



# **PME 42**

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Editors: Ewa Bergqvist, Magnus Österholm,  
Carina Granberg, and Lovisa Sumpter

**Volume 5**

**Oral Communications, Poster Presentations**

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# **ORAL COMMUNICATIONS**



# EVOLUTION OF UNDERGRADUATE STUDENTS' MATHEMATICAL REASONING

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In undergraduate mathematics education, little explicit attention is given to mathematical reasoning (Bergqvist & Lithner, 2012). It is therefore interesting to investigate undergraduates' mathematical reasoning evolution during their studies. We investigated the mathematical reasoning characteristics of twelve students over three years of bachelor studies in mathematics. Each year, the first author administered individual task-based think-aloud interviews in which the students solved three indefinite integrals such as  $\int \sqrt{x^2 - 9} dx$ . The tasks used are similar to the ones in first year calculus text books, but are known to be difficult to many students. The interviews comprised additional questions on the task solving process. Interview transcripts were analysed to determine reasoning foundations (superficial or mathematical); level of metacognition (deep or shallow); whether the student recalled the solution strategy or instead reasoned in a novel way; whether errors were made and whether the solution was successful. Reasoning is mathematically founded if strategies are based in intrinsic mathematical properties (Lithner, 2008). Deep metacognition comprises activities as sub-goaling, planning ahead, strategy modification, monitoring of the global solution process. Shallow metacognition concerns regulation based upon mere observations (e.g. 'this strategy does not work'), and local monitoring (i.e. of subsequent steps).

We report about the reasoning by two students (Henrike and Lisa) on the task stated above. In year 1, both students' reasoning on this task was superficially founded and unsuccessful. Henrike used shallow metacognition, novel reasoning, and made no errors. Lisa however, used deep metacognition, recalled an (inadequate) method and made an error. In year 3, Henrike's reasoning characteristics on this task were similar to year 1, while Lisa's reasoning had changed: mathematically founded, novel, almost successful, no error. Again she used deep metacognition.

These results illustrate how students' mathematical reasoning at undergraduate level can evolve, and that some students may need support. We observe that in Lisa's reasoning (which evolved), use of deep metacognition preceded mathematically founded reasoning. This finding indicates that to stimulate mathematically founded reasoning, its interaction with use of deep metacognition may play an important role.

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# PRE-CALCULUS COLLEGE STUDENTS' QUANTITATIVE REASONING IN THE CONTEXT OF REAL-WORLD SITUATIONS

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Technological advances and emerging changes in today's world require people to be capable of quantitative reasoning in order to interpret, represent and analyze relationships in real-life situations around them, and make conscious decisions in their personal, academic and professional experiences. Quantitative reasoning includes mental actions like comprehending the context of phenomena or a situation (i.e. a runner in a race), constructing quantities of the conceived situation (i.e. elapsed time and distance taken by the runner), determining relationships among the quantities (i.e. relationships between elapsed time, distance taken by the runner, and average speed of the runner), and analyzing how quantities change together (Moore & Carlson, 2012). Quantitative reasoning enables individuals to develop an understanding of how quantities, quantitative relationships, mathematical formula, and procedures are applied to and solve real-life problems. Therefore, quantitative reasoning has a notable value in everyday life as well as almost all academic areas, especially in mathematics education and STEM fields. The purpose of this study is to investigate pre-calculus college students' quantitative reasoning in the context of real-world situations. In line with this aim, an open-ended problem-solving test -including quantitatively rich and open situations- was administered to 400 pre-calculus college students. After the analysis of data, the researchers purposively selected, for conducting clinical interviews, 20 students who were thought to have different ways of reasoning. The data were analyzed qualitatively by using an open coding approach and axial coding approach. The findings indicate that a majority of the pre-calculus college students encounter difficulty making sense of real-world situations. These students express commonplace ideas suggested by the context of the problem rather than reasoning quantitatively. For this reason, the students are unable to construct quantities and determine relationships among the quantities in the problems. Moreover, they are unable to make use of quantitative structures and their meanings when constructing arguments about problem situations. On the other hand, only a few students ( $< 25$ ) who reason quantitatively-including making sense of quantities, determining relationships among the quantities and justify their ideas in terms of quantitative meanings- demonstrate productive problem-solving behaviors. The study suggests that students' productive problem-solving behaviors are closely related to their quantitative reasoning.

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# WHY DO INSTRUCTORS PREFER NOT TO PRESENT PROOFS OF SOME THEOREMS?

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Although many studies conducted on the roles of proof sometimes emphasize similar and sometimes different roles it can be said that they are all agree with the importance of proof in mathematics education. The questions about why proof should be presented in lessons or about why students are expected to attend proving activities also continue to take place in literature with many answers (e.g., Fukawa-Connelly, 2016). However, it is also known that instructors who are teaching proof-based mathematics courses don't present proofs of all theorems presented in courses. As for the questions about which proofs are not presented or about which pedagogical reasons these preferences rely on, they point to some parts of the researches that needs to be clarified. Therefore, this study aims to investigate why instructors prefer not to present proofs of some theorems, and how they explain this preference to their students.

In this study, which is a part of larger study, seven instructors who were teaching undergraduate level proof-based mathematics courses were interviewed about their proof presentation as well as video observations of each classroom were gathered throughout 3-6 weeks. The courses observed were Analysis, Linear Algebra, Topology, Abstract Algebra and Geometry. Based on the analysis of the interview data, it was determined that the instructors had 19 different reasons for preferring not to present some proofs. These reasons were evaluated under 5 categories: proof itself, time, department, measurement and evaluation, and pragmatic view. In addition, some arguments, that instructors used when they were explaining to their students why they didn't present proofs of some theorems, were those: the proof is very long or difficult or easy or similar to a previous proof that was presented by instructor or it must be done as a homework. It was also observed that they had continued without any explanation in presentation of some theorems.

Consequently, it can be said that the instructors preferred not to present proofs of some theorems with various pedagogical reasons, and they mostly explained why they didn't present them to their students. In the presentation, further results obtained from interview and observations will be discussed in detail.

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# ELEMENTARY PRE-SERVICE MATHEMATICS TEACHERS' CONCEPTIONS OF ALGEBRA

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According to Kaput (2008), school systems mainly focus on narrow view of algebra like manipulation of formalism. However, to develop algebraic thinking in the classroom, teachers play the key roles (Blanton & Kaput, 2005). The present study focuses on investigating elementary mathematics pre-service teachers' (PSMTs) conceptions of algebra.

The participants were eight PSMTs who were required to take the Methods of Teaching Mathematics (MoTM) courses in their third year in a four-year teacher education program. The data were collected through hour-long semi-structured interviews that were adapted from studies that had a focus on algebra (e.g., Stephens, 2008). PSMTs' conceptions of algebra were examined through asking them to describe algebra and having them evaluate tasks and corresponding student responses as algebraic or not with their reasons. The pre-and post-interviews were conducted before and after the MoTM courses. Kaput's framework (2008) provided existing categories, emerging codes were noted and applied across the interviews.

The initial pre-interview findings of the PSMTs' conceptions of algebra were found mostly aligned with the Kaput's (2008) second aspect (manipulation of formalism). Also, the majority of PSMTs' conceptions of algebra were not consistent. The findings of the post interviews and how the PSMTs' conceptions of algebra changed will be shared during the presentation.

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# GRICE'S RELEVANCE MAXIM FOR INVESTIGATING UNDERGRADUATES' ARGUMENTATIONS

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Since 1945, research on argumentation has produced a rich choice of papers from different perspectives. In 1996 van Eemeren et al. linked the theory of argumentation to the pragmatic theory of speech acts, acknowledging that argumentation involves the production of a piece of text too. So it should be analyzed by taking into account not just its propositional content (i.e. the factual and logical components of its meaning), but also its relationship to the context as a speech act. Communicative success is achieved if the speaker chooses her words in such a way that the hearer will, under the circumstances of utterance, recognize her communicative intention (Grice, 1957). This paper aims to examine undergraduates' argumentations to justify their answers to elementary calculus problems involving the recognition of relationships among graphs, verbal texts and formulas. The coherence of the students' argumentations will be analyzed respect to the Grice Cooperative Principle (CP), in particular respect to Relevance Maxim. Some relationships between the registers used by the student (colloquial or literate) and the correctness of the arguments and of the answer will be also considered. The experiment involved 88 freshman Engineering students and was carried out within a basic mathematics course. The protocols have been classified into two groups according to the relevance of the premises from the standpoint of CP. Concerning the first group (characterized by right answers and relevant premises), we observed that for some protocols the argumentations provided refer to implicit knowledge and theoretical references, related to algebraic/analytical and graphical knowledge, although correct, do not show any relationship between them and with the argumentation, but they seemingly are meant just to display theoretical knowledge. Using the Grice's CP lens, we can affirm that the argumentation is consistent and relevant with the answer, but it makes some hidden assumptions. Concerning the protocols of the second group (characterized by wrong answers and relevant premises), we can affirm that response and motivation are consistent, but the arguments adopted seem not relevant related to the task, since the students use wrong theoretical recalls. This research is partially supported by Ministry of Education, University and Research under the National Project "Digital Interactive Storytelling in Mathematics: a competence-based social approach", Prin 2015, Prot. 20155NPRA5.

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# CHALLENGES OF MATHEMATICS TEACHERS WITH PROOF: OSCILLATING BETWEEN DIFFERENT EXPECTATIONS

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Proofs have central roles in mathematics, e.g. roles of validation, explanation and discovery. Zaslavsky et al. (2012) identify five categories of needs for proof among students and teachers: for certainty, for causality, for computation, for communication and for structure. However, the needs for proof among mathematics in-service teachers have not been pursued deeply as has been the case with the students. In our pilot research study we explore through interviews how three in-service mathematics teachers look at the notions of proof and proving and how their constructions of conceptions of proof and proving is being shaped through their participation in a professional development course. Our research question is: *How do in-service mathematics teachers conceive and construct notions of proof and proving as they navigate between different expectations within their professional contexts?*

Here we present some results on one in-service mathematics teacher with ca. 18 years of teaching experience in grades 8 to 10. In 2017/18 it is the first time that she teaches mathematics to a whole class of students. For her, one motivation of working with reasoning and proofs is the aim that students would not need to memorize mathematical results if they can provide reasons for these results. Secondly, she considers proofs as seeing connections, e.g. work with number patterns can engage students with reasoning and could lead to formal proofs and proving. However, she identifies that students do not see the need for establishing a generality of these patterns. These results point out the tensions and challenges a mathematics teacher faces while handling different demands emanating from mathematical discipline, proofs as understood and perceived by the students and proving as prescribed by the curriculum guidelines. I.e. there are different needs on different sides, as is outlined in Zaslavsky et al. (2012). Research also points out normative ideas associated with proof and emotional investments concerning teaching and learning of proof (e.g. Stylianides & Ball, 2008). Further research is needed to investigate how proofs can become genuine parts of the meaning formations among learners and teachers.

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# CROSS-SECTIONAL ANALYSIS OF STUDENTS' ANSWERS TO A REALISTIC WORD PROBLEM FROM GRADE 2 TO 10

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Since the 80s and 90s of the last century, a number of investigations have revealed that students tend to exclude their real-world knowledge when solving mathematical word problems that seem to be solvable by routine steps. A superficial strategy (term borrowed from Verschaffel and De Corte, 1997) consists of searching for numerical data in the text of the word problem, selecting and executing one or more of the basic arithmetic operations, and providing a numerical answer which is usually the result of the arithmetic calculations. The aim of the current research is to reveal a developmental trend in students' realistic answers by means of using a simple routine-like word problem. Besides exploring the rate of realistic reactions in different age-groups, our sampling procedure enabled for investigating whether there are “crystallization points”, i.e. classes where the majority of the students provide realistic solutions. The sample consisted of 1346 students from six age-groups from grades 2 to 10. Two versions of a simple arithmetic word problem (the Pocket Money problem by Ambrus, 2016) was used in the investigation. Students from grade 2 to grade 6 received the simpler version with smaller numbers, and students from grade 6 to grade 10 received the version with large numbers (the sample of the 6<sup>th</sup> grade students was halved retaining the possibility to reveal any potential effect the magnitude of the numbers may have caused.) The results show that there seems to be a growing tendency in the rate of realistic answers, but even in the higher grades it always remains below 4%. There were only three classes where at least 4 students gave realistic answer; two in Grade 10, and one in Grade 8. They formed a minority in their classrooms against those whose answer could be categorized as non-realistic.

This research has received support from the Content Pedagogy Research Program of the Hungarian Academy of Sciences (MTA-ELTE Complex Mathematics Education Research Group).

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# GENDER ISSUES IN THE KANGAROO CONTEST

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A gender gap in mathematics performance and the underrepresentation of women in science, technology, engineering and mathematics (STEM) careers is still a real issue of educational debates and practice (Hyde et al., 2008). Do gender-mixed mathematical competitions contribute to reduce the gap or to wide it? Gender inequity is particularly evident in data related to the number of girls that participated in the International Math Olympiad (Hyde & Mertz, 2009). Does this pattern exist in early school grades competitions? We started investigating gender issues in the context of the Virtual Mathematical Marathon by studying students' participation and performance (Applebaum et al., 2013). We found that girls and boys showed similar patterns regarding the decision to remain in the competition regardless of the results in previous rounds.

In this paper, we analyze gender differences based on data from the 2016 Israeli competition, as part of an International Kangaroo Contest, which aims to attract as many students as possible, with the purpose of showing them that mathematics can be interesting, beneficial and even fun. 345 participants (234 boys and 111 girls) aged 7-12 took part in the final test that consisted of 24 problems for 2<sup>nd</sup>-4<sup>th</sup> graders and 30 problems for 5<sup>th</sup>-6<sup>th</sup> graders. All tasks were multiple-choice, ordered according to increasing difficulty (Easy – Average – High) and were more challenging than those students solve in their classrooms. While investigating two research questions: (1) "Does girls' performance differ from that of boys? If so, how does it differ across the grades and difficulty levels?" and (2) "What are the gender-related patterns across the different types of tasks?", we found that in all grades and in all levels of difficulty, boys seem to exhibit greater success in solving problems, yet significantly only for 2<sup>nd</sup>- and 5<sup>th</sup>-grade. We also found that in 7 tasks out of 30 (5<sup>th</sup> grade), girls were more successful than boys. The results require deeper analysis of the differences in students' performance, as well as in types of tasks that solved better by girls.

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# MATHEMATICS TEACHER'S PROBLEMS IN THE PROCESS OF PEDAGOGICAL REASONING FROM THE PERSPECTIVE OF CURRICULUM MAKER; THE CASE OF AN ELEMENTARY SCHOOL TEACHER IN THE PHILIPPINES

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Teachers' problems, such as teacher-centered instruction and a lack of knowledge for teaching have been pointed out in the Philippines; however, it has not been made clear yet how those problems are related. Therefore, the purposes of this study involved describing how teachers' problems occur in the process of pedagogical reasoning, comprehension, transformation, instruction, evaluation, and reflection (Shulman, 1987), and to determine the underlying problems from the perspective of curriculum maker (Clandinin & Connelly, 1992).

The sample for this study was one single female teacher who was teaching in the sixth grade in elementary school in the Philippines. The methods were lesson observation, a researcher-designed short test for students, questionnaires and interviews with the teacher. The survey determined the teacher's problems leading to curriculum differences among five levels of curriculum, which are: intended curriculum, teacher-intended curriculum, enacted curriculum, and teacher-recognized attained curriculum, as well as attained curriculum.

It is revealed that the learning content in attained curriculum were less than the teaching content in intended curriculum. Among the example of the mechanism causing this situation, one was a lack of curriculum knowledge leading to miss the significant point to foster students' conceptual understanding of proportion in the process of transformation. Secondly, she made an assessment through the test that was focused on instrumental understanding. The most important finding is that she did not recognize the problematic situation judging from her response to the questionnaire and interviews. For example, concerning the teacher-recognized attained curriculum and attained curriculum, she had confidence that all students understand proportion, even if the students had difficulty obtaining the conceptual understanding of proportion.

In conclusion, the range of coherence from intended curriculum to attained curriculum, according to her recognition, created an obstacle in terms of noticing the students' misconception and improving her teaching skills in order to be curriculum maker.

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# STRATEGIES OF SOLUTION OF GEOMETRIC PATTERN PROBLEMS AS TRAITS OF MATHEMATICAL GIFTEDNESS <sup>1</sup>

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*Algebraic thinking* allows students operating with variables and unknowns without needing formal symbolic alphanumeric expressions, and it enhances the mathematical abilities of students, particularly of mathematically gifted students (gifted students hereafter). *Geometric pattern problems* (gp problems hereafter) are a context adequate to develop algebraic thinking in primary school. A description of gp problems can be found in Rivera (2013) and the references therein. García-Reche, Callejo, & Fernández (2015) have described students' resolution strategies.

Gifted students present unusual problem solving abilities, compared to ordinary students of the same age or grade, like identifying patterns, generalising, and inverting mental procedures, all them necessary to progress in algebraic thinking (Miller, 1990). However, there is little research reporting gifted students' behaviour when solving gp problems. In this direction, we wish to identify solution strategies characteristic of gifted students, since they would be helpful *traits of giftedness*. Our goal is, among students in grades 4 to 6, to identify differences between solution strategies i) among the grades, and ii) among good gp problem solvers and less successful ones.

We present results from the answers of 118 students in grades 4 to 6 to several gp problems. We consider as traits of giftedness the solution strategies by students who solved correctly all the gp problems posed. Most good gp problem solvers in grade 4 always used functional strategies, in grade 5 combined recursive and functional strategies, and in grade 6 used functional strategies. The other students combined functional and proportional (incorrect) strategies in grades 4 and 6, and used functional strategies in grade 5, many times incorrectly.

The analysis of data from our experiment let us conclude that i) the absence of proportional incorrect strategies appears as a trait of giftedness in algebraic thinking, and ii) the higher the grade, the more efficient became all students, but iii) the less successful students in all grades had difficulties identifying functional strategies.

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# APPLYING JAPANESE DIDACTIC PRACTICES IN A SWEDISH CLASSROOM

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Mathematics teaching practices are shaped by a number of factors, which often originate from conditions and constraints outside of the school. The didactic theory, that is, explicit and shared knowledge and principles related to the teaching of mathematics, differs in different cultures and may often be taken for granted within the community of teachers. This case study investigates to what extent a specific theory and practice of mathematics teaching - the Japanese *structured problem solving* - can be transferred and applied in a new context (Sweden). The main research question is:

- To what extent can Japanese lesson plans based on *structured problem solving* be applied in the Swedish mathematics classroom?

Structured problem solving involves a specific lesson structure that emphasizes the process of mathematical thinking and students' attitude towards engaging in mathematical activities (Stigler & Hiebert, 1999). My longitudinal study was based on observations of 25 lessons in grade 7 and 8 in a Swedish lower secondary school, and on interviews with the teacher. Also, the national curricula were studied to compare the relevant conditions in the two countries. For the analysis, the concept of the ecology of didactic and mathematical organisations from the anthropological theory of the didactic (Chevallard, 1999, quoted in Bosch & Gascón, 2006, p. 60) was applied.

The results show that the cognitive and epistemological aspects of the approach promote the development of a rich mathematical organisation within the sections of arithmetic and algebra, also in Sweden. However, there were limitations to applying the Japanese lesson plans of geometry, because of significant differences in the ecologies of the national curricula between the countries. The Swedish teacher did not continue using the approach after the project ended. There are following possible reasons for it: In Japan, much more than in Sweden, there is clearly shared theoretical knowledge about the didactic practices teachers use in designing lessons, using the structured problem solving approach. One example is the professional terms which describe the teachers' didactic techniques in the curriculum. A variety of occasions for teachers' professional development, like open lessons, function to disseminate these didactic theories, which justify the teachers' use of their didactic techniques.

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# SIMILARITIES BETWEEN MATHEMATICAL PROBLEMS FROM THE PERSPECTIVE OF PRIMARY STUDENTS

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Many researchers emphasise the high importance of analogizing for problem solving but also its great difficulty especially for young students (e.g. Wareham, Evans, & Rooij, 2011). Because the construction of similarities between mathematical problems or situations is a major element of analogizing, the following research questions seem important: Based on which criteria do young students assess mathematical problems as similar, and by which criteria similarly solvable problems are identified? Among others, these questions are investigated in a large interview study with students from grade 3 to 6.

## METHOD

By now, we have conducted 50 partially standardized guided interviews. Every student was presented one of four problem sets consisting of a source problem (SP), problem C with similar context, problem S with similar structure and problem N with similar numbers and order of numbers. After working on the first problem, the students were asked to peruse the other problems, to assess which problems were similar to SP and to determine the most and least similar problem to SP. After that, the students should decide and justify which problem is similarly solvable to SP.

## IMPORTANT RESULTS

This approach allows gaining detailed information about criteria for similarity judgements of young students. By now, we could identify three different concepts of similarity used by students: 1.: a *structure-oriented concept* in which structural similarities between mathematical problems are recognized very quickly and also used in problem solving; 2.: a *surface-oriented concept* in which structural features play a subordinate role compared to superficial similarities; and 3.: a *variant concept of similarity* in which students switch between different characteristics levels depending on the requirements. Asked for similar tasks, students focus on superficial features; but if they are to choose similarly solvable tasks, they consider structural features.

In our view, the last two concepts require special attention, as they may reveal misunderstandings and misconceptions in teaching situations.

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# TEXTBOOK APPROACHES AND PISA EXAM QUESTIONS

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Textbooks are one of the tools that is used worldwide in the mathematics classroom. It has considerable relevance in the teaching and learning of students. Textbooks not only influence what students learn, but how they learn and can perform on exams. The Programme for International Student Assessment (PISA) gives an opportunity for students to practice what they learned. This study makes an analysis to answer how the different approaches of questions regarding the Pythagorean theorem are noted in textbooks from Brazil (BR) and Taiwan (TW), and how they relate to PISA questions.

This study is situated on PISA 2015 results. The PISA exam emphasizes in its framework mathematical modeling, which in turn involves problem-solving of real-world problems (extra-mathematical). This modeling process is built upon the components of formulate, employ, interpret, and evaluate (Stacey & Turner, 2015). Analyzing textbooks proposed-problem approaches support this research in understanding the opportunity for learning in textbooks. Consequently, resulting in successful ways which relate to how students are required to solve mathematical problems on PISA exams.

Findings show that both textbooks have presented questions and worked examples with different approaches. In BR, a total 98.4% of problems are intra-mathematical compared to 76.7% in TW. Only 1.6% of BR problems are extra-mathematical compared to 23.3% in TW. TW textbook has provided more opportunity for practice in questions that give students opportunity to develop important characteristics of the PISA modelling process (extra-mathematical problems). Therefore, it is found a lack of extra-mathematical questions in BR that provide students with problems on the Pythagorean theorem. Students not only apply the Pythagorean theorem formula, but also interconnect this topic with other subjects. In conclusion, it gives students a broader idea of how to solve problems based on formulate, employ, interpret and evaluate (the ultimate goal of PISA framework).

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# REPRESENTATION OF INDETERMINATE QUANTITIES IN FUNCTIONAL CONTEXTS BY THIRD GRADE STUDENTS

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This research is part of a broader project, based on the proposal of *early algebra*, the main purpose of which is to explore the algebraic abilities displayed by primary school students in tasks involving a functional relationship. Algebraic thinking consists in reflecting on indeterminate quantities analytically and it is possible to think about this without having the symbols to express it (Radford, 2018). Our goals in this communication are two. First, to describe how third-grade students represent indeterminate quantities when expressing functional relationships. Second, to illustrate how students make use of the letter in such representations when this one is proposed as a representation for the independent variable of the function. We qualitatively analysed the answers given by 24 students (8-9 years-old) during three consecutive work sessions of a teaching experiment.

The main results show us that the students represent the variables involved with letters, numbers or both, making use of different meanings that they assign to the letters. They can understand the idea of variability and link it with undetermined quantities. Given a literal representation for the independent variable, when having to represent the dependent variable, sometimes they use the order of the alphabet to replace the letters with numerical values. This result coincides with previous research findings. However, in our study we also got evidences of other use of this alphabetic order, as a criterion to choose the letter to represent the dependent variable and leave the value of both variables undetermined. Other times although they assign a unique value to the letter, they indicate that such value can vary according to the chosen letter. Recognizing thus a certain variability. When interpreting the letter as an indeterminate value, students say that it can be "the number that you want". Consequently, to represent the dependent variable they use the same letter, a different letter, or they write any number. The use of numbers could be interpreted as a static view of the letter, however, the students argue that this number is only an example.

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# THE COMPLEXITIES OF ENACTING MULTIPLE REPRESENTATIONS WHEN TEACHING FRACTIONS

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Fractions are considered to be a complex concept due to their multiple interpretations, with Hodgen, Küchemann, Brown, and Coe (2010), amongst others, identifying five subconstructs: part-whole, quotient, operator, ratio and magnitude. They argue that learning is enhanced when magnitude is emphasised alongside other interpretations, whereas in England the part-whole orientation dominates. Moreover, developing a broad understanding of fractions requires the flexibility to engage with multiple representations, but that flexibility is hard to achieve (Hodgen et al., 2010).

A larger video study in English secondary schools is exploring how teachers adapt their teaching for students with differing attainment. This paper reports the analysis of one lesson on fractions, drawing on the lesson observation framework developed in the wider study (Baldry, 2017). The teacher planned to use multiple representation to develop understanding; the focus of their attention was traced as the lesson unfolded, with the aim to explore the complexities of teaching with multiple representations.

Initially, students were asked how fractions could be represented; language associated with part-whole, quotient, ratio and magnitude were heard. The teacher selected facets of part-whole representations to highlight, including the unit whole. Then diagrams representing four-fifths were displayed, including a bar partitioned into five, a square split into twenty and a number line. When the students were asked “why four-fifths”, “out of” comments were heard first, indicating part-whole orientations, but then they offered decimals, indicating magnitude, and division. During these latter stages, the teacher redirected discussions back to “four out of five”, treating other contributions as unsatisfactory. Despite the teacher’s stated aim to use multiple representations, in enactment the teacher’s discourse shaped the lesson trajectory towards a part-whole orientation; with this orientation dominating English classrooms, it appears that intent may not be sufficient to shift practice towards integrating multiple representations.

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# ANALYSIS OF MATHEMATICALLY GIFTED STUDENTS' ANSWERS TO COGNITIVELY DEMANDING SCHOOL TASKS

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Research shows that challenging tasks make mathematically gifted students (gifted students hereafter) struggle and engage in high order thinking (NCTM, 2014). There is a need to develop tools to help teachers and researchers design and evaluate the adequacy of tasks to that objective. The *model of cognitive demand* is one of such tools, that is producing interesting results. We present a part of a research project<sup>1</sup> aimed to analyse the cognitive demand levels achieved by gifted students when solving *rich tasks* based on ordinary school contents.

A *rich task* is formed by a series of questions where the first ones are within reach of all students, and the successive questions require deeper use of mathematical contents and more complex reasoning. The *model of cognitive demand* evaluates the complexity of students' reasoning while solving tasks. It characterizes four levels of increasing cognitive demand (Benedicto et al., 2017): 1) *memorization*, 2) *procedures without connections*, 3) *procedures with connections*, and 4) *doing mathematics*.

We present results from an experiment based on rich tasks about polygons. We analyse the cognitive demand levels of the answers by 7 gifted students (aged 11-13), 15 ordinary students in primary grade 5 (aged 10-11) and 50 in secondary grade 7 (aged 12-13) to one task. We analyse the 75% of students who answered the task.

The task had three parts. Part 1: draw and count the diagonals from a vertex in 3- to 5-sided polygons. All the students answered it in the 2<sup>nd</sup> level. Part 2: count the diagonals from a vertex in 3- to 7-sided polygons and state a general rule for any polygon. Some students answered it in the 2<sup>nd</sup> level and others combined 2<sup>nd</sup> and 3<sup>rd</sup> levels. Part 3: count all the diagonals in 3- to 7-sided and 20-sided polygons, and state a general rule for any polygon. 13% of 5<sup>th</sup> graders, 56% of 7<sup>th</sup> graders and 86% of gifted students solved the last question showing the 4<sup>th</sup> level of cognitive demand.

We conclude that i) rich tasks are useful to identify gifted students in ordinary classrooms, and ii) the cognitive demand levels allow differentiate trajectories of problem solving between gifted and ordinary students, and between different gifted students.

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# READING STYLES: GOALS, PRIOR KNOWLEDGE AND TEXT

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In Berger (under review), I describe five styles of reading a mathematics textbook. The five reading styles are: close reading in which explicit mathematical connections are made within and outside the actual text; close reading when the connections are not explicit; scanning in which the reader focuses on appropriate keywords and is able to productively use what she reads; skimming in which the reader seeks specific keywords but does not productively use what she reads; avoiding in which the reader focuses on procedures and avoids all proofs and definitions. Reading is regarded as a transaction between the enacted curriculum (what the reader says or writes as she reads) and the written curriculum (the text) (Rosenblatt, 1982). In this presentation, I build on this research and argue, through examples, that a reading style can be regarded as appropriate, or not, if the *goal* of the reading (self-study, revision, doing mathematical exercises based on the text, etc.), the *prior knowledge* of the reader, and relevant aspects of ‘*form of address*’, i.e. genre, voice, medium, structure and look of the text (Remillard, 2012), are taken into account.

I discuss the transcripts of two contrasting students, Paul and Abby, who have been video-recorded (separately) while studying out loud from a chapter on logarithms from a precalculus textbook. They are both participants in a self-studying mathematics course for preservice mathematics teachers. In the textbook, the text is frequently punctuated with Warnings alerting the reader to common misconceptions. These Warnings, speaking as they do to the reader, are an important component of the voice of the text.

Paul scans the text and ignores all Warnings. If Paul’s prior knowledge of the topic were thorough, his reading style may be appropriate. However since Paul’s prior knowledge is weak and he is reading for self-study, his reading style (scanning) is inappropriate to the activity. In contrast, Abby reads closely with connections and she reads all Warnings. Despite good prior knowledge, Abby’s reading style is appropriate to self-study. However it may be unwarranted for revision or just doing exercises; in that case, scanning may be more appropriate for her purpose.

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# HOW TO OVERCOME FRAGMENTATION IN PRE-SERVICE TEACHER EDUCATION IN MATHEMATICS AT UNIVERSITY LEVEL

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Specifically in mathematics, students in pre-service teacher education in Germany often experience their study program to be fragmented and separated from what they expect to be relevant for teaching mathematics in school. Therefore, they often consider university mathematics meaningless. To overcome this fragmentation, a design study on university level is conducted through three iterative design cycles to exemplarily make university students experience the necessity of linking university content knowledge with pedagogical content knowledge (see Loewenberg Ball, Thames, & Phelps 2008).

In the project Spotlight-Y, the lecture of complex analysis is related to a seminar about task design in order to guide the students in preparing and conducting an experimental exploration day on phenomena related to complex analysis for high achieving students at grade 12. An aim of the study is to identify conditions and ways of how pre-service teacher students may link university subject matter knowledge with pedagogical content knowledge (ibid.). This aim is achieved by collecting data of the university students' reflections on the linkage of knowledge in their teaching experience. About 20 students per cycle are involved in the data collection. The Oral Communication will present this study with its theoretical and methodological framework as well as some data and preliminary results from the first cycle. One interesting result already explains the observed fragmentation: a *connection between the two kinds of knowledge* requires a high amount of additional resources invested by the university students, and hence, cannot be built automatically in general. Thus, fragmentation seems to result from the lack of experiencing the joining of different kinds of knowledge in the study program. Further results and insights will assist to understand better the challenges and the difficulties the study program provides for the pre-service teacher students, to extract and test teaching strategies and, on this basis, to improve the study program for mathematics teacher education.

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# PROFESSIONAL ONLINE LEARNING COMMUNITIES IN MATHEMATICS: A CASE STUDY OF THE ISRAELI VHS

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The continual growth in using technology as a platform for professional learning communities has formed unique opportunities in educational systems. In light of research that showcases how online learning provides individualized and differentiated instruction (Archmbault et al., 2010) and develops more attuned pedagogical practices (Pierce & Stacey, 2010), we wanted to see how the VHS professional community—understood here in terms of professionals’ discussion of their work both in the classroom and outside it (Seashore et al., 2003)—engages in and through the VHS environment to foster and sustain a learning community.

This study explored learning opportunities that were generated within and between the VHS teacher-tutor and tutor-tutor communities. VHS electronic records of post-session reflections and peer observations were analyzed and coded deductively to identify and analyze TPACK-related (Chai et al., 2013) learning opportunities and inductively or identify emerging themes.

It was found that the VHS professional community not only attends to and extends TPACK-related insights through processes of crystallization of mathematics-related use of technology, pedagogy, and content knowledge but also to turns to students’ perceptions of self (Sfard & Prusak, 2005) as users and doers of mathematics. This may suggest a need to adjust the TPACK model to include a small-case ‘i’ for learners’ discursive construction of self as users and doers of mathematics. Using a TPACKi model may turn effective in retaining more students in mathematics.

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# PEER TUTORS' AWARENESS OF AFFECTIVE FACTORS

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Tutoring by nature is a complex social interaction (Colvin, 2007; Topping, 1996). Tutoring at the undergraduate level in mathematics is under-studied, but research suggests that tutoring, particularly for first-generation college students, can be a key factor in student success (Colver & Fry, 2016; Mills, Tallman, & Rickard, 2017). In this study, four undergraduate mathematics peer tutors were observed tutoring in a mathematics learning center at a US university and interviewed about their tutoring beliefs and practices – both in repeated stimulated recall interviews following particular tutoring episodes and in a final semi-structured interview. The participating tutors all expressed an awareness of affective factors in their interactions with students and that affective factors played an important part in student outcomes of tutoring.

Tutors indicated that they saw themselves as a more knowledgeable peer and highlighted that affect matters in one-on-one tutoring interactions, both for their own sake and for the sake of students' greater learning and better outcomes. One tutor saw themselves in "... the role of the cheerleader or the positive affirmation." A second tutor echoed that sentiment, adding that relating to a student's struggle is important, "Sometimes it helps if a student is really struggling to be like 'yeah, that was really hard for me as well...' Giving them that comfort, you can do it, you've got to be encouraging all the time." One tutor tied affective factors specifically to her goal in tutoring: "in reality what I want to do is a little bit more ambitious. I want to get these students to like math, to like learning math."

The tutors who participated in this study show an awareness that more than merely cognitive factors influence their interactions. Further analysis will be directed at understanding the source of this awareness and I will share particular examples of how it influences decisions within a tutoring interaction.

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# PROSPECTIVE TEACHERS' EXPLANATIONS FOR INTEGER WORD PROBLEMS

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Based on Leinhardt (2009), mathematical instructional explanations involve verbal explanations, drawings or tools, and equations. When solving negative number addition problems, prospective teachers (PTs) draw pictures, use explanations and write out number sequences, or draw number lines (Almeida & Bruno, 2014). We further investigated: How do PTs' verbal explanations, equations, and drawings align to integer subtraction and missing addend problems and each other?

Fifteen PTs from a US university solved eight word problems (e.g., Kyle has -2 points. Jill has 9 points. How many more points does Kyle need to get to catch up to Jill? Equation:  $9 - -2$  or  $-2 + \underline{\quad} = 9$ ) during think-aloud interviews. PTs were also encouraged to show their thinking on paper. We examined PTs' verbal and written materials, coding each for consistency with the problem and alignment to each other.

J01 initially wrote down an inconsistent equation for  $9 - -2$ :

Nine minus two, which would be sev[en], (wrote  $9 - 2 = 7$  and crossed it out). Actually, I'm going to do the timeline, um, so negative two, negative one, zero, one, two, three, four, five, six, seven, eight, nine (drew a number line from -2 to 9 with hops over each space)...she needs eleven points to catch up to the winner.

She was able to use a drawing to rethink the problem. A03's drawing for  $9 - -2$  not only aligned with her explanation but provided clarification of her strategy:

(Drew number line marked with -2, 0, 9). Well, he needs two points to *get back* to zero (wrote "2" in space between -2 and 0), and then nine points to *get back* to Jill (wrote "9" in space between 0 and 9)...eleven points cause you add them.

A03 did not say -2, and her use of *get back* could be confusing; however, her drawing clarified that she added the distances -2 to 0 and 0 to 9. PTs' number line drawings, though used rarely overall, helped illuminate their explanations. Future work should focus on helping PTs connect visual representations to their explanations.

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# CHILDREN AND ADULTS UNDERSTANDING RANDOMNESS, SAMPLE SPACE AND COMPARISON OF PROBABILITIES

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According to Bryant and Nunes (2012), probabilistic reasoning involves four cognitive demands: 1) understanding randomness, 2) working out the sample space, 3) comparing and quantifying probabilities and 4) understanding correlation. In this sense, the two studies here presented aim to investigate the understanding of the first three of these cognitive demands by young children (Study 1) and by adults that attend basic schooling (Study 2). Both studies used clinical interviews: the first study with 36 children (in 1st, 3rd and 5th grade) and the second study with 24 adult students (in three distinct stages of schooling). Quantitative and qualitative analyses were performed of the instruments used that involved varied probabilistic situations. The children in the first study solved problems in the context of two games and the adults of the second study solved word problems in appropriate contexts. In terms of the effect of schooling, it was observed that the 5th grade children gave similar answers to the 1st and 3rd grade children, although they presented more coherent arguments to justify their answers. Schooling effects amongst the adults were observed only between the group with less schooling and the other two groups. Both studies, thus, indicate that a very superficial teaching of probability has happened and little impact has occurred in students' learning. Difficulties concerning randomness (less than 50% of correct answers in both studies) were related to not understanding when events had the same chance to occur. Concerning sample space better performance was observed (more than 40% and up to 100% of correct answers), especially when students identified impossible events, very probable events and improbable events. Errors occurred when students were not systematic in listing elements of the sample space or when they were unsure if the order of listing consisted, or not, of distinct elements. The main difficulty in the comparison of probabilities was not considering the proportional character of probabilistic situations. This led to very poor performance: with children, in questions of equal and different probability, presenting sometimes 20% or less of correct answers and around 40% of correct answers amongst adults. The two studies show that much has to be improved in the teaching of probability so that all basic education students – children and adults – have the opportunity to develop their probabilistic reasoning and deal with everyday situations of chance and risk.

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# ON STUDENTS' UNDERSTANDING OF IMPLICIT DIFFERENTIATION, BASED ON TWO LENSES: APOS AND OSA

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Although there are many research studies about derivatives, we could find few research about implicit differentiation. For this reason, and also due to the importance of implicit differentiation procedure in Calculus to differentiate implicitly defined functions with one or several variables, more research is needed for this notion. In the Didactics of Mathematics there is a tendency to make networks between different theories; in particular, between APOS and OSA (Onto-semiotic approach) in relation to Calculus notions (Borji, Font, Alamolhodaei & Sánchez, 2018).

The goal of this research is analysing students' understanding of implicit differentiation after they know and learn the derivative of explicit functions. In order to attain this goal, first, we posed tasks of implicit differentiation. Second, we analysed the mathematical activity following the methodology of OSA, which consists of the analysing of mathematical practices and then considering notations, definitions, procedures, propositions and arguments that were activated during these practices. Third, we made a genetic decomposition (GD) based on APOS. Fourth, we used mental constructions of the GD and the analysis of mathematical activities of OSA to characterize the development of implicit differentiation schemas in terms of intra, inter and trans levels. Fifth, we interviewed 25 students from an Iranian university and classified students' schemas in terms of levels of development of the implicit differentiation schema. Results showed that students have difficulties, especially in terms that are composite functions where the inner function contains both  $x$  and  $y$ . Networking APOS and OSA helped us to better describe students' understanding –students that cannot apply the procedure because they do not identify  $y$  as a function of  $x$  and do not have an object conception of the composition of functions (intra level); students that not always consider  $y$  as a function of  $x$ , for example use the product rule for terms which are the product of  $y$  and a function of  $x$  properly (inter level); students at trans level consider  $y$  as a function of  $x$  and can properly use both the product rule and the chain rule, and can generalize their understanding for computing  $y''$  in an implicit function.

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# AN EXAMINATION OF A PRESERVICE MATHEMATICS TEACHER'S REASONING ON CONSTRUCTION TASKS IN A DYNAMIC GEOMETRY ENVIRONMENT

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This paper is a part of an on-going research project which aims to explore instrumental genesis of preservice mathematics teachers working on construction tasks in a dynamic geometry environment (DGE). It has been indicated that constructing geometrical figures in a DGE improve students' reasoning skills and evolve their mathematical explanations (Jones, 2000). In this sense, the design of appropriate construction tasks in classroom settings is of crucial importance. In this work, we address a research question: what reasoning processes are displayed by a preservice mathematics teacher while working on circle-related constructions in GeoGebra?

The data collection included task-based interviews with a preservice mathematics teacher, Gonca (pseudonym, 22 years old female), who was enrolled in mathematics education program at a state university in Turkey. Although Gonca was familiar with the basic use of the GeoGebra, she had no previous experience with GeoGebra while solving construction problems. She had some misconceptions on a number of properties related to circles. The interviews consisted of two different tasks focusing on her misconceptions and were video recorded. The first task was related to constructing a perpendicular line, and the second was about finding the centre point of a circle. During the task based interviews, screen recorder software was also used to record Gonca's activities with the use of GeoGebra. The collected data was analysed according to cognitive model of geometrical reasoning (Duval, 1998). Findings indicated that Gonca eliminated her misconceptions regarding the relationships between the centre of a circle and chord by the use of the perpendicular bisector tool. She also discovered the difference between the centre of circumscribing circle and the centre of inscribed circle. The research suggests that dragging and measurement tools in DGE mediated the participant's perceptual recognition and discursive apprehension. Research is supported by Eskisehir Osmangazi University Research Projects Council.

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# SECONDARY STUDENTS' LOGICAL REASONING ABILITIES

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Logical reasoning is of great importance, in everyday life situations and in various professions, not only to verify reasoning and argumentation, but also for making informed decisions (Halpern, 2014). Therefore, it is important that secondary school students explore all kinds of reasoning. However, little research has been done on logical reasoning abilities of secondary school students. In attempting to address this gap, we investigated students' logical reasoning abilities in formal and non-formal contexts with a specially designed written test.

To do that, we developed a literature-informed framework, which consists of four categories of logical reasoning. We classified the context (formal or non-formal) and the way of reasoning within the given context (formal or informal). Formal contexts are situations with formalisations, such as (logical) symbols and/or mathematical rules, where non-formal contexts are situations in ordinary language. Both contexts can be approached formally, informally, or with a combination of both ways by switching back and forth. Our hypothesis is that one's reasoning will benefit from the use of formalisations, in particular if used in non-formal contexts.

To examine students' abilities, we administered a written test to 56 secondary school students (12<sup>th</sup> graders) who had not received prior training in logical reasoning. Test items concerned syllogisms, logical implications (based on: Stanovich, West, & Toplak, 2016) and newspaper articles. Responses were scored on correctness and were also analysed in relation to the kind of formalisations used.

Results showed that most students prefer to use informal reasoning only: depending on the item, 45 to 96%. Also, students made many mistakes in their reasoning process and found it difficult to make strong and useful formalisations, such as formal reasoning schemes and visualisations. However, all students who used formalisations in a syllogism within a non-formal context (11/56) came to a correct conclusion.

The results support the idea that paying attention to the use of proper formalisations when teaching logical reasoning is useful for the development of reasoning abilities. The results, which will be presented in detail in the presentation, serve as an important input for the development of an intervention, which is the next phase of our research.

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# RESEARCH IN MATHEMATICS EDUCATION SEEN AS AN ACTIVITY: TEACHER PROFESSIONAL DEVELOPMENT IN FOCUS

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Assuming research in Mathematics Education as an activity (Leontiev, 1983) implies labelling it as a formative factor. In the formative research field, professional development is understood as a project focused on “monetizing its reconstructive potential based on ethical, political and sociocultural commitment” (Araujo & Moura, 2008, p. 98).

Adopting the formative research as methodological procedure demands highlighting two essential aspects (Moura, 2017); first: the main concern does not lie on whether results are susceptible to generalization, but on whether other contexts and subjects can be generalized to them; second: conceptual rigor of the research means building knowledge rather than giving opinions about a given context. The main question addressed in the current study is: How do formative aspects of research in Mathematics Education seen as an activity emerge in investigations developed by teachers members of a research group?

The current research was a theoretical study developed to analyze four master’s dissertations written by Basic Education teachers participating in a research group. The analysis of these dissertations indicated that the subjects’ professional development process was fulfilled. Teachers seeking to become researchers in the Mathematics Education field managed to restructure their practice and, consequently, their activity. Thus, the research activity embodies a quite creative character and becomes part of the subjects’ humanization, as well as of their professional development process, fact that turns the research into an activity.

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# **A SECONDARY SCHOOL MATHEMATICS TEACHER'S PROFESSIONAL DEVELOPMENT IN DIFFERENTIATED INSTRUCTION: A CASE STUDY**

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The substantial gap of learning achievement in mathematics has been a critical issue in Taiwan. Meeting students' learning needs and closing the learning achievement gap is the priority of 12-year basic education in mathematics in Taiwan. The reformed mathematics curriculum emphasizes the importance of differentiated instruction (DI) as one of the means to address the issues. However, most of the secondary school teachers in Taiwan were not obtained any training in DI in preservice teaching programs. Thus, this case study explored a secondary mathematics teacher's professional development in DI with an aim to suggest a way of improving in-service teachers' teaching practice.

The researcher adopted the theory of DI (Tomlinson, 2001) and the theory of formative assessment (Black & Wiliam, 2009) to improve the case teacher' DI. The researcher and the case teacher collaborated to design teaching materials for DI. Data collection includes lesson transcripts, meeting recordings, interview transcripts, and lesson recordings from applying an observational protocol for DI. Research findings suggested that collaborating to design teaching materials, implementing them in the classroom, and reflecting the effect on student learning is a practical way for improving DI. The teacher's lesson focus changed from catching up lesson progress to students' learning progress. His view on teacher role changed from an instructional director to a host of classroom discourse. The teacher appreciated the conduct of the protocol which contributes to his lesson planning and reminds him the critical elements of implementing DI.

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# HOW TO HELP AN EXPERIENCED TEACHER TO TAKE THE FIRST STEP SUCCESSFULLY IN CHANGING THEIR MATHEMATICS TEACHING

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Many people know that Taiwanese students are of high achievement in math. However, the results of PISA and TIMSS also show that there is a huge gap between high and low level of students, and students' self-confidence and interest of learning math are unbelievable low in comparison to the other countries. The educational challenge now is how to help more students make sense of math information in learning rather than pursue fantastic teaching methods. This challenge means the teachers must change their teaching. They should pay more attention to establish an friendly environment of promoting more students engaging in math activity. However, it is a difficult work for an experienced teacher to change teaching style. Our previous study showed that even the teachers have strong motivation to learn new teaching method, they will fall back to original teaching and oppose to change if their first attempt of change do not succeed. This study bases on the story of Sherry, an well-educated and experienced elementary school teacher, her effort on changing her math teaching to be "game-playing" approach. We report how a teacher educator helps her to take the first step successfully, including the students' achievement get better and low achieved students can engage in math learning joyfully. Because of the first successful experience, Sherry become self-confident to change and implement more "game-playing" teaching in her math lessons. Sherry's learning and implementing process confirms the ARCS model of motivational design (Keller, 2009), say attention, relevance, confidence, satisfaction the for promoting and sustaining motivation in the Sherry's teaching design and implement process. By reflection in the teacher educator's intervention in helping Sherry, we find some crucial points of effective teacher educating. Including (1) to play the role of cooperative designer rather critical mentor, (2) to apply a theory as the focus of pedagogical thinking, including set the goal of learning and interpreting the students' performance, in this case we only use "mental object" (Freudenthal, 1983) as the foundation theory of "game-playing" teaching approach, (3) to question "how will the students do and think" rather "how will you do and think". The detailed story and teacher educator's intervention will be report in the conference.

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# IMPORTANT FACTORS CLASSIFYING MACAO'S RESILIENT AND NON-RESILIENT STUDENTS IN PISA 2012 MATHEMATICAL STUDY FOR INFORMED POLICY MAKING

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This study seeks to find out the most important factors classifying Macao's resilient and non-resilient students in the Programme of International Student Assessment (PISA) 2012 mathematical literacy study (OECD, 2013). In PISA perspective, resilient students are those who can overcome the obstacles due to their disadvantaged home status and are able to achieve very well internationally (OECD, 2013).

In PISA 2012, sampled students, parents, teachers, schools responded to tests and questionnaires. After a review of pertinent literature, responses to questions asked in the various questionnaires, after appropriate scaling and validation procedures, would be used to explain resilience in student mathematical literacy performance.

There are 4 steps to achieve the aim of this study. First, according to PISA's definition, Macao's academic resilient and non-resilient students were identified (OECD, 2013). Second, Macao's resilient and non-resilient students (552 vs 273 respectively) were combined as a group. Third, a data mining tool named Classification and Regression Trees (CART) was applied to examine the hundreds of variables and scales so as to find out the most important factors proved able to make a distinction between the resilient and its non-resilient counterparts (Breiman, et al., 1984). Fourth, the factors uncovered were discussed for purposes of informed policy making.

The most important factor found was practices of grade repetition policy. For those who have not been grade repeated, resilient students were found to have better mathematics work ethic, whereas for those who have been grade repeated, in order to become resilient it is important to be equipped with favourable mathematics efficacy.

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# IMPLEMENTING STANDARDS-BASED MATHEMATICS: TOWARD IMPROVING CONCEPTUAL UNDERSTANDING FOR MIDDLE SCHOOL IMMIGRANT STUDENTS

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Today, one in five children in the United States is the child of immigrants, and by 2040 it is projected that one in three children will fit this description (Suarez-Orozco, 2009). The literature surrounding immigrant students in U.S. schools shows that immigrant students face barriers that their native born peers do not, and still are expected to achieve at the same rate (Freeman & Crawford, 2008).

This study contributes to a growing body of evidence that carefully structured instructional strategies increase immigrant students' opportunities to learn with conceptual understanding. Standards-based mathematics teaching approaches are inquiry-based and student-centered, where mathematical discussions are central and align with equity goals. This study reports on qualitative analysis of a descriptive case study of seven immigrant students in a constructivist teaching experiment to document students' conceptual understanding of key ideas of linear functions prior to and after participation in Standards-based mathematics curricula as intervention (Cobb & Steffe, 2011).

Findings from the teaching experiment revealed that immigrant students benefitted from Standards-based mathematics teaching approaches that emphasized multiple representations; and that process-oriented questions coupled with opportunities to discuss with peers led to increases in more conceptual responses. Pedagogical and curricular implications will be discussed in the presentation.

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# EXAMINING STUDENTS' INFORMAL MODELING STRATEGIES TO SOLVE APPLIED MATHEMATICS PROBLEMS

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Mathematical modeling is the process of using mathematics to analyze functional relationships in real-life situations. The mathematics education community has called for particular studies to examine the role that informal representations (e.g., idiosyncratic charts and diagrams) play in modeling problem situations (Ärlebäck & Doerr, 2015). This paper reports on a study of pre-service middle-grades mathematics teachers (PMMT) that examined how they used informal modeling methods to solve mathematics problems. The study incorporated a theoretical framework (Lesh and Zawojewski, 2007) that views modeling as a type of problem solving through which students interpret situations to form initial conceptual models that then evolve through iterative cycles of testing and revising the model.

Participants came from a college Mathematics course for PMMTs (N=15) at a four-year university in the USA. Data included results of the students' performance on a mathematics placement exam and written records from individual interviews. The analysis identified: (1) informal modeling strategies students used to solve problems; and (2) roles that the student's informal modeling strategies played in the solutions.

Students demonstrated modeling strategies ranging from simple lists, tables and external diagrams to more sophisticated strategies that involved the use of inferred internal diagrams. The results confirm the initial hypothesis that informal representations play an important role in how students model and solve mathematics problems. In addition, these results offer some support to the idea that when students interpret a problem situation, rather than starting with an external diagram, they might be better served to engage in increased reflection (and develop an internal diagram) prior to initiating solution activity.

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# PROMOTING MATHEMATICS' TEACHING THROUGH HANDS-ON SCIENCE EXPERIMENTS

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Science education should be used to perform interdisciplinarity because it promotes learning of other curricula subjects (e.g. Abell & McDonald, 2006). Relating mathematics with science has been widely advocated by several authors, but it is not an easy goal to achieve (e.g. Baxter, Ruzicka, Beghetto, & Livelybrooks, 2014).

This study aims to contribute to research by presenting a case study a primary teacher, who participated in a collaborative Mathematics and Science Continuing Professional Development program, and developed mathematical tasks related to electrical circuits. With a qualitative methodology, data collection results from classroom observations and an individual portfolio presented by the teacher in the end of the program.

Josefina asked students to bring to the class old batteries. Students organized the batteries according to their sizes and models. Also, they used a multimeter to measure the potential difference in Volts of the batteries and of biological batteries (e.g. fruit). With the obtained measurements they worked organization and processing of data.

Zehetmeier, Andreitz, Erlacher and Rauch (2015) sustains that innovations should be appropriated by teachers and transformed into their own practice, to have real effects. We argue that this is what happened to Josefina who created and implemented mathematical tasks, promoting interdisciplinarity with science experiments. Findings of our research shows that it is possible to innovate teachers' mathematical tasks through a collaborative teachers' professional development context.

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# AN EXPLORATION OF HIGH SCHOOL FRENCH IMMERSION STUDENTS' COMMUNICATION DURING A COLLABORATIVE MATHEMATICS PROBLEM-SOLVING TASK

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Communication is important for both language and mathematics learning. Grounded in sociocultural theory (e.g., Vygotsky, 1978) and the mathematics education register (e.g., Pimm, 1987), this research aims to describe, interpret, and understand how students communicate, that is, how they use and attend to language and mathematics, as they work collaboratively on problem-solving tasks in their second language.

Twenty-two Grade 9 (15 years old) French immersion mathematics students and their classroom teacher participated. Materials included a problem-solving task that required reading, writing, oral interaction, hands-on modelling, and graphing. Data were collected via classroom-based audio recordings, fieldnotes, student work, and post-hoc interviews. Data were coded using frameworks I developed for language- and mathematics-related episodes (Barwell, 2009; Moschkovich, 2007; Swain, 2001); this process was extended through an in-depth discourse analysis (Gee, 2014).

Results suggest that student talk dominated the activity (rather than teacher talk), was mainly about the mathematics at hand, and involved mathematical problem-solving strategies. Various kinds of language-related episodes were coded, especially regarding lexis and use of the first language. The mathematical episodes mainly involved expanding during mathematical explanations. Key discussion points stemming from the findings relate to task influence, the role of collaboration, and the intertwined nature of language and mathematics. With a view toward valuing multiple discursive practices (“translanguaging”) for mathematical problem solving, theoretical and practical implications will be presented based on the salient findings.

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# A TEST TO MEASURE EARLY NUMBER SKILLS PROGRESS AMONG 4 TO 6 YEARS OLD CHILDREN

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Given their importance for later school achievement, the *Mathplay* research project aims at fostering parental involvement in mathematics education and thus early number skills through a play-based approach. To this end, traditional mathematical games have been implemented in class or in class *and* in families whereas a control group has continued its usual teaching practices. In order to measure the progress made by 4 to 6 years old children in early number skills across these conditions, a measurement instrument has been created. Inspired by existing tests, this one has been conceived for the characteristic of the current project.

Early number skills are widely considered as the development of the number words sequence, enumeration ability, logico-mathematical operations, arithmetic strategies and (de)composition of numbers (Jordan, Kaplan, Ramineni & Locuniak, 2009). These skills are largely interrelated and some authors consider them as hierarchically related according to the level of competencies (Krajewski & Schneider, 2008).

Thirty-seven items were constructed around 11 situations. The research is conducted in four countries (Luxembourg, Belgium, France and Switzerland) and the test has been administered to 725 children (mean age = 5;1). On the basis of this data, the current communication deals with the presentation of the test, its psychometric characteristic and some improvement suggestions following the first results.

The items discrimination index show that the overall sensitivity of the test is good. A principal component analysis has been performed and reveals that the various number skills are rather underpinned by a single common factor. The internal consistency of the whole scale is very good ( $\alpha = .922$ ). With the prospect of the provision of the test for research purposes, shallow modifications of some items will be suggested.

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# TO ADD OR TO MULTIPLY? AN INTERVIEW STUDY ON PRIMARY SCHOOL CHILDREN'S PREFERENCE FOR ADDITIVE OR MULTIPLICATIVE RELATIONS

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Learning to reason multiplicatively is a pivotal goal in primary math education. Despite the omnipresence of multiplicative problems such as “A car of the future will be able to travel 8 miles in 2 minutes. How far will it travel in 6 minutes?”, children erroneously solve them additively ( $2+4=6$ , so  $8+4=12$ , Kaput & West, 1994). Other children erroneously solve additive problems multiplicatively, such as “Ellen and Kim are running around a track. They run equally fast but Ellen started later. When Ellen has run 4 laps, Kim has run 8 laps. When Ellen has run 12 laps, how many has Kim run?” ( $4\times 3=12$ , so  $8\times 3=24$ , Van Dooren, De Bock, & Verschaffel, 2010). Recently, children's *preference* for resp. additive or multiplicative relations has been raised as an explanation for these errors, besides lacking procedural or conceptual abilities.

So far, children's preference was measured by collectively administered tests consisting of problems *open* to both additive and multiplicative relations (20 and 30 are equally valuable and correct in Figure 1). The present study aimed to *characterise* this preference by means of semi-structured interviews in which 145 fifth and sixth graders who did and did not demonstrate a preference were stimulated to consider the alternative answer in such open problems.

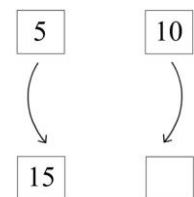


Figure 1:  
Open problem.

Results revealed that children who preferred additive or multiplicative relations gave their preferred answer in a rather intuitive way.

They experienced difficulties in justifying it and initially did not realize that there was an alternative answer. Moreover, this preference was resistant to change evoked by presentation of the alternative answer. Especially children with a multiplicative preference more often explicitly called the alternative answer incorrect and stayed convinced of their preferred answer during the interview. Hence, instruction might aim at making children aware of their preference, at remedying and at preventing the future development of preferences.

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# EXAMINATION THE HORIZONTAL MATHEMATIZATION PROCESSES OF SECONDARY SCHOOL STUDENTS ACCORDING TO THE REALISTIC MATHEMATIC EDUCATION: THE EXAMPLE OF PROBABILITY SUBJECT

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Realistic Mathematics Education (RME) is a domain-specific instruction theory for mathematics education. In RME mathematics was not the body of mathematical knowledge, but the activity of solving problems and more generally, the activity of organizing matter from reality. These periods are called ‘mathematization’ (Freudenthal, 1968). Treffers (1987) formulated the idea of two ways of mathematizing in an educational context by distinguishing ‘horizontal’ and ‘vertical’ mathematization.

The purpose of this study is to examine the horizontal mathematization processes of secondary school 8th grade students with an activity on the topic of probability in the theoretical umbrella of Realistic Mathematics Education. This study is qualitative in nature and makes use of the case study method. For this case study, we purposefully selected 35 of 8<sup>th</sup> grade students in the same class in a secondary school. To document and analyse the process of developing students’ horizontal mathematization processes, “a story that involves subjects in the field of probability in context problems” was arranged by the researchers. Horizontal mathematization processes were examined as three different elements in this study: 1. Describing mathematical terms in a context problem: 2. Expressing numbers in different types: 3. Solving a problem situated in daily life.

Consequently, the students were more successful at subjects based on interpreting situations related to daily life, rather than definitions of clearly mathematical activities. In fact, the main purpose of the RME approach is to support informal learning as opposed to formal learning. In this context, the present study concludes that preparing activities that will help students face everyday problematic situations may be influential for developing positive attitudes towards mathematics by ensuring active participation in the mathematic classes.

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# USING SELF-LEARNING GUIDE TO SUPPORT STUDENT TALK IN A LARGE-SIZE CLASS

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To better facilitate student talk in the large-size mathematics class (Jin & Cortazzi, 1998) and allocate more time for student talk, a learning material called self-learning guide was developed by some teachers and then adopted in most of the classroom in China. The self-learning guide is a carefully designed worksheet including various questions and tasks, allowing students to preview the contents, consider mathematics questions and solve some mathematics tasks before the lesson. To understand more about the functions of self-learning guides in mathematics class, this study aims to answer the following two questions: (1) what opportunities were students given to talk mathematics? (2) how did the self-learning guide function to support students' talk?

The participants are a male mathematics teacher and 46 year eight students in an urban public school in Nantong, Jiangsu province, China. A whole unit of 10 consecutive lessons was video-recorded. The unit topic of the lesson sequence is *Quadratic functions*. By examining the lesson sequence, it is expected to document a wider variety of possible ways of using the self-learning guides in the classroom. In order to examine how the lesson time was allocated in the class teaching, classroom activities were divided into three categories, namely teacher-dominated activities, teacher-student interaction and student-dominated activities. To analyze the how students' talk was supported by the self-learning guide, the time allocated for student talk and the number of Chinese characters in student talk was examined.

It finds the self-learning guide helps to make use of lesson time in a more efficient way. The majority of the number of Chinese characters spoken by the students was observed to be connected with the use of the self-learning guide. Two models of using the self-learning guide to support student talk are identified and discussed. This study makes some contributions to the research community. Firstly, this study presents a Chinese mathematics class in which rich student talk could be observed and thereby reveal some changes that is happening in the context of curriculum reform in mainland China (Zhao, Mok, & Cao, 2016). Secondly, this study provides practitioners with some insights regarding the possible ways of establishing a discourse-rich community in a large-size class.

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# A SECOND GRADER'S SHIFTING DESCRIPTIONS OF TIME ON AN ANALOG CLOCK

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Time underlies aspects of everyday life and informs scientific explorations of *how things work*. Yet despite both mathematical and everyday applications, many students have difficulty understanding mathematical properties of time units (Earnest, 2017; Kamii & Russell, 2012; Piaget, 1969). We present an illustrative case study of an 8-year-old second grader, Mela, solving elapsed time problems on an analog clock and experiencing conflict in her own responses as she interprets intervals of time. The analysis of Mela's problem solving illuminates the varied ideas—both accurate and inaccurate—children apply to time. Perhaps more importantly, this case provides a glimpse of how a student came to resolve conflicting ideas that led to productive shifts in her description of time and use of the tool.

We address the following research question: How do time ideas related to unit and interval emerge in problem solving as a student reasons with the analog clock? Data reported here come from a larger study (Earnest, 2017) that included students in Grade 2 ( $n = 72$ ) and 4 ( $n = 72$ ) from six elementary schools in urban, suburban, and rural areas in the New England region of the United States.

Results illuminate important aspects of time related to intervals for which many students likely need further support. As Mela worked with the clock, she offered ideas related to various conventions and procedures that at times were incompatible and, across explanations, reflected shifts in thinking as Mela reasoned with intervals on the clock.

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# LEARNING COMPLEX SPATIAL VERBS IN A BILINGUAL MATHEMATICS CLASS

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A study in progress is investigating children's learning of spatial language in Mawng, an Indigenous Australian language. The initial objective was for students to gain a firmer foundation in spatial language and thinking through use of their first language in a formal school context. However, since preliminary investigations show that some children may not be fluently acquiring Mawng, due to the pressure of English, the motivation now also includes a language maintenance aspect.

Spatial language varies considerably between languages. Word complexity may affect the ease and time of acquisition of spatial terms in different European languages (Johnston & Slobin, 1979). One question of interest is whether the occurrence of spatial terms as complex verbs delays their acquisition. In Mawng, for example, *kiwraka* 'in front' is a transitive verb that agrees with tense and subject noun class. Example 1 is an instruction to put a rock (land class) in front, in the future tense.

1. Kumanyi kungutpanyi ta waryat **angpanuraka** tuka mutika?

*'Can you put the rock in front of the car?'*

Because *kiwraka* agrees with both the subject and the object, there may be more clarity about what or whom is in front of what than in a language such as English where this is only indicated by word order, but children have many forms of the word to learn.

The project is a teaching intervention conducted by a Mawng speaking assistant teacher, with support from the researcher, with daily lessons over approximately two weeks. There is a pre-intervention and post-intervention assessment in both Mawng and English, in which the children (ages 5-8 years) place objects in response to verbal instructions and describe object arrangements. The learning activities are being developed collaboratively and include activities where children place themselves or toys in locations or spatial relations in response to instructions, and barrier games where children sit side by side with a barrier between them, and one describes an assembly of objects to another to recreate. The study is designed to show which spatial terms are learnt when in each language and to measure the effect of first language instruction in output in both languages. It is hoped the results will further knowledge about children's bilingual learning of spatial terminology.

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# FINGER PATTERNS AS MEANS TO EXPERIENCE NUMBERS' PART-PART-WHOLE RELATIONS

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Contemporary teaching strategies for early arithmetic problem solving suggest that understanding part-part-whole relation is necessary for developing number facts and successful arithmetic skills (Baroody, 2016). A prerequisite for children to experience numbers' part-part-whole relations is furthermore the understanding of the cardinal and ordinal aspects of numbers. In an intervention study conducted in Swedish preschool with 5-year-olds we investigated how their learning developed through participation in an intervention, based on the idea of experiencing numbers and their part-part-whole relations.

An 8-month intervention program was designed based on a conjecture that children can develop number sense and arithmetic skills without using counting strategies. By using finger patterns as means to structure number relations (Neuman, 1987) we anticipated that the ability to discern numbers' part-part-whole relations would be developed.

For the purpose of this study, one of the participating preschool groups (8 of the total 65 children) was chosen for closer investigation of the development of experiencing and handling numbers and additive relations.

Individual task-based interviews were conducted with all children at three times: before, after and delayed. Data were coded according to how the children expressed number meaning and approaches to solving the tasks.

It was found that the children initially had problems differentiating the cardinal and ordinal aspects of numbers and could generally not find strategies to solve simple arithmetic tasks (number range up to 10). In a follow-up study one year after the intervention it was found that the children had developed their understanding of cardinality and the part-part-whole relation which provided them with tools to solve the tasks. It can be concluded that the structural approach offers an alternative to the commonly recommended counting approach (counting single units). Furthermore, the learning outcome give support to our conjecture that finger patterns work as tools to enable number structure to be experienced.

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# LEVELS OF GENERALIZATION AND THE SOLO TAXONOMY

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This paper compared two theoretical perspectives to analyze students' responses in pattern generalization tasks: The structure of learning outcomes (SOLO) taxonomy and Radford's taxonomy regarding levels of generalization. While the former is of general nature and predominantly cognitive, the latter is specific to forms of algebraic thinking and corresponds to a cultural-semiotic perspective.

According to Biggs and Collis (1982), an increased use of applicable elements, operations and relationships in a mathematical task produces a response of higher structural complexity. The five SOLO levels correspond to the usage of: (1) no important feature of the task; (2) one important feature of the task; (3) various important disjoint features of the task; (4) all related and relevant features of the task; and (5) all important features in relation to abstract principles. In Radford's taxonomy, the generalization reaches a higher level as the process of generalization changes from: (1) noticing a commonality; (2) extending a local commonality to subsequent terms of the pattern; (3) going beyond a step-by-step approach and dealing with any particular step of the pattern; (4) dealing with generic objects that cannot be perceived by our senses; and (5) expressing the generalization through symbols (Radford, 2003).

A logical comparison as well as illustrative and typical examples of students' responses revealed a high degree of similarity between the two sets of levels. We are not suggesting that Radford's levels of generalization be ignored even though there is a logical correspondence between the levels. The relationships between the two taxonomies are not as simple as they appear because of differences in the theoretical perspectives of each taxonomy. Nevertheless, in some studies, adopting the SOLO taxonomy as an analytical lens may be a more palatable as a methodological simplifying move. Thus, students' responses to pattern generalization tasks can be characterized in a way that is compatible with Radford's levels without necessarily deriving students' behavior indicative of Radford's levels for every pattern generalization task. In conclusion, the findings in this paper suggest using a multidisciplinary approach to capture various aspects of generalization.

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# HOW DO PROSPECTIVE MATHEMATICS TEACHERS DEFINE THE BASIC CONCEPTS IN NUMBERS AND OPERATIONS CONTENT AREA?

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Defining can be considered as one of the most important mathematical activities such as proving and problem solving. Knowledge about mathematical definitions can also be regarded as the core element of content knowledge. Researches in educational area indicate that teachers' and prospective teachers' knowledge of mathematical definitions effect their didactic decisions (e.g. Zazkis & Leikin, 2008). The definitions effect teaching methods, the sequence of topics to be learned, and which theorems and proofs are covered. The definitions of mathematical concepts are also important to prevent students' misconceptions, to enable students to interpret these concepts, and to develop an understanding about them. However, there is not only a unique definition for each mathematical concept. Various definitions can be used for a concept (Shield, 2004).

It can be said that number and operations content area, one of the content areas mentioned in NCTM, is quite important because it includes both number sets and fractions, decimals, proportions etc. to be regarded as basic concepts. Prospective teachers' definitions about various concepts were examined in this study which is a part of a larger dissertation conducted to investigate prospective mathematics teachers' content knowledge concerning numbers and operations content area. For this purpose an open ended scale including definitions of 24 concepts (prime, number, ratio, absolute value, scientific notation etc.) was applied to 200 prospective elementary mathematics teachers, 50 of whom from each grade. Furthermore, semi-structured interviews were conducted with eight of these students. The definitions of prospective teachers were examined in terms of correctness (appropriate, inappropriate, rigorous, not rigorous), richness, minimality and hierarchy criteria.

It was observed that prospective teachers had definitions that vary even in basic concepts. In particular, the lack of necessary or sufficient conditions in the definitions of the number sets showed that prospective teachers had difficulty in understanding the hierarchy between the sets. Moreover, it was noticed that although necessary or sufficient conditions were provided, some of the definitions were not rigorous enough.

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# TOWARDS A FRAMEWORK FOR UNDERSTANDING THE CHOICE AND USE OF EXAMPLES IN TEACHER EDUCATION MULTILINGUAL MATHEMATICS CLASSROOMS

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Examples that teachers choose and use are fundamental to what mathematics is taught and learned and what opportunities for learning are created. Research has shown that the type of examples that are used by the teacher in teaching mathematical concepts can either constrain or enable learners' access to mathematical knowledge (Goldenberg & Mason, 2008). The fact that most classrooms in South Africa are multilingual presupposes that examples chosen in multilingual classrooms, how these examples are worded, and how language is used, play an important role in how multilingual learners learn mathematics.

I bring together three frameworks which have been used separately by researchers. The emergent framework consists of a three-pronged approach to understanding the exemplifying practices within teacher education (TE). It consists of an amalgam of variation theory (Marton & Booth, 1997), Mortimer and Scott's (2003) notion of meaning making as a dialogic process, and the notion of interacting identities within TE (Essien, 2014). I argue that while variation theory provides perspective into the choice of examples by the teacher educator, Mortimer and Scott's framework provides a tool for how language is used to engage with these examples in practice, and finally the framework on interacting identities provides perspective on how the different interacting identities in TE are (co-)constructed. I show how these three frameworks work together in examining the choice and use of examples in mathematics TE classrooms. Preliminary findings on the use of the new (amalgamated) framework show that it can be a tool for understanding what a "good" (use of) example in multilingual pre-service TE mathematics classrooms should look like.

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# PUZZLE-BASED LEARNING IN UNIVERSITY MATHEMATICS: STUDENTS' PERSPECTIVES

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In this talk, we present an overview of a project aimed at integrating puzzles, paradoxes, sophisms and provocations (later for convenience called just 'puzzles') into university mathematics courses during traditional lectures. By a *puzzle* we mean a non-standard, non-routine problem with a counterintuitive answer and a surprise solution presented in an entertaining way. Normally a puzzle looks deceptively simple and doesn't require specific knowledge (domain free). The intention of using puzzles in teaching and learning is to engage students' emotions, creativity and curiosity and also enhance their critical thinking skills. The theoretical considerations of the project were based on the Puzzle-Based Learning concept (Michalewicz & Michalewicz, 2008) that has become increasingly popular worldwide and Guilford's well-established model of creativity (Guilford, 1959). The impact of the pedagogical strategy was evaluated via questionnaires, interviews and class observations involving 137 students from four groups at two universities. Each lecturer used puzzles in their lectures on a regular basis (2-3 a week) during a semester. The students were able to choose to solve them individually or in small groups in class. It took only 7-10 minutes a week and was not part of the course assessment. The vast majority of the participants reported that integrating puzzles in their courses helped them to improve their problem-solving (91%) and generic thinking skills (92%). Also, 82% of the participants commented on other benefits from using this pedagogical strategy – primarily increasing motivation and enhancing creativity. The project was an extension of a pilot study by Klymchuk (2017) and was supported by a grant from Ako Aotearoa New Zealand National Centre for Tertiary Teaching Excellence.

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# HOW CAN WE PROMOTE PRE-SERVICE TEACHERS' REFLECTION ON STUDENTS' MATHEMATICAL THINKING?

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The authors are tutors on a post-graduate initial teacher education programme. We have noted that our pre-service teachers (PST) often focus on classroom management and student engagement in their reflective feedback on lessons but there is little focus on students' mathematical thinking. Literature tell us that when PSTs collaboratively plan and teach this can lead to greater dialogue and reflection on teaching practices and an increase in discussion which is specifically about teaching and learning (Gardiner and Robinson, 2009). Jacobs et al. (2010) also reported a developing professional noticing of students' mathematical thinking as a result of collaborative teaching.

We undertook this study to explore whether a group debriefing following a collaborative planning and teaching experience could facilitate deeper and more effective reflection on their students' learning of mathematics. 26 PSTs of mathematics, organised into six teaching teams, jointly planned and taught three lessons. Clear guidance was given that they should pay close attention to what the students did, said and wrote down, so that in the lesson debriefing they could discuss the mathematical learning which they had noticed. Two of these teaching teams were observed while teaching and recorded by the authors during the debriefing sessions. The data from the recordings was analysed thematically using grounded theory.

In analysing the recordings we noted that although there was discussion on classroom management and student participation and engagement, there was less productive discussion about student reasoning. It was evident that PSTs rarely built on the reflections of their peers. Indeed there were instances during the debriefing sessions when a discussion about student reasoning in mathematics was derailed and diverted into a discussion of classroom management. However the PSTs did often question their peers about contingent teaching decisions made during the lesson and the impact of those decisions was discussed and often validated. These findings suggest that we need to explore further strategies which target the development of PSTs' confidence and skills in reflecting on the mathematical thinking and learning of their students.

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# CHARACTERIZING TEACHER FOLLOW-UP MOVES

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This presentation will describe patterns in a teacher’s talk moves during whole-class discussions about fractions problems in a 5<sup>th</sup> grade classroom (10-11 year-old students) in the U.S. We build on Pierson’s study (2008), which found that two dimensions of teacher follow-up moves – *responsiveness* to student thinking and the level of *intellectual work* requested of the students – supported conceptual understanding in mathematics. We examined videotapes of whole-class discussions focused on mathematical tasks, such as identifying the quantity represented on a card (Fig. 1) and sharing one’s reasoning and justification for various solutions.



Figure 1: “Dot Card” showing two and two-thirds. “What quantity do you see?”

Using work on teacher talk moves (Michaels & O’Connor, 2015) as a framework, we identified three phases in each discussion episode that reflected the goals of sharing, deepening, or analyzing students’ reasoning. Episodes began with an *eliciting*, or brainstorming, phase. Next, the teacher engaged in a public back-and-forth discussion with a student during a *probing* phase. Finally, the teacher orchestrated an *interpreting* phase where students analyzed one another’s’ reasoning.

Results show that responsiveness and intellectual work varied depending on the phase (Fig. 2). Levels of responsiveness increased as the discussion progressed while higher levels of intellectual work were most evident in the interpreting phase.

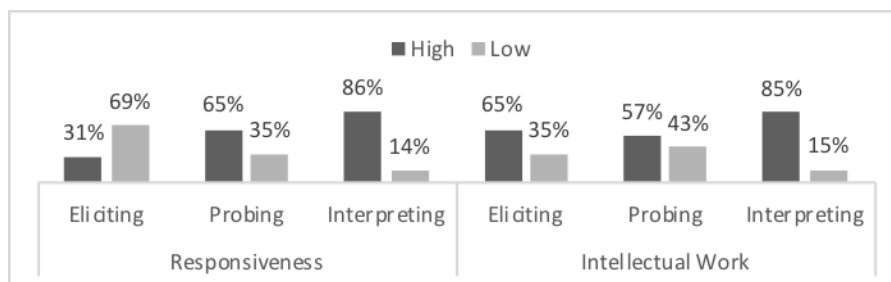


Figure 2: Levels of responsiveness and intellectual work within each phase.

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# ERROR ANALYSIS AND METACOGNITION OF COLLEGE STUDENTS

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Error analysis has been utilized to improve student achievement by investigating common errors, finding explanations for these errors and making them target in remediation (Herholdt & Sapire, 2014). However, it is yet to be explored and studied in relation to metacognition which has a substantial effect on mathematics performance.

This study aims to explore the relationship between the student error description ability and metacognitive skills among college students. The student error description ability is assessed using a questionnaire where students describe their errors in terms of actions not done and absence of application of cognitive skills. An adaptation of the Metacognitive Awareness Inventory by Schraw and Dennison (1994) was used to measure the abilities of students related to metacognition. The following components of the inventory were included in the study: Metacognitive Knowledge and Metacognition Regulation. Metacognitive Knowledge has declarative knowledge, procedural knowledge and conditional knowledge as subcomponents. Planning, comprehension, monitoring, debugging strategies and evaluation were included as subcomponents of Metacognitive Regulation.

A sample of thirty-three (33) students was taken from the population of first-year college students who were taking up a Precalculus course. These students were also grouped according to mathematical ability. The findings indicate that among the high ability students, error description ability is correlated to all the subcomponents of Metacognitive Knowledge ( $r = 0.61$  to  $0.72$ ). Such relationship is not that strong with the subcomponents of Metacognitive Regulation. Also, these results are not seen among average and low ability students.

It can be hypothesized that there is a positive relationship between student error description ability and metacognitive knowledge among students with high mathematical proficiency. In the presentation, further results will be discussed and explained in detail.

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# GAP THINKING DOES NOT FULLY EXPLAIN SOME FRACTION COMPARISON DIFFICULTIES

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Even a simple fraction comparison task (“which fraction is larger,  $\frac{2}{3}$  or  $\frac{5}{7}$ ?”) displays an important number of possible dimensions of item variation. One such dimension concerns the congruency relation between the magnitude of the fractions and the magnitudes of the fractions’ components. In congruent items the larger fraction has the larger numerator and denominator, whereas in incongruent items the larger components are those in the smaller fraction (e.g. see Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013). However, as noticed by Gómez and Dartnell (2015; see also DeWolf & Vosniadou, 2015), congruent items in which the two fractions have different numerator and denominator turn out to be more difficult than their incongruent counterparts. One possible explanation for this difference is gap thinking, since this strategy can only fail in items that are congruent and lack common components: If somebody uses gap thinking systematically, scores for these congruent items will be lower than the corresponding incongruent ones.

To investigate if gap thinking is the cause of this pattern of results, we presented a fraction comparison task (where the usefulness of gap thinking was carefully controlled) to 50 Chilean prospective secondary math teachers. Results confirmed that congruent items with no common components were more difficult than similar incongruent ones (70% vs. 86% correct), although this difference cannot be fully attributed to gap thinking because both performance in items where gap thinking leads to the incorrect and to the correct answer (resp. 65% and 75%) were lower than performance in incongruent items. Therefore, gap thinking alone cannot explain the relatively low performance in congruent items, and novel hypotheses are needed.

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# PROSPECTIVE TEACHERS' STRATEGIES TO SOLVE NON-DECIMAL ADDITION PROBLEMS

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The presented study investigates prospective elementary teachers' (PTs) understanding of the place value system. It focusses on whether or not PTs are working merely on a well-known procedural level or if they are able to reflect on their mathematical knowledge gained at school to work on an enhanced structural level. Our theoretical framework is based on concepts of Activity Theory (Leont'ev, 1979) and investigates ZPD applications within the SRCK model of teacher professional development (Dreher et al., 2016).

The study examines how conceptual understanding of place value can be promoted by working with multi-digit carries in long addition problems in non-decimal number systems. 227 students were assessed in performing and understanding standard multi-digit addition algorithms in the decimal number system. Similar to Thanheiser's (2009) and Murawska's (2013) results, the majority of students lacked a thorough understanding of the algorithm. To refine the data, 21 selected students were asked to take part in a qualitative think aloud video study, in which they described and discussed their thoughts regarding the addition of four multi-digit numbers in the base 2 number system.

Results indicate that PTs transfer their prior knowledge from addition problems in the base 10 number system to addition problems in the base 2 number system. We were able to identify 7 strategies each of them indicating a different level of understanding the place value system in general.

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# THE MATHCITYMAP APP – A GAMIFIED MATH TRAIL EXPERIENCE

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The British Department of Education and Skills considers out-of-classroom learning activities a good way to “nurture creativity, develop skills, improve attitude to learning, stimulate and improve motivation” (DfES, 2006). The idea of math trails (cf. Shoaf et al., 2004) implements learning outside in the context of mathematics classes. The MathCityMap (MCM) app combines the concept of math trails with the possibilities of modern smartphones, e.g. virtual maps, GPS, automatic feedback, a stepped hint system and gamification. Fuchs and colleagues (2014) define gamification as the application of game elements (such as points and leader boards) in a non-game context to manipulate the behaviour of users towards a certain goal. One goal of the MCM gamification system is to increase motivation to work on the math trail tasks.

From April to July 2017 a study with 426 ninth graders has been conducted to examine the impact of gamification on the above-mentioned goal. The students were divided into three groups with modified app versions: G0 – no gamification, G1 – points gamification, G2 – leader board gamification. After a short introduction to the MCM app, the participants received a prepared smartphone and walked in groups of three on a math trail with a focus on solid figures (cylinders and cones) for 75 minutes. Finally, they were asked to fill in a questionnaire on intrinsic motivation. Additionally, the app collected data on user behaviour and movement.

The G2-students reported a statistically significant higher value at the motivational scales than the G0 group ( $p < .001$ ). There was no significant difference between G0 and G1. Furthermore, groups in the G2 setting completed 3,8 tasks per hour compared to 3,1 tasks (G1) and 2,7 tasks (G0). The results of the G2 group can be explained by competitive elements that the leader board adds to the math trail activity. These lead to more engagement and an increased motivation. Whereas the G1 group performed comparable to the G0 group and thus did not meet the goals of the gamification system.

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# CHANGES IN WHOLE CLASS DIALOGIC ARGUMENTATION DURING FIVE MATHEMATICS LESSONS

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Recent studies have examined mathematical argumentation from the classroom discussion perspective (e.g., Conner, Singletary, Smith, Wagner, & Francisco, 2014). However, longitudinal studies investigating students' development in argumentation discussion are needed. The aim of this case study is to find how students' dialogic argumentation changes when a 7<sup>th</sup> grade class participates in a program on dialogic argumentation. The teacher of the class participated in a professional development program and taught one dialogic argumentation lesson per month. In this paper, we focus on the whole class discussion after a small group assignment in the first five lessons. The lessons were video recorded and coded for student-student dialogic moves. The moves indicating a high level of dialogue were elaborating, challenging and questioning other students' ideas. Other, less dialogic moves were commenting other students' ideas or responding to other students' moves. We also identified episodes where students described support for their claim and interpreted whether the described support included articulation of reasoning or competing claims.

We found that the number of students' dialogic moves increased noticeably from first (10 dialogic moves) to the fifth (81 dialogic moves) mathematics lesson. The moves indicating high level of dialogue increased from 5 to 31 moves. The fourth lesson deviated from the increase tendency, as it did not include dialogic moves. We did not find such a distinct change in the number of episodes where students described support for their claim. The second lesson included the highest number of these episodes. Also, the number of articulated reasoning was highest in the second lesson. Only a few episodes included competing claims.

The results suggest that the class had improved in dialogic dimension of argumentation whereas such a development did not exist in the dimension of structural elements of argumentation. The result was expected as our aim was to first increase dialogue in the class and then increase the structural elements. Thus, this is an indication of the program working in case of this class. The results also show that some of the differences (e.g., lack of dialogue in lesson 4) may be related to the lesson design.

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# INSERVICE TEACHERS' PERSPECTIVES OF PARALLELOGRAM DEFINITIONS

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Researchers described criteria for logical necessities of definitions, as the criterion of equivalence. Other criteria are part of general culture but not necessarily follow from logical perspective, as the criterion of minimality. Past research showed that generally mathematic educators prefer mathematical definitions to be minimal (e.g. Van Dormolen & Zaslavsky, 2003). On the other hand, educators, in certain cases, also prefer a non-minimal definition (e.g., Zaslavsky & Shir, 2005).

The present research examines mathematics teachers' conceptions of geometric definitions. This is especially needed due to the little research on teachers' conceptions of geometric definitions, which is a basic aspect of their ability to teach geometry.

Sixty-two in-service mathematics teachers participated in this study. These teachers study for completing their M.Ed. (Master of Education) degree in mathematics education in an academic college for teachers. We used an open-ended questionnaire for data collection. To analyse the data, we used the constant comparison method in order to arrive at categories of the definition properties.

The research results indicate that the participating teachers were not aware of some mathematical definitions. The participating teachers conceived the mathematical definition as associated with the name of the geometric concept. Moreover, the participating teachers conceived the mathematical definition as distinct from the mathematical property. In addition, the participating teachers conceived the mathematical definition in light of their understanding the sufficient conditions issue of the definition.

Based on the above results, we propose that mathematics teachers participate in an educational program that targets mathematical definitions and teachers' conceptions of them. This program would constitute a platform for discussing the definitions' features, in addition to the impact of mathematics teachers' conceptions of definitions on their teaching and their students' learning. This would make these conceptions sounder and help the participating teachers reflect on their own definitions, as well as on their geometric teaching.

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# WHAT IS CRITICAL IN ORDER TO LEARN THE AVERAGE RATE OF CHANGE?

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The concept of rate of change seems to be a prerequisite for a conceptual understanding of calculus (Thompson & Carlson, 2017). However, a fundamental concept in calculus, the derivative, is often understood by students as a procedural differentiation, rather than as a concept of its own (see e.g. Jukic & Dahl, 2012). Teaching and learning about the derivative typically require the rate of change, as it is often described as the endpoint of a move from the average towards the instantaneous rate of change. In the light of Thompson & Carlson (2017), i.e. considering rate of change to be a central concept to open up the field of calculus, we therefore focus on what is critical to learn to calculate the average rate of change, prior to calculus.

We describe an ongoing study where two teachers and one teacher-researcher used learning study and variation theory (Marton, 2015) to explore what lower secondary students (ages 14-15) need to learn to be able to express average rate of change as a quantity of its own. The setup was a series of three lessons, each comprising nine students from a mixed-ability class working in groups with tasks on rate of change. The communication between students in each group was video-captured and analysed in relation to the aspects focused on by the teacher. Data is still being analysed.

Preliminary results indicate not only the importance of using different pairs of graphical intervals (i.e.  $dy/dx$ ) to calculate the average rate of change. Also, the way such pairs are connected to a constant rate of change seems to cause differences in students' learning. Another important result concerns highlighting the starting points of the intervals, especially in contrast to the origin. Finally, data suggest that the variation of which rate is actually calculated (i.e.  $dy/dx$  or  $dx/dy$ ), is necessary to grasp the rate of change conceptually. We believe that these results can be useful in teaching practices, and support an appropriate design of lessons and tasks about rate of change.

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# PRE-SERVICE MATHEMATICS TEACHERS' EVOKED CONCEPT IMAGES OF NUMBER SYSTEMS

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One part of mathematics teacher's professional knowledge is horizon content knowledge (HCK) that includes knowledge of the connections between academic mathematics and school mathematics (Jakobsen, Thames, & Ribeiro, 2013). However, prior research literature indicates that in mathematics teacher education such connections often remain vague (e.g., Moreira & David, 2008). Although there is a trend towards designing courses for pre-service teachers addressing this issue, there is still a lack of research evaluating the development of HCK during such courses.

The current study examined Finnish pre-service teachers' (N=30) concept images (Tall & Vinner, 1981) of number systems during a course that was designed especially to enhance participants' HCK. The concept images that emerged in participants' learning diary entries were examined in relation to the three worlds of mathematics framework. The framework divides mathematical thinking into the embodied world and the symbolic world of mathematics emphasized in school mathematics, and the formal world of mathematics emphasized in academic mathematics (Tall, 2008).

The current study showed that participants' evoked concept images included a strong bias towards the symbolic world of mathematics laying less stress on embodied and formal worlds of mathematics. These results support prior research literature that implies that development of instructional designs are still needed to help prospective teachers to consolidate their HCK. One key element of such development is enhancing the interplay between non-formal and formal aspects of mathematical thinking.

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# IMPROVING GRADE 4 MULTIPLICATIVE REASONING IN SOUTH AFRICA: FINDINGS FROM AN INTERVENTION STUDY

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Studies within the South African landscape have shown a strong prevalence of learners using diagrammatic or iconic models based on unit counting and repeated addition for multiplicative problems (Ensor et al., 2009). Multiplicative reasoning (MR) is central to middle grades mathematics and does not develop naturally but rather, requires deliberate instruction (Siemon et al., 2005). Outcomes from earlier studies in the Wits Maths Connect–Primary project indicated that intervention lesson sequences had potential to contribute to supporting primary learners’ working on additive and multiplicative relation-based problems. This paper reports on the impact of an intervention to improve learners’ MR in one Grade 4 class in South Africa.

The dataset comprised test papers (pre-and post-tests) completed by 61 Grade 4 learners from one school in Johannesburg, South Africa. In eight weekly sessions, work with the intervention group focused on developing and using the ratio-table as a model of multiplicative situations, and a model for problem-solving in these situations. A control group determined whether the intervention had any impact.

Findings revealed that, prior to intervention, learners’ solutions to problems presented limited multiplicative models and were predominantly confined to traditional algorithms. Overall mean scores in the pre-test revealed that both groups of learners intervention and control – initially scored poorly – below 40%. However, after the small-scale intervention, a mean gain of 20.85%-points was achieved by the intervention group with an emerging take up of ratio-table models. In contrast, limited or no changes in performance were seen in the control group (mean gain of 4.76%). This suggests that the ratio table and the broader approach introduced in the intervention lessons have supported intervention learners to explore and develop their MR in ways associated with improved performance.

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# EXAM VS. SELF-ASSESSMENT – STUDENTS’ ACHIEVEMENT GOAL ORIENTATIONS IN TWO DIFFERENT ASSESSMENT MODELS

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In this research, we study how replacing the final exam with self-assessment in an undergraduate mathematics course is connected to students’ achievement goal orientations. Achievement goal orientations (Niemivirta, 2002) represent a person's general tendency towards favouring certain types of goals and outcomes in an achievement context, such as studying. The goal orientations are considered a rather stable feature of a student's motivational mindset, together with e.g. self-efficacy beliefs and personal goals. Niemivirta’s model has been applied in education research, but almost exclusively below higher education level and also not in the context of a single course.

Replacing the exam with self-assessment is part of a new assessment model called DISA (Digital Self-Assessment), designed for large undergraduate mathematics courses. In the DISA model, students award themselves their final course grades. The final grade is validated by automatically comparing it to the work the student has done during the course. The self-assessment is based on learning objectives given by the teacher. Students' reflection skills are supported throughout the course with external feedback and self-assessment exercises (cf. the cyclical self-assessment process; Yan et al., 2017).

In the autumn 2017, we conducted a comparative study on a large DISA course. All the participants (n = 303) were taught with the DISA model during the course, but at the end of the course, half of the students assessed their own grades, and the rest took a final exam. After the course, students' achievement goal orientations were measured with a validated questionnaire (Niemivirta, 2002). Preliminary analysis shows that in the self-assessment group, mastery-intrinsic orientation was more pronounced than in the control group. It seems that although the orientations are usually stable, some variation can arise in the context of radically different assessment regimes.

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# FOSTERING CRITICAL THINKING THROUGH MATHEMATICAL PROBLEM SOLVING BASED ON THE PERSPECTIVE OF CRITICAL MATHEMATICS EDUCATION

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The objective of this paper is to develop mathematics lessons that can enhance critical thinking skills, and to identify the effectiveness of classroom practice. This research employed a socially open-ended problem (Baba, 2009) as a method and the perspective of critical mathematics education (Skovsmose, 1994) as a theoretical framework. One of the developed lessons “Purchasing of a car” was analyzed in this paper. It targeted fifth-sixth, and eighth grade students. In particular, the lesson was recorded with a video camera for a protocol analysis, and the worksheets used during the class were also analyzed. The main problem in the lesson is the following:

A teacher is considering purchasing a car. The price and fuel consumption of the “gasoline car,” “hybrid car,” and “electric car” that the teacher is considering purchasing is as follows: The teacher’s annual driving distance is set as 12000 km, the gasoline price is set at 120yen/L, and the electricity price is set at 12 yen/kwh.

	Gasoline car	Hybrid car	Electric car
Vehicle price	1.8 million yen	2.1 million yen	2.4 million yen
Fuel consumption	15km/L	32km/L	8km/kwh

The teacher says, “I want to buy a gasoline car because it is the cheapest.”

Now, from these three types of cars, which one would you recommend to the teacher?

Students displayed their critical thinking, which consisted of mathematical ideas and social values such as publicness, morality, and ethics. In the above problem, mathematical tools such as tables, formulas, and graphs were used by junior high school students, and mathematical tools such as tables and formulas were used by elementary school students. They were used to express various social values and mathematical models based on the values. This suggests that the lesson practice using socially open-ended problems proved to be successful in terms of fostering critical thinking skills in students.

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# TEACHERS' MATHEMATICAL KNOWLEDGE OF DEFINITIONS IN GEOMETRY

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Mathematical definitions are the cornerstones of mathematical concepts. They provide an accurate description of the concept and determine the necessary requirements for distinguishing between objects belonging to the defined category and those that do not belong to the defined category (Hershkowitz, 1989). In this study, we used a theoretical framework that combines Schulman's theory (Shulman, 1986) with Fischbein's (1993) theory to test teachers' instructional knowledge.

This study focuses, in the context of the definitions, on a geometric subject in which secondary schools are highly involved: special lines in the triangle. This study relates to three special lines in the triangle: altitude, bisector and median. Teaching geometry requires the use of definitions and there is great importance to the teacher's comprehensive knowledge of the essence of the mathematical definition and with the mathematical definitions of the concepts taught in the school. Therefore, it is very important to examine the mathematical knowledge of secondary school teachers about mathematical definitions and to promote knowledge about this subject.

The study found that most teachers produce and identify the definitions of concepts. However, in some of the statements they used inaccurate terms. Regarding the characteristic of minimalism, it was found that a significant proportion of the teachers refer to the length of the definition, the clarity of the definition and the instructional aspect, rather than the requirement that a mathematical definition should not contain parts derived from other parts of the definition. The main findings of the study strongly recommend that training and professional development of teachers include an in-depth reference to the characteristics of mathematical definitions. It is important to address, among other things, the critical characteristics of the concept and the minimalism characteristic.

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# PROMPTS, TECHNOLOGY AND PROBLEM SOLVING

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In this communication we present an analysis of situations that arise when solving problems using GeoGebra. The goal of our research is to identify those events that take place during a problem-solving scenario that relies on technology, and to generate prompts based on these events. A prompt is an event that has occurred in mathematics classroom and raises issues that illuminate the mathematics understanding that would be beneficial for secondary teachers (Heid, Wilson, & Blume, 2015, p. 3).

Data were collected from a Problem-Solving Workshop implemented during nine two-hour sessions of the course *Mathematics for Teaching* for undergraduate mathematics students. In this workshop students solve the tasks in pairs, using GeoGebra. In this Oral Communication we show an in-depth analysis of the work done by a pair of students while solving the *Equal chords* problem (Figure 1).

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*Equal chords:* There are two circles with centers at A and C. Two lines are drawn from each center that are tangent to the other circle. The points where these tangents intersect with the circles define two chords, IJ and KL. Prove that the lengths of these two chords are the same.

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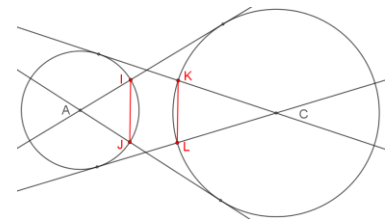


Figure 1: The Equal chords problem

We identify three prompts related to: (i) tangent lines, (ii) arc lengths and (iii) a no typical configuration for similar triangles. They depend on how the episodes in which technology is used to solve problems are framed: *Comprehension Episode*, *Problem Exploration Episode*, *The Searching for Multiple Approaches Episode* (Santos-Trigo & Camacho-Machín, 2013). These prompts serve as resources for promoting the development of Mathematical Understanding for Secondary Teaching, offering opportunities to reflect on the mathematics related to them.

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# USING IMAGERY WITH GEOGEBRA TO ASSIST WITH MATRIX MULTIPLICATION: A COLLABORATIVE PROJECT

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A project “Catalyst” explored the design of computer-based tasks by teacher-researchers and student partners to support Foundation Studies students (FS) in the learning of matrices. It built on theory of imagery and animation to evoke students’ mental images of matrix manipulation and support their understanding of matrix algebra. Tasks were designed in which students manipulated matrix elements represented using GeoGebra to simulate arithmetic of matrices.

Following a lecture on matrices, three tutorial sessions (each 50 minutes) were held in a computer laboratory with screen-capture software, capturing screen, mouse movements and nearby conversation. Each FS student attended one of these tutorials.

The GeoGebra file showed two matrices, with randomly generated positive numbers, which were to be multiplied together. All but one value of the answer matrix were shown. The task was to enter in the remaining number. Animation used colour and movement to guide operations and sliders to vary positions and difficulty of a task. Tasks were initially trialled, through three iterations, with former FS students who were recruited as partners in the project and who participated in task design, initiating tasks and providing feedback.

A real-time analysis of each screen-capture was carried out where the video was played in real time and a factual summary of what happened was noted along with timings. Rich conversations, related to the task or reflecting upon the visual imagery, were transcribed and coded in a grounded analysis. A focus was taken on the developing success, or otherwise, of the students on matrix multiplication tasks and, in particular, the role of the visual support offered from the design of the GeoGebra file.

Comments from the FS students, on their use of the tasks were all positive. Some students suggested improvements to the tasks. However, students’ success with the use of the visual imagery in earlier tasks did not always translate into success with later tasks where matrices were larger and there was no visual support.

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# THE "TEAMS" PROFESSIONAL DEVELOPMENT PROGRAM FOR ENHANCING EXPLORATIVE INSTRUCTION

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The TEAMS (Teaching Exploratively for All Mathematics Students) professional development (PD) aims to enhance explorative instruction in elementary and middle school Israeli classrooms: to minimize reliance on external authority, strengthen students' reliance on mathematical reasoning and acquaint them with norms for talking about mathematical concepts. This is done by selecting cognitively demanding tasks, asking students assessing and advancing questions and using Accountable Talk® during discussions. The PD included eight 4-hour sessions in which teachers worked in small groups on cognitively-demanding problems followed by a discussion about practices promoting explorative instruction.

Research goals were to examine: a. the level of explorative instruction in videoed lessons, b. differences between regular lessons, recorded at the beginning of the year, and TEAMS lessons implemented based on practices learned in the PD, c. differences between elementary and middle school lessons. Participants included 27 middle school and 22 elementary school teachers who were provided a video camera and asked to video 5-6 lessons during the year. A total of 210 lessons were analysed using Stein and her colleagues' (Stein, Correnti, Moore, Russell, & Kelly, 2017) Quadrants coding scheme: Written Cognitive Demand (CD) of the task, Enacted CD, Explicit Attention to Concepts (EAC), and Students Opportunities to Struggle (SOS).

Results show that teachers generally maintained the CD of tasks when provided with tasks selected by PD leaders, while lowering SOS when they selected their own tasks. Regular lessons scored relatively high in EAC and low in SOS, while TEAMS lessons exhibited low EAC and high SOS. EAC was higher in middle school while SOS was higher in elementary school. We interpret the maintenance of CD as indicating success of the PD. Higher EAC found in middle school lessons may be related to the differences between teachers' mathematical background in middle and elementary school levels. Higher SOS in elementary schools is coherent with the stress in Israeli elementary school curriculum on "exploration" which is less evident in middle-school curriculum. Lowering of CD in self-selected tasks points to the importance of scaffolding task-selection during initial stages of the PD.

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# HOW DO STUDENTS VISUALIZE FRACTIONS?

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Problems may be solved in a variety of ways. To get insights into students' problem-solving behavior, data gathered during the solution process can be examined. When presenting problems on touchscreen devices, the finger movements on the touchscreen during the solution process may be recorded and analyzed afterwards to see how the solution emerges. In addition, this so-called finger tracking may reveal what students think about when solving problems (cf. Freeman, Dale, & Farmer, 2011). In particular, when asking students to create visualizations, finger movements may help in understanding students' perceptions of mathematical concepts.

Visualizations play an important role in the acquisition of rational number concepts (Lesh, Post, & Behr, 1987). In our study, we asked 144 six graders to create visualizations of fractions during iPad-assisted classroom instruction in their first lesson on the topic. In each task, the whole (a bar or a circle) had no partition (thus addressing a *measurement* concept of fractions). Finger trajectories during the solution processes were recorded and saved on the iPad. Two researchers categorized the 2901 solution processes in three categories independently after the intervention,  $\kappa = .61$ .

The analysis revealed that students' finger movements show repeating patterns. A repeated measures ANOVA revealed a significant interaction between the shape of the whole and the appearing patterns,  $F(1, 143) = 14.43$ ,  $p < .001$ . Students tended to *correct their solution* more when the whole is represented by a bar and tended to give more *immediate responses* when the whole is represented by a circle. Furthermore, a significantly higher proportion of students used a *dividing process into equal parts* on the bar than on the circle,  $p = 0.035$ . This pattern may speak for a fallback onto counting strategies, which are often used in pre-partitioned visualization tasks.

The results show that finger tracking may be used to get insights into how students solve fraction visualization tasks and how they conceptualize fractions. Furthermore, they indicate that their approach to these tasks differs depending on the shape representing the whole, when a measurement concept of fractions is addressed.

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# BACKWARD TRANSFER EFFECTS WHEN LEARNING ABOUT QUADRATIC FUNCTIONS

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While there exists a long history of *transfer of learning* mathematics education research, *backward transfer* research has rarely been conducted. Our research addresses this gap. We define *backward transfer* as “the influence that constructing and subsequently generalizing new knowledge has on one’s ways of reasoning about related mathematical concepts that one has encountered previously” (Hohensee, 2014, p. 136). Our definition of backward transfer is based on and extends Lobato’s (2006) *actor-oriented transfer* perspective on the transfer of learning.

We are examining backward transfer in four algebra classrooms studying quadratic functions. Our *research questions* are the following: (a) What changes are observed in students’ previously-established ways of reasoning about linear functions after students complete a unit on quadratic functions; and (b) What classroom processes are connected to changes in ways of reasoning about linear functions?

The participants for our study are 105 high school students four Algebra classrooms at two schools and the four teachers. One school uses a traditional algebra curriculum and the other uses a reform algebra curriculum. Instruction in all four classrooms are being video recorded during their entire quadratics units. The day before the start of the unit, students completed a linear function pre-test. The day after the unit, students will complete a linear function post-test (isomorphic to the pre-test). A subset of students 4 per class) are interviewed about their responses to the pre-test and post-test. The pre/post-test results and interviews address the first research question. The videos of the quadratics instruction address the second research question.

Preliminary analysis of pre-tests, interviews and quadratic lesson observations suggests some of our participants’ understanding of y-intercepts from linear functions got muddled when learning about roots and y-intercepts from quadratic functions. Also, some participants appeared to revert back to an *action view* of functions (from a *process view*). These preliminary findings suggest that when algebra students learn about quadratic functions, their prior knowledge of linear functions can be significantly upended or destabilized. Future research is needed to inform teacher and curriculum developers on how to mediate these backward transfer effects.

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# INTERACTIONS OF KNOWLEDGE AND BELIEFS IN MATHEMATICS PROFESSIONAL DEVELOPMENT

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This research is grounded in literature related to beliefs and knowledge of teaching mathematics. As has been indicated in the research, there is a specialised knowledge of mathematics that is important for teaching mathematics effectively (e.g., Silverman & Thompson, 2008). There is also a growing body of work around how beliefs influence teaching practices (e.g., Wilkins, 2008). This report adds to these research paradigms by examining the interplay between a teacher's beliefs and knowledge of mathematics in how professional learning is taken up during professional development experiences. For this paper, I look at a portion of the entire three-year phenomenological case study on using professional learning groups to support the development of grade 6 to 10 mathematics teachers. The data includes interviews, field notes, classroom observations, and meeting transcripts. This research reports on two of the teachers within the group who were members for the last two years of the study as examples of the larger themes within the entire data set. The research question for this portion of the study was: In what ways do the knowledge and beliefs of mathematics affect the participation and learning of teachers in professional development?

“Wesley” was an experienced secondary teacher who firmly believed his mathematics knowledge was stronger than all the other members of the group. “Blaine,” an elementary teacher, had less experience and felt that his knowledge was weak. Wesley believed that mathematics was about learning rules and procedures, and Blaine believed it was about exploring problems and discussion. Stemming from his weak knowledge of mathematics, Blaine barely spoke or shared; whereas, Wesley attempted to monopolize the meetings. Blaine's beliefs encouraged him to try new things as well as gain valuable mathematics knowledge, while Wesley felt the professional learning groups were too rigid and did not try anything until a catalyst shook his beliefs. This research highlights the difficulties in professional development and notes that to encourage real growth, both the knowledge and the beliefs of the teachers needed to be identified and taken into account, as can be seen in the journeys of the two teachers.

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# DEVELOPMENT OF A SCALE FOR IDEAL MATHEMATICS TEACHING BEHAVIOR AND A COMPARISON BETWEEN TAIWAN AND MAINLAND CHINA

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Investigating student opinions on ideal mathematics teaching behaviours can inform researchers and teachers about student needs and then refine the teaching to reflect student-centred view (Kane & Maw, 2005). A comparison among various countries/regions can provide opportunities for participants to reflect on their mathematics education from outsiders' perspectives, which otherwise may not be able to be accessed by insiders. Thus, this study aims to (1) develop the scale of ideal mathematics teaching behaviours and (2) compare Taiwanese and Chinese secondary school students' perspectives on ideal mathematics teaching behaviours.

To develop the scale, Taiwan research team first developed a questionnaire composed of 151 dichotomous items through literature review, consulting researchers and expert teachers, and a survey on 238 secondary school students using open-ended questions. The questionnaire was used to survey 4514 students and then shortened through factor analysis. The shortened questionnaire was finally administered to 837 students to confirm the factor structure. The questionnaire was validated by the Chinese team through group discussions in which researchers and expert teachers participated, a survey on 150 secondary school students, and interviews with 12 students. Finally, all the items were retained, and the questionnaire was administered to 2477 students. To compare Taiwanese and Chinese students' perspectives, t-test and Cohen's rule for effect size were used to analyse students' scores of their agreements on the subscales.

The findings include that (1) the scale of ideal mathematics teaching behaviours consists of six subscales: demonstrating teaching skills, conducting meaningful explanation, arranging student activities, providing motivations and math values, employing multiple assessments, and using concrete representations; (2) the biggest difference between Taiwanese and Chinese students' perspectives was on arranging student activities, and the difference of the scores was significant ( $p < .01$ ) and reached a medium effect size ( $d = 0.31$ ). This may show that Chinese students value student activities to a higher degree than Taiwanese students do. It is also possible that, comparing with Taiwanese students, Chinese students desire student activities more since they have fewer opportunities for student activities.

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# TEACHERS' USE OF EXAMPLES AND ACCOMPANYING MATHEMATICAL DISCOURSE: THE CASE OF DERIVATIVE

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Students' conceptions of the derivative concept have been extensively studied. There is also some research on teaching of derivatives, but it is not yet as extensive as the research on student learning about derivatives. The increasing attention to examples stems from the central role that examples play in learning and teaching, in general, and in mathematics and mathematical thinking, in particular. Examples constitute a fundamental part of a good explanation - a building block for good teaching, they are inherently connected to explanations and mathematical discourse (Leinhardt, 2001). This study aims to examine the examples and mathematical discourse teachers at the university level offer as they elaborate derivative concept in their classrooms.

The empirical data in this study consists mainly of videotaped lectures and examples given by two teachers in beginning-level calculus classrooms at a university of technology in Taiwan. It took both teachers ten lessons to teach the concept of derivatives. The transcribed lectures were then analyzed, using Sfard's commognitive Framework (2008) with its four components of mathematical discourse (words, visual mediators, narratives and routines) to try to distinguish the discursive patterns characterizing the teachers' respective discourses of examples. In this paper, the categorizations of the examples and of the construction and substantiation routines used by the teachers is presented.

The findings indicate that the types of examples offered by the two teachers include start-up examples, exploration examples, illustrating examples, counterexamples, and extending examples. There are significant differences in the way the two teachers use examples and do mathematics in their lectures. These differences present themselves on the level of discursive routines. Moreover, attention to examples accompanying mathematics discourses is not only useful for analysis across a range of lessons for teachers; it also connects discursively with discourse in ways that speak more directly to practice. In the presentation, further results will be discussed in detail.

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# CHILDREN'S PERFORMANCE OF RECOGNIZING ANGLES OF INCLINATION, TURNING, AND ROTATION

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Angle concepts involving static and dynamic aspects are frequently introduced to third- and fourth-grade children (Ministry of Education [in Taiwan], 2010). However, how well third- to fourth-grade children apply their angle-related knowledge received from the angle instruction formally given in school mathematics for justifying angles embedded in inclination and rotation contexts, which are not directly instructed, remains underexplored. This study aimed to explore the variations in children's performance of angle recognition across angle contexts. The participants were 40 third- to fourth-grade children who were recruited from two public elementary schools in Taipei, Taiwan. The participants received the static and turning concepts of angles in the first semester of third grade and the first semester of fourth grade, respectively, based on the regular teaching schedules of school mathematics. The data were collected from the end of the participants' second semester of third grade to the end of fourth grade via the worksheets and interviews. The results showed that the regular instruction of textbook lessons given in fourth-grade did facilitate the children's performance as a whole. However, after the instruction, 42% and over 50% of the children still failed to identify angles of inclination and rotation, respectively. The results of a one-way ANOVA with angle context as the repeated measure showed significant differences in the children's performance among the three angle contexts,  $F(2, 78) = 17.20, p < .001, \eta^2 = .31$ . The results suggested that the levels of difficulty in identifying angles of inclination and rotation are higher than those of angles of scale (turnings). The reasons given by the children who did not believe that angles were embedded in the inclination and rotation contexts included: (a) a lack of angle arms, (b) the music boxes were round without sharpness, and (c) no angles exist in circular turnings. The findings support Mitchelmore and White's (2000) perspectives that children mentally classify corners, openings, and turns as being distinct from each other. Seeing that third- to fourth-grade children are unlikely to transfer spontaneously the definition of two-line angles and turning concepts of angles to various angle contexts where the attributes of an angle are not explicitly integrated, encouraging students to compare and link various angle situations (or representations) is a fundamental base to help them construct a general abstract angle concept.

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# PRE-SERVICE TEACHERS AS LEARNERS OF PEDAGOGICAL CONTENT KNOWLEDGE IN COMBINATORICS

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The study is part of broader research into scaffolding pre-service teachers' (PSTs') learning of pedagogical content knowledge (PCK; Hill et al., 2008) that would enable them to develop their future students' combinatorial thinking. We characterized a PST's level of combinatorial content knowledge (CK) using Lockwood's (2013) model of combinatorial thinking. We conceptualized PSTs learning of PCK in the sense of Vygotsky's (1978) Zone of proximal development (ZPD). The research question is following: What are the differences in the learning of PCK between PSTs with a high, medium and low CK? We developed and tested a research tool and a scoring system to capture PSTs' CK and PCK in domain of combinatorics. The research tool consisted of three combinatorial problems with six connected tasks each. Each combinatorial problem and its first connected task were used to measure level of CK. The remaining connected tasks measured PSTs' level of PCK. Furthermore, the information about past pedagogical experience of PSTs was collected through interview. Ten PSTs specialising in secondary mathematics in their final year of study participated in the research. We observed that PSTs who demonstrated high CK and high PCK ( $n = 3$ ) were those with rich experience with math education (regular tutoring of students struggling with math and thesis in math education). Those who scored high in CK and medium or low in PCK ( $n = 2$ ) were PSTs with medium (only tutoring) or poor experience with math education. No discernible effect of experience on PCK was observed in PSTs with medium/low CK ( $n = 5$ ). This indicates that pedagogical experience scaffolds learning of PCK only if PST has high CK. Further research is necessary to substantiate this hypothesis. In-service teachers especially novice teachers should become the matter of our concern as well.

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# APPROPRIATING INSCRIPTIONS AND MANIPULATIVES

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This presentation will describe how 4th-graders appropriated mathematical tools, T-charts (for recording groups of ten and units) and manipulatives (Unifix cubes). We used an academic literacy in mathematics framework (Moschkovich, 2015) to examine appropriation as students modeled base-ten addition. The framework proposes three integrated components; mathematical proficiency, mathematical practices, and mathematical discourse (including symbolic systems, language, objects, and inscriptions).

The unit of instruction, The T-shirt Factory (Fosnot, 2007), contextualizes lessons in the simulation of a factory and student management of the ‘inventory’, modeled by Unifix cubes. T-charts were an inscription promoted as a means to “organize the T-shirts” and to support conceptual understanding of place-value by prompting students to list all possible ways to decompose numbers into bundles of ten and individual units. Directions asked students to “Bundle the T-shirts” in as many ways possible (see Figure 1). In the example below, 38 could be represented on a T-chart with three rolls of tens and eight units or with two rolls of tens and eighteen units.

	Number of Rolls	Number of Loose T-Shirts
Original problem		38
Student response	3	.8

Figure 1: Sample problem: “Bundle the shirts. How many ways can you find?”

Our analysis examined student discourse in small groups. The presentation will show how students appropriated discourse practices and tools, including everyday terms (‘loose items’), visual inscriptions (T-Charts), and concrete objects (e.g., Unifix cubes). Many students used the T-chart for organizing numbers, not as a tool to think about decomposition, reflecting a procedural focus. Although they modeled the number of shirts using their manipulatives, they used the phrase “loose items” with a situated meaning that included ALL loose units available, instead of the intended meaning as items left over after making groups of ten, which would reflect cardinality of units after counting all groups of ten.

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# THE COMPREHENSION OF RELATIONAL CONCEPTS (<, >, =) BY PRE-SERVICE AND PRESCHOOL TEACHERS – PERSPECTIVE OF “QUANTITY”

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The early development of symbolic reasoning in children should allow them to properly use mathematical symbols later in formal math. Teaching mathematics to pre-schoolers today requires professional knowledge. Unfortunately, studies conducted in recent years indicate that teachers assigned with teaching preschool mathematics do not have adequate knowledge (Sarama et al., 2016). This may stem from negative personal experiences or a lack of appropriate training in college. They often use the knowledge and experience they bring from daily life, meaning that they might not always give the correct mathematical importance to the symbol. Symbolic reasoning means the ability to grasp the meaning of a symbol representing an object or idea, without having an expression in the symbol itself. The goal of the study was to understand how pre-service and preschool teachers understand and use the mathematical symbols <, >, and = when comparing figures and shapes of different sizes and thicknesses. Using both quantitative and qualitative methods, we examined a study population of 71 pre-service teachers attending a course for teaching mathematics to pre-schoolers and 149 preschool teachers.

The results of this study show that preschool teachers feel that mathematical symbols may be used in different ways, depending on context: sometimes with respect to the quantity and sometimes to the shape or size of graphical images and they did not restrict them only to their mathematical significance. The majority of participants did not answer the questions correctly, with a significant difference between how the two groups validated their answers, indicating that the participants do not correctly understand that mathematical symbols should only be used in the mathematical context. Teachers must be made aware that the signs “<, >, and =” must be used only in the mathematical sense. Preschool teachers who incorrectly see quantity as a graphical concept and do not see the mathematical significance will, most likely, pass on this misconception to the children. This might lead the children to think that the size of the number or graphical object is what determines the relationship and which symbol to use (Hassidov & Ilany, 2017).

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# THE MULTIPLE FUNCTIONS OF SCAFFOLDING IN SUPPORTING STUDENTS' TASK COMPLETION

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This oral communication focuses on the notion of Scaffolding (Wood et al. 1976). Wood et al.'s (1976) analysis focuses specifically on the supportive moves the teacher makes to enable the students to complete the task. Here we focus more on the processes and practices involved in the completion of the task, and how different moves support different practices.

Research drawing upon the idea of scaffolding has largely taken a socio-cultural approach (Moschkovich, 2015), specifically exploring the role of scaffolding within Vygotsky's ZPD. This work generally views scaffolding as a teaching strategy whereas the conversation analytic approach adopted in this project (CA) considers it as a social interaction between a teacher and learner which makes it an ideal approach for examining the contingency of the teacher's moves on what her students do.

The data used in this paper comes from the Talk in Mathematics project, a collaborative research project between 9 mathematics teachers and 3 researchers focused on developing students' mathematical talk in the classroom. The project involved the teachers videoing themselves and then meeting regularly over a period of 2 years to share short clips from these videos and explore different aspects of their practice and their students use of mathematical language. The data analysed here comes from the videos of students who are low attaining in mathematics and were transcribed using Jefferson transcription enabling a more detailed analysis of not just what was being said, but also how it was being said.

We focus on three mathematical practices that arise in the interactions: using the precise language associated with the topic; categorising and classifying units of measurement; and students use of deductive reasoning within their talk. We show how scaffolding turns are contingent upon the responses given by students, but also offer examples of how this scaffolding fades and how responsibility is transferred in ways that are also contingent upon interactions with students.

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# MODIFYING THE TRADITIONAL PUBLIC VIEW OF MATHEMATICS

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The goal of this paper is to make the mathematics education community aware of the importance of modifying the traditional public view of mathematics. Generally, people find mathematics to be a difficult, mysterious, unpleasant, and elitist computational discipline with unchangeable rules and a collection of facts to be memorized. These widespread views contradict the new orientation of educational reforms in mathematics. As long as these misconceptions remain common within the educational community, mathematics reform will continue to face considerable difficulties. For instance, as long as parents believe their children are doing well in mathematics just because they can do the computations quickly and precisely, home support for reforms will probably remain minimal (Nanna, 2016).

Adapting the variation theory of learning (Marton, 2015), it can be said that the traditional perceptions of mathematics are a result of deeply-rooted ideas people habitually share and take as truth in their awareness of this subject. According to this theory, people can make changes in their perceptions (i.e., see a known situation in a different way) if these natural attitudes are broken and restored in the same instant by interacting with patterns of variation such as contrasts (see opposites and differences).

Within this framework, action research was carried out to help parents see mathematics differently from their traditional way. As part of an event for parents at a private middle school, a 6-hour workshop was conducted for 21 parents over two evenings. The activities of the workshop aimed to show that mathematics is not an unchangeable collection of rules, but rather is a growing body of knowledge and a way to understand patterns and order. Such activities as “ $1+1$  not necessarily always to be 2” and “math in nature”. Dynamic visualization tools were used.

The activities were designed to help the parents first recognize their own perceptions about mathematics and then experience alternative activities to test and possibly contradict their perceptions. An assessment tool was used to determine perceptions of mathematics before and after the workshop, and the results indicated a significant change. The research methods and results will be clarified in the presentation.

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# HOW DO PRE-SERVICE TEACHERS APPLY THEIR KNOWLEDGE FOR TEACHING MATHEMATICS AND ECONOMICS?

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In addition to models of teachers' professional knowledge, research in teacher education currently focusses more sophisticated approaches taking into account teachers' ability to use their subject-specific knowledge for mastering instructional demands (Blömeke et al., 2015). However, it is so far largely neglected if the ability to use knowledge in teaching situations, which can be modelled as teachers' *action-related competence* (AC; Lindmeier, 2011), is specific for the subject to be taught. It is hence an open question whether teachers need different skills and abilities to use knowledge for teaching mathematics than for teaching e.g. economics. Considering this, the present study approaches the following research question: *What is the intra-individual relation between mathematics and economics pre-service teachers' AC in teaching mathematics and in teaching economics?*

To investigate this, we conducted a study with 6 pre-service teachers trained in both, mathematics and economics, and which showed profound outcomes in knowledge tests of both subjects. In a multiple case study, we analysed their responses to video-based teaching situations in both subjects as indicators for knowledge application.

The results show that that one of the 6 selected teachers did not show differences between both subjects in applying subject-specific knowledge. For the other 5 cases, we observed noticeable qualitative differences in how knowledge is enacted for dealing with teaching situations in Mathematics and Economics instruction. Based on this empirical evidence, we conclude that the ability to use subject-specific knowledge in teaching situations may indeed have to be learned in and for a particular subject. This finding also underpins previous hypotheses regarding the subject-specificity of teacher competences which, however, have not been confirmed empirically yet.

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# RELATIONSHIPS BETWEEN MATHEMATICS TEACHER EMOTIONS AND INSTRUCTIONAL APPROACHES

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Emotions are ‘at the heart of teaching’ (Hargreaves, 1998, p. 835). A body of qualitative studies suggest that teachers experience various positive and negative emotions during teaching (Sutton & Wheatley, 2003). These recurrent emotional experiences are reported to be interactive with teachers’ teaching practices (e.g. Hargreaves, 1998; Sutton & Wheatley, 2003). How do senior secondary teachers’ approaches to teaching mathematics relate to their emotional experiences in mathematics classrooms?

This study adopted a Teacher Emotions Scale (Frenzel et al., 2016) and an approach scale to investigate mathematics teachers’ three emotions (enjoyment, anger and anxiety) in teaching and two instructional approaches (transmission-oriented and constructivist-oriented). 220 senior secondary mathematics teachers participated in the questionnaire in Guangdong province, China. The confirmatory factor analysis and structural equation modelling were used to validate the instruments and examine the relations. The goodness-of-fit indices obtained were as follows:  $\chi^2 = 438.026$ ,  $df = 340$ ,  $p < .001$ ,  $\chi^2/df = 1.288$ ,  $RMSEA = .036$ ,  $CFI = .950$ ,  $TLI = .944$  and  $SRMR = .061$ , indicating that the hypothesized 5-factor model had a good fit for the data. Factor loadings of items were also above 0.4. The Cronbach’s  $\alpha$  reliabilities are enjoyment, 0.848; anger, 0.863; anxiety, 0.762; transmission approach, 0.761; and constructivist approach, 0.820. The results showed that Chinese mathematics teachers in high schools experienced more enjoyment than anger and anxiety, and constructivist approaches were more frequently adopted than transmission approaches. Moreover, the two approaches to teaching predicted teachers’ three emotions differently.

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# TEACHERS' AND STUDENTS' PERCEPTIONS OF THE GAP BETWEEN SECONDARY AND TERTIARY MATHEMATICS

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The problems facing students in the transition to university mathematics are influenced by mathematical, didactical as well as social aspects, which have often been studied separately. By taking an integrated view of the transition problem, Jablonka, Ashjari and Bergsten (2017) identified five dimensions of research that highlight mismatches in criteria for what constitutes mathematics at upper secondary and tertiary levels. These dimensions are: (1) change in expected learning habits and study organisation, (2) different teaching formats and modes of assessment, (3) differences in pedagogical awareness of teachers, (4) curriculum misalignment, and (5) changes in level of formalisation and abstraction. The purpose of our on-going study is to analyse, both from a student and a teacher perspective, if any of these dimensions influence the transition problem more than any of the others, and whether different groups of students are affected differently by the dimensions.

An online questionnaire is used, with six to nine questions per dimension, where each question addresses an issue of a potential gap between upper secondary and tertiary mathematics education. The questionnaire has been distributed at one university, and has so far been answered by nine teachers of first-year undergraduate mathematics courses and 49 first-year engineering students. The questionnaire is being distributed to another 420 similar students enrolled at the same university. Similar data will be obtained from other universities in a subsequent phase. By quantifying and normalising all answers in the questionnaire, we constructed a measure for the perceived size of the transition gap for each of the five dimensions, with 0 for no identifiable gap and 1 for the strongest indication of a perceived gap. Among the teachers, the largest gap is perceived in the 5<sup>th</sup> dimension (0.70 on the normalised scale, as opposed to 0.41-0.59 for the other dimensions). However, the students do not perceive the gap in this dimension as equally severe (0.55), also not in relation to the other dimensions (0.36-0.58). For the 4<sup>th</sup> dimension, teachers also perceive a larger gap than the students (0.48 compared to 0.36), while the first three dimensions are more similar between teachers and students. In conclusion, university teachers and students seem to perceive the gap between secondary and tertiary mathematics differently, and particularly with respect to some dimensions, students do not identify as large gap as the teachers do.

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# PROMOTING SECONDARY STUDENTS' SHIFTS TO COVARIATIONAL REASONING: THEORY NETWORKING AND DIGITAL TASK DESIGN

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We investigate the question: How do secondary students shift to covariational reasoning when interacting with digital task sequences involving linked animations and graphs? Networking Thompson's theory of quantitative reasoning (Thompson & Carlson, 2017) and Marton's variation theory (Marton, 2015), we designed digital task sequences and analyzed students' reasoning. We report results of a small scale, qualitative study (n=13) investigating shifts in students' covariational reasoning.

Drawing on Thompson's theory of quantitative reasoning, we posited three critical aspects of students' covariational reasoning: their conceptions of attributes as being possible to measure; their conceptions of change in attributes; and their conceptions of a relationship between attributes. Drawing on Marton's variation theory, in the digital task sequences, we designed dynamic graphs in which students could vary individual attributes, then both attributes, within and across different backgrounds.

We conducted a series of three individual clinical interviews with each student. The first served as a preassessment. The second and third incorporated the digital task sequences. Our analysis built from description to inference to explanation. We inferred students were engaging in covariational reasoning when they worked to represent and/or describe relationships between simultaneously varying attributes.

Four students engaged in covariational reasoning in the preassessment. By the third interview, eight students engaged in covariational reasoning. Students' conceptions of how graphs represent the direction and nature of the motion of objects in linked animations (e.g., up/down, curving/straight) impacted their covariational reasoning.

Students can think that dynamic graphs should resemble physical features of the motion of objects in linked animations (e.g., if objects in linked animations move up/down, so should dynamic graphs). To promote students' covariational reasoning, provide students with opportunities to sketch and interpret different dynamic graphs with physical features distinct from the motion of objects in linked animations.

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# PROSPECTIVE TEACHERS' LEARNING OPPORTUNITIES TO POSE MATHEMATICAL MODELING PROBLEMS

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This study aims to examine learning opportunities prospective teachers (PTs) experienced and recognized as they engaged in posing and revising mathematical modelling problems. Research shows that when learners engage with problem posing, they develop problem solving ability and positive attitude toward mathematics, and connect mathematical ideas (e.g., Brown & Walter, 1990). Teaching and learning of mathematical modelling is one of the focal aspects of mathematics education (Lesh & Kelly, 2000), but little is known about learning opportunities in the context of posing and revising mathematical modelling problems.

The participants were 38 PTs enrolled in three sections of a problem-solving mathematics course at a Midwestern university in the U.S. Using multi-tiered design-based research framework (Lesh & Kelly, 2000), we engaged the PTs in activities designed to strengthen their understanding of mathematical modelling. Three times during the semester, small groups of PTs posed, solved and revised authentic problems – Model-Eliciting Activities. Data included audio-recordings of individual interviews, 75 written mathematical modelling problems posed by PTs (i.e., 25 original problems and their successive revisions), peer feedback PTs provided to one another, and their individual reflections on their learning experiences.

Framework for posing mathematical modelling problems, developed from the literature on mathematical modelling, guided our qualitative data analyses. A majority of PTs were ultimately successful in developing rich mathematical modelling problems. Analyses of PTs' reflections, their original tasks, and task revisions document the evolution of PTs' understanding of mathematical modelling over the course of the semester. We will discuss changes in characteristics of PTs' problems by highlighting PT's ability to use mathematical modelling problems to address diverse social issues, introduce multiple mathematical perspectives, and facilitate problem-solvers' meta-cognitive capabilities.

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# THE LANDSCAPE OF EARLY ALGEBRA TEACHING AND LEARNING

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Since the 1990s, early algebra has progressively developed into a research area and teaching practice in several countries. There seems to be agreement among researchers that algebra has a place in the elementary school curriculum, but that early algebra is not the same as (traditional) algebra early (Carraher, Schliemann, & Schwartz, 2008). However, there also seems to be differences in the views of what constitutes algebra and algebraic thinking, and these differences may have implications for teaching algebra in the elementary grades (Stephens, Ellis, Blanton, & Brizuela, 2017).

This project aims to characterize and discuss approaches to teaching 6-12 year old students algebra. The research questions are: What characterizes the approaches to early algebra teaching and learning given in the literature, and which similarities and differences are there between these different approaches?

In order to answer these questions I have conducted a literature review with a hermeneutic approach (Boell & Cecez-Kecmanovic, 2014). I have categorized the included studies in four types of categories: (1) algebraic content, (2) algebraic activities, (3) role of syntax, and (4) the role of extra-mathematical contexts. This categorization makes it possible to draw a landscape, which is useful to characterize entry points to early algebra teaching and learning, and to describe differences between the approaches in the review.

I found that early algebra is generally based on students' work with arithmetic and/or functional relationships through activities that aims at students' discovery of general relations, problem solving including mathematizing and/or relational thinking. I also found that the main differences between different approaches relate to the role of representations and contexts. In the presentation, I will elaborate on these findings in detail.

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# OPPORTUNITY TO LEARN MATHEMATICS AFFORDED BY TWO COMPETENT TEACHERS IN THEIR LESSONS

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The circumstances that allow students to engage in and spend time on academic tasks such as working on problems, exploring situations and gathering data, listening to explanations, reading texts, or conjecturing and justifying have been labelled opportunity to learn (National Research Council, 2001, p.334). Watson (2007) examined opportunities to act mathematically afforded by tasks teachers enacted during their lessons.

This research study examines the opportunities to learn mathematics afforded by two competent mathematics teachers in Singapore, T1 and T2. In the context of the study, the locally-defined ‘teaching competence’ is a composite of teaching experience, student achievement in mathematics, and recognition by peers as exemplary teachers of mathematics. The sources of data are the video-records of the sequences of lessons enacted by the teachers and student interviews both of which adopts the complementary accounts methodology developed by Clarke (1998). Watson’s analytical instrument (see 2007, p, 119) that allows one to capture student mathematical engagement during enacted classroom practices was adopted for analysis of the data.

The findings of the study showed that there were similarities and also differences in type of tasks used by the teachers and also the ways the tasks were enacted. Both teachers used tasks to enact content-focussed instruction guided by their instructional objectives. T1 used the tasks to evoke thinking and misconceptions and develop a deeper understanding of content knowledge while T2 focussed on application of conceptual knowledge and development of procedural fluency.

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# STUDENTS' CONCEPTION OF SPANNED SPACE AND ITS RELATION TO CONCEPTION OF LINEAR INDEPENDENCE

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Several studies suggest that use of visual images helps students learn linear algebra concepts. But, there are also studies (cf. Sierpinska, 2000) indicating that using visual images is sometimes problematic. In order to understand this issue better, it is important to study students' understanding of linear algebra concepts in the context of geometric vectors. With this aim, this study focused on the concepts of linear independence and spanned space.

We conducted a semi-structured interview with 17 first-year undergraduate engineering students. The participants were selected based on the results of a paper-based test conducted with 71 students. The test consisted of ten questions with pictures, and asking them to determine the linear independence of two to four geometric vectors in space. The participants in the interview consisted of the two groups: 7 students who answered all questions correctly (Group A), and 10 students who missed the question in Figure 1, but answered all other questions correctly (Group B). In the interview, students were asked to explain their thought processes while answering each question, and explain their answers in terms of a spanned space.

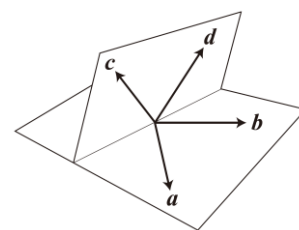


Figure 1: A question in the test

The obtained data were analyzed according to APOS theory in order to identify students' conceptions. The difference between the two groups was in the conceptions of linear (in)dependence of four geometric vectors and a spanned space of three linearly independent geometric vectors. Most students in Group A had Process/Object conceptions, while most students in Group B had Action conceptions only. No difference was found in conceptions of linear (in)dependence of two to three vectors, and spanned space with one or two dimensions. Students in Group B with only Action conceptions had difficulty in imaging that linearly independent three geometric vectors in space span the whole space, and hence they could not notice the linear dependency of the vectors in Figure 1. The results of the study will contribute to understanding the limitation of effectiveness of using visual images in teaching and learning linear algebra concepts.

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# RE-VIEWING MATHEMATICAL PRACTICE

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This a study that focuses on four preservice-teachers' observations of digital video recordings of their own teaching practice over the course of their student teaching semester. The study is designed in order to register: a) changes in preservice teachers' understanding of the key aspects of productive mathematics classrooms, and b) signs that they are changing their own teaching in relation to those aspects. The study is inspired by recent research on "responsive pedagogy," where the teacher consciously attends to and responds to students' thinking, fosters equitable student participation, and promotes student agency (e.g. Lampert & Graziani, 2009); and in alignment with "noticing," (e.g. Mason, 2002; van Es & Sherin, 2008), a set of teacher dispositions and behaviours involving observation, description, analysis and interpretation of teaching practice.

This study will focus on changes in how prospective teachers identify and enact the elements of a "powerful mathematics classroom," and for purposes of identifying and analysing these, we will use Schoenfeld's "Teaching for Robust Understanding in Mathematics Scoring Rubric" (Schoenfeld, 2014).

Data is collected over the course of one semester, and consist of: (a) video recordings and transcriptions of each group and individual meeting; (b) students' personal classroom logs; (c) my own field notes. Transcripts of each videotaped discussion is analysed according to PST's responses to the videos they have watched. Transcripts of videos--along with the participants' reflective logs--will be thematized using a grounded theory approach (Strauss & Corbin, 1998). Special attention will be given to changes over time in participants' approach to the analysis and interpretation of what they notice and to *identify important dimensions of "powerful" classrooms*--and to their growing capacity to enact these understandings in their teaching.

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# TEACHERS' QUERIES ABOUT ALGEBRA TEACHING

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There are various views on what teachers need to know to teach algebra successfully, with a growing interest in developing ways of assessing mathematical knowledge for teaching (Ball et al, 2008; McCrory et al 2012). This study takes the perspective of practicing teachers, investigating what areas of knowledge they feel a need to develop when teaching algebra. Based on the relevance principle our results can inform teacher education for pre-service as well as in-service teacher training.

Data consists of six video recorded focus group discussions collected as part of a larger project about introductory algebra in grades 6 and 7. Four consecutive algebra lessons were recorded in five Swedish and four Finnish classrooms. The teachers (n=11) were then asked to choose episodes from their own lessons and pose questions to discuss with colleagues in focus group interviews.

In the analysis, four topics of algebra were identified based on the object of learning in the episodes; variable expressions; equivalence; equation solving; algebra as a field of knowledge. Each question was categorized according to what aspects of the content the question was about; representation; procedure; conceptual understanding of variable; choice of examples; teaching approach. Using the Mathematical Knowledge for Teaching framework (Ball et al, 2008), we found that 8 out of 16 questions related to teaching, 4 out of 16 to students, and two questions tightly connected teaching with student understanding. Most of the questions concerned aspects of variables known as problematic from previous research (Bush & Karp, 2013).

The discussions were further analysed using a framework developed by McCrory et al (2012), producing examples of how teachers use content knowledge when *trimming* and *decompressing* school algebra. Our tentative results indicate that teachers can identify their own knowledge gaps when they start reflecting on specific content in authentic teaching situations, and that video recordings are useful tools to initiate fruitful discussions. Further results will be discussed in the presentation.

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# HOW DENSE ARE RATIONAL NUMBERS?: A NEW MATERIALIST APPROACH TO THE NUMBER LINE

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This study aims to empirically examine the power of the number line in learning number concepts. The authors analyze a situation where students plot points on a line and name them using rational numbers in a multi-touch dynamic geometry environment from the new materialist view. From the new materialist perspective (de Freitas & Sinclair, 2013), physical matter is not inert but bears the *virtuality*, such as mobility. Mathematical activity entails *actualizing the virtual*, through which new mathematical concept is created. In that, mathematical concept is also considered material - instead of being abstract – and as a part of material body-assemblage. It is when learner attends to the virtual and forms new body-assemblages that a new mathematical concept emerges, and hence learning occurs. In this light, the number line has the power to create new mathematical concepts. Furthermore, dynamic geometry environment could facilitate the actualization of the virtual of geometric shapes (Sinclair, de Freitas & Ferrara, 2013).

The pilot study includes four 5<sup>th</sup> grade students in elementary school level. They have not previously learned or obtained any knowledge about neither the number line nor rational number. In a task using *GeoGebra*, the students were asked to play game plotting points on a line and naming them with rational numbers in accordance with previously plotted points. Touchpad keyboards with multi-touch sensor were provided to the individual student, and they were allowed to discuss with each other. The analysis focused on students' hand gesture and utterance in their body-assemblage.

The result shows that the students have obtained an intuition which would lead to the concept of the density of rational numbers through the actualization of the virtual in point, line and space while forming a hand-touchpad-line assemblage. Using touchpad, the students constantly zoomed in and out to solve the task. They later reported that “the game would not end” since they will always find room and plot another point whenever they “stretch.” During the conversation, their gestures resemble the hand shape of operating touchpad. In the presentation, further detail will be discussed.

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# THE INTERNATIONAL CLASSROOM LEXICON PROJECT: IDENTIFYING AND DOCUMENTING KOREAN MIDDLE SCHOOL MATHEMATICS CLASSROOM PRACTICES

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Different naming systems in different culture describe how community members view the phenomena of mathematics classrooms (Stigler & Hiebert, 1999) as well as confirm the need for the members to crystalize suitable professional languages for reflective practices involving noticing as teacher expertise (Sherin, Jacobs & Philipp, 2011). The International Lexicon Project has documented pedagogical naming of middle school mathematics classroom in nine different countries: Australia, Chile, China, the Czech Republic, Finland, France, Germany, Japan and the USA. The Korea team has recently joined in this international project and started to document Korean lexicon lists that middle school mathematics teachers name pedagogical phenomena of classrooms in Korea. This paper reports how the Korea lexicon team has investigated and created lexicon lists and characterized a pedagogical naming system in middle school mathematics classrooms in Korea as a part of the International Lexicon Project. As other teams have done in each country (see Mesiti, Clarke, Dobie, White, & Sherin, 2017), the Korea team consisting of two mathematics educational researchers and three secondary mathematics teachers has begun creating pedagogical terms and phrases used in our school communities while watching a set of video stimulus of middle school mathematics classrooms. The iterative processes of watching video stimulus, creating pedagogical terms and phrases, discussing and refining the created terms, and categorizing the terms continued until new terms did not emerge. We currently have six preliminary categories of Korean lexicons: preparation, lesson structure, instruction-learning activity, assessment, improving teaching practice, explicit strategy related to mathematics. We are now in the process of conducting local validation across the country. In the PME session, we will provide the finalized Korean lexicon lists with video examples and discuss the unique features of Korean mathematics classrooms.

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# REFLECTION OF A BILLIARD ACTIVITY UPON REASONING PROCESSES IN TEACHING QUADRILATERALS

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Geometry requires making associations that open doors to generalization, making transition between two and three dimensions, determining the variable and invariable features of objects, dividing geometric structures into pieces and doing regulation again. Students' giving meaning to the concepts and reasoning processes regarding these requirements constitutes the focal point for mathematics teachers.

Duval (1999) considered reasoning processes regarding geometric concepts as perceptual, operative, sequential and discursive and stated that there is no phase or order between these processes. In this respect, it is thought that perceptual apprehension has an important place in geometry teaching for students in terms of stating their views about the figure, recognizing the features of the figure and creating a new figure by putting forward the pieces of the figure. With respect to perceptual process, quadrilaterals could provide an appropriate teaching environment. Even if the hierarchical definitions made by the students for quadrilaterals are correct, it is obvious that they experience difficulty recognizing the quadrilaterals related to the definitions. The fact that quadrilaterals, which has been a popular research subject in recent years, poses a problem for students could also be associated with the theoretical framework. In this respect, the framework formed by Duval regarding ways of apprehending a figure is thought to provide an effective teaching environment for giving meaning to quadrilaterals. In this respect, the present study focused on in-class reflections of Duval's reasoning processes by examining a billiard activity including three sub-questions regarding elementary school 7<sup>th</sup> grade students' classification of quadrilaterals. The initial findings related to the activity demonstrated that the students did not take the features related to angles into account while defining the quadrilaterals; that they made decisions in accordance with the shapes of the billiard tables and thus failed to reveal the features of quadrilaterals correctly. In process of teaching, the activity was supported with multiple representations and dynamic geometry softwares, and it was observed that the students made transition from perceptual apprehension to discursive apprehension.

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# INVESTIGATING EIGHT GRADERS' WAYS OF THINKING ASSOCIATED WITH PROBLEM SOLVING IN CONTEXT OF MATHEMATICAL LITERACY

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Mathematics which should be taught in schools, comprise of two crucial parts as ways of thinking (WoT) and ways of understanding (WoU) in DNR framework (Harel, 2008). When WoU is defined as products of a mental act, WoT represent the characteristics of the same mental act. Due to the view of achievement in school mathematics emphasize solving problems correctly, it can be said that WoU is more valued than WoT. In addition to this, mathematical literacy is defined as formulating, using, and interpreting mathematics in various context in PISA frameworks. As a mental act problem solving in context of mathematical literacy should be supported by means of students' WoT. Due to these reasons, the purpose of this study is to investigate and support eight graders' WoT associated with problem solving in mathematical literacy context in a learning environment that is designed based on teaching with variations for supporting the mathematical literacy competencies. With this purpose, a pre-paper-pencil test was administered to 237 eighth graders. After the analysis of data quantitatively, seven students who are thought to have different reasoning competency levels according to competency scheme (Turner & Adams, 2012) were chosen for the second stage of the study. In this stage, the teaching experiment process in which the data was collected through pre and post paper-pencil tests -including PISA mathematical literacy items-, clinical interviews and four teaching episodes is conducted with seven students. The findings indicate that eight graders' WoT associated with problem solving in context of mathematical literacy divided into three categories. The less desirable WoT consists of calculation, daily life experience and preference based problem solving. The second group WoT consists of memorizing based and impulsive problem solving. The most desirable WoT, which is thought to be called as analytical problem solving, consists of conceptual and contextual problem solving. Moreover, it can be said that eight graders' WoT supported to a certain degree. Further results will be discussed in the presentation.

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# CRITICAL INCIDENTS SHAPING TEACHERS' CONCEPTIONS OF ARGUMENTATION IN PRIMARY EDUCATION

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Argumentative structures – as an aspect of social interaction – are fundamental for enabling learning in classroom settings. As such, argumentation has been widely examined focusing primarily on students' argumentation or activities and interactions in the mathematics classroom. However, few studies on mathematical argumentation discuss the specifics of teachers' role, which plays a vital part in initiating argumentative processes (Peterßen, 2012).

This exploratory case study aims to examine teachers' conceptions of mathematical argumentation in order to gain insight into reasoning culture in primary mathematics education. More concretely, we focus on critical incidents leading to turning points which shape teachers' thinking, and their actions in classroom (Chapman, 2017). The following research questions guided the study: What conceptions do primary teachers hold with respect to mathematical argumentation? What incidents from their practical experience do they consider influential in shaping their conceptions?

Data collection tool included a guideline for a semi-structured interview. The sample comprises three 4<sup>th</sup> grade primary teachers, whose interviews were analyzed using qualitative content analysis. Results show that the teachers emphasized distinct functions of argumentative structures in the classroom, which highlight primarily didactical aspects of argumentation. Explanations included the conception that argumentation processes support mathematical comprehension or serve particular diagnostic purposes. For shaping these conceptions, especially incidents within a teaching context, such as irrational and implausible student solutions, can be found as critical. In the presentation, further results will be discussed in detail.

In summary, the findings of the study underpin the influence of teaching experience on future teachers' thinking, and actions in the classroom. Since the results are based on a small sample and identifying themes of critical incidents concerning the conception of argumentation is of relevance, further research in this area is needed.

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# NON-ROUTINE PROBLEM POSING SKILLS OF PROSPECTIVE MATHEMATICS TEACHERS

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Teachers, having an important role in the implementation of problem posing into the curriculum (Gonzales, 1996), have to develop skills in problem posing. However, many studies have reported that problems generated by students and teachers are mostly cognitively undemanding and textbook-like (Crespo and Sinclair, 2008). In addition, Lavy and Shriki (2007) reported that prospective teachers tend to focus on common posed problems. Therefore, this descriptive study was especially focused on "non-routine" problems and aimed at investigating prospective teachers' performance in non-routine problem posing.

43 prospective math teachers participated in the study. Before the study was carried out, participants had a one-term elective course about problem solving. At the end of term, all participants were asked to pose one non-routine problem and to hand in to the researchers as written texts. Through document analysis, all problems were analyzed according to five criteria: *type*, *contextualization*, *problem's source*, *complexity* and *strategy use*. Findings showed that most of the non-routine problems posed by participants had only one answer but multiple solution methods. Almost half of the problems were contextually original when compared to problems which have been confronted previously. Six participants posed problems that have high complexity in terms of linguistic and syntactic. As to strategy use, 17 participants posed problems requiring multiple strategy use. Most preferred strategies were *make a systematic list*, *guess and check* and *draw a diagram*, while *make a table* and *simplify the problem strategies* were not used at all. Based on these findings, it can be said that prospective teachers have potential to pose non-routine problems. However, this potential can be developed by allocating more time to these kinds of problems in math teacher training so that prospective teachers can have their students pose non-routine problems in future as well. Studies including more participants from different age and grade levels will be more illuminating.

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# ESTABLISHING REPRESENTATIONAL MEANING OF GESTURES – LEARNING FROM THE DEAF CLASSROOM

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Gestures can be a powerful didactic means in the mathematics classroom, especially in classrooms where spoken language provides only limited access to communication. However, in order to be used as shared means of communication about a mathematical idea, the content of a gestural representation and its relation to the mathematical idea represented need to be learned first. It is still an open question how gestures can be used by the teacher in the mathematics classroom to become actual shared representations that reflect mathematical meaning.

Answers to this question can be found by looking into the specific setting of the deaf mathematics classroom, using the gestural language of sign language. Mathematical signs often represent aspects of the underlying mathematical ideas in an iconic or metaphorical manner and can thus get a twofold referential function; as linguistic signs by which is referred to a mathematical idea when communicating about it and as visual representation that potentially recalls specific aspects of the mathematical concept. However, this relationship between the sign and the aspects represented in it needs to be established actively in order to become perceived as such, hand in hand with the development of the knowledge on the mathematical concept itself (Krause, 2018).

In a re-analysis of two teaching situations in an all-deaf geometry classroom (Krause, 2018), I trace the development of gestures and signs across three referential levels: direct indication to inscriptive signs (L1), ephemeral supplementation within a fixed inscription (L2), and free gestural representation without direct physical reference to inscription (L3) (Krause, 2016). This way, I reconstruct how the teacher establishes representational meaning in his signs for axis symmetry and point symmetry.

In the analyses we see the extensive use of the teacher's gestures on L1 and L2 to introduce defining and mapping component of the two kinds of symmetry (axis & folding/mirroring; point & rotating) and how he refines these gestures until they blend into the respective component of the signs on L3. I will discuss how examples like this can help us develop methods for using gestures as representational means in general.

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# INITIATING STUDENT DISCOURSES WITH SILENT VIDEO TASKS IN MATHEMATICS CLASSROOMS

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For learning to happen, conversation is important. At least, if we think of cognition as a process of communication, (Sfard, 2010). Silent video tasks can act as a conversation starter, even in a mathematics classroom where students usually are silent. In silent video tasks teachers invite students to watch a short mathematics film with no text or sound. It, in turn, is discussed in groups and students record their commentaries (voice-over). The activity can be implemented either as an introduction to a new concept or as a summary to review previously learned topics. In the follow-up lesson, students receive feedback on their solutions, possible imprecise language use can be addressed to clarify concepts, and preferably some selected solutions are shown and discussed with the entire class.

Teachers in four upper secondary schools in Iceland assigned a silent video task in their classes in fall 2017. I was interested in seeing whether the task would awaken teachers' interest in practices described by Liljedahl (2016) as a *thinking classroom*. Teachers were interviewed three times; before and after assigning the silent video task, and after the follow-up lesson. Their students answered two short online questionnaires; one after completing the task and another after the follow-up lesson. All teachers expected that it would be difficult for students to handle the assigned technology and were positively surprised by their students' technological expertise. The silent video task was easy to implement even for teachers who were used to be the only one talking in class. It offered students an opportunity to listen to explanations from others; even making some students realize that once they can explain to others what they have learnt, they have improved their understanding. For some teachers the task was a powerful tool to initiate discussions and they all noted that it helped to break up their normal teaching routines. Still, because the task did not directly prepare students for their final tests, they expressed that they would unlikely use it again. In the presentation further results will be discussed.

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# PRE-SERVICE SECONDARY MATHEMATICS TEACHERS' READING OF CURRICULUM MATERIALS

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It has been known that there is difference between written curriculum and enacted curriculum. To look into the enacted curriculum, researchers has emphasized interpretations and decisions made by the teacher in order to envision and plan instruction (Remillard & Heck, 2014). Also, for pre-service secondary mathematics teachers' education, it is necessary to investigate their reading of curriculum materials (Land, Tyminski & Drake, 2015). Because, it is related with their beliefs about teaching and learning, so it will influence their teaching practices.

This study investigates pre-service secondary mathematics teachers reading of curriculum materials. Especially, we focused on specific content; conditional probability. Because, many researchers pointed out understanding of the concept of conditional probability is difficult (Gras & Totohasina, 1995) and there is not enough research which investigated reading of curriculum materials with respect to specific mathematics content (Land et al., 2015).

This study carried out a survey to examine pre-service secondary mathematics teachers reading of curriculum materials. The questionnaire was developed according to previous research (Land et al., 2015). It focused on how they understand the text of curriculum materials and make their own meaning about curriculum materials. By interview with some pre-service teachers, we analysed the reading of curriculum materials in more depth and confirm the interpretation of the researchers.

The pre-service secondary teachers read curriculum materials differently: 1) descriptive reading; 2) evaluative reading; 3) interpretive reading. Different reading showed different pre-service secondary teachers' orientations and meaning making of curriculum materials. It will provide implications for pre-service secondary teachers' education with regard to conditional probability.

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# GERMAN PROSPECTIVE TEACHERS' UNDERSTANDING OF PLACE VALUE

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The inadequate understanding of place value is, beside the consolidated counting strategy, one main reason for difficulties in arithmetic. However, it is not only school children who struggle understanding place value. Prospective elementary teachers (PTs) also lack an understanding of the principles underlying our number system and the algorithms we use (e.g. Ball, 1988, Thanheiser, 2018, Zazkis & Khoury, 1993). Many PTs struggle when asked to explain the algorithms conceptually. Hence, they will not be able to explain these mathematical concepts to their future students. In this study, we examined PTs' understanding of place value at the beginning of their teacher training studies at two German University. Our research questions are:

- What kind of justification do the student teachers give for 'the little ones'?
- Are there any differences between the PTs (regional, cultural, etc.)?

Our sample comprised 2115 PTs at the beginning of their teacher training studies at the University in Saarbruecken and in Potsdam. The written survey included five questions focussing on 'the little ones' in the written addition and subtraction algorithms, the bundling algorithm of subtraction, as well as the understanding of bundling in general. Some answers suggest that students often do not see 'the little ones' as a representation of value (e.g. 10 or 100, depending on the place) but rather as an aid to carry out the standard algorithms. Furthermore, the PTs show a lack of thorough understanding with regard to the process underlying those standard addition and subtraction algorithms. Interestingly, regional differences are less significant.

Based on the results, we were able to categorize the PTs answers to find an empirical typification.

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# SUPPORTING QUALITY OF LEARNING IN UNIVERSITY MATHEMATICS: A COMPARISON OF TWO INSTRUCTIONAL DESIGNS

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The aim of this quantitative study is to articulate the role of instructional designs on students' quality of learning in undergraduate mathematics from the perspective of the Students' Approaches to Learning (SAL) tradition. As well illustrated in a literature review by Baeten, Kyndt, Struyven, & Dochy (2010), students who apply a deep approach to learning perceive the teaching-learning environment more positively than students applying a surface approach to learning. Therefore, to better understand the role of instructional designs in the quality of learning, the study compares students' approaches to learning and their experiences on the teaching-learning environment in two undergraduate mathematics courses using different pedagogical approaches. The first course functions within a traditional lecture-based framework, and the second course is implemented with Extreme Apprenticeship, a novel student-centred teaching method.

The HowULearn instrument was used to collect five-point Likert scale data. HowULearn is a research-based instrument widely used at the University of Helsinki for feedback, research, and instruction enhancement purposes (Parpala & Lindblom-Ylänne, 2012). The analysis is based on the same cohort of students in the two course contexts (N=91). Students are clustered based on their deep and surface approaches to learning, and three clusters are identified: students applying a deep approach, students applying a surface approach, and students applying a context-sensitive surface approach. The results show that the more student-centred course design succeeds in supporting more favourable approaches to learning and promotes more positive experiences of the teaching-learning environment. In addition, all three clusters benefit from the more student-centred course design, with students applying a context-sensitive surface approach benefiting the most. Overall, the results suggest that it is possible to promote the quality of learning in university mathematics with instructional designs that besides content, take a holistic approach to the learning environment.

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# STORYTELLING AS A MEANS TO COMMUNICATE MATHEMATICS AND TO ENGAGE STUDENTS EMOTIONALLY

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One way of sharing delight between teacher and students, is via stories which act to connect, orientate and support understanding. As part of research into developing a means to study how expert teachers affectively communicate mathematics (Lake & Nardi, 2014), my research has identified storytelling as an important characteristic for engaging students in mathematics. I identify advantages of spontaneous storytelling as a connective, aligning cultural practice, and I apply a structure for identifying the characteristics of storytelling to a teaching episode. Fineman (2000) describes storytelling as an art form that evokes empathy, sharing basic features. These are that norms and boundaries need to be established within a story and that dynamic tensions should appear. Thirdly, a story should include capacity for growth or movement. Finally, a ‘good’ story holds future possibility. These four features offer a definition for episodes of teaching as illustrative of storytelling. One teacher, Adam, who works with students aged 14 to 15 creates ‘the sheep’ story. Upon realising that his students are unclear as to the meaning of ‘n’ in an expression such as  $3n+4$ , and following a few probing questions (that later he says seemed ineffective), he begins to construct the story by setting the boundary expectations of ‘not student’ behaviour (fantasy role). In this illustration, the students do not know where this story will go (dynamic tension). Nevertheless, the students trust that it will entertain (shown by student willingness to happily bleat from their positioning as fantasy sheep). Adam is an experienced teacher who has the skills to manage students who might (and in this case, do) attempt to divert from the story for other purposes. Adam is the initiator of the story, a story intended to move students forward in their mathematical understanding. His language has a strong future orientation, often using the future predictive in the form of ‘you will’, a form that is appropriate for keeping suspense and for meeting a requirement for a story to include growth or movement and future possibility. One implication from the wider research is that spontaneous storytelling in this form is unexpectedly common and can be a source of delight and as such is a powerful form of emotional mediation between students and mathematics.

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# PROOF BY INDUCTION – A COMPARATIVE STUDY

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This comparative study aims to explore university students' understanding of proof by induction (PMI), and how teaching and textbooks influence the outcome of students' learning. Here, we present an analysis of 90 solutions to an exam task dealing with PMI. The same task was given at one university in Norway and one in Sweden. Hence, we also compare solutions from students at the two universities. We have also observed the lectures introducing PMI, and analysed the course literature at the respective university. The task on the exams was:

Prove by induction that  $\left(1 + \frac{1}{1^2}\right)\left(1 + \frac{1}{2^2}\right)\left(1 + \frac{1}{3^2}\right)\cdots\left(1 + \frac{1}{n^2}\right) < 3 - \frac{1}{n}$  for all integers  $n \geq 3$ .

We used content analysis to indicate what was included in each student's solution. By a first review, we identified categories, which then were used to describe the solutions and to identify similarities and differences between the two countries.

The analysis is ongoing. In this paper, we focus on three issues in the solutions. Firstly, the induction basis. A possible difficulty is that it here should be at  $n = 3$ , while many examples have  $n = 0$  or  $n = 1$  as starting point (Stylianides, Stylianides, & Philippou, 2007). However, our data do not confirm this as a difficulty in this task, just very few students did not use  $n = 3$  as induction basis. More problematic was for both student groups to set up a correct initial inequality. Almost half of the Norwegian students used just one factor on the left hand side of the inequality, and a fifth of the Swedish students made the same mistake. Secondly, the possible use of a dummy-variable in the induction step, e.g.  $n = k$  (Ernest, 1984). Three times as many Norwegian students used a dummy. A possible explanation is that a dummy was introduced in the Norwegian textbook but not in the Swedish. However, in lectures the notion  $n = k$  was used in both countries. Thirdly, challenges related to algebraic expressions (cf. Ernest, 1984). As already mentioned, several students had problems setting up the induction basis. We also found similar problems with setting up the induction hypothesis and the inequality for  $n = k + 1$ . Further, due to algebraic mistakes a third of the students in both groups did not manage to correctly finish the proof after a correct start of the induction step. Further data collections and analyses will enable a deepening of our knowledge concerning the influence of teaching and textbooks on students' understanding of PMI.

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# MIDDLE SCHOOL STUDENTS' PROPORTIONAL REASONING

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Proportional reasoning is required in many mathematics topic areas such as scale, probability, and measurement as well as real world and everyday tasks and activities (e.g., scaling recipes), making it a major component of numeracy. Despite its prevalence throughout the curriculum and being fundamental to many day-to-day activities, students typically do not do well on proportional reasoning tasks (Dole, Hilton, Hilton, & Goos, 2015; Van Dooren, De Bock, & Verschaffel, 2010). It takes a long time to learn and the learning process is not straight forward; students struggle to understand proportional reasoning in upper elementary school and then tend to overuse it in middle school, thus demonstrating that it is not only hard to achieve but also problematic to distinguish when it is applicable (Van Dooren et al., 2010).

In an ongoing quantitative study, Swedish middle school (grade 4–9) students' understanding of proportional reasoning is investigated by a diagnostic instrument which measures both the students' knowledge and their use of additive or proportional reasoning (Hilton, Hilton, Dole, & Goos, 2013). The test items are spread over several contexts, both everyday situations and different mathematical topic areas, such as numbers and geometry. Preliminary results are hypothesized to show that the Swedish students' overall performance is similar to earlier international studies. Additionally, since the test also provides an orientation of the students' reasoning, it allows for a comparison of context and the students' type of reasoning, which offers insights into students' reasoning in different proportional situations. These findings might underpin guidelines for teaching of proportional reasoning in order to support students' struggle to acknowledge when proportional reasoning is appropriate and when not.

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# THE EFFECTS OF A PEDAGOGY COURSE ON PRE-SERVICE MATHEMATICS TEACHERS' BELIEFS AND CONFIDENCE ABOUT TEACHING ALGEBRA

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As research has provided ample evidence of the association between teachers' beliefs, confidence, knowledge, and practice (Beswick, 2012), changing teachers' beliefs and enhancing their teaching confidence appear to be a viable way to engender change in their behavior and practice. This is particularly relevant for pre-service teachers studying at university who have considerable opportunities for professional learning.

This study examined any change in beliefs and confidence of 24 pre-service mathematics teachers (PMTs) after they participated in a 18-hour six-session pedagogy course about algebra. The overarching aim of the course was to develop their conceptual understanding and discuss effective instructional design frameworks for various algebra topics. The PMTs completed an identical pre- and post-questionnaire with 19 items adapted from Day and Hurrell's (2013) study that measure beliefs and confidence about teaching algebra. The first ten items measure teachers' beliefs about their effectiveness as a teacher of algebra on a 5-point Likert scale (1: Strongly Agree to 5: Strongly Disagree). The next nine items measure teachers' confidence with respect to teaching of algebra on a 5-point Likert scale (1: Very confident to 5: No Confidence).

Using the Wilcoxon signed-rank test with the asymptotic method, it was found that there were significant differences in ratings on four items of the belief measures and seven items of the confidence measures from pre- to post-questionnaire with medium to large effect sizes. It is plausible that the PMTs still hold many central and stable beliefs about teaching algebra that are less likely to be modified in a relatively short time. On the other hand, it is clear that their confidence is significantly enhanced. Teacher educators should help PMTs to strengthen beliefs that are compatible with the curriculum goals and dispel beliefs that hinder student learning of algebra. They should also endeavor to raise PMTs' confidence in teaching algebra.

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# STUDENTS' CONSOLIDATION THROUGH PROBLEM POSING AND SOLVING ACTIVITIES

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The purpose of this study is to explore what pedagogical way contributes students' consolidation of their newly formed mathematical knowledge. Consolidation is a process in which newly formed mathematical knowledge becomes familiar to students in a flexible manner. In the course of consolidating mathematical knowledge, students create connections between the newly constructed knowledge and established knowledge (Monaghan, & Ozmantar, 2006). However, there has been little interest in pedagogical strategies for students to make these connections flexibly. Problem posing is a way to motivate the development of students' mathematical knowledge (Lavy, & Shriki, 2010). Hence, in this study, we investigated students' consolidation through two activities: problem posing and solving problems.

Participants were 27 undergraduate students in a course of complex analysis. Two activities were carried out in groups of three. The first activity was problem posing using the concepts of complex analysis, and the second was solving the posed problems. The activities of two groups were selected for videotaping and post-activity interviews. All the discussions and interviews were transcribed. We analyzed students' working process based on the five conditions of consolidation: immediacy, self-evidence, confidence, flexibility, and awareness (Dreyfus, & Tsamir, 2004).

We could find students' immediacy, self-evidence, and confidence, flexibility, and awareness to their mathematical knowledge. In the process of problem posing and solving as a team, students became able to make direct connections between previously constructed knowledge and newly constructed one. Also, they were able to use mathematical concepts clearly and confidently when solving problems rather than posing a problem. In addition, frequent use of mathematical concepts during activities contributed to students' flexible use of the concepts. Students even recognized new concepts that they had not learned yet. These characteristics show that students have consolidated their knowledge through problem posing and solving activities.

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# EXPLORING HOW A REFLECTION INSTRUCTIONAL APPROACH INFLUENCED A STUDENT'S LEARNING OF MATHEMATICAL PROBLEM SOLVING

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Reflection has great influence on students' mathematics learning (Freudenthal, 1994; Wiggins & McTighe, 1998). When students reflect on their problem solving processes, they are encouraged to deepen their mathematical thinking and build connections between ideas. Although students' learning of mathematical problem solving may be promoted by reflection, it was found that students rarely reflected on their problem solving processes (Charles, Lester, & O'Daffer, 1987; Jacobbe, 2007; Polya, 1973). The purpose of the study was to explore how a reflection instructional approach influenced an eighth-grade student's learning of mathematical problem solving.

The participant of the study was an eighth-grade high-ability student in mathematics. A reflection instructional approach was developed in the study to encourage the student to reflect on his problem solving processes after solving problems. During the instruction, the student was asked to solve mathematical problems, and after solving problems, he was shown correct solutions to judge or correct his solutions on his own. Then he was asked to identify key points he missed out or got wrong in his solutions. Data for this study consisted of pre and post testing, observation notes, and interview protocols during the instruction and after the pre and post testing. The results of the study showed that the student not only improved his problem solving performance from pretest to posttest but also improved his metacognitive ability. To determine the effectiveness of the reflection instructional approach, more studies should be implemented with more participants. Studies should also be undertaken to develop other ways to promote students' reflection in mathematical problem solving.

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# WELL-DEFINED AND ILL-DEFINED INITIAL STATE AND GOAL STATE OF TASKS AND THEIR EFFECT ON TASK DIFFICULTY

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For assessing mathematical competence of students in the first weeks at university, the question raises how to design tasks. Aspects besides the content may affect difficulty. For instance, Newell & Simon (1972) characterized problems with their initial state and goal state. Both can be well-defined or ill-defined. The initial state may contain exactly the information, which is needed for the solution of the problem, or there may be additional information (which is not necessary for the solution). A task may contain its goal state (Explain *that* statement A is true.) or not (Decide, *if* statement A is true and explain why.). We asked, how variations concerning the initial state or goal state of a task affect its difficulty.

We developed four variations of nine mathematical tasks. The variations contained the same mathematical problems, but differed in a well-defined or ill-defined initial state and goal state. Altogether, 450 students in their first semester at university answered them. After Rasch-scaling, we compared the item difficulty.

There was a significant main effect of the well-defined or ill-defined initial state in two out of nine tasks ( $p < .01$ ). For one of these, the variations without additional information were more difficult, for the other one the variations with additional information. Significant main effects for the well-defined or ill-defined goal state were also found: In three tasks ( $p < .045$ ) variations including a clear aim were easier.

Altogether, in most of the presented tasks variations in the initial state and the goal state do not affect the difficulty of the tasks significantly. These results are in line with our pre-study (Lehner & Reiss, 2017) and are especially interesting in the light of Lehner, Döring, & Reiss (2016): Although additional information are in focus of attention, this does not affect the difficulty of the items.

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# THE JOY OF LEARNING MATHEMATICAL MODELLING: NETWORKED LEARNING COMMUNITY AS AN ACTIVITY SYSTEM

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Mathematical Modelling (MM) is a complex process. To develop students' intrinsic motivation to pursue MM, teachers may need to first engender in students the joy of learning. In addition, the question of developing teachers' competencies in teaching MM through a Networked Learning Community (NLC) has yet to be attended. In Singapore, the impetus to conduct MM arises from the inclusion of real world context problems in the national examinations. The study examines the teachers' implementation of the Spread the Disease Problem (SODP) to engender joy of learning MM as an activity system. The research questions are: (1) what impact has the SODP on the joy of learning mathematical modelling? And (2) how are teachers' experiences in teaching MM mediated in the NLC environment?

Hasan and Kazlauskas (2014) explained that Cultural-Historical Activity Theory (CHAT) is about 'who is doing what, why and how'. Our study was conducted using a mixed method action research. Data collected included notes of NLC meetings, email correspondences and surveys to assess students' appreciation of MM. After the implementation of SODP in one school, 130 out of 160 Grade 8 students completed the google form on five questions using the 4-point Likert scale. The result is 1) students' enjoyment ( $M = 3.02$ ,  $SD = .54$ ); 2) importance of MM, ( $M = 3.15$ ,  $SD = .52$ ); 3) better understanding of probability ( $M = 3.08$ ,  $SD = .54$ ); 4) application of math concepts to real world, ( $M = 2.95$ ,  $SD = .60$ ); and 5) willingness to participate in similar learning experience ( $M = 2.96$ ,  $SD = .61$ ). The internal consistency of the survey data was assessed using Cronbach's alpha (5 items,  $\alpha = .839$ ). Two open ended questions are how could this learning experience be enhanced? And what other learning experiences might I like? The results showed that students' enjoyment was high and this indicated that there may be a positive impact of the SODP on the joy of learning MM. Using CHAT, we will present how teachers' learning and competencies in implementing MM could be mediated by the division of labour, collaboration, sharing of resources, observations of the implementation of SODP in classrooms and sharing of experiences in the NLC.

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# RELATIONSHIPS BETWEEN TRAIT EMOTIONAL INTELLIGENCE AND DIFFERENT MATHEMATICAL SKILLS IN ELEMENTARY STUDENTS

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There has been a surge of studies examining the role of personality in academic performance. Emotional intelligence can predict scholastics performance. In particular, empirical findings show that associations between trait Emotional intelligence (trait EI) and academic achievement were modest (Mavroveli & Sánchez-Ruiz, 2011). However, since most studies were conducted with academic achievement, there is hardly any research evidence on the relationship between different mathematics skill and trait EI. This research investigates the associations between trait EI and different mathematics skill, such as performance in calculation fluency, math operation, and math reasoning. Pupils completed the Trait Emotional Intelligence Questionnaire-Child Form (TEIQue-CF). Two measures of mathematics were adopted from Wechsler individual achievement test (WIAT-III). Finally, math reasoning was assessed with the math standard achievement test, which was based on the National Standards for Mathematics Curriculum of China. The sample comprised 326 children (161 boys and 165 girls) between the ages of 8 and 12 ( $M = 9.50$ ,  $SD = .95$ ) attending three primary school students. The findings indicate that high trait EI scores related to calculation fluency ( $r = .22$ ,  $p < .001$ ), math operation ( $r = .31$ ,  $p < .001$ ), and math reasoning ( $r = .26$ ,  $p < .001$ ). To further examine the predictive efficacy of the trait EI, the nine sectors were entered into a regression equation for three mathematics tasks whilst controlling for gender. For the calculation fluency scores, after entering gender into the regression equation ( $F [1,324] = 2.02$ ,  $p = .16$ ;  $R^2 = .006$ ) the TEI scores were observed to predict a significant amount of variation ( $F [10,315] = 5.24$ ,  $p < .001$ ;  $R^2 = .14$ ). With regard to the regression concerning reasoning skills, after entering gender into the regression equation ( $F [1, 324] = 7.74$ ,  $p < .01$ ;  $R^2 = .02$ ) the TEI scores were observed to predict a significant amount of variation ( $F [10,315] = 8.27$ ,  $p < .001$ ;  $R^2 = .21$ ). With regard to the regression concerning operation skills, after entering gender into the regression equation ( $F [1, 324] = 2.629$ ,  $p = .106$ ;  $R^2 = .008$ ) the TEI scores were observed to predict a significant amount of variation ( $F [10,315] = 8.154$ ,  $p < .001$ ;  $R^2 = .206$ ). It can be concluded that a positive relationship between trait EI and mathematical skills can be confirmed already among elementary school children. Moreover, Trait EI holds important and multifaceted implications for the development of mathematical skills. Trait EI should be consciously and constructively developed in primary children. In the presentation, further results will be discussed in detail.

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# COMMUNICATING BIRTHDAYS' IN PRESCHOOL A WAY FOR CHILDREN TO UNDERSTAND TIME

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This presentation has the overall aim to describe how preschoolers communicate birthdays to understand time changes and mathematical concepts related to time. The questions posed in this presentation is: How can children's understanding of time be challenged in preschool? And, how can preschool teachers use children's concrete events or activities to understand time?

Previous research has shown that preschool children who spontaneously use and understand numbers, number and quantity in everyday life are better at solving mathematical problems in school (Hannula & Lehtinen, 2005). It has also been shown that young children's mathematical communication can be developed if teachers integrate mathematics in playful activities (Hye Young & Reifel, 2011; Klibanoff et al, 2006).

This study is based on a sociocultural perspective where children's understanding of mathematical concepts can be developed through social interaction (Wertsch, 2007). Thirty-one children between 3-6 years and three preschool teachers participated and they were observed under the duration of 10 months in daily activities in preschool. Their interaction was documented with video camera, photos and field notes.

The analysis of empirical data shows that children use time concepts when they for example compare their birthdays in interaction with others. It also shows that a supportive environment where children can be challenged to use time concepts can contribute to their understanding of time and time changes.

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# NOTICING STUDENT GENERALIZATIONS AND JUSTIFICATIONS: DOES TASK CONTEXT MATTER?

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Building on Jacobs, Lamb, and Philipp's (2010) description of *professional noticing of students' mathematical thinking*, i.e., attending-and-interpreting students' thinking, and responding to students, our study draws attention to the nature of tasks that have the potential to support the development of professional noticing skills of pre-service teachers (PSTs) preparing to teach grades 1-8 mathematics. Focusing on PSTs' ability to notice mathematically significant aspects of student-generated justifications, we examined possible impact of tasks that engage PSTs in analysing video-records of students' work, and tasks that engage PSTs in analysing written artefacts of students' work, on PSTs' professional noticing skills. Our work contributes to the discussion about enhancing PSTs' ability to identify significant aspects of students' mathematical thinking, interpret the validity of students' reasoning, and respond to students in a way that connects to students' thinking. We engaged 15 PSTs in analysing justification and generalization strategies of elementary school students who reasoned about figural patterns. Student work was presented in form of (a) video-records of group interactions (2 tasks), and (b) written artefacts (2 tasks). Depending on the task, PST examined three or four elementary school students' responses. We thus identified and examined 60 attending-and-interpreting and 60 responding segments across PSTs' analyses of the video-tasks, and 45 attending-and-interpreting and 45 responding segments across PSTs' analyses of the written artefacts tasks. Scores on attending-and-interpreting were significantly higher at the 0.05 level for the written artefacts tasks compared to the scores on the video tasks ( $M = 1.444$ ,  $SD = 0.559$  &  $M = 0.95$ ,  $SD = 0.484$  respectively;  $z = 2.104$ ). The same was true for PSTs' scores on responding ( $M = 0.889$ ,  $SD = 0.411$  &  $M = 0.533$ ,  $SD = 0.471$ ;  $z = 2.476$ ). Our qualitative results document different characteristics of PSTs' practices of attending-and-interpreting, and responding depending on the context of the task: analysing video-records or written artefacts of student work. The results highlight the potential of different tasks to promote noticing skills. Implications for teacher education will be discussed.

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# PROBLEM SOLVING IN THE EYES OF EGYPTIAN TEACHERS: A CASE STUDY AT A NATIONAL CURRICULUM SCHOOL

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An ongoing debate in the literature has been to define and classify problem solving tasks (Stein, Smith, Henningsen, & Silver, 2000). Depending on internal factors such as the teachers' mentality and background, the cultural context of the school and external factors such as stakeholders' expectations and agendas, problem solving tasks could be perceived and executed by teachers in different ways (Goodson, 2000). This case study aims to cast some light on the power dynamics associated with teachers' views about a key mathematical activity (problem solving) in a cultural and curricular context (the national curriculum schooling in Egypt) where the activity might not be in line with daily instructional practices. The research question is twofold:

- Within the framework of national curriculum schooling in Egypt, how do individual mathematics teachers view mathematics problem solving tasks and how do they perceive the suitability or barriers of adopting those tasks into the context of their daily instruction?

The data collection comprised a focus group of three mathematics teachers teaching at a national curriculum school. The discussion was triggered by a task sorting activity. Teachers were asked to single out tasks that they would associate with problem solving or with their local teaching context respectively, and to share and discuss their choices. A thematic analysis inductively unfolded and contrasted patterns in the focus group transcripts. Results showed that teachers seemed to favour adopting tasks that establish a connection to daily life, to a pre-studied mathematical concept or that relate to a new idea that students have not seen. These three features have also been associated to problem solving tasks. In their local setup, however, teachers seem to be constrained to adopting repetitive reproduction tasks (Stein et al., 2000). This is due to internal limitations such as the school policy and other external restrictions such as parents' demands, curriculum requirements and time constraints. The same analysis has been replicated across more teacher focus groups and similar results have been identified. Consistency in findings implies a need to re-assess the gap between teacher desires and contextual limitations as will be more elaborately discussed at the conference.

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# STIMULUS OF PROBABILISTIC THINKING BY ENGAGING CHILDREN AND PRIMARY TEACHERS IN GAME INVENTION

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Recent researches highlight the importance of problem posing in mathematics teaching and learning (Singer, Ellerton & Cai, 2015). On the other hand, researchers of educational psychology highlight the importance of games during children's mathematical learning processes (Nikiforidou, Pange & Chadjipadelis, 2013). With these frameworks, we have researched the stimulus of probabilistic thinking in children by engaging them in game invention, and we are currently broadening this research to consider primary school teachers.

Our research questions are: how to use playful situations to stimulate children's probabilistic thinking, and how to improve teachers' mathematical and didactic knowledge of probabilistic thinking. Looking for answers to these questions, we did a case study with five 6 to 10-year-old children, and we are currently working with six in-service primary teachers. In each session, we play a previously structured card game on decision-making with each one of them. Then, we ask him/her to invent another game by modifying the previous one, and we play it with him/her. We observe and compare their emotional reactions, as well as the relation between their decisions and probabilistic thinking, both at level of System 1, which is fast, automatic and intuitive; as well as System 2, which is slower, analytical and where reason dominates. (Kahneman, 2011). We complement the information with interviews.

We have verified that this game invention dynamic on decision-making generates positive emotions, making it more attractive and motivating, which helps children to develop their probabilistic intuition and teachers to improve their mathematical and didactic knowledge. Accordingly, it seems that this game invention dynamic has a significant impact on the development of children and teachers' probabilistic thinking. Moreover, it has a positive effect on the strengthening of creativity, self-efficacy, self-esteem, the ability to ask questions and the enjoyment of mathematics.

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# INSTRUCTOR CREATION OF LECTURE NOTES WITH DIGITAL TEXTBOOKS

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Digital open source textbooks eliminate financial barriers to education, particularly in countries in which education costs are high, as they are distributed at no cost to students. For this reason, they are becoming more ubiquitous in tertiary education. However, empirical research on the uses of such textbooks by instructors is limited, so affordances for student learning have not been realized yet.

We report on a study, *Undergraduate Teaching and Learning in Mathematics with Open Software and Textbooks* (UTMOST, Beezer et al., 2016), that seeks to describe how instructors use two digital open source textbooks, *First Course in Linear Algebra* and *Abstract Algebra: Theory and Applications*; specifically, when instructors create the lecture notes they present to students. We discuss the answers to the question, How do instructors use digital textbooks in creating lecture notes? To the best of our knowledge there is limited research on the creation of lecture notes.

To this effect we traced the processes of instrumentation and instrumentalization of the documentational approach (Gueudet & Trouche, 2009); in particular, we studied the development of the document *lecture notes*, focusing on the use of digital open source textbooks and other resources in its creation. Observational and interview data were collected during weeklong visits to seven instructors' test-sites in four states; we video recorded lesson planning, lessons and three interviews with each instructor.

We found that all instructors created their lecture notes maintaining the notation and definitions presented in the textbook, even when they presented proofs different than those of the textbook coming from other resources. They used the dynamic features of the textbooks (e.g., computing cells) depending on their knowledge of and familiarity with those features. Across the seven instructors, we found a continuum of textbook use for the creation of *lecture notes* from extensive to minimal. In the presentation, we will draw on cases of use of dynamic features of the textbooks to illustrate the various processes of instrumentation and instrumentalization which help explain the continuum of textbook use in producing the *lecture notes*.

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# THE $\varepsilon$ - $\delta$ DEFINITION FOR ONE VARIABLE REAL FUNCTION REVISITED

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Typically, the practice in mathematics departments is that definitions of fundamental concepts are presented to students with limited discussion on their intuitive underpinnings and a delineation of their structure. As stated in Parameswaran (2010) some mathematicians emphasize that “equivalent reformulations of definitions enrich understanding”, but there is limited research evidence on this for concepts at the advanced mathematics level. This study examines to what extent students of mathematics can build up alternative formulations of the  $\varepsilon$  -  $\delta$  definition of the limit for one variable real functions, a concept that pertains essentially to the structure of the reals and the axiom of completeness.

As a case study account, we present the results from only two of the participants in our fieldwork that we considered interesting to our investigation. The questions were designed in a *scaffolding* mode. The students were first asked to ‘scan’ the  $\varepsilon$  -  $\delta$  definition and then were lead towards an alternative definition, based on the interval of  $\delta$ ’s: the subjects were asked to consider the function  $h(\varepsilon)$  that designates the length of the interval  $(x_0 - \delta, x_0 + \delta)$  or otherwise the ‘greatest’ possible value of  $\delta$  for a particular  $\varepsilon$  and via this function to express the condition for the existence of a limit at a point. The function  $h(\varepsilon)$  underlies the effect on  $\delta$  when  $\varepsilon$  is decreased and indicates the ‘rate’ of convergence.

The first student was following a Master of Sciences program in the Didactics of Mathematics; the second was an undergraduate currently following a Real Analysis course. Both students showed a good understanding of the standard  $\varepsilon$  -  $\delta$  definition and the character of  $\varepsilon$  and  $\delta$  as parameters. When considering the set  $S_\varepsilon$  of  $\delta$ ’s, both gave an adequate proof of the fact that  $S_\varepsilon$  is an interval. However, they did not seem to have a clear picture of what happens at the ends of the interval. Their answers to the core question on the significance and form of  $h(\varepsilon)$  betrayed an ‘instrumentalist’ logic; either as a ‘base’ for investigating the limit, or as a way to find the biggest interval. Neither of them gave an explicit expression for  $h(\varepsilon)$ , that necessarily would involve a supremum or an infimum. In the talk, certain questions and answers will be discussed. Despite the limitations in practice of this approach, we believe that it enhances the interiorization of pivotal mathematical concepts and thus their use in proving and problem solving.

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# STUDENTS' DIFFICULTIES WHEN SOLVING SPATIAL TASKS

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Several studies have already pointed out the need to investigate student spatial thinking by analyzing student spatial discourse (cf. Levinson, 1996). This stand point differs from previous studies about spatial ability who tended to focus on assessment of spatial ability using pencil-and-paper tests (e.g., Linn & Petersen, 1985). In this study, an analysis of student spatial language in the solving of spatial tasks should provide results to the following research question: Which difficulties do students encounter when describing spatial objects? In contrast to previous models of spatial ability, recent models emphasize the role of language as an important medium for the externalization of (mental) spatial abilities. As Levinson (1996) states, an analysis of spatial language, which is the language about space and its objects, enables the investigation and understanding of the underlying spatial concepts.

In the reconstruction method, the research method applied in this present study, two students are assigned two different roles, the describer and the builder. Sitting back-to-back, the describer is required to instruct the builder what actions to do and the builder is required to interpret and perform these instructions. In the case of this study, the describer obtained a pre-designed spatial object (see Figure 1) and were required to describe it to the builder who translated the instructions into spatial actions on manipulatives (building cubes). At the end, the students compared the original object (of the describer) with the constructed object (of the builder). The participating sixth-grade students were video-recorded and the data was transcribed and analyzed for difficulties during the solving process.

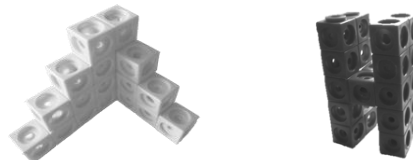


Figure 1: Spatial Object A (left) and Spatial Object B (right).

The results show that different types of difficulties can arise when students solve spatial tasks. Whereas some students had problems during the verbalization of spatial actions or the decoding and interpretation of spatial language, other students showed difficulties during spatial orientation or during the perception of spatial actions or of properties of objects.

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# THE CONVERSION OF PERSPECTIVES ON ANGLES THROUGH THE OPERATION OF SIGNS AND ATTENTION

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The purpose of this study lay in analyzing the *shifting of students' attention* through which students learn to justify the sum of the exterior angles of a polygon in accordance with Deleuze's perspective on signs and perception.

Signs as defined by Deleuze can be viewed as materials that break down learners' macro-perception and form new micro-perception (Deleuze, 1993). Here, macro-perception signifies perception that learners can consciously recognize, perception that follows existing habits or rules, and perception that is structural. Micro-perception is perception that cannot be sensed consciously but is recognized unconsciously, perception that stimulates new creation, and perception that shocks and confuses learners by breaking down existing habits and rules (Deleuze, 1993). Attention is considered a primary operation of perception and is an important perceptual function leading to mathematical awareness (Watson & Mason, 2005). So, adopting Deleuze's perspective above, the present study defines *macro-attention* as pre-learning attention that can be recognized by learners, follows existing habits and rules, and is structural and *micro-attention* as attention that is unconscious, is free from existing habits and rules, is accompanied by shock and confusion on the part of learners, and induces changes in their actions.

The present study analyzed the case of a classroom session at a middle school where students succeeded in converting their *perspectives on angles* through the types of attention. Students had the habit of viewing angles solely from the perspective of static shapes. But after students walk along the tape in the shape of a square and triangle, their macro-attention oriented to interior angles is converted to attention oriented to micro-attention to exterior angles. As a result, they expressed, through language, gesture, and tape, the meanings of exterior angles, which he had not learned previously. These results support previous studies about the relation between agential materials and perception (de Freitas, 2016). In the presentation, further detail will be discussed.

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# **THE INFLUENCE OF MATHEMATICS TEACHERS' COMPETENCIES, ATTITUDE AND INTEREST ON THEIR PROFESSIONAL BEHAVIOR**

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Mathematical Competence for teaching secondary school teachers according to Wilson & Kim (Camacho-Machin, Moreno, & Afonso, 2016) is structured around three dimensions that are: 1) Mathematical Ability; 2) Mathematical Activity; 3) Mathematical work of teaching. The attitude of teachers according to Patra and Metch (2011) is almost as important as students' attitudes toward mathematics. Teachers are key to improving mathematics education. Interest is a sense of attraction and attention accompanied by confidence in the usefulness of something that encourages a person to seek or obtain it. In the theory of Fishbein and Ajzen (1975), interest is strongly influenced by two variables, namely: attitude variables and subjective norms.

This research used correlational quantitative approach with ex post facto design and only explore the facts that have occurred in the subject itself through the questionnaire. The subject of this research is 52 mathematics teachers who come from several areas in west java Indonesia and become participants of training and education of teacher profession. Mathematics teachers were given training and workshops on mathematics learning for 10 days. On the last day of the training, they were asked to conduct a learning simulation to be commented upon and assessed.

Based on the research findings can be concluded that of the three aspects that exist in mathematics teachers, then the most influence on their professional behavior is the competencies of teachers. Mathematics teachers' competencies consisting of pedagogical, social, personality and professional competence or content knowledge, can have an impact on the mathematics teachers' professional behavior.

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# THE ROLE OF INTERACTIVE WHITEBOARDS TO SUPPORT A GROWTH MINDSET IN ALGEBRA LEARNING

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Algebra is a well-known area of difficulty for many students, in particular understanding the core concepts *equivalence* and *variable*. Understanding these concepts is essential for succeeding with algebra and mathematics in general. To stimulate algebraic reasoning at the elementary level is considered essential for in-depth algebra learning (Carraher & Schliemann, 2016). Further, opportunities to convince and reason, as well as experiencing mathematics as a creative subject seeking for connections and patterns, are all factors emphasised as important for developing a growth mindset (Boaler, 2016), crucial for deep learning.

Interactive whiteboards (IWB) have, through affordances such as multiple visualisations and the ability for movement and animation, important potentialities for supporting the development of mathematical concepts and improve understanding (De Vita, Verschaffel & Elen, 2014). This study investigates how whole-class conversations based on visualisations and animations on an IWB can support a growth mindset in algebra learning, with particular emphasis on algebraic reasoning connected to equivalence and variable. As part of a larger Norwegian project *ARK&APP*, all the algebra lessons of 23 5<sup>th</sup>-grade students and their teacher were video-recorded for three consecutive weeks in December 2014.

The oral communication will present findings from the study, emphasizing how visual representations and animations on the IWB can support learning of equivalence and variable through exploring core ideas of what constitute developing a growth mindset; in particular the possibility to reason and to explore, and the nature of the questions asked – by the students and the teacher. The presentation will further illustrate how the teacher facilitated for learning through building a bridge between visual representations related to contexts that are relevant to students' everyday life, the classroom dialogue, and the core concepts.

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# A COMMOGNITIVE ANALYSIS OF GRADE 12 LEARNERS' PARTICIPATION IN A SITUATED MATHEMATICAL ACTIVITY

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There are generally two kinds of activities that are used in mathematics classrooms: those perceived to have visible connections to mathematics and those that are far-removed from mathematics. In this paper we report on an analysis of learners' experiences of an activity of the second type: the Kangaroo and Crossing the River activities. Mathematical process skills such as communication, representing, connecting, reasoning and problem solving (NCTM, 2000) appear to be the mostly displayed skills in the contextualized challenging game tasks such as these.

Completing a challenge successfully lies in developing strategies that are often based on the intuition of those engaging in it. The combination of individual learner's strategies into collective acquisition is seen as a complex process. As such by focusing on observable discursive acts, a dialogue was documented among a class of 69 grade 12 learners from a high school in Pretoria in South Africa on how they thought one would solve these challenges. A theory of thinking as communicating (Sfard, 2008) informed our analysis of learner dialogues. The analysis showed that learner interactions were dominated by word use followed by visual mediators, routines and narratives. Learners made limited use of routines that resulted in endorsable narratives. We argue that although this was the case, the Kangaroo and Crossing the River activities appeared to have provided new experience of learning mathematical concepts linked to number patterns, algebra and functions among learners. Although the Kangaroo and Crossing the River tasks appear far-removed from mathematics, they are viewed as more appealing and appropriate for encouraging participation of all learners, and hence enable learners to engage in more desirable experiences that are critical for gaining access to mathematics.

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# EXPLORING PRE-SERVICE MATHEMATICS TEACHERS' SUBJECT MATTER KNOWLEDGE OF SCHOOL MATHEMATICS CONCEPTS

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To make teacher preparation and professional development effective, it is important to explore possible competencies and deficiencies in teachers' knowledge. By focusing on Pre-service mathematics teachers' subject matter knowledge (SMK) in this paper, we conceptualize the notion as containing both the knowledge of the content and knowledge for teaching. With specific reference to Ball, Thames and Phelps (2008) model for Mathematical knowledge for teaching, this paper focuses on pre-service teachers' competences of Common Content knowledge (CCK) and Specialised Content Knowledge (SCK) of high school mathematics.

The data presented here is from the first phase of the project, which focused on pre-service teachers' competencies of solving school maths problems and analysing errors and misconceptions in learners' responses. The sample comprises of 180 pre-service teachers, across three Teacher Training Institutions (TTT) in the Province of KwaZulu-Natal, South Africa. Data was collected using activity sheets consisting of three set of questions on functions. The analysis involved generating quantitative data showing their competences and qualitative data revealing the competencies across concepts. Lastly, it involved the categorisation errors evident in some of the responses. The categorisation of errors was informed by Siyepu's framework (2013).

The results reveal that while many PMTs' become competent with subject matter knowledge of university mathematics their subject matter knowledge of school maths is weakening. Therefore, there is a need for TTI to ensure the development of pre-service teachers' subject matter knowledge of school mathematics concepts. In the presentation, more results will be discussed in detail.

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# PROOF COMPREHENSION OF UNDERGRADUTE STUDENTS

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Proving is one of the major activities in mathematics. In the first semester of a mathematic study program, students show serious problems understanding written, correct proofs. According to Mejia-Ramos et al. (2011), proof comprehension includes besides understanding on a local basis (e.g. in terms of the meaning or the logical status of its statements), understanding on a global basis (e.g. in terms of the proof's high-level ideas, its main components or modules). However, there is limited research of undergraduate students' proof comprehension in general, and particularly which factors influence proof comprehension. Therefore we focus on the question:

- Do learning prerequisites (like school achievements) correlate with proof comprehension?

In reference to a similar approach by Hodds et al. (2014), we developed a proof comprehension test in number theory using the assessment model for proof comprehension given by Mejia-Ramos et al. (2011). Our sample consists of 114 students slightly before the first semester of their university course (e.g. business mathematics, mathematics, or computer science) at a German university. The students completed the test in the middle of a mathematical bridging course, a course to support the transition from school to university. The test includes a number theory proof and 11 open-ended or multiple-choice items. Additionally, the students reported their mathematics, German, and overall school grades.

The reliability of the test ( $M = 6.35$ ,  $SD = 2.37$ ,  $Max = 11$  points) is medium. The proof comprehension measured by the test correlates with the grade in the final exams and with the most recent school grade in German, but not with the most recent school grade in mathematics. This is an unexpected result and may relate proof comprehension to reading comprehension, which inspires new ideas for developing support courses for first-years. Additional results will be discussed in the presentation.

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# IN-SERVICE MATHEMATICS TEACHERS' VIDEO-BASED NOTICING OF 3D PRINTING PENS 'IN ACTION'

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This study aims to contribute to existing literature by engaging in-service mathematics teachers in a video-based noticing activity for professional development, in particular in the area of technology integration. In particular, I explore teachers' noticing of the educational potential of an emergent technology: a handheld "3D Printing Pen", which enables one to draw 'in space', and to touch, transform, and interact with the 3D-printed models during the meaning-making process (Fig. 1). This study adopts Star and Strickland's (2008) observation categories, i.e. classroom management, classroom environment, communication, mathematical content, and tasks to analyse four in-service mathematics teachers' noticing upon watching videos capturing an actual mathematics lesson integrating the 3D Printing Pens for teaching "properties of prisms". After watching each of the nine episodes, semi-structured interviews were conducted, in which I analyse what the participants generally identified as important or noteworthy in the video. In addition, the participants engaged in refining the lesson upon watching the episodes, which prompted a thematic analysis for delving deeper into the participants' interpretations and decisions in relation to the use of 3D Printing Pens for teaching and learning mathematics.



Figure 1: 3D Pens 'in-action'.

Results suggest that the teachers initially attended to the physical appearance of the student-drawn 3D models, i.e. sturdiness and size, but they soon realised that the drawing processes with 3D Printing Pens afforded students' visualisation of 3D solids. Hence, there was an important shift from the product to the process of drawing that enabled the teachers to attend to mathematical ideas in robust ways that would enhance the teaching and learning of mathematics. In particular, the teachers' discourse change was facilitated by re-enacting the drawing process, either through gesturing or drawing diagrams on paper. The findings that the teachers attended to classroom management and later mathematical content suggest a synergy of efficiency and innovation within teachers' noticing for realising the transformative potential of emergent technologies.

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# NUMBER ESTIMATION, WORKING MEMORY AND QUANTITATIVE REASONING: A STUDY WITH 3<sup>rd</sup> AND 4<sup>th</sup> GRADE SCHOOL CHILDREN

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Number estimation has been the focus of many studies into mental number representations, especially in young children. There is evidence that both number estimation (e.g. Link, Nuerk, & Moeller, 2014) and working memory are correlated (e.g. Lee & Bull, 2016) with mathematical achievement. However, little is known about the relation between number estimation ability and working memory capacity.

Therefore, the research aimed to analyse the relations between the performance of a group of students in number estimation, working memory and quantitative reasoning tasks. The sample was composed of 143 3<sup>rd</sup> and 4<sup>th</sup> grade students ( $M = 9.8$  years old,  $SD = .74$ ) from two public schools in Porto Alegre/Brazil. The students were evaluated using two number line estimation tasks: number-to-position (NP) and position-to-number (PN), working memory tasks involving the four components (phonological loop, visuospatial sketchpad, central executive and episodic buffer) and a quantitative reasoning task, which assessed the students' abilities in solving additive and multiplicative reasoning problems. Number estimation and quantitative reasoning tasks were applied separately in groups of 10 students, lasting 30 minutes per group. Working memory tasks were administered individually.

We found significant relations between number line estimation ability, working memory capacity and quantitative reasoning skills, specifically the highest levels of correlation were found in the central executive (NP:  $r_s = -.189$ ,  $p < .05$ ; PN:  $r_s = -.257$ ,  $p < .01$ ) and episodic buffer components (NP:  $r_s = -.190$ ,  $p < .05$ ; PN:  $r_s = -.218$ ,  $p < .01$ ) and additive reasoning (NP:  $r_s = -.378$ ,  $p < .01$ ; PN:  $r_s = -.394$ ,  $p < .01$ ). Even though there was an association of working memory and number estimation, quantitative reasoning presented a higher correlation with number estimation.

Thus, the results may contribute to the discussion on the role of number estimation in mathematical achievement and the influence of working memory on number estimation ability.

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# HOW DOES VIDEO ANALYSIS OF TEACHING LEAD TEACHERS TO ACTION, OR TO CHANGE?

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## PURPOSE AND RESEARCH QUESTIONS

There has been an increase in the use of video for teacher reflection. Related studies reported that video was a beneficial feedback method for teacher reflection (e.g., Powell, 2005). However, there is not a synthesis of practices and processes (nor a framework) for effective video analysis that encourages teachers to change their practices. The purpose of this study is to introduce a guiding framework for using video to support teacher change in hoping to help educators make more informed decisions as they establish their own video analysis processes. The study was guided by the following research questions: (1) What are the dimensions of video analysis process that seem to lead teachers to change their practices? (2) What are the stages of implementation of video analysis process that seem to lead teachers to change their practices?

## METHOD

Our video analysis process was carried in the context of a professional development program (PD) as a learning community (Buffum & Hinman, 2006) where 16 secondary mathematics teachers were provided with 10 sessions, each for 6 hours, over 8 months. Major activities of the PD included learning teaching analysis protocols and analysing videos of their own teaching and other teachers' teaching. Data collection included pre- and post-surveys, focus interviews, recorded session discussions, and written work.

## RESULTS

Some of the dimensions of video analysis process that encouraged teacher change were having a focus for analysis, analyzing both individually and with participating teachers and/or researchers, allowing 'learning time' between recordings. Putting a substantial amount of efforts in developing lesson plans (individually and collaboratively) that reflect the results of analyses appeared to be one of the significant stages of the video analysis process.

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# INFLUENCE OF AN OUT-OF-SCHOOL MATHEMATICS MENTORING AND TUTORING PROGRAMME ON PARTICIPANTS' IDENTITY FORMATION

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The National Council of Teachers of Mathematics (NCTM, 2000) argues that: "...students' understanding of mathematics, their ability to solve problems, and their confidence in, and dispositions toward mathematics are all shaped by the teaching they encounter in school" (pp.16-17). This is because mathematics learning, it is argued, involves a process of socialization requiring that individuals learn particular ways of knowing and doing mathematics as valued in a particular learning environment (Cobb, Gresalfi, & Hodge, 2009). Therefore, different learning environments can lead to different ways of knowing and valuing mathematics. As such, it is entirely possible for a student to experience a change in his or her disposition towards mathematics.

In highlighting the potential of such environments to support a positive turnaround in students' mathematics socialization process, Walker (2012) also called for a departure from studies which focus on "inadvertent" spaces to those "intentional" spaces that contribute in strong ways to mathematics socialization and talent development for larger groups, particularly for underserved students" (p.68). The study reported in this paper addresses this concern by examining the influence of an intentionally-designed mathematics mentoring and tutoring programme known as Prepare2Nspire (Covington Clarkson, Ntow, & Tackie, 2014) on high school students' mathematics socialization and hence, identity formation process. The research questions are: How are normative identities as doers of mathematics constructed within selected P2N communities and its influence on P2N participants' developing mathematical identities?

Using an interpretive framework developed by Cobb, et al. (2009), video-taped sessions, questionnaires, and semi-structured, reflective interviews data were coded to identify themes related to the normative tutoring obligations and the resulting personal identities. Findings indicate that a combination of factors such as the normative tutoring obligations and access to particular resources were influential in participants' mathematics identity formation.

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# PRIMARY STUDENTS EMOTIONS TOWARDS MATHEMATICS

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Several studies show that students' interest in and motivation towards mathematics declines from around early primary years and that this decline seems to be consistent for the whole compulsory school period (e.g. Blomqvist, Elamari, & Sumpter, 2012; Hannula, 2012). Affective factors, how you think and feel about mathematics, have an impact on student achievement, therefore it is important to investigate how students think and feel about mathematics and mathematics education. In this study, a questionnaire instrument containing a four-step likert scale formatted as the choice of face-cards developed by Blomqvist et al. (2012) were replicated as an interview instrument for making semi structured interviews with complementary follow up questions. The aim is to study students' expressions of their affective relations to mathematics education. This presentation focus on the following research question: what emotions towards mathematics are expressed by primary students?

The analysis was made using thematic analysis (c.f. Braun & Clarke, 2006) comparing with previous research (Blomqvist et al., 2012) but also with an inductive approach searching for subthemes. The initial findings indicate that students express negative emotions to feelings of stress or fear in relation to (a) not getting the right answer, (b) not having enough time to finish tasks, or (c) not getting the help they need. Students generally describe these negative emotions as related to themselves. The positive emotions are of three types; (1) joy, (2) challenge or (3) a type of double negative as the absence of some stressant – most often of category (a) or (b) above. In relation to Blomqvist et al. (2012) these findings display complex and nuanced descriptions that uncovers a group of students that expresses a spectrum of emotions, each emotion connected to one in a variety of factors.

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# A TOOL FOR PLANNING AND ORCHESTRATING MATHEMATICAL DISCUSSIONS

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Orchestrating a mathematical discussion is an important skill for teachers to develop, since mathematical discussions make up powerful classrooms (Schoenfeldt, 2014). However, that skill is neither innate nor easy to convey to pre-service teachers. This study reports on the development of a communication competence framework based on research that stresses the importance of mathematical discussions (e.g. Franke, Kazemii & Battey, 2007, Kazemii & Hintz, 2014). The aim was to create a useful tool for planning, conducting and analysing mathematical whole class discussions.

Using a lesson study approach, a framework was developed and included as a course element in four different mathematics education courses for pre-service teachers. It was tested and revised through an iterative process, using video observations of lectures and workshop activities, as well as written reflections and examination tasks.

Our study resulted in a framework describing the interplay between two main aspects of mathematical discussions: *mathematical objectives* and *talk moves*, as follows: First, the planning of a whole class discussion starts with a focus on the mathematical objective (*strategy sharing, focussing, comparing, justifying*). Then, pupils' strategies and questions are anticipated and the teacher orchestrates a discussion making use of specific talk moves. Finally, the mathematical objectives are revisited when analysing the discussion and assessing what learning has occurred. A significant breakthrough in the development of the framework was when we made a clear distinction between moves to initiate student actions (*think quietly, talk in pairs, describe, reason, repeat, add on, revise*), and moves indicating teacher actions where new mathematical input is added (*revoice, challenge, question*). The final version of the framework proved much appreciated by the students in helping them to conceptualise and decompose mathematical discussions. The framework contributes to teacher education by making pre-service teachers aware of basic components of a mathematical discussion.

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# DIFFERENCE AND REPETITION – INSTRUCTIONAL EXAMPLES

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Within the educational design-based research paradigm (McKenney, Nieveen & van den Akker, 2006), this study designed and evaluated a professional development (PD) intervention for grades 7–9 and upper secondary school teachers. The PD intervention was designed to support teachers in the teaching of first-degree functions. Using the theory of variation (Marton, 2015) and Deleuze's (1994) philosophical concepts as the main interpretative framework, the study investigates what is involved in repetition and the nature of its interiority. The key objective of the research study was to evaluate the effectiveness of the design of the PD intervention and to generate design principles that can be used by other researchers developing PD interventions for building generalization concerning the concept of first-degree functions in a particular context. By means of a mixed methods research approach, data was collected in a three-year longitudinal study that involved two teachers from secondary schools and two teachers from upper secondary school. The focal point for data analyses was the examples that teachers had planned to use and had been used in the classrooms. Qualitative data was analysed using Deleuze (1994) frameworks. The findings revealed that the intervention had a positive impact on participant teachers' construction of examples for building generalization concerning the concept of first-degree functions. The groups progressed in the development of their choice of instructional examples at different paces and levels of advancement. The potential of repetition in the teaching and learning of school mathematics relies on the process of producing examples, which involves design and analysis. Design may include: (a) implementation of a known procedure to generate the successive terms in, for example, an arithmetic or geometric sequence or (b) modification of a given procedure to use a new but related example. Analysis is the process of determining the differences in repetition. This paper argues that philosophical research is needed to enhance theoretical understanding of how mathematics teachers learn and develop.

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# TEACHERS' ABILITIES TO IDENTIFY PROPORTIONAL SITUATIONS

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There is considerable evidence that determining whether a situation is directly proportional is difficult for students (e.g., Van Dooren, De Bock, Janssens, & Verschaffel, 2008). Such research is only starting to emerge about teachers (e.g., Izsák & Jacobson, 2017).

This talk will focus on one aspect of our research on teachers' abilities to discern situations that are proportional from those that are not. For this study, we look at results from two samples. One included 179 6<sup>th</sup> to 8<sup>th</sup> grade teachers who completed a written assessment that included nine items in which they were asked to determine whether a situation was a direct proportion. The second sample included 32 teachers who completed the same assessment plus an additional 11 items examining their ability to discern proportional relationships.

Our results report teachers' abilities to properly identify situations that are directly proportional. They also look at the kinds of underlying mathematical structures in each situation to examine which structures are more or less likely to be incorrectly identified as being proportional. Specifically, we talk about linear relationships, inverse relationships, and exponential relationships. We found that teachers are generally quite good at recognizing situations that are proportional, but often misidentify those that are not. Most notably, Linear Relationships were often misidentified. This has implications for teacher development as it helps identify structures that are confused with proportions and highlights the need for considering the mathematical structures of problems in teacher education.

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# RESEARCH SITUATIONS IN ELEMENTARY SCHOOL

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Since 1991, the French “Maths à Modeler” has designed didactical “research situations for the classroom” (RSC) with problems coming from the ongoing mathematical research thanks to a collaborative work between mathematicians and didacticians. The aims of an RSC are to put students in the role of a mathematical researcher. We use RSC as a way to identify pupil’s potential reasoning processes, which are little explored in mathematics education (e.g. Stylianides, 2007). In order to analyze and implement RSC, we use the Theory of Didactical Situations, in particular the concepts of *devolution*, *didactical contract* and *didactical variables* (Brousseau, 1997). An RSC has to fulfil six criteria (derived from Ouvrier-Bufferet, 2009). Some are close to those of problem solving, imply a specific contract in the classroom and put pupils in a research activity. Firstly, there are only local ending criteria and possibly no final ending: an answered question often leads to a new question. Secondly, the students should manage their research themselves (didactical variables are left to the students, while the teacher can set the other ones). Thirdly, the problem should be easily understood. Fourthly, initial strategies without notional pre-requisites exist. Note that the last two criteria make the devolution of the problem easier. Fifthly, several strategies are possible to enable the research process and the emergence of mathematical skills and knowledge. Sixthly, at the end of the research process, students are invited to share their results (e.g. through posters in their school). We will illustrate these features with a variation of the *Pentomino Exclusion Problem* (Golomb, 1965), develop the reasoning processes that can be mobilized by pupils (4<sup>th</sup> & 5<sup>th</sup> grades), and then underscore perspectives for the teaching of research processes and proof at elementary school.

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# CONSTRUCTING AXES OF SYMMETRY AND VERTEX FOR QUADRATIC FUNCTIONS THROUGH QUANTITATIVE REASONING

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As many of the properties of the quadratic functions can be transferred to the polynomial functions and other functions, it is crucial to learn quadratic functions (Metcalf, 2007). Ellis (2011) stated that reasoning about quantitative relations could support students' flexible thinking in terms of quadratic functions as in various functional approaches. However, the existing literature lacks research studies deeply focusing on what the students understand about the quadratic functions (Nielsen, 2015). It is of importance for the students to construct the quantities of the graph, namely the parabola, along with algebraic expression to provide the students with conceptual understanding of the quadratic functions. The present study is a part of the design-based research in which an instructional sequence was designed for teaching quadratic functions. The purpose of the study is to enable students to construct the quantities of the axes of symmetry and vertex through quantitative reasoning. With this aim, two 10<sup>th</sup> grade students worked in pairs on the tasks triggering quantitative operations to construct vertex and axes of symmetry via a teaching experiment. In this activity including a real-life situation about throwing a basketball, students firstly converted the parabola, which represented the path of the ball, to the algebraic expression. Both the starting and ending points of the ball were at the same height. Thus, it became easier for the students to construct the axes of symmetry. The students comprehended that the distances of the points that were at the same height on the parabola to the axes of symmetry were equal to each other through additive comparison of these distances. They formed the meanings related to the vertex and axes of symmetry without any numerical procedures by means of the quantitative reasoning. Based on the results of the study, it is thought that real life situations are crucial for students' construction of these quantities and for preventing their tendency to rote learning approaches.

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# PROOF TRIPOD: PROOF IMAGE, ENLIGHTENMENT & FEELINGS

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As proof is one of the fundamental elements of mathematics, difficulties in proving have led to detailed discussions of this process in mathematics education. Thus, many theoretical frameworks that interpreted this process with different perspectives have been presented. Kidron and Dreyfus (2014) have introduced Proof Image, which is one of these perspectives, by associating different frameworks that explain the individual's learning process. It can be thought that the detailed analyses of sub-dimensions of this framework, which provides a new perspective on proving process by blending cognitive and affective aspects, will contribute to exploration of proving process. In proving activities students are expected to construct a new knowledge and to be enlightened due to it. It is also clear that all these activities and knowledge construction process can't be considered as independent from individual's feelings. In this regard, the present theoretical study aims at giving an insight into the following headings: 1) Sub-dimensions of proof image 2) The relation between knowledge construction process and enlightenment 3) Feelings in proving process.

As a result of the examination of these headings together, the cognitive dimension of the proof image was thought as a cyclical structure including the personal understanding, logical links, dynamism, and entity. It could also be said that the relations, established within the mentioned structure let the individual experience the insight (Aha!) experiences and enlightenment moments. At this point, the insight experiences can be defined as choosing of the structures that are available in the mind, however, the latter can be defined as giving meaning to the situation. These concepts were interpreted in the context of a jigsaw metaphor, thus choosing the piece corresponding to a void in the puzzle was considered as "Aha!" experience and the image, that appears in the puzzle when this piece is placed, was considered as enlightenment. On the other hand, it can be said that the feelings are discussed in different meanings in the literature, yet they are usually restricted in affective domain. However, in this study, feelings were adopted in the sense of Clore (1992) as "*all internal signs from bodily, cognitive or affective situations that provide a usable information about these situations*". In this context, the feelings accompanying this process were elaborated, and the affective dimension of proof image was reinterpreted.

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# SECOND-GENERATION MIGRANT STUDENTS FROM SINGAPORE: A PRELIMINARY STUDY OF THEIR MATHEMATICS PERFORMANCE IN AUSTRALIA

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While it is well-known that East Asian students have been topping international mathematics assessments such as TIMSS and PISA, it is not as well-known that the performances of many other economies in these tests have also been driven by East Asian students studying in these places. For example, despite being born and raised in a Western country with an “average-performing” (Jerrim, 2015, p. 16) education system, Australian students of East Asian descent still obtain scores consistent with their respective home countries at the top of PISA and TIMSS rankings. The case is also true for Singapore migrant students. While studies had been conducted to understand the contributing factors for the mathematics achievement of these students, there is a lack of literature on the mathematics achievement of Singapore students studying in education systems overseas. This study aims to inform this gap, by investigating the factors which could have contributed to the mathematics achievement of second-generation Singapore migrant students (SGSMS) in Australia. Here, SGSMS refers to students who were born and raised in Australia and had at least one overseas born parent (Jerrim, 2015). Thus, they would have experienced Australia’s educational system, its curriculum and institutional structures.

The data source was SGSMS students based in Melbourne, Australia. The theoretical framework adopted was based on a multilevel conceptual framework adapted from the Ecological Systems Theory proposed by Bronfenbrenner (1979). Data from the participating SGSMS were collected through semi-structured interviews. Factors contributing to the high mathematics achievement of SGSMS are conceptualized in accordance with Glaser and Strauss’ Grounded Theory. A positive relationship between family/cultural level factors and the mathematics scores of SGSMS was found. The findings are expected to shed light on the potential of adopting available cognitive and affective resources in contributing towards a more effective mathematics education process.

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# PROSPECTIVE MATHEMATICS TEACHERS' SELF-EFFICACY BELIEFS ON USING HISTORY IN MATHEMATICS TEACHING

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This study investigates the prospective mathematics teachers' (PMTs) self-efficacy beliefs on using history of mathematics (HM) in teaching mathematics. Using HM in teacher training program is a way of facilitating the future teachers' experience that creates and develops the mathematical concepts in an integrated way; and they will be able to apply it in their future classroom teaching/learning activities (Avital, 1995). Using HM is a teaching approach in mathematics education that utilizes the primary and secondary sources of history of mathematics (Tzanakis & Arcavi, 2000). In this study, the self-efficacy beliefs were used regarding the context of using HM. The self-efficacy beliefs consist of two dimensions: efficacy expectation and outcome expectancy. Efficacy expectation is an individual's beliefs in his/her capability to execute a behavior successfully, whereas outcome expectancy is his/her beliefs that the behavior will result in specific consequences (Bandura, 1977). The study measured the PMTs' self-efficacy beliefs through the adapted survey research instrument: *Behavior Scale* with the purpose of administering to 305 respondents in Phase I; and 80 classes of 8 PMTs were observed using *Classroom Observation Checklist* in Phase II. The validity of the instruments was assured by the scrutiny of 5 experts in mathematics education; and reliability was measured by Cronbach alpha ( $\alpha = .829$ ,  $N = 305$ ) to assess the internal consistency of the *Behavior Scale* items. Both the Likert data were analysed through the descriptive and inferential statistics. The findings derived from the analysis of both types of data showed that the PMTs had high sense of self-efficacy beliefs regarding the context of using HM in their classroom teaching behavior. The finding indicates that the PMTs' higher self-efficacy belief is a strong belief in their capability to teach mathematics effectively in their classroom. To strengthen the argument for using HM in pre-service programs, it is recommended that an expanded investigation might be conducted among in-service teachers.

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# STUDENTS' INTRODUCTION TO THE AXIOMATIC FOUNDATION OF MATHEMATICS THROUGH GEOMETRY

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Euclidean Geometry enables students to be introduced to the axiomatic foundation of mathematics, as it constitutes a coherent system of axioms, definitions and theorems that can be understood by them. Relevant literature recognizes the contribution of geometry to this understanding (Herbst et al., 2010; Boero, 2007), but there is very limited research for the development of teaching approaches related to this issue.

The present study focuses on a teaching intervention aiming at supporting 43 high school students to understand the role of axioms, theorems and definitions in Euclidean Geometry. Based on previous studies (cf. Jahnke & Wambach, 2013; De Villiers, 1998; Mariotti & Fischbein, 1997) a sequence of proving activities were given to them; in each task the students had to identify and discuss the content of each proposition as well as its status and role in the proof. The participants were pre- and post-examined in relevant tests and their performance was compared to that of a control group. The results indicated that the students participating in the teaching intervention improved their understanding related to the role of the axioms and the function of theorems and definitions in different geometric proofs. In general, however, they encountered difficulties to comprehend the general structure of an axiomatic system.

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# UNCATEGORIZABLE CASES IN DEVELOPING A FRAME FOR ANALYZING DIFFERENT MEANINGS OF THE CONCEPT OF THE VARIABLE MEDIATED BY MATHEMATICAL TASKS

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Pupils' studies in arithmetic can form a good foundation for understanding algebraic concepts and principles, if arithmetic is taken as a starting point for generalizing and sense-making discussions. One of the most important concepts in algebra is the concept of the variable, which can have many different meanings depending on its context. Finnish elementary school textbooks include a rich collection of tasks that have the potential of sociocultural mediation of the different meanings of the concept.

This study began by using Usiskin's analysis (1999) of the different ways variables are used in school algebra, and then combined it with Carraher and Schliemann's (2007) view of varying variables in equation solving. A frame was constructed for content analysis on the different meanings of the concept of the variable, which the tasks in elementary school textbooks have the potential to mediate. When developing the frame, it was tested using a sample of six books that were taken from three textbook series, one book from each grade. There were tasks that could not be categorized, and which, therefore, required additional research. The data in this presentation consists of those tasks, and the interest is in the type of them. The tasks were classified pursuant to the data, and nine classes of tasks were identified—five of which helped to expand the frame. In the tasks that required quantitative reasoning, the role of the variable was like what is used in generalized arithmetic. This was referred to as the general value, and a quantitative thinking category was added to the frame. The tasks from three other groups indicated that a second, more abstract category should be established for functions, functions as a correspondence, with the meaning of the varying variable in that context. Finally, simple inequalities fit in the category of equations. The frame developed can be useful for analysing the roles of the variable in curriculum materials.

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# USING MANIPULATIVES TO TEACH FRACTIONS IN CANADIAN AND CHINESE ELEMENTARY SCHOOLS

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The use of manipulatives in mathematics teaching is often prescribed as an efficacious strategy. The major theoretical rationale for the use of manipulative materials has been attributed to the works of Piaget, Bruner, and Dienes who suggested that children's concepts evolve through direct interaction with the environment, and materials provide a vehicle through which this can happen (Post, 1981). Our earlier study shows that Canadian and Chinese teachers use different manipulatives when teaching the same mathematical concept for the purpose of facilitating students' understanding. A typical example is that, for teaching the mathematical concept of regrouping, in the Canadian school, Base Ten Blocks is widely used as a basic tool for students to count, while in the Chinese school, students use sticks to learn how to regroup and count (Peng, Ezeife, & Yu, in press).

Under a seven-year broader Reciprocal Learning Partnership Project between Canada and China, this study further documents the differences in the use of manipulatives for teaching Fractions in a pair of Canadian and Chinese elementary schools. Research data were collected through direct and indirect interactions between the pair of research schools, including Skype meetings notes, video clips, mathematics textbooks and formal and informal conversations with teachers.

Results show that the Canadian and Chinese schools differ in their use of manipulatives and their Fraction models. In the Canadian school, various concrete manipulatives are widely used, whereas pictorial manipulatives widely used in the Chinese school. The area model is dominantly used in both schools, and the length and discrete models are frequently used in the Canadian school, both of which are far less used in the Chinese school. The reasons for the differences are discussed.

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# INTEGRATING INSTRUMENTS TO CAPTURE PRESERVICE TEACHERS' BELIEFS ABOUT MATHEMATICS AND ITS TEACHING

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Beliefs are regarded as a central point in the work of teachers as they are formed in a system that structures their knowledge, perceptions and decisions during their classes and affect how they perceive and interpret their experiences (Voss et al., 2013). Therefore, an important goal of initial teacher education is the development of beliefs, and dispositions toward efficient teaching practices. In this context, it would be desirable to have a baseline on the type of beliefs students have when they start their teacher training. The aim of this research is to explore beliefs about mathematics and its teaching among students first-year of primary teacher training program and to identify emergent beliefs from the Chilean context.

The complexity of the belief system makes necessary to use methodologies that allows an effective access to it (Erkmen, 2012). We highlight the use of different collection methods that promote the decision making, the activation of the episodic memory and the use of resources that intermediate the answers of the individual (e.g. images, or affirmations). We elaborated an interview with an in-depth multi-method design, focused in three lens: students' declaration of 5 principles about the teaching of mathematics, their reflections on a math class video clip and the relations with the 5 principles, and their biographical experiences during their schooling. Interviewees were 16 first-year teacher students from 8 universities. The findings were classified in three groups: the nature of mathematics, teaching and learning, and self-perception. The most frequently declared beliefs were related to the need for the teacher to motivate students, using material and experiences close to them and giving them opportunities to reflect on their own knowledge. Moreover, six of the participants highlight the role of families in the process of learning.

The design of the interview, based on the contrast of own principles and experiences and the analysis of real math classes, allowed access to the complex and personal belief system in participants own cultural and social context.

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# **A TROUBLE AT THE INTERFACE OF DIFFERENT DISCIPLINARY FIELDS (MATHEMATICS EDUCATION, PSYCHOLOGY AND COGNITIVE SCIENCES): DYSCALCULIA**

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Research about dyscalculia still must be lightened (Lewis & Fisher, 2016). Various interpretation perspectives exist but the dominant perspective focuses on individual's cognitive characteristics. We question the place of mathematics education in these researches and the way to reconcile the different points of view to a better understanding of mathematical learning disabilities (MLD). The compartmentalization of approaches makes the communication between teachers and speech therapists difficult. We therefore consider the reconciliation of approaches through the creation of a mathematical difficulties detection tool that would facilitate the exchanges between these two types of professionals by proposing a common inventory of child's difficulties that can be used by each of them.

To develop this tool, we conducted an analysis of different existing tests designed to evaluate mathematical basic skills at the entry to basic school. To ensure the diversity of theoretical foundations, we selected diagnostic tests from research in numerical cognition, but also tests used in school designed on elements from mathematics education. Analysis based on criteria from mathematics education and numerical cognition allows us to precisely identify tasks and variables used in each test.

With this method, we have highlighted some biases of numerical cognition tests in relation to what is taught and to mathematics education knowledge (non-evaluated elements or questioning of the choices of variables). These elements confirm the interest of mathematics education in research about MLD, especially regarding the diagnosis. Moreover, further to this analysis, we can identify the tasks and variables used according to the theoretical frameworks (mathematics education and numerical cognition). This provides us the bases to build our detection tool which will be more detailed during the conference.

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# CONCEPTIONS AND BELIEFS OF THE PROFESSORS OF NUMERICAL METHODS IN THE ENGINEERING DEGREES AT ECUADORIAN UNIVERSITIES

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Nowadays numerical methods constitute a useful mathematical tool in the engineering field, which is reinforced by the computational development. For this reason, there exists an interest in examining how numerical methods are taught. According to Pepin (1999) professors' pedagogy is to be analysed and understood considering the cultural context as well as the professors' conceptions and beliefs. In most studies, beliefs about mathematics teaching and learning do not allude to any specific mathematical domain, process, or topic (Zhang and Morselli, 2016), being mainly focused on primary and secondary teachers' beliefs.

The main goal of this study is to make an approach to the conceptions and beliefs that some numerical methods professors have in engineering degrees at the universities of Ecuador, due to the lack of preliminary studies. To this effect, a collective case study has been chosen, which covers a total of twelve cases. In them, representativeness of the three categories in which the Ecuadorian universities are classified today as well as their geographical representativeness are sought. A semistructured interview, a class observation and the syllabus were analysed in order to obtain an accurate picture of the professors' conceptions and beliefs and how they influence their teaching practice.

The results of the collected information in five of the cases suggest that the analysed professors tend to the instrumentalist conception of numerical methods as well as to traditional conceptions and beliefs about the teaching and learning of numerical methods. Likewise, some deficiencies appear in the teaching practice, especially in the case of the university in the lowest category. It urges a reflection upon the need to improve the teaching of numerical methods in Ecuador, especially with respect to the professor's mathematics training.

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# ASSESSMENT FOR LEARNING IN FIRST SCHOOL YEARS

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Learning is understood as a complex process of situated activity, which requires an active participation of the learner. This process is performed through the interactions that students establish with the tasks propose to them. To be effectively a means of learning, it is necessary that students understand these tasks, execute what is requested, and achieve what is expected (NCTM, 2014). The formative assessment is an enabling context for this process, as an intentional process of learning support (Wiliam, 2007). One of its essential components is to help students to identify and correct their mistakes by reflecting on their own action (Santos & Cai, 2016). Although these ideas are theoretically accepted, teachers' practice is far from this reality (Santiago, Donaldson, Looney, & Nusche, 2012). The perspective that young students do not have enough maturity to self-regulate is still strong. The present study aims to understand if, through formative assessment practices, students at the beginning of their schooling are able to reflect and regulate their learning, that is, to identify and correct their errors in mathematics.

Our starting point was three studies conducted in two classes, respectively with five and seven-years-old students developing portfolios, and a third one with seven-years-old students, developing quiz. All studies used an interpretive methodological approach supported by student interviews, participant observation of classes and documental evidence.

The first results pointed out that these students are able to reflect on their work, understand their mistakes and use the cues given by their teachers through feedback. Portfolios seem to potentiate the development of reflection and quizzes the regular conceptual and processual fluency (NCTM, 2014). The teacher's role is fundamental in order to achieve these positive results, namely to help overcome some difficulties that students still manifest.

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# REFLEXIVE ABILITIES OF THE 9TH GRADE STUDENTS AND "MANY-VALUED" GEOMETRIC PROBLEMS

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Reflexive ability as an ability to comprehend one's experience, knowledge, evaluations is a psychological condition of thinking activity. Development of intellectual type of student reflexing (ability to analysis of mental activity) is the key for successful development and learning math. But the ability to realize reflexive control does not develop without purposeful training. The mechanism of inclusion of such analysis in the cognitive processes is the situation of choice. It promotes to take more effective control of the behavior and thinking (Glasser, 1989). Therefore, as an instrument for developing reflexive students' abilities while studying mathematics, we proposed geometric problems (we called them "many-valued" problems) in which the multi meaning of each problem component creates a situation of choice: the methods of solution; the selection of a specific set of conditions, each of them leads to its specific answer, including the non-existence of the given figure. Such problems generate interest and surprise among students. We worked out sets of problems and the requirements for working with them. We distinguished three levels of development of reflexive ability in teaching geometry, in accordance with certain types of reflexing: extensive, intensive, constructive. The research questions are:

1. Will the level of development of students' reflexive ability be changed if "many-valued" task are used in studying geometry in accordance with certain requirements? 2. Will the transfer on other school subjects be carried out?

The study involved 147 students in control classes and 228 – in experimental. Unlike the control classes the students in the experimental classes solved the "many-valued" problems within the studied topic at the lessons of geometry (twice a week). The study used the Pearson criterion. The findings indicate that the effectiveness of the formation of levels 2 and 3 of reflection of students' reflexive ability in the experimental classes is statistically significantly higher than in the control ones ( $47.45 = 2\chi$ ,  $df = 1$ ,  $p < 0.01$ ). The number of students with the same level of reflective abilities differed by 1-2% while working with geographical and with mathematical material. "Many-valued" problems include tasks the solution of which involves operating the object mental image without a visual basis This type of tasks and further results will be discussed in detail in the presentation.

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# PARENTS' ATTITUDES TOWARDS MATHEMATICS, HOME MATH-RELATED ACTIVITIES AND THEIR EFFECTS ON PRESCHOOLERS' SKILLS IN EARLY NUMERACY

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Little is known about parents' beliefs regarding mathematics and how parents approach preschoolers' mathematics teaching (Cannon & Ginsburg, 2008). Research highlighted the importance of early home math-related activities to understand how numeracy develops later in primary school but little is said on the mechanisms through which young children develop these skills and knowledge (Missall, Hojnoski, Caskie & Repasky, 2015).

The aim of this study is 2-fold: 1) to describe parents' attitudes towards mathematics and parent-reported home math-related activities and 2) to understand the relation between these constructs and specific preschoolers' numeracy-related skills.

This study is part of a larger research project (designed and implemented in Luxembourg, Belgium, France and Switzerland), entitled *MathPlay*, which aims at developing early number competencies (counting, conservation ability and magnitude comparison, (de)composition numbers) both at school and at home. Our play-based approach is using selected and adapted traditional math games, well known by families, in order to especially target the competencies mentioned above. The research design includes one experimental group (EG) with two treatment conditions (X1 - games at school, X2 - games at school and at home) and one control group (CG). Before the intervention, a total of 275 parents (coding in progress) filled out a questionnaire to investigate their beliefs regarding 1) the preschool numeracy competencies and 2) parental involvement. Data is being analysed. Descriptive results will illustrate how deep parents are involved in home math-related activities. Furthermore, we will analyse if a statistically significant relation exists between this construct and parents' attitudes as with preschoolers' skills in early numeracy.

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# STUDENTS OF RELIGION STUDYING SOCIAL CONFLICTS THROUGH SIMULATION AND MODELLING - A PILOT STUDY

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Social scientists are increasingly studying social phenomena by using simulation and modelling (SAM). By creating a virtual world, they can run social experiments, discover underlying causalities or explore future scenarios (Clark & Fossett, 2008). Colleagues at our university use SAM for the study of religious conflicts, refugees, and tolerance (Shults et al., 2017). We started a project to introduce undergraduate students of religion to SAM as an approach to study social phenomena, anticipating a limited understanding of the mathematics needed for creating SAM. The goals were to study whether participants were able to reason quantitatively about the simulated phenomena, understand the effects of the different variables (sliders), the role of assumptions in mathematical models, the way in which SAM can illuminate social mechanisms (e.g. how tolerance between citizens still can lead to segregation), and the extent to which they could imagine using SAM in the future.

Cultural-historical Activity Theory (Engeström, 1999) was employed to frame participants as learners, but also as future social workers or research colleagues, and to frame learning as participating, initiating, communicating, and negotiating meaning, and intentions (Lerman, 2001). We organised a seminar whereby students of religion used Schelling's segregation model to examine social inclusion and exclusion. Video recordings of these sessions were transcribed and analysed, focusing on the nature of the communication and quantitative reasoning among participants. Some students showed a keen interest in using SAM in their future research. Moreover, participants who were worried about their mathematical skills discovered that this lacuna hardly hindered them.

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# STUDENTS' EXPECTATIONS CONCERNING STUDYING MATHEMATICS AT UNIVERSITY

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Students experience the transition from school to university in mathematics as a difficult passage. This phenomenon may be related to the high demands at university because of mathematics as a scientific discipline (Clark & Lovric, 2009). This unfamiliar mathematics is presented in lectures to students who need self-regulation strategies to make the content accessible. According to person-environment fit theories, for successful learning processes it is necessary that *students' expectations* and the *real demands* fit together (Lubinski & Benbow, 2000). To achieve this fit, we develop a 90 min-workshop before university starts which has two aims: (1) to support study choice; (2) to close the gap between expectations and real demands. Here, we focus on the second aim: in which way does the participation in such a workshop change students' expectations concerning studying mathematics at university?

The *workshop* consists of three parts: In the first part, students rate given prejudices and correct statements regarding a mathematics study. These selected statements are probably in line with their own expectations. In the second part, they watch a video in which tutors and lecturers inform students about major changes between learning mathematics at school and at university. Then, they repeat rating the prejudices and correct statements to bring together their expectations and the real demands. In the last part, students may ask the tutor, who conduct this workshop as a role model, concerning her own study experience. Before and after the workshop, students state their expectations in an open-ended format. This workshop took place in two mathematics courses at school (all together  $N = 30$  students, 17-18 years old).

Before this workshop, many students expect that to be successful in university, one need to be interested in mathematics and to be able to think logically. After the workshop, students put a bigger emphasis on engagement in the learning process. We conclude that these students underestimate the demands in a mathematics study program before the workshop. More results concerning students' expectations will be presented. The results are limited to a small sample. In further studies, we will conduct more workshops and analyse if students' participation has an impact on students' study choice and maybe on their study activities in the first semester.

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# CONFLICT OR COMPROMISE: PRE-SERVICE MATHEMATICS TEACHERS' EXPERIENCES OF SCHOOL AND UNIVERSITY MENTORS

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The practicum experiences of pre-service teachers (PSTs) take place under the supervision of experienced school mentors, whose responsibility it is to monitor the development of the PSTs in real-world conditions, all the while learning to teach in context. In addition, university-assigned mentors visit the PSTs in order to clinically supervise and assess the extent to which the students are mastering their professional practice according to the measures promoted in the university setting (Nyaumwe & Mavhunga, 2005). Thus, the role of the mentors is to support, encourage, coach, give feedback and initiate student teachers into teaching.

Crucially, the university and school mentors operate in different institutional spaces where educational ideologies and beliefs may differ: the university is usually a more progressive space, while the schools often prefer traditionalism (Hudson, 2007). This beliefs disjuncture has the potential to lead to conflicting messages conveyed in the mentors' feedback to the pre-service teachers (Nyaumwe & Mavhunga, 2005).

In this study, 4 PSTs were tracked during their practicums. Each PST was interviewed via semi-structured interviews and asked about their mentoring experiences. In addition, the mentors' feedback sessions with the students were recorded. Transcriptions were analysed using Hudson's (2007) 5-factor model to analyse and describe the work of mentors, in particular the oral and written feedback.

One PST in particular, received patently contradictory messages from his school and university mentors: while the school mentor objected to the PST's use of algebraic and graphical representations in the same lesson on exponents, his university mentor strongly encouraged this practice. Such results suggest that the mentorship space is a site of conflict between the school and university mentors, in terms of the ideologies they promote and the demands they make on PSTs. This type of conflicting feedback could impact negatively the emerging practice of the pre-service teacher.

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# SHIFTS IN MATHEMATICS INTERACTIONS BETWEEN GRADE 8 STUDENTS FROM AN ENACTIVIST PERSPECTIVE

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While many studies (e.g., Chapman, 2004) focus on the interaction of the students and teacher in mathematics classrooms, there is little focus on how mathematics emerges from the details of their interaction. In this study, the research question is: How is the emergence of mathematical knowing shown in the details of the shifts in interactions between students?

To understand how the interactions in detail amongst students within a mathematics classroom occur, I adopted an enactivist position, in which “every act of knowing brings forth a world” (Maturana & Varela, 1992, p. 26), a mathematics world from each student that can be triggered by the interactions with others. The analysis carried out considered two levels of observation based on Rosch (1978), basic-level, close to actions (e.g., categorisation of questions, answers used) and superordinate level, from analysis of transcripts, of how the actions are interrelated. Making such distinctions is part of my enactivist approach.

I present the analysis of two transcripts that come from video-recordings and observation notes of a mathematics teacher and their 23 students (aged 13-14 years). Students were working in their usual way on solving problems and also engaged in a mathematical modelling task that was new to them. These observations are part of a large project characterising the emergence of mathematics in interactions.

The study shows a number of shifts, for instance, that when the students were interacting, there is a space, a basic-level distinction, which I have called an interval of waiting, triggered by an intervention (for instance, a question or statement) made by one student to another. At a superordinate level, seen through the transcripts, the “interval of waiting” allows the distinction of a “new start” to be made in the actions performed in the emergence of mathematical knowledge. Ongoing analysis will look at how basic-level and superordinate shifts allow me to observe the different foci of attention of the students solving the same mathematics problem.

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# **SUPPORTING UPPER SECONDARY SCHOOL STUDENTS' INTEREST IN PLAYFUL LEARNING AND PEER TUTORING IN A NEW LEARNING ENVIRONMENT**

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In Finland, international (e.g. PISA and TIMSS) and national studies have caused a concern for young people's interest in mathematics. For example, decline in secondary school students' interest in school subjects have been widely reported (Hidi, Renninger & Krapp, 2004). By organizing an interesting teaching situation, it is possible to awake students' situational interest that can lead to longer-lasting interest (Krapp & Prenzel, 2011). This study is part of a bigger design-based research. The aim is to train upper secondary school students to guide primary school students in the playful learning environment *Pulmaario* to provoke and support their interest in mathematics. Designing a new model to organize math activities makes it also possible to spread the activities to municipalities further away.

In the *Pulmaario* learning environment, we are trying to provoke students' interest in mathematics via playful learning and peer tutoring. This study represents the results from the first cycle of the design-based research that answers to the research question: what the upper secondary school students find interesting in playful learning and peer tutoring.

The sample of this case study comprises seven upper secondary school students who were working as instructors in the *Pulmaario* learning environment in autumn 2017. The research data includes interviews and learning diaries of the students. The students were interviewed with semi-structured interview before and after the *Pulmaario* workshop meetings. The learning diaries were written after each workshop meeting. The analysis of the data will be based on the method of inductive content analysis. This study gives new information about what upper secondary school students find interesting in playful learning and peer tutoring. The results will be used to design the second cycle of this design-based research.

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# TECHNOLOGY BASED ASSESSMENT OF EARLY NUMERACY AND LATER MATHEMATICS ACHIEVEMENT

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Since later mathematics achievement is predicted by early numerical skills (Jordan et al., 2009), it is essential to measure and develop these skills at school entry. Due to the rapid technological development technology-based assessments can be realized with younger students as well (Csapó, Molnár, & Nagy, 2014). However, while reliability and construct validity of these new online tests can be checked instantly, little is known about their predictive validity. The aim of the study is to compare the results of a technology-based early numeracy assessment at school entry with later mathematics achievement.

4277 first grade students participated in the study, their mean age was 7.09 at the first assessment. The online early numeracy test comprised of 40 items. Its reliability (Cronbach's  $\alpha=.89$ ) and structural validity ( $\chi^2= 5089.56$ ;  $p<.001$ ; CFI=.928) were verified (Mean=82.6%p; SD=15.5%p). The online mathematics achievement test with three subtests (mathematical thinking, application, and discipline) comprised of 50 items (Cronbach's  $\alpha=.94$ ; Mean=50.0%; SD=22.8%). The early numeracy test was administered in computer laboratories of primary schools in the first months of school, and it was followed by the mathematics achievement test six month later.

We found positive correlation between the online early numeracy test and the mathematics achievement test ( $r=.53$ ;  $p<.001$ ). The results of structural equation modelling showed that early numerical skills have significant effect on later mathematics achievement (standardized regression coefficient=.63). Model fit was good ( $\chi^2= 363.41$ ;  $p<.001$ ; CFI=.972). Results of our study showed that our online early numeracy test is a reliable and a valid instrument in terms of construct and predictive validity. Due to the advantages of technology-based assessment such as automatic scoring or group testing our online instrument could be used in everyday school practice for identifying learning drawbacks in early mathematics.

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# **BASIC IDEAS FOR ADDITION IN PRIMARY EDUCATION TRAINEES**

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Focusing on primary education initial teacher training, the objective of this work is to determine the Basic Ideas (BI) that primary education trainees have or acquire about the way in which addition is taught, and what BI they use to teach addition. The importance of BIs come from their three dimensions that come from the subject's experience: (1) the construction of meaning from a mathematical concept, (2) the generation of a representation of such concept, and (3) the capacity of applying the BI in the real world (vom Hofe & Blum, 2016). The BIs for addition (Blum and others, 2004) are: Adding S-C-S; Joining S-C-S; Joining S-S-S; and Completing S-S-C. The Adding action can be observed in the adding of stones in a bowl; there are five stones in the bowl (S), three are added (C) so eight are in the bowl (S). The Joining action can be observed when there are 3 stones in a group and 5 in another group (S), then these two groups get together (C), and resulting in a set of 8 stones (S). The Joining S-S-S is a mental joining of two sets of concretely objects. The Completing S-S-C can be observed in the strategy used for comparison between sets, the initial set, 5 squares and the final set of 8 squares.

This is a mixed and non-experimental research. The questionnaires have 12 multi-items scales for the quantitative data and four open questions for the qualitative data. The survey was applied to students of general primary pedagogy in a Chilean university. 48 subjects are first year students who have never participated in mathematics didactic class; and 47 subjects are senior students who have had at least one mathematics didactic class. The results show that the qualitative differences in the senior students' BI in the category out of the four BI in addition are the way in which the BI is used in context. Less than half of the first-year students are able to use two out four BI in the first addition class. Two senior students did not change their beliefs and kept the traditional classes approach without BI for teaching addition. For the students who considered the BI for the addition, the use and type of concrete materials take a leading role in math class.

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# RESOLVING COGNITIVE CONFLICTS ABOUT FUNCTIONS: PRESERVICE SECONDARY MATHEMATICS TEACHERS' INTERACTIONS WITH TASKS

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Preservice secondary mathematics teachers (PSMTs) can deepen their understanding of mathematical concepts they will teach by developing *sensitivity* and *awareness* to subtle aspects of these concepts (Mason, 2008). This learning can be facilitated by eliciting cognitive conflict—dissonance between PSMTs' current understandings and the concepts being explored (Watson & Mason, 2007). Using a design experiment, we created instructional materials for PSMTs to elicit cognitive conflict (CC) about functions. Participants were 23 PSMTs enrolled in an inquiry-oriented course designed to deepen their understanding of school mathematics.

We report on the CCs that arose in one lesson and how PSMTs resolved these. CCs that emerged included (a) whether a discrete list of correspondences could constitute a function and (b) whether the relation  $y = -x^2$  was a function. Elements of the learning environment afforded opportunities for PSMTs to resolve their CCs: Tasks built from PSMTs' prior knowledge of functions, provided several examples and representations, and offered shared learning experiences to which PSMTs referred in later discussions. In addition, the inquiry environment—including small group discussions, instructor facilitation, and whole-class discussions focused on PSMTs' thinking—supported the resolution of CCs. The results provide insight into how PSMTs can deepen their understanding of functions and the means by which their learning can be supported in undergraduate courses (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003).

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# CHALLENGES AND OPPORTUNITIES: LISTENING TO NEWLY ARRIVED STUDENTS' MATHEMATICAL STORIES

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In Sweden, *newly arrived students* often start school in preparatory class for gradual integration into regular classes. The government secures the right to get education and tutorial support in their mother tongue while also providing Swedish language instruction. In the Swedish student-centred mathematics education, characterized by participation and communication, language plays a great role in teaching and learning. It is believed that several languages strengthen each other and support better learning (Lindberg, 2009). However, the tension between language learning and mathematics learning generates the challenge of ensuring that the mathematics is not sacrificed by the emphasis on multiple language learning.

In research in multilingual settings with newly arrived students, students' perspective is rarely considered. This study focuses on newly arrived students' perspective on their integration into regular mathematics education. It investigated: How do they use mother tongue in mathematics? What challenges and opportunities do they experience when transitioning from preparatory to regular class? Semi-structured interviews with 4 sixth graders recently integrated into regular education were carried out. Inductive content analysis (Elo & Kyngäs, 2008) and story-telling (Jørgensen, 2017) were used to analyse the material.

The students' stories point to the importance of being integrated in ordinary mathematics education at an early stage. They wanted to be challenged to *dare to speak* Swedish and *make an effort* to understand Swedish mathematics, particularly *problem solving* because it differed the most from what they knew. In regular classes, they felt they got access to *more mathematics* and *more Swedish*. They developed their own strategies to advance both in Swedish and mathematics. This increased their learning and motivation. The students' stories question the accepted belief on the desirability of using multiple languages to learn mathematics.

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# APPROACHES TO MODELING BY MATHEMATICS TEACHER EDUCATORS: THE PERCEPTION OF STUDENT TEACHERS

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In the last two decades, research has been paying attention to the impact of modeling by teacher educators, considering modeling as an intentional practice aimed at developing future teachers' professional learning. However, this research has approached modeling in a fragmented way, without recognizing that it is a situated practice, mediated by the teacher educator, and related to student teachers' educational objectives and actions (Goizueta et al., 2017). The current research investigated student teacher perceptions of teacher educator instructional practices for teaching mathematics. Data was collected through a Likert questionnaire from a sample of student teachers from six method courses belonging to pre-service teacher education programs for secondary school mathematics teachers in Chile. The instrument inquired into how student teachers see different instructional practice to teach mathematics according to the types of modeling defined by Lunenberg, Korthagen & Swennen (2007); cluster analyses were employed to explore the data. Results from this exploratory study identify four distinct clusters that represent four different perceptions of approaches to modeling according to the conceptual model proposed in this study. C1 represents modeling with the focus on linking theory and school teaching practices. C2 characterizes implicit modeling with emphasis on teaching instructional practices for teaching mathematics. C3 portrays modeling with emphasis on sharing the pedagogical reasoning underpinned on teaching mathematics. And C4 represents modeling with the focus on transferring to the school classroom the mathematics teaching practices learned in the pre-service teacher education classroom. The conclusions suggest that modeling by teacher educators should be enacted in a more structured way, highlighting the pedagogical reasons for doing some mathematics and instructional practices. In this way, teacher educators will contribute to developing more sophisticated initial teaching models among student teachers (Rojas & Deulofeu, 2015).

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# INVESTIGATING THE STUDENT PERSPECTIVE ON SCHOOL MATHEMATICS IN THE PROCESS OF CHANGE

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It is a challenge for mathematics education to develop classrooms where students maintain positive relationships towards mathematics after the first school years (cf. Lewis, 2016). However, most studies are concerned with either classroom practices or students' views and there is not much research evidence on how these two domains are connected over time. This case study took a long-term perspective investigating why some mathematically skilful/capable students develop negative attitudes to and abandon the study of mathematics.

The study was situated within a three-year-long action research project in a Swedish-speaking lower secondary school in Finland. The first author closely followed the action-research of the teachers with a particular focus on the student perspective. The design and methodology of the study was emergent, embedded in social practice theory (Wenger, 1998) and with meanings denoted to school mathematical experiences as its unit of analysis. The primary case record material was recurrent interviews with three students: five individual interviews during lower secondary school and one adult interview. The interviews were analysed in several phases, firstly, by meaning categorization and concentration. Secondly, essential aspects in the interviews were noted through an inductive approach. Lastly, the interviews were analysed holistically to understand how each student made sense of his or her school mathematical experiences. The analysis focused specifically on students' alignment, development of identities and imagination in relation to the practices in which they took part.

In general, the reform-related activities had the potential to open up for increased participation for all students. Yet, students did not always see the connection between these activities and the development of mathematical knowledge. Moreover, the invisible classroom traditions prevailed in parallel with the new activities and the students considered this as the legitimate mathematics classroom. Tentativeness, wrong answers, misinterpretations or extended interpretations of textbook tasks were not considered by students to be characteristics of a space where every student could belong and participate. For some students this situation contributed to the development of negative identifications towards mathematics.

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# TEACHER LEARNING IN THE CONTEXT OF LESSON STUDY: ONE TEAM, FIVE MATHEMATICS TEACHERS

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Research shows that Lesson Study (LS) has positive effects on teacher's knowledge, beliefs and instructional practice (e.g. Lewis & Perry, 2014). However, not all teachers say they benefit from LS. To gain more insight into teacher learning in the context of LS, we choose a model of Lewis et al. (2009), including four LS features and three mechanisms through which LS improves instruction, to which we added a conditional part based on the Reasoned Action Approach (Fishbein, 2008). This theory posits that teachers' positive intention is a condition to perform LS. We added also other LS conditions such as interpersonal and structural factors. On the basis of the combined model, we investigated the learning of five teachers who participated in one LS-team performing two LS-cycles.

The leading questions in this research are: What are self-reported effects of participation in a LS team? To what extent are self-reported effects observed in regular mathematics classes? Which conditions foster or hinder teacher learning in the context of LS? Data sources were pre- and post LS-interviews, pre- and post-video recordings of classroom practices in regular mathematics classes, and a questionnaire after each LS-cycle.

Results show that all teachers reported positive effects on their beliefs and knowledge (e.g. about the teaching of 'similar triangles'). Some teachers were able to implement changes in a concrete way in their instructional practice in regular mathematics classes (e.g. changes in the structure of the math class). Other teachers could not mention concrete examples of changes. There seemed to be a relationship between teacher's intentions (such as positive disposition towards LS and voluntary participation) and effects on knowledge and beliefs and changes in instructional practice. The detailed study of one team of five teachers made clear that teacher learning in the context of LS depends on many factors such as individual, interpersonal and structural factors.

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# REVISITING AND GROWING PRIOR KNOWINGS FOR LEARNERS EXPERIENCING MATHEMATICS DIFFICULTIES

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Learners are constantly positioned through narratives (Wagner & Herbel-Eisenmann, 2006). The prevailing narratives in North American classrooms for learners experiencing mathematics difficulties (MDs) tend to be around expectations of low ability and deficiency (McDermott, Goldman & Varenne, 2006). Siegler (1996) argues that in order to advance an alternative discourse, researchers should consider processes of change and the fluid, non-linear movement and moments of change. This paper reports on a finding from a larger study exploring the recursive changes and growth in understanding of zero by learners experiencing MDs. The participants in the study were between the ages of nine and eleven years old. Each learner participated in an individual task-based interview session, as well as five subsequent group intervention sessions. The data were recorded, analysed, and mapped utilising the Pirie Kieren Theory for the Dynamical Growth of Mathematical Understanding (PK Theory) (Pirie & Kieren, 1994). Here, I report on the similar shapes of the three mappings made from the initial task-based interview sessions. The mapping of all three participants showed multiple recursive movements of revisiting prior understandings (Primitive Knowing in PK theory), each time, before they made forward movements in order to attempt to construct or actually construct mathematical images (Image Making and Image Having in PK theory). Commonality between these multiple recursive revisitings was: (i) learners required “thickening” their prior knowings before creating a blend (Fauconnier & Turner, 2002), (ii) some knowings were very embedded and required constant revisiting, and (iii) some knowings were the opposite, they were so flimsy with little support, that participants attempted to attach them indiscriminately to new learnings.

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# VALIDATING A QUESTIONNAIRE: CAPTURING THE WAY IN WHICH BELIEFS ABOUT MATH AND STUDENTS' ABILITIES INFLUENCE TEACHERS' ACTIONS IN PROBLEM SOLVING

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The use of Likert scale items to measure beliefs is questioned since it may amplify problems related to social desirability (cf. Di Martino & Sabena, 2010). We argue that the use of Likert scale items may give teachers opportunities to respond to them ideally, not realistically. Therefore, we developed a questionnaire using rank-then-rate items for studying teachers' beliefs on their practice (TBTP). In addition, we also consider students' mathematical abilities as a social context in the classroom in the TBTP. In this paper, we present the final validation of the TBTP.

The TBTP contains ten rank-then-rate items grouped into three themes: (1) the nature of mathematics, (2) the teaching and learning of mathematics, and (3) the practice of problem solving. Each item has three statements, which are – in this order – always associated with the instrumentalist view, the Platonist view, and the problem-solving view described by Ernest (1989), respectively. To answer an item, a respondent firstly ranks the three statements of the item and then rates them.

We have tested the TBTP with a large sample of teachers, and the results show that the TBTP is valid and reliable (reported in Safrudiannur & Rott, 2017). However, since we also need to ensure that the TBTP allows for a valid representation of beliefs and practices, we evaluate the convergent validity of the TBTP as the final validation. We invited four teachers to respond the TBTP, and then we interviewed them and observed their lessons of teaching problem solving.

The results of the evaluation show the consistency between the four teachers' responses to the TBTP with their interviews and lessons. These results confirm the validity of the TBTP. Moreover, we also remark that since we consider teachers' beliefs about students' math abilities in the TBTP, the TBTP seems able to explain the inconsistency between teachers' beliefs of the nature of mathematics and their practice.

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# CONSIDERATIONS OF CREATIVITY BY PRE-SERVICE TEACHERS OF MATHEMATICS IN THEIR MASTER'S DEGREE FINAL PROJECTS

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Creativity is an important process that may be developed from early school years (Sriraman, 2005). However, Mathematics at school are rarely creative. Our study focuses on pre-service teachers' interpretation of the importance of creativity. In the work placement of the master's degree in Teaching in Secondary Schools (specialisation in Mathematics) the pre-service teachers design and implement a learning sequence. Later, in the master's degree final project (MFP), they analyse the implementation using the didactic suitability criteria of the Onto-Semiotic Approach (Breda, Pino-Fan & Font, 2017). Previous research show that the teachers' analysis of their own practice affects the development of other professional competences. We assume that the didactic analysis also helps pre-service teachers to develop their conception of creativity. Our research questions are: 1) Do pre-service teachers refer to creativity in their MFP? 2) Which aspects of the sequences are related to creativity?

We took a sample of 198 MFP of the master's degree from 2009-2010 to 2014-2015. First, we searched for comments about creativity in the MFP and registered them. Then, we contextualized the comments in the MFP and found which aspects of the implemented sequence creativity is related to. We classified the comments depending on the aspect that they refer to, based on the didactic suitability criteria (Breda et al., 2017). 112 MFP include references to creativity. In the comments, creativity is associated with different aspects: the type of tasks proposed, for example, the resolution of open-ended problems (epistemic suitability); the students interaction in cooperative learning situations (interactional suitability); the use of technology and other resources (media suitability); the development of civic responsibility and critical thinking (ecological suitability); and the assessment of the learning sequence (cognitive, affective and interactional suitability). In conclusion, more than a half of the pre-service teachers mention creativity in their MFP, although they implicitly assign different levels of importance to it. Our study also reveals a high variety of aspects related to creativity. This work is part of the research projects EDU2015-64646-P (MINECO/FEDER, UE) and REDICE16-1520 (ICE-UB).

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# THE TEACHER'S ROLE IN WHOLE CLASS DISCUSSION IN DEVELOPING THE QUANTITATIVE REASONING

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In a context of a project focused on the development of flexible calculation and quantitative reasoning in primary students we aim understand how teacher's role influences that development. In this presentation we will present the resolution and discussion of a task involving the quantitative difference in a 2<sup>nd</sup> and a 3<sup>rd</sup> grade classes, and analyse how quantitative additive reasoning (Thompson, 1993) is evident in the way students explored and discussed the task and how the teacher's role in questioning the students may influence that development. Within a qualitative approach, the data collection was done through the participant observation, complemented with field notes and videotaping of the classes.

The quantitative reasoning involves reasoning about relationships between quantities. Thompson (1993) connects the notion of quantity to the idea of measure, clarifying that the reasoning does not depend on their measures, that is to say, on their numerical value. Therefore, teachers should propose questions that lead to discussions of quantities, not numbers. During the whole class discussions, teachers should encourage students to explain and justify their reasoning and resolutions, thus developing their understanding (Ruthven, Hofmann, & Mercer, 2011).

Our results show that how teachers develop the process of communication in classroom, namely the negotiation of meaning, seems to overcoming the challenging situation relative to quantitative difference and it seems to be influenced by the degree of teachers' ease in leading with the situation involving losses. For us, this situation is intrinsically linked to the role the teacher played in the whole process, putting the appropriate questions so that the students feel challenged to advance in their mathematical thinking, namely their quantitative reasoning.

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# IS MATHEMATICS ME? CHINESE HIGH SCHOOL STUDENTS' IDENTITY IN MATHEMATICS

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Students who regard themselves as capable mathematics learners are more likely to enter mathematics fields (Cass et al., 2011). Meanwhile, mathematics identity is seen as an important factor influencing students' such perceptions (Cribbs et al., 2015). On the other hand, Asian students continuously demonstrated excellent performance in mathematics in many international comparisons, while many of them didn't identify themselves as math persons. Regarding students' mathematics identity, most studies appeared to be confined to students in the West with nearly none on their Eastern peers.

Adopting Cass et al.'s (2012) framework for students' identity, this study intends to examine a group of Chinese high students' mathematics identity and explore its relation with four important factors (i.e., interest, functionality, extracurricular experience and competency). Accordingly, three research questions are addressed: 1) What's Chinese tenth graders' mathematics identity level and what factors contribute to their perceptions? 2) Are there gender differences in students' mathematics identity? 3) How four factors influence students' mathematics identity?

A 35-item questionnaire on a four-point Likert Scale was constructed for this study, covering identity and four related factors. About 372 tenth graders (186 boys and 186 girls) from one Chinese public high school attended the survey in early 2018.

The results show that the tenth graders' mathematics identity is overall at the moderate level ( $M = 2.57$ ,  $SD = 0.72$ ) with boys at a significantly higher level,  $t(203) = 5.11$ ,  $p < .001$ ,  $d = 0.78$ . Students' math interest ( $r = .69$ ) and competency ( $r = .84$ ) have strongly positive correlations with their identity, while functionality is negatively correlated ( $r = -.24$ ). The MANOVA reveals a significant difference between the genders across interest, functionality, extracurricular and competency,  $F(4, 344) = 12.41$ ,  $p < .001$ ; Wilk's  $\Lambda = 0.87$ , partial  $\eta^2 = 0.13$ . A further path analysis illustrates that competency has a significantly positive effect and functionality has a significantly negative effect on students' math identity, while the influence of interest and extracurricular is insignificant. More discussions and implications will be included in the full paper.

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# HOW ORDINARY LANGUAGE INFLUENCES THE FORMULATION OF STATEMENTS WITH QUANTIFICATIONS

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The word usage in ordinary natural language may influence how the statements and proofs are formulated and understood by students in the mathematical discourse (Mejia-Ramos & Inglis, 2011). This study aims to investigate an influence of ordinary language, used in a specific country, in the formulation of quantified statements to be proven. To achieve this aim, we focus on the linguistic aspects of universal and existential quantifications to be formulated in a given statement (so-called AE and EA statements (e.g., Dubinsky & Yiparaki, 2000)), by examining Japanese prospective mathematics teachers' difficulties related to such formulations.

In the Japanese language, universal and existential quantifications used in ordinary language are rarely made explicit in the written form at secondary level. Most of students do not encounter quantifiers written in ordinary language (any, all, etc.) or mathematical symbols (" $\forall$ " or " $\exists$ ") based on predicate logic, until they reach the undergraduate level (Shinno et al., 2018). To investigate their difficulties or gaps associated with their interpretations of statements involving multiple quantifiers, we conducted a pilot study targeting 47 undergraduate students who have already learnt mathematical statements with multiple quantifiers. The questionnaire includes both AE and EA statements formulated by two different language forms: mathematical and English forms. The participants were asked to translate each statement into Japanese form and to judge whether a given statement was true or false. The results show that most of them fail to interpret EA statement (by English form) in their own language. Regarding this difficulty, we discuss some grammatical characteristics of the Japanese language which may affect how they engage with AE/EA statements.

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# INVESTIGATING STUDENT STEM BELIEFS WITHIN A TRANSDISCIPLINARY ROBOT BUILDING PROJECT

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Mathematics education researchers call for more equitable representations of science, technology, engineering and mathematics from studies aiming to advance STEM learning (English, 2016). English (2016) advocates for a better understanding of the reciprocal relationship between mathematics and other STEM disciplines and also suggests a heightened presence of engineering within integrated STEM education. Others too, envisage that STEM learning will go beyond sparking short term student interests and support connection making “within, between and among” STEM disciplines, promoting knowledge, skills and thinking processes (Honey et al, 2014, p.19). Designing STEM learning experiences effective for engaging student interest and promoting thinking is challenging. Studies seeking to elicit student beliefs about STEM within transdisciplinary projects offer insights for future learning experiences.

This qualitative study (interviews and observations) uses a multidimensional engagement framework comprising three types of engagement: behavioural (participation, involvement); emotional (feelings, values, interest); and cognitive (goals, self-regulation, metacognition) (Fredricks, Blumenfeld, & Paris, 2004). Although studies of student engagement typically take place in classrooms and investigate single subject areas, this study investigates student beliefs and engagement in an extra-curriculum interdisciplinary STEM project. Data was elicited from 12 secondary students throughout the building of several heavy weight (110kg) non-autonomous electro-mechanical robots with the findings presented in three categories. *Responses to STEM* revealed: shifting perceptions of STEM subjects; reciprocity between robot and academic learning; and influences on A-level/HE course choices. *Engineering practices* revealed: holistic project approaches; systems thinking, problem solving and visualising; and learning from mistakes. *Cognition and learning* revealed: high levels of cognitive engagement; and promoting factors such as persistence, self-efficacy, interest, value and student autonomy.

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# STUDENTS' AND MATHEMATICIANS' ACCEPTANCE CRITERIA FOR MATHEMATICAL PROOFS

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Research has repeatedly revealed that students struggle with mathematical proofs, especially during the transition to university mathematics. In parts, this can be related to an enculturation process during which students acquire the local, sociomathematical norms regarding the acceptance of mathematical proofs. Several publications cover acceptance criteria for proofs from a philosophical, a theoretical, and an empirical perspective (e.g., Dawkins & Weber, 2016; Heinze, 2010). Still, research so far mostly examined mathematicians' research practice and did not cover the acceptance criteria for proofs used in university teaching.

The present study compares acceptance criteria for mathematical proofs in an educational setting described by beginning university students, university students in their first to third semester, and mathematicians. For this, a set of proof validation tasks were administered via (online) questionnaires, each containing both closed and open questions regarding the correctness of the purported proofs and the reasons for the acceptance or the rejection of the purported proofs within the tasks. The tasks were based on a theorem from elementary number theory and embedded into a university teaching setting. Open answers were coded using a deductive, theory-based approach, complemented by the inductive creation of additional categories. Interrater-reliability shows acceptable agreement ( $\kappa > .70$ ).

First results show that mathematicians are highly consistent regarding the correctness of the purported proofs, while students are significantly less accurate. Substantial differences between the reasons for or against the acceptance of the purported proofs between the different subsamples were identified. For example students referred more to individual criteria when validating proofs, such as a feeling of understanding. Moreover, mathematicians seem to adapt their acceptance criteria specifically for the research and teaching contexts, downplaying individual criteria in teaching practice. It might be that individual criteria, such as understanding the proof, serve as a “weak heuristic” for novices as well as for experts, when other criteria are hard to apply.

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# TEACHERS' JUDGMENT ACCURACY OF TASK DIFFICULTY RELATED TO FUNCTIONS

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Reasoning with functions is considered as a specific way of thinking in relationships, interdependencies and changes (Vollrath, 1989). Hence, it is not only relevant in school contexts but also in students' daily lives. Empirical studies indicate that learners have various problems in this domain (e. g., Nitsch, 2015). In order to support their students' learning, teachers should know about such problems and hence be able to accurately judge the difficulty of classroom tasks. As a measure for the accuracy of teachers' judgments of task difficulty, Hadjidemetriou and Williams (2002) analyzed the correlation between task difficulty estimated by teachers and empirical solution rates of the corresponding tasks. They report of a medium correlation of 0.40.

To investigate if such findings do also apply for our learning contexts and hence if teachers need to be supported in that regard by a professional development, we investigated the correlation between empirical solution rates of tasks related to functions and solution rates of the corresponding tasks estimated by teachers. Therefore, 80 students (36 female) aged between 12 and 16 years ( $M = 13.7$ ,  $SD = 1.0$ ) responded to a test related to functions (cf. Nitsch, 2015). Additionally, their four mathematics teachers were asked to estimate the solution rates for seven of these tasks.

Within the four classes, two of the correlations were positive ( $r_1 = 0.07$ ;  $r_2 = 0.38$ ) and two showed a negative direction ( $r_3 = -0.60$ ;  $r_4 = -0.67$ ). There are two main sources for these low correlations: 1. The teachers mainly overestimated solution rates for "traditional" tasks such as changes between equation and graph or reading slope and y-intercept from an equation. In contrast, they underestimated solution rates for tasks that they had rarely focused in their classrooms such as changes between graph and situation. 2. Concerning the "traditional" tasks, the teachers wrongly expected that the students would have less difficulty with reading slope and y-intercept from an equation than with changing between graph and equation. These results do not only indicate systematic problems in teachers' judging accuracy related to functions but also show shortcomings of teaching culture as tasks with real-life-context appear to be neglected.

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# STORYTELLING AND LEGITIMATION IN SOCIAL JUSTICE MATHEMATICS

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Nowhere is the distinction between the intended and received curriculum as important as in social justice mathematics education. Social justice mathematics scholarship has focused primarily on curricular examples and on teachers' implementation of the approach (Gutstein, 2006). As important as these curricular and case studies are, we need more critical attention to how students receive and process social justice mathematics topics (Esmonde, 2016). This paper reports on a social justice activity in which twelve students in a university class modelled the spread of gender inclusive speaking habits, that is, the habit of using gender neutral pronouns like *they* as a term of reference for a single person who does not identify themselves as *she* or *he* (Whipple, Staats, & Harrison, n.d.). Our university is promoting the use of persons' preferred pronouns, so that the activity was an opportunity for students to use mathematics to reflect on social justice action in process around them.

The activity asked students to develop a mathematical solution, to write a critical reflection on the validity of the mathematical model, and to write a story or personal experience to illustrate how and whether social change can happen in the way the mathematical model describes it. We use Van Leeuwen's (2008) critical discourse analysis of discursive modes of legitimation to understand how students positioned their arguments in their personal stories. Van Leeuwen's framework, broadly speaking, uncovers varied ways of expressing authority, morality, and naturalizing rationalizations. Summative results show that students used richer, more complex ways of legitimizing their opinions when they wrote personal stories compared to their more purely mathematical reflections. We will provide several close analyses of students' storytelling about social change to highlight the ways that students receive social justice mathematics teaching can be challenging and unexpected.

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# TEACHERS' GROWTH: PROPORTION AND THE 5 PRACTICES

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This study investigates the impact of a two year professional development program (PD) on 24 middle school teachers' knowledge of student proportional reasoning and facilitation of productive mathematical discussions. Teachers' knowledge of rational numbers is fragile (Ball, Lubienski, & Mewborn, 2001), making it challenging for them to teach for deep conceptual understanding. An important aspect of teaching for understanding is facilitating meaningful mathematical discussions (Franke, Turrou, & Webb, 2015), which is challenging for many teachers (Stein, Engle, Hughes, & Smith, 2008). Therefore, student thinking about proportions, selection of tasks that promote proportional reasoning, and implementation of these tasks were essential parts of the PD. We analysed classroom observations, Mathematical Knowledge for Teaching (MKT), Fraction and Proportion Thinking Inventory (FPTI), and teachers' self-reports on growth in content knowledge, knowledge of student thinking, and implementation of the five practices (Stein et al., 2008).

Our findings suggest that teachers' content knowledge and knowledge of student thinking increased substantially. FPTI showed a significant change in teachers' knowledge of students' thinking. Over 80% of teachers reported growth in knowledge of both content and student thinking. All teachers reported some or substantial change in their implementation of the five practices. For example 76% of teachers reported growth on "monitoring", 96% on "anticipating", and 88% on "connecting". The MKT reported positive gain, albeit not statistically significant. The observation protocol (RTOP) showed a significant change in two of the five domains. While only some differences were statistically significant, all were positive.

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# **“INCLUSION” AND “DISABILITY” IN THE MATHEMATICS CLASSROOM: THE CASE OF VISUALLY IMPAIRED PUPILS**

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“Inclusion” and “disability” are conceptualised differently by different models in educational research (Nardi, Healy, Biza & Fernandes, 2018). Our two-phase study, consisting of an exploratory phase (Phase 1) and an experimental phase (Phase 2), examines “inclusion” and “disability” in mainstream primary mathematics classrooms with visually impaired pupils in the UK. In Phase 1, we investigate the inclusion and disability discourses of teaching staff (teachers and support staff) and how these are experienced by visually impaired pupils in the mathematics classroom. In Phase 2, we and the teaching staff will design and trial mathematics lessons that aim to address issues identified in Phase 1 in order to examine shifts in the pupil and staff perspectives on “inclusion” and “disability” and to explore benefits that may have arisen in the trials, for all pupils. In both phases, we collect data through classroom observations and interviews of teaching staff and pupils and our data analysis brings together elements of Vygotskian sociocultural theory, discourse analysis and embodied cognition (Nardi et al, *ibid.*). Here we draw on data from three primary classrooms to showcase preliminary Phase 1 findings. Our analysis suggests that “inclusion” is understood by all teaching staff as achieved when the visually impaired pupils’ needs are considered in mathematics lessons. Support staff and teachers though understand “inclusion” somewhat differently: the former see this as adaptation to the needs of individual pupils and the latter as adaptation as well as through universally designed practices. Furthermore, teachers report limited training on inclusion and we observe that the often more nuanced training offered to support staff may also be linked to institutional understandings of “inclusion” which are closer to those of special education in mainstream settings. As for “disability”, all staff consider visual impairment as a form of disability and all, but one, describe disability as socially constructed, rather than an individual’s physical limitation attributed to impairment. Their emphasis on factors which enable the visually impaired pupils to fully participate in all class activities suggests willingness by these staff to deconstruct the notion of “disability”. Sample evidence of how this enabling is enacted will come in our presentation from a lesson to Year 5 pupils on division.

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# THE INSTRUMENTAL GENESIS IN THE DEMONSTRATION OF THE PYTHAGOREAN THEOREM

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Using the graphing calculator, we developed a teaching experiment that allowed to demonstrate the Pythagorean Theorem by two processes: through the similarity of triangles and later with the area of the squares built on the sides and the hypotenuse of a triangle rectangle.

We seek to understand the instrumental genesis (Rabardel, 1995) played by the mediating artifact, graphing calculator, in the student activity system (Engeström, 2001), developing use schemes and instrumented action schemes (Drijvers & Trouche, 2008).

Using a methodology of qualitative research of an interpretive nature, the experiment was realized in the school year of 2017/18, in an 8th year class, with 23 students. In this communication we will discuss the performance of two students. The techniques used to collect the data were based on students' written reports, direct observation by the researcher and images of the graphical representations of the calculator.

The dragging technique inherent in the dynamic geometry software of the graphing calculator facilitated the construction of mathematical knowledge because it allowed the students to verify that their conjectures were confirmed in other triangles and squares of different dimensions. In resolving the tasks, the students showed use schemes and instrumented action schemes. In some situations, the instrumented action schemes created by students later became use schemes for these same students (Rabardel, 1995). In the presentation, further results will be discussed in detail.

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# GENDER AS A CULTURAL CONSTRUCT: THEORETICAL ISSUES

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Gender equity in mathematics education has been described as a “well-explored area of feminist research, theory and reform in education” (Abu El-Haj, 2003, p.403), where several studies used the same instrument to study gender. However, when trying to replicate a study made in two different countries and two different groups of respondents, we failed (Nortvedt & Sumpter, 2017). The results indicated limitations in the questionnaire although following ‘good practice’. This limitation is not new: Johnston and Dunne (1996) concluded that the changes of gender roles and participation have not been addressed by gender research, and that research instead works with a limited conceptualisation of gender. Such a limitation becomes even more problematic when studying groups in different contexts. In Nortvedt and Sumpter (2017), we argued that it is not just a question of intercultural differences but also that within groups different interpretation of gender can occur. Hence, there is also an intra-cultural dimension that needs to be addressed when developing questionnaires aiming to study perceptions about gender differences. Here, the aim is to present the results from a literature review of how gender can be operationalized in questionnaire research in mathematics education that take into account both these two dimensions. Preliminary results show that the amount of research that consider both these dimensions is sparse, and most research assume that gender is a cultural-neutral construct and there are little opportunities for the respondents to demonstrate knowledge about gender beyond the male –female dichotomy such as nuances in gender symbolism.

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# MAPPING THE DEVELOPMENT OF PROBABILISTIC REASONING IN CHILDREN

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In recent years there has been growing interest in young children's ability to reason probabilistically (e.g., Denison & Xu, 2014). Binary choice tasks have long been a popular means to investigate this (e.g., Falk, Yudilevich-Assouline, & Elstein, 2012). This study presents the design and evaluation of a novel binary choice task allowing to explore the development between the age of 5 and 9 of the abilities (1) to distinguish certain from uncertain events and (2) to compare two probabilities.

The sample comprises 177 Flemish children from different classrooms in the last year of kindergarten up to third grade of elementary school. Ninety-seven were boys. Probabilistic reasoning is not included in the school curriculum in these grades. The task was presented as a computer game in which children were introduced to a blindfolded bird that had to pick a berry from one of two boxes. They were asked to help the bird by selecting the box with the greater likelihood for picking a desired black berry. In a first subtask (5 items) one of two presented boxes contained only desired berries, while the other box contained a mixture of desired and undesired (white/green) berries. So, children had to distinguish certain from uncertain events. In a second subtask (24 items based on Falk et al., 2012) both boxes contained a mixture of desired and undesired berries and probabilities had to be compared.

Results showed a good internal consistency for the first subtask ( $\alpha = .87$ ). The smaller internal consistency for the second subtask ( $\alpha = .62$ ) is due to its multidimensionality: The impact of different item characteristics which will be discussed during the presentation. The ability to recognize a certain event significantly improved with grade,  $F(3, 173) = 4.28, p = .006$ . Likewise, the ability to compare probabilities significantly improved with grade,  $F(3, 173) = 20.27, p < .001$ . Based on these results, together with the absence of floor or ceiling effects, the novel task seems reliable and valid for investigating individual trajectories of probabilistic reasoning skills in children.

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# INVESTIGATION OF FINNISH AND GERMAN 9<sup>TH</sup> GRADE STUDENTS' PERSONAL MEANING TO MATHEMATICS

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Learners long to assign personal meaning, understood as personal relevance (Vollstedt, 2011), to mathematical contents and learning mathematics. Personal meanings embody students' subjective motives to deal with mathematics in a school context. This research project focuses on a comparison of personal meanings students from Finland (FIN) and Germany (GER) assign to (learning) mathematics.

Finland has been renowned for its high performing students in international comparative studies. However, in PISA 2015 Finnish students' mathematics performance (low correlation with their socio-economic status) had rapidly decreased compared to the former PISA studies, and Finland is now just barely ahead of Germany (OECD, 2018). On the other hand, Germany has parallel schooling systems and an early selection to tracks according to students' academic performance while Finland has a comprehensive education for all (grades 1-9). Based on these facts a comparison between Finnish and German students' personal meanings would be fruitful to consider them in order to plan the mathematics lessons accordingly. Hence, the research question is:

- Where are differences and similarities between patterns of personal meaning found within the samples from Finland and Germany?

The sample comprises 237 Finnish and 188 German 9<sup>th</sup> graders. The survey consists out of 17 scales that are based on Vollstedt's (2011) theory of personal meaning. The German version was translated into Finnish. Using IRT partial credit models, the psychometric properties of the scales were found to be good. The estimated variances ranged from 0.49 to 5.73 (FIN) and from 0.58 to 5.30 (GER) with most values around or above 1. Scale reliabilities ranged from an acceptable .65 to a very good .88 (FIN) and .68 to .85 (GER). As statistical procedure, differential item functioning (DIF) analysis was conducted to compare the two groups' (FIN and GER) answers. In the presentation, further results will be discussed in detail.

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# EFFICIENCY OF TEST-ENHANCED LEARNING IN TEACHING NUMBER THEORY

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Retrieving information from memory after an initial learning phase enhances long-term retention more than restudying the material; an advantage referred to as the (retrieval practice) testing effect (see Karpicke et al., 2014). The testing effect has been demonstrated with a variety of practice tests, materials, and age groups (Dunlosky et al., 2013).

Test-enhanced learning is a method which uses active recall of the information during the learning process. It has been proved to be efficient concerning learning texts or foreign words, but these experiments were principally carried out in laboratory-environment on psychologist undergraduate students. The topic of this presentation is an experiment on the efficiency of test-enhanced learning used for teaching Number Theory in university. The experiment was carried out at the Eötvös Loránd University, Budapest with real students on real maths lessons. Subjects of our experiments were six groups of undergraduate pre-service maths teachers. Three groups of the six were learning Number Theory using the testing effect and the other three groups were learning the original way. The experimental and the control groups were statistically indistinguishable by their pre-knowledge and by math anxiety tests. All groups were learning exactly the same topics also they had exactly the same lecturer. The experimental groups and the control groups learned the same concepts and wrote the same midterm and final tests. We compared the results of the two types of group.

The score of students on both tests learning by the testing effect were more than 20 percent higher than the results of the control group. A delayed test was written 10 weeks after the semester. The experimental group scored 25 percent above the control group and had showed a massive knowledge of the material.

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# PRE-SERVICE KINDERGARTEN TEACHERS' EXPERIENCES ABOUT TEACHER-GUIDED MATHEMATICAL ACTIVITIES

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The early development of mathematical skills is of utmost important, according not only to teachers' and lay people's experiences and beliefs, but well-documented results have been summarized in for instance a meta-analysis conducted by Duncan and Magnuson (2011).

Fostering mathematical thinking by means of teacher-guided activities may be implemented in several kinds of scenarios. As the time for starting school is also different from the nation. In Hungary since the 363/2012 Decree of the Government on the National Curriculum for Education in Kindergartens contains only a fairly general statement on mathematical abilities, the main aim of the current investigation was to explore how kindergartens implement activities that aim for fostering mathematical abilities, how they embed the development of mathematical thinking in either purely mathematical or other types of teacher-guided activities.

This research aimed to explore the current Hungarian practice kindergarten have in fostering mathematical thinking. A questionnaire was administered to pre-service preschool teachers (N = 253) in the last semester of their study after having completed an eight- to ten-week long practical training. The questions concerned three foci: (1) What is the frequency of different teacher-guided activities? (2) Which mathematical topics were addressed during the eight-week long training? (3) Did they encounter mathematical activities during other types of teacher-guided activities?

The results point out to the potentials in both enhancing the frequency of mathematical activities and widening the topics covered within mathematics: a surprising underrepresentation of geometrical contents have been revealed.

This research has received support from the Content Pedagogy Research Program of the Hungarian Academy of Sciences.

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# DEVELOPMENT OF A COMBINED TEST ASSESSING SIXTH AND SEVENTH GRADERS' MATHEMATICAL COMPETENCE OF ALGEBRA

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This paper is a part of on-going large scale research project aims to contribute middle school mathematics teachers' professional development through web-based hypothetical learning trajectories (HLT). We focus on the notion of HLT, which can be considered as a heuristic tool in teachers' professional development (Simon, 2014). In the project, more precisely, in-service teachers will develop their own HLTs through web-based modules and they will field-test their designs. In this proceeding, we will present a combined algebra test that can be used for data triangulation to assess the effectiveness of the HLTs. Therefore, we consider a research question, 'How to develop a combined algebra test to assess middle school algebra HLTs?'

The combined test includes two major parts for the contexts of sixth and seventh grade mathematics curriculum (MEB, 2013): *multiple choice items*, and *open-ended tasks*. In the development of the test items and tasks, three main steps were followed: (i) textbooks and curriculums were reviewed and a blueprint was prepared, (ii) a pool of questions was constructed, (iii) items and tasks were reviewed by a group of (ten) mathematics education researchers. Initial form of the measure included 24 test items and 5 open-ended tasks for the sixth grade; and of 28 test items and 5 open-ended tasks for the seventh grade. The measure was fulfilled by an extensive group of the students from five different schools. Exploratory factor analysis with tetrachoric correlation matrix, confirmatory factor analysis,  $p$  and  $d$  indexes and reliability analyses for the test are under progress. We expect a valid and reliable combined test for assessing sixth and seventh graders' mathematical competence of algebra.

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# INVESTIGATING THE NATURE AND DEVELOPMENT OF A PROSPECTIVE MATHEMATICS TEACHER'S PROFESSIONAL NOTICING SKILLS DURING FIELDWORK EXPERIENCE

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Integrating theoretical knowledge with school-based practical knowledge is an important goal for prospective mathematics teachers (PSTs). A traditional way of helping PSTs integrate theories learned in university teacher preparation and practice in fieldwork experience is use of reflections. One of the frequently used theoretical constructs in understanding how PSTs develop professionally by engaging in reflections has been professional vision (Goodwin, 1994), which refers to how professionals notice and make sense of situations. The aim of this study is to investigate the nature and development of a PST's professional vision skills, more specifically, professional noticing of students' mathematical thinking in the context of a teacher education program which has a strong focus on fieldwork experience.

This study utilized the self-study method which helps practitioners learn from their own practice (Loughran, 2007). Although investigation of PST reflections is common in teacher education research, a PST's inquiry into her own professional development in collaboration with a teacher educator is rare and has a potential to contribute to literature in understanding PST learning. The authors of this study (a PST and a teacher educator) analyzed the PST's professional noticing of students' mathematical thinking skills by using a framework developed by van Es (2011) and conducting a thematic analysis through the weekly reflective journals across 3 semesters. Additional data sources were semi-structured interviews conducted with the PST's mentor teachers from school sites. The initial results of data analysis indicated that the PST's reflections showed a variety of different levels of noticing students' mathematical thinking (*baseline, mixed, focused, and extended*) and that the development of noticing was not linear. The extended level of professional noticing occurred simultaneously with the themes of "caring for students" and "teacher identity." In the presentation further results on the nature and development of professional noticing skills will be discussed in detail. The results of the study have implications in understanding teacher reflections and learning in the context of practice-oriented teacher education programs.

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# CONFLATING DISCOURSES: UNIVERSITY STUDENTS' EXAM RESPONSES ON INJECTIVE AND SURJECTIVE FUNCTIONS

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The study we draw on here (Thoma & Nardi, 2017) aims to offer insight into mathematics undergraduates' transition from school to university mathematics, focusing on their scripts in the final Year 1 examinations (22 students, 6 exam questions). Here, we present students' responses to an exam question which asked to provide the definitions of an injective and a surjective function, and examine whether two functions ( $g: \mathbb{R} \rightarrow \mathbb{R}$  where  $g(x) = 1/(1+\sin^2(x))$  for  $x \in \mathbb{R}$  and  $h: \mathbb{Z} \rightarrow \mathbb{Z}$  where  $h(n) = 3n$  for  $n \in \mathbb{Z}$ ) are injective, surjective, both or neither. Following a discursive approach, Sfard's (2008) theory of commognition, we analyse students' scripts in terms of their engagement with recall (e.g. recalling definitions) and substantiation (e.g. proving and justifying) routines. Our analysis sets out from the observation that, unlike the UK schools these students have been attending, at university, students are asked to provide definitions using universal and existential quantifiers and logical expressions such as *modus ponens* ("if p then q") and logical equivalence ("if and only if"); and, to pay attention to the domain and codomain of the functions under investigation. Our commognitive analysis identifies evidence in the student scripts which suggests lack of awareness of these required discursive shifts. For example, in four scripts there is problematic use of equivalence in the definition of injective function; in ten scripts there are partial justifications regarding the injectivity of  $h(n) = 3n$ ; and, in seven scripts we noted conflation of justifications for a function's surjectivity in  $\mathbb{Z}$  with those used for functions in  $\mathbb{R}$ . Specifically, this conflation is visible in those scripts that include graphs of continuous functions, where the function is defined in  $\mathbb{Z}$  and students use calculus procedures and terminology as if the functions were defined in  $\mathbb{R}$ . We identified analogous evidence of conflating discourses of different mathematical domains in the student responses to all other exam questions. We see this evidence of students' uneven transition from school to university mathematics and we recommend that teaching at school and university addresses explicitly and systematically the distinctive differences between these discourses.

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# WHEN IS A PATTERN NOT A PATTERN? DEVELOPING PATTERN AWARENESS IN YOUNG CHILDREN

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Papic, Mulligan, and Mitchelmore (2011) define pattern as any “replicable regularity” (p. 238) and identify two contexts for patterns that are significant for mathematical development but also appropriate for young children. The first context is within a single object where the components are consistently related, as with an array of dots. The second context is within an ordered set of objects where there is a consistent relationship between one component and the next, as in a repeating pattern.

In this study we investigate how young children interpret a triangular six-dot pattern, which adults might normally interpret as three rows of one, two and three dots, with the possibility to extend as a ‘growing pattern’ by adding more rows.

In this study, 26 children aged between 36 and 62 months were assessed on their pattern knowledge, in terms of copying and extending the triangular six-dot pattern (Mulligan & Mitchelmore, 2015). As we reviewed the children’s responses, we noticed that these did not fit in easily within Mulligan and Mitchelmore’s (2015) levels and so we reanalysed the data to produce new levels.

We found that only four children copied the pattern correctly and none were able to extend it. Using Mulligan and Mitchelmore’s (2015) classification scale, most children were classified as prestructural or emergent. We realised that gradations were possible within the classifications of prestructural and emergent: children represented none, one, two or three of four possible elements of the pattern, the dots, the rows, the shape and the number. Whereas adults might interpret this pattern as ordered rows of dots, children were seeing it as composed of numerous elements, which made it difficult to copy and impossible to extend. This suggests that, when presenting patterns to young children, we need to be aware of the number of elements they may be perceiving and control this range so that they can focus on key aspects.

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# DEVELOPING EARLY NUMBER COMPETENCIES THROUGH GAMES PLAYED IN SCHOOL AND AT HOME

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Most research studies show that initial number knowledge constitutes the basis for understanding several major principles of the number system. Moreover, early number competencies are considered as strong predictors of mathematics outcomes at the end of the first grade and even later on (Jordan, Kaplan, Ramineni & Locuniak, 2009). In preschool, the development of these competencies should obviously not give rise to formal learning but be developed through significant activities. However, while everyday situations offer significant contexts, they are still not sufficient to develop the basic number competencies necessary for first grade children (Cannon & Ginsburg, 2008). Adults have to create opportunities to learn mathematical competencies. Mathematical games can meet this requirement.

The MathPlay project aims to develop early number competencies through games implemented in school and/or at home. According to the literature, early number competencies consist in counting, conservation ability, magnitude comparison, and (de)composition of numbers. To develop these competencies, we decided to adapt 8 well-known mathematical games. To evaluate the effect of this play-based approach on the development of number competencies, an instrument was created and validated. A quasi-experimental research design (pre/post-test) was implemented in four countries: Luxembourg, Belgium, France and Switzerland. Seven hundred and twenty-five children from all these countries participated in this study. They were divided into one control group and one experimental group with two treatment conditions, X1 (games in school), and X2 (games both in school and at home). The post-test was administrated at two moments: just after the intervention and 8 weeks later. Data is currently being analysed. This communication will present the first outcomes and will answer our research question: ‘what is the immediate and deferred effect of this play-based approach (X1-X2) on students’ number competencies?

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# EFFICIENT AND FLEXIBLE USE OF SUBTRACTION BY ADDITION IN MULTI-DIGIT SUBTRACTION IN ELEMENTARY SCHOOL CHILDREN

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Subtraction problems can be solved via various mental computation strategies, including the subtraction by addition (SBA) strategy, in which one determines how much should be added to the subtrahend to get to the minuend (e.g.,  $343-187=$ \_\_ via “ $187+13=200$ ;  $200+143=343$ ; leading to  $13+143=156$ ”). This strategy can be differentiated from the direct subtraction (DS) strategy, in which the smaller number is taken away (e.g.,  $343-187=$ \_\_ via  $343-100=243$ ;  $243-80=163$ ;  $163-7=156$ ). Despite the assumed efficiency of SBA on small-difference subtractions, current mathematics instruction focuses on the perfect mastery of DS (Selter, Prediger, Nührenbörger, & Hussmann, 2012).

We investigated 32 4th- and 33 6th-graders’ use of DS versus SBA when mentally solving subtractions up to 1000, in terms of the four parameters of the model of strategy change (repertoire, distribution, efficiency and flexibility) and relying on the choice/no-choice method (Siegler & Lemaire, 1997). All participants solved multi-digit subtractions in one choice condition (choice between DS or SBA on each item) and two no-choice conditions (obligatory use of either DS or SBA on all items). We distinguished between two types of subtractions, i.e., subtractions with a small (e.g.,  $504-476=$ \_\_) versus a large difference (e.g.,  $616-28=$ \_\_) between the minuend and the subtrahend.

Most 4th- and 6th-graders reported SBA to solve the items in the choice condition. In both grades, SBA use was more accurate and faster in the no-choice conditions than DS, particularly on small-difference problems. Furthermore, 4th- and 6th-graders flexibly fitted their strategy choices to both numerical item characteristics and individual strategy efficiency characteristics. These findings add to our theoretical understanding of children’s strategy acquisition and challenge current mathematics instruction practices focusing heavily on DS.

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# IMPROVING IN-SERVICE TEACHERS' PROBLEM POSING SKILL BY MEANS OF DIDACTIC REFLECTION

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Recently, problem posing (PP) in mathematics teacher education is becoming more interested in empirical researches (Felmer, et al., 2016). Generally speaking, findings of researches related to mathematics teacher education highlight the importance of mathematics teachers' skill to pose or modify problems with didactic purposes (e.g. Tichá & Hošpesová, 2013). Likewise, this skill involves aspects of didactic reflection, which we call *didactic analysis* in our study. In this research, we use the theoretical notion of didactic analysis proposed by the onto-semiotic approach to mathematical knowledge and instruction (Godino, Batanero & Font, 2007). This notion has been incorporated in a PP strategy that we developed for posing mathematical problems with didactic purposes. Thus, the new PP strategy consists of an Episode, didactic Reflection, Pre problem and Post problem (ERPP). By using ERPP strategy and taking the quadratic function as mathematical object, our research aims to improve in-service mathematics teachers' problem posing skill by means of didactic reflection.

For the purposes of this research, a PP workshop was carried out with 17 in-service teachers, which comprised in solving and posing tasks with individual and group work stages. Certainly, the PP tasks were done taking into account the ERPP strategy. The research methodology consisted in case studies in order to analyze the problem posing and solving tasks of 2 participants. The problems posed by these teachers were classified, using an ad-hoc categorization, and then analyzed, using the content analysis and expert judgment techniques. As a consequence of our qualitative analysis, we have some evidences to claim that the ERPP strategy allows the improvement of in-service teachers' problem posing skill with didactic emphasis in a school context. Finally, we believe that the use of ERPP strategy will contribute to improve the benefits of problem posing, both in mathematics teacher education and training programs. Further results will be discussed in detail during the presentation.

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# SHAPE PATTERNING TASKS: THE ROLE PLAYED BY COLLABORATION IN SUPPORTING CHILDREN'S THINKING

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Generalisation is considered by many to be a highly significant component of algebraic thinking. In particular, constructing general terms for shape patterns supports children in reasoning algebraically about covariance and rates of change (Rivera & Becker, 2011). The research presented in this paper involved groups of children engaging with patterning tasks, wherein they were asked to describe, extend and construct general cases for each of three shape patterns. As generalisation is not mentioned within the current Irish Primary School Mathematics Curriculum, the patterning tasks were potentially novel for the children involved in this research. It was thus necessary to conduct the research within a setting which allowed children to tease out their understanding of the tasks presented, by affording opportunities for them to discuss their thinking, and to work collaboratively (Mueller, Yankelewitz & Maher, 2012). The research questions are:

What strategies did the children employ in seeking to construct general terms from shape patterns? What factors impacted on the strategies the children adopted, and the success they experienced in their constructions of general terms?

A random sample of 42 children was selected from a total of seven third grade classes from two schools. Each child participated in an hour-long task-based group interview with two or three peers. The findings indicate that the children demonstrated a broad range of interconnecting strategies, drawing from both figural and numerical observations and applying recursive, explicit, whole object and counting strategies. Collaboration, and the exchange of ideas was a key support in the progress made by some, but not all, children, and findings suggest that some collaborations may not lead to insight for some children. In the presentation, examples will be given of when collaboration was or was not supportive for specific children, including for example one case when peer interrogation of a flawed whole object approach supported one child in refining her understanding of the pattern structure.

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# 4<sup>TH</sup> GRADERS' WORKING ON A FUNCTIONAL CONTEXT: GENERALIZATION LEVELS AND INFLUENCE OF STIMULI

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In the last decades studies about the early algebra proposal have provided evidences of elementary students' algebraic skills since early ages. Some of these studies (e.g. Blanton & Kaput, 2011) address algebra from a functional approach. In this context, one of the main focus of interest is the students' ability to generalize a relationship between covarying quantities and how it is expressed by the students.

Within this approach, in our study we analyze the generalization levels shown by eight 4th grade Spanish students while working, during an interview, on a task based on the functional relationship  $x+2$ . The task had an inductive structure and asked about the relation between the hours of stay of a car in a parking lot and the money to be paid. At the same time we study how the stimuli made by the interviewer (e.g. suggest, summarize information, redirect, etc.) influence the manifestation of generalizations.

The results indicate that all the students manifested several generalization levels while solving the task. We detect facility to express the functional relationship by referring to specific numbers. When being asked about indeterminate quantities, they manifested generalization by means of referring to generic examples or using verbal expressions to allude indeterminacy. However, most of them required the interviewer's stimuli. Four students accepted the use of a symbolic representation of the relationship when suggested and were able to explain its meaning or reproduce it with another letter.

Our results evidence that the proposal of a functional task with inductive organization was useful to structure students' ideas and reasoning with the final purpose of expressing the generalization of the relationship involved. At the same time they show that stimuli like suggesting processes or redirecting observations were determinant to obtain different answers from the students, to consolidate their ideas, and to provoke and promote the mobilization between generalization levels.

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# A FRAMEWORK FOR STUDENTS' UNDERSTANDING OF INVARIANCE IN PROPORTION PROBLEMS ACROSS GRADES

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Understanding proportionality involves being able to identify the multiplicative relation between two quantities and recognising situations where this relation remains invariant. In the research literature, this invariant is often referred to as an intensive quantity. Both children and adults have been reported to face difficulties in understanding such quantities (Nunes & Bryant, 2015). In particular, students' understanding of an intensive quantity like taste may be linked to their understanding of the invariance of ratios (Harel, Behr, Lesh, & Post, 1994).

This project investigates middle-school students' understanding of invariant ratios in direct proportion contexts, as well as how this understanding changed with grade level. This was done using a paper-and-pencil test followed by interviews. The test contained comparison problems in three different everyday contexts to elicit students' use of invariant ratios and intensive quantities in their explanations. 14 students of Grade 6, 17 students of Grade 8, and 18 students of Grade 10 from a public school in India participated in the study.

Based on a bottom-up analysis of the test responses, we propose three categories of understanding among students: (a) *pattern seekers*, who used constant sum, difference, product, or ratio strategies, but lacked awareness of the intensive quantity; (b) *formal thinkers*, who seemed to identify the invariant in a formal way without being able to associate a meaning with it; and (c) *sense-makers*, who were able to identify the appropriate invariant in the context and make sense of it (Nunes & Bryant, 2015) in the real world. We classified the majority of Grade 6 students in our study as pattern seekers, and the majority of Grade 8 and 10 students as formal thinkers. We found very few sense makers in our sample, with none in Grade 6.

Interviews with three Grade 8 students indicated that their understanding appeared to be dependent on problem contexts and the kind of calculations required by the problem. We plan to further validate and refine this framework by investigating the understanding of invariant ratios among high school students and teachers.

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# MIDDLE SCHOOL STUDENTS' REASONING PROCESSES ON SYMMETRICAL FIGURES THROUGH A DYNAMIC GEOMETRY ENVIRONMENT

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This paper is a part of a teaching experiment conducted within a PhD research aiming to research middle school students' reasoning processes in a dynamic geometry environment (DGE). According to the relevant literature, it is stated that a DGE provides a number of affordances for students to use heuristic, visual and explorative approaches (Hanna, 2000) and to implement abductive reasoning process (Arzarello et. al, 2002). On the other hand, considering Instrumental Genesis approach, students' utilization schemes on a DGE are critical factors for their reasoning processes. In line with this, in this study, we focus on a research question: which reasoning processes and utilization schemes occur in 7<sup>th</sup> grade students' DGE aided proofs?

The data was obtained through two video-recorded clinical interviews and one teaching episode conducted between clinical interviews. The participants of the study consisted of four 7<sup>th</sup> grade students who were familiar with use of a DGE before. During clinical interviews, we presented three tasks that required students to prove whether two congruent polygons given in a DGE are symmetric according to a hidden line. The collected data were analysed through qualitative thematic analysis.

The results revealed that all participants firstly used dragging method for investigating relationships between dependent and independent figures visually. After conjecturing about the symmetrical relationship, certain DGE tools were used with the aim of proving existence (or non-existence) of symmetry line through heuristic, abductive or deductive approaches. After the teaching episode in which students evaluated their strategies, it was observed that deductive reasoning processes made progress in their new strategies that refer to relationships between symmetry line and perpendicular bisectors of line segments constructed between symmetric vertices of given polygons.

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# THE WORLD AROUND PYTHAGORAS: INTERPRETATION, LOGIC AND REASONING CONSIDERATIONS

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The psychological literature on reasoning has provided us in the last fifty years, as one of its characteristic themes, a whole body of evidence that the interpretation and use of the logical connectives is very commonly in disagreement with the meaning prescribed by Classical logic. Similar trends are present in a growing educational literature on reasoning in Mathematics (e.g. in Hoyles & Küchemann, 2002). Our study focusses on determining which specific aspects of the interpretation of conditional mathematical statements deviate from the classical “material implication” and under which conditions this happens. Are these interpretations purely circumstantial, arbitrary, or is there any logic behind this understanding and use? To address these issues we will (1) extend, with a different format and different content, empirical results already documented in the literature, and (2) conduct, along the lines of Stenning & van Lambalgen (2008) the analysis of the answers obtained using Logic Programming (LP) tools, some of whose salient relevant features we will briefly present.

As a paradigmatic example we will describe how the interpretational process occurs on an apparently obvious conditional statement (for second year undergraduate students, mean age=19 years,  $n=178$ ), namely Pythagoras’ Theorem. We will report answers to multiple choice questions which show, e.g., high (68%) endorsement rates for the Affirmation of the Consequent deduction schema. A qualitative argumentation analysis of the written justifications provided by participants confirms the robustness of phenomena as this one, among others to be discussed in the presentation. From this evidence, we shall argue that, psychologically, logic is not fixed, not given from the outset once and for all, but obtained only at the end of an interpretational process which is embedded in a “web of beliefs”: a “world” with its inner logical coherence. From this perspective, talking of a “child logic” in contrast to a “math logic” (O’Brien, et al., 1971) may be misleading in treating this form of reasoning as a “poor man’s logic” in the absence of full acquisition of Classical logic, and not as a phenomenon that can be characterized, explained, and even justified on its own and which is not at all restricted to a certain developmental stage.

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# TEACHERS' PERSPECTIVES TOWARD CURRICULUM

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When U.S. teachers are dissatisfied with their curriculum they often choose to adapt or supplement their materials. The deluge of free, easily-accessed online materials entices teachers to select convenient options, but these may not necessarily yield desirable student learning outcomes. To study how teachers develop skills for managing and selecting options, we consider two perspectives toward curriculum articulated by Choppin, Roth McDuffie, Drake, and Davis (2015). Curriculum viewed as a *delivery mechanism* presents mathematics as a static body of knowledge to be remembered and diminishes the teacher's role to a transmitter. Curriculum viewed as an *epistemic device* engages the teacher and the students in mathematical dialogic situations, and creates opportunities for legitimizing the mathematics.

This work builds on previous work of the Measure Up project (MU), which uses students' knowledge of generalized quantities to introduce and build concepts (Dougherty, 2008). A set of MU lessons about place value were adapted and used in a summer course for sixteen students, 7-8 years old. Students participated in the three-week course, for 1.5 hours per day. Lessons were designed to serve a dialogic purpose and support conceptual learning of place value with quantities counted in different bases. Five teachers, each with 5-10 years of teaching experience, studied the curriculum and observed lesson enactment. In addition to learning about MU, the teachers were introduced to design-based research and improvement science methods.

Although teachers contributed to retrospective analysis to the development of the curriculum, they rarely explored issues that addressed the dialogic nature of the curriculum. They instead reflected on lesson enactment and student actions. This suggests that although the approach to the mathematics was strikingly different, teachers were unfamiliar with viewing the potential epistemic terrain of the curriculum. We believe targeted activities could be designed to help teachers reflect on their own biases toward curriculum and the role curriculum plays in instruction.

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# TEACHERS' KNOWLEDGE AND THEIR STUDENTS' MISCONCEPTIONS ABOUT THE EQUAL SIGN

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Students' understanding of equations, equality and the equal sign has been the focus of many research studies, usually motivated by problems related to the transition from arithmetic to algebraic thinking (Tossavainen et al., 2012). Such studies concluded that many students lack a relational understanding of the equal sign. A limited number of studies have been conducted to determine teachers' understanding of these concepts, or how this might have influenced students' understanding of these concepts. No South African curriculum document explicitly mentions the concept of equality or the equal sign, or provide guidance to ensure that students understand this concept. Neither do South African textbooks explicitly promote the relational conception of the equal sign. We asked the following research questions: (1) What misconceptions do Grade 6 students have of the equal sign? (2) What is the nature of teachers' Mathematical Knowledge for Teaching (MKfT) of the equal sign? (3) Could the nature of teachers' MKfT of the equal sign possibly influence students' misconceptions of the equal sign? The sample consisted of 57 grade 6 students, and two grade 5 and one grade 6 teacher teaching Mathematics. We used a teacher questionnaire; one focus group teacher interview (FGI); a student questionnaire; and one-on-one interviews with six students. Results indicate that most students have a *flexible operational* (level 2) or a *basic relational* (level 3) understanding (Matthews et al., 2012) of the equal sign. Teachers' questionnaires revealed low levels of MKfT of the equal sign which limited their ability to identify, correct, prevent or reduce misconceptions of the equal sign. Significantly however, during the FGI teachers revealed an *implicit* awareness of the relational conception of the equal sign, and acknowledged that their students display a largely operational conception. Through discussion, teachers gradually became *explicitly* aware of the equal sign as a relational conception, and that textbooks' and their own teaching might encourage students' operational view of the equal sign.

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# USING FORMATIVE ASSESSMENT TO SUPPORT STUDENT DEVELOPMENT IN LOWER SECONDARY MATHEMATICS

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The new curriculum in Finland highlights the role of formative assessment in student evaluation. While students are expected to take an active role in every step of the learning process from planning to evaluation, as well as to recognise the role of affect and metacognition in their learning, teachers are expected to continuously discuss with the students about their learning processes, working habits and behaviour, give supportive yet realistic feedback, and regularly inform both students and their parents about these issues (FNBE, 2016). These challenges guide a small pilot study aiming at developing three students' mathematical thinking in lower secondary school.

In a previous project, I developed a tool for studying students' mathematical thinking through problem solving and view of mathematics (see Viitala, 2018). In the pilot study, the tool is used by a mathematics teacher in short learning discussions with the students for two purposes: identifying individual students' strengths and weaknesses in mathematics, and in setting and evaluating individual long-term goals for mathematics learning. In the classroom, students are supported to reach their learning goals through formative assessment, simultaneously helping them to develop mathematical thinking.

In helping students to reach their learning goals, the teacher had to modify their normal classroom activities. First, she increased problem solving to support students in solving word problems. Second, she had to rethink classroom norms and learning space to promote peer collaboration in the classroom. Furthermore, self-evaluation helped the students to concretise the progress in achieving their learning goals. The pilot data will be analysed using the analytical framework developed together with the tool (Viitala, 2018) and the results of the experiment will be discussed in the conference.

Finding adequate learning goals with the students was easy in the short pilot study. When extending the study over a school year, the role of the teacher and the tool is emphasised. In classroom, the main study should include regular problem solving to enhance the use of the tool and strengthen the results of mathematical development. Also, the students are required to actively take responsibility of their own learning.

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## PRIMARY SCHOOL MATH: PROPORTIONS AS A STARTING CONTEXT?

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Sound proportional reasoning is often considered as an ultimate goal for teaching middle school math. In our previous study, students (11-12 years old) worked in pairs, with the goal to arrive at and maintain the value of the composite parameter. In spite of a high efficiency in teaching ratio-based concepts, switching from one ratio-based concept to another (e.g., from density to concentration) requires repeating some of the steps. We assume that the main obstacle here is a significant limitation of students' number concept at its earliest form: whole numbers are not considered as ratios in primary school. However, it is crucial for further progress to understand a whole number as a ratio (Dole et al., 2012). Such introduction should be done even before children deal with numbers – at the beginning of the 1st grade. In our study we redesigned this part of primary school Mathematics curriculum according to the principles of Activity Theory and Developmental Instruction (Davydov, 1986) and then tested it within the educational design framework.

Currently we are at the first year of 6-years experiment so we present the results only from one classroom and focus mostly on qualitative analysis. The data sources include video recordings of the classroom processes and students' materials. Early results show that using proportions as a starting point in teaching mathematics has a great potential in terms of mathematics modelling, problem-solving, motivation, and engagement. Despite significant individual differences, students are able to (1) predict results of their partner's work and thus interiorize their hands-on activity into mental actions; (2) recognize the importance of measurement in contrast to guess work; and (3) get a first-hand experience about why it is important to write down the results of measurement. This introduction opens a direct way to the concept of number that we expect to be understood as a ratio and thus lead towards fluent understanding of proportional reasoning.

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# LANGUAGE CHOICE AND MEANING IN PREDICTION

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There is a reciprocal relationship between language and conceptualization: language repertoires are necessary to convey an idea, and the language used to describe an idea shapes the way people conceptualize it. This reciprocity motivated our cross-sectional longitudinal study to investigate children's language repertoires, focusing on language for conjecture and prediction in English-medium and French Immersion instructional contexts in an Anglophone region in Canada. The English-medium classrooms included students for whom English is an additional language. Students worked in groups in class and we interviewed the groups shortly thereafter to extend the group work. At the end of each interview, we asked participants about the meaning of things they said in their group work and in the interview. We were guided by our desire to see language as a resource; we attend to the language repertoires of the students to understand their perspectives on prediction, not to identify deficits in their knowledge but rather to afford us new perspectives on situations involving prediction. Our primary analytical approach was to map language strategies for prediction. In the presentation we will describe the tasks used for the group work in the research and some brief excerpts of student interaction to set the context for the major findings.

First, though not a goal of the research we found that non-trivial narrative contexts for mathematical tasks, along with pair and group structure, seeded considerable talk amongst students. Second, context-grounded language allows students (and ourselves) to talk about uncertainty without technical language. Third, we identified differences among prediction situations—students did not see these situations in the way we had expected. Fourth, refuting a claim from Rowland (2000), we found the students in our study avoiding uncertainty language for conjecture. They stated their conjectures with simple assertions (e.g., 'it will be six'). Then, when testing their conjectures, they said no or yes (or similar words). Fifth, participants across the grade spectrum in our research (starting at Grade 3) and even operating in a new language had language repertoire for making distinctions about levels of certainty. Sixth, participants tended to rely on their most familiar language patterns (especially patterns resembling their first language) to make such distinctions. (This research was supported by a Social Sciences and Humanities Research Council of Canada grant entitled "Students' language repertoires for investigating mathematics," PI: Wagner.)

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# EXPLORATORY STUDY ON THE RELATION BETWEEN THE UNDERSTANDING OF CORRELATION AND ASSOCIATION

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Both the correlation between two numerical variables and the association between two categorical variables are applied to real problems. Prior research has shown the misconceptions about the correlation and explored factors which can influence the judgments of the association in contingency tables. This study aims at exploring whether the understanding of the correlation between two numerical variables in scatter plots can predict the judgements of the association in contingency tables.

The aim of this study not only provides insight into the possibility of learning transfer but also contributes the development of mathematical curriculum in schools. We use two questions to measure students' understanding of the correlation tendency and strength in scatter plots (Liu, Lin, & Tsai, 2011) and eight questions to measure students' judgment of the association in contingency tables (Yang, Chu, & Liu, 2018) are surveyed to 1188 eleventh-graders who had learned the Pearson correlation in school.

We suppose that the understanding of the correlation tendency in scatter plots is a more effective indicator for differentiating students' judgment of the association in contingency tables than the understanding of the correlation strength in scatter plots. However, the result only shows that the understanding of the correlation strength in scatter plots is significant ( $t = -2.04, p = .04$ ). A possible reason is that students use an alternative conception that the shape in scatter plots looks like a line, rather than the concept of comparing the scattered points on the four quadrants of the coordinate system, to judge the correlation tendency. Furthermore, it is suggested to investigate whether the understanding of the Pearson correlation formula is a more effective indicator for differentiating students' judgement of the association in contingency tables than the understanding of the correlation strength in scatter plots.

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# PRESENTATION OF FRACTIONS IN PRIMARY MATHEMATICS TEXTBOOKS: THE COMBINATION OF HISTORICAL ORDER AND “TRANSCENDENT RECURSION”

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In this study we analyse the historical-genetic principle and transcendent recursion mode of fraction understanding. According to the development of fractions found in history, Park, Güçler & McCrory (2013) classify fractions into “Part-whole”, “Measurement”, “Division”, “Set-theory”. For these four milestones, “part/whole” denotes a relationship between two integers as part of a whole, and “measurement” is interpreted as a ratio of two integers in the concept of commensurability, neither of which is free from the category of the concept of integers. “Division” treats fractions as multiplicative inverse. Each rational number set in the sense of “set theory” exists in the form of an independent number, indicating the successful expansion from an integer system to rational number system.

Pirie and Kieren (1994) proposed a model of growth of mathematics understanding by combining theories of epistemology, epistemological barriers, conceptual definition, conceptual representation, multi representation, operational concepts and structural concepts. It is known as transcendent recursion model in China (Li & Zhang, 2002). Based on the analysis of multiple meanings of fractions and guided by both the historical-genetic principle and transcendent recursion model, the conclusion may be made that the presentation of fractions in textbooks should take into account both the development of fractions in history and the folding back in “transcendent recursion”.

Finally, when presenting fractions, the primary mathematics textbooks should not only introduce different concepts in order, but also attach importance to the organization and design of textbooks concerning the “transcendent recursion”, thus helping students with understanding of fractions.

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# DESIGN AND EVALUATION OF A WORKSHOP TO DEVELOP TEACHERS' COMPETENCE IN DESIGNING MATHEMATICAL GROUNDING ACTIVITIES

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Although Taiwan is one of the highest achieving countries in TIMSS and PISA, it is notorious for its students' low interest and confidence in mathematics. In response to this problem, a project titled Just Do Math has been launched in Taiwan. The project focuses on developing students' fundamental prerequisite mathematical ideas before learning in regular classes, providing students with manipulative representations that they can refer to in future learning, and employing gamified learning activities to increase students' motivation. The project involves multiple levels of math education, including a teacher professional development (TPD) program aimed at cultivating teachers' competence in designing learning tasks that reflect the project's themes; such tasks are named mathematics grounding activity modules (MGA modules). This paper describes the design of this TPD program and evaluates its effectiveness.

The TPD program design comprises five stages: theory learning, evaluating the MGA modules, designing the MGA modules, teaching experiments, and reflection and refinement. During these stages, participating teachers must design and repeatedly refine their MGA modules according to suggestions from teacher educators and their peers, feedback from their students, and theories pertinent to meaningful learning, representations, and motivations.

To evaluate the effectiveness of the TPD program, a questionnaire containing 46 5-point Likert scale items was developed according to the ARCS Model of Motivational Design (Keller, 1987). The sample included 80 primary and secondary math teachers in the program. The findings revealed that (1) the teachers' attention was grabbed and sustained by the program because they wished to inspire their students in mathematics learning, and the program provided pertinent learning opportunities; (2) the teachers confirmed that the program was relevant by providing both interior and exterior motivations; and (3) teachers' evaluative confidence and satisfaction scores regarding their ability to design MGA modules and module quality were significantly lower than those regarding attention and relevance. Teachers' confidence and satisfaction affect their motivation and the effort they are willing to exert into their learning; therefore, improving these facets should be a primary focus for the project.

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# THE ROLE OF REFLEXIVITY IN THE EXPRESSION OF TEACHERS' IDENTITIES IN TEACHING MATHEMATICS

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This short oral communication seeks to highlight the structural and cultural properties and powers (mechanisms) that condition the emergence and expression of teachers' identities in teaching Grade 3 mathematics. It also foregrounds how teachers draw on their personal power, that is reflexivity, to 'act back' on these structural and cultural mechanisms. I follow Archer (2000) to posit that teacher identity refers to the manner in which they express their teaching roles. Because individual teachers have varying dispositions and personal characteristics, they perform their common teaching roles differently from one another. Central to understanding teacher identity is the notion of reflexivity, that is, an internal conversation which enables persons to consider, express and transform that which they care about, and their roles as teachers (Archer, 2000). While everyone is a reflexive being, we exercise our reflexivity in different ways.

The empirical data that is used is drawn from classroom observations and interviews generated over a period of six months in four Grade 3 teachers' classrooms. Data from the observations and interviews suggest that despite recent changes in the prescribed roles of teachers tracked through successive policy documents, there has been little change in the way in which teachers express their identities. Also, teachers' modes of reflexivity have remained static, thus constraining their ability to 'act back' on these new roles. The research suggests that these modes of reflexivity promote reproduction of existing structural and cultural mechanisms and of established teacher identities.

Through the process of identifying the structural and cultural emergent properties that condition the expression of teachers' identities in teaching Grade 3 mathematics, and their modes of reflexivity, one is able to understand why the expression of teachers' identities are continually reproduced, despite changes in the roles of teachers. This in turn enables one to consider how one can begin to shift teachers' identities to assume more progressive teacher roles. This short communication advances knowledge on teacher identity by suggesting that teachers' personal emergent property and power of reflexivity guides the *choices* they make. The implications of this research are that teacher education institutions need to develop pre-and in-service courses that recognise teachers, their current teacher identities, and work together with them to modify these identities. Careful consideration should be given to teachers' modes of reflexivity and whether it is possible to disrupt the stance of teachers toward more 'progressive' systemic roles.

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# TEACHER QUESTIONING: COMPARING MULTI-MEDIA PLATFORMS IN INITIAL TEACHER EDUCATION

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Teachers' pedagogical strategies for posing questions are important for students' learning. In mathematics, teachers' questioning strategies may influence students' developed understanding of mathematical concepts, and students whose teachers ask them to explain and justify their mathematics tend to have higher mathematics achievement than students whose teachers do so less frequently. Researchers have also observed a relationship between teachers' questioning strategies and their pedagogical content knowledge. Such findings have led to interest in studying and improving preservice teachers' (PSTs') questioning skills, but studies tend to examine PSTs' practices as demonstrated through a single medium, such as a written account or a rehearsal. In contrast, the present study examined how PSTs approximated teacher questioning through the use of two different animated mediums. Our research questions were, *How do PSTs illustrate questioning strategies using one of two multi-media platforms (LessonSketch or GoAnimate) to approximate teaching? How do PSTs' illustrations of questioning compare across two platforms?*

In this mixed methods study, we used a convergent parallel design to create and analyze a task that was implemented across four US institutions with 99 PSTs using one of two multi-media platforms: GoAnimate or LessonSketch. To examine questioning strategy type, we adapted Hufferd-Ackles, Fuson, and Sherin (2004) *Levels of Math-Talk Learning Community* rubric to evaluate questioning sequences in the shorter timeframe of PSTs' one-minute animations. We also analysed the number of teacher utterances, use of visual information, and format of classroom interaction.

Results indicated that PSTs who used GoAnimate included significantly more utterances than those who used LessonSketch, yet the level of questioning was statistically equivalent across both mediums. PSTs who used GoAnimate tended to convey small group interactions, whereas LessonSketch depictions typically portrayed whole class interactions and included additional visual information. We will discuss these findings and their implications for teacher educators to select and align use of such multi-media platforms for use in practice-based teacher education. Our findings indicate that one platform is not superior to the other, but rather that each platform may be particularly suited to specific instructional aims.

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# SELF-REFLECTION AS A PEDAGOGICAL TOOL FOR PRE-SERVICE ELEMENTARY TEACHERS' GROWTH IN REASONING ABOUT MEASUREMENT

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Pre-service elementary teachers (PST)s often enter university studies with procedural, inflexible understandings of the mathematics they will be expected to teach, especially in the domain of measurement (Batturo & Nason, 1996). Their mathematical understanding is fragile and it is important for teacher educators to provide opportunities for self-reflection in learning. The act of reflection has long been considered a critical component of both constructivist teaching as well as problem-centered learning (Wheatley, 1992). Reflection allows the mathematical thinker to distance himself/herself from the mathematics and consider the rationale behind a way of reasoning and if there are other potential paths to solve the task.

In this presentation, I will discuss the incorporation of reflection in a mathematical content course for PSTs and follow three PST case studies, at varying levels of understanding of measurement, through several data points to explore both their content knowledge of area measurement over time and their self-awareness of their mathematical thinking. In the presentation the following research questions will be addressed: (1) How did PSTs conceptualize area measurement and reason through a tiling task prior to instruction? (2) What changes in mathematical knowledge and reasoning, if any, did the PSTs describe when asked to reflect on prior thinking? (3) In what ways did their reflection on the task align with their mathematical choices when asked to solve a similar task later in the semester?

Using a descriptive coding process, I analysed both the PSTs' mathematical strategies and their own understanding of their mathematical thinking across two assessments and reflections on these assessments. Findings indicate that for all cases, no matter the skill level, the act of reflexivity in learning was important because it allowed PSTs to encounter ways in which they had grown, to witness changes in their conceptualization of the topic such as recognizing and discussing various approaches to the task, and to help build confidence in their ability as a learner.

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# PERSONAL CHARACTERISTICS OF TEACHERS THAT INCREASE THEIR INCLINATION TO ENACT PEDAGOGICAL PROBLEM-SOLVING ACTIVITY

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The broader study from which this data was selected examines whether there are personal characteristics that teachers possess that incline them to pedagogically problem solve when implementing mathematical problem solving activity in class. Mason and Spence (2000) point to something more than knowing-about what to do to be able to act ‘in the moment’ in class: “The central problem of education is that knowing-about does not in itself guarantee knowing-to, as teachers have attested throughout the ages” (p. 135). This study examines personal characteristics of a Year 1 teacher who demonstrated knowing-about and knowing-to as he elicited children’s thinking about links between diagrammatic and numerical representations of sharing and notions of remainders during a problem-solving task. The question posed is “Are there personal characteristics that inclined this teacher to pedagogically problem solve ‘in the moment’?” A post-lesson video-stimulated teacher focus group and teacher interview provided data that enabled analysis of this teacher’s orientations to the successes and ‘not yet successes’ he encountered in class. It was found that this teacher possessed the same personal characteristic ‘optimism’ (Seligman, 1995) that Williams (2014) found was possessed by students who were inclined to problem solve mathematically. In particular, the optimistic dimension pervasive-specific was found to be critical for this teacher’s progressive modifying of his questions as he worked to elicit children’s thinking and kept trying different variations of questions when he did not initially succeed (‘not yet knowing-to’ as specific). He did not give up, because he had ‘taken in’ his ability to succeed in such situations as an attribute of self (success as pervasive). This finding has implications for teacher education and teacher professional learning because optimism can be built (Seligman, 1995). The question thus becomes, how can we build the optimism of teachers to increase their inclination to know-to in the moment. Doing so should increase teacher capability in implementing mathematical problem-solving and progressing student learning.

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# INVESTIGATING THE RELATIONSHIP BETWEEN EPISTEMOLOGY AND GOAL-ORIENTATION IN LEARNING STATISTICS

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The ability of formulating personal point of view (e.g. interpreting and making inferences from data or diagrams) is an important characteristic in developing individual's statistical literacy, reasoning and thinking. However, few research explores how students' epistemological belief is related to goal-orientation in learning statistics, which is thus the main purpose of this study.

As for epistemological belief, we referred to the four dimensions: certainty, simplicity, source, and justification, in Hofer and Pintrich's (1997) research, to design the questionnaire for measuring secondary students' epistemological belief in statistics. As for goal-orientation, we revised the questionnaire of goal orientation towards learning science in Chinese, developed by Tsai, Ho, Liang and Lin (2011), to measure secondary students' goal orientation towards learning statistics.

The participants in this study were 488 tenth-graders from five senior high schools in Taiwan. They had been taught about basic statistical topics including probability and statistics as well as data analysis and statistical diagrams. The questionnaires were delivered as a paper-and-pencil test.

Through item analysis and exploratory factor analysis, three factors of simplicity, non-justification as well as exploration were identified and different from Hofer and Pintrich's (1997) research. Two factors of mastery as well as performance goal orientations were identified and the same structure as found in Tsai et al.'s (2011) study.

Moreover, the hypothesis that students who inclined to the epistemology of exploration more likely held mastery goal orientation and student who inclined to the epistemology of simplicity or non-justification more likely held performance goal orientation, and how the findings can contribute to statistical instruction will be discussed.

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# EXPLORING A MATHEMATICS TEACHER'S DESIGNING OF ACTIVITIES INTEGRATING MODELLING AND READING TASK

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In mathematics, to cultivate students' 21st century skills, both modelling and reading play important roles because they can provide students opportunities and support them to solve life situation problems (Blum, 2002; Adams, 2003). However, little is known about teachers' design process of activities. Hence, we aimed at investigating factors influencing a teacher's decision in designing activities integrating model eliciting with reading tasks. This study is a case study conducted by one of teachers, Lily (pseudo name), who had taught for more than 20 years and kept trying innovative teaching. Data were collected from field-notes, documents generated during the design process and in-depth interviews by the graduate. The stages of analysing are shown in Figure 1.

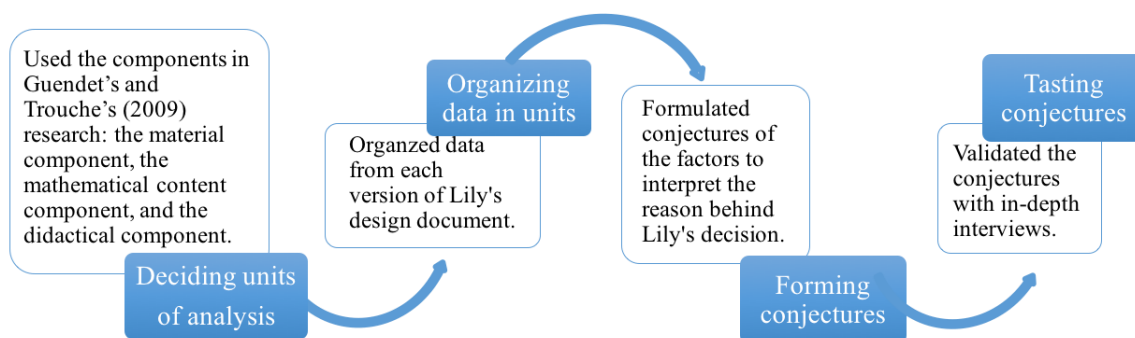


Figure 1: Stages of analysis.

Three factors which mainly influenced Lily's decisions in the process of developing the documents were (1) her intention to facilitate students' advantages but not to suppress students' weakness, (2) her knowledge about modelling, and (3) her belief about how to teach well. Lily set up learning goal according to her intention which was less acknowledged in literature but was a key to influencing teachers' designing. We also provided Lily criteria for evaluating activities to scaffold her to realize her intention in designing. Furthermore, Lily's belief was transformed after finding students' engagement had been enhanced. We will discuss more about these factors intertwine to influence Lily's designing.

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# PROFESSIONAL DEVELOPMENT OF A MATHEMATICS TEACHER THROUGH WEB-BASED EDUCATIONAL PORTAL

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Web-based educational portals allow the creation of virtual campuses independent of time and space for use in education and training processes. Teachers' professional development could be elaborated through such web-based implementations (Kao & Tsai, 2009). This research aims to investigate how a middle school mathematics teacher prepares hypothetical learning trajectories (HLT) to be used in teaching and learning algebra through a web-based educational portal, and how he develops his pedagogical content knowledge while he refers to designed learning trajectories.

The web-based educational portal was created with a project funded by Scientific and Technological Research Council of Turkey, and the portal includes an extensive information regarding HLT, i.e., theoretical background, design steps, affordances etc. More precisely, the portal aims to support teachers' professional development providing a specific environment them to interact with other teachers and researchers. In the present paper, we focus on the case of Atakan (pseudonym, male and 27 years old), who had a five-year teaching experience and had no experience on learning trajectories.

Data were gathered through clinical interviews, web-based portal logs, video recordings of Atakan's lectures and his reflections after employing trajectories. The collected data underwent a thematic analysis to conceive his practices. Results revealed that, before referring to the notion of HLT, Atakan did not prepare a detailed lesson plan except some small preparations, and he not only did not prepare such plan considering students' pre-knowledge and misconceptions, but also did not ask carefully-designed questions that would reveal students' mathematical thinking, and that he generally followed conventional teaching techniques. After considering the notion of HLT, Atakan paid attention to the feedbacks given through the web-based educational portal that he made progress in the pedagogical content knowledge and he developed a number of professional teaching skills; one appeared as constructing a hypothesis on students' learning and reasoning steps.

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# DEVELOPING MATHEMATICAL THINKING POWERS: THE CASE OF USING A COMPUTER ALGEBRA SYSTEM IN DIFFERENTIAL EQUATIONS

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All students can learn to think mathematically, yet this has not been widely achieved in mathematics instruction. Using a computer algebra system (CAS) in courses such as differential equations (DEs) can be particularly powerful to develop mathematical thinking (MT) powers due to the facilities provided for the calculation and visualization of solutions (Rasmussen & Kwon, 2007; Zeynivandnezhad & Bates, 2017). This study examines the MT powers fostered while students work with a CAS in DEs. The study was conducted in two parts: a teaching experiment with an undergraduate DEs class of 37 chemical engineering students in a public university in Malaysia and task-based interviews with six students from the course. The theory of instrumental genesis and prompts and questions (Mason, Burton & Stacey, 2010) informed the design of tasks in the parts of the research. Instrumental genesis explains the interaction between student and a CAS. The researcher in the teaching experiment part collaborated with a lecturer with the MT approach and students completed tasks to construct appropriate instrumented action schemes. The task-based interviews took place after students completed the course without the students worrying about exams. Qualitative analysis of the transcribed interviews based on Mason et al. (2010) showed that the students applied a wide range of mathematical thinking powers such as specializing, imagining and expressing powers. The appropriate choice of the problem, integrated with prompts and questions, can lead to explicate MT powers. Real life problems need more use of MT powers than problems that are straightforward and procedural in nature. Knowledge of DEs and the Maxima commands had effect on utilization schemes to use mathematical thinking powers.

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# WHAT IS IMPORTANT IN MATHEMATICS LEARNING: PERSPECTIVE FROM THE CHINESE MAINLAND STUDENTS

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Understanding how to teach and learn mathematics effectively have been investigated for decades around the world. Recently, a social-cultural approach was adopted to investigate and compare what are emphasized to optimize effectiveness during mathematics classroom teaching within different countries (Seah & Wong, 2012). Different education systems embrace and emphasize different approaches in teaching and learning mathematics. With the increasing interest in studying the Chinese learners' phenomenon (Wong, 2004), as a part of the international study named What I Find Important (in mathematics learning) study (WIFI study), this study investigated what the Chinese Mainland students valued in their mathematics learning in the context of the new curriculum reform. Values in mathematics education are the deep affective qualities that education fosters through the school mathematics and express the extent to which we value aspects of classroom norms and practices that relate to the teaching/learning of school mathematics (Bishop, 2001). By the use of the WIFI questionnaire, data is collected by ten continuum dimensions items and an open-ended, scenario-stimulated responses section. A total of 2,518 students in the Chinese mainland expressed what they thought important in their mathematics learning. Results showed that students tended to value process, pleasure, ability, facts and theories, recalling and exploration compared to the opposed dimension in the scale. They also valued objectivism, openness and progress of mathematics. Significant differences were found among different age and gender. Primary (vs. secondary) students tended to value ability, effort, diligence, formula and memory more. With the increasing of age, they began to value knowledge and thinking in their learning. Boys tended to value, ability, rational understanding and creating more than girls, and girls (vs. boys) tended to value exploration mathematics more. With such knowledge, the research can provide insights and deepen our understanding of students' mathematics learning environment and help teachers design their teaching more effectively.

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## **POSTER PRESENTATIONS**





# GEOMETRIC THINKING LEVELS AMONG COLLEGE OF EDUCATION STUDENTS

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Geometry is a key area of math. Reviewing the curriculum of primary and secondary school indicates that geometry is one of the major academic subjects, and it is considered one of the most difficult areas of mathematics to pupils.

Quite a few studies conducted in recent decades reported the difficulties encountered by pupils that learn geometry. One of the main reasons for these difficulties is the gap between the level of teaching and learning abilities to the level of pupils' understanding. The pupils are low-leveled geometric thinkers, while the teachers are trying to provide them with high-leveled knowledge (Mayberry, 1983; Senk, 1985).

Students that received, in the mathematics department at academic college, specialized elementary and junior high School curriculums are committed to studying various courses in geometry. In our experience at college of education, we meet students that have difficulty at learning geometry.

In order to make teaching more effective and efficient, we conducted a study that examined the level of geometric thinking of the students who want to be math teachers and come to learn in college of education. To this end, a questionnaire was comprised of 15 questions that examine the first three levels of geometric thinking by the Van Hiele theory (Usiskin & Senk, 1990). The questionnaire was given to students who specialize in mathematics program for primary and secondary school (N=84). The conclusion obtained from the study is that a significant proportion of the students received in the mathematics department at academic college control only at the lowest level. In order to qualify students to the third level, at least, we need to teach them geometric during the first semester of learning.

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# THE RELATIONSHIP BETWEEN COGNITIVE AND METACOGNITIVE STRATEGY FOR CONSTRUCTION OF GEOMETRY PROOF

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In advance studies of geometry proof, there is a research which has investigated the structural relationship of the cognitive and metacognitive reading strategy uses for reading comprehension (e.g. Yang, 2012). This study has focused on the cognitive and metacognitive strategies as reading strategy. In addition, it has clarified the structural relationship between these strategy uses and, moreover how these strategy uses are related to the reading comprehension of a geometry proof. These results are of great interest for reading comprehension. However, these strategy use are restricted to reading comprehension of geometry proof.

We also should discuss the relationship between cognitive and metacognitive strategies not only reading, but also constructing. I think reading and constructing are mutually related in the similar way of writing and reading in language. Therefore, I focus on the proof construction of geometry in this study. The research questions are:

- What is the relationship between cognitive and metacognitive strategy uses in the proof construction of geometry?
- How do these strategy uses enhance and decline proof construction of geometry?

To assess the ability to construct proof and such strategies, I firstly have been making questionnaire about the cognitive and metacognitive strategy and the test about the ability to construct proof. Then the structural relationships among cognitive, metacognitive strategy and ability to construct proof of geometry will be confirmed by structural equation modelling (SEM) for junior high school third graders to improve learning state of proof construction of geometry. The reason for selection third graders is that they seem to use such strategies than the second graders.

By investigating structural relationship using SEM, I will clarify the current state of the strategy use for construction of geometry proof, and characterize cognitive and metacognitive strategy uses for geometry proof of junior high school students.

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# “THINK, DRAG, COMMUNICATE” - A SCAFFOLDING TOOLKIT TO FOSTER ARGUMENTATION AND PROOFS

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It is widely recognized reasoning and argumentation as fundamental mathematical capability. Students should be able to create arguments that support, refute or qualify a mathematical claim. An argument consists in a deductive text produced according to specific socio-mathematics norms which regulate communication among mathematicians (Mariotti, 2006). In the above perspective, we assume communication as both educational tool and objective, and language is seen as *means of objectification*, that is an artefact “intentionally used by individuals in social processes of meaning production, in order to achieve a stable form of awareness” (Radford, 2002). The paper presents a digital scaffolding artefact, the Interactive Semi-open Question (ISQ), which allows to make language an object, consisting in virtual language tiles, that can be manipulated in order to construct answers to open-ended questions. The student can choose, drag and juxtapose some of the available language tiles so passing from freely thinking to formulate statements according to shared textual conventions. Our research questions are: (RQ1) Can ISQ language manipulation help students in moving from argumentation as a process (reasoning) to argumentation as a product (deductive text)? (RQ2) Can ISQ language manipulation foster the achievement of a stable form of awareness of their mathematical thinking? Pilot experiences involving students from 10<sup>th</sup> grade high school are ongoing, using ISQ within geometrical proofs and for supporting conjectures in explorative context. In both the cases, there is the emergence of the argument, which often is completely missing during freely thinking (RQ1). Moreover, students are fostered to be aware of their knowing of mathematical concepts (RQ2), especially when applied in geometrical proofs. In this respect, the design of the tiles is crucial, as the availability of well-designed tiles can allow the student’s own misconceptions to become evident. This research is part of the Italian National Project “Digital Interactive Storytelling in Mathematics: a social approach based on competence”, Prin 2015, Prot. 20155NPRA5.

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# **SCAFFOLDED TRANSFERENCE FROM BASIC MUSICAL NOTATIONS TO BASIC FRACTIONS: A STUDY ON THE DEVELOPMENT OF PRESERVICE TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE**

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An and Tillman (2014) connects between music and mathematics in Vygotsky's social interactions using pedagogical content knowledge (PCK). Shulman (1987), as cited by Johnson (2017) defines PCK as the blending of content and pedagogy into the understanding of teaching and learning of mathematics in adaptive instructions. The researchers narrowed down Schulman's seven categories of teacher knowledge to form a conceptual framework of three (content knowledge, pedagogical knowledge, and pedagogical content knowledge) in fractions. Despite studying music and dance, Ghanaian preservice teachers cannot transfer this knowledge to improve upon knowledge of teaching of fractions. The researchers therefore explore how preservice teachers can transfer knowledge with the following research questions:

1. How does PCK improve preservice teachers' knowledge in fractions?
2. To what extent does PCK help transfer music to fractions?

The exploratory quasi-experimental (pretest-posttest non-equivalent control group) design explored two groups of 75 preservice teachers. The experimental group was administered by the lecturer utilizing music-mathematics techniques and the control group was administered by the lecturer using only standard mathematics lectures in order to control for internal threats to validity, abide by ethical issues, ensure the internal consistency of data instruments with Kuder-Richardson 20 formula, and analyzed with independent t-test and one-way analysis of variance.

The independent-samples t-test scores showed PCK improved preservice teachers' knowledge in fractions ( $p > .05$ ). Equally, ANOVA results show that PCK helped the preservice teachers to transfer music methods to fractions.

We concluded that social interactions as concepts in Vygotsky's theory did not only improve preservice teachers' knowledge for teaching fractions but also brought music joy, fun and variety of plays to develop the teaching and learning of fractions.

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# TWO SENSES OF UNIT WORDS AND IMPLICATIONS FOR TOPICS RELATED TO MULTIPLICATION

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Research on reasoning about topics related to multiplication, and quantitative reasoning more generally, has long emphasized the importance of units. At the same time, we are not aware of mathematics education research that has considered whether students might understand the meaning of unit words in different ways. We identify two meanings that students might have, a measurement meaning and a classification meaning. These two meanings for unit words lead to two different ways that students might interpret how numbers describe quantities. We discuss how the classification meaning for unit words could cause difficulties for learners in topics related to multiplication, for example by leading students to formulate equations with the well-known “reversal error” (e.g., see Lucariello, Tine, & Ganley, 2014).

## HOW MIGHT STUDENTS INTERPRET NUMBERS AND UNIT WORDS?

Suppose a student is considering a set of 5 identical paperclips. What does the phrase “5 paperclips” mean to the student? If the student interprets the unit word “paperclips” as implying a measurement unit of 1 paperclip and interprets the “5” as the result of measurement with that unit, then we say that the student is interpreting “5 paperclips” in a magnitude-measurement sense. This sense is needed to interpret a multiplier as how many groups one is considering (e.g., 5 groups of 2 grams). In contrast, the student might interpret the unit word “paperclips” as standing for the set itself and providing classification information about the set. The student might also consider the “5” alone—without its unit word “paperclips”—as standing for the set itself and providing size information about the set. If so, we say that the student is interpreting “5 paperclips” in a signifier-classifier sense. With this interpretation, the student might form a kind of identification between the “5” and the “paperclips,” as in “these 5 are paperclips.”

A student who takes a signifier-classifier view might interpret fractions as labels, for example by interpreting “ $\frac{1}{3}$  cup” as referring to a cup that is  $\frac{1}{3}$ . Or, if there are 5 equal parts of yellow paint and the total amount of yellow paint in the 5 parts is  $Y$  liters, the student might formulate the expression  $5Y$  for the yellow paint, interpreting the entire expression as stating that “the 5 parts are yellow.” Similarly, the student might interpret  $\frac{1}{8} X$  as “ $\frac{1}{8}$  is  $X$ ” rather than as “ $\frac{1}{8}$  of  $X$ .”

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# CHILDREN'S CONCEPTIONS OF TIME

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Time is an elusive and abstract phenomenon, and strongly related to social, cultural and personal experiences (e.g. Michel, Harb & Hidalgo, 2012). In order to communicate about time, we often use metaphors (Lai & Boroditsky, 2013). Many studies since Piaget (1946/1969) have investigated children's conceptions of duration, sequence etc. Children's own thoughts about what time actually *is*, is however far from well explored.

This poster examines children's visual and linguistic conceptions about time, and how children make sense of this abstract phenomenon through metaphoric reasoning in their own drawings and verbal accounts. The results are part of a larger study on the role of metaphor in early mathematics teaching and learning.

18 elementary school children, age 8–9, were posed the question “What is time?” and asked to draw and write their conceptions. The children's drawings were inductively analyzed.

17 out of 18 drawings expressed the measurement of time, with drawings of analogue or digital clocks. 1 student drew a time-line of seconds. 7 of the clock drawings were complemented with other sketches and written statements; “Time is numerals”, “Time has always existed”, “Time is invented by man”, “The clock is *not* time!”, “Time can slow down or speed up”. Other students drew pictures of the four seasons, an infinity symbol, hurrying adults, or the life-span of a person from infancy to old age.

Measurement of time is close at hand as an explanation of what time is for the students. However, the drawings, and especially the students' written statements, revealed interesting thoughts on what time is, beyond how it may be measured. These results are consistent with Michel et al (2012). More descriptors than metaphors were used, and objective versus subjective perceptions were both present.

In the presentation I further discuss how these results can provide constructive entry points for teaching about time.

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# MATHEMATICAL LEARNING PROCESSES WITH VARYING TYPES OF MATERIAL CONDITIONING

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In the past decades, several studies were conducted to analyse the role of typical material in the learning process. Materials can help to visualise mathematical thoughts. In some situations, the material becomes a link to the strange culture (culture of mathematics), in which experts (for example adults) demonstrate handling the material with reference to the task and enable children to interpret mathematical issues (Vogel, R. 2014). However, the authors could hardly find any research that observe the potential of digital and non-digital material conditioning learning environments to support mathematical learning processes of primary school children. To close this lack of attention, the presented study *MatheMat* concentrates on mathematical learning processes of primary school children dealing with mathematical topics that are varying in types of material conditioning. The aim of the presented research is to analyse which type of material accounts the best fitting between learner and mathematical topic, and therefore supports mathematical learning processes. The study takes place in the context of learning environments which enable children on different mathematic learning levels to work on the same mathematic topic individually or cooperatively (Vogel, R. 2014). The digital and non-digital designed learning environments are equally textured to offer a comparison of the learning process. The presented study *MatheMat* focuses on the learning processes of primary school students in grade 3 and 4. Peer interactions of these children with the different digital and non-digital material will be video-taped and qualitatively analysed for example with help of qualitative content analysis (Mayring 2015). In the poster presentation, it should be discussed that different types of material conditioning have varied potential for the mathematical learning processes depending on the emergent interaction. In addition, a selectively learning environment dealing with the subject sorting data will be discussed in detail and first results from the pre-study will be shown.

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# INTERDISCIPLINARY ACTIVITIES FOR AN INCLUSIVE MATHEMATICS EDUCATION

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The main goal of this research is to analyse how interdisciplinary activities increase the motivation of teenagers in risk of social exclusion for learning mathematics. The research is framed within the *Anaquiños Matemáticos (Mathematical Bits)* socio-educational program. This program is carried out outside classroom and proposes interdisciplinary activities for stimulating mathematics learning by means of a collaborative-based methodology. Such activities entail solving problems related to real life situations, applying and integrating knowledge from different school subjects, using hands-on materials as well as technology and games in collaboratively environment (Blanco, Gorgal, Salgado and Diego-Mantecón, 2017).

This is an experimental research in which 15 teenagers, aged 12 and 13, developed interdisciplinary activities outside the classroom during a two-years period, two hours a week. Although the teenagers attend regular lessons in the Spanish secondary education system, they are in risk of exclusion due to family-based factors (Vermunt, 2005). To assess the impact of these interdisciplinary activities, we used pre- post-interviews, as well as classroom observations during their lessons in the regular system and questionnaires to their mathematics teachers. The results revealed positive changes including better academic performance in mathematics, a more positive attitude towards this subject, and an increasing participation in the classroom.

## Acknowledgments

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# ELEMENTARY STUDENTS' INTEGER COMPARISONS

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Solving number comparisons involving negative integers is challenging because students have to reinterpret what *more* means. In terms of the framework theory for conceptual change, students' mental models for whole number constitute their initial mental models of number (Vosniadou, Vamvakoussi, & Skopeliti, 2008). Therefore, when they first encounter negative numbers, they interpret them as positive numbers. With positive numbers, number order and values align; the magnitude of whole numbers increases (gets *more*) as students count *higher* in the numbers (or get *closer to 10* from zero). However, with negative numbers, numbers with higher magnitudes are farther from 10. This study investigates the role of phrasing (which integer is higher versus closest to 10) on students' understanding of integer values.

This study involved 73 grade 5 and 132 grade 2 students in the mid-western United States. Students analysed worked examples during two small-group sessions. We measured changes in integer knowledge on two pre- and post-test tasks: a counting backward in negatives task and integer comparisons where students identified the integer closest to 10 (8 items) and identified the highest score (8 items).

On the pre-test, students overall had better performance when choosing which integer score was highest than they did on which integer was closest to 10. These results flipped by the post-test, perhaps because of the integer number path representation students examined during their small-group sessions. Irena, a second grader, said, "The lowest [negative] numbers are the closest to 10... [Negative one] is closer to 10 than negative nine and negative eight." However, she thought -6 was higher than -2 "because if you go way down [in negatives], it's like higher," suggesting that she, like others, continued to struggle with magnitude versus order with integer values.

## Acknowledgement

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# A LEARNING THEORY FRAMEWORK TO GUIDE 'GROWTH MINDSET' INTERVENTIONS

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Social psychology research confirms that learning can be strongly influenced by what students believe about their academic ability. According to Dweck (2006), students with a 'growth mindset' believe academic ability can increase with appropriate effort and are more likely to persevere following setbacks, embrace challenges and ask questions. In contrast, students with a 'fixed mindset' believe that beyond a basic level, they are not able to do much to change their academic ability. Fixed mindset beliefs are linked to behaviours that can lead to reduced learning.

The potential impact of a growth mindset on mathematics achievement has resulted in calls for the development of interventions to encourage growth mindsets and discourage fixed mindsets. A teacher looking to develop and/or use such an intervention will be influenced by their understanding of how learning happens. Therefore, a framework that shows how the behaviours associated with fixed and growth mindsets can be viewed through the perspectives of different learning theories would be useful for developers and users of growth mindset interventions.

Through an inductive process drawing on writings about Dweck's theory of growth and fixed mindsets, behaviours characteristic of growth and fixed mindsets were contextualised in terms of the four learning theories—behaviourism, constructivism, communities of practice and connectivism. For example, *choosing challenges* is a growth mindset characteristic while *avoiding challenges* that may expose weaknesses is a fixed mindset characteristic. Through a **behaviourist** lens, success with challenges may be achieved from practice with similar types of problems. Growth mindsets may be encouraged by giving greater rewards for more challenging problems and allowing multiple quiz attempts until success with a challenge has been achieved. Through a **connectivist** lens, challenges can be resolved by connecting to information resources (e.g. people, videos). Growth mindsets may be encouraged by a teacher favouring resources that include more challenging problems and the use of a variety of methods of solving problems in the topic.

The consolidation of the different views from the four learning theories of the behaviours characteristic of growth and fixed will help practitioners to design and implement interventions for developing growth mindsets to fit their dominant learning theory.

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# STUDYING TEACHERS' BELIEFS ON TEACHING WHOLE NUMBERS WITH THE APPLICATION OF NINE TIMES TABLE

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This study aims at investigating a Chinese way of treating multiplication table from teachers' perspective. The study combines with semi-structured interviews and classroom observation which shows that teachers' lesson structure is tightly connected with their beliefs on multiplication table.

Multiplication table has been argued to play crucial roles in learning whole number multiplication (Ebner & Kothmeier, 2012). This study then investigates how teachers view multiplication table and in which way their beliefs influence the teaching of whole numbers multiplication. The data collected in the study is from classroom observation of lessons on whole number multiplication and semi-structured interviews with four primary mathematics teachers who are teaching multiplication and labeled as T1, T2, T3 and T4. The recorded four interviews and four lessons has been transcribed and analyzed.

Specific characteristics of Chinese mathematics teachers have been recognized from the study. The four mathematics teachers all put highly attention on multiplication table. Therefore, their beliefs on 'nine times table' strongly influence their lesson planning and lesson structure. Teachers' instruction on multiplication tends to begin from introducing how multiplication oral formulas are composed, which is the rule of forming Chinese pithy formulas. Teachers also have students memorize the 81 basic facts in nine times table. They agree the importance of memorization in learning but also argue the difference from rote memorization and context based understanding or memorization. They also help students to explore other strategies, such as inference calculation, counting in pairs, inference, finding patterns. In a word, efficient and flexibly retrieval has been seen as the sign of acquiring multiplication facts.

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# REALIZATION TREES: A COMMOGNITIVE LENS

Matteo Caponi and Giulia Lisarelli

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We adopted Sfard's commognitive lens (Sfard, 2008) that describes mathematical discourse as involving continuous transitions between a signifier and its realizations. These relations can be visually represented through *realization trees*. For this study we designed 5 lessons, introducing functions to 8<sup>th</sup> grade students through dynagraphs (Goldenberg et al., 1992). Students worked in pairs on pre-designed GeoGebra files. We analyzed the formation of a new mathematical object in students' discourse through the reconstruction of their realization trees, in order to gain an insight into their growth and richness as resulted from the sequence of activities.

We introduce realization trees two students exploring the dynagraph of  $f(x) = -x/2 + 3$ . The reconstruction of their realization trees for the signifier "relation between variables" (which they called A and B) sheds light onto several differences, although they developed within the same discourse. For example, Tommaso's discourse seems to be more situated and he does not realize the relation in algebraic terms. The realization trees also show how the students deeply influenced each other's discourse: Flavia induced the creation of a new branch in Tommaso's tree (Figure 1).

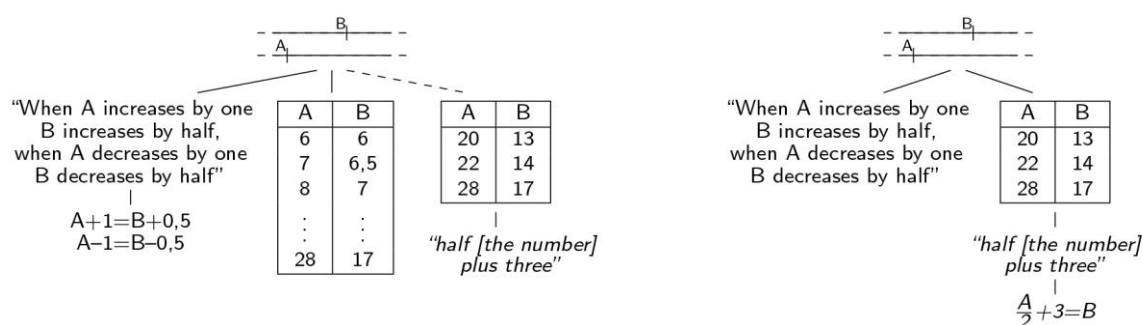


Figure 1: Realization trees for the relation between variables of Tommaso (left) and Flavia (right).

The example presented above sheds light onto how useful realization trees can be as tools for analyzing the discourse of students who are introduced to a new mathematical object. In particular, studying the number of ramifications and the depth of each branch can be insightful as feedback on an experimented sequence.

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# MAPPING ATTENTIVENESS TO STUDENT THINKING

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From a progressive formalization perspective, teachers' ability to elicit, analyse, and respond to student thinking is a fundamental aspect of classroom practice (Freudenthal, 1973). We use the term *attentiveness* to describe this ability and are developing assessments to examine how teacher preparation programs influence its development (Carney, Cavey, & Hughes, 2017). One assessment being developed measures teacher candidates' ability to analyse and respond to a secondary student's thinking in relation to the learning intention(s) of quantitative reasoning tasks. Developing an attentiveness construct map (Wilson, 2005) is a key component of the development and validation process. This involves articulating increasingly sophisticated ways of demonstrating attentiveness (i.e., a construct map) based on theory, examining the hypothesized construct map in relation to empirical evidence, followed by further refinement of the construct map. Our research questions are (a) What categories of attentiveness emerge within and across mathematical tasks and item prompts? (b) In what ways are the categories similar and different across mathematical tasks and item prompts?; and (c) What evidence supports the ordering of these categories into a construct map?

The sample comprises responses from 17 mathematics teacher education candidates collected prior to and following an intervention designed to increase their attentiveness. Responses were de-identified and blinded in terms of timing of assessment (pre vs. post). Categories were developed using a grounded theory approach.

Our primary finding is that increasing attentiveness is characterized by greater specificity of the (a) analysis of a student's process or understanding, and/or (b) response to the student by the teacher candidate in relation to the mathematical intention of the task. Based on the results, the hypothesized construct map for teacher candidates' attentiveness to student thinking in quantitative reasoning contexts was further refined for use in our assessment development process and more broadly as a tool for teacher education programs.

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# TEACHING MATHEMATICS THROUGH FINANCIAL CONTEXTS: ARE TEACHERS COMFORTABLE WITH FINANCIAL CONCEPTS?

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Since 2016, Québec, Canada, incorporates Financial Literacy (FL) in their secondary mathematics curriculum through Financial Mathematics. This incorporation follows an international trend of recognizing the importance of FL education ever since the global economic and financial crisis of 2008. However, we have yet to understand how to support mathematics teachers to develop their FL content knowledge for teaching (Ball et al., 2008). Therefore, we conducted a study (questionnaire and focus groups) with 36 secondary mathematics teachers from Québec (grades 7-11), to investigate their perceptions on teaching FLE in their classes. The questions focused on their knowledge to teach FLE and their needs.

Our findings suggest that teachers are more comfortable teaching the following financial concepts: taxes (36 teachers), consumption (32), budget (34), credit (27), saving (32). These concepts require applications of mathematical concepts such as: percentage, interest, linear functions. Nonetheless, the teachers were less comfortable teaching financial concepts that require deeper mathematical reasoning, generally related to actuarial sciences and stochastic thinking. These financial concepts include inflation (14), interest period (15), retirement (18), time value of money (10), stock market (06). We observe that they are mainly related to the mathematical concepts of annuities, subjective probability, statistical inference. These results, that we intent to discuss in our poster presentation, aligned with the findings of Ju, Moon, Park & Jung (2017) and Sawatzki & Sullivan (2017), show that teachers need more opportunities to develop their knowledge in terms of the pedagogy related to FLE.

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# STATISTICS IN PRIMARY AND MIDDLE SCHOOL: HYPOTHESIS FORMULATION, DATA ANALYSIS AND CONCLUSIONS

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In the teaching of statistics, hypotheses formulation, data analysis and the drawing of conclusions based on them constitute phases of the investigative cycle. Experiencing these phases of statistical research is fundamental for the statistical literacy of the citizen. This research aimed to investigate the understanding involved in hypothesis formulation. For that, a diagnostic test was carried out with all students from two classes (one from the 5th grade and the other from the 7th grade - 10 and 13 aged students) from two public schools in Recife / Brazil. The test had two (2) activities proposed in the same sequence: formulation of hypotheses based on a research question presented; presentation of real data on this issue for interpretation and subsequent comparison between hypotheses created and data; conclusions and predictions from the data. It should be emphasized that the activities presented real data, since they provide a critical reading of the student's reality (Allmond & Makar, 2014).

It was verified that both students of the 5th and the 7th grade were able to formulate hypotheses (approximately 82%). Regarding the comprehension of information presented in a graph (activity 1), 71% of the students had succeeded, and in the table representation (activity 2), 65% of the students were able to perform adequately. In addition, the students were able to confront their hypotheses with the real data presented in the graphical representations and reanalyze their answers. However, when asked to evaluate a conclusion that could not be made from the data presented, most of the students in both years presented difficulties, which demonstrates the need for teaching to focus on the ability to lead students to make decisions from the data collected. Besides that, using probabilistic vocabulary when making predictions has also proved difficult for both groups of students.

Exploring all the skills highlighted in this study in a systematic way is fundamental, once students since the 5th grade show ability to understand the relationship between the different phases of investigative cycle, which will enable them to become, in fact, citizens.

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# DO MATHEMATICS TEACHERS EXPERIENCE A PROFESSIONAL DEVELOPMENT PROGRAM IN FORMATIVE ASSESSMENT ANY DIFFERENTLY THAN OTHER TEACHERS?

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Formative assessment (FA) is a teaching approach that has gained a lot of attention in recent years, and research indicates that FA is an efficient way to improve student achievement (e.g., see Hattie, 2009). FA is about using frequent assessment to identify student learning needs and to adapt the teaching practice to meet these needs (Black & Wiliam, 2009). FA is also becoming an increasingly popular approach in Sweden, with several municipalities investing considerable time and money to train teachers in FA. This study examines one municipality, where teachers in all subject areas were trained in FA. There are studies indicating that there might be a difference in what characteristics in professional development are suitable for mathematics teachers compared to other teachers (Hodgen & Marshall, 2005). In this study, I examine if and how mathematics teachers experience the quality of a professional development program (PDP) in FA differently than teachers in other subjects. Mixed groups of teachers from different school subjects and school years worked together in the PDP. Data was collected through questionnaires conducted at three different occasions, to get data from beginning, during and after the PDP. All secondary school teachers (school year 7-9) within the municipality were invited to answer the questionnaires. I here present results from four questionnaire questions that concerned the teachers' beliefs about their ability to use high quality FA in their teaching practice after the PDP. The mean value for each of the questions were calculated, as well as a combined mean value for all four questions. A t-test was used to see if there was any statistical significance ( $p < 0.05$ ) in the differences shown in the mean values. The mean values for the mathematics teachers are lower than for the other teachers on almost all questions in all three questionnaires, but statistically significant only for two questions. When analyzing all four questions together, the mean values for mathematics teachers are lower than for the other teachers. This would indicate that the mathematics teachers had a lower expectancy of being able to use high quality FA after the PDP.

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# A STUDY ON ELEMENTARY SCHOOL TEACHER TREATING STUDENT ERROR EXAMPLES IN FRACTION OPERATION

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This study attempted to explore how elementary teachers handle perceived error examples made by students when fractions operation, including how to interpret and respond to actions, design math problems in response to student learning, expand students' learning experience, and promote effective teaching skills. The researcher used Isik and Kars' (2012) fraction error patterns, selected three fraction multiplication questions provided 56 primary school teachers to notice on the professional development workshop. Both quantitative statistics and qualitative descriptions used to analyze the knowledge of the teachers. The conclusions are as follows: Teachers provide good explanations regarding student error examples in multiplying fractions and can explain the mathematical thinking of students from multiple perspectives. The types of written explanations include 1. understanding problem structure and linguistic relationships, 2. fraction unit conversion, 3. imperfect processes for multiplying fractions, and 4. errors in calculation strategy thinking. Most of the teachers used multiple strategies to approach flaws in student thinking with regard to multiplying fractions. The types of responses include 1. becoming familiar the fraction concepts and computing abilities of the students, 2. interpreting the linguistic meaning of units, 3. repeating calculation exercises, and 4. employing diverse tutoring models. Most of the participating teachers found designing problems is difficult: many teachers simply changed the numbers or context to create new problems and focused on procedural calculation problems. The types of problem design included 1. graphs, 2. text description, 3. diverse scenarios, 4. calculation exercises, and 5. learning trajectories. The researcher formulated relevant suggestions based on the study findings to enhance the professional development and effectiveness of teaching mathematics.

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# YOUNG CHILDREN CAN DISTINGUISH FAIR SITUATIONS FROM UNFAIR ONES IN PROBABILITY CONTEXT

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There has been substantial research on young children's probabilistic thinking (e.g., Fischbein, Nello, & Marino, 1991). Previous some research informed that classroom instruction probability for young children may be inappropriate; however, some research confirmed that young children can perform well through designed program probabilistic thinking. This study was to describe and predicted how young children think in probabilistic situations about fair and unfair. This research based on the classroom teaching experiment. The instruction activities concerned with the distinction between fair and unfair games. The aims to examine average 5.8 years old young children whose performance of probability judgments can be promoted after invention teaching. All 28 children attended the kindergarten classroom participated in the classroom teaching experiment. Invention involved six classes an hour in per weeks via teachers formulated questions and group discussion to practice probability judgement that can be addressed with data and collect, organize, and display relevant data to answer them. To understand efficient invention of probabilistic thinking, four tasks included four spinners compare in pair to make the game fair. Each spinner has different ratio ( $3/4:1/2$ ,  $2/3:1/2$ ) sector of white and black colours, and two bags include four different number chips ( $3:2$ ;  $2:2$ ) with harts and diamonds shape; In the procedure of testing, children were asked to predict the most probable outcome from four tasks. Correct and complete each one gets one score. Data collection and analyses with video record and statistics. Before and after invention, subjects were assessed to distinguish "fair" probability situations from "unfair" ones. The result indicated young children improved their judgement of probability. They were significantly different performance of pre-test form post-test on tasks. Pre-test  $M=.39$ ;  $M=1.12$   $t(28) =4.033$ ,  $P=.000$ . The finding responded suggestions of Fischbein and Gazit (1984) that probabilistic thinking can be improved through instruction, and the claim of Kafoussi (2004) 5 years old children can do probability judgments overcome their subjective interpretations and develop a primitive quantitative reasoning in probabilistic tasks. Furthermore, over 80% subjects compare correct fair and unfair tasks of probability judgement and the finding will be discussed further.

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# MATHEMATICS IDENTITY AMONG CHINESE STUDENTS

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Empirical research has explored educational, psychological and cultural reasons behind the “high mathematics achievement but negative attitudes” phenomenon among Chinese students. However, the past research has more emphases on mathematics success, rather than the competitive and struggling sides of mathematics learning. Besides, there exists little sociological investigation of this phenomenon. The dialectic relationship between individual and community is unclear. Such research gap calls for an investigation into the development of mathematics identity in a Chinese cultural context (e.g. Darragh, 2016).

For investigating the impact of both mathematics success and struggles on identity, the research is conducted in a competitive elite high school in a metropolitan city in China. The definition of mathematics identity includes agency and ownership, which is drawn upon Schoenfeld’s (2018) TRU framework. The research has generated a theoretical framework examining how the local mathematics practices shape the ways that individuals in which build their mathematics identities; how social agents (e.g. student, teacher, parent, private tutor and the state) support the local mathematical practices; and what the underlying support structure is.

The framework is built through a one-year-long qualitative study, which uses observations and interviews to collect stories of mathematical experiences of students, teachers, parents and community members, who are selected by purposeful and snowball samplings. The study conducts thematic and narrative analysis of interview scripts and fieldnotes.

The study discovers mathematics identity is built in relation to the affordances provided by 3 local practices - repeated practices, summary and categorization, and self-investigation of mathematics. They are in a decreasing amount of accessibility. The more positive feedback from practices gives rise to the more positive identities. The underlying support structure for developing mathematical practices is the unidimensional elite status hierarchy, based solely on test scores (Chiang, 2017).

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# THE ANALYTICAL WORKING SPACE USED BY STUDENTS OF PEDAGOGY IN MATHEMATICS WHEN SOLVING A MODELLING TASK

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With interest in the training of mathematics teachers in the area of analysis and considering the theoretical framework called Mathematical Working Spaces (ETM) (Kuzniak, Tanguay & Elia, 2016) different perspectives open to go in depth with the form of mathematical work such as modelling (Montoya Delgadillo, Viola & Vivier, 2017). The aim of this research is to characterize the *personal* ETM and identify the different levels of transposition between what has been taught and what is placed into practice. To achieve this, a series of didactic situations are being proposed. In this work we present the problem of the gutter, which has two phases. In the first, the gutter has perpendicular edges, the solution shows a use of a real variable by the position of the fold (where the variable  $x$  is a length). However, some students realize that in the problem there are more variables at play, such as the inclination of the fold of the gutter (with variable  $\alpha$  as an angle), or the height of the channel. These variables are considered in the study of the problem presented in the second phase, as well as, the case where the channel has two different folds (not identical or symmetrical). Results show that most students develop in the first phase an ETM for the functions of a real variable, however the problem can be considered as two different exercises because of the didactic contract, each one with a single real variable. Indeed, most of students worked with this same ETM in both phases. But, from the modelling point of view, it is natural to deal with two, or more, variables. This requires changing to an ETM of functions in several variables, hence with different signs and knowledge, which is less mastered by the students. Then, solving a modelling situation allows students, besides they are few, to adopt a critical position with respect to the conditions of the problem and change ETM in a natural way.

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# YOUNG CHILDREN'S VISUAL – SPATIAL THINKING IN INTERPRETING SPATIAL VOCABULARY

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The purposes of this research are to understand: how the four distinct modes of thinking that support a theoretical model for understanding visual – spatial thinking (Costa et al., 2010) are suitable for young children and what can support the learning of young children on interpreting what they see and think about spatial relationships.

Actions, words, and gestures of the children can provide evidence about how they are thinking about mathematical ideas. Mathematical thinking of young children is often linked to children completing activities using recognizable mathematical content and can involve conjecturing, justifying, and interpreting.

To explore these understandings, we implemented two studies and examined the results. The first study had the research question, *how do two-years-old children interpret spatial vocabulary?* The research question for the second study was *what mathematics learning opportunities the use of pattern blocks can offer to five-years-old children?* For both studies we adopted a qualitative research methodology under the interpretative paradigm. We also adopted the perspective on early math of Clements and Sarama (2009) and a sociocultural perspective on learning and development.

The poster will focus on the findings of the studies in looking for discussion about their importance for the visual-spatial thinking model.

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# MATHEMATICS LEARNING AND APHANTASIA

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The purpose of this qualitative case study was to explore how students with aphantasia, or a “blind mind’s eye (Zeman, Dewar, & Della Sala, 2015)”, experienced mathematics in the classroom, how that experience affected their beliefs about their mathematics ability, and the strategies these students used to learn mathematics. The research questions addressed by this study were as follows:

- How do students with aphantasia describe their experiences in the mathematics classroom?
- What are the perceptions of students with aphantasia about themselves and their abilities in mathematics?
- What cognitive strategies do students with aphantasia use to learn mathematics to compensate for lack of mental imagery ability?

The sample consisted of two secondary school students who self-identified as having aphantasia. Participants completed an online survey consisting of items from three existing instruments: Vividness of Visual Imagery Questionnaire; Visual Imagery Questionnaire; and Fennema-Sherman Mathematics Attitude Scales. Subjects were interviewed individually using a semi-structured interview protocol. Data was analysed using discourse analysis within the framework of Dialogical Self Theory (Hermans, 2008) and Self-Efficacy. Participants described feeling isolation and anxiety in the classroom, and used spatial strategies like “positioning” and “building” to replace visual imagery.

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# SECOND GRADERS WITH MATHEMATICAL LEARNING DISABILITIES USE MORE FINGER-COUNTING ON SMALL PROBLEMS BUT LESS ON LARGE ONES

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Persistent use of finger-counting is widely considered to be an immature counting strategy which is more prevalent and lasts longer in children with mathematical learning disabilities (MLD). Nonetheless, Geary, Hoard, Byrd-Craven and Desoto (2004) found that while MLD first-graders did effectively use more finger-counting than their typically developing (TD) peers on problems with sums under ten, they used it less frequently when facing problems such as  $16+7$ . Instead, MLD children were guessing more often. This very interesting result regarding arithmetic strategies is based on data collected through observations and self-verbal reports, whose validity is debated in the literature (Lucidi & Thevenot, 2014). It can also be argued that the large problems used by Geary and colleagues were unfamiliar to children of this age and that this could have favored guessing. Therefore, the current study, based on behavioral indications only, aims at replicating and further understanding this finding among children solving arithmetic problems that are more appropriate to their age.

Fifty-eight TD and 24 MLD children were administered single-digit additions with sums up to or above ten at the middle of their second-grade (mean age = 7;6 years old). Results showed that whereas the MLD children did not differ from their peers in the overall frequency of finger-counting, they used it more frequently for small problems but less for large ones. Nevertheless, it was not found that they were guessing more often since their overt strategies accuracy did not differ from TD children. Conversely, when using finger-counting, they made more errors than their peers. In parallel, MLD children also struggled when having to quickly compare Arabic numbers. Thus, MLD children lower amount of finger-counting on large problems is rather interpreted as a consequence of their finger-counting inaccuracy and/or of their difficulty to assess the magnitude of the problems.

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# PROSPECTIVE TEACHERS' CONCEPTIONS OF MATHEMATICAL DEFINITIONS: ARE DEFINITIONS ARBITRARY?

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As part of a broader study in which prospective middle school mathematics teachers' conceptions of mathematical definitions are investigated, 23 (junior and senior) prospective teachers responded to a written task related to the definition of trapezoid. The name "trapezoid" in mathematics literature is alternatively given to two different set of quadrilaterals: those having *at least* one pair of parallel sides or those having *exactly* one pair of parallel sides. Defining in mathematics is arbitrarily naming concepts (Vinner, 1991). Therefore, neither of these definitions is inherently *true* or *false*; that is, they do not have *truth values* (Edwards & Ward, 2008). In the current study, participants were asked to review these two definitions, by avoiding the use of "true/false" and "correct/incorrect" in the wording of the questions. The first question was framed through a middle school student's words: "Searching about trapezoid, I came across two different definitions. What do I do now?" After the participants worked on possible explanations for such a student question, they were presented the explanation *that both definitions, in fact, existed in mathematics literature*. The second question asked participants to interpret this information in terms of its possible implications *about* mathematical definitions. Analysis of the responses to the first question revealed that majority of the participants labelled one of the definitions as "true" and the other one as "false" (n=21). Two of the participants indicated that defining trapezoid was a controversial issue among mathematicians. In answering the second question, most of the participants maintained the idea that only one of the two definitions could be "true" (n=10). Seven of them expressed their inference as "the same concept can be defined in different ways"; which could be a correct interpretation for equivalent definitions, but it was not the case in our study. Others did not provide any relevant interpretations. Findings suggest that prospective teachers might assign unquestionable truth values to definitions of mathematical concepts; hence arbitrariness of mathematical definitions needs to be addressed in their preparation.

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# ON-LINE PLATFORM AS AN ARTEFACT FOR TEACHING AND LEARNING MATHEMATICS

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The study results from the monitoring of a large project whose aims are to understand how teachers use Khan Academy Platform (KAP) for teaching mathematics and how the students learn. This poster intended to characterize teachers' orchestrations when they conducted the mathematical activities of their students, using KAP in learning environments.

The theoretical constructs is based on activity theory (Engeström, 2001), professional knowledge of teachers and students' learning. With activity theory, we intend to broadly frame the actions of the various stakeholders in the project. Thus we can characterize the actions of the various actors in the integration of the technological tool in use. We also seek support for the processes of instrumentation and instrumentalization (Rabardel, 1995), reinforcing the semiotic power of the artefact.

We assumed an interpretative paradigm and the participants were teachers and their students. Data were collected through observation of classes, researcher's diary based on field notes and interviews to the teachers.

The results show that the teachers play several roles in orchestration (Drijvers et al, 2013) when they use the KAP in the classroom and they work collaboratively each other and also with their students. The gamification characteristic of KAP is important to involve and motivate students in mathematical activities and the social interactions in the classroom can be fruitful.

This work is supported by national funds through FCT - Foundation for Science and Technology in the context of the project UID/CED/02861/2016

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# COMPARING TEACHER QUESTIONING SEQUENCES IN CHINA AND AUSTRALIA

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Teacher questions are viewed as significant to stimulate students' mathematics thinking and understanding. Some other researchers distinguished the concept of "good questions" from the instructional practice "good teacher questioning" (Aizikovitch-Udi, Clarke, & Star, 2013). In other words, the interactional sequence in which the teacher asks his or her questions and responds to students' answers has been argued to make a difference.

This study was designed to investigate the ways in which teachers used questioning sequences in consecutive mathematics lessons in China and Australia. Four teachers (two teachers from China and another two from Australia) were selected and labelled as CHN1, CHN2, AUS1 and AUS2. A unit of consecutive lessons was video recorded for each participating teacher. Altogether 28 lessons were recorded (6 lessons respectively for the teacher CHN1, AUS1, and AUS2; 10 lessons for the teacher CHN2). All the episodes of teacher-student interactions were transcribed and analyzed. The teacher questioning sequences were identified and categorized according to the roles played by the teachers and students during the questioning sequences. Eventually, three types of questioning sequences were identified: the *leading* sequence, the *facilitating/probing* sequence, and the *orchestrating* sequence.

Cultural specialities were identified in the purposes of questioning sequences used by the participating teachers. For the teachers CHN1 and CHN2, the usages of three different types of questioning sequences varied across the consecutive lessons and this variation was paralleled by the changes of lesson topics within the teaching unit. By contrast, for the teachers AUS1 and AUS2, the systematic arrangement of distributing the three types of questioning sequences could be found in each lesson within the teaching unit. These cultural specialities could be attributed to teachers' different ways of planning and preparing mathematics instruction in China and Australia. By comparing the questioning practices across the consecutive lessons in two different cultural settings, this study helps to understand how different ways of pedagogical planning may shape the teachers' practices in a unit of mathematics lessons.

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# GRADE 3/4 STUDENTS' MISCONCEPTIONS OF CUBES: A LANDSCAPE OF SOURCES OF ERRORS

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Most of the research relating to children's geometric reasoning has been interested in geometrical concept knowledge of two-dimensional shapes, whereas few studies have investigated children's knowledge and visualization of three-dimensional objects, in detail. In previously presented research, we introduced a construction task to investigate young children's knowledge of geometrical solids. First results, namely children's construction strategies and products in this task, were interpreted according to the Van Hiele framework and indicated a wide variety in Grade 3 students' geometrical concept knowledge on solids (Reinhold & Wöller, 2016). Moreover, we identified errors and misconceptions on cubes that provoked us to widen our perspective and to search for more general aspects of cognitive development that we considered could influence (mis)conceptions on geometrical objects.

This poster reports on part of an international collaboration of Australian and German colleagues investigating Grade 3/4 students' development of geometrical conceptual knowledge. The results of our case studies with Australian students highlight frequent misconceptions, such as attention to the number of blocks rather than properties of a cube (when using rectangular prisms) or a focus on the top face being square (rather than all faces being square).

In excerpts from three Australian case studies, which are introduced in detail on the poster, we interpret students' responses when responding to the interview tasks in relation to four major categories (coordination and integration, visualization, class inclusion, flexibility and stability). As indicated in our literature review, the categories used for our interpretation are not entirely new to this field of research. However, identifying new interrelations of these categories, the way we suggest this with reference to our data, contributes to further understanding students' solutions, errors and (mis)conceptions.

The bi-national cooperation which led to the research presented in this paper was funded by a grant provided by Monash University, Melbourne, Australia.

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# CAN VISUALIZATIONS (RE)ACTIVATE CONCEPTUAL KNOWLEDGE OF CENTRAL MATHEMATICAL CONCEPTS? - AN EXPLORATORY STUDY

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It is widely acknowledged that using visualizations can foster students' conceptual knowledge regarding many central mathematical concepts (e.g., Presmeg, 2006). In particular when new mathematical concepts are introduced in primary or lower secondary school, visualizations are used in mathematics classrooms. Usually, later on in secondary school there is a focus on symbolic representations and visualizations are often neglected. At the same time, it is often noted by practitioners as well as researchers that upper secondary students lack conceptual knowledge of core concepts of lower secondary mathematics. Thus, the question as to whether visualizations can (re)activate such conceptual knowledge merits attention (Atagi et al., 2016). When such research questions are focused, usually the content domain is restricted to functions or fractions, where there is already a broad base or research on visualizations. However, it is well-known that visualizations can have very different functions and play different roles in different content domains (e.g., Presmeg, 2006). Hence, we should be careful with generalizing findings of studies focusing on these prominent content domains. Consequently, this exploratory study focuses on three different content domains to explore whether visualization prompts can (re)activate corresponding conceptual knowledge of 11<sup>th</sup>-graders. In a paper-pencil test,  $N = 136$  students in grade 11 were asked to explain three concepts of lower secondary mathematics (addition of fractions, distributive law, percentage calculation) to a fictitious younger student. One third of the students did not get any additional prompt (version A), one third was prompted to use visualization for their explanation (version B) and one third was given a suitable visualization which they were asked to use for their explanation (version C). The data analysis (ANOVA) yielded more correct and more conceptual explanations for the groups with visualization prompts than for group A regarding the fraction task, more correct answers regarding the percentage task, but no differences between the groups regarding the distributive law task.

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# THE MATHEMATICS OF TIME AND TIME MANAGEMENT

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Recent research has revealed various challenges students encounter related to time (Earnest, 2017) but has yet to consider potential ramifications of such challenges beyond elementary mathematics. A conjecture of the present work is that challenges found among elementary students related to representations of time are consistent with challenges older students encounter when grappling with time-related ideas. In particular, the present research explores how undergraduate students reason with representations of time when engaging in the practical skill of time management.

Time management is a life skill with which many individuals struggle, and in fact students' poor time management contributes to poor performance in university coursework (Thibodeaux, Deutsch, Kitsantas, & Winsler, 2017). In particular, one's time management may be well supported when utilizing mental or written representations of time (Ancona & Chong, 1996) in which time units are explicit, such as a weekly schedule with which one may evaluate chunks of time.

This exploratory study considers the following research question: How do university students draw upon time representations to manage time? The study features individual interviews with undergraduates ( $n = 7$ ) as well as graduate students working as tutors for those undergraduates ( $n = 7$ ). Interviews explored the representational tools they draw upon as to manage their time. All interviews were audio recorded and transcribed.

Findings indicate that tutors draw upon rich and mathematical time representations to support undergraduates' time management. Findings further suggest that one's understanding of time representations supports productive time management, whereas individuals that do not integrate such representations struggle with time management and college success. The poster presentation will invite PME participants to discuss the role of time representations in time management, and will create opportunities to discuss the role of the topic of time across K-16 mathematics.

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# AN ANALYSIS OF TEACHERS' APPRAISAL OF THE USE OF EXIT CARDS IN MATHEMATICS TEACHING

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Exit cards are small simple cards which teachers use to elicit information from students on their gains and challenges in any given lesson. From empirical evidence (Keeley & Tobey, 2011), the use of exit cards holds a lot of benefits for mathematics teachers and their students. However, since these studies were conducted in other countries, there is hardly any empirical evidence of either mathematics teachers' or students' experiences of the use of exit cards in the study area. The conceptual framework for this study is the Mathematics Assessment Instrument and Learning (MAIL) cycle adopted from Keeley and Tobey (2011).

This study was designed to train teachers on the use of exit cards and elicit information from the teachers on the benefits and challenges inherent in the practice after they have used it for ten weeks of teaching. The study addressed three research questions which were focused on how teachers appraised, what teachers see as the major benefits as well as what teachers see as the major challenges of the use of exit cards. An eclectic approach to research was adopted for this study in order to take care of the two goals of the study. The sample for the study consisted of 50 middle basic education level Mathematics teachers. The instrument for data collection had five sections (A-E). A and C were open ended while sections B, C and D were Likert-typed scales of four response options having ten, twenty and five, items respectively. Data collected were analyzed using descriptive statistics.

The findings indicate that all (100%) the teachers of have positive appraisal of the use of exit cards in Mathematics teaching. "Activate thinking and engage students in learning" is one of the major benefits while "students not being able to express themselves in writing" is one of the major challenges of the use of exit cards in Mathematics teaching. This study concludes that the use of exit cards should be promoted but should have inbuilt mechanical for forestalling the challenges inherent in the practice.

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# STUDENTS' UNEXPECTED WAYS OF ENLARGING FIGURES

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The poster reports empirical results on students' ideas of the geometric concept of similarity based on Brousseau's Tangram task (1997, p. 177; Hodgen & Küchemann, 2013) which engages students actively in diverse individual ways of enlarging figures by uniform scaling. For grasping students' ideas, I refer to Vergnaud's construct of theorems-in-action that describe propositions that are individually "held to be true" (Vergnaud, 1998, p. 168). For the case of enlarging figures in the Tangram task, these are rules for enlarging sides and angles (e.g., adding a constant summand to each length or keeping the size of the angles).

Research question: Which theorems-in-actions can be identified in students' individual ways of enlarging figures for the Tangram task?

The analysis is based on video data of 23 groups, each with 3 students from German comprehensive schools aged around 13 years. The Tangram task was the first task for them on enlarging figures. For the explorative analysis, relevant episodes were identified in the video data and transcribed. The qualitative analysis of the transcripts with respect to students' theorems-in-action followed inductive category formation.

The analysis expands Brousseau's observation of students enlarging figures by adding constant summands to each length: 8 identified theorems-in-action that will be visualized on the poster show the various rules students held to be true while solving the task. For example, students only enlarging vertical sides, enlarge all sides by a individually developed pattern, or enlarge the figure by keeping relations between sides in the original the same in the enlarged version. Interestingly, no student thought about enlarging by multiplying with a constant factor, which is often the only way displayed in German textbooks. These results inform the refinement of the teaching-learning-arrangement. Furthermore, the study contributes to expanding the little research on similarity (Jones & Tzekaki, 2016).

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# CORRELATION AND REGRESSION PROBLEMS IN THE SPANISH HIGH SCHOOL TEXTBOOKS

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Main content in new curricula for high school in Spain and other countries are correlation and regression, which are fundamental statistical ideas that expand the previous knowledge about univariate distributions and mathematical functions and extend functional dependence to random situations (Engel & Sedlmeier, 2011). Previous research is mainly focused on students understanding of correlation and regression (Estepa & Batanero 1996; Zieffler & Garfield, 2009).

The aim of this research was to analyse the tasks characterizing the problems used to present correlation and regression in high school Spanish textbooks directed to Social Sciences students. In a sample of eight mathematics high school textbooks directed to Spanish Social Sciences students published by editorials of prestige and wide diffusion in Spain we performed a content analysis of 2166 problems. The variables taken into account in the analysis were the following (a) *type of problem*: reduction or representation of bivariate data; analysis of dependence between variables; and fitting a function to the data; (b) *strength and sign of correlation*; (c) *type of fitting function*: linear, logarithmic, exponential or other type of function; and (d) *context of data*. Our results suggest important differences in the problems proposed by the different textbooks and some biases in the distribution of the variables analysed, in particular there is a tendency towards direct, strong and linear relationship and a high percentage of problems with no context. It is important that teachers complement these types of problems with a wider variety of strength, sign and type of association, as well as with a variety of contexts, as it is suggested in the curricular guidelines.

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# CAN PROBLEM FIELDS ENGAGE ALL STUDENTS IN MATHEMATICAL PROBLEM SOLVING?

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(Inter-)national standards in mathematics advocate problem solving for all students. However, problem solving is often reserved for high achieving students only. This is due to mathematical problems, which are accessible to high achieving students only. Pehkonen (1995) suggested that successful implementation of problems solving can be realized through the use of open-ended tasks, so called problem fields, that are tailored to student's individual interest, problem solving preferences, and abilities.

The purpose of the study was to analyze problem solving abilities of low, average and high achieving students with respect to their *fluency* (how fluent the solver is in creating a large number of ideas), and *flexibility* (how flexible the solver is in seeing things from different points of view and using many different strategies) (Laine, Näveri, Ahtee, & Pehkonen, 2014) when working on “Paper squares” problem field.

Data sample comprised of two grade 5 classes ( $n = 41$ ), which were clustered in three groups on the basis of their mathematical performance; high achieving students with grade 1 ( $n_{gr1}=11$ ), average achieving students with grade 2 ( $n_{gr2} = 19$ ) and low achieving students with grades 3 or 4 ( $n_{gr3/4}=11$ ). The students' answers were coded depending on the complexity of their solutions in four categories. The number of solutions differed especially between the high achieving ( $M_{gr1} \approx 12.27$ ) and low achieving students ( $M_{gr3/4} \approx 7.55$ ), whereas the difference to average students ( $M_{gr2} \approx 8.11$ ) was unexpectedly small. The groups differed particularly with respect to found solutions. Nearly all pupils found the extreme figures, namely “cross” ( $n = 37$ ) and “straight line” ( $n = 32$ ). The solution figure “staircase” was found by high and average achieving students only. Similarly, we found a tendency towards vertical as opposed to horizontal figures among the average and low achieving students. In addition, high achieving students exhibited flexibility in their solutions.

In summary, the results show discrepancies in the problem-solving behavior between the high and average/low achieving students. Above all, it seems that the high achieving students have a clear advantage when it comes to changing perspective and using different strategies. The students did not differ greatly with respect to their fluency. Thus, the problem field allowed all students access to problem solving.

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# PROBLEM SOLVING FOR ALL: EVALUATION OF A PROBLEM SOLVING TEACHING CONCEPT FOR PRIMARY EDUCATION

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The aim of the DiPa-project (Differentiated development of problem solving abilities) is to develop, and enhance the problem solving competence of all learners – as advocated by (inter-)national standards. Drawing from previous research (e.g., Sturm & Rasch, 2015) we developed a teaching concept for primary grade students based on implicit and explicit heuristic training using problem fields (e.g., Pehkonen, 1995). Particular in this teaching concept is the *second exploration phase*; after the phase of intuitive problem solving (first exploration) students get insights into classmates' solutions, followed by further work on the problem field applying the newly introduced strategies (second exploration). Its evaluation is the focus of the poster.

Two grade 5 classes ( $n = 40$ ) and one teacher participated in the study. Data collection tools included students' written work (solutions of both exploration phases when working on one subtask of the problem field "Paper squares"), and a semi-structured interview with the teacher.

The findings indicate that 19 students engaged with the subtask in the second exploration phase, namely low-achievers (grade 3 or 4,  $n_{3/4} = 6$ ), average-achievers (grade 2,  $n_2 = 9$ ) and high-achievers (grade 1,  $n_1 = 4$ ). Each performance group increased its number of solutions: low-achieving students from 2.8 to 9.5, average-achieving students from 4.4 to 9.1, and high-achieving students from 6 to 12.5 solutions between the two exploration phases. Thus, the low-achievers found nearly the same or more solutions compared to other performance groups. The benefits of the second exploration phase were also confirmed by the teacher. In comparison to the high-achievers many low-achievers had no possibility to overcome the problem barrier. After getting insight into the classmates' work they often changed their approaches and adopted their strategies. We hypothesize that the second exploration phase is a key point in providing all students – especially low-achieving students – with problem solving opportunities. Further evaluation of the problem solving teaching concept will be presented on the poster.

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# OVERLAPPING CIRCLES – USING PARTICIPANT GENERATED INFLUENCE MAPS AS AN INTERVIEW TOOL

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Interviews form a key element of many qualitative research studies. They have formed an important data collection method for a longitudinal qualitative study currently under way in the UK, which is looking at how personal factors and factors related to their school context combine to influence the trajectories of early career primary teachers' teaching of mathematics. A particular research objective has been to understand the teachers' perspective but getting participants to think deeply about the questions being asked and give fully reasoned responses can prove difficult.

The influence map was developed to help each participant to visually and verbally articulate their perceptions of the influences on them as a teacher of mathematics. Potential influences include their teaching context and personal reflection on practice, their own background as a learner of mathematics and their mathematical subject knowledge of various forms (e.g. Ball, Thames and Phelps, 2008).

It is suggested in the literature that participatory techniques such as the use of visual data can provide additional and complementary data to verbally answered interview questions (e.g. Wall, Higgins, Hall and Woolner, 2013). They can also be motivating for the participants and give them a greater voice in the research process.

The influence map uses circles of translucent coloured plastic and labels relating to potential influences; participants matched the size of circles to the size of perceived influences to indicate the ranking order of these influences on their development. With flexibility beyond that of more rigid visual structures such as Q-sorting, participants were encouraged to overlap the circles to show relationships between the influences, thus creating a visual map, and whilst doing so to verbalise their rationale.

Deeper and broader responses have resulted, giving richer data than using conventional interviewing techniques, whilst this approach has proved interesting and motivating for the participants. New insights have emerged as to how early career teachers see the influences on their development as teachers of mathematics, and this study has provided further evidence of how visual data can enhance qualitative research.

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# STRATEGIES FOR FRACTION COMPARISON WHEN SELECTING THE LARGER OR THE SMALLER ONE

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A successful understanding of fractions requires mastering a diversity of aspects associated to them (e.g., parts of a whole, ratios, quotients; Kieren, 1976). Emphasizing one or another aspect may lead to different solution strategies for typical fraction tasks, highlighting the importance of looking at the strategies underlying students' answers.

Previous research has investigated a simple fraction comparison task, where students are presented with pairs of fractions and asked to select the larger one (e.g. Gómez, Jiménez, Bobadilla, Reyes, & Dartnell, 2014; Van Hoof, Lijnen, Verschaffel, & Van Dooren, 2013). Several of these studies have focused in the interplay of the magnitude of the fractions' components and the magnitude of the fractions, showing for instance that congruent pairs (those in which the larger fraction has the larger components as well, e.g.  $1/3$  vs.  $6/7$ ) are easier to answer than incongruent ones (those in which the larger fraction has the smaller components, e.g.  $2/3$  vs.  $4/9$ ).

We present data from 299 Chilean adult participants who compared fractions according to either the usual "which one is larger?" task or the variant "which one is smaller?". We looked at the prevalence of different strategies such as those based on congruency ("larger components-larger fraction") and gap ("smaller gap-larger fraction"). Results show that, despite the select-larger group obtained significantly better scores in average, participants in both groups used these types of strategies with a similar likelihood (although gap strategies were slightly more probable in the select-larger group). We discuss how small changes in the task may affect overall group performance and/or preferred solution strategies.

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# **ALICANTE - BARCELONA - HELSINKI: STUDENTS' MATHEMATICAL BACKGROUND AND REQUIREMENTS TO ENTER A PRIMARY TEACHING DEGREE**

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We situate our research among those studies that analyse the factors that influence the development of mathematical knowledge for teaching during initial teacher education. Some of these factors are the institutional characteristics and national context in which teacher education takes place, together with the individual characteristics of future teachers (Blömeke & Delaney, 2012). International studies in this area have tended to focus on the organization, curriculum, processes, and outcomes of training (Blömeke & Delaney, 2012; Li, 2012). However, little attention has been paid to the mathematical knowledge that students bring to teacher education programs as an individual characteristic that might influence their subsequent achievements.

We explore the interplay between the formal requirements that allow access to primary teaching programs in Finland and Spain and the prior mathematical knowledge that applicants bring with them. We use a test to compare the mathematical knowledge background of students entering primary teaching programs at the University of Alicante (ALI), University Autònoma of Barcelona (BCN), and University of Helsinki (HEL) (386, 254, and 116 participants respectively) in September 2016, before they had started any undergraduate mathematics or mathematics education courses.

Our main results were: a) overall, HEL students performed better than BCN students, who in turn performed better than ALI students; b) ALI and BCN students' answers to the various test questions followed a similar pattern, which differed from the pattern seen in HEL student responses; and c) when the admission criteria applied at HEL were superimposed on the data obtained in ALI and BCN, the apparent gaps between levels of performance narrowed significantly. At present there exist no studies demonstrating that students initiating a university degree are better prepared in Finland than in Spain. Our research suggests that the differences observed between ALI, BCN and HEL can be attributed to the access criteria and number of students admitted in the teaching programs in Finland and Spain.

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# THE IMPACT OF NUMBER-PAIRING ON STUDENTS' IDEAS ON HOW TO EVALUATE NUMERICAL EXPRESSIONS

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Without the mathematical conventions, like the rules for the order of operations, there are different ways in which numerical expressions can be evaluated. Often these are described in terms of mal-rules of errors (see e.g. Blando, Kelly, Schneider & Sleeman, 1989) and are considered as structural errors. Linchevski and Livneh (1999) showed that many students, while being inconsistent in the computational aspects, can be very consistent in these structural errors.

With the aim of looking for students' ideas when evaluating numerical expressions, we analysed 235 students' (11-12 years old) solutions to longer numerical expressions (three to six operations in each) and identified three main 'ideas' that students seem to have: evaluation by ordering operations, sequential evaluation (left-to-right), and evaluation by number pairing. Whereas the first two are frequently studied, the idea of number pairing is much less well-described. We present instances of three qualitatively different types of pairing, as they emerged from written numerical solutions to four different arithmetical expressions (e.g.,  $2 \times 3 \times 5 - 4 \times 2 + 6$ ).

We find that the pairing mechanism can come in qualitatively different types. Three different types are described with examples from the data:

(I) Pairing as a way to cluster numbers throughout the entire expression. This means that the number-pairs are formed independent of the operation within, or between, the pairs. In the example above, three pairs are typically formed:  $(2 \times 3) \times (5 - 4) \times (2 + 6)$ .

(II) Every operation triggers a new pair. Eventually all these pairs are added together. Hence, in the example above five operations give rise to five pairs as  $(2 \times 3) + (3 \times 5) + (5 - 4) + (4 \times 2) + (2 + 6)$ . Every number in the expression, except from the first and the last, is included in two pairs, and hence operated with twice.

(III) Pairing are delimited to terms. For instance, as  $2 \times 3 \times 5 - 4 \times 2 + 6 = (2 \times 3) + (3 \times 5) - 8 + 6 = 6 + 15 - 8 + 6$ . This type of pairing seems to be a hybrid idea between I and II, or, possibly, an adaption to the rules for the order of operations.

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# PAPER CUTTING THE WORLD: FOSTERING OBJECT RECOGNITION IN GEOMETRY TEACHING THROUGH DISSECTION AND REARRANGEMENT OF SHAPES

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During the visual perception of an object, the human mind takes the object apart into individual elements so that it is ultimately perceived as a particular arrangement of mental components (Biedermann, 1987). Object recognition does not require all components to be visible, e.g. we often identify a partly covered object just as easily as its uncovered version (Franke & Reinhold, 2016). So far, little is known whether and how object recognition works when the individual components are completely restructured, possibly into a new object even.

In order to challenge and examine student's abilities in object recognition, an activity was designed that will ask students to dissect the image of an object into individual components by using the art of paper cutting, which makes the components lose properties such as colour and structure and keep only their shape and size. After an optional seriation of the generated paper pieces to uncouple them from their previous position, students will subsequently reposition and fix the pieces into another (fantastic or realistic) arrangement. These arrangements are shared among the group whose task it ultimately is to guess the original object. Conducting and observing this method will allow for study questions such as:

- Does object recognition work with rearranged components?
- Which role do amount or shape of the components play?
- Does having experience themselves in dissecting and rearranging make student's object recognition faster?

Next to object recognition abilities, the described method is expected to foster an understanding of the part-whole-relationship in shapes and objects and to support geometrical thinking by creating and reviewing own materials rather than playing with prefabricated ones.

Paper cutting and rearranging shapes require craft skills as well as geometrical fantasy. The method is thus to be located in between mathematics and art, which is expected to appeal to students with a high interest in design as well as arts and crafts.

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# PROSPECTIVE TEACHERS' EXPLORATION OF *CINDERELLA.2,8* WITH PRIMARY CHILDREN

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Eight (4 groups of 2) prospective teachers (PT) developed interactive instructional lesson plans using the tools of *Cinderella.2,8* (a computer-based interactive geometry program) and explored them with 48 German primary school children from grade 1 to 4 in 2017 (Table). Objective of this study is to observe the interactions of the children with the artifact based on “Artifact-Centric-Activity Theory” (Ladel & Kortenkamp, 2016) and draw inferences for the professional development of PTs. The activity is carried out through an artifact, here a computer with *Cinderella.2,8*, and is orientated to an object, here the interactive geometrical learning environment. Assignments and lesson plans by PTs and observation schedules are used to analyze the collected data.

Description of Instruction	Observation
Group 1, N=11, PT1 & PT2, Plane figures, 90 Minutes	All the children could draw the plane figures. Problem with mouse to move the figures.
Group 2, N=12, PT3 & PT4, Plane figures, 90 Minutes	Some children had problem with circle size/ separating geometric figures using tools.
Group 3, N=8, PT5 & PT6, Plane figures, 45 Minutes	Time was short but children enjoyed the work. Used <i>press-release-sequence</i> tools.
Group 4, N=17, PT7 & PT8, Varignon parallelogram, 90 Minutes	Children could conjecture the properties of Varignon parallelogram using <i>drag</i> mode.

Table: Observations of the interactions of the children with the artefact.

The children worked individually and engaged themselves with the tasks matching heterogenetic performance level; their motivation was evident in all the sessions. Some PTs were initially skeptic about children using the tools of *Cinderella.2,8*. Specifically, each of the 11 first graders could draw the figures which was “surprising” to PT1 and PT2. For most children, drawing geometrical figures on computer is a “play” activity although some children had difficulty to coordinate their eyes with hand as they move the mouse (Ladel & Kortenkamp, 2016, p. 237). Tasks challenging children’s reasoning skills are barely used (PT7 & PT8). The PTs justified their choices of tasks by giving institutional or social reasons which are not epistemological or didactical.

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# FUNDAMENTAL TASK TO GENERATE THE IDEA OF *REDUCTIO AD ABSURDUM*

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It is well-known that *Reductio ad Absurdum* (RA) as a type of mathematical proofs is difficult for many students, though they can make easily arguments using contradictions in extra-mathematical contexts (cf. Freudenthal, 1973). Focusing on this phenomenon, Antonini (2003) designed a task that can help the generation of the proof by contradiction, based on the notion of *non-example*. The aim of our research is to further advance such works, in particular to find other favourable conditions that allow students to spontaneously develop RA in the process of investigating certain mathematical tasks. In this presentation, we report a task we designed in this respect (given below) and the results of a teaching experiment in a Japanese upper secondary school with 40 grade 10 students (15-16 years old) who had never learnt RA.

The task requires to check several conditions ( ${}_8C_2 = 28$  combinations); 16 of them are sufficient conditions to make a quadrilateral a parallelogram. In the experiment carried out as a form of inquiry (cf. Hamanaka & Otaki, to appear), a group of students was trying to construct a counterexample based on two presupposed conditions and noticed that some of them are not possible (for example, (c) and (f)). We identify here a germ of RA. Our claim as a conclusion after the analysis of empirical data is that the task requiring students to judge the true or false of their own conjectures could be a fundamental situation to generate the idea of RA.

There is a quadrilateral ABCD with its diagonals crossing at M. Find combinations of two out of the following eight conditions as many as you can, each of which makes this quadrilateral ABCD a parallelogram: (a)  $AB = CD$ , (b)  $AD = BC$ , (c)  $\angle A = \angle C$ , (d)  $\angle B = \angle D$ , (e)  $AM = MC$ , (f)  $BM = MD$ , (g)  $AB \parallel DC$ , (h)  $AD \parallel BC$ .

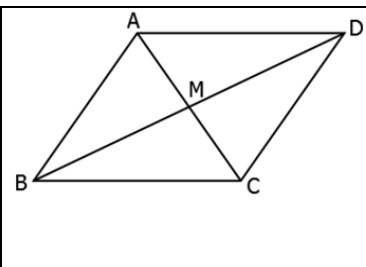


Figure: Reductio ad Absurdum (RA) task.

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# A CASE STUDY ON FACILITATING STUDENTS' APPRECIATION OF AESTHETIC QUALITIES OF MATHEMATICAL OBJECTS

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Although appreciation of the aesthetic qualities of mathematical objects has important meaning, the literatures have found that general learners can not appreciate these qualities (e.g. Tjoe, 2016). However, there are not enough studies that consider on how to facilitate learners' appreciation of these qualities.

The purpose of this study is to clarify how to facilitate learner's appreciation of these qualities. For this purpose, the author defines the aesthetic qualities of mathematical objects by three viewpoints based on the theory by Takeuchi, Japanese aesthetician: 'the form' (e.g. symmetry), 'the whole', and 'one's feeling of the vastness' of the mathematical objects. This framework is well suited to Poincaré's definition. In addition, based on this framework, the author identified a method for facilitation of learner's appreciation of them: (i) to promote learner's understanding of differences arising in the mathematical object with or without 'the form', (ii) to make the learner find 'the whole' of the mathematical object that establishes 'the form', (iii) to make the learner compare the mathematical objects that clarified (i) and (ii) with the mathematical objects that are not so.

This study carried out teaching experiments to investigate effects of above method. Participants are 3 pairs of high school students belong grade 10. They asked to identify what figure fulfills the condition that the distances of the two ways are equal (see Fig. 1, one way is  $A \rightarrow C \rightarrow B$ , the other is  $A \rightarrow P \rightarrow D \rightarrow Q \rightarrow B$ ). As results, 3 pairs, through consideration on some figures like Fig. 1, Fig. 2 and Fig. 3, identified proposition between lengths of sides as 'the form', and they also identified the figures that fulfill above condition like Fig. 4. Then, they felt 'the vastness' of figures that fulfill above condition.

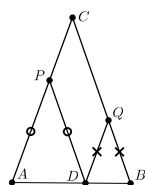


Fig. 1

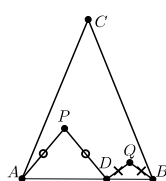


Fig. 2

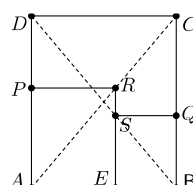


Fig. 3

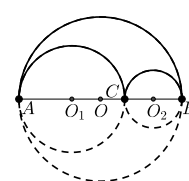


Fig. 4

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# THE NOTION OF PROJECTILE MOTION – A CASE STUDY

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This study adds to research on the use of mathematics in physics classrooms at upper secondary school. The aim is to look closer into what types of transfer do the teacher and textbook set up for the pupils with respect to ways of reasoning from other physics contexts as well as from mathematics. The frame for analysis is an analytical model based on relations made between Reality, Theoretical models and Mathematics (Redfors, Hansson, Hansson & Juter, 2016). Horizontal and vertical transfer is defined as mappings of new information to an activated known structure and as the creation of a new structure in the learner's mind, respectively (Rebello, Cui, Benett, Zollman & Ozimek, 2007). Transfer occurs within mathematics and physics and also between the topics.

We will focus on a physics lecture (40 min, video recorded) in a 3rd year class. When reasoning movement of charged particles in electric fields the teacher stresses horizontal transfer from mechanics and projectile motion. The procedure used is focused on analysing movement in “x direction” and “y direction” separately, not explicitly relating movement to the field direction. Whereas the argumentation in the textbook is based on movement in relation to the existence of a field direction. When considering velocity, the main focus is in both cases on a framework where the components of velocity is central.

The tangent of a curve is a notion the students in the present study are quite familiar with from their courses in mathematics, which makes an opportunity for transfer from a mathematics context to help understanding physics. However, the notion of tangent is not used in the textbook or by the teacher in relation to velocity. Using the vector concept in this way would require students and teachers to perform a vertical transfer. This has been shown hard for both students and teachers. However, introducing this way of reasoning had made use of an opportunity for structural use of mathematics – an opportunity overlooked by both teacher and textbook.

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# DRUG CALCULATIONS IN NURSING EDUCATION: IS MATHEMATICS A PROBLEM?

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This study concerns the teaching of drug calculations in nursing education. It is part of a larger study and focuses on the first year of a three-year nursing program when the students are introduced to drug calculations. The students who attended the first year on the program was divided into smaller groups. We followed one group where the lecture and problem-solving session was video recorded.

It is well known that drug calculations are a critical component in nursing practice. Nurses need to do drug calculations correctly and as part of their nursing education must take a drug calculation test obtaining no errors in the results. However, in spite of drug calculation tests many adverse events occur in nursing practice (e.g., Røykenes & Larsen, 2010). Studies of nursing practice show that mathematics enters practices in a rich variety of ways and that it is not advisable to avoid the complexity of a situation by only using standard methods to capture its visible arithmetic and teach it (Coben & Weeks, 2014). To restrict the teaching to an elementary use of mathematics will not cover all the knowledge that is actually relevant to practice. In routine use, mathematical reasoning can be almost invisible and many artefacts in the nursing profession often depends on this invisibility. But at times nurses will need to understand underlying mathematical models to sort out what is happening or what has gone wrong (Pozzi, Noss & Hoyles, 1998).

The results of the current study show that the teaching of first-year students did not support conceptual understanding of mathematics including discussions about mathematical reasoning or relevant mathematical concepts. Instead, the students were advised to forget their previous mathematical skills – in particular if they felt insecure about mathematics – and apply “safe” methods with a strong focus on instrumental use. For example, in drug dose calculations a triangular arrangement of dosage ( $d$ ), concentration ( $c$ ) and volume ( $v$ ) was used in relation to the “formula”  $d=cv$ , instead of reasoning about how to solve an equation. Discussions about the use of mathematics and underlying models were absent in the teaching.

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# ATTUNING TO THE MATHEMATICS OF DIFFERENCE: EXAMPLES OF TEACHERS' HAPTIC CONSTRUCTIONS OF NUMBER

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The CAPTeaM project (Challenging Ableist Perspectives on the Teaching of Mathematics) involves teachers and researchers from the UK and Brazil in reflecting upon the practices that enable or disable the participation of disabled learners in mathematics (Nardi, Healy, Biza & Fernandes, 2018). We do so through developing and trialling situation-specific tasks that engage teachers with incidents of disabled learners' participation in classroom mathematics discourse (Type I) or invite teachers to solve mathematical problems in small groups, some members of which are temporarily deprived of sensory or communication channels (Type II). We collect data through video recordings of group work and plenary discussions as well as written responses to the tasks. Our data analysis is underpinned by tenets of sociocultural (Vygotsky, 1997) and embodied cognition (Gallese & Lakoff, 2005) theory and focuses on themes such as deconstructing the notion of the normal mathematics student/classroom and attuning mathematics teaching strategies to student diversity. Here, we sample our analyses through showcasing four strategies devised by participants engaged with a Type II multiplication task ("counting fingers"; "tracing the sum"; "negotiating signs to indicate place value"; "decomposing") and discussing how their haptic constructions of number successfully circumvented barriers created by the temporary removal of hearing or sight from their communicational means.

## Acknowledgements

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# QUALITY OF 1<sup>ST</sup> YEAR PRESERVICE TEACHERS' SELECTION OF TASKS, INTENDED AS A BASE FOR INTERACTION

Petra Hendrikse

Katholieke Pabo Zwolle

When during interaction, former made tasks (and upcoming tasks) are addressed, learning will be increased. Orchestrating such interaction demands a high-level of expertise, among others on selecting proper tasks. The first 3 of Stein, Engle Smith & Hughes (2008) 5 principles on orchestrating interaction, literally contain the word 'task'. To judge the quality of the tasks the IQA Mathematics assignment rubric for the potential of the task (Boston, 2012) can be used.

Designing and choosing challenging tasks should be taught. In this research we collect data on the task selection skills of 1<sup>st</sup> year pre-service elementary school teachers after they've had a short introduction on a sub-set of criteria of these kind of open, challenging tasks. The research questions are:

- What is the quality of tasks selected by preservice primary school teachers, regarding 6 criteria taken from the description of 'the potential of the task' from the IQA?
- Which quality criteria correlate to each other?

The sample contains 118 tasks selected by 118 students at the start of their education after they had a short introduction to some of the criteria mentioned in the description of 'the potential of the task' (rubric 1 in the IQA). In this study we scored each task on each of these 6 criteria (cr 1 to cr 6 in text below).

First analyses show that only 5 (out of 118) tasks don't require mathematical activity (cr 1). More often than not the underlying task is appropriate for the specific group (cr 2). The results on the criteria 'applying a broad general procedure' (cr 3), 'having the potential to engage in creating meaning' (cr 4), 'focus on mathematical understanding' (cr 5) and 'ambiguity about what needs to be done' (cr 6) are less homonymic. Several criteria correlate (5 correlations, ranging from  $r=0.56$  to  $0.71$ , all with  $p < 0.01$ ).

The results indicate that several task selection criteria need more attention during education. Introducing them in one meeting, is insufficient for some students.

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# DEVELOPING PROSPECTIVE MATHEMATICS TEACHERS' TPACK IN A DIDACTICS COURSE

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A strategic use of educational technology in classrooms led to important changes in mathematics education and can support students' meaningful learning. Recognizing that teacher's decisions ultimately impact students' experiences as learners, there is an ongoing debate on how to improve the preparation of prospective teachers to effectively integrate technology into mathematics teaching and learning. Pre-service teacher education must help them to develop technological pedagogical and content knowledge (TPACK) that is a distinct type of knowledge that relies on the interconnection and intersection of content, pedagogy and technology, which are the components of teacher knowledge needed for teaching with technology in specific content areas (Koehler & Mishra, 2008). This poster presents the results of a study aiming to analyse the development of Portuguese prospective teachers' TPACK in a context of a teacher education experiment, carried out in a didactic of mathematics course for basic and secondary education (grades 7<sup>th</sup> to 12<sup>th</sup>), and based on tasks that promote, in an articulated way, knowledge of content, pedagogy skills, and technology skills. The TPACK Development Model of Niess (2013) was adopted to analyse the data collected from the work of 6 pre-service teachers on the tasks solved as required course work. Although prospective teachers showed variability in their levels of TPACK for different knowledge components, after the course all of them developed their TPACK levels, from recognizing to accepting, adapting and exploring. Valuable insights and suggestions for pre-service mathematics teacher education programs are also offered.

## Acknowledgments

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# DOING MATHEMATICS PLAYING NINE MEN'S MORRIS

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Games can be seen as a joyful activity, and also include meaningful and inspiring mathematics. The mathematics education literature about games usually focuses on students' performance motivation improvement (Mousolides & Sriraman, 2014). But what do students *do* mathematically when playing a game? The *Nine Men's Morris* game is potentially very rich mathematically, as shown by Gasser (1996) and Boyd & Hirunthanakorn (2012) computer-based analysis. In this poster, I present this game and discuss what some students were doing mathematically while playing it.

The Nine Men's Morris game starts with an empty board and 9 stones per players, which, in turns, players put or move on the game board. The goal of the game is to place 3 consecutive stones along any line of the board to make a 'mill'. Each time a player makes such alignment, he removes a stone from his opponent. A player wins when his opponent can no longer move stones or can no longer make mills. What can be said about the nature of the mathematical activity observed when young students play this game? What kind of reasoning and the strategies do they use?

This study follows a data collection in which I filmed pairs of Grade 1 (6-7 years old) students playing under the supervision of their teacher. Analysing the data, I found evidence of deductive and inductive reasoning when students try to predict what their opponent will do next. I also observed combinatory reasoning when the students consider several moves ahead, and even some form of probabilistic thinking when they had to choose among moves. Ideas related to graph theory can also be observed when students weigh the advantages of different moves.

From a theoretical perspective, attending to what students' do mathematically during a game, as opposed to what they might be "learning" from it, offers a different point of view to understand the student's mathematical experiences. I will briefly discuss the epistemological foundations and methodological challenges that comes with this focus (Maheux & Proulx, 2015), and the contribution made by this study in terms of how playing the game shapes and is shaped by mathematics/mathematical activity.

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# DEVELOPING UNDERSTANDING OF NUMBERS IN DIFFERENT APPROACHES TO MATHEMATICS IN KINDERGARTEN

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Research has revealed the importance of mathematics in early childhood education. Developing understanding of numbers in young children is a crucial prerequisite for the mathematical progress through elementary school (Jordan et al., 2009). Various research was implemented in early childhood mathematics education also with the focus on specific approaches (Stebler et al., 2013). However, there is still a lack of knowledge regarding approaches to support understanding of numbers in less advanced young children.

The qualitative study targets to observe learning trajectories of less advanced children in the last six months of kindergarten, with a special focus on different approaches to mathematics in early childhood education. It aims evolving data-based hypothesis on different approaches to foster less advanced young children's abilities in understanding of numbers. We chose 22 less advanced children (based on a standardized test) from ten different kindergarten classes, which were clustered in four different approaches: training program, game-based approach, specific group actives and solely providing material during free-play. We interviewed all children three times in the last six months of kindergarten. The activities of early mathematics education were also conducted during that time. The semi-standardized interview intended gathering data regarding different aspects of number knowledge and understanding like counting, subitizing, composing and decomposing quantities and early addition.

The data analyses aimed to reveal the degree of children's numerical abilities as well as their development. Therefore, a qualitative content analyses was conducted which allowed to describe learning trajectories and to develop different types. The results show distinct advantages of the game approach. Children, who attended kindergarten classes where mathematically substantial games were regularly provided, showed the highest degree of development. Whereas, considerably less development was observable in children who attended kindergarten classes with the other approaches. Especially less advanced children who attended classes with the free-play approach showed hardly any development regarding understanding of numbers.

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# ELEMENTARY TEACHERS' TAKE-AWAYS FROM SUMMER MATH INSTITUTES IN A THREE-YEAR PROFESSIONAL DEVELOPMENT PROJECT IN RURAL APPALACHIA

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Research on teacher learning emphasizes the potential impact of mathematics focused professional development on teachers' knowledge about teaching mathematics (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). Through the presentation of open tasks, we sought to support teachers' development of content understanding and pedagogy. In addition to prompting teachers' engagement with math-focused tasks, we positioned teachers as experts about their students through a *decentering* approach that focused on three critical questions for teachers: How would your students respond to this task? What difficulties would students experience? What supports would address these difficulties? The research focused on two questions: 1) Did teachers view these questions as important questions to consider in the planning process? 2) Did teachers view their participation in discussions, involving these questions, positively?

A two-week Summer Math Institute took place during each year of the project. The project team collected surveys during each Institute. These findings are based on the year three survey data analysis and offer a glimpse of participants' ideas related to the two research questions. 83% of participants viewed open tasks positively. 78% of the teachers noted that the discussions around the task were helpful because the discussions emphasized thinking about individual students. Open responses on the survey indicated that 76% of the participants mentioned that the questions encouraged them to think about assets and challenges that students bring to mathematics. In regards to the second research question, 59% of the teachers noted that they "had a lot to say" during the discussions since the focus was about their students. Over half felt that they participated the most during these discussions. Some of the teachers commented that they were "teaching" the facilitators and other members of the professional development project about their students. 76% of the participants indicated that the most beneficial aspect of the professional development was engaging in the discussions around imagined implementation of tasks in their classrooms. Based on these findings, we propose that this decentering approach is productive in supporting teacher voice in professional development and seriously considering students' challenges and assets in task implementation.

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# PROSPECTIVE TEACHERS' CONCEPTIONS OF MATHEMATICS

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A teacher's beliefs impact choices made within a classroom (e.g., Handal & Herrington, 2003). Wilkins (2008) added to this discussion by noting that not just beliefs, but attitudes towards mathematics also had an impact on classroom practices. As such, this research investigates how teacher education candidates (TECs) perceive mathematics at the beginning of a teacher education program in order to create dialogue around perceptions of mathematics.

This research project was part of a larger study into conceptions of mathematics of TECs and how to support development during an elementary education program. The data set used in this project came from an anonymous first day of class activity around the TECs initial perspectives on mathematics. The idea was to write the first (up to) five words or phrases about "mathematics." The data was turned into an image, and then a thematic analysis (Braun & Clarke, 2006) was performed. A total of 366 phrases were the data points in this study. After removing all duplicates and forms of the same words (i.e., add, adding), a total of 253 unique phrases were analysed for themes.

Five major themes were evident in the data sample: positive comments, negative comments, neutral comments, difficulty of mathematics, and mathematics content terms. 71 of the unique statements were negative statements about mathematics (42.9% of the total phrases recorded). Positive statements accounted for 12.6% of the total number. Phrases related to difficulty (i.e., hard, easy, challenging) were coded separately since they are not inherently positive or negative (33.9%). Only 2 instances of the word "easy" were recorded, and 33 of the word "hard."

This data is problematic where limited time is spent in mathematics courses within a teacher education program, and the challenge becomes changing these perceptions of mathematics in this short amount of time. If we want to enact lasting changes in mathematics classrooms, then we need to consider how to support moving past initial feelings around mathematics in order to create more positive classroom environments.

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# TASK DESIGN TO SCAFFOLD LOW-ACHIEVERS IN LEARNING SYSTEM OF LINEAR EQUATIONS WITH TWO UNKNOWNNS

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Developing algebraic concepts and using those concepts to solve problems have been recognized as cognitively demanding for students, especially for low-achievers. Many researchers have proposed different instructional approaches to help students learn algebra (e.g., Kieran 2007). In this study, we reported the design of an instructional task and demonstrated how it can well scaffold low-achievers in understanding system of linear equations with two unknowns and the elimination method by substitution. The fundamental idea for the task design is the structural mapping between the manipulative activities and the problem-solving process. In other words, each reasoning step in solving the system of linear equations with two unknowns can be structurally matched and comprehensively recognized by the manipulative activities with concrete representations. Additionally, the task also involves several other characteristics that can scaffold student learning of algebra, including (1) activity for low-achievers to generate various examples that can facilitate the understanding of the meaning of variables and their relationships in linear equations; (2) the use of concrete representations (e.g., head and legs of dog) that help low-achievers easily observe the mathematical relationships; (3) the use of well-structured tables that scaffold low-achievers in conjecturing mathematics relationships in correspondence with the system of linear equations and each problem-solving step; (3) the use of manipulative objects (e.g., dog) to represent the system of linear equations and problem-solving steps; (4) the use of reading comprehension strategy to facilitate low-achievers in connecting between the manipulative activities and the problem-solving process by substitution method. A case study with twenty-five 7<sup>th</sup> grade low-achievers was implemented in a remote-area middle school in Taiwan. Qualitative and quantitative data analyses showed that all the participating low-achievers could conjecture mathematics relationships by the scaffolding of manipulative activities. 73% of low-achievers were able to construct the pre-algebra equation into another equivalent one, which is the key to the substitution method (e.g., transforming “heads of birds + heads of dogs =10” into “heads of birds=10-heads of dogs”). By the use of manipulative objects and the reading comprehension strategy, 80% low-achievers can understand each problem-solving step by substitution method.

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# HOW DO STUDENTS CLASSIFY THE COMMON PROPERTIES OF TWO GEOMETRIC SHAPES: AN ANALYSIS OF CHILDREN'S WRITTEN RESPONSES

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According to van Hiele's model of the development of geometric thinking, the design of geometry curriculum for senior elementary students in Taiwan is critical for a successful transition from the analysis stage to the informal deduction stage. Crowley (1987) suggested such activities that encourage students to observe, identify, and induce the common properties of geometric figures and classify them are beneficial for the transition. However, little research has done to understand how children make senses of classes of shapes, and how they identify and communicate the properties of different geometric shapes. Walcott & Mohr & Kastberg (2009) analysed the written reports of 900 year four students' description of the common and different properties of two given figures to understand how middle grade elementary students make sense of classes of shapes. Similarly, Hsu (2006) asked 265 year six students to list the differences of two figures to understand what were the salient features of a figure. The purpose of this study was to understand (1) how year six students identify and communicate their perception of the properties of two geometric shapes; (2) the easiness of a property being identified. The participants were 265 year six students from four elementary schools in Taiwan. They were asked to answer two questions, each comparing two geometric shapes and listing as many common properties as they could. Three researchers then analysed the students' written responses in terms of correctness, types of properties identified which related to two out of Duval's (1998) three independent cognitive process of geometrical activities: visualization and reasoning (naming and description). The results showed that (1) based on NAEP's scoring criteria to similar problems, students list, on average, 2.3 common properties and achieving 37.7% correct. (2) Some of the frequency of the properties mentioned was very different from Hsu's (2005) study.

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# VALIDITY OF NATIONAL HIGHER EDUCATION ENTRANCE EXAMINATION MATHEMATICS TEST SCORES IN CHINA

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The National Higher Education Entrance Examination (NHEEE), commonly known as *Gaokao*, is important in China for high school graduates because the scores could determine not only what university they can enter, but also their future career (Davey, Lian, & Higgins, 2007). However, very few studies have investigated the validity of these scores. The present study adopts Messick's (1995) definition of validity, which puts the interpretation and fair use of the test scores as the most important component of validity. The following research questions guided this study:

- (a) Is there evidence that the NHEEE mathematics test scores can be used to measure the intended constructs?
- (b) Using modern measures of score reliability, is there evidence that the NHEEE mathematics test scores measure the intended constructs?

637 Grade 11 students in the science concentration/track from a high school in a suburban area in China were tested using the 2014 NHEEE mathematics test, which is consisted of 12 multiple-choice items, 4 short-answer questions, and 8 open-ended questions. For dichotomy items, a 0-1 scoring scale was used. For the open-ended questions, a 0-4 scoring scale was used. Rasch model was applied to address the research questions.

The item-level scores revealed that overall multiple-choice items (Items 1-12) and short-answer questions (Items 14-16) are quite easy for the participants. Of the five compulsory open-ended items, item 17 is the easiest and item 21 is the most difficult. Of the three student self-selected items, item 23 is the easiest followed by item 24 and item 22.

Item difficulty measures obtained from Rasch model ranged from -3.13 to 3.32 logits ( $M = 0.00$ ,  $SD = 1.55$ ). The person ability measures ranged from -1.57 to 3.55 logits ( $M = 1.21$ ,  $SD = 0.78$ ). The person-item map indicated that 11 items (46%) (items 1-3, 5-10, & 14-15) were too easy, 10 items (42%) (items 11-12, 16-20, & 22-24) are at the intermediate level, and one item is too difficult for the participants. The results implicated that more complex items are needed to measure high ability students.

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# UBUNTU INFUSED IN THE CONCEPTUAL UNDERSTANDING OF GEOMETRIC CONCEPTS

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Geometric structures through pattern, quantity, structure and space, shape our world. According to (Fujita & Jones, 2003) Geometry has a dual nature, the theoretical and an area of practical experience. Scheerens (2012, p134) proposes “the recognition of the importance of teachers as resourceful practitioners” who are committed and cooperate with the organizational needs. The research question was: How can Ubuntu as a philosophy be used to foster the conceptual understanding of geometric concepts? A convenient sample of two mathematics teachers participated in a qualitative third phase of a longitudinal study. Data was collected through classroom observations, semi-structured interviews together with reflections on their journal entries

The observed practice after interventions with Teacher 1 revealed the use of integration of selected instructional materials to extract indigenous knowledge on geometric concepts from processes of building traditional huts. Reflections on her journal revealed growth in knowledge and skills gained through research on how huts are built. Teacher 2: Used expansion of beehive nets (a local trade in the area) to share knowledge on polygons. Semi-structured interviews revealed the teachers’ experience of interactive participation with deep conceptual analysis of geometric concepts. All the contexts used were shared through Ubuntu that gave value into indigenous local practices that were familiar to students. The activities were structured in ways allowing students to explore, explain, extend, and evaluate their progress observation. Based on the results, the learning environment created enabled students to support and challenge one another’s strategic thinking in arguments with acceptance of others’ diverse ideas. Teachers are encouraged to engage in reflective practice to discover innovative ways of geometry presentation. Through Ubuntu teachers can improvise, adjust and use other contexts derived from the learners’ culture and or environment.

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# AN INVESTIGATION OF COGNITIVE AND META-COGNITIVE MATHEMATICAL MODELLING COMPETENCIES OF 7TH GRADE STUDENTS

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Mathematics education has much to gain from including mathematical modelling activities in the classroom. According to Niss, Blum and Galbraith (2007, p. 19), modelling can make ‘fundamental contributions’ to a student’s development of mathematical competencies. The literature on mathematical modelling presents one with a variety of modelling cycles that explain and clarify the process of mathematical modelling. Considered the categorization of Biccard (2010) of modelling competencies as cognitive (understanding, simplifying, mathematising, working mathematically, verifying, interpreting, presenting and arguing), meta-cognitive (using informal knowledge, planning and monitoring, a sense of direction) and affective (beliefs about mathematics), we decided to use these competencies in this study. The development of the selected competencies was documented from a qualitative approach. Of particular interest in this study was the development of competencies in students stereotyped as “strong” and “weak”. Eight 7th grade students were used and they worked in two distinct groups of four students each: one group of students considered mathematically ‘strong’ and one group classified as mathematically ‘weak’. They worked for 6 weeks solving three modelling problems. At the end of each session, video transcripts, written work and researcher notes were merged into determining an index for each competency. These were graphed for a visual representation and the narrative characterization of competency development accompanied the graphical representation. Findings in terms of learner growth in cognitive and meta-cognitive competencies are presented in this paper. Results revealed that the strong group commenced the program with greater initial competencies particularly in *a sense of direction, planning and monitoring, understanding, simplifying, mathematising, working mathematically, and arguing* competencies. The weak group also showed competency development in *a sense of direction, planning and monitoring, simplifying, mathematising, interpreting, and arguing*.

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# CODING IN MATHEMATICS EDUCATION: THE APPLICATION OF CODING ON MATHEMATICS EXPLORATION

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Around the world, the latest change in the primary and secondary education is the introduction of computer programming in the K-12 classroom to develop computational thinking of students, which is a skill that involves solving problems, designing systems, and understanding human behaviour (Wing, 2006). In our country, the coding education is scheduled to adopt in the subject of Information of the Revised 2015 Middle School Curriculum. But it is well-known that mathematics education using coding should affect the mathematical thinking including computational thinking positively (Calao, Moreno-Leon, Correa, & Robles, 2015). From that viewpoint, to adopt coding in mathematics learning can be regarded as one of the new teaching methods. As mathematics professors working for training the secondary school mathematics teachers, we thought that it should be necessary to introduce the coding to our students so that it can be used in teaching mathematics. So, we tried to provide our students with the experience to explore mathematics using coding.

The topic in mathematics to explore using coding program ‘Scratch’ is Bertrand’s Paradox, which is the problem to find the probability that a random chord in a circle has length exceeding the length of a side of an inscribed equilateral triangle (Anagnostopoulos, 2006). In this paper, we gave the solutions which were different from Bertrand’s three solutions. We analysed the solutions with mathematical strictness and then simulated solutions using Scratch. We knew that to visualize the results of experiment enabled students to understand the reason for various solutions of Bertrand’s Paradox and the Principle of Indifference in geometrical probability deeply. This shows us the possibility of applying coding to mathematics learning and exploration. Also we got the implication about how to introduce and to implement coding education for training the secondary school mathematics teachers.

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# ANALYSES OF CHANGE IN YOUNG CHILDREN'S CAPABILITY OF COMPARATIVE JUDGMENT BASED ON RECTANGLES DRAWN ON A TABLET

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How and when can children compare the sizes of areas correctly? Children's ability to compare sizes develops in a U-shaped manner (Maratsos, 1973). Kawasaki et al. (2016) examined age-related changes of thirty kindergartners and thirty elementary school pupils' in their performances and strategies in comparing sizes. Next, Kawasaki et al. (in press) asked twenty-six kindergartners to draw bigger or smaller rectangles than the one presented on a tablet computer, by using the application developed by us. Results showed that the percentage of correct answers exhibits a U-shaped development in children. These findings are based on data not from longitudinal but from cross-sectional studies. Therefore, we decided to conduct longitudinal investigations over 3 years with children who were three years old in June 2017. Accordingly, by using our application, we elucidated the developmental process of their capability in drawing tasks. This study was expected to report the survey results of the nine 3-year-old children in June and December of 2017. The following results were found when the results of June and December were compared:

- More answers of each task were observed to be correct in December than in June.
- No difference in strategy of drawing bigger rectangles was observed.
- Differences in strategy of drawing smaller rectangles were observed. In December, all subjects drew rectangles with both sides shorter.

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# LOST IN TRANSLATION: IMPLEMENTING INTERNATIONAL RESEARCH IN SWEDISH MATHEMATICS EDUCATION

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Translation is always an issue for researchers in a small country like Sweden when reporting their results internationally. Specific difficulties appear in translating school mathematical discourse, since there are two parallel translation processes going on, one for the natural language and one for the mathematics. Also, exchanging ideas across research communities in different parts of the world is constrained by differences in culturally specific discourses (Clarke, 2013). This paper brings to attention yet another aspect of translation – that of implementing internationally reported mathematics education research in Swedish education practices. Based on experiences from a lesson study aimed at constructing a research based communication competence framework to be used as a tool in teacher education (Gallos-Cronberg et al, 2018), and experiences from translating American teaching materials into Swedish (Fosnot & Dolk, 2001), several different translation problems have surfaced.

- 1) Similar words do not have a one-to-one correspondence: *Whole Numbers* and the literal translation *Hela Tal* do not cover the same number domain. The three words *number*, *numeral*, *digit* may seem to correspond to the Swedish words *tal*, *nummer*, *siffror*, but do not mean exactly the same thing and are used differently.
- 2) Some words do not exist in Swedish school mathematical discourse: There are not two separate words for *rate* and *ratio*. There are no words for the mathematical activities of *conjecturing* and *justifying* separating them from *hypothesizing* and *proving*. There are no words for *array*, *subitizing*, *unitizing*, or for the concept of *big idea*.
- 3) Idiomatic expressions differ: when talking of a fraction like  $\frac{3}{4}$  Swedes do not say *3 over 4* but always *three fourths*, which effects proportional reasoning.

These, with more examples, will be presented on the poster. The results are of value for Swedish researchers with an ambition to make research reported in English applicable in a Swedish education context, and calls for a discussion of how to create and align relevant terminology within our community.

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# TEACHERS' VIEWS ABOUT CHARACTERISTICS OF PROFESSIONAL DEVELOPMENT ACTIVITIES

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In his overview article, Lipowsky (2010) describes characteristics of effective professional development (PD) activities on the base of existing empirical studies. Among other, PD activities have shown to be effective if they referred closely to the teachers' classroom practice and if they had a specific focus in mathematics education. This highlights the importance of specific characteristics of PD activities, including specific task formats for professional learning: For example, in the light of these findings, reflection tasks using representations of practice (Buchbinder & Kuntze, 2018) have a high potential for mathematics teachers' professional learning, and how they are embedded in PD activities can be expected to be crucial for their effectiveness.

For the acceptance of PD activities and the recruitment of participants, teachers' views about PD activities and about their characteristics may play a decisive role. However, research in this area is still scarce (e.g. Kuntze et al., 2008). In particular, to our knowledge, teachers' views about specific formats and task types used in PD have hardly been investigated so far. Correspondingly, our study focuses on this research need and explores teachers' views related to such characteristics of PD activities. The study uses a questionnaire instrument in which teachers are not only asked to give their preferences of topics for PD activities in open and in standardized formats, but the main part of the instrument focuses on teachers' views about different methods used in PD activities and other PD characteristics. Moreover, the teachers are asked to evaluate specific professional learning tasks, also tasks which use representations of practice.

Using this approach, the study can provide insight into teachers' views about PD activities in a broad sense. The results have high relevance for practice, as the teachers' perception may provide orientation for design decisions related to PD programs.

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# COMPARING CRITICAL ASPECTS IN LESSONS OF EXPERIENCED TEACHERS WITH VARIATION PEDAGOGY

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In a study about identifying characteristics of Chinese mathematics pedagogy, we compared lessons taught by two experienced teachers on the same topic in grade 5, dividing fractions by a whole number. We analysed the lessons based on variation theory of learning (Marton, 2015), with a focus on what critical aspects of the object of learning were being considered by each teacher. We found that even when the teachers were working in a rather standardised environment with prescribed materials, there was substantial difference in their interpretations of critical aspects for seemingly the same object of learning. This case study leads to discussion of the nature of critical aspects and its relation to meaning and purpose associated with an object of learning.

From our previous experience in working with teachers, it is not easy to tell how we should agree on a set of critical aspects. Teachers may be too conscious of what to vary and how to vary in a lesson, instead of asking carefully what a set of critical aspects should be chosen. Although good use of variation is generally preferred, more important is how a concept is understood in terms of suitably identified dimensions of variation.

In this study, good tasks were designed in both lessons to bring about useful contrast and generalisation but leading to different ways of seeing the same mathematical content. In one lesson, the newly learnt division algorithm was considered as a strategy for tackling difficult computation. In another, it was mainly considered as a means to re-present multiplicative / partitioning relations.

The same learning unit, together with similar teaching materials and pedagogical approaches have been reported and analysed elsewhere such as Han et al (2017) and Li et al (2009). By further comparing with these researchers' analysis, we elaborate our view about the significance of identifying critical aspects in shaping the objective of a lesson and creating meanings in the long-term development of concepts.

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# LEARNING MATHEMATICAL PROBLEM SOLVING BY DOING

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Dewey (1944), a famous American philosopher, claimed that children learn by doing. As problem solving has become an indispensable part in today's mathematics classrooms (National Council of Teachers of Mathematics, 2000; Mayer, 1992; Silver, 1982; Zeitz, 2006), it is important to improve students' learning of mathematical problem solving. Little, however, has been done to investigate how to enhance students' learning of mathematical problem solving by means of doing. The purpose of the study was to explore how a "Doing It" activity influenced a third-grade student's learning of mathematical problem solving.

Data for this study were comprised of the student's think-aloud protocols while solving two mathematical problems with contrast problem structures, problem solving performance on both of the problems, and interview protocols during the "Doing It" activity. In the "Doing It" activity, the student was asked to perform the contexts of the problems with real objects. Polya (1973) claimed a four-phase problem solving model which consisted of "understanding the problem", "devising a plan", "carrying out the plan", and "looking back" (p. xvii). The data were analyzed based on the four phases of Polya's problem solving model. The findings of the study indicated that through the "Doing It" activity, the student not only improved his understanding of the problems but also developed his ability in devising a plan to solve the problems. More studies are needed with a larger group of students to understand better the effect of the "Doing It" activity on students' learning of mathematical problem solving.

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# VISIBLE AND INVISIBLE AFFORDANCES OF A MATHEMATICS TEXTBOOK

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This presentation reports on a part of a larger study investigating teacher-textbook relationships in a context of under-resourced schools where the print textbook remains the most accessible resource for teaching (Leshota, 2015). Analysis of 20 lessons from 7 teachers using the same prescribed textbook showed all teachers mobilising elements of the content of the textbook in contrast to only one teacher adopting the embedded instructional approach emphasised by the textbook.

In the lessons on transformations of functions, all the seven teachers mobilised definitions, examples and exercises from the textbook, but only one teacher adopted the sequencing of the presentation formats (Valverde et al., 2002) as suggested in the textbook. The textbook adopted an approach which began with an activity that encouraged engagement with concepts before explanations or worked examples could be offered. This investigative approach opened up opportunities for learners to generalise properties of a particular parent function and its transformations. The remaining teachers omitted the starting activity and offered a didactic approach in which the properties of the parent function and its transformations were ‘told’ to learners; which did not provide opportunities for learners to build generalisations.

This result illuminates an important aspect of teachers’ mobilisation of the textbook: that not all the affordances (Gibson, 1979) of the textbook are visible to the teacher. The content is, and therefore easily perceptible. However, the instructional approach in the textbook is not so easily perceptible and needs to be mediated (Wertsch, 1998). This indicates the visibility of the content versus the invisibility of the approach, with implications then for both pre-and in-service teacher education.

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# INTERPRETATION AND CONSTRUCTION OF BAR CHARTS IN THE ADULT EDUCATION

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Understanding quantitative data requires people's ability to critically interpret and evaluate statistical information, related to data, arguments or phenomena, that can be found in various contexts, as well as the ability to discuss or communicate their reactions to such information (Gal and Ograjensek, 2017). The objective of this work is to present the results of a study, as part of a doctoral thesis, about adult students' interpretation and construction performance of graphs. 95 EJA students, of which 45 were enrolled in module III of the initial years, and 50 in module V of the final years, performed two activities of bar graph interpretation and two of construction. In the interpretation activities, students answered four questions, elaborated based on the categorization of Curcio (1989). In construction activities, students were asked to construct two bar charts from a table. The presentation order of the activities was controlled. In interpretation performance, we have observed greater difficulties in level 2 - read between the data (24.4% accuracy in the initial years and 34% in the final years). The performance in levels 1 - read the data and 3 - read beyond the data was high in both modules. However, module V had obtained better results in level 1 - read the data, while module V and module II presented similar results in level 3 - Read beyond the data. In the construction of graphs, we verified that there was an increase as a function of literacy, with 60% of students in module III and 84% of students in module V having built graphs. Considering the order of presentation's effect, results indicated that starting with building bar charts was harder for students in module III, while the same difficulty did not occur for module V. Important aspects to understanding a bar chart were not included at the time of construction, such as the title and the naming of the axes. Difficulties with proportional construction of the scale on the ordinate axis and the abscissa axis' description of the categories were also observed, especially in module III. These results suggest the need for a more systematized work in the classroom with the advancement of literacy and the importance of school teaching's role in providing students with understanding of charts, a primary task for understanding reality.

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# THE CONSTRUCTION OF THE MEASURE OF MATHEMATICAL ARGUMENTATION

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Argumentation plays a crucial role in discovering mathematical knowledge, such as verifying and fostering conceptual development (Stylianides, Bieda, & Morselli, 2016). However, research on measuring student mathematical argumentation by an instrument to collect large scale data is limited in mathematics education. The present study seeks to construct a measure for assessing students engaging in collective argumentation in mathematics classrooms. We purposely sampled 1176 six graders. A measure of MA was developed through two stages. Stage 1 was aimed at developing two constructs as the framework of the measure, deciding the number of the items and revising the statements of the items for clear. One construct consisted of four factors: classification, formulation, validation, generalization, and justification (Lin & Horng, 2017). Each factor had two subcomponents. Stage II was for evaluating the reliability and validity of the measure by item analysis, confirmatory factor analysis (CFA), and reliability analysis. The statistical results show that the skewness index of the 15 items ranged from -1.533 to 1.015 and the kurtosis index ranged from -2.006 to 0.351. The 15 items did not violate the basic assumptions of normal distribution, so that the CFA can be carried out.

The results show that the difficulty of the items ranged from 0.30 to 0.74 and the discriminatory ranged from 0.25 to 0.69. The reliability coefficient of the 15 items was .613. The correlation coefficients of the four factors were .91, .84, .96, and .83, respectively. The four factors were effectively predicting mathematical argumentation. The measure of mathematical argumentation has high reliability and validity. The structure of measuring MA consisted of four factors: formulation, validation, generalization, and justification. The CFA was further employed to test overall goodness-of-fit,  $\chi^2=120.245$   $p < .01$ , CFI=.933, RMSEA=.025, TLI=.918. The correlation coefficients of the four factors were .91, .84, .96, and .83, respectively. The four factors were the stable structure in the MA measure. The measure was recommended to be used for teachers teaching argumentation in classrooms and for researchers whose research area focuses on argumentation.

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# THE INVESTIGATION OF READING AND COGNITIVE COMPONENTS OF ITEM DIFFICULTIES FOR MATHEMATICAL LITERACY TEST

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The mathematical cognitive components have been supported by many studies that can clearly prove to help students conceptualize mathematical problems (e.g. Embretson & Daniel, 2008). In addition, there is an interactive relationship between mathematics and reading, and students' meaningful reading in solving mathematical problems will help them to know the application of the strategy (e.g. Yang, 2012). The purpose of this study is to explore the relationship between reading and cognitive component and item difficulty of the PISA 2012 mathematical literacy test. According to the responses of Taiwanese students in the PISA 2012, a model of reading and cognitive components of the PISA 2012 mathematical literacy items for Taiwanese students are proposed. All items are coded with reading and cognitive components, such as number of words, problem-solving steps needed, properties of number, narrative translation, and graphic information, cognitive categories (reproduction, connection, and reflection) etc. The relationship between the components and item difficulty is obtained by the multiple regression analysis. This study also examines whether there are discrepancies in interpretations of different content fields and mathematics processes.

Overall, the results show that the cognitive categories significantly affect the difficulty of item, followed by the number of steps to solve the problem, indicating that the familiarity of cognitive thinking has a greatest impact and the second is the cognitive complexity of the item. However, the number of words in the item are not significant. For 15-year-old students, the lower level cognitive components of reading, such as number of words, only cause the proportion of item difficulty variance explained about 1%, it shows that for the adolescent students, the core of reading in problem-solving lies in the understanding of the relationship, rather than low-level encoding, decoding, and other components.

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# MATHEMATICS TEACHERS' IDENTITY WORK IN THE CONTEXT OF LESSON STUDY

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This poster presents results coming from a larger research project focused on mathematics teachers' professional development while they engage in Lesson Study. The project is being developed in a collaborative working group that gathers together teachers from different school levels, prospective teachers, and researchers. In this context, the poster presentation focuses on how two experienced secondary mathematics teachers perform identity work as they engage, for the first time, in a LS. In using the notion of identity work, we conceive the teacher as being ascribed and becoming inscribed by diverse and competing discourses and as being constantly negotiating multiple and fluid meanings of self and other (Chronaki & Matos, 2014). In this way, we aimed at analysing, firstly, how did the teachers use the discourses associated with the LS to position themselves as mathematics teachers. Secondly, we sought to understand how they combined and articulated such discourses for producing understandings about themselves as mathematics teachers. The research adopts a qualitative methodology. The collected data were written descriptions and audio recordings of the group meetings, emails, and WhatsApp conversations. Data analysis involved the selection and analysis of critical episodes centred on the teachers' identity work. We used triangulation and respondent validation to strengthen our analysis' reliability and validity. Our analysis shows that, as the teachers journeyed through the LS cycle, the discourses associated with two phases were particularly relevant in terms of positionality and production of self-understandings: the discourses concerning the planning phase (i.e. to understand themselves as teachers that plan a lesson in detail, and to understand how such planning enriches their teaching practice) and the discourses concerning the role of classroom observation during the implementation phase (i.e. to understand themselves as observers, and to negotiate their role as observers). Our results underline the conflicts experienced by the teachers but also the potentialities of LS as a context for negotiating their professional identity.

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# AN EXAMINATION OF STUDY MOTIVATION CLASSIFYING HIGH- AND LOW-PERFORMING STUDENTS IN PISA 2012 MATHEMATICAL LITERACY STUDY

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This study examines the intertwining relationships of mathematics study motivation classifying high- versus low-performing Macao students in the Programme of International Student Assessment (PISA) 2012 study (OECD, 2013).

## RESEARCH FRAMEWORK

According to Deci and Ryan's (2000) Self-determination theory, the study motivation examined is not dichotomised but may be manifested in the form of a continuum ranging from the more extrinsic to the intrinsic, postulating that high-performing students are having integrated regulation or entirely intrinsically regulated.

## METHOD

There are two steps to achieve the aim of this study. First, according to PISA's definition, Macao's high- and low-performing students are identified and the two groups of students (767 vs 328 respectively) are combined as a group for the ensuing statistical analyses. Second, a data mining tool named Classification and Regression Trees (CART) is applied to examine the eight Likert-type items of the two scales of study motivation (intrinsic vs extrinsic) in PISA 2012 so as to find out the most important factors making a distinction between the high- and low-performing students.

## RESULTS

The most important factor found is *Interest in Mathematics*, which is an intrinsic motivation. The other secondary important factors reveal that the group of students who are *entirely intrinsically motivated*, or who are of *integrated regulation* are prone to be high-performing students. On the contrary, the group of students who are *not that intrinsically motivated* but are of *identified regulation* are not destined to be low-performing students; those who are of partial *identified regulation* are the worst.

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# MATHEMATICS TEACHERS' EDUCATION WITHIN A TECHNOLOGY ENHANCED LEARNING ENVIRONMENT IN PROJECT FTE-LAB

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Immersion of student-teachers in digital habitats (Wenger, White & Smith, 2009) may represent an important step to improve the quality of their practices (Matos, Pedro & Pedro, 2017). We aim to find answers to the question ‘how does immersion in technology enhanced learning spaces stimulate innovative ways of designing teacher education’ taking up the idea that the boundaries between living, learning and working will blur in the future and this will result in the creation of flexible multiuse spaces that may accommodate different activities and serve different learning purposes. This research articulates the piloting of experiments with student-teachers in real secondary school classes as part of initial teacher education program at the University of Lisbon with the analysis of the complex relationships between the three dimensions of the TPACK model (Mishra & Koehler, 2006), its affordances, its constraints and its interactions.

It is presented (i) the rationale that supports the pedagogical assumptions made, (ii) the model for immersion of student-teachers in the practice of teaching with reference to the strategic role of the learning scenario (Matos et al, 2017), (iii) data collection methods (observation, interview to the student-teacher and questionnaire to pupils), and (iv) data analysis (narrative analytical account of the teaching practice, roles assumed by the student-teacher and the pupils). Finally, we present two key results: (a) the crucial role of the structuring resource (learning scenario) used by the teacher in planning activities, and (b) the way the immersive digital habitat stimulates student-teacher reflection and supports the design of teaching activities.

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# THE EFFECT OF EXPOSING STUDENTS TO MATHEMATICS-NEWS-SPAPSHOTS ON THEIR IMAGE OF MATHEMATICS

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Mathematics is a constantly evolving discipline, with over 100,000 new items added to the Mathematical Reviews database each year. Nevertheless, school mathematics curriculum usually does not reflect its ever-growing nature. Consequently, students tend to perceive mathematics as a stagnated domain with no room for creation. This has an implication on students' choice of future career in mathematics or math-intensive fields. In many countries, this has developed into an economic, social and national concern (Noyes & Adkins, 2017). Given the above, the question that troubled us was whether it would be possible to change school students' image of mathematics through exposing them to contemporary mathematics.

Considering students' limited mathematical background and the need to cover the mandatory curriculum, we developed a series of Mathematics-News-Snapshots (MNSs) in the form of 30-minute Power Point presentations, each revolving around a result published in the past 3-4 decades and its history (Amit & Movshovitz-Hadar, 2011). Mathematics teachers who took part in dedicated training courses present the MNSs in their classes. Currently, we are at the second year (out of three) of a longitudinal study, focused on the gains of over 1000 high school students from their exposure to MNSs. Initially, students were asked to respond to a questionnaire aimed at identifying their views of mathematics. At the end of the first year they responded to the same questionnaire plus questions about the MNSs and their impact. Implementing a process of open and axial coding (Corbin & Strauss, 2008), preliminary results indicate that after the first year, students are more likely to view mathematics as a dynamic domain, recognize human aspects related to it and believe that the MNSs had widened their horizon and developed their mathematical thinking. We still need to examine possible cumulative effects.

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# A COMPARATIVE ANALYSIS OF MATHEMATICS CURRICULUM BETWEEN JAPAN AND THE UNITED STATES

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The purpose of this study is to investigate the characteristics of mathematics curriculum between Japan and the United States. For this purpose, I analyzed curriculum standards and textbooks in the case of the Triangle Sum Theorem (TST, the sum of interior angles of a triangle is 180 degrees). The reason why I selected TST is that this is one of the most important theorem in school geometry. The research questions is: *How do the curriculum differ in Japan and the USA about core geometry?*

The theoretical framework to analyze is based on the three domains of mathematical activity (Noddings, 1985). The first is the informal domain, in which we consider things in the concrete situations or the real world. The second is the formal domain, in which we deal with formal objects or procedures such as algorithms, propositions, and proofs. The third is the metadomain, in which we consider the second domain in order to critique and discuss formal objects or procedures themselves. Using this framework, I analyzed Japanese curriculum standard: the *National Course of Study*, and the USA standard: the *Common Core State Standards*. Further, I analyzed the textbooks based on these curriculum standards.

The results were: In Japanese curriculum, TST was appeared in Grade 5 and intended pupils to find out it by inductive reasoning. It was appeared again in Grade 8 in relation to proof and intended students to understand the difference between experiments and proofs. This is the activity in metadomain because of considering about the functions of proof (especially justification) from a higher level. In the USA curriculum, TST was appeared in Grade 8 and intended students to establish it by informal argument such as patty papers and geometry software. It was appeared again as one of the theorems to prove in high school geometry. The results suggest that Japanese curriculum tend to emphasize the justification about a general object in lower secondary education. On the other hand, the USA curriculum tend to emphasize the informal argument through several ways in lower secondary education. In this way, by using the framework of the three domains of mathematical activities, I could clear that the characteristics of mathematics curriculum between Japan and the USA. From now on, analyses of other theorems will be needed to generalize the conclusion of this paper.

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# WHAT IT MEANS TO BE INTRODUCED TO MATHEMATICS: AN EXPOSITION OF SECONDARY SCHOOL STUDENTS FROM AN INTERVENTION STUDY

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This paper reports on the analysis of data collected from mathematics intervention program offered to 1245 grades 10-12 students from three selected secondary schools in one province in South Africa. An open-ended questionnaire task was used to collect data. This investigation is anchored within the interpretive paradigm of qualitative research and makes use of a perspective to mathematics education which takes into account of the need to provide thick descriptions of the “lived experiences” of mathematics teaching and learning (van Manen, 1997). With this perspective in mind, the purpose of our investigation was to give voice to the participants in the program by listening to them, come to understand their experiences in mathematics. Hence, we asked the following research question: What are the lived experiences of grades 10-12 students in school mathematics prior to and after the intervention? Employing an inductive approach to data analysis we uncovered several themes and key of these were: nature of mathematics & conceptions in mathematics linked to introduction of mathematics; effective pedagogic practices that are teacher/learner centred, and learners’ identifications/identities. Some of these themes are consistent with previous research findings in mathematics education (Ernest, 2002). However, the notion of “what it means to be introduced to mathematics and by who” is a theme that seems to uniquely describe the voices of the students participating in the intervention program. The argument being put forward in this paper is that it is contextually critical to consider and understand how students first get introduced to mathematical concepts. This is because, as revealed in the analysis of students’ narratives, how one is introduced to mathematics on the first encounter lays an inert foundation for access that is likely to lead to success or lack thereof in mathematics.

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# NUMBER ESTIMATION ABILITY IN 3<sup>rd</sup> AND 4<sup>th</sup> GRADE SCHOOL CHILDREN

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Number estimation is a process of translation between quantitative representations, in which one is approximate, and another is numerical. Studies have described the importance of number estimation in math achievement (e. g. Moore & Ashcraft, 2015). Therefore, this research aimed to evaluate the number estimation ability of a group of 3<sup>rd</sup> and 4<sup>th</sup> graders. For this, two number line estimation tasks were applied, the one in which the students achieved better accuracy, and the distribution model that best fit their estimations were identified. The sample comprised 143 3<sup>rd</sup> and 4<sup>th</sup> grade students ( $M = 9.8$  years old,  $SD = .74$ ) from two public schools in Porto Alegre – Brazil. The students were evaluated in number-to-position task (NP), which consists of estimating the position to a given number on a number line bounded from 0 to 100, and position-to-number task (PN) which consists of estimating a number to a given mark on a number line also bounded from 0 to 100. The children had to make their estimates in a notebook containing one number line per page. The task was applied in groups of 10 students, lasting approximately 30 minutes for each student group.

The results indicated that there was a significant difference between the accuracy of 3<sup>rd</sup> and 4<sup>th</sup> grades students (NP:  $U = 1827$ ,  $p < .05$ ; PN:  $U = 1767.5$ ,  $p < .05$ ). Thus, the experience and increase in schooling raise the accuracy in the students' performance in these tasks, as found in previous studies (e. g. Siegler & Opfer, 2003). It was also possible to verify relation between the children's achievement in both number line estimation tasks ( $r_s = .66$ ,  $p < .01$ ), that is, the students who had better accuracy in the NP task also showed better accuracy in the PN task. Regarding the distribution of the estimations, it was found that in both tasks children's mean accuracy best fitted the Linear Model. Also, children tended to perform equally well or poorly in both tasks.

Furthermore, the results provide evidence about number estimation ability of 3<sup>rd</sup> and 4<sup>th</sup> graders, that can be used to assess children's number understanding and to assist the development of basic math skills. However, future research is needed to accompany the development of number estimation ability from the first grade and to verify the students' strategies used throughout each school grade.

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# GENDER AS A CULTURAL CONSTRUCT: METHODOLOGICAL ISSUES IN QUESTIONNAIRE DEVELOPMENT

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Validating attitudes questionnaires is challenging yet vital in bringing mathematics education research forward (Nortvedt & Sumpter, 2017). A number of different existing questionnaires are often translated and used in new research, using different frameworks than the original studies did; for example, the Attitudes Toward Mathematics Inventory, the seven C's and the teacher-student and pedestrian questionnaire exemplify this tendency. In validating a questionnaire, researchers mainly focus on analysing student responses to verify the measurement model, arguing that a unified model implies a uniform understanding of the measured attitude constructs (see, for instance, Khine & Afari, 2014). Cultural differences in the response patterns may, for instance, be treated through the application of vignette methods. However, when constructs, such as gender, are conceived differently across or within cultures, further development may be necessary, improving how questions are both framed and validated; for instance, more qualitatively-oriented validation efforts could be employed in addition to the currently applied factor analyses and IRT methods (Nortvedt & Sumpter, 2017).

This poster will display outcomes from validating efforts in a project where a questionnaire for a cross-country and cross-teacher education programmes research project aiming at investigating gendered beliefs about mathematics education in Norway and Sweden. Following a pre-pilot in autumn of 2016 survey questions were altered from traditional Likert-scale statements to matrix questions (framed by a narrative describing recent belief developments in society and mathematics education). In 2018, task based interviews were conducted with mathematics teacher students to investigate whether the culturally framed matrix questions, rather than the more traditional Likert questions, allow students to demonstrate their awareness of a range of culturally rooted differences in attitudes towards boys' and girls' abilities to learn mathematics, which are held by different stakeholders in society. Sample questions and transcripts from interviews will be used to illustrate the survey development.

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# ARTISANS, MACHINES AND INFORMAL LEARNING: THE CASE OF FIBRE MATHEMATICS

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The term *fibre mathematics* refers to the mathematical activity of weavers and textile artists. This poster seeks to expand our knowledge of how mathematics is pursued differently through material media by focusing on *fibre mathematics*. Contributing to a growing body of research that seeks to understand how mathematical thinking happens through the body, this research draws primarily on *inclusive materialism* as a way of rethinking the entanglement of mathematics and matter, and as a means of opening up discussion about diverse learning environments (de Freitas & Sinclair, 2014). This methodology is supplemented by the work of Gilbert Simondon (1958/2017). Rather than treating technical objects as merely tools with use-value, Simondon's analysis points toward the distinctive mathematical potential of loom technologies and artistic technicity.

The research project pursues the following research questions: In what ways does the loom create a rich learning environment? What kind of mathematics pedagogy is entailed in informal textile studios and workshops? Data collected through interviews, participant observation and research-creation conducted in the weaving studio of a craft school in the eastern United States will be used to analyze a complex problematics of loom-body-concept.

Preliminary findings from "first-person" methods (Roth, 2012) indicate the loom serves as an experimental field for the generation of a diverse set of "weaving problems," which incorporate trial and error techniques and set the stage for sensorial shifts in the discernment of new units and future building blocks. An analysis of the mathematical nature of this inventive process will be shared alongside the personal descriptions research participants give about the mathematics in their work.

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# EPISTEMIC NETWORK ANALYSIS AS A LENS TO UNDERSTAND TEACHER KNOWLEDGE OF PROPORTIONS

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Literature on teacher knowledge consistently refers to the importance of the coherence of teachers' knowledge. However, such coherence has rarely been defined or investigated. In this poster, we will present the findings of our study of the coherence of teachers' knowledge related to proportional reasoning.

To study coherence, we rely on knowledge in pieces (KiP; diSessa, 1988) as it allows us to conceive of understandings as being comprised of fine-grained knowledge resources and posits that learning involves developing more knowledge resources and connections between knowledge resources, allowing them to be invoked in a wide variety of situations. We refer to a well-connected body of knowledge resources as being coherent. We also rely on Epistemic Network Analysis (ENA; Shaffer, Collier, & Ruis, 2016), which provides a visual way of capturing the connections between knowledge resources. ENA uses a 2D projection of high dimensional space to show co-occurrences of knowledge resources.

Using ENA and KiP, we were able to code responses to two interviews (one think-aloud and one clinical interview) for each of 32 in-service middle grades teachers in which they were asked to solve proportions and comment on others' reasoning about proportions. We then created graphs of the co-occurrences of the knowledge resources they used. By separating the teachers according to their relative mathematical strength as determined by an assessment, we then looked for patterns in the groups to highlight knowledge resources that were well-connected and those that were not well-connected. The poster examines these patterns and makes suggestions for implications around teacher learning.

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# GRADUATE STUDENTS' CONCEPTUAL UNDERSTANDING OF SHIFTING BETWEEN REPRESENTATIONS

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This research focused on characterizing graduate students' conceptual understanding of representations and their ability to shift between representations during problem solving. Tall (2013) defines three long-term developments that together generate coherent mathematical reasoning. The first refers to the properties of functions, the second to operations on functions, and the third (*crystalline*) to reasoning that requires shifting and creating meaningful connections between all representations. Discussing the heuristic strategies used in solving similar problems advances the problem-solving process as it develops from the stage of building connections on the formulation level to the algorithm level and then to the heuristic level. Heuristic connections testify to the enhanced ability of learners to perceive all features of mathematical concepts and to apply them in solving unfamiliar problems (Ovadiya, 2014). The current study sought to apply the levels defined by Tall (2013) to the solution of differential calculus problems. The research questions are: 1) what considerations do graduate students in mathematics education use while solving mathematics problems entailing shifting between representations? 2) What can the students' problem-solving process reveal about their perceptions in the context of representations and shifting between representations? The research population comprised twelve students enrolled in a one-semester course as part of a master's degree program. The research instruments included dialogues discussing the solution of 30 mathematical problems that entailed shifting between representations, the researchers' reflection log, questionnaires in which students reflected on their solution of each problem, and final course summaries. The prose discussing the use of SBR in solving calculus problems was classified into two themes. The first theme—*dynamic operational perception of SBR*—includes reaching the solution using a variety of techniques and algebraic operations. This theme reflects a non-crystalline perception of the representations and thus less success in problem-solving. The second theme—*dynamic crystalline SBR perception*—encompasses the ability to offer a variety of considerations and connections and to move progressively and dynamically between graphical and symbolic representations or from one graphical representation to another, thus reflecting successful problem-solving, crystalline perception of the representations and SBR. The findings of two themes promote the notion that teachers should be trained in SBR and specialized content knowledge.

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# GENERALIZING THE IDEAS REGARDING TRANSLATION OF A PARABOLA BY A MATHEMATICAL MODELING ACTIVITY

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A parabola, the graph of a quadratic function, is drawn by relating it with the parameters of the quadratic function. It is thought that the students who connect the different representations of the quadratic function with each other can think the changes related to the parabola and the algebraic expressions in the case of the translations of the parabola. Thus, examining the students' ideas about translations of a parabola presents evidences related to their conceptual understanding. Eraslan (2005) stated that the students had procedural understanding while making the inferences related to the translations of parabolas without comprehending the quantities and making relations among them. Additionally, Zazkis, Liljedahl and Gadowsky (2003) examined how the students coped with the difficulties about horizontal translations of quadratic functions and achieved that the students did not think about the functions by tending to memorize their knowledge. In this direction, we focused on the students' conceptual understanding by triggering for them to work on a real-life task. The purpose of this study is to examine the students' conceptual understanding and generalizations related to the translations of the parabolas by a task in a real-life context. The data of the study were collected by the clinical interviews with two 10th grade students and the clinical interviews were recorded by a video camera. In these clinical interviews, the students worked on the task in which the chain guards representing the limited parabolas in the seaside was related with the translations of the parabola. The transcriptions of the clinical interviews were analyzed in terms of the students' understanding and generalizations of the changes in the parameters and in the algebraic representation in the case of the translations of the parabola. The students made connections about the concepts instead of remembering their knowledge while modeling the real-life context. As a result of the study, it was revealed that the real-life context supported the students' generalizations and conceptual understanding regarding the translations of the quadratic functions.

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# THE EFFECT OF COMMUNICATIVE MULTI-MEDIA MATERIALS OF MATHEMATICS WORD PROBLEM

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Many research addressed that the instructional technology and media benefit mathematics learning (Smaldino, Russell, Heinich & Molenda, 2005) and mathematics word problems (Voyer, 2011). Voyer (2011) assessed the effect of the situational model included different types of the word problem statements and then found that pupils who give greater situational and explanatory information in a problem have greater success in solving the problem.

This study aims to improve low achievers' performance of problem solving by manipulating a set of multi-media animation of word problem included situational and explanatory information. The multi-media materials consisted of ten types of word problems which were designed by the author. The experimental design was conducted to analyze the students' performance, learning attitude and motivation on solving math word problems. 6<sup>th</sup> grade low achievers from two schools in the same school district were designated to the control group and the experimental group. The mathematics performance test and questionnaire of mathematics learning attitude were designed by the author. Two groups of students were tested to compare their word problem performance and mathematics learning attitude. In addition, the students were given a semi-structured interview form with five questions.

The study finds out that the interactive communicative multi-media materials benefited the math performance, attitude and motivation of the low achievers in the experimental group. Students from the experimental group showed the higher performance on solving word problems than the control group. In the interview, students reported that the materials attracted their attention in the math classroom, and most low achievers reported that they change their views of mathematics because of the materials. They liked mathematics more after the class than before.

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# SPANISH PRIMARY MATHEMATICS PROSPECTIVE TEACHERS' GENERALIZATIONS IN FUNCTIONAL THINKING

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Generalization is one of the highlighted processes in primary education programs since the generalization of numerical patterns and regularities to describe functional relationships form the basis for later study of functions (Blanton, Levi, Crites, & Dougherty, 2011). However, research documents students' difficulties when they come across generalization in sequences because the transition from the particular to the general takes time (Kieran, 2007), which shows the importance of providing students with adequate teaching situations to develop their functional thinking. Guided by the previous theoretical context, our study aims to analyze how Spanish primary mathematics prospective teachers (PTs) express and identify generalization in tasks that require functional thinking. The data were collected by a questionnaire applied at the beginning of the school year of 2017/18 to 94 Spanish PTs that were attending the 1<sup>st</sup> year of the degree in primary education teaching. Results indicate that only few PTs were able to express generalization in a functional way for a geometric pattern. One third of the PTs were able to interpret the generalization of a relation among two quantities provided in an algebraic expression but almost half of the participants considered co-variation of two quantities for several values without generalizing it for all set of values. These findings provide information about PTs reasoning concerning generalization of functional relationships and bring into light the main difficulties they encounter, which are issues that are worth considering when designing mathematical content courses for prospective primary teachers.

The poster begins with a presentation of the study including the aims, context and methodology. The focus is then on some examples of the students' work to document the results.

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# VISUAL REPRESENTATIONS OF MULTIPLICATIVE RELATIONSHIPS BY STUDENTS LEARNING MATHEMATICS

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Recent studies (e.g., Moyer-Packenham, Ulmer & Anderson, 2012) associate improvement in students' mathematical achievement with the integration of visual models and schematic drawing in the process of teaching and learning. However, while some studies focus on using visual representations mostly for teaching number sense and number fluency; other studies (e.g. Dougherty & Slovin, 2004) extend its use to problem solving. In order to support students who are struggling with mathematics, we used Davydov's idea of importance of mathematical relationships in mathematical knowledge development. According to Davydov, the very concept of number represents a multiplicative relationship between two quantities, one of which is measured by using the other as a unit of measurement.

We currently conduct a two-year study working with secondary school students (age 12-14) who are struggling with mathematics to improve their problem-solving skills. Specifically, we integrate several types of visual representations of multiplicative relationships in the learning process. We started with individual interviews with students to understand whether and how they may use visualisations to support their reasoning. We then introduced particular visual representations designed to support understanding of multiplicative relationships. Throughout the first year of the study, we observed positive changes in students' use of visual representations to support their reasoning in problem solving.

Our poster will present the theoretical background of a relational approach to problem solving; the ways multiplicative relationships can be visually represented; and the analysis of students' visual representations and mathematical reasoning.

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# THE SOCIAL PROCESS OF STUDENTS' COLLABORATIVE PROBLEM SOLVING IN CHINA

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It has been widely accepted by the researchers and practitioners that collaborative learning could help to develop students' cognitive and social skills. In China, collaborative and interactive ways of learning is recommended in the mathematics curriculum reform and mathematics teachers are expected to facilitate students' collaborative learning. But the large-size class in China makes it challenging for teachers to cater to the complex processes involved in collaborative learning. This study investigated the social processes of students' collaboratively solving mathematics problems in China, aiming to better understand students' collaborative learning in the large-size classes and thereby help teachers to develop students' collaborative problem-solving abilities and to optimize students' collaborative learning.

Six mathematics teachers from three schools in Beijing participated in this study and each teacher taught a year-seven class with about 36 students. Students were divided into small groups with 4 or 6 members. And an open-ended mathematics task in real-life context was given to student groups to solve collaboratively. Each group's collaborative problem-solving process was video recorded and altogether the videotapes of 48 small groups' collaborative problem solving were collected and analyzed. The analytical framework for the process of collaborative problem solving was adapted from Kumpulainen and Mutanen (1999), and Hoek and Seegers (2005). The interaction episodes were transcribed and categorized into different modes: collaborative (including tutoring, argumentative and cumulative), conflict, dominative, confusion and individualistic.

It finds that the mode of collaborative talk took up the majority of students' talk in the small groups observed, suggesting that Chinese students were willing to engage in group talk and express their thinking. However, students were observed to be easily distracted by the background information in the mathematics task, which was likely to result in a superficial discussion of the mathematics task and unsuccessful work. This study contributes to better understand the complex process of collaborative learning in large-size classes.

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# BIOGRAPHY, EMOTION AND MOTIVATION IN MATHEMATICS STUDIES: DESIGN OF A COURSE FOR STUDENT TEACHERS

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Recent research on mathematics teacher education indicates the benefit of enriching academic mathematics courses for prospective teachers by school related content knowledge (SRCK, Dreher et al. 2018) to show the relevance of academic mathematics studies for their future profession. A first semester arithmetic course, which emphasizes SRCK, was evaluated inter alia regarding students' emotion and motivation. Interviews with students point to obstacles for them to engage and succeed in the course, which can be traced back to own negative experiences with math at school. Low motivation and self-confidence go along with unreflected beliefs about mathematics, corresponding with the partly inappropriate image of mathematics in society. This evidence was impulse to design the course "Encounters with Mathematics" that aims to overcome these obstacles focusing on specific facets of SRCK.

The course, on the one hand, creates an environment for reflecting one's own mathematical learning biography, the image of math in society and personal beliefs about what mathematics is. Thereby students' self-regulation, motivational orientations and beliefs/values/goals are addressed — aspects of teachers' professional competence according to the model of Baumert & Kunter (2013). On the other hand, the course explicitly teaches math-specific working methods and approaches, one of the facets of SRCK, giving students the opportunity to experience themselves as competent in doing math and deepening into the process-oriented nature of mathematics.

Research questions are: What are the effects of the course on the motivation and self-efficacy of students in their math studies? What are its effects on beliefs of students concerning mathematics? An ongoing case study includes pre- and post-surveys on emotion and motivation, questionnaires, observations of the course as well as interviews with individual students and lecturers. Expected results are increases in students' motivation and self-efficacy as to their math studies as well as a shift in their beliefs on the nature of mathematics towards a more process-oriented understanding.

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# PROBLEM SOLVING TO DIFFERENTIATE GRADES OF MATHEMATICAL TALENT

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Mathematics education researchers have focused on mathematically talented students' behaviour. Three types of studies have been carried out most frequently: identification, description, and intervention. We present results related to identification. Standardized psychological tests are widely used to evaluate general giftedness, but some studies show that, to evaluate mathematical talent, solving a set of selected mathematical problems is more reliable than standardized tests (Benavides, 2008).

We present results from an ongoing research project aimed to analyse the behaviour of 3 secondary school students (A, B, C) with diverse grades of mathematical talent: A is very talented, B is quite talented, and C is less talented than A and B, although he has a very good performance in school mathematics. Student A was in grade 8, and B and C were in grade 7. Based on the types of proofs proposed by Marrades and Gutiérrez (2000), we analysed their solutions to a set of proof problems with different characteristics (topic, complexity, type of proof).

The students participated in a problem-solving workshop that lasted six 90-minute sessions. They solved 12 problems including variants and generalizations of the statements, proposed to induce them to make more complex and sophisticated solutions. The sessions were video-recorded, with a researcher being the teacher. Each student felt confident, along the experiment, with a different kind of proof: Student A used examples to help organize deductive *thought experiment* proofs. Student B made mainly empirical *generic example* proofs, but he sometimes relied on specific examples (*naïve empiricism*). Student C relied often on examples (*naïve empiricism*), from which he induced (sometimes wrong) general rules, and other times he produced *crucial experiment* proofs.

We conclude that, to identify mathematical talent, a set of carefully chosen problems, which allow students to use examples and empirical reasoning as well as abstract deductive reasoning, may be a good tool.

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# SIX YEARS OLD PUPILS' KNOWLEDGE ABOUT RECTANGLES

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This study aims to know what kind of knowledge Portuguese six year old pupils' use to recognize rectangles. The theoretical framework draws on the notion of mental construct of a concept, which initially, includes mostly visual images based on perceptual similarities of examples, related to sensorial cognition (Fischbein, 1999). It also draws on the notion of intuitive knowledge, which is related to intuitive thought and based on figural aspects or visual prototypes, without considering attributes or properties of shapes (Clements et al., 1999). This knowledge can be related to impressions; previous experiences; and structural schemata, behavioral strategies that create the possibility of assimilation and interpretation of information and the adequate reaction to different stimuli (Fischbein, 1999). It may also be related to major theories of concept formation: classical view, where a set of defining features are shared by all examples, and the prototypical view, related to prototypes (Smith & Medin, 1981).

The study follows a qualitative approach. Data were collected through clinical interviews (videotaped and transcribed), carried out by the first author to four pupils.

Participants were all six years old and belong to the same 1<sup>st</sup> grade class, constituted by 21 pupils, of an elementary private school near Sintra, a small city at the Lisbon area.

Regarding the pupils' answers, the results show that they used visual prototypes; impressions, including nonexamples; previous experiences; and structural schemata, without considering attributes or properties of rectangles. Their knowledge seemed to be related to general characteristics of intuitive cognition, as a direct and self-evident intuition and an intrinsic certainty. All the pupils considered a partitive type of classification, mentioned the square as a nonexample of a rectangle but never as a particular case of it.

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# DIAGNOSIS OF BELIEFS AND KNOWLEDGE THAT PRE-SERVICE PRIMARY TEACHERS HAVE ABOUT SCHOOL MATHEMATICS, ITS LEARNING AND TEACHING

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Since the creation in Chile of the Professional Teaching Development System in 2016, training programmes for teachers are forced to take a diagnosis test at the beginning of the programme, in order to establish levelling and monitoring actions for teaching students that need it. In this context, it is extremely important to have specific information on what prospective teachers know and believe about mathematics learning and teaching. Therefore, we have designed and validated an instrument to identify beliefs and knowledge about school mathematics at the beginning of their professional training process. Using a mixed methodology, we built an instrument and validated it by its application to 511 first-year primary school teaching students (about one third from 2017 national registration) in 14 Chilean universities. The statistical validation process considered the analysis of factors and clusters for belief items, in addition to Classical Test Theory analysis of tests, factors, and internal consistency, and the Item Response Theory for the items of School Mathematics Knowledge (CME, from its acronym in Spanish). We got a psychometrically validated instrument consisting of 57 items to assess beliefs regarding teaching and learning, expectations and achievements, and the nature of mathematics, and 40 items to assess CME that considers school mathematics contents and abilities (Martinez et al., 2018). One of the most interesting results was the possibility to look into the relationship between the construct of beliefs and of knowledge, for which correlations between latent variables of beliefs items and CME items were studied. Significant correlations between beliefs about teaching and learning and the abilities on school mathematical content areas were identified. The complete sample's self-perception shows significant relationships with the performance shown in the CME part of the instrument.

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# EMBODIED FRACTIONS: CONCEPTUAL DIFFICULTIES IN THE LIGHT OF GROUNDING METAPHORS

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Fractions and rational numbers are known to be hard to both teach and learn, as there are many conceptual difficulties concerning fractions. For example, pupils may interpret the entirety of a picture as the whole (Mack, 1990), or seeing a part as a fourth as long as the whole is divided in four parts, regardless of the size of the parts (Ball, 2007). A recent study has revealed additional difficulties: Seeing fractions as divisions may hinder pupils to recognise one of the parts as  $\frac{1}{4}$ , and claim that it is the partition that is  $\frac{1}{4}$ . The role of numerator and denominator can be mixed up, or the denominator may be seen as the remaining parts, resulting in a picture of 2 fifths to be named  $\frac{2}{3}$ . Pupils can also claim that a fraction has a specific representation, for example that it should be the upper right fourth of a circle that should be shaded, in order for the picture to represent one fourth. One possible reason for misconceptions is stereotypical or restricted use of representations of rational numbers, especially area models (Zhang, Clements & Ellerton, 2015). However, if the number line is introduced, there is a risk that the difficulties are transferred to the new representation. In the recent study, some pupils saw the number line as a whole, and place one half at the centre, regardless of the part of the line visible.

In this study, we relate conceptual difficulties concerning fractions to Lakoff and Núñez (2000) four grounding metaphors for numbers, by analysing the underlying metaphors of visual models used by pupils when the difficulties manifest. The results give implications for the introduction of fractions in the early years of elementary school. Our poster will present how misconceptions can manifest in area models and on the number line, how these misconceptions are related to the metaphor implicitly used in the models, and suggested activities where metaphors aid the understanding of fractions.

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# CHANGING PARADIGMS IN PROBLEM SOLVING: AN EXAMPLE OF A PROFESSIONAL DEVELOPMENT WITH ELEMENTARY SCHOOL TEACHERS

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This poster describes a three-year professional development (PD) that aims to support elementary school teachers (Grades 1-3) as they change their teaching paradigm, from operational to relational, in regard to problem solving on additive structures. In the *operational paradigm*, word problems are mostly treated as a tool to learn mathematical operations. In the *relational paradigm*, the focus is on the mathematical relationships between quantities or measures in a given problem. The concept of relationship is seen as “the law of composition by which the relation between two elements determines a unique third element as a function” (Davydov, 1982, p. 229). In partnership with a school board, we used a design experiment (Cobb, Confrey, DiSessa, Lehrer & Schauble, 2003) to investigate the applications and limitations of Davydov's theory. Since the relational paradigm is foregrounded as an alternative for procedural approaches to additive structures, we wanted to investigate the transition in the teachers' epistemologies as we create an opportunity for them to test its potential for teaching elementary students. We worked with four Grade 2 teachers throughout the whole project and had five Grade 1 and Grade 3 teachers joining the project at a later stage. Our experiment consisted of a PD program comprised of 15 iterations throughout a period of three years. In each iteration, we proposed a new activity to be enacted by teachers in their classrooms. We created the activities to emphasize the nature of the relational paradigm, to generate mathematical discussions about problem structures, and to gauge the mathematical development of children in elementary grades. Our results show that the participating teachers not only change their vision of additive structures, moving from the operational to the relational paradigm but also identify the new paradigm as more effective in teaching additive structures in word problems. Furthermore, our results indicate that teachers change their paradigms in different ways, therefore revealing a continuum spectrum from the operational and relational paradigms. Future research has yet to explore how such a spectrum is composed and what steps comprise the paradigm change.

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# **SUPPORTING MIDDLE-SCHOOL STUDENTS' SPATIAL ORIENTATION BY USING ORIENTEERING ACTIVITIES IN REAL-WORLD ENVIRONMENT**

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Spatial orientation is an important component of spatial skills and it deals with spatial objects, their positioning in the real world and spatial relationships. Spatial orientation provides students with substantial opportunities to better understand the real world they live in and make an inference about some facts about the world, since it includes mapping skills, navigation strategies (i.e. landmark strategies, Euclidean strategies and cognitive maps) and implication of movement (Bergqvist, 2015). Orienteering activities are a good way to use such as map-and-compass skills, decision making and strategy development as well as to apply children's spatial orientation. For this reason, orienteering activities may support students' spatial orientation. The purpose of this study is to investigate seventh graders' spatial orientation in a learning environment which is designed based on real-world orienteering activities. 10 middle-school students enrolled in a 6-month teaching experiment which is focused on using real-world orienteering activities. The data of the study is collected by a pre- and post-test of spatial orientation, two map-reading tasks, two map-drawing tasks, map-tasks associated with using cardinal directions, clinical interviews focusing on solving these tests/tasks and nine teaching episodes (i.e. orienteering activities using a variety of navigation strategies) conducted with the participant students. The data were analyzed qualitatively by using an open coding approach, axial coding approach and conceptual analysis. The results of the study indicate that the learning environment which is designed based on real-world orienteering activities supports middle-school students' spatial orientation from the aspects of reading map, interpreting map, using map-and-compass skills and developing different navigation strategies in order to find a target location. On the other hand, some aspects of this competency such as drawing map, inferring map and using cardinal directions are not supported sufficiently. The most notable result is that navigation strategies used by students show the change from undesirable to desirable types based on real-world orienteering activities. At the beginning, many students were only able to use landmark strategies. At the end, the participants preferred to use Euclidean strategies instead of landmark strategies, and most of them were masterfully able to use both strategies. As a consequence, it is seen that there is a remarkable movement from landmark strategies to Euclidean strategies.

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# SELF-RESPONSIBILITY IN MATHEMATICS LEARNING OF MACAO STUDENTS IN PISA 2012 MATHEMATICS STUDY

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This study examines the intertwining relationships of *attributions to failure in mathematics* and the associated *work ethic* of Macao's high versus low-performing students in the Programme of International Student Assessment (PISA) 2012 mathematics study (OECD, 2013).

## RESEARCH FRAMEWORK

*Self-responsibility in mathematics learning* is conceptualized as student's response to his/her *attributions to failure in mathematics* (e.g. *bad guesses, not good at mathematics problems, teacher did not explain well*) through one's efforts to enhance conditions of learning and schoolwork (e.g. *work hard on homework, keep work organized*). It is postulated that student performance is higher if attribution is internal and the self-responsibility is associated with effective work ethic in one's learning.

## METHOD

There are two steps to achieve the aim of this study. First, according to PISA's definition, Macao's high- and low-performing students are identified and the two groups of students are combined as a group for the ensuing statistical analyses. Second, a data mining tool named Classification and Regression Trees (CART) is applied to examine the Likert-type items of the two rating scales of *attributions* and *work ethic* in PISA 2012 so as to find out the most important factors making a distinction between the high- and low-performing students in mathematics (Breiman, et al., 1984).

## RESULTS

The most important factor found is *bad guesses*, which is *not* an internal attribution to failure. For those students who are *not* likely to make bad guesses, they are found to be more self-responsible in their mathematics learning. Through work hard on homework and insisting that teacher explains well they are prone to be high-performers.

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# TEACHERS PROMOTING ACTIVE STUDENT DIALOGUE THROUGH LISTENING AND QUESTIONING ACTIVITIES

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When students actively listen and interact with each other, their opportunities for developing reasoning and communication abilities in mathematics increase (Sjöblom, 2015). However, they are not always given opportunities to work dialogically during mathematics lessons, and there is little research in upper secondary classrooms about how teachers can promote student-to-student interaction. This poster presents an educational design research project (McKenney & Reeves, 2012), in which the intervention focuses on teachers' work to make multilingual students interact while working with mathematical problem solving. The design and analysis build on the inquiry co-operation (IC) model (Alrø & Skovsmose, 2004) and Fuentes' (2009) framework on student communication. The aim of the intervention is to find ways for teachers to promote and support student interaction, so that students engage in all dialogic acts in the IC-model, and hence listen actively to each other and ask/answer questions. It is expected that the design will need to take into consideration how students perceive group work, how they understand the purpose of questioning and listening, what role language and/or multilingualism plays in interaction, and how teachers can cooperate with other teachers to develop their teaching. Sjöblom (2015) provided results on these issues concerning the students; so now teachers are in focus.

The poster presentation has the aim of discussing insights on: 1) What kind of language do students need in order to participate in fruitful student-to-student interaction, and what do teachers need to promote this? 2) How to design tasks and support means for students, such as problem-solving strategies or communicative roles, in order to make students listen and ask mathematical questions? 3) What can other possible frameworks offer to continue the investigation?

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# CAN VALIDATING PROOFS HELP TO CONSTRUCT PROOFS?

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Today, it is widely acknowledged that handling mathematical proofs incorporates more activities than constructing proofs, for example comprehending or validating (purported) proofs (e.g., Selden & Selden, 2015). Thus, these activities may be suitable for learning about mathematical proof and developing mathematical proof skills. Still, research on the relation between these activities, for example examining if proof validation is beneficial for proof construction, is scarce (Powers, Craviotto, & Grassl, 2010) and robust empirical data is still missing.

To address this lack of data, we conducted an experimental study with  $N = 93$  undergraduate mathematics students aiming to assess the benefits of working on proof validation tasks for students' success in proof construction tasks and vice versa. Students were randomly assigned to two conditions, one requiring students to work on proof validation tasks and subsequently on proof construction tasks, one with the reversed sequence of both activities. For this, pairs of tasks were created based on common proofs from undergraduate real analysis and linear algebra. For each of these tasks three versions were created: a proof construction task as well as a correct and an incorrect proof for according proof validation tasks. The versions of the tasks were randomized, so that one partner task was included in the construction and one in the validation scale. We expected students of both conditions to benefit from working on the first partner task, when working on the subsequent partner task. Several possible reasons for the expected short-term effects can be given, such as the transfer of main solution ideas or of certain aspects of the according problem-solving spaces.

However, first results indicate that students were not able to benefit from working on the proof validation or proof construction tasks for the subsequent tasks. Accordingly, short term transfer effects between the partner tasks appear to be minimal, indicating that prior results (Powers et al., 2010) are consequence of other, more robust long-term effects and are likely dependent on the instructional implementation of the tasks and possibly the shared discussion of the proofs.

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# SENSUOUS SYNTAX: COLORING CONJECTURES WITH SOUND, SILENCE, GESTURE AND GRAMMAR

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Multimodal discourse analysis in mathematics education has shown that gesture, sounds created through gesture, prosody, and rhythm can be coordinated to express mathematical understanding (Bautista & Roth, 2012; Radford, 2014). Though this *sensuous cognition*, “all our relations to the world (hearing, perceiving, smelling, sensing, etc.), are an entanglement of our body and material and ideational culture” (Radford, 2014, p. 350). This paper suggests that the syntax that students use in sentences expressing mathematical conjectures is also sensuous. While sentence syntax may seem like the abstract, mental frame that is used to describe language, it is as immanent and alive as gesture, recognizable to speakers through rhythms and patterns of sound. Furthermore, when students offer conjectures over several sentences, they use repeated patterns of syntax to assert, adjust, hedge and nuance their mathematical ideas (Staats, 2007, 2017).

This oral communication presents close transcription of two early undergraduate students solving an algebra problem on the perimeter of a string of  $n$  hexagons. One student noticed that when an additional hexagon is appended to a string, two interior sides must be removed from the perimeter. The student expressed this property several times to his doubtful partner, using elaborate layers of repeated syntax, pause, intonation changes, gesture, and writing. The key idea is that repetitions of words and syntax highlight key mathematical ideas, but the cohesive, logical relationship among these ideas is amplified through multimodal means, in particular, through prosody, silence, gesture and writing. The scenario suggests that sensuous syntax—the multi-sensory analysis of syntactic patterns across multiple sentences—helps us notice more fully the subtle tools that students use to convey their mathematical thinking.

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# SUBJECT MATTER ANALYSIS: THE CASE OF CRYPTOGRAPHY WITH ELLIPTIC CURVES

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Mathematics education has roots in psychology, empirical education research, and (abstract) mathematics (Hußmann et al., 2016); regarding the latter, it deals with examples, and applications (Freudenthal, 1980). These aspects stand in relationship to and cannot be divided from each other.

A main part of lessons in mathematics is to extend students' topical knowledge and understanding of mathematical backgrounds. Likewise important for lessons in mathematics is to structure the students' knowledge and the guidance of students through lessons of mathematics by teachers. In the German tradition, the aspects of topical knowledge and didactics of mathematics are combined to *Stoffdidaktik* or *subject matter analysis* (e.g. Hußmann et al., 2016; Wittmann, 2014).

In recent years, the catalogue of topical knowledge to be taught in mathematics lessons has been thinned out; however, some topics could be back on the agenda in the coming years (cf. Wittmann, 2014). This begs the question how to develop didactically prepared learning environments for mathematical contents in the sense of *subject matter analysis* for secondary school both from students' and teachers' perspective.

The organization of such a development will be described by reference to courses with students at upper secondary schools and teachers in advanced training and research. We present the development and the results of mathematical content, prepared in the sense of *subject matter analysis*, for upper secondary students: cryptography under the application of elliptic curves. We show how students can discover the abstract mathematical topic of algebraic structures on curves by starting from the well-known concept of functions. During the self-regulated examination, the students detect difficulties in cryptography with elliptic curves and explore sophisticated new sections of mathematics. The presented approach might be assigned to other topics of secondary or high school mathematics education.

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# THE DANGER OF PICKING THE WRONG DRESS

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A possible classification splits math problems into three classes: dressed-up word problems are one of the three types besides modelling problems and intra-mathematical problems (Blum & Niss, 2007). We can distinguish them by their strength of connection to reality. Contexts have several benefits (Van den Heuvel-Panhuizen, 2005). We call a word problem a dressed-up problem if for a purely mathematical problem a real life imitating context is created to make it a word problem. By solving modelling problems we create connection between real and mathematical world. In (Schukajlow et al., 2012) an idealized process of solution for a modelling problem is presented.

In this study we would like to point out how dangerous is it to dress up mathematical problems. We go back to the principle of De Lange (1995). The problem designer is not only dressing up the problem, but he is the solution designer, as well. We show three examples where the intended solution does not solve the problem, because the dressing changes the context and changes the problem itself. This happens especially when we want to give the context a modelling flavour. We demonstrate these hazards with a problem from analysis, combinatorics and geometry selected from Hungarian high school textbooks.

Our conclusion is that when dressing up a problem, then the validation process of the modelling cycle might fail. Hence, for dressed-up problems a more thorough reviewing process should be required.

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# EFFICIENCY OF TEST-ENHANCED LEARNING IN TEACHING ELEMENTARY GEOMETRY

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Retrieving information from memory after an initial learning phase enhances long-term retention more than restudying the material; an advantage referred to as the retrieval effect (for reviews, see Delaney et al 2012; Karpicke et al., 2014). Test-enhanced learning is a method which uses active recall and retrieval of the information during the learning process. The efficiency of the retrieval effect has been demonstrated with a variety of practice tests, materials, and age groups. These experiments were mostly carried out concerning learning texts or foreign words under laboratory circumstances.

The topic of this presentation is an experiment on the efficiency of test-enhanced learning used for teaching geometry in a regular maths class. Subjects of our experiments were a study group of grade 9 in an underprivileged vocational school. The control groups were one other study group from the same school and two study groups of grade 9 in an elite secondary school, ranked in top 10 in Hungary, learning the same topic. The experimental group wrote a 5-minute-test after each class on the topic learnt that day. The control groups learned the same concepts in the traditional way. The curriculum allowed 10 classes for this material in the vocational school and 17 classes in the elite school. All four groups wrote the same final test when they finished the topics. The results of the students learning using the testing effect were high above the results of the control group in the same school and they achieved similar results as the control groups in the elite high school. This case study is a good indication that the retrieval effect might be helpful in mathematical environment as well.

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# POSITIONING OF MATHEMATICS WITHIN INTEGRATED STEM EDUCATION: THE VIEWPOINT OF LEARNING MATHEMATICS

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The aim of this study is to clarify mathematical activities necessary in learning activities in integrated STEM education from the viewpoint of learning mathematics. Integrated STEM education can nurture the required qualities and abilities to build a sustainable society (e.g. 21st century skills) through making use of knowledge and skills of science, technology, engineering and mathematics in a realistic context integrally. However, each discipline in STEM education is not equal (English, 2016).

I will summarize the role of mathematics in integrated STEM education and point out problems from the viewpoint of learning mathematics. In this study, learning mathematics does not mean only applying knowledge and skills, but also acquiring new theories and ideas related to mathematics.

According to the conceptual framework for integrated STEM education (Kelly & Knowles, 2016), each discipline is expressed as scientific inquiry, technical literacy, engineering design, mathematical thinking. That is, the role of mathematics is interpreted as a thinking tool for doing scientific and engineering practice. Actually, many teaching materials related to STEM education are expected to apply mathematics as a thought tool for problem solving. However, in mathematics learning, the application of mathematics is not the end by applying mathematics, but by looking back on what mathematical knowledge was used and how mathematical knowledge it also leads to learning mathematics such as organizing and integrating (Ikeda, 2017). From this point of view, learning activities in integrated STEM education are strongly focused on applying mathematics and there is little assumption of activities to look back on applied knowledge. It is important to review knowledge and skills, to organize and integrate mathematical knowledge. Therefore, not only applying mathematics, but also looking back on the applied knowledge should be placed within integrated STEM education.

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# HOW WOULD STUDENTS DECIDE IF A LINE IS STRAIGHT?

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Many geometry curricula spend much time on technical issues such as computation of angles and areas as well as proof and proving, rather than on developing students' understanding of fundamental concepts. For example, when students learn how to draw lines using rulers and construct figures using straightedge and compass, the concept of straightness is seldom explained. The concept is introduced operationally and the students are assumed to grasp what it means automatically.

In order to help students learn basic concepts meaningfully, we have developed a hands-on plane geometry curriculum that covered most traditional topics using paper folding. The material is organized surrounding Huzita's axioms of paper folding (Huzita, 1989). The purpose of this study is to explore the prior concepts of students on straightness and how they learn the concept through activities.

## Methodology

A teaching experiment was conducted to test out the new curriculum in a ten-day twenty-hour course organized during the summer of 2017. Nine eighth graders from a rural junior high school in Taiwan were invited to participate. A pretest was administered on the first day of class and the concept of straightness was covered on the fifth day. The medium of instruction was inquiry oriented. The class was taught by a teacher with two participant observers and the lessons were videotaped.

## Results

The pretest revealed that all students were unclear about the concept of straightness. They provided tautological answers or explaining a straight line as contrary to a curved one. We provided two lines, one straight and the other not, for students to determine if they were straight. All participants were engaged in exploring how to use paper folding as a reasoning tool. All eventually found at least one way to make the decision.

## Conclusion and discussion

Basic geometrical concepts, no matter how intuitive, need to be taught meaningfully. But instruction needs not reduce to demonstration. As seen from this study, students can discover for themselves through paper folding under minor guidance.

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# PRIMARY SCHOOL TEACHERS' ENJOYMENT (OR NOT!) OF MATHEMATICS AND ITS TEACHING

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Teachers' attitudes and beliefs about Mathematics influence significantly their teaching of the subject (Hannula, Kaasila, Laine, and Pehkonen, 2005). Accordingly, teachers with positive past mathematics experiences develop positive feelings about the subject and they are able to teach it effectively. On the contrary, negative experiences and attitudes usually lead to ineffective instructional methods.

We attempted to fill a gap in the aforementioned literature by focusing on the community of Greek primary school teachers. The aim of our research was to investigate their beliefs and attitudes towards mathematics, the quality of their previous learning experiences, the quality of their teaching and whether there are any links amongst these elements. Ten primary school teachers, five males and five females, were chosen to take part in the study. The 'structured interview', consisting of open-ended questions, was used as the main research tool whereas the 'open-ended classroom observation' provided supplementary information. The resulting data were analysed by adopting a 'grounded theory' methodology.

Surprisingly, our findings are not in line with the relevant literature. In general, the teachers with positive school experiences had positive attitudes towards Mathematics as well. However, not all of them seemed to enjoy teaching the subject. Some of them taught it in a very traditional behaviourist way, unable (or unwilling) to share their knowledge and love for mathematics and to provoke their pupils' motivation.

In conclusion, we want to stress, that enjoyment of mathematics does not necessarily mean enjoyment of its teaching. This is an unexpected but alarming issue. Hence, it becomes imperative to research further the causes of such a discrepancy.

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# GROUNDING THE BIG IDEA OF FACTORING QUADRATIC TRINOMIALS WITHIN A LEARNING ACTIVITY

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As we all know that mathematics concepts and skills would be grasped by learners' active participation and reflect on their actions when they engage in learning activities. Therefore, how to develop a meaningful learning activity in mathematics classroom which can motivate students' interest and utilize mathematical thinking to solve problems is a significant issue for mathematics education community. In this study, our purpose aims to develop a learning activity which can provide students opportunities to get insight the big ideas of factoring quadratic trinomials through perceptual manipulatives. Base on Bruner's three modes of representation (Bruner, 1977), we designed a learning activity from real-life situation, which transformed abstract mathematics concept (e.g. factoring quadratic trinomial) into the relationship between the length and the area of the assembled rectangle. Students were guided to use enactive, iconic, and symbolic representations respectively to express their thinking and results in the learning activity. In order to find out effects of the learning activity and revise its weakness, the design research was adopted and the data were collected preliminarily through classroom observation. The main results reveal that the big idea and skills of factoring quadratic trinomials were embodied and abstracted by students themselves without teachers direct demonstrate the mathematical concepts and skills during students engaged in manipulating in the learning activity. In other words, students can master the technique to derive two integer numbers from its sum and multiplication by assembling rectangles in the learning activity. Furthermore, students can acquire deep understanding of mathematics concepts by exploring within multiple representations (enactive, iconic, and symbolic). Since information technology have the possibilities providing multiple representations and embodied mathematics concepts by virtual manipulatives (Ainsworth, Bibby, & Wood, 1997). It would be worth to investigate how student perform and their cognitive load in a computer micro-world environment, especially with the function of dynamic linking and direct acting on mathematics objects in touchscreen.

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# INVESTIGATING THE MATHEMATICS TEACHING EFFICACY BELIEFS OF STUDENT TEACHERS

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Belief in one's ability to enact change is pivotal to motivation, and thus behaviour (Bandura, 1993). Mathematics teaching efficacy beliefs encompass the extent to which a teacher believes that his/her teaching is capable of bringing about change in the mathematical understanding of his/her pupils, and the extent to which teaching in general supports the learning of mathematics for all children. Bandura (1993) posits that teachers with a low sense of efficacy are less inclined to support children in responding constructively to challenge and may thus undermine children's sense of self efficacy in relation to mathematics. This research study seeks to explore the Mathematics Teaching Efficacy Beliefs of student teachers as part of a longitudinal design-research study, whereby research findings will guide planning for future modules. The Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) of Enochs, Smith and Huinker (2000) was employed to measure the overall efficacy beliefs of teachers, and also the beliefs within two subscales, the Personal Teacher Efficacy, and Teaching Outcome Expectancy. A convenience sample of 40 undergraduate students participated in a questionnaire based upon the MTEBI with additional questions relating specifically to the content of a Mathematics Education module recently completed by the students. The sample was drawn from a year group of 440 students by invitation. Many responses to statements of the MTEBI reflected inconsistencies in students' self-efficacy beliefs, with 24 students declaring that they will not teach mathematics as well as other subjects, even if striving to do so (a further 11 were uncertain). However, only 6 students believed that they would not be capable of supporting a child who was struggling to understand. In relation to the efficacy of teaching in general, 34 students agreed that increases in pupils' mathematics achievement are due to the effectiveness of the teacher. In contrast, 25 were uncertain or did not agree that pupil underachievement reflected ineffective teaching, possibly echoing Bandura's contention that teachers with low efficacy beliefs are less likely to foster constructive responses to challenge. More detailed findings will be presented in the poster, including findings from the next phase wherein a second sample of 18 students participated in focus group interviews.

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# STUDENTS' REASONING IN THE CLASSROOM: AN APPROACH FOR ANALYSIS

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Lithner's (2008) reasoning framework differentiates students' reasoning based on whether or not it is novel to the students, and whether it contains arguments that are grounded in intrinsic mathematics. Experimental studies applying that framework have shown that the type of reasoning students use during practice will influence their performance on later tests (e.g., Jonsson, Norqvist, Liljekvist, & Lithner, 2014).

Building on this important finding, we argue that the research field would benefit from a better understanding of how reasoning is related to making progress when solving mathematical tasks. This study addresses this challenge by means of developing an approach to study how reasoning is related to making progress when solving mathematical tasks. Additionally, whereas the experimental studies were carried out in a laboratory setting as set out by researchers, the current study was carried out in students' natural habitat, the classrooms, and the tasks students were working with were selected by their teacher.

We built on the same reasoning framework as in the abovementioned studies (Lithner, 2008) to characterize students' reasoning as well as the tasks' reasoning potential.

The reported approach is based on analysis of four transcribed group conversations of upper-secondary school students when solving tasks, pictures of their notes, and clarifying post-interviews with the students. The approach consists of the following six steps: 1) analysis of crucial mathematical aspects of the task that the students have to solve, 2) break down students' solution into subtasks, 3) characterize reasoning potential of subtasks, 4) characterize students' reasoning when solving subtasks, 5) determine progress of subtask compared to previous subtask, 6) plot in a visual representation. Applying this approach helped to form first hypotheses as to the relationship between students' reasoning and making progress when solving mathematical tasks.

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# VALIDATING A METHODOLOGICAL INSTRUMENT TO FOSTER CREATIVITY IN SOLVING OPEN-ENDED TASKS IN MATHEMATICS TEXTBOOKS

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The objective of this paper is to discuss a methodological instrument to foster creativity when (re)designing open-ended tasks for 11-12 year old students in an elementary mathematics textbook. Previous literature suggest that *creativity* is associated with mathematical activity such pattern recognition, mathematical connections, conjecturing, etc. (Sánchez & Fiol, 2016). Many teachers seek to foster students' creativity as a way for them to identify alternative strategies for problem solving to develop their mathematical abilities. Wertheimer (1959) found that *productive thinking* based on re(structuration) of mathematical objects, and their representations, led students to better understand propositions, definitions, and procedures. More recently Ward, Finke and Smith (2013) claimed that students must be confronted towards challenging situations to help them to restructure their cognitive schemes when finding unexpected answers, because those answers have the potential to create contexts for meaningful understanding. In a recent work we analysed *creative mathematical thinking* (CMT) as a set of different dimensions (Sala, Barquero, Font & Giménez, 2017), including: (a) openness, generalization and versatility, (b) problematization and inquiry, (c) combining representations, (d) exploring and conjecturing, (e) connectness, (f) validation and (g) emotion. In this paper we analyse CTM drawing on data from open-ended tasks solved by 11-12 years old students. We use the *didactical suitability* instrument created by Godino and colleagues to discuss how CMT is fostered through the textbook tasks. We conclude that using this tool may support teachers to re(design) open-ended tasks based on a consistent and detailed analysis, validated by researchers.

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# MATHEMATICS IN THE SWEDISH FRITIDSHEM CURRICULUM: A POLICY ENACTMENT PERSPECTIVE

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In 2016, the *fritidshem* in Sweden obtained a new curriculum, where mathematics has been explicitly introduced. Fritidshem—literally “freetime home” or afterschool—has been an offer for children in school age at the end of the school day, mainly based on play and socialization. The curriculum now demands a clear focus in different subject areas, such as mathematics, to complement school. This change poses a big challenge for the people involved in fritidshem because the idea of learning in activity by playing that dominates fritidshem stands in contrast with the strong logic and tradition of school mathematics. The tensions between these two logics will impact the meanings that *fritidshem mathematics* will get in practice.

This problem is studied as an issue of *policy enactment* (Ball et al., 2012), or the active process where people appropriate and reconfigure the meaning of policies in the context of their institutional practices. Two research questions are explored: 1) which tensions do practitioners experience when enacting the policy? and 2) which meanings of fritidshem mathematics as activity emerge in practice? To analyze the policy enactment process in relation to mathematical activities (Bishop, 1988) two case studies in two fritidshem institutions were conducted, during the period of 2016-2017. Observations and interviews with practitioners were included.

The findings emphasize that the practitioners value the specific role of fritidshem to foreground and prioritize the beauty of an activity-based mathematics offered through play and games. The tensions that the practitioners experience are rooted in a discourse that positions them as unimportant actors in the process of policy enactment and practitioners in the educational system. Bishop’s six mathematical activities helped to identify the mathematics that emerged in different contexts and discourses in the practice of fritidshem. Mathematical interactions were evident in spontaneous and playful activities, derived from the students’ interest. We consider that further research projects in mathematics education are important for framing the role of mathematical activities arising from the student’s interest in the practice of fritidshem.

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# CULTIVATING MATHEMATICS PRACTICE IN A COMPUTATIONAL THINKING INFUSED ROBOTIC ACTIVITY

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The emerging field of Science, Technology, Engineering and Mathematics (STEM) education provides students a fascinating and unique learning opportunity. Yet, the role of mathematics and how it might facilitate student STEM learning and thinking in this unique learning opportunity are rarely made explicit. In our study, we investigate ways in which students making sense of quantities and their relationships, applying the mathematics they know in problem situations, and using mathematical language to communicate with each other precisely in computational thinking infused STEM inquiries. In this presentation, we share findings on students' mathematical practice while participating in robotic activities.

The robotic activities are designed to help students learn about programming using Lego Mindstorms EV3 and to apply engineering design process into a specific context – designing a Lego robot that detects water in a simulated Mars model. In order to successfully program the robot to complete the task, students need to first determine the measurements of how a robot moves in terms of directions, distances and angles of turning and detects objects in a Mars simulated model, and then convert these measurements into a set of data needed for programming. Eighteen students from grades 4<sup>th</sup> - 6<sup>th</sup> participated in eight-week robotic activities, and worked in small groups of 2 or 3 students in each group. We video recorded students' teamwork to analyse their interactions and ways they communicate their mathematical and computational thinking with each other. Sfard's (2008) discursive framework served as a tool to analyse students' mathematical discourse. In this presentation, we share findings of Kyle and Dan's (pseudonyms) mathematical discourse on how they visualize the paths in which the robot moves, their course of actions of finding and comparing the measurements on distances and of determining the angles of turning. The discourse analysis on students' mathematical thinking and practice shows the similarities and differences in Kyle and Dan's approaches to solve problem and shed a light on their understanding of mathematics concepts from their used of words such as distance, angle, turn, etc. In our analysis, we have found that there are extensive evidences of mathematics contents and practices applied in these robotic activities. Kyle and Dan's mathematics practices in this robotic activity provide an example of how mathematics serves as a tool for students in their STEM learning.

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# ACTIVITY FOR SPATIAL REASONING DEVELOPMENT OF ELEMENTARY SCHOOL CHILDREN

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The aim of this study is to propose geometry activities to develop the spatial reasoning of young children (eight years old). Three geometry activities focus on improving children's understanding of how a plane moves or stands vertically, or how two or more surfaces stand while rotating in 3-D. Through the second and third activities, children were encouraged to fold, bend and rotate faces with the help of one of their classmates. As a result, they found out about six different shapes of 2-D flat patterns. The eight-year old children were able to visualize and physically fold, rotate, and transform the planes. This experience of physically manipulating the planes will have a positive influence on the spatial manipulation in their ability to visualize the spatial manipulation (dynamic spatial reasoning).

## THREE ACTIVITIES DEVELOPING THE SPATIAL REASONING

In the first activity, the children learn how to construct a 3-D solid by connecting 2-D faces. In this activity, the central focus is on constructing and transformations from 2-D flat patterns to 3-D solid. In the second activity, the children find out how planes move by cutting a solid along the sides. In this activity, the central focus is on bending, rotating and transformations from 3-D solid to 2-D flat patterns. In the third activity, the children discover different shapes of 2-D flat patterns while attaching or detaching faces. In this activity, the central focus is on folding, bending, rotating and transforming between 2-D flat patterns and 3-D cube.

## THE RESULTS

Before participating in these activities, the children only knew the two kind of 2-D flat patterns. As a result, they observe various movements of the faces and found out six different shapes of 2-D flat patterns. The eight-year old children were able to visualize and physically fold, rotate, and transform the planes. This experience of physically manipulating the planes will have a positive influence on their spatial reasoning.

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# DEVELOPMENT OF A MEASUREMENT ESTIMATION TEST FOR LENGTH, AREA, AND VOLUME

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Estimation tests were developed to investigate estimation strategies in mathematics education (Siegel et al., 1982) and frontal lobe functions in psychology (Peretti Wagner et al., 2011). Most estimation tests do not include any deliberations about the development of their tasks. It seems problematic that numerical estimation, measurement estimation, and computational estimation were mixed up as well as all kinds of measures. Therefore, there is a need for a carefully developed estimation test which is based on mathematics educational and psychological knowledge and which contains a stable structure of items.

This research project is focused on measurement estimation of length, area, and volume. Although estimation strategies are well explored, only several basic approaches of structuring estimation tasks as such exist. Bright (1976) distinguishes eight kinds of (measurement) estimation, which were based on the variation of the pre- or absence of object, unit, and possible answers.

Aim of this project is the development of a well-structured task framework. This will include both the essential comprehension of *what* an estimation test is and the fundamental knowledge of *how* estimates are done. The poster shows 84 item categories, which differentiate deeper than Bright's distinctions.

Some categories could be excluded by theoretical reasons, others by practical reasons while constructing parallel test versions. For example, a touchable object and unit allows concrete measurement (which should be avoided). The poster will show all judgements about the categories in detail and visualize them by examples.

During the presentation, the choice of item categories for the parallel test versions could be discussed. Furthermore, results of the pilot-study planned in May 2018 could create hypotheses of the correlation between length, area, and volume.

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# PLAYING A COOPERATIVE GEOMETRY BOARD GAME: WHO BENEFIT? WHEN AND HOW?

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While learning math can be frustrating for many students, playing games, in contrast, is perceived as fun and entertaining. Games are often used as a strategy to motivate children or as a reward. Surprisingly, combining education and games is often regarded as a “dear killer” (Treher, 2011). Parents thought games are not learning; students thought educational games generally don’t have “game-feeling”. Obviously, we need evidence of learning from games and educational games need to take game elements into account. Asgari and Kaufman (2009) identified nine such elements for reference.

The purpose of this study was to design a cooperative geometric board game, focusing on geometric properties of polygons (parallelogram, triangle and trapezoid), and examined whether it facilitated learning and whether there were ability differences in the effects. In addition, it examined whether different ways of mixing the geometric shapes would affect how students benefit from it. Two types of mixing the geometric shapes were examined: single shape (SS) and multiple shapes (MS). A total of 236 year six students participated in this study (129 in the game groups and 107 in the control group). The students in the game groups were divided into high and low achievement groups based on their midterm exam and then randomly assigned to either the SS or the MS group. All students took a pre-test and a four-week post-test. The game groups played the game in a small group of four (two high and two low achievement) during the three weeks between the two tests. Four different games were played using the same board game. Gain scores were computed by subtracting the pre-test score from the post-test score. The results showed that the two game groups were superior to the control group in gain scores, suggesting the game effect, and no difference between the two game groups, suggesting no game type effect. In addition, the results show that ability difference was observed only in difficult test questions, not easy and moderate questions. Furthermore, this study also found that the students’ most favourite game was also the one ranked the most difficult one. How to motivate students to challenge difficult tasks lied in the design of the game mechanics.

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# THE EFFECT OF EXPERTISE LEVEL IN GEOGEBRA ON TECHNOLOGICAL AND PEDAGOGICAL AND CONTENT KNOWLEDGE AMONG IRANIAN SECONDARY MATHEMATICS TEACHERS

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Quality of teachers undeniably is a key factor to increase educational standards in the subjects that they have growing rate of failure such as mathematics. Mathematics teachers are currently facing a lot of challenges on how to teach and similarly students are addressing issues on how to learn (Fennell, 2018). Teachers are the ultimate key to improve schools and educational change. However, high quality of teaching needs high quality of teacher education program. Great number of teachers and students are ignoring technological domain and paying attention to content and pedagogical domain (Zeynivandnezhad & Bates, 2017). This study focuses on the investigation of relationships between demographic information and TPACK's components. Survey based on Alshehri (2012) with 162 secondary mathematics teachers had been conducted. Multivariate analysis of variance (MANOVA) was used to compare the TPACK's components according to demographic information such as age, gender, years of experience and level of expertise in GeoGebra. Findings showed that there were no significant differences between participants in terms of age, gender, and years of experiences. However, there was a significant difference of TPACK's components among participants excepting CK, PK and PCK components according to level of expertise in GeoGebra. These findings highlight the value of technological knowledge to enhance integrating of digital tools in mathematics instruction. Accordingly, emphasis on technology knowledge along with pedagogical knowledge are put forwarded in teacher training.

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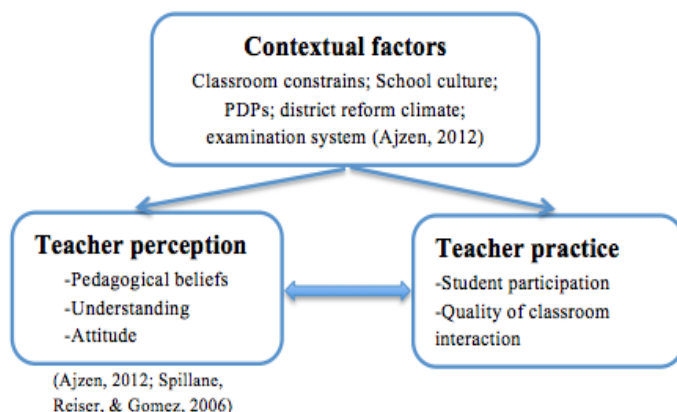
# TEACHERS' PERCEPTIONS AND ENACTMENT OF THE MATHEMATICAL CURRICULUM REFORM IN CHINA

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The current mathematical curriculum reform in China began in 2001 and is the 8<sup>th</sup> and the most fundamental one since the foundation of People's Republic of China. However, there are few empirical studies conducted to examine how teachers perceive and enact the reform initiatives. This study aims to fill this gap through a longitudinal case study, last from 2012-2018.

Literature reveals that reforms are not always implemented as expected, as the complex interaction between various individual and contextual factors. To capture these factors, the theory of planned behavior (Ajzen, 2012), which includes attitudes, social norms, perceived behavior control and the theory of sense-making (Spillane, Reiser, & Gomez, 2006), which shows the complexity of human's sense-making process are considered as the theoretical lens of this study. The theoretical framework is presented in the figure.



Data collected in the present study include 64 videotaped lessons, semi-structured interviews of 6 math teachers, as well as field notes. The results show both consistencies and inconsistencies in teachers' perceptions and practices. Furthermore, inconsistencies are mainly due to those teachers' limited understanding of the reform ideas and constraints of contextual factors

(e.g., school support, students, professional development programs, workload and examinations). In addition, it argues that the influence of contextual factors on teachers' enactment of the reforms is mediated by their beliefs, attitudes, and understandings, and thus varies between teachers.

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