3)-(5) were analyzed qualitatively. The PCK instrument with a sample of nine hypothetical solutions (some correct and some with mathematical limitations) was administered and analyzed. Teacher profiles were developed after researchers completed analysis of the aforementioned instruments completed by participants.

The results of this pilot study identified main components of teachers' professionally situated knowledge as well as their misconceptions & preferred strategies. The profiles of teachers' professionally situated knowledge in geometry were created using radar diagrams with the following axes, teacher's: 1) geometric knowledge, 2) knowledge of student challenges and conceptions, 3) pedagogical knowledge of appropriate instructional strategies, 4) knowledge of proper use of manipulatives and technology, and 5) knowledge of potential mathematical extensions. Completed marker teacher profiles and remaining results will be shared during the presentation.

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DIRECTION STRATEGIES OF GERMAN 2ND-GRADERS USING DIFFERENT MAPS

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Abilities to orient oneself including using maps are regarded important for elementary school children. The competencies of spatial orientation are linked to mental activities and language (Li & Gleitman 2002). Although German textbooks (e.g. Wittmann & Müller 2013, p. 84) present simple maps to learn directions and describing paths there are different spatial reference systems which teachers are not always aware of so that misunderstandings could arise. There are three types of spatial reference systems: object-centred (intrinsic), viewer-centred (relative) or environment-centred (absolute). Only viewer-centred or object-centred reference frames are used with maps (Levinson 2003). The research questions are:

- What is the understanding of path descriptions regarding different kind of maps?
- How do German 2nd graders describe paths?

Within the research project two different kinds of maps were used: map of the school environment and a map of a Manhattan-like city with only horizontal and vertical roads.

The sample comprises 200 German 2nd graders, who had to answer a paper – and – pencil test. Depending on their results 45 children were additionally individually interviewed.

The first analysis of the given answers seems to support the following results: With a viewer-centred reference system it is easier to start with a Manhattan-like map. Whereas with an object-centred reference frame it is easier to follow known directions in an environment map.

In the short oral presentation, the results will be additionally discussed for the learning perspectives of elementary school children and the teaching perspectives as well.

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PLATFORM 3: WHERE FOREGROUND MEETS BACKGROUND

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We always looked with admiration at those stairs at that station leading to platform no. 3, beaming with groups of boys sitting on the steps, studying, and asked ourselves “why do they study at this place?” Located on one side of a crowded suburban railway station in Mumbai, platform 3 remains vacant almost the whole day and serves to different groups of students who come here to study any time of the day, in the platform or in the stairs' shade. Our interest was to understand why those boys were studying at such a non-typical place and in what ways do their foregrounds guide their decision. Based on a pilot interview that the authors made to gather elements aiming at our research study, the second author visited the place again a few months later and had a focus group discussion with one such group.

We met teenage boys studying and discussing in groups. Most of them come to Platform 3 for preparing for some competitive exams like those of engineering or business management entrances, school-end or University exams. We met 5 boys preparing for the ongoing Grade 12 exams. They all belonged to immigrant families from North India living in a densely populated low-income settlement close by. They spoke about their current studies as gateway to future welfare. Interestingly, their aspirations and foreground (Skovsmose, 2012) have a direct relation with their father’s profession. However, none of them mentioned their mothers or sisters as examples to follow, but males in their families. In the caste-based socially stratified Indian context, children's social location often guides and limits their foreground.

Adopting an eclectic exploratory approach, we argue that such impromptu places of collective learning, motivation is often guided by the prospects of inter and intra peer group learning and tends to redefine children's foregrounds who handle their financial disadvantaged conditions to come out of their background, empowering themselves and achieving autonomy (Freire, 1996). According to Skovsmose (2012), statistics suggests that background defines possibilities and frame foregrounds but it reveals only tendencies as background is fixed and foregrounds are widely open. We will analyse and present the data looking into the influence of platform 3 students' backgrounds on their foregrounds and the role that Mathematics plays in this process.

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IDENTIFYING OPPORTUNITIES FOR MATHEMATICS LEARNING IN PICTURE BOOKS USING A FRAMEWORK

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Recent research documents the benefits of using picture books to promote mathematics learning (Anderson, Anderson & Shapiro, 2005). Learning-supportive characteristics have been identified through expert review utilising an explicit framework (Van den Heuvel-Panhuizen and Elia, 2012). Depending on the type of picture book, mathematics can be presented in different ways and can produce different kinds of interactions between teacher and student. An exploratory descriptive study comprising three phases, initially investigated how teachers classified and evaluated picture books for mathematics learning using a new framework. Three questions guided the research:

1. What potential do picture books (of different types) have for facilitating the development of mathematical concepts in young children?
2. What are authors' intentions when mathematical concepts are incorporated into picture books?
3. How do picture books facilitate children's engagement with mathematical concepts?

Teachers (28 novice and experienced educators) completed 123 surveys of selected picture books containing perceived, explicit or embedded mathematical content. Each book was classified according to these three types and then evaluated on a five point Likert scale according to the seven categories of the framework including features such as mathematical meaning, content and reasoning.

The findings confirmed teachers' perceptions that there are qualitative differences between the three types of picture books. Experienced teachers were more consistent in their classification but were not necessarily adept at distinguishing mathematical characteristics utilising the framework. Moreover, teachers reported that the framework, rather than the classification of picture books into explicit, perceived and embedded types, aided their identification of mathematical features that would not have been evident without its use.

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THE INTENTIONAL INTERVENTION: INVOKING EARLIER UNDERSTANDINGS IN A CALCULUS CLASS

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This paper presents some initial findings from a project that is exploring the potential for a theoretical framework—folding back (Martin, 2008)—to be enacted as a pedagogical tool in the classroom. Here, we focus on Mort, a high school teacher, working with a calculus class on the concept of vectors in three-dimensional space and identify how he intentionally provides opportunities for the students to fold back.

The NCTM 'Learning Principle' (NCTM, 2000) states "students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (p. 20). However, it does not speak directly to specific pedagogical actions that might occasion such building. The conceptual construct of folding back offers a way to characterise how learners can build a connected understanding through returning to and 'thickening' earlier existing images for a concept. Our study introduces teachers to the idea of folding back and, through video, traces its use as a pedagogical strategy in their lessons. Mort explicitly invokes the prior knowledge that students have relating to lines in two dimensions, focusing on the key notion that a line is defined by its slope and any point on it. However, he does not merely remind students of this prior knowing but instead takes time to explore what will be problematic about this definition in the move to three dimensions (i.e., that slope and point will be insufficient to fully define a line). In doing this he is making explicit, and working with, what McGowan and Tall (2010) refer to as 'problematic met-befores'—prior knowings that are insufficient for a new context. Mort deliberately and intentionally takes students' existing understandings and offers a space to thicken these—to think more generally about the location and direction of a line, in a way that will help connect the subsequent use of vector equations to what they already know. What is powerful and important here is that an engagement with the theoretical framework of folding back encourages Mort to not merely review earlier ideas, but take specific met-befores and thicken these to promote understandings that are both more general in nature and conceptually connected.

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MEANINGS OF SINE AND COSINE AS EXPRESSED BY NON-COMPULSORY SECONDARY SCHOOL STUDENTS

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Trigonometry is conceptually rich and fuses arithmetic, geometry and algebra. Although other topics related to trigonometry have been thoroughly studied, not only is there little research about what makes trigonometry hard, but also about the intuitive ideas which students have of trigonometric concepts. Researchers have proposed different representations of the notions of sine and cosine (Brown, 2005). The goal of the study was to identify and characterize the meanings of "sine" and "cosine" and establish two interpretative frameworks: one to describe representations, using the rising complexity sequence of contents used; and another one to describe the understanding of sine and cosine concepts by students, using the structure of the responses, the interpretations made by the students and the history of trigonometry. The theoretical framework is based on Frege´s semiotic triangle based on reference, sign and sense. Concretely, this study belongs to the tradition that considers the notion of representation developed by Kaput (1987). Moreover, we assume that history is useful to determine the origin of some notions, to compare different representation systems, to discover basic problems, and to create frameworks.

The case study involved 74 students in the first year of non-compulsory secondary school, following a Science and Technology track. Data sources were two questionnaires presented as two different options. Both quantitative and qualitative methods were used to analyse the questionnaires. The method selected is the analysis of content.

The study revealed that students have a diversity of meanings for sine and cosine: angle, angle of a triangle, ratio between side of a triangle, and length. The participants in the study gave priority to the representations as length (51.22%). In addition, several cognitive obstacles were identified: wrong connection between the "line system" and the "ratio system"; non-enough understanding of the unit circle and, little knowledge about the notions of sine and cosine as a coordinate.

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HANDS-ON INEQUALITIES

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In this study students manipulated transparent squares of different colours to represent solution sets for systems of linear inequalities. Data shows significant improvement in performance and understanding of graphing of linear inequalities.

Preschool children primarily learn using manipulatives. According to Piaget's cognitive-developmental theory, children use manipulatives to learn about and understand their world (Berk, 2005, p. 20). An interesting study done on the development of formal operations in logical and moral judgment, merely 30% of adults fully attain conceptual thinking while 15% think only in tangible ways (Kuhn, Langer, Kohlberg & Haan, 1977). Results in another study show that pre-service math teachers, not only are able to understand Mathematics better using manipulatives, but their math anxiety levels decreases (Vinson, 2001).

In this study, students used manipulatives to represent the (x,y)-plane solutions of systems of linear inequalities. Each student had a set of transparent squares of different colours, which they used to attempt for possible "shadings" of their solutions. The pre-test showed that students were somewhat familiar with the systems of inequalities, however their overall performance was low. The worksheet that they worked on during this study included systems from two to four linear inequalities. Students graphed the boundary lines first, placed the transparent squares on the correct sides of the lines to represent the solutions, and then checked the inequalities for the covered points. This resulted with overlapping transparencies which represented the common solution in the changed or darkest colour (depending on the colours used). On the post-test students' average scores increased 29% on systems with two inequalities, and 31% on systems with four inequalities. Statistical analysis showed significant improvement in students' performance, as well as improvement on conceptual understanding. The additional survey showed that students enjoyed these manipulatives as a learning tool.

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ON STUDENTS' UNDERSTANDING OF PARTIAL DERIVATIVES AND TANGENT PLANES

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While understanding the role of partial derivatives and tangent planes in the description of the behaviour of functions of several variables is important both in advanced mathematics and in applications, there is hardly any published research about how students understand them and how their difficulties can be explained. Previous research (McGee, 2014) has shown that understanding representations for 3D slopes as the rate of change from one point to another and as a rate of change associated with movement on a plane appears to be necessary in order to understand concepts in differential multivariable calculus. Our investigation builds from this and other studies and applies APOS theory (Arnon et al, 2013) to study the questions: What are students' conceptions of partial derivatives and tangent planes? What are the main mental constructions involved in learning these concepts? A conjecture (called a genetic decomposition, GD) of the mental constructions (in terms of the constructs of APOS: actions, processes, objects, and schemas) that may be used to build an understanding of these topics was initially established. Activity sets, consistent with the conjectured constructions in the GD, were then used in pilot sections of the multivariable calculus course. Semi-structured interviews using 6 multi-task items were conducted and data was collected to investigate whether students showed evidence of the conjectured constructions in the GD. Students (18) were chosen from pilot and control sections (9 each), their only difference being, essentially, their use of the activity sets.

Results show that students in the pilot sections were more likely to show the mental constructions conjectured in the GD. Results also suggest ways to refine the GD and to improve the activity sets. In summary, the results show the different conceptions of partial derivatives and tangent planes for students in the control and pilot sections, thus answering the first research question and also providing an answer to the second one as support of the GD proposed and showing evidence that helps to refine it.

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ANALYSING OF THE ASSESSMENT TASK OF SCHOOL MATHEMATICS AT A PRIVATE UNIVERSITY: FOCUS ON FRESHMAN UNDERSTANDING OF THE LINEAR FUNCTIONS

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A system for educating elementary school teachers has been set up recently in many private universities in Japan. Improvement of pre-service mathematics teacher education in such private universities is a serious challenge. Many Japanese freshmen studying elementary education in private university have had little opportunity for studying fundamental school mathematics. Previous studies suggested that a variety of approaches could lead to positive learning outcomes for pre-service teachers (see, e.g. Rowland et al., 2001). A variety of approaches is necessary to improve pre-service mathematics teacher education in Japan, such as designing effective curricula of mathematics at the university level, supporting the study of mathematics, teacher training, and so on.

The purpose of this study is to investigate the private university freshman students' understanding of school mathematics and to explore various teaching approaches for them. For this purpose, freshmen in the private university (N = 139) were given a set of assessment tasks. One of the tasks involved determining characteristics of the graph of a quadratic function. The result was that most of the subjects had difficulties in describing the characteristics of the graph of the quadratic function using mathematical terms (Masuda, 2013). This paper will analyse the results of additional assessment tasks of both linear and quadratic functions, comparing the difficulties subjects encountered with each type of function.

The results showed two major difficulties in understanding the linear function as follows: (i) About 80 % of subjects could not understand the "ratio of change" of the linier function using tables; and (ii) Most of them could not identify a situation in daily life in which the relation of two quantities changing simultaneously was a linear function. It also revealed a common difficulty with both linear and quadratic functions: understanding functional relations using mathematical terms.

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INVESTIGATING INSTRUCTORS' VIEWS ON THE ROLE OF CALCULUS

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The calculus reform movement called for calculus to become "a pump that feeds more students into science and engineering, not a filter that cuts down the flow" (Renz, 1987, p. 267). During this time, there was a lot of discussion as to the goals and roles of calculus in the institution with Strang (1987) noting that instructors are often unaware of the purpose of calculus. A large corpus of research exists about the importance of instructors' beliefs and the relationships between their beliefs and instructional practices (Cross, 2009). Despite the importance of teacher beliefs and the large paradigm shift in calculus, teachers' beliefs regarding the calculus course have not been empirically studied.

As part of a larger study on successful calculus programs, twenty-four instructors from five bachelor's granting institutions were interviewed. The instructors were asked about their opinions on calculus as well as what they wanted their students to "get out" out of their course and assignments. A grounded theory approach was used to analyze the interviews for instructor beliefs on the role of calculus. The results indicated that instructors largely viewed calculus as a course for exposing students to mathematics. Only one instructor mentioned calculus as serving a gatekeeper role. The *pump* rather than *filter* emphasis may reflect the type of sample (instructors from successful calculus programs) or might capture a larger effect of the calculus reform movement. During the presentation, further analysis and additional roles of calculus will be discussed. These will include instructors' views on the role of calculus in mathematical development as well as content and mathematical practice goals.

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STUDENTS' NON STANDARD REASONING – A MEDIATOR FOR REFLECTION IN MATHEMATICS TEACHER EDUCATION

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In mathematics teacher education there are only few approaches using mistakes and non standard reasoning as learning opportunities (Tulis, 2013), while, from our prospective, one of the core aspects of teaching practice should concern teachers' knowledge and ability to give sense to students' production. Such knowledge would allow teachers to develop and implement ways to support students to build knowledge starting from their own reasoning, even when such reasoning differs from the ones expected by the teacher (Ribeiro *et al*., 2013). Aiming at accessing and developing such knowledge and ability, we have been developing tasks for teachers training that ask them first to solve problems and, afterwards, to give sense to students' productions answering such problems. These tasks were used, at the moment, both as a tool to observe and deepen prospective teachers' mathematical knowledge for teaching (MKT) (Ball *et al*., 2008) mobilized by this prompt, as well as to support, in our lectures, the development of prospective teachers' MKT. Using such tasks in our lectures we, as educators, live a transformative experience from participating into a co-learning community with our students (prospective primary teachers). Part of these lectures were video recorded and we will discuss a fragment of these video-recorded discussions (in a context using fractions) with an "*inquiry* in practice" perspective (Goodchild *et al*., 2013). In particular, we will discuss and reflect upon some prospective teachers' mathematical epiphanies and some of the mathematical issues arising from their unforeseen observations. Moreover we will discuss also some of our own believes (and knowledge build) arising from the later observation of the discussion and our interaction during it. Finally we will argue that this kind of work grounded on discussing non standard students' reasoning could represent core aspects for mathematics teachers education field, meant as co-learning community.

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INQUIRY ACTIVITIES IN MATHEMATICS TEACHER EDUCATION

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Dialogue is the form of communication between teacher and students, and among students, which aims at learning in landscapes of investigation (Alrø & Skovsmose, 2004). In this context, the participants are engaged in making discoveries and put many dialogic acts in action (getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging, and evaluating). This scenario can be brought into teacher education programs in order to facilitate the learning of being engaged in dialogue by prospective teachers. Understanding this process of learning is the main objective of the doctoral research I have been developing. Assuming an interpretative and pragmatic research approach (Denzin & Lincoln, 2003), I have set up a meeting between the concept of dialogue and some prospective teachers in a teaching practice course through dialogue activities, as for example the following inquiry activity: *what is the smallest number of sticks necessary to use in order to construct a square of squares?* In this situation, the prospective teachers could experience the dialogue among them. During the process, it was possible to identify many inquiry gestures: surprising, intending, thinking aloud, listening, advocating, visualising, perceiving, experimenting, challenging, and discovering (Milani & Skovsmose, in press).

After the inquiry activities, the prospective teachers, the supervisor teacher and I discussed what had happened in terms of kind of communication and results that we have achieved in the inquiry processes. In those moments of reflection, the dialogic acts and the inquiry gestures were stressed and the prospective teachers showed their reasoning on realizing this kind of activity in their teaching practice at schools. The development of inquiry activities showed itself as a possibility for the prospective teachers' learning of being engaged in dialogue.

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SOLUTIONS STRATEGIES SOLVING PROBLEMS REFERRING TO VELOCITY OF 4TH GRADERS

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Among the application problems at the end of elementary mathematics education we find problems relying on the concept of velocity (questions of movements where the notions of distance and time play the main role). These problems serve as an exploration of environment and are not taught systematically within the subject matter neither of mathematics nor of physics (Nachtigall, 1987). Mathematically this kind of problem means a mathematical anticipation of the secondary level since velocity is a compound magnitude of distance and time, more precisely a ratio.

It is of interest what students' cognitive processes arise with physical concepts used in math classes. Students on the elementary level can solve these problems using only arithmetic modelling (Henn, 2005).

Questions arise what kind of understanding 4th graders have concerning velocity and what enhancement these problems possess for them. Also, what is the teachers' understanding of physical concepts? Research questions are:

- What is the students' understanding of velocity problems?
- Which strategies do they use solving these problems?

The sample comprises 60 students of German 4th graders of various competency levels. They were given several velocity problems which they had to answer with paper and pencil.

The results show that the concept of velocity needs more detailed explanations in classes than teachers think they have the time to present. Also the representations of the solutions are more complex and require therefore more verbal explanations since the understanding of ratios is limited.

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FLUENCY: A PRODUCT OF CONNECTED REPRESENTATIONS

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Representations are indicators of the student thinking process, but they can also mask incomplete understanding. Student work may be a physical or pictorial manifestation that serves as a record of a solution process, or it may represent simply the end product of a solution. Gravemeijer (1999) calls this the distinction between models of thinking and models for thinking. Moreover, student work may show multiple representations that are superficial re-presentations of the same thinking, but shown in different modalities. Fluency can be defined as the use of multiple representations that engage different mental models (Lesh et al., 2003) and demonstrate meaningful connections, or translations between the representations (Cramer, 2003).

The goal of this study was to examine students' use of representations and evaluate fluency by making inferences about mental models. It is part of a larger study of two sixth grade classes as the teacher adopts a reform curriculum. Samples consisted of a set of six fraction comparisons, including directions to explain reasoning. Analysis revealed that student strategies based on equivalent fractions (common denominator) were 96% accurate, while area representations of a partitioned whole were 76% accurate. One might conclude that models that include area representations are less constructive, however the area models were not wholly inaccurate: they were just not precise enough for these comparisons. On the other hand, student work showing equivalence strategies often revealed overreliance on the strategy as rote process.

Students who engaged multiple representations showed evidence of translating between the representations. Students who utilized mental models that appeared to include a variety of interpretations of a fraction also were successful. While small, this study suggests that fluency be characterized by the strategic use of a variety of representations and by the ability to make translations between them. It also suggests that accuracy in a single representation is not an adequate demonstration of fluency.

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ACADEMIC LITERACY IN MATHEMATICS FOR ENGLISH LEARNERS

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This paper uses a sociocultural theoretical framework to provide an integrated view of academic literacy in mathematics for English Learners. Mathematics instruction for English Learners requires an expanded definition of academic literacy in mathematics as more than low-level language skills (i.e. vocabulary) or mathematical skills (i.e. arithmetic computation). A sociocultural approach to mathematical discourse (Moschkovich, 2007) assumes that mathematical discourse is more than vocabulary, discourse is embedded in mathematical practices, and meanings are situated in participation in mathematical practices. This view of academic literacy in mathematics includes three integrated components: mathematical proficiency (Kilpatrick, 2001), mathematical practices (Cobb et al. 2001), and mathematical discourse (Moschkovich, 2007). An analysis of a discussion in an 8th grade bilingual mathematics classroom illustrates how these three components are intertwined, how meanings are situated, and how participants use hybrid resources. The analysis is guided by these questions: 1) How are students participating in mathematical proficiency, mathematical practices, and mathematical discourse? 2) What resources (modes, sign systems, and registers) do students use to communicate mathematically? 3) How are situated meanings for words and phrases negotiated and grounded?

The analysis shows a) how the three components of academic literacy in mathematics appear together during student activity, b) how learners negotiate situated meanings for words and phrases, c) how these meanings are grounded in the classroom sociocultural context and coordinated with different perspectives of inscriptions, and c) how learners draw on multiple resources—modes of communication, symbol systems, as well as both everyday and academic registers.

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EVOLUTION OF AN ALGEBRA CURRICULUM

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Teachers typically use curriculum to guide instruction and shape the nature of activities. In this study, a curriculum was developed to support student learning of sixth-grade algebra in an urban elementary school in a western state. The goal was to document the learning trajectory (Simon, 1995) that emerged with regard to how students learn algebra. This report documents how the curriculum was shaped to support student learning and lessons learned from this process.

A teaching experiment using design research (Cobb, 2000) was conducted over four weeks. The curriculum was modified daily based on teacher interviews, analysis of video recordings of teaching episodes, and student work. The researcher collaborated with the classroom teacher who taught the researcher-designed curriculum and consulted with the research team before, during, and after the process of refinement of the instructional unit. The rationale for making modifications to the curriculum and how it was adapted and modified to support student learning was documented. The data was analysed using a constant comparison method (Corbin & Strauss, 2008). The findings reveal that the curriculum was modified to communicate the mathematical concepts and mechanisms to support student learning to the teacher. This process involved synthesizing research into brief information bullets that communicated complex ideas into understandable and practical ideas. Although each lesson was situated in a context, it became evident that the big mathematical ideas needed to be foregrounded and explicit. The tasks were designed around the mechanisms to shift student thinking and possible student misconceptions were highlighted and remedies were suggested. A revised version of the curriculum, based on retrospective analysis, visually emphasizes key ideas and mechanisms of shifting student thinking through questions that facilitate small and whole class mathematical discussions. The curriculum was ultimately designed to support teaching through a standards-based, conceptual approach to support the development of a learning trajectory. Additionally, the tasks were designed to promote understanding and connections among mathematical ideas.

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EFFECTS AND AFFORDANCES OF VIRTUAL MANIPULATIVES

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BACKGROUND

Technologies like virtual manipulatives (VMs) contain multiple dynamic mathematical representations thereby promoting students' *representational fluency* by linking different forms of representation (Zbiek, Heid, Blume, & Dick, 2007). Virtual environments allow students to observe relations between actions and virtual objects (Durmus & Karakirik, 2006). For over two decades, researchers have documented the effects of virtual manipulatives in mathematics instructional treatments. The purpose of this study was to utilize a meta-analysis method to synthesize this research base.

RESEARCH QUESTIONS

1) What are the effects of VMs on student achievement as an instructional treatment in mathematics? Sub-questions examined comparisons with physical manipulatives, mathematical domains, and grade levels. 2) What virtual manipulative affordances provide empirical evidence that they promote student learning?

METHODS

There were two methods used in this meta-analysis. The first was a quantitative synthesis computing effect size scores from 66 empirical research studies that reported student achievement results. The second method was a conceptual analysis of VMs affordances to identify empirical evidence that the affordance promoted learning.

RESULTS & DISCUSSION

The results of the averaged effect size scores yielded a moderate effect for virtual manipulatives compared with other instructional treatments. There were additional large, moderate, and small effects when virtual manipulatives were compared with physical manipulatives and textbook instruction, and when the effects were examined by mathematical domains, grade levels, and study duration. The results of the conceptual analysis revealed empirical evidence that five specific interrelated affordances of virtual manipulatives promoted students' mathematical learning.

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STRUCTURING REPRESENTATIONS OF CONTINUOUS DATA THROUGH INVESTIGATIONS IN GRADE 1

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The study of data modelling with elementary students involves the analysis of a developmental process beginning with children's investigations of meaningful contexts: visualising, structuring, and representing data and displaying data in simple graphs (English, 2012; Lehrer & Schauble, 2005; Makar, Bakker, & Ben-Zvi, 2011). A 3-year longitudinal study investigated young children's data modelling, integrating mathematical and scientific investigations. One aspect of this study involved a researcher-led teaching experiment with 21 mathematically able Grade 1 students. The study aimed to describe explicit developmental features of students' representations of continuous data.

The teaching program adopted a design-based research approach that enabled experimentation with data representation skills. Investigations included topics such as *Melting ice* and *Growth of chickens*. Allowing young students to create their own graphs and construct a uniform scale was a basis for developing concepts of frequency, range and variation. Data collected included children's' work samples, written accounts, and researcher's observations. Analysis utilized iterative refinement cycles, comparing prior learning with new structural features (Lesh & Lehrer, 2000). Features of structural development were classified: pre-structural, emergent, partial, structural and advanced based on prior analyses (Mulligan & Mitchelmore, 2013).

Children's line graphs showed structural features such as grid lines and symbols that reflected student's individual forms of representation. Pedagogical strategies can be designed to reflect the fine-grained structural development of graphs including explicit attention to spatial structuring and collinearity, coordination of interval and scale leading to the concepts of range and variation.

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CHILDREN'S USE OF MODALITY IN PROBLEM SOLVING

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From a pragmatic viewpoint, language has evolved because of its functions in making meaning out of a given environment (Halliday & Matthiessen, 2004). The primary tense expresses what is present at the time of speaking, for example 'it is' or 'it isn't'; whereas modality expresses certainty or possibility, for example 'it has to be' or 'it can be.' Although the study of pragmatics in mathematics language has been carried out (e.g. Rowland, 2000), children's use of modality has not been studied, even though much mathematical language relies on the use of modality both deontic (the necessity or possibility of acts) and epistemic (the speaker's beliefs).

A task was developed (Figure 1) to support six year-old children's mastery of number facts through collaborative problem solving. The task required the children to complete a rectangle by placing ten frame cards that were more or less than the previous one according to a given value (e.g. two more or two less). The rectangle was to be closed.

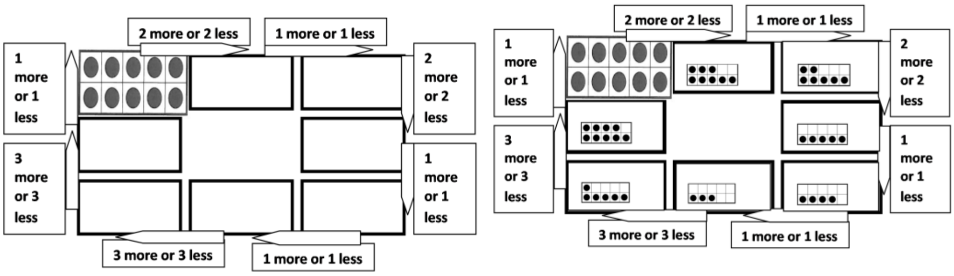

Figure 1: The More or Less task showing an incomplete and a completed version

Analysis of the children's dialogue indicated that initially the children used the primary tense with statements such as "What about we do..." When the children realised that the possibilities were limited ("But there's no bigger number...[than ten]"), then modal terms were used with deontic statements such as "So we have to..." and epistemic statements such as "It'll have to be one less" and "That one has to be nine..." This spontaneous use of modality suggested the children were reasoning by conceiving possibilities and reflecting on these. Reasoning is a semantic process. It depends on understanding the meaning of the premises and selecting modals to express a state of knowledge. The children's use of modality in solving this problem suggested a change in control of their state of knowledge.

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USING THE CENTRAL CORE THEORY TO STUDY PROBABILISTIC MISCONCEPTIONS

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We studied the conceptions of a group of math teachers regarding the concept of independence of events using the Central Core Theory. Our results show that all fourteen participants possess correct mathematical knowledge; however their interpretation of independent events using natural language is the base of their confusion between the independence of events and mutually exclusive events.

THEORATICAL BACKGROUND

Our work is based on the Canevas of reasoning (Rouquette & Flament 2003) that is based on the theory of Central Core (Abric, 1994). This technique allowed us to classify all the elements that are related to the same concept into two layers: the central core, which is the main force behind all decisions and the peripheral surface.

RESEARCH GOALS

The research aims at understanding how can teachers possess the correct math and at the same time show confusions between independent and mutually exclusive events.

METHOD

The study is an empirical qualitative research, based on four clinical interviews with each of the participants. Interviews consisted of open-ended questions and followed the assessment-related tasks that participants performed.

RESULTS

We identified three different conceptions related to the concept of independence of events that differ by the correctness of their central element and the peripheral elements.

CONCLUSION

The technique that we used brings a new perspective to qualitative research in math education especially for the studies of misconceptions.

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EXAMINING LIMITS WITH THE CONTENT ENAGEMENT MODEL (CEM)

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The Content Engagement Model (CEM) is applied to the concept of limit to consider the interplay between individuals' level of engagement (e.g., procedural, conceptual, or contextual) and their primary modes of engagement (e.g., visual or analytic).

Individuals' interactions with mathematical concepts are impacted by, and in turn impact, their understanding of the content. Recently, research has focused on the nature of individuals' engagement (Gresalfi & Barab, 2011). The CEM describes the progression from a procedural focus on rote execution of algorithms to a conceptual focus on why algorithms work to a contextual focus that considers the mathematical, social, and physical situations while selecting appropriate strategies and interpreting results. The CEM emphasizes that visual or analytic modes of reasoning may be used, but that they are more closely intertwined in higher levels of engagement.

Research has described students' procedural understanding of limits—which may be understood in terms of engagement that emphasizes finding $\lim_{x\to c} f(x)$ by visually or analytically evaluating $f(x)$ at a sequence of inputs which get closer to c or by rote procedures, such as direct substitution. The CEM sheds light on why students do not readily connect graphical and analytic interpretations of limit (Williams, 1991), suggesting procedural engagement does not afford opportunities to make these connections. With conceptual engagement, individuals focus on understanding how procedures relate to determining the limit value. The CEM suggests a lack of conceptual engagement may prevent the relationship between the limit value and nearby function values from being established and helps explain students' reluctance to change their notions about limits (Williams, 1991) in terms of lack of contextual engagement where reasoning about the mathematical context (e.g., type of function) and the corresponding assumptions can occur. A student engaging contextually might consider counterexamples to common misconceptions (e.g., limit equals function value) by considering the mathematical contexts (i.e., continuous functions) under which this might be true or false. When students are not engaged contextually, they are more likely to ignore troublesome counterexamples rather than situating them in a broader perspective of when particular relationships will hold.

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A THEORETICAL FRAMEWORK FOR THE EXAMINATION OF THE ROLE OF BILINGUAL LEARNERS' LANGAUGES WHEN ENGAGED IN CONCEPTUAL MATHEMATICAL LEARNING

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The significant role of language in mathematics teaching and learning is not a new phenomenon. However, little investigation has been undertaken in relation to the specific role of bilingual learners' different languages when engaged in mathematical learning (Barwell et al., 2007). Differences exist between any two languages. Dominance in one language over the other is common among bilinguals depending on the use and function of each language. Grosjean's (1999) concept of monolingual and bilingual as a *continuum of modes* facilitates an understanding of bilinguals using their languages independently and together depending on the context/purpose. Accordingly, there is an imperative need for a coherent and integrated framework to investigate whether differences in languages, and their use, by bilingual mathematical learners have a differential impact upon cognitive mathematical processing. Conceptual mathematical learning necessitates the development of higher order mathematical thinking as a critical component. The effectiveness of such learning facilitates a secure foundation of mathematical competence and confidence.

This research project will present five key integrated theoretical concepts that inform our framework for investigation - mathematics registers (specific characteristics of each language), psycholinguistics, sociolinguistics, mathematics content/conceptual activity and pedagogical practices. These theoretical concepts are underpinned by the perspective of discourse practices as both cognitive (thinking, signs, tools, meanings) and social (arise from communities) and are thus embedded in practices (Razfar, 2012). In the presentation, an example of a mathematical interaction will be utilized to demonstrate the application of the theoretical framework.

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HAVING A PASSION FOR MATHEMATICS: THE ROLE OF MATHEMATICS SPECIALIST TEACHERS IN GRADES 6-8 CLASSROOMS

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Questions of what mathematics should be taught and who should teach it have, historically, never been fully agreed upon by the various educational stakeholders. The debate is especially prevalent on the matter of the mathematics education of elementary (K-8) school children, where the notion of mathematics specialist teachers is front and centre (Allen, 2010; Li, 2008).

The research question for the study informing this paper is: What perceptions do individuals, involved in mathematics teaching and learning, hold of the use of mathematics specialist teachers in Grades 6-8? In other words, to study from multiple perspectives the phenomenon of middle years mathematics teacher specialist and how various educational stakeholders (school administrators, grades 6-8 classroom teachers, grades 6-8 students and their parents, as well as pre-service teachers) perceive it.

Data is currently being collected through surveys administered to these stakeholders, where schools have been selected at random from the two main divisions in Regina, Saskatchewan as well as rural divisions close to Regina. Schools range from those comprising grades K-8 to K-12, as well as one middle school catering to students in grades 6-8 exclusively. Surveys ask general questions about experiences teaching and learning mathematics, as well the perceived benefits and shortcomings of educating and hiring mathematics teacher specialists for middle years (grades 6-8) students.

Through a phenomenological analysis of data, preliminary results show that mathematics teacher specialists are considered a valuable resource and a desirable direction for grades 6-8 mathematics education. For example, one pre-service teacher responded that, if possible, she would choose to be a mathematics teacher specialist because "math has always been a passion of mine." From this research, we hope to make recommendations to teacher education programs for the creation of mathematics teacher specialist programs for middle years' teacher candidates.

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SECONDARY SCHOOL STUDENTS' SENSE-MAKING OF CIRCLE GEOMETRY CONCEPTS

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This paper is based on an ongoing larger project that is investigating secondary school students' meanings of circle geometry concepts prior to and after formal instruction. The focus here is on the nature of their pre-formal instruction meanings, factors influencing the meaning construction, and use of the meanings in problem solving. Meaning and meaning construction are interpreted from a constructivist perspective (von Glasersfeld, 1996). Thus, they involve the students' sense-making of the concepts based on their own way of relating to them. Pre-formal instruction meaning is how they make sense of the concepts before learning them in the classroom. Understanding this meaning is important from the constructivist perspective of teaching that must take students' meaning into consideration in helping them to learn mathematics with understanding (NCTM, 2000). The specific focus on geometry is also of importance to current curriculum perspectives (McCrone, King, Orihuela, & Robinson, 2010).

Participants for this case study were 20 grade 9 students in an advanced-placement class taught by an experienced teacher. Data sources for the larger project included interviews about meanings and a contextual mathematical task before and after formal instruction by the regular teacher and observations of this instruction. The interviews and task were based on the circle-geometry concepts in the grade 9 curriculum that included central angles, chords, tangents, arcs, and inscribed angles. Analysis involved a qualitative, emergent approach of identifying themes of the students' sense-making based on their recurring ways of thinking of the concepts occurring in the data.

Findings indicated meanings for circle-geometry concepts that were mathematical and real-world related, e.g., shape of, part of, or actual real-world objects; visual or spatial representations; and location. Students' real-world experiences were key influences on their sense-making of the concepts. Their meanings created obstacles to their problem solving. These meanings and influences will be explained with representative samples of the students' thinking. Findings have implications for instruction in relation to possible types of meanings to unpack and make connections in students' learning.

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FACILITATING PROSPECTIVE TEACHERS' FRACTION NUMBER SENSE DEVELOPMENT THROUGH PROBLEM SOLVING AND PROBLEM POSING

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One goal of mathematics content courses for prospective teachers (PTs) is to develop aspects of their mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008). While much research has focused on PTs' understandings of fraction operations, less attention has been paid to their fraction number sense, which lays groundwork for making sense of the operations. The little research there is suggests that PTs' fraction number sense is generally limited (e.g., Yang, Reys, & Reys, 2008), but offers few ways to help improve it.

The purpose of this study is to examine an approach that uses problem solving and problem posing in order to help PTs develop fraction number sense. While problem solving is a common activity in content courses, Crespo (2003) suggests that problem posing is an important yet under-utilized activity in teacher education.

We designed a sequence of tasks in which PTs co-created several reasoning and sense-making strategies for comparing fractions, applied these strategies to other problems, and posed problems intended to elicit specific strategies. The presentation will focus on results from one of the author's implementation of the tasks with 24 PTs. The results suggest that having PTs move back and forth between problem solving and problem posing helped them develop the intended reasoning and sense-making strategies, although some of the strategies were more challenging for PTs to understand than others. In the presentation, the results will be discussed in greater detail.

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IMPROVING TEACHER EDUCATION BY IMPROVING THE EXPERIENCING OF THE OBJECT OF LEARNING

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In this presentation, the intention is to make a contribution to the existing research on algebra learning by presenting a new theoretical approach, variation theory. From a variation theory perspective (Marton & Tsui, 2004), learning is seen as a function of how the learner's attention is selectively drawn to critical aspects of the object of learning. A critical aspect is the capability to discern aspects presented, for example in algebraic structures by experiencing them. To experience a rational expression is to experience both its meaning, its structure (composition) and how these two mutually con¬stitute each other. So neither structure nor meaning can be said to precede or succeed the other. If these aspects are not focused on in a teaching situation or in textbooks, they remain critical in the students' learning. The chosen theory will be exemplified by presenting results from a case study (included in a longitudinal study) concerning the simplification of rational expressions.

The research questions in this article are: (1) What aspects are discerned by the students when simplifying rational expressions?; (2) Does classroom instruction using the variation theoretic approach have any positive impact on students' learning of factorising rational expressions? If so, what is the magnitude of the impact?; (3) How does classroom instruction using this approach impact students' learning of factorising rational expressions?; (4) What actually happens inside the classroom when the variation theory approach is used?

The presentation is based on data collected, during a 3-year period, in a development project. The analysis is grounded in 30 exercises and 12 written reports which refer to simplifying rational expressions. Initial analysis entailed coding student responses for types of discerned aspects and the teachers report for types of focused aspects.

The findings suggest that developing an understanding of the students' critical aspects can be a productive basis in helping teachers make fundamental changes to their instructions and improve the mathematical communication in the classroom.

The communication in the classroom succeeds or not depending on the opportunities offered in the classroom to work out the meaning of the whole by knowing the meaning of the simple parts, the semantic significance of a finite number of syntactic modes of composition, and recognizes how the whole is built up out of simple parts.

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TIME AS A CONDITION FOR CHANGE IN TEACHER PRACTICE

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This report is concerned with identifying situations where time can act as a condition for change in teacher practice. Lack of time is often seen as an obstacle for change in teachers' practices, but my data indicate that it is not always so. The research question addressed is: *When can time be a condition for change in teacher practice?*

I report from a PhD-study on teachers' perception of what makes a 'good' task in vocational school mathematics lessons; the study employs a design based research methodology. In the Norwegian school system "the comprehensive school" is an important concept, which results in diverse classes in vocational schools. This report focuses on two teachers in these schools who were offered help designing mathematics tasks they would want to use in the classroom and which they felt would improve their students' learning, which gives unique data on teacher-driven changes in practice. The collaboration lasted a school year, and all conversations between the researcher and teachers were audio recorded. A total of 20 hours and 49 minutes of audio recordings have been used in the analysis; the recordings consist of interviews, and discussions about the tasks, refining tasks and evaluation of implementing the tasks.

The data were analyzed using techniques from grounded theory (Corbin & Strauss, 2008) and time emerged as a recurring issue. This led me back to previous research on time and the segments of data relating to time have been analyzed using Assude's (2005) theoretical timescales as analytic categories; didactic time, time capital and the pace of a course.

The findings show that in a diverse class, the difference in learning pace between students can condition change in the teacher's practice. Given that teacher change is mostly viewed as a slow and difficult process, identifying teachers who are so clearly expressing that they want change, can be of great value to further research. This research also points out that we should be careful at characterizing a class as one unit, but need to consider the individuals within as well.

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DEVELOPING CONCEPTUAL UNDERSTANDING OF SEQUENCES AND LIMITS BY WORKSHOPS FOR UNIVERSITY STUDENTS

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Convergence of sequences is an important concept where many students develop misconceptions or limited individual concept images (e.g., Tall & Vinner, 1981). There also exist suggestions for how to introduce this concept in order to build better conceptual foundations. Przenioslo (2005) offers a concept for a teaching sequence where secondary school students are encouraged and supported in creating a formal definition on their own. We have built on these ideas and developed an intervention for German first year university students, to study the research question: Does an intervention where German university students create the formal definition of the limit of a sequence on their own enhance their understanding of this concept? The concept of Przenioslo was adapted for a starting workshop and supplemented by a follow-up workshop. In order to evaluate the intervention, both, the experimental group consisting of the ten participants of the workshop and the control group formed by all 77 other students attending the class "Analysis 1" took two tests: one test assessed the previous knowledge before the first part of the intervention took place and the second one assessed the understanding of the concepts of sequences and limits and was used after the second part of the intervention took place. Both tests' data are analysed according to Item Response Theory. When regarding the whole "post-test", it is not possible to establish a significant effect. But in respect of a subtest consisting of those items of the "post-test" that directly assess the known misconceptions regarding the concept of limit, which the workshop aimed at, in fact there is a significant effect even though it is much smaller than the influence of the previous knowledge tested by the "pre-test". This can be shown by computing a generalized linear model with two covariates: the score in the "pre-test" (η^2=.235, p<.001) and the attendance at the workshop (η^2=.083, p=.012). Thus, it was possible to show a significant effect of the attendance in the workshop even though it is a small one. In order to get more insight in the students' development of understanding of the concept of convergence while exploring the definition, the data collected during the intervention will be analysed.

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TEACHERS' DEVELOPING TALK ABOUT THE MATHEMATICAL PRACTICE OF ATTENDING TO PRECISION

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Attending to precision is an important aspect of mathematical thinking, whether it be an awareness of the precision of measurements and calculations or a concern for the precision of mathematical communication. In the U.S., attending to precision has been explicitly included as a Standard for Mathematical Practice within the *Common Core State Standards for Mathematics* (2010). This study addresses the following question: How do mathematics teachers talk about the practice of attending to precision? The analysis of teacher talk is based on a sociocultural and sociolinguistic (Halliday & Matthiessen, 2003) perspective wherein the ways that individuals know and understand an idea is inseparable from the ways they communicate about it in various contexts. In this study, the general context is a mathematics teacher discourse community and specific contexts are discussions of readings, discussions of mathematical tasks, and discussions of classroom practices and interactions.

The eight participating mathematics teachers (grades 5–12) were involved in summer study sessions focused on the *Common Core* mathematical practices. Data consisted of recordings of the group discourse and artifacts of teachers' written communication about attending to precision. Thematic discourse analysis (Herbel-Eisenmann & Otten, 2011) was used to reveal the semantics and the patterns across contexts within the teachers' talk.

Findings include general thematic patterns of the teachers' talk about attending to precision. For example, many of the teachers tended to focus their talk on number and estimation. The practice was also often situated in a discourse of decision making. A smaller number of teachers exhibited more complex thematic maps around attending to precision, including connections to both number and mathematical communication.

The mathematical practice of attending to precision has not been examined as extensively as other practices, such as problem solving or reasoning. This study provides insight into middle and high school mathematics teachers' understandings of this important practice that is expected to be widely implemented.

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FOSTERING GEOMETRIC THINKING OF ELEMENTARY MATHEMATICS TEACHERS THROUGH LESSON STUDY[1]

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Geometry is a field of learning having a great place in school mathematics. Despite this, the fact that students are unsuccessful at this field shows the necessity of performing activities for fostering geometric thinking of teachers and development of knowledge for teaching geometry. Considering this necessity in this study it was aimed to foster geometric thinking of elementary mathematics teachers.

Participants of the study comprise of five mathematics teachers from four different elementary schools. These participants attended to the seminars about fostering geometric thinking based on the Geometric Habits of Mind (GHOM) framework (Driscoll, DiMatteo, Nikula, Egan, Mark & Kelemanik, 2008) for a month held by the researchers. Following these seminars, lesson study model was applied on geometry classes with attending teachers for three months. By this means, teachers had the opportunity of observing, discussing and reporting lessons they planned with their colleagues in lesson studies in live classroom environment (Murata, 2011).

Data sources of the study made up of records of pre and post-lesson study interviews with teachers, video records of classroom practices and planning and discussing meetings, and lesson observation forms filled by the teachers during the classroom practices. Data were analyzed by qualitative data analysis methods. During the study, the progress of the geometric thinking of each participant was observed. Moreover, it was found out in the process that indicators of "Reasoning with Relationships" and "Generalizing Geometric Ideas" GHOMs were more than "Investigating Invariants" and "Balancing Exploration and Reflection" GHOMs. In the end of the study teachers pointed out that the lesson study model enhance geometric thinking of each other and it enabled them to see different point of view for teaching geometry.

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[1] This study was supported by Anadolu University Scientific Research Projects Commission under the grant no:1308E325.

AN EXAMINATION OF THE PROBAILITY KNOWLEDGE OF IRISH OUT-OF-FIELD MATHEMATICS TEACHERS

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Teacher quality is believed to be one of the most important factors affecting student learning, and research has demonstrated that students learn more from skilled and experienced teachers who know what and how to teach (Darling-Hammond & Youngs, 2002). However, out-of-field mathematics teaching is prevalent in the Irish context and accordingly, a two-year part-time Professional Diploma in Mathematics for Teaching (PDMT) has been established nationally to up skill these out-of-field teachers.

This research project aims to examine the content and pedagogical knowledge of out-of-field teachers, specifically related to probability, on commencing the PDMT (September 2013). This knowledge was assessed by a paper-and-pencil test administered during the programme induction. The TEDS-M Conceptual Framework (Tatto et al., 2008) underpinned the construction of test items. All items are aligned to key strands (Statistics and Probability, Geometry and Trigonometry, Number, Algebra, Functions) and associated sub-topics of the Irish post-primary curriculum, which teachers are required to teach and assess.

The sample comprises of 202 out-of-field teachers from across Ireland who are enrolled in the PDMT. Findings indicate wide variations and significant areas of weakness in out-of-field teachers' conceptual understanding of topics in probability. In particular, only 25% of teachers correctly answered the question on 'choice'; 64% correctly answered the question on 'combinations'; and 36% correctly answered the question on 'chance'. Results also indicate underdeveloped pedagogical knowledge regarding the teaching, learning and assessment of probability.

Overall, such findings have significant implications for understanding areas in which mathematics teachers need support and for designing effective continuing professional development (CPD) programmes. In the presentation, these results and others will be discussed in detail.

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STUDENTS' STRATEGIES AND DIFFICULTIES FOR FINDING THE PROBABILITY OF COMPOUND EVENTS

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The probability of compound event in school mathematics is related to a two-stage or two-dimensional experiment, and it has been discussed as a central topic in teaching and learning probability (Rubel, 2007). Nevertheless, there is little discussion about the strategies or reasoning that students apply to the exploration of the probability of compound events (Rubel, 2007), so we aim to identify how students' exploration into compound events progresses and find difficulties students have with these explorations, focusing on students' strategies and reasoning.

The participants of this study were six eleventh-grade students who showed slightly over average levels of mathematical achievement. The students participated in experimental class with designed tasks, and the object of these tasks was to find the probability of compound events which consisted of two-stage or two-dimensional events and replacement or non-replacement events. The main data for this study were obtained from video documentation of and student worksheets from an experimental class, and grounded theory techniques were employed to analyse collected data.

We found the students' two major strategies for finding the probability of compound events. (a) The students found the probability by illustrating every possible outcome systematically. (b) The students found the probability by the multiplicative and additive rule. The students applied both of these two strategies to verify their strategies and solutions. They compared and supported plausible solutions when they used multiple strategies in order to find an appropriate solution or verify the validity of a solution. We also identified reasons for students' difficulties for finding the probability of a compound event. The students believed that the first enumerative strategy (a) can be applied to every probabilistic situation and always results in the same answer obtained using the second probabilistic strategy (b). They also considered that the denominator of the probability always corresponds to the total number of possible outcomes of an event.

These reasons for students' difficulties are crucial since these are closely related to students' unawareness of the equiprobability of a sample space. Based on these results and many paradoxes related to the equiprobability of a sample space, we will discuss the complementary roles and hierarchical relationships between the enumerative strategy and formal probabilistic rules.

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CHARACTERISTICS OF THE INSTRUCTOR'S DISCOURSE IN A GUIDED-REINVENTION CLASSROOM SETTING

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Research has reported students' difficulties understanding a formal definition of limit and attempts to make the definition accessible to students. One such attempt is a guided reinvention in which students reinvent a formal definition of the limit through a carefully designed teaching experiment (TE). This study explores features of an instructor's discourse in conducting a guided reinvention TE focusing on the ways he interacted with students' definitions using the Commognitive lens (Sfard, 2008).

The data came from a calculus II classroom at a public university, where the teacher -researcher (TR) conducted a five-day guided reinvention TE with three groups of students. The activity of one group of four students who volunteered for this study (TE group) was videotaped. During TE, TR asked students to generate example and non-example graphs of sequences converging to 5 and to write a definition by completing the statement, "A sequence converges to 5 provided," and then guided them through iterative refinement process (IRP) in which students test their definition on the graphs, identify problems, resolve them, and revise the definition.

Analysis of TR's discourse showed that in the beginning of TE, TR mainly interacted with the reinvention process by explaining what definitions should do (e.g., including examples, excluding non-examples) and telling students what to do next to move them forward in IRP. Then, in the middle of TE, TR mainly interacted with students' definitions by asking them to illustrate the definitions on the graphs by focusing on the words and notations in their definition. TR often produced conflicts to help TE group see problems in their definition when it included words that describe the behaviour of dots on the graph (e.g., ultimately approach) rather than the values that they represent and when it did not reflect their illustration on graphs (e.g., drawing ε first). Towards the end of TE, TR mainly supported TE students' decisions by approving their work and provided solutions by letting non-TE groups present their definitions. TR also confirmed the nature of words and notations by expanding TE groups' work (e.g., "error bound' means any error bound" or "n_c depends on your choice of ε".)

The results highlight changes in the instructors' discourse in guided reinvention especially about aspects of a formal definition that students struggled with. Prompts for such aspects confirmed by the instructor could be carefully designed in future TE.

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PERSISTENCE OF PROSPECTIVE TEACHER NOTICING

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Mathematics education researchers are recognizing a teacher's ability to notice as an important asset to student-centered instruction (Sherin, Jacobs, & Philipp, 2011). As part of a larger study, three prospective mathematics teachers (PTs) were engaged in targeted noticing activities during an early field experience (as described in Stockero, 2014) As a post-assessment, the PTs analyzed a 12 minute video for mathematically important moments a teacher should notice; two years later, they analyzed the same video at the conclusion of their student teaching experience. Two researchers coded the video using a framework, identifying seven key moments to notice. The PTs' analyses at the two points in time were compared to the researchers' to understand changes in the PTs' noticing. The goals were to gain insight into the effectiveness of the intervention and the extent to which time in a classroom affected PTs' noticing.

The data showed that two of the PTs marked fewer instances after student teaching than after the early field experience. PT1 identified three (42.9%) key moments at the end of the field experience, but only two (28.6%) at the conclusion of student teaching. PT2 identified four (57.1%) and two (28.6%) key moments, respectively, at the same points in time. PT3, in contrast, tagged more instances at the end of student teaching than after the early field experience—six (85.7%) after the field experience and seven (100%) after student teaching.

The data suggests that the teacher noticing intervention alone did not allow the PTs' noticing skills to further develop over time; that is, it did not promote generative learning (Franke & Kazemi, 2001). The results point to a need for more structured noticing activities during student teaching to maintain the PTs' noticing skills.

Acknowledgement

This material is based upon work supported by the U.S. National Science Foundation (No. 1052958). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

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HOW SMALL-GROUP MATHEMATICAL PROBLEM SOLVING SUCCEEDS OR FAILS: FROM THE ANGLE OF COORDINATION

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Small-group mathematical problem solving has been studied from varying perspectives such as helping-related behaviors, status, and rudeness (Webb, 2010). However, there is still a paucity of research carried out from the perspective of coordination. In this connection, this study addresses the issue by adopting and adapting a tripartite framework of coordination (Barron, 2000) as follows: a) shared task alignment; b) mutual monitoring behaviors; and c) mutuality of exchange. The purposes of this study are to identify the differences behind successful and unsuccessful mathematical problem solving in relation to the three aspects of coordination and to explore how those differences, if any, influence problem solving.

In two 8^{th}-grade classes from a Chinese public school, a two-month-long small-group cooperative learning intervention was conducted in mathematics classroom instruction. Near the end of this intervention, two groups (four students in each) were drawn from the two classes. Divided by performance in previous group work of mathematical problem solving, one group was classified as the "successful" group, the other "unsuccessful". The conversations of both groups solving a word problem were videotaped and then processed through discourse analysis. As an end to triangulation, semi-structured interviews were conducted and analyzed.

Results show that for the group differences, compared with the unsuccessful group, more members in the successful group were found to have contributed to discussion, more high-level mutual monitoring behaviors occurred, and the group members had equal opportunities to express themselves at their will. For the influences, high-level of shared task alignment could lead group members to the co-construction of solutions. High-level mutual monitoring behaviors were helpful to orientate group members toward the accuracy-check of each other's thinking. High-degree mutuality of exchange ensured that helpful ideas could be considered by group members.

The findings indicate that students in the successful group had stronger awareness and better performance in sharing, monitoring, and exchanging ideas than those from the unsuccessful group. Additionally, coordination has crucial impacts on small-group mathematical problem solving.

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CHARACTERISTICS OF COMPREHENSION PROCESSES IN MATHEMATICAL MODELLING

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Due to the increased working on mathematical problems with larger parts of text in mathematics classrooms text comprehension has become a central prerequisite for a successful problem solving (Prediger & Özdil 2011). Especially when constructing the situation and real model within the mathematical modelling process mathematical and linguistic competencies are relevant. The first step consists of comprehending the real situation by reading the task. Reading competence it defined as a cognitive and active (re-)construction of information in which an interaction of processes guided by know-ledge and processes guided by text features is necessary (Rayner & Pollatsek 1989)

This study within the SITRE research project focuses how processes of understanding can be empirically described when working on a mathematical modelling task and which factors may influence these processes. A laboratory study was conducted with 50 seventh graders. In individual sessions the pupils worked on three mathematical modelling tasks using the thinking aloud method. This method was used to reconstruct the process of understanding in the analysis. Furthermore mathematical competence and reading competence were collected. To take reading competence and modelling competence into consideration the tasks were systematically varied with regard to modelling and linguistic aspects. To analyse these solving processes firstly the approach in the several process steps (inspired by the modelling cycle of Blum and Leiss (2007)) was rated and secondly the chronology and proportion of time of the several process steps were identified.

Exemplarily three results can be shown. As a methodological result the development and application of the thinking aloud method can be classified as successful because almost all of the pupils showed such a good performance in this method that their data could be used. A remarkable result lays in the achievement of classifications concerning the processes of understanding. The study could reveal that the reading strategy of taking notes had positive effects on the construction of an adequate situation and real model.

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CONSTRUCTION OF VIDEOGAMES FOR MODELING LEARNING BY ENGINEERING STUDENTS

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We will present some results of a research project where university engineering students (in their 5th year of studies) are asked to construct videogames as a means to help them develop their conceptions and skills related to the modeling of physical systems. For example, students had to build a game modeling liquid water behavior.

Learning how to model is important for engineering students, because modeling, which requires students to produce adequate mathematical representations, is the first step in designing a device, system, product or process. The theoretical framework of the study is based on the constructionism paradigm (Papert & Harel, 1991), which considers that learning is facilitated when the learner engages in the active construction and sharing of external objects. Thus the importance of providing learners with opportunities and a context to build and engage with software or hardware artifacts that are meaningful to them. In our proposal students engage in the design and programming of videogames in order to promote modeling learning. In the building of video games for modeling processes, we integrate sets of activities whose design takes into consideration the six principles of Model-Eliciting Activities (MEAs) (Hamilton, Lesh & Lester, 2008): Reality, model construction, model documentation, self evaluation, model generalization, simple prototype. During the past semester, a group of 13 engineering students engaged in the construction of a videogame for modeling water behavior. The results indicate that, through the MEAs and in programming the videogame, students engaged in producing a working model that was meaningful for them and gained a deeper understanding of all the elements involved in the modeling process: That is, they not only had to produce mathematical representations of the physical phenomena for their model, but also understand these in the context and dynamics of the game they were building, establishing relationships (within the technological restrictions of the game engine) between the computational objects (which also had to be attributed values such as form, color and size), such as those that simulate water particles, and their physico-mathematical properties such as gravity, density and potential and kinetic energies.

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DRAWING INFERENCE FROM DATA VISUALISATIONS

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The importance of statistical education coupled with the emergence of powerful visualisation tools—has led to some reconceptualization of teaching statistics (Ridgway, Nicholson, & McCusker, 2013). Students need to become familiar with reasoning about multiple variables, covariation between multiple variables, and the use of complex visualizations to represent quantities in new ways, using intuitive visual artefacts. However, we know little from empirical research about how students interpret the multivariate view of data within data visualization tools and even less about students' ability to meaningfully construct data visualisations that highlight important aspects of data. 43 students of grade 8-9 (2 classes) were asked to select appropriate quantities represented in the data to construct visual representations that would help them to make informed decisions about the following task: "Preparing to live in an unknown country in the future: Which is the best country to live and work abroad? Consider different variables that impact on your decision." Students were paired to discuss how to visually represent data sets as they engaged within the Gapminder data visualization tool that has built-in access to large global datasets regarding economy, education, energy, environment, health, infrastructure, population, society, and work. The results suggest that the collaborative task challenged students to construct visual structures that highlight relevant information for their analysis task. In their justification efforts, students revisited their specific kinds of inferences while using complex data visualization tools, built on them and took new actions based on the needs of their exploratory tasks. When the students deal with exploratory tasks that require making sense of large collections of data, the students often accomplished to gain insight into big data by implementing a few principles that would make their data exploration easier: (1) explore the trends of fewer variables on the instantiated data structures, (2) students' exploration of data visualisations revealed "trade-offs" where there was no clear decision, leading students to a cycle of invention and revision of further visual representations of data. In summary, we can claim that students' engagement in seeking insights in multivariate data visualizations would help students develop mental models of possible relationships between multiple variables that could give them a stronger conceptual basis for considering the formal statistical analysis.

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VARIABLES INFLUENCING THE COMPETENCIES OF MATH FIRST-YEAR STUDENTS AT THE UNIVERSITY

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Dropout is still a problem of great importance in mathematical studies at universities. An especially important role in dropout plays the transition from school to university. Therefore it is important to know, which competencies the students posses at the beginning of their studies. These competencies and dropout are influenced by a lot of different variables (Rasmussen & Ellis, 2013). In this study four of them are further investigated. The first investigated variable is gender, which is known to be an important factor on students' knowledge. Since the school system in Germany is highly dependent on the federal state, a comparison of two federal states is conducted. The third variable represents the duration of school duration, which is lowered in the two federal states investigated, from thirteen to twelve years. At last the time between school leaving and starting at university investigated. Overall the effects of the four mentioned variables on the knowledge of beginning university students are investigated.

To investigate the research questions a test focused on declarative knowledge was used. The test items were developed by comparison of school curricula and content of the first lectures at university. The sample comprises 336 students, who took the test in one of two consecutive years and was conducted as beginning of a prep course. The two years correspond to the changing in school attendance of the federal states.

The test was modelled with the Rasch model and an analysis of variance was conducted. Two of the variables were found to have an significant effect on the test performance, male students performing better then female students - $F(1,334)=5.648$; $p=0.018$; $\eta 2=0.018$ - and having a time gap between leaving school and starting studies at the university lowers the test performance - $F(1,334)=6.213$; $p=0.013$; $\eta^2=0.020$. Also an interaction effect between duration of school attendance and federal state – $F(1,334)=8.227$; $p=0.004$; $\eta^2=0.026$ was found, indicating that in one federal state the students with thirteen years of school attendance showing better performance, while the opposite was true in the other state. While the main effects could rather be expected, the interaction effect is very interesting, showing a federal state, in which the students did not perform worse in the test while lowering the time of school attendance by one year.

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PEDAGOGY OF RISK IN THE CONTEXT OF HIGH SCHOOL MATHEMATICS

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My research explores ways in which mathematics educators can foster secondary school students' understanding of risk. Specifically, I investigate students' interpretation, communication, and decision making based on data involving risk in the classroom setting. I then consider ways in which curriculum and pedagogy can assist in the development of the teaching and learning of risk. Students' learning is studied within the pedagogic model of risk (Levinson et al., 2012).

A qualitative case study is presented in order to explore an inquiry-based learning approach to teaching risk in two different grade 11 mathematics classes in an urban centre in Canada - an all-boys independent school (23 boys) and a publicly funded religious school (19 girls and 4 boys). The students were given an initial assessment in which they were asked about the safety of nuclear power plants and their knowledge of the Fukushima nuclear power plant accident. The students then participated in an activity with the purpose of determining the empirical probability of a nuclear power plant accident based on the authentic data found online. The purpose of the second activity was to determine the impact of a nuclear power plant accident and compare it to a coal power plant accident. The findings provide evidence that the students possess intuitive knowledge that risk of an event should be assessed by both its likelihood and its impact. The study confirms the Levinson et al. (2012) pedagogic model of risk in which individuals' values and prior experiences together with representations and judgments of probability play a role in the estimation of risk. The study also expands on this model by suggesting that pedagogy of risk should include five components, namely: 1) knowledge, beliefs, and values, 2) judgment of impact, 3) judgment of probability, 4) representations, and 5) estimation of risk. These components do not necessarily appear in the instruction or students' decision making in a chronological order; furthermore, they influence each other. For example, judgments about impact (deciding not to consider accidents with low impact into calculations) may influence the judgments about probability. The implication for mathematics education is that a meaningful instruction about risk should go beyond mathematical representations and reasoning and include other components of the pedagogy of risk. The study also illustrates the importance of reasoning about rational numbers (rates, ratios, and fractions) and their critical interpretation in the pedagogy of risk.

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OPPORTUNITIES FOR LEARNING: EXPLORING PRESERVICE TEACHERS' ENGAGEMENT WITH MATHEMATICAL TASKS

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This paper reports the findings of an exploratory study examining the mathematical and pedagogical engagement of preservice secondary teachers. Eight teachers registered in a math methods class in the first term of a post-baccalaureate teacher education program were invited to work in individual or paired problem solving sessions. The "think aloud" problem-solving sessions were video recorded and partial transcripts were analysed for emerging themes and patterns. The study investigates: 1) how preservice teachers approached the mathematics tasks; 2) what preservice teachers conceived as good mathematical problems for students; and 3) how discrepancies that emerged between their thoughts and actions while approaching open-ended and non-routine mathematical tasks could be utilized to create transformative learning spaces.

Findings indicate that most teachers tried to "solve" the mathematical tasks by drawing on previously learned facts, rules, formulae, or theories. Challenges in switching the "teaching hat" with a "learner's hat" were also evident. For example, rather than explaining their own approaches to solving the mathematical tasks, they explained how they would present the tasks to others. Although invited to use manipulatives, role play, or develop simulations, most chose to work with paper and pen. Contrary to how preservice teachers approached the problems, they described good mathematics problems for students as ones that can be solved using multiple approaches, lead to the use of mathematical representations, required critical thinking, and were appropriately difficult for the learner. Discrepancies in the thoughts and actions exhibited during these sessions create a space for teachers, and teacher educators, to reflect on pedagogical practices in mathematics classrooms. Such reflective experiences could serve as catalysts for changing teachers' underlying beliefs and conceptions about mathematics and may help in strengthening the mathematical understandings and pedagogical knowledge that is essential to create rich mathematical experiences for their students (Boston, 2013).

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COHERENCE OF THOUGHT STRUCTURES OF MODELLING TASK SOLUTIONS AND TASK DIFFICULTY

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Although there is a broad consensus about the need of mathematical modelling tasks in everyday school life, several studies show that the amount of modelling tasks in school is low (e.g. Jordan et al., 2006). Concentrating on the teachers' perspective there exist some special characteristics of modelling tasks which may withhold teachers to use modelling tasks. For example, multiple solution approaches and their varying task difficulty complicate the development of modelling tasks and not least its assessment.

The project MokiMaS develops an instrument to determine the task difficulty of modelling tasks on the basis of thought structures of real student solutions building on Cognitive Load Theory (Sweller, 2003). Considering thought operations of student solutions as difficulty generating characteristics provide information about task difficulty and can be taken as a basis for a well-founded rating scheme.

During a pilot study six short modelling tasks were developed and tested. The main solution approaches have been identified ensuing from which thought structures and accordingly levels of difficulty have been set up. The main study now aims at validating the thought structure analysis empirically with a sample of 1800 student solutions of ninth graders (15 years of age) of German grammar schools. The key question is whether empirical results verify the theoretically determined levels of task difficulty and thus, the thought structure analysis. The findings to date indicate that there is an affirmative coherence between student performance and theoretically determined level of difficulty. Students show a better performance when dealing with theoretical low-level tasks than with those rated as high-level. Teacher interviews show that the thought structure-based rating schemes are closely orientated by their daily assessment pattern and hence justify a good applicability in everyday school life. The analysis of the remaining student data will then show to what extent the specific levels of difficulty have a general significance. In the presentation further results will be discussed in detail.

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PRIMARY TEACHERS CONCEPTIONS ABOUT SOME ASPECTS OF ALGEBRAIC THINKING

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Special attention has been recently given to the area of early algebra. Referring to young children, Kieran (2007) speaks about this area as being a way of thinking, much beyond a set of knowledge and techniques. The young child to be successful in Algebra, needs to develop six types of skills: generalization, abstraction, analytical thinking, dynamic thought, modelling and organization.

The present study asked fifty primary school teachers about their conceptions and practices while teaching this subject, in particular with respect to the equal sign and proportionality. A semi-structured interview was conducted according to a guide that was elaborated to address the research questions. This guide was constructed from a literature review on the topic concerned that illustrates the relevance of the questions to ask teachers. Results show that teachers agree with the existence of an early algebra although there is a considerable distance between them and what is considered by the international scientific community with regard to the identification of the abilities involved in algebraic thinking and the activities that can foster it. The meaning of the equal sign was said to be worked in class both as indicating a relationship between quantities and as pointing to a result. Equivalence relationships were not given much attention, as important as they really are, as mentioned by Welder (2008). With regard to activities fostering proportional reasoning the answers were very scarce, which should be a concern since it is considered to be an essential component of algebraic thinking. Curcio and Sydney (1997) had already mentioned that the challenge is to look for ways of exploring problem solving involving children in proportional reasoning. In general, teachers report practices which can foster at least some aspects of an algebraic thinking.

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PROSPECTIVE TEACHERS' SPECIALIZED KNOWLEDGE ON PROBLEM POSING

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Internationally it is more or less accepted that problem solving is one of the core aspects in and for developing mathematical reasoning. At the same level should be problem posing, as good problems need to be posed to allow solvers to develop their problem solving ability and thus mathematical knowledge. Being teachers' knowledge a crucial factor in students' learning, and the fact that prospective teachers (PTs) need to acquire a deep understanding of mathematical concepts, we perceive problem posing as a way to access (and develop) problem posers' specialised mathematical knowledge. Focusing on PTs knowledge, a set of tasks was elaborated in the context of operations with integers and fractions (here we discuss a particular example of a division having as a result an infinite repeating decimal). Those tasks required for PTs first to give an answer to the proposed operation and afterwards pose a problem that could be solved using such expression – justifying also the grade level they consider to be more adequate for such a problem to be posed. Those sessions were audio and video recorded, but here we will focus mainly on PTs written productions. Such tasks were elaborated and implemented in our courses not only as a starting point for accessing and developing prospective teachers' knowledge, reasoning and argumentation but also, complementary, as a way to better understand the content of some different sub domains of Mathematics Teachers' Specialized Knowledge (MTSK) (Carrillo *et al*., 2013).

We will discuss some results concerning methodological and theoretical aspects both on the conceptualization and implementations of the tasks with implications for research and training. The results allow expand Leung and Silver's (1997) categorization on problem posing, and contributed for a deeper understanding of the content of MTSK. Finally we will reflect upon the implications of such work on the elaboration of our own MTSK as teachers' educators and researchers.

Acknowledgement: This paper has been partially supported by the Portuguese Foundation for Science and Technology (FCT) and by the Pró-Reitoria de Pós-Graduação – UNESP.

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FORMATIVE ASSESSMENTS IN CONNECTED CLASSROOMS

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This qualitative case study research considered the use of technology to support the formative assessment process by two high school mathematics teachers. Results of the seven emergent categories of question types used to formatively assess students during instruction will be shared.

Formative assessments can increase student achievement (Black & Wiliam, 1998). However, challenges of incorporating formative assessments into instruction include the length of time needed to collect, assess, and provide feedback to students (Black & Wiliam, 1998). These challenges are easily overcome by using technology to solicit responses from all students. Connected classroom technology can also result in improvements in student achievement as teachers instantaneously collect, manage, and analyse data received from all students (Roschelle, Penuel, & Abrahamson, 2004).

This case study research considered two high school mathematics teachers, from one urban and one suburban district, who used technology, specifically the TI-Nspire™ Navigator™, as a formative assessment tool. The purpose of this research is to determine how mathematics teachers use the Navigator system to formatively assess their students.

Qualitative research methods were used to collect and analyse data. Specifically, audio-recorded classroom observations, and pre and post-observation interviews were transcribed. Codes and categories were grounded in the data and constant comparative method was used to analyse each successive classroom observation and interviews with a focus on formative assessments using technology.

Results indicated that teachers primarily used the *Quick Polls* feature of the Navigator system to formatively assess their students. Seven different categories of question types emerged that were used to formatively assess students during instruction. The most prominent were a check for understanding and recall of prior knowledge. Examples from each category and how teachers used the feedback will be shared. In sum, the use of the Navigator system affords the teacher a variety of formative assessment options and questions that can be used to seek feedback from all students in a networked classroom resulting in data-based instructional decision making.

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PATTERN GENERALIZATION ACTIVITY USING VIRTUAL MANIPULATIVES WITH STUDENTS OF LOW PERFORMANCE

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Here we report about the implementation of a scenario of learning elaborated especially for 7th grade students of low attainment at school. A specific teaching design looked for incorporating the use of digital devices into the classroom for the learning of algebra in regular middle school classrooms, based on the manipulation of figural patterns and for student recognition of some associated visual templates (Giaquinto, 2007; Rivera & Becker, 2008). According to Rivera (2010, p. 300), meaningful pattern generalization involves the coordination of two interdependent actions, abductive–inductive action on objects, and symbolic action in the form of an algebraic generalization. In this work we used virtual manipulatives (see http://nlvm.usu.edu/en/nav/vlibrary.html) distinctly to what Rivera (2010) and Rivera & Becker (2008) accomplished before, to allow a digital direct and visual manipulation of figural templates to separate, coloured and counting distinctive elements of the pattern figures. We searched for enabling low attainment 7th grade students to go forward in the solution of figural pattern generalization tasks, usual in algebra learning. Empirical observations were accomplished with 90, 7th grade, low attainment students, situated in 10 different regular classrooms. Each group was allowed to work with us in an equipped classroom and during ten special sessions of one hour of practical work at school. Students were working alone, each one with one computer, and with specific instructions at hand. We obtained detailed descriptions of students' use of the virtual manipulatives when they solve the mentioned tasks. In general we verified that students determined a unit of measure associated, in each of the stages of the figural pattern, with the number of the corresponding stage, and according to the sequence of patterns given. Using virtual manipulative functionality students of low attainment were able to establish hypothesis about the whole number of components of the figures that did not appeared in the given figural patterns, showing an abductive stage in the process of generalization, and also could advance on testing its assumptions (inductive stage) to confirm the validity of the rule they have advanced around what was defining the given pattern.

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EXPLORING MENTOR TEACHERS' EVALUATION OF CPD DESIGN ASPECTS

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Manifold variables are discussed in the literature in the field of continuous professional development (CPD), and teacher learning is often conceptualized differently. But at least there is some consensus on key design principles as continuous professional development programs should be competence-oriented (P-CO), participant-oriented (P-PO), case-related (P-CR), should stimulate cooperation (P-SC), contain various instruction formats (P-IF), and foster (self-) reflection (P-SR) (cf. Garet et al., 2008; Desimone, 2011). Based on these principles, a CPD course provided by the German Center for Mathematics Teacher Education (DZLM), in which teacher trainers were engaged for one year, is evaluated. In particular, we consider the following research questions:

- What relevance do teacher trainers assign to the DZLM design principles?
- To what extend do the DZLM design principles become relevant for the own work as teacher trainer?

Participants in our study were 12 teacher trainers from Germany who were immersed in the CPD course. The qualification program was divided into two modules each of which dealt with different topics of mathematics education also addressing the participants' role as teacher trainer. We collected quantitative data by a questionnaire delivered to teachers six month after the CDP course. In addition, we conducted qualitative interviews to explore teacher trainers' individual understanding of the design principles. Our findings show that teacher trainers value the DZLM CPD design principles in hierarchic order: The participants rate P-PO and P-CO as highly relevant for designing an effective CPD environment. In comparison, P-IF and P-CR are considered minor relevant. Taking a closer look on explanations given in the interview study we encounter two interesting phenomena: First, teacher trainers estimate 'practice orientation' as key for designing effective CPD courses which is emphasized by referring to a combination of P-PO and P-SR features. Second our results stress the importance of an overarching structure that frames the design principles in general. That is, teachers particularly value the alteration of theory and practice phases of the CPD course and consider this aspect as most relevant for their own work as trainers who provide CPD for teachers.

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INTELLIGENT DIALOGUES WITHIN TERTIARY EDUCATION PUPILS: WORKING WITH PARAMETERISED MODELING ACTIVITIES

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Intelligent Dialogues (DiT) is a system with which it is possible to simultaneously display on a computer screen a chat window and a window with a microworld, dynamically hot-linked to each other. While users work in the microworld, they can enter into dialogue with the system in natural language (Rojano & Abreu, 2012). Here we report outcomes from a study which aims to investigate the role of feedback, by way of the *DiT* system, in parameterized modeling activities of phenomena of the physical world, carried out by tertiary education students in a spreadsheet microworld. Specifically, we analyze the role of said feedback at critical modeling points, such as *comprehension of the phenomenon* being modeled, *building up the model*, and *prediction of the phenomenon's behavior*. Theoretical elements from the *learning with artificial worlds* perspective (J. Ogborn et al, 1994) were adopted for the design of the modeling activities. Three pairs of 16-17 year old students volunteered to participate in four modeling sessions, working in the activities of *Pollution of a lake* and *Molecular Diffusion*. Analysis of the video material from the sessions shows that: 1) the DiT feedback was particularly crucial in the *building up the model* episodes, and 2) the role of intelligent support in the form of dialogue was a meaningful complement to the feedback in the microworld, which confirms what other authors have hypothesized about natural language potentially being an important factor for feedback and learning.

Acknowledgments

To Conacyt (Ref. no. 168620), and participant students and teachers from the CCH Vallejo School in Mexico City.

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LEARNING OPPORTUNITIES IN PRIMARY MATHEMATICS TEACHER EDUCATION

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The results of research on the factors that influence school performance have concluded that teacher quality is the most important factor (Mourshed, Chijoke & Barber, 2010). However, the elements that characterize a "good education" not yet clearly defined, although studies related to TEDS-M have found a link between disciplinary and pedagogical knowledge that show novice teachers and the learning opportunities they had in their initial training programs (Tatto & Senk, 2011).

This paper is part of a research project that seeks to build a diagnostic tool of learning opportunities that have the primary teachers in their initial training. This instrument is a Likert scale questionnaire about the presence of the learning opportunity (1: never to 4: often), designed from the crossing of two categories, (a) the relationship theory - practice, and (b) the two main types of professional knowledge: discipline and teaching. The questionnaire consisted of 82 general and specific math questions, which addressed the issues of numbers and operations, algebra, geometry and data analysis and probability. These questions are classified into 4 types, based on the above dimensions: Theoretical-Discipline, Theoretical-Teaching, Practical-Discipline, Practical-Instruction. The pilot questionnaire was administered to 27 students from 3 different universities in their final years of training.

In the pilot questionnaire, students report more disciplinary learning opportunities, independent of mathematical subject, both theoretical (mathematical concepts and procedures) and practical (development of mathematical skills) (57% disciplinary learning opportunities with rating 4 versus 23% of teaching). Regarding curricular topics, students report more learning opportunities in the areas of Numbers (53%) and Geometry (49%) versus Algebra (33%) and Data Analysis and Probability (42%).

These results show an initial training focused on discipline and not in professional skills related to teaching, and focused on mathematical topics with more weight in the school curriculum.

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A RESEARCH-BASED LEARNING TRAJECTORY OF FUNCTION

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This paper presents a research-based framework that describes learning trajectories of function in four key domains of the concept: *Algebraic Representations, Graphical Representations, Linking Representations*, and *Equivalent Relationships* developed from a study of secondary school students understanding of function in Philippines. The study combined the process-object view (Breidenbach et.al, 1992) and property-oriented view (Slavit, 1997) of function to identify and describe 'big ideas' or 'growth points' towards acquiring an object notion of function. The following research questions guided the development of the framework: (1) What are the 'growth points' in students' developing understanding of function? (2) Is there a typical learning trajectory for these growth points in each of the domains of function?

The research approach was in two phases. The first phase was exploratory and interpretive in order to select and develop the range of tasks, identify a range of students' strategies, and formulate the descriptions of the initial growth points. The second phase involved 444 Year 8, 9 and 10 students from three high performing public schools in the Philippines. The data were collected twice from the same students, the first at the beginning of the school year, and the second five months later, to gain insights into students' movement, if any, in the levels. The main data analysed for this purpose came from written responses, solutions and explanations of the students to the tasks.

The study identified four growth points under Algebraic Representation and has been reported elsewhere (Ronda, 2009), three under Graphical Representations, five under Linking Representations, and three under Equivalent Relationships. The three growth points under Equivalent relationships include GP1: Two functions are equivalent if they have the same set of values or transforms into the same representations; GP2: Two functions are equivalent if they share some properties; and, GP3: Two functions are equivalent if they have the same invariant properties. The framework can serve as analytic tool for describing understanding, assessing learning and for pedagogy.

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BELIEF STRUCTURES ON MATHEMATICAL DISCOVERY – FLEXIBLE JUDGMENTS UNDERNEATH STABLE BELIEFS

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Personal epistemological beliefs are seen as an important predictor as well as a product of learning processes (e.g., Hofer & Pintrich 1997). Nevertheless, research on personal epistemology is confronted with theoretical issues as there is conflicting evidence regarding its coherence and the context-dependence of epistemological beliefs (Greene & Yu 2014). Furthermore, there are methodological issues regarding the techniques of measurement. Especially self-report instruments like commonly used questionnaires with rating scales lack validity (cf. Stahl 2011; Greene & Yu 2014).

We conducted 16 semi-structured interviews on the topic of "mathematical discovery" with pre-service and in-service teachers of mathematics as well as with professors of mathematics. We present excerpts showing that interviewees can hold the same beliefs (as measured on a rating scale) with substantially differing argumentative backgrounds and varying degrees of sophistication. For example, the students B.G. and C.P. both stated that "mathematical discovery is justified inductively". B.G., on the one hand, had unreflected beliefs and was only able to say that she could not imagine mathematicians to get to their results by only reasoning deductively. C.P., on the other hand, showed very sophisticated beliefs and was able to give reasons (mathematical discourse) and examples (four color theorem) for her position; she was even able to defend her position against the possibility of wrong hypotheses.

These results support the claim that the categories frequently used in questionnaires to measure epistemological beliefs are of limited use and have to be differentiated, because the same answer could be given for very differing reasons (cf. Stahl 2011). We argue that these issues can partly be resolved by not only measuring beliefs but also the degree of sophistication with which beliefs are being hold. An according theoretical framework is given by distinguishing between relatively stable "epistemological beliefs" and situation-specific "epistemological judgments" (Stahl 2011).

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CONTENT KNOWLEDGE AND COMMITMENT TO INQUIRY

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The relationship between mathematics teachers' subject matter knowledge and their teaching performance is complex, but there is general consensus that secure content knowledge facilitates good teaching and student progress (e.g. Baumert et al, 2010). Data from an earlier study (Rowland et al., 2001) with 164 prospective elementary school teacher participants support the association between content knowledge and teaching quality. However, 'anomalous' cases were identified, and in particular 4-5% of the sample were judged to teach mathematics well *despite* low scores on the audit of content knowledge. The goal of the current investigation is to identify characteristics of such individuals' teaching that might explain the anomaly.

The focus of this paper is a lesson in which a student teacher, Bríd, invited a 'Fifth Class' (age 10-11) in Ireland to consider the problem of sharing 6 pizzas among 8 children (Corcoran, 2008). Student-pairs offered multiple solution strategies. Bríd proceeded to orchestrate a class discussion in which two outwardly-distinct solutions were reconciled. Bríd had stated that she was apprehensive about teaching mathematics, and her content audit score was significantly below the mean for the study cohort (N=123); in particular, her response to an item on fraction concepts was not secure. The factor, evidenced in the account above, that seemed to facilitate a lesson rich in mathematical connection and cognitive challenge was a strong *commitment to inquiry* on the part of the teacher. We note that Wilkins' (2008) elementary teacher participants' (N=481) content knowledge was negatively related to the use of inquiry-based instructional practice. Whilst one might compensate for the other, a combination is likely to offer optimal conditions for effective instruction.

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EXPLORING PRESERVICE TEACHERS' MATHEMATICAL AND PEDAGOGICAL ENGAGEMENT

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This paper explores how preservice teachers mathematically and pedagogically engage with a non-routine math problem. We examine preservice teachers' solutions to a specific task, their suggested adaptations for using the same task with secondary students, and what they notice about the task's pedagogical potential for teaching and learning mathematics. During video-recorded, semi-structured interviews, six preservice teachers of a secondary school mathematics teacher education cohort were asked to solve a double-pan balance word problem requiring them to find four different weights necessary to be able to measure any whole number amounts from one to 40 grams. They were also asked to consider the appropriateness of the problem for high school students. Preservice teachers' thinking was explored using a think-aloud protocol while solving the problem. Partial transcripts of the videos were then examined using thematic analysis.

The results of this study indicate that preservice teachers were able to solve the problem with most relying on laborious guess and check strategies to do so. Although the problem offers interesting and rich mathematical connections, no preservice teacher explored the problem for more in-depth patterns. They focused mainly on getting a solution or how they thought students would find a solution over exploring the potential mathematics in the problem. Preservice teachers suggested the problem would need to be adapted before it could be offered to students. Adaptations included: reducing the complexity, changing the context, providing instructions on how to approach the problem, and scaffolding the problem with similar but simpler problems. Following Watson (2008) the results of this study suggest that preservice teachers may find it challenging to examine the mathematical aspects of non-routine tasks for themselves before considering how they would present such tasks to their future students. The mathematical aspects, pedagogical aspects, and preservice teachers' interpretations of non-routine tasks are important to consider when determining how teacher educators might better support beginning teachers in learning to navigate through the mathematics and pedagogy of non-routine problems.

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INFLUENCES ON PRE-SERVICE MATHEMATICS TEACHERS' LEARNING ACTIONS

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My research aims at a better understanding of learning actions of pre-service mathematics teachers, studying in the context of German higher education.

In recent years international mathematics education literature states a demand to consider the socio-political perspective (e.g. Pais & Valero, 2012). I am integrating the socio-political dimension using subject-scientific learning theory (see Holzkamp, 1995; Roth & Radford, 2011). Students are conceptualized as subjects within societal and political structures, able to consciously react to their environment, according to what is reasonable from their standpoint. Students` learning actions are based on their premises for action that depend on the possibilities offered by their particular societal context. Hence students have a choice to act. The following research questions arise: (1) What spaces of possibility are available for pre-service mathematics teachers learning activities? (2) What are relevant premises for learning actions to the students?

Eight pre-service mathematics teachers, with different educational backgrounds and at different stages of their current studies, participated in the study. Data was obtained from semi-structured interviews, supported by the assembly of a Mind Map done by the students. Initial data analysis, based upon methods of grounded theory (Strauss & Corbin, 1990), exhibited written exams to be the preferred mode of examination, even though assignments are perceived to foster long-term learning in contrast to the written exam preparation style described by students as "bulimic learning". The ongoing study will result into an empirically grounded theory of pre-service mathematics teachers' learning actions, and therefore contribute to the improvement of teacher education.

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THE IMPORTANCE OF VARIATION AND INVARIANCE WHEN DISCERNING ASPECTS OF THE DERIVATIVE

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Zandieh (2000) describes the concept of derivative as multifaceted and instead of asking if a student understands it, one should try to describe which aspects the student is aware of. An answer to the question in general seems to be the process of differentiation and the ability to perform a procedure (e.g. Jukić & Dahl, 2012). The aim of the current study was to expand this to also include a discernment of the derivative in the graphical representation.

The study comprised 68 students at three different programs (science, engineering, social science) at the Swedish upper secondary school and the method in use was Learning Study (e.g. Lo, 2012). Learning Study is an iterative process where the same content is taught to several groups of students and the aim is to find out how subtle changes in the treatment of the content influence students' learning. The content of the study was the relationship between a graph and its derivative graph and the lessons were designed and analyzed by means of Variation Theory (ibid.). According to Variation Theory, learning is equivalent to discern critical aspects of a phenomenon and for this to occur; the patterns of variation and invariance are determinant.

Students' ways of discerning the derivative were evaluated via pre-, post-, and delayed posttests. The results primarily indicate that a qualitative graphical discernment was associated with: 1. Absence, both written and verbally, of algebra during the lessons i.e. form of representation invariant. 2. A variation of graphs. Even if not intended, teachers' verbal comparison with differentiation rules or an invariance of graphs (e.g. only polynomial) led to a procedural thinking which in turn impeded the discernment in the graphical representation. Extracts from the lessons and students' ways of reasoning at the tests will be presented, compared and discussed at the presentation.

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CREATIVITY AND MATHEMATICS ACHIEVEMENT: EXPLORING CHILDREN'S MATHEMATICAL THINKING AND ACHIEVEMENT VIA PEDAGOGICAL DOCUMENTATION

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Education systems worldwide recognize the value of creative and critical thinking for students' success academically, and in their lives in general. Moreover, early achievement in mathematics has been established as a leading indicator of academic achievement across subject areas (Duncan, 2007). The present paper develops a framework of six dimensions of creativity through a review of psychology literature on critical and creative thinking to explore the role of creativity within mathematical thinking. Creativity is often defined as novel and appropriate solutions to authentic problems. Although throughout the literature the dimensions of creative thinking have been identified differently, there are some common components to creative and critical thinking across the existing literature (Kozbbelt, Beghetto, & Runco, 2010). The following six components have been more frequently operationalized in the critical and creative thinking literature, and will be explored in the present study: Flexibility and Openness, Perseverance, Fluency, Originality, Initiative, and Collaboration. Within the context of education, Smith and Smith (2010) suggest an iterative approach which involves: (1) Defining creativity and bringing it into the classroom; (2) exploring the utility of creativity for academic growth; (3) demonstrating the effectiveness of approaches to use creativity to both enhance and encourage it. The following study serves to explore the first of these approaches to define creativity and explore creativity within the mathematics classroom. Video pedagogical documentation of approximately 50 pairs of students engaged in collaborative work in mathematics is analysed with this framework of creativity and related to measures of students' achievement in mathematics. The findings of this paper will contribute to a greater understanding of the relations between student creativity and achievement, and will be used to further inform educator professional learning.

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GESTURE BLENDS IN AN IWB-MEDIATED CLASSROOM

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This study explores the role of gesture in the learning of mathematics on an interactive whiteboard (IWB) through the framework of conceptual blending. Using video data of Grade 6 students and their teacher, one exemplary gesture-illustrative episode is presented and analyzed. The episode reveals how gestures were used by the teacher on the IWB to relate multiple representations of an angle.

This research contributes to the understanding of how gestures are embodied in an IWB-mediated mathematics classroom. Through qualitative analysis, this research explores the question: What are the roles of gesture in the teaching of mathematics in an IWB-mediated environment? The extant literature connects the use of gestures to improved knowledge retention and development of problem solving strategies (Cook, Mitchell, & Goldin-Meadow, 2008). As well, IWB technologies have been linked to enhanced whole-class discussions, improved visual demonstrations, and classroom management (Bruce, McPherson, Sabeti, & Flynn, 2011).

In the analysis of gesture use, many factors must be considered including the nature of the mathematics problem, its context, and the available tools, such as an IWB. Thus, Fauconnier and Turner's (2002) framework of *cognitive blending* informs this analysis by relating that cognitive processing and meaning-making involves multiple conceptual spaces being 'blended' together to create a new conceptual space.

Using video data from a Grade 6 classroom, one of four gesture-illustrative episodes emerging from this research will be analysed in this presentation. In this episode, the teacher employs complementary gestures representing two ways of conceptualizing an angle. This episode also reveals one instance where gestures used by the teacher were related to actions for: a) creating line segments and rotating objects in the IWB software, and b) identifying angles in other dynamic geometry software packages. This study reveals that gesture use is related to, and informed by, the learners' environment and their associated experiences in learning mathematics. As a budding research area, implications of these results on the design and practical use of IWB and other touch-mediated technologies will also be considered.

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A FRAMEWORK FOR ADAPTING CAOS AND MKT MEASURES

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Teacher knowledge of statistics and knowledge for teaching statistics are compulsory elements for providing statistics instruction as currently recommended in the fields of both mathematics and statistics education. However, literature suggests that there is little known about teachers' knowledge of statistics and the pedagogical knowledge needed for teaching statistical content (Shaughnessy, 2007). Moreover, what is known needs to be further studied. One of the impediments to conducting such research is the availability of appropriate instruments to assess teachers' knowledge of statistics. Turkey presents an appropriate context for this research because "data analysis and probability" has become one of the main mathematical strands in school curricula and the research investigating Turkish teacher's statistics knowledge is limited. Although two internationally well-known surveys exist—*Comprehensive Assessment of Outcomes in a First Statistics Course 4* (CAOS; see DelMas, Garfield, Ooms, & Chance, 2007), and *Mathematical Knowledge for Teaching* (MKT) *Middle School Probability, Data Analysis, and Statistics* (see Hill, Schilling, & Ball, 2004)—they have not been adapted and validated in the context of Turkey. The purpose of this study is to introduce a conceptual framework for *adaptation processes* (all steps involved in adapting instruments from original form to validation of the adapted instruments via field testing in Turkish form) and present the findings from its implementation. In this study, double forward-translation with reconciliation design was employed and then adapted measures were field tested for validation to implement the framework. For validation, both of the adapted measures were administered with randomly selected (n=121) mathematics teachers from public middle schools of a mid-sized province in Turkey. Multiple strategies were employed for the psychometric analysis, and the validity of the instruments was established. The findings indicate that the conceptual framework provided a structure and guidance for adaptation processes to establish both interpretive and procedural equivalence (Johnson, 1998). Psychometric analysis produced same reliability index (Cronbach's alpha) of 0.78 for both measures.

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A 100-YEAR RETROSPECTIVE ON RESEARCH FOCUS AREAS AND DOCTORAL PROGRAMS IN MATHEMATICS EDUCATION

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Historically research efforts have struggled to look back at doctoral programs and the specific research focus areas that have influenced the field of mathematics education (Kilpatrick, 1992). One report on doctoral graduates in mathematics education revealed that more than 150 different institutions of higher education reported awarding at least one doctorate with an emphasis in mathematics education (Reys & Dossey, 2008). This short oral presentation will address the following research questions:

- What have the predominant research focus areas been over time and in what ways have they been affiliated with specific academic institutions?
- In what ways have the leading academic institutions and dissertation advisors contributed to developing generations of mathematics teacher educators?

The data compiled for this ongoing study spans over a century of information. The framework for studying this retrospective involves the evaluation and statistical analysis of doctoral granting institutions, doctorates in mathematics education including insights regarding dissertation advisors as well as the associated research focus areas. The results of cluster sampling techniques have established "peer" institutions in terms of developing doctorates and these findings in fact have shifted in recent decades. Other results indicate institutions that have been impactful in areas such as conceptual understanding and algebra differ from those influential in geometry/visualization, and likewise from teaching and learning with technology.

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VIEWS OF MATHEMATICAL PROCESSES: A FRAMEWORK

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This study focuses on teacher leaders who presented at state-level mathematics conference, and how they viewed mathematical processes.

BACKGROUND

The NCTM (2000) and the Common Core State Standards for Mathematics in the United States emphasize the importance of content and process, advocating classrooms that include problem solving, communication, connections, reasoning and proof, and representations. Teachers and teacher educators still struggle to change classroom practice and are seeking understanding of mathematical processes. Many choices exist for professional development, including NCTM-affiliated conferences, at which teachers may receive varying messages. We investigated the research question, “How do teacher-leaders view the mathematical processes of communication, connections, problem solving, representation, and connections?”

RESULTS AND DISCUSSION

Data included interviews, observations, and session transcripts. Using Merriam’s (2009) three phases of analysis (description, interpretation, and theorizing), we developed *An Emergent Framework for Views of Mathematical Processes* from the data we gathered. We found that participants’ views of mathematical processes varied in the way they attended to sense-making. In the Participatory view, participants focused on getting students’ attention and making mathematics fun and interesting. In the Experiential view, participants wanted students to be involved in mathematical explorations and discourse. In the Sense-making view, the participants focused on students making sense of mathematics through construction. During our session, we will share our framework and how it emerged, along with implications for research and practice, particularly in methods courses.

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INVESTIGATING DYNAMIC PATTERNS OF STUDENT ENGAGEMENT IN MATH CLASS

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In recent years, the concept of engagement has been the focus of many research studies and reports. Studies show that when students are solving a challenging mathematics problem, their affect (emotions, attitudes, beliefs, and values) may influence the nature of their engagement throughout the session. The type of engagement that students experience can be important for their mathematical learning. For learning and doing mathematics, technology, at least potentially, can assist students' problem solving, exploration of mathematical concepts, representations of ideas, participation and general engagement. Researchers (Schorr & Goldin, 2008) have found that SimCalc technological resources can provide affordances that influence middle school students' engagement in mathematics. SimCalc can be described as representationally innovative technology, situated in a genre of software called "dynamic mathematics".

The goal of this study is to characterize the types of engagement that occur in middle school students as they use SimCalc MathWorlds ® software and how it relates to their mathematical learning. Video and audio data of four children during 5 classes, together with their written work, survey data, pre/post math test scores, and students' retrospective stimulated recall interviews were analyzed for evidence of the nature of their engagement according to a coding scheme devised based on Goldin, Epstein, Schorr and Warner (2011) developed theoretical idea of *engagement structures*. *Engagement structures*, as they describe them, are idealized recurring highly dynamic affective patterns that have been inferred from observed behaviors, survey instruments, video data, and interviews with students from various studies. The results of the analysis will contribute to our understanding of the nature of "in the moment" engagement of urban students while working in a SimCalc MathWorlds ® environment.

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TO WORK COLLABORATIVELY ON LANDSCAPES OF LEARNING DESIGN ABOUT SOCIAL PROBLEMATICS

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The research “Learning Milieus as a Proposal from the Critical Mathematics Education for In-Service Mathematics Teachers” (done by the research group EdUtopía –EdUthopy-) allowed building through a collaborative way with mathematics teachers, some teaching proposals that enable some students to strengthen their sociopolitical training. The theoretical background was based on the collaborative work approach, in which Boavida & Ponte (2002) propose valuing the professor’s knowledge as well as the teacher’s knowledge as equally important and the work between these two collectives establish common goals. Besides the project foundations framed in the Critical Mathematics Education; the previous theoretical perspective regards students as sociopolitical subjects and declares six sorts of landscapes of learning (Skovsmose, 2000), surface from the combination between the type of activity and the kind of reference in which the activity is developing.

The methodology was derived from collaborative work. Weekly meetings with teachers were done during the project; these sessions were recorded, and the classes in which teachers compelled the landscapes as well.

In accordance with some results, some landscapes of learning were applied in high school math classes, specifically in seventh and ninth grade. These landscapes were designed with the school’s math teachers by means of collaborative work and considering some of the students’ social problematic detected in conjunction with the teachers. In particular, insecurity, trash, mining, hooligans and future perspectives were the social problematic identified by the social mapping technique and the principal landscape was made to be related with mining. It involved proportion as a mathematical object and it was applied to seventh grade students. The landscape application carried changes in mathematics, mathematics learning and the role of students and teacher conceptions. It generates more student participation and their questions about some enterprise’s landlords real intentions whose enterprise mine the land where students live.

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THE ROLE OF VISUALIZATION IN THE TEACHING OF MATHEMATICS – SECONDARY TEACHERS' BELIEFS

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Visualization, "as both the product and the process of creation, interpretation and reflection upon pictures and images" (Arcavi, 2003), plays an important role in mathematics learning. In students' learning, visualization either could act as a mathematical object itself or as a learning tool to support the understanding of mathematical concepts or as a strategy for problem solving. In spite of this important role of visualization and of the teacher's role in such learning (Presmeg, 2014), we know little about mathematics teachers' knowledge and beliefs regarding the use and functions of visualization that impact on their classroom practices (Calderhead, 1996).

This study explores secondary teachers' beliefs regarding the benefit and functions of visualization in the classroom. The qualitative approach is based on semi-structured interviews involving 12 secondary teachers. The interviews address their use of visualization in teaching fractions, algebra, functions and calculus as well as their beliefs concerning mathematics and mathematics teaching. The interview transcripts are analysed according to grounded theory (Glaser & Strauss, 1967). In open and axial coding we developed categories concerning the function and relevance of visualization for the teachers. In addition, we examined the interviews using theoretical codes, e.g. concerning the relevance of the change between different visual representations.

Findings indicate that although all teachers use visualization in the classroom they differ in their beliefs regarding the roles of visualization in the teaching of mathematics. Different types seem to emerge: E.g. the teacher who uses visualization mainly as a deductive method, the teacher who uses visualization predominantly as a memory aid for applying mathematical procedures or the teacher who considers it as the only way to achieve students' understanding. Initial indications suggest that their use of visualization is related to their confidence in the students' capabilities. At PME, profiles of beliefs regarding the role of visualization in the classroom will be discussed in detail.

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MEASURING STUDENTS' CONCEPTUAL KNOWLEDGE IN SECOND SEMESTER CALCULUS

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The goal of the study was to discuss a new concept inventory measuring conceptual knowledge in second semester calculus (CUIC2) as well as to examine the relationships between performance in second semester calculus, Epstein's Calculus Concept Inventory, CUIC2, and spatial ability as measured by the PSVT:R. The results show correlations between performance on the four measures, and the potential for CUIC2 to be used as a tool for improving instruction.

Calculus is a gateway course for many science, technology, engineering, and mathematics (STEM) majors, and differences in calculus performance have strong influence on students' career choices. Haciomeroglu et al. (2013) found a significant correlation between first semester calculus performance and spatial reasoning ability. We sought to determine if there was a similar correlation with second semester (single-variable integral) calculus. In addition, we wanted to develop a measurement of conceptual knowledge in second semester calculus.

A total of 137 students (almost exclusively STEM majors) were recruited from three sections of second semester calculus at a small university. Calculus performance was measured by Calculus Concept Inventory (CCI) (Epstein, 2007), our Conceptual Understanding Instrument for Calculus (CUIC2), and second semester calculus grades. We included scores on the Purdue Visualization of Rotations Test (PSVT:R) as a measure of spatial ability.

We found statistically significant correlations between CUIC2, CCI, PSVT:R, and second semester calculus grades ($p<0.01$), confirming our hypothesis of a correlation between spatial ability and second semester calculus performance. In addition, an analysis of responses to items within CUIC2 identified common student misconceptions, providing an avenue for potentially improving instruction.

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EXPLORING GEOMETRIC TRANSFORMATIONS BY PROSPECTIVE TEACHERS WITH DYNAMIC GEOMETRY ON TOUCHSCREEN DEVICE

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Studies have shown prospective elementary teachers' difficulties in understanding various concepts related to transformations (e.g., Harper, 2002). In addition, technological devices have reportedly provided much flexibility in the understanding of transformations. However, few studies have focused on how digital technologies may contribute to teacher candidates' understanding of geometric transformations.

We assume mathematical concepts can and should be dynamically represented, and we strongly believe in embodied cognition. Accordingly, to investigate the nature of prospective teachers' understanding of geometric transformations one must observe ways in which they interact, manipulate, interpret, generate and embody representations of geometric transformations using dynamic geometry. Our study explored the conjecture that more flexible treatment of geometric transformations enables greater understanding of those concepts (using dynamic geometry software in which the correlated points on an iPad can be continuously transformed).

Four prospective student teachers who were taking an introductory mathematics course and attained the lowest score (0 to 4 out of 10) in a transformation quiz voluntarily contributed in this study. Participants were asked to work on four *Geometer's Sketchpad*™ tasks involving two-dimensional transformations (viz., reflection, translation, rotation and scaling). The tasks were designed based on the course syllabus and textbook. Interactions were videotaped and analysed. Taking Radford's "sensuous cognition" as our theoretical framework, we explored the semiotic coordination of speech, body, gestures, symbols and tools (Radford, 2012).

We found a high degree of sensuous and body engagement in digital touch-based interaction in the emergence of geometric transformation understandings. Where language and artifacts predominated we detected some interesting compact semiotic nodes along with coordinated gesture and utterance enabled by the tool. In the presentation, our theoretical framework and results will be discussed in further detail.

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TRANSFORMATION OF STUDENTS' VALUES IN THE PROCESS OF SOLVING SOCIALLY OPEN-ENDED PROBLEMS

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Bishop (1991) pointed out the importance of research on values in mathematics education. In a previous study (Shimada & Baba, 2012), we identified three values – mathematical, social, and personal – that we should cherish in mathematics education, and developed three "socially open-ended" problems[1], "Hitting the target," "Room assignment," and "Cake division," in which students' values play an important role. At the end of the previous research, the issue of how students appreciated others' values and transformed their own values in the classroom interaction remained for further study. The objective of the present research is to study this remaining issue: the transformation of students' social values. Here, "social values" represent those related to morals and ethics, such as equality, fairness, and so on, which appear in the process of problem-solving. According to this design, Shimada carried out three lessons with 4th graders at a private elementary school in Tokyo in March 2013. This paper focuses on the lesson using the problem "Hitting the target." At the beginning, some values remained implicit. The students were given explanations about other students' values and interacted with each other. At the end, the students may or may not have transformed their values and chose values that they liked and/or thought to be important after knowing others' values. We analyzed classroom interaction and compared written responses both at the beginning and end of the class. We identified four characteristics of students' mathematical solutions or models and transformation of values: implicit values become apparent through comparison with others' values; there are various mathematical models with the same values; there are both students who transform their values and those who do not; and there are students who did not transform their values but changed their mathematical models.

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[1] A socially open-ended problem is a type of problem (Baba, 2010) which has been developed to elicit students' values by extending the traditional open-ended approach (Shimada, 1977).

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FOSTERING QUANTITATIVE OPERATIONS IN JUSTIFICATIONS OF STATEMENTS ABOUT FRACTIONS OF QUANTITIES IN PROSPECTIVE ELEMENTARY TEACHERS

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It may be tempting to justify solutions to problems about fractions by rules of formal *numerical operations* (e.g., "cross-multiplication"). But in the context of fractions of quantities (e.g., ½ of 3 kg) it would be more appropriate to use *quantitative operations* (Thompson, 1994), especially in teaching. The goal of our research was therefore to design and experiment with a sequence of teaching interventions that would foster justifications based on quantitative operations in prospective elementary teachers.

Our research methodology was Didactic Engineering with Theory of Didactic Situations as a general framework, except that we did not attempt to construe quantitative justifications as an "optimal solution" in an action situation, but only expected them to emerge as a *cultural need* in a learning community. In terms of content, the 8-weeks teaching sequence was an adaptation to teacher training of Davydov et al.'s (1991) "measurement approach" to fractions. It was experimented in a group of 37 prospective elementary teachers. Data consisted mainly of class observations, and participants' responses to assignments and tests. Data analysis was framed by the cK¢ model of mathematical conceptions (Balacheff, 2013). In this presentation we will focus on one of the four dimensions of this model: "operators."

The results were encouraging. For example, in justifying statements such as (1) "14 yards is 8/7 of 12 ¼ yards" or (2) "5 ½ kg is both 2/5 and 4/10 of 13 ¾ kg", 54% of the 37 participants used quantitative operations in justifying (1) and 62% – in justifying (2), and all justifications were correct; 32% used numerical operations in (1) and their solutions would be correct if they were answers to the question: justify why "the *number* 14 is 8/7 of the *number* 12 ¼." The higher occurrence of quantitative operations in justifications of statement (2) (equivalence of fractions) suggests that such problems are more effective than problems of type (1) in initiating the "cultural need" to use quantitative operations in reasoning about fractions of quantities.

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LEARNING TO RECOGNIZE STUDENT REASONING: AN INTERVENTION FOR PRE-SERVICE SECONDARY TEACHERS

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There is a paucity of research examining teachers' attention to student reasoning at the secondary level. While preservice secondary-school (PSS) teachers have the mathematical content knowledge to produce justifications to problems, we explore whether PSS teachers can identify the arguments posed by students. From the perspective of the value of studying videos of students' reasoning (Maher, 2008) we investigate whether PSS teachers' ability to recognize students' reasoning improves after an intervention that called for studying videos of students' justifications and working on similar tasks. Two sections of 3rd year PSS majoring in mathematics, one an experimental (EG) and the other a comparison group (CG), were subjects (Palius & Maher, 2011). Video pre/post assessments of student reasoning were given to both groups and coded for partial/complete recognition of arguments: cases and induction. Data were coded and scored with inter-rater reliability 0.90. The EG group's intervention consisted of five, one hour problem-solving sessions followed by the study of videos of students working on the same problems. Pre-assessment data indicated that the two groups were initially comparable with 23.1% of the EG and 27.3% of the CG participants identifying argument descriptions. Post-assessment data showed that 38.5% of EG increased in the number of complete student argument descriptions, in contrast to none of the CG. Fisher's Exact Test was used to test the hypothesis of no difference in growth between EG and CG groups supporting the alternate hypothesis of higher growth for EG at the 0.0152 level. The study gives promising support for interventions on learning about reasoning using video.

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ARGUMENTATION IN DISTRIBUTED DYNAMIC GEOMETRY

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Argument is a central mathematical activity and recent work has focused on understanding naive argumentation and developing more sophisticated ways of reasoning and arguing (Harel, 2008). In our work, we explore how argumentation is mediated by wholly-online dynamic geometry environments. The foci for the current study involved understanding both the structure of argumentation in online collaborative sessions and how this structure evolves over time.

The data for this work comes eight collaborative Virtual Math Teams with Geogebra (VMTwG) sessions over one month. VMTwG allows students to engage collaboratively in construction, argumentation and proof at a distance. Data was recorded using the VMT Replayer, which replays both discussion and geogebra actions exactly as they occurred. Data was analyzed using Toulmin's argumentation scheme (1969), describing data, claims, warrants, and backing. We then created summary memos describing the interaction patterns, summarized at the level of the session, and developed conjectures regarding stable patterns.

Two primary interaction patterns emerged as a result of our analysis: (1) A focus on verifying the data used to support claims (ie. seeing it for themselves), with the relationship between the data and claims closely tied to "what they see." In these cases, little clarification or explanation regarding the relationship between data and claim was sought. Second, we observed more complex arguments, where students' articulated complex abductive conjectures, included more and wider use of warrants, and actively questioned the validity of data, the appropriateness of warrants and clarity of claims. Across the sessions there was a general shift in the role of data and structure of the argumentation from less to more mathematical. The proposed session will include detailed descriptions of each of these patterns, including excerpts from data. In addition, we will discuss our current investigations that focus on the relationship between discourse and type of task and the particular affordances and constraints to argumentation in this technologically mediates space.

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"THIS MULTIPLIED BY THAT"; THE ROLE OF SPEECH AND GESTURES IN STUDENTS' MEANING MAKING PROCESS

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This paper reports on low achievers' potential to contribute substantially to collaborative processes in heterogeneous groups during the students' work with mathematics. The research question we aim to answer is: how are students' speech, gestures and actions related to their collaborative meaning making process?

We designed the project in a communicative tradition building on work by Sfard and Kieran (2001). We carried out an intervention in a grade 8th classroom. During one week, students were working on a sequence of pattern problems. The final task dealt with the Skeleton Tower, a task from Swan (1984). We offered the students the possibility to represent the problem with artefacts (cubic blocks). The question posed in the task can easily be understood. However, the task is complex, inviting for different approaches and offering many opportunities for communication and thinking.

The concept *semiotic node* developed by Radford (2009) was used to analyse the students' use of speech, gestures and actions in their meaning making processes. Our analysis shows how Kim (a below average achiever) transforms his speech. First, he uses *this* and *that* supported by gestures and the use of artefacts to explain his idea to transform the tower into a rectangle. Later he repeats the expressions, now explicitly using the words *bottom* and *highest*. Both this transformation and his contribution are supported by the availability of artefacts.

Our findings indicate that low achievers can inspire high achievers' thinking, and access to artefacts is a crucial element. The use of artefacts, gestures and speech are intertwined with thinking in the meaning making process. It even seems, that the complexity of the problems is a factor to facilitate low achievers' active contribution, because complex tasks invite for discussion and thinking, and avoiding that some group members immediately know the solution.

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MAKING MATHEMATICAL CONNECTIONS IN PRACTICE

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Making mathematical connections is a process necessary for students to develop a conceptual understanding of mathematics (e.g., Hiebert & Carpenter, 1992; NRC, 2001). To that end, discussions across the literature in mathematics education assume teachers should make mathematical connections in their instruction to support students' learning of mathematics (e.g., Boaler & Humphreys, 2005). However, researchers have yet to explicitly examine the mathematical connections teachers make in practice. For this reason, this present study examines the kinds of mathematical connections three secondary mathematics teachers make in their teaching practice.

Grounded with an interpretive framework, a multiple-case study design was used to study the kinds of connections made in practice. Primary data sources were in-depth, semi-structured interviews and classroom observations. I videotaped each instructor teaching an entire mathematics unit, and I created full transcripts for each classroom observation. To begin analysis, I defined a mathematical connection as a relationship between a mathematical entity and another mathematical or nonmathematical entity, where A is related to B, and identified mathematical connections across the transcripts. Using an inductive and iterative coding scheme, I developed a framework to characterize the mathematical connections made in practice.

The findings of this study comprise a framework that provides categorizations for the different relationships existing between A and B as a means to describe the kinds of mathematical connections made in practice. The teachers in this study made various kinds of mathematical connections for and with their students. The findings also include reasons for some of the possible differences in the kinds of connections made, such as differences kinds of problems used during instruction, the amount of student thinking present during instruction, and the mode of instruction.

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ELEMENTARY PRESERVICE TEACHERS' IDENTIFICATION AND MODIFICATION OF EDUCATIVE FEATURES OF MATHEMATICS CURRICULAR MATERIALS

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We present research on how preservice elementary teachers (PSTs) attended to and modified the educative features of textbook lessons presented in both traditional and standards-based curriculum materials. According to Sherin, Jacobs, and Philipp (2011), teacher noticing compasses two main processes—attending to particular events in an instructional setting and making sense of those events. We consider these processes applicable to the ways PSTs interact with curriculum materials in anticipation of classroom practice. The 135 PSTs were provided 10 mathematics lessons (3 lessons from standards-based and 7 lessons from traditional textbooks) and were asked to review them and choose one to create a detailed lesson plan. They were **then** asked to identify strengths and weaknesses of their lessons and explain any modifications they might make to their lesson plan. Analyses of the PSTs' written responses revealed that of the provided lesson plans, over 75% of the PSTs selected traditional 1st and 2nd grade lesson plans, dealing with addition and subtraction. The most common reasons for selecting a lesson were personal, targeting the grade level they wanted to teach. Lessons were also chosen based on their educative values. The strongest trend within this category was an expressed belief that the concept taught in their lesson plan was fundamental to mathematical understanding. When it came to strengths and weaknesses of their lesson plans, the most common appraisal of lesson plans was as a resource for teaching. Specifically, the built-in differentiation supports of traditional textbooks were highly valued among traditional lesson plans. The lack of this feature in a reform lesson plan was criticized by the PSTs. Specific modifications varied, but adding group work or modifying manipulatives were the most common practices. A smaller portion of the PSTs modified the instructional qualities, such as concepts and strategies. Finally, assessment resources received minimal attention, with most of the focus on summative assessment. This study highlights the value of having prospective teachers examine and reflect on curricular materials to help develop and assess their pedagogical and curriculum knowledge.

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FEATURES OF MATHEMATICALLY IMPORTANT MOMENTS THAT ENHANCE OR IMPEDE TEACHER NOTICING

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In recent years, much research has focused on *professional noticing of children's mathematical thinking* (Jacobs, Lamb, & Philipp, 2010). To understand instances of student thinking that teachers ought to notice due to their potential to support mathematics learning, Stockero and Van Zoest (2013) identified five types of *pivotal teaching moments* (PTMs). We build on this work by examining how three instance characteristics might affect prospective teacher (PT) noticing: (a) instance PTM type, (b) the form of student thinking, and (c) the clustering of instances in close proximity.

During a field experience course, PTs analysed and discussed classroom video to develop an understanding of types of instances that are important for a teacher to notice. The researchers coded the classroom video by PTM type and compared their coding to the PTs' to understand how instance characteristics affect noticing. The analysis revealed that PTs noticed 32.6% of noticing opportunities. Their noticing by instance type was fairly consistent (33.3% to 38.6%), with the exception of inappropriate justifications and contradictions, which were noticed at rates of 22.6% and 15.2%, respectively. Three instance types included both student statements and questions. Within these types, PTs' noticing rate for questions was 36.6%, only slightly better than their overall rate for these types (33.3%). Instances within clusters were noticed 25.8% of the time, compared to 34.6% for non-clustered instances.

The low overall noticing rate highlights a need to incorporate targeted noticing activities in teacher education programs. The findings that some instance types were noticed less frequently than others and that noticing is lower within clustered instances have implications for video selection and suggest a need to provide opportunities to make sense of what is important in complex classroom interactions.

Acknowledgement

This material is based upon work supported by the U.S. National Science Foundation (No. 1052958). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

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DIDACTICAL HANDLING OF A GAP BETWEEN A TEACHER'S INTENTION AND STUDENTS' MATHEMATICAL ACTIVITY

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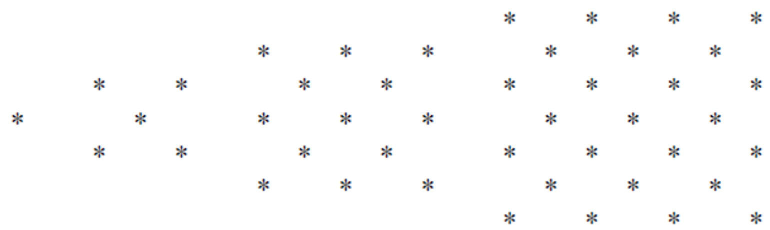
The concept of *milieu* (Brousseau, 1997) models the elements of the material and intellectual reality on which the students act when engaging with a mathematical task. The research question addressed in the reported project is: *What features of the milieu constrain students' establishment of algebraic generality in a given shape pattern?*

Research participants were a group of three student teachers and a teacher educator of mathematics. The data are transcripts of student teachers' (video-recorded) engagement with a task on algebraic generalisation of a shape pattern, with teacher intervention. This is the pattern they were supposed to generalise:



The task was to find first a recursive *formula*, then an *explicit formula* for the general element of the sequence mapped from the shape pattern. From the numerical values of the elements of the pattern, the following recursive formula was established by the student teachers: $(n-4)+s_{n-1}=s_n$. In their attempt to find an explicit formula (expressed in the task as 'a connection between position and numerical value of the elements'), the student teachers calculated the difference between the numerical value and position of elements, $f(n)-n$. This inadequate approach was however not noticed by the teacher. In an attempt to help them to progress, the teacher changed the milieu by directing attention towards type of growth of the sequence at stake. This I interpret as a *metamathematical shift* (Brousseau, 1997), where the original task is substituted by a discussion of the logic of its solution.

In the reported episode there was no focus on connections between recursive and explicit formulae (neither on arithmetical nor iconical properties). This condition is a weakness in the milieu for pattern generalisation (Lannin, Barker, & Townsend, 2006).

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TECHNOLOGY USE AS IN IMPORTANT ISSUE IN THE STRATEGY AND EDUCATIONAL MATERIAL DESIGN

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In the research project we report at this short oral session are interested in determining what use of research knowledge is in the incorporation of educational technological tools. Fullan & Stiegelbauer (1991) mention that when teachers are asked to use new technological tools to facilitate the learning process is necessary to identify an attitude change from the teacher in the following directions: beliefs, attitudes or pedagogical ideologies content knowledge in their specific area, pedagogical knowledge in their teaching practices, and strategies, methods or different approaches to adapt practices. Particularly, in this time we studied the interaction between teachers and researchers in an online seminar with the theme of how the representations could be used by the students when constructing a mathematical concept and played a significant could be role and that they are part of their conception. (Hitt, 2006). We analyzed 163 entries in the seminar discussion forum with grounded theory (Trinidad, et al, 2006) and between the main results in the category of beliefs in the use of technology, we find two trends, those who do not use it for context difficulties (lack of resources and technologies) and those who think that they must use technology once you have learned the math. This first result points towards other questions that follow in our research: Which would be the minimum knowledge needed to design methods and strategies with new technologies? What would that be the theoretical framework in the design of materials and strategies that incorporate the use of technological tools?

Acknowledgements

The work discussed in this paper was supported with funding from the Research and Graduate Studies Secretariat of the National Polytechnic Institute, project 20140544.

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SEQUENCING THE MATHEMATICAL LEARNING PROGRESSION THROUGH VERTICAL ARTICULATION DURING LESSON STUDY

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With the recent release of the *Common Core State*, researchers and mathematics educators are looking at the standards to help teachers in mapping out the learning progression that will guide the sequence of mathematical concepts that is crucial in building mathematical understanding. The notion of learning progressions are important for teachers to understand as a guiding post for analyzing the learning progression and to tailor their teaching sequence so that extends previous learning while avoiding repetition and large gaps (Confrey, 2012). This leads to these important questions: 1) How did the vertical lesson study help teachers sequence the learning progression? 2) What type of mathematics knowledge for teaching is critical for teachers as they map out the learning progressions in teaching early algebra? This research focuses on case study of a vertical team and the analysis of a research lesson that was taught in three different grade levels. The lesson study team consisted of a third grade teacher, two 6th grade teachers, two 8th grade teachers and a special educator. The data sources included video clips from the research lessons, student work, teacher reflections from the Lesson Study, and researchers' memos. The three recurring themes from the analysis of the research lessons included: a) Vertical articulation of standards through the use of a problem with multiple entry points; b) Developing and anticipating modelling strategies—Teaching and learning through multiple representations; c) Teachers learning about student progression of mathematical ideas. Some important implications to promoting the learning progression, including giving teachers an opportunity to re-experienced algebra as learners. Teachers learned to grapple and solve algebraic problems while analyzing students learning progression and discussing previous and future learning goals, which allowed teachers to make deeper vertical connections.

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TENSIONS IN ENACTED SOCIOMATHEMATICAL NORMS WHEN WORKING ON VOCATIONAL ORIENTED TASKS

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Students in vocational programmes in grade 11 in Norway have to attend a mathematics course, which is supposed to be directed towards the students' future vocations. This presentation reports from a study focusing on enacted sociomathematical norms in mathematics lessons while students were working on vocational oriented tasks. Sociomathematical norms are described as ''normative aspects of mathematical discussions that are specific to students' mathematical activity (Yackel & Cobb, 1996, p. 461)''. The research question is ''which tensions emerge between the vocational context and the class' sociomathematical norms when students work on vocational oriented mathematical tasks?''

Together with the mathematics teacher I developed vocational oriented mathematical tasks for a class consisting of 15 boys in the technical production education program. The mathematics lessons were video recorded during 11 lessons over 6 months, data reduced and transcribed. The social and sociomathematical norms of the class were identified and coded from the classroom transcriptions with codes expressing possible norms. Then the situations were reanalysed by studying interactions between classroom participants. If a possible norm was broken or challenged in an interaction, the participants' reaction would decide if the norm would be kept, revised or restated. Enacted sociomathematical norms such as how accurate one should be, what counted as an efficient solution method, and how answers should be explained using mathematical concepts and statements were identified. When the students engaged with vocational orientated mathematics tasks they experienced tensions between the expressed sociomathematical norms of the teacher and their experiences from vocational training lessons. For instance the students often truncated partial answers to nearest mm, used 3.14 as π and rounded answers, while the mathematics teacher did not round partial answers. There were also tensions between the pupils and the teacher in what counted as a mathematical explanation of the solution of the task. The pupils often argued their explanations while referring to their vocational training lessons. In vocational practice lessons a correct solution of the problem, regardless of method, was sufficient, but in the mathematics class explanations were expected, and the mathematics teacher preferred a gradual move towards use of algebraic methods in the solution. These tensions lead to cases in which the students questioned the vocational relevance of the mathematics tasks.

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WHEN MATH TEACHERS TALK: CONNECTING 'PEDAGOGY' AND TEACHING PRACTICES

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This project is part of a larger study that seeks to describe mathematics teaching practices across Canada. In this paper, we are reporting on data gathered over two years with four Ontario Grade 7 mathematics teachers. We seek to understand their views of important components of mathematics teaching, teaching dilemmas, and ways that they negotiate their understanding of teaching and their practice of teaching.

Our study takes an enactivist approach. Recalling Maturana's statement that "everything said is said by an observer" (1987), we focus on teachers' observations of their own and others' teaching. We seek to understand teachers' pedagogy and teaching practices and the ways in which these interact. In this study, we use the term "pedagogy" to mean the ways teachers describe what they believe should be happening in classrooms. Pedagogy may be seen as a set of implicit cultural practices that teachers believe will facilitate student learning (Anderson-Levitt, 2002). We use the term "teaching" to mean the observable practices of what teachers actually do in the classroom.

The first phase of our data collection includes three video-recorded lessons of each of the four Grade 7 teachers. The second phase includes three group meetings with the four participants during which time they observed and discussed the classroom video recordings. The thematic analysis of this second phase is the focus of this paper.

The participants describe a variety of practices they observe recurring in their teaching, and through discussion they connect these practices to pedagogical understandings of teaching mathematics. These understandings are also evident when they discuss aspects of pedagogy they value and are working on improving. In our analysis of these discussions, one prominent theme that we will present is the importance of student voice and the ways this is demonstrated in practice through the use of classroom discussion, student representations and models, and student explanations of their thinking. The teachers also acknowledge that this is an aspect of their practice that they would like to improve, and they note that the recurring group discussions of their teaching provide further insights to assist with this.

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ON THE DIFFICULTY WITH DECREASING AND INCREASING QUANTITIES BY A FRACTION

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INTRODUCTION

For quite a while, we noticed that decreasing and increasing quantities either by a fraction or a percentage is a difficult item type for many students in Taiwan, from primary levels to graduate schools included. We found few studies that directly tackled this type of problems (Petit, Laird & Marsden, 2010). Hence, a study was launched of which only the part related to the performances of a group of primary students were reported here.

METHOD

Several problems were given to over 80 fifth and sixth graders from a rural district in eastern Taiwan. A sample item is as follows: *Mary took a piece of ice from the freezer and let it melt to water. When she compared the volumes of both ice and water, she noticed the volume of ice decreased by 1/12 after it was completely melted. She then put the beaker of water into the freezer. What change would she found in the volume of ice when compared to that of water? Would the volume increase or decrease, and by how much?* Nine students were interviewed in depth and all students were given self assessment assignments that were intended to enable them to identify their own errors.

RESULT AND DISCUSSION

It was found that they encountered a variety of difficulties, ranging from misinterpreting the problem, misunderstanding the part-whole relationship, mixing up the multiplicative structure with the additive structure (Harel & Confrey, 1994), improper application of the concept of conservation, and the following of the same A same B intuitive rule (Tirosh & Stavey, 1999). These difficulties are also partly related to the situation in which the problem was phrased. Educational implication derived from the findings will be discussed in the full version of the paper.

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IDENTIFYING WHAT IN STUDENTS' MATHEMATICAL TEXTS?

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School mathematics, in particular problem-solving activities, often involves the reporting of mathematical work in written form. In mathematical texts, produced by students, a range of different modes of communication (language, symbols and images) can be used. Morgan (1998) argues that in a school situation the mathematical text is most commonly addressed to a teacher, and her research shows that teachers often assume these texts to be transparent and unproblematic records of students' intentions and hence their mathematical understandings. Barmby et al. (2007) are among those who argue against this by stressing that, assessing students' mathematical understanding is a complex enterprise that requires multiple sources of information.

This study examines students' written mathematical communication in school context and is guided by the question; *what can a mathematical text say?* A sample of 553 mathematical texts, produced during a problem-solving activity, by around 300 students aged 7-12, was collected and analysed. The analysis adopts a multimodal approach in which the semiotic resources used by students are identified in graphic units. The interests that motivate the graphic units are categorised *as interpreting the problem, describing a calculation or exploration, and stating an answer or result*.

The findings show that a large number of semiotic resources are used, across the sample. Images, the most widely used resource, are used for a number of different purposes. In the texts students display very different ideas on the *what* and *how* of mathematical communication. Ideas range from providing an ignorant reader with explicit and detailed descriptions of the problem, strategies and calculations along with a clearly stated result, to a combination of text fragments that require quite a knowledgeable reader. When studied closely the texts often reveal sophistication in response to different ideas on the purpose of the problem-solving activity as well as the purpose of communication. These different ideas suggest that students' texts say more about their understanding of *how* and *what* to communicate in mathematics than they say about their mathematical understanding.

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TRAINING EFFECTS ON THE DEVELOPMENT OF TEACHERS' DIAGNOSTIC COMPETENCES

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Teachers' diagnostic competences – i.e. the ability to accurately assess students' learning processes and outcome– are a precondition for effective teaching. Still, research hints at deficiencies in this area. This study aims at examining how teachers' diagnostic competences in a specific subject matter area (mathematical functions) can be developed through a teacher training. Since existing instruments to measure diagnostic competences (e.g. Buchholtz et al., 2012) are not sensitive enough to evaluate a specific training course, we developed a questionnaire.

In a control-group design, 26 secondary teachers attended the training on three meetings during six weeks. Both groups received the same training as a treatment with the focus on pedagogical content knowledge (Shulman, 1986) for diagnosing students' competences with respect to functions. By providing the experimental group with information on their own students' achievement, we examined whether this component additionally supports the development of diagnostic competences.

To capture the effects of the teacher training, we developed an online questionnaire containing open and closed items to be filled out before and after the training. It covers the two competence dimensions of accurate judgment of a) student achievement and b) the diagnostic potential of tasks. By means of qualitative analysis following grounded theory, we developed categories to describe teachers' diagnostic judgments. These were applied by two raters with satisfying agreement.

With this instrument, we identified different types of competence development and established *diagnostic profiles* for each participant. Effects of the training are noticeable when comparing diagnostic profiles before and after the training. Some of the most prominent types are: (1) A teacher has the same diagnostic profile before and after the training but additionally applies pedagogical content knowledge after the training. (2) A teacher's diagnostic profile is *analytical* before, but additionally contains *descriptive* aspects after the training. The detailed test structure and different types of competence development will be presented at PME.

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STUDENTS' ALGEBRAIC REASONING IN CLASSROOM USING LESSON STUDY AND OPEN APPROACH

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Algebraic reasoning is a gatekeeper for students in their efforts to progress in mathematics (Greenes et al., 2001). It is a process in which students generalize mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways (Kaput & Blanton, 2005). This study conducted within the context of school participating in the project of teaching professional development through Lesson Study and Open Approach, launching by Center for Research in Mathematics Education (CRME) in 2013-2014 academic year. Lesson Study and Open Approach, as innovations, were implemented for providing the students with the opportunities to use algebraic reasoning in mathematics classroom. This paper aimed to survey 7th grade students' algebraic reasoning in this context. Teaching experiment and case study was used as research methodologies. Video analysis was used to analyze students' algebraic reasoning. The partial findings showed that students used algebraic reasoning during the 2nd and 3rd phases of Open Approach (i.e. students' learning by themself, whole class discussion and comparison). For future research, the teachers' use of students' algebraic reasoning for improving the lesson should be study.

Acknowledgements

This research is supported by Center for Research in Mathematics Education, Khon Kaen University, Thailand.

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PROCESS-OBJECT CONFLICTS IN STUDENT PERCEPTIONS OF THE INDEFINITE INTEGRAL AS A CLASS OF FUNCTIONS

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The definite and the indefinite integral are key concepts, which, even though denoted by the same symbol, are epistemologically different. The definite integral is a real number, whereas the indefinite integral is a set of functions with the same derivative. While students' perceptions of the definite integral have been the focus of several studies, the indefinite integral remains a much understudied concept. Our study examines a part of students' concept image (Tall & Vinner, 1981), their personal definitions of the indefinite integral, and offers a categorisation of the students' definitions according to Sfard's (1991) triad of conceptual development (*interiorisation, condensation* and *reification*).

Participants (Thoma, 2013) were ten mathematics undergraduates with various levels of performance on the formal examination of the Integral Calculus course. They completed a questionnaire consisting of nine tasks designed to examine perceptions of definite and indefinite integrals. They were then interviewed. One of the tasks aimed to explore students' perceptions of the constant C in the expression: $\int 2x dx = x^2 + C$. Analysis suggested that four students had a *reified* image of the indefinite integral. These students wrote and spoke meaningfully about the indefinite integral as an object and as a set of functions with the same derivative. The other six students' responses showed evidence of a *condensed*, not yet *reified*, image (inverse process of differentiation, associations with one function). These students had yet to realise the importance of the constant C in the indefinite integral expression. Interestingly though, evidence of an emerging *reified* image of the indefinite integral as a set of functions materialised during the interviews with two of these students (as they engaged with the graphical representation of the concept). This small-scale study provides some insight into students' concept images of the indefinite integral, especially in relation to the ontological shift implied in the transition from definite to indefinite integral.

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A METHODOLOGY FOR INVESTIGATING TEACHERS' MATHEMATICAL MEANINGS FOR TEACHING[1]

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We argue that a focus on teachers' mathematical meanings rather than on their mathematical knowledge holds greater potential for connecting what teachers know, what they do, and what students learn.

We designed an instrument, *Mathematical Meanings for Teaching secondary mathematics* (MMTsm) to focus on teachers' mathematical meanings because meanings, by their very nature, are implicative of action.

We ground the design of assessment items and methods for interpreting teachers' responses in a theory of meaning based on ideas of assimilation and schemes (Piaget & Garcia, 1991; Thompson, 2013; Thompson, Carlson, Byerley, & Hatfield, in press). This theory provides a coherent system for addressing relationships among teachers' understandings, meanings, and ways of thinking (Thompson, et al., in press; Thompson & Harel, in preparation). To design items that have the potential to reveal teachers' mathematical meanings, we begin with epistemic models of meanings teachers might hold with regard to the mathematical idea being targeted. Examples of items and scoring rubrics will be discussed in the presentation, as will results from applying this methodology.

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[1] Research reported in this article was supported by NSF Grant No. MSP-1050595. Any recommendations or conclusions stated here are the authors' and do not necessarily reflect official positions of the NSF.

COMPETING PHILOSOPHIES OF MATHEMATICS: VIEWS FROM THE OUTSIDE AND INSIDE

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More than 20 years ago Ernest (1991) argued that philosophies of mathematics education profoundly influence the way curriculum is constructed and enacted. He claimed that the hierarchical content-driven view of mathematics prevailing in schools was both a product and a reification of absolutist philosophies of mathematics, whereas more progressive, learner-centred approaches built on and suggested relativist philosophies such as social constructivism. These two seemingly dichotomous views on the nature of mathematics have been, if not at the heart of the so-called *Math Wars*, at least significant contributing factors. In this theoretical presentation I suggest that neither absolutist nor relativist philosophies of mathematics alone can account for the nature of mathematical discovery and the connectedness of mathematical knowledge. Rather absolutism and relativism are flip-sides of the same coin and arise from outside and inside views of mathematics respectively.

Viewed from the outside mathematics appears unified, undisputable and inanimate. This is the typical view presented in traditional school mathematics classrooms. Viewed from the inside mathematics appears segmented and open to question, a product of human reasoning (Latour, 2013). This is the typical view espoused in more reform-oriented classrooms. While recent curriculum documents such as the *Common Core State Standards* (CCSS, 2010) have attempted to describe both perspectives through the inclusion of both proficiencies and content descriptions, they have failed to articulate a clear philosophical basis. Hence I argue that the valued reforms run the risk of being derailed by political or partisan agendas, particularly those arising from perceived inadequacies in student achievement in national and international tests.

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PLACE-VALUE UNDERSTANDING FOR STUDENTS WITH LEARNING DISABILITIES

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Place-value is a fundamental concept that underlies the structure of our number system. Despite being a basic concept of mathematics it is complex and multifaceted, and contains the following four elements: base-ten numeration, place-value numeration, counting, and the flexible composition and decomposition of numbers (Van de Walle, Karp, & Bay-Williams, 2010).

Few studies have investigated the place-value knowledge of students with learning disabilities, and those that have present contradictory viewpoints about it, possibly because these different studies investigated different facets of place-value. In order to try and resolve these contradictory view points, this study investigated all four elements of place value while addressing the research question: What do 4th grade students with learning disabilities (LD) understand about whole-number place value?

In this study, fifteen fourth grade students with LD participated in individual clinical interviews. The interviews examined the students' understanding of place-value using 3 types of problems: counting tasks, mental arithmetic, and word problems. The students' responses were analyzed for accuracy, strategy use, and errors.

There was a great diversity in the students' understanding of place-value, and five different profiles emerged for what students understood and did not understand about place-value: (1) Good place-value understanding; (2) Reliance on algorithms; (3) Counting errors; (4) Beginning place-value understanding; and, (5) Concatenation of digits. These profiles reflected a range of understandings across the four aspects of place-value.

Students with LD's difficulties with the different facets of place-value affected their ability to correctly solve the tasks. Therefore when studying or teaching students with learning disabilities we need to assess all aspects of their place-value understanding, as well as their calculation and problem-solving skills.

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PROSPECTIVE MATHEMATICS TEACHERS' PERCEPTIONS OF MATHEMATICAL MODELS

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Mathematical Modeling (MM) has become increasingly integrated into school mathematics curricula. Thomas & Hart (2010) suggest that students' experiences with MM as learners of mathematics and their orientation toward MM are linked. Therefore, it is important to develop an understanding of the prospective mathematics teachers' (PMTs) knowledge of and ability to utilize the MM process to solve real world problems. This research adds to the research conversation by examining: How does focusing on MM in a methods course impact PMTs perceptions of MM?

The study examines 20 prospective mathematics teachers (PMTs) perceptions of MM before and after a methods course that contained focused instruction on MM. MM is the mapping of an extra-mathematical (real world) problem to a mathematical domain. In the mathematical domain actions are untaken, outcomes are determined, and these outcomes are mapped back to the extra-mathematical domain giving a solution (Niss, Blum & Galbraith, 2007). Pre- and post-course data were collected concerning PMTs perceptions of what is a model and the MM process.

PMTs responses to questions about MM were open coded. Themes were developed using the constant comparison method (Glaser & Strauss, 1967). Five themes emerged (percent response before course, percent response after course): (1) use models to understand mathematics (50%, 0%), (2) use models to showing the value of mathematics (35%, 0%), (3) MM involves high cognitive demand (15%, 12.5%), (4) MM mathematizes real world events (25%, 94%), (5) MM uses mathematics to find real world answers (15%, 44%). Two of these categories (4 and 5) represent the traditional view of MM; the PMTs' perspective of MM before the course was not in line with these views since only 25% recognized the role of mathematization in MM and 15% recognized MM as using mathematics to find real world answers. After instruction, these two categories increased to 94% and 44% respectively, evidence that courses with a MM focus can lead to increased understanding of MM for PMTs.

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A STUDY ON EYE MOVEMENTS OF ADDING AUXILIARY LINES IN GEOMETRIC PROOF'S FIGURES: THE EFFECT OF ILLUSTRATING THE AUXILIARY LINES

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The aim of this study is to investigate the effect of adding auxiliary line(s) in a figure for completing a geometric proof by means of the eye-tracking technology and the approach of thinking aloud, especially focusing on the influence of the change of reading focus of the given geometric proof texts or figures with or without auxiliary lines in order to grasp whether adding auxiliary lines could lower ones' cognitive load. Literatures about eye movement associated with text reading point out that the initial processing stage of a lexical process is measured by means of the first fixation duration (FD) or the first gaze duration(FGD or GD) (Juhasz & Rayner, 2003).

The results show that: (1) the total FD of all subjects on the figure area of the texts with auxiliary lines are significantly greater than the text area statistically. Epelboim and Suppes (2001) also found that when the necessary auxiliary line is not given in the figure, the experts could construct it in their mind as there appears the track of the auxiliary line through eye scans, while the novices do not have such a moving trajectory; (2) during reading the text with auxiliary lines by means of thinking aloud, the time ratios of FD of all subjects of the major area of interest are significantly greater than the ratios of non-major area of interest statistically. It reveals that the effect of the auxiliary lines might be helpful to construct and link the spatial relationship of geometric elements in each message.

Gal and Linchevski (2010) considered that the visual perception and knowledge representation (VPR) of the geometry contain three consecutive stages, and the comprehension difficulties of geometric illustrations may also occur in the three stages. Therefore the message needs to be re-read due to the limitations of working memory, which suggests that the design and teaching of auxiliary lines in geometric proofs be worth of further studies.

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VIRTUAL MANIPULATIVES' AFFORDANCES INFLUENCE MATHEMATICAL UNDERSTANDING

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THEORETICAL BACKGROUND

When considering mathematics apps, educators and app developers must evaluate both mathematics content and technological *distance*, which is the gap between a student's understanding of how to interact with the app and how the app requires the student to interact with it (Sedig & Liang, 2006). Many mathematics apps include virtual manipulatives (VM). Moyer-Packenham and Westenskow's (2013) recent meta-analysis of VMs suggests that their affordances may influence students' learning of mathematics. However, qualitative research about how specific affordances affect students' learning of mathematics is lacking. This project examined how students' experiences with affordances of a VM mathematics iPad app revealed, concealed, and developed mathematical understanding.

METHODS AND FINDINGS

Thirty-two students aged 7-8 years old interacted with six VM mathematics iPad apps in 45-minute video-recorded clinical interviews. Learning progressions were created to describe mathematical knowledge required by each app. Video analysis involved identifying the learning progression stage and affordances evident in every interaction with the app. Three cases were selected based on their demonstration of how affordances of the app MotionMath Zoom influenced students' mathematical understanding. The first student fluently iterated strategies to demonstrate and develop his mathematical understanding. The second student's struggle to interact with the app partially concealed his mathematical understanding. The third student barely attempted to bridge the technological distance, concealing much of his mathematical understanding and hindering its development. Preliminary results suggest that students experience VM mathematics apps' affordances differently depending on the appropriateness of the mathematical content and technological distance. Future analyses will examine additional students and apps, and correlations among demographic factors and students' interactions with the apps.

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OPTIMIZATION GAME: A METHOD FOR PROMOTING AUTHENTIC MATHEMATICAL COMMUNICATION

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Although students are expected to learn authentic mathematics in general, the meaning of the term "authenticity" is not quite clear in mathematics education research. Weiss, Herbst, and Chen (2009) indicated that the term "authenticity" has at least four meanings based on the following: (1) whether classroom mathematics is rooted in a real-world context (AM_W); (2) whether the intellectual precision of classroom mathematics is as high as that of mathematics as an academic discipline (AM_D); (3) whether student practices resemble those of working mathematicians (AM_P); and (4) the theoretical stance of always regarding students as mathematicians (AM_S). Each authenticity seems to have educational value. It is therefore worth experiencing some authenticities simultaneously if possible. What situations allow this to occur? The present study explores this research question.

For this, the potential of an activity style referred to as the "optimization game" is examined. The game was designed to simultaneously provide three dimensions of authenticity (AM_S, AM_P, and AM_D). It is a team-based competition for discovering better solutions to a mathematical problem compared to other teams. As there is no requirement for the best solution, this increases the possibility that more proofs and refutations of the validity of the solutions occur, thus deepening students' mathematical knowledge.

The authors conducted a test game with four Japanese undergraduate students to test the game operation and verify whether the anticipated advantages were gained. Through the analysis of the video-recorded data of the test game, we found that (1) the game style has the potential to promote AM_S, interpreted as private discussions within each team, but (2) it does not have enough potential to promote AM_P and AM_D, interpreted as certain types of public discussions among teams, especially like those described by Lakatos (1976). The reason that the result was disappointing seems to be that the role of the teacher in the game was not well-defined. Thus, further studies are needed to elucidate the possible role of the teacher as the impetus for breakthrough ideas on improving solutions.

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MULTICULTURALISM IN MATHEMATICS EDUCATION

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Maths teachers, especially those working in secondary schools, feel the necessity for training and materials which reflect the needs of their classes in terms of linguistic and cultural differences. Their pupils from minority cultures and/or those with a migrant background encounter even more difficulties than their native classmates in acquiring fundamental maths skills. A team of seven European partners in the EU-funded project M³EaL investigates the situation regarding maths teaching with respect to these issues, and develop training materials for pre-service and professional development courses.

THEORETICAL BACKGROUND

Several studies (e.g. Barton, Barwell and Setati 2007) show a need of mathematics teachers for input with respect to linguistic and cultural differences in their classrooms. Learning a new language and culture at the same time as you learn mathematics places double burden and challenges on immigrant pupils (Norén, 2010).

RESEARCH TOOLS AND METHODS

We developed a questionnaire to map teachers' experiences and attitudes towards teaching in multicultural settings in six European countries. This Short Oral will present the situation in Austria. The four-part questionnaire intended to find out basic information about the teacher, the school and the social background, the teachers' prior experiences, and the support available for teachers working in multicultural settings. Attention is paid to the situations specific for working in such conditions as well as to support of any kind that such a teacher has and/or would like to have.

RESULTS

About 83% of the teachers taught mathematics to migrant students at some point in time. Almost none of their schools offer a special programme for such pupils. Typical issues that have been reported by teachers were difficulties of such students in understanding complex word problems and in expressing mathematical connections with a satisfying degree of exactness. The majority of teachers with migrant students wished for concrete didactic units from various cultural backgrounds.

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MULTIPLE REPRESENTATIONS USED BY RWANDAN PRIMARY TEACHERS IN MATHEMATICS LESSONS

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The Multiple Representations in teaching are important to change students understanding and expressing their ideas through different ways. This research aimed at analyzing multiple representations used by Rwanda teachers in mathematics lessons. In particular, Lesh Translation Model was used to identify teacher's representation in their planning, use of them in teaching and interact with students, as well as translations among the used representations. In order to identify the representations and their use analysis of lesson plans, lesson observation and teacher's interviews were used .The study was administered to 10 fifth grade teachers. Data was analyzed qualitatively and results from the themes show that Rwandan primary mathematics teachers in the visited schools have narrow conception about representations, use and translations among the used representations.

Representations are essential part of learning and doing mathematics. When teachers use the variety of representations to make learning more meaningful to students it will motivate students to see the relevance of studying mathematics and apply what they learnt in their daily life. Although 97% of Rwandan primary teachers qualified, most of them use chalk and talk teaching method and don't give student time to use their ideas. The use of representations allows teachers to explain mathematical concepts in ways that are more accessible to students. The main question that needed to be explored was how teachers use representations and create mathematical activities that would give students an opportunity to demonstrate their ability. Referred to representations proposed by Lesh, the language and symbolic representations dominate the lesson plans. Teachers used representations to demonstrate the steps in procedure as clearly as they can, and students practice applying procedures. There is a little or no discussion of student's ideas. Teacher's language dominates classroom interaction, talking most of the time pupils talk much less, in most cases only in response to teacher prompts. Teachers faced challenges in translating or connecting the mathematical content to a context that had meaning for students, and using symbolic representations basically looking for correct responses; what they did not look for, however, was how the students came up with the answers and what the answers meant to the students.

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GRADE 3-4 STUDENTS' FORMS OF REASONING

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Noticing, generalising and justification are three important components for students reasoning (Carpenter, Franke & Levi, 2003; Ellis, 2007; Lobato, Hohensee & Rhodehamel, 2013). However there are diverse views on what it means exactly for learners, its place in intended mathematics curricula and its visibility in mathematics classrooms. The *Mathematical Reasoning Professional Learning Research Program* [MRPLRP] was conducted to investigate elementary children's reasoning and teachers' perceptions and teaching of reasoning. In this Short Oral Communication we present findings on children's reasoning. Variation theory, including application of dimensions of variation (Lo, 2012) was used to analyse children' reasoning from a demonstration lesson conducted in Grade 3-4 classrooms in three Australian primary schools and one Canadian elementary school. The demonstration lesson used the problem "What else belongs?" (Small, 2011). It required comparing and contrasting, noticing common properties, forming a conjecture about common properties and justifying this conjecture.

Analysis of the children's reasoning resulted in clarification and elaboration of classes and levels of reasoning developed by Ellis (2007) and Carpenter et al. (2003). We found that Ellis' (2007) generalising actions were better understood and applied when analysing how children conducted comparing and contrasting and what they noticed. We also further developed levels of justification to distinguish between explaining and convincing. The framework for children's reasoning that emerged from the analysis of the task that involved noticing common properties will be presented along with illustrations of children's reasoning for each element of this framework evidenced during their work on the tasks. The potential for using this framework for analysing children's reasoning for tasks concerning noticing and generalising relations will be discussed.

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INHIBITING THE NATURAL NUMBER BIAS IN RATIONAL NUMBER TASKS: TOWARDS A COMPREHENSIVE TEST INSTRUMENT

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Understanding rational numbers is an essential part of mathematical literacy. Still, research has shown that learners have many difficulties with them. This is often - at least in part – attributed to the "natural number bias", namely the tendency to inappropriately rely on natural number properties in tasks with rational numbers (Ni & Zhou, 2005). The natural number bias has been described in three aspects: density, size, and operations (e.g. Vamvakoussi & Vosniadou, 2004). Because there is no single study that investigated how these three aspects are related, we created an instrument that measures 6th graders' ability to inhibit the natural number bias in each of these three aspects. Doing so we wanted to answer the question whether children's rational number reasoning – more specifically their ability to inhibit the inappropriate use of prior natural number knowledge - can be described by a model with one latent variable. This would mean that students who have a better rational number understanding in one aspect would also perform better in tasks with other aspects, or that a certain level of understanding in one aspect is a precondition for gaining understanding of another aspect. Participants were 230 6th graders from eight different elementary schools in Flanders who solved the paper-and-pencil test with no time restriction.

The reliability of the test was high (Cronbach's alpha = .86). By conducting an IRT-analysis, the Rasch model, a two-parameter logistic model (2 PL model) and a three-parameter logistic model (3 PL model) were fitted and compared. Based on an ANOVA on the Bayesian Information Criterion-scores, the results showed that the 2 PL model (BIC= 10461,46) had a significantly ($p<.001$) better fit for the data than the Rasch model (BIC=10473,66) and the 3 PL model (BIC=10637,72). The 2 PL model showed that the density items have the highest difficulty levels, while size items have the lowest difficulty levels. The operation items have varying difficulty levels.

By finding a fitting IRT-model, we can conclude that 6^{th} graders' rational number reasoning can be summarized by a model with one latent variable. More specifically, we found that a good understanding of size is a precondition for gaining understanding in operation and both understanding in size and in operation form a precondition for gaining understanding of density.

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GEOMETRIC ACTIVITIES OF CHILDREN IN MATHEMATICAL SITUATIONS OF PLAY AND EXPLORATION

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Children are confronted with spatial objects in different situations of their everyday life. This is not unusual since all people are spatial beings. Spatial objects are represented in the mathematical world in the form of different model types that focus on the typical properties of the objects, in the form of drawings and descriptions. The development of geometric thinking takes place in the context of these different representation formats (Battista, 2007). The focused research question is:

- Which mathematically understanding of spatial objects can be reconstructed from the activities (including language activities) of children in geometric situations of play and exploration?

The results presented here, refer to the geometric situation of play and exploration, called "solid figure", which were carried in the context of the longitudinal study "erStMaL" (early Steps in Mathematics Learning, located in the Center for Individual Development and Adaptive Education of Children at Risk (IDeA)). In "erStMaL" the situations of play and exploration have been conducted with children (between 4th and 10th years) in tandems or in smaller groups with a guiding adult in six waves of the survey. The situations have their conceptual origin in different mathematical domains. In the situation "solid figure" different representation formats of spatial objects are at the children's disposal. Further the situation is developed in its respective expression over the time however the basic structure remains constant. Four children have been chosen for the analysis presented here. With these children the geometric situation was carried out in each of the six waves of the survey (altogether 18 videos). The selected videographed situations are analysed by methods of low-inferent video analyses and the developed method of "explication analyse" according to Mayring. Briefly some of the results are mentioned here: already young children create relationships between the different mathematical representation formats and their everyday life by picking up central mathematical properties of the spatial objects; the children can switch between 2-dimensional and 3-dimensional representations of spatial objects.

Acknowledgement: This research was funded by the Hessian initiative for the development of scientific and economic excellence (LOEWE).

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ON THE PERSONAL MEANING OF GEOMETRY: A THEORETICAL APPROACH TOWARDS A NEGLECTED RESEARCH AREA

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One of the questions posed time and again by students is the one for the meaning of learning mathematics or dealing with mathematical concepts. Hence, one of the challenges posed for mathematics education is to find convincing answers to the questions for meaning. It is possible to differentiate between *personal meaning* (personal relevance of a mathematical object or action) and *objective meaning* (collectively shared meaning) respectively. We focus on the personal meaning of students only. They show a relation with fundamental ideas, basic concepts and mental models as they offer for example a relation between mathematics and applications in life, or they provide orientation in the vast body of mathematical theory. Thus, they may help students constructing personal meaning.

With a focus to geometry we can ask what may be personally meaningful to students in this concrete subject area. On the one hand, we expect to get a research topic, which is directly linked to geometrical concepts etc., and on the other hand, we expect to learn from such a reduced target domain about the overall feasibility of research into the personal meaning of mathematics as a whole. Our decision to narrow down to geometry results from the assumption that geometry offers a great potential to develop personal meaning as geometry is omnipresent and geometric shapes can be seen everywhere. In addition, geometry offers the opportunity of self-controlled activities and immediate identification with the process and product of learning, e.g. by the artistic element of compass and straight-edge constructions.

Although there are no empirical studies, literature offers some ideas that might be meaningful for students. Bender (1993) identified three areas, where students may develop personal meanings of geometry: the practical use of geometry, representation and visualisation, and the theoretical aspect of geometry. Instead of looking for personal meaning of geometry, one can also invert the question: Are there general, human concepts and/or ideas, which are closely related to geometry? The approach of *embodied cognition* offers here e.g. image schemas that relate to spatial relations.

In addition to these theoretical considerations, empirical studies are necessary to shed more light on this neglected research area to come up with convincing answers to the students' question for meaning.

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UNDERSTANDING ISSUES IN MATHEMATICAL PROBLEM SOLVING AND MODELLING: LESSONS FROM LESSON STUDY

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At the heart of our concern, and research, has been to understand how, in classroom learning, students might develop capabilities towards mathematical literacy (Steen, 2001). We have sought to gain insight into how to teach students to be able to apply mathematics effectively to solve problems that arise in a range of different contexts. We have, therefore, researched professional learning communities in which teachers work together and learn from each other, informed by ‘knowledgeable others’ who have research-informed expertise. Working to an adaptation of the Japanese lesson-study model (Fernandez & Yoshida, 2004) we asked the general questions:

- What does progress look like in students’ learning in relation to problem solving and modelling competencies?
- How do we support students in making this progress?

More specifically, here we focus on how students’ development of mathematical representations may assist them with structuring and supporting their mathematical thinking.

We developed case studies of 30 research lessons that were carried out within nine schools collecting data that comprised of videos of lessons and post-lesson discussions, students’ work and observer notes. Across lessons our analysis shows that, as expected, students use a variety of approaches when working on modelling tasks. However, often the validity of some of the models being formulated was flawed.

We found this to be a common occurrence with too little time and priority in teaching being dedicated to supporting students in (i) simplifying the reality of the problem context and (ii) developing a mathematical structure that represents, or maps onto, the simplified reality. More specifically we found that many students had little insight into how a change of a quantifiable factor in the reality of the context necessitates a change of a variable in the mathematical structure (model), and vice versa: that is, how variation of a factor in the mathematical model has implications for the reality it represents. We have concluded that a pedagogical approach that focuses on asking students to develop a model that could have repeated application, allowing for variation of a key factor, may have the potential to force this issue in the classroom.

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DISCURSIVE RENDITION OF VAN HIELE LEVELS

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The Van Hiele theory (1959) is a well-known framework to understand students' geometric thinking at different levels. The discursive nature of the theory suggests that thinking as communication made distinct by its repertoire of admissible actions and the way these actions paired with re-actions, and therefore, it is possible to view each of the van Hiele levels of thinking as its own geometric discourse (Sfard 2008).

The study examines the usefulness of viewing the van Hiele theory as five levels of geometric discourses. In particular, based on the van Hiele theory and Sfard's framework, this study develops a model that describes each van Hiele level as a level of geometric discourse with respect to *word use, routines, endorsed narratives* and *visual mediators*. Pr-service elementary teachers' geometric thinking was explored in the context of quadrilaterals. The study started with sixty-two pre-service teachers who participated in a van Hiele geometry pre-and post-tests, and then recruited twenty of them for the pre- and post interviews.

Aligning pre-service teachers' test results with analyses of their geometric discourses from the pre-and post- interviews, it shows the variations of geometric discourses at the same van Hiele level among different prospective teachers, as well as changes in geometric discourses within a van Hiele level for the same pr-service teacher (Wang, 2011). The examination of each van Hiele level with four characters of discourse (*word use, routines, endorsed narratives* and *visual mediators*) demonstrated how thinking is communicated through what pre-service teachers *say* and *do*. The findings show that discursive analysis allows us to see the details of these teachers' geometric thinking across van Hiele levels, as well as within a level. The discursive lens not only enhances our understanding of the range inherent in each van Hiele level, but also demonstrates the benefit of revisiting van Hiele theory with discursive lens in order to better understand how students' geometric thinking develops.

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MATHEMATICS KNOWLEDGE OF A SECONDARY MATHEMATICS TEACHER: A CASE STUDY

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Researchers have found that mathematics teachers in elementary and secondary schools need a certain level of understanding of the subject matter. Monk (1994) studied the subject area preparation of secondary mathematics teachers, and found that five undergraduate mathematics courses is a "cut-point". "The addition of courses beyond the fifth course has a smaller effect on pupil performance, compared with the effect of an additional math course up to and including the fifth course" (p. 130). The prior research employed quantitative methods to measure teachers' knowledge or directly measure some specific mathematics knowledge in elementary and low secondary grades. Few research studies directly measure the depth and breadth of the mathematics knowledge of secondary school teachers. This study examines what are an exemplary teacher's substantive and syntactic structures, case knowledge, strategic knowledge (Shulman, 1986), and horizon content knowledge, and how he uses this knowledge.

An exemplary teacher was selected for this study based on his experience in presenting workshops to provincial and national mathematics teachers as well as numerous recommendations that he is an exemplary teacher. The researcher interviewed the teacher, observed his classroom and shared student work to be analyzed for error detection. A rubric of mathematics knowledge was designed to assist in data analysis. The findings show this exemplary teacher has profound knowledge in mathematics content and student thinking methods. He understood the connections, coherences and extensions from an elementary, secondary and undergraduate mathematics perspective as well as providing numerous applications in real life problems and other subjects to his students. He identifies the flaws and missing core concepts of the Ontario provincial curriculum framework and provides the necessary supplementary documents for student learning in order to prepare them for higher-level mathematics. He can divide a complex mathematics tasks into simple subtasks and synthesize the solutions for the whole. Teacher understanding of fundamental mathematics knowledge can help them to diagnose misconceptions, make better pedagogical decisions about teaching and enhance student achievement, thinking and learning skills.

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RELATING PRESERVICE TEACHER NOTICING WITH MATHEMATICAL KNOWLEDGE FOR TEACHING

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Research has shown that *teacher noticing* is a critical element for effective teaching (Sherin, Jacobs, & Philipp, 2011). Informed by research suggesting that expertise in *teacher noticing* can be developed (Miller, 2011), this paper's aim is to discuss a mixed methods study and hypothesize the findings based on preliminary data collection of approximately 200 elementary and middle-school preservice teachers' (PST) development of *teacher noticing* (attending to student thinking, interpreting student understanding, and responding to student work) in a math content course.

The study was conducted in 2014 over a 14-week period at a state university in southern United States. Data was collected through pre- and post- Mathematical Knowledge for Teaching (MKT) assessments of content knowledge (CK) and knowledge of content and student (KCS), and three writing assignments (WA), where each WA packet included a task and artifacts of elementary or middle school student work on that task. The quantitative data for this study (the MKT instruments) is analyzed using pre-post t-tests and the qualitative data (the three WAs) will be coded using grounded theory and a rubric designed to capture the PST's level of attention to student thinking. In our data analysis, we examine (1*) teacher noticing* through the WA (2) pre- and post-CK and KCS and (3) relate PST's change in *teacher noticing* to the changes in their CK and KCS. For the WA data, we have preliminary evidence that PST are improving their noticing of student thinking but whether this growth is sustained throughout the semester is uncertain. Because the course focuses on content, with particular emphases on the mathematical equivalence of multiple strategies, we expect that PST will likewise improve their noticing of student strategies in the WAs. Therefore, our hypothesis is that these two will grow together.

Upon analysis of all of our data, we discuss the implications of this study with regard to fostering connections between components of *teacher noticing* and the mathematical knowledge for teaching.

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QUANTITATIVE REASONING, RATE AND VISUALIZATION OF THREE DIMENSIONAL SURFACES

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The purpose of this paper is to explore in what ways physical, three-dimensional surfaces might support multivariable calculus students' ability to a) visualize surfaces and b) reason about quantities necessary to measure directional rate of change.

As recent work has shown, students' thinking about function and rate of change in the context of two or more variable functions goes beyond just natural extensions of existing frameworks (Martinez-Planell & Trigueros, 2013). Much of the work in this area has pointed to two central issues: a) how do students conceptualize, measure, and interpret quantities (and relationships between them) when there are three or more quantities, and b) how do they visualize the functions with which they are working to identify and measure these quantities (Weber & Dorko, 2014). The purpose of this paper is to report on a study that explored the following question: *What is the role of quantitative reasoning and visualization in multivariable calculus students' thinking about rate of change in a direction (i.e. directional derivative)?*

We interviewed six students enrolled in multivariable calculus at a mid-size university in the northwestern United States. The purpose of Phase 1 of the interviews was to gain insight into the students' thinking about rate of change without a specialized instructional intervention. The purpose of Phase 2 of the interviews was to use whiteboard surfaces to understand in what ways students' ability to visualize the surfaces in space could help them focus on issues of direction.

The overarching theme of our results is that the three dimensional surfaces allowed students to conceive of direction and the quantities needed to measure it in way that was not possible in their formal exposure to these ideas in their multivariable calculus course. Our results illustrate the importance of visualization of surfaces in space for a) conceptualizing of the need to consider direction of rate of change and b) determining what quantities to measure to quantify rate in a particular direction.

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INVESTIGATING CO-TEACHING OF PROFESSIONAL DEVELOPMENT FOR K-8 TEACHERS

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Recent reports and research have supported a need for elementary and middle school teachers to have strong and relevant mathematical content knowledge and to engage in ongoing professional development that is content-based, discipline-specific, and focused on the actual knowledge needed to carry out the work of teaching mathematics. While neither the number of mathematics courses teachers take nor the degrees or certifications they attain predict student learning (National Mathematics Advisory Panel, 2008), there is evidence that teachers' performance on assessments that measure their mathematical knowledge for teaching (MKT) do correlate with student learning (Hill, Rowan, & Ball, 2005).

This report describes results from an in-depth qualitative look at the instructional model of an 80 hour professional development course for K-8 teachers focused on a series of problem sets organized around mathematical themes that are central to the K-8 curriculum. The course has the unique feature that it is co-taught by a mathematician and a mathematics educator, with the roles of the two instructors delineated by the structure of the curriculum. Our investigation focused on video analysis of the interactions between the two co-facilitators, the way in which each facilitator handled the content of the curriculum, the instructors' facilitation skills, and how these features impacted participant engagement in doing mathematics.

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MATHEMATICAL ACHIEVEMENT FOR GRADE 8 STUDENTS IN TAIWAN WITH IAEP INSTRUMENT

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To focus on cultivating student's mathematics ability and high level of mathematical thinking instead of memorizing operational rules, Taiwan enforced new Grade 1-9 curriculum in 2004 (Lin &Tsai, 2006). In order to evaluate the effects of curriculum reform, Taiwan adopted the research instruments of International Assessment of Educational Progress held by Educational Testing Service in 1991 to investigate Taiwanese Gr.8 students' mathematical achievement. This study aimed to figure out the item response model which fits the data well and to explore Taiwanese Gr.8 students' possession situation of mathematical abilities.

The instrument obtained 76 math-related items in five mathematics content areas: number and operations (NO), measurement (ME), geometry (GE), data analysis, statistics, and probability (DS), and algebra and functions (AF). Besides, they were divided into three different levels of cognitive processes: conceptual understanding (CU), procedural knowledge (PK), and problem solving (PS). The participants included 1840 Gr. 8 students in 51 classes from 50 schools. The research design and practice followed the procedure as IAEP, 1991.

This study employed the models of item response theory to analyze the data. Among them, two-parameter logistics model fits the data the best. In order to investigate the students' mathematical ability from IAEP framework, this research computed the mean of item difficulty in three categories of cognitive process and five domains of content area. According to ANOVA test with Scheffe's method, there's no significant difference among the average item difficulties of three cognitive processes and of five content areas. It shows that students in Taiwan have balanced ability on CU, PK and PS, and equivalent ability on NO, ME, GE, DS and AF. However, this study found that, even for Taiwanese students, who outperform students from other countries participated in TIMSS and PISA on mathematics, still experienced several learning difficulties. For example, quite a few students can not apply procedures appropriately when mathematics symbols are involved; even the concepts related are simple and well-known for students. A few students cannot correctly distinguish the order of decimals of which the concepts they learned in primary school. The new schema which they obtained in lower secondary school influences their judgement. These situations are worth of further exploration and should be a warning to mathematics teachers.

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STUDENTS' EXPERIENCES AND ACHIEVEMENT IN THREE FLIPPED LARGE UNDERGRADUATE CALCULUS COURSES

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Our flipped classroom model combines the use of instructor-made video lectures outside class and the use of i>Clickers, peer-instruction and just-in-time teaching inside class. The studies on the use of peer instruction (Crouch & Mazur, 2001) and just-in-time teaching (Simkins & Maier, 2010) show dramatic effects of these techniques on improved conceptual understanding and problem solving skills. Our study examines the impact of our flipped model on students' experience and learning.

We investigated three first-year calculus courses conducted by experienced senior instructors in two cycles of implementation, with two courses (enrolments of 246 and 224) in the first cycle and one course (enrolment of 346) in the second cycle. Data were collected from student questionnaires on their perceptions of the flipped classroom experience (twice during the semester with an overall response rate of 50% at mid-term and 38% at term-end) and students' achievement scores (two mid-terms and the final exam).

Our initial findings indicate that students can be sceptical about the pedagogy when first exposed to it; however when the instructor stays committed to the method students' views turn much more positive at semester end. Our respondents reported a number of advantages of the video lectures including the ability to learn at their own pace, to focus better, to watch anytime and anywhere, to learn in privacy, and to learn content in depth. Compared to the traditional lecture class, they also reported that they were more enthusiastic about their class, paid more attention, and were more active with their learning in the flipped class. They perceived the instructors as more interactive, and in general believed the flipped class helped them learn better. Students' perceptions in above accounts held true across all performance levels. In terms of achievement, there was no significant difference in students average exam grades as compared to previous offerings of the two courses in our first round of implementation; in contrast, for the course offered in our second round of the implementation there were significant gains in terms of grades. However, such results should not be generalized since we are still at the initial stage of our experiment. Further investigations are needed to verify these initial results through future iterations of implementation across settings and instructors.

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EASING EPISTEMOLOGICAL ANXIETY WITH MODELS: A CASE STUDY IN SOUTH KOREA AND THE U. S.

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Numerous studies have examined student mathematical epistemologies and how the use of different representations can remedy deeply held anxieties about mathematics (e.g., Wilensky, 1997). Mathematical epistemological anxiety often manifests when a learner engages in rote problem solving without meaningful understanding. Operating under such epistemological frames can be counterproductive to learning. Previous studies indicate that the use of agent-based modelling (ABM) can help students gain a deeper qualitative understanding of various phenomena. However, most of these studies have been conducted in the U.S., and not much qualitative work has been done to examine the differences between cultural groups on this matter. Thus, the purpose of this study is to examine how epistemological anxiety manifests in students from different countries, and the ways in which multiple representations (and specifically ABM) can serve as a remedy for such anxieties across cultures.

The sample consists of student volunteers from South Korea (n=58) and the U.S. (n=32), in grades 9 to 11. Students come from high-performing schools that are located in suburbs of large metropolitan cities. Semi-structured interviews and a survey (containing PISA items) were conducted, focusing on students' beliefs towards mathematics, and mathematical problem solving and learning. Both statistical and qualitative content analyses were employed.

The survey results indicate that, across multiple variables, students' background characteristics remain quite comparable. However, one key difference is that the Korean students reported that they use more metacognitive strategies as elaborative strategies than their U.S. counterparts, and that they are more worried about getting poor grades ($p<0.05$). This result is reflective of the national trends, and interview data also provide evidence for this.

The analyses of interviews suggest that the Korean students exhibit deeper epistemological anxiety, but generate more trial solutions to the problem at hand than the U.S. students. In both Korean and U.S. samples, students who are welded to the formulae, or more confident in their solutions, are less likely to value the models; students who have difficulty generating formulae, or who exhibit more epistemological anxiety, are able to generate qualitatively rich understanding through the use of models. This study suggests that the use of models can leverage students' intuitive understanding and ease their anxieties across cultural groups, albeit with some differences. Nuances of these findings will be discussed in the presentation.

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MIDDLE SCHOOL STUDENTS' MATHEMATICAL REASONING EMERGING THROUGH DRAGGING ACTIVITIES IN OPEN-ENDED GEOMETRY PROBLEMS

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Solving open-ended geometry problems under Dynamic Geometry Environment (DGE) has been reported to allow students to explore their conjectures and further contribute to their reasoning processes. In particular, dragging activity in DGE is a powerful tool to visually represent geometrical invariants in the midst of varying components of a geometrical configuration, which plays a key role to formulate the students' conjectures (Arzarello et al., 2002). In the present study, we analyze students' mathematical reasoning in their dragging activities in terms of abduction, induction and deduction, and attempt to build a bridge between their empirical exploration in Geometer's Sketchpad (GSP) and deductive proof of Euclidean geometry.

For our qualitative case study with four Korean 9th grade students, four semi-structured interviews, each of which was conducted in two hours, were video-recorded. Field notes, the students' activity sheets and GSP files were also collected as part of major data. The results of the analysis are as follows.

The students utilized 'abduction' to adopt their hypotheses, 'induction' to generalize them by examining various cases and 'deduction' to provide warrants for the hypotheses. In the abduction process, 'wandering dragging' and 'guided dragging' seemed to help the students formulate their hypotheses, and in the induction process, 'dragging test' was mainly used to confirm the hypotheses. Several difficulties during the dragging activities were also identified in their solving processes such as misunderstanding shapes as fixed figures, not easily recognizing the concept of dependency or path, and trapping into circular logic.

In conclusion, for students' smooth transition from experiential justification to deductive reasoning through their dragging activities, we, as mathematics teachers, need to encourage them to flexibly embrace diverse hypotheses in abduction process, which be followed by their feeling logical necessity for proof leaping from an experimental level to an axiomatic level. In the presentation, further results and implications will be discussed in detail.

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2014. In Oesterle, S., Nicol, C., Liljedahl, P., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 6, p. 268. Vancouver, Canada: PME.

A CASE STUDY OF IN-SERVICE MIDDLE SCHOOL MATHEMATICS TEACHER'S USE OF KNOWLEDGE IN TEACHING ALGEBRA WITH DYNAMIC SOFTWARE

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Teaching mathematics with a Dynamic Software (DS) in schools is emphasized for students' learning and thinking. Nevertheless, teachers are in an ongoing developmental process to adapt this technological integration into their classroom practices. Therefore, how teachers use their mathematical knowledge for teaching with technology in classrooms is an open issue for investigating teachers' knowledge and practices (Doerr, 2004). This study is a part of ongoing dissertation that analyzes teachers' knowledge of algebra for teaching and their use of knowledge in classroom context. In this paper, the focus is a teacher's use of knowledge for teaching slope of line with DS (GeoGebra) in one 8th grade classroom, 2 class sessions, in a public school. The teacher, Oya, designed the activities using GeoGebra and the researchers gave technological support when needed. Planning interviews before the lesson and exit interviews after the lesson were recorded. Planning and exit interviews and field notes were used for data triangulation. Researchers videotaped all classroom sessions and took field notes. Teacher's mathematical practices in teaching the related mathematical knowledge for teaching mathematics were analyzed using constant comparative methods through the knowledge for teaching algebra framework (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012).

Analysis of the data indicated that the teacher's *bridging* the slope of a line and linear equation was displayed in transition across graphics, algebra and spreadsheet view of GeoGebra. Oya intentionally *trimmed* the meaning of the slope of a line as a rate of change in quadrant 1 coordinate grid in a real-life problem with connecting trace of points, slider of y-variable, and dynamic text of slope calculation. Oya also *unpacked* the meaning of slope as rate of change by constructing a slider that make connection across the slope value, the slope of a line and equation of a line. In conclusion, bridging and decompressing practices revealed the interconnection between knowledge of teaching algebra and knowledge of teaching with technology.

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INFLUENCES OF A MATHEMATICAL CULTURE COURSE ON MEDICAL UNIVERSITY STUDENTS' MATHEMATICS BELIEFS

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In this paper, we propose a liberal-arts course of mathematics, with an emphasis on the culture and history of the discipline, and examine its influences on the mathematics beliefs of medical university students in Taiwan. The rationale behind this study is that medical practitioners in Taiwan have high social influences due to the country's colonial history, and if medical practitioners perceive mathematics in a more humanistic way, then so may the general public in the long run. This study used a single-group pretest-posttest design. Research tools of this study include: (1) a liberal-arts mathematics course with an emphasis on history and culture, and (2) a 20-question Likert-scale questionnaire used in the pre-test and the post-test, designed and modified according to the framework proposed by Op't Eynde, De Corte, & Verschaffel (2002). The questions were separated into two aspects, aiming to investigate students' beliefs about the *nature* and the *values* of mathematics. A total of 100 students took the pre-test, participated in the teaching experiment, and finally took the post-test. One of the authors was the lecturer in the teaching experiment. In the course of the teaching experiment, named "Mathematical Thinking in the Multicultural Contexts", students were exposed to elementary and advanced mathematical topics presented in their historical contexts. There were also examples of distinct approaches to the same problems by scholars in different civilisations, such as comparing Liu Hui's (3rd C. China) work and Euclid's *Elements*. Students were also required to make artistic creations related to mathematics. After the pre-test, the experiment and the post-test, the results showed that part of the students' beliefs did change. In the aspect of the nature of mathematics, after taking the course, the students were more prone to believe that "generalisation" was a method of thinking in mathematics; however, the results also revealed that the course did not clarify for students the difference between the context of justification and the context of discovery. As for the values of mathematics, students were more prone to believe that creativity and sensibility to beauty were important values of mathematics.

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TEACHERS' MEANINGS FOR FUNCTION NOTATION[1]

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Students' understanding of functions and its importance has been a central issue of mathematics education research (DeMarois & Tall, 1996). However, there is little research on students' or teachers' understandings of function notation per se. The research aims to investigate teachers' meaning of function definitions that use function notation, and further how these associate with teachers' mathematical meanings of function concept.

We focus on what teachers see as representing an output and as representing a function definition. Thompson (2013) described function definitions that use function notation as including the name of the function, the variable that represents a value at which to evaluate the function, and a rule that says how to determine the function's output given the input. The left-hand-side represents the output of function V. This is because we can use "$V(u)$" to represent the output of function V regardless of whether we know its rule of assignment. The left-hand-side (output), "=", and the rule of assignment constitutes the definition of V using function notation.

We administered 39 items to 100 high school teachers in the Midwest and Southwest United States, 14 of which related to the concept of function. The results suggest that teachers who think that a function's rule of assignment represents its output avoid using function notation to represent varying quantities. Additionally, over half of the teachers who attend to a function's rule of assignment as constituting its definition are unaware of inconsistent use of variables in a function definition. Based on these results, we hypothesize that attention to the rule of assignment creates an inability to use function notation representationally. The use of function notation to represent the output of a function is not meaningful to teachers who fixate on a function's rule of assignment because $f(x)$ does not convey to them any information for how to assign values. Attention to this hypothesis in future research could clarify the source of many difficulties already reported in research on teaching and learning functions.

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[1] Research reported in this article was supported by NSF Grant No. MSP-1050595. Any recommendations or conclusions stated here are the authors' and do not necessarily reflect official positions of the NSF.

ENGAGING A STUDENT'S INTENTIONALITY IN THE CONSTITUTION OF MATHEMATICAL OBJECTS THROUGH A NON-ROUTINE TASK: A HUSSERLIAN APPROACH

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This study is using a Husserlian approach to investigate the initial stages of one student's thinking as he engaged deeply with a group non-routine mathematical task presented to a class of prospective teachers. In the described stages of Ivan's case, objects taken as self-given (in a phenomenological approach) led to the surfacing of empirical and mathematical objects from his investigation in his group, and his exploration that followed at home. The teacher, who limited his interaction to strictly operational issues concerning the task, described it as follows:

> Here we've got a small precinct [the teacher drew one small square on the board] and basically you've got a policeman standing there, [he added a 'cop' at the bottom right corner] *who is able to see two blocks along*. [he added three more squares to form a larger square] … How many policemen would you need to see along every street, in that configuration?... So the question is, as the precincts get bigger how does the number of policemen you need increase?

Ivan's preoccupation with symmetry, and the different strategies that the other members of his group followed did not allow him to come to a consistent pattern until the end of the classroom investigation, although he had discovered asymmetrical configurations that yielded better results than his symmetrical ones. At home, Ivan put next to each other his symmetrical and asymmetrical drawings and decided to concentrate on grids with the same height. The analysis of Ivan's "break through moment", which took place when he discovered "a logical order to the number of cops per column" manifested the three key features of Husserlian *intuitions* (*immediacy, intentionality* and the *feeling of certainty*). The phenomenological analysis from the point of view of Ivan's *intentional origins* revealed how the *diagonal asymmetrical pattern* that emerged in Ivan's consciousness was separated from the student's world-as-lived in three stages. The significance of the analysis is that—through this example—it uncovers the capacity of open, indeterminate tasks such as the one given, to engage students deeply in the processes of object constitution, through separation from each student's lived experience, to the reflecting, positing experience of communicable mathematical objects. Finally, implications for learning and teaching are suggested through the analysis of the three stages of Ivan's intuition.

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AFFORDANCES OF PROCESS AND OBJECT VIEWS FOR COORDINATING POLYNOMIAL REPRESENTATIONS

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This exploratory investigation considers the affordances of strategies used by secondary students to coordinate multiple representations of polynomial functions.

This analysis, from a larger study of student learning in calculus, examines strategies used by 37 upper secondary students to identify matched function representations. A "process" view of function (Moschkovich, Schoenfeld, & Arcavi, 1993) aligns with the strategy of systematically matching ordered pairs across representations, whereas an "object view" focuses on a function's global features such as slope, shape, and degree (Bell & Janvier, 1981; Leinhardt, Zaslavsky, & Stein, 1990).

Using a think-aloud protocol, we presented students with 12 pairs of representations (e.g., a graph and an equation) and tasked them with identifying whether the representations were the same function. 1282 utterances were coded for strategies including matching ordered pairs (including the intercepts) and evaluating the direction or degree of the function. Identifying matches can be accomplished by exhaustion or by evaluating global features of a function, and we expected students who used global features of the functions to be more successful than those who matched ordered pairs of points. We also expected these "object-focused" students to use such strategies consistently, because such strategies are efficient.

The results challenged our expectations. Out of the 11 students who attained a perfect score, all used some version of point matching, and five checked more ordered pairs than necessary on over half of the items. However, these same students were also more likely than less successful peers to use "global" features such as evaluating the function's degree and magnitude of coefficients. One possible interpretation is that this stage of students' expertise involves using both more- and less-efficient methods.

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FOSTERING CHILDREN'S UNDERSTANDING OF UNIT FRACTIONS

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Area models are regarded as important representations in the teaching and learning of fractions. Analysis of curriculum documents revealed that for many decades US children have been suggested to mainly adopt area-model approaches when learning fractions (NCTM, 2000). However, the overemphasis on area model approaches is likely to restrict student thinking about the range of problem-solving approaches available for fraction tasks, and hinder their future conceptual development (Gould, 2008). The purpose of this research was to investigate the mathematical activities that might profitably be used for advancing the conceptual knowledge and skills of fifth graders regarding the unit fractions of 1/2, 1/3 and 1/4.

The sample comprises 40 end-of-fifth-grade students who had been taught fractions concepts through a curriculum greatly emphasizing area-model approaches to teaching fractions, along with their normal mathematics teacher. The intervention including 6 activities was completed in 5 sessions. The activities were associated with a variety of different models like perimeter, capacity, linear, and discrete models. Three parallel forms of tests—pre- and post- teaching and retention tests that required students to provide written or pictorial responses to 28 items incorporating various representations of fractions—were administrated, together with pre- and post- teaching interviews conducted to further explore students' knowledge of unit fractions.

Student mean performance improved in an educationally significant way immediately after the intervention, and most of what the students learned was retained during the summer break of 4 months (The mean scores were 18.3 on pre-test, 22.9 on post-test, and 21.8 on retention test). At the pre-teaching stage, the students performed very well on area-model tasks (at least 80% correct), although on average more than 30% of them failed to correctly answer the questions where fractions were linked to non-area-model scenarios. At the post-teaching stage, the students performed much better on those non-area-model questions, and on average, more than 76% of the answers given were correct.

The results suggest that the emphasis on the importance of area-model representations may not generate a well-balanced conceptual understanding of unit fractions, and more attention may be drawn to multiple-embodiment approaches.

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ADVANCED MATHEMATICS AND PRACTICAL TEACHING AMONG SENIOR SECONDARY SCHOOL TEACHERS

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Whether advanced mathematics is essential for teaching elementary mathematics has been a popular topic since the era of Felix Klein. Despite the criticism on the big gap between the practical teaching of the mathematics teacher and advanced academic mathematics, Hill and his colleagues (2008) attempted to unpack the mathematics for teaching based on the model originally developed by Shulman (1986) as well as vast empirical research on this topic. However, since most studies were conducted among perspective teachers or among teachers in lower grades, there is hardly any research on the relationship between advanced mathematics knowledge and senior secondary school mathematics teaching, in which advanced mathematics is most probably applicable. In addition, the measurement of teachers' teaching in the overwhelming majority of the past studies is interview, which can only represent teachers' "report teaching".

This proposed research aims to examine the relationship between advanced mathematics and practical teaching among senior secondary teachers. Advanced mathematics knowledge is assessed by a paper and pencil test, which will be conducted for a sample comprised of around 100 teachers in senior secondary schools. Cases with weak and strong advanced mathematics will be identified through the test and lessons given by those teachers will be videotaped. A coding protocol based on the Knowledge Quartet (Rowland, Huckstep, & Thwaites, 2005) will be developed to give a quantitative description of teaching. The correlation between teachers' advanced mathematics knowledge and their teaching will accordingly throw light on the necessity or otherwise of advanced mathematics for teaching senior secondary mathematics. Moreover, the results will also inform the exploration on the structure of mathematics teachers' knowledge from the perspective of pragmatism. In the presentation, results of the pilot study will be discussed in detail.

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POSTER PRESENTATIONS

IMPROVING MATHS TEACHING FOR PRE-SERVICE PRIMARY TEACHERS – RESULTS FROM THE KLIMAGS PROJECT

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International studies like TEDS-M demonstrated that German pre-service primary teachers show relative weaknesses in their mathematical content knowledge. Accordingly, the main goal of our project is to examine and to improve the development of performance in the mathematical content knowledge (here, we only report about the arithmetic part) of primary students in their first academic year.

In a recent study we have analysed the performance of two student groups (N=69+62). Both groups' performance was measured before and after a lecture on arithmetic (the control group participated in 2011/12, the experimental group in 12/13 resp.), using the same self-developed tests. The analysis of the control group tests showed unsatisfactory results especially for the content areas of divisibility rules and the positional notation system. Corresponding to these findings and to conclusions of Duval (2006) as well as Schneider and Artelt (2010) we developed an intervention for these content areas that refers to the simultaneous mobilization of three modes of representations and to corresponding metacognitive monitoring. Our research question was: Does the intervention have a positive influence on students' performance in the respective content areas and/or in arithmetic in general?

To examine the influence of our intervention, a reliable performance test (EAP/PV>.69) with two dimensions (concerning the intervention directly or not) was used. A two-dimensional Rasch analysis was conducted to examine the student's latent ability. The subsequent ANOVA with "group" as within-subject contrast and two categorical factors (dimension and time) of students' ability shows a significant influence of the intervention in dimension one over the time ($F(1)=10.335$; $p<0.01$; $\eta^2=.074$). Another set of two ANOVA for each dimension with "group" as within-subject contrast and just one categorical factor (time) confirms that while the development in dimension one is significant ($F(1)=8.342$; $p<0.01$; $\eta^2=0.061$) there is no significant difference in dimension two.

The results indicate that our intervention had a positive effect on the performance in the treated content areas but none on arithmetic in general. Thus, similar interventions will be necessary in other content areas.

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DEVELOPMENT OF GEOMETRIC THOUGHT AMONG FIFTH TO EIGHTH GRADERS

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Dina van Hiele-Geldof and Pierre van Hiele (1957) notice the difficulties faced by the students in learning geometry, so they suggest the Van Hiele model for geometric thinking. The model consists of five hierarchical sequential levels which represent the development of geometric thought. These levels are: recognition, analysis, ordering, deduction, and rigor (Khasawneh, 1994). This research aims to explore the geometric thinking levels among 5th to 8th graders, and to investigate the differences between their geometric thinking levels due to grade level and geometric concept. This study is driven by the following questions: 1) What are the geometric thinking levels among 5th to 8th graders? 2) Do students' geometric thinking levels differ with respect to their grade levels? And 3) Do students' geometric thinking levels differ with respect to different geometric concepts?

This descriptive survey research was conducted with a population of 5th to 8th grade public education students in the city of Irbid/Jordan. Two-stage cluster sampling was used in selecting the 600 participants. To build the test, performance tasks were identified based on the description of the first four levels of Van Hiele's model of geometric thought. The test was based on three geometric concepts: the angle, the triangle, and the rectangle. The final version of the test included 39 items; apportioned equally across the three concepts. The score on each item was either 0 or 1.

A passing criterion was assigned to each level. In all the four criteria the student's score was considered to be 1, and other than that, it was considered to be 0. Based on four accepted performance patterns (Crowley, 1990), the students' percentages on the geometric levels were: 51.3%, 16.0%, 15.2%, and 24.5% respectively. The Chi-square test for independence indicates significant differences in classifying the students on geometric thinking levels due to grade level, χ^2 (9, n=400) = 54.92, $p < .05$. Three scores were assigned to each student according to his/her level of thinking on each geometric concept. A repeated-measures ANOVA was used to compare the means of students' scores on each concept, in each grade separately. The results indicate that the classifications of 5th graders on geometric thinking levels with respect to the angle concept were significantly better than those of the triangle and the rectangle concepts. The classifications of 6th, 7th, and 8th graders with respect to both the angle and the triangle concepts were significantly better than those of the rectangle concept.

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WHAT DO YOU KNOW AND WHERE DOES IT FAIL?

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We frequently explore how students' existing knowledge forms a basis for future understanding. Here, we instead look at places where students' existing mathematical ideas break down, and how this 'failure' of a previously understood concept thickens students' mathematical understanding. We combine initial findings of two research studies undertaken by the authors, one at the secondary and one at the primary level.

This research employs the Pirie-Kieren Theory for the Growth of Mathematical Understanding and, in particular, the notion of 'folding back' (Martin, 2008) – the idea that a student seeking to expand their mathematical understanding will return to prior knowledge to examine and modify their existing ideas about the concept. This process implies that when learners revisit earlier images and understandings for a concept, they carry with them the demands of the new situation, using these to inform their new thinking at the previous layer, leading to what may be termed a 'thicker' understanding of the concept.

Recent research on neuroplasticity suggests that students' brains experience more growth and form more new connections when analysing 'mistakes' and attempting to understand information that challenges their pre-existing understandings. Dweck (cited in Boaler, 2013) proposes that when students think about why something is wrong, new synaptic connections are sparked that cause the brain to grow.

We conducted two studies, one in a secondary course on vectors and calculus, and one in an elementary school unit on integers. Our studies introduced teachers to the idea of folding back and, through video, traced its use as a pedagogical strategy in their lessons. In both classes, we found teachers using elements of folding back as a way of thickening their students' understanding. However, rather than using this strategy only to emphasize sameness (i.e. instances where prior images and understandings are applied in a more complex situation), we noticed both teachers also using this strategy to emphasize difference. We examine, among others, a situation where a calculus teacher challenges students' understanding of slope in R2, exploring how it breaks down in R3, and a situation where an elementary teacher questions students notion of numbers as being primarily to represent concrete objects and challenges how this breaks down when we begin to consider negative integers.

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PROBABILITY AND GAMBLING ABUSE

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Steinbring (1991) highlights the unavoidable intertwining between theoretical concepts and concrete experiments in learning probability. If the aim is to contrast young students' tendency to gamble, it is necessary to also take into account typical misconceptions related to probabilistic thinking (Fischbein & Gazit, 1984). Following Pirie and Kieren (1994), we view mathematical understanding as a process of constantly changing recursive growth, as a dynamic and active process of negotiating and re-negotiating one's world whereby the abstract can never be severed from the concrete. In teaching activities to prevent gambling abuse, where understandings are inevitably concerned with actions, emotions, expectations and will around games, it is this interplay between *concrete* and *abstract* that becomes especially significant. Thus, considering the relationship of the individual with the tools and practices of betting, with his peers, and with experienced gamblers is essential.

We focus on "probability-for-gambling", indicating the difference between simply understanding concepts in probability *per se* and understanding them in the context of gambling. Our research question is: which are the distinctive features of probability-for-gambling? A teaching experiment in 6 classrooms of grade-12 (17 years old) students shows the central role of emotions and expectations. The students' starting positive emotional disposition towards the possibility of discovering "the best strategy" to win a game develops into engagement in tasks that allow the students to access probabilistic modelling and specifically to become aware of the (un)fairness of the game. First results show that probability-for-gambling not only considers the fundamental role of emotions in the learning process, but mostly resorts to the initial positive emotional disposition and transforms it throughout the teaching activity to reach emotional and cognitive awareness about both games and probability. This is a first step towards identifying other distinctive features of probability-for-gambling.

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HIGH SCHOOL TEACHERS' USE OF CALCULATORS IN CLASSROOM SETTINGS

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Since the 1980s mathematics education literature in the U.S. has reflected a trend towards technology integration in mathematics classrooms. This culminated in the NCTM's Principles and Standards for School Mathematics (NCTM, 2000) explicitly including a *Technology Principle*. However, it is an undeniable fact that there have been also many voices against the use of calculators in teaching and learning mathematics. Therefore, it is valuable to investigate how calculators are currently being incorporated into mathematics classrooms.

This research project aims to examine how high school mathematics teachers incorporated calculators -scientific and graphing- into college preparatory math courses, including Algebra I, Geometry, Algebra II, and Pre-Calculus/Trigonometry, and whether they allowed calculator use on tests. So, the research questions are:

- To what extent do high school teachers use calculators in classroom instruction?
- To what extent do high school teachers use calculators in classroom mathematics assessments?

A survey questionnaire containing 15 items was given to 29 high school teachers from five schools in a Midwestern state in the U.S. Non-probabilistic, purposive sampling was utilized for quantitative data collection. The data generated by the survey questionnaire were analyzed by means of descriptive statistics to uncover general tendencies about mathematics teachers' use of calculators in classrooms and in evaluation.

The findings indicated that calculator use was a common practice in high school mathematics instruction and relatively high percentages of testing items on classroom tests required use of both scientific and graphing calculators in Algebra II and Pre-Calculus/Trigonometry. The data also suggest that school policies explicitly determine calculator use in instruction and in assessment.

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A COEVOLUTION OF SCHOOLS AND PROBLEM POSING

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In the last few decades, several authors and researchers have studied problem posing. First, to examine problem posing some researchers have studied the nature of problems posed by teachers and students and their ways to solve them (Silver & Cai, 1996). In other studies problem posing is the process of proposing and sharing questions in class, allowing students to engage in an authentic mathematical activity. By posing their own problems and trying to solve them, students and teachers do mathematics together (Brown & Walter 2005). Another way to address problem posing is by giving problems without *a priori* expectations and being attentive to how students approach problems, thus figuring out the nature of students' mathematical work (Proulx, 2013).

As these views aim to offer an alternative view of how mathematics can be done, an overarching concern is problem posing's viability in schools: *What does it require from schools?* In our work, we question the dominant context of mathematical learning and teaching (e.g. curriculum). In this way, the NCTM (2000) promotes problem posing as a way to favor mathematical work through student's mathematical creations. However, the arrangement of mathematical content within the curriculum requires students and teachers to explore *predetermined mathematics*. There is also a stress between *emergent* mathematics and mathematics which are culturally accepted or even defined. In this poster, we highlight some characteristics of the educational context that are conceived as promoting problem posing and the challenges of reaching problem posing without *denaturing* it. We are talking about a way of doing mathematics in appropriate contexts where action and experience develop a particular way of being (Maturana & Varela, 1987) and where, at the same time, problem posing interacts and evolves in favourable ways in relation to students' learning and teachers' work in mathematics.

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CCS PEDAGOGY: TEACHERS' AND STUDENTS' VOICES

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The implementation of the Common Core Standards (CCS) and the added implications of new testing will impact classroom teachers' practices and their students. A meta-synthesis of research studies (Au, 2007) showed that the effects of high-stakes testing most often included a narrowing of curricular content as well as a fragmentation of content, and resulted in teachers choosing to teach using a transmissive practice (p. 263). Au's analysis of 49 qualitative studies described how, in such an environment, many of the classrooms suffered from the very opposite of the goals of the new CCS documents: that content should be integrated and that pedagogy should be student-centered. This Critical Participatory Action Research (Kemmis & McTaggart, 2005) study asked how high school teachers and students perceived the implementation of a student-centered lesson and what were the qualities that made it effective and engaging. The seven participating teachers from a small semi-rural high school described each of their lesson features, and 80 of their students explained what they found engaging about the lesson and why. Data from lesson plans, journals, and focus groups described what teachers identified as the important features: that the lesson they designed allowed students the time to struggle and to make sense of a problem, and that it allowed them to work in groups with hands-on or authentic tasks. Students indicated that they most liked the hands-on and interactive features of the lesson and the opportunity to communicate and cooperate during group work. Answers to these questions can generate new knowledge about what teachers and students value and how they experience pedagogical shifts during curriculum implementation. The immediate challenges are to ask how teachers can guard against *teaching to the test*, and how they can best be supported in their journey to implement more student-centered lessons within the existing climate of the CCS standards and testing. The poster includes reflective quotations from students and teachers, engaging student-centered tasks and assessments, a number of colorful samples of student work, and a chance to engage and interact with the tasks.

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FRUSTRATING ENGAGEMENT: UNIVERSITY STUDENTS' EXPERIENCES WITH PEDAGOGICAL CHANGE

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In order to improve student understanding of mathematics and the likelihood of students continuing in STEM fields, a College Algebra course was redesigned to reduce lecture time and increase student engagement in mathematical reasoning and exploration. Such changes are based on conclusions that lectures are problematic for students (Seymour & Hewitt, 1997) and that prompting students to reason mathematically during class time will increase their understanding and appreciation of mathematics (Ganter & Haver, 2011). While many universities have restructured mathematics courses with similar goals and structure, little research has aimed to comprehend how students negotiate such potentially significant changes to their learning experiences (Star, Smith, & Jansen, 2008). The study described in this proposal aimed to answer the questions: (1) What do students feel is important for their learning of mathematics; (2) How do those views conflict with the structure of the redesigned course; and (3) How do students reconcile their preferences for learning with the structure they are offered in the redesigned course?

Students enrolled in the redesigned College Algebra course were recruited to participate in focus groups every two weeks during the ten-week term. Questions probed their experiences in the course, conceptions of mathematics, and views of quality mathematics teaching and learning. Interview transcripts were analysed using an iterative process of defining and refining themes (Braun & Clarke, 2006). Findings from this study include students': (1) preferences for a procedural approach to mathematics teaching and learning; (2) appreciation of doing mathematical work with their peers; (3) resistance to deeper conceptual thinking about mathematics; and (4) frustration with the exploratory nature of the student engagement model.

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A SOCIOCULTURAL APPROACH TO UNDERSTANDING IDENTITY AS A TEACHER OF NUMERACY

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All teachers have an important role to play in developing the numeracy capabilities of students, but can only do this if they see themselves as teachers of numeracy. The aim of this research is to understand how teachers develop such an identity, in order to assist in the design of professional development that supports teachers to recognise and exploit numeracy learning opportunities in subjects across the curriculum.

This study is being conducted over a two-year period and involves eight teachers from two Australian schools. The study is non-interventionist; however, the teachers were recruited because of their participation in a larger study. This poster presents a preliminary case study of Karen, a science teacher, and illustrates how an adaptation of Valsiner's (2007) zone theory, such as that used by Goos (2005), can be used to describe and analyse identity as a teacher of numeracy. Empirical data collected through semi-structured interviews about Karen's background, professional context, and classroom practices are mapped onto her Zone of Proximal Development (ZPD), Zone of Free Movement (ZFM), and Zone of Promoted Action (ZPA). Karen's classroom practices and personal conceptions of numeracy are described in terms of the numeracy model developed by Goos (2007).

Karen's potential to learn about developing students' numeracy capabilities through science (ZPD) is supported by her beliefs about the relationship between numeracy and science. However, she may lack the appropriate knowledge because this was not part of her university studies. Karen's personal conception of numeracy is fairly narrow and her classroom practice is mediated by her beliefs about students. Despite having occasions to develop her capacity to utilise numeracy learning opportunities in science (ZPA), Karen's capacity to fully exploit these will depend on how she resolves affordances and constraints identified within her professional context (ZFM). Development of case studies for all the teachers in the study will enable identification of similarities and differences through cross-case analysis.

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LEARNERS' USE OF RESOURCES IN A LEARNER-CENTRIC ENVIRONMENT

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This research explores learners' use of certain curriculum resources (textbooks, class discussions) in a pre-calculus mathematics course for in-service high school mathematics teachers (the learners). The underlying theoretical assumption of the research is that learners' use of curriculum materials involves an interaction between the learner and the materials. This is analogous to Remillard's (2005) assumption that teachers' use of curriculum materials involves an interaction between the teacher and the materials. The course consisted of eleven weekly three-hour sessions led by a colleague of mine. There were sixteen learners (the teachers), many of whom had weak mathematics content knowledge (CK). Based on the assumption that adequate CK is essential for high school mathematics teaching and that teachers need to constantly and independently learn new CK in line with curriculum changes, the focus of the course was on self-study of mathematics using a prescribed pre-calculus textbook as the primary resource. Class time was used for discussions and mathematical activities around the self-studied content. I call this a learner-centric environment.

This poster describes a case study within the research. Data regarding two learners' use of the textbook and class discussions for mathematics learning were gathered via weekly structured reflection sheets and semi-structured interviews. I was the interviewer. The two learners were purposively chosen according to prior CK. Teri has a Science degree in mathematics and good CK; Jim has an education degree with courses in mathematics education and weak CK. Analytic categories for resource usage were inductively derived from the data. Data analysis shows that the textbook was used to access new CK, to revise or deepen previously learnt CK or to make connections between mathematical concepts. It also alerted learners to ideas which needed further clarification. Class discussions were used as a forum to which learners brought mathematical ideas from the textbook for clarification and consolidation, or as a lecturer-driven forum in which the lecturer highlighted key ideas from the self-studied mathematics topic. In this case study, the use of class discussions complemented the use of the textbook and the nature of this relationship was contingent on the learner's prior mathematical experience: Teri used both the textbook and class discussions to broaden and deepen her mathematical knowledge and to make connections; Jim used the textbook primarily to access new knowledge and relied on class discussions to clarify and consolidate this knowledge.

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A COVARIATION CONTEXT TO EXPLORE QUADRATIC GROWTH

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Given the widespread difficulties students experience as they learn about functions, it is important to develop methods for helping students develop an understanding of functional relationships. This five session (1 hour each) teaching experiment aimed at providing one 8th grader with an opportunity to explore quadratic growth using a covariation approach in the context of growing rectangles. The student was provided with tasks designed to investigate how quadratic understanding develops (based on Ellis, 2011). All interviews were video-recorded and the relevant portions were transcribed for analysis.

The student investigated the areas of proportionally growing rectangles (see Figures 1 & 2 for a proportionally growing square). In this context, the student was able to notice patterns in both the rates of growth and the differences of rates of growth. When asked to predict the graph of a proportionally growing square, the student displayed a deep understanding of quadratic growth by revealing the attributes of the growing square within the graph $A=L^2$, such as the change in y (rate of growth) as the sum of the previous area plus a constant difference in the rate of growth of 2, see Figure 3. The poster will describe how she went from a previous misconception to a fuller understanding of quadratic growth and draw inferences for learning quadratics in general.

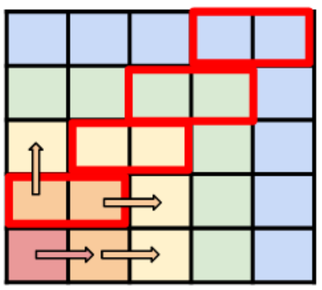

Figure 1: The border grows by the prior border plus 2

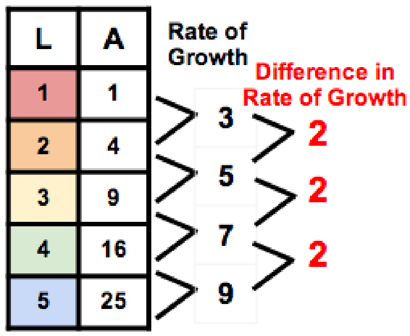

Figure 2: A length and area table with rates of growth

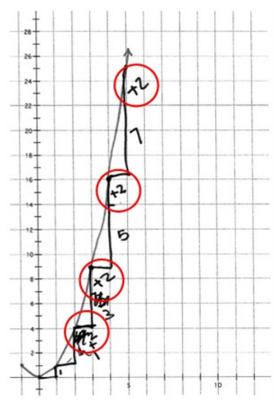

Figure 3: Student's graph of the area of $\mathbf{L}^2$

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EMBODIMENT AND ARGUMENTATION THEORIES: TIME AXIS IN PERIODIC PHENOMENA

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The teaching of trigonometry has been the subject of studies in mathematics education. According to the literature and our professional experience, high school students and teachers present many difficulties with trigonometric graphs in the Cartesian plane. This fact has motivated this study, whose purpose was to investigate and analyze the evolution of mathematical discourse of high school students discussing graphs of periodic phenomena, while interacting in a specific environment.

This environment, named Context Interactive Learning, is composed of tasks, digital technologies, and the interactions of participants with the tasks and technologies. Our research focuses on participant dialog. The tasks and the development and use of technologies were conceived on the basis of our assumptions about the importance of discursive and sensory-motor experiences, according to the Theory of Embodied Cognition (Nunez, 2000) and Argumentative Strategy Model (Castro & Bolite Frant, 2011), within a perspective that mind and body are inseparable

The methodology adopted was Design Research, performed in three cycles, each composed of following phases: elaboration, implementation, hypothesis testing, and modification. Research data was produced in a variety of forms, using audio and video recordings, transcripts, photographs of the chalkboard and whiteboard, and researcher notes.

The results showed that articulating theories used for preparation and analysis of the tasks and the use of technology were fruitful. Students discussed the nature of trigonometric graphs and periodic phenomena, mentioning various properties of the graphs. They participated in an interactive environment in which their debate was initially triggered by the researcher, based on corporeal explorations with applets and calculators with motion sensors. The study showed how students' concepts of trigonometry develop through embodied explorations, how tasks provoked students to discuss the variable of time and the decomposition of circular motion, that showed how these are important for comprehension of the nature of trigonometric graphs and their coordinates.

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MODELING AUSTIN'S PROPENSITY TO COORDINATE UNITS

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In this poster, we report on the results of on-going efforts to extend radical constructivist teaching experiment methodology (Steffe & Thompson, 2000) to model the variability in an individual student's mathematics[1]. The resulting *third-order model of students' mathematics* incorporates the role of the teacher-researcher as an observer, modeller, and interventionist complicit in the constructions made by his students (Boyce, 2013). The purpose of our efforts is to study Steffe's (2001) *reorganization hypothesis*: that fractions emerge from reorganizations of the operations and schemes constructed for whole number units. To analyze the reorganization process, we adopt the construct of *propensity to coordinate* units to consider the stability of a student's units coordinating activity.

Our poster illustrates methods and results from a 9-session teaching experiment with 2 sixth-grade students, Austin and Jane. Transcription of the teaching sessions was separated into chunks, and each chunk was coded qualitatively for mathematical and communicative context and dichotomously for Austin's coordination of units. We constructed a plot of Austin's propensity to coordinate units that did not incorporate contextual discrimination using the baseline algorithm described in Boyce (2013). We then identified contextual codes and incorporated them into the algorithm using optimization techniques. Modification of the algorithm did not make important qualitative changes in Austin's units coordinating activity more salient. Hence, we interpret the baseline algorithm output. Results include illustration of a benefit of engaging with whole number units immediately prior to fractions tasks.

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[1] This research was supported by the National Science Foundation, Grant DRL-1118571.

DEVELOPING STATISTICAL LITERACY: THE CASE OF GRAPHS WITH PRESERVICE TEACHERS

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Statistical literacy is a key issue in the orientations of the current mathematics curricula. It comprises the capacity of interpreting, critically evaluate and make judgments concerning statistical information and messages based in data or chance phenomenon (Gal, 2003, p.16). Research shows that both students and teachers experience many difficulties with statistical literacy, namely when the situations in study appeal to a more sophisticated use of statistical knowledge, associated with the interpretation and attribution of meaning to statistical information, expressed as numbers or graphs, or when the situations require assessment and critical judgment of conclusions derived from statistical studies (Shaugnessy, 2007).

This study focuses on the statistical literacy of preservice primary teachers, namely in what concerns their use of statistical graphs for interpretation and critical representation. It is our purpose to describe the graphs that these students used when they were asked to represent and to give meaning to a particular situation, in the context of their regular classes of Didactics of Mathematics. We produced a case study of a class of 16 students from the 3th year of a 1st cycle in Elementary Education, based on the content analysis of the written productions of the students concerning the solutions of some statistical tasks with graphs. In the poster we illustrate several responses of the preservice teachers, namely the graphs that they produced by their on to represent the situations in study proposed in the tasks. We conclude that a large part of the preservice teachers was not sure about what kind of graph to use and how to draw it. But, in spite of this, the preservice teachers elaborated some creative graphs trying to give meaning to the situations, although some of them were incorrect.

Acknowledgements

This study is supported by national funds by FCT – Fundação para a Ciência e Tecnologia through the Project Developing Statistical Literacy (contract PTDC/CPE-CED/117933/2010).

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A DETAILED STUDY OF FOUR MATHEMATICS TEACHERS' EXPERIENCES IMPLEMENTING FORMATIVE ASSESSMENT

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This study investigates the processes in play when mathematics teachers try to implement formative assessment (FA) in their teaching practice. The study has two parts: a case study following four mathematics teachers in their teaching and professional development over a period of three semesters, trying to understand why they make the changes they make in their teaching practices; and a survey study.

FA is a teaching approach that has gained a lot of attention in recent years, and research indicates that FA is an efficient way to improve student achievement. FA is also becoming an increasingly popular approach in Sweden, with several municipalities investing considerable time and money to train teachers in FA. Earlier research has identified several characteristics that are important for in-service training to be successful, but this study also aims at understanding other things that might affect the implementation of FA in teaching practices (e.g. teachers' motivation, and school culture). This study is an ongoing study aiming at contributing to the understanding of the implementation process; what happens and why. The study is mainly based on cases of four secondary school mathematics teachers. Several different methods for data collection are used: classroom observations, interviews, informal discussions. Data is also collected about the in-service program about FA that all four teachers are attending. The methods for data collection in the study aim at creating a coherent picture of each individual teacher's work situation in order to gain a deep understanding of why the teacher does what s/he does, and of the effects of their choices for the FA implementation. The in-service program is based on the FA framework of Black & Wiliam (2009), and so is the analysis of the teachers' classroom work. The Expectancy-Value Theory of achievement motivation (Wigfield & Eccles, 2000) is used for the analysis of why the teachers implement certain FA-practices and not others. It is easy to underestimate the support it takes to develop new complex practices. We hypothesize that, despite the long in-service training, limited implementation of FA will be achieved. Characteristics of decisive support are an expected outcome of the study.

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INTERACTIVE FORMATIVE ASSESSMENT AND QUICK POLLS: EXPLORING MATHEMATICS TEACHERS' PRACTICES

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Texas Instrument (TI) Navigator System provides wireless communication options with TI N-spire Computer Algebra System (CAS) handhelds that provide teachers an opportunity to take a snapshot of students' understandings by posing different types of questions and collecting students' responses instantly. Research studies have been conducted that show how such systems increase student engagement, interactions between students and teachers, and promote multiple representations. However, limited research has been done investigating how mathematics teachers use the results of QP in terms of questioning, academic feedback, and assessment abilities. To close the gap in the literature, this study was conducted in order to reveal what teachers do after collecting students' responses in their actual teaching. Initial findings show that the teachers were not able to use the results of QPs efficiently in order to provide sufficient instructional responses.

INTRODUCTION

The TI Navigator system seems to hold substantial promise as an interactive formative assessment as long as teachers use this system appropriately. The purpose of assessing students formatively was given as a means for revealing the gap between students' and desired performance, and then, closing it accordingly (William, 2011). Therefore, teachers are supposed to complete the follow-up formative assessment cycle by evaluating students' responses, issuing academic feedback, and adjusting instruction when there is a need. According to Black and William (1998), when teachers complete the cycle effectively, formative assessment improves learning. However, there is limited research on whether teachers who use QPs complete the follow-up formative assessment cycle or not. The data of this study is constituted by semi-structured interviews with three teachers and consistent observations of their lessons throughout the 2012 and 2013 school year (A total of 14 videotaped lessons). The data of the qualitative case study was analysed by utilizing open and axial coding systems (Gibbs, 2007) to reveal recurrent words and phrases, which formulated categories and themes.

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A CASE STUDY OF ADAPTING TEXTBOOK BY A MATHEMATICS TEACHER

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Over the past ten years, most of the teachers who teach five and six grades of mathematics are either junior or substitute teachers in Taiwan. Thus, how to adapt the mathematics textbook becomes more critical, when the lessons get more difficult and teachers are less experience. Moreover, as solid geometry usually gets less attention in mathematics education in Taiwan, the purpose of this study is to explore how a fifth grade teacher adapts the mathematics textbook and what factors affect the adaptation for teaching polyhedron.

This study collects the data by employing the method of teaching observation, in depth interview and focus group to observe a primary school teacher who has twenty years teaching experience and long term textbook editing experience. The types of textbook adaptation refer to the types by Sherin and Drake (2009), inclusive of create, replace, and omit. The influential factors are analyzed based on 'framework of components of teacher-curriculum relationship' by Remillard (2005) from three elements- teacher, curriculum and context.

This study found four types of textbook adaptation adopted by the teacher in the unit of polyhedron, inclusive of addition, replacement, omission and reorganization. Addition is to add new activities or questions which are not shown in textbook. Replacement is to replace activities or questions provided in textbook with different ones. Omission is to remove activities or questions arranged in textbook and conduct nothing related to the curriculum in the corresponding lesson. Reorganization is to adopt materials provided in textbook but to teach in different order. The factors which may affect the adaptation are originated from three aspects below. The first one is the representation of the textbook, to be precise, which includes representations of physical objects, representations of tasks, and representations of concepts. The second factor is teacher's personal knowledge and habit which includes mathematics knowledge, knowledge of students' mathematical learning, mathematics pedagogical content knowledge, and personal habit and perspective. The last one is the context which includes time, quantity of teaching aids, and support from the experts.

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STUDY OF GESTURES, VERBAL PRESENTATION AND ALGEBRAIC THINKING

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This study researched how students generalize graphic presentation, in order to understand how they utilize words and gestures at each phase of generalization; how they identify links between problems and recognize the equivalence of generalized equations, and how their understanding of algebra evolves. We conducted a case study of two sixth grade students from a public elementary school in southern Taiwan, using video recordings, interviews and writing to collect data, which was then analysed qualitatively. We found that

1. In the concept phase, students form rules by using visual graphics to compare changes in objects and item numbers, in accordance with the common elements of iconic, deictic and metaphoric gestures and perceptions.
2. In the linking phase, most students use deictic and metaphoric gestures and semantics, along with numbers and equations, to identify relationships in the problem structure.
3. Students employ strategies such as verifying metaphoric and computational outcomes, and comparing equation structure and object relationships, to comprehend the equivalence of different equations.
4. Students continue to develop their understanding of algebraic concepts based on item numbers, numbering of symbols and the unknowns of relationship diagrams.

This study provides recommendations on diverse permeation and the relationship between development of algebraic concepts and symbol indication, which can serve as reference in advancing algebraic thinking.

 2014. In Oesterle, S., Nicol, C., Liljedahl, P., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 6, p. 296. Vancouver, Canada: PME.

A STUDY ON STUDENTS' COMPREHENSION OF VARIATION WITH DIFFERENT DATA REPRESENTATIONS

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Statistical literacy and statistical thinking start from being aware of data variation in the true environment. Although it may not be easy for children to understand mathematical measuring and modelling for variation, but they probably can use their own experiences and knowledge to sense the variability in life. The variability can be represented by different kinds of data representations, such as pictures, graphs, numbers, etc. By manipulating these data representations, students' comprehension and application of variation can be observed. What we are interested in is whether different kinds of data representations affect children's comprehension of variation?

In order to explore the above question, we design an assessment with a framework of two dimensions，one is data representations (verbal descriptions, pictures, bar graphs and numbers) and the other is grades (4^{th}-6^{th}). "Consideration of Variation" proposed by Wild and Pfannkuch (1999) is used to define the comprehension of variation. The problems in the assessment are not only showed with words but also films that screen objects with characteristics of variation in the true environment, such as oranges with a variety of sizes and colours in the supermarket. This is for the ideas that children may sense the variation through vision and then store their thinking of variation in mind. Different kind of data representations is for stimulating their thinking of variation in mind, so we can observe their comprehension of variation.

A total of 827 students of grade 4-6 from Taipei City, New Taipei City, Keelung and Taoyuan County are selected to participate in this survey. From the analysis of ANOVA, we find that there is no interaction between data representations and grades. Students in grade 6 have the best performances significantly, however students between grade 4 and 5 have no significantly different performances. Verbal descriptions and numbers are respectively easiest and most difficult representations for students significantly. Furthermore, the homogeneity of students' answers among grades for each problem in the assessment is analyzed by Chi-Square Test and the results are for a hierarchy framework of the comprehension of variation. Interviews are also conducted for children's misconceptions of variation.

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MASTERY-LEVEL TRENDS OF NUMBER-AND-OPERATIONS AND ALGEBRA FOR U.S. AND KOREAN 8TH GRADERS

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Researchers use data from international comparative studies to examine different practices in educational system (Jacob et al., 2006). TIMSS indicates that the average scores of number-and-operations and algebra in the U.S. are significantly lower than 8th graders in high achieving Asian countries. Understanding these discrepancies is important in relation to the CCSS-M. Due to the lack of detailed information on specific mathematics skills from TIMSS, identifying students' mastery levels of specific mathematics cognitive skills led to the following research questions:

- How have the mastery level of mathematics skills in number-and-operations and algebra for eighth grade students in the U.S. changed over the period 2003, 2007, and 2011 and how does it differ from that of students in Korea?

In order to assess changes in mastery levels of mathematics skills for eighth grade students in the U.S. and Korea, this study analysed trends over time. More than 1,000 students' responses in the U.S. and 600 responses in Korea were collected for each year. Specific mathematics skills were identified based on the CCSS-M. A cognitive diagnosis modelling (Henson, Templin, & Willse, 2009) was applied to compare the students' mastery levels of each mathematics skills over time. A two-proportional z-test was used to compare the values from 2003 to 2011.

The results showed that, for U.S. eighth grade students, there was a significant increase in the mastery level of mathematics skills related to evaluating algebraic expressions and understanding ratio concepts from 2003 to 2011 (min $|z| = 2.19$, $p = .028$). However, eighth grade students in the U.S. had significantly lower mastery levels than their Korean counterparts in each of mathematics skills (min $|z| = 3.19$, $p < .01$).

Based on these results, we recommend that researchers and educators in the U.S. introduce early algebra in the curriculum, build a stronger foundation of number-and -operations, and avoid students' overreliance on calculators to meet the CCSS-M.

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INFLUENCES OF BLACK-BOX APPROACH TO PRESERVICE TEACHERS' DESIGNING GEOMETRIC TASKS

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In contrast with the recommendation by CCSS-M, geometric tasks with dynamic geometry environments still emphasize rote memorization and skills rather than conceptual understanding. The black-box approach is a new type of task, in which a pre-constructed figure satisfies certain conditions to maintain geometric invariants when using the drag mode, is challenging to students because it "requires a link between the spatial or visual approach and the theoretical one" (Hollebrands, Laborde, & Sträßer, 2008, p. 172). Through the theoretical lens of the Mathematical Tasks Framework (Stein, Grover, & Henningsen, 1996), this study aims to examine how secondary mathematics preservice teachers make choices or creations of high and low cognitive demand geometric tasks and how the role of their technology use is changed before and after the exposure of black-box approach.

To examine influence of the black-box approach to design geometric tasks, we conducted a qualitative case study with three secondary mathematics preservice teachers. This study conducted semi-structured pre-interviews, followed by the lessons regarding the black-box approach. Finally, we examined the preservice teachers' preparation of geometric tasks in their lesson plans along with post-examination interviews.

The examination of preservice teachers' lesson plans found that they changed the creation and selection of geometric tasks from emphasizing textbook-like procedural tasks to providing black-box tasks or real-life context problems, which require high cognitive demands. Through the post-interviews, the participants also built positive attitudes towards using the dynamic geometry software for geometric tasks with high level thinking. Findings of this investigation may provide guidelines for integrating technology with designing geometric tasks, which requires high cognitive demands, in teacher educators and researchers.

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IF NOT HERE THEN WHERE? DEVELOPING METACOGNITION IN A GEOMETRY CONTENT COURSE OF ELEMENTARY PRESERVICE TEACHERS

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Current calls for mathematics teacher education emphasize the importance of specially designed courses that provide experiences for teachers to develop the habits of mind that they will in turn develop in their students (CBMS, 2012).

Because these habits of mind require the sharing mathematical thinking and reasoning it is important that students must be aware of how what they are thinking and how they are regulating those thoughts, thus emphasizing the importance of metacognition. This study employs the pillars presented in Lin's (2001) framework for designing metacognitive activities to answer the following question: Is there alignment between the textbook's message for developing metacognition, the intended message sent by the instructor, the enacted classroom activities, and the message received by the students?

The qualitative data was collected from two sections of a geometry content course for elementary education majors (n=23 and n=28) and included the required course textbook, instructor interviews, classroom observations, and open-ended surveys with the students. Analysis was conducted by first identifying examples of the four principles in the data, then comparing the information from each data source.

Findings emphasize that even though students are provided frequent opportunities to self assess, the enacted activities conducted by the teacher provide fewer opportunities for the remaining three principles, thus limiting the thoughts to be used as objects for class discussion. These findings emphasize the importance of the instructor and their ability to create the class norms that lead to the development of the four principles. Further, studies should continue to explore the impact of the mixed messages being sent from this course on students' future opportunities to learn and develop their pedagogical knowledge.

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BUILDING SUCCESSFUL MATHEMATICS LEARNING COMMUNITIES

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School mathematics presents the subject as a study of rules with mastery being an individual's ability to memorize rules and reproduce them effortless. Students who are not successful in such a teaching environment are likely to assume an identity of inferiority (Stinson, 2013) and give up studying mathematics in the face of challenges. Using the ideas of mindsets by Dweck (2007), a mathematics learning community, Prepare2Nspire (P2N), was designed to provide mentoring and content support to eighth and eleven graders using a cascading mentoring and tutoring structure. The rationale for this model is that through the sharing of personal experiences as mathematics learners, mentors and tutors ("mentutors") will nurture in their mentees and tutees ("mentutees") a change in mindset and attitude towards mathematics.

This current study investigated the influence of "mentutors" on "mentutees" mathematics achievement and their attitude towards mathematics after participating in P2N. There were 15 undergraduates, 45 eleventh graders and 90 eighth graders in the program. Preliminary results show that such a learning community is welcoming to participants who tend to fall through the cracks in most mathematics classrooms. Also, participants reported the joy of observing their mentors use different methods and strategies in solving tasks in contrast with what usually happens in traditional mathematics classrooms where there is usually only one way to perform tasks – the teacher's way. The *Prepare2Nspire* Model has one undergraduate who works with three eleventh grade students, while each eleventh grade student is paired with two eighth grade students.

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USING COOPERATIVE LEARNING STRATEGIES TO ENHANCE PRESERVICE TEACHERS' UNDERSTANDING OF FRACTIONS

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Since fractions are an essential mathematical concept and a challenge for students to understand, preservice teachers must be prepared to teach them. Research has established that for some teachers, their prior learning experiences and understanding of fractions may be inadequate for the task (Thompson, 1992). Such learning experiences often emphasize procedures over conceptual understanding. Vygotsky's theory (1982) supports the use of social interaction to promote learning of mathematics through small-group cooperative learning. Within a group, the more expert members often assist the novice members in gaining understanding and expanding their zone of proximal development. The aim of this study is to explore the benefits of cooperative learning to enhance preservice teachers' understanding of fraction concepts.

Fifty-three elementary preservice teachers were assessed on their ability to answer questions involving fraction concepts. Despite most having had four years of high school mathematics, and being enrolled in a highly selective undergraduate institution, the pretest revealed their limited understanding of fractions. Two cooperative learning strategies (Jigsaw and Random Reporting) were implemented over two eighty-minute periods. Each small group contained at least one high-performer from the pretest, and the lessons focused on the items garnering the fewest correct responses on the pretest including the definition of a fraction, fraction models, and division of fractions.

Results indicate significantly improved performance on three of five posttest items, but the ability to explain the reasoning behind the invert and multiply rule for division of fractions remained elusive for twenty-four (45%) of the participants. Cooperative learning played a partial role in the improved posttest performance, but the lack of mastery of the ability to explain division suggests that increased time and attention must be provided to insure the type of understanding needed to successfully teach fraction concepts. Further results will be showcased at the presentation.

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INDIGENOUS MATHEMATICS AT THE EDGE: CULTURALLY BASED MATHEMATICS UNITS

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Ethnomathematics provides a locus from which to engage issues of equity and quality in education. PREL received National Science Foundation (NSF)[1] funding for a study focused on ethnomathematics and culturally diverse learners in ten Micronesian indigenous language communities with the aim of informing research and deepening practice for teachers and leaders who work in these contexts. In 2009 PREL implemented Project MACIMISE (Mathematics and Culture in Micronesia: Integrating Societal Experiences), a collaborative effort between PREL and UHM.

The goals of the Project were to (a) develop and assess local mathematics curriculum units for Grades 1, 4, and 7; (b) rediscover/uncover the indigenous mathematics of ten Micronesian language communities; and (c) build local capacity by offering advanced degree opportunities to local mathematics educators. The dynamics of how these goals are being achieved consisted of three phases: (1) to educate local mathematics educators to be *collectors* and, (2) *documenters* of mathematical practices found in Micronesian cultures, and (3) to have the twenty-one *Macimisers* (the term used to designate participants) selected, create and implement curriculum units based on the practices uncovered.

As of 2014, Macimisers had developed seventeen culturally based mathematics curriculum units, pilot tested them in first, fourth, or seventh grade classrooms, and selected seven units for an experimental/control impact study currently underway (LaFrance, 2013). On the poster, pictorial examples will be provided of the culturally based mathematics that have been developed.

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[1] Project MACIMISE is supported by a NSF grant (#0918309). The material in this paper is based on work supported by that grant. The content does not necessarily reflect the views of the NSF or any other agency of the US government.

TECHNOLOGY USE OF STUDENT TEACHERS IN MATHEMATICS CLASSROOMS

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This study focuses on student teachers' technology use during a larger project. We believe understanding how new teachers incorporate technology in the classroom provides evidence to improve the field of education. In order to understand the nature of student teachers' technology use, we adopted Dick and Hollebrands' (2011) classification of technology use as 'conveyance' and 'mathematical action' in instruction. Conveyance technologies are used to display material or transfer knowledge while mathematical action technologies require dynamic interaction and provide students learning by reasoning and understanding (Dick & Hollebrands, 2011). Our research questions are;

- What types of technology do student teachers utilize during mathematics' lessons?
- How do student teachers utilize technology during mathematics' lessons? Do they use technology in a conveyance manner or mathematical action manner?

Data was gathered from NSF granted Iterative Model Building Project (IMB). We used lesson observation protocols by looking at ratings on the items about material use during instruction and related notes about specific use of technology. We also looked at observers' field notes and student teachers' interviews to have a better understanding of the nature of their technology use.

The data included a total of 79 student teachers and 146 lessons. Only 42 student teachers used technology in 68 lessons. The results showed that the types of technology used in the lessons were interactive whiteboards, visual presenters, the Internet, videos, tablets and electronic flash card devices. The findings suggested that technology was exclusively used in a conveyance manner during mathematics lessons. Thorough analysis using constant case comparison tables created from the different data sources of protocols, field notes, and interview transcripts provided support in finding no instances of mathematical action technology. We found that the lack of mathematical action technology in student teachers' lessons is particularly concerning considering the capabilities technology has to learning mathematical ideas in a dynamic manner.

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ENGAGING UNDERGRADUATE MATHEMATICS STUDENTS IN INFORMATION PROBLEM SOLVING: A CROSS-CASE ANALYSIS

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This poster reports on a research study that explores what happens when information problem solving (Walraven et al., 2008) is introduced into undergraduate mathematics courses. In recent years, there has been a strong push for the integration of problems that require students to seek out and evaluate information into instruction in the disciplines. In particular, students have been shown to be able to construct superior arguments when they are given the opportunity and means to seek out and evaluate the credibility of information sources (Wiley et al., 2009). However, most of this work has been focused on science classrooms and so this study seeks to answer the following research questions: 1) How do undergraduate students in a general education mathematics course work with information-based problems? 2) How does the instructor negotiate the work transacted for these problems?

My study consists of a cross-case analysis of three courses taking place at a community college (n = 17), a public research university (n = 22), and a regional public university (n = 14) respectively. The author collaborated with all three instructors to develop information problem solving activities tailored to each class. All of the activities included an in-class component (e.g., a discussion or debate) that was audio- and video-recorded. Discourse analysis focused on argumentation was primarily used to analyse the data. I will be presenting results in the form of a mid-range theory describing how the students related to the information-based problems, and a taxonomy of the types of arguments that they constructed about the credibility of the sources that they used with a particular focus on how they employed their mathematical knowledge in these arguments. I will also present findings focused on how the instructors managed these activities. Most strikingly, I found that the instructors of these courses engaged in practices that discouraged sophisticated credibility judgments and encouraged the students, instead, to focus their attention on the presence or absence of specific mathematical objects.

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KNOWLEDGE OF FEATURES OF LEARNING MATHEMATICS AS PART OF MTSK

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This poster presents some advances made in the construction of the model of professional knowledge, Mathematics Teacher's Specialised Knowledge (MTSK), which is being developed by the research group for mathematics education based at the University of Huelva, Spain. We focus mainly on specifications for the subdomain, Knowledge of Features of Learning Mathematics (KFLM), for which we provide examples that serve as the basis for discussion of the model and the subdomain as part of this specialised knowledge.

Inspired by the idea of Specialized Content Knowledge – SCK (Ball, Thames & Phelps, 2008), as a knowledge uniquely "useful to" and "necessary for" the mathematics teacher, Carrillo, Climent, Contreras and Muñoz-Catalán (2013) present an analytical model for studying mathematics teachers' knowledge, MTSK, which was developed through the analysis of some difficulties detected in MKT (Flores, Escudero & Carrillo, 2013) and firmly grounded in thorough theoretical work and empirical studies into knowledge and professional development.

We now focus our attention on the knowledge about learning mathematics which the teacher has or requires, which, in MTSK, is located in the subdomain KFLM. This refers to the teacher's knowledge about the features of learning inherent to a particular mathematical content or mathematics in general and knowledge about theories of mathematics learning gained from professional experience or from research into the generation of theoretical frameworks which are able to account for the process of construction of mathematical knowledge. Despite the focus is not on the students' knowledge as principle actors in the learning process, but instead on the learning features derived from their interaction with the mathematical content and on the features of the mathematical content itself as an object of learning, the knowledge about the student learning process is consider as part of this subdomain.

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EXPLORING LEARNERS' MATHEMATICAL IDENTITY AMONG NEWCOMERS USING A FOUR-PART ANALYTICAL MODEL

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Learners' mathematical identity has been recognized as a dominant and determining factor in learning school mathematics (e.g., Sfard & Prusak, 2005). Although substantial research has established a strong association between identity and school mathematics, mostly within the sociocultural framework (e.g., Vygotsky, 1978), there is a paucity of relevant research that focuses on teenage newcomers as they integrate into a new educational system thus creating a lacuna in our understanding of newcomers' developing identities as learners of mathematics. This research aims to investigate learners' mathematical identity among teenage newcomers from Israel.

The research design in the current research comprises of three interview contexts: a family interview with at least one dyad of a parent and a teenager per family, an individual interview with the teenager and the parent, and an all-parent and an all-teenager focus groups. This research design allows participants to respond to and further develop emergent issues that relate to their experience of mathematics. Following a dissemination of *A Call for Participants*, seven newcomer families contacted the researcher. To date, seven family interviews and 16 individual interviews were conducted. Ivanič's four-part analytical model from the field of academic writing was adopted to analyse the data. This analytical model offers four constitutive referents for the investigation of identity that include: an autobiographical self, a discoursive self, an authorial self, and socioculturally available selves. Each of these different aspects of identity expression will be discussed and supported by examples from the data.

Preliminary findings show how identity work in the context of school mathematics among teenage newcomers changes the past and is directed by it. Specifically, how teenage newcomers produce portrayals of themselves as learners of mathematics and what sense they attribute to these portrayals will be highlighted.

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EXPLORING LOGARITHMS USING NUMBER LINES

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If mathematics is seen as a subject of arithmetical operations (see van Oers, 2001) student could develop what Skemp (1976) called instrumental understanding and if that is the case, the meaning and structure of the logarithm concept might be lost.

This presentation will report the findings from an intervention study that used learning activity (see Davydov, 2008) as a theoretical framework, both for design and analyses. In this paradigm concept are not presented in "ready-made" form, e.g. $y = a^x \Leftrightarrow x = \log_a y$ where $y > 0$, so the aim of the study was to develop a new model for logarithms. The construction of the model was inspired by the original idea that Napier constructed in 1614, exactly 400 years ago, by using two number lines. In order to see if students could use the model to operate with logarithms, some learning tasks were constructed as well. If taken, the intention was that they should help the students to unfold and single out some of the unique properties of logarithms.

Learning study (see Marton & Pang, 2006) was used as a research model to be able to develop the lesson iteratively. The sample comprises 150 upper secondary students and the intervention took place in six different classes with six different teachers. Data from the lessons were collected by filming, to analyse students learning actions, and by short interviews, to ask students about the meaning of logarithms.

The analyses showed that the constructed model for logarithms seemed to be helpful for the students, as very few of them over-generalised mathematical rules, e.g. used the distributive law, or separated log-expressions, e.g. adding log expression part by part. On the other hand, many students suggested square rooting as an inverse to exponential expressions, when they searched for the relationship between the numbers on the two number lines. This could be interpreted as a new kind of over-generalisation, but further research about this issue is suggested.

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PRE-SERVICE TEACHERS' COMPETENCE OF ANALYSING THE USE OF REPRESENTATIONS IN MATHEMATICS CLASSROOM SITUATIONS

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Due to the abstract nature of mathematics, it is not possible to access mathematical objects without the use of representations. Dealing with representations and changing between them is a highly demanding cognitive process – a lack of support in reflecting and making connections between different representations can lead to difficulties in students' understanding (Duval, 2006). Against this background, analysing classroom situations with respect to the use of representations can be considered as a key aspect of teachers' expertise. Not only the question to what extent teachers attend to significant events (Sherin, Jacobs & Philipp, 2011), but also whether they are able to analyse the classroom situations by drawing on professional knowledge related to the use of representations, are core aspects of an important competence component of mathematics teachers.

The poster presents a current project which focuses on relevant professional knowledge related to the use of representations and aims at developing an evidence-based hierarchical model of the competence of analysing the use of representations in classroom situations. In a first step, we currently develop a test instrument comprising both of text and video vignettes. These vignettes present authentic classroom situations in which the use of representations calls for analysis. The competence of analysing will be assessed with pre-service teachers and in-service teachers in both a pilot and a main study. The teachers will be asked to analyse the use of representations in the presented classroom situations in open and multiple-choice question formats. The format of the questions will be designed on the basis of a pre-study carried out with 31 teacher students, which indicated that different question formats had a substantial influence on the quality of the pre-service teachers' answers. The challenge is to design the questions so as to encourage a focus on the use of representations without, however, suggesting a specific view or particular criteria for expected aspects of analysis based on relevant professional knowledge.

The poster presents an overview of the theoretical background, research interest and design of the planned study alongside with results of the pre-study and examples of classroom situations serving as possible scripts for the planned video vignettes.

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SYMBOL SPACING AND ITS INFLUENCE IN SCHOOL CHILDREN'S CALCULATION OF MIXED-OPERATION ARITHMETIC EXPRESSIONS

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Learning the order of operations when calculating the result of a mixed-operation expression like 2+3×5 is surprisingly difficult despite its apparent simplicity. Even in university, sizable numbers of students may incorrectly report that 10+5/5 equals 3 (Pappanastos, Hall, & Honan, 2002). The search for teaching strategies to improve learning has led researchers to evaluate the use of explicit, unnecessary brackets as a means for improving students' retention of the correct order of operations. Results in this direction, however, are not consistently positive (e.g. Gunnarsson, Hernell, & Sönnerhed, 2012). An alternative approach to brackets is to use symbol spacing for grouping operations (e.g. 2 + 3×5 *vs*. 2+3 × 5, Landy & Goldstone, 2010).

Here, we investigate how symbol spacing affects calculations made by school children. More precisely, we presented an arithmetic questionnaire to 60 Italian 5th graders and 115 Chilean 7th and 8th graders. Symbols in each expression were positioned with larger spaces surrounding either the first or the second operation (e.g. 2 + 3×5, 2+3 × 5). The data show that students split naturally into two main groups: those who calculated from left to right regardless of the operations involved, and those who calculated consistently multiplications before additions. In addition, for both countries the group of students who calculated from left to right showed a statistically significant modulation of behaviour depending on symbol spacing: the probability of answering correctly to items like 2+3 × 5 and 3 × 5+2 was lower than for items like 2 + 3×5 and 3×5 + 2. The presence of this bias across both age groups suggests that symbol spacing may be a useful tool for supporting the learning of the order of operations.

This research was funded by the Chilean programs CONICYT PIA (grant CIE-05) and CONICYT PAI/Academia (grant 79130029), the Italian Ministry of Health (grant F-2009-1530973), and the NEURAT project from Università di Padova.

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THE PRE-SERVICE TRAINING OF PRIMARY TEACHERS. WHAT PLACE IS THERE FOR VARIATION AND COVARIATION?

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Functions play an important role in the curricula of school mathematics in many countries and they are necessary for the understanding of other mathematical notions. However, research has identified many difficulties that students encounter when learning functions at school. To overcome some of these difficulties, Kieran (2004), among other, recommends the introduction of functional relationships, and especially the notions of variation and covariation, from elementary school. Nevertheless, research has also identified some gaps in the knowledge to teach in primary school teachers. Taking into account these two elements (the recommendations to introduce the notions of variation and covariation in primary school and teachers' gaps in their knowledge to teach), we are interested in investigating whether the pre-service training program allows future primary teachers to be aware of the importance of the notions of variation and covariation for their future practice.

Our research analyses the structure of the pre-service training program of primary teachers in our university, together with the syllabi of the mathematics education courses of this program, following the anthropological theory of didactics (TAD, Chevallard 1999). This theory attempts to achieve a better understanding of the choices made by an institution to organise the teaching of mathematical notions, as well as the consequences of these choices on the learning achieved. We then try to establish what elements of the pre-service teachers' *personal relationship* with the notions of variation and covariation seem to be a consequence of the choices of the institution to train them.

In this poster, we discuss the data concerning the first part of our research: to identify the elements that characterise the *institutional relationship* with the notions of variation and covariation as a prelude to functions, in the institutions of pre-service training of primary teachers. Our data seem to indicate that these institutions might neglect the importance of pre-algebra and functional relationships to train future primary teachers, which could have consequences in their knowledge to teach, as well as in their ability to follow the recommendations of research to introduce variation and covariation in primary school.

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FORMATIVE MATHEMATICS ASSESSMENT IN UPPER SECONDARY SCHOOL

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The purpose of this research is to examine how students experience high quality feedback, how they react to the feedback and how they use it.

Providing feedback on student learning is a key strategy in formative assessment (Hattie et al., 2006; Black & Wiliam, 1998). Students need to know (1) the goal, (2) their current level, and (3) the gap between those in order to advance towards the goal (Hattie & Timperley, 2007). In addition to be given information about (1)-(3) above, the students also need to engage actively with the feedback. Students are, however, often not satisfied with the feedback they receive and do not always use it. In order to facilitate students' use of their feedback, in this study a sample of upper-secondary mathematics students received high quality feedback (i.e. non-evaluative, supportive, timely and specific, see Shute, 2008). How the students experienced this high quality feedback, how they used it and what they learnt from it was then investigated.

The data collection was done in the following steps. First, an intervention test with two calculation problems was given to the students. The answers were then handed in and processed based on theories of formative assessment and feedback. The students were given the high quality feedback and, in order to catch their first impressions, asked how they perceived it. The next step was the regular teacher's test – to examine potential progress resulting from feedback. Finally, in depth interviews were conducted in order to study students' experiences of the high quality feedback.

As initial results show that the students tend to limit their view on assessment as whether a mathematics question is answered correctly or wrongly, they see feedback as an extracurricular activity and not as a learning situation. The results suggest that it is essential to integrate the feedback in the day-to-day instruction in order to increase the likelihood for it to be used by the students.

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EXPLORING UNIVERSITY STUDENTS' LEARNING STRATEGIES IN MATHEMATICS: REFINING THE LIST QUESTIONNAIRE

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There are numerous projects that attend to the transition from secondary to tertiary education in mathematics, some focussing on learning strategies. Cognitive development theories are often used to understand how learning higher mathematics works, e.g. by the PME working group on *Advanced Mathematical Thinking* founded in 1985. The role of affect, motivation and beliefs has broadened the view on learning processes and has contributed to reach a deeper understanding. Learning strategies mirror both cognitive and affective aspects and therefore are well worth studying. Similar to the English MSLQ, a seminal approach for studying learning behaviour is given by the German LIST questionnaire (Wild & Schiefele, 1994). While studying the effects of learning strategies, a new research goal emerged: shortening LIST while keeping the factor structure comparable to the original, and therefore its potential for describing learning behaviour and significant changes therein.

First-year engineering students (N=1123, 76.54% male, M_{age} = 20.65 years, 21.64% with a first language different from German) were questioned in a pre/post design. Confirmatory component analysis (CFA) of the original 69-item LIST questionnaire had identified *Metacognition* as problematic. Cronbach's α were calculated for each original scale, including the *scale if item deleted* option. Thus, each scale was repeatedly shortened. CFA and principal component analysis (PCA) of the resulting 40-item questionnaire prompted further deletions, resulting in a 32-item questionnaire with acceptable model fit (RMSEA= .0601) that yielded nine factors (varimax rotation, pairwise exclusion of cases, KMO= .807, all KMO values ≥ .648, $\chi^2(496)=7985.894$, p= .000) that exactly match all original scales but *Metacognition*. Together these factors explain 60.48% of variance. Reliability holds in the separate surveys (out of the 36 alphas, all but one are above .6), and the potential for describing pre/post developments with *t*-tests is comparable to the original. Some initial assumptions about the impact and measurability of metacognitive behaviour seem incorrect, so it might still be advisable to keep the *Metacognition* items in the questionnaire for further tests, or to use other forms of procuring data.

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IS 1 SQUARE METER EQUAL TO 10000 SQUARE CENTIMETERS?

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A central aspect of measurement is unit iteration that is the repetition of a single unit. For this purpose students have to be aware (1) of fitting units together without gaps or overlaps and (2) of unit conversion, which includes an understanding of the inverse relationship between the chosen unit and the number of units (Lehrer, 2003). Taking into account that these principles of measurement are already taught in primary school it is assumed that students are able to solve tasks like "Change 1 m^2 to cm^2" in secondary school at the latest. Unfortunately it's not as simple: Students and even pre-service teachers still have difficulties visualizing unit structures (Outhred et al., 2003). Therefore two interventional studies are conducted focusing on the following research questions: (RQ I) Is it possible to foster students' unit conversion performances by unit structure instruction in a short period of time for the sizes area and length? (RQ II) Does knowledge of benchmarks support converting units more effectively?

Referring to RQ I within an interventional study (taking place in January 2013) 60 pre-service teachers of an experimental group have been trained in unit conversion in comparison to 60 pre-service teachers of a control group not having been trained. Results contrasting these two groups point out that it's possible to foster students' unit conversion performances in a short period of time: Students of the experimental group scored higher in a posttest (8 items, α=.79) than students of the control group ($t(104)=4.28$, $p<0.01$, $d=0.84$). Furthermore regression-analyses with performances on a verbal intelligence test and treatment-condition as predicting-variables and students' posttest performances as depending variables hint at a significant influence of the treatment condition. Nevertheless there are still students not being able to convert units for the sizes length and area at the end of this study. That's why – referring to RQ II – within a second study (taking place in May 2014) two different interventions to foster pre-service teachers' unit conversion performances will be tested: The unit structure intervention (see RQ I) will be compared with a mixed intervention based on unit structure and benchmark instruction. It is expected that knowledge of benchmarks can support converting units more effectively than knowledge of unit structure.

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HIGH SCHOOL STUDENTS' REASONING ABOUT PROPERTIES OF SOLIDS USING DGS

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In this report, we examined high school students' uses of dynamic geometry software (DGS) while solving 3-dimensional geometry tasks. The data were collected from two geometry classes. In the first class, students used DGS ***constructions*** *in which they were able to drag objects. In the second class, students used DGS* ***drawings*** *in which properties of objects were not maintained when a point was dragged. We illustrated cases in which students did not attend to the properties of 3D objects using a DGS drawing or construction.*

OUTLINE

Building upon previous research, Hollebrands and Smith (2009) described *drawing, construction, figure*, and *diagram* using DGS. Hollebrands and Smith defined a DGS *drawing* as "a process that involves the use of "freehand" tools to create a geometrical object," and focus on its perceptual characteristics (p. 221). In this case, students may work on a *diagram* without "attending to the properties or the perceptual characteristics of the visual representation of a geometrical object" (p. 222). However, DGS *drawing* does not necessarily have the properties of geometric object as opposed to a DGS *construction*. In this report, we schematized the aforementioned definitions together as shown in Figure 1, and illustrate cases in which students used a DGS construction and drawing for solving 3D geometry tasks.

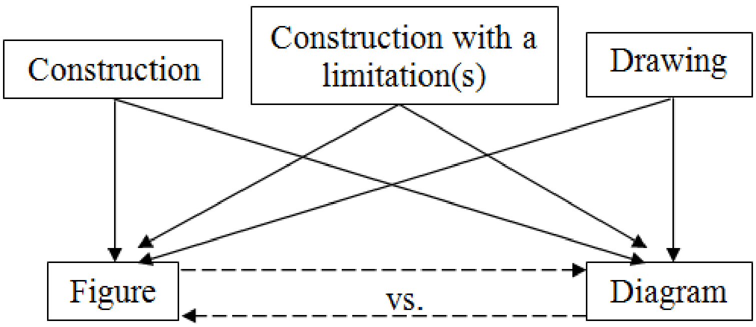

Figure 1

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A MODEL OF PROFESSIONAL LEARNING GROUPS

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Professional learning groups have been widely used as a form of professional development in mathematics and have resulted in a variety of impacts on teaching practices (Kajander & Mason, 2007). This poster discusses a framework for examining teacher growth in such groups. The model was created by exploring a professional learning group which was successful in supporting teacher development.

SUMMARY

There is a range of characteristics of professional learning groups, and thus the impact that they have on teacher growth (Kajander & Mason, 2007). Professional learning groups vary from groups which work together to develop and explore new ideas in mathematics education, to groups which maintain their current practices by simply developing new tests to be used in their classrooms. This research focused on the characteristics that were vital for a professional learning group to encourage teachers in using reform-based pedagogy in their mathematics classrooms.

This research focuses on a three year case study of a professional learning group comprised of grades 7 to 10 mathematics teachers. After exploring existing research of the characteristics of professional learning groups (i.e. Hord & Sommers, 2008) as well as research in mathematics education, the features of the case study were expanded to create the model.

Although differences from characteristics of other professional learning groups became apparent during the case study, the most important was the need for a "leader" in the group. In order for conversations in the professional learning group to stay focused on exploring research-based pedagogy, one or more of the members needed to believe in these strategies as well as be able to encourage discussions to focus on them.

This model was created to explore features that were important for a mathematics professional learning group to be successful in order to support teacher growth.

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THE COMMON CORE IN APPALACHIA: A TALE OF THREE TEACHERS

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The Common Core State Standards for Mathematics (CCSSM) have now become law in most all of the United States. "Mathematics teachers will direct and determine the extent to which any curriculum changes in mathematics affect student learning." (Reys, 2011) In addition, teachers of rural, poor students face added difficulties in providing an equitable education for their students. (Harmon and Smith, 2012) Much research goes into the big picture changes in curriculum, assessment, and instruction; however, what is known of the specific experiences of teachers in adjusting to change? This research is an attempt to understand the effects of implementation through the unique lens of three rural Appalachian mathematics teachers.

RESEARCH AND FINDINGS

Structured interviews were conducted with three high school teachers who practice in rural Appalachian counties with child-in-poverty rates between 31% and 48%. Interview questions focused on three main areas: effects of implementation on classroom instruction, curriculum and assessment; teacher preparation for implementation; and any perceived equity issues involving the rural poor. The interviews were transcribed, coded by key words, and analysed for emergent themes. The major changes these teachers saw revolved around the increased rigor and depth of the standards. In all three cases, the schools have added an algebra class between Algebra I and Algebra II to cover the required content. Common planning and common assessments have also been implemented in each school, putting teacher collaboration as a central focus of scheduling. Lecture time has decreased to give way to practice, but project-based learning is rare. The preparation undergone by each of the teachers varied from attending state-sponsored meetings, recently completing an undergraduate teacher certification where the Common Core was emphasized, and having no assistance at all. Equity of resources was still a problem, but all teachers felt that the new standards would put their students on par with those from other areas.

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STUDENTS' RELATIONAL REASONING ABOUT MEAN, MODE AND MEDIAN

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This study examines students' relational reasoning regarding the three measures of central tendency. Using mixed methods we analysed 138 students' responses on 4 pre- and post-problems. Results show students were more likely to reason about mean and mode, but had limited view of data distribution.

The concept of measures of *central tendency* (a.k.a. *measures of average*), with three standard measures, has been included in curricula as a part of data handling. The Common Core State Standards in Mathematics includes this concept in Grade 6 curriculum as an entry point to inferential statistics. Research has documented students' developmental growths in this area (e.g. Watson & Moritz, 2000). However, much of past research has placed emphasis on children's understanding of arithmetic mean. There is a paucity of empirical research that captures how students understand the three measures - mean, mode and median- together in relationship to distribution.

This study aimed to examine students' relational reasoning about mean, mode, and median, as well as obstacles that prevent them from establishing relational reasoning. We used the SOLO taxonomy (Structure of Observed Learning Outcomes) to rank students' thinking based on their responses to the constructions of data sets given two or three measures of central tendency from Prestructural, Unistructural, Multistructural, and finally to Relational levels (c.f. Watson & Moritz, 2000). Data was collected from 138 sixth grade students from nine schools. Instrumentation consisted of both pre- and post-tests. The pre-test problem required students to reason about two measures while the post-test required analysis of all three. Mixed methods of nonparametric Friedman's test and qualitatively examining students' work were conducted.

Based on the results of the Friedman's test, the growth of relational reasoning from pre-test to post-test was significant, χ^2 (1) = 42.05, $p < .001$. A close examination revealed that the gain dominantly arose from Prestructural or Unistructural level to Multistructural level. Students were more likely to reason between mean and mode, but not median together with mean and mode. Two obstacles were present in students' solutions: 1) students were confounding mean as the mid-point of the data set; 2) students had limited visions of data sets which were sets of small, positive integers with no extreme values.

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CONCEPT IMAGES OF INNER PRODUCT IN THE MINDS OF TWELFTH GRADE STUDENTS IN TAIWAN

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When students solve mathematics problems, the concepts or thinking evoked in their minds are not the same as those taught by their teachers. The model of concept image and concept definition raised by Tall and Vinner (1981) provides us directions to seek out explanations for the phenomenon. Inner product is an important topic in Taiwanese senior high school mathematics curriculum; however, the studies regarding students' concept images of inner product still lack. This study aims to probe into the concept images of inner product possessed by students in Taiwan.

A questionnaire with dichotomous items and open-ended items was administered to obtain quantitative and qualitative data pertaining to students' concept images of inner product. The sample includes 149 twelfth grade students in two senior high schools in Taiwan. A content analysis and inductive analysis were performed to analyze students' responses to the open-ended items.

The findings indicate that the concept images for the definition of inner product which the students aroused were of three types: (1) the algebraic type, the images were described with an algebraic formula $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\theta$, (2) the graphic type, the images were described based on the graph which illustrated the inner product $\vec{a} \cdot \vec{b}$ as the multiplication of the projection of $\vec{a}$ on $\vec{b}$ and the length of $\vec{b}$; (3) the coordinate type, the images were described with the equation relating to coordinates $(x_1, y_1) \cdot (x_2, y_2) = x_1x_2 + y_1y_2$. Almost all students (95%) could arouse the concept images of the algebraic type; however, only 70% of the students had the concept images consistent with the concept definition of inner product in mathematics. A total of 82% students possessed the concept images of the coordinate type. Only 38% of the students could arouse the concept images of the graphic type, and this type was almost only evoked by students with advanced mathematics level. The concepts of vectors are prerequisite knowledge for learning inner product. However, this study found that certain students possessed misconceptions toward vectors. One fourth of students still had difficulties in distinguishing vectors from scalar quantities even after learning inner product.

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A MODEL OF COGNITIVE DOMAINS IN MATHEMATICS AND SCIENCE BASED ON CONDITIONAL PROBABILITIES

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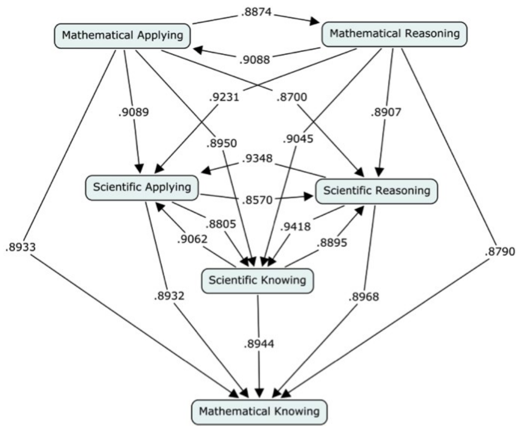
The purpose of this study is to investigate a relationship among cognitive attributes of mathematics and science using generalized DINA (G-DINA; de la Torre, 2011) as an analysis method. For targeted attributes, three cognitive domains from the TIMSS assessment framework (Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009) - knowing, applying, and reasoning in mathematics and science - were adopted. Data used in this study were the responses of 11,673 fourth-graders in mathematics and science standardized assessment conducted between 2006 and 2010.

A probability of students' mastery of each attribute, given by G-DINA, was used to compute conditional probabilities of students' attainment of two attributes. Conditional probabilities over .85 were selected to determine necessary conditions of attaining each attribute. We established the prerequisites for attaining each cognitive skill based on the necessary conditions; "If P is a necessary condition of Q, then there are admissible circumstances in which the absence of Q is a consequence of the absence of P" (Sanford, 1976, p. 204).



As a result, attaining mathematical knowing is a necessary condition for mastery of all the other cognitive attributes in mathematics and science where mathematical reasoning and applying are correlated. It is noted that three interrelated science attributes are necessary to master mathematical applying and reasoning simultaneously. These results imply that students' performance in science can significantly influence on mathematical applying and reasoning, and mathematics knowledge impacts science learning. This study signifies the needs to investigate interdisciplinary approaches to teach and learn mathematics and science.

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PERCEPTIONS OF "SIMPLE PARAMETRIC EQUATIONS"

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Many pupils find it difficult to recognize that a letter represents a number and do not know how to work with symbolic values (Kieran, 2012). Problems with parameters test the solver's knowledge and understanding of mathematics and help to identify his or her weak points. The study objectives are to examine and identify the difficulties of high school students and pre-service student teachers in solving equations with parameters. In addition, to find out if there are significant differences between students and student teachers in regards to solving such equations. Which procedures are used and which difficulties are encountered in solving equations with parameters. We chose to present some equations in non-standard ways. For example, having to express "*a*", usually perceived as a parameter, in terms of "*x*", usually perceived as a variable (Ilany, 1998). We investigate how subjects solve those equations as well as the differences between the students and the student teachers.

Examples of equations investigated in the study:

1. Find x (in terms of a) in $5x^2+8ax-4a^2=0$ (fairly standard equation, with variable x and parameter a)
2. Find b (in terms of x) in $x^2+6bx+5b^2=0$ (less standard as it is required to express b in terms of x, after re-arranging the equation "properly").

The study population consisted of 115 mathematics student teachers in their third and fourth year of studies, and 133 twelfth grade students of high mathematics level in four secondary schools. A questionnaire was designed for the study and administered to both groups. To understand the subjects' solution process, five students and six student teachers were interviewed. The purpose was to clarify and prod further the findings of the questionnaire. In addition, open observations were made while the subjects were tackling the questions. Quantitative analysis was carried out via descriptive statistics, χ^2 tests and t-tests. Qualitative analysis was carried out by observations and interviews. The results were then analysed according to the emerging criteria. Our findings suggest that equations containing parameters are more difficult to solve than equations without parameters. The difficulties were found in the letters that should be expressed and in the arrangement of the equation. The results of this study would help teachers to understand the difficulties encountered by the students and to teach the subject more effectively.

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WHO SHOULD TEACH PROSPECTIVE MATHEMATICS TEACHERS: MATHEMATICIANS WITH EDUCATION BACKGROUND OR ONLY EDUCATION EXPERTS?

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The Fundamental goal of teaching programs is to develop teachers' knowledge because much research literature indicates that those teachers who have more teaching knowledge would choose more effective teaching instruction to benefit their students than those who would not. However, it is not clear whether mathematics teachers should be trained by teacher educators (specialist in education) or teacher educators as in mathematics specialists. Which one of them often selects teaching instructions led him/her to develop teachers' knowledge? An appropriate theoretical framework for this paper is related to Shulman's classification of teacher's knowledge. Shulman (1986) divided teachers' knowledge into three groups: subject matter content knowledge, pedagogical content knowledge and curricular knowledge. A teacher educator and a mathematician, who had experience teaching prospective teachers for more than a decade in a large Canadian university, participated in this study. The participants' class-lectures were observed several times. A two hours interview was conducted with the mathematician, however, because of limited time; no interview was run with the teacher educator. The interview was audio recorded and then the recording was transcribed and analysed based upon Shulman's (1986) framework. The observations were also reviewed and analysed.

The results of the mathematician interview and the class observation of both participants show that they intended to employ their rich mathematics background to support students designing open-ended mathematics tasks that have a number of ways of being approached and no automatic solutions. The class observation of both participants and the interview conducted by the mathematician also illustrated that the main goal of their teaching instruction is the broadening of teachers' horizon about mathematics concepts (going beyond the mathematics facts written in the textbooks). It appeared that the main aim of both the teacher educator and the mathematician was developing teachers' pedagogical knowledge (according to Shulman's (1986) framework).The results of this study show that there have been no differences in choosing instructional design for the teacher educator and the mathematician. In other words, they both intend to develop teachers' pedagogical knowledge.

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HOW THE SHAPE OF GRAPH EVOKES DIFFERENT DISCOURSES: CASE OF CONTINUOUS FUNCTIONS

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Studies on understanding of ‘continuity’ have reported a number of concept images held by university students and pre-service teachers about continuous functions such as ‘a graph that can be drawn in one stroke’ or ‘uninterrupted curve’ (Vinner, 1987; Tall & Vinner, 1981). In addition, in a text book analysis the authors found that there are mainly two different definitions used for both ‘continuity at a point’ and ‘continuous function’. The focus of this paper, which is part of a bigger study, was motivated by an observation made of university first year students facing confusion when they have to decide the continuity of a function that is undefined on an interval. The study was hence guided by the research question ‘how does the ‘size’ of undefined part (single point, finite points, finite interval and infinite interval) in a function affect students’ choice of continuity of the function?’

Thirty six first year university students answered a questionnaire where they chose whether six functions (A to F) given in graphical form were continuous or not. The graphs had different point/s, intervals undefined and all the functions were *not* continuous according to the definition they had learnt. Sfard’s (2008) commognitive framework was used to analyse the data. Only three students were consistent throughout in their choices. But the majority of students’ responses changed quite drastically from the function that was not defined at 3 points (C) to the function that was not defined in a finite closed interval (D) and also from D to E (function defined on a closed finite interval) and from E to F (function defined on an infinite open interval). These changes in the discourse could be hypothesized to be evoked by the different features present in the graphs. These could be identified as ‘discourse framers’: “factors that activate certain discourses while inhibiting some others” (Sard, 2008, p. 209). This shows the sensitivity and dependence of discourse procedures on certain specificities of situational stimuli.

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A COURSE FOR COACHES OF MATHEMATICAL RESILIENCE

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"The construct mathematical resilience indicates a positive approach to mathematics that allows people to overcome affective barriers presented when learning mathematics." (Lee & Johnston-Wilder, 2014). It includes the belief that mathematics is personally worth studying, recognition of struggle as part of learning mathematics, a growth mindset, conjoint agency, and an orientation to access a range of available support. Creation of a new course for coaches in mathematical resilience (who support students rather than teach mathematics) gave opportunity for a small- scale design-research study. The research questions were 'would the course enable personal mathematical resilience?' and 'would participants be positioned to help others develop resilience when learning mathematics?'

The participants were 12 women who worked as trainers for apprentices, mainly hairdressers and health care workers, who were required to increase their knowledge of mathematics. The participants self-identified as anxious about mathematics. The course used known good practice in teaching mathematics, such as inclusion, discussion and investigation, and also Egan's skilled helper coaching model.The ten sessions focused on an aspect of coaching and of learning mathematics. The leaders identified specifically with either coaching or mathematics. The focus was on creating an effective learning environment with ground-rules for safe learning, alongside learning mathematics where the participants could practice their growing resilience. Emotional aspects of learning mathematics were discussed using a growth-zone model (Lugalia et al., 2013), which fostered explicit awareness of feelings when in the growth zone and management of panic when in the danger zone. Peer coaching enabled learning to support others' mathematical learning and understanding of how to enable learners to remain longer in their own growth zones.

Data were an initial and final survey, field notes, appreciative enquiry interviews, and portfolios. There is strong evidence that participants gained personal mathematical resilience, managing their own anxiety, and developing resilient ways of learning. Both participants and their employers requested further training, and proposals are in train for larger courses and courses at more advanced levels.

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ACTIVITY THEORY AND MATHEMATICAL RESILIENCE

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"The construct mathematical resilience indicates a positive approach to mathematics that allows people to overcome affective barriers presented when learning mathematics." (Lee & Johnston-Wilder, 2014). A course in coaching for mathematical resilience was perceived as highly successful by all participants, and further courses are being planned. A retrospective analysis of the course was undertaken using activity theory with the aim of identifying its critical aspects in order to facilitate the replication of the course with less experienced leaders.

A mapping of Engestrom's (1987) structural model onto the course highlighted several critical aspects of the course. Mediating artefacts were important, in particular a growth zone diagram enabling participants to identify their emotional responses. Also vital was the explicit negotiation of the rules governing the system, and making visible and challenging a particular cultural belief, that of fixed mathematical ability. Another critical aspect was the development of conjoint agency within the community.

Most significant, however, proved to be the way in which potential tensions informed the planning and running of the course. Experience with a problematic earlier intervention with would-be coaches enabled the course designers to explicitly introduce an unusual division of labour in leading the course. The 'coach' took responsibility for the emotional safety of students whilst the 'teacher' focussed on mathematical ideas, enabling the students to both learn mathematics and learn to handle the stress engendered by doing so. Participants were fully aware at the start that they would experience a tension between their desire to avoid mathematics, and their desire to manage their anxiety in order to work more productively with their students. This suggests that a major requirement for future course leaders would be the ability to enable participants to identify and deal constructively with the tensions that arise when learning mathematics.

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MATHEMATICAL DISCOURSE AT TRANSITIONS

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Studies in second-language acquisition emphasise that the mastering of particular linguistic abilities relevant for a school context has a significant influence on the success of children in school. In this context, Cummins (2000) distinguishes between "Cognitive Academic Language Proficiency" and "Basic Interpersonal Communicative Skills". According to Cummins, the language used by teachers in class is disengaged from situations and expresses dense information. Especially in a subject like mathematics, where greater complexity goes hand in hand with a higher level of abstraction, the ability to receptively understand abstract language disengaged from situations, and to productively apply it, seems almost invariably linked to increased mathematical competencies. If we follow this line of argument, then as children move up through school the mathematical discourse used by teachers should show a change towards a discourse that is disengaged from situations. The German school system separates children by academic level after the fourth year in school – compared to other countries very early; we should therefore be able to reconstruct a change in the form of discourse at this transition point. Recent studies, however, have shown that primary school mathematics teaching provides no introduction to such context-independent linguistic forms of negotiation, even shortly before secondary school (Schütte 2011 et al.), and that everyday knowledge expressed in everyday language does not necessarily stand in opposition to high-level subject learning (Moschkovich 2002). The proposed study would adopt a longitudinal design to address the following questions: How does mathematical discourse change in different school years and after the transition to secondary school? How are children introduced to the mathematical discourse expected in school?

The planned project will make video recordings of everyday mathematics classes in Years 3–6. With a "mixed methods" approach, qualitative video data will be mutually supplemented by quantitative achievement data.

It is hypothesised that the discourse changes in "hidden" ways, and that children are given no explicit introduction to discourse practice.

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PRE-SERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' WAYS OF THINKING AND PEDAGOGICAL APPROACHES IN PROBLEM SOLVING PROCESS

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Problem solving, a fundamental skill of school mathematics, have been studying for more than half of a century. However, some recent studies emphasize that the field of problem solving should be reexamined with different point of views. For instance, English, Lesh and Fennewald (2008) emphasize that problem solving is seen as isolated from development of mathematical concepts and thus there is not a considerable development in this area. Therefore, they stress that is the time to try out alternative approaches in this area. According to Harel (2008) problem solving approaches are ways of thinking in problem solving process. He argues that the most important point in problem solving is the ways of thinking. Since mathematics teachers play a significant role in developing students' problem-solving skills, ways of their thinking are important in that context. The purpose of this study is to investigate, within the framework of DNR (Harel, 2008), pre-service middle school mathematics teachers' ways of understanding and thinking, and pedagogical approaches as well as the relationships among them. This is a qualitatively designed study in which the data was collected through clinical interviews with four pre-service middle school mathematics teachers. Data was analyzed qualitatively by using content analysis technique. The results of the analysis indicated that pre-service mathematics teachers' ways of thinking were fell into two categories. Participants with the first way of thinking attempted to solve the problems by looking for relevant relationships among the given quantities, by looking for patterns or by using trial and error approaches. On the other hand, the pre-service teachers with the second way of thinking tended to solve the problems by using prior mathematical knowledge, definitions, concepts and formulas. This study also revealed that ways of thinking and particularly proof schemes played effective role in pre-service middle school mathematics teachers' pedagogical approaches.

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CHARACTERIZATION OF EXPERT PROVING BEHAVIOR: THE SEARCH FOR AND USE OF COUNTEREXAMPLES

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The importance of proof has long been emphasized by both mathematics teacher education organizations and professional mathematics organizations (e.g., CBMS, 2001). Moreover, the process of proving is also indispensable to the act of doing mathematics. At the higher academic levels (graduate and professional mathematics), proving can be considered as the way in which the truth of a claim is established or realized (Hanna, 2000). However, the research on proof has routinely focused more on the production of a finished and valid proof and less on the purposes behind the provers' use of their demonstrated mathematical knowledge during the process of proving. This study focuses on how and why provers use the mathematical knowledge they call on in the service of proving a mathematical statement. The main research question guiding this study is: In what ways are expert and novice provers of mathematics similar and different in their use of the mathematical knowledge they call on during the process of proving a mathematical statement?

Five novice provers (undergraduate students of mathematics) and five expert provers (advanced PhD students of mathematics) were each given the same five mathematical statements to validate or refute. The participants were asked to think-aloud as they attempted to construct arguments to validate or refute the statements. Using grounded theory methods, claims were generated about how expert provers were similar to and different from their novice counterparts with regards to the use of their mathematical knowledge. The analysis suggests that both the expert and novice provers recognize the implication of finding a counterexample on the validity of the task statement. The expert provers persevere in the search for a counterexample regardless of whether they believe one exists or not. They place an intrinsic value on the search for a counterexample by stating that even a failed search for a counterexample provides them insight into why the statement may be true. However, the novice provers did not seem to value the search for counterexamples. They would not search for counterexamples, unless they possessed a priori knowledge of the existence of one. In the presentation, more details (including examples from data) will be presented.

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PROFESSIONAL DEVELOPMENT OF MATHEMATICS TEACHERS EDUCATORS IN MALAWI

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When Malawi introduced free primary education in 1994, enrolment increased by 50%, from 1.9 million in 1994 to 2.9 million in 1995 (Kazima & Mussa, 2011). As a result of this reform and high population growth, the primary school enrolment was at 3.6 million in 2008 and in 2013 estimated at 4 million. This large increase in pupil enrolment has put tremendous pressure on the already limited resources, including provision of qualified teachers, and consequently the current average teacher : pupil ratio in Malawi primary schools is 1 : 88 (Ministry of Education, 2013).

We report on base line data that informed the development of a five-year project of improving quality and capacity of mathematics teacher education in Malawi (Kazima & Jakobsen, 2013). 29 mathematics teacher educators at four different teacher colleges in Malawi answered a questioner about different aspects of prospective teachers' mathematical knowledge for teaching (MKT) – with a special focus on the subject matter knowledge. Our findings indicate that prospective teachers struggle with the same topics as Malawi pupils – as reported in two reports by The Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ II and SACMEQ III).We also find that what prospective teachers struggle least with in College, is reported to be most difficult for them to teach at schools. We discuss these findings with insights from a focus group discussion conducted as a follow up with some of the teacher educators.

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SELF-EFFICACY BELIEFS OF COLLEGE ALGEBRA STUDENTS WITH LEARNING DISABILITIES

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I present the findings of a mixed-methods pilot study which examined the relationship between self-efficacy and performance for college algebra students with learning disabilities. Most work with self-efficacy, mathematics, and students with learning disabilities is quantitative in nature. Although studies have shown that students with learning disabilities tend to over-estimate their abilities, very little work has been done to explain this difference. This mixed methods study will attempt to answer this question by analysing the students' explanation of their thought process for rating their ability at a certain level.

Calibration of self-efficacy beliefs of student with learning disabilities to actual performance has been studied by multiple researchers (Alvarez & Adelman, 1986; Schunk, 1985). In each of these cases, the authors quantified the students' self-beliefs as accurate or overestimated. In the studies that involved mathematics, it was found that students with learning disabilities are typically overconfident in their predictions for tasks that were considered to be of medium difficulty and tended to be more accurate for tasks that were seen as very easy or extremely difficult (Klassen, 2008).

In this study, ten college algebra students with learning disabilities assessed their self-efficacy for 10 math problems. They were then interviewed and asked to reflect and explain why they rated themselves at a certain level for each problem. The students then completed an assessment of 10 similar math problems to determine their actual performance. Calibration bias was then calculated by subtracting the student's actual performance from their predicted confidence. This data was compared to the student's discourse to discover patterns that explain the over confidence. The expected results include: that calibration bias does exist and that the patterns of discourse which emerged will help to explain this bias.

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CONCEPTUALIZATION OF PEDAGOGICAL CONTENT KNOWLEDGE (PCK) FOR TEACHING MATHEMATICS IN UNIVERSITY LEVEL

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Pedagogical content knowledge (PCK) first has introduced by Shulman (1986) as a "missing paradigm". Shulman (1987) described PCK as a special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding. Although, PCK is an interesting concept which has absorbed many eyes, its nature is complicated. Besides, many research have been done about PCK in school level mathematics and specially school mathematics teachers education, but little has been done in mathematics education in university levels. Therefore, the problem in this study is: what does PCK for teaching mathematics in university level mean?

This research is done through qualitative research methods. Data gathered mainly through semi structured interviews about experiences of teaching from respondents od this research who were mathematics professors and PhD students from two main universities in Iran. Data were analysed through coding and making theme. The process of gathering data has continued until the saturation of findings from analysis of data. In total, 19 PhD students and 8 professors cooperated in this research.

Data analysis showed that we could explain concept of PCK in a model with 4 main elements as "mathematics syntactic knowledge", "knowledge about students", "knowledge about mathematics curriculum planning" and "knowledge about creating an influential teaching-learning environment". Each of these theme is explained through sub-themes. Moreover, 3 contextual themes which influence on these main themes were found as "nature of subject", "professor's features" and "terms of learning atmosphere". This model provides a starting point for discussion about PCK for teaching mathematics in university level.

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CHARACTERISTICS OF CLASSROOM PARTICIPATION IN A MENTOR-MENTEE TEACHING METHOD

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In a communicational approach to cognition, mathematical learning can be seen as a change in students' ways of communicating mathematical activities (Sfard, 2008). In this kind of theoretical perspective, students' active participation in communication activities is required for effective learning. This study focuses on two conceptual categories – instructional scaffolding and classroom interaction – to investigate characteristics of classroom participation in effective mathematics teaching. The term "instructional scaffolding" is a metaphor to describe different types of assistance offered by a teacher to support students' learning activities. Appropriate scaffolding techniques can make classroom interactions active and thus contribute to student engagement. However, in the communicational approach to cognition, classroom interaction for improving student engagement is more important than instructional scaffolding. This is because instructional scaffolding cannot be successful without context-dependent classroom interaction. In order to characterize instructional scaffolding and classroom interaction, a teacher who helped students be engaged mathematics learning for 16 years and her 36 middle school students were recruited to receive a year-long mentor-mentee teaching method. Representative video clips of classroom instruction were collected and classroom discourse was analysed.

The teacher offered two types of instructional scaffolding and used two prominent techniques for classroom interaction to encourage student participation in class activities. In the case of instructional scaffoldings, a game method that raised student interest was used to select a presenter. Competition for bonus class participation points was employed by asking "Is there any other method to solve the same problem?" In the case of classroom interaction, peer mentoring activities seemed to expedite classroom participation because peer-to-peer communication made classroom communication more natural and easy, peer support made learning difficulties more understandable and resolvable, and positive peer relationships helped some mentees to feel that their frustrations were accepted and understood. In addition to the mentor-mentee interactions, the teacher's two different questioning strategies (i.e., open and encouraging questions for effective and ineffective mentoring activities respectively) in teacher-student interaction improved classroom participation by fostering vibrant, dynamic, and efficient interactions.

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ANALYSIS OF LEVEL OF REFLECTION OF INSERVICE MATH TEACHERS IN REFLECTIVE JOURNAL WRITING

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According to Schön(1983), teachers' reflection is defined as teachers' thinking process for finding alternatives as looking back their own teaching, analyzing problematic events and actions, and making decisions. The extent of understanding the complexity of teaching and learning environment can be criterion of the level of reflection (Bain et al, 1999; Davis, 2006). Reflective journal writing is considered as one of efficient tools for enhancing teachers' level of reflection. This study investigates the change of level of reflection from math teachers' journal writing based on integration of the four aspects of teaching, 'learners and learning', 'subject matter knowledge', 'assessment', 'instruction' which were suggested by Davis (2006).

Participants were three middle school math teachers with teaching experience of less than five years. The teachers wrote one journal entry each week in naturalistic setting which means no feedback to teachers. According to the four aspects of teaching, contents of the journals were coded within entry line by line, analyzed which aspects were included and integrated in, and scored from 1 to 4.

Changes of the two scores according to the time for each teacher showed that three teachers' level of reflection were developed. Teacher A's inclusion scores were generally higher than integration scores but both of the scores increased later so that teacher A had come to understand the complexity of teaching, which implies development of the level of reflection. Teacher B integrated more various aspects of teaching as time progressed. In the 1st and 2nd weeks, the inclusion scores were higher than integration one, but the inclusion and integration scores tended to be same from the 3rd week. Teacher C included and integrated various aspect in journal entry from the beginning. But he didn't integrate some aspects which were included in journal entry. After the 5th journal entry the progress of inclusion and integration score were the same, which means teacher C had come to integrate all aspects included in journal entry.

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SOLVING MULTIPLICATION AND DIVISION WORD PROBLEMS WITH DECIMAL FRACTIONS: FOCUS ON EFFECTS OF PROPORTIONAL REASONING AND METACOGNITION

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It is widely known that many students have difficulty solving word problems of multiplication and division with decimal fractions (Greer,1992). Researchers have described various factors that are presumed to be associated with the difficulties they have in solving word problems. To give the appropriate operation for word problems, one need not only to develop these conceptions, but also to develop many other problem solving abilities. This paper focuses on proportional reasoning and metacognition as significant factors in solving word problems.

The purpose in this paper is to investigate the effects of proportional reasoning and metacognition on students' ability to solve word problems of multiplication and division with decimal fractions. A test used in the investigation was consisted of 4 multiplication and 4 division problems, 4 proportional reasoning problems (Lamon,1992), and 12 meta-cognitive questionnaires (Fortunato et al.,1991). The subjects who participated in the investigation were 256 students from 3 different elementary schools in Japan. The pool of subjects included 83 students in Grade 4, 83 students in Grade 5, and 90 students in Grade 6.

As the effect of proportional reasoning, mean scores such as multiplication, division, and proportional reasoning increased in Grade5, but these are not changeable in Grade 5 and 6. 'Grade 4 and 5' and 'Grade 4 and 6' were significant, but 'Grade 5 and 6' is not. The correlation coefficient of 'multiplication and proportional reasoning' and 'division and proportional reasoning' were high in Grade 6.

As the effect of metacogniton, the mean score of this didn't have a remarkable change in terms of grades. 'Grade 4 and 5' and 'Grade 5 and 6' were significant, but 'Grade 4 and 6' is not. The correlation coefficient of multiplication and metacognition was high in only Grade 4. But the correlation coefficient was low in other Grades.

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CHARACTERIZING YOUNG CHILDREN'S MATHEMATICAL ARGUMENTS

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Recently there has been an increased interest in proof in the elementary grades in terms of considering the nature of young children's mathematical arguments. Mason (2008) suggests that young children have a natural inclination to prove and to convince one another of their ideas, and that this should be nurtured early in their schooling. It has been found that young children use various approaches to justify their ideas, including appealing to authority, using examples, and applying reasoning based on a representation or story context (Schifter, 2009). The purpose of this study is to characterize the nature of these arguments and to determine if there is a developmental trajectory that exists in the early grades.

Data was gathered from a long-term professional development project that involved six school districts where teachers were regularly videotaped. From this collection, four lessons from each grade level (K-2) were selected for analysis, for a total of 12 lessons. Segments were identified where students provided explanations. These were transcribed and coded for argument type and analysed for the extent to which those explanations were convincing. As of this date, half of the lessons have been analysed.

One important finding from this preliminary analysis was that young children's representation-based arguments were more convincing when they utilized their own representations versus those introduced by the teacher. Secondly, there was no instance found of appealing to an authority until second grade. This occurred when a student used an alternative algorithm for subtraction, got -2 in the ones place, and added it to what was left in the tens place (20) by subtracting the 2. When asked to justify this, she said, "Because Ms. Voss said that when you get to negative numbers, it's like saying 'take away' whatever number from that." These findings contribute to research on proof by suggesting a developmental sequence in how young children learn to use representations to argue when they first enter school and progress through the lower elementary grades.

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THE EFFICACY OF PROFESSIONAL DEVELOPMENT

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This study examines the efficacy of the Learning and Teaching Linear Functions (LTLF) professional development materials on teachers' mathematical knowledge for teaching linear functions (Seago, Mumme, & Branca, 2004). Learning and Teaching Linear Functions are modular, video-based professional development materials designed to enable teachers to deepen their specialized content knowledge by understanding ways to conceptualize and represent linear functions within their teaching practice. The best practices for supporting such professional development involve providing experiences that are intensive in focus and extensive in duration and that are practice-based (Cohen & Hill, 2001). The theoretical frame for the LTLF video case materials is adapted from the work of Deborah Ball and colleagues (Ball & Cohen, 1999) that incorporates research on both teaching and learning. The content of the video case materials focuses on the interactions between the teacher, the content (in this case, linear functions), and the students, within the context of an authentic classroom environment. A primary research question guided the study: Do teachers participating in the LTLF professional development exhibit greater increases in their mathematical and pedagogical knowledge for teaching linear functions?

The sample consisted of 34 treatment and 32 control Algebra 1 teachers drawn from California, U.S.A. middle schools. The treatment teachers participated in a one-week summer institute. To assess teachers' mathematical knowledge for teaching linear functions, an "artefact analysis" task was used, which asks teachers to solve a mathematics problem and provide written responses about (a) a 5-minute video clip of 6th grade students presenting solutions to a linear function problem and (b) three specific samples of student work (each representing a different typical student error). Although no pre-intervention differences were apparent between treatment and control teachers, at post-test, treatment teachers were substantially more likely to (1) indicate an understanding of students' potential than control teachers on the student work task (46.9% vs. 7.4%) and (2) focus on the mathematical content of student work than their counterparts in the control group (78.1% vs. 44.4%). There was also a greater tendency for treatment teachers to use evidence to justify their inferences with regard to student work and analysis of the classroom video (46.9% vs. 29.6%), and these differences were statistically significant at conventional levels.

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CONTEXT-DEPENDENCY BETWEEN MATHEMATICS AND SCIENCE: CLASSIFYING THE QUESTIONS BY USING STUDENTS' RESPONSE PATTERNS

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Mathematics and Science are taught separately in many countries, including Zambia. Students are expected to construct the meaningful knowledge by connecting mathematics and science. However, students got different answer in the same two types of tests, providing different context between mathematics and science. It is known as context-dependency. Although context-dependency between mathematics and science has been recognised, little is known about types of questions and factors that cause context-dependency. The objective of this study is to examine the types of questions that cause context-dependency by using students' response patterns in the tests and identify the factors that caused context-dependency.

One hundred and sixty-five students in Grade 12 at high school in Zambia were chosen for this study. Their academic performance was average among high schools in Zambia. The research was conducted by using the same two types of tests about function providing different context between mathematics and science. The instruments were developed based on previous studies (e.g. Ishii, Minowa, & Hashimoto, 1996). The two tests were conducted on the different dates in order to avoid the influence of the first test. Context-dependency was studied by using chi-square test. Types of questions that caused context-dependency were studied by using correspondence analysis, classifying the questions caused context-dependency by using students' response patterns in the test. In each classification, factors that cause context-dependency were identified through students' used solving methods and interview.

The result showed that the five questions out of twelve caused context dependency. These five questions were classified into three categories based on students' response pattern. Three factors that caused context-dependency were identified: unit in science context, different formulae in each context, and the difference in nature of the subject, concrete or abstract by analysing their solving methods and interviewing. In order for students to be able to apply the concept of function in both mathematics and science, teachers need to understand the same concept is taught in a different way.

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DESIGNING AND IMPLEMENTING PROFESSIONAL DEVELOPMENT PROGRAM OF MUTI-TIERED TEACHER COMMUNITY

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This research has an intention of developing a professional development program. We have perspective that a teacher is a reflective practitioner (Schön, 1983). Learning in situated learning theory is understood in participating course on the discourse of community of practice. It means that learning take place in same context in which it is applied(Lave & Wenger, 1991). We propose a professional development model based on the theories. Conceptually, it is discrimination from existing programs in terms of *participation subjects, contents* and *methods*. Participation subjects are teacher communities of same school and contents of this program include some processes which observe and reflect their own classes with theories. In terms of training methods, it has a structure which needs active participations. It takes three steps: 1.In the first attendance-based program, teachers learn theories. They have experiences relative to the theories. 2. In the school visiting program, teacher community plans their classes, open class, observes and analyses classes with mentors (teacher, educator or researcher). 3. In the second attendance-based program, teacher communities share their own outcomes in the field.

The developed mathematics teacher professional development program recruited participation unit that was consisted of 3 or 4 teachers in same school. The participants were totally 28 teachers that consisted of 12 teachers in 4 elementary schools, 16 teachers in 5 high schools. Also, there were 18 mentors to support each school. In this sense, it can be called multi-tiered teacher community professional development program. The program had progressed for about 4 months(2013.9.28~2014.1.4).

Through the program, the teachers improve their teaching competency. Also, the operation ability of teacher learning community was improved. Learning community culture has been formed in each school. It shows ability that the explorative learning community can be operated voluntarily although the program finished. Furthermore, community shared corporate responsibility about open class. They recognize open class as a new method to improve community teaching ability than a tool to evaluate individual teaching ability.

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WHAT'S THE USE OF SCIENCE? SCIENTIFIC CONTEXTS IN TEXTBOOKS PROBLEMS

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Mathematical concepts, especially in high school textbooks, are commonly approach referring to scientific phenomena for the "concreteness and usefulness" they provide to mathematical ideas (e.g. Bossé et al., 2010). But what is science's contribution in mathematics problems? In this poster, we present details of our textbooks analysis and considerations regarding mathematics teachers' scientific preparation.

Analyzing mathematics textbooks recently published in Quebec (Canada) reveals an increasing presence of scientific concepts as we progress through high school. Examining 12 textbooks, the proportion of such problems is about 15%. Scientific phenomena are thus significantly present on the quantitative side (reaching up to nearly 100% when considering only "context problems" in the chapters on vectors or functions). Concerning the *qualitative* aspect of this representation, our analysis leads to the observation of numerous *inadequacies*, from a scientific perspective, in the presentation of those notions: In many cases, the science behind the problem is incorrect. One example is a problem involving fireworks trajectories, inaccurately described as parabolas. This raises questions, especially in regard with the *scientific* preparation of mathematics teachers (e.g. Koirala & Bowman, 2003) who eventually have to use these problems: Is scientific inadequacy in mathematics textbooks an important issue? Should we prepare teachers to deal with it? What orientation could we take in this preparation? To investigate these questions, we began conceptualizing the *relations* between scientific phenomena and mathematical concepts in regard with the possible use of textbooks problems involving both. We found that, epistemologically speaking, the relation between scientific phenomena and mathematical concepts is far from obvious (e.g. Einstein, 1926). Notions such as modelization, mathematization, representation or application allude to different understandings of those relations along the (trans-)disciplinarity spectrum. Presenting those perspectives and some examples, we offer to discuss the concept of *conceivability* as a way to "get around" problems involving scientific concepts, somehow regardless of their scientific inadequacies.

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SOLVING DIVISION PROBLEMS IN NARRATIVE AND ARGUMENTATIVE TEXTS

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The present study aimed to observe the effect of text types (narrative and argumentative) on the performance of students solving different division problems (partition and quotition) in the early years of schooling. Previous studies (Correa, Nunes & Bryant, 1998; Skoumpourdi & Sofikiti, 2009) have pointed performance differences by type of treated division. Seventy eight students (mean age: 11 years, 5 months) in fifth grade of Elementary School were asked to answer comprehension questions (Questions 1 to 5) and to solve division problems (Questions 6 to 10) from narrative and argumentative texts. In the narrative text 87% of the answers and in the argumentative text 83% of the answers were correct in the text comprehension items (Questions 1 to 5). Concerning the division problem questions (6 to 10), in the narrative text students were correct in 68% of the items and in 62% of the items in the argumentative text. A Wilcoxon two-sided test showed significant differences in performance in comprehension questions and division questions, both in the narrative text ($p < .001$) and the argumentative text ($p < .001$). By comparing the responses in partition problems (Question 8 in both texts) with those of the quotition problems (Question 10 in both texts), it was found that participants presented correct answers in the following way: partition – narrative text: 34%, argumentative text: 25%; and quotition – narrative text: 27% and argumentative text: 23%). The Wilcoxon two-sided test confirmed that no significant differences were observed in performance in partition or in quotation problems, neither in the narrative text ($p < .001$), nor the argumentative text ($p < .001$). The results show that reading comprehension was evident in both text types, but it was not enough to guarantee a good performance in division problems, what goes against the notion that the student who cannot solve math problems is not good at text comprehension.

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WHAT TEACHERS THINK ABOUT STUDENTS' ERRORS IN PROBLEM SOLVING

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Analysing errors in problem solving situations has long interested researchers in the field of the psychology of mathematics education. Three perspectives have been identified. The first refers to errors in the context of assessing students' knowledge, the second refers to errors as a teaching strategy, and the third examines how teachers interpret students' errors when solving mathematical problems (Esteley & Villareal, 1996). This study investigates how 12 pre-service and 12 in-service teachers interpret the errors made by elementary schoolchildren when solving problems of multiplicative structure. Six cards were presented, each one containing a word-problem and an incorrect solution procedure. On three of the cards the problems involved Cartesian product while on the other three the problems involved isomorphism of measures. Each participant was asked to interpret the error presented in the cards. Four types of interpretation were found: no interpretation of the error (Type 1), error as a consequence of the use of an incorrect operation (Type 2), lack of comprehension of the language of the word-problem (Type 3), and lack of conceptual understanding of the multiplicative relations needed to solve the problem (Type 4). The pre-service teachers tended to consider the errors in the Cartesian product problems to be conceptual in nature, and the errors in the isomorphism of measures problems to be both linguistic and conceptual. On the other hand, the in-service teachers tended to interpret errors in both types of problems as being strictly conceptual. The conclusion is that the teachers' education plays an important role in the way they interpret errors in mathematical problem solving situations, as does the type of problem being solved. Thus, it is important to include in the competencies required to teach mathematics, knowledge about how to deal with students' errors. (Peng & Luo, 2009).

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PRESENTING VIDEO DATA TO CONNECT RESEARCH AND PRACTICE: PEDAGOGY IN MIDDLE SCHOOL MATHEMATICS

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This poster presentation will illustrate how analysis summaries of video data collected for classroom-based research in middle school mathematics have been structured for use in teacher education and professional learning activities. Connecting research and practice in this way is one aspect of a large scale study that addresses the questions: (i) How does middle school mathematics pedagogy differ across regions of Canada? (ii) How can differences in pedagogy establish the basis for further research into regional differences in mathematics achievement?

The enactivist framework (Maturana & Varela, 1992) that guides this research prompts a fine analysis of both observable and implicit cultural practices within a self-sustaining social system like in a mathematics classroom or professional learning setting. Data were collected from focus group interviews and classroom observations and the review of lessons by teachers and researchers was similar to the TIMSS video study (Hiebert et al., 2003). The focus here is on presenting an analysis of the classroom video data in a format that will be useful in professional learning activities.

Video data collected from lessons in four middle school mathematics classrooms will be represented in the poster. These summaries are structured in an accessible format to illustrate lesson sequences (e.g., who is involved in conversations, the mathematics that is addressed, time spent on each aspect of the lesson), transcription of student and teacher contributions (e.g., teacher prompting and questioning, student contributions), reference to video clips for a visual of key episodes, and images of classroom artefacts (e.g., student work, physical arrangement of the classroom).

Using classroom-based data in this way provides a window into classroom practices and can be used to stimulate discussion in both research and practice about observable teaching practices and the emergence of implicit teaching and learning culture in middle school mathematics classrooms.

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PRE-SERVICE TEACHERS' UNDERSTANDING OF FORMAL PROOF IN A GEOMETRY CONTENT COURSE

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This study examined 52 elementary pre-service teachers' understanding of formal proof while proving six geometry problems using triangle congruence. The results suggested that pre-service teachers showed four types of common misconceptions.

Both the Principles and Standards for School Mathematics (NCTM, 2000) and the Common Core State Standards for Mathematics (CCSSI, 2010) have emphasized that students at all grade levels should be able to follow mathematical arguments and "by the end of secondary school, students should be able to produce mathematical proofs – arguments consisting of logically rigorous deductions of conclusions from hypotheses – and should appreciate the value of such arguments" (NCTM, 2000, p. 56). Thus, to successfully respond to these standards, elementary teachers who teach upper elementary or middle school grades need to have enough understanding of deductive proofs to be able to provide their students with relevant experience that could be naturally extended to formal proof later on.

The purpose of this study was to investigate how elementary pre-service teachers (PSTs) proved geometry problems and what common misconceptions they showed in the process of formal proof. For this, a two-hour test including six geometry problems that could be proved using triangle congruence was administrated at the end of a semester-long geometry content course to 52 PSTs who intended to teach upper elementary or middle school grades. The data were then analysed using a content analysis method (Krippendorff, 2004).

The results suggested that the PSTs showed four types of common misconceptions when formally proving the given problems using triangle congruence: (1) using unnecessary information that was not related to drawing a conclusion; (2) using their prior knowledge without considering the given information; (3) using a conclusion as a given item of information; and (4) simply listing properties related to the problem from their own knowledge without proving. Implications for ways in which formal proof can be effectively taught to PSTs are discussed.

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THE EFFECT OF MULTIPLE SOLUTIONS ON STUDENTS' FLEXIBILITY

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OECD (2012) indicated the importance "to present and discuss evidence and policies in mathematics and science education that can lead to better skills in thinking and creativity" (p. 16). Literature indicated that flexibility is an essential component of creativity (Guilford, 1967; Leikin, 2007; Silver, 1997). Little, however, was done to enhance students' flexibility in today's mathematics classrooms (Akita & Saito, 2013). The purpose of this study was to investigate the effect of multiple solutions on eighth-grade students' flexibility.

The subjects for this study comprised two eighth-grade classes in a school. One class served as the experimental class, and the other class served as the control class. The experimental class received instruction in multiple solutions and the control class received traditional instruction for eight classes, respectively. Each student was given the pretest before and the posttest after they received either instruction in multiple solutions or traditional instruction. They were required to solve each problem on these two tests in multiple solutions. Based on Leikin (2007), flexibility scores were computed for each student on both tests by adding up the number of different solutions the student used to solve the problems on the tests. The findings indicated that the experimental class improved their flexibility scores from pretest to posttest while the control class didn't. The results of this study suggested that students' flexibility could be promoted and multiple solutions could serve as a way to promote their flexibility.

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AN EXPLORATION ON USE OF COGNITIVE-METACOGNITIVE STRATEGIES FOR MATHEMATICAL PROBLEM SOLVING

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Cognitive and metacognitive strategies are important to problem solving (Schoenfeld, 1992). Montague (1997) developed a cognitive-metacognitive model of mathematical problem solving which identified three metacognitive strategies of self-instruction, self-questioning, and self-monitoring associated with each of seven cognitive processes, including read, paraphrase, visualize, hypothesize, estimate, compute, and check. Studies have shown that students who received cognitive-metacognitive strategy instruction based on Montague's model improved their mathematical problem solving performance (Lee & Yang, 2013; Montague, 1997). Little, however, was known about how students engaged in cognitive-metacognitive strategies while working on Montague's model. The purpose of this study was to explore a sixth-grade student's use of cognitive-metacognitive strategies while working on Montague's model. Metacognitive-Strategy Worksheet (MSW), developed in Lee and Yang (2013) and based on Montague's model, was used in this study to support the student's engagement with Montague's cognitive-metacognitive strategies.

Data for this study consisted of the participant's written work and think-aloud protocols while working on nine MSWs. A coding scheme was developed based on Montague's model to identify the student's use of cognitive-metacognitive strategies in the protocols. The percentage agreement was 89%. A chi-square test was conducted on frequency of the student's cognitive-metacognitive strategies use, and the findings indicated that there was a statistically significant (χ^2 =47.208, p=0.00) difference in the student's total use of self-instruction, self-questioning, and self-monitoring among the seven cognitive processes. The student self-instructed, self-questioned, and self-monitored most often when hypothesizing and least often when estimating. More studies should be implemented with a larger group of students to understand better students' use of cognitive-metacognitive strategies while solving problems.

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U.S. PRE-SERVICE TEACHERS' PERCEPTIONS OF ACADEMIC LANGUAGE

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With the national rollout of edTPA (formerly Teacher Performance Assessment) that champions teaching with academic language, there is a renewed interest in academic language and related pedagogy in the U.S. Using the edTPA's framework on academic language, we asked: To what extent do pre-service teachers (PSTs) at middle and secondary grades understand academic language? A survey was used to measure the basic perception of academic language. The participants of the study included forty-two PSTs at middle and secondary level in two large state universities in southern U.S. states.

About 54% (n=22) of PSTs indicated that academic language is academic words or styles used in scholastic writing. With regard to PST's experience of learning mathematics with academic language, some PSTs (n=28) mentioned that they could relate to academic language to address the needs of English language learners. The majority of them (n=31) failed to describe their learning mathematics with academic language.

The findings of the study demonstrate that the emerging snapshot of the PST is someone who has little understanding of academic language, thereby struggling with developing teaching repertories that engage students in the use of academic language (Gottlieb & Ernst-Slavit, 2014). The PSTs believe vocabulary instruction addresses academic language while identifying acquisition of key vocabulary as the significant outcome of teaching with academic language. It follows that most strategies mentioned by the participants to use academic language in instruction have to do with increasing vocabulary. There is little evidence that PSTs plan to use writing activities to introduce classroom discourse in instruction (Cazden, 1988).

The mathematics teacher educators (MTEs) in the U.S. have begun to examine ways to support teachers in creating lessons with academic language in mathematics class (Kersaint, Thompson, & Petkova, 2009). More efforts are necessary to propose and validate the effectiveness of developing knowledge and skills for teaching with academic language.

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COMPARING U.S. AND TAIWANESE PRESERVICE TEACHERS' SOLVING TRIANGULAR ARITHMAGON PROBLEMS

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The study aimed to investigate pre-service teachers' knowledge and computational skills by using Triangular Arithmagon. Triangular Arithmagons refers to polygons having a number on each vertex and a box number on every side so as to make the number on each box equal to the sum of circle number next to it (Macintosh& Quadling, 1975). Triangular arithmagons proves to be very helpful to enhance student's problem solving techniques. Moreover, triangular arithmagons can also be solved by establishing and implementing linear equations.

Teachers' knowledge has been widely explored and discussed in research. However, empirical research assessing their knowledge in computational skills is limited. The students of Asia showed a remarkable performance in cross-cultural mathematical studies as compared to the North Americans (Schmidt, Wang, McKnight, & Curtis, 2005). More studies are needed to compare pre-service teachers' knowledge in computational skills and problem solving between different countries in order to gain insights into ways to strengthen teachers' knowledge in mathematics that can benefit both teachers and students.

Participants included 65 pre-service teachers from three schools in the U.S. and Taiwan. The Triangular Arithmagons Test (TAT) was used to measure pre-service teachers' performance in whole number, fractions, and decimals operations, each of which included level-1 (basic) and level-2 (advanced) tests. MANOVA analysis was performed to compare the performance between teachers from the U.S. and Taiwan. Results indicated that overall, pre-service teachers in Taiwan outperformed those in the U.S., especially on the advanced-level tests. Pre-service teachers in the U.S. seemed to have poor ability of solving complex operation problems. This study not only confirms the outperformance of Taiwanese pre-service teachers over the U.S. teachers in computational skills and fluency for advanced operations, but also provides evidence that adds to the limited research about teachers' knowledge in computational skills in cross-cultural settings.

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AN INVESTIGATION OF THE ERROR TYPES AND THE LITERACY OF CONSTRUCTING STATISTICAL CHART FOR SENIOR HIGH SCHOOL STUDENTS

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In nowadays society, learning probability and statistics is an important topic of modern mathematics education, and statistical literacy is gradually being regarded as an important skill for citizens. The ability to interpret and critically evaluate messages that contain statistical elements, termed statistical literacy, is paramount in our information rich society (Gal, 2003). Not only the ability of judgment and comment on data, but also the cognitive comprehension of statistical charts we needed in this information generation. However, for few students, learning statistics is quite difficult and easy to produce some misconceptions. Teachers should understand and correct students' misconceptions, and guide students using the right conception and principle of problem-solving procedure to the right way of learning targets. This study used the constructed-response items of statistical charts to investigate the error types in the learning process of senior high school students. The results showed that there were 43% examinees with proficient statistical literacy, and about 68% examinees with basic knowing about statistical charts. The data analysis revealed that, the most common error type in constructed-response items were comprehension errors (61%), mathematical processing errors (19%), and in transforming them into wrong mathematical procedures (14%), respectively. For the lower-proficiency level groups, the comprehension error was dominant in constructed-response items of statistical charts. However, for the higher-proficiency level students, the comprehension error rate was decreased and the mathematical processing error rate was relatively increased.

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REFLECTION ON THE INTERPRETATION OF INTERACTION BY EXPERTS FROM THREE ACADEMIC FIELDS

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Quantitative information is widespread, and statistical data are being increasingly presented to add evidence to various reports. Therefore, statistical information must be correctly understood to achieve precise decision-making (Ben-Zvi & Garfield, 2004). Interaction is an important aspect of statistical literacy. During interactions, identifying relationships among variables, a process of scientific thinking, is necessary (Zohar, 1995). This study investigates experts' concepts concerning interaction to expand on the core components of interaction for statistics learning and teaching. Our research questions are as follows: **How do experts interpret interaction? What are the main components of interaction as defined by experts?** In total, 16 experts from different academic fields participated in the survey: six statisticians, six medical scientists, and four physicists. Two texts that presented the concept of interaction were used as semi-structured interview questionnaires, the topics of which were medicine and physics. The results indicated five components by which experts interpreted the concept of interaction: (1) using the concept of the "variable" to explain interaction, (2) revealing causal relationships, (3) indicating the conditions for interaction, (4) pointing out the mechanism of variables interacting with each other, and (5) confirming the formation of new products through interaction. Fig. 1 reveals that all experts referred to components 1 and 2. Furthermore, the statisticians mentioned component 3 and both the medical scientists and the physicists emphasized components 4 and 5.

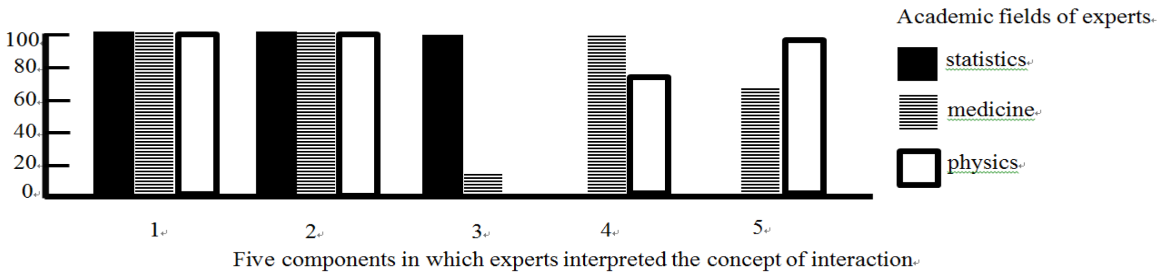

Figure 1: Distribution percentages for each interpretation component by academic field.

Experts had divergent perspectives of components 3, 4 and 5. A reflection on the diverse range of ideas from the experts will be used to suggest methods for compiling statistical materials in math textbooks for high school courses in Taiwan.

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EXAMINING THE DIFFERENCES OF SOLVING SYSTEMS OF LINEAR EQUATIONS BETWEEN FINNISH AND TAIWANESE TEXTBOOKS

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The purpose of this study was to investigate the difference and problem types between Laskutaito textbooks of Finland (WSOY, 2009) and Kung Hsuan textbooks of Taiwan (Kang Hsuan Educational Publishing Group, 2012). The content analysis method was used to code and analyze the textbooks in this study. This study adapted a vertical analysis which the problems in the textbooks were analyzed by its (1) teaching sequence (Hong & Choi, 2014), (2) application types (real-life situation or not), and (3) representation forms (purely mathematical, visual, verbal and combined) (Zhu & Fan, 2006). Results shows that the main difference between Finnish and Taiwanese textbooks is that the Finnish textbooks put more emphasis on the connection between graphs and equations (graphical approach) while the Taiwanese textbooks highly focus on algebraic methods (algebraic approach). The Finnish textbooks include more application problems (24.43% > 17.18%) when compared to the KH textbooks. Regarding the representation form, although the prevalent representation forms of both countries in the total result are the pure mathematical form (TW: 76.07%, FI: 61.09%), the Finnish textbooks include more problems with visual representations (visual + combined form: 15.38% > 8.58%). The Finnish textbooks also have higher percentage in the verbal form (23.53% > 15.34%). From the more algebraic approach in the Taiwanese textbooks design, students may develop better symbolic and abstract thinking. However, students, who are exposed to Taiwanese textbooks, learn how to graph equations after the algebraic methods are introduced but there is no content about solving linear systems graphically. Graphs are only used to support the algebraic solution methods. In contrast, the Finnish textbooks start from solving linear systems graphically and then introduce how to solve linear systems algebraically.

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SOLVING RUDIMENTARY AND COMPLEX MATHEMATICAL TASKS

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In this research, I discuss a type of task specifically designed to foster students' ability to flexibly apply their existing mathematical knowledge and skills in problem solving situations, which I have come to call ‘rudimentary and complex mathematical tasks’ (RCMTs). In particular, I look at students’ problem solving processes when working collaboratively on such tasks. Given that modelling tasks and RCMTs share some common goals, modelling cycles are used as a framework for analysis in this study to describe students’ behaviours during RCMTs.

This study involves a group of grade 9 (age 13-14) mathematics students (n = 29) enrolled in a high school in a middle class neighborhood in western Canada. At the time of this study, these students have just over a year of RCMT experiences. Students were assigned RCMTs to be completed in groups of two to three. All students were asked to pay attention to their thoughts and approaches used as they worked on the problem in order to help them accurately describe their RCMT processes. Data include in class observations, field notes, class discussions and impromptu interviews that focus on their actions taken to solve the task. The class discussions and impromptu interviews were transcribed immediately as these conversations happened.

Results indicate that students’ RCMT process in this study parallels Kaiser’s modelling cycle in general (Borromeo Ferri, 2006). However, while the modelling cycle is useful in providing us with a general structure of the RCMT process, it does not account for all that was observed in the task.

Compared to what is suggested in the modelling literature, students spent more time and energy to understand the situation and to investigate factors involved in the solution, and less time in mathematization during RCMTs. Also, students constantly validated and evaluated their own ideas during RCMTs, except when the ideas come from their peers. Another difference between the modelling process and what was observed during RCMTs lies in the stages of the modelling cycle. While literature suggests the process of creating a real model and mathematization as distinctive stages, data show that these stages overlap each other. Finally, since RCMTs focus on the flexible use of rudimentary mathematics, mathematization and the resulting mathematical models are downplayed in the process.

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PROSPECTIVE ELEMENTARY TEACHERS' THINKING PROGRESSION ON DIVSION

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This poster reports the findings of 45 prospective elementary teachers' works on writing and visualizing division problems from a trajectory perspective. The following research question is answered: What kind of semantic structures do prospective teachers use to interpret symbolic expressions of division? The findings can help mathematics education community recognize prospective teachers' understanding and their future teaching tendency in division.

THEORETICAL FRAMEWORK

The theoretical framework for this research is built on specialized content knowledge (SCK) proposed by Ball, Thames, and Phelps (2008) and Piaget's theory of assimilation and accommodation. SCK refers to mathematical knowledge and skills needed uniquely by teachers. In this research, prospective teachers' SCK will be determined through analysing the semantic structures that they used to interpret symbolic expressions of division. Learning about whole numbers has been assumed to be different from learning about non-whole numbers (Siegler, Thompson, & Schneider, 2011). Piaget's theory of assimilation occurs when individuals use their existing schema to respond to new conceptions or phenomenon. Piaget's theory on accommodation, on the other hand, asserts that one's existing schema must change in order to suit more complicated and challenging conceptions or phenomena. While progressing from whole-number division to non-whole division, it was unknown whether or not prospective teachers would use their prior semantic structures for whole-number division to interpret the more complicated and challenging non-whole division, or accommodate it with new semantic structures.

METHODOLOGY AND FINDINGS

Participants of this research comprised of 45 prospective elementary teachers from a teacher preparation program in the U.S. For data coding and analysis, division problems created by participants were classified into two main semantic structures: partition division and measurement division. Most prospective teachers explained whole-number division as partition division, but the percentage declines while progressing from whole number division to division with a fraction as a divisor.

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TOWARDS A RECURSIVE MATHEMATICS CURRICULUM

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This research explores what a recursive high school mathematics curriculum might be. Although mathematics curricula may be designed with a recursive quality, my teaching experience in Canadian high schools suggests to me that the lived mathematics curricula tend to be linear: mathematics teachers often rush through topics without enough emphasis in revisiting the topics learned before from different perspectives, resulting in a fragmental view of mathematics knowledge. To enhance student development, a nonlinear curriculum centred on recursion is a promising direction (Doll, 1993). A recursive mathematics curriculum seems to fit the development of mathematical understanding, which, based on Pirie and Kieren's (1994) model, follows a recursive path. Given the importance of the process of recursion, more research is needed about recursive mathematics curriculum.

Complexity thinking (Davis & Sumara, 2006) is a useful theoretical framework for this study. The concept of recursion is derived from Doll's (1993) postmodern perspective of curriculum and shaped by Bateson's (1979/2002) theory of mind, Bruner's (1962) spiral curriculum, Pirie and Kieren's (1994) recursive theory of mathematical understanding and complexity thinking. In this study I use hermeneutical inquiry because of the observed associations between recursion and hermeneutics. I interpret the concept of recursion and re-view "reviewing" in mathematics classes as a particular instance of rethinking the curriculum as recursive. I suggest that daily teaching and learning activities in a recursive curriculum can be built around recurrent re-views centering on relations and differences. I will explore in-service high school mathematics teachers' interpretations of recursive mathematics curriculum in the next stage of the research.

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THE RELATIONSHIP OF TEACHER BELIEFS TO SUCCESS IN IDENTIFYING STUDENTS' MATHEMATICAL REASONING

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Our research investigates the relationship between teachers' beliefs about learning and teaching and their success in identifying children's mathematical reasoning from video. Logistic regression analysis for pre and post-assessments on teacher beliefs and teacher success in identifying students' reasoning from video showed that the teacher beliefs aligned with NCTM Standards and were strong predictors of success in recognizing students' reasoning from video for all teacher groups. The intervention consisted of a modified lesson study model, with teachers engaged in cycles of problem solving using tasks and videos from longitudinal studies (Maher et al., 2010; NCTM, 2000). The videos are stored on the open access Video Mosaic Collaborative (VMC) which is located at http://www.videomosaic.org. The question we are addressing is how, if at all, are teacher beliefs related to their growth in recognizing student reasoning from video? This research builds on previous research where we analysed the growth in beliefs and ability to notice reasoning through video based interventions (Maher, Landis, & Palius, 2010; Maher, Palius, & Mueller, 2010).

Exploring the relationship of teacher's beliefs about learning and teaching and growth in recognizing student reasoning from video is important in gaining a better understanding of the obstacles that impede teacher noticing of students' mathematical behavior from video.

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INCOMING: THE UNINTENTIONAL "COMING" OF TECHNOLOGY IN MATHEMATICAL DOINGS

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Research in mathematics education increasingly emphasizes the importance of tools and technology in mathematical doings (e.g. Hoyles & Lagrange, 2010). This poster illustrates the concept of *incoming* technology as a new way to conceptualize tools and their use when doing mathematics. Briefly put, tools and technology are generally considered as "means to an end" in relation with teaching, learning or simply "doing" mathematics. But tools also *exceed* intentions in various ways, bringing in more than what we expect (e.g. Rabardel, 1997). More so, tools even precede and preform intentionality, including those of mathematics educators. Based on data gathered through my studies and theoretical reflections these provoked, the concept of *incoming* technology was developed to positively address the inevitable unintentional "coming" at the heart of tools and their use when doing mathematics (Maheux, 2014). Instances of such unintentional "coming" of technology are found in the many ways tools present possibilities that are unrelated to the mathematical activity for which they where designed. Students playing games or storing cheat sheets in their graphing calculator are one class of examples. Another is in how tool use is also synonymous with potential breakdown, for example when hardware malfunctions and software crashes or glitches: geogebra online forum shows many 'mathematically wrong' constructions. Thinking with *incoming*, such interferences are seen as occasions to step back and reconsider what we do, question intentions, and what we take for granted. Beside this, it also became clear that the divergence characteristic of *incoming* is also at the heart of *invention*. Tools are one source of the positive surplus by which we discover things that were not intended. For example, I observed pupils serendipitously finding regularities in decimal development of whole numbers divisions while we invited to "play" with calculators, and older students coming across a convergence when repeatedly running a sin (ANS) command. Finally, the concept of *incoming* also brings forth the situated nature of technological outbreaks. What is once failure or discovery for someone is not necessarily in different circumstances or for somebody else, and attitude is determinant in the way one respond to technological provocations. New questions relate to how we prepare for the unpredictable so we can welcome it in the most productive way.

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PROBLEM POSING AS A MEANS FOR DEVELOPING TEACHER COMPETENCES

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Using both textual and graphical elements, we will present experiences about problem posing carried out by teachers. These experiences show that problem posing in class episode contexts is a way of contributing to the development of teachers' didactic and mathematical competences.

The problem posing research and its relation to the problem solving process has led to new research about incorporating problem posing in teacher training programs (Ellerton, 2013). Following this line of thinking, we have designed a sequence of activities with the purpose of motivating pre-service teachers and in-service teachers to create math problems and to reflect on their didactical aspects.

The sequence of activities designed for our workshops is: a) We give a short presentation on problem posing, including some examples of problems created in previous workshops. b) We present a class episode of teacher P, which includes a previously designed problem and also teacher P' students reactions when solving this problem. c) We ask participants to: i) solve the given problem; ii) pose problems by modifying the given problem to make the solution easier and to help clarify the students' reactions (these problems we call "*pre-problems*"); iii) pose problems by modifying the given problem in a way that challenge teacher P's students beyond correctly solving the given problem (these problems we call "*post- problems*"). d) The problem posing should be carried out individually at first and then in groups. e) The problems created by a group should be also solved by other groups. f) The explanation of the solutions is part of a socialization process with all the participants.

The analysis of teachers' activities following this sequence shows that they contribute to the teachers' development of didactic and mathematical competences. Problem posing provides opportunities in which these two competences interact creatively.

We have observed that when teachers have experiences on didactic analysis and mathematical connections through class episode analysis in processes of creating new problems, they improve their didactic and mathematical competences.

Acknowledgement

This study was funded by research project EDU 2012-32644 and research project 53133-IREM-PUCP 2013.

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PEDAGOGICAL CONTENT KNOWLEDGE OF MATHEMATICS PRE-SERVICE TEACHERS IN THAILAND

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For decades, Thai government has attempted to solve a problem of students' mathematical achievement by improving teaching quality based on an assumption that teachers' mathematical knowledge is a key aspect for improving the teaching quality (Hill, Ball, & Schilling, 2008). Thus, in 2008, Thailand participated in TEDS-M study in order to acquire information about the mathematical knowledge of pre-service teachers. After the results of TEDS-M were officially published, teacher preparation institutes have developed new teacher preparation programs in order to enhance pre-service teachers' pedagogical content knowledge (PCK) parallel with content knowledge. Thus, a replication study of teachers' knowledge is needed in order to investigate the quality of the new programs. Since the new programs increase the emphasis on the PCK, this study focuses on the measure of PCK.

The samples were 34 pre-service teachers in the last year of a mathematics teacher preparation program in a teacher university in Thailand. They were selected by snowball purposive sampling. They completed a test on PCK. The test consisted of 19 items, which were the released PCK items used in TEDS-M 2008. These items illustrate three sub-domains of PCK in TEDS-M framework (see Senk, Peck, Bankov, & Tatto, 2008). The collected data were scored based on TEDS-M scoring guide and calculated for total scores, mean scores, and mean percentages. The mean percentages were compared with those from TEDS-M 2008 by one-sample *t*-test.

The findings indicate that PCK of the samples and that of Thai pre-service teachers in the TEDS-M are not significantly different in both primary and secondary school levels ($t = 1.48$, $p = .15$; $t = 1.14$, $p = .26$). Comparing with the TEDS-M's international mean, the primary and secondary school PCK of the samples of this study is significantly lower ($t = 3.50$, $p = .001$; $t = 2.54$, $p = .02$). It seems that new preparation programs cannot improve pre-service teachers' PCK. Thus, the appropriateness of the teacher preparation program should be considered in the future research.

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TWO DIFFERENT KINDS OF MAPS

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The understanding of spatial relationships as a part of spatial ability can represent problems that lead to knowledge of necessary tool for orientation in everyday life situations. On the elementary level spatial relationships are used to enhance children's understanding of spatial abilities and also using maps. In particular, specifying locations and describing spatial relationships with concepts as top, bottom, right and left can be used for path describing (NCTM 2000). Thus relations of position are a topic in math classes. Examining locations is a precondition for geometry and other mathematical concepts (Reys et. al 2004). German textbooks present two different kinds of maps: maps of the environment and maps of a Manhattan-like city with only horizontal and vertical roads (e.g. Wittmann and Müller 2013, p. 84). There are three types of spatial reference systems: object-centred (intrinsic), viewer-centred (relative) or environment-centred (absolute). Only viewer-centred or object-centred reference frames are used with maps (Levinson 2003). One of the research questions is:

What is the understanding of German 2nd graders of path descriptions regarding different kinds of maps?

The sample comprises 200 German 2nd graders, who had to answer a paper – and – pencil test. Depending on their results 45 children were additionally individually interviewed.

The first analysis of the given answers seems to support the following results: With a viewer-centred reference system it is easier to start with a Manhattan-like map. Whereas with an object-centred reference frame it is easier to follow known directions in an environment map.

The poster will present the study with its theoretical background, main results and selected examples of the pupils. Additionally perspectives are given for further discussion of elementary school teaching.

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MAPPING TONGAN BILINGUAL STUDENTS' LANGUAGE USE AND THEIR GROWTH OF MATHEMATICAL UNDERSTANDING

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This study discusses how bilingual students' acts of "language switching" appear to influence, or are influenced by, the process of mathematical understanding. The key question about *why, how*, and *when* do these bilingual students switch languages in communicating their mathematical understanding with others is central to this study.

Video recordings of small groups of junior high school students in Tonga, which is a small island country in the South Pacific, were collected as part of my doctoral study (Manu, 2005). Analysis of the Tongan students' language switching in these video recordings results in the categorization of four main forms of language switching. These forms are identified, categorized, and developed from the data to provide a language for describing and accounting for the particular way Tongan students switch languages. The study employs and thus demonstrates the power of Pirie and Kieren's (1994) theory as a language for, and a way of, examining understanding in action, of interpreting the observed cues, and of counting for any evidence of "growth of understanding" even within a bilingual context.

In this poster presentation, various "mappings" of a selected number of Tongan bilingual students' growing understanding of a specified topic for a particular task, using the Pirie-Kieren Model, are presented to illustrate the results and key findings of the study. Among these are the following: one, that Tongan students' language switching influences their understanding of mathematics, and thus shapes the way they conduct their mathematical activities; two, that the effect of growth of mathematical understanding on language can be observed through these bilingual students' language *of*, and *for*, understanding mathematics; three, that growth of mathematical understanding can occur without language switching, and vice versa; and lastly, that a group's language-use plays a significant role in their *collective* growth of mathematical understanding, and vice versa. The findings from this study are arguably applicable to other similar bilingual situations that involve individuals using words with no direct or precise translation between two 'distance' languages.

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OBSERVING TEACHERS: THE MATHEMATICS PEDAGOGY OF QUEBEC FRANCOPHONE AND ANGLOPHONE TEACHERS

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Large-scale international assessments reveal a considerable range in student achievement in mathematics across Canada, even between the Francophone and Anglophone communities. Although some factors have been claimed to contribute to these differences (e.g. Anderson et al., 2006), no comparison of teaching pedagogies, which we expect would impact student achievement, has been made between regions of Canada. The goal of this new nation-wide study is to describe regional differences in mathematics teaching and underlying pedagogies in Canada, and to relate these differences in student achievement. In our poster, we report preliminary results of this study done in Quebec. Our research question is: How do the mathematics teaching pedagogies vary between Quebec Francophone and Anglophone teachers?

To define pedagogy, we refer to what Tobin et al. (2009) call "implicit cultural practices of teachers, by which we mean practices that though not taught explicitly in schools or written down in textbooks reflect an implicit cultural logic" (p. 19).

The sample comprises of 4 Quebec Francophone and 4 Quebec Anglophone grades 7-8 mathematics teachers who volunteered. This study consists of discussing via focus groups pedagogies used in 3 types of mathematics lessons previously video-recorded by each teacher: one they felt was typical, one they felt was exemplary, and one introductory lesson on fractions. Each video was edited, keeping the best 15-20 minutes. The teachers met with the research team in separate focus groups where they watched the edited videos of their lessons, and defined the important practices for each type of lesson.

Results, obtained by identifying emergent themes for each type of lesson, reveal similar pedagogies between the 2 groups. We assure that the 2 linguistic communities developed a shared pedagogy because they share the same curricula. Some important differences however emerged. The Anglophone teachers prioritize differentiated instruction, while Francophone teachers emphasize the synthesis of the content.

These results only allow for very preliminary conclusions regarding the differences in regional pedagogies in Canada and their connection to student achievement.

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INVESTIGATING THE DIFFICULTIES OF MIDDLE SCHOOL MATHEMATICS TEACHERS WITH LESSON STRUCTURE

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The relationships among teachers' preparedness, the way of presenting concepts, lesson structure, and students' learning are endorsed by common sense, but the structure of the lesson has been overlooked in the literature over time. There is a need to explore what kind of problems mathematics teachers have with the structure and pace of the lesson. The aim of this study is to investigate difficulties of in-service middle school mathematics teachers with the lesson structure in their actual teaching. The results of initial data analysis demonstrated that teachers had problems with the disproportionate division of instructional time and lacked differentiation for students who progress at different learning rates.

INTRODUCTION

Students' learning depends upon the preparation, alertness, reflection, and dispositions of their teachers. Although having a well-designed lesson plan is important, it is not sufficient by itself for effective instruction. The mastery of teachers in the effective implementation of designed lesson plans is very important because

> "[W]hen teachers provide high structure by communicating clear expectations and framing students' learning activity with explicit directions and guidance, these instructional acts support students' engagement by keeping students on task, managing their behaviour, and avoiding chaos during transitions." (Jang, Reeve, & Deci, 2010, p. 588)

This qualitative case study was conducted to explore the difficulties that teachers had with lesson structure. A total of 73 videotaped lessons of 29 grades 6-8 mathematics teachers were observed by trained math coaches and university personals and scored using a rubric. *Open coding* was used to divide the recurrent words and phrases into categories. Further analysis, *axial coding*, was made by examining categories and comments of the individuals to develop themes (Corbin & Strauss, 2008).

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GAINING FROM PARICIPATION: CO-CONSTITUTED LEARNING OPPORTUNITIES IN MATHEMATICS LESSONS ON FUNCTIONS

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Voigt (1996) studied the relations between social interaction and mathematics learning and emphasized the negotiation of mathematical meaning in classroom processes as a necessary condition of learning. However, interaction between a teacher and the students could also be understood as something reducing the learning opportunities in a content perspective. According to previous research results (Emanuelsson & Sahlström, 2008), there seems to be a price of participation.

This on-going empirical research further investigates the learning opportunities in relation to student participation in the constitution of the mathematical content. The theoretical perspective is Variation Theory and one of the key concepts used is *the space of learning* (e.g. Marton & Tsui, 2004), which addresses the potential for learning in a lesson. The main research question is: how are the learning opportunities of the lesson content influenced by the interaction between students and the teacher?

The study was conducted in secondary and upper secondary school and the sample involves 19 mathematics lessons with 15 groups of students aged 16-18 years and their 13 teachers. The equation of the straight line was introduced in all of the lessons, which were videotaped and later analysed. The study investigates what the students could gain by active participation in constituting the content of the lesson. The poster will be illustrating students' alternative ways of seeing the equation of a straight line and learning opportunities in different lessons will be compared and discussed.

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“SQUARE ROOT” BUT WHAT DOES IT DO?: INQUIRY-BASED LEARNING WITH YOUNG CHILDREN IN MATHEMATICS

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This poster describes research that combined two approaches – discursive psychology and multimodal data analysis – in order to deepen our understanding of what constitutes algebraic reasoning in early mathematical thinking. Taking a broad view of discourse highlighted the flow of algebraic reasoning within and between the children, revealing the structure of a six-week-long inquiry.

RESEARCH DESIGN

In this study I frame algebraic reasoning as *a social rather than a cognitive practice*, using discursive psychology (Edwards & Potter, 1992) to explore six hours of video recordings generated through a “learning experiment” (Francisco & Hähkiöniemi, 2012) in order to answer two questions: *How do young children construct a mathematical inquiry? How do young children express mathematical generalization?*

Acknowledging the multiple ways children use to communicate mathematical meaning (Kaput, 1999) I applied multimodal analysis to video, transcriptions and child-produced artefacts using Transana software. Multiple passes generated 136 clips coded for social action (e.g. *doing knowing*,) and mode of communication(e.g. verbal).

MAIN FINDINGS

A seven year old challenged another’s conjecture with two verbal expressions that bridged arithmetical and algebraic reasoning. A five year old produced a gestural demonstration of the function of square root and a six year old produced a conjecture in verbal, numerical and written forms. These examples extend Davis (1996) by suggesting that while unconventional, the children’s reasoning was not unformulated.

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TEACHABLE AGENTS IN MATHEMATICS EDUCATION

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A new notion of the use of technology in students' mathematical activities has been suggested lately, a notion based on the use of so called *pedagogical agents* (http://aaalab.stanford.edu/). The overall idea is to let students learn by teaching the computer, in this case, a specially created *Teachable agent* (TA) (Blair et.al, 2011). A student teach his TA and then assess its knowledge by asking it questions or by getting it to solve problems. The TA uses artificial intelligence techniques to generate answers based on what it was taught by the student. Depending on TA's answer, the students can revise their agents' knowledge, and, in this way, their own (Brophy et. al, 1999).

This study investigates the potential of TA:s in supporting students' understanding of mathematics. The study focuses particularly on the integral, a mathematical concept that has been shown to be dificult to deal with for many students.

The first step in the study is to construct a hypothetical model of students' problem solving trajectories – based on their solutions of integral tasks formulated with a reference to some specific missconceptions they have about the integral concept identified in previous research. The second step is to try a model through cyclic empirical testing in order to increase its usability and validity. The third step is to conduct a ”Wizard-of-Oz”-based sequences in which students teach their TA to solve integral tasks. The researcher, here acting a TA, based on a particular trajectory that student's responce is related to, comes with a feedback - a critical question that aims to cause a cognitive conflict (Tall & Vinner, 1981) in the student's mind, in order to make him reflect and, if needed, to restructure his concept image of integrals.

Based on the model with problem solving trajectories and the ”Wizard-of-Oz”-experiments I hope to be able to suggest a prototype of TA that might support students' understanding integrals. Another hope is that the methodological framework that have been risen in this study, will be helpful for a future development of TAs aimed for students' work with wide range of mathematical concepts.

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MEANING OF PROFESSIONAL LEARNING FROM MATHEMATICS TEACHERS' PERSPECTIVES

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Teaching is generally regarded as a complex and demanding profession that requires a mixture of subject knowledge together with both theoretical and practical education knowledge, skills and understanding (White, Jaworski, Agudelo-Valderrama, & Gooya, 2013). With this view about teaching, these authors propose the idea of "teachers learning from teachers". Professional learning is therefore seen as empowering teachers, considering teachers as actively involved and allowing them to develop a sense of ownership of their personal growth (Roesken, 2011). With this stance, the aim of this study was to explore the meaning of professional learning from mathematics teachers' perspectives.

The data were collected through the semi- constructed interviews with eight secondary mathematics teachers with either MSc or BSc in mathematics in which, none of them had any experience in teaching other school subjects, and only, different mathematics subjects. The eight teachers volunteered to participate in the study. All the interviews were transcribed and coded line-by-line. The interviews were analysed and categorized according to the grounded theory methodology.

The analysis of the data revealed that one of the main concerns of the practicing mathematics teachers for their professional learning, is to become more knowledgeable and skilled with the various methods of teaching different mathematics content. To enhance their professional learning and empowering themselves, they expressed that the proposed idea of "teachers learning from teachers" is promising.

The result of this study showed that in mathematics teachers' view, "different teaching methods are equivalent to professional learning". The meaning of the professional learning depends on the expectations of teachers. However, mathematics teachers had consensus on this idea that to facilitate mathematics teachers' professional learning, they should have genuine concerns and real problems about their own teaching and learning, as well as their students' learning.

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SIXTH GRADE IRANIAN STUDENTS ENGAGE IN MATHEMATICAL MODELLING ACTIVITIES

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Modelling problems help students for developing mathematical ideas and concepts as well as generating novel solutions. Students' progress in these problems is identified through exploring and searching. Within few past decades, in various countries, the process of teaching mathematics has strikingly advanced toward modelling problems in classrooms (Niss, Blum & Galbraith, 2007). The current results have been collected from an investigation into the activities of 300 Iranian students, in grade 6 (age 12). These students divided into experimental and control groups. There were only pre-test and post-test for control group. But in experimental group, there were teaching intervention with focusing in mathematical modelling plus pre-test and post-test. The students experienced seven sessions of intervention education with four different modelling activities, which were instructed to them.

One of the intervention modelling problems was **House purchase loan modelling activity:** Ali's father wants to receive a loan of 15 million tomans from two governmental and private banks, namely Maskan and Gharz Alhasaneh respectively, in their city for the purpose of purchasing an apartment. Every one of these banks holds their own conditions. With respect to the situation of Ali's family and conditions of each of the banks, write your proposed solution. This problem was the third intervention problem, introduced in this study and done in the intervention classes.

This paper displayed an example of the loan issue, a social and mathematical process with which all families are engaged and about which children have heard, but with which they have been in contact very little. The students exhibited the procedures of interpretation; presentation of hypotheses, argument, and use of the material learned earlier, justification of self arguments, proper decision-making, and action to resolve the problem, and criticism over each other's solutions. The modelling activity of house purchase loan concentrated on advanced mathematics, such as proportion, percentage, multiplication, and division of decimals and large numbers. The students declared their models and opinions quite well and conferred with each other on the approaches they had chosen for the loan and attempted to elaborate their arguments. For analysing of students solution, we use Ludwig and Xu (2010, p. 80) theoretical framework.

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POEM AND CANDY USED AS AN ALGEBRA MANIPULATIVE

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In this study, students memorized a poem to identify various functions and used candy discs to graph them. Data shows significant improvement in performance, and hence understanding, of identification and graphing of functions.

Mathematics is the study of quantity, structure, space, and change. Algebra is a key part of mathematics; unfortunately, sixty-percent of nationwide college-bound freshman are not proficient in Algebra (ACT, 2005). This means that these students need to complete remedial Elementary and/or Intermediate Algebra classes in order to be admitted into the university system. By introducing an abstract approach when learning Algebra concepts, specifically when learning to identify and graph functions, can we improve the learning experience and results for these at risk students? The activity described in this abstract was designed to test that hypothesis.

Our research was conducted with two remedial Intermediate Algebra classes held at California State University, Channel Islands in Southern California in the mornings, which were taught by two different teachers on the same days. Students were typical students coming from California high schools. All of the students were freshman.

During this study, a poem and manipulatives were used to teach Intermediate Algebra concepts which did result in an improvement in performance on tests. The manipulatives used in the study were candy discs, which had been numbered on one side and included names (with students' name from class) on the other. Students were asked to create the various types of functions mentioned in the poem using the candy on graph paper.

After comparing the pre and post testing, the students included in the test group improved their overall test score by almost 35%. Both the poem and candy manipulatives were well received by the students as a learning tool.

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MATHEMATICAL MODELLING: RUNNING IN CIRCLES?

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Mathematical modelling activities are often within educational settings considered as processes. The concept of a process meant here can be understood as cyclic or linear. Contemporary modelling discussion concerning educational purposes emphasizes the cyclic aspect and distinguishes the “real” and the “mathematical” world (Henn, 2003).

However there are conflicting views (Meyer et al., 2010) that are worth to be discussed and some interesting questions arise:

- Is there a “real world” without mathematics?
- Should we stress the “vertexes” or the “edges” of the graph representing a modelling activity?
- Are there representations of cognitive processes that are different from a simple cyclic scheme but do include cyclic aspects?

These questions and more have been discussed with first-year students seeking the teacher profession for primary and secondary level within the framework of a calculus course. They were given several modelling tasks related to classical applications of calculus like physics and social sciences and economy as well.

The evaluation of short mathematical essays resulted in interesting findings about students’ conceptions of mathematical modelling and science in general and gives reason to reconsider the schemes mentioned above.

In conclusion the poster shows some approaches on how to improve students’ competencies relating to economic modelling and compares the relevant modelling skills of first-year students in both economy and mathematics education.

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DEVELOPING DEICTIC EXPRESSION AND JOINT ATTENTION IN GROUP WORK WITH YOUNG CHILDREN

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Studies in exploratory talk have suggested that, where students engage critically and constructively with each other, learning can be enhanced (Mercer and Sams, 2006). Two episodes, with three six year old children, illustrate that the development of constructive engagement requires a social reciprocity, not just in sharing actions and behaviours but in sharing understanding. This sharing of understanding is examined through a theoretical lens based on joint attention and deictic expression.

Joint attention is a fundamental communication function within language acquisition (Bruner, 1981). Indexicality is intrinsic to developing joint attention, as one participant directs the attention of another towards an intention. Indexicality becomes deictic when the intention is not just to influence behaviour, but to influence understanding. Deictic terms (such as the demonstratives 'this' and 'that') are used alongside gestures to point to an idea.

Prior to the first episode the children had not worked in groups independently of the teacher. In the two months between the two tasks the teacher encouraged the children to listen and agree with each other. The children were asked to order pattern dot cards to show two more dots each time. Reflection on the spatial patterns in seeing the difference of two, would encourage children to use relationships between numbers rather than rely on counting.

In the first episode the children acted on the problem in positioning cards, but did not speak to each other or direct each other to their intentions. Two months later one of the children pointed to the dots on the 3 card and the 5 card to show the relationship to the others, and used deictic terms in referring to '*this* line', '*this* one', and 'two more on *it'*. The children were in agreement that five was two more than three, and the suggestion "I think seven" for the next card was agreed.

Analysis of the two episodes indicated that there was a shift in the children's use of intentionality, and that this shift was evidenced through deictic expressions. A social reciprocity was developing as the children established joint attention, not just to influence behaviour, but to influence understanding.

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SUPPORTING PRE-SERVICE TEACHERS OF MATHEMATICS IN THEIR FIRST YEAR OF UNIVERSITY

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The transition from high school to university mathematics causes many difficulties for students. Reasons for that are often the overall view on mathematics (e.g., the axiomatic deductive character of university mathematics) and the learning culture that both differ greatly from secondary schools (for an overview, see Gueudet, 2008). To bridge the gap between the secondary and tertiary level mathematics, it is particularly necessary that students have the opportunity to repeat basic mathematical theories and methods deeply, and learn how to adapt *concept image* and *concept definition* (Vinner, 1991, p. 68) appropriately. Furthermore, pre-service teachers of mathematics need to develop their pedagogical content knowledge that is important for their future profession as a high school teacher. For that reason they should learn to connect high school and university mathematics, and how to present and communicate mathematical contents. Therefore, the TUM School of Education in Munich (Germany) now offers innovative tutorials that aim at supporting students of mathematics education in the above-mentioned aspects of their mathematics educational training. These tutorials complement the basic lectures of linear algebra and calculus.

We investigated the students' views on the relevance of the tutorials by delivering evaluation sheets at the end of the summer term of 2013 (linear algebra N=30, calculus N=25). The students were asked to indicate whether the tutorials actually met their needs in their first year of university studies, and which of the tutorials' contents were in their opinion most important. The results showed that there was high agreement of over 70% that the tutorials actually enhanced the students' abilities to adapt content image and concept definition as well as to repeat basic mathematical content. According to the students, the two most important contents of the tutorials were "adapt concept image and concept definition" and "foster mathematical communication and presentation" (over 85% agreement). These results demonstrate the relevance of the content knowledge and the pedagogical content knowledge in the students' view. In addition, the evaluation suggests that, all in all, the tutorials are a promising element to support students of mathematics education in their first year of university.

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PRE-SERVICE MATHEMATICS TEACHERS PERCEPTIONS OF ACTION RESEARCH AND ROLE IN TEACHER DEVELOPMENT

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There is growing recognition in teacher education for the need to recognise the hectic and demanding nature of teaching and consequently the need for personalising professional development (Wilkins & Wood, 2009). The development of action research within teacher education programme is driven by a desire to empower pre-service teachers through facilitating an experience that demonstrates the potential that they can be influential within their own classrooms/school environment (Whitehead & McNiff, 2006). Action research can enhance the integration of reflection, research and action, and accordingly improve mathematics teaching and learning. Moreover, through introducing action research into mathematics teacher education it should promote the development of autonomous professionals engaged in continuous professional development. Central to this research project is the view of action research as improving practice, not just examining it (Carr & Kemmis, 1986). It is undertaken by practitioners and views teachers as researchers

The sample comprises of 24 pre-service mathematics teachers in their final year of study of a four-year undergraduate degree in post-primary mathematics teaching. All students completed a detailed questionnaire (quantitative and qualitative questions) and 8 of the students participated in a focus group interview. Findings indicate that the majority of students found undertaking an action research project to be of immense benefit for their own learning and development, and their pupils learning and development. Moreover, they felt that action research is essential for professional development and contributes to informed self-reflection. This is the first study to be undertaken in the Irish context in relation to utilising action research within mathematics teacher education and thus the finding emerging can help inform policy and future practice. Action research affords an opportunity for pre-service teachers to develop an awareness of their own preconceptions, to question them and to change their practice in the classroom.

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MATHEMATICS FOR ALL: THE EFFECTS OF ROBOTICS ON CHILDREN'S MATHEMATICAL LEARNING AND PHYSIOLOGICAL WELL-BEING FOR CHILDREN WITH ACUTE LYMPHOCYTIC LEUKEMIA (ALL)

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One of the most difficult and unpublicized areas of mathematics education is the education of students with pediatric cancers. Children with cancer and acute lymphocyctic leukemia (ALL) in particular are among the most vulnerable of mathematics students, receiving little to no meaningful mathematics instruction during their months and years of hospitalizations. Many return to school where they are expected to have maintained and retained growth coincident with the progress of the normal child. Constructionist theory proposes hands-on activities that promote three-dimensional thinking and visualization by applying mathematics skills and strategies to real-world problems that are relevant, epistemologically meaningful, and personally meaningful (Papert, 1980).

In a 24 week ethnographic study, I investigated an intervention for children with ALL using the Lego Mindstorms EV3 and WeDo Robotics kits, and a tangible-graphical programming language, Creative Hybrid Environment for Robotics Programming. This study asked: What effects do robotics in a hospital setting have on children's mathematical learning, physical health, and socio-emotional well-being?

Data were collected pertaining to (1) robotics use and mathematical content knowledge; (2) socio-emotional well-being (including learning motivation and engagement) through the use of robotics; and (3) data about physiological well-being (i.e., observable measures of anxiety such as reduction in heart rate or lowered blood pressure). In particular a 3-year-old, 14-year-old, and 16-year-old child's involvement with shape, space, and measure will be presented as case studies. The data were examined with a Critical Realist Activity Theory theoretical perspective (Nunez, 2014). Findings indicate the robotics leveraged and enriched the student's mathematical content knowledge and heuristic knowledge. Average mean arterial, systolic, and diastolic blood pressures decreased during chemotherapy and other procedures when the children were engaged in robotic activities, and all children indicated the robotics tasks provided their first happy consciously mathematical experiences.

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DIAGNOSIS OF STUDENTS' LEARNING DIFFICULTIES IN THE FIELD OF FUNCTIONAL RELATIONSHIPS

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Recent studies revealed that students show crucial difficulties when translating between different forms of representations of functional relationships (Adu-Gyamfi, Bossé & Stiff, 2012). These difficulties, especially misconceptions, could unable successful further learning. The aim of our study is thus to provide a diagnostic tool which helps teachers to identify such typical learning difficulties at an early stage. Therefore, we analyse frequently occurring error patterns for the purpose of identifying common misconceptions. Based on the results of the project HEUREKO, in which the general competence structure in the field of functional relationships was examined (Leuders et al., 2009), we structured the subject area considering translations between different forms of representations: graphical-algebraic (GA), situational-algebraic (SA) and graphical- situational (GS). As the aim of this study is to develop an online tool with automatic feedback, most items are designed in a multiple choice format. The distractors of the multiple choice items are designed based on common systematic errors reported in previous research. In order to find consistent error patterns we considered at least two items of each translation and function type.

In a pilot study, we explored whether the distractors of the multiple choice items yielded the expected systematic errors and misconceptions in order to develop a standardized feedback and identified the most common error patterns with the aim to develop a whole diagnostic tool. 24 tasks were used with a test time of 40 minutes. The pilot study sample was of 95 German students of two ninth-grade classes and two tenth-grade classes of a comprehensive school. 16 students took part in diagnostic interviews in order to get deeper insights in the thinking process of the students. We identified seven error patterns which occurred among more than 10% of the students. For example, considering the translation SA, 17% of the students showed an error pattern due to a sign error when displacing the parabola on the x-axis. The evaluation of the interviews revealed that students who showed this error only used their intuitions and argued within the situation. The results of the ongoing main test will help to hypothesize to what extent the examined misconceptions occur in German classes.

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A CONTINUITY OR DISCONTINUITY FROM WHOLE NUMBER TO DECIMAL FRACTION AND FRACTION IN ELEMENTARY SCHOOL IN JAPAN

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Though a lot of previous studies had examined the nature of number sense or mental representation for whole number in young children and adult, we had a tiny knowledge about a developmental path from whole number to the other number in childhood. Seigler et al. (2013), pointed out five future questions to examine. One of these questions is what relations connect whole number and fraction knowledge. We examine that are there continuity from whole number to decimal fraction and from whole number to fraction or not in this study.

The participants in this study were one hundred 3rd graders, one hundred twenty-two 4th graders, and eighty-nine 5th graders children. They were asked to answer a magnitude decision task for each three numbers; whole number, decimal fraction and fraction. A magnitude decision task was carried out on an original iPod application. Eight number pairs were included in each three numbers.

An ANOVA for simple response time of magnitude decision task showed 4th=5th<3rd for whole number and decimal fraction, but 4th<5th<3rd for fraction. These results indicated the fraction was difficult for 3rd graders and the fraction whose different denominator and different numerator was difficult for 5th graders. Figure 1 showed that a correlation coefficient between whole number and decimal fraction was very strong and significant ($p<.01$) for all grades, but a coefficient between whole number and fraction was relative weak and it was not significant for 5th graders. These results indicated that there is continuity from whole number to decimal fraction, but there is discontinuity from whole number to fraction. And those suggested children did not use same number sense for whole number and fraction.

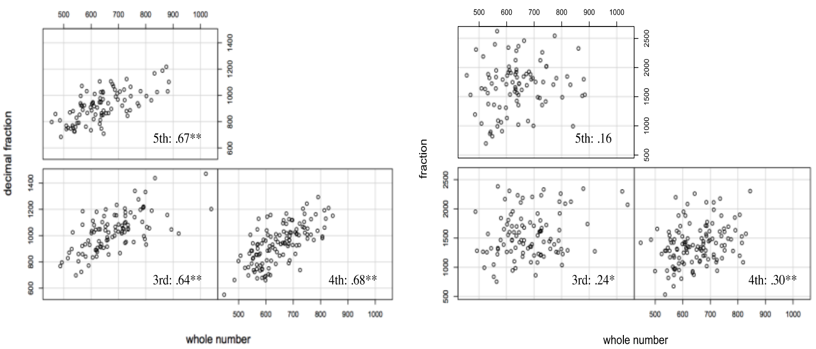

Figure 1

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INVESTIGATION AND COMPARISON OF CRITICAL ASPECTS

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Despite the increased interest in people with deeper mathematical knowledge, there is a constant stream of new articles which indicates that students have unsatisfactory knowledge in mathematics. Because equations and functions are often conveyed in symbols, oral and written communication about mathematical ideas is recognized as an important part of mathematics education. The main questions in this presentation are: How can theory help us to understand and support students' developing mathematical learning? It is possible to understand the mechanism for an effective communication which may lead to students' understanding pattern and structure, the logical analysis, and calculation with patterns and structures when working with algebra and functions during the classrooms lessons?

The focus will be on explaining why communication succeeds, and how it succeeds, by having as basis variation theory (Marton & Tsui, 2004). The general idea of success is this: a communicative event is successful just if the terminal state corresponds to the initial state. This implies that, the communication in the classroom succeeds or not if the aspects of the content supposed to be treated in the classroom (the intended object of learning) are the same as or different from the aspects of the content of the teacher's representation (the enacted object of learning), and if the aspects of the content of the teacher's representation (the enacted object of learning) are the same as or different from the aspects discerned of students, i.e. the content of the student's representation (the lived object of learning).

The study was performed in two classes, selected from the Natural Science Program in upper secondary school, in Sweden. In both classes, the same textbook was used. A total of 45 students, 16 years old (25 males, 20 females) and two teachers (Anna and Maria) took part in the study.

These studies indicate the process of meaningful interaction among the intended, enacted and lived objects of learning is an indication of whether the communication in the classroom is successful or not. The communication succeeds if teachers are constantly working with an iterative process in which they can discuss with each other and reflect on the implementation of lessons in relation to what students discern and what dimensions of variation are created in the classroom.

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ANALYZING A PROBABILISTIC CURRICULUM: THE CASE OF A JAPANESE JUNIOR HIGH SCHOOL TEXTBOOK

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This paper reports a part of results of our curriculum research in which we attempt to clarify primary factors on difficulty of learning probability. In this paper, we analyze the most popular 8th grade mathematics textbook in Japan (MHS2; Okamoto, et al., 2012), because Japanese students encounter officially probability for the first time at 8th grade. Our conceptual tool for textbook analysis is *discourse* in the Commognitive Theory of Learning proposed by Anna Sfard (2008). Results of discursive analysis are summarized as a following table, in which three italicized words emphasize that the signs are signifiers. These results suggest that, in Japanese junior high school curriculum, the word uses of *probability* are different between in defining context and calculating one. According to a terminology of Ian Hacking (1975/2006), probability in the former context is *aleatory* on one hand, and in the latter is *epistemological* on the other hand. Aleatory interpretations regard probability as objective nature of the physical world, while probability in epistemological views is related to belief or logic. Consequently, it seems that this fundamental conflict between definition and calculation of probability in the curriculum is responsible for difficulty of learning probability.

Discursive indicators	Results of analysis
Word use	*Probability*, *Equally likely*, *Tree diagram*
Visual mediator	Figure of concrete objects, Table of frequency, Graph of relative frequency, Tree diagram, Figure of combination, Table of combination, Table of outcomes
Endorsed narrative	Frequentistic definition (i.e. aleatory interpretation)
Routine	Combinatorial calculation (i.e. epistemological procedure)

Table 1: Results of discursive analysis on probabilistic contents in MHS2

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IMPROVING THE TEACHING OF MATHEMATICS IN ELEMENTARY SCHOOLS BY USING LOCAL LANGUAGES AND CULTURAL PRACTICES

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Elementary education is dependent on the understanding of teachers to build on cultural knowledge and to transit to school mathematics without dysfunction and loss of identity (de Abreu, Bishop, & Presmeg, 2002). We report on the early stages of a three year study which is exploring how best to identify and use cultural mathematical proficiencies to assist young students to transition to school mathematics in Papua New Guinea (PNG). The project uses a design research methodology to design and refine guidelines to assist elementary teachers to recognise and use these cultural mathematical proficiencies, and to develop vernacular phrases for school mathematics.

We present the guidelines in the form of interrelated key principles. These guidelines were developed from past research into PNG mathematics such as Lean's (1992) categorisation of counting systems and extended by others such as Matang and Owens (2014). The guidelines have been trialled in case study workshops, and refined and used in three Provinces with different cultural ecologies. The workshops use ICT technology to assist delivery. Initial responses of the elementary teacher participants in the workshops have been highly positive. The participants' responses inform the refining of the workshop for subsequent deliveries. They are also informing the development of an interactive user-friendly guide which will allow the professional learning to be delivered to more teachers in PNG Provinces. Follow-up questionnaires and student interviews are being used to evaluate the effects of changes to teaching practices.

Acknowledgements

The project is funded by an Australian Development Research Award 2013-15. The views expressed in the publication are those of the authors and not necessarily those of the Commonwealth of Australia.

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TEACHING APPROACHES IN READING COMPREHENSION OF MATHEMATICAL WORD PROBLEMS IN TAIWAN

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Mathematics performance and reading skills have been found to be closely related. Since most studies conducted with either students' performance (e.g. Jordan, Kaplan, & Hanich, 2002) or with instructional methods to solve word problem, little is known about how teachers in Taiwan teach students to comprehend the word problem.

This proposal aims to examine what is the most preferable instructional approach for teachers in Taiwan to foster comprehension in math word problem. Accordingly, the research questions are: (1) Which approach for teaching comprehension in math word problems are popular? (2) Are there any significant differences in teaching approaches among teachers of different grades and different years of experience?

In order to assess teaching approaches, we developed a teaching approach analysis questionnaire. The questionnaire consists of two parts: the direct instruction (DI) and the constructivist instruction (CI). Questions of each part are subdivided into four dimensions proposed by Mayer (1992); that is, problem translation, problem integration, solution planning and solution execution.

The sample comprises 192 teachers teaching from 3rd to 6th grade (13 schools from 7 counties/cities). According to the results of questionnaire, teachers are divided into 4 teaching approaches: DI, CI, DI & CI and neither CI nor DI. The findings indicate that 78% of teachers prefer to teach word problem comprehension not only through DI, but also through CI. In addition, there is a significant difference among years of teaching experience. Teachers with more than 21 years of experience prefer to teach students through CI & DI. This result may be explained by some studies of teaching beliefs of teachers with different years of teaching experience. However, there is no significant difference in teaching approaches among different grade teachers.

In summary, using both DI and CI is the most popular approach to teach word problem comprehension. Moreover, the more the years of experience teachers have, the more the teachers prefer to use both CI and DI in their math class.

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MATHEMATICS TEACHER LEADERSHIP: A SUSTAINABLE APPROACH TO IMPROVE MATHEMATICS EDUCATION

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Research on teacher leadership began in the 80s and has continued to gain importance to this day. This poster presentation will describe the impact of a mathematics leadership institute funded by the National Science Foundation. The development and implementation of the institute was informed by research on the key components of effective professional development for teachers (e.g., Desimone, 2009) and on York-Barr and Duke's (2004) conceptual framework for leadership programs. The institute's goals were to increase teachers' math content knowledge and improve their leadership skills and teaching practices. Seventy-nine math teachers with three years or more of teaching experience from two large urban school districts were recruited. Lead teachers participated in the institute during one of three cohorts. Each cohort attended two consecutive summer programs and then received extensive academic-year support throughout the institute's existence. Each cohort began the institute the summer after the preceding cohort had completed its second summer program.

Data for this study included lead teachers' content test scores; results of surveys administered to lead teachers, their colleagues, and administrators at the conclusion of the institute; and classroom observations of lead teachers. Content test scores were analyzed quantitatively (ANOVA - repeated measures) to measure differences in content knowledge from pre- to post-tests. Gains on content tests were statistically significant ($p < .001$, Cohen's $d > 1.35$). Survey questions were dichotomous (yes/no) demonstrating agreement to given statements. Percentages of agreement to survey questions were calculated and reported. Agreement with statements about the institute's impact on lead teachers' leadership skills was above 72%. Finally, lead teachers were observed using a classroom observation protocol. Aggregated data on observed lessons were generated. In each of the four major categories of the observation protocol (procedural knowledge, propositional knowledge, lesson implementation, and classroom culture), 70% or more of lessons observed demonstrated exemplary practices. Results indicated that the institute positively impacted lead teachers' understanding of math concepts, equitable instructional strategies, and collaborative leadership skills. The poster will present the complete results and provide opportunities for discussion in further detail.

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A COMPARISON OF INSTRUCTIONAL SEQUENCE IN INTELLIGENT TUTOR-ASSISTED MATH PROBLEM-SOLVING INTERVENTION PROGRAM

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There is a lack of intervention approach supporting low-achieving students to overcome the challenge of mathematics reform particularly in mathematics reasoning skills. Thus, it is crucial to examine possible ways that can address multiplicative reasoning skills for struggled students. Math and special education researchers developed an intelligent tutor, PGBM-COMPS (Xin, Tzur, & Si, 2008), for nurturing multiplicative reasoning and problem solving of students with learning disabilities/difficulties (LD). The PGBM-COMPS was built on constructivist view of learning (PGBM) and Conceptual Model-based Problem Solving (Xin, 2012). The intelligent tutor has four modules (A, B, C, & D). Module A consisted of *multiplicative double counting* (mDC) and *same unit coordination* (SUC). Module B consisted of *unit differentiation and selection* (UDS) and *mixed unit coordination* (MUC) tasks. Module C had *quotative division* (QD) and Module D *partitive division* (PD) tasks. Pilot field-testing studies using PGBM-COMPS indicated that students struggled in the MUC task, as it involved two-step problems.

The purpose of this study was to compare the effects of the two instructional sequences of the PGBM-COMPS (i.e., Modules A-B-C-D or A-C-D-B) on students' reasoning and problem solving. Participants were 18 third and fourth graders with LD from a mid-western elementary school. They were assigned to one of the two comparison conditions, A-B-C-D or A-C-D-B sequence. Students worked with the PGBM-COMPS intelligent tutor for about 36 sessions throughout the semester. An analysis of variance (ANOVA, 2 groups × 3 times of testing) was conducted with repeated measures on time (pretest, posttest, one follow-up test) using the Multiplicative Reasoning (MR) test (Xin, Tzur, & Si, 2008). The result indicated that students in both conditions improved their word problem solving performance at a similar rate. However, the revised sequence of instruction (i.e., A-C-D-B) allowed students with LD to acquire the quotative and partitive division skills before dealing with the MUC tasks, which involve both additive and multiplicative tasks. Thus, it seems that the A-C-D-B instructional sequence can better provide accommodation for students with LD.

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FOSTERING AUTONOMY IN COGNITIVE DISABLED STUDENTS THROUGH MATHEMATICS

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It can be said that autonomy and the ability to socialize and enjoy culture are among the basic rights of each person, those which the school is institutionally required to foster, and that these rights are obviously possessed by all people including those with any form of disability. Mathematical education must contribute to the cultural development of the citizen, in order to allow him/her to take a conscious part in social life, showing critical and independent ability. According to Flòrez and Troncoso (1998), from the educational point of view, integration does not mean keeping different pupils together in the same class without applying to each of them the content and methods he/she is in need of. In reality there are two important variables: the huge differences existing between different types of difficulty, which mean that what is suitable for one may not be so for another, and the progressive changes in conditions, needs and skills brought about by growth of the pupil.

This work analyses some mathematical concepts which encourage the acquisition of independent competences: knowing how to spend money, use public services, ask for help, do one's shopping. After being turned into basic mathematical goals, some basic mathematical concepts aiming at the acquisition of autonomy, such as the concept of number or the spatial orientation, have been put into practice. Some particular cases of secondary school disabled students will be presented, together with the successful experimentation of learning paths such as the case of M. aged 19, affected by Down syndrome and attending the 3rd year of the Technical Agricultural Institute, who had considerable difficulty in recognizing and using numbers higher than one thousand, and the case of G. aged 20, attending the 2nd year of the secondary school of Arts, with a borderline psychotic personality, who needed to reinforce his spatial organization and orientation abilities as a means of fostering personal autonomy and boosting his self-esteem.

Personal autonomy represents an important achievement for every young person, therefore the importance of the activities based on the above-mentioned methods is evident, since they are meaningful and useful to everybody. Once more, it is necessary to acknowledge the ability of mainstream schools to branch out along unexpected paths when teaching to disadvantaged pupils and addressing such complex skills as mathematical ones.

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REASONING DUALITY IN SOLVING ADDITIVE WORD PROBLEMS: HOW TO MEASURE STUDENTS PERFORMANCE

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It is well known that students' difficulties in solving simple additive word problems are related to the type of mathematical reasoning they apply (*Riley, Greeno, & Heller, 1984*). Research shows that students' success in solving problems of different categories can differ significantly. We can distinguish two paradigms in which additive problem solving and related reasoning can be seen: the Operational paradigm, where problem is understood as a task to apply operations (Brissiaud, 2010), and the Relational paradigm (Davydov, 1982), where the main goal is to understand the system of quantitative relationships described in the problem. Students' performance in problem solving improves gradually throughout the learning experience. It is important to determine whether this improvement in performance is due to the procedural knowledge of operations or to an enhanced understanding of quantitative relationships. In cases of large screenings of students' performance, such as written tests, how can we know the quality of reasoning behind the average performance? In the scope of the collaborative study conducted in six second grade elementary school classrooms, we developed and implemented a teaching approach to promote relational reasoning in the context of additive word problems. The approach has been implemented over the past two years. Among other methods of analysis of students' reasoning (e.g. individual interviews), we used the written test. We calculated the average success and the gap between success in solving easy and difficult problems. Our data shows: a) on average, the experiment groups perform better in problem solving than control groups; b) in the control groups, there is an important gap in success between easy and difficult problems, a gap which does not exist in the experimental groups. We will present the teaching approach and measurements indicating the presence of the relational reasoning in students' performance.

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TRAJECTORIES OF TEACHER EFFICACY AND ITS SOURCES

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Mathematics teaching and learning courses in teacher preparation programs provide the opportunities for preservice teachers to consolidate their knowledge of mathematics into mathematical knowledge for teaching (e.g., Ball, Hoover Thames, & Phelps, 2008). An important outcome of these courses is the changing of teachers' beliefs about what it means to teach so that others can learn. The purpose of this study was to inquire into secondary school preservice mathematics teachers' teacher efficacy (a teacher's belief in her or his capability to affect student outcomes) and the interactions of teacher efficacy with the sources of teacher efficacy (mastery experience, vicarious experience, verbal persuasion, and affective and physiological states) and teachers' concerns (Self, Task, and Impact – their perceived problems or worries of the teaching and learning of mathematics).

This mixed method study explored three complete teacher preparation program years with an aggregate of 100 preservice teacher participants. The Teachers' Sense of Efficacy Scale (TSES) (Tschannen-Moran & Woolfolk Hoy, 2001), and open response question data were collected four times each year. The TSES contains three subscales; instructional strategies (IS), student engagement (SE), and classroom management (CM). A combination of repeated-measures ANOVAs over time and constant comparative analysis were used in the analyses of the data for each year.

In all three program years, the findings indicate that teacher efficacy demonstrated significant increases, with details such as, SE decreased in the first quarter, IS was highest at the end, and CM was the lowest at the start. Throughout each program any combination of concerns with Impact was consistently least present. Additionally, Mastery Experiences appeared rarely at the beginning of the program but became a dominant source at the end. Conversely, the Affective/Physiological source was higher at the beginning with a large drop right away, ending low. Further details will be presented in the poster.

Inferences from these analyses suggest opportunities for the positioning of course and program aspects in relation to student readiness for learning new skills, and there may be improved benefits for inservice teacher learning with immediate and ongoing teacher professional learning in a programmatic and intentional manner.

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DIFFERENT REPRESENTATIONS IN MATHEMATICS TEACHING WITH TECHNOLOGY

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The main focus of this poster is the teacher's representational fluency in a context of graphing calculator use. The conclusions reached point to a more intensive use of some representations over the others, suggesting that technology turns numerical or tabular representation into two different representations.

One of the features of the graphing calculator is to allow access to multiple representations, which makes it possible to establish or strengthen links in a way that would not be possible without the support of technology, articulating the numerical or tabular, symbolic or algebraic and graphic representations and enhancing the development of a better understanding of functions. In this sense it is crucial to understand the teacher's *Knowledge for Teaching Mathematics with Technology* - KTMT (Rocha, 2013) and the way the teacher's representational fluency is part of this knowledge. The study presented in this poster focus on the teacher's representational fluency, looking for an understanding about how the teacher balances the use of different representations, paying attention to the sequence by which representations are used and to the frequency of such use. The study adopts a qualitative and interpretative methodological approach, undertaking one teacher case study (in the part presented here). Data were collected by semi-structured interviews, class observation, and documental data gathering. Data analysis consists of data interpretation, considering the problem studied, and the theoretical framework. The conclusions reached point to the close link between teacher's representational fluency and teacher's knowledge of teaching and learning, emphasizing the importance of teacher's KTMT. The conclusions also suggest a prevalence of algebraic and graphic representations, with a preference to go from algebraic to graphic representation and a marginal use of tables. Nevertheless the numerical representation is often used, suggesting that numerical and tabular should be consider as different representations. Thus this study raises questions about the implications of technology over the representations used, suggesting that is the use of technology that turns numerical and tabular into different representations.

The poster adopts a visual approach, highlighting the main results using some calculator screens in relation to the tasks proposed by the teacher.

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STUDENT EVOKING TEACHER FOLDING BACK: NEGOTIATING UNDERSTANDING OF INTEGERS IN THE CLASSROOM

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This research is part of a larger study on the use of folding back (Martin, 2008) as a purposeful tool by teachers to increase student understanding. The theoretical framework of folding back is an integral part of the Pirie-Kieren theory of mathematical understanding. In folding back, a person realizes that previous mathematical understandings require extension, revision or discarding to include or in favour of new understandings. Through the process of integrating new understandings, the person "thickens" their previous understandings. In this instance, we examine a case study of a grade seven teacher, Rick, collaborating with his students to fold back and thicken his own understanding of integers.

This analysis contributes to research in understanding of integers (e.g. Bishop, Lamb, Phillip, Whitacre, Schappelle & Lewis, 2014) and research in pedagogical content knowledge (e.g. Ball, Thames & Phelps, 2008). Research has demonstrated that topics in the area of integers are often difficult to conceptualize and learn. Difficulties can range from calculating with integers to understanding the magnitude of negative integers. Pedagogical content knowledge incorporates both a teacher's pedagogical practice and their knowledge of the subject in order to teach. Here, Rick expanded both his practice and understanding of integers through his interactions with his students. We follow Rick as he creates open-ended tasks for his students to introduce concepts in integers. As the students share their discoveries of their findings about integers, they introduce new understandings of integers Rick had not previously considered. In our poster, we analyze both the discursive process between the students and the teacher, and how the teacher changes artifacts (e.g. classroom charts) in his classroom to conform to his new "thickening" understanding of his previous ideas about integers.

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‘FOLDING BACK’: A TEACHER–RESEARCHER’S PERSPECTIVE

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The Pirie-Kieren Theory for the Growth of Mathematical Understanding is a widely accepted framework for examining students’ reasoning with mathematical concepts (Pirie and Kieren, 1994), and this study aims to examine the application of this framework to classroom instruction and pedagogy. In particular, the significant facet of ‘folding back’ (Martin, 2008) within the Pirie-Kieren Theory is considered as a pedagogical tool to facilitate students’ growth of understanding when faced with challenging and novel concepts. In ‘folding back’, students leverage prior knowledge to follow a non-linear trajectory towards developing new knowledge and understanding, while concurrently strengthening their prior knowledge.

In this study, ‘folding back’ was explicitly employed to support students’ understanding of equations of lines in a senior high school mathematics course. Given students’ prior extensive experiences with equations of lines on the Cartesian plane, ‘equations of lines in three-space’ was selected as an appropriate topic to explore ‘folding back’. A sequence of two lessons was collaboratively designed to intentionally direct students to utilize their existing layers of understanding about equations of lines in order to extend their thinking about lines to three-space.

Four hours of video data were collected for this sequence of lessons from two classrooms. The data analysis focuses on the visible learning outcomes of students engaging in the practice of ‘folding back’. Reflecting specific episodes from the video data, this poster will focus on the perspective of the teacher-researcher involved in this study. We consider the positive outcomes as well as challenges that may emerge in planning and employing this approach. The goal of this preliminary work is to reflect on the teaching practice of facilitating learning mediated by intentional ‘folding back’ in mathematics classrooms. Accordingly, this presentation will demonstrate how intentional ‘folding back’ assisted in connecting students’ existing understanding to a new context, and how the experience of facilitating ‘folding back’ for students also served to ‘thicken’ the understanding of the teacher.

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SPATIAL REASONING IN MATHEMATICIANS: ROLE IN UNDERGRADUATE AND RESEARCH MATHEMATICS

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Research over the twentieth century has shown there is a strong correlation between spatial reasoning and mathematics achievement (Uttal et al., 2013). Some studies have looked at the direct link between spatial reasoning and K-12 mathematical content areas, like geometry or arithmetic (Cheng & Mix, 2012). But what is the role of spatial reasoning in advanced mathematics? Many studies have looked at spatial abilities in children, but the world of mathematicians' practice remains largely unexplored in this regard. In particular, my research questions are: 1) How do mathematicians from different fields reason through spatial tasks? 2) How does spatial reasoning connect to mathematicians' a) own research and b) undergraduate courses they teach?

Uttal et al. (2013) provided a classification for types of spatial skills along two dimensions: extrinsic/intrinsic or dynamic/static. With this framework, I conducted think-alouds/interviews with three (N=3) young mathematicians. Mathematicians' reasoning was captured through the think-aloud, and the connections to their research and teaching captured through subsequent interview questions. The think-aloud portion consisted of three tasks: mental rotation, cross section of three dimensional objects, and a perspective task. All of these tasks were classified as different skills according to Uttal et al.'s (2013) framework. Data consisted of transcripts of audio recordings and field notes. This research is currently in the pilot phase, but transcript data will be analyzed using grounded theory techniques. I hypothesize that there will be some similarities in reasoning across all mathematicians but also major differences based on subfield of mathematics. Algebraists, geometers, and analysts may have different ways of thinking and see vastly differing roles for spatial reasoning in their area. This works extends the research on mathematicians' practice and explores deeper how spatial reasoning and mathematics are intertwined.

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CULTURAL PRACTICES OF COUNTING: AN EYE TRACKING STUDY OF COUNTING STRATEGIES IN ADULTS AND PRESCHOOL CHILDREN

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According to Vygotsky (1934/2001) the distinctiveness of a child development is the pre-existence of “ideal forms”, which are special cultural ways of how to perceive reality and perform actions. At the same time an “ideal form” is a result of the development. These ideal forms appear in a child psyche “at first as a form of collective behaviour of a child, forms of collaboration with others” (ibid, p. 90) and then they are transformed into individual functions. Following the cultural-historical approach, Radford (2011) claims that perception needs transformation in order to become theoretical: a child should be involved in cultural practices by an adult through the embodied system of gestures and intonations.

We investigated how cultural ways of perception of the number line emerge in children through interaction with an adult who serves as a carrier of the ideal form of counting. Four child-parent pairs (collecting data is still in progress) took part in three different activities: counting by an adult, counting by child while being taught by her/his parent and counting by a child alone. During all activities we recorded eye movements by SMI RED eye tracker and gestures and speech by video camera.

Our data revealed two main strategies of localization on the number line (also found by Schneider, et al., 2008): counting up to the number and counting from midpoint. But qualitative analysis of eye paths allowed distinguishing of four other strategies. At the teaching stage we analysed a correspondence between gestures, oral instructions and eye-movements. We saw that the adults transformed their practices in order to teach and then a variety of children reactions appeared: to be involved in an adult practice and then follow it for his own; not being affected by offered strategy or to consciously prefer his own strategy as a more reliable.

The conclusion is that ideal form of adult counting needs to be transformed in a special practice in order to be taught and even then a unique collaboration should appear for a child to approach a given strategy.

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ADOLESCENT STUDENTS' PERCEPTIONS OF MATHEMATICS AND SCIENCE AS A GENDERED DOMAIN

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Research within mathematics and science education has suggests that male are more likely than female students to implicitly associate mathematics and science with males (e.g., Cvencek, Meltzoff, & Greenwald, 2011). We concur that this stereotype becomes internalized by individuals through the socialization process, and is exacerbated as students progress through school (Bem, 1981). In this study, we examined to what extent, if any, female and male adolescents perceive mathematics and science as a female, male, or gender-neutral domain.

To address the research question, 232 students (130 females, 102 males) in grades 5-8 were administered the *Who and Mathematics* instrument (Forgasz, Leder, & Kloosterman, 2004), and a revised *Who and Science* instrument. There were 30 items on each instrument, including prompts such as "Enjoy mathematics" and "Find science difficult," respectively. One-sample t-tests were conducted on each question to determine whether the mean scores for male and female participants were significantly different from neutral (No difference between girls and boys). Since 30 one-sample t-tests were conducted, a Bonferroni adjustment was made to control for Type I errors ($p \approx .002$). This analysis was conducted separately for each instrument.

The findings on the mathematics instrument indicate a significant difference from neutral for females on 15 items, eleven indicating a perception that girls were more likely than boys to believe with the item and four indicating a perception that boys were more likely than girls to believe with the item. Males scored significantly different from neutral on one of the items and in the direction that boys were more likely than girls to believe or act in agreement with the item. The findings on the science instrument are similar in nature. In the presentation, results will be discussed in more detail. The study contributes to the evolving discourse and understanding of the gendered attitudes and beliefs towards mathematics and science.

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SYNTACTIC PARALLELISM IN MATHEMATICS DISCOURSE: BRIDGING DISCURSIVE AND COGNITIVE SCIENCE PERSPECTIVES

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As students refine mathematical ideas through discussion, they may repeat segments of sentences. Repetition can occur across several speakers' statements, or in monologues, as in the statement: "We thought that the birth rate is just going to keep on growing/and also that life expectancy is going to be higher/so therefore the growth of population will increase no matter what." Here, the student repeats the progressive verb form "is going…infinitive phrase" as a means of posing a deductive argument. Sociolinguists note that syntactic repetition can serve many social functions, including demonstrating involvement, establishing shared units of meaning, and modifying interpretations. In mathematics conversations, repetition may signal an emerging, informal, collective argument.

Cognitive scientists have also documented syntactic repetition experimentally as a means to describe mechanisms of language production and comprehension. Syntactic repetition in dialogue is stronger when it co-occurs with lexical and semantic repetition, though these combinations are not necessary; in multi-person conversations, both an addressee and a listener will subsequently repeat syntactic structures (Branigan et al., 2007). Order of operation calculations may influence the structure of a speaker's sentences (Scheepers et al., 2011). Repetition-based methods are beginning to be used in second language research on teaching and learning. Research on repetition is significant to educators because repetition may provide evidence of, and facilitate, collective learning.

The authors—a mathematics educator with interest in discursive methods and a cognitive scientist—will analyse several examples of repetition in transcribed mathematics conversations, highlighting interpretive differences in the two disciplines. Themes addressed will include emergence of shared meaning, theories of production and comprehension of sentences, recursion in representations, and potential contributions to education. We will consider the degree to which cognitive science perspectives on repetition can extend discursive research in mathematics education.

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FRACTIONS: WHAT DO 7TH GRADERS KNOW AND HOW CAN WE HELP WEAK STUDENTS

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Fractions is one of the most difficult topics in elementary school mathematics. Many students have great difficulties solving fractions problems – they approach them with little meaningful understanding and make errors. Poor knowledge of fractions poses a challenge for middle school teachers, who must teach algebraic equations with fractions. On the other hand, when fractions is taught in a meaningful way in elementary school, children develop rich and solid understanding (Empson & Levi, 2011). The research goals are:

- To determine 7th graders' knowledge of fractions and to document their errors.
- To find out if a short remedial instruction program can improve weak students' knowledge of fractions to a level sufficient for learning algebra.

In the first part of the study, 229 7th grade students from one school in Israel were tested on their knowledge of fractions, using a national exam. In the second part, three very weak students were chosen for further diagnosis and remediation. The 3 students were interviewed individually with a dynamic assessment instrument. They participated in a 10 lesson remediation unit. The main emphasis of the lessons was to lay a foundation for understanding, through continuous use of drawing representations and using word problems. The students were tested at the end of the study and their work was documented throughout the unit.

The test showed that 30% of the students got a score of less than 60%. The students made a variety of errors, using procedures without meaning and confusing procedures from different operations. Examples:

$1-\frac{4}{5}=\frac{1}{1}-\frac{4}{5}=\frac{3}{4}=\cancel{1\frac{1}{4}}$

$1-\frac{5}{6}=\frac{6}{1}-\frac{5\times1}{6}=\frac{31}{6}=\cancel{5\frac{1}{6}}$

$\frac{4}{5}:\frac{3}{4}=\frac{5}{4}\times\frac{3}{4}=\frac{15}{16}$

$\frac{1}{4}+\frac{7}{12}=\frac{11}{13}$

$3\frac{1}{3}+\frac{1}{2}=\frac{10}{3}+\frac{1}{2}=\frac{11}{6}=1\frac{5}{6}$

$3\frac{3}{1}+\frac{5}{1}=\frac{3}{10}+\frac{5}{1}=\frac{3+5}{10+1}=\cancel{\frac{8}{11}}$

$\frac{1}{3}+\frac{5}{12}=\frac{1}{3}\times\frac{12}{5}=\boxed{\frac{4}{5}}$

The remediation led to a large change in the students' level of knowledge, with more significant understanding and scores of 87-92% on the post-test. Moreover, they could solve algebraic addition and subtraction equations with fractions. The 7th graders showed a low level of meaningful fraction knowledge and difficulties using the procedures. It is possible in a short time to bring weak students to a good knowledge and understanding that allow them to learn equations with fractions. Elementary schools need to teach fractions more meaningfully and to help students who confuse procedures to understand them.

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EPISTEMOLOGICAL BELIEFS AND STUDENT CHOICES

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The influence of the epistemological beliefs and views individuals have on mathematics on the way they employ this science has often been the focus of research. Liu and Liu (2011), among others, investigate the connection between physics and mathematics. They classify mathematical beliefs as focusing on *local nature, empirical nature* or *creativity and imagination*, contrast *discovered* and *invented* beliefs as well as *socio-cultural* and *scientific aspects*. This grouping is associated with a perception of mathematics based on Bermy et al. (2007) divided into the *opinion about the topics of mathematics*, the *application of mathematics in society*, the *meaning of mathematics for nature* and *the purpose for the application of mathematics*. The research question is how these constructs are manifested in the decisions students make (when confronted with different topics) and their explanations thereof.

The study investigates the relationships between students' epistemological beliefs and their perception of mathematics. The intervention consisted of four supplemental courses, two devoted to coding theory, and one each to mathematical aspects of cosmology and particle physics. 22 students of grade 11 and 12 in German high schools were taught for six months by the author. In between lessons, they had to study the mathematical and application-oriented background of the subject matter. Having covered the mathematical background, there was a choice between several project topics for which students had to prepare presentations.

Afterwards, semi-structured interviews were conducted by the second author. Students' beliefs and their perceptions of mathematics were classified according the abovementioned framework. Thus, a surprising interdependence of beliefs and perception of mathematics was revealed: A group of students assumed mathematics as a science assisting applications, but chose abstract mathematical topics like elliptic curve cryptography. What is more, another group who understood mathematics as abstract thought construct chose topics in applied mathematics. These choices of topics seem to contradict the aforementioned classifications, but are explicable when looking deeper. Most students see applications and abstract mathematics as depending on each other in a balanced system.

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EDUCATIONAL INTERFACES BETWEEN MATHEMATICS AND INDUSTRY – REPORT ON AN ICMI-ICIAM-STUDY

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The first joint study organised by the International Commission on Mathematical Instruction (ICMI) and the International Council for Industrial and Applied Mathematics (ICIAM) looked into "Educational Interfaces between Mathematics and Industry". This EIMI-Study started with a Discussion Document (see Damlamian & Sträßer, 2009), had intensive discussions during a Study Conference in Lisbon in October 2010 (for the proceedings see Araújo et al. 2010) and is now condensed in the Study Book recently published (see Damlamian et al. 2013). The process brought to light the following main issues:

- Mathematics in education and industry share certain properties, but are also different – especially in terms of the importance of applications.
- Communication between academia, industry and education is hampered by different jargons in different institutions. The differences can be bridged by the use of metaphors, which are understood in the respective institutions.
- A comparison specifically between industry and educations shows that these institutions follow different time lines, have different goals and have different ways to learn.
- To allow for communication and cooperation between industry and education "boundary objects" and the de-greying of black boxes, which contain and sometimes hide mathematics, can be helpful in this process.
- In both institutions, modelling of extra-mathematical situations is the standard way to cater for the "rest of the world" outside mathematics. In this process, information technology can play a major role.
- There is a wide variety of activities in use to develop and strengthen links between education and industry (like modelling weeks and workshops). But there is no institution focussing on research on educational interfaces between mathematics and industry.

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MATH TEACHER MINDSET AND CLASSROOM PRACTICES

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In the United States it is common to believe that math ability is innate – some people have it, others do not, and there is little a person can do to change her basic math ability (Stevenson, Chen, & Lee, 1993). Such a belief aligns with Dweck's (2006) implicit theory of intelligence (or *mindset*), which identifies people's beliefs about math ability as falling along a spectrum, ranging from a *fixed mindset* (believing that math intelligence or ability is innate and limited) to a *growth mindset* (believing math intelligence or ability is malleable and can develop through hard work and effort). Research has repeatedly examined students' mindsets, finding that students with more of a growth mindset tend to have higher math achievement (Dweck, 2006). However, little research has been conducted to examine math teachers' mindset. This study seeks to address this gap in the literature by asking: (1) How do we reliably measure math teacher mindset? (2) How do math teachers' mindsets relate to their self-reported instructional practices?

Surveys were administered to 40 math teachers from middle schools (6^{th}-8^{th} grade) serving diverse student populations in California at the beginning of the school year (September 2013). The survey consists of the following five constructs: (1) Mindset: 6 items (2) *Expectations*: 3 items, (3) *Nature of math*: 6 items and (4) *Classroom practices*: 4 items.

Findings from this study validated a multiple item survey instrument to reliably measure math teacher mindsets. Reliability testing of the survey items in the mindset construct yielded a significant Cronbach's alpha of 0.74. Findings also suggest that math teachers' mindsets are related to their views about the nature of math and self-reported instructional practices. Correlation analysis revealed several significant relationships. For example, math teachers with more of a fixed mindset were: (1) less likely to hold fixed views of the nature of math ($\beta = .367$, $p = 0.02$), meaning they were more likely to view math as a set of procedures, (2) more likely to have the expectation that some students would not make much progress or succeed in math ($\beta = .384$, $p = .015$) and (3) less likely to report viewing mistakes as important for learning math ($\beta = -.355$, $p = .025$). Additional findings, implications, and future research will be addressed in greater detail in the poster presentation.

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TEACHING INTERVENTIONS IN MATHEMATICS

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At the upper secondary school level in Norway poor completion rate is a problem (less than 70 %), and students' weak performance in mathematics in both lower and upper secondary school is identified to be one of the underlying factors (Falch, Borge, Lujala, Nyhus & Strøm, 2010). This poster presentation reports from the early stages of a research project in the northern part of Norway, Teaching Interventions in Mathematics (TiM). The project has a focus on utilising teachers' craft knowledge (Ruthven & Goodchild, 2008) and seeks a close collaboration between researchers and teachers. TiM aims to identify teachers' wishes and expressed needs for lesson plans to help low achieving students in mathematics in lower secondary school, and from this insight design task-based teaching interventions.

TiM will be carried out in three phases; the first and second phases are exploratory, focusing respectively on students' and teachers' perspectives at the different school levels. The third phase from 2015 onwards is based on active collaboration with teachers to design, implement and analyse tasks in the mathematics classroom. The data collection presented in this poster is conducted in the two first phases. In the first phase, six students from both lower and upper secondary school are interviewed about their perceptions and experiences of mathematics in school. These interviews are analysed and form the background for the second phase which includes semi-structured interviews with three teachers working at lower secondary school. All interviews are transcribed and analysed using techniques from grounded theory (Corbin & Strauss, 2008).

Expected results from the two ongoing phases in TiM are main categories and characteristics of interventions that teachers claim to be helpful for low achieving students' learning in mathematics. This will give a foundation for designing and implementing tasks together with teachers in the third phase.

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KINDERGARTEN TEACHERS’ KNOWLEDGE OF CYLINDERS

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Learning about three-dimensional geometric shapes is an inherent part of many kindergarten curricula. In order to support this learning, it is important for teachers to be knowledgeable of these shapes. Fischbein (1993) considered the figural concepts an interesting situation where intuitive and formal aspects of mathematics interact. The image of the figure promotes an intuitive response. Yet, geometrical concepts are derived from formal definitions. Thus, concept images are not always in line with the formal concept definitions and knowledge of geometric definitions is essential (Vinner, 1991). Our study investigates preschool teachers' concept images and concept definitions for cylinders. In addition, we investigate teachers’ use of correct and precise mathematical language, and reference to critical and non-critical attributes.

Forty-five practicing kindergarten teachers were asked to fill out a questionnaire requesting them to first define a cylinder and then identify a series of figures as an example or non-example of a cylinder. In choosing the figures, both mathematical and psycho-didactical (i.e., whether or not the example or non-example would intuitively be recognized as such) dimensions were considered. All of the teachers identified the long thin cylinder standing up and the long thin cylinder lying down, as examples of cylinders and all teachers recognized the sphere as a non-example of cylinder. However, 36% of the teachers misidentified a short coin-like cylinder as a non-cylinder, 22% misidentified a “cone” with its top cut off as a cylinder, and 44% misidentified a cylinder-like figure cut on a slant, as a cylinder. These results were in line with teachers’ definitions. Nearly 75% of the teachers wrote that a cylinder has two bases which are circles, but only one teacher mentioned that the circle bases of a cylinder must be congruent and none of the teachers mentioned that the bases must be on parallel planes. Furthermore, several teachers referred to the cylinder as a long object, one calling the cylinder a pipe and another comparing it to a toilet paper roll. Knowledge of concept definitions and critical attributes can affect the examples and non-examples teachers choose to present to children and in turn influence the concept images children develop of these figures.

Acknowledgement

This research was supported by The Israel Science Foundation (grant No. 654/10).

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MAT²SMC AND MSC4ALL – TWO EUROPEAN COOPERATION PROJECTS FOSTERING MATHS-SCIENCE-COLLABORATION

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In the EU-funded projects MaT²SMc and MSc4All, teams from seven European countries strive to develop, improve and distribute teaching materials with the goal of fostering collaboration of mathematics and science teachers to motivate students' learning in these subjects. The poster will present the teams, preliminary research results regarding teachers' needs in these fields, and pilot versions of the teaching materials.

THEORETICAL BACKGROUND

While materials for teaching in schools are aplenty, and a lot of them are published, they are usually very specific to one subject matter. One can find many mathematics materials with a science context, though the context is mostly arbitrarily chosen to demonstrate certain points in mathematics, not to foster learning in science as well. Also, many science materials contain mathematics of some sort; however the mathematics is mainly used to effectively and swiftly make calculations for the science content, and not to learn or strengthen abilities in mathematics. Fairly often this leads to compartmentalization, i.e. the feeling of many students that "now is the Mathematics lesson, it does not have anything to do with any other [e.g. Science] lesson, or the real world" (see e.g. Mandl, Gruber, & Renkl 1993), and the inability to combine several fields of knowledge or to apply knowledge in one field to another field (see e.g. Renkl, Mandl and Gruber 1996).

METHODOLOGY AND RESULTS

We developed and distributed a questionnaire to find out what would be required by teachers to allow effective collaboration between maths and science teachers. The overwhelming answer was: Materials that are useful for both at the same time. We developed test materials (topics: aviation, blood chemistry) that are now in the piloting phase in schools. They will be presented at the poster, along with results from piloting, and input from teachers with respect to usefulness of such materials.

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STUDENTS' EVOLVING REPRESENTATIONS OF MATHEMATICAL STRUCTURES

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This report describes students' movement from personal notations and models to elegant use of mathematical symbols that shows their deep understanding of advanced counting ideas. Furthermore, students demonstrated mathematical proficiency as defined by the Common Core State Standards Initiative by decontextualizing specific problems in combinatorics in order to extract the mathematical structure and then contextualizing the symbolic forms by referring back to specific problems.

MOVING BETWEEN CONCRETE AND SYMBOLIC REPRESENTATIONS

Representations play a key role in students' building of mathematical ideas (Maher, 2005). The Common Core State Standards Initiative (CCSS) specifies that mathematically proficient students should have:

> the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols … without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process, in order to probe into the referents for the symbols involved. (CCSS, 2012)

Working from detailed analyses of video and meta data collected over several years, this report traces the evolving representations of a group of students who participated in a long-term study of the development of mathematical ideas (Maher, 2005), as they worked on problems in combinatorics in middle school and high school. By high school, they were able to use their deep understanding of several problems in combinatorics in order first to construct Pascal's Identity by decontextualizing specific problems and then to contextualize Pascal's Identity by referring back to specific problems in combinatorics.

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MATHEMATICS TEACHER EDUCATORS USE OF TECHNOLOGY

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Teacher educators' role in terms of increasing pre-service teachers' (PSTs) awareness and attitude of technology use is indisputable. Through teacher education program, educators play an influential role for PSTs' future practice. For this reason, being a good model in terms of utilizing technology in a meaningful way is critical but not easy for teacher educators. Carefully designed instruction to illustrate PSTs how to integrate technology is needed. The Association for Mathematics Teacher Educators (AMTE) in their Technology Position Statement emphasized, "Mathematics teacher preparation programs must ensure that all mathematics teachers and teacher candidates have opportunities to acquire the knowledge and experiences needed to incorporate technology in the context of teaching and learning mathematics" (2006). Thus, the purpose of the study is to explore the practice of mathematics teacher educators' technology use in their courses. The study will seek answers for the following research questions: What technologies do mathematics teacher educators use in their courses? How do they use technology in their courses? How does use of technology differ related to the types of courses that these educators teach?

The data sources of this multi-case study are semi-structured interview with four faculty members in the Mathematic Education department in a large Midwest state university. The data was analyzed by using qualitative methods to identify which kind of technology was being used, how educators integrate technology in their courses and how their practice differs in relation to the kinds of course they teach.

The preliminary findings of the study suggest that these teacher educators use technology as a content (teaching how to use specific technology in mathematics) and/or supportive tool (as educational technology that help for delivering the instruction) for teaching. The finding of this study would help future researchers to understand educators' role for PSTs' TPACK development and also to argue that differences on PSTs TPACK development may be related to educators' practice of technology.

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3RD GRADERS CONSTRUCTION OF DIVISION AND DIVISIBILITY KNOWLEDGE ONLY DEPENDING ON A KEY NOTION

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There is an increasing trend to include proof in mathematics curricula worldwide as an important component in mathematics education at all school levels. However, it is more common to see studies supporting the belief that proof should be taught through a Euclidean geometry course (Cabassut et al, 2012). Our insight is more focused to use justification as a means for constructing new knowledge, in particular knowledge in the arithmetic field and more specifically to construct knowledge related to divisibility.

This research aims to analyze how a specific notion can help students justify their knowledge construction of divisibility. According to this, we define a simple concept and we make division and divisibility concepts depend on it (Lay, 2009). We also give a sequence of tasks that allow us to measure the proof production level acquired by the students (Bieda et al, 2011). Through this sequence of tasks we analyse the 3rd graders knowledge construction. In the poster we will show analysis of representative dialogues or tasks taken from the knowledge construction process.

The sample comprises 24 third graders (7-8 year old students) from a public school in Peru. The intervention took place through 14 sessions. Each session was videotaped and transcribed. The analysis of the information collected is mainly qualitative.

Our findings suggest that using a key notion allows students not only to learn about division and divisibility in a natural way, but also allows them to present justifications in terms of this concept (our "key" notion).

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SPECIALISED KNOWLEDGE OF A LINEAR ALGEBRA TEACHER AT THE UNIVERSITY LEVEL

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There are various models proposed to study the specificity of the mathematics teacher knowledge. We explore the specialised knowledge of Linear Algebra teacher under the *Mathematics Teacher's Specialised Knowledge* (MTSK) (Carrillo, Climent, Contreras, & Muñoz-Catalán, 2013) emerging model, which involves a restructuring of the *Mathematics Knowledge for Teaching* (MKT) (Ball, Thames, & Phelps, 2008) in order to overcome some difficulties found in analytic work with it (Flores, Escudero, & Carrillo, 2013). Our work focuses on the deep and supported mathematical knowledge having a teacher who teaches Linear Algebra in a University of Ecuador, on the topic *Matrix and Determinants*.

The methodology used in this research is qualitative and interpretative, based on a case study design. To analyse the data, where we focus on the teacher's knowledge related to the *Knowledge of topics* (KoT) category, we propose five categories that describe the teacher's knowledge about: *Phenomenology, Properties and Fundamentals, Representation Registers, Definitions* and *Procedures*. It is our interest to contrast with the data of completeness from these categories, obtained from the reflection on the research literature. The preliminary results show that the teacher knows in depth the algorithms associated with the topic as well as its foundations. The perceived KoT allows the teacher to warn the students of possible mistakes and difficulties, by choosing examples that demonstrate features of the content and boundaries of some properties. The poster will display the model used with all its subdomains, emphasizing the KoT and the evidence of the categories used. This work is a contribution that goes into detail about the nature of an aspect of that mathematical knowledge from the mathematics teacher, which guides us in the creation of stronger professional development programs in this topic.

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PROMOTING THE UNDERSTANDING OF REPRESENTATIONS BY THIRD GRADE STUDENTS

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We aim to understand how two third grade teachers promote students' learning of mathematical representations and the development of students' reasoning in working with a graph representation in the classroom.

All mathematical reasoning requires the use of representations. The NCTM (2000) indicates that representations help students to interpret, organize and understand the information given in a problem, to figure out how to get the answer, and to monitor and evaluate their work. Referring specifically to graphical representations, Curcio (1987) suggests three levels of comprehension: (i) reading the data; (ii) reading between the data; and (iii) reading beyond the data. With Stylianou (2010), we assume that students' learning is strongly influenced by teachers' practice, in particular by the way they use mathematical representations. Bishop and Goffree (1986) indicate that teachers should provide students with opportunities to learn different types of representations. In addition, teachers may seek to understand their students' reasoning, observing the representations that they use while solving a task (NCTM, 2000).

This poster presents part of a qualitative research study on the practice of primary school teachers concerning their work with mathematical representations. The participants are two third grade teachers, Rui and Catarina, from a school cluster in the surroundings of Lisbon. Data was gathered by video recording during class observations and it was analysed trough content analysis. The results show that, to promote the understanding of the graph, the teachers stressed how to read the data and, to promote the development of students' reasoning, they explored the questions involving reading between the data, in both cases mostly by questioning the students.

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DESIGNING STUDENT ASSESSMENT TASKS IN A DYNAMIC GEOMETRY ENVIRONMENT

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Many institutions and teachers have introduced the use of digital technologies into the teaching and learning of mathematics, but there is hardly any research on task design for the evaluation of the mathematical learning developed through the use of technology. If digital technology is never part of the assessment tasks, students (and perhaps even teachers) will not see how to effectively use it in mathematics as a learning goal. This paper is a report from a pilot study of a project which aims to explore the affordances of digital technologies to design innovative assessment tasks in a DGE, and then to analyze teachers' comments and opinions on the tasks, in order to understand if the exposure to different types of tasks involving pre-made sketches might change their approach to technology-based assessment.

Laborde (2001) describes a case study on teachers designing tasks for a DGE, and uses four different categories to drive teachers' tasks. Laborde states it is easier for teachers to adapt paper-and-pencil tasks for a DGE, but much more difficult to create novel technological tasks different in nature from what one might do with paper-and-pencil. Sinclair (2003) suggests some interesting characteristics that a task designed in a DGE should have depending on the aim of the task. Digital technology asks for rethinking assessment in mathematics, because teachers have the opportunity to design tasks that enable certain mathematical thinking that is not accessible with paper-and-pencil tasks, and students can show different kinds of abilities and knowledge, and they have the possibility to continue learning while they are taking a test.

My research is driven by the wish to identify which kinds of tasks teachers might be more interested in using. The results of the first part of the project consist on some tasks I designed for the iPad to assess student knowledge on circle geometry. I used (Laborde, 2001) as framework to design the tasks, and the hints of (Sinclair, 2003) to implement the sketches in Sketchpad. In the presentation, the tasks will be illustrated in detail and further results on the features of evaluating tasks will be discussed.

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RELATIONSHIP BETWEEN LEARNING ATTITUDE AND MATHEMATICAL ACHIEVEMENT

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The main concern of this study is to analyse the relationship between learning attitude and mathematics performance of first grade students of certain institute of technology in Taiwan. First, we collected students' average mathematics scores from 2009 to 2012 academic years and analysed the quantitative data. The statistical results show that there are obvious differences between male and female students' mathematics scores. In order to explore the factors of these differences, the researcher-made questionnaire, entitled "Students' mathematical learning attitude of institute of technology", was used to examine the students' attitude of learning mathematics. It consisted of 30 items including 5 facets of learning method, learning habit, learning desire, preparing exam and teachers' encouragement.

Samples for the study included 150 male and 150 female, first grade students of certain institute of technology. To analyse the quantitative data, author collected the learning attitude of students and their average mathematics scores on the first semester of 2013 academic year, and used mean, standard deviation, t-test and correlation analysis to do it. Based on the empirical data, researcher preliminarily addressed some results about the investigation. Those include that there are obvious differences ($t=-2.4589$, $p<0.05$) in mathematics scores between male and female students, and scores of male students are inferior to females'. Furthermore, the learning attitude of male students is inferior to females' in mathematics class ($t=-2.0411$, $p<0.05$). At the same time, better learning attitude will obtain higher mathematics scores ($r=0.2315$, $p<0.05$).

Author thought that there appeared to be some implications and suggestions about the investigation. First, traditional concept about mathematical performance is that male students are better than female, so gender difference is not the main element to cause the difference of mathematics performance. Next, there is significant difference about attitudes of learning mathematics between male and female and female are superior to male. Finally, there is significantly statistical difference between attitudes of learning mathematics and mathematical performance, that is, students who have better learning attitude toward mathematics will get higher scores. Mathematics is a very important part of fundamental curriculum, so it is necessary to improve the teaching quality of teacher and enhance the learning achievement of students. There are some suggestions for teacher as fostering students' desire of seeking knowledge, increasing their opportunity of touching mathematics, respecting their opinion and giving them praise appropriately, and so on. Owning no good learning attitude leads to worse mathematics scores generally, so we suggest for students that they should make mathematical calculation every day, preview and review mathematics exercises, ask questions in case of having doubt right now, discuss with classmates actively, and so on.

 2014. In Oesterle, S., Nicol, C., Liljedahl, P., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 6, p. 404. Vancouver, Canada: PME.

INFLUENCE OF ACADEMIC EXPECTATIONS ON THE MATHEMATICS ACHIEVEMENTS OF 8TH GRADERS IN TAIWAN

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Eighth-grade students in East Asian countries, including Taiwan, outperformed their Western counterparts in mathematics achievements, based on international comparison studies such as TIMSS and PISA. Several researchers have sought to determine the reasons involved. A primary reason frequently mentioned is the shared Confucian sociocultural values and practices in these countries. In Confucian heritage cultures, students are expected, by their families and even themselves, to demonstrate exceedingly high educational performance (Tan & Yates, 2011). The factors related to these academic expectations have been considered to affect the mathematics achievements of students. However, empirical evidence on these circumstances in Taiwan has yet to be obtained.

This study examined the relationships between eighth-grade students' mathematics achievements and factors related to academic expectations in Taiwan. Data were collected by readministering the International Assessment of Educational Progress questionnaire to 1840 eighth-grade students at 50 schools in Taiwan. The questionnaire comprised 76 mathematics items in three cognitive domains, which were conceptual understanding (CU), procedural knowledge (PK), and problem solving (PS), and 28 items addressing student learning environments and affective characteristics.

Item response theory was applied to estimate the mathematics achievement scores for CU, PK, and PS. For each domain, the two-parameter logistic (2PL) model fit the data optimally. We adopted the scores of the 2PL models and transformed the scores to a scale at a mean of 500 points and a standard deviation of 100 points for each domain. The findings indicate that parents' expectations played a critical role in the students' mathematics achievements. The students whose parents hoped for them to master mathematics significantly outperformed those whose parents did not, by a score of approximately 1 standard deviation for all three domains. The students whose families cared about their mathematics learning significantly outperformed those whose families did not, by 19 to 26 points for each domain. Attending cram school is also a factor in student mathematics achievement, representing an additional effort in mathematics learning; however, this could indicate the will of the parents or students themselves. The scores for the students who attended cram school were significantly higher than those for the students who did not, by 60 to 70 points for each domain.

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EVIDENCE-BASED CURRICULA FOR INSERVICE TEACHERS' PROGRAM

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This study examines the curricula of the first year an ongoing mathematics professional development program for grade eight inservice teachers. The meaning of curriculum in this paper is that a specific set of instructional materials to support learner improvement (Clements, 2007). Four full-day workshops were delivered for the participants. The design of the project curricula was based on literature of prior research on mathematics knowledge and pedagogical content knowledge of mathematics teachers, school culture of collaboration and education system context in this large urban area. The content of the curricula was to meet the areas that teachers identified through an attitudes and beliefs survey. The theoretical framework of this study is based on Tyler' (1949) view that the curriculum set goal or goals, and then select learning experience to be useful in attaining objectives, and organize of materials for effective instruction. The research questions will examine whether the curricula reach the goal of program design and effects of content and activities delivered. The participants were 29 eighth grade mathematics teachers from eight different schools from a large urban school district in Ontario. The data sources were from a Likert-like scale survey with 20 questions at the beginning and end of the program, an exit survey after each workshop, face-to-face interviews, and a final survey about the effects of the program. A mixed quantitative and qualitative research methodology was used to collect and analyze the data. The result shows that the curricula materials meet the program goals. The participants modified their mathematics knowledge, pedagogical content knowledge and attitudes toward mathematics teaching. The quantitative and qualitative results give evidence that research-based curriculum is an effective way to enhance the professional knowledge of teachers. The integration of technology with subject matter learning, and general pedagogic strategy is beneficial to help professional development of teachers.

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SEMIOTIC REPRESENTATIONS IN THE LIMIT CONCEPTUALIZATION IN HIGH SCHOOL TEACHERS

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In the process of limit conceptualization the use of semiotic representations is important because math operations performed on objects depend on the representation system utilized. In this sense, Lagrange (2000) warns that the representation of a concept is not exempt from technical associated with it. According to the theory of semiotic representations (Duval, 1998), to build the concept of a limit is necessary to link coherently at least two systems of representation by three cognitive functions: identification, internal treatment and conversion. We consider that there is still much to learn about this concept from the point of view of the teachers' conceptions. The research questions are:

1. What semiotic representation systems utilize high school teachers to represent the limits?
2. What are the characteristics of conceptualization possessed by high school teachers on the concept of limit?

Eleven high school mathematics teachers answered a questionnaire with eight questions in different systems of representation and that were taken from other research. Representations used in questionnaire responses were analyzed as proposed by Duval (1998) consider two levels: first analyzed the content of a representation in a given record; and second, the functioning of the record chosen to represent the limit. The results show that teachers utilized in a differentiated way the various representations to solve limits problems, which allowed us to characterize different levels of conceptualization: *Closed Static Conceptualization* basically utilize a single representation; *Closed Dynamic Conceptualization* utilize two representation systems that relate consistently but just in one direction; *Static Transitive Conceptualization* utilize more than two representations, but exhibit difficult to relate them in a consistent manner; *Dynamic Transitive Conceptualization* use multiple representations, however only put them into coherent in pairs relationship; and, *Reversible Conceptualization* use multiple representations and operated coherently.

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AN INTERNATIONAL COMPARISON OF THE RELATIONSHIP BETWEEN MATHEMATICS AND READING ACHIEVEMENT: FOCUSING ON PISA SURVEY

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This study is undertaken to better understand the relationship between mathematics and reading achievement focusing on PISA survey. In PISA, the mathematics and reading literacy are examined continuously every three years from the year 2000. Therefore, the relationship between them can be analysed including its secular changes. The research on the relationship between mathematics and reading ability has currently focused by researchers of mathematics education (e.g, Máire & John, 2009).

In this study, the approach of international comparison will be adopted. The countries which are performing high mathematical literacy tend to also have higher performance in reading literacy. However, the relationship between them within the country is not always cleared. In order to make this point clear, we will focus on not only the indexes of reflecting the level of students' performance of mathematics and reading literacy such as the mean score, but also the indexes of reflecting the relationship between two variables have to be considered to capture the domestic feature of countries, for instance, such indexes are correlation coefficient and regression coefficient and so on.

In this study, there are two points of view. One is to focus on test score. By using Hierarchical Linear Model (HLM), the international comparison of mathematics literacy test score will be conducted through controlling reading literacy test score. Another one is to clarify the answer pattern of each country by focusing on item difficulties based on Items Response Theory (IRT).

As a result, in the countries have gotten higher level mathematics literacy, the students' mathematics and reading performance are more related domestically than in lower performed countries. In addition, the differences of the level of reading literacy test score in higher performed mathematics literacy countries come to a head markedly on the items of open-constructed response. On the other hand, regarding lower performed countries, we could not make mention the same situation. The results suggest that the relationship between mathematics and reading achievement is different between higher and lower performed mathematical literacy countries.

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UNCOVERING CONCEPTUAL MODELS OF INTEGERS

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Isaac, a Grade 8 student, reflected about learning negative integers, "It's harder because you haven't used negative numbers your whole life. You've been using positive numbers your whole life." Students' limited experiences with negative integers may be one reason for the difficulty in understanding negative integers and connecting them to their world. Also, students may find negative integers challenging because of the lack of physical materials to model the negative integers (e.g., Peled & Carraher, 2008). Researchers have shown that students do not intuitively use negative integers from contexts, like the owing and borrowing of money (e.g., Whitacre et al. 2012). This study was conducted with six Grade 8 students, ages 12 to 14, in the Midwest US. We sought to understand how students make sense of the integers, particularly negative integers, and how they connect them to contexts. The students posed stories for ten open number sentences involving addition and subtraction of integers during a task-based interview. The stories were analyzed using grounded theory according to the context utilized and the broader ways of thinking that the students used (Strauss & Corbin, 1998). Results indicated that students often used unconventional contexts in their stories for integers (e.g., good/bad deeds). Conceptual models, or ways of reasoning, about integers emerged from the students' stories and will be discussed at the poster session (i.e., bookkeeping, counterbalance, translation, relativity, and rule). These conceptual models point to mathematical ways that students use the integers. Each of the conceptual models that emerged from this study will be described in depth at the poster session. An implication from this study is that different types of thinking may be promoted by the use of different contexts.

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EFFECTS OF MATH TEACHERS' CIRCLE WORKSHOPS ON MATHEMATICAL KNOWLEDGE FOR TEACHING

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Math Teachers' Circle (MTC) is a professional development program for middle grades teachers in the United States that focuses on developing content knowledge, problem-solving skills, and mathematical habits of mind by engaging the participants in learning mathematics through problem solving, under the direct guidance of professional mathematicians. We examined the impact of MTC participation on mathematical knowledge for teaching, as measured by the Learning Mathematics for Teaching (LMT) instrument. The number concept and operations subscale of the LMT was administered to approximately 260 teachers who participated in intensive four to five day summer MTC workshops. Overall results indicated an average increase of .31 standard units (.65 SD) in teachers' LMT scores, with a p-value of less than .000000001. This increase in LMT scores is comparable to that observed by Bell and her colleagues (Bell et al., 2010) in their study of Developing Mathematical Ideas, an intensive professional development program that focuses specifically on elementary and middle grades mathematics content.

Qualitative data augments this, providing information on who participates in MTCs, why they choose to do so, and how they perceive that it affects them as both a teacher and a learner of mathematics. We are also conducting in-depth, multi-year case studies at three sites that include classroom video observations and interviews.

On the poster, we present our quantitative work in detail, as well as discuss further supporting qualitative results and ongoing case studies of participating teachers.

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2014. In Oesterle, S., Nicol, C., Liljedahl, P., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 6, p. 410. Vancouver, Canada: PME.

CONFRONTING ACTUAL INFINITY VIA PAINTER'S PARADOX

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Gabriel's horn is the surface of revolution formed by rotating the curve $y = \frac{1}{x}$ for $x \geq 1$ about the x-axis. The resulting solid was discovered and studied by Evangelista Torricelli in 1641. He showed that this solid, though infinite in length, has finite volume. Philosophers and mathematicians in the seventeenth century found this result very counter intuitive (Mancosu and Vailati, 1991). Calculating the volume and surface area of this solid are given in Stewart (2011) as exercises. Painter's paradox is that the inner surface of the Gabriel's horn, though has infinite surface area, can be painted with a finite amount of paint by pouring a π amount of paint into the horn, π is the volume of the horn, and then emptying it!

In mathematics education research paradoxes have been used as a lens on student learning. This study explores the specific challenges faced by a group of calculus students in resolving the Painter's paradox with a focus on their conceptions of infinitely small. Participants in this study were 12 undergraduate students studying calculus at a university in Canada using the textbook by Stewart (2011). At the time of the study they were familiar with integral calculus techniques in calculating volumes and surface areas of surfaces of revolution. They were presented the Painter's paradox with detailed mathematical justifications of computing volume and surface area of the Gabriel's horn, and asked to respond in writing what they thought of the paradox. Later they were interviewed and audio recorded. Using reducing abstraction by Hazzan (1999), and platonic and contextual distinction by Chernoff (2011) as the main theoretical frameworks their responses were analysed. Like the seventeenth century philosophers and mathematicians, our participants struggled to reconcile with the infinitely long Gabriel's horn having a finite volume. Decontextualizing the paradox from its apparent real life context turned out to be difficult for most of the participants. Conceptions of *infinitely thin* seem to have emerged in some students' thinking in their attempts to reduce abstraction of a finite volume having an infinite surface area.

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STEM EDUCATION IN FORMAL AND INFORMAL ENVIRONMENTS: THE REAL WAY

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This research concerns how well sixth grade students interacted with a project-based curriculum called REAL (Realistic Explorations in Astronomical Learning) as they engaged in integrated mathematics and science learning in a formal classroom setting and in an informal after school setting. A quantitative approach was used with this semester-long study to expose how focused experiences with REAL affected male and female students' STEM self-efficacy and facilitated students' spatial-scientific understandings within both formal and informal arenas. Significant gains were made on a Spatial Orientation Test for students in both the formal and informal settings and on mathematical self-efficacy domain items for females in the formal setting.

THEORETICAL FRAMEWORK

Previous research on the effectiveness of the REAL integrated math and science curriculum in formal classroom settings has shown students significantly improved their understanding on the mathematics related to *periodic patterns, geometric spatial visualization*, and *cardinal directions*. Significant improvement on understanding these topics is noteworthy since students have had historically difficult times comprehending the spatial mathematics needed to understand physical phenomena such as lunar phases and seasons and have often harbored numerous misconceptions (Zeilik & Bisard, 2000). Although research studies on project effectiveness has been shown in the formal setting, limited research has been presented thus far regarding students' understanding within an informal environment. The following research reports significant gains in understanding spatial topics that have been particularly resistant to change, namely, *spatial projection*.

RESEARCH METHODS AND FINDINGS

The question for this study is: How will students' development of spatial-scientific content and STEM self-efficacy compare in formal and informal settings? In order to gauge students understanding, content and efficacy surveys were administered pre/post REAL implementation within formal and informal settings. Significant gains were made on the Spatial Orientation Test (SOT) by students in both the formal and informal settings. In terms of the STEM self-efficacy survey, only females in the formal setting showed significant gains on math self-efficacy. Although not significant, students in the informal settings showed increases on math efficacy items.

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FUNCTIONAL THINKING AND COGNITIVE ABILITIES IN THE EARLY GRADES

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There is a growing interest for understanding the links between cognitive development and early algebraization (Carraher, Schliemann, Brizuela, & Earnest, 2006). This study is part of a project that addresses this issue from a cognitive psychology perspective. We explored links between *functional reasoning* (i.e., children's understanding of the simultaneous variation of two quantities), academic skills, and cognitive abilities known to influence mathematics learning, such as inhibition, memory and numerical estimation.

Participants were 13 grade 5 students (7 girls, *Mean* age = 10.57 years). Measures included the *Functional Thinking Assessment* overall score (FTA, McEldoon & Rittle-Johnson, 2010), performances in calculation, arithmetical problem solving and reading, and cognitive measures including: Percentage of perseverative errors in a Card Sorting task (CS) similar to the Wisconsin Card Sorting Test (inhibition), forward digits recall (FD, memory), and median estimation error in a number-to-position task (MEE, numerical estimation). Considering the small sample size, data were analysed non-parametrically with Spearman's rank correlation method (ρ).

The FTA scores were negatively correlated with MEE ($\rho = -.61$, $p < .05$), and positively correlated with FD ($\rho = .68$, $p = .01$) and arithmetical problem solving ($\rho = .58$, $p < .05$). Although sizeable, correlations with calculation ($\rho = .48$) and reading ($\rho = .54$) were marginally significant ($p < .10$), presumably due to the small sample. The correlation with CS was both small and non-significant ($\rho = -.22$, *ns*). Functional reasoning might not only be associated with academic skills, but also with cognitive abilities such as memory and numerical estimation, although not with inhibition. Future research will aim to verify these results with a larger sample, and to identify the ways in which these and other cognitive abilities might participate in the acquisition of functional reasoning.

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MATHEMATICS TEACHERS' INTERPRETATIONS OF TEACHING MATERIALS: MULTIPLE SOURCES OF MEANING

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This paper is a report from a pilot study of a project which aims to investigate mathematics teachers' interpretations of teaching materials. Herein, an interpretation is viewed as a meaning-making process influenced by reading goals and sources of meaning. The principal research question to guide the investigation was 'which sources of meaning can be found when mathematics teachers interpret familiar textbooks with different reading goals'. This investigation can contribute to explaining why one mathematics teacher can generate different interpretations of the text. In order to facilitate teachers' interpretations, we asked the case teachers to interpret the beginning, intermediate and last paragraphs as to the topic of similar figures in a mathematics textbook used by them. Moreover, we used a semi-structured individual interview with three guiding questions as reading goals, which were designed on the basis of the three components of content knowledge, student thinking and teacher instruction (Zoest & Bohl, 2002). As to data analysis, we initially identified teachers' responses to each question and each paragraph according to three main sources of meaning: extracted from *the text*, extended from *the reader*, and derived from *the integration* of the reader and the text. Then, interview data were used to elaborate each source of meaning. Lastly, we contrast the teachers' sources of meaning and their characteristics of interpreting the three paragraphs. The results showed that the two teachers' sources of meaning changed with reading goals. Moreover, it was found that changing sources of meaning implied having different interpretations, but taking the same source of meaning did not necessarily imply having the same interpretation. Both teachers tended to adopt the *reader* source to identify students' difficulties (student thinking) in understanding each paragraph. Nonetheless, they took the *text* source, the *integration* source or multiple sources to make meaning of what is instructed (content knowledge) and how to instruct it (teacher instruction). This study sheds light on ways to improve teachers' interpretations of teaching materials. In particular, results from this study provide some basis for formulating hypotheses that teachers' learning to interpret teaching materials by shifting multiple sources of meaning may be conducive to transforming them into learning materials. If teaching materials can create opportunities for teachers to learn, it is worthy of further justification.

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COMMUNAL ASSESSMENT OF PROOF: UNDERGRADUATES' DEVELOPMENT OF PROOF-WRITING CRITERIA

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Undergraduate mathematics students' perceptions of proof are a concern for many mathematics educators. A significant and vital skill of proof is the ability to *negotiate* the validity of presented arguments within the student's community of practice (Inglis & Alcock, 2012). This study asks, how do students' evaluations of other students' proofs affect their perceptions of proof as communal negotiations?

This mixed methods study was conducted with four professors at four universities with four undergraduate mathematics education classes (two secondary content courses, two secondary methods courses) across the United States with a total of 60 undergraduate students. A pre-test was given to the students on their perception of proof along with an activity called the "Sticky Gum Problem" problem that required justification. In class, students evaluated five selected classmate's arguments and composed a list of proof-writing criteria based on their evaluations. A communal rubric for evaluating proof was then constructed by the class. The students took a post-test where they evaluated and rewrote their own solution to the "Sticky Gum Problem" and reflected on how they valued proof.

Qualitative results demonstrated that many of the students had never been given the opportunity to evaluate a proof. Students commented on how their only experience with reflecting on the evaluation of proof came from professors critiquing their proofs. Quantitative results showed classmates' evaluation of the selected students proof was *not* correlated with selected students' evaluation of their proof (r =.331, p = 0.167), demonstrating that the classroom perceptions of proof differed from the student's perceptions. Thus there is a disparity between how students perceive their proof and how the classroom community perceives the same proof.

The results of this study identify a significant gap in mathematics undergraduate education of proof. This study showed that students' perceptions of proof as a communal negotiation was a novel notion for many students and their own perceptions of proof varied significantly from a classrooms understanding of proof. This study offers a method of communal assessment of proof as a means to shed light on the need for students to negotiate validity of arguments not only with themselves, but with their mathematical community.

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CREATING AN ACTIVE LEARNING CLASSROOM THROUGH "REFORM-BASED" TEACHING MODE

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Current research recognizes that traditional high school mathematics teaching in china is mainly led by teachers. For students, they are seldom provided the opportunities to interact with teachers or exchange thinking with other students during the teaching and learning process (Yang, 2004). Under these circumstances, students are not actively involved in learning, but passively sitting through lectures. Active math learning has been shown to have great potential in enhancing engagement in the learning process (Kyriacou, 1992), with decades of success in developed countries (Prince, 2004). Contrarily, in traditional teaching system, memorization, repeated practice and sheer hard work have still been emphasized as the main path for Chinese students to understand the knowledge and concepts introduced by teachers in class (Zhang, 2005). This has resulted in the widespread dissemination of passive learning, which directly influences students' psychological and long-term personal development (Zhang, 2005).

Acknowledging of the importance of mathematics education reform in China, this paper is a report from an ongoing research, which aims to explore the relationship between reform-based teaching modes and active learning. The principal research question is 'how Chinese teachers' participation in the theory of active learning affects their own pedagogy'. Through an action research framework, this research is studding three collaborating Grade 10 mathematics teachers who are using theories of active learning as a basis for working with the challenges of reform-based teaching during the period of the research project (Jan. 4th – May 11th, 2014) in a Wuhan high school. The research employs the characteristics of qualitative research for data collection and analysis, including class observations, interviews and tests. Interview sessions will be conducted with teachers and students. Three interviews are going to be conducted throughout the action research project: one interview with participating teachers before the tryout of reform-based teaching, and the other two interviews after the tryout of reform-based teaching, with the students and participating teachers respectively.

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TEACHING FRACTIONS: ISSUES IN THE USE OF AREA MODELS

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Area-model representations seem to have been dominant in the teaching and learning of fractions, especially in primary-school mathematics curricula. For decades US elementary teachers have, when teaching fractions, adopted an area-model approach (NCTM, 2000). However, children who had been taught fractions almost always by the area model approach were less likely to facilitate deeper, more transferable, understandings of fractions. The over-emphasis on area models could hinder student conceptual development of fractions (Gould, 2008). This research was intended to investigate and improve the knowledge of the unit fractions 1/2,1/3 and 1/4 of students.

There were 40 participating end-of-fifth-grade students who had followed an area-model fractions curriculum. They were randomly allocated to two groups (Group 1 and Group 2) before being taught fractions lessons. The intervention took place in five sessions including 6 hands-on activities in which unit fractions were associated with parts of boundaries (perimeters), capacities, lengths of ribbon, number lines, and discrete objects. Data for the pre-post-retention-test comparison was collected by parallel tests with 28 items. Students needed to provide written or pictorial responses to the items and responded to pre- and post- interviews with 12 questions. Six hypothetical populations were defined in order to examine twelve pairs of null and research hypotheses by *t*-test analyses. Results indicated that at the pre-teaching stage, the students were adept at partitioning regional models, but they did not cope well with questions for which unit fractions were embodied in non-area-model scenarios. Their performance significantly improved after the intervention and most of what the students learned was retained (Group 1 had means of 18.0, 23.2, and 21.7, and Group 2 had 18.5, 22.6, and 21.8, respectively on pre-teaching, post-teaching, and retention tests.) The results of the hypothesis testing revealed that group mean scores were significantly higher after the interventions than before the interventions, and there were no statistically significant differences between the post-teaching and retention mean scores. The main difference in mean performances could be attributed to effects of the instructional interventions. The domination of area-model representations in the teaching and learning of fractions seems not to help students develop sound conceptual understandings of fractions, and more emphasis may be placed on multiple-embodiment approaches.

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