PME 2025 48

SANTIAGO, CHILE

MAKING SURE THAT MATHEMATICS EDUCATION
RESEARCH REACHES THE CLASSROOM

PROCEEDINGS

of the 48th Conference of the International Group
for the Psychology of Mathematics Education

JULY 28 TO AUGUST 2, 2025
SANTIAGO, CHILE

EDITORS

Claudia Cornejo, Patricio Felmer, David M. Gómez, Pablo Dartnell,
Paulina Araya, Armando Peri, Valeria Randolph

GENERAL CONTRIBUTIONS

Plenary Lectures, Plenary Panel,
Research Forums, Working Groups,
Publication Reports, Oral Communications, Poster Presentations

Universidad de Chile

Universidad de O'Higgins

48th Conference of the International Group for the Psychology of Mathematics Education (PME)

July 28 to August 2, 2025

Santiago, Chile

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PREFACE

We are pleased to put in your hands the Proceedings of the 48th Annual Conference of the International Group for the Psychology of Mathematics Education. PME is one of the most important international conferences in mathematics education and brings together educators, researchers, and mathematicians from all over the world. This year, we had 367 participants attending the conference, coming from 42 different countries, presenting 327 contributions.

PME 48 was co-organized by Universidad de Chile (Santiago, Chile) and Universidad de O'Higgins (Rancagua, Chile), and it was hosted by the Faculty of Physical and Mathematical Sciences of Universidad de Chile. This event constituted an important milestone for the Latin American region and for Chile, since in the past the PME presence in Latin America had only reached Mexico and Brazil. An exception to this was the organization of the First PME Regional Conference in 2018, held in Rancagua and organized by Universidad de O'Higgins.

The theme of PME 48 was "Making sure that mathematics education research reaches the classroom." This theme reflected the challenges of translating mathematics education research into actionable recommendations for the mathematics classroom. We believe that, while this dialog is important all around the world, it is critical for Latin America, where resources and funding for mathematics education research and mathematics education in general are scarce. Nonetheless, we acknowledge that not every research project in mathematics education must be forcefully connected to the classroom. We hope that PME 48 was an opportunity for the community to engage with this theme and reflect on the research and practice connection in our field.

We had the honor of receiving plenary speakers and panelists from different countries, representing a diversity of educational contexts and theoretical perspectives, reflecting the diversity of the PME community. The PME 48 scientific program includes the traditional PME individual presentation formats (Research Reports, Oral Communications, and Poster Presentations) and group presentation formats (Research Forums and Working Groups). In addition, this year, a new individual presentation format has been introduced: Publication Reports.

Moreover, you might have noticed that, starting from PME 48, the structure of the PME Proceedings changed because Research Reports will now be indexed in Scopus. Accordingly, these Proceedings are structured into three tomes: one tome containing all the general contributions, and two tomes containing all Research Reports.

The conference was organized with the invaluable support and expertise of the Foundation for Mathematical Modeling (FMM), to whose team we are deeply grateful.

We also thank the support received from Universidad de Chile through the Faculty of Physical and Mathematical Sciences (FCFM), the Center for Mathematical Modeling (CMM), and the Center for Advanced Research in Education (CIAE); from Universidad de O'Higgins (UOH); from the Chilean Society of Mathematics Education (SOCHIEM); and from the Chilean Mathematics Society (SOMACHI). Finally, we acknowledge the support of two funding sources from the Chilean National Agency of Research and Development (ANID) through the BASAL fund for centers of excellence: FB210005 (CMM), ANID/PIA/ Funds FB0003/ Support 2024 AFB240004 (CIAE), and from the international collaboration fund ANID/FOVI 230206. We also received support from Universidad de O'Higgins institutional funds.

Our Local Organizing Committee worked hard with the objective of providing all our visitors with a memorable conference, from catering to excursions. As for the scientific program, we thank all of you for making wonderful and stimulating contributions, registered in these Proceedings.

Greetings from Santiago!

Patricio Felmer — Universidad de Chile

David Maximiliano Gómez — Universidad de O'Higgins

PME 48 co-chairs

THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION (PME)

HISTORY OF PME

The International Group for the Psychology of Mathematics Education (IGPME) is an autonomous body, governed as provided for in the constitution. It is an official subgroup of the International Commission for Mathematical Instruction (ICMI) and came into existence at the Third International Congress on Mathematics Education (ICME 3) held in Karlsruhe, Germany in 1976. Its former presidents have been:

Efraim Fischbein, Israel	Fou-Lai Lin, Taiwan
Stephen Lerman, UK	Nicolas Balacheff, France
Richard R. Skemp, UK	João Filipe Matos, Portugal
Gilah Leder, Australia	Kathleen Hart, UK
Gerard Vergnaud, France	Barbara Jaworski, UK
Rina Hershkowitz, Israel	Carolyn Kieran, Canada
Kevin F. Collis, Australia	Peter Liljedahl, Canada
Chris Breen, South Africa	Markku Hannula, Finland
Pearla Nesher, Israel	

The current President is Wim Van Dooren (Belgium). The President-Elect is Oh Nam Kwon (Korea).

THE CONSTITUTION OF PME

The constitution of PME was adopted by the Annual General Meeting (AGM) on August 17, 1980 and changed at the AGM on July 24, 1987, on August 10, 1992, on August 2, 1994, on July 18, 1997, on July 14, 2005, and on July 21, 2012. PME decided to seek charitable organization status under UK law and the new constitution related to this change was accepted by the Annual General Meetings in 2018 and 2019. The name of the Charitable Incorporated Organisation ("the CIO") is the international Group for the Psychology of Mathematics Education. The objects of the CIO are to advance the field of mathematics education for the public benefit by:

- Promoting and stimulating research.
- Organizing regular educational conferences around the world.
- Supporting regional workshops around the world in general, and in underrepresented regions of the world in particular.
- Collaborating with organizations with similar aims.

- Facilitating cross-disciplinary discussion and the sharing of information and research with an international emphasis.
- Helping scholars from different parts of the world establish collaborative networks to further our collective understanding of how to improve mathematics education in their respective countries.
- Providing grants to help bring scholars from under-represented regions of the world to our annual meeting.
- Disseminating our research for the benefit of improving mathematics education at the classroom, school, district, and national levels.
- Disseminating our research for the benefit of improving students' experiences with mathematics education at the classroom, school, district, and national levels.
- Disseminating our research for the benefit of improving the preparation of mathematics teachers at the university level around the world.
- Providing access to our research publications to the public.

All information concerning PME and its constitution can be found at the PME website: www.igpme.org

PME MEMBERSHIP AND OTHER INFORMATION

Membership is open to people involved in active research consistent with the aims of PME, or professionally interested in the results of such research. Membership is on an annual basis and depends on payment of the membership fees. PME has between 700 and 800 members from about 60 countries all over the world.

The main activity of PME is its yearly conference of about 5 days, during which members have the opportunity to communicate personally with each other during working groups, poster sessions and many other activities. Every year the conference is held in a different country.

There is limited financial assistance for attending conferences available through the Richard Skemp Memorial Support Fund.

A PME Newsletter can be found on the PME website. Occasionally PME issues a scientific publication, for example the result of research done in group activities.

WEBSITE OF PME

All information concerning PME, its constitution, and past conferences can be found at the PME website: www.igpme.org

PROCEEDINGS OF PREVIOUS PME CONFERENCES

Previous PME proceedings can be found in digital form at the PME website: www.igpme.org

HONORARY MEMBERS OF PME

Efraim Fischbein (Deceased)

Hans Freudenthal (Deceased)

Joop Van Dormolen (Retired)

PME ADMINISTRATIVE MANAGER

The administration of PME is coordinated by the Administrative Manager. Starting from August 2025, this role will be assumed by:

Dr. Peter Gonscherowski

email: info@igpme.org

BOARD OF TRUSTEES OF PME

Members of the Board of Trustees (BoT), formerly known as the PME International Committee, are elected for four years. Every year, four members retire and four new members are elected. The BoT is responsible for decisions concerning organizational and scientific aspects of PME. Decisions about topics of major importance are made at the Annual General Meeting (AGM) during the conference. The BoT work is led by the PME President, who is elected by PME members for three years. Before becoming President, the President-Elect works together with the past President and the Board of Trustees for one year.

President

Wim Van Dooren (Belgium)

President-Elect

Oh Nam Kwon (Korea)

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THE PME 48 CONFERENCE

Two committees are responsible for the organization and execution of the PME 48 Conference: The International Program Committee (IPC) and the Local Organizing Committee (LOC).

In addition, we thank all the reviewers and the IPC for the detailed review work that has led to the selection of the contents of these proceedings.

THE INTERNATIONAL PROGRAM COMMITTEE (IPC)

LOC representatives

David Maximiliano Gómez	Universidad de O'Higgins (Chile) PME 48 co-chair
Pablo Dartnell	Universidad de Chile (Chile)
María Victoria Martínez Videla	Universidad de O'Higgins (Chile)
Farzaneh Saadati	Universidad de Chile (Chile)

PME representatives

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Oh Nam Kwon	Seoul National University (Korea) PME President-Elect
Juan Manuel González Forte	Universidad de Alicante (Spain)
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Valeria Randolph	Universidad de Chile
Paola Ramírez	Universidad Católica del Maule
Eder Pinto	Universidad de O'Higgins
Cristián Reyes	Universidad de Chile
Felipe Leiva	-

REVIEW PROCESS OF PME 48

RESEARCH REPORTS (RR)

Research Reports are intended to present empirical or theoretical research results on a topic that relates to the major goals of PME. Research Reports should state what is new in the research, how it builds on past research, and/or how it has developed new directions and pathways. Some level of critique must exist in all papers.

The number of submitted RR proposals was 214, and 117 of them were accepted. Of those not accepted as Research Reports, 61 were invited to be resubmitted as Oral Communications (OC) and 35 as Poster Presentations (PP). After acceptance, 7 Research Reports were withdrawn because of inability to participate in the conference. Altogether, 110 Research Reports are expected to be presented at PME 48.

As in previous years, every RR submission underwent a fully independent double-blind peer review by two to four international experts in the field. The reviewers' reports served as input for the PME 48 International Program Committee to decide acceptance for the conference. Accepted Research Reports are indexed in Scopus: https://www.scopus.com/sourceid/21101194210

PUBLICATION REPORTS (PR)

Publication Reports are intended to present already submitted, accepted, or recently published empirical or theoretical research results on a topic that relates to the major goals of PME. Publication Reports should state what is new in the research, how it builds on past research, and/or how it has developed new directions and pathways. Some level of critique must exist in all submissions and authors are encouraged to also go beyond what is already published wherever possible – so that the presentation is also of interest to those that are already aware of the published work.

The number of submitted PR proposals was 8, and 4 of them were accepted. All of them are expected to be presented at PME 48.

Similarly to RR proposals, every PR submission underwent a fully independent double-blind peer review by two to four international experts in the field. The reviewers' reports served as input for the PME 48 International Program Committee to decide acceptance for the conference. Accepted Publication Reports' abstracts are included in the conference proceedings.

ORAL COMMUNICATIONS (OC)

Oral Communications are intended to present smaller studies and research that is best communicated by means of a shorter oral presentation instead of a full Research

Report. They should present empirical or theoretical research studies on a topic that relates to the major goals of PME. Reports should describe on which past research it builds, give a concise overview of the new results attained, and what the main implications of these results are.

The number of submitted OC proposals was 140, and 101 of them were accepted. Of those not accepted as OC proposals, 33 were invited to be resubmitted as Poster Presentations (PP). Including resubmissions of Research Reports as Oral Communications, 142 OCs were accepted. After acceptance, 8 Oral Communications were withdrawn because of inability to participate in the conference. Altogether, 134 Oral Communications are expected to be presented at PME 48.

POSTER PRESENTATIONS (PP)

Poster Presentations are intended for information/research that is best communicated in a visual form rather than an oral paper presentation. There is no formal oral presentation associated with Poster Presentations, although there can be a time allotted, after sufficient display time, during which presenters will be available by their posters for discussion with conference participants. Posters should present empirical or theoretical research studies on a topic that relates to the major goals of PME. Reports should describe the research context of the study presented, the study itself, and its implications, with a particular focus on the visual aspects of the presentation.

The number of submitted PP proposals was 44, and 39 of them were accepted. In the end, considering resubmissions of Research Reports and Oral Communications as Poster Presentations, 71 PPs were accepted. After acceptance, 4 Poster Presentations were withdrawn because of inability to participate in the conference. Altogether, 67 Poster Presentations are expected to be presented at PME 48.

COLLOQUIA (CQ)

The goal of a Colloquium is to provide the opportunity to present a set of three Research Reports that are interrelated in a particular way (e.g. they depart from related or contrasting theoretical stances, use identical instruments or methods, or focus on closely related research questions), and to initiate a discussion with the audience on the interrelated set.

There were no CQ proposals submitted to PME 48.

RESEARCH FORUMS (RF)

A Research Forum is a format of group presentation undertaken by a group of at least five researchers. The goal of a Research Forum is to provide PME members with a comprehensive overview of the state of the art on a topic where substantial research

has been undertaken in the last 5-10 years and that is of ongoing interest for the PME members. The researchers are expected to be experts with respect to the research topic and offer coherently structured activities during 2 slots of 90 minutes each, as well as a comprehensive contribution (30 pages) to the conference proceedings. A Research Forum is not supposed to be a collection of individual research presentations (see Colloquium format), but instead is meant to convey an overview of research, highlighting contemporary academic debates and perspectives in the field. In contrast to the Working Group format for evolving research topics, Research Forums are considered to mark significant advances with respect to established research topics.

The number of submitted RF proposals was 2, and both of them were accepted.

WORKING GROUPS (WG)

A Working Group is a format of group presentation undertaken by a group of at least two researchers. The aim of a Working Group is that PME participants are offered the opportunity to engage in exchange or to collaborate with respect to a common research topic (e.g., start a joint research activity, share research experiences, continue or engage in academic discourse). A Working Group may deal with emerging topics (in the sense of newly developing) as well as topics that are not new but possibly subject to changes. A Working Group needs a clear goal (outcome) and a strategy to reach this goal in collaboration with the participants. It must provide opportunities for contributions of the participants that are aligned with the goal (e.g., share materials, work collaboratively on texts, discuss well-specified questions). A Working Group is not supposed to be a collection of individual research presentations (see Colloquium format), but instead is meant to build a coherent opportunity to work on a common research topic. In contrast to the Research Forum format, meant to present the state of the art of established research topics, Working Groups are considered to involve fields where research topics are evolving.

The number of submitted WG proposals was 17, and 10 of them were accepted.

SEMINARS (SE)

The goal of a Seminar is the professional development of PME participants, especially new researchers and/or first comers, in different topics related to scientific PME activities. This encompasses, for example, aspects like research methods, academic writing or reviewing. A Seminar is not intended to be only a presentation but should involve the participants actively. PME can give a certificate of attendance to participants of the Seminar.

The number of submitted SE proposals was 2, and none of them were accepted.

LIST OF REVIEWERS FOR PME 48

In the process of reviewing Research Reports and Publication Reports, 223 researchers from 40 different countries and regions took part. The PME 48 International Program Committee is grateful to all of them for their support.

A

Akkoc, Hatice (Turkey)

Alatorre, Silvia (Mexico)

Alcock, Lara (United Kingdom)

Alsalim, Lyla Ali (Saudi Arabia)

Anderson, Judy (Australia)

Andrà, Chiara (Italy)

Antonini, Samuele (Italy)

Arcavi, Abraham (Israel)

Areti, Panaoura (Cyprus)

Armstrong, Alayne Cheryl (Canada)

Asghari, Amir Hossein (United Kingdom)

Ayalon, Michal (Israel)

B

Baccaglini-Frank, Anna (Italy)

Bakos, Sandy (Canada)

Ball, Lynda (Australia)

Barwell, Richard (Canada)

Beisiegel, Mary (United States of America)

Bempeni, Maria (Greece)

Bennison, Anne Bernadette (Australia)

Beswick, Kim (Australia)

Bikner-Ahsbahs, Angelika (Germany)

Bingolbali, Erhan (Turkey)

Biza, Irene (United Kingdom)

Boero, Paolo (Italy)

Bose, Arindam (India)

Botelho da Costa Mamede, Ema Paula (Portugal)

Bruder, Regina (Germany)

C

Cao, Yiming (China)

Carrapiço, Renata Carvalho (Portugal)

Chang, Yu Liang (Taiwan)

Changri, Narumon (Thailand)

Cheng, Ying-Hao (Taiwan)

Chernoff, Egan J. (Canada)

Chimoni, Maria (Cyprus)

Chorney, Sean (Canada)

Choy, Ban Heng (Singapore)

Coles, Alf (United Kingdom)

Cooper, Jason (Israel)

Csikos, Csaba (Hungary)

D

Da Ponte, Joao-Pedro (Portugal)
Damrau, Milena (Germany)
Di Martino, Pietro (Italy)
Diez-Palomar, Javier (Spain)
Ding, Rui (China)
Doerfler, Willi (Austria)
Downton, Ann Patricia (Australia)
Dreher, Anika (Germany)
Dreyfus, Tommy (Israel)
Dyrvold, Anneli (Sweden)

E

Edmonds-Wathen, Cris (Australia)
Edwards, Laurie (United States of America)
Eichler, Andreas (Germany)
Ekol, George L. (South Africa)
El Mouhayar, Rabih Raif (Lebanon)
Erens, Ralf (Germany)
Essien, Anthony (South Africa)
Evans, Tanya (New Zealand)
Even, Ruhama (Israel)

F

Fellus, Olga (Canada)
Fernandez, Ceneida (Spain)
Ferrari, Pier Luigi (Italy)
Font, Vicenç (Spain)
Francisco, John (United States of America)
Fritzlar, Torsten (Germany)

G

Gates, Peter (United Kingdom)
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Getenet, Seyum Tekeher (Australia)
Giberti, Chiara (Italy)
Gildehaus, Lara (Austria)
Glass, Barbara (United States of America)
Göller, Robin (Austria)
Gomez, David Maximiliano (Chile)
González-Forte, Juan Manuel (Spain)
Gooya, Zahra (Iran)
Graven, Mellony Holm (South Africa)
Gutierrez, Angel (Spain)

H

Halverscheid, Stefan (Germany)
Hanazono, Hayato (Japan)
Hannula, Markku (Finland)
Hattermann, Mathias (Germany)
Hegedus, Stephen (United States of America)
Heinze, Aiso (Germany)
Helliwell, Tracy (United Kingdom)
Helme, Rachel Elizabeth (United Kingdom)
Hernandez-Martinez, Paul (Australia)
Hershkowitz, Rina (Israel)
Heyd-Metzuyanim, Einat (Israel)
Hino, Keiko (Japan)

Hodkowski, Nicola (United States of America)

Horne, Marj (Australia)

Hosein, Anesa (United Kingdom)

Hunter, Jodie (New Zealand)

I

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Ingram, Jenni (United Kingdom)

Inprasitha, Maitree (Thailand)

Ivars, Pedro (Spain)

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Jiang, Chunlian (Macau SAR)

Johansson, Helena (Sweden)

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Kageyama, Kazuya (Japan)

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Khan, Steven K. (Canada)

Kieran, Carolyn (Canada)

Kim, Ok-Kyeong (United States of America)

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Komatsu, Kotaro (Japan)

Kortenkamp, Ulrich (Germany)

Kosiol, Timo (Germany)

Krainer, Konrad (Austria)

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Krawitz, Janina (Germany)

Kron, Stephanie (Germany)

Krummenauer, Jens Oliver (Germany)

Krzywacki, Heidi (Finland)

Kuntze, Sebastian (Germany)

Kuzu, Taha (Germany)

Kwon, Oh Nam (Korea)

L

Leikin, Roza (Israel)

Lerman, Stephen (United Kingdom)

Liljedahl, Peter (Canada)

Lin, Fou-Lai (Taiwan)

Lin, Yung-Chi (Taiwan)

Lindmeier, Anke Maria (Germany)

Llinares, Salvador (Spain)

M

Mamona-Downs, Joanna (Greece)

Matos, João Filipe (Portugal)

Mellone, Maria (Italy)

Mesa, Vilma (United States of America)

Mesiti, Carmel (Australia)

Metaxas, Nikolaos (Greece)

Miragliotta, Elisa (Italy)

Misailidou, Christina (Greece)

Mizoguchi, Tatsuya (Japan)

Moreno, Mar (Spain)

Moutsios-Rentzos, Andreas (Greece)

Mulligan, Joanne (Australia)

Murphy, Carol (Australia)

N

Nakawa, Nagisa (Japan)
Nama, Samaher Ahmad (Israel)
Nannini, Bernardo (Italy)
Nardi, Elena (United Kingdom)
Neria, Dorit (Israel)
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Novotna, Jarmila (Czech Republic)
Ntow, Forster (Ghana)

O

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Öçal, Mehmet Fatih (Turkey)
Oesterle, Susan Michele (Canada)
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Okumuş, Samet (Turkey)
Orellana, Claudia Andrea (Australia)

P

Pantziara, Marilena (Cyprus)
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Pierce, Robyn (Australia)
Pinto, Marcia (Brazil)
Pittalis, Marios (Cyprus)
Planas, Nuria (Spain)
Potari, Despina (Greece)
Prediger, Susanne (Germany)
Prodromou, Theodosia (Australia)
Psycharis, Giorgos (Greece)

R

Rasslan, Shaker[9] (Israel)
Reid, David A. (Norway)
Rivera, Ferdinand Dychitan (United States of America)
Roberts, Nicky (South Africa)
Robertson, Sally-Ann (South Africa)
Rubel, Laurie H. (Israel)

S

Saadati, Farzaneh (Chile)
Sabena, Cristina (Italy)
Safuanov, Ildar (Russia)
Sakonidis, Haralambos (Greece)
Sánchez, Alicia (Spain)
Sanchez-Matamoros, Gloria (Spain)
Scheiner, Thorsten (Australia)
Schindler, Maike (Germany)
Schukajlow-Wasjutinski, Stanislaw (Germany)
Schwarts, Gil (Israel)
Seah, Wee Tiong (Australia)
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Shimizu, Yoshinori (Japan)
Shinno, Yusuke (Japan)
Soendergaard, Bettina Dahl (Denmark)
Sommerhoff, Daniel (Germany)
Spagnolo, Camilla (Italy)
Spinillo, Alina Galvao (Brazil)
Staats, Susan Kimberley (United States of America)

Stewart, Sepideh (United States of America)

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Sumpter, Lovisa (Sweden)

Surith, Dhanya (United Arab Emirates)

Suurtamm, Chris (Canada)

Sztajn, Paola (United States of America)

T

Tabach, Michal (Israel)

Thomas, Michael O. J. (New Zealand)

Thornton, Stephen John (Australia)

Tirosh, Dina (Israel)

Tortora, Roberto (Greece)

Triantafillou, Chrissavgi (Greece)

Tso, Tai-Yih (Taiwan)

Tzekaki, Marianna (Greece)

U

Ufer, Stefan (Germany)

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Voica, Cristian (Romania)

Vos, Pauline (Norway)

W

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Wang, Ting-Ying (Taiwan)

Watanabe, Koji (Japan)

Weigand, Hans-Georg (Germany)

Williams, Gaye (Australia)

Wu, Lan-Ting (Taiwan)

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MAKING SURE THAT MATHEMATICS EDUCATION
RESEARCH REACHES THE CLASSROOM

PLENARY LECTURES

THE DESIGN AND IMPLEMENTATION OF INTEGRATED STEM STUDENT LEARNING EXPERIENCES: CHALLENGES FOR MATHEMATICS TEACHERS AND SCHOOL LEADERS

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Integrated science, technology, engineering and mathematics (STEM) curriculum has been promoted as a strategy to improve student engagement and increase STEM workforce participation. However, authentic integrated STEM learning requires the use of real-world scenarios where knowledge, skills and practices from the STEM subjects can be used to solve engaging and challenging problems. What challenges arise as teachers and/or researchers co-design integrated STEM learning experiences that foreground mathematics? What approaches have teachers used to design and implement integrated STEM curriculum that foreground mathematics? These questions are addressed through analyses of papers from a recent ICME-15 topic study group and from the reported experiences of teachers in schools that created new integrated STEM learning opportunities for their students. Results affirmed the many challenges associated with this work, particularly at the school and learner levels but also provided some examples of how challenges can be addressed.

INTRODUCTION

Designing an integrated curriculum requires purposefully connecting school subjects to provide students with a more engaging and holistic learning experience (Beane, 1995). Given the promotion of integrated STEM curriculum for school students, the design task for teachers is to connect knowledge, skills and practices from science, technology, engineering and mathematics, while maintaining the integrity of the separate subjects (English, 2016; Honey et al., 2014). Thus, teachers need knowledge of each of the subject areas and an understanding of how best to design integrated STEM problems that allow students opportunities to select and use their STEM knowledge as appropriate (Anderson & Tully, 2024; Reinholz et al., 2018). Although some argue integrated STEM curriculum involves solving problems that require the use of all four STEM subjects, others accept that the purposeful combination of knowledge and practices from at least two is sufficient (Roehrig et al., 2021a), but should one of the subjects be mathematics?

Considering the contribution of each of the STEM subjects to an integrated curriculum helps to highlight the importance of mathematics. Several scholars have argued mathematics is foundational to the other STEM subjects (e.g., Honey et al., 2014). Based on the etymological origins of the disciplines, Snyder (2018) proposed science and mathematics provide foundational knowledge, engineering provides a bridge between foundational knowledge and technological development, and technology

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provides the tools, materials and techniques to create artefacts. Further, Snyder suggested it is through technology and engineering that mathematics and science can be applied to real contexts. However, Lyons (2018) argued that the discipline of mathematics provides much more than tools or applications in the service of the other subjects as it is central and fundamental to an integrated STEM curriculum. This argument becomes more compelling when considering the mathematical competencies required to solve engineering problems – thinking, reasoning and modelling mathematically; posing and solving mathematical problems; representing mathematical entities; handling mathematical symbols and formalism; communicating in, with and about mathematics; and making use of aids and tools (Alpers et al., 2013). Given the importance of including mathematics in an integrated STEM curriculum, I use the term 'authentic' if mathematics is foregrounded in STEM problems.

In her plenary address at PME39 in Hobart (Australia), English (2015) raised some of the challenges and opportunities of a STEM agenda for mathematics education in schools. Her concerns that "mathematics is in danger of being overshadowed, in particular by science" (p. 1-1) were not ill-founded. Several studies have identified the predominance of science and engineering in many integrated STEM student learning opportunities (e.g., McLure et al., 2022). Other research has sought to find examples of rich mathematics learning experiences in integrated STEM curriculum (e.g., Baldinger et al., 2020). While few, they do exist. This became evident at the recent ICME-15 conference held in Sydney (Australia) in July 2024. As the co-chair of the 'mathematics and interdisciplinary education/STEM education' topic study group (TSG3.16), I learned how 71 researchers designed and implemented various aspects of an integrated STEM curriculum as they worked with practicing teachers, preservice teachers (PSTs), university students and school students. The inclusion of mathematics in the interdisciplinary approaches was considered necessary for the TSG3.16 papers to be accepted for presentation at the conference but this did not necessarily mean mathematics was foregrounded. However, the papers demonstrated a diversity of approaches to integrated STEM curriculum design and implementation; knowledge worthy of sharing with the mathematics education community.

In this paper, I draw on the literature, as well as the papers and presentations from ICME-15 and data from my own research with schools into integrated curriculum to highlight the challenges for mathematics teachers, mathematics leaders (particularly in secondary school contexts), and school leaders (particularly in primary school contexts) as they design and implement an integrated STEM curriculum. The research questions considered in this paper are:

> What approach have teachers, mathematics leaders and researchers used to design and implement integrated STEM learning experiences for students that foreground mathematics?
>
> What challenges arose as mathematics teachers and mathematics leaders designed and implemented integrated STEM curriculum in schools?

LITERATURE REVIEW

Challenges to Integrated STEM Curriculum Design and Implementation

In this paper, 'curriculum' refers to "the major organizing concept in considering how educational opportunities are provided to students and the factors that influence how students use these opportunities" (Mullis, 2017, p. 4). Mullis describes three aspects of curriculum – *the intended curriculum* (the written or prescribed curriculum), *the implemented curriculum* (the curriculum as teachers enact it in classrooms) and *the attained curriculum* (student learning outcomes). The focus in this paper is how teachers become curriculum designers and use the intended curriculum to develop a new integrated STEM curriculum for implementation in classrooms (Anderson & Tully, 2024). Hence the intended curriculum includes curriculum documents and mandates from education authorities while the implemented curriculum encompasses teaching programs, plans and practices.

Challenges to the design and implementation of an integrated STEM curriculum in schools can be categorised using an ecological framework (Goos et al., 2020). In their review of the literature into interventions aimed at addressing gender imbalance in STEM education, Goos et al. identified nested spheres of influence acting on each of four levels from the outer level of society, to school, family and the learner at the centre. Society includes communication of social and cultural norms, evident in mandated policies and practices frequently built on traditional views of schooling. School level interventions concern school leader and teacher implementation of the curriculum, assessment, the use of role models, and perceptions of, and attitudes towards STEM. Family influences involve parent/guardian beliefs, attitudes, knowledge and encouragement. Learners encompasses students' interest, engagement, motivation, enjoyment and self-perceptions, as well as negative stereotypes and STEM identities. In relation to the design and implementation of an integrated STEM curriculum, school teachers and leaders typically focus on school practices and approaches that meet the needs of students while also engaging parents and community, particularly if they are viewed as partners and potential resources in the design and delivery of curriculum (Watson et al., 2022).

Using the same framework for the design and implementation of an integrated STEM curriculum approach in schools, challenges can be identified at each of the levels of society, school, family and learner. At the level of society, policies related to siloed mandated curriculum with subject-specific high stakes assessments tend to drive curriculum implementation rather than enabling the development of a vision for an integrated curriculum (Falloon et al., 2020). At the level of the school, developing an integrated STEM curriculum offers challenges related to teachers' knowledge and understanding, school leadership, school organisation, and connecting to community. Teachers require the knowledge, capacity and self-efficacy to collaboratively create a new curriculum and suitable problems from separate, subject-specific curriculum documents (Falloon, et al., 2020). It appears from several studies that designing an

integrated STEM curriculum in schools is frequently led by teachers invested in creating richer learning opportunities in science and/or technology/engineering education (Anderson et al., 2023). Hence it is unsurprising that studies have reported science or engineering design are frequently the starting point of integrated curriculum design (McLure et al., 2022). While Falloon et al. (2020) advocated any STEM subject could be a starting point for integrated STEM curriculum design, this appears to be rarely the case for mathematics.

While teachers are aware of the challenges to designing an integrated STEM curriculum and report issues with pedagogy, school structures, student capability, resources, and assessment requirements (Margot & Kettler, 2019), they may be less aware of the impact of their beliefs. Wang et al. (2011) found secondary school teachers with different STEM subject matter expertise had distinctive perceptions about STEM integration that led to dissimilar emphases in their STEM lessons. In their study, the science teacher thought problem solving was the main purpose of STEM integration, the mathematics teacher thought it was to bring a real-world context to learning mathematics, and the engineering teacher thought it was for students to develop skills such as independence and team work. Although the three teachers in this study were from the same school and worked collaboratively to design STEM lessons, they each delivered the lessons separately and with different emphases. This study raises further challenges with designing and implementing an integrated STEM curriculum in secondary settings – who designs and who delivers the curriculum?

Successful integrated STEM curriculum design and implementation involves school principals and subject-specific STEM leaders communicating a clear vision for STEM education in the school, trusting teachers to take risks, and creating a school culture that builds individual and collective teacher efficacy (Geiger et al., 2023). School structures and organisation need to become more flexible including timetabling, use of specialist classrooms and other larger spaces to combine classes, and allow for team teaching (Tytler et al., 2019). Connecting with local community members helps to identify and make connections with STEM workplaces, STEM professionals and STEM role models (Hobbs & Kelly, 2020).

At the level of the family, community members have expectations that teachers use the curriculum to select or design suitable instructional materials to meet students' needs and since they are likely to be unfamiliar with integrated STEM curriculum, they need to become partners in the new enterprise (Watson et al., 2022). At the level of the learner, there are challenges in addressing students': self-efficacy, self-perception and expectancies (Krajcik et al., 2022); attitudes, interest and enjoyment (Weiselman et al., 2020); engagement and motivation (Tytler et al., 2023); STEM identities, particularly through societal stereotypes and the influence of significant others (including family) (Watson et al., 2022); and experiences of STEM role models and careers education (Holmes et al., 2018).

Given the challenges at each of the levels of society, school, family and learner, school teachers and leaders need to consider which challenges should become the focus of their integrated STEM curriculum approach. An analysis of the school's policies and practices (often historical and taken for granted) would seem to be a necessary starting point for the development of a new integrated STEM curriculum (Bybee, 2018), accompanied by the development and communication of a clear vision for school change (Geiger et al., 2023). However, there are further challenges for teachers and leaders of mathematics in schools including the creation of authentic mathematics learning opportunities within an integrated STEM curriculum that foregrounds, and maintains the integrity, of mathematics.

Challenges to Designing and Implementing an Authentic Integrated STEM Curriculum that Foregrounds Mathematics

In her reflection chapter, Goos (2024) identified three perspectives on the role of mathematics in an integrated STEM curriculum – "mathematics as a goal, mathematics as a tool, and mathematics as an integrative bridge between the STEM disciplines" as might be the case with mathematical modelling (pp. 341-342). She suggests all three perspectives can make a useful contribution to the learning of mathematics but when mathematics is a 'goal' then it becomes foregrounded.

To foreground mathematics, there appear to be three key challenges for mathematics teachers and mathematics leaders in schools as they design and implement an authentic integrated STEM curriculum:

1. achieving **curriculum coherence** that generates productive mathematics learning,
2. creating an integrated STEM curriculum with **mathematics as a primary focus**, and
3. **designing STEM experiences that foreground mathematics**.

Each of these challenges is discussed in this section with examples drawn from the literature that help to demonstrate the potential to foreground mathematics in integrated STEM curriculum design.

The approaches schools have taken to design and deliver an integrated STEM curriculum have been diverse, typically informed by teachers' perceptions, interests or prior experiences (Rennie et al., 2018; Tytler et al., 2019). Design may begin with the content and processes of one STEM subject and then create connections with one of more of the others (Falloon et al., 2020; Roehrig et al., 2021a), or through a problem or design challenge set in a context (Rennie et al., 2018). Delivery in secondary school contexts can be more challenging and may be within each of the STEM subjects and taught by just one of the STEM subject teachers or by combining subjects so that teams of teachers share delivery (Tytler et al., 2019; 2023). However, without a set of research-informed criteria for evaluating a quality integrated STEM curriculum

program, teacher-led design may lack direction and coherence, particularly for mathematics.

The first key challenge involves **curriculum coherence** to generate productive mathematics learning. Curricular coherence is an important consideration in determining the quality of integrated STEM curriculum (Guzey et al., 2016). Beane (1995) defines a coherent curriculum as "one that holds together, that makes sense as a whole, and its parts, whatever they are, are unified and connected by that sense of the whole" (p. 3). Not only do connections between the STEM disciplines and to the real-world context need to be clear and explicit (English, 2016; Honey et al., 2014), these connections also need to develop concepts through a coherent curricular storyline within a unit (Beane, 1995). Roehrig et al. (2021b) noted the importance of how concepts are sequenced and linked to one another, both within and across lessons and across different time scales, because "unrelated ideas are less meaningful [for students] than those that are richly interrelated, and as such, learning is impacted by the coherence of ideas" (p. 4).

The creation of a 'storyline' within the curriculum can be established using a driving question as in project-based learning (Krajcik et al., 2022) or an engineering design challenge from a design brief for a prospective client (Roehrig et al., 2021b). Depending on the context of the problem, science or engineering may become the focus thus relegating mathematics to a support role (i.e. as a tool or as an integrative bridge) although English (2024) argues mathematics can be foregrounded as illustrated in her example of grade five students designing a mini-golf course. Students used their knowledge and understanding of the areas and perimeters of 2-D shapes to help position the shapes on the golf course. Rather than focusing on the product, attention was drawn to the design processes and the further development and understanding of core mathematical concepts. In this case, students needed to understand the structure of arrays and grids, conservation of area, and seeing how length units produce area units, that is, mathematics became a goal.

What of the coherence of mathematics learning and the balance between multi-disciplinary and disciplinary learning? Ongoing concerns have been expressed about whether students can learn mathematics deeply within an integrated STEM program, and whether the mathematics is at the appropriate level. However, Tytler et al. (2024) demonstrated how connecting mathematics and science in a grade 2 classroom led to students successfully engaging with variability and average, and developed predictive reasoning that was beyond grade level expectations. By exploring the flight of paper helicopters, the grade 2 students developed a "command of investigative design, ideas about force and motion in relation to flight, and statistical constructs of variation in measure, and data organization and modeling including measures of central tendency" (p. 33) and hence learning new mathematics was one of the goals of the lessons.

To address concerns about meeting all mathematics curriculum learning expectations, Bybee (2018) proposed the school curriculum should address both the development of

the content and practices in each of the STEM disciplines as well as opportunities for solving problems involving the knowledge and practices across the STEM disciplines. Similarly, Siemon et al., (2019) used the challenges of developing students' multiplicative thinking to highlight the importance of using research-informed trajectories and targeted teaching in the middle years and concluded curriculum "is neither all integrated or all discipline specific … the truth lies in a creative and purposeful amalgam of the two" (p. 95). The challenge is to determine which aspects of the mathematics curriculum are best learnt through an integrated STEM curriculum approach, although Watson et al. (2020) offer one solution. They argue statistics offers a mechanism for integrating the STEM subjects because of its potential to assist in decision making based on data collection.

The second key challenge involves the creation of an integrated curriculum in which **mathematics is a primary focus**. In their review of research from 2013 to 2018 at the secondary school level (learners aged 12-18), Baldinger et al. found only 32 of 4027 publications (<1%) were identified as having mathematics as a primary focus, which occurred when:

> … the setting was primarily a mathematics classroom, learning goals were primarily mathematical, the research questions were focused primarily on mathematics, or the study findings were primarily focused on ideas related to mathematics teaching and learning (p. 74).

Of the 32 publications with mathematics as a primary focus, 19 studies occurred in classroom settings, mostly in engineering or technology spaces, seven occurred in science, and only two in mathematics classrooms. Twenty articles blended mathematics with other subjects with only three positioning mathematics as the primary subject. However, a wide range of mathematical content was required to solve the problems including algebra, number, geometry, and probability and statistics, and a range of mathematical practices were evident including communication, representation, generalisation, and proof/justification. Eleven studies attended to conceptual understanding as well as procedural fluency and mathematical practices. Finally, the review identified four common themes—frequent attention to communication, task authenticity, inquiry, and learning in informal spaces. Along with well-designed problems, each of these themes provided opportunities for rich mathematics learning.

Looking across stages of schooling, Stohlmann (2018) identified 21 papers published between 2008 and 2018 with mathematics as the focus of an integrated STEM curriculum—elementary (5), middle school (12), and high school (4). His analyses of the curriculum approaches resulted in the proposal of an s.t.e.m. (support, topics, emphasis, and mathematical content) model. The 'support' component identifies positive student outcomes including the development of 21st century competencies and mathematical modelling as recommended by Maass et al. (2019). Typical 'topics' in integrated STEM curriculum involved investigating global and environmental issues

as recommended by Bybee (2018) and Maass et al. (2019) through the development of responsible citizenship. The 'emphasis' for integrated curriculum implementation typically involved engineering design challenges, mathematical modelling, and game based or open-ended mathematics integrated with technology. Stohlman (2018) argued the connections between the STEM subjects must be focused, meaningful and relevant to students and should build on natural connections. This view aligns with Baldinger et al. (2020) who recommended

> … if mathematics learning outcomes are identified as central alongside learning outcomes in other STEM disciplines, then mathematical practices of proof and justification, creating and analyzing representations, and generalization may be visible in integrated STEM curricula (p. 71).

Not all researchers agree that to foreground mathematics, it is essential to identify mathematical connections before students begin integrated STEM tasks. For example, Way et al. (2024) pose the following questions:

> Could it be that the current emphasis on maintaining the visibility and integrity of mathematics learning within STEM activities is counterproductive and diminishes the value of integrated STEM education? Would it be better to view STEM projects as opportunities to develop critical numeracy skills rather than learn new mathematics? Or would it be better to ask from 'inside' the project what mathematics will serve the needs of the inquiry community, rather than attempting to insert mathematics from 'outside' the project community?" (pp, 21-22).

However, there is a danger that if we leave the potential for integrating mathematics to serving "the needs of the inquiry", the use of mathematics will remain a 'tool' rather than becoming a goal through the development of mathematical content and skills or mathematical competencies.

The third key challenge is **designing STEM experiences that foreground mathematics**. Stohlmann (2018) recommended the use of tasks based on an engineering design challenge, mathematical modelling, or game based/open-ended mathematics integrated with technology. Maass et al. (2019) advised three interdisciplinary approaches that focus on the development of twenty-first century skills, mathematical modelling, and education for responsible citizenship. Common to these recommendations is mathematical modelling, an approach advocated by Goos (2024) as an 'integrative bridge' between the STEM disciplines. Regardless of the approach adopted to foreground mathematics, it is critical to obtain clarity about the design of 'good' of interdisciplinary STEM problems (Goos et al., 2023).

As a starting point, Reinholz et al. (2018) compared quality disciplinary and interdisciplinary problems and concluded the latter needed to have real-world connections, reinforcement of conceptual understanding, a low floor and high ceiling, multiple solution paths, and opportunities to build dispositions of professionals in the discipline. Further, Reinholz et al. noted that within a *discipline*, a good problem helped students learn about concepts, practices, procedures and dispositions, and a good

interdisciplinary problem helped students learn these four dimensions across multiple disciplines, typically using "a contextual intersection that is interesting and relevant to multiple disciplines" (p. 48). Krajcik et al. (2022) argued this can be achieved through cross-cutting concepts that are big conceptual ideas such as patterns, systems and conservation.

Many real-world challenges aimed at managing and finding solutions to climate change, biosecurity and pandemics, require significant input from mathematicians and those working in the mathematical sciences (Burkhardt et al., 2024). Finding problems suitable for school students that address global environmental issues and are accessible for the students' age and stage of learning can be challenging, particularly problems that require the use of mathematics beyond counting, tallying, measuring, or the use of basic statistics. Examples of suitable tasks were presented at the recent ICME-15 conference in Topic Study Group 3.16.

RECENT TRENDS IN INTEGRATED STEM CURRICULUM DESIGN

While the proceedings from ICME-15 remain unpublished, a brief review of the submissions to the *Mathematics and Interdisciplinary education/STEM Education* TSG reveals recent trends in the research, providing further evidence of ongoing challenges to the design of integrated STEM curriculum. Researchers from twenty-four countries were represented in the 71 papers and posters with contributions examining a range of aspects of the design and implementation of an integrated STEM curriculum in schools or university settings (see Table 1 for the focus of research contributions). Using the Goos et al. (2020) framework, almost all contributions addressed the design, delivery and/or evaluation of STEM curriculum for school or university students, or teacher learning and hence encompassed the school/institution and learner levels. None specifically focused on the levels of society or family.

Examining the frequencies of the main foci of the research in Table 1 reveals 33 submissions had a focus on approaches to designing integrated STEM curriculum in schools (noting several papers could have been classified into more than one category). Stohlmann's (2018) five supports were evident across the research, particularly developing students' mathematical modelling abilities, 21st century competencies, and confidence and engagement. In relation to the topics, numerous papers used a context to promote STEM integration including global issues, environmental issues and engineering design challenges although several contributions highlighted the use of local issues (e.g., Steffensen, 2024) and cultural connections (e.g., Orey et al., 2024) as important to engage and make connections with students' knowledge and interests. The three emphasis categories of engineering design, modelling and using technology to solve problems were also evident. The mathematical content topics in the thirty-three school-based papers were spread across key organisers of the mathematics curriculum with most focusing on data collection and analyses (8), geometry (8), measurement (6), and proportional reasoning (4).

Focus of the Research	Frequency
Researching integrated STEM curriculum approaches in schools:	**33**
Coding and/or robotics	8
Modelling	8
Projects and/or PBL	7
Sustainable Development Goals and/or sustainability	6
Engineering design challenges	4
Researching challenges to design and implementation	**38**
Conducting curriculum reform or textbook review	6
Investigating teachers' or PSTs' perceptions of STEM	5
Analysing the mathematics in teacher-designed STEM tasks	5
Evaluating lessons connecting STEM content and practices	5
Proposing a framework of integrated STEM	3
Exploring assessment of integrated STEM	3
Using connections between university level mathematics with STEM	3
Reviewing preservice teacher education programs for integrated STEM	2
Improving teachers' knowledge through professional learning	2
Evaluating after school STEM education programs	2
Critiquing the purpose of STEM Education	2
TOTAL	**71**

Table 1: Frequencies for main research foci of TSG 3.16 contributions at ICME-15

The remaining 38 papers explored challenges to the design and implementation of integrated STEM and/or STEAM education in school and university settings. While most papers reported on just the STEM subjects, several adopted a broader interdisciplinary perspective, or STEAM education approach, that included other subjects such as art or geography. Five papers presented a critique of STEM education (2), or proposed a framework for STEM education (3). Six papers examined system level challenges including curriculum design and evaluation with evidence of impact across schools and school systems (e.g., Spreitzer et al., 2024). Five focused on university mathematics and preservice education programs, while two examined the impact of after-school STEM programs. Others investigated teachers' or PSTs perceptions of STEM/STEAM education (5) and the design of professional learning (2), while a further group analysed the mathematics content of teacher-designed integrated STEM tasks (5) or lessons designed to closely align the content of mathematics with another STEM/STEAM subject area (3 with art, 2 with physics). Finally, three papers explored the assessment of integrated STEM curriculum.

Regarding the connections between mathematics and the other STEM/STEAM subjects, 20 referred to STEM rather than a specific focus on particular subjects, and 15 referred to STEAM. For these papers, the descriptions of problems typically

involved project-based learning or an engineering design challenge where mathematics was a tool or an integrative bridge. For example, grade 8 students were required to 'build a ramp for a new school hall' (Sri Padmi et al., 2024).

A further group of 29 papers described specific links between mathematics and one other STEM/STEAM subject – science (10), technology (9), engineering (2), geography (1), art (4). For example, grade 11 students were introduced to graphing using GeoGebra, they reviewed basic transformations of functions, and were then required to design artworks using graphs of functions (Bautista et al., 2024). Three papers connected three of the STEM subjects. For example, grade 6 students were required to plan a house, build a model, and then 3D print it (Békési et al., 2024). The remaining seven papers had a strong emphasis on mathematics with some connections to other STEM practices rather than other STEM content areas. For example grade 7 students were required to 'write a program (using Scratch) that calculates the sum of two integers' (Abdollahpour, et al., 2024).

From the papers and presentations in TSG3.16 at ICME-15, several key challenges were identified arising from researching integrated STEM curriculum in education settings including:

- identifying the relationship between mathematics and the 'real world', between mathematics and the other subject disciplines, and how those relationships are being conceptualised;
- identifying explicit connections between subjects and foregrounding that to students;
- having horizon knowledge to build student capability both within the disciplines as well as across the disciplines;
- understanding what knowledge is best explored through connected learning experiences versus the mathematics knowledge that is best learnt without connections; and
- designing preservice education programs that connect disciplines in relation to what is promoted, what is permitted, and what are the goals of the program from a mathematics teacher educator perspective.

These challenges have arisen in earlier research but it was clear from the presentations at ICME-15, we may not have progressed as we would wish in supporting teachers to design and implement integrated STEM curriculum, particularly for mathematics.

A PROFESSIONAL LEARNING PROGRAM SUPPORTING TEACHER-DESIGNED INTEGRATED STEM CURRICULUM

My research with colleagues over several years has identified similar challenges when working with teachers and school leaders to design and implement integrated STEM curriculum (e.g., Anderson & Tully, 2024; Tytler et al., 2023). While our professional learning program, *the STEM Teacher Enrichment Academy*, did not set out to have teachers design programs that foregrounded mathematics, we wanted the teacher-

designed integrated STEM curriculum to be authentic in enabling students to engage with, and learn how to use their knowledge and understanding of the STEM subjects to solve real-world problems, and to understand how each of the STEM subjects contributes to such solutions. The following provides insights from our work, and presents new analyses seeking to identify programs that foreground mathematics.

The Academy program was designed to support teams of teachers from participating schools develop and implement integrated STEM programs suitable for their students. Informed by research, the year-long program was content focused, involved active learning, supported collaboration, used models of effective practice, provided coaching and expert support, offered feedback and reflection, and was of a sustained duration (Darling-Hammond et al., 2017). Because of differences in curriculum, school structures, and teachers' breadth and depth of knowledge of the STEM subjects, the secondary (grades 7 to 12) and primary (Kindergarten to grade 6) school programs were delivered separately. From 2015 to 2019, 11 secondary and 5 primary programs were delivered to teachers from a total of 70 primary and 120 secondary schools in NSW, Australia. Schools represented a diversity of locations, size, socio-economic status (SES), education sector (i.e., public, Catholic or independent), and single-sex/coeducational contexts. The 963 participating teachers had from three to 33 years of teaching experience, with many in leadership roles. Some schools had already begun their integrated STEM curriculum journey before joining the program but most were just beginning.

Each **primary school program** involved up to 14 schools, with each school sending teams of two to four teachers depending on size. Face-to-face, multi-day meetings were held at the beginning, middle and end of the school year, and teachers were supported by academic mentors. Since all teachers had a breadth of knowledge of school curriculum requirements across the STEM subjects, sessions tended to focus on developing deeper content knowledge and connecting the subjects in more authentic ways. While many of these teachers were familiar with integrated curriculum, they were less familiar with integrated STEM curriculum and unaware that their attitudes to STEM had the potential to influence their students' attitudes and aspirations. To enable the gradual introduction of the program to their students, they were introduced to open-ended integrated STEM inquiry learning at three levels (Way et al., 2022) with exemplar tasks presented for grades 3 and 4. Level 1 involved short tasks to develop students' STEM skills such as collaborative group work with assigned roles, active-listening, quick construction challenges, or collecting, displaying and interpreting data. Level 2 were more substantial tasks to develop design and/or inquiry processes such as "designing a wind-powered car within a unit of work on forces" and "hypothesis testing and data-based decision making" (p. 4). Finally, Level 3 were "sustained multi-disciplinary projects to solve authentic problems or complete student-driven inquiries" (p. 4). Teams from each school developed their integrated STEM curriculum program and trialled tasks between each session of the Academy program with a presentation of their achievements at the final 'showcase' session of the year.

Each **secondary school program** began with a three-day face-to-face immersive experience where up to 12 participating schools sent a team of six teachers (typically two each of mathematics, science, and technology/engineering). Each school had a 'team leader' charged with coordinating the program and facilitating the delivery of the program within the school, particularly when there were usually many more teachers in each subject faculty. During the initial session, teachers were provided with advice and guidance about working collaboratively with separate curriculum documents to create integrated STEM learning experiences, strategies for addressing rigid timetables and school structures, and approaches to implementation including sample problems, lessons and units of work. However, because schools varied widely in context, size and experiences, it was necessary that each school designed a unique program that would meet their students' needs and capitalise on connections with local STEM businesses and professionals. After working intensively with academic mentors during the initial three-day session, teachers refined programs and began to trial the program with a targeted group of their students and classes – typically grades 7 or 8 where the curriculum is more flexible and teachers feel they have less pressure from high-stakes external examinations. Throughout the course of the year, academic mentors visited schools to provide additional support, discuss issues, and make suggestions for overcoming challenges in program implementation.

To determine the impact of Academy programs, data were collected from schools using a variety of methods – initial school integrated STEM curriculum plans, a final report on outcomes and future intentions, the final showcase presentation slides, school programs and student work samples. Teachers, their students, and parents completed questionnaires before and after the program. For further detail about questionnaire design and analyses, see Anderson and Tully (2020, 2024) and Anderson et al. (2019). In a sample of schools, interviews were conducted with principals, curriculum leaders, STEM team leaders, participating teachers, parents and groups of students (Anderson & Tully, 2021; Anderson et al., 2023; Tytler et al., 2023). For this paper, school reports, showcase presentation slides, programs and student work samples were examined in a sample of participating primary and secondary schools for evidence of integrated STEM approaches that foregrounded mathematics. The following section presents a summary of the data analyses for both primary and secondary schools.

SEEKING EVIDENCE OF INTEGRATED STEM CURRICULUM APPROACHES THAT FOREGROUND MATHEMATICS

To answer the research questions, the reports, presentations, programs and student work samples from a sample of 20 primary and 30 secondary schools were analysed to determine the approaches taken to designing an integrated STEM curriculum program and establish how schools foregrounded mathematics. In particular, analyses sought to identify the level of coherence of the integrated STEM curriculum, particularly the balance between disciplinary and multidisciplinary knowledge and practices, whether mathematics was a focus (i.e. a goal or a tool), and the type of problem/task design.

Primary School Approaches and Challenges to Integrated STEM Curriculum Design

The 20 primary schools included in the sample were representative of the 70-participating schools in relation to location, size, SES, and education sector. All reported following the approach advocated by Way et al. (2022) of beginning with short STEM tasks to build skills and then gradually progressing to larger projects. Some small schools designed their integrated STEM curriculum for the whole school, although most introduced their program with classes from grades 3 and/or 4. Sustained project work typically focused on science content with an engineering design challenge that addressed one of the following real-world themes:

- improving the school environment, e.g., updating the playground for students with a disability; designing and creating a sensory garden (8 schools – 1, 5, 7, 11, 14, 16, 18, 19);
- improving recycling and/or sustainability within the school and/or community (5 schools – 3, 4, 6, 8, 20);
- using technology to address a personal need (4 schools – 10, 12, 15, 17); and
- addressing a local community issue such as improving ecosystems or preparing for floods (3 schools – 2, 9, 13).

The projects captured the imagination of students and the local community with several reports highlighting the role parents played in visiting the school, participating in showcase events, and learning about sustainability. For example, the parents of participating students from school 3 visited the school to design wind-powered cars like the students had during lessons (Anderson et al., 2019). The parents from school 20 attended a workshop at the school to learn how students had repurposed old tee-shirts as reusable shopping bags. Understanding the role of the STEM subjects in solving real-world challenges was also evident when STEM professionals were invited to visit schools (3, 4, 14, 16, 17, and 20) to share expertise and provide input to project work.

In relation to the three key challenges to foregrounding mathematics, **curriculum coherence** was evident through the storylines and driving questions frequently used in the primary school projects. For example:

> What urban habitats protect bird nests from predation? (school 13)

Others provided a focus but allowed groups of students the flexibility to pose more specific inquiry questions or to design creative solutions to problems. For example,

> What could we do to make our school more sustainable? (school 20)

Most schools implemented the integrated STEM program during allocated science curriculum time of up to 2 hours per week. This meant that for many schools, mathematics specific lessons were still conducted each week. As such, schools

recognised the need for integrated learning experiences as well as content specific mathematics learning as recommended by Bybee (2018) and Siemon et al. (2019).

While mathematics was typically used as a tool to collect data or to measure distances and time, there was evidence from some schools' reports that teachers set out to include mathematics as a primary focus. For example, the teachers at school 16 suggested the students "research the wants and needs of students who use the playground" to ascertain what improvements could be made to better meet their needs. Students learnt about designing good questions to collect useful information, collating and analysing the data, and strategies to present the results to convince an audience that the recommendations were viable. Similarly, the teachers from school 5 used an investigation for students to learn about collection, collation and analyses of data as they posed the question:

> How do we solve the problem of bike, scooter and skateboard storage at our School?

School 4 was explicit about using the initial project work as an opportunity to learn about data collection and displays as well as using data to inform decision making. In a grade 1 and grade 2 composite classroom, the students decided they wanted to address the amount of discarded waste materials in the playground after lunch. They were curious about recycling and whether they could get recycling bins in the school to reduce waste. With support from the teacher they sorted rubbish to ascertain which articles could be recycled and which could not. After visiting the local council to learn more about recycling, the students obtained the recycling bins for the school and then set out to convince other classes to use the correct bins as well as eliminate single-use plastics by promoting the use of recyclable bees-wax paper which they made with support from community members. The students also used mathematics to determine:

- How big each piece of bees-wax paper needed to be to make it suitable for wrapping a sandwich as well as other lunch food items.
- How many metres was needed to create enough for all students in their class.

A principal of a small rural school (14) set out with a different STEM agenda. She participated in the Academy program with her three teachers to ensure she showed leadership and commitment to the project and could offer the necessary support to her team. She provided time for the STEM team to meet and plan, she actively engaged with the local community, and promoted the school's success beyond the community. The school's project involved re-developing an area within the school grounds, that was overgrown and unkempt. This choice was influenced by two factors: the area already had interesting features and safe trees, and the local botanical gardens had offered to relocate a boardwalk from the gardens to the school. With only three classes in the school, each group of students discussed the natural flora and fauna that existed within the school environment and, with the help of their teachers, considered what they needed to include in the garden space to make it an accessible place for everyone to enjoy. Students from the older grades, worked collaboratively in groups to create

scale-drawings of new designs using Google Maps, and then created scale models. They investigated what animals and plants were local to the area and, since the school had a high percentage of Indigenous children, the group was particularly interested in planting native plants with input from an Indigenous elder from the community. In science, the topic 'all living things' included research into the different vegetation and plant species in the area, and an investigation into which insects and native animals' habitats already existed. Mathematics included number operations, measurement (length, area, mass, volume), scale drawings (ratio, symmetry) budgeting and planning for the mathematics level appropriate at each grade.

Adding to the principal's commitments to provide better outcomes for students, she also saw it as the school's role to "send our girls off to high school knowing that they can do [STEM project work] and not be fearful of it". She expressed concern that high school teachers have low expectations of girls' capacity to think and emphasised that "we want to get the point across that they can think and they can think hard and they can think well". It was considered important that the girls experienced success while also recognising that failure was an opportunity to learn and that moving forward into high school "they will jump in and have a go and they won't be fearful".

Regardless of teacher confidence and with the support of STEM Academy mentors, the teachers identified contexts that were relevant to their students lived experiences, designed problems that engaged and interested their students, and made links to subject-specific content in the syllabuses to satisfy mandatory curriculum requirements. Most integrated STEM curriculum programs focused on science with opportunities for students to use mathematics as a tool. However, for some schools, project work did enable the learning of new mathematics, particularly for statistics but also for measurement and geometry. As most schools still had dedicated mathematics learning time in the school timetable, there was evidence of both disciplinary and multidisciplinary experiences that contributed to the development of students' mathematical learning.

Secondary School Approaches to Integrated STEM Curriculum Design

The 30 secondary schools included in the sample from the 120 participating secondary schools represented a diversity of locations, size, SES, and education sectors. Approaches to integrated STEM curriculum design and delivery varied although almost all schools (26) implemented their teacher-designed program in grades 7 and/or 8. In some schools, the program involved a small number of classes. For example, in school 6, the two enrichment classes in grade 7 explored how individual diet impacted their health, reported the results to school executive members, parents and the canteen supervisor, and recommended improvements to the canteen menu. While in other schools, the program was implemented across the whole grade thus introducing challenges as the integrated STEM curriculum approach needed to be embraced by all teachers including those who had not attended the Academy program.

Like the primary schools, some schools began with small tasks to build STEM skills and dispositions, such as team work, collaboration, communication, and design thinking processes. Tasks such as designing and making a water rocket, a bridge, and a parachute to drop an egg from a building, as well as coding a Sphero to traverse a path, provided opportunities for students to build 21st century skills. For four of the sample schools (10, 13, 18 and 19), their integrated STEM program involved completing similar tasks during a one-day 'STEM Bootcamp' or 'immersion program'. School 13 reported trialling these tasks as impetus for designing a new integrated STEM subject to be implemented in two 50-minute lessons per week for all grade 8 classes the following year. They indicated "classes will be taught by two teachers from the STEM faculties with all classes based in a newly refurbished STEM Laboratory". This approach to STEM suggests it was to become another 'siloed' subject on the timetable and may not be offering the best approach to integrated curriculum design with genuine collaboration and hence integration across STEM subject boundaries.

All other schools embraced larger projects requiring greater inter-faculty collaboration. Sustained project work typically involved science and/or engineering design challenges set in a context that was relevant to students' interests and experiences. Projects could be grouped into the following themes:

- improving the school environment, e.g., updating the playground; designing and building new storage units; designing a new school library using shipping containers (4 schools – 5, 17, 21, 24);
- improving recycling and/or sustainability within the school and/or community (11schools – 2, 3, 11, 15, 20, 22, 25, 26, 27, 28, 29);
- addressing a local community issue such as preparing for floods (3 schools – 7, 9, 16);
- investigating personal fitness/health/survival, e.g., healthy eating and exercise programs (4 schools – 1, 6, 23, 30); and
- designing solutions to challenges using expertise from community members and/or STEM professionals, e.g., designing a dream home, a solar car (4 schools – 4, 8, 12, 14).

Building a team of cross-curriculum teachers was challenging given differences in knowledge and pedagogical approaches used by the three-main curriculum subject areas of science, mathematics and Technology and Applied Sciences (TAS) taught in these schools. During the Academy program, we observed that in some schools, teachers within a STEM team had not worked together previously and had little knowledge of how other subject areas designed and implemented their curriculum but they were keen to learn and on return to their schools, requested opportunities to participate in peer observation of classes, sharing of resources, and common meeting times (Anderson & Tully, 2021; Tytler et al., 2019).

While teachers reported working in cross-curriculum collaboration on task design, in 18 of the 30 schools, delivery of the integrated STEM program involved subject-

specific teachers implementing specified tasks with students during regular science, TAS or mathematics lessons. Only five schools specifically mentioned team teaching in their final reports with a frequently sited challenge as finding spaces within the school to combine classes. On the one hand, without opportunities to team teach to observe teachers in other subject areas as they facilitate and support inquiry learning and teach unfamiliar content from science or TAS, mathematics teachers may be limited to teaching fragmented components of projects in isolation. On the other hand, not disrupting the timetabling of regular mathematics lessons meant that in all schools, specific mathematics learning of the curriculum continued uninterrupted throughout the year allowing for coherence of mathematics learning within an amalgam of both disciplinary and interdisciplinary learning.

Because all secondary school students had participated in project-based learning (PBL), particularly in TAS and to a lesser extent in science, many STEM teams designed an integrated STEM curriculum program that built on familiar approaches. While in the sample of 30 schools, leadership of the STEM team was spread evenly across the three main curriculum areas of science, TAS and mathematics, the mathematics teachers reported STEM project ideas were usually generated by the other two subject areas so they were left to identify the mathematics content that would be relevant and appropriate for the grade level of the participating students. This meant that mathematics was typically used as a tool with the most frequently reported mathematics learning outcomes including measurement, statistics, financial mathematics, rates and ratios, and algebraic modelling.

As mathematics teachers were less familiar with PBL, designing and implementing an integrated curriculum program was viewed as an opportunity to explore new pedagogical approaches to learning. The team leader from school 28 reported the benefits from the experience:

> Whilst TAS teachers, and possibly some of our science teachers involved in the program are accustomed to design-based projects that are based on guided inquiry practices, I feel that our engagement in the STEM Academy enabled our mathematics teachers to experience a different style of pedagogy that does not usually come naturally in a mathematics classroom. Consequently, all staff involved in the program are now much more experienced and have an insight into guided inquiry-based learning where the students direct the learning and outcomes of the design problem provided, rather than a traditional teacher directed learning approach.

One secondary school adopted a different approach to designing an integrated STEM curriculum based on earlier experiences noticing the benefits of PBL for improving the attitudes of their students in TAS lessons. School 5 introduced an integrated STEM curriculum across the whole of the school year for grade 7 students with projects to be completed in each of four terms. While the three leaders of the STEM subjects in the school were all very keen to embrace the program, some of the teachers felt less confident as one early career mathematics teacher noted:

> … it was okay to not exactly know where we're going, because STEM isn't like a maths syllabus or a science syllabus, … it's sort of a still unknown space and people interpret it differently … I'm finding it quite challenging, but I'm enjoying the challenge.

With substantial support from the school's leadership, curriculum planning and implementation were led by the Deputy Principal, a qualified mathematics teacher. The STEM team met regularly to create a scope and sequence using themes from the science curriculum. They then mapped content from the mathematics and TAS curriculum areas that enabled connections with the science theme. An overview of the Term 4 scope and sequence (Table 2) indicates that students were to create a project that involved designing a new, improved garden area in the school grounds. Weeks 4 and 5 were set aside for annual examinations and are therefore not included.

Mapping the content from the mathematics curriculum to each of the four projects across the year revealed several content topics did not appropriately connect with any of the projects. That meant some mathematics content was taught separately without connections to a project as indicated in Table 2 as "grade 7 mathematics program". Project descriptions included specific expectations for each of the STEM subjects although for mathematics they were less open-ended than those for science and technology. In addition, marking criteria and rubrics were developed for each project that clearly indicated project expectations for each STEM subject.

<table>
<tr><td colspan="4">Term 4</td></tr>
<tr><td>Weeks</td><td>1 to 3</td><td colspan="2">6 to 10</td></tr>
<tr><td>Project</td><td colspan="3">Better Parks & Gardens</td></tr>
<tr><td>Science</td><td>Rocks/Water Cycle
Working Scientifically</td><td>Rocks/Water Cycle
Working Scientifically</td><td rowspan="2">Refine
Resolve
Present</td></tr>
<tr><td>Technology</td><td>Landscape Design
(Built environments – Design, Model, Implement, Establish)</td><td>Landscape Design
(Built environments – Design, Model, Implement, Establish)</td></tr>
<tr><td rowspan="2">Mathematics</td><td>Geometric Figures, Area</td><td colspan="2">Grade 7 Mathematics Program – Angle relationships, Integers</td></tr>
<tr><td>Working Mathematically</td><td colspan="2">Working Mathematically</td></tr>
</table>

Table 2: Subset of the Term 4 STEM scope and sequence for School 5

The *Better Parks and Gardens* project required students in groups to redesign an allocated area of the school playground that met particular criteria including:

- Technology – a themed garden area that included shade, paths, borders, planting and soil, seating, water and drainage;
- Mathematics – a shaped garden with at least three triangles and quadrilaterals forming a composite figure with the requirement to calculate its area to contribute to an environmental impact statement; and

- Science – an environmental impact statement including water requirements and how weathering and erosion affect the site.

Group reports were submitted for assessment against an assessment rubric for each subject. To challenge students critical and creative thinking in mathematics, they were encouraged to "think bigger" in each of the projects with the instruction to:

> Identify another interesting way that mathematics applies to your project. Describe this clearly and show how you made use of this mathematics. Don't forget to think bigger – don't describe mathematics that should have been mastered in primary school!

The approach to integrated STEM curriculum design and implementation used by school 5 made visible the links between science, TAS and mathematics. The use of opportunities to learn mathematics in both disciplinary and multidisciplinary ways ensured the curriculum was coherent. The storyline of environmental sustainability was woven throughout the year in each of the four projects where students were required to explore the potential of living on Mars, investigate the sustainability of growing crops locally, and analyse approaches to health, well-being and longevity. While mathematics was frequently used as a tool, it was foregrounded through the project descriptions and assessment rubrics. As reported by Tytler et al. (2023), students from school 5 reported being more engaged in mathematics and were more interested in pursuing the STEM subjects in the future.

During the final presentations from the STEM Academy schools, many reported the challenges they had encountered while implementing an integrated STEM curriculum at their school. For example, school 23 reported their challenges as "half the team left for various reasons, buy in from teachers not part of the team, follow through by teachers, and time" although they also noted positive outcomes including "support from the principal and funding, more people wanted to join as others left, reusability of activities in the future as we embed into programs, and greater engagement by students". Other schools mentioned the challenges of locating appropriate learning spaces, upskilling of teachers and finding suitable, economic resources.

CONCLUSION

While there are many challenges to developing an integrated STEM curriculum for school students, school leaders and teachers have continued to embrace the potential benefits of giving their students a more holistic learning experience where they can use knowledge and practices from several subjects to solve problems (Krajcik et al., 2022; McClure et al., 2022; Rennie et al., 2018). Regardless of whether the curriculum design involves a single day experience, a longer-term project, or an embedded integrated curriculum, students have responded positively to the opportunity to work collaboratively on STEM projects connected to their experiences and interests or to local community issues (Anderson et al., 2019; Tytler et al., 2023). Such student responses have meant school leaders and teachers have been willing to work to overcome challenges and to challenge the status quo of only delivering a curriculum based on siloed subjects.

Additional challenges have been evident for mathematics leaders and mathematics teachers in schools as they seek to ensure the visibility of mathematics within STEM projects, particularly as science and engineering do dominate integrated STEM curriculum design (English, 2015; McClure et al., 2022). Ensuring curriculum coherence (Roehrig et al., 2021b), mathematics as a primary focus (Baldinger et al., 2020), and foregrounding mathematics in STEM task design (Stohlmann, 2018) require extra attention if they are to be realised. This approach is not for the faint hearted, particularly in secondary school contexts where mandatory curriculum and assessment regimes dominate the landscape. However, the desire to foreground mathematics is not impossible as was evident in some of the papers and presentations at topic study group 3.16 from the recent ICME-15 conference and in some of the schools from the STEM Academy program. While mathematics was frequently employed as a 'tool', this role is still important Goos (2024) and demonstrates to students that mathematics *is* useful and is foundational to solving many local, national and international problems that have engaged scientists, engineers and mathematicians for many years.

Regardless, challenges at the society, school, family and student levels remain when we consider the potential of an integrated STEM curriculum in schools. Researchers continue to explore the possibilities, particularly working collaboratively with teachers as they design and implement new curriculum approaches in their schools. The potential for impact is heightened when partnerships are formed between researchers, school leaders, teachers, community members and STEM professionals. Such research must continue to make the greatest impact on the learner.

References

Abdollahpour, K., Fadaee, M. R., & Riepour, A. (2024). *Establishing a reciprocal relationship between mathematics and technology in STEM education through computational thinking*. Paper presented at the fifteenth International Congress of Mathematics Education conference (ICME-15). Sydney, Australia.

Alpers, B. A., Demlova, M., Fant, C., Gustafsson, T., Lawson, D., Mustoe, L., et al. (2013). *A framework for mathematics curricula in engineering education: A report of the mathematics working group*. European Society for Engineering Education (SEFI).

Anderson, J., & Tully, D. (2021). Factors which sustain integrated STEM curriculum approaches in secondary school settings. In D. Anderson, M. Milner-Bolotin, R. Santos, & S. Petrina (Eds.), *Proceedings of the 6th International STEM in Education Conference (STEM 2021)* (pp. 20–26). University of British Columbia, Vancouver, Canada, July 5–9. University of British Columbia. https://doi.org/10.14288/1.0402129

Anderson, J., & Tully, D. (2024). Creating new STEM opportunities in schools: Teachers as curriculum designers. In Y. Li, Z. Zeng & N. Song (Eds.), *Disciplinary and interdisciplinary education in STEM: Changes and innovations* (265-294). Springer.

Anderson, J., Tytler, R., & Williams, G. (2023). Supporting curriculum innovation in integrated STEM for secondary teachers in Australia. *Research in Integrated STEM Education*, *1*(1), 30-59. https://doi.org/10.1163/27726673-00101001.

Anderson, J., Wilson, K., Tully, D., & Way, J. (2019). Can we build the wind powered car again?" Students' and teachers' responses to a new integrated STEM curriculum. *Journal of Research in STEM Education*, *5*(1), 20-39. https://doi.org/10.51355/jstem.2019.61

Baldinger, E. E., Staats, S., Covington Clarkson, L. M., Contreras Gullickson, E., Norman, F., & Akoto, B. (2020). A review of conceptions of secondary mathematics in integrated STEM education; Returning the voice to the silent M. In J. Anderson & Y. Li (Eds.), *Integrated approaches to STEM education: An international perspective* (pp. 67-90). Singapore: Springer Nature.

Bautista, G., Prodromou, T., & Lavicza, Z. (2024). *Function art as a teaching tool in STEAM education.* Paper presented at the fifteenth International Congress of Mathematics Education conference (ICME-15). Sydney, Australia.

Beane, J. (1995). Curriculum integration and the disciplines of knowledge. *The Phi Delta Kappan*, *76*, 616–622.

Békési, B., Houghton, T., Ulbrich, E., & Lavicza, Z. (2024). *From crafting to 3D-printing at small and great scale.* Paper presented at the fifteenth International Congress of Mathematics Education conference (ICME-15). Sydney, Australia.

Burkhardt, H., Pead, D., & Stacey, K. (2024). *Learning and teaching for mathematical literacy: Making mathematics useful for everyone.* New York: Routledge.

Bybee, R. W. (2018). *STEM education: Now more than ever*. National Science Teachers Association Press.

Darling-Hammond, L., Hyler, M., Gardner, M., & Espinoza, D. (2017). *Effective teacher professional development.* Learning Policy Institute.

English, L. D. (2015). STEM: Challenges and opportunities for mathematics education. In K. Beswick, T. Muir, & J. Fielding-Wells (Eds.), *Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education (Vol. 1)* (pp. 1-1 to 1-16). Hobart, Australia: PME.

English, L. D. (2016). STEM education K-12: Perspectives on integration. *International Journal of STEM Education, 3*(3), 1-8.

English, L. D. (2024). Designing with mathematics in fifth-grade STEM education. In Y. Li, Z. Zeng & N. Song (Eds.), *Disciplinary and interdisciplinary education in STEM: Changes and innovations* (pp. 149-168). Springer

Falloon, G. W., Hatzigianni, M., Bower, M., Forbes, A., & Stevenson, M. (2020). Understanding K-12 STEM education: A Framework for developing STEM literacy. *Journal of Science Education and Technology*, *29*, 369-385. https://doi.org/10.1007/s10956-020-09823-x

Geiger, V., Beswick, K., Fraser, S., & Holland-Twining, B. (2023). A model for principals' STEM leadership capability. *British Educational Research Journal, 49*(5), 900–924. https://doi.org/10.1002/berj.3873

Goos, M. (2024). Reflecting on the contribution of mathematics to STEM education. In J. Anderson & K. Makar, *The contribution of mathematics to school STEM education: Current understandings* (pp. 337-349). Springer

Goos, M., Carreira, S., & Namukasa, I. K. (2023). Mathematics and interdisciplinary STEM education: Recent developments and future directions. *ZDM – Mathematics Education*, *55*, 1199-1217. https://doi.org/10.1007/s11858-023-01533-z

Goos, M., Ryan, V., Lane, C., Leahy, K., Walshe, G., O'Connell, T., O'Donoghue, J., & Nizar, A. (2020). *Review of Literature to identify a set of effective interventions for addressing gender balance in STEM in early years, primary and post-primary education settings.* Department of Education. https://assets.gov.ie/96986/f05f7b2f-e175-442e-85e9-4a2264391843.pdf.

Guzey, S. S., Moore, T. J., & Harwell, M. (2016). Building Up STEM: An analysis of teacher-developed engineering design-based STEM integration curricular materials. *Journal of Pre-College Engineering Education Research (J-PEER), 6*(1), Article 2. https://doi.org/10.7771/2157-9288.1129.

Hobbs, L., & Kelly, H. (2020). STEM into industry – Brokering relationships between schools and local industry. *Curriculum Perspectives*, *40*(2), 247–255.

Holmes, K., Gore, J., Smith, M., & Lloyd, A. (2018). An integrated analysis of school students' aspirations for STEM careers: Which student and school factors are most predictive? *International Journal of Science and Mathematics Education, 16,* 655-675. https://doi.org/10.1007/s10763-016-9793-z

Honey, M., Pearson, G., & Schweingruber, H. (2014). *STEM integration in K-12 education: Status, prospects and an agenda for research.* Washington, DC: National Academies Press.

Krajcik, J. Schnieder, B., Miller, E. A., Chen, I., Bradford, L., Baker, Q, … & Peek-Brown, D. (2022). Assessing the effect of project-based learning in science learning in elementary schools. *American Educational Research Journal*, *60*(1), 70-102. https://doi.org/10.3102/00028312221129247

Lyons, T. (2018). Helping students make sense of STEM. *Teaching Science*, *64*(3), 37–43. https://doi.org/10.3316/informit.841550904446404

Maass, K., Geiger, V., Romero Ariza, M., & Goos, M. (2019). The role of mathematics in interdisciplinary STEM education. *ZDM Mathematics Education*, *51*(6), 869-884.

Margot, K. C., & Kettler, T. (2019). Teachers' perceptions of STEM integration and education: A systematic literature review. *International Journal of STEM Education*, *6*(2), 1-16. https://doi.org/10.1186/s40594-018-0151-2

McLure, F. I., Tang, K., & Williams, P. J. (2022). What do integrated STEM projects look like in middle school and high school classrooms? A systematic literature review of

empirical studies of iSTEM projects. *International Journal of STEM Education*, *9*(73), 1-15. https://doi.org/10.1186/s40594-022-00390-8

Mullis, I. V. S. (2017). Introduction. In I. V. S. Mullis, & M. O. Martin (Eds.). *TIMSS 2019 assessment frameworks.* TIMSS & PIRLS International Study Center

Orey, D. C., & Rosa, M. (2024). *Polysemic interactions of STEM education: An ethnomodelling approach through glocalisation.* Paper presented at the fifteenth International Congress of Mathematics Education conference (ICME-15). Sydney, Australia.

Reinholz, D., Slominski, T., French, T. A., Pazicni, S., Rasmussen, C., & McCoy, B. (2018). Good problems within and across disciplines. *Journal of Research in STEM Education*, *4*(1), 37–53. https://doi.org/10.51355/jstem.2018.34

Rennie, L., Venville, G., & Wallace, J. (2018). Making STEM curriculum useful, relevant and motivating for students. In R. Jorgensen & K. Larkin (Eds.), *STEM education in the junior secondary: The state of play* (pp. 91-110). Springer Nature. https://doi.org/10.1007/978-981-10-5448-8_6

Roehrig, G. H., Dare, E. A., Ellis, J. A., & Ring-Whalen, E. (2021a). Beyond the basics: A detailed conceptual framework of integrated STEM. *Disciplinary and Interdisciplinary Science Education Research*, *3*(1), 1-18. https://doi.org/10.1186/s43031-021-00041-y

Roehrig, G.H., Dare, E.A., Ring-Whalen, E., & Wieselmann, J. R. (2021b). Understanding coherence and integration in integrated STEM curriculum. *International Journal of STEM Education*, *8*(2), 1-22. https://doi.org/10.1186/s40594-020-00259-8

Siemon, D., Banks, N., & Prasad, S. (2019). Multiplicative thinking: A necessary STEM foundation. In T. Barkatsas, N. Carr, & G. Cooper (Eds.), *STEM education: An emerging field of inquiry* (pp. 74–100). Koninklijke Brill.

Snyder, M. (2018). A century of perspectives that influenced the consideration of technology as a critical component of STEM education in the United States. *The Journal of Technology Studies*, *44*(2), 42–57.

Spreitzer, C., Krainer, K., Muller, F. H., & Kollosche, D. (2024). *Integrated STEM education in Austria: Curriculum and evaluation.* Paper presented at the fifteenth International Congress of Mathematics Education conference (ICME-15). Sydney, Australia.

Sri Padmi, R., Rahmawati, U., & Danoebroto, S. W. (2024). *Disability friendly environment with ramp: Integrated STEM activity for 8th grade mathematics classroom*. Paper presented at the fifteenth International Congress of Mathematics Education conference (ICME-15). Sydney, Australia.

Steffensen, L. (2024). *A task about MarshLands and the associated controversies: ENGAGing students as responsible citizens*. Paper presented at the fifteenth International Congress of Mathematics Education conference (ICME-15). Sydney, Australia.

Stohlmann, M. (2018). A vision for future work to focus on the "M" in integrated STEM. *School Science and Mathematics*, *118*, 310–319. https://doi.org/10.1111/ssm.12301

Tytler, R., Anderson, J., & Williams, G. (2023). Exploring a framework for integrated STEM: Challenges and benefits for promoting engagement in learning mathematics. *ZDM – Mathematics Education*, 55, 1299-1313. https://doi.org/10.1007/s11858-023-01519-x

Tytler, R., Mulligan, J., White, P. J., & Kirk, M. (2024). Promoting effective interactions between mathematics and science: Challenges of learning through interdisciplinarity. In Y. Li, Z. Zeng & N. Song (Eds.), *Disciplinary and interdisciplinary education in STEM: Changes and innovations* (33-62). Springer.

Tytler, R., Williams, G., Hobbs, L., & Anderson, J. (2019). Challenges and opportunities for a STEM interdisciplinary agenda. In B. Doig, J. Williams, D. Swanson, R. Borromeo, P. Ferri, & Drake (Eds.), *Interdisciplinary mathematics education: The state of the art and beyond* (pp. 51–81). Springer.

Wang, H., Moore, T. J., Roehrig, G. H., & Park, M. S. (2011). STEM integration: Teacher perceptions and practice. *Journal of Pre-College Engineering Education Research*, *1*(2), 1-13. https://doi.org/10.5703/1288284314636

Watson, J., Fitzallen, N., & Chick, H. (2020). What is the role of statistics in integrated STEM education? In J. Anderson & Y. Li (Eds.), *Integrated approaches to STEM education: An international perspective* (pp. 91-115). Singapore: Springer Nature.

Watson, S., Williams-Duncan, O. M., & Peters, M. L. (2022). School administrators' awareness of parental STEM knowledge, strategies to promote STEM knowledge and student STEM preparation. *Research in Science & Technological Education*, *40*(1), 1-20. https://doi.org/10.1080/02635143.2020.1774747

Way, J., Cartwright, K., Bobis, J., Anderson, J., McMaster, H., & Attard, C. (2024). Yesterday, today and tomorrow in mathematics education research. In. C. Mesiti, W. T. Seah, B. Kaur, C. Pearn, A. Jones, S. Cameron, E. Every & K. Copping (Eds.)., *Research in mathematics education in Australasia 2020-2023* (pp. 13-28). Springer.

Way, J., Preston, C., & Cartwright, K. (2022). STEM 1, 2, 3: Levelling up on primary schools. *Education Sciences*, *12*(11), 827. https://doi.org/10.3390/educsci12110827

Wieselmann, J. R., Dare, E. A., Ring-Whalen, E. A., & Roehrig, G. H. (2020). "I just do what the boys tell me": Exploring small group student interactions in an integrated STEM unit. *Journal of Research into Science Teaching*, *57*: 112-144. https:// doi.org/ 10.1002/tea.21587

MATHEMATICAL PROBLEM POSING: PROGRESS AND POTENTIAL

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Based on a Problem-Posing Based Learning (P-PBL) instructional model, this paper provides an overview about the progress on research in the teaching and learning of mathematics. The model provides a guide to study each step from the design of problem-posing situations and prompts to understanding problem-posing processes as well as how teachers should handle students' posed problems in the classroom. The P-PBL instructional model is also a way show the implementation of problem posing in the classroom and its instructional flow. The progress of problem-posing research is presented along each component. The paper ends with a discussion of six areas of research to realize the potential of mathematical problem posing.

In 1993, Edward Silver was invited to deliver a plenary at PME 17 titled "On Mathematics Problem Posing." The paper based on the plenary was then published in *For the Learning of Mathematics* (Silver, 1994). In his plenary, he discussed six themes of problem posing as: (1) a feature of creativity or exceptional mathematical ability, (2) a type of inquiry-oriented instruction, (3) a prominent feature of mathematical activity, (4) a means to improving students' problem solving, (5) a window into students' mathematical understanding, and (6) a means to improve students' disposition toward mathematics. There are three conceptions of problem posing: Problem posing as a learning goal, cognitive activity, and instructional approach (Cai, 2022). Silver's Theme 1 is primarily related to problem posing as a learning goal; Themes 3 and 5 are primarily related to problem posing as a cognitive activity; and Themes 2, 4, and 6 are primarily related to problem posing as an instructional approach. As there were very few empirical studies in mathematical problem posing, the plenary and subsequent paper set up a research agenda for problem-posing research. There have been nearly 1,800 Google Scholar citations by March 15, 2025; Figure 1 shows the number of citations in each year from 1994 to 2024. The conventional literature refers to journal papers and book chapters, whereas the grey literature includes theses, conference papers, working papers, and so forth.

Reflecting the growth of problem-posing research as well as the foundational nature of Silver (1994), there has been a clear increase in the number of publications citing Silver over the past 30 years, with the increase accelerating from 2009 to 2022 (Hwang et al., in press). This blooming of problem-posing research has *also* been reflected in journal special issues (Cai et al., 2024; Cai & Hwang, 2020; Cai & Leikin, 2020; Singer et al., 2015), books (e.g., Cai & Leikin, in press; Felmer et al., 2016; Sarikaya et al., 2023;

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, pp. 29-53. PME.

Singer et al., 2015; Toh et al., 2023), and meta-analyses (e.g., Ran et al., in press).

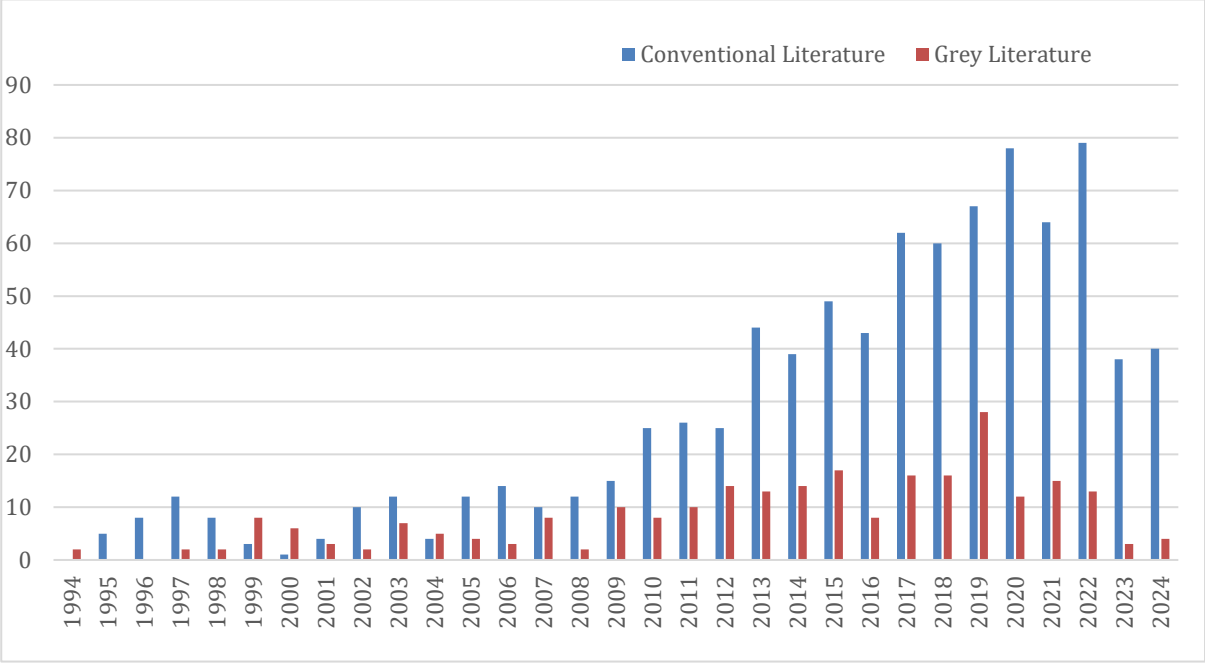


Figure 1: Distribution of Publications Citing Silver (1994) by Publication Year and Type (1994-2024)

In this plenary, I provide an overview of problem-posing research with a focus on its progress and potential since Silver (1994). To provide some context, I first briefly describe the history of the teaching and learning of mathematical problem posing, following which I lay out three important ideas: problem posing, problem-posing tasks, and the problem posing-based learning (P-PBL) instructional model. In this paper, the P-PBL instructional model will serve as a basis for providing an overview of the progress and potential of problem-posing research. There are three reasons for my use of the P-PBL instructional model in this manner.

The first reason is my firm belief that research should inform practice. Cai et al. (2017) noted that "educational researchers have lamented the failure of educational research to have a transformative effect on educational practice despite repeated reform efforts" but posited that the "first step is to better understand the fundamental reasons for the divide between research and practice—in other words, to define the problem" (p. 2). Using this P-PBL instructional model is a way to not only show the implementation of problem posing in the classroom and its instructional flow but also to call for research to inform practice along the model's various components.

The second reason is that this model can be used to conduct research. The model provides a guide to study each step from the design of problem-posing situations and prompts to understanding problem-posing processes as well as how teachers should handle students' posed problems in the classroom. Research needs to be conducted along each component.

Finally, the third reason is that the model provides a guide for actual implementation of problem posing in the classroom. The instructional model itself is an integrated model including all the necessary steps for teachers to implement problem-posing instruction in the classroom. Steps 1 and 2 are in the planning stage of a lesson and Steps 3 and 4 are in the delivery stage of the lesson.

A BRIEF HISTORY OF THE TEACHING AND LEARNING OF MATHEMATICAL PROBLEM POSING

There had been problem-posing work, though sparse, before Silver's plenary (Silver, 1994). Pólya's (1945) framework emphasizes "looking back" to generate new problems, whereas Butts (1980) highlighted the impact of problem formulation on motivation and conceptual understanding. However, problem posing did not gain widespread attention in mathematics education until Brown and Walter's *The Art of Problem Posing* (1983), which introduced pedagogical techniques like the "What-If-Not" strategy to encourage students to create their own problems. They emphasized the importance of re-positioning students as problem *posers*, not only as problem *solvers* (Han & Cai, in press), and they provided specific examples of how to pose mathematical problems for students, demonstrating the What-If-Not strategy as one of the most comprehensive and feasible approaches for those who want to become better problem posers.

Kilpatrick (1987) further advanced the field by advocating for research on how problems are formulated rather than just solved, arguing that problem posing should be both an instructional goal and approach. He observed that most students would say that the only sources of mathematics problems are teachers and textbooks. In K-12 education, students rarely have opportunities to pose their own problems and then solve them. In discussing Kilpatrick's impact on problem-posing research, Silver (in press) acknowledged Kilpatrick's historical milestone in research related to problem posing as well as the personal impact on Silver's problem-posing research. The impact was also felt in the National Council of Teachers of Mathematics (NCTM) curriculum standards. By the late 1980s, problem posing had become an explicit focus in mathematics curricula, and NCTM incorporated problem posing into their 1989 and 1991 standards, promoting it as a means to deepen student learning. This shift reflected increased increased research interest both in the United States and worldwide (Christou et al., 2005; Ellerton, 1986; English, 1997; Tichá & Hošpesová, 2013). As problem posing gained recognition, it became an essential component of mathematics education, influencing both curriculum development and pedagogical considerations worldwide.

If the work of Brown and Walter (1983) drew our attention to using problem posing to teach mathematics in the classroom and the work of Kilpatrick (1987) called for problem-posing research and explicitly for posing problems as an object of research, then the work of Silver (1994) mentioned above set a research agenda for problem-posing research (Cai et al., 2023).

PROBLEM POSING, PROBLEM-POSING TASKS, AND THE P-PBL INSTRUCTIONAL MODEL

Problem posing refers to the process of generating new problems or reformulating existing ones (Cai & Hwang, 2020). Unlike problem solving where students are expected to solve problems given to them by teachers, problem posing shifts the emphasis to students' creative and explorative agency, using the information provided and their prior knowledge to devise problems themselves. The problem-posing process has been shown to foster students' creativity (Ayvaz & Durmuş, 2021; Bicer et al., 2020; Khalid et al., 2020), deeper mathematical understanding (Cai & Hwang, 2002; Kapur, 2015), and problem-solving skills (El Sayed, 2002; Hirashima et al., 2008; Suarsana et al., 2019). Research has also shown that students of all levels of mathematical proficiency are capable of posing mathematical problems, making this a low-floor and high-ceiling activity (Cai & Hwang, 2021).

In a party, guests enter when the doorbell rings.

The first time the doorbell rings, 1 guest enters.

The second time the doorbell rings, 3 guests enter.

The third time the doorbell rings, 5 guests enter.

The fourth time the doorbell rings, 7 guests enter.

The pattern keeps on going in the same way.

On the next ring a group enters that has 2 more persons than the group that entered on the previous ring.

<table>
<tr><th>Problem-solving task</th><th>Problem-posing task</th></tr>
<tr><td rowspan="4">A. How many guests will enter on the 10th ring? Explain or show how you found your answer.
B. How many guests will enter on the 100th ring? Explain or show how you found your answer.
C. 299 guests entered on one of the rings. What ring was it? Explain or show how you found your answer.
D. Write a rule or describe in words how to find the number of guests that entered on each ring.</td><td>For his student's homework, Teacher Li wanted to make up three problems based on the above situation: an easy problem, a moderate problem, and a difficult problem. These problems can be solved using the information in the situation. Help Teacher Li make up three problems and write these problems in the space below.</td></tr>
<tr><td>The easy problem:</td></tr>
<tr><td>The moderately difficult problem:</td></tr>
<tr><td>The difficult problem:</td></tr>
</table>

Figure 2: The Doorbell Task.

Problem-posing tasks include both a problem-posing situation and a prompt. The problem-posing situation is the context in which problem posing is to take place. For example, consider the following Doorbell task context from Cai and Hwang (2002; see

Figure 2). The left side is a problem-solving task and the right side is a problem-posing task. In the problem-posing task, the problem-posing prompt is the part of the task that gives explicit direction or guidance for what students are expected to do during problem posing. The prompt in this example asks students to pose three mathematical problems: an easy, moderately difficult, and difficult problem. However, problem-posing prompts can vary considerably, ranging from general requests like "Pose as many problems as you can think of" to more specific requests to pose problems similar to or different from the given context or to pose a series of problems varying in difficulty levels.

The P-PBL instructional model (Cai, 2022; see Figure 3) outlines the four steps involved in teaching through problem posing: (a) the teacher introduces a problem-posing situation; (b) the teacher provides a problem-posing prompt alongside the situation; (c) students pose problems individually or in groups; and (d) teachers address the posed problems in alignment with learning objectives, guiding students in solving selected problems. The P-PBL model applies to the use of one problem-posing task.

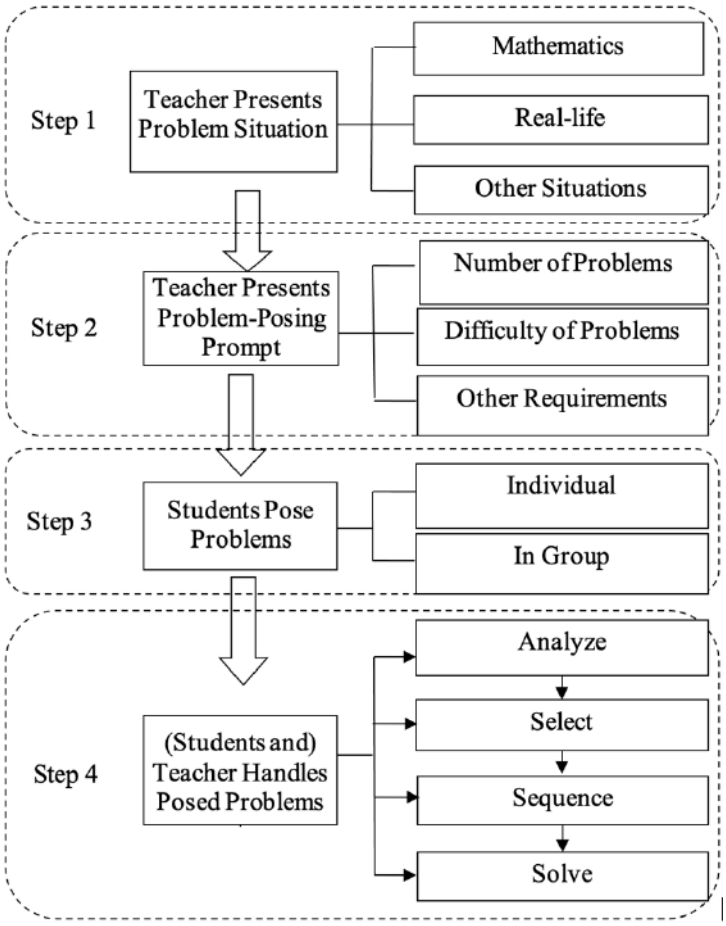


Figure 3*:* The P-PBL Instructional Model (Cai, 2022).

The first two steps, the teacher introducing the problem-posing situation and the teacher providing a problem-posing prompt with the situation, are typically presented together; however, they are intentionally separated here to highlight the significance of

considering both the situation and the prompt when designing problem-posing instruction. Step 3 involves students actively engaging in problem posing, either independently or collaboratively, and Step 4 focuses on how teachers handle posed problems in the classroom. This model shows an instructional flow when implementing problem posing in the classroom. It also serves as a guide for empirical studies. In the following sections, I use this model to show the progress and potential of research related to the teaching and learning of mathematics.

PROBLEM-POSING SITUATIONS

Different task formats can potentially influence the products of problem posing (Rosli et al., 2013; Yao et al., 2021). Task formats can vary based on a variety of factors: the context (e.g., realistic or mathematical), openness in terms of how much structure and information is provided to problem posers, the difficulty levels, number of strategies and types of reasoning invoked, the order of tasks presented, the presence of graphical or diagrammatic information, the amount of information provided, and so on. Yao et al.'s (2021) study of preservice teachers' conceptual understanding of fraction division through graphical representation and problem posing found that the provision of a conceptual cue related to the meaning of fraction division increased the likelihood of the preservice teachers' exhibiting conceptual understanding of this important topic.

More specifically, Yao et al. investigated preservice teachers' mathematical understanding using the division expression 1¾ ÷ ½, chosen for its conceptual and computational demands. Participants completed four tests that differed in task order and the inclusion of a conceptual cue. Each group of preservice teachers completed six fraction division computation problems, a drawing problem-solving task, and a problem-posing task. Group 1 completed the drawing task first, followed by the problem-posing task, whereas Group 2 completed them in reverse order. Group 3 received a conceptual cue with the drawing task first ("1¾ ÷ ½ means how many one-halves are in 1¾") to test its effect on conceptual understanding, whereas Group 4 received the conceptual cue in reverse order. Although all problem-posing tasks seemed to elicit the preservice teachers' conceptual understanding, a large number failed to exhibit conceptual understanding even when they performed the procedural calculations correctly. The results also showed that the inclusion of a conceptual cue significantly improved preservice teachers' ability to exhibit conceptual understanding in their drawings, with 79% of Groups 3 and 4 showing conceptual understanding compared to only 17% and 28% in Groups 1 and 2. Task order did not significantly impact the type of understanding exhibited in the drawings. Regarding the problem-posing task, most preservice teachers across all groups demonstrated conceptual understanding through at least one posed problem, with Group 3, which received the cue before the task, showing the highest rate of conceptual understanding.

Other studies have analyzed specific characteristics of problem-posing situations and contexts (Baumanns & Rott, 2024; English, 1997, 1998; Leung & Silver, 1997;

Walkington & Bernacki, 2015; Silber & Cai, 2017; L. Zhang et al., 2022). Table 1 shows some of these characteristics and the major findings from these studies.

Study	*Problem-posing situations*	*Major findings*
Baumanns & Rott (2024)	*Unstructured versus structured, open versus closed, patterns versus geometry contexts*	*"The influence of the situation and prompt is small and topic-specific . . . [and] self-efficacy is significantly lower in unstructured situations with an open prompt than in the other tasks." (p. 1)*
English (1998)	*Formal (i.e., using mathematical symbolism such as 12 – 8 = 4) versus informal (i.e., free of mathematical symbolism such as "Bob has 5 apples, which is 4 more than Karen. How many apples does Karen have?") contexts*	*"The children interpreted the formal number sentences in terms of the basic change and part-part-whole situations and were reluctant to pose problems that did not correspond to the semantic structure of a sentence." (p. 98)* *"Informal contexts promoted greater diversity in problem type, with children posing some compare and equalize problems. . . . Of the informal contexts, the non-goal-specific situations appeared to be the least effective in encouraging problem diversity." (p. 99)*
English (1997)	*Routine (computational / procedural) versus non-routine (involving reasoning processes such as deduction)*	*Nonroutine problems were the "most and least favored by the children. They avoided the combinatorial problems because they considered them to be too structurally complex or too time consuming…[and] gravitated towards the deductive problems, which contained more information and demanded more complex reasoning processes… It was the structural elements of the problems that appeared to influence their choices." (p. 209)*
Leung & Silver (1997)	*Presence or absence of specific numerical information*	*"Problem-posing performance was better when the task contained specific numerical information than when it did not" (p. 5)*
Walkington & Bernacki (2015)	*Involvement of personalized information (i.e., pertaining to*	*"An intervention that combines personalized problem-solving and problem-posing has the potential to increase*

	students' out-of-school interests and hobbies)	*students' interest in and perception of the utility of learning mathematics." (p. 186)*
Silber & Cai (2017)	*Free (open mathematical situation) versus structured (require poser to match predetermined operations of given situation)*	*"While the task format had limited impact on the complexity of problems posed, PSTs in the structured-posing condition may have more closely attended to the mathematical concepts in each task, and may have also impacted their process of posing problems than those in the free posing condition." (p. 1)*
L. Zhang et al. (2022)	*Presence or absence of specific numerical information, with or without context*	*"Students' performance was better when the task contained specific numerical information (i.e., the task with context) than when it did not (the task without context) during the problem-posing stages of understanding tasks and constructing problems." (p. 497)*

Table 1: Findings From Studies Analyzing Different Characteristics of Problem-Posing Situations

Based on the studies in Table 1, structured tasks with specific numerical information have been shown to improve problem-posing outcomes, as students find them easier to navigate and construct meaningful problems (Leung & Silver, 1997; L. Zhang et al., 2022). Whereas informal contexts encourage diverse problem types, students tend to rely on familiar mathematical structures in formal settings (English, 1998). Additionally, nonroutine problems requiring deeper reasoning processes evoke mixed reactions—students often favor deductive challenges but shy away from combinatorial ones due to their perceived complexity (English, 1997).

Personalization plays a crucial role in fostering interest and perceived usefulness of mathematics, as integrating students' hobbies and real-world contexts enhances motivation (Walkington & Bernacki, 2015). However, the effect of problem openness varies by ability level—high-modelling potential tasks increase interest for students with lower mathematical competence but reduce engagement for those with higher competence (Krawitz et al., 2024). Structured problem-posing conditions also help students focus on mathematical concepts, potentially guiding their thought processes more effectively than open-ended approaches (Silber & Cai, 2017). Collectively, these findings suggest that problem-posing tasks should balance structure and openness while incorporating personalization to optimize student engagement and learning.

PROBLEM-POSING PROMPTS

Problem-posing prompts can elicit different cognitive and affective responses. Researchers have studied the effect of the openness of prompts, meaning the degree to which they provide scaffolding and guidance to the problem posers, on problem-posing performance and affect (Cai et al., 2024). Cai et al. (2023) studied the impact of using prompts that asked problem posers to pose a few different problems, some with and others without additional requirements. The Pose-EMD prompt asked students to pose easy, somewhat difficult, and difficult problems, whereas the Pose-SD prompt asked students to pose problems that were similar to and different from a provided sample problem. For example, Table 2 shows the Animal Task problem situation, which was used in Cai et al. (2023) with two different prompts: *One problem that can be answered based on the information in the table below is, "How many times as fast is a spider compared to a tortoise?"*

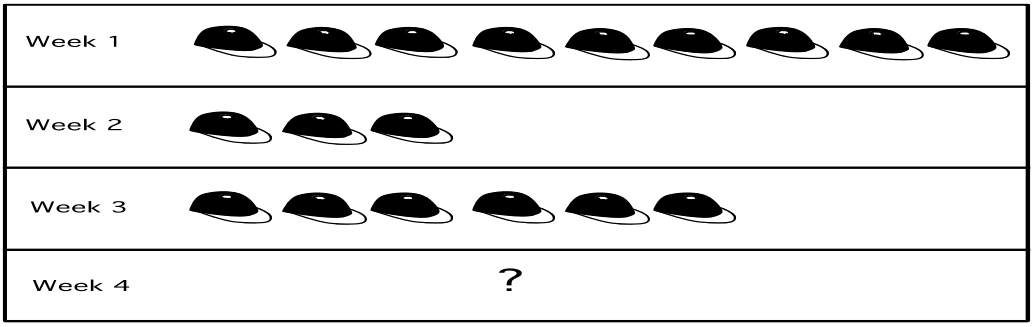
Animal	Crawling speed
Snail	50 meters per hour
tortoise	1000 meters per hour
Spider	6000 meters per hour

Table 2*:* Animal Task Situation.

In this situation, a sample problem was given. Two different prompts were used with this situation. The first was to pose "two more problems that can be solved using the information in the table" (Pose-2). The second prompt was to pose "a similar problem and a different problem that can be solved using the information in the table" (Pose-SD). Although there were no differences across groups in terms of number of responses, students tended to pose linguistically and semantically complex problems in response to the problem-posing prompt specifying additional requirements, suggesting that students were prompted to explore the mathematical relationships more deeply when asked to pose problems that were different from the sample problem.

Chen et al. (2024) extended the work of Cai et al. (2023) by using three prompts with four problem-posing tasks involving the arithmetic average. One of the tasks was the following Hat Average problem-posing task (Figure 4).

Angela is selling hats for the Mathematics Club. This picture shows the number of hats Angela sold during the first three weeks. After Angela sells some hats in Week 4, the <u>average</u> number of hats sold is 7.



Prompt A: Pose three different mathematical problems that can be solved based on the situation.

Prompt B: Pose one easy problem, one moderately difficult problem, and one difficult problem that can be solved based on the situation.

Prompt C: Pose three different mathematical problems based on the situation to challenge your classmates to solve.

Figure 4: Hat Average Problem-Posing Task.

A group of sixth-grade students were randomly assigned to one of the three prompts. All students in the three prompt conditions were able to pose appropriate problems based on the situations. However, students in the Prompt B and Prompt C conditions were likely to pose more problems involving statistics. Students in the Prompt C condition were likely to pose application problems of the average in a statistical sense—that is, the statistical understanding of the arithmetic average—whereas students in the Prompt A condition tended to pose more problems involving a procedural understanding of the arithmetic average and Prompt B students had a higher percentage of posed problems involving a conceptual understanding of the arithmetic average.

Han et al. (2024) conducted interviews with middle school teachers to probe their views of prompts with and without additional specifications, in particular a prompt that merely asked for students to pose three problems without additional requirements (Prompt A) and the abovementioned Pose-EMD prompt (Prompt B). In their responses, the teachers focused on different characteristics of problem-posing prompts, such as the prompts' accessibility and promotion of critical thinking. Although teachers thought Prompt A, being more open and less specific, would be more accessible to students, they acknowledged that Prompt B had greater potential for promoting critical thinking as well as its greater scaffolding in providing explicit direction for students.

Despite their potential accessibility, unstructured or open-ended prompts might also lower self-efficacy, particularly when there is a lack of clear guidance (Baumanns & Rott, 2024). In a study with preservice teachers, Baumanns and Rott (2024) tested eight problem-posing tasks mixing different features of situations (unstructured vs. structured), prompts (open vs. closed), and contexts (patterns vs. geometry). They found that teachers' self-efficacy was significantly lower for unstructured situations

with the open prompt. Thus, confidence in problem posing is perhaps dependent on prior experience with the activity, potentially suggesting that greater guidance and scaffolding might modulate a problem poser's developing sense of self-efficacy.

STUDENTS POSING PROBLEMS

Based on students' posed problems, researchers have inferred the processes of problem posing (Cai et al., 2024). For example, Silver and Cai (1996) found that students tend to pose related problems. Some researchers have directly investigated problem-posing processes either individually or in groups (see a review in Cai & Rott, 2024). Recently, Cai and Rott (2024) proposed a general problem-posing process model which has four components: Orientation, Connection, Generation, and Reflection (Figure 5).

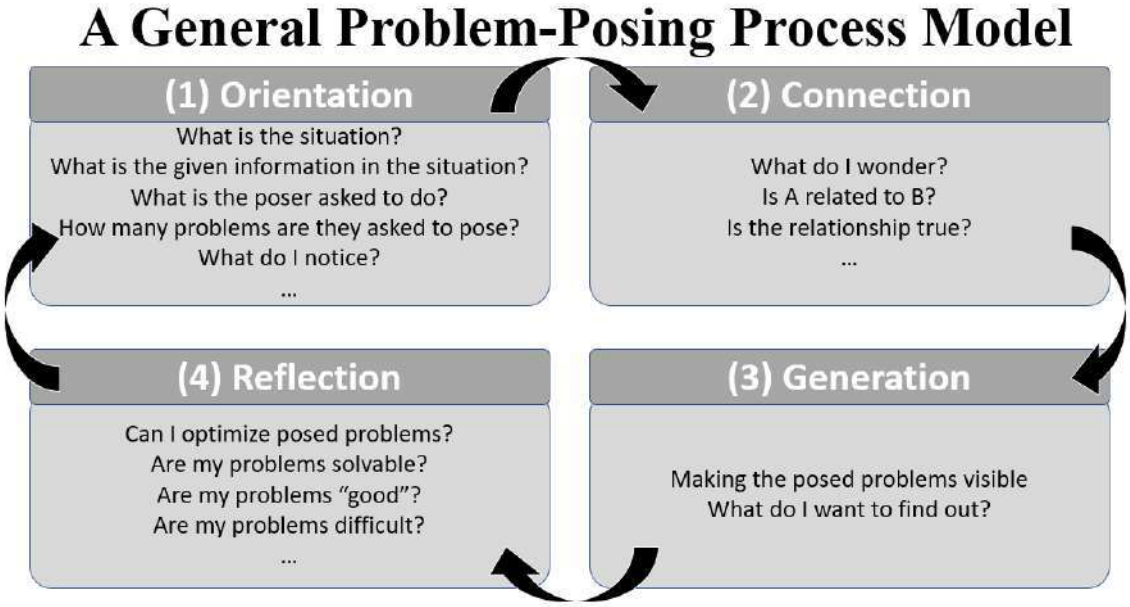


Figure 5: Graphical Representation of a General Problem-Posing Process Model (from Cai & Rott, 2024).

Orientation

Orientation refers to the process of grasping a problem-posing task, aligning with Pólya's initial stage of understanding a given problem. As previously mentioned, a problem-posing task consists of both a situation and a prompt (Cai & Hwang, 2023). The situation can be mathematical or related to real life, and the objective of this phase is to comprehend both elements. Several studies, including L. Zhang et al. (2022), have acknowledged the significance of this stage. During orientation, problem posers may consider questions such as:

What is the situation?

What is the given information in the situation?

What is the poser asked to do (prompt)?

How many problems are they asked to pose?

What do I want to find out?

What relationships are involved in the task?

For instance, in the graph-based task below (see Figure 6), various factors can be analysed, such as time periods, regions, and the number of orphans. This phase involves interpreting the data and understanding the requirements for posing problems.

The graph below shows the number of orphans whose parents died due to AIDS in 2002 and 2010 in the entire world, in Africa, and in Asia. Pose five different mathematical problems that can be solved based on the data in the graph (you don't have to solve them).

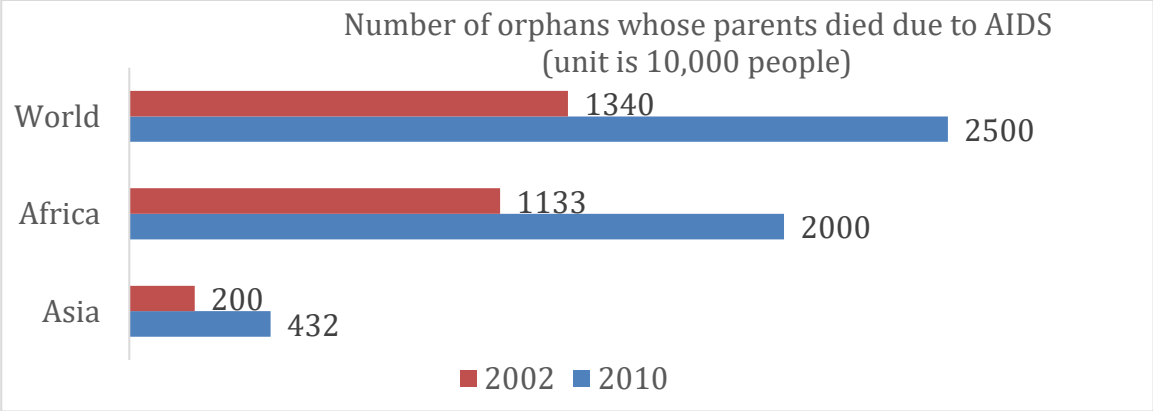


Figure 6: Example of Graph Problem-Posing Task.

Another example involves a function-based task (see Figure 7). In this case, the orientation phase focuses on understanding the linear equation, including what the variables x and y represent as well as the significance of the constants 5 and 6. The task requires not only formulating a real-world scenario based on the function but also generating related mathematical problems.

Given a linear function y = 5x + 6, what real-life scenarios can you think of which can be represented by the function? Please write down one real-life scenario.
Based on your real-life scenario, pose two different mathematical problems that can be solved using y = 5x + 6.

Figure 7: Example of Linear Function Problem-Posing Task.

Connection

The second stage, connection, involves exploring relationships within the problem-posing task. After developing an initial understanding, problem posers begin to consider connections and formulate hypotheses. They might ask themselves:

What do I wonder?

Is A related to B?

Is the relationship true?

Connections can take different forms, such as linking mathematical representations to real-world contexts, as seen in the linear function example. In the graph-based task (Figure 5), connections may involve analysing time trends, regional differences, or variations in the number of orphans. A potential question could be: "Has the number of orphans in Africa increased or decreased compared to Asia between 2002 and 2010?"

For the function-based task, connections relate to real-world applications and the mathematical representation of the function, including concepts like slope and intercept. Additional information might be incorporated to deepen connections, such as predicting the number of orphans in 2022 based on past trends.

Generation

The third phase, generation, involves externally representing the connections identified in the previous stage. Whereas orientation and connection are largely internal processes, generation requires articulating and structuring problems so that others can engage with them. Similar to the execution phase in problem solving, this stage is where the poser formulates specific problems.

In some cases, problem posers may frame their questions as conjectures that need to be tested or proven. For instance, in the graph task, they might create questions that highlight relationships or trends in the data, allowing others to explore and analyse them.

Reflection

The final phase, reflection, involves reviewing and refining the posed problems. This process can occur at different points—after generating a single problem, when transitioning between problems, or upon completing the entire set of problems. Similar to the reflection phase in problem-solving (e.g., Schoenfeld, 1985), this step serves as a metacognitive check on the overall problem-posing process.

By reflecting on their work, problem posers can assess their approach and make adjustments where needed (Jung & Magiera, 2021). This stage helps ensure clarity, relevance, solvability of posed problems, and completeness in the problems they generate.

Focus-Group Discussion of the General Problem-Posing Process Model

To evaluate the appropriateness of the four phases of the problem-posing process, I conducted two focus-group discussions with three mathematics education researchers in each group. In the first group, one was very familiar with problem posing and had been involved in problem-posing research for the preceding 15 years; the other two were post-docs working on a problem-posing project and had only just begun work in problem posing over the preceding several months. In the second focus group, one had been involved in problem-posing research for 5 years and the other two had started problem-posing research about 2 years prior.

Group 1 participants only received the graph task (Figure 6), whereas Group 2 participants received both the graph task and the linear function task (Figure 7). Each group was asked to pose mathematical problems related to each task with no time limit. Participants took 10 minutes and 8 minutes for the first and second tasks, respectively. Each member in each group explained their problem-posing process, following which the four phases of the problem-posing process were reviewed, and the intent of the focus-group discussion was provided. The verification of the four problem-posing process phases was based on two perspectives: the focus-group participants' initial discussion of their problem-posing process and, after being told the four phases, their reflection on their processes and the appropriateness of the four phases to describe their thinking process. Both focus groups clearly confirmed the appropriateness of the four phases of the problem-posing process. The following is from one participant with respect to the graph task:

> So to me, like once I read it, I need to pose the five different problems. So I saw the graph, and I see the differences. So I thought like, Oh, I can make some problems just simply saying like it was the difference between 2010 and 2002 for each of these three [locations]. So I simply made it three questions. And then once I noticed the differences, and then I was thinking like, Oh, then, what's the What's the country? Or which part of the world um, as the most orphans' parents died to AIDS, and then the last one was like the rate regarding which one grows faster than. I am simply done with the five questions.

Opportunities to Pose Problems Individually

When teachers initially learn to teach mathematics through problem posing, students are often quickly called to "shout out" the kinds of problems they pose right after a problem-posing task is presented. In this practice, the problem posing may involve only a small number of students in the classroom, leaving the majority of the students without enough time to think through possible problems. When implementing problem posing in the classroom, it is recommended that teachers give each student opportunities to pose problems individually. This is because learning is an individual knowledge-construction process, and doing so allows students to use their own interpretation of the problem-posing task as well as their knowledge and background experience. Given opportunities to engage in thinking mathematically through problem posing individually, it is possible that students may pose quite different problems. The learning environment of P-PBL provides a natural setting for students to each present various posed problems to their group or class and learn mathematics through social interactions, meaning negotiation, thereby reaching shared understandings.

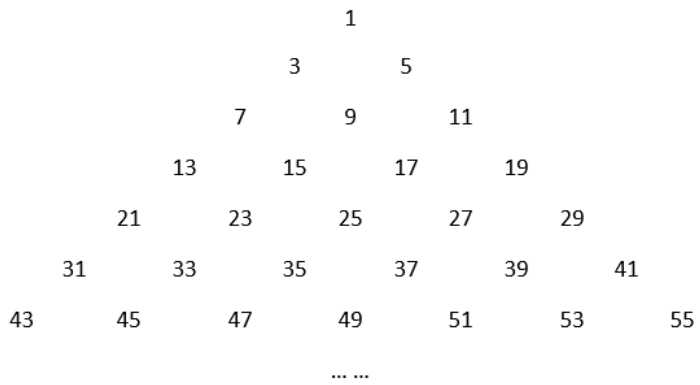
HANDLING STUDENTS' POSED PROBLEMS

One of the biggest challenges in P-PBL is handling students' posed problems. Although there has been wide recognition of the promise of P-PBL, little attention has been paid to the specifics of handling students' posed problems (Bonotto, 2013; Cai et al., 2024; Cankoy, 2014; Toluk-Uçar, 2009; H. Zhang & Cai, 2021). In problem-solving research, there has been extensive discussion about how to handle various

strategies employed by students when they solve problems (Cai, 2003; Smith & Stein, 2011). The main idea is to discuss, compare, and contrast the various strategies among groups or as a whole after students present their individual solution strategies. In recent years, I have engaged in research about how to handle students' posed problems in the classroom.

In 2021, we analysed 22 P-PBL teaching cases published in Chinese teaching journals (Zhang & Cai, 2021). A teaching case includes major elements of a lesson and related analysis, but it is not a transcribed lesson. Teaching cases include narratives describing instructional tasks and related instructional moves for the tasks. Cases also include information about the thinking underlying major instructional decisions, as well as reflections on and discussions of those decisions. H. Zhang and Cai (2021) found a common teaching flow across the 22 teaching cases: Present a problem-posing task, students pose problems, teachers guide students to solve some of the problems in the class, and end with a summary and reflection. In addition, across the 22 cases, we observed a pattern in how the teachers dealt with students' posed problems. The pattern was based on the teachers' effort to address the instructional goal(s) for the lesson. Teachers mainly focused on the posed problems that were closely aligned with the lesson goals. Among those posed problems closely aligned with the lesson goals, teachers would focus on moderately difficult problems, and some of these problems were selected to be solved during the lesson. For rather easy problems, teachers would quickly go through some of them. For very challenging problems, teachers would often assign them as homework or as instructional tasks to be discussed in the next lesson.

In my own teaching with both preservice and in-service teachers, I have tried to follow the four practices (analyze, select, sequence, and solve) to handle posed problems. The four practices of handling teachers' posed problems are described for preservice teachers in Cai et al. (2015) and for in-service teachers in Cai et al. (2024) using my P-PBL lesson with the Odd Number Pattern task (Figure 8). I briefly recap the four practices here; readers can find details in Cai et al. (2024). I first gave teachers time to individually work on the problem-posing task involving the Odd Number Pattern. After teachers posed their required problems individually, they were asked to discuss their posed problems in groups and then write down their posed problems on the board. If there were responses that overlapped, they only needed to write those problems once. If a response was unclear to other teachers, they would make the response clear. These activities helped teachers to analyze their posed problems.



The pattern continues in the same way.

Figure 8: Odd Number Pattern Task.

Then I selected three problems to solve: (1) The row number is the same as the number of numbers in the row; (2) How can I use a perfect square (the middle of the odd rows) to determine their row number? (3) Find the sum of the numbers in the first 100 rows. The first problem was chosen because almost all teachers had posed this problem and in fact had found the solution already. The fact that the row number is the same as the number of numbers in the row is an important characteristic for the Odd Number Pattern. The second problem was chosen because about one half of the teachers had posed this problem but were puzzled by identifying the middle number when the number of numbers in a row was even, such as the fourth row with four numbers (13, 15, 17, and 19), meaning the middle number is "invisible." The third problem was chosen because the solution to it involves a number of interesting results, such as that the sum of the first m odd numbers is m^2 and the sum of the first n integers is $[n(n+1)/2]$. The solution to the third problem could lead to very interesting results: $[n(n+1)/2]^2 = 1^3 + 2^2 + 3^3 + \ldots + n^3$.

In Ma and Cai (2024), we described a teaching case to illustrate the four practices of a teacher handling students' posed problems in the classroom. The cases involve a review lesson after a unit on percentage. The problem-posing task used was the following: *With the following numbers and symbol, please pose one easy problem and one difficult problem involving percentages: 40; 50; %.* Usually, in reviewing the percentage unit, teachers present different types of percent-related problems for students to solve. In this lesson, the teacher used the problem-posing task to ask students to use 40, 50, and % to pose one easy problem and one difficult problem involving percentages. I mention this case to emphasize not only the importance of analyzing students' posed problems but also the ways of analyzing students' posed problems. In particular, this lesson used two particular approaches to analysing students' posed problems: classification and clarification.

As indicated in Park and Leinhardt (1995), percentage problems can be classified into different categories. In this lesson, students were first asked to explain why their posed problems were easy or difficult. They were then asked to classify percentage problems.

Through classification, students could compare various posed problems and then reach the goal of solving one problem implying solving many problems of the same type. For example, one of the posed problems was as follows: *There are 40 girls in the class, and the number of boys is 50% of the number of girls*. Another posed problem was: *There are 50 boys in the baseball team, and the number of girls is 40% of the number of boys. How many girls are there? How many boys are there?* Students classified these as the same because both problems ask for a percentage of a certain number. By classifying the problems, teachers and students were able to know which type of percentage problems were missing.

In this lesson, one group of students presented the following problem: *There are 40 boxes of bananas in the supermarket, which is 50% more than the boxes of apples. How many boxes of apples are there?* They chose this problem to solve in the lesson and found that the answer was close to 26 but not a whole number. The teacher and students discovered the issue of a non-whole number of boxes of apples. The teacher encouraged the students to modify the problem so that the answer would be a whole number, which makes more sense for a real-life situation. Different students offered modifications, such as the following:

> Student 1: You can change the question to "There are 40 boxes of bananas in the supermarket, and the boxes of apples are 50% more than that of bananas. How many boxes are the apples?" The calculation is $40 \times (1 + 50\%) = 40 \times 1.5 = 60$ (boxes).
>
> Student 2: It can also be "The supermarket has 40 boxes of bananas, which is 50% of the boxes of apples. How many boxes are there of apples?" The calculation is $40/50\% = 80$ (boxes).

Summary of Handling Students' Posed Problems

In summary, how to handle students' posed problems is a very critical step in implementing P-PBL in the classroom. Research so far has already given us some direction for what we need to do. The four practices (analyse, select, sequence, and solve) seem to work well. In this section, I would like to summarize the previous discussion into three main points.

The first is the importance of analysing students' posed problems by students and the teacher. It is very clear that students can pose very different problems. Some students even pose unexpected responses such as statements or irrelevant problems. Thus, the analysis step is critical. Analysis can be based on classification as well as clarification of the problems. Such practices could not only help students structure their problems based on different classifications but also examine the appropriateness of each of their posed problems. Our recent study has shown that even those unexpected responses like nonmathematical problems or unclear responses can be very useful in the classroom (Ran et al., in press). Thus, the analysis stage could help students see how such unexpected responses could be modified to become more desirable mathematical problems.

Second, there is no way that teachers can discuss or solve all the problems students pose. Current research suggests that, based on learning goals, teachers need to select the kinds of problems to be solved. For example, even though a problem could be very interesting and mathematically sound, if the problem is irrelevant to the lesson learning goals, teachers might decide to assign that problem as homework instead of discussing it in the lesson.

Finally, although it sounds natural to sequence the problems from simple to more difficult, it may occasionally make more sense to sequence them from difficult to simpler. In any case, it is very important to select problems to solved. Solving problems not only helps students learn further with respect to related mathematical ideas but also reinforces the problem-posing aspects of the lesson.

FUTURE RESEARCH

The progress in problem-posing research over the past 30 years is extremely encouraging. At both ICME-14 and ICME-15, problem posing has explicitly been part of a topic study group (Problem posing and solving in mathematics education), which was not the case in earlier ICMEs. In order to reach its potential, continuous effort is needed to advance problem-posing research. To end this paper, I list six promising areas of research.

Problem-Posing Situations and Prompts: Design and Implementation

The future of research on problem-posing situations and prompts remains open to exploration in a variety of ways. Specifically, I see four main potentially fruitful areas for research (Cai et al., 2024). First, more research is needed to understand the complex interplay between different task variables and their impact on problem-posing processes and products. In particular, we need to understand how students interpret problem-posing prompts. Second, there is a need to develop a framework for designing problem-posing tasks. Creating a framework for categorizing different types of problem-posing situations and prompts could help educators design more desirable tasks. Currently, there are very few problem-posing tasks in textbooks, making this effort even more important. Third, there is a need for research on integrating problem posing into classroom instruction. Research in this area could inform the development of strategies for incorporating problem posing into mathematics classrooms to enhance student learning and engagement. Finally, there is a need to integrate problem posing into the research on the teaching and learning of mathematical modelling (Cai et al., in press). Cai et al. (in press) documented an unexpected apparent scarcity of problem posing in the mathematical modeling literature (less than 6% of tasks using in modeling research require problem posing) and provided some recommendations to better position problem posing as an integral part of research related to the research on the teaching and learning of mathematical modeling.

Problem-Posing Processes

Whereas problem-solving research has established cognitive and affective frameworks, problem-posing research still lacks a comparable theoretical foundation. Future research on problem-posing processes should focus on gaining a deeper understanding of how students engage with problem-posing tasks by using interviews and advanced technological methods, such as eye tracking, to analyse their cognitive and affective processes. In-depth interviews could help researchers gain a clearer understanding of how students engage with problem-posing tasks by revealing students' reasoning, the connections they make between mathematical concepts, the challenges they encounter during problem formulation, and their affective aspects while engaging with problem-posing tasks. Beyond interviews, incorporating advanced technologies such as eye-tracking methods could further enhance research by offering real-time data on students' cognitive and emotional engagement with problem-posing tasks.

Students' Affect

Future research should explore how problem posing can be effectively integrated into mathematics instruction to foster not just students' cognitive but also affective development. This includes investigating how teachers can explicitly address students' enjoyment, curiosity, and motivation when engaging in problem-posing activities and how such activities can become a natural part of learning rather than merely a task assigned by instructors. The interplay between affect and problem posing as a research tool and instructional goal remains underexplored, particularly in understanding how positive affect can serve as a trigger for mathematical curiosity and problem formulation. Furthermore, studies should examine cognitive and affective mechanisms that contribute to the creation of elegant and original problems and determine whether these mechanisms can be explicitly taught. By integrating cognitive and affective dimensions in both instructional practice and research methodologies, mathematics education can develop more effective strategies for nurturing students' problem-posing skills and intrinsic motivation.

Teachers Learning to Teach Mathematics through Problem Posing

Future research should explore effective ways to support teachers in integrating problem posing into their mathematics instruction. Although studies suggest that simple modifications, such as removing the final question from textbook problems, can transform typical problem-solving tasks into problem-posing tasks, more investigation is needed into how teachers can implement these strategies systematically. Additionally, research should further develop instructional models, such as the P-PBL model mentioned earlier in this paper, to provide structured guidance on incorporating problem posing into other areas of mathematics, such as mathematical modelling. A key area for future study is understanding how teachers learn to make instructional decisions around problem posing and what professional development approaches best

prepare them for this shift. Researchers should also examine the challenges teachers face in adapting problem-posing strategies and how these challenges can be addressed through targeted support and resources.

Measuring Teacher Change in Teacher Learning

To better understand how teachers develop expertise in problem posing, future research should focus on how their instructional thinking and decision-making evolve over time. Investigating how teachers internalize and apply problem-posing strategies in their classrooms can provide insight into the learning process. Studies should examine how different professional learning experiences—such as case-based learning and collaborative reflection—support teachers in refining their instructional practices. Additionally, research should identify key indicators of teacher growth in problem posing, including shifts in their beliefs about mathematics instruction, their ability to assess and respond to student-generated problems, and their confidence in facilitating open-ended discussion in P-PBL setting. Developing robust assessment tools to measure these types of learning will be essential for designing more effective professional development programs. There is also a pressing need for research that rigorously examines how teachers' professional learning in problem posing translates into improved classroom instruction and student achievement.

Impact of Problem-Posing on Students' Learning

Although existing studies suggest that problem posing provides insight into students' understanding, more experimental studies are needed to determine the impact of different dimensions of problem posing on students' learning. Further investigation is needed into how various types of problem-posing tasks—such as structured, semi-structured, and free problem posing—shape students' conceptual development. Research should also examine how problem-posing experiences impact students' ability to generalize mathematical concepts, transfer knowledge across different problem contexts, and develop metacognitive strategies. Additionally, longitudinal studies are needed to determine how sustained engagement in problem posing affects students' overall mathematical proficiency and their attitudes toward mathematics and learning mathematics.

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REFERENCES

Ayvaz, U., & Durmus, S. (2021). Fostering mathematical creativity with problem posing activities: An action research with gifted students. *Thinking Skills and Creativity, 40*, 100846. https://doi.org/10.1016/j.tsc.2021.100846

Baumanns, L., & Rott, B. (2024). Problem-posing tasks and their influence on pre-service teachers' creative problem-posing performance and self-efficacy. *The Journal of Mathematical Behavior*, *73*, 101130. https://doi.org/10.1016/j.jmathb.2024.101130

Bicer, A., Lee, Y., Perihan, C., Capraro, M. M., & Capraro, R. M. (2020). Considering mathematical creative self-efficacy with problem posing as a measure of mathematical creativity. *Educational Studies in Mathematics, 105,* 457–485. https://doi.org/10.1007/s10649-020-09995-8

Bonotto, C. (2013). Artifacts as sources for problem-posing activities. *Educational Studies in Mathematics*, *83*, 37–55.

Brown, S. I., & Walter, M. I. (1983). *The art of problem posing.* Franklin Institute Press.

Butts, T. (1980). Posing problems properly. In S. Krulik & R. E. Reys (Eds.), *Problem solving in school mathematics* (pp. 23–33). National Council of Teachers of Mathematics.

Cai, J. (2003). Singaporean students' mathematical thinking in problem solving and problem posing: An exploratory study. *International Journal of Mathematical Education in Science and Technology, 34*(5), 719–737.

Cai, J. (2022). What research says about teaching mathematics through problem posing. *Éducation et Didactique*, 31–50.

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in U.S. and Chinese students' mathematical problem solving and problem posing. *Journal of Mathematical Behavior, 21*, 401–421.

Cai, J., & Hwang, S. (2020). Learning to teach through mathematical problem posing: Theoretical considerations, methodology, and directions for future research. *International Journal of Educational Research*, *102*, 101391. https://doi.org/10.1016/j.ijer.2019.01.001

Cai, J., & Hwang, S. (2021). Teachers as redesigners of curriculum to teach mathematics through problem posing: Conceptualization and initial findings of a problem posing project. *ZDM–Mathematics Education, 53*(6), 1403–1416. https://doi.org/10.1007/s11858-021-01252-3

Cai, J., Hwang, S., Jiang, C., & Silber, S. (2015). Problem posing research in mathematics education: Some answered and unanswered questions. In F. M. Singer, N. Ellerton, & J. Cai (Eds.), *Mathematical problem posing: From research to effective practice* (pp. 3–34). Springer. https://doi.org/10.1007/978-1-4614-6258-3_1

Cai, J., Koichu, B., Rott, B., & Jiang, C. (2024). Advances in research on mathematical problem posing: Focus on task variables. *The Journal of Mathematical Behavior, 76*, 101186.

Cai, J., & Leikin, R. (in press). Research in mathematical problem posing: New Advances and Directions. Springer.

Cai, J., Su, M., & Hwang, S. (in press). Making problem posing an integral part of the teaching and learning of mathematical modeling. *Journal for Research in Mathematics Education*.

Cai, J., & Leikin, R. (2020). Affect in mathematical problem posing: Conceptualization, advances, and future directions for research. *Educational Studies in Mathematics*, *105*, 287–301.

Cai, J., Morris, A., Hohensee, C., Hwang. S., Robison, V., & Hiebert, J. (2017). Clarifying the impact of educational research on students' learning. *Journal for Research in Mathematics Education, 48*(2), 118–123.

Cai, J., Ran, H., Hwang, S., Ma, Y., Han, J., & Muirhead, F. (2023). Impact of prompts on students' mathematical problem posing. *The Journal of Mathematical Behavior*, *72*, 101087. https://doi.org/10.1016/j.jmathb.2023.101087

Cai, J., & Rott, B. (2024). On understanding mathematical problem-posing processes. *ZDM–Mathematics Education*, *56*(1), 61–71.

Cankoy, O. (2014). Interlocked problem posing and children's problem posing performance in free structured situations. *International Journal of Science and Mathematics Education*, *12*(1), 219–238.

Chen, T., Sun, Q., & Cai, J. (2023). Impact of prompts on the performance of elementary school students' mathematical problem posing involving arithmetic average. *Journal of Mathematics Education, 5,* 28-34 (In Chinese).

Christou, C., Mousoulides, N., Pittalis, M., Pitta-Pantazi, D., & Sriraman, B. (2005). An empirical taxonomy of problem posing processes. *ZDM—Mathematics Education, 37*(3), 149–158.

Ellerton, N. (1986). Children's made-up mathematics problems: A new perspective on talented mathematicians. *Educational Studies in Mathematics*, *17*(3), 261–271.

El Sayed, R. A. E. (2002). Effectiveness of problem posing strategies on prospective mathematics teachers' problem solving performance. *Journal of Science and Mathematics Education in Southeast Asia, 25*(1), 56–69.

English, L. D. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics, 34*(3), 183–217. https://doi.org/10.1023/A:1002963618035

English, L. D. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education*, *29*(1), 83−106. https://doi.org/10.5951/jresematheduc.29.1.0083

Felmer, P., Pehkonen, E., & Kilpatrick, J. (2016). *Posing and solving mathematical problems: Advances and new perspectives.* Springer Cham.

Han, J. P. & Cai J. (in press). Teaching mathematics through problem posing: Contributions of Brown and Walter's book "The Art of Problem Posing" in problem-posing research. In J. Cai & R. Leikin (Eds.), *Research in mathematical problem posing: New Advances and Directions*. Springer.

Han, J. P., Hwang, S., Muirhead, F., & Cai, J. (2024). Exploring middle school teachers' views about problem-posing tasks. *The Journal of Mathematical Behavior, 73*, 101140.

Hirashima, T., Yokoyama, T., Okamoto, M., & Takeuchi, A. (2008, June). An experimental use of learning environment for problem-posing as sentence-integration in arithmetical word problems. In B. P. Woolf, E. Aïmeur, R. Nkambou, & S. Lajoie (Eds.), *The proceeding of 9th International Conference on Intelligent Tutoring Systems* (pp. 687–689). Springer.

Hwang, S., Zhu, Y., & Cai, J. (in press). Footprints through a landscape of mathematical problems: Edward Silver and his research on mathematical problem posing. In J. Cai & R. Leikin (Eds.), *Research in mathematical problem posing: New Advances and Directions*. Springer.

Jung, H., & Magiera, M. T. (2021). Connecting mathematical modelling and social justice through problem posing. *Mathematical Thinking and Learning, 25*(2), 232–251

Kapur, M. (2015). The preparatory effects of problem solving versus problem posing on learning from instruction. *Learning and Instruction, 39*, 23–31. https://doi.org/10.1016/j.learninstruc.2015.05.004

Khalid, M., Saad, S., Hamid, S. R. A., Abdullah, M. R., Ibrahim, H., & Shahrill, M. (2020). Enhancing creativity and problem solving skills through creative problem solving in teaching mathematics. *Creativity Studies, 13*(2), 270–291. https://doi.org/10.3846/cs.2020.11027

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123–147). Erlbaum.

Krawitz, J., Hartmann, L., & Schukajlow, S. (2024). Do task variables of self-generated problems influence interest? Authenticity, openness, complexity, and students' interest in solving self-generated modelling problems. *The Journal of Mathematical Behavior*, *73*, 101129. https://doi.org/10.1016/j.jmathb.2024.101129

Leung, S. S., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal*, *9*(1), 5–24. https://doi.org/10.1007/BF03217299

Ma, Y., & Cai, J. (2024). Teaching mathematics through problem posing: Four practices for handling students' posed problems. *Revemop, 6*, e2024009. https://doi.org/10.33532/revemop.e2024009

Parker, M., & Leinhardt, G. (1995). Percent: A privileged proportion. *Review of Educational Research, 65*, 421–481.

Pólya, G. (1945). *How to solve it*. Princeton University Press.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Author.

Ran, H., Cai, J., Hwang, S., Han, J., Ma, Y., & Muirhead, F. (in press). Effects of engaging in problem-posing interventions on learners' cognitive mathematics outcomes: a comprehensive meta-analysis. *Journal for Research in Mathematics Education*.

Ran, H., Cai, J., Muirhead, F., & Hwang, S. (in press). An analysis of unexpected responses in middle school students' mathematical problem posing from the perspective of problem-posing processes. *Educational Studies in Mathematics.*

Rosli, R., Goldsby, D., & Capraro, M. M. (2013). Assessing students' mathematical problem-solving and problem-posing skills. *Asian Social Science, 9*(16), 54–60.

Sarikaya, D., Baumanns, L., Heuer, K., & Rott, B. (2023). Problem posing and solving for mathematically gifted and interested students: Best practices, research and enrichment. Springer Fachmedien Wiesbaden. https://doi.org/10.1007/9783658410612

Schoenfeld, A. H. (1985). Making sense of "out loud" problem-solving protocols. *The Journal of Mathematical Behavior, 4*(2), 171–191.

Silber, S., & Cai, J. (2017). Pre-service teachers' free and structured mathematical problem posing. *International Journal of Mathematical Education in Science and Technology, 48*(2), 163–184.

Silver, E. A. (in press). Problem formulating as a gateway to problem posing: A retrospective look. In J. Cai & R. Leikin (Eds.), *Research in mathematical problem posing: New Advances and Directions*. Springer.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19–28.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521–539.

Singer, F. M., Ellerton, N. F., & Cai, J. (Eds). (2015). *Mathematical problem posing: From research to effective practice*. Springer.

Smith, M., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. National Council of Teachers of Mathematics.

Suarsana, I., Lestari, I. A. P. D., & Mertasari, N. M. S. (2019). The effect of online problem posing on students' problem-solving ability in mathematics. *International Journal of Instruction*, *12*(1), 809–820. http://doi.org/10.29333/iji.2019.12152a

Tichá, M., & Hošpesová, A. (2013). Developing teachers' subject didactic competence through problem posing. *Educational Studies in Mathematics, 83*(1), 133–143.

Toh, T. L., Santos-Trigo, M., Chua, P. H., Abdullah, N. A., & Zhang, D. (2023). *Problem posing and problem solving in mathematics education: International research and practice trends.* Springer Singapore.

Toluk-Uçar, Z. (2009). Developing pre-service teachers understanding of fractions through problem posing. *Teaching and teacher education*, *25*(1), 166–175.

Walkington, C., & Bernacki, M. (2015). Students authoring personalized "algebra stories": Problem-posing in the context of out-of-school interests. *The Journal of Mathematical Behavior*, *40*, 171–191. https://doi.org/10.1016/j.jmathb.2015.08.001

Yao, Y., Hwang, S., & Cai, J. (2021). Preservice teachers' mathematical understanding exhibited in problem posing and problem solving. *ZDM–Mathematics Education*, *53*(4), 937–949.

Zhang, H., & Cai, J. (2021). Teaching mathematics through problem posing: Insights from an analysis of teaching cases. *ZDM–Mathematics Education, 53*, 1–13.

Zhang, L., Cai, J., Song, N., Zhang, H., Chen, T., Zhang, Z., & Guo, F. (2022). Mathematical problem posing of elementary school students: The impact of task format and its relationship to problem solving. *ZDM–Mathematics Education, 54*, 497–512.

WHAT DOES PREPARING OUR BOYS AND GIRLS TO INTERACT WITH ARTIFICIAL AGENTS MEAN? PERSPECTIVES FROM EARLY STATISTICAL EDUCATION IN TODAY'S CLASSROOM

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In this plenary talk, I propose the development of computational and statistical thinking beginning in the early grades of school. I will present two problem-solving learning sequences that use games and storytelling and are designed to develop each of these modes of thinking. These sequences include tasks that have been carefully planned, piloted, and tested in the classroom. I also propose that lesson study acts as a bridge between research and collaborative practice, enabling us to recognize new paths and confront barriers.

INTRODUCTION

I am honored to give a plenary lecture at PME 48 in my country, along with my colleagues and friends. I have reviewed my recent studies and experiences as a mathematics educator and researcher to prepare this talk. My research areas include teacher education and professional development, textbook development and research, and topics in elementary mathematics, mainly early statistical thinking and, more recently, computational thinking.

Twenty years from now, the world will be radically different, as it will be populated by knowledgeable artificial agents. We will be living in an ecosystem that is diametrically different from anything previously seen in our history. This unpredictable future creates a tremendous educational challenge for us today (Araya, 2024). Given that we have created an entity that is more intelligent and powerful than humans, we face urgent choices today because nonhuman intelligence could threaten our very existence if we do not establish adequate control mechanisms; the repercussions could be devastating, not only for humanity but also for the entire ecological system (Harari, 2024).

In this era of artificial intelligence (AI), it has become necessary to study new teaching and learning processes that allow us to understand how students work with humans and artificial agents and to anticipate and explore these new forms of interaction.

What does engaging with artificial agents mean?

This type of engagement involves interaction with artificial intelligence systems that have been designed to respond, adapt, or collaborate with humans on various tasks.

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These agents can take different forms, such as virtual assistants (Siri, Alexa), chatbots, educational robots, recommendation systems, or generative AI models.

One key aspect of interaction with artificial agents is data-driven interaction and algorithms, which process information and generate responses on the basis of previous data. One example would be an educational chatbot that adjusts its explanations based on student responses.

Communication skills and natural language comprehension are also essential, as many agents use natural language processing to interpret prompts and respond to them in human language. For example, a student who queries ChatGPT about a math concept and receives a structured response.

Other key aspects are decision-making and adaptability. Some AI systems, such as an intelligent tutor that adjusts the difficulty level of exercises on the basis of user performance, are capable of learning and improving through interaction.

Furthermore, artificial agents can influence human decision-making by suggesting options or prompting confident choices, which necessitates the development of critical thinking. An example would be a video recommendation system that reinforces or shapes content consumption patterns.

Finally, the ethical dimension is crucial in this relationship between humans and artificial agents. Understanding the biases, limitations, and social impacts of AI enables a more informed and responsible use of these technologies.

Therefore, preparing children to interact with AI involves more than simply the use of artificial agents. Understanding how these agents work and ensuring their ethical and effective use in different contexts is necessary. Such preparation also requires using AI agents and interpreting their responses critically. To achieve this goal, the development of a critical stance during early childhood education seems essential, as AI generates its responses on the basis of statistical patterns, which may contain errors, biases, or misinterpretations. Recommendation algorithms on social media and internet searches can limit user exposure to different perspectives or interests. AI is based on data, and that data may be incomplete or have been manipulated.

This conference aims to explore how to prepare children to interact with AI, especially the design of playful classroom tasks that foster the development of statistical and computational thinking.

What thoughts do children need to develop to interact critically with AI?

Computational thinking

CT is becoming the new literacy of the 21st century (Wing, 2011). Furthermore, the educational benefits of computational thinking serve to improve intellectual skills and are transferable to any domain (Wing, 2014). In particular, recent studies have shown

that unplugged activities effectively foster this type of thinking during early stages of development (Del Olmo-Muñoz et al., 2020; Su & Yang, 2023; Wu & Su, 2025).

Unlike other approaches that are focused exclusively on programming, the model proposed by Shute et al. (2017)—adopted in this study as a theoretical framework for computational thinking—has an expanded scope through its conception of ways of thinking and acting that transcend the instrumental use of digital tools. This model conceptualizes computational thinking as a multidimensional process that includes decomposition, abstraction, algorithm design, debugging, iteration, and generalization.

We have adapted a framework (see Figure 1) that shows the set of shared concepts of computational thinking (CT) and mathematical thinking (MT): problem solving, modeling, abstraction, algorithms, data analysis and interpretation, and statistics and probability. We have included abstraction and algorithms at the intersection of CT and MT, as they are fundamental to mathematical reasoning and problem solving. Algorithms provide a structured framework for decomposing problems, identifying patterns, and constructing step-by-step solutions, which couple abstract mathematical concepts with actionable computational strategies, where data serve as both the medium and the outcome.

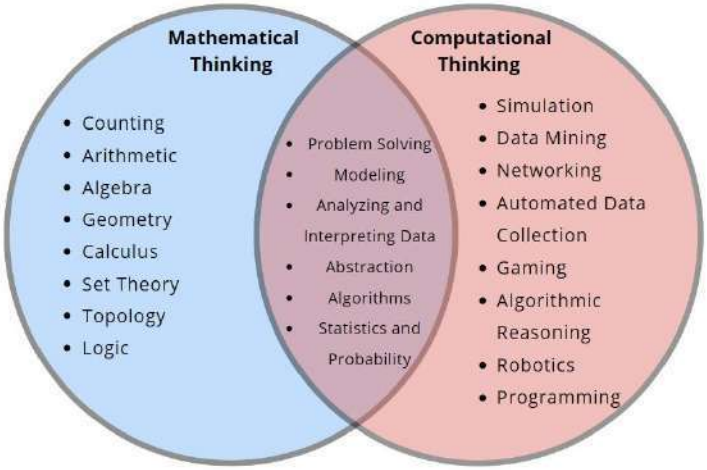


Figure 1: Similarities and differences between CT and mathematical thinking. Adapted from Sneider et al. (2014).

By working with a population of kindergarten and early elementary school students, we were able to focus on certain areas at the intersection of CT and mathematical thinking and to borrow foundational content and procedures from geometry and arithmetic.

About statistical thinking

With respect to statistical thinking, recent research has shown that young children can actively engage in data modeling and express evidence-based inferences (e.g., English, 2012; Estrella et al., 2020, 2023a, 2023b, 2024a, and 2024b; Mulligan, 2015), which challenges the notion that such engagement is not possible in the early school years. We have found that critical reflection and the refinement of data representations strengthen students' metarepresentational competencies, particularly among those who have had the opportunity to develop methods of representing the data they have collected (Estrella et al., 2017; Estrella, 2018). Furthermore, students' critical reflection on and refinement of their data representations enhanced a wide range of metarepresentational competencies, suggesting that student-led statistical work with data can contribute to a more critical stance and the development of comprehensive statistical concepts from an early age (Mulligan, 2015).

Some connections between CT and ST

The development of a typical statistical attitude in students, such as skepticism, involves questioning, critically analyzing, and refusing to accept claims without sufficient evidence. These elements can be considered key elements of CT and statistical thinking (ST), although they are manifested differently within these two disciplines. ST, on one hand, is based on data interpretation, inference, and uncertainty management, thus statistical skepticism involves questioning the quality of data, the assumptions underlying models, the robustness of results, and whether conclusions are justified and based on data evidence. CT, on the other hand, is focused on systematic problem solving through processes such as decomposition, abstraction, debugging, and algorithm development. A critical stance in CT becomes substantiating the validity of the proposed solutions, rigorous debugging, and optimizing algorithms according to the problem constraints.

In recent years, our research has sought to foster the development of ST and CT, among many other mathematics-related approaches that are promoted in school curricula. Through data literacy, a direct connection is established between CT and ST; this connection enables students not only to become familiar with data-driven procedures and the information necessary for decision-making but also to cope with uncertainty.

We recognize that these two approaches complement each other. CT prioritizes solving a problem by refining each step, whereas ST focuses on questioning and drawing conclusions from data under uncertain conditions.

From a playful perspective

The incorporation of games and storytelling into the mathematics classroom reduces student anxiety and can also benefit teachers. By transforming the classroom into a playful and planned space, teachers can approach content more confidently, which reduces their math anxiety. This reduction is key, as some researchers have reported that math anxiety affects teachers' ability to promote inclusive classroom learning environments (Mizala et al., 2015).

In grades K-2, games and designed storytelling also enable the development of other modes of thinking, such as spatial thinking, which allows children to perceive the properties of objects, such as their location, dimensions, and relationships with each other; these activities provide students the ability to visualize objects, that is, to mentally model and move objects. Furthermore, children apply spatial thinking in everyday activities, such as formulating procedures, navigating maps, or issuing instructions (Bati, 2022; Wing, 2006, 2008). Some authors even suggest that the training of children's spatial skills effectively improves their math performance (Atit et al., 2022; Farran et al., 2024).

Olfos and Isoda (2025) suggested that using playful tasks in the classroom motivates and facilitates automation processes that reduce the students' cognitive loads, thereby freeing up mental resources that the students can devote to achieving deeper mathematical conceptualization.

METHODOLOGICAL ELEMENTS: DESIGN RESEARCH

As a group of early statistical learning researchers, we have adopted a research approach to create key tasks and plan the learning sequence moments for both the "Golden Pearls and Pirates" game and the "Climbing a Binary Tree" storytelling games. The use of these games enabled the creation, refinement, and improvement of new learning situation tasks through the intervention of iterative design and analysis cycles among both teachers (Cobb et al., 2017) and researchers.

Using the two developed games, we studied the ways that students can develop their problem-solving skills through collaborative play. From a qualitative perspective involving the use of interviews, we explored how these games and their associated tasks serve to support learning activities and help children learn aspects of statistical and computational thinking.

While the Golden Pearls story/game is oriented toward developing statistical thinking—addressing data, samples, repeated sampling, and inferences—and the Binary Tree story/game is focused on aspects of computational thinking, such as decomposition, abstraction, algorithm design, and debugging. These two games integrate aspects of both types of thinking, which demonstrates their interrelationship.

For both learning sequences, four and six researchers shared multiple meetings, although at different times, and drafted the problem and tasks internally at first, and then with the teachers. They subsequently made revisions and improvements based on feedback after the interventions and reflection sessions.

Below, I present two playful learning sequences, their storytelling and theoretical frameworks, and the aspects of thinking they seek to develop.

Learning sequence for the development of ST in students through data modeling in grades 1 - 2: Golden Pearls and Pirates

Several tasks in this sequence are focused on deciding which bottle is most likely to "be the most valuable" (the bottle that possibly has the greatest number of gold pearls), and they require students to argue for their choice by comparing and describing the behavior of random data samples (boards or diagrams) and express levels of uncertainty regarding risk levels according to the results of the repeated samples, which are obtained from each bottle (Figure 2).

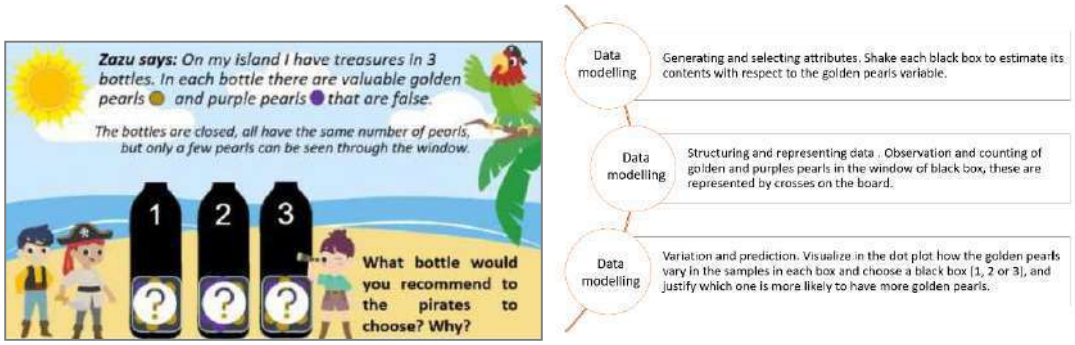


Figure 2: Storytelling activity [golden pearls and black boxes] with aspects of data modeling (English, 2012).

The four stages of the learning sequence (see Table 1) begin by encouraging students to formulate a conjecture about which of the three bottles (the three black boxes) is the most valuable based on the highest proportion of gold pearls that are visible through a small window in each bottle. The students then test their conjecture by shaking the bottles to mix their contents. Because they cannot see all the pearls inside the bottle but rather only those that appear in the window, they must conduct four trials per bottle to observe how many gold pearls emerge in each sample.

In each attempt, students record the number of pearls observed by marking an "X" on tables designed for this purpose. Thus, four empirical samples are generated per bottle. The results of all the groups are subsequently shared in the plenary session and represented in a dot plot, allowing a graphical visualization of the behavior of the gold pearls in each bottle. The session concludes with an analysis of the dot plots. Finally,

students must make an inference that goes beyond the collected data. On the basis of the evidence collected, they must advise the pirates on which bottle to choose (1, 2, or 3), using the language of uncertainty. Through this process, they are able to perceive the variability in the number of gold pearls observed in each attempt, evaluate whether their initial conjecture is valid or should be discarded, and make an informal inference by the data obtained for each bottle (Estrella et al., 2023, 2024a, 2024b).

Table 1 specifies the aspects of informal inferential reasoning (IIR) expected to be developed at each sequence moment.

	Moment 1 Storytelling and exploration of random artifacts	**Moment 2** Task 1a: Sample collection and data recording	**Moment 3** Task 1b: Data analysis and inference formulation	**Moment 4** Task 1c: decisions Making under uncertainty
Central tasks	Contextualize the story of pirates and pearls, present random artifacts and generate initial hypotheses. Three static bottles [random artifacts] with different proportions of gold and non-gold pearls) are presented and asked: "Which bottle do you think might have the most gold pearls?", inviting discussion of strategies to identify the bottle that has the most gold pearls inside through the windows. 	Using the three bottles as a "black box" and conducting the experiment in which each bottle is shaken to obtain a sample of gold pearls. The students, organized in trios and assigned specific roles (counter, secretary and shaker), repeat this process for each bottle, recording at each trial the number of gold pearls obtained. 	Analyze the dot plots (constructed in plenary) for each bottle and, from the samples obtained, identify trends in the distribution of gold pearls. Formulate hypotheses and make inferences about which bottle might contain the highest amount of gold pearls in future samples. 	Communicate results using uncertainty language to argue and justify which bottle is more probably to contain more gold pearls. Give advice to pirates about the best choice, based on interpretation of the data and regularities identified in the samples.
IIR expected	Data as evidence: experiencing static sample variation.	Data as evidence: recognize variation between samples and observe some regularity.	Data as evidence: recognizing the uncertainty of the situation; variation in samples and regularity in gold pearls.	Generalize using data as evidence and levels of uncertainty: express an informal statistical inference integrated to the context of the situation.

Table 1: Core tasks of the four moments in the ST learning sequence and the expected informal inferential reasoning.

IIR involves a series of statistical concepts that support inference, which requires several experiences with data to establish conceptual interrelationships (Makar et al., 2011). The repeated sampling technique enables a sample to be obtained from a population and additional samples to be generated under the same conditions. The repetition of this process results in an empirical sampling distribution, which can help students understand the variability in the data and reduce the uncertainty in their estimates. Ben-Zvi et al. (2015) noted the importance of constructing the sampling distribution because students can experience and better understand the process involved rather than simply being presented with the final result. The simulated repeated sampling technique makes it possible to generate empirical sampling distributions, which is key to analyzing and developing statistical inferential reasoning from an informal approach.

One of the main strengths of the playful approach to randomized experiments is the high level of interest it generated among the students who actively participated in the

creation, collection, and exploratory analysis of authentic data. Conducting this experiment with random artifacts to obtain samples allowed students to advance their understanding of randomness from an inferential perspective. Through repeated sampling, these students observed how random behavior tends to become more consistent with multiple repetitions while the variability of results when the number of repetitions is limited was also demonstrated (Estrella et al., 2024b).

Learning sequence that seeks to develop aspects of CT in students in grades 1 - 2: Climbing a binary tree

Owing to the nature of the tasks, the definitions of the CT components used to analyze task content are taken from a systematic review (Shute et al., 2017), leaving out two (iteration and generalization), as shown in Figure 3.

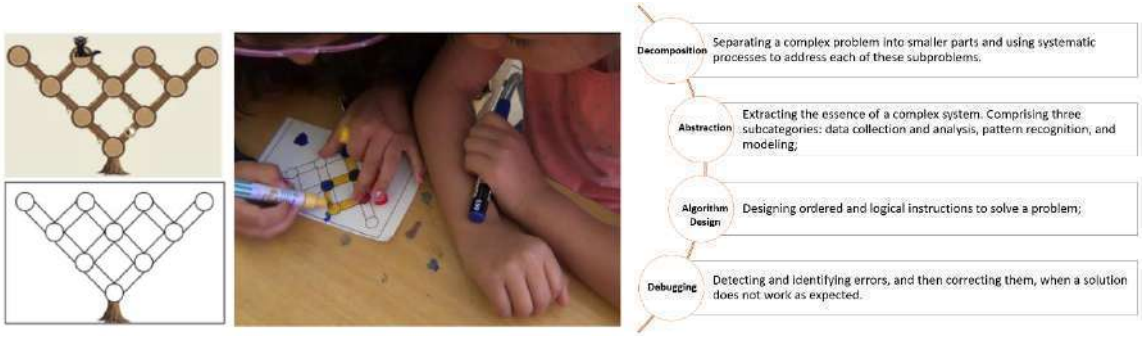


Figure 3: Storytelling activity [binary tree] with CT components (Shute et al., 2017).

The four moments of the learning sequence (see Table 2) begin by motivating corporality and activating students' visual-spatial skills.

This activity, which is framed within a playful setting, enabled the simultaneous activation of several CT components. Decomposition was demonstrated by dividing the problem into smaller steps; abstraction was demonstrated by identifying movement instructions and selecting optimal routes. Algorithm design was essential when constructing a logical sequence of instructions with cards. Debugging was performed when errors were detected in the proposed routes, allowing students to correct and adjust their strategies.

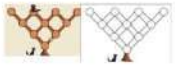
	Moment 1 Activation of visual-spatial skills	Moment 2 Task 1a: Replicate and build a binary tree	Moment 3 Task 1b: visualize and map route	Moment 4 Task 1c: building and debugging instructions
Central tasks	Recognize directions (right, left) and movements (forward) through verbal activities and the use of instruction cards. 	Storytelling of a meeting between two characters in a binary tree, and with board pieces. (1) co-replicate the projected binary tree, identifying the location of the characters. (2) co-construct the binary tree by tracing possible routes of encounter: (i) in pairs, on a printed sheet that has only clues - trunk, decision points and arrival point, without and with nests; and then, (ii) in plenary, verify the proposed routes. 	Visualize a tree with more branches and paint possible meeting routes in it, using a printed sheet and pencils. Colors are assigned to distinguish directions and movements of the proposed route: at each decision point (red for right, blue for left) and yellow on the branch to move forward.  Both, pairs and in plenary, trace the meeting route(s), review and verify the proposed meeting routes. 	Build sequences of instructions with direction and movement cards that allow the characters to meet, and debug these sequences. In pairs and in plenary, with a projected binary tree, build sequences of instructions with cards that allow routes of meeting of the characters and review these routes. Replicate and debug two sequences of instructions from cards, and fix the sequence by positioning the card on the wrong card.
CT component	decomposition	debugging and algorithm design	debugging and algorithm design	debugging and algorithm design

Table 2: Core tasks of the four moments of the CT learning sequence and aspects of the expected CT.

To contextualize the students' interventions in this sequence, the following episode exemplifies how the students applied algorithm design and debugging.

In the plenary session, a first-grade student verified the instructions on the cards:

> Teacher 01: [To the plenary] What is happening with this sequence?
>
> Class: [students respond in chorus] It is wrong!
>
> Teacher 01: What we need to do is fix this sequence.

[After the students construct their sequence, the teacher checks each card up to the last decision point]

> Teacher 01: What does the person who made this [the projected sequence] decide? To go to the right [at the first decision point], is that decision, okay?
>
> Student 07: No, because it should go to the left.

In pairs, the second-grade students identify two routes and discuss them with each other, while the teacher marks one of the routes on the board.

> Teacher 02: Is there another possible route?
>
> Class: Yes! [Students respond in chorus as everyone checks their routes.]
>
> Teacher 02: [After reviewing the route, asks] Why couldn't [mom] go left here?

> Course: Because there is a nest, an obstacle.
>
> Student 09: [naming the directions on the cards] it was… right, right, left.
>
> Student 10: And I move forward! [Remembering that they must indicate the card to move forward] and it can also be… right, move forward, left, move forward, right, and forward.

Both episodes show that children develop fundamental CT skills through playful, concrete, and unplugged activities. Through play, they learn to design sequences, detect and correct errors, evaluate different solutions, and recognize patterns. The inclusion of tangible elements, such as tree image pieces, cards, and shapes, serve to facilitate the translation of children's ideas into a concrete medium, making the thought process behind their problem solving visible through the symbol cards.

How do we ensure that the results of this research reach the mathematics classroom?

These playful experiences promote fundamental cognitive skills that are useful for today's life and prepare children to interact critically with artificial agents. In the first learning sequence, the children were primarily able to work with data and variability, identify and compare patterns, and express inferences in situations of uncertainty; in the second sequence, they were able to decompose and compose sequences of instructions and verify and debug their solutions. Thus, both learning sequences—statistical thinking and computational thinking—offer experiences that integrate the statistical and computational foundations that determine the behavior of artificial agents, ranging from their inferential capacity to the apparent objectivity of their processes and results.

Our research design has been developed in two complementary dimensions based on a problem-solving approach that incorporates games and storytelling. From a theoretical perspective, this research enables us to better understand how to foster certain types of thinking in the teaching and learning processes. From a practical perspective, considering today's real-life classrooms, we have focused on working with lesson study groups to design resources and materials for teaching mathematics, engaging teachers as task (re)designers, and implementing their lesson plans in real contexts within their schools (Estrella et al, 2020; Felmer, 2023; Isoda and Olfos, 2012; 2021).

As the first lesson study group in Chile (www.estudiodeclases.cl), with more than 15 years of existence, we have promoted this methodology and applied it in several Chilean schools with the intension of promoting greater teacher reflection and planning conducted collaboratively among colleagues who learn together. The model that has emerged from our local experience lasts for two months, involves eight weekly

sessions (see Figure 4) and comprises two cycles with a twice-implemented lesson plan and a third improved version (Estrella et al., 2022; Estrella and Olfos, 2023).

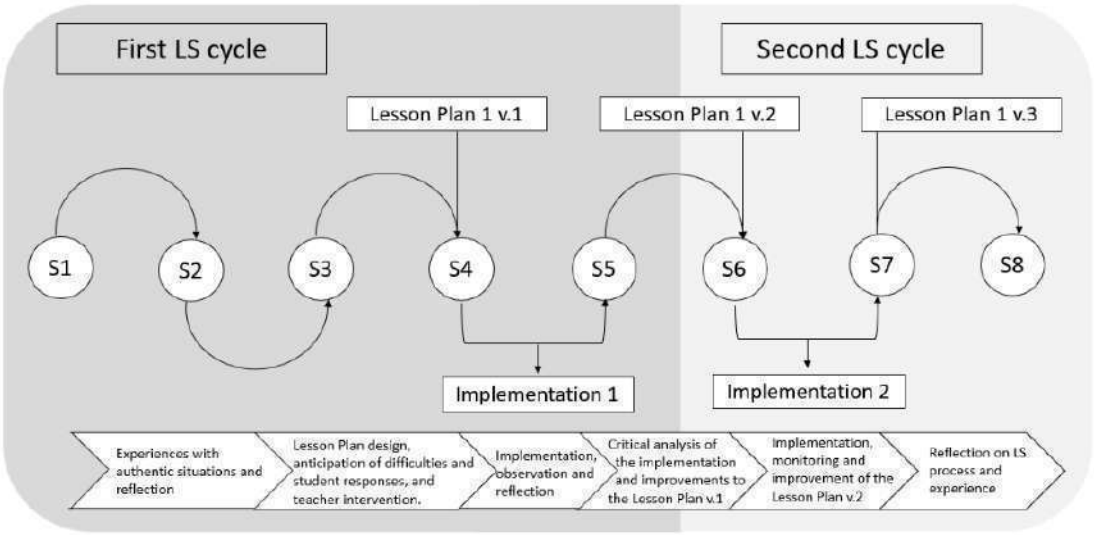


Figure 4: Eight-session, two-cycle lesson study model.

Although the lesson plan and proposed tasks for each of the learning sequences presented were initially piloted in small groups of children, their implementation has been carried out on a larger scale through work in primary schools with teachers participating in lesson study groups and supported by researchers.

In general, the results of our recent research revealed that, after involvement in teaching practices based on the lesson study model, teachers in training, those in practice, and even teacher trainers all had a high perception of the effectiveness of lesson study, stating that they have made greater observations and critical reflections on their teaching practices and were more aware of their students' thinking (Estrella and Olfos 2023; Ramírez and Estrella, 2024; Reyes-Bravo and Estrella, 2025).

CLOSING REMARKS

As argued, artificial agents are increasingly present in everyday life, which poses both opportunities and risks. For children to interact with AI critically rather than passively, the development of CT and ST from an early age appears necessary. Developing these thinking skills in the mathematics classroom, ranging from problem-solving through games and storytelling to interacting with humans—in pairs and groups—allows children to progressively acquire a deeper understanding of how to break down complex problems, analyze patterns, make data-driven decisions, and understand AI decision making.

The ST learning sequence using games and storytelling has motivated children to answer a question of interest, collect data, represent that data, and attempt to

understand majority behavior and its variations. These children were also able to engage in data modeling and make inferences beyond the collected data in situations of uncertainty. The CT learning sequence using games and storytelling enabled children to create and apply process instructions logically and effectively, thereby developing abstraction, problem decomposition, algorithm design, and instruction debugging skills.

The learning sequences in the "Golden Pearls" and the "Binary Tree" exercises demonstrate the possibility, of designing tasks that promote the development of statistical and computational thinking from an early age. Both activities enabled children to actively engage in data analysis, decision-making under uncertainty, problem decomposition, and the construction of simple algorithms, which are all essential elements in the understanding of how artificial agents work. Engaging children in playful situations through storytelling and collaborative problem solving is the foundation for the development of an initial understanding of how data-driven decisions and the logic underlying intelligent systems operate. These experiences help prepare students for educational environments that are geared toward the era of artificial intelligence and big data and promote classroom training that addresses current challenges.

By bringing these, and many other, learning sequences into mathematics classrooms through lesson study groups, we have observed that this methodology leads to a high level of appreciation for the process, thereby reducing teacher resistance to change and strengthening teacher commitment to quality education. However, in lesson study groups, specific barriers to integrating these new topics in primary education have been detected. Among these barriers, highlighted by the teachers themselves, are a lack of adequate materials in primary education and the mistaken perceptions that children cannot engage with complex data. The latter leads to two limitations: on the one hand, teachers might limit their teaching practices in advance, and on the other hand, this could reduce children's learning opportunities.

Recognizing the true scope of children's abilities and providing them with more fruitful learning experiences are essential to overcoming these limitations. The early integration and application of concepts and procedures in playful environments allows children to develop their potential and engage in deeper conceptual learning. Such early integration also helps teachers recognize the importance of teaching and developing new, future-oriented ways of thinking.

References

Araya, R. (2024). AI as a Co-Teacher: Enhancing Creative Thinking in Underserved Areas. In A. Kashihara, B. Jiang, M. Rodrigo & J. Sugay (eds.), *Proceedings of the 32nd International Conference on Computers in Education. Asia-Pacific Society for Computers in Education*.

Atit, K., Power, JR, Pigott, T., Lee, J., Geer, EA, Uttal, DH, ... & Sorby, SA (2022). Examining the relations between spatial skills and mathematical performance: A meta-analysis. *Psychonomic bulletin & review*, 1-22.

Bati, K. (2022). A systematic literature review regarding computational thinking and programming in early childhood education. *Education and Information Technologies, 27*(1), 2059–2082.

Ben-Zvi, D., Bakker, A. & Makar, K. (2015). Learning to reason from samples. *Educational Studies in Mathematics, 88*, 291–303. https://doi.org/10.1007/s10649-015-9593-3

Cobb, P., Jackson, K., & Sharpe, C. (2017). Conducting design studies to investigate and support mathematics students' and teachers' learning. In J. Cai (Ed.), *Compendium for research in mathematical education* (pp. 208–233). NCTM. Common Core State Standards.

del Olmo-Muñoz, J., Cózar-Gutiérrez, R., & González-Calero, J. A. (2020). Computational thinking through unplugged activities in early years of Primary Education. *Computers & Education, 150*, 103832.

English, L.D. (2012). Data modeling with first-grade students. *Educational Studies in Mathematics, 81*, 15-30.

Estrella, S. (2018). Data representations in Early Statistics: data sense, meta-representational competence and transnumeration. In A. Leavy, A., M. Meletiou, & E. Paparistodemou (Eds.). *Statistics in Early Childhood and Primary Education – Supporting early statistical and probabilistic thinking*, (pp. 239-256). Springer.

Estrella, S., Zakaryan, D., Olfos, R., & Espinoza, G. (2020). How teachers learn to maintain the cognitive demand of tasks through Lesson Study. *Journal of Mathematics Teacher Education, 23*, 293 – 310.

Estrella, S., Méndez-Reina, M., Salinas, R., & Rojas, T. (2023a). The Mystery of the Black Box: An Experience of Informal Inferential Reasoning. In G. Burrill, L. De Oliveira and E. Reston, *Research on Reasoning with Data and Statistical Thinking: International Perspectives. Advances in Mathematics Education* (pp. 191-210). ISBN 978-3-031-29458-7. Springer.

Estrella, S., Méndez-Reina, M., & Vidal-Szabó, P. (2023b). Exploring informal statistical inference in early statistics: a learning trajectory for third-grade students. *Statistics Education Research Journal, 22*(2), 1-16. https://doi.org/10.52041/serj.v22i2.426

Estrella, S., Morales, S., Méndez-Reina, M., Vidal-Szabó, P., Ramírez, B., & Mondaca-Saavedra, A. (2024a). Design of a hypothetical learning trajectory to introduce informal statistical inference in primary school. *RELIME, Revista Latinoamericana de Investigación en Matemática Educativa, 27*(1), 11-42.

Estrella, S., Morales, S., Méndez-Reina, M., Vidal-Szabó, P., and Mondaca-Saavedra, A. (2024b). Collective argumentation in elementary school: An experience of reasoning with data in an online Open Class. *International Journal for Lesson and Learning Studies, 13*(3), 205-220.

Estrella, S., Morales, S., Olfos, R., & Salinas, R. (2022). Estudio de e-Clases en Chile: cambios percibidos por profesores que diseñan, mejoran e implementan una tarea que desarrolla el razonamiento inferencial informal desde PK-3. En A. Richit, J. da Ponte y E. Soto, *Estudos de aula na formação inicial e continuada de professores*, (pp. 89-119). Editora Livraria da Física. ISBN: 978-65-5563-266-8.

Estrella, S., Olfos, R., Morales, S., & Vidal-Szabó, P. (2017). Argumentaciones de estudiantes de primaria sobre representaciones externas de datos: componentes lógicas, numéricas y geométricas. RELIME, *Revista Latinoamericana de Investigación en Matemática Educativa, 20*(3), 345-370.

Farran, E. K., McCarthy, S., Gilligan-Lee, K. A., Bates, K. E., & Gripton, C. (2024). Translating Research to Practice: Practitioner Use of the Spatial Reasoning Toolkit. Gifted Child Today, 47(3), 202-215.

Felmer, P. (2023). Collaborative problem-solving in mathematics. *Current Opinion in Behavioral Sciences, 52*, 101296.

Harari, N. (2024). *Nexus: A Brief History of Information Networks from the Stone Age to AI*. Fern Press.

Isoda, M., & Olfos, R. (2012). *El Enfoque de Resolución de Problemas: en la enseñanza de la matemática a partir del Estudio de Clases*. Ediciones Universitarias de Valparaíso.

Isoda, M., & Olfos, R. (2021). *Teaching multiplication with Lesson Study: Japanese and Ibero-American Theories for Mathematics Education.* Springer. ISBN: 978-3-030-28560-9.

Makar, K., Bakker, A., & Ben-Zvi, D. (2011). The Reasoning Behind Informal Statistical Inference. *Mathematical Thinking and Learning, 13*(1–2), 152–173.

Mizala, A., Martínez, F., & Martínez, S. (2015). Pre-service elementary school teachers' expectations about student performance: How their beliefs are affected by their mathematics anxiety and student's gender. *Teaching and Teacher Education, 50*, 70-78.

Mulligan, J. (2015). Moving beyond basic numeracy: data modeling in the early years of schooling. *ZDM Mathematics Education, 47*(1), 653–663.

Olfos, R., & Isoda, M. (2025). Intergroup and Intragroup Differences in Addition Fluency and Strategies for Adding 3 to 5 at the End of First Grade in Chile and Japan. *Journal of Southeast Asian Education*. (In press).

Ramírez, B., & Estrella, S. (2024). Perceptions of Lesson Study Effectiveness in Initial Teacher Training. *Revista Zetetiké, 32*, e0240010-e0240010.

Reyes-Bravo, M., & Estrella, S. (2025). Percepciones de formadoras de profesores en Estudio de Clases sobre sentido numérico. *REXE, Revista de Estudios y Experiencias en Educación, 24*(54), 68-87.

Shute, V. J., Sun, C., & Asbell-Clarke, J. (2017). Demystifying computational thinking. *Educational Research Review, 22*, 142-158.

Sneider, C., Stephenson, C., Schafer, B., & Flick, L. (2014). Teacher's Toolkit: Exploring the Science Framework and NGSS: Computational Thinking in the Science Classroom. *Science Scope, 38*(3), 10–15.

Su, J., & Yang, W. (2023). A systematic review of integrating computational thinking in early childhood education. *Computers and Education Open, 4*, 100122.

Wing, J.M. (2006) Computational Thinking. *Communications of the ACM, 49*, 33-35.

Wing, J. M. (2008). Computational Thinking and Thinking about Computing. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 366*, 3717e3725.

Wing, J. M. (2011). Research notebook: Computational thinking—what and why? *The Link Magazine, 6*, 20–23.

Wing, J. M. (2014). Computational thinking benefits society. *Communications of the ACM, 49*(3), 33–35.

Wu, S. Y., & Su, Y. S. (2025). The effect of learning computational thinking skills through educational board games on students' cognitive styles, cognitive behaviors, and learning effectiveness. *Asia Pacific Education Review*, 1-14.

FROM THE LABORATORY TO THE CLASSROOM AND BACK: REFLECTIONS ON THE RELATION BETWEEN MATHEMATICS EDUCATION RESEARCH AND PRACTICE

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Research done from the perspective of educational psychology – and particularly cognitive psychology – is often perceived as being quite disconnected from the practice of mathematics education, partly because of the methodologies it relies on and the tasks it uses, which often do not have a very strong ecological validity. In this plenary paper and lecture, I will argue that this research does have an important value. I present studies from a range of topics in mathematics education in a range of ages, that rely on methodologies and psychological frameworks that at first sight seem quite distant from the real mathematics classroom, and would be considered as "fundamental research" rather than applied research. Still, I will try to show that these studies do deliver implications that could most likely not be achieved through studies that are situated in or much closer to the mathematics classroom.

INTRODUCTION

The theme chosen for the PME 48 conference is "Making sure that mathematics education research reaches the classroom". I am particularly pleased with the choice of this theme. The very reason why we all conduct research in mathematics education is that we want the insights that we gain through our research to improve the teaching and learning of mathematics as it occurs in the classroom (and outside it). Still, the theme implicitly points at a long-lasting concern, which is that there is a gap between the research community and insights that are gained in that community and the everyday practices that take place in the classroom.

Whether real or not, this "research-practice" gap in the field of mathematics education is something that needs close reflection. In my plenary talk, I wish to do so from the specific perspective that I have taken in the past 25 year that I was involved in IGPME. My research background is that of an educational psychologist, with a heavy focus on cognitive psychology. When I stood as a candidate for election in the International Committee of IGPME in 2012 in Taiwan, I explicitly stated that I thought the "P" in PME needs sufficient attention. I did not intend to diminish the importance of other approaches and perspectives on mathematics education that through the years have found their way to our community, but wanted to stress that psychological research still has its place. At that time, I did so after receiving reviews of submitted research

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, pp.71-93. PME.

reports being "too experimental", "only providing quantitative data", and "ignoring the complexities of real classrooms". When I stood for election as president of IGPME in 2021, I once more stressed maintaining the diversity of research approaches and perspectives as one of the many ongoing challenges in the PME community.

Research done from the perspective of educational psychology – and particularly cognitive psychology – is often perceived as being quite disconnected from the practice of mathematics education as it takes place in real classrooms. This perception partly originates because of the methodologies it relies on, and because the tasks used and the circumstances in which the research takes place often do not have a very strong ecological validity. In this plenary paper and lecture, I will argue that this research does have a value to practice that may not immediately be clear, and delivers insights that cannot easily be achieved through research that takes place in the actual classroom practice. The contribution is intended to be provocative: not provocative for its own sake, but thought-provoking. I will present studies from a range of topics in mathematics education in a range of ages, that rely on methodologies and psychological frameworks that at first sight seem quite distant from the real mathematics classroom, and would be considered as "fundamental research" rather than applied research. Still, I will try to show that these studies do deliver implications that could most likely not be achieved through studies that are situated in or much closer to the mathematics classroom.

I will even go a step further in the provocation, and present studies of a particular nature: studies that are unusual or trivial at first sight, and that may even make people initially laugh, but then also make people *think*. The importance of this kind of research is shown by the fact that a special award is devoted to it: the Ig Nobel prize.

(MATHEMATICS) EDUCATION RESEARCH AS THE HARDEST SCIENCE OF ALL

If we look at empirical research in the natural sciences, one can distinguish a two-fold goal. On the one hand research is expected to reveal the causal mechanisms that are present in nature and that help explaining and predicting various phenomena. This kind of research can be termed as "fundamental" research. On the other hand, this increased knowledge and insight in causal mechanisms helps humankind to achieve certain goals that it considers (or that some consider) desirable: houses that get heated with less CO2 production, a virus that is no longer spreading as quickly across the globe, crops that survive better in climate change conditions, but unfortunately also goals such as weapons becoming more lethal.

The expectations for educational research are essentially the same, and this also holds for mathematics education research. On the one hand, we expect mathematics education research to provide deep theoretical insight in phenomena that occur when learners acquire mathematical insights, and when teachers or other educational actors such as technology attempt to instigate such learning processes in learners. On the other

hand, we expect that this increased knowledge helps us to achieve goals that we consider (or that some consider) desirable. In other words: to improve educational practice.

It is generally accepted that the contribution of research in the natural sciences to improving our daily lives has been very successful. However, this is less the case for educational research. My predecessor in the chair of educational psychology at our institute, Erik De Corte, stated about three decades ago that

> "After 100 years of systematic research in the fields of education and educational psychology, there is, in the early 1990s, still no agreement about whether, how, and under what conditions research can improve educational practice." (Weinert & De Corte, 1996, p. 43).

A quote in the same line and from the same period from Glaser, Lieberman, and Anderson (1997, p. 25) is that

> "One continuing dilemma for educational research as we move toward and into the 21st century will be how the research and scholarship that we do are ever going to find their way into practice. We've had various models of the proper relationship between research and practice. None of the models work very well."

CHALLENGES IN STUDYING MATHEMATICS EDUCATION IN LABORATORY SETTINGS

One of the most important reasons for the distinction between the natural sciences and educational sciences, and in their (perceived) relative success is probably the difference in their complexity. Berliner (2002) provokingly called educational research "the hardest science of all", thereby referring to the importance and the enormous influence of contexts, the ubiquity of interactions between numerous variables, and changes in the social environments and educational goals that make findings obtained in one decade useless or irrelevant in the next.

Let me stress that I am not claiming that natural sciences do not address complexity. But in its research, the complexity can be handled more easily by simply controlling for this complexity. When I participated as a committee member in job interviews where we were seeking good researchers in mathematics education, it occurred several times that my colleagues from the exact sciences departments (who do not conduct educational research themselves) used expressions like "Mathematics education research obviously is not rocket science". While I initially experienced such expressions a denigrating or even belittling our discipline of mathematics education, by now I have learned to react to these colleagues by telling them that I totally agree: "Indeed, you are right. It is much harder than rocket science!"

Phillips and Dolle (2006, p. 278) provide clear a argument for the substantially larger complexity of educational research. They argue:

> "Natural phenomena often can be taken out of in situ and located in the artificially simple setting of the laboratory where they can be manipulated for study, and this can be done without completely converting them into useless artifacts".

And also, the reverse mechanism is true: Findings that occur in the laboratory can be generalized rather directly to the natural world. Take the example of electrical current through copper wires. This phenomenon occurs in real life across the world, and it is of an enormous complexity and diversity. Still, it can be studied in a laboratory setting in which the experimenter gets rid of all or nearly all of the complexity except for the variables he or she is specifically interested in. The electrical current will not behave differently by the very fact that it is being investigated, and the principles and laws that are discovered in such a highly simplified experimental setting still hold in the complex real life.

Experimental and cognitive psychologists often try to follow the same paradigm, studying individual (human or animal) subjects aiming to discover mechanisms underlying their behavior. They do so by conducting research in highly controlled settings which I call laboratory-like settings. Research in such settings has revealed various well-established and widely replicated psychological mechanisms such as the structure and functioning of our working memory, cognitive biases in decision making, and the way in which attention and perception work. In some cases, these discoveries required research in actual laboratories, i.e. dedicated rooms in universities or other research institutes, in which specific laboratory equipment was available. In other cases simple paper-and-pencil data collections were sufficient to reach this goal, but the setting was nevertheless laboratory-like: That the behavior was studied not in its natural habitat, but taken to a dedicated setting in which one tries to control as many variables as possible, except for the ones that one intends to study. The researcher follows pre-determined, fixed and standardized procedures, tries to minimize his or her influence on the situation apart from experimental manipulations, and can thus be considered as an outsider to the situation to which the participant is subjected rather than as a true participant in the situation. Typically, social interaction among research participants is considered undesirable unless it is explicitly an object of study, and it can be controlled.

A lot of research in mathematics education also has these characteristics, and can thus be considered "laboratory-like". Surely, research in actual laboratories is not so common in mathematics education, but the research paradigm still very often resembles the aforementioned features. This can even be the case when studies take place in the actual classroom, for instance when learners are working on carefully designed test sheets that experimenters have developed, and that contain specific experimental features. Learners received standardized instructions, work individually, often for a specific amount of time, and social interaction is to be avoided, with a possible exception of a whispered conversation with the experimenter.

As I explained above, educational research (and thus also mathematics education research) is challenging, with one main reason being that unlike phenomena studied in the natural sciences, educational phenomena cannot be simply taken out of their context and studied as if they would still be the same as the real phenomenon occurring in classrooms. I will provide two examples of mathematics education research which took place in a setting that has features of the laboratory. And I will show that this approach indeed creates challenges: The intended phenomenon does not lend itself very well for being studied in very controlled laboratory-like settings, as its nature is affected by that setting.

Example: Suspension of sense-making when solving word problems

A convincing example of how a laboratory-like setting may lead to completely different observations than a more life-like setting is the line of research initiated by Lieven Verschaffel and Brian Greer on the so-called "suspension of sense-making" while solving word problems (Verschaffel et al., 1994, 2000, 2009). In a series of studies, they offered upper primary school children word problems with a "twist": Proper answers require considerations based on real-world knowledge rather than a simple straightforward application of some arithmetic operations. An example of such a problem is the following:

> 450 soldiers need to be bussed to their training site. Each army bus can hold 36 soldiers. How many buses are needed?

The problem asks for a division (450 : 36) but then a proper interpretation of the answer is required (i.e. rounding the outcome 12.5 up to the answer "at least 13 buses"). However, a large majority of learners either rounded down (as they usually do) or simply answer 12.5 buses. Various other problems with a non-straightforward link between the word problem and reality were used, and studies were replicated worldwide (Verschaffel et al., 2000). However, it turned out that the very specific setting in which the studies took place (written tests in a very controlled setting with a lack of interaction between students and between students and their teacher) had a very decisive impact on the tendency to give unrealistic answers.

Defranco and Curcio (1997), for example, first gave pupils the above-mentioned bus problem in which the correct solution had to be rounded up. In the individual written test setting, only two out of twenty pupils gave a realistic answer. When the same twenty pupils were given a more authentic version of the same task, in which they were asked to actually make a phone call to a taxi company to order minivans for a school party, 16 of them did do the correct rounding. Similar effects of bringing students in more authentic contexts were found in numerous other studies (e.g., Reusser & Stebler, 1997; Van Dooren et al, 2005). Learners thus show different behavior depending on the context, a finding central to situated cognition. Regarding word problem solving, Jean Lave (1992) rightfully argued "The activity of solving word problems and the contents of word problems in school are not the same as 'the same' activity or contents embedded in other systems of activity in other parts of life". Her comment on

behaviour in the classroom setting holds by extension for the setting of research, while that is often overlooked. In line with the idea of a ‘didactical contract’ (Brousseau, 1997), Greer (1997) introduced the term 'experimental contract' to stress the importance how students construe what is happening when being tested or interviewed, and how this construal influences their responses.

Example: Intuitions in mathematical reasoning

A second example I want to address relates to mathematical intuitions. In his pioneering work in this domain, Fischbein defined intuition in mathematics and science as an immediate cognition that exceeds the given facts, as "a theory that implies an extrapolation beyond the directly accessible information" (Fischbein, 1987, p. 13). In his work, he characterized intuitions by self-evidence, extrapolativeness, coerciveness and globality. In the past decade, we have conducted studies in multiple mathematical content areas (e.g., understanding the size of decimals and fractions (Gonzalez-Forte et al., 2022; Obersteiner et al., 2015; Van Dooren et al., 2015; Van Hoof et al., 2018), interpreting box plots (Abt et al., 2024, 2025; Lem et al., 2013, 2015), solving non-proportional word problems (Fernandez et al., 2012; Van Dooren et al., 2005, 2009). As I will elaborate later in this paper, we could demonstrate that students had specific intuitions that indeed impacted their reasoning. By means of reaction time studies in very laboratory-like settings, we were even able to show that although students performed very well, expert mathematicians’ thinking was still impacted by the intuitions in the sense that experts needed slightly more time to solve problems in which the intuition would lead them to a wrong answer than to solve problems where the intuition led to the correct answer.

However, one feature of intuitions was impossible to study in a laboratory-like manner: the fact that intuitions are coercive, or the experience of certainty that a problem solver has when following an intuitive way of reasoning. Based on anecdotal evidence and indications from qualitative interviews with students, we were able to document that student often showed great certainty about the intuitive solution they obtained, even when the answer was incorrect (De Bock et al., 2002). However, our attempts to systematically and quantitatively measure the certainty that students experienced when answering generally did not work, and hence was never published. The reason was that by repeatedly asking students how certain they are, they get suspicious. They started thinking that there must be a reason why this question about certainty is posed so repeatedly. On the one hand this led them to start doubting about their answers. On the other hand, we saw more correct answers appearing, simply by asking students about their certainty. The very fact that we attempted to systematically measure something (the certainty with which students give an answer) immediately has affected the nature of that answer and its accompanying certainty. The answer no longer occurred in its natural environment and the essence of it has changed.

So, to sum up, I am very well aware of the fact that the laboratory-like studies that I have conducted in the past decades have substantial limitations. Certain phenomena

are completely different when studied outside the natural classroom setting, or are affected by the very fact that researchers attempt to measure them. Still, in the rest of this contribution, I will develop the argument that laboratory-like studies also have a lot of potential, as (if problems as described above do not arise or do not manifest themselves that strongly) they are able to reveal phenomena that would be impossible to systematically demonstrate in more natural classroom settings, while these phenomena are still very relevant to these natural classroom settings. I will provide a series of cases in an attempt to convince the readers, stemming from a variety of domains, and largely referring to work that I had the pleasure to conduct with many colleagues that I have met in the PME community.

MATHEMATICIANS READING PROOFS

Weber and Mejia-Ramos (2011) conducted a study in which they investigated the strategies research mathematicians engage in when they read mathematical proofs. They interviewed a group of research mathematicians about the practices they engaged in, and analyzed the way in which they determined if mathematical arguments were correct. In a follow-up survey in a much larger group of respondents, these researchers came to similar conclusions (Mejia-Ramos & Weber, 2014). Generally speaking, the self-reports by the respondents indicated that when reading proofs they appealed to the authority of other mathematicians who read the proof, and did line-by-line reading, but also a kind of "modular reading" in which they tried to get an overall sense of the proof structure before proceeding to the line-by-line processing. The implications drawn from these findings are that mathematics instruction should stimulate students to engage in justification and argumentation activities in a manner that is similar to what these research mathematicians report.

However, Inglis and Alcock (2012) have conducted an eye-tracking study in which they compared undergraduate students and mathematicians regarding the way in which they read proofs. As such the study was not intended to have direct practical implications, but rather to get a deep understanding of the differences between these groups. However, in the eye-tracking patterns, Inglis and Alcock found much less of the "modular reading" that Weber and Mejia Ramos were referring to. They found no evidence of initial skimming: participants generally did not have any fixations on the last lines of purported proofs until approximately half way through their reading of the proofs.

This resulted in a debate between both groups of researchers, discussing mainly what could be concluded from two very different sources: self-reported findings of mathematicians describing their proof reading behaviour as it naturally occurs (in which 92% of mathematicians indicate that they skim proofs first to get the main idea, Mejia-Ramos & Weber, 2013) or eye-tracking data from mathematicians reading relatively short proofs in an unnatural research setting.

In an attempt to settle the debate, Inglis and Alcock (2018) asked mathematicians to read research papers that were drawn from their own fields, and that the mathematicians were planning to read. This methodology added substantially to the ecological validity of the study, as proofs were relevant for the participants, and were much longer than the proofs in the original studies (up to 53 pages long). Again, eye patterns were tracked while reading the data, and once more Inglis and Alcock found no evidence of skimming in the reading patterns.

What I see as an important take-home message from this is that self-report data about behaviour occurring in natural settings need to be treated with great caution, even when a study takes place in a population such as research mathematicians; a group that can be expected to have good metacognitive skills. The reasons why self-report data and actual behaviour differ are numerous: participants (even research mathematicians) may not always be fully aware of the cognitive processes that take place, or may not be able to adequately report them; but as they are aware of the fact that they are being investigated (and are usually aware of the goal of the study they participate in), they may also (consciously or unconsciously) give socially desirable responses. Hence, findings obtained through self-report data findings deserve to be validated through triangulation, using more objective methods, even when these unavoidably require less ecologically valid research circumstances.

To further show the importance and potential of this research line with eye-tracking data, Inglis' and Alcock's (2012) reveal important differences between students and mathematicians in proof reading: The latter exhibited many more attempts to identify the implicit logical relations between successive lines in proofs. This finding led to a further series of experiments – some of which were still heavily lab-based – in which self-explanation training was found to be very effective in improving students' proof comprehension (Hodds, Alcock, & Inglis, 2014). While the research line started off with the aim of gaining fundamental understanding and settling a methodological debate on self-reported behaviour versus behaviour as revealed by eye-tracking, the self-explanation materials that these researchers have developed have by now found their way in mathematics textbooks.

THE INTUITIVE APPLICATION OF PROPORTIONAL METHODS TO NON-PROPORTIONAL WORD PROBLEMS

Proportionality is a major tool to interpret a large range of everyday-life situations and to understand mathematical and scientific phenomena. It therefore receives substantial attention in the mathematics curriculum. However, research from the past two decades has shown that students' growing familiarity with proportional solution methods goes along with a tendency to apply them also in situations where this is inadequate. This phenomenon has been demonstrated in different mathematical domains, such as elementary arithmetic, geometry, and probability (an overview, see Van Dooren et al., 2008). Consider the following word problem:

> Ellen and Kim are running around a track. They run equally fast but Ellen started later. When Ellen has run 4 laps, Kim has run 16 laps. When Ellen has run 8 laps, how many has Kim run?

Many upper elementary school children erroneously give the proportional answer "8 x 4 = 32" instead of the additive answer ''8 + 12 = 20'' this problem, and this number increases from fourth to sixth grade (Van Dooren et al., 2005). The phenomenon of the over-use of proportionality has many explanations, but one of them is that the idea of proportionality has become an intuition, in the aforementioned sense as described by Fischbein (1987). In the field of cognitive psychology, intuitive modes of thinking are recognized (they are often called "heuristic thinking") and are opposed to analytical thinking processes. The famous "dual process theory" is a clear example (Evans, 2008; Kahneman, 2000; Sloman 1996). In recent years, suggestions have been made to apply the insights and methodologies of these cognitive psychological frameworks also to problems in the field of mathematics education (Gillard et al., 2009a; Leron & Hazzan, 2006).

In line with this suggestion, Gillard et al. (2009b) have conducted a series of studies to investigate the intuitive nature of proportional errors to problems such as the example above. They asked university students to solve a series of such word problems, some of which having a proportional solution while others had a non-proportional solution. Word problems were offered on a computer in a carefully controlled setting. Students in the control condition had no time limit, while students in a time pressure condition were forced to respond within 17 seconds. It was found that students performed much worse on the non-proportional problems in the time pressure condition than in the control condition (and gave a lot of proportional answers instead), while performance on the proportional problems was not affected. Gillard et al. saw this as an indication that the proportional error is indeed intuitive in nature, as this kind of answer can still be produced under time pressure, while the correct (analytic) reasoning cannot be successfully conducted under time pressure.

In a next experiment, Gillard et al. (2009b) gave the same word problems, but this time, students' executive resources were experimentally with an attention-demanding secondary task that needed to be accomplished simultaneously to solving the word problems. Students were given the dot memory task (e.g., Miyake et al., 2001) in which a three-by-three matrix containing three to four dots was briefly presented, which the participants had to memorize and reproduce after having solved the word problem (see Figure 1). In the "load" condition, the matrix was filled with complex four-dot patterns which are expected to heavily burden working memory, while in the control condition the pattern was a three-dot pattern in a single row, which can be stored in working memory in a single chunk. Analogously to the findings of the time pressure study, Gillard et al. found that in the load condition, accuracy on the non-proportional problems was heavily affected (resulting in more proportional errors) while performance on proportional problems was not affected. The authors again saw these

findings as an indication of the intuitive nature of proportional errors, as intuitive processes are less demanding of working memory.

The studies by Gillard et al. (2009b) are typical laboratory studies, administered in a setting very unlike the typical classroom context in which proportional and non-proportional word problems are typically given by teachers and solved by learners. The studies were done with university students, on a laptop in an individual setting, in return for course credit, and the studies administered time pressure or an experimentally burdened working memory in a way that would never occur in the classroom. Still, this experimental approach enabled them to demonstrate that cognition while making proportional errors has the characteristics of intuitive reasoning.

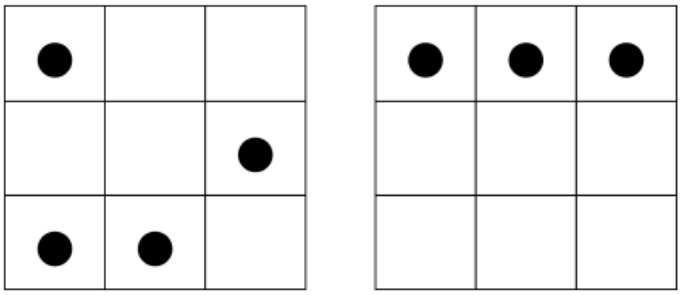

Figure 1. Dot memory task administered in the load (left) and control (right) condition

When discussing results about our work on the role of intuitions, and especially when discussing the effects of time pressure (or reaction time differences) or the experimental burdening of working memory, I frequently get comments about why this at all would be relevant for mathematics education and the mathematics classroom. Responses then typically refer to the operationalisation of working memory load, or the way we administered time pressure. Comments are usually based on the assumption often is that in the mathematics classroom, students are expected to engage in deep analytical thinking rather than superficial and fast intuitive thinking, and thus that students take all time and commit all available working memory capacity. I want to argue that the reality is different. We must be aware that students are not always fully cognitively engaged in the tasks we give to them in the mathematics classroom (let alone in mathematics education research, in which they often participate under subtle force). Now and then, students will also be simultaneously engaged in other things than the problem at hand, thus their working memory may not be fully available. Moreover, answers obtained by intuitive reasoning will often be also correct. And also, time constraints are not that uncommon in actual mathematics classrooms: After collective instruction moments, teachers often let students work individually or in dyads, but at some point in time they return to a collective discussion.

In such a system, it is beneficial that one has managed to finish the assigned tasks. In this respect, following up on the "rationality debate" and its relevance for mathematics education (Leron & Hazzan, 2006) one can argue that rational/analytic thinking is not always required nor economical in mathematics classroom settings, and that quick, superficial, intuitive reasoning is often more rewarding. For this reason, Todd and Gigerenzer (2012) introduced the term "ecological rationality" to denote the kinds of intuitive and thus "irrational" reasoning that students may engage in.

INTUITIVE MISINTERPRETATIONS OF HISTOGRAMS BY STUDENTS AND EXPERTS

Histograms are a popular way to represent the distribution of a quantitative variable, both in scientific publications and in popular media. Research points out that people often misinterpret them (Lem et al., 2012). For instance, when comparing two histograms, they think that the histogram in which the "jumps" from one bar to the other are larger also has a larger variation. This is illustrated in Figure 2. Other misinterpretations relate to confusing histograms with bar charts, or seeing the x-axis as representing a timescale (delMas et al., 2005).

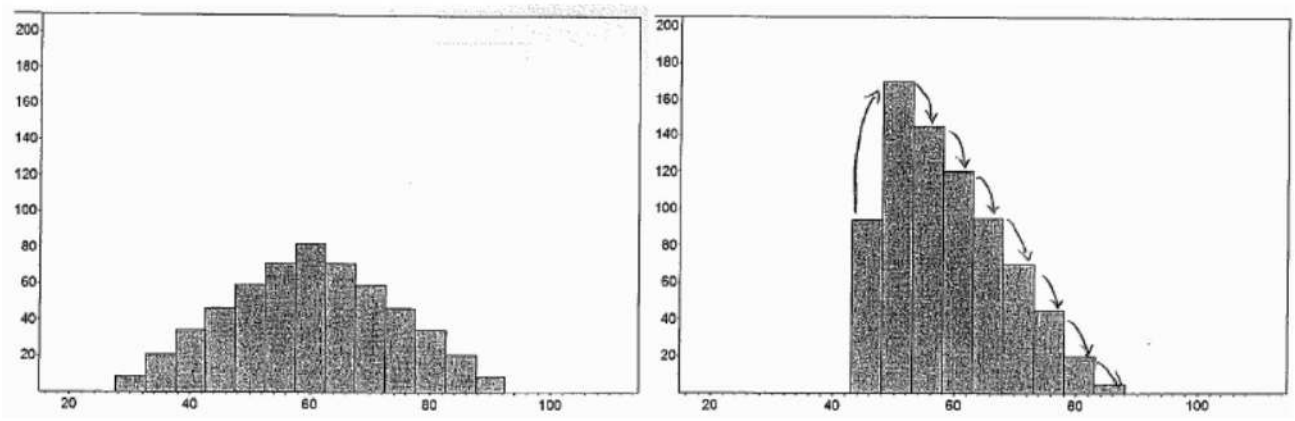

Figure 2. Illustration of "larger jumps imply larger variation" misinterpretation in histograms (taken from Lem et al., 2013, p. 13)

An extensively investigated misinterpretation of histograms relates to the comparison of two means: Students think that the histogram with the higher bars has also has a higher mean, while the height of the bars as such does not indicate the mean (Lem et al., 2012). For instance, in Figure 3, students would argue that the right histogram has a higher mean, while in fact both histograms have the same mean.

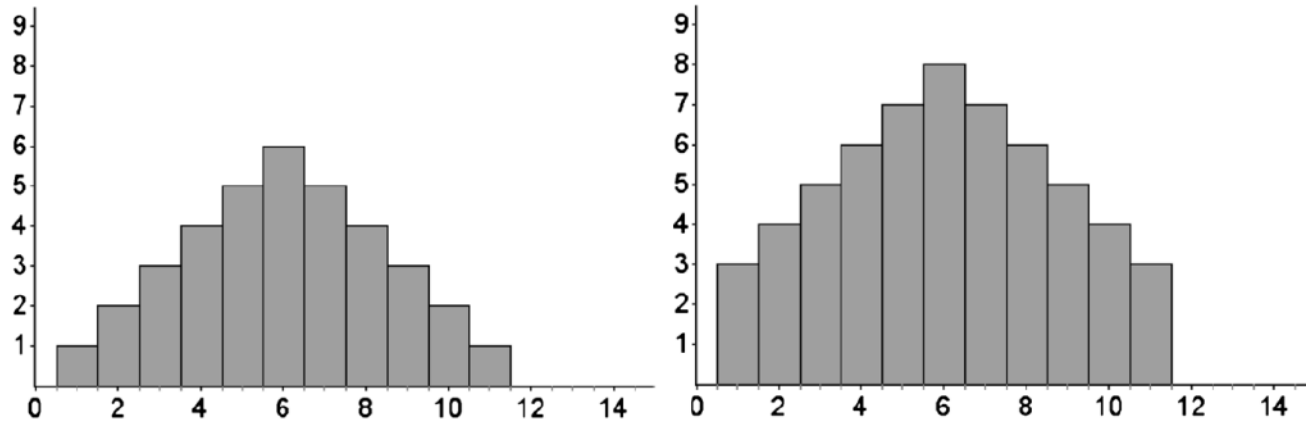

Figure 3. Illustration of "higher bars imply larger mean" misinterpretation in histograms (taken from Lem et al., 2012, p. 558)

The misinterpretation is often attributed to the ide that most graphs are designed according to the principles described by Tversky (1997), in which the vertical dimension is perceptually more salient than the horizontal one, while the horizontal dimension needs to be considered to compare the means.

Lem et al. (2014) also framed the misinterpretations of histograms as being intuitive in nature: The knowledge to achieve a correct interpretation of histograms is present in students who misinterpret them, but they nevertheless fail on items where focusing on the horizontal dimension leads to a wrong answer. They conducted a series of laboratory-like studies with results that indeed showed this intuitive nature.

In a first step, they asked university students to conduct a series of histogram comparison tasks on a computer, thereby collecting not only students' accuracies but also their reaction times. They found that for histogram comparisons where the intuitive answer was also the correct one (in the literature often termed "congruent items"), accuracies were much higher than on items where this was not the case (called "incongruent items"). Moreover, reaction times to give correct answers to congruent items were significantly shorter than those for correct answers to incongruent items, which Lem et al. (2014) take as an indication that in incongruent items students have to overcome a predominant intuitive response tendency in order to give a correct answer.

In a follow-up study reported in the same paper, they wanted to investigate the strength of the intuition. In the first study, students all knew how to come to a correct comparison of the means in the two box plots: This was established to a declarative and procedural knowledge test. However, in the actual experiment they still failed in many cases. And if they succeeded, there were still signs in the reaction time that they needed to overcome their initial intuitive response. Hence, it was worth investigating whether this intuition was a weak one that could easily be overcome, or that it would be more deep-rooted. To study this, Lem et al. gave the same task as in the first study to similar students, but in two new conditions: a time pressure condition in which

responses were required in 4 seconds (a time limit so strict it would not allow analytic processing of the items at all), and an analytic condition with unlimited solving time and a warning that graphs may be misleading (with an example of other misleading graphs). Overall, very similar results were found as in the first study, with nearly no differences between the time pressure condition and the analytic condition. These findings can be taken as an indicator that the intuition that was studied is an extremely strong one, that is not overruled by stimulating analytical thinking, and that is equally strong under a very strong time pressure as it is in a situation with no time pressure at all.

In a last follow-up study, the same histogram comparison task was given to a group of experts. Generally, it is assumed that experts focus more on structural principles of a task whereas novices focus more on surface features (e.g., Rabinowitz & Hogan, 2002). One could thus expect that experts no longer are sensitive to the intuitive processing of histograms. Lem et al. (2014) invited a group of experts, consisting of students in a master of statistics programme, university researchers and professors in statistics, and researchers actively using quantitative research paradigms (mainly researchers in educational effectiveness). Also in this group, a lower accuracy on some incongruent items was found (although less pronounced), and also longer reaction times to correct answers in incongruent items than to congruent items were found.

While these studies occurred in rather artificial settings (a computer environment that registers reaction times to long series of carefully controlled item stimuli offered in different experimental conditions), they have important implications, as they show the intuitive nature of specific misinterpretations that are widely present in students, and show their particular strength, as they even occur in experts.

THE ROLE OF ILLUSTRATIONS IN SOLVING WORD PROBLEMS

Above, I already referred to the research line that demonstrated that upper primary school students often do not make realistic considerations when solving word problems, even when this is necessary (Verschaffel et al., 2000, 2009). Various attempts to lead students to make more realistic considerations have only had moderate success. Such attempts include warnings in the test (Yoshida et al., 1997), working in dyads and suggesting to reword the problems (Mellone et al., 2017), problem posing (Chen et al., 2007) or using humor (Van Dooren et al., 2018). Sometimes a small positive effect was found, sometimes not even a significant one, but in all studies the comments in the discussion referred to the fact that the obtained effects were much smaller than expected, and even not educationally relevant.

Dewolf et al. (2014) conducted a series of studies focused on a specific type of intervention that is very common in mathematical textbooks, and that had the potential to increase the number of realistic considerations made by students: provide illustrations that show the realistic complexity of the word problem. For instance, they used the following word problem:

> A man wants to have a rope long enough to stretch between two poles 12m apart, but he has only pieces of rope 1.5m long. How many of those pieces would he need to tie together to stretch between the poles?

In one condition, the word problems were accompanied by illustrations that were especially designed to illustrate the realistic complexity. The illustration that was designed for the rope problem is shown in Figure 4. In the other condition, the word problem was given without the illustration. Contrary to what was expected, results obtained with a large group of fifth and sixth graders showed no difference between both conditions. Even when the realistic complexity was emphasized by placing a marker in the accompanying illustration and when this was pointed out explicitly in the test booklet, no effect was found (Dewolf et al., 2014).

In an attempt to gain a deeper understanding of the absence of the effects of illustrations (and even of stimulating their use), Dewolf et al. (2015) gathered eye-tracking data with higher education students who were offered the same word problems with the illustrations. In a follow-up experiment, they even forced the students to look at the illustration and process it before they received the word problem. The most important finding of these studies was that students only scarcely look at the illustrations, and that even when they do so, there was no visible effect on the number of realistic reactions. In discussing these findings, Dewolf et al. (2015) refer to the fact that even if the illustrations are looked at, this is usually done in a very superficial manner, similarly as when the illustration would have a merely decorative role. Indeed, many illustrations in textbooks are only decorative and not really functional to gain a better understanding of the realistic situation that a word problem is referring to, which may have created a belief in students that illustrations by default do not serve a meaningful purpose in a word-problem solving process.

Figure 4. Illustration accompanying the “rope” word problem (Dewolf et al., 2014, p. 338)

Once more, an eye-tracking study conducted in very controlled circumstances and in a sample of higher education students (while the tasks were initially designed for fifth and sixth graders) reveals important insights about mechanisms that underly a task that occurs frequently in mathematics classrooms (word problem solving) and in which a fundamental problem frequently occurs (the fact that learners do not include their real-world knowledge). Other research techniques that would be more embedded in natural classroom settings would most likely not be able to reveal these processes.

"WEIRD" STUDIES IN MATHEMATICS EDUCATION

I would like to spend the last pages of this plenary paper for the discussion of a few studies that are unusual in several ways. They are conducted in a way that is very far from the natural mathematics classroom setting, and yet they are provocative. They provide some insights that seem important for that setting despite the unusual methodology.

As I explained above, I have selected two studies that seem a bit odd at first sight, and the very idea to conduct them may initially make the reader laugh, but after a while it may also make them think. Such studies have characteristics that are shared with studies that are eligible for an Ig Nobel award (improbable.com/ig/about-the-ig-nobel-prizes/).

That sounds Greek to me! Offering word problems in another language

Earlier in this paper, I already explained learners' tendency to apply proportional methods to non-proportional word problems. In previous studies, we used additive (thus non-proportional) problems to demonstrate that from 4th grade on, children increasingly tend to provide proportional answers (Van Dooren et al., 2005). The following problem was frequently used:

> Ellen and Kim are running around a track. They run equally fast but Ellen started later. When Ellen has run 4 laps, Kim has run 16 laps. When Ellen has run 8 laps, how many has Kim run?

Percentages of students answering "32 laps" to this problem increased from around 20% in Grade 4 until more than 50% in Grade 6. So, a more complex (multiplicative) solution is applied by a substantial number of students, while a simpler (additive) solution is correct for this kind of word problems, and this phenomenon gets stronger with age. This kind of error has been observed in numerous other studies (Fernandez et al., 2012; Van Dooren et al., 2009, 2010), and was also seen in adult students (Gillard et al., 2009).

A combination of possible explanations is brought forward to explain the phenomenon. As the number of errors increases in the period when proportionality is systematically taught and practiced in school, it seems that along with the acquisition of the ability to solve proportional word problems, the overgeneralization of this skill also starts to occur. A reference is often made to the tendency of learners to apply the most recently acquired strategies to whatever problems they have to solve, which is one of the

undesired sociomathematical norms or "rules of the game" when solving word problems that learners learn to follow (Verschaffel et al., 2000). Other complementary explanations refer to learners' tendency to engage in superficial modelling strategies when solving word problems, which are also partly a consequence of teaching practices. Often, there is a strong focus on the technically correct and fluent execution of certain skills and procedures, without explicitly and systematically questioning whether that skill or procedure is applicable. We would argue that argued instruction in proportionality often leads students to initially acquire a kind of "routine expertise" (defined by Hatano, 2003, as "simply being able to complete school mathematics exercises quickly and accurately without much understanding," p. xi) instead of an "adaptive expertise" ("the ability to apply meaningfully learned procedures flexibly and creatively," p. xi). A particularly salient element here is that proportional reasoning is mainly exercised by means of missing-value problems. Most proportional reasoning tasks that students encounter are formulated in this missing-value format (Cramer et al., 1993). At the same time, nonproportional problems stated in a missing-value format like the ones used by Van Dooren et al. (2005) are very rare.

The routine nature of students' proportional answers to word problems is convincingly shown in a study by Degrande et al. (2016), although it used a very unusual method: word problems were offered in Greek. The aforementioned "runner" problem was offered to children as follows:

> Η Έλεν και η Κιμ τρέχουν γύρω από μια πίστα. Τρέχουν εξίσου γρήγορα αλλά η Έλεν ξεκίνησε αργότερα. Όταν η Έλεν έχει τρέξει 4 γύρους, η Κιμ έχει τρέξει 16 γύρους. Όταν η Έλεν έχει τρέξει 8 γύρους, πόσους έχει τρέξει η Κιμ?

Children in this study were third to sixth graders from Flanders, Belgium. They were Dutch-speaking and hence could not understand the word problems. Degrande et al. (2016) simply included some word problems in Greek in a larger test with a variety of mathematical problems, preceded by the sentence: "This word problem is in Greek. Try to solve it anyway".

Evidently, as learners could not understand these word problems, it made no sense to evaluate the answers in terms of correctness. Still, it was possible to investigate how many children - even in the situation where they have absolutely no idea about the contents of the word problem – would be prepared to "play along the rules of the game of word-problem solving" (Verschaffel et al., 2000) and thus provide a numerical answer, and how many of them would do so by giving a "proportional" answer.

Results pointed out that about 60% of third to sixth graders were prepared to give a numerical answer. Younger children often simply added the three numbers in the word problem, but the number of proportional answers strongly increased with age, with up to 41% proportional answers in sixth grade.

Degrande et al. (2016) took the study a step further. They already showed that merely offering a piece of incomprehensible text including some numbers leads students of

specific ages to generally come up with the same solution. But what influence to the numbers in this incomprehensible text have on this behavior? To figure this out, Degrande et al. used a variant of the Greek "runner" problem in another mathematical test which was given to the same students one week later. The variant was as follows:

> Η Έλεν και η Κιμ τρέχουν γύρω από μια πίστα. Τρέχουν εξίσου γρήγορα αλλά η Έλεν ξεκίνησε αργότερα. Όταν η Έλεν έχει τρέξει 4 γύρους, η Κιμ έχει τρέξει 14 γύρους. Όταν η Έλεν έχει τρέξει 6 γύρους, πόσους έχει τρέξει η Κιμ?

The word problem is identical to the previous one; only the numbers in it were purposefully manipulated: If learners wanted to apply proportional calculations now, they would need to work with non-integer ratios (to go from 4 to 6, they need to multiply by 1.5, and to go from 4 to 14, they need to multiply by 3.5). After this manipulation, the number of children who were prepared to give a numerical answer to the Greek word problem did not change meaningfully, but the number of children giving a proportional answer decreased: no children in 3rd or 4th grade gave such answers, but still more than 20% of sixth graders gave a proportional answer, even though it required quite complex calculations.

Evidently, offering children word problems in another language – and even using an alphabet system that they are not familiar with – is not a common thing to do in the mathematics classroom. It is an experimental stimulus in the purest sense of the word, and some called it a miracle that learners were prepared to take part in the study at all. As a side note, it also took quite some effort to convince the ethical committee to give approval for this study.

Still, despite the strange nature of the study, this approach also shows in a clear way how learners process word problems, or better: how they often do not process them at all. Even when they are offered word problems they do not understand, they have a sense of what is probably expected from them, and the generally think alike. And even a seemingly very subtle manipulation of the problem such as changing its numbers drastically affects what "solution" learners come up with. Such behavior is undeniably shaped by the classroom practices that children have engaged in, and the results therefore make us think about the desirability of these classroom practices, in which children can indeed quite easily predict which operations or combinations of operations they need to apply to word problems: the most recently learned ones, or the ones suggested by the problem formulation, and even the numbers in the problem may give a good hint.

Pigeons are better than humans in solving the Monty Hall dilemma

When thinking about thought-provoking laboratory studies in mathematics education, I felt it would be nice to include also a study involving animals: One can barely take more distance from the natural classroom setting than by looking at how non-humans solve "mathematical tasks". Plenty of psychological studies have shown that also non-human animals have certain mathematical abilities, such as discriminating small

numerosities (e.g., Gelman & Gallistel, 1991). However, I have chosen a more complex task – even a notoriously difficult one – involving conditional probabilistic reasoning: the Monty Hall dilemma.

The Monty Hall dilemma – also known as the Three Doors problem – is a notorious brain teaser, adapted from the popular TV game show *Let's make a Deal* (Friedman, 1988). The dilemma works as follows: In a TV game show, a contestant is shown three doors. Behind one of the doors, there is a car. The other two doors each hide a mock prize: a goat. The contestant initially chooses one of the doors. Next, the show host (who is aware of the location of the car) opens one of the non-chosen doors to reveal that there is a goat behind it. Then, the contestant is asked whether they want to stay with the initially chosen door, or wants to change to the other unopened door.

The most optimal strategy in this dilemma is to change from the initially chosen door to the remaining unopened door. The probability that the initially chosen door hides the car is 1/3, while the probability that the remaining unopened door hides the car is 2/3. However, that solution is very counterintuitive. Many people tend to think that the chance of winning the car is equal for the two doors (e.g., Stibel et al., 2009).

Numerous studies have been conducted to understand the difficulties people have in understanding the Monty Hall dilemma and in providing a correct response to it (for an overview and a systematic review of explanations that were identified in research, see Saenen et al., 2018). Explanations relate to not understanding basic probabilities at all and the illusion that one can control outcomes, a failure to take into account that the show host's behavior is dependent (the show host knows where the car is, and depending on the initially chosen door often has only one door he can open), the equiprobability bias (thinking that when two doors are left, both must have the same chance), the anticipation of regret (which is experienced as worse when the game host has convinced the contestant to switch than when the contestant stayed with the initial host) and many more.

Saenen et al. (2019) gave their systematic review the title "Why humans fail in solving the Monty Hall dilemma". The title suggests that non-human animals might be better in solving the dilemma. And this indeed turns out to be the case.

Herbranson and Schroeder (2010) sought to compare the decision-making abilities of pigeons (*Columba Livia*) and humans (*Homo Sapiens*) when confronted with the Monty Hall problem. To achieve this, they designed an experiment in which pigeons were trained to engage in a modified version of the dilemma. The experiment featured a three-key setup, with each key representing one of the three doors in the Monty Hall scenario. When a pigeon pecked a key, it effectively "chose" a door. A computer program, acting as the game show host, would then reveal an empty outcome behind one of the two unchosen keys. The pigeon then had the opportunity to either stick with its initial choice or switch to the remaining key. If the pigeon selected the correct key (i.e., the one with the hidden reward), it would receive a food reward.

A separate group of human participants was given an analogous computerized version of the task, requiring them to make choices using a computer interface. Both pigeons and humans were given ample opportunity to learn from their experience, as they did over 100 trials spread out in several training sessions.

The results were striking: pigeons learned to switch doors approximately 96% of the time, aligning closely with the mathematically optimal strategy. In contrast, human participants only switched about 36% of the time, demonstrating a strong tendency to adhere to their initial choice despite the statistical disadvantage of doing so.

The study's findings suggest that pigeons, through reinforcement learning, adapt more effectively to probabilistic scenarios than humans. This discrepancy is attributed to fundamental differences in decision-making strategies. Pigeons, unlike humans, do not experience the same cognitive biases that lead to errors in probabilistic reasoning. Instead, they rely on trial-and-error learning to maximize rewards. Over repeated trials, pigeons quickly identified the optimal strategy and consistently applied it. Humans, on the other hand, exhibited cognitive biases that interfered with their ability to adopt the correct strategy.

Does that make pigeons smarter than humans? Definitely not. The findings show the power of behaviorism and simple reinforcement learning, a mechanism that also has relevance for mathematics education (think of the need for automatization and memorization of multiplication tables), but is not the only mechanism in learning mathematics, and definitely not the mechanism for this particular setting. While we would probably be able to also teach humans to perform optimally in the Monty Hall dilemma by reinforcement learning, this would not meet the educational goals that we want to achieve by engaging learners in the dilemma.

What the results also point out and what is more important for mathematics education is the insight that humans frequently struggle with probabilistic reasoning in everyday situations. Humans – despite their superior cognitive capabilities – are often hindered by biases and irrational decision-making tendencies, but these are inherently part of the attempts they undertake to gain a deep understanding of the situations they encounter. And this is exactly what we want to achieve in mathematics education. Errors, biases, and intuitions are inherently part of learning mathematics and mathematical problem solving. The most important challenge is to turn these errors into meaningful learning experiences that have a long-lasting impact.

CONCLUDING THOUGHTS

In this paper, I have presented a number of lines of research, in which laboratory-like studies that take place in settings very distant from the natural classroom take an important role. I want to stress that generally the research lines also include studies that are closer to the relevant classroom practice, but the crucial studies are often the laboratory ones. These laboratory-like studies often have serious shortcomings in terms of ecological validity: The administered tasks are carefully controlled and look very

different from the way in which they would occur in the classroom, often studies administered long series of very similar items, stimuli were very often offered on a laptop, and in several cases even with participants sitting in front of an eye-tracker, even with a fixated head. The experimental circumstances generally did not allow social interaction (and if so, it was with an experimenter who follows a standard protocol), and samples were often from a very different population than the intended population of school students: university students, expert researchers, or even pigeons.

I have tried to argue that these studies reveal important insights in the reasoning about mathematical tasks, and in some of them even about the way in which mathematics is learned. Several of these insights would not have been achieved – or at least the available evidence would not be that strong – if only studies in ecologically more valid settings would have been conducted. I do not wish to suggest that studies in ecologically valid settings have no value; on the contrary. But research into the psychology of mathematics education needs to rely on a range of paradigms and research techniques.

Evidently, because of the lack of ecological validity, several of the studies that I presented cannot be directly translated to the classroom. Often, this is used as an aspect of criticizing these studies. However, that direct translation was never the intention. Rather, the studies aimed at gaining theoretical understanding, at revealing thinking and learning mechanisms, pointing at task features that impact our thinking and learning, and so on. These insights are important for the classroom, and a thorough understanding of these – to be disseminated by including them in teacher education and professionalisation activities – may help all professionals who take part in shaping what happens in the mathematics classroom, including teachers and developers of learning materials to improve the mathematics education practice.

References

Abt, M., Leuders, T., Loibl, K., Strohmaier, A., Van Dooren, W., & Reinhold, F. (2024). How can Eye-Tracking Data be Used to Understand Cognitive Processes when Comparing Data Sets with Box Plots? *Frontiers in Education*, *9*.

Abt, M., Loibl, K., Leuders, T., Van Dooren, W., & Reinhold, F. (2025). Understanding student errors in comparing data sets with boxplots. *Educational Studies in Mathematics*.

Berliner, D. C. (2002). Comment: Educational Research: The Hardest Science of All. *Educational Researcher*, *31*(8), 18–20.

Chen, L., Van Dooren, W., Chen, Q., & Verschaffel, L. (2007). The relationship between posing and solving arithmetic word problems among Chinese elementary school children. *Research in Mathematical Education.*

Degrande, T., Verschaffel, L., & Van Dooren, W. (2016). Proportional word problem solving through a modeling lens: A half-empty or half-full glass? In: P. Felmer, E. Pehkonen, J. Kilpatrick (Eds.), *Posing and Solving Mathematical Problems: Advances and New Perspectives* (209-229). Switzerland: Springer International Publishing.

delMas, R., Garfield, J., & Ooms, A. (2005, July). *Using assessment items to study students' difficulty reading and interpreting graphical representations of distributions.* Paper presented at the Fourth Forum on Statistical Reasoning, Thinking, and Literacy, Auckland, New Zealand.

Dewolf, T., Van Dooren, W., Ev Cimen, E., Verschaffel, L. (2014). The impact of illustrations and warnings on solving mathematical word problems realistically. *Journal Of Experimental Education*, *82* (1), 103-120.

Dewolf, T., Van Dooren, W., Hermens, F., Verschaffel, L. (2015). Do students attend to representational illustrations of non-standard mathematical word problems, and, if so, how helpful are they? *Instructional Science*, *43* (1), 147-171.

Evans, J. St. B. T. (2008). Dual-processing accounts of reasoning, judgement and social cognition. Annual Review of Psychology, 59, 255-278.

Fernández, C., Llinares, S., Van Dooren, W., De Bock, D., & Verschaffel, L. (2012). The development of students' use of additive and proportional methods along primary and secondary school. *European Journal of Psychology of Education*, *27* (3), 421-438.

Gallistel, C. R., & Gelman, R. (1991). Subitizing: The preverbal counting process. In W. Kessen, A. Ortony, & F. I. M. Craik (Eds.), *Memories, thoughts, and emotions: Essays in honor of George Mandler* (pp. 65–81). Lawrence Erlbaum Associates, Inc.

Gillard, E., Van Dooren, W., Schaeken, W., & Verschaffel, L. (2009). Proportional reasoning as a heuristic-based Process: Time constraint and Dual-task considerations. *Experimental Psychology*, *56* (2), 92-99.

Gillard, E., Van Dooren, W., Schaeken, W., Verschaffel, L. (2009). Dual-processes in the psychology of mathematics education and cognitive psychology. *Human Development*, *52* (2), 95-108.

Glaser, R., Lieberman, A., & Anderson, R. (1997). "The vision thing": educational research and AERA in the 21st century. Part 3: Perspectives on the research–practice relationship. Educational Researcher, 26 (7), 24–25.

González-Forte, J.M., Fernández, C., Van Hoof, J., & Van Dooren, W. (2022). Incorrect ways of thinking about the size of fractions. *International Journal Of Science And Mathematics Education*.

Herbranson, W. T., & Schroeder, J. (2010). Are birds smarter than mathematicians? Pigeons (*Columba livia*) perform optimally on a version of the Monty Hall Dilemma. *Journal of Comparative Psychology, 124,* 1–13.

Inglis, M., & Alcock, L. (2012). Expert and Novice Approaches to Reading Mathematical Proofs. *Journal for Research in Mathematics Education JRME*, *43*(4), 358-390.

Inglis, M., & Alcock, L. (2018). Watching mathematicians read mathematics. In: E. Bergqvist, M. Österholm, C. Granberg, L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education (Vol. 2)*.

Kahneman, D. (2000). A psychological point of view: Violations of rational rules as a diagnostic of mental processes. Behavioral and Brain Sciences, 23, 681-683.

Lem, S., Kempen, G., Ceulemans, E., Onghena, P., Verschaffel, L., & Van Dooren, W. (2015). Combining multiple external representations and refutational text: An intervention on learning to interpret box plots. *International Journal of Science and Mathematics Education*, *13* (4), 909-926.

Lem, S., Onghena, P., Verschaffel, L., & Van Dooren, W. (2013). External representations for data distributions: in search of cognitive fit. *Statistics Education Research Journal*, *12* (1), 4-19.

Lem, S., Onghena, P., Verschaffel, L., & Van Dooren, W. (2013). External representations for data distributions: in search of cognitive fit. *Statistics Education Research Journal*, *12* (1), 4-19.

Lem, S., Onghena, P., Verschaffel, L., & Van Dooren, W. (2013). The heuristic interpretation of box plots. *Learning and Instruction*, *26* (4), 22-35.

Lem, S., Onghena, P., Verschaffel, L., & Van Dooren, W. (2014). Interpreting histograms. As easy as it seems? *European Journal of Psychology of Education*, 1-19.

Leron, U. & Hazzan, O. (2006). The Rationality Debate: Application of cognitive psychology to mathematics education. Educational Studies in Mathematics, 62, 105-126.

Mejia-Ramos, J.P., & Weber, K. (2014). Why and how mathematicians read proofs: further evidence from a survey study. *Educational Studies in mathematics, 85*, 161–173.

Mellone, M., Verschaffel, L., & Van Dooren, W. (2017). The effect of rewording and dyadic interaction on realistic reasoning in solving word problems. *The Journal of Mathematical Behavior*, *46*, 1-12.

Miyake, A., Friedman, N. P., Rettinger, D. A., Shah, P., & Hegarty, M. (2001). How are visuospatial working memory, executive functioning, and spatial abilities related? A latent-variable analysis. Journal of Experimental Psychology: General, 130, 621-640.

Phillips, D.C., & Dolle, J.R. (2006). From Plato to Brown and beyond: Theory, practice, and the promise of design experiments. In: L. Verschaffel, F. Dochy, M. Boekaerts, S. Vosniadou (Eds.), *Instructional psychology: past, present and future trends*, (277-292). Oxford: Elsevier.

Saenen, L., Heyvaert, M., Van Dooren, W., Schaeken, W., & Onghena, P. (2018). Why Humans Fail in Solving the Monty Hall Dilemma: A Systematic Review. *Psychologica Belgica*, *58* (1), 128-158.

Sloman, S. A. (1996). The empirical case for two systems of reasoning. Psychological Bulletin, 119, 3-22.

Stibel, J. M., Dror, I. E., & Ben-Zeev, T. (2009). The collapsing choice theory: Dissociating choice and judgment in decision making. Theory and Decision, 66, 149–179.

Todd, P. M., & Gigerenzer, G. (2012). What is ecological rationality? In P. M. Todd, G. Gigerenzer (Eds.) & ABC Research Group, *Ecological rationality: Intelligence in the world* (pp. 3–30). Oxford University Press.

Tversky, B. (1997). *Cognitive principles of graphic displays.* Paper presented at the Association for the Advancement of Artificial Intelligence Workshop on Diagrammatic Reasoning. Cambridge, MA.

Van Dooren, W., De Bock, D., Evers, M., & Verschaffel, L. (2009). Students' overuse of proportionality on missing-value problems: How numbers may change solutions. *Journal for Research in Mathematics Education*, *40* (2), 187-211.

Van Dooren, W., De Bock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2005). Not everything is proportional: Effects of age and problem type on propensities for overgeneralization. *Cognition and Instruction*, *23* (1), 57-86.

Van Dooren, W., De Bock, D., Janssens, D., & Verschaffel, L. (2008). The linear imperative. An inventory and conceptual analysis of students' overuse of linearity. *Journal for Research in Mathematics Education*, *39* (3), 311-342.

Van Dooren, W., Lehtinen, E., & Verschaffel, L. (2015). Unravelling the gap between natural and rational numbers. *Learning and Instruction*, *37*, 1-4.

Van Dooren, W., Lem, S., De Wortelaer, H., & Verschaffel, L. (2018). Improving realistic word problem solving by using humor. *The Journal of Mathematical Behavior*, *53*, 96-104.

Van Hoof, J., Degrande, T., Ceulemans, E., Verschaffel, L., & Van Dooren, W. (2018). Towards a mathematically more correct understanding of rational numbers: A longitudinal study with upper elementary school learners. *Learning & Individual Differences*, *61*, 99-108.

Verschaffel, L., De Corte, E., & Lasure, S. (1994). Realistic considerations in mathematical modeling of school arithmetic word problems. *Learning and Instruction*, *4*, 273-294.

Verschaffel, L., Greer, B., Van Dooren, W., & Mukhopadhyay, S. (Eds.) (2009). *Words and worlds. Modelling verbal descriptions of situations*.. Rotterdam/Boston/Taipei: SensePublishers.

Weber, K. & Mejía-Ramos, J. P. (2013). On mathematicians' proof skimming: A reply to Inglis and Alcock. Journal for Research in Mathematics Education, 44, 464-471.

Weber, K., & Mejia-Ramos, J.P. (2011). Why and how mathematicians read proofs: an exploratory study. *Educational Studies in Mathematics,* 76, 329–344.

Weinert, F. E., & De Corte, E. (1996). Translating research into practice. In E. De Corte, & F. E. Weinert, International encyclopedia of developmental and instructional psychology. Oxford, UK: Elsevier Science.

Yoshida, H., Verschaffel, L., & De Corte, E. (1997). Realistic considerations in solving problematic word problems: Do Japanese and Belgian children have the same difficulties? *Learning and instruction*, *7* (4), 329-338.

MAKING SURE THAT MATHEMATICS EDUCATION
RESEARCH REACHES THE CLASSROOM

PLENARY PANEL

MATHEMATICS EDUCATION RESEARCH MUST BE USEFUL FOR THE CLASSROOM

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The theme of the PME-48 conference, Making sure that mathematics education research reaches the classroom, implicitly acknowledges a concern: not all mathematics education research finds its way into classroom practice. This raises several fundamental questions: Which research fails to reach the classroom? Why? And should all research be expected to do so? It is therefore fitting that the plenary panel engages with these significant issues by debating the following motion: Mathematics education research must be useful for the classroom. This paper presents the debate as structured for the purposes of this publication. Following an introduction by the Chair, Tanya Evans, each panellist's arguments are presented individually. Anthony Essien and Salomé Martínez Salazar argue against the motion, while Maitree Inprasitha and Demetra Pitta-Pantazi argue in support. We hope this debate will stimulate ongoing dialogue and encourage the mathematics education research community to critically engage with this important issue—one that is central to the relevance and influence of research in our field.

INTRODUCTION (TANYA EVANS)

To debate the motion *"Mathematics education research must be useful for the classroom"*, it is first necessary to ground our discussion in the fundamental question: *What do we mean by mathematics education research?* Without a shared understanding of what mathematics education research entails, discussions about its usefulness—particularly to classroom practitioners—risk becoming either vague or polarising.

A useful frame for exploring this question is offered by Inglis and Foster (2018), who conducted a seminal study providing a comprehensive and systematic overview of mathematics education research over the past five decades. They analysed the full text of all articles published in two leading journals of the field—*Educational Studies in Mathematics* and the *Journal for Research in Mathematics Education*—from their inception to the present. Using a statistical modelling technique known as topic modelling, they identified prominent topics and traced their evolution over time. Importantly, Inglis and Foster drew on Lakatos's (1978) methodology of scientific research programmes to interpret their findings. The central idea in Lakatos's account is that the base descriptive unit of research is not, as Popper (1959) maintained, the individual hypothesis or even theory, but rather the research programme—a broader and more durable structure that guides the development of knowledge over time.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, pp. 97-118. PME.

Lakatos (1978) described a research programme as comprising three main components: a "hard core" of foundational assumptions, a "protective belt" of auxiliary hypotheses, and a "heuristic" that governs how research progresses. The hard core defines the programme's core commitments and is shielded from falsification. The protective belt can be modified to accommodate anomalies from new findings, allowing the hard core to remain intact. The heuristic consists of guiding principles for selecting methods, investigating phenomena, generating data, and developing theories within the programme.

Importantly, Lakatos used this framework to distinguish between *progressing* and *degenerating* research programmes. A progressing programme generates new insights and explains more phenomena over time. A degenerating one, in contrast, responds to anomalies only through post hoc adjustments to its protective belt, without generating new predictive power. Over time, researchers may abandon a degenerating programme in favour of one that is progressing.

Inglis and Foster (2018) applied this framework to the history of mathematics education research, illustrating how various research programmes have risen and declined over time. For example, the constructivist research programme, which dominated the 1980s and 1990s, was founded on the hard-core assumption that knowledge construction is an individual process aimed at helping learners make sense of the world. According to this view, individuals receive sensory input, filter it, and actively organise it into mental schemas. Learning occurs when people construct new knowledge by creating new schemas or reorganisig existing ones. Rather than passively absorbing information, knowledge is actively constructed by the individual. The programme's heuristic was based on Piagetian clinical interviews and small-scale qualitative studies focused on how students make sense of mathematical ideas (Piaget & Cook, 1952).

By the late 1990s and early 2000s, a shift was underway: the so-called "social turn" in mathematics education research, documented by Lerman (2000), signalled the rise of a new sociocultural research programme. This programme rejected the individualist assumptions of constructivism and instead asserted that thinking and learning are fundamentally social processes. As Vygotsky (1986) famously put it, "the true direction of the development of thinking is not from the individual to the social, but from the social to the individual" (p. 36). Consequently, the heuristic shifted from individual interviews to analyses of classroom discourse, peer interaction, and the broader social and cultural contexts of mathematics learning (Inglis & Foster, 2018).

Rather than replacing the earlier paradigm entirely, the sociocultural turn ushered in a period of theoretical diversification. As Inglis and Foster (2018) observed, multiple research programmes—such as those based in semiotics, embodied cognition, and didactical theories—now coexist and often compete for attention. From a Lakatosian perspective, this theoretical pluralism can be seen as a vital indicator of disciplinary

maturation. Different research programmes challenge each other by offering rival explanations and methodological innovations.

Crucially, the interaction between competing research programmes should enable researchers to uncover anomalies, refine protective belts, and drive the ongoing evolution and vitality of the field. However, as Dreyfus (2006) warned, theoretical proliferation without empirical challenge can also lead to fragmentation. He criticised the field for "inventing theories at a pace faster than we produce data to possibly refute our theories" (p. 78). In other words, the discipline risks hosting degenerating programmes that persist unchallenged—not because they are progressing, but because they are insulated from critique. To the extent that different research programmes fail to engage with one another, mathematics education research may lack the kind of productive tension that Lakatos considered essential for progress.

An illustrative example of the need for cross-programme dialogue lies in the relationship between the sociocultural research programme and the experimental psychology programme—so termed by Inglis and Foster—which is closely related to what is now often referred to as the *Science of Learning* (National Academies of Sciences, 2018)). The latter is characterised by a hard-core commitment to developing and testing theories through the manipulation of experimental stimuli to reveal underlying psychological processes—whether cognitive, behavioural, social, or developmental. Instructional approaches associated with this programme—such as explicit instruction, mastery learning, spaced retrieval practice, memorisation of arithmetic facts, and time-limited practice—have produced robust evidence of effectiveness (e.g., Fuchs et al., 2021; Gersten et al., 2009; Kirschner et al., 2006; McNeil et al., 2025). From a sociocultural perspective, however, these methods may appear to conflict with its hard-core assumptions, particularly the view that learning is constructed through social interaction and participation. Yet such findings constitute potential anomalies—ones that should prompt reconsideration of aspects of the protective belt of sociocultural theory (Inglis & Foster, 2018).

Conversely, experimental psychology might benefit from insights generated within sociocultural programme. For instance, research on micro-identities (e.g., Wood, 2013) shows that students' mathematical identities can shift rapidly in response to subtle contextual cues. This finding challenges a key assumption of many cognitive experiments: that individual traits and behaviours are relatively stable over the course of an experimental task. If micro-level identity shifts influence learning outcomes, then experimental psychology may need to refine its methods and assumptions to accommodate such variability (Inglis & Foster, 2018).

This brief account demonstrates that mathematics education research is not a monolithic endeavour but a diverse and evolving field comprising multiple research programmes with competing claims and heuristics. Against this backdrop, the question of usefulness—central to this plenary panel—becomes more complex. What counts as "useful" depends heavily on the epistemological commitments of the research

programme in which the work is situated. For instance, a programme grounded in experimental psychology may value generalisable findings about cognitive processes, often derived from controlled studies with individual learners. By contrast, sociocultural research may prioritise rich, contextualised understandings of classroom interaction, student identity, or institutional discourse— insights that might not be readily transferable to other settings, but that can deepen our understanding of mathematics education more broadly. To ask both programmes to produce the same kind of "useful" knowledge is to ignore their differing assumptions, aims, and methods.

The theme of the PME-48 conference—Making sure that mathematics education research reaches the classroom—implicitly acknowledges a concern: that some research in mathematics education has limited impact on practice. But this prompts deeper questions. Which types of research struggle to reach classrooms, and why? Should all research be expected to do so? And perhaps most fundamentally, is usefulness to classroom practice a necessary criterion for mathematics education research to be considered valuable?

This plenary panel paper addresses these questions through a structured debate of the motion: *Mathematics education research must be useful for the classroom*. Following this introduction, the arguments of the four panellists are presented. Anthony A Essien and Salomé Martínez Salazar argue against the motion, while Maitree Inprasitha and Demetra Pitta-Pantazi argue in support. The perspectives they offer are shaped by the research traditions, methodological tools, and theoretical assumptions they have engaged with—elements that align in various ways with different research programmes as described by Lakatos (1978). Framing the debate in these terms highlights how underlying assumptions about knowledge, methods, and aims influence what is considered "useful" in mathematics education research. Rather than seeking resolution, this discussion aims to illuminate the productive tensions across programmes and to stimulate further reflection on how diverse forms of research can contribute meaningfully to our field.

References

Dreyfus, T. (2006). Linking theories in mathematics education. In A. Simpson (Ed.), *Retirement as process and concept: A festschrift for Eddie Gray and David Tall* (pp. 77-82). Durham University.

Fuchs, L. S., Bucka, N., Clarke, B., Dougherty, B., Jordan, N. C., Karp, K. S., Woodward, J., Jayanthi, M., Gersten, R., & Newman-Gonchar, R. (2021). Assisting students struggling with mathematics: Intervention in the elementary grades. Educator's Practice Guide. WWC 2021006. *What Works Clearinghouse*.

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, *79*(3), 1202-1242. https://doi.org/10.3102/0034654309334431

Inglis, M., & Foster, C. (2018). Five decades of mathematics education research. *Journal for Research in Mathematics Education*, *49*(4), 462-500. https://doi.org/10.5951/jresematheduc.49.4.0462

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist*, *41*(2), 75-86. https://doi.org/10.1207/s15326985ep4102_1

Lakatos, I. (1978). *The methodology of scientific research programmes: Philosophical Papers* (J. Worrall & G. Currie, Eds. Vol. 1). Cambridge University Press. https://doi.org/DOI: 10.1017/CBO9780511621123

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (Vol. 1, pp. 19-44). Ablex.

McNeil, N. M., Jordan, N. C., Viegut, A. A., & Ansari, D. (2025). What the science of learning teaches us about arithmetic fluency. *Psychological Science in the Public Interest*, *26*(1), 10-57. https://doi.org/10.1177/15291006241287726

National Academies of Sciences, Engineering, and Medicine. (2018). *How People Learn II: Learners, Contexts, and Cultures*. The National Academies Press. https://doi.org/doi:10.17226/24783

Piaget, J., & Cook, M. (1952). *The origins of intelligence in children* (Vol. 8). International universities press. https://doi.org/doi:10.1037/11494-000

Popper, K. (1959). *The logic of scientific discovery*. Routledge.

Vygotsky, L. S. (1986). Thought and language (A. Kozulin, trans.). In: Cambridge, MA: MIT Press.

Wood, M. B. (2013). Mathematical micro-identities: Moment-to-moment positioning and learning in a fourth-grade classroom. *Journal for Research in Mathematics Education*, *44*(5), 775-808.

PITFALLS OF CONSEQUENTIALISM IN MATHEMATICS EDUCATION (ANTHONY A ESSIEN)

Mathematics and mathematics education research

Should mathematics be taught solely for its practical value to humanity? Should research in mathematics education be deemed useful only in as much as it informs classroom practice? An answer in the affirmative would suggests the consequentialist perspective of mathematics education and mathematics education research where the outcome or results are prized above the inherent benefit that accrue to a particular activity or act, in the present case, research in mathematics education. Speaking on the irrationality of mathematics education research, Herbert Wilf (2011) made a radical statement that “Research in mathematics education is generally hopelessly unintelligent. The field as a whole is not far from worthless.” My own position is very

different from Wilf's (2011) fundamentalist position. I argue rather against solely viewing mathematics education research though the consequentialist lens.

The risks of consequentialism in mathematics education research

As its name suggests, consequentialism is a philosophical approach that prioritises the consequences or outcomes of actions when determining their value. It asserts that the overall consequences of an action encompass both the action itself and all that it brings about. Central to consequentialism is the idea that the outcomes of our actions possess inherent value and should serve as the main standard for value judgement. This view emphasises the importance of results above the methods used to achieve them.

One significant challenge with consequentialism, however, is that it does not specify which types of consequences are considered good. A parallel can be drawn with the statement: *Mathematics education research must be useful for the classroom*. What type of research does this imply, and by who? As an example, for a very long time before the early 1970s, research in mathematics education led the mathematics education community into believing that due to dissipation of stock of available intellect resulting from knowing more than one language, bi/multilingualism has a negative cognitive effect. For example, research by MacNamara in the early 1960s on the mathematics achievement of learners from English speaking homes when taught arithmetic in the medium of Irish falls into this category. MacNamara's sample consisted of 1,084 learners from 119 schools in Ireland. Each of the learners was given tests in problem arithmetic and mechanical arithmetic (see for example, Macnamara, 1977). The conclusion MacNamara drew from this study was that Irish bilingual education, and bilingualism in general, has a negative consequence (Baker, 1988). Today, with a plethora of further research in the field of language and mathematics (see Essien and Adler, 2025), the epistemic potential of the multiplicity of languages is acknowledged. Clearly, the work of MacNamara which used research results to argue for the compartmentalisation of human brain was deemed 'useful' in classroom practices, and in deciding on bi/multilingual policies at a time, albeit, with devastating consequences.

Mathematics has a value in itself, so does mathematics education research

Mathematics possesses inherent worth, offering a distinct lens through which to understand the world by emphasising logic, abstract thinking, and the pursuit of knowledge independent of immediate practical use. In the words of Ernest (2016), "[I]rrespective of its worldly origins and applications, like many crafts and practices, mathematics takes on a life of its own". Mathematics engages the mind in a rich intellectual experience, nurturing critical thinking and problem-solving while encouraging the exploration of complex ideas. The process of engaging with mathematics can be deeply fulfilling, providing a sense of achievement and insight into the structures underlying various aspects of life. Furthermore, developing mathematical skills sharpens logical reasoning and analytical thinking—abilities that are valuable across academic disciplines, daily life, and professional fields.

By extension, mathematics education research is intrinsically valuable as it enhances our comprehension of how mathematical thinking develops and is best taught. While it serves practical purposes in refining teaching practices and curricula, its broader significance lies in promoting intellectual growth, fostering equitable learning environments, and reinforcing the role of mathematics as a vital component of human inquiry and progress.

Stifling of intellectual exploration

A consequentialist view of mathematics or mathematics education research can potentially stifle intellectual exploration, innovation and transdisciplinarity. While the value of research for classroom practice cannot be denied, placing a requirement that mathematics education research MUST be useful for the classroom undermines the broader purpose of scholarship and is a disregard for the multifarious dimensions inherent in mathematics education as a field. A consequence of this is that theoretical advancements are also stifled.

To the above also lies the question of immediate practicality vs long term applicability. The danger of consequentialism in mathematics education research lies in the fact that research would focus on the immediate mathematics education problems, and research on these. History has taught us that many breakthroughs in science and mathematics began with abstract investigations. Focusing on the immediate problem has the potential of stifling future innovations and in mathematics education research, this could translate into stifling critical research that could challenge dominant paradigms if these are not immediately applicable to classrooms. For instance, research on feminism in mathematics education may not offer immediate practical utility for the classroom, but could expose systemic inequalities.

Conclusion

In summary, mathematics is valuable not only for its practical applications but also for its intrinsic intellectual qualities, its role as a foundation for other disciplines, and its contribution to the development of essential cognitive skills and ethical values. In the same vein, while classrooms applicability is an important facet of mathematics education research, it cannot be the sole criterion of its value.

References

Baker, C. (1988). *Key Issues in Bilingualism and Bilingual Education*. Clevedon: Multilingual Matters.

Ernest, P. (2016). Mathematics and Values. In B. Larvor (Ed.), *Mathematical Cultures* (pp. 189-214). Birkhäuser, Cham. https://doi.org/10.1007/978-3-319-28582-5_12

Essien, A.A. & Adler, J. (2025). Language in mathematics education in Africa: Chronicling recent growth and leadership in the field. *ZDM Mathematics Education*. 57(4). https://doi.org/10.1007/s11858-025-01698-9

Macnamara, J. (1977). On the relation between language learning and thought. In J. Macnamara (Ed.), *Language learning and thought*. Academic Press.

Wiff, H. (2011). Some critiques of mathematics education research articles. *Intellectual Mathematics*. Retrieved from https://intellectualmathematics.com/blog/some-critiques-of-mathematics-education-research-articles/

SHOULD MATHEMATICS EDUCATION RESEARCH ALWAYS BE USEFUL FOR THE CLASSROOM? (SALOMÉ MARTÍNEZ SALAZAR)

In this paper, I argue that restricting research in mathematics education to approaches or areas deemed directly useful for classroom practice narrows the scope of inquiry and undermines disruptive academic efforts essential for advancing knowledge in any discipline. My arguments are, of course, shaped by my own academic trajectory. A significant part of my career has been dedicated to research in mathematics. From that perspective, it is difficult for me to imagine imposing on a discipline such a heavy burden as that of being 'useful.' Nevertheless, the discussion is far more nuanced, particularly given that mathematics education as a discipline is focused on the study of the teaching and learning of mathematics.

Indeed, several scholars have argued that education, as a discipline, should be directly oriented toward improving practice rather than primarily generating theoretical knowledge. This claim has been justified on the basis *of the distinctive character of human social action or, alternatively, by appealing to the kind of 'interest' that ought to guide educational—and perhaps all social—research* (Hammersley, 2003). Given the well-documented concern that mathematics educational research has had too little impact on improving classroom teaching and learning (Hiebert et al., 2002), it may seem tempting to restrict the focus of mathematics education research to that which provides a useful knowledge base for teaching. However, doing so entails significant risks.

The necessity of freedom for research endeavors

Abraham Flexner, founder of the Institute for Advanced Study in Princeton, argued in his influential essay "The usefulness of useless knowledge" that many of the most significant scientific breakthroughs have originated from research unconcerned with immediate practical application. His central claim is that the freedom to pursue knowledge for its own sake is not a luxury, but a prerequisite for genuine intellectual progress. Although Flexner was likely referring to fields such as mathematics, physics, and the humanities, the spirit of his argument should extend to educational research as well. If we accept mathematics education as a scientific discipline grounded in the pursuit of knowledge, it too requires the freedom to explore foundational and theoretical questions aiming to understand the nature of mathematical thinking, teaching, and learning —regardless of their immediate applicability—if it is to contribute meaningfully to long-term understanding (Schoenfeld, 2000).

The unpredictable nature of knowledge development makes it unreasonable to constrain research questions, stimuli, or approaches to what is presumed to be 'useful' in the short term (Hammersley, 2003). Knowledge production is not a linear process; it requires open-ended exploration and can follow uncertain trajectories over extended periods (Flexner & Dijkgraaf, 2017).

The need for interdisciplinary research

Requiring short-term utility from research can restrict interdisciplinary approaches. According to Repko and Szostak (2020), for two research areas to be effectively integrated and produce new knowledge, genuine interdisciplinarity is essential: one that goes beyond merely combining perspectives, and instead involves the development of shared theoretical frameworks and clearly defined goals. This process demands ongoing dialogue that helps overcome linguistic, conceptual, and methodological barriers in order to establish a common language (Klein, 2017). It also requires epistemological and methodological flexibility, enabling researchers to adapt their approaches and open new avenues for exploration and integrated knowledge (Newell, 2001).

An example of the potential of interdisciplinarity for advancing toward a more comprehensive understanding of phenomena involved in mathematical learning is proposed by Leikin (2018). In her work, she addresses the potential advantages of connecting cognitive neuroscience with mathematics education, even though she acknowledges that they are still *tangent areas*, pointing out that *even though a relatively large number of neurocognitive studies have addressed numerical cognition, these studies are rooted in cognitive psychology and are not connected to the findings of mathematics education research.* Particularly, the use of technical terminology and the fact that many studies are performed in laboratory settings, which are not similar to classroom settings, make cognitive neuroscience research studies have little impact on the processes of learning primary mathematics in school (Leikin, 2018). Furthermore, Leikin points out that there is still a small number of studies in cognitive neuroscience that explore brain processing associated with more advanced mathematical concepts and that *these are rarely connected to theories in mathematics education*. The lack of applicability of cognitive neuroscience to the classroom is also stressed by Bowers (2016), who points out that there are no current examples of neuroscience motivating new and effective teaching methods, particularly due to the fact that characterizing children's cognitive capacities through behavioral measures is easier than using brain measures.

Nonetheless, Leikin argues that neurocognitive investigation can enrich mathematics education by helping to test conclusions drawn from behavioral research in mathematics education. For this purpose, neurocognitive research goals and questions can be informed by results from behavioral research, while behavioral studies can guide neuroscientific research through task design and interpretation of findings (Leikin, 2018).

Although this type of interdisciplinary research has the potential to influence the development of mathematics education, its significant applications in the classroom remain distant. Many programs that aim to bring neuroscience closer to teachers are based on neuromyths—for example, the belief that students learn better when information is presented in their preferred learning style (Privitera, 2021). Moreover, it is often teachers' own interest, combined with their lack of specialized training in neuroscience, that may partly contribute to the persistence of these neuromyths in education (Dekker et al., 2012). Thus, time and careful preparation are necessary before findings can be useful for teaching practice.

The risk of narrowing the scope of research in mathematics education.

A crucial question for our debate is what conditions must be met for research to inform teaching practice. From an organizational science perspective, direct knowledge exchange—without intermediaries—between research and practice is straightforward only when both domains share purposes, common understandings, and a low need for innovation (Carlile, 2004). In education, these conditions are not met: researchers and teachers pursue different objectives, reforms are constant, and teachers are neither dependent on nor typically resourced or motivated to apply evidence in their daily practice (Brown & Zhang, 2016). Therefore, translating research findings into the classroom demands considerable effort. Educators need to critically engage with research, focus on pressing questions, and reflect on their practice in light of empirical evidence. Moreover, practitioners have no opportunity to shape educational research, which limits its relevance for those responsible for implementing policy in teaching and learning contexts (Cain & Allan, 2017; Ball & Forzani, 2007).

Recognizing these challenges, Hiebert et al. (2002) propose building a professional knowledge base for teaching rooted in practitioner-generated insights—rich, context-sensitive findings capable of meaningfully impacting classroom instruction. They further advocate for a research-and-development system that integrates teachers' and researchers' expertise and unique skills, ensuring that practitioner knowledge is valued, shareable, and verifiable.

However, understanding teaching practice requires research questions to be framed and answered, which demands a theoretical perspective (Mason & Waywood, 1996; Hiebert & Stiegler, 2023). Theory provides the language to frame, describe, and discern phenomena. Thus, any research paradigm employs theory to formulate questions, specify methodology, define a coherent view of study objects, articulate consistent data collection and analysis approaches, and support the validation of findings (Mason & Waywood, 1996). Accordingly, fundamental research must not be underestimated. Advancing knowledge in mathematics education requires, for example, that researchers deepen theoretical concepts related to computational thinking, equity, and justice and develop new frameworks to study emerging classroom phenomena. Mathematics education—a discipline intimately linked to the vicissitudes

of a changing society—demands that researchers continually question their prevailing methods and frameworks.

Conclusion

Framing mathematics education research in terms of its classroom utility not only narrows the scope of what we understand as research in education but also shifts its value from the generation of knowledge to a mere instrument for achieving an external goal. However commendable that goal may be, this utilitarian approach contrasts the freedom of inquiry that characterizes research in other scientific disciplines.

While it is of utmost importance that research in mathematics education informs educational practice, it also serves broader purposes. For the field to thrive intellectually, it must preserve space for foundational, interdisciplinary, and disruptive inquiries—work that may not have immediate classroom applications but enriches the discipline in ways that may ultimately benefit educational practice.

References

Ball, D. L., & Forzani, F. M. (2007). What makes education research" educational"?. *Educational Researcher, 36*(9), 529.

Bowers, J. S. (2016). The practical and principled problems with educational neuroscience. *Psychological review*, *123*(5), 600-612.

Cain, T., & Allan, D. (2017). The invisible impact of educational research. *Oxford Review of Education*, *43*(6), 718–732.

Carlile, P. R. (2004). Transferring, translating, and transforming: An integrative framework for managing knowledge across boundaries. *Organization science*, *15*(5), 555-568.

Dekker, S., Lee, N. C., Howard-Jones, P., & Jolles, J. (2012). Neuromyths in education: Prevalence and predictors of misconceptions among teachers. *Frontiers in psychology*, *3*, 33784.

Flexner, A. & Dijkgraaf, R. (2017). *The usefulness of useless knowledge.* Princeton University Press.

Hammersley, M. (2003). Can and should educational research be educative?. *Oxford Review of Education*, *29*(1), 3-25.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one?. *Educational researcher*, *31*(5), 3-15.

Klein, J. T. (2017). *Typologies of interdisciplinarity: The boundary work of definition*. En R. Frodeman (Ed.), *The Oxford Handbook of Interdisciplinarity* (pp. 21-34). Oxford University Press.

Leikin, R. (2018). How can cognitive neuroscience contribute to mathematics education? Bridging the two research areas. In *Invited Lectures from the 13th International Congress on Mathematical Education* (pp. 363-383). Springer International Publishing.

Mason, J., & Waywood, A. (1996). The role of theory in mathematics education and research. In *International Handbook of Mathematics Education: Part 1* (pp. 1055-1089). Dordrecht: Springer Netherlands.

Privitera, A. J. (2021). A scoping review of research on neuroscience training for teachers. *Trends in Neuroscience and Education*, *24*, 100157.

Newell, W. H. (2001). *A theory of interdisciplinary studies*. Issues in Integrative Studies, 19, 1-25.

Repko, A. F., & Szostak, R. (2020). *Interdisciplinary Research: Process and Theory*. Sage Publications.

Schoenfeld, A. H. (2000). Purposes and methods of research in mathematics education. *Notices of the AMS*, *47*(6), 641-649.

MATHEMATICS EDUCATION RESEARCH EMBEDED IN CLASSROOM PRACTICE (MAITREE INPRASITHA)

The historical relationship between mathematics education and research

The history of mathematics education and mathematics education research reveals an important but often unfulfilled promise. Mathematics education as a field has always been concerned with a fundamental question: how can mathematics be taught and learned in ways that are meaningful to students? This question led to the emergence of mathematics education research as a systematic inquiry to inform classroom practices (Kilpatrick, 1992; Schoenfeld, 2000; Sierpinska & Kilpatrick, 1998). The very existence of our community is predicated on the belief that research should enhance mathematics teaching and learning. Looking at the historical development of mathematics education research, we can observe various paradigm shifts in approaches, methodologies, and focus areas.

The establishment of organizations like the International Commission on Mathematical Instruction (ICMI) in 1908 and its affiliated study groups, along with respected journals such as Educational Studies in Mathematics (first published in 1968) and the Journal for Research in Mathematics Education (1970), all share the stated aim of supporting mathematics teaching and learning. The International Group for the Psychology of Mathematics Education (PME), established in 1976, was founded precisely to bridge psychological research on mathematical thinking with pedagogical practices in classrooms (Kilpatrick, 1992). An examination of these journals' mission statements reveals consistent commitments to "linking theory and practice in mathematics education" (ESM) and "improving mathematics education through research" (JRME). The Mathematics Teacher Educator journal, established more recently in 2012, explicitly states its purpose as "contributing to the improvement of mathematics teacher education".

However, Lester (1994), in his comprehensive review in the Journal for Research in Mathematics Education, outlined how research paradigms have evolved over

decades—from behaviorist approaches through cognitive perspectives to sociocultural frameworks—each with their strengths and limitations. Yet despite these developments, a persistent critique has remained: much of this research fails to significantly improve mathematics teaching and learning in schools. Despite these laudable aims and the proliferation of research output over the past five decades, international assessments such as PISA (Programme for International Student Assessment) show that mathematics achievement worldwide has not significantly improved in the last 20 years. This suggests a fundamental disconnect between the growing body of mathematics education research and actual classroom practices. We must ask ourselves: Why has our research, despite its volume and methodological sophistication, not led to more substantial improvements in mathematics classrooms globally?

The disconnect between research and classroom practice

As a mathematics teacher in a Bangkok secondary school before becoming a mathematics teacher educator in 1987, I observed firsthand the gap between research and practice. This disconnects stems from several issues that our mathematics education community must address.

First, researchers often position themselves as external observers, adopting perspectives from social science that privilege theorizing over practical applications (Isoda, 2025, forthcoming). While theoretical frameworks are essential, they sometimes become ends in themselves rather than means to improve classroom practice. Mathematics education research has increasingly valued theoretical contributions and methodological innovations over tangible improvements in teaching and learning.

Second, the contexts in which research is conducted often differ significantly from the everyday realities of regular classrooms. Research settings frequently involve ideal conditions, special interventions, or highly motivated participants that do not reflect typical teaching situations. This creates an implementation gap where findings, while valid in research settings, prove difficult to translate into everyday practice. Guo et al. (2023) highlights the challenges of integrating simulation-based teaching with traditional methods and the tension between generative interactions and structured classroom culture. These contradictions illustrate the difficulties teachers face in applying research findings in practice, suggesting that effective co-design processes can help bridge this gap by fostering collaboration and perspective sharing between teachers and researchers.

Third, the language and communication of research findings are frequently inaccessible to practitioners. Research reports are written primarily for other researchers, using specialized terminology and focusing on issues that may not align with teachers' immediate concerns. This creates a two-community problem (Huberman, 1990) where researchers and practitioners operate in separate discourse communities with limited meaningful exchange.

Fourth, traditional research dissemination models assume a linear transfer of knowledge from researchers to practitioners, especially fundamental values whereby research merely *informs* practices. This model fundamentally misunderstands how teachers develop their practice, which typically occurs through collaborative reflection, experimentation, and adaptation within professional communities rather than through reading research reports.

As a result of these disconnects, we find ourselves in a situation where mathematics education research has grown substantially in volume and sophistication but has had limited impact on everyday classroom practices. This leads us to question not just the usefulness of our research but its very purpose and value.

Japanese lesson study: research embedded in lived classrooms

One approach that bridges research and practice is Japanese Lesson Study, which gained international attention following Stigler and Hiebert's influential book *"The Teaching Gap"* (1999). Lesson Study represents a research approach deeply embedded in classroom practice, where teachers collaboratively plan, observe, and refine lessons (or classes) in a systematic cycle of inquiry. Many researchers employed Lesson Study as research methodology, such as Berkics (2024), who defines Lesson Study as a research methodology in educational settings by facilitating systematic, cooperative, and critical examination of teaching practices. It effectively addresses research questions while enhancing teaching practices, as demonstrated by its successful implementation in familiarizing students with model-based reasoning (Jansen et al., 2021). Arévalo et al. (2024) highlights Lesson Study as a valuable research methodology in educational settings, demonstrating its effectiveness in enhancing teachers' practical knowledge and understanding.

The global interest in Lesson Study has grown tremendously since 1999, with publications on the approach increasing from fewer than 10 English-language articles prior to 2000 to hundreds of publications annually in recent years. The establishment of International Journal for Lesson and Learning Studies (IJLLS) in 2012 is evident of the growing interest around the world. This growth also reflects recognition of Lesson Study's potential to connect research with improvements in classroom teaching.

What distinguishes Lesson Study is its situation within "lived classrooms"—real educational contexts where the complexities and challenges of everyday teaching are confronted directly. Unlike research approaches that abstract from classroom realities, Lesson Study embraces these complexities as essential to understanding and improving mathematics teaching and learning.

Key features that make Lesson Study valuable for connecting research and practice include:

1. Research is conducted *by* teachers, not just *on* teachers, repositioning practitioners as active knowledge producers rather than passive research subjects.

2. The research process is collaborative, involving multiple stakeholders including teachers, administrators, and university-based educators in a community of practice.
3. The focus remains firmly on student learning and thinking, grounding all inquiry in observable evidence from actual classroom interactions.
4. Knowledge is developed through iterative cycles that integrate planning, action, observation, and reflection, creating a continuous improvement model.
5. The research findings are immediately applicable, being developed in the same context where they will be used.

This approach represents a fundamentally different relationship between research and practice—one where the boundaries between researcher and practitioner blur, and where the ultimate value of research is judged by its contribution to improving teaching and learning. This is a paradigm shift from *research just informing practice* to *practice* (i.e., of Lesson Study) as *research.*

The TLSOA model: transforming research in Thai classrooms

Since 1999, I have been engaged in a longitudinal study transforming Japanese Lesson Study for use in Thai classrooms through what we have termed the Transformative Lesson Study incorporating Open Approach (TLSOA) model. This work has provided concrete evidence of how research can be genuinely useful for classroom practice when it is embedded within a school-based professional learning community.

The TLSOA model positions the school as a research site where multiple stakeholders—including teachers, administrators, university researchers, and teacher educators—collaborate to improve mathematics teaching and learning. The "Open Approach" serves as a thematic framework for implementing Lesson Study, emphasizing open-ended problems and multiple solution pathways. This approach has demonstrated significant benefits for both students and teachers. Students develop 21st-century thinking skills through engagement with open-ended mathematical problems and collaborative problem-solving. Teachers experience continuous professional development within a supportive community, developing deeper mathematical knowledge and pedagogical skills through collaborative planning, observation, and reflection (Inprasitha, 2022).

What makes this approach particularly valuable is that research occurs in authentic classroom settings, with researchers and practitioners working together on the "same front line" to understand and improve mathematics teaching and learning. The complexity of classroom practice is not reduced or simplified but becomes the central focus of collaborative inquiry.

The Open Class component of TLSOA creates a continuum for professional learning across multiple levels of the education system. Pre-service teachers, in-service teachers, school administrators, teacher educators, and researchers all participate in

observing and analysing lessons, creating a shared space for developing practical knowledge about mathematics teaching and learning.

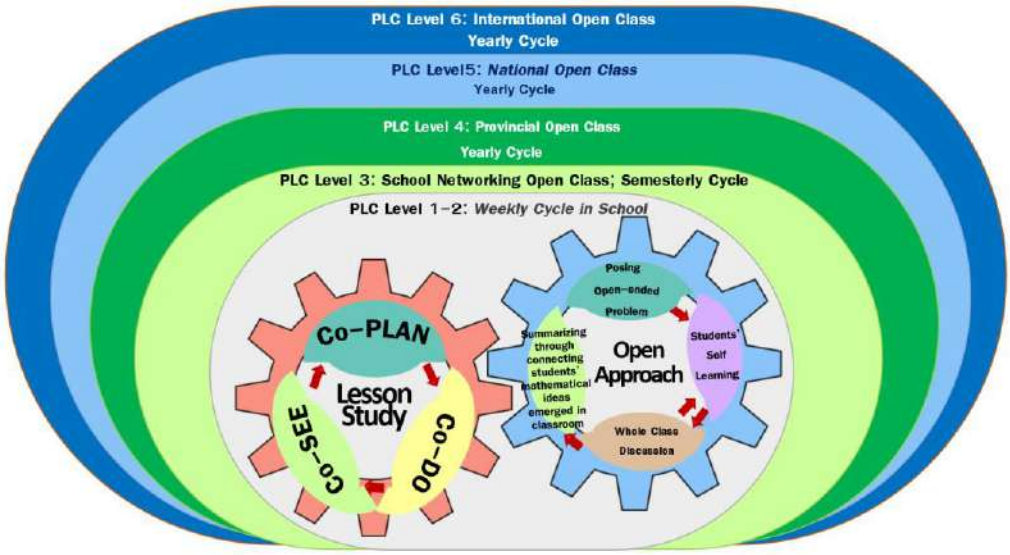

Figure 1: TLSOA Model: Open Class as a Continuum of Multi-Level Professional Learning Communities (adapted from Inprasitha, 2022)

Our longitudinal research has documented how this approach leads to sustainable improvements in classroom practice. Rather than research findings sitting unused in academic journals, knowledge generated through TLSOA is immediately implemented in teaching practices and continues to evolve through ongoing cycles of collaborative inquiry.

Conclusion: towards research that serves the classroom

Mathematics education research must be useful for the classroom—this is not merely a preference but an ethical imperative. If our research does not ultimately improve the teaching and learning experiences of students and teachers, we must question its purpose and value.

The history of mathematics education research shows that despite good intentions, much research has failed to significantly impact classroom practice. To address this disconnect, we need to reconceptualize the relationship between research and practice in several ways:

1. Research should be conducted *with* teachers rather than *on* teachers, recognizing practitioners as essential partners in knowledge generation.
2. The lived realities of classrooms must be the central context for research, embracing rather than simplifying the complexities of teaching and learning.
3. Knowledge development should occur within professional learning communities that span traditional boundaries between schools and universities.

4. The value of research should be judged primarily by its contribution to improving teaching and learning practices rather than by theoretical sophistication alone.

The Japanese Lesson Study approach and our TLSOA model in Thailand offer concrete examples of how research can be genuinely useful for classroom practice. These approaches demonstrate that when research is deeply embedded in the realities of classroom teaching, it can lead to sustainable improvements in mathematics education.

As a community of mathematics educators and researchers, we must commit to ensuring that our research serves its ultimate purpose: improving the mathematical experiences and outcomes of students in classrooms around the world. This requires not just new methodologies but a fundamental reorientation of our values and priorities as a field. Research that is not useful for the classroom is research that has forgotten its most fundamental purpose.

References

Arévalo, V. M. S., Pérez, P., Sgreccia, N., & Gómez, E. S. (2024). Contributions of Lesson Study to the reconstruction of teachers' practical knowledge within a virtual setting context. Case study at Universidad Nacional de Educación (Ecuador). *International Journal for Lesson and Learning Studies*. https://doi.org/10.1108/ijlls-02-2024-0046

Berkics, E. (2024). Lesson Study, a methodology to reconstruct knowledge. *Sociedad & Tecnología*, *7*(2), 240–251. https://doi.org/10.51247/st.v7i2.449

Guo, W., Wu, B., & Li, D. (2023). Collaborative design of a simulation-based math classroom: Contradictions and solutions between teaching and research. *International Conference on Computers in Education 2023 Proceedings.* https://doi.org/10.58459/icce.2023.1387

Huberman, M. (1990). Linkage between researchers and practitioners: A qualitative study. *American Educational Research Journal, 27*(2), 363-391.

Inprasitha, M. (2022). Lesson study and open approach development in Thailand: A longitudinal study. *International Journal for Lesson and Learning Studies*, *11*(5), 1-15. https://doi.org/10.1108/IJLLS-04-2021-0029

Isoda, M. (2025, forthcoming). Transformative lesson study incorporated with open approach: From Japanese perspective. In *Proceedings of the 11th Thailand Society of Mathematics Education Conference (TSMEd 11)*. Thailand Society of Mathematics Education.

Jansen, S., Knippels, M.-C. P. J., & van Joolingen, W. R. (2021). Lesson study as a research approach: a case study. *International Journal for Lesson and Learning Studies*. *10*(3), 286-301. https://doi.org/10.1108/IJLLS-12-2020-0098

Kilpatrick, J. (1992). A history of research in mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 3-38). Macmillan.

Lester, F. K. (1994). Musings about mathematical problem-solving research: 1970-1994. *Journal for Research in Mathematics Education, 25*(6), 660-675.

Schoenfeld, A. H. (2000). Purposes and methods of research in mathematics education. *Notices of the American Mathematical Society, 47*(6), 641-649.

Sierpinska, A., & Kilpatrick, J. (Eds.). (1998). *Mathematics education as a research domain: A search for identity.* Kluwer Academic Publishers.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* Free Press.

MATHEMATICS EDUCATION RESEARCH MUST BE USEFUL FOR THE CLASSROOM (DEMETRA PITTA-PANTAZI)

During this panel discussion, I argue that Mathematics Education Research (MER) must be useful for the classroom—not merely as an academic pursuit but as a transformative force. To support this position, I briefly refer to the nature of research in mathematics education and the concept of today's classroom. Then, I present arguments about why research should be useful in the classroom.

The nature of inquiry within mathematics education

MER has two core purposes: pure research, which is to understand mathematical thinking, teaching, and learning, and applied research, which is to use such understanding to improve mathematics instruction (Schoenfeld, 2003). It can be argued that prioritizing the applied research risks narrowing its intellectual scope, limiting its broader contributions to the field and society, and reducing it to immediate classroom utility. However, even the most 'pure' MER ultimately seeks to inform and enhance mathematical teaching and learning in classrooms. This objective underpins our scholarly endeavors across the diverse landscape of MER, establishing a unifying thread throughout seemingly disparate methodological approaches and theoretical frameworks.

Kilpatrick (2008) asserted that MER's theoretical and practical needs should be addressed scientifically rather than pragmatically to ensure that research remains useful for the classroom. Peter Taylor's question, "What's it all for?" (Kajander, 2020, p. 776), highlights the essential purpose of MER. If research is to fulfill its true purpose, it must provide tangible benefits for educators and students, ensuring classrooms become centers of effective and meaningful learning.

The nature of the classroom

Classrooms are evolving rapidly due to social and technological changes. They are diverse and exist in various settings, such as homes, hospitals, museums, streets, and other informal settings. They may be in-person or online, synchronous or asynchronous. Classrooms involve multiple partners, including parents/guardians, informal schools, media creators, and textbook authors. The classroom model has changed and will change even more. Given this fluidity, MER must be useful for all these types of classrooms to address the various educational and global challenges.

Why research must be useful for the classroom

Below, I highlight some arguments as to why mathematics education must be useful for the classroom. They are organized into two categories: academic and societal.

Academic arguments

Address critical educational needs

MER must address the most pressing challenges faced by educators and students. Alignment with these needs ensures that research yields meaningful and applicable classroom outcomes. As Cai et al. (2017) stated, "If research is not solving teachers' problems, why would teachers want to use the research findings?" (p.345), highlighting the necessity for research to target areas requiring urgent improvements.

Bridge the gap between theory and practice

The traditional divide between research and practice has limited the impact of valuable findings on classroom instruction. MER gains credibility and advances rapidly when its theoretical constructs are regularly tested against the complex variables of actual classroom settings. MER must be useful for the classroom to bridge the gap between theoretical findings and teaching practices. Without a direct impact on practice, research remains an abstract exercise with little influence on student learning.

Improve pedagogical strategies

Research that directly informs practice helps teachers design and implement effective strategies. This symbiotic relationship between research and the classroom strengthens effective teaching strategies and, at the same time, improves research methodologies and theoretical frameworks, creating a continuous cycle of improvement.

Enhance the dynamic exchange of knowledge and concerns between researchers and practitioners

If MER is useful for the classroom, it will strengthen the collaboration between researchers and practitioners. Lasting improvements in education occur when practitioners actively define problems and develop context-specific solutions (Tyack & Cuban, 1995). Practitioners offer invaluable insights about their classrooms, while researchers contribute their expertise in literature, methodologies, and related topics. Strengthening the collaboration between practitioners and researchers enhances the understanding of the challenges that practitioners face (Cai et al., 2018). This collaboration grounds research in classroom realities and makes teaching more research-informed. Such synergies create a dynamic exchange of knowledge and concerns, allowing practitioners and researchers to refine their roles and enhance their impact.

Facilitate complex decision-making in educational environments

Educators work in complex, unpredictable classrooms requiring both theoretical knowledge and practical experience. Research provides the frameworks and evidence-

based approaches to enhance decision-making and effectiveness. Referencing established findings helps practitioners justify pedagogical choices while preparing them for difficult or unusual situations by offering potential solutions.

Encourage more teachers to participate in research

Research designed for classroom utility has a broader impact on educational systems. When teachers see its benefits, they are more inclined to engage in research and encourage other teachers to do the same. This contributes to a shared body of knowledge that benefits the entire profession (Cai et al., 2018). The collective engagement creates a ripple effect, refining instructional practices across diverse educational settings and amplifying the research's impact on education.

Address different students' needs at different locations

Researchers can design appropriate instructional activities and adapt teaching to the needs of different students in different educational contexts. This adaptability ensures that research findings remain relevant and effective across different learning environments. Additionally, research useful for the classroom helps identify variables (e.g., social, personal, technological) that influence learning outcomes, leading to more accurate insights. In this way, research informs practice, provides valuable data on student learning, and leads to continuous advancements in students' learning.

Identify potential issues before they become a widespread problem

Research is useful not only because it sheds light on current problems but also helps detect emerging issues. By collecting and analyzing data across classrooms, researchers can pose and investigate new questions about teaching and learning. This forward-looking approach allows mathematics education to evolve and address emerging challenges. Identifying potential issues early enables research to serve a preventive function, helping educators develop proactive rather than reactive strategies.

Create a cumulative knowledge base

For research to have a lasting impact on classroom practice, it must contribute to an accessible, shared, and evolving knowledge base. As mathematics learning builds cumulatively, educational research should systematically build on previous findings rather than produce isolated studies. The NCTM (n.d) emphasizes that research-practice knowledge must be continuously updated with rigorous methods and reported in ways that can be shared, replicated, and adapted across educational settings. This ensures that valuable insights reach educators rather than being confined to academic journals. By promoting both theory and practice, research creates the conditions for ongoing progress in education. As findings accumulate, they enhance our understanding and promote current practice, theory building, and future investigations.

Support teacher professional development

MER supports teachers' professional development by enhancing teachers' knowledge for teaching, improving classroom practices, and fostering reflective teaching. It also promotes the integration of new technologies and innovative teaching methods.

Societal arguments

The view that MER must be useful for the classroom extends beyond mere academic discussions, emphasizing the need to recognize its broader societal role. When research is disconnected from classroom realities, it may produce important but isolated theoretical frameworks that cannot address the challenges teachers and students face. Thus, I will point out that MER, useful for the classroom, can serve society in at least three key ways: (a) build students' resilience and address global issues, (b) promote equity, and (c) influence policies and public opinion.

Build students' resilience and address global challenges

In today's evolving educational landscape, particularly post-pandemic, research for the classroom is more urgent than ever. Mathematics education fosters complex reasoning skills addressing global challenges like climate change, public health, and economic and world crises. MER explores how classroom implementation equips learners with essential skills while promoting a more resilient educational system, helping students see mathematics not as an isolated academic subject but as a tool for understanding and impacting their communities.

Rapid technological and social changes demand flexible and forward-thinking education. Students need curricula and adaptable learning approaches emphasizing fundamental principles and transferable skills rather than rigid content that may become obsolete. Research-based teaching fosters more effective learning environments that meet current and future demands. It empowers individuals to engage meaningfully with data, recognize patterns, and make informed decisions.

Promote equity

MER can identify systemic barriers that impede students' access to quality mathematics education and propose solutions that promote equity (Cai et al., 2018). It explores how classroom implementation can promote a more equitable educational system appropriate for all students, regardless of their backgrounds. This approach helps tackle broader social issues such as race, gender, class, and identity. It transforms MER from a purely academic endeavor to a means for fostering social change.

Influence policies and public opinion

Research restricted to academic circles has limited classroom impact. Systematic research is useful for the classroom and provides valuable insights to help educational systems make informed decisions and changes. To be truly helpful, MER must influence the broader educational landscape, shaping policies related to curricula, educational practices, assessment, and resource distribution. The NCTM emphasizes

the importance of communicating research to stakeholders to enact systematic, large-scale improvements. Effective MER that is useful for the classroom is more likely to reach a wider audience and influence public opinion.

Conclusion

MER, which is useful for the classroom, provides multiple benefits. It offers theoretical foundations and evidence-based strategies that inform practice, while classroom experiences generate new insights and questions that drive further research. A commitment to make mathematics education useful for the classroom is essential for professional responsibility and ongoing educational and social improvement. By fostering MER that is useful for the classroom, we can enhance the quality of mathematics education, improve student learning outcomes, impact the whole educational system, and ultimately support a more resilient future for all.

References

Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V., & Hiebert, J. (2017). Making classroom implementation an integral part of research. *Journal for Research in Mathematics Education, 48*(4), 342–347. doi:10.5951/jresematheduc.48.4.0342

Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V., & Hiebert, J. (2018). Reconceptualizing the roles of researchers and teachers to bring research closer to teaching. *Journal for Research in Mathematics Education, 49*(5), 514–520. doi:10.5951/jresematheduc.49.5.0514

Kajander, A. (2020). The mandate of scholarly mathematics education research: Moving ourselves forward. *Canadian Journal of Science, Mathematics and Technology Education, 20*(4), 775–779. https://doi.org/10.1007/s42330-020-00121-7

Kilpatrick, J. (2008). The development of mathematics education as an academic field. *Proceedings of the Symposium on the Occasion of the 100th Anniversary of ICMI* (Rome, 5–8 March 2008), 25–39.

National Council of Teachers of Mathematics. (n.d.). *Linking mathematics education research and practice: A position of the National Council of Teachers of Mathematics.* Retrieved from https://www.nctm.org/Standards-and-Positions/Position-Statements/Linking-Mathematics-Education-Research-and-Practice/

Schoenfeld, A. (2002). Purposes and methods of research in mathematics education. In D. Holton, M. Artigue, U. Kirchgräber, J. Hillel, M. Niss, & A. Schoenfeld (Eds.), *The teaching and learning of mathematics at university level* (Vol. 7, pp. 221–236). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47231-7_22

Tyack, D., & Cuban, L. (1995). *Tinkering toward utopia: A century of public school reform.* Harvard University Press.

MAKING SURE THAT MATHEMATICS EDUCATION
RESEARCH REACHES THE CLASSROOM

RESEARCH FORUMS

NEW DIRECTIONS IN THE DISCOURSE ON MATHEMATICS TEACHER PROFESSIONAL DEVELOPMENT

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In recent years, the discourse on the professional development of mathematics teachers and their competencies has developed strongly. Departing from the original focus on cognitive aspects of teacher competence, the discourse includes situation-specific skills such as teacher noticing, and emphasizes the need to develop culturally sensitive approaches. Moreover, multilingual perspectives on mathematics learning and learners have received considerable attention, often in connection with equity-oriented approaches on teacher professional development. Approaches focusing on new technological developments have also been proposed. The discourse has also returned to its roots questioning the paradigmatic differences between expertise- and competence-oriented perspectives. In this Research Forum, we explore these new directions in the discourse on mathematics teachers' professional development. Specifically, we address five key areas: research on expertise, equity-oriented approaches to teachers' professional development, culturally sensitive approaches focusing on teacher noticing, and finally, new technological approaches.

AIMS AND BACKGROUND

This Research Forum aims to provide insight into new directions in the discourse on the professional development of mathematics teachers. Teacher professional development is usually understood as a continuous process of reflection, learning, and action to enhance teacher competence, encompassing professional knowledge and situation-specific skills, leading to the improvement of teaching practices that positively impact students' learning.

Research on teacher professional development has gained considerable momentum, particularly focusing on mathematics education (König et al., 2023). However, the field today is shaped by a variety of perspectives, some of which are paradigmatically different, which has implications for how teacher professional development is conceptualized and thus studied. Although there is a global consensus that teachers

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, pp. 121-150. PME.

play an important role in providing adequate learning opportunities for their students and thereby influence student learning, it remains unclear to what extent teachers' professional competencies and professional development influence their students' learning gains and emotional development. On the one hand, recent reviews of research from the past decades have emphasized more intra-systemic changes, such as the need to incorporate new technological approaches (Kosiol et al., 2023). On the other hand, changes that go beyond current educational systems and challenging tendencies of marginalization of certain groups of students can be identified, calling for, among other things, equity-oriented approaches to teachers' professional development or approaches that are responsive to multilingual mathematics learning processes and learners (Planas & Abtahi, 2024). Furthermore, in the current discourse, the understanding of mathematics education as, at least in part, culture-independent with similar characteristics worldwide has been challenged, and the call for culturally sensitive frameworks can be identified (Louie, 2018). Finally, the dominance of competence-oriented research (Yang & Kaiser, 2022), particularly in the context of mathematics teacher professional development, has been challenged by calls for more performance-oriented approaches putting the individual teacher in the foreground, as is the case in expertise-oriented approaches to teacher professional development.

In the following contributions these issues are discussed in detail, focusing on innovative approaches in this domain of research and their potential to advance the broader field. In detail: (1) Research on expertise in mathematics instruction; (2) Culturally sensitive perspectives on teachers' professional development: teachers noticing; (3) Equity-oriented approaches in the professional development of teachers; (4) Linguistically-responsive professional development for mathematics teachers in multilingual classrooms; and (5) New technology-oriented approaches to teacher professional development.

RESEARCH ON EXPERTISE IN MATHEMATICS INSTRUCTION

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Research on expertise in mathematics instruction has traditionally focused on the knowledge teachers need to teach effectively, a focus that has yielded valuable insights over the years. However, the acquisition of knowledge by teachers does not always translate into effective teaching practices. A shift toward examining teachers' practices both in and out of the classroom has provided a critical perspective for understanding what teachers do and how they can improve their teaching. In this overview, we will explore research from these two areas and discuss how research on expertise in mathematics instruction can evolve further by recognizing teaching as a multifaceted profession.

KNOWLEDGE-ORIENTED RESEARCH ON THE ESSENTIAL KNOWLEDGE FOR EFFECTIVE TEACHING

Teachers play a crucial role in developing and implementing effective instruction in the classroom. For decades, research has focused on understanding what teachers need to know and be able to do to enhance their teaching. Existing literature highlights the importance of *knowledge* in the acquisition and development of expertise in knowledge-intensive fields, such as mathematics and mathematics instruction (Li & Kaiser, 2011). However, the significance of knowledge extends beyond simply the quantity of coursework completed; it also includes the depth and structure of that knowledge (e.g., Ball et al., 2008). Knowledge-oriented research has made substantial contributions to mathematics education and teacher development over the years. This knowledge-based approach to teachers' expertise has also been employed in large-scale international studies, such as the Teacher Education and Development Study in Mathematics (TEDS-M; Blömeke et al., 2014), which measures three types of knowledge: mathematics content knowledge (MCK), mathematics pedagogical content knowledge (PCK), and general pedagogical knowledge.

Although understanding the different types of knowledge specific to mathematics instruction is important in teacher education, it is not sufficient for helping teachers develop expertise in mathematics instruction. There are several factors related to the complexity that go beyond discipline-based knowledge categorizations, which are briefly summarized below.

1. The first complexity is that many teachers are unaware of what they don't know or what additional knowledge they need (e.g., Li & Kulm, 2008). A recent review of the mathematics training needed for elementary school teachers summarized extensive research showing teachers' limited understanding of mathematics topics beyond algorithmic procedure (Li & Howe, 2021). Despite

this, many teachers often consider themselves competent and well-prepared to teach mathematics (Li & Kulm, 2008). This overconfidence in their mathematics preparation makes it even more challenging to address the areas in need of improvement.

2. The second complexity is that teachers often view mathematics as a static collection of concepts and skills, without fully understanding what it means to *do* mathematics or to *think* mathematically (Goldenberg et al., 2021). Li and Schoenfeld (2019) illustrate this with the example of teaching and learning long division. Specifically, Brownell (1945) emphasized that arithmetic should not only be taught as a set of procedures but also as a meaningful system of thinking. He provided examples, such as the division, $8\overline{)576}$, where instead of merely teaching students how to follow the procedure, teachers should ask questions like: What does the 5 of 576 mean? Why must 57 be the first partial dividend? Do you actually divide 8 into 57, or into the 57…'s? etc. These questions encourage deeper understanding, rather than just rote memorization of steps. Brownell's approach, which promotes a focus on sense-making, has since become a widely accepted principle in mathematics education.
3. Recognizing the existence of different types of knowledge is important in teacher education. It is equally crucial that the connections between these types of knowledge are clearly identified and learned in teachers' training. Without these connections being specified, teachers are left to make them on their own after learning isolated pieces of knowledge. For instance, the concept of Mathematics Conceptual Knowledge for Teaching (MCKT) has been proposed as key topic-based knowledge comprising three important components (both content and pedagogy) that should ideally be taught together in the same course. MCKT emphasizes a deep, systematic understanding of mathematics knowledge, along with associated pedagogy, to empower teachers in their ongoing development of expertise (Li & Howe, 2017).
4. Knowledge-based research on teachers' practices has also led to the identification of important areas beyond disciplinary knowledge. For example, studies have explored mathematical practices in the home, community, and workplace, where culture, language, affect, and environment also play significant roles. The concept of "funds of knowledge" has become important for teachers to learn how to connect students' everyday practices with those in the mathematics discipline (e.g., Civil, 2016). Additionally, the notion of knowledge has been expanded to include contextual knowledge (Mishra, 2019), which encompasses everything from a teacher's awareness of educational resources to their knowing and understanding of related policies at the school, state, and national levels.

Even with the expansion of knowledge types, the process of developing expertise cannot be reduced to acquiring a set of static knowledge and skills. It is not a one-off act. Collaborating with others and reflecting on professional practice in mathematics

instruction should be seen as vital opportunities for developing expertise – a perspective that emphasizes the knowledge growth and moves beyond the static characterization of knowledge and expertise.

PRACTICE-BASED RESEARCH ON WHAT TEACHERS NEED TO ENGAGE IN FOR TEACHING IMPROVEMENT

There have been several dynamic areas of research on teachers' professional development, including communities of practice, lesson study, and technology integration. For example, Horn and her colleagues (e.g., Horn & Little, 2010) analyzed how conversation routines and interactions within professional communities help foster workplace learning and practice improvement. Similarly, Ding and her colleagues (Ding et al., 2023, 2024) conducted literature reviews of lesson study in mathematics education from 2015 to 2022. Lesson study, originally from Japan and China, has been adopted as professional learning gathering in many other countries and has contributed to positive changes in mathematics teachers' knowledge and beliefs.

With the growing integration and use of technology in education, there is an increasing need for teachers to develop new prerequisite skills and competencies to effectively use these technologies in mathematics instruction. Studies on technology integration in mathematics education are on the rise (Chiu et al., 2025), and such studies can contribute valuable insights to the kind of knowledge and skills that teachers need in teaching mathematics with technologies.

TEACHING AS A MULTIFACETED PROFESSION

In the book *Expertise in Mathematics Instruction* (Li & Kaiser, 2011), several researchers highlighted different ways to portray the various roles that teachers can play in the complex activity of teaching. For example, Russ and her colleagues (Russ et al., 2011) identified four distinct roles that teachers may assume in the classroom: teachers as diagnosticians (referring to their ability to understand and interpret students' thinking and strategies), teachers as conductors (referring to their ability to organize classroom discourse around mathematical ideas), teachers as architects (referring to their skills in selecting and identifying cognitively challenging tasks), and teachers as river guides (referring to their experience and improvisation in guiding the instructional flow). These roles reflect different aspects of expertise in mathematics instruction, requiring teachers to adapt to diverse tasks and contexts as needed. While some roles can be effectively addressed through knowledge-based characterizations, others may not be as easily captured by this approach. For example, the role of "teachers as river guides" might be more aptly illustrated through situated cognition and research on noticing, rather than relying solely on knowledge structure. This complexity underscores the need for further research into the nature of expertise and how it can be characterized in the context of mathematics instruction.

CULTURALLY SENSITIVE PERSPECTIVES ON TEACHERS' PROFESSIONAL DEVELOPMENT: THE ROLE OF TEACHER NOTICING

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We demonstrate how an adapted teacher noticing instrument reveals cultural influences on teachers' professional competence. The findings show that teacher noticing is deeply shaped by cultural contexts, underscoring the importance of culturally sensitive approaches in professional development design.

INTRODUCTION

Since many countries, including Chile, have their own teacher evaluation systems, establishing a comprehensive framework for assessing teacher Professional Development (PD) on a large scale is essential for ensuring consistency, comparability, and effectiveness in supporting teachers' overall growth and competence, both through formal programs and informal learning experiences. This was a key focus of the TEDS-M study, in which 16 countries participated by assessing pre-service mathematics teachers' competencies, including knowledge and beliefs. The study laid the groundwork for large-scale competence measurement using cognitive and affective dimensions (Kaiser et al., 2015). Its follow-up study, TEDS-FU, extended this by incorporating a video-based assessment of situation-specific skills, such as noticing, which is an important domain of skills for mathematics teachers. These domain-specific skills are context-dependent, as highlighted in the literature (Blömeke et al., 2016), and culturally sensitive, as demonstrated in studies by Yang et al. (2019) and Saadati et al. (2024). Because these competencies cannot be observed directly, their assessment relies on a well-developed framework capable of capturing them with sensitivity and adaptability.

CULTURAL INFLUENCES ON TEACHER NOTICING

Teacher noticing, a multifaceted concept, has been conceptualized and studied through various perspectives, shaping research in this domain over the past two decades (König et al., 2022). Efforts to develop suitable assessment instruments for teacher noticing have encountered challenges due to the relatively nascent nature of the field and its rapid expansion in recent decades (Jacobs, 2017). Reviewing existing literature, it is evident that most empirical studies employ qualitative approaches to explore deeply the nature and development of noticing skills but lack scalability for assessing large cohorts of teachers (König et al., 2022). Standardized instruments exist but vary in suitability for capturing noticing skills, particularly in different educational contexts (Dreher & Kuntze, 2015; Jacobs & Empson, 2016). Thus, some international studies have expanded competence assessment by integrating video-based tools, like the

TEDS-FU instrument, to measure teacher noticing in its inherently context-dependence (Blömeke et al., 2016; Kaiser et al., 2015; Yang & Kaiser, 2022). The instrument was adapted to other contexts, like Chile and China, carefully considering cultural circumstances and validating the applicability of such instruments within local teaching practices, emphasizing the need for culturally sensitive adaptations (Yang et al., 2019; Saadati et al., 2024).

Adapting Teacher Noticing Instruments and Comparison Studies

Adapting Existing Frameworks. Adapting existing frameworks requires careful consideration of cultural and contextual nuances, particularly when applying instruments across diverse educational settings. Regarding the video-based noticing instrument from TEDS-FU, the collaboration among three countries taught us that terms and pedagogical concepts from the original German framework were not always directly applicable to the other context. For example, specific video vignette scenarios in the instrument had to be reinterpreted to better align with Chilean teachers' more traditional educational practices, focusing less on active teaching methods (Saadati et al., 2024). Additionally, some German educational terms used in the instruments were difficult for Chilean teachers to understand. One item, for instance, referred to linguistic-cultural heterogeneity, an issue typical for the German context but not for Chile or China (Saadati et al., 2024; Yang et al., 2019). The cases of Chile and China illustrate how instruments developed for a specific context, such as TEDS-FU for the German context, can benefit from carefully customizing local educational practices and further emphasize the need for culturally sensitive adaptations as well as the importance of context-sensitive frameworks in teacher noticing.

Examples from Chile, China, and Germany. Using the TEDS-FU instrument, a three-country comparison study was conducted to assess teacher noticing of in-service mathematics teachers from Germany, Chile, and China, revealing significant differences in the professional competence that could be explained, in part, by cultural influences. By identifying which aspects were systematically easier or more difficult for teachers in each country, country-specific strengths and weaknesses were highlighted. The results show distinct patterns of strength in noticing among teachers from the three countries, reflecting cultural and educational differences. German teachers demonstrate strengths in subject-specific areas, particularly with mathematical representations, modeling, and task criteria like openness. In contrast, Chilean teachers focus more on general pedagogical aspects, such as classroom management, student well-being, and teacher behavior. Chinese teachers show a mixed achievement pattern that has some similarities with both German and Chilean teachers. They show significant strengths in identifying and describing mathematical errors and helpful student contributions from a subject-specific perspective but also show strength in more general pedagogical aspects, such as effective use of teaching time or classroom management, although to a lesser extent than Chilean teachers. Some of these results align with international empirical findings (e.g., Bell et al., 2020; Blömeke

et al., 2014), and some patterns warrant further analysis and discussion. In particular, the item format plays an important role in shaping teachers' responses, with different countries systematically performing better on different item formats. Moreover, the study also reveals significant differences between novice and experienced teachers' performance across contexts, suggesting that opportunities to learn in teacher education and professional practice influence the development of teacher noticing and differ from country to country. In Germany, novice teachers perform particularly well, while in Chile and China, experienced teachers significantly outperform their novice counterparts. This indicates that learning opportunities are distributed differently in the three systems, with professional practice playing a more crucial role in developing situation-specific skills in the Chilean and Chinese contexts. This illustrates how cultural paradigms and educational traditions, as part of the professional context, shape teacher noticing and ultimately inform the development of culturally sensitive frameworks for teacher competence.

Implications for Professional Development

Based on this work and its findings, the implications for professional development are multifaceted, offering significant potential for enhancing teacher education and continuous professional development. The validation of adapted noticing instruments in different cultural contexts presents opportunities for cross-cultural research in this field. By comparing findings across countries and cultures, researchers can gain a deeper understanding of how cultural differences influence teacher competencies, particularly by noticing and identifying more effective ways to measure them. This can enrich the global discussion about what constitutes effective teaching and how best to assess it. Incorporating cultural sensitivity into the design and application of instruments for measuring teacher noticing, this study underscores the importance of equipping teachers with not only content and pedagogical knowledge but also the contextual awareness necessary to adapt to diverse educational settings. This approach fosters a more holistic view of teacher development—one that goes beyond formal PD programs and includes everyday teaching experiences. In fact, as the research suggested, providing teachers with opportunities to develop their noticing skills can improve their sensitivity to classroom dynamics, ultimately benefiting student learning outcomes (Yang & Kaiser, 2022).

In summary, these findings highlight the complexity of comparing teacher noticing across cultural contexts and emphasize the need for culturally sensitive frameworks in mathematics education. Due to its situational specificity, teacher noticing is inherently shaped by cultural influences, and instruments that do not account for these differences may fail to capture essential aspects of teacher competence. Therefore, constructs, theoretical frameworks, and assessment instruments must be carefully adapted rather than directly transferred across cultural settings, avoiding simplifications to ensure a more accurate and holistic understanding of teacher noticing. At the same time, cross-cultural comparisons offer valuable opportunities to identify specific strengths within

different educational systems. Acknowledging these differences ensures that professional development is effective and culturally relevant, so more research on the design and development of culturally sensitive PD programs that address specific contextual needs could further improve teacher education and its impact on teacher competence.

EQUITY-ORIENTED APPROACHES TO TEACHERS' PROFESSIONAL DEVELOPMENT

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Equity-oriented approaches to teachers' professional development (PD) have seldomly considered ways to build on each other. In this paper we propose an approach to teachers' PD that brings together the concepts of status and funds of knowledge. We illustrate what this could look like through three practices in teaching mathematics that are often the focus of PD: the use of tasks; student participation; and assessment. We offer this intersectional approach to teacher PD as a starting point to creating powerful learning opportunities.

INTRODUCTION

The chapter by Civil et al. (2020) presents first a review of equitable teaching practices and then goes more in depth into three approaches, complex instruction (CI) (Cohen & Lotan, 1997), funds of knowledge (González, et al., 2005), and developing mathematical inquiry communities (Hunter & Hunter, 2018). At the end of the chapter, the authors call for "collaboration and cross pollination of diverse approaches" (p. 266). In this paper we address this call by proposing an equity-oriented approach to teachers' professional development (PD) that brings together some key ideas found in the three approaches just mentioned. In particular we focus on attending to issues of status and valuing families' funds of knowledge. A guiding premise in the approach we propose is the need for PD that makes a more explicit bidirectional connection to community / families' funds of knowledge, not only in terms of their mathematical knowledge but also their values and cultural ways of being (Civil & Hunter, 2015). In what follows we describe three practices of mathematics teaching that could be strengthened when intersecting complex instruction and funds of knowledge frameworks to support teacher PD.

PRACTICE 1: USING COMMUNITY CENTERED MATHEMATICS TASKS

Central to Complex Instruction is the idea of group-worthy tasks (Featherstone et al, 2011; Harper & Crespo, 2020). These are tasks that require multiple smartness and collaborative learning strategies. In recent work with a group of mothers and teachers

(Varley Gutiérrez et al., 2024) we contrast implementation of a culturally based task that drew on the mothers' funds of knowledge with a CI task that centred on somewhat typical school mathematics content. This experience leads us to consider, what may be the potential of professional development for teachers that brings together funds of knowledge and CI? What we propose is to use group-worthy tasks that reflect the knowledge of the local school community, that is tasks that reflect the families' funds of knowledge. This would involve teachers learning with and from the local community and ideally engaging with parents (and students) as co-designers of tasks (Beltrán-Grimm, 2023). Part of the co-design would involve combining teachers' expertise with the school curriculum with caregivers' expertise with the community and with how their children learn best, to then develop group-worthy tasks that focus on multiple strengths. For example, in Civil et al. (2020), we discuss a culturally based activity where the mothers provided key information for the development of the task. A co-design approach would have teachers and mothers working together to design group-worthy problems around this task.

PRACTICE 2: BROADENING STUDENT PARTICIPATION

Here the questions to consider are, what does it mean to participate in a mathematical discussion? What forms of participation are valued? Whose ideas are heard? Teachers who use complex instruction strategies in their mathematics classroom use group norms, roles, and group talk protocols to more evenly distribute and share the students' participation within their groups. However, in some classroom contexts these strategies are not enough. For example, in our local contexts we often work with students whose home language is different from the language of instruction. Thus, an over-emphasis on verbal participation may prioritize the voices of students who are most comfortable with the language of instruction. Or an emphasis on individual participation may go against the experience of students from communities that emphasize collaboration (Hunter & Hunter, 2018; Rogoff et al., 2017). The work of CI on status seeks to address issues related to participation but we argue that teachers need to know their school community and get to know their students' participation patterns, as some of them may already bring in expertise in collaboration. As Civil and Hunter (2015) write, we should "learn from and build on the students' cultural ways of being" (p. 308).

PRACTICE 3: RE-IMAGINING MATHEMATICS ASSESSMENT

Because what schools and teachers value is communicated with the assessments that are used to evaluate students' growth and progress, re-imagining teaching practices associated with assessments is also important (Cohen & Lotan, 1997). In particular, the overly individualistic mathematics assessments typically used in the mathematics classroom can be transformed into group-based projects and assessments. We propose that PD at the intersection of complex instruction and funds knowledge can provide a foundation to support teachers to re-imagine their classroom assessments, including the type of feedback they provide students. Considering families and caregivers as collaborators and partners in the process of assessing student learning is one way that

teachers can bridge the classroom-home divide. Creating assessments that provide students opportunities to engage in cycles of revisions can include families and community experts in the process of supporting students to revise their mathematical work. This reimagining of assessments as a collaborative endeavor and as a community engaged practice can transform the mathematics classroom into spaces that value and welcome collaborative approaches to learning and the local knowledge of the school community.

CONCLUSION

Ultimately the goal of equity-oriented approaches to teachers' professional development is to create opportunities for teachers to reflect on their current teaching practices and begin the process of reconsidering and transforming long-held practices and beliefs that create inequitable learning opportunities for students. As Killi and Xenofontos (2024) point out, teachers, and particularly teachers working with immigrant, minoritized communities need to be supported in their professional needs so that they can in turn better support their students. We argue that most approaches miss opportunities to bridge the school-home divide that is offered by PD that focuses on families' and communities' funds of knowledge. As part of their "equity-driven principles" (p. 310), Abtahi and Planas (2024) call for a practice of "collaboration across groups with diverse life experiences…" (p. 310). In our proposed approach, we view teachers and caregivers working together as fulfilling this practice. Bringing together features of complex instruction and of funds of knowledge focuses on teachers noticing and acting on the strengths of their students, families, and community members. This approach to PD extends calls to (re)humanize mathematics classrooms into spaces that are culturally sustaining (Zavala & Aguirre, 2024) to teacher PD. It is also consistent with calls for developing anti-deficit framing and noticing as described by Louie, et al. (2021). These calls are grounded in more expansive ecological perspectives on teaching and learning that are not always reflected in teacher PD. Our proposed equity-oriented approach is a first step towards addressing these calls.

LANGUAGE-RESPONSIVE PROFESSIONAL DEVELOPMENT FOR MATHEMATICS TEACHERS IN MULTILINGUAL CLASSROOMS

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In this paper, we propose an approach to professional development work with teachers that considers the importance of language responsiveness in mathematics teaching with multilingual students. We present this approach in articulation with the Discipline of Noticing and how it helps us – as teacher educators and researchers – to draw on the professional knowledge and expertise of teachers. We exemplify a moment of gaining awareness of language(s) and language responsiveness in mathematics teaching with multilingual students. A teacher shared mathematical-linguistic challenges faced by a student, and the teacher educator gave value to focused talk around this experience.

INTRODUCTION

In our research collaborations, we design, conduct and discuss professional development work with mathematics teachers that is language responsive because it reflects on the uses of language(s) for mathematics teaching and on the linguistic demands involved in this teaching and in students' mathematics learning. We take what we have learned from multilingual and equity research (Adler, 2001; Civil et al., 2019; Moschkovich, 2010) and put this in combination with insights from the Discipline of Noticing (Mason, 2002) regarding the ways in which we choose to talk about mathematics teaching with the teachers. Núria's research trajectory started with a focus on mathematics classroom research and language diversity and recently moved into issues of language(s) in mathematics teacher education and the Discipline of Noticing, while Jenni started with a focus on mathematics classroom interactions drawing on the same underlying framework as the Discipline of Noticing. During the RF session, we will discuss our emphasis on developing teachers' sensitivity, awareness and professional knowledge for mathematics teaching that is language-responsive.

Given our diverse backgrounds, we are particularly interested in the notion of language responsiveness in mathematics teaching, intersecting the work of Lucas and Villegas (2011) and Erath et al. (2021), which aims to support students' learning of mathematics, including students that do not have the language of instruction as a dominant home language and those with socioeconomic status and linguistic capitals historically undervalued in the school culture. Taking this notion of language responsiveness in mathematics teaching and assuming that all mathematics classrooms are linguistically and socially diverse, the focus in our work with the teachers is to facilitate their thinking and implementation of lesson tasks and discussions that contribute to leveraging the linguistic repertoire and participation of students while

learning specific mathematical topics. In all this, work around identifying the linguistic demands of content teaching and around scaffolding accordingly the acts of teaching are strategic. This understanding of what becoming language-responsive may mean for mathematics teaching in practice is at the basis of our understanding of how we, as teacher educators and researchers, can support such a process in spaces of collaboration with and learning from teachers.

In a project in Spain (Planas & Alfonso, 2023; Planas et al., 2024), some of the workshops with mathematics teachers raised the many challenges that teachers face in supporting mathematical discussions in their multilingual classrooms with students who face both the linguistic demands of mathematics and the teaching in a language that is not a home language for them. Influenced by the Discipline of Noticing, we see teachers' professional expertise at the centre of the process of enacting language responsiveness in mathematics teaching. In our role as teacher educators, we do not 'teach' the teachers but work with them on issues of sensitivity and awareness regarding the linguistic demands of their mathematics teaching and for the students' learning. A similar project in England combined this focus on supporting mathematical discussions with a topic-specific collaborative analysis of the linguistic demands involved when working with key mathematical concepts in tasks that the teachers use with their students.

An important feature of the Discipline of Noticing is the way in which it is possible to enhance and amplify the possibility that, in the future, teachers will become aware of actions to enact that they might not have previously been aware of, through noticing an opportunity within a teaching situation. In our projects, the realisation of the Discipline of Noticing is about raising the teachers' awareness of the linguistic demands involved in the teaching of mathematical topics such as linear equations, probability, angles and proportional reasoning. We draw on the teachers' expertise in mathematics teaching and their experiences of interacting with students facing a variety of linguistic challenges in their lessons. These challenges are, of course, more than just linguistic, since linguistic communication in classroom interaction is necessarily alongside non-linguistic forms of meaning-making like gestures and visual displays. Hence, we also want to support teachers in their discussion of linguistic demands in relation to the broader communicational demands of mathematics teaching and learning.

A MOMENT OF GAINING AWARENESS OF LANGUAGE(S) AND LANGUAGE RESPONSIVENESS IN MATHEMATICS TEACHING

The extract below illustrates a moment of development of awareness, in one of the workshops on proportionality in Barcelona. The awareness work was around linguistic demands and language responsiveness in mathematics teaching with multilingual students, and it was mediated by sharing and contrasting the understanding of a real teaching experience. The teacher educator (TE) and a secondary-school teacher (T) talked about aspects of the linguistic, communication and social dimensions of

participating in mathematical discussions. The language of the conversation was Catalan, and we use brackets only for translations that can facilitate the understanding of the mathematical-linguistic challenges involved in the original situation.

TE: So, you say these students participate less in the discussions, why do you think this is the case?

T: Because they struggle more. Proportional [*proporcional*], for example… they can say *a* and *b* are in disproportion [*a i be estan en desproporció*], and that is not what you expect, and it is difficult to understand what they want to say.

TE: You mean that they do not say *a* and *b* are not proportional [*a i b no son proporcionals*]. Being in disproportion does not sound mathematical, I see what you mean, but you can follow from there and still discuss proportionality with them all. Because you can guess what they want to say, can't you?

T: Well, yes, I can guess, but sounds more like disproportionate [*desproporcionat*], not the right something, like proportional and non-proportional.

TE: I am curious. How many of you, how many of you can guess what a student who talks about *a* and *b* being in disproportion may want to say? Is this student comparing *a* and *b*, and saying that *a* and *b* are proportional?

The workshops are contexts for educating awareness and for anticipating language-responsive mathematics teaching by reflecting on habitual actions and interpretations. The experience of the teacher in the example was used as a tool for discussion on how language-responsive mathematics teaching is about focusing on participation in mathematics rather than on linguistic accuracy in any particular language. Awareness was worked on around the fact that we view the challenges of multilingual students' participation in mathematics through our challenges in understanding this participation, as distinct from what we might expect for monolingual students, and through our challenges in refraining from the habit of searching for linguistic accuracy in the first place. By raising, interacting with and discussing mathematics teaching experiences with multilingual students, we take the time to consider their mathematical and pedagogical appropriateness. We thus work on becoming aware of habitual actions, discerning details and recognising relations between aspects of situations and enactable teaching actions that are responsive from a mathematical-linguistic perspective.

The strategy of discerning and valuing alternative teaching actions is helpful in overcoming the habit of focusing on linguistic accuracy over mathematical reasoning in multilingual classrooms. Alternatively, we could choose to understand the structures of meaning and reasoning that are communicated (Adler, 2001; Ingram et al., 2024; Moschkovich, 2010). Mathematics teachers in multilingual classrooms are subjected to habits or self-narratives, which may be difficult to interrogate. There are also habits about what is 'proper' communication in mathematics and 'proper' language use in the classroom context, which need to be made visible and discussed in relation to how they

influence the shaping of the opportunities for different students to participate in the mathematical discussions. The workshops provide alternatives to habits associated with viewing "difficult to understand what they [multilingual students] want to say", hence associated with ways of enacting communication and language use in school mathematics. The teachers work on seeing that they have a multitude of choices in responding to a student's reasoning that "*a* and *b* are in disproportion", and these choices may differ in how they encourage or support students' participation in mathematics and in strengthening the opportunities of mathematical discussion and learning for all.

FINAL REMARKS

Responsiveness itself is a notion underlying our research collaborations. Language-responsive mathematics teaching focuses on being responsive to the language resources, the diverse uses of language(s), and communication by students learning mathematics. Analogously, the Discipline of Noticing is similarly about being responsive to teachers' awarenesses, attention, and expertise. It is not about a teacher educator or researcher anticipating or planning a focus, but it is about responding to the different linguistic demands the teachers, the teacher educators, and the researchers notice and express an awareness of in the moment of the professional development. This is based on the principles within the Discipline of Noticing of working with awareness in order to act consciously and perhaps differently when teaching mathematics.

RECENT TECHNOLOGY-ORIENTED APPROACHES ON TEACHERS' PROFESSIONAL DEVELOPMENT

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In this paper, we describe the role of technology in teachers' professional development (PD). Focusing on professional competence as an outcome of PD, we present recent models and results on teachers' technology-related professional competence as well as formulate desiderata of research in this context. In addition, technology as a tool in PD to support access to PD and integrate rich representations of practice is addressed.

INTRODUCTION

Educational technology relates to digital hard- and software that can support teaching or learning. Since its potential benefits have been hypothesized for decades, our discussion focuses on current trends that can inform teaching and learning. Technology may play a twofold role in teachers' professional development (PD). Teachers can learn *about technology* and its use in their profession, and PD can include learning *with technology*. Regarding the first, it has been argued that teachers play a pivotal role for integrating technologies into teaching and learning mathematics and that an effective PD is a prerequisite for successful technology integration (Tabach, 2021). Hence, it is necessary to (i) characterize (technology-related) teacher characteristics relevant for this integration as desirable outcome of PD in terms of advancing instructional quality or student learning, (ii) investigate how PD must be designed to foster these relevant teacher characteristics and (iii) explore how technology can advance PD effectiveness.

TECHNOLOGY-RELATED PROFESSIONAL COMPETENCE AS PD GOAL

The key role of teachers' professional competence has long been theorized (Tabach, 2021) and it has been shown empirically that teachers' PD influences the effectiveness of technology integration for student achievement (Hillmayr et al., 2020). Hence, it is necessary to clarify to which extent teachers' professional competence needs to be conceptualized as specific to technology. Analytic approaches conceptualize teachers' competence by cognitive and motivational dispositions. For technology-related aspects of professional knowledge the Technological Pedagogical Content Knowledge framework has been widely used (TPACK; Mishra & Koehler, 2006). It extends the well-established distinction of content knowledge, pedagogical knowledge and pedagogical content knowledge with technology-related dimensions, e.g., knowledge how to use technology to represent and work with mathematical contents (Technological Content Knowledge) and knowledge how to teach mathematics using technology (Technological Pedagogical Content Knowledge). However, it largely lacks subject-related specification. Teachers' motivation to integrate technology is mostly modelled by their self-efficacy, attitudes, and values. Holistic approaches

describe the real-life demands teachers need to master in their professional practice. For example, the DigCompEdu framework describes demands such as creating and modifying digital resources or to plan for and implement digital devices and resources in the teaching process (European Commission et al., 2017). To come to a subject-specific conceptualization, it has been proposed to conceptualize technology-and-content related knowledge along specific contents (e.g., linear functions), technologies (e.g., dynamic geometry software) as well as established conceptualizations of PCK (Kosiol & Ufer, 2024). Current developments, e.g. focusing on the role of Artificial Intelligence (AI; Sperling et al., 2024) for learning, may be included in this perspective.

A main challenge here is to measure technology-related knowledge, beyond the widespread use of self-reports with unspecific references to technology or mathematics showing limited validity (Kosiol & Ufer, 2024). Recently, test instruments have been proposed that aim to measure teachers' technology-related knowledge posing standardized demands situated in authentic situations (e.g., Kosiol & Ufer, 2024; Seifert & Lindmeier, 2024). This will increasingly allow to evaluate the effects of PD programs from an analytic perspective covering the range of different goals, comprising technology-related knowledge, self-efficacy, and values.

PD OF TECHNOLOGY-RELATED PROFESSIONAL COMPETENCE

Although technology-related dimensions of professional knowledge have been modelled as separable dimensions, this separability is subject to debate. First results hint that distinct dimensions are separable but highly (Seifert & Lindmeier, 2024). Since only a few experimental studies on the development of this knowledge are available, it is an open question whether the different dimensions require separate learning opportunities. Research on PD programs to foster teachers' technology-related competence in mathematics education is limited resulting in mostly qualitative evidence regarding key factors of effective PD (e.g., Tabach, 2021): Potential characteristics of effective programs seem to be practice-based approaches where teachers' experience the use of technology for learning and teaching mathematics, and the implementation of professional learning communities so that teachers can co-construct new knowledge in a reflective process. However, a person-centred approach on teachers' technology-related competence may reveal differential needs of teachers to be taken into account when designing PD for effective technology integration.

TEACHERS' PD WITH TECHNOLOGY

Independently of whether PD focuses on technology use in the classroom or not, different potentials of technologies to increase the impact of PD programs have been discussed in the past. First and foremost, digital technology—in particular social media platforms—may help to *inform teachers* about current trends and PD opportunities. However, there is little evidence to which extent this takes place and is actually used by teachers in different countries. Beyond this, technology-based online or blended learning formats are expected to *increase availability and accessibility* of online PD programs for teachers over a whole country. It is unquestionable that knowledge

transfer, for example, by instructional videos may be very efficiently implemented in online systems. Yet, this affordance must be balanced against teacher engagement and depth of processing, if such online courses show little facilitator presence or direct interaction. To this end, Borba and Llinares (2012) theorize the potential of digital tools for *teacher networks, communication, and collaboration*. However, such communities often require *specific tools* to share and to discuss ideas and experiences (e.g., to promote noticing and online discourse about videos, Larison et al., 2024). Moreover, it stays open to which extent such networks require support by researchers or subject specialists providing background knowledge or guiding the group's joint development.

Beyond these more organizational aspects of PD design, technology may allow to include *rich representations of practice* aiming to stimulate joint deep processing and application of concepts introduced in a PD program. Given appropriate support, externally provided or own classroom videos may support teacher noticing and reasoning (Fischer et al., 2022; Larison et al., 2024). Recently, simulations as more interactive representations of practice have been discussed as a tool in pre-service and in-service PD (Schwarts et al., 2024). AI-based technology holds promises for reducing the efforts to create and adapt such rich stimuli or to integrate tailored feedback based on participants' actions. However, specific learning support (e.g., cues or prompts), might be required initially to support learning from such practice representations (Fischer et al., 2022). Finally, technology-based and online PD offer affordances in terms of *adaptability and personalization* of PD components. This starts with participants' freedom to choose time and location for processing of asynchronous components of online or blended PD. Furthermore, participants may choose among alternative materials or activities. Such adaptability is, however, prone to choices misaligned to participants' actual needs. Participants' self-regulation plays a role in such cases. Recent approaches aim at personalizing cases, stimuli, or learning support based on teachers' dispositions or current actions in a simulation directly, or provide recommendations to participants (e.g., Fischer et al., 2022).

Regarding the state of research, many studies in the field provide design reports and in-depth explorations of implementation of such tools by PD facilitators and use by participants, only a few studies systematically investigate the mechanisms tackled and effects caused by these tools. Such research also requires stronger theoretization of the expected mechanisms, by which technology exerts effects on teacher learning.

CONCLUSION

Concluding, we see that technology results in challenges for teachers' PD, needing to support them in continually adapting to new mathematical technologies, that students are required to master, or subject-specific or general educational technologies that may augment student learning. Learning to which extent teachers' professional competence is specific to a type of tools, and to which extent technology-independent dimensions of competence can be transferred to teaching with or about technology, is a major open question. The answer may guide our efforts to implement effective, but also efficient

PD for technology use in the classroom. For this purpose, and beyond, technology also holds a promise for PD, letting us expect a broader, deeper, and more personalized engagement with PD contents by teachers—provided we understand the mechanisms and effects of specific technology types in PD theoretically and empirically.

TRANSFORMING MATHEMATICS TEACHER PROFESSIONAL DEVELOPMENT: EQUITY, EXPERTISE, AND TECHNOLOGY-DRIVEN APPROACHES

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Mathematics teacher professional development (PD) has expanded to incorporate multiple approaches, including expertise-based, equity-driven, and culturally responsive strategies. It also emphasizes linguistic adaptability and technology integration, reflecting the evolving needs of modern classrooms. However, challenges remain in implementation, scalability, and empirical validation. For instance, PD programs often struggle with sustained teacher engagement due to time constraints and competing professional responsibilities. Additionally, the lack of infrastructure and funding limits the scalability of effective PD models, particularly in under-resourced educational systems. Empirical validation is further complicated by inconsistent assessment frameworks, making it difficult to measure long-term impacts on teaching practices and student outcomes. This paper critically examines these developments, identifying strengths and limitations across five key areas: teacher expertise, equity in PD, culturally responsive teacher noticing, linguistically adaptive instruction, and technology integration. Additionally, practical constraints such as policy limitations, teacher resistance, and inadequate institutional support are discussed. Drawing on international comparative research, this review proposes sustainable PD models tailored to diverse educational contexts. Findings emphasize the necessity of context-sensitive, data-driven, and scalable PD frameworks that align with the realities of contemporary classrooms.

INTRODUCTION

Mathematics teacher PD plays a crucial role in enhancing instructional quality and student achievement. While traditional PD models emphasized content knowledge (CK) and pedagogical content knowledge (PCK), contemporary research underscores the importance of adaptive decision-making, equity awareness, and cultural responsiveness. Moreover, with increasing linguistic diversity and technological advancements, PD must prepare teachers to navigate complex educational environments.

Despite these advances, many PD programs fail to produce sustained improvements due to weak empirical foundations, ineffective implementation, and lack of institutional support. This paper critically examines the evolution of mathematics teacher PD across five key areas: expanding teacher expertise, equity-oriented PD, culturally responsive teacher noticing, linguistically adaptive instruction, and technology integration. It also explores challenges in implementation and provides evidence-based recommendations for sustainable, impactful PD models.

EXPANDING TEACHER EXPERTISE

Recent research challenges knowledge-based PD models, arguing that effective teaching extends beyond CK and PCK to include dynamic interpretation of student thinking. Teacher noticing—recognizing and responding to key instructional moments—is increasingly acknowledged as a vital skill. Research by Jacobs, Lamb, and Philipp (2010) highlights how teacher noticing contributes to effective instructional decision-making by helping educators anticipate student misconceptions and adjust their strategies accordingly. For instance, studies in mathematics education have shown that teachers who develop strong noticing skills are better equipped to engage students in meaningful discussions and foster deeper conceptual understanding. Comparative studies highlight the role of teacher noticing in professional growth, emphasizing how expertise evolves over time.

One widely studied approach is lesson study, a collaborative PD model that fosters deep professional reflection. Research indicates that lesson study enhances instructional strategies and positively impacts student achievement. However, its success is contingent on institutional support, structured collaboration, and sufficient teacher time, raising concerns about scalability in resource-limited settings.

ADDRESSING EQUITY IN MATHEMATICS PD

Equity-driven PD seeks to bridge systemic gaps in mathematics education, particularly for underrepresented student populations, who often face disparities in access to quality instruction, culturally relevant curricula, and language support. In some contexts, marginalized students receive fewer opportunities for advanced coursework due to systemic biases, while in others, linguistic barriers prevent meaningful engagement with mathematical concepts. Addressing these challenges requires PD programs that equip teachers with strategies to recognize and counteract these inequities in diverse educational settings.

An example of a successful equity-driven PD model is the 'Complex Instruction' framework, which empowers teachers to address status differences in the classroom and create more inclusive mathematical discussions. Similarly, the 'Cognitively Guided Instruction' approach has demonstrated success in supporting diverse student populations by encouraging teachers to build on students' existing mathematical reasoning.

However, in many educational contexts, equity-driven PD remains aspirational rather than practical due to limited teacher training and institutional resistance. A major

challenge is the theoretical nature of equity discussions in PD; programs must shift towards sustained, practice-based training that enables teachers to implement equity principles in real-time classroom scenarios. Equity-driven PD seeks to bridge systemic gaps in mathematics education, particularly for underrepresented student populations, who often face disparities in access to quality instruction, culturally relevant curricula, and language support. In some contexts, marginalized students receive fewer opportunities for advanced coursework due to systemic biases, while in others, linguistic barriers prevent meaningful engagement with mathematical concepts. Addressing these challenges requires PD programs that equip teachers with strategies to recognize and counteract these inequities in diverse educational settings.

INTEGRATING CULTURAL AND LINGUISTIC RESPONSIVENESS

Cultural and linguistic diversity significantly impacts mathematics instruction. Research indicates that teacher noticing varies across cultures, necessitating culturally responsive PD tailored to specific educational contexts. Similarly, linguistically adaptive PD is critical in multilingual classrooms, where language barriers can hinder mathematical understanding.

Translanguaging strategies and discourse-based teaching methods have demonstrated effectiveness in supporting multilingual learners. However, many mathematics teachers lack formal training in linguistic pedagogy, resulting in missed opportunities for student engagement. PD must integrate linguistic responsiveness as a core component rather than an ancillary skill, and longitudinal studies are needed to validate its effectiveness.

THE ROLE OF TECHNOLOGY IN PD

Technology-enhanced PD offers personalized learning experiences through AI-driven analytics, virtual simulations, and video-based lesson analysis. The Technological Pedagogical Content Knowledge (TPACK) framework provides a model for integrating technology, pedagogy, and content knowledge in PD. However, research suggests that merely incorporating technology into PD does not guarantee improved teaching practices.

To maximize the benefits of technology-enhanced PD, programs should incorporate adaptive learning platforms that adjust content based on teachers' progress. Additionally, real-time feedback mechanisms, such as AI-driven virtual coaching, can provide immediate instructional support and reinforce PD learning outcomes.

Successful implementations, such as blended PD models in China and Singapore, combine in-person training with online modules and AI-powered simulations. Despite their success, these models require significant investment in infrastructure and policy alignment, which poses challenges for scalability in resource-constrained environments. Future research should explore cost-effective strategies, such as open-access digital tools and localized implementation frameworks, to make technology-driven PD more accessible on a global scale. Technology-enhanced PD offers personalized learning experiences through AI-driven analytics, virtual simulations, and

video-based lesson analysis. The Technological Pedagogical Content Knowledge (TPACK) framework provides a model for integrating technology, pedagogy, and content knowledge in PD. However, research suggests that merely incorporating technology into PD does not guarantee improved teaching practices.

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OVERCOMING IMPLEMENTATION BARRIERS AND FUTURE DIRECTIONS

Ensuring teacher well-being is crucial for sustaining PD engagement. High workloads and burnout often limit teachers' ability to fully participate in PD initiatives. Schools and institutions should implement workload management strategies, such as designated PD time and incentives, to encourage ongoing participation.

School leadership plays a pivotal role in fostering a culture that supports continuous PD. Principals and instructional leaders should facilitate collaboration, allocate dedicated PD time, and actively participate in training programs to reinforce the adoption of new methodologies.

Despite promising advancements, mathematics PD faces persistent challenges, including teacher resistance to new methodologies, policy misalignment, and scalability constraints. Future research should prioritize long-term, data-driven studies that track PD effectiveness. AI-driven personalized PD could address scalability issues by tailoring content to individual teachers' learning needs. Additionally, sustained mentorship programs and professional learning communities should be integrated to ensure continuous professional development. Furthermore, governments and educational institutions must reframe PD as an ongoing process rather than a series of isolated workshops by implementing long-term mentorship programs, integrating PD into teacher evaluation systems, and securing continuous funding for PD initiatives.

Governments and educational institutions must reframe PD as an ongoing process rather than a series of isolated workshops by implementing long-term mentorship programs, integrating PD into teacher evaluation systems, and providing continuous funding for PD initiatives. Additionally, policies should incentivize collaborative professional learning communities and ensure access to technology-driven PD resources tailored to diverse teaching contexts. Strengthening policy frameworks, securing institutional investment, and fostering teacher agency will be critical in building sustainable, impactful, and empirically validated PD initiatives. Despite promising advancements, mathematics PD faces persistent challenges, including teacher resistance to new methodologies, policy misalignment, and scalability

constraints. Future research should prioritize long-term, data-driven studies that track PD effectiveness. AI-driven personalized PD could address scalability issues by tailoring content to individual teachers' learning needs. Additionally, sustained mentorship programs and professional learning communities should be integrated to ensure continuous professional development.

CONCLUSION

Mathematics teacher PD is undergoing significant transformation, incorporating expertise-based, equity-driven, culturally responsive, linguistically inclusive, and technology-enhanced approaches. However, critical barriers to implementation must be addressed to fully realize the potential of these frameworks. Moving forward, empirical research, AI-driven innovations, and policy interventions must play a central role in shaping scalable and context-sensitive PD models. By integrating research-based insights with practical applications, mathematics education can become more effective, inclusive, and adaptable to diverse learning environments worldwide.

LOOKING AHEAD

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In recent years, due to the influence of societal and political changes, the rapid development of technology, and international mathematics curriculum reform, the discourse on mathematics teacher professional development has evolved significantly, which incorporates diverse perspectives such as expertise-oriented approaches, equity-driven strategies, culturally responsive frameworks, linguistically adaptive practices, and technology-enhanced methodologies described in this paper. However, as the field continues to expand, several critical areas warrant further exploration to address emerging challenges and leverage new opportunities. Below, we outline key directions for future research in mathematics teacher professional development, emphasizing the need for interdisciplinary collaboration, empirical validation, and context-sensitive implementations.

INTERDISCIPLINARY AND INTEGRATED APPROACHES TO TEACHER EXPERTISE AND COMPETENCE

While research on teacher competence has traditionally focused on cognitive aspects, such as mathematics content knowledge and mathematics pedagogical content knowledge, future studies should first employ a broader conceptualization of teacher competence which includes both cognitive and situational aspects of teaching expertise, in particular, teacher noticing, for a fuller understanding of mathematics teacher professional competence (Depaepe et al., 2020). However, the clarification

and distinction of competence and expertise has not been thoroughly explored; the question, what makes a teacher an expert teacher, how to characterize high teaching expertise, are currently underexplored. In the seminal work by Berliner (1988) descriptions of expert behavior have been developed, however, the question, how to define and especially measure reliably superior performance in professionally representative task remains currently unclear (Ericsson, 2018). Further research seems to be necessary, for instance, insights from cognitive science, to find out how knowledge restructuring processes, characteristic for expertise development (Boshuizen et al., 2020) take place and how they can be fostered, for example by deliberate practice.

EQUITY-DRIVEN PROFESSIONAL DEVELOPMENT IN DIVERSE AND CROSS-CULTURAL CONTEXTS

Equity-oriented professional development has attracted researchers' interests, particularly in addressing systemic disparities in mathematics education. However, there is a pressing need for more empirical research on the long-term impact of equity-driven professional development programs, especially in under-resourced and multilingual contexts. Future studies should focus on developing scalable and sustainable models that can be adapted to diverse or cross-cultural educational systems. Research should also explore the role of teacher agency in implementing equity-driven practices, as well as the impact of school leadership and policy frameworks on the success of such initiatives. Furthermore, the intersection of equity with other dimensions of professional development, such as technology integration and cultural responsiveness, remains underexplored and offers fertile ground for future inquiry.

CULTURALLY RESPONSIVE AND TECHNOLOGY INTEGRATED TEACHER PROFESSIONAL COMPETENCE FRAMEWORK AND INSTRUMENTS

Teacher professional competence, especially teacher noticing, as a culturally situated practice, requires further investigation to understand how cultural and contextual factors influence teachers' professional competence. Future research could employ cross-cultural comparative studies to identify commonalities and differences in teacher professional competence across diverse educational systems. Additionally, the development of culturally sensitive assessment instruments and frameworks for teacher professional competence could enhance the validity and applicability of professional development programs in multicultural classrooms. Moreover, the integration of technology in mathematics teaching may also change or bring new meaning and requirements to various dimensions of the existing teacher professional competence framework, beyond the scope of well-known frameworks. Therefore, future research should consider developing a specific framework for such kind of teacher professional competence and instruments as well. For example, research could explore how teacher noticing evolves in response to professional development interventions, particularly those that incorporate technology or AI tools.

IMPACT OF PROFESSIONAL DEVELOPMENT ON TEACHER PROFESSIONAL COMPETENCE AND STUDENT LEARNING OUTCOMES

As discussed earlier, even though teacher professional development has been widely accepted as a critical influence on the growth of teacher professional competence and students' mathematics learning outcomes as well, there is a lack of studies that have tried to empirically investigate the link between teacher professional development, teacher professional competence, teaching practice/quality, and students' learning outcomes (König et al., 2023). Future studies could consider to empirically investigate the effectiveness of professional development, particularly cross-cultural studies between different contexts (Yang & Kaiser, 2024). Additionally, longitudinal studies could track the development of teacher professional competence over time, examining and comparing how different professional development models contribute to sustained professional growth.

GLOBAL AND INTERNATIONAL COLLABORATION AND KNOWLEDGE EXCHANGE

Finally, the globalization of education in general, and of mathematics education in particular, necessitates greater collaboration and knowledge exchange among mathematics education researchers, policymakers, and practitioners within and across different countries and regions. Future research should prioritize international comparative studies that examine the transferability of professional development models of mathematics teachers across diverse cultural and educational contexts. Collaborative research networks could facilitate the sharing of (best) practices, resources, and innovative approaches to mathematics teacher professional development. Additionally, with the assistance of technology or AI, the development of open-access digital platforms and repositories could support the dissemination of research findings and professional development materials at a global scale.

CONCLUSION

The future of mathematics teacher professional development lies in the integration of diverse perspectives, the adoption of innovative technologies, and the alignment of policy and practice. By addressing the gaps identified in this research forum, the field can move towards more effective, inclusive, and sustainable professional development models that empower teachers to meet the challenges of mathematics education in the future. As we continue to explore these new directions, it is necessary that research remains grounded in empirical evidence, responsive to the needs of diverse educational contexts, and committed to the ongoing professional growth of mathematics teachers worldwide.

REFERENCES

Abtahi, Y. & Planas, N. (2024). Mathematics teaching and teacher education against marginalisation, or towards equity, diversity and inclusion. *ZDM - Mathematics Education*, *56*, 307-318.

Adler, J. (2001). Teaching mathematics in multilingual classrooms. Springer.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: what makes it special? *Journal of Teacher Education, 59* (5), 389-407.

Bell, C., Castellano, K., & Klieme, E. (2020). Classroom Management. In OECD (Ed.), *OECD Global Teaching InSights: A video study of teaching* (pp. 46–68). OECD Publishing.

Beltrán-Grimm, S. (2023). *No solamente ellos aprenden, aprendemos nosotros también*: Latina mothers' experiences with co-design approaches in creating a math activity for their children. *Journal of Latinos and Education*, *23*(3), 1089-1108.

Berliner, D.C. (1988). *The development of expertise in pedagogy*. American Association of Colleges of Teacher Education Publications.

Blömeke, S., Busse, A., Kaiser, G., König, J., & Suhl, U. (2016). The relation between content-specific and general teacher knowledge and skills. *Teaching and Teacher Education, 56*, 35–46.

Blömeke, S., Feng-Jui, H., Kaiser, G., & Schmidt, W. H. (2014) (Eds.). *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn. TEDS-M Results.* Springer.

Blömeke, S., Hsieh, F.-J., Kaiser, G. & Schmidt, W. H. (Eds.) (2014). *International perspectives on teacher knowledge, beliefs and opportunities to learn*. Springer.

Borba, M. d. C., & Llinares, S. (2012). Online mathematics teacher education: overview of an emergent field of research. *ZDM - Mathematics Education, 44*(6), 697-704.

Boshuizen, H., Gruber, H., & Strasser, J. (2020). Knowledge restructuring through case processing: The key to generalise expertise development theory across domains? *Educational Research Review*, 29:100310.

Brownell, W. A. (1945). When is arithmetic meaningful? *The Journal of Educational Research, 38*(7), 481-498.

Chiu, T. K. F., Li, Y., Ding, M., Hallström, J., & Koretsky, M. D. (2025). A decade of research contributions and emerging trends in the *International Journal of STEM Education. International Journal of STEM Education, 12*:12.

Civil, M. (2016). STEM learning research through a funds of knowledge lens. *Cultural Studies of Science Education, 11*, 41-59.

Civil, M. & Hunter, R., (2015). Participation of non-dominant students in argumentation in the mathematics classroom. *Intercultural Education*, *26*(4), 296-312.

Civil, M., Hunter, R., & Crespo, S. (2019). Mathematics teachers committed to equity. In D. Potari & O. Chapman (Eds.), *International handbook of mathematics teacher education: Knowledge, beliefs, and identity in mathematics teaching and teaching development* (Vol. 1, pp. 243–274). Brill.

Civil, M., Hunter, R., & Crespo, S. (2020). Mathematics teachers committed to equity: A review of teaching practices. In D. Potari and O. Chapman (Eds.), International handbook of mathematics teacher education (Vol. 1, *Knowledge, beliefs, and identity in mathematics teaching and teaching development* (2nd edition)) (pp. 243-273). Sense Publishers.

Civil, M., Stoehr, K. J., & Salazar, F. (2020). Learning with and from immigrant mothers: Implications for adult numeracy. *ZDM Mathematics Education, 52*(3), 489-500.

Cohen, E. G. & Lotan, R. A. (Eds.) (1997). *Working for equity in heterogeneous classrooms: Sociological theory in practice*. Teachers College Press.

Crespo, S. (2020). Complex Instruction in Mathematics Teacher Preparation. In M. Peters, (Ed.) *Encyclopedia of Teacher Education*. Springer.

Depaepe, F., Verschaffel, L., & Star, J. (2020). Expertise in developing students' expertise in mathematics: bridging teachers' professional knowledge and instructional quality. *ZDM Mathematics Education, 52*(2), 179–192.

Ding, M. et al. (2023). A scoping review of mathematics teachers' learning and professional growth through lesson studies. *Asian Journal for Mathematics Education, 2*(4), 492-510.

Ding, M. et al. (2024). A review of lesson study in mathematics education from 2015 to 2022: implementation and impact. *ZDM – Mathematics Education, 56*(1), 87-99.

Dreher, A., & Kuntze, S. (2015). Teachers' professional knowledge and noticing: The case of multiple representations in the mathematics classroom. *Educational Studies in Mathematics, 88*(1), 89-114.

Erath, K., Ingram, J., Moschkovich, J., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning: A review of the state of development and research. *ZDM – Mathematics Education, 53*, 245–262.

Ericsson, K. A. (2018). An introduction to the second edition of the Cambridge handbook of expertise and expert performance: Its development, organization, and content. In K. A. Ericsson, R. R. Hoffman, A. Kozbelt, & M. A. Williams (Eds.), *The Cambridge handbook of expertise and expert performance* (pp. 3–20). Cambridge University Press

European Commission, Joint Research Centre, Redecker, C., & Punie, Y. (2017). *European Framework for the Digital Competence of Educators: DigCompEdu*. Publications Office of the European Union.

Featherstone, H., Crespo, S., Jilk, L., Oslund, J., Parks, A., & Wood, M. (2011). *Smarter Together: Collaboration and Equity in the Elementary Math Classroom*. National Council of Teachers of Mathematics.

Fischer, F., Bauer, E., . . . Fischer, M. R. (2022). Representational scaffolding in digital simulations – learning professional practices in higher education. *Information and Learning Sciences, 123*(11/12), 645-665.

Goldenberg, E. P., Cuoco, A. A., & Carter, C. J. (2021). The missing element for teachers: Learning what mathematics *is*. In Y. Li et al (Eds.), *Developing mathematical proficiency for elementary instruction* (pp. 51-87). Springer.

González, N., Moll, L., & Amanti, C. (2005). *Funds of knowledge: Theorizing practices in households, communities, and classrooms.* Routledge.

Harper, F. K., & Crespo, S. (2020). Learning to collaborate while learning mathematics. *Mathematics Teacher: Learning and Teaching PK-12*, *113*(10), 800-811.

Hillmayr, D., Ziernwald, L., Reinhold, F., Hofer, S. I., & Reiss, K. M. (2020). The potential of digital tools to enhance mathematics and science learning in secondary schools: A context-specific meta-analysis. *Computers & Education, 153*, Article 103897.

Horn, I. S. & Little, J. W. (2010). Attending to problems of practice: Routines and resources for professional learning in teachers' workplace interactions. *American Educational Research Journal, 47*(1), 181-217.

Hunter, R. & Hunter, J. (2018). Opening the space for all students to engage in mathematical talk within collaborative inquiry and argumentation. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas, & D. Wagner (Eds.), *Mathematical discourse that breaks barriers and creates space for marginalized learners* (pp. 1-21). Sense Publishers.

Ingram, J., Abbott, A., Smith, K., Planas, N., & Erath, K. (2024). Experienced teachers talking about their mathematics teaching with linguistically disadvantaged learners. *Journal of Mathematics Teacher Education, 27*, 785–808.

Jacobs, V. R. (2017). Complexities in measuring teacher noticing: Commentary. In E. O. Schack, M. H. Fisher, & J. A. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 273-279). Springer.

Jacobs, V. R., & Empson, S. B. (2016). Responding to children's mathematical thinking in the moment: An emerging framework of teaching moves. *ZDM - Mathematics Education, 48*(1-2), 185–197.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202. https://doi.org/10.5951/jresematheduc.41.2.0169

Kaiser, G., Busse, A., Hoth, J., König, J., & Blömeke, S. (2015). About the complexities of video-based assessments: Theoretical and methodological approaches to overcoming shortcomings of research on teachers' competence. *International Journal of Science and Mathematics Education, 13*(2), 369–387.

Killi, E. K. & Xenofontos, C. (2024). Teaching mathematics to children of immigrant backgrounds: Primary teachers' perspectives from Norway. *Journal for Theoretical and Marginal Mathematics Education*, *3*(1), 1-19.

König, J., Heine, S., Kramer, C., Weyers, J., Becker-Mrotzek, M., Großschedl, J., ...Strauß, S. (2023). Teacher education effectiveness as an emerging research paradigm: a synthesis of reviews of empirical studies published over three decades (1993–2023). *Journal of Curriculum Studies*, 56(4), 371–391.

Kosiol, T., & Ufer, S. (2023). Teaching mathematics with technologies: Profiles of teacher characteristics. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 195–202). PME.

Kosiol, T., & Ufer, S. (2024). Teachers' self-reported and actual content-related TPACK – new results on their relation and gender differences. *Computers and Education Open, 7*, Article 100205.

Larison, S., Richards, J., & Sherin, M. G. (2024). Tools for supporting teacher noticing about classroom video in online professional development. *Journal of Mathematics Teacher Education, 27*(2), 139-161.

Li, Y. & Howe, R. (2017). Developing pre-service teachers' mathematics conceptual knowledge for teaching. In Kaur, B., Ho, W. K., Toh, T. L., & Choy, B. H. (Eds.). *Proceedings of the 41st Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 84-87. PME.

Li, Y. & Howe, R. (2021). Toward a thinking-oriented training in mathematics for elementary school teachers. In Y. Li et al (Eds.), *Developing mathematical proficiency for elementary instruction* (pp. 13-49). Springer.

Li, Y. & Kaiser, G. (Eds.) (2011). *Expertise in mathematics instruction*. Springer.

Li, Y. & Schoenfeld, A. H. (2019). Problematizing teaching and learning mathematics as "given" in STEM education. *International Journal of STEM Education, 6*:44.

Li, Y., & Kulm, G. (2008). Knowledge and confidence of pre-service mathematics teacher: The case of fraction division. *ZDM - Mathematics Education, 40,* 833-843.

Louie, N. (2018). Culture and ideology in mathematics teacher noticing. *Educational Studies in Mathematics*, 97(1), 55–69.

Louie, N., Adiredja, A., & Jessup, N. (2021). Teacher noticing from a sociopolitical perspective: the FAIR framework for anti-deficit noticing. *ZDM - Mathematics Education*, *53*, 95-107.

Lucas, T., & Villegas, A. M. (2010). A framework for preparing linguistically responsive teachers. In T. Lucas (Ed.), *Teacher preparation for linguistically diverse classrooms: A resource for teacher educators* (pp. 55–72). Taylor & Francis Group.

Mason, J. (2002). *Researching your own practice. The Discipline of Noticing*. Routledge.

Mishra, P. (2019). Considering contextual knowledge: The TPACK diagram gets an upgrade. *Journal of Digital Learning in Teacher Education, 35*(2), 76-78.

Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054.

Moschkovich, J. (Ed.) (2010). Language and mathematics education: Multiple perspectives and directions for research. Information Age Publishing.

Planas, N., & Abtahi, Y. (2024). Decentring research on mathematics teacher education: Why does it matter. *Journal of Mathematics Teacher Education, 27*(5), 743–754.

Planas, N., & Alfonso, J. M. (2023). Secondary-school teachers' noticing of aspects of mathematics teaching talk in the context of one-day workshops. *The Journal of Mathematical Behavior*, *71*, 101084.

Planas, N., Alfonso, J. M., Arnal-Bailera, A., & Martín-Molina, V. (2024). Mathematical naming and explaining in teaching talk: Noticing work with two groups of mathematics teachers. *ZDM–Mathematics Education, 56*, 1211–1222.

Rogoff, B., Coppens, A., Alcalá, L., Aceves-Azuara, I., Ruvalcaba, O., López, A., & Dayton, A. (2017). Noticing learners' strengths through cultural research. *Perspectives on Psychological Science*, *12*(5), 876–888.

Russ, R. S., Sherin, B., & Sherin, M. G. (2011). Images of expertise in mathematics teaching. In Y. Li & G. Kaiser (Eds.), *Expertise in mathematics instruction*. (pp. 41-60). Springer.

Saadati, F., Larrain, M., Bastian, A., Felmer, P., & Kaiser, G. (2024). Adapting German instruments and validating their use to measure Chilean mathematics teachers' professional competence. *Journal of Curriculum Studies, 56*(4), 413-431,

Schwarts, G., Herbst, P., & Brown, A. M. (2024). Harnessing Asynchronous Digital Simulations of Problem-based Lessons to Support Mathematics Teachers' Professional Development: A Design-based Approach. *International Journal of Science and Mathematics Education.*

Seifert, H., & Lindmeier, A. (2024). Developing a Performance-Based Assessment to Measure Pre-Service Secondary Teachers' Digital Competence to Use Digital Mathematics Tools. *Journal für Mathematik-Didaktik, 45*, Article 25.

Sperling, K., Stenberg, C.-J., McGrath, C., Åkerfeldt, A., Heintz, F., & Stenliden, L. (2024). In search of artificial intelligence (AI) literacy in teacher education: A scoping review. *Computers and Education Open, 6*, Article 100169.

Tabach, M. (2021). Competencies for Teaching Mathematics in the Digital Era: Are We Ready to Characterize Them? In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 32-47). PME.

Varley Gutiérrez, M., Napp-Avelli, C., Quintos, B., Salazar, F., Turner, E., & Civil, M. (2024). Families and teachers doing mathematics tasks together: Considering funds of knowledge and positioning. *Mathematics Teacher Educator*, *13*(1), 282-296.

Yang, X., & Kaiser, G. (2022). The impact of mathematics teachers' professional competence on instructional quality and students' learning outcomes. *Current Opinion in Behavioral Sciences*, *48*, 101225.

Yang, X., & Kaiser, G. (2024). Teachers' Professional Competences in Western and Eastern Countries: Findings from Comparative Studies. In M.A. Clements, B. Kaur, T. Lowrie, V. Mesa, & J. Prytz (Eds.), *Fourth International Handbook of Mathematics Education* (pp. 103-135). Springer.

Yang, X., Kaiser, G., König, J., & Blömeke, S. (2019). Professional Noticing of Mathematics Teachers: A Comparative Study Between Germany and China. *International Journal of Science and Mathematics Education, 17*(5), 943-963.

Zavala, M. R., & Aguirre, J. M. (2024). *Cultivating mathematical hearts: Culturally responsive mathematics teaching in elementary classrooms*. Corwin Press.

CURIOSITY IN MATHEMATICS: THEORETICAL, METHODOLOGICAL AND PRACTICAL LENSES

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Despite its fundamental importance, mathematical curiosity has received limited attention in mathematics education research. Knuth (2002) defines it as both the desire to learn mathematics and the drive to explore mathematical concepts, particularly through problem-solving and problem-posing activities. The broader psychological understanding of curiosity emphasizes its role as an intrinsic motivator for acquiring new information, linking it to cognitive engagement, including intellectual, epistemic, and information-seeking pursuits (Gruber et al., 2019). This suggests that mathematical curiosity is associated with knowledge gaps that students are motivated to overcome. However, this definition alone does not provide mathematics educators neither with a complete understanding of the nature and structure of mathematical curiosity, nor how it develops and can be fostered. While research interest in mathematical curiosity is emerging (Leikin, 2023), overall research on curiosity in mathematics education remains limited. This research forum aims to draw the PME community's attention to this important topic.

INTRODUCTION

Roza Leikin and Demetra Pitta-Pantazi

Curiosity is acknowledged as an important factor in children's development and learning process (Gruber, et al., 2019). Although it is recognized as a vital factor of knowledge construction (Piaget, 1952) and has received extensive attention in educational literature, there has been a limited number of theoretical, methodological, and empirical publications in mathematics education, where domain-specific approaches are needed to leverage its potential

Curiosity is an elusive concept that resists simple definition (Jirout & Klahr, 2012). Psychological literature characterizes it as the intrinsic desire to acquire new information, which is fundamental to learning and memory in everyday life (Gruber et al., 2019). It is both inherent to children's development and develops through social interactions (Engel, 2011).

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, pp. 151-180. PME.

There exists a notable contrast between trait curiosity (a personality characteristic) and state curiosity (temporary states affecting cognitive development) (Gruber et al., 2019). Curiosity can be interpreted as either a long-term trait or a short-term state triggered by environmental variables (Lamnina & Chase, 2019).

The affective dimension of curiosity appears in definitions describing it as a need for knowledge/information or the motivation to pursue it through exploration (Grossnickle, 2016). Harackiewicz et al. (2016) add that curiosity connects to psychological states of attention and affect toward specific objects/topics, or an enduring predisposition to reengage over time. This perspective distinguishes curiosity from interest, which is driven by goals of engagement and enjoyment with particular objects or phenomena.

Peterson and Cohen (2019) challenge the traditional domain-general view of epistemic curiosity, arguing that this framework inadequately tracks learners' progress within specific domains like mathematics. While epistemic curiosity has been broadly understood as a general desire for knowledge that triggers positive emotions, heightened arousal, and exploration (Grossnickle, 2016), mathematical curiosity requires special attention from mathematics education researchers due to its domain-specific nature. The developmental trajectory of mathematical curiosity is intimately linked to the learning of mathematics and the development of mathematical knowledge and skills. Recognizing this specificity is essential for establishing curiosity-driven mathematical instruction. Specifically, researchers can develop more effective frameworks for fostering curiosity in mathematics education, ultimately supporting students' long-term mathematical literacy and problem-solving abilities.

Previous works by Knuth (2002) and Posamentier and Lehmann (2014) have provided practical examples of curiosity-evoking mathematical activities. However, these contributions, while valuable, lack a comprehensive research framework for understanding mathematical curiosity (Leikin, 2023). Both cognitive and affective dimensions of curiosity are particularly noteworthy, as curiosity manifests in the drive for knowledge acquisition and exploratory behavior (Grossnickle, 2016; Peterson & Cohen, 2019). Despite the importance of this conceptual distinction, "there has been very little research on curiosity and, therefore, our understanding of how curiosity impacts learning is relatively poor" (Gruber et al., 2019). Although "Children's interests and explorations are the mechanisms that underlie authentic learning" and curiosity should be cultivated in schools, it is often "almost completely absent from classrooms" (Engel, 2011). Thus, developing a comprehensive framework for understanding mathematical curiosity is crucial for educators to design effective instructional strategies that foster curiosity, and enhance learning.

Following the International ISF Research Workshop “Mathematical curiosity as means for the developing of mathematical proficiency in a wide range of students” (Leikin, 2023), this Research Forum aims to catalyze discussion within the mathematics education community about the nature and nurture of mathematical curiosity and its

vital role in mathematical learning and teaching. The goal is to foster an international dialogue that deepens our understanding of mathematical curiosity while exploring how it can be developed across diverse student populations.

The presentations and discussions at the Research Forum will draw distinctions between domain-general and domain-specific curiosity, with mathematical curiosity representing a crucial domain-specific form that encompasses the fundamental mechanisms of authentic mathematical learning (Peterson & Cohen, 2019). While several works explored curiosity in general terms, this Research Forum aims to address mathematical curiosity as a distinct phenomenon that warrants cultivation in mathematics education settings.

During the first session of the research forum, we will focus on theoretical and historical perspectives on curiosity.

Roza Leikin and Demetra Pitta-Pantazi will open the session with an introduction (see section introduction above) that provides an overview of the state of the art of research on curiosity in mathematics education by linking this research to psychological research on general curiosity.

Alexander Karp will provide a historical analysis addressing the question "How have attitudes towards curiosity changed in mathematics teaching?" He will argue that mathematics educators for centuries were not concerned with arousing students' curiosity, and draws our attention to the changes that took place in the awareness of the importance of fostering curiosity. Stanislaw Schukajlow, Pia Gödeke and Janina Krawitz will analyze mathematical curiosity from the affective perspective, examining its characteristics, conceptual overlaps, and relationships. Using Pekrun's (2006) control-value theory of achievement emotions and Loewenstein's (1994) information-gap theory, they will emphasize the significance of curiosity as both an emotional and motivational construct. The authors will explore curiosity as an affective construct and critically examine mutual relationships between curiosity and interest. Rina Zazkis will apply a cognitive lens to mathematical curiosity as related to intellectual needs (Harel, 2013). Through a reflective account of several mathematical incidents that triggered her personal curiosity and resulted in investigations and unexpected solutions, she will draw conclusions on how curiosity in mathematics can be related to curiosity in mathematics teaching and in mathematics education research.

During the second session, the research forum participants will be exposed to examples of research on curiosity-driven mathematical instruction.

Demetra Pitta-Pantazi and Roza Leikin will outline questions that can open new directions in mathematics education research, such as:

- Which instructional approaches can arouse curiosity in learning mathematics?
- How can problem solving or problem posing activities develop curiosity, and are they more effective when curiosity-driven?

- What is the role of teachers in evoking mathematical creativity in their students or impeding learners' curiosity when studying mathematics?
- Do technological tools support mathematical curiosity, and how do they do so?

Jinfa Cai and Victoria Robison will examine mathematical curiosity exhibited in and nurtured through problem posing. Based on the three dimensions of mathematical curiosity suggested by Peterson and Cohen (2019) - knowledge gaps, heightened arousal, and question-asking - they will demonstrate how and why mathematical problem posing is a powerful mathematical activity based on and leading to mathematical curiosity. Patricio Felmer and Boris Koichu will explore curiosity triggered by collaborative problem solving settings for undergraduate engineering students. The researchers will examine students' conceptions about curiosity and suggest their categorization. Hui-Yu Hsu will describe curiosity reflected in students' metacognitive behaviors taken during problem solving, through focusing on 4th grade and 8th grade students solving of high cognitive-demand problems. Her findings reveal notable differences between the two grade levels, highlighting developmental differences in metacognitive approaches and providing insights into the intertwined relationship between curiosity and mathematical problem solving.

AN ATTEMPT AT HISTORICAL ANALYSIS: HOW ATTITUDES TOWARDS CURIOSITY HAVE CHANGED IN MATHEMATICS TEACHING

Alexander Karp

The fact that scientists are inquisitive is commonplace today. Therefore, there is a universal belief that curiosity should be encouraged from a young age. A modern Russian textbook on methods of teaching mathematics (Stefanova & Podkhodova, 2005) says directly that one of the motives of learning activity is that students attempt to satisfy their curiosity (p. 29). But this has not always been so. The purpose of this section is to briefly trace how attitudes toward curiosity in studying, and, just as importantly, toward its formation, have changed. Of course, this section will be limited to some examples and observations (there is no room for a systematic review).

Even in antiquity, curiosity in learning and the pursuit of knowledge were usually sought after. Aristotle (1983) wrote that learning is the most pleasant thing. Later, St. Augustine wrote: "Hinc satis elucet maiorem habere vim ad discenda ista liberam curiositatem quam meticulosam necessitatem" (Confessions I, 14), that is, "Hence it is perfectly clear that free curiosity has greater power for learning than coercive discipline." But curiosity itself was still considered a sin and, Thomas Aquinas (1225-1274) had to ask the question "utrum vitium curiositatis possit esse in cognitione intellectiva" (Summa Theologiae, question 167), i.e., "Can the vice of curiosity regard intellective knowledge?" It was further explained that since intellective knowledge is

a pure matter and not contaminated by anything evil, the answer was “no.” Still, the word aroused suspicion.

People of the Middle Ages were often portrayed as impersonal beings who completely followed the prevailing norms, which is, of course, inaccurate (Gurevich, 2005); yet it is hard to imagine a medieval monk trying to arouse curiosity in the students of a monastic school in process of studying certain properties of numbers (admiration for the Lord's wisdom is another matter). At least we have no evidence to suggest that such things occurred.

Even though the 15th-16th centuries are often considered to be the time of “love of calculations,” neither then, nor even later, when large centralized national states began to emerge, building their armies and needing mathematics for that, they did not talk much about curiosity in the educational process—they talked about its usefulness.

Take, for example, the textbook published in Russia in 1703, the famous *Arithmetic* of Leontius Magnitsky (1703). This book was used in the educational process for almost half a century, and many famous people, such as the great natural scientist Mikhail Lomonosov (1711-1765), called it the gateway to their scholarship. As Magnitsky himself writes, the book is brought from different dialects into the Slavic language and united into one treatise—that is, it reflects not so much Russian experience, but European. It is addressed to wisdom-loving Russian adolescents and people of every rank and age. The meaning and value of learning, however, are expressed in the poetic preface: “Love wisdom and science, and wealth will come to you in your hands” and it is further explained that arithmetic is useful for merchants, and craftsmen, and masters of other arts. In another place Magnitsky, however, writes that he who does not care about the descent of sciences is deprived of the delicate ornaments of life, but what distinguishes the book is not the section of rhetorical exercises which opens the book but the way in which Magnitsky presents the material.

In that book, however, hardly anything is designed for the inquisitive. For example, the second section of the first part opens with an explanation (which probably should have been memorized) of what addition is: addition is two or many numbers in one assembly. Then addition tables are given (displaying the results of adding the first ten positive integers), then it is shown how to add the multi-digit numbers 532 and 46 (no explanation is given as to why this is so), and so on. This style is quite revealing and typical of textbooks of the time.

It seems that it was only after Pestalozzi (1746-1827) with his attention to the inner life of children and their feelings that we can begin to talk about the importance of awakening and developing curiosity in pupils. Both theorists and practitioners of education thought about curiosity.

“Teacher of German Teachers” Adolph Diesterweg (1790-1866) in his *Guide for the Instruction of German Teachers* (Diesterweg, 1956) recommended, “At each stage, mention separate parts of the material to follow, and without allowing for any

significant gaps, present separate ideas from it to arouse the schoolchild's curiosity, without, however, satisfying it completely!"

This kind of going ahead does indeed excite curiosity, as it does the story of unknown lands to be seen in the future, but Diesterweg does not give a concrete method of how to do it here. The American Warren Colburn (1793-1833) proposed a special technique in his arithmetic textbooks—it consisted of asking questions. The technique was not new at all; however, these questions were not aimed at reproducing what had been learned, but at analyzing what was being studied. Colburn (1823) did not speak directly about awakening curiosity but saw his task as awakening students' independence even in learning calculations, which for centuries had been considered to be learned by rote: students "should be allowed to pursue their own method first, and then they should be made to observe and explain it, and if it was not the best, some improvements should be suggested" (p. 4). The questions in the book are usually not difficult—it opens with "How many thumbs have you on your right hand?"- but it is important that they are often based on the experience of the students and are therefore manageable for them, and at the same time, they offer some new perspective on that experience and therefore may be of interest to them.

One might ask here, what awakens curiosity? Today, it is sometimes thought that there is no better way to do this than to talk about how math is useful in some applications, with an emphasis on how much money it makes. The American mathematics educator Jacob William Albert Young (1865-1948) argued strongly against this approach in his much reprinted and widely translated book *The Teaching of Mathematics in the Elementary and the Secondary School*:

> Utility often acts powerfully in arousing the interest of the adult; utility alone is never the child's standard. When he appeals to it, what is the "use" of this or that, he is sophisticated; he is not expressing his own views, but is appealing to the adult point of view. He never asks, "What is the use of playing ball?" and this fact points to what is the child's standard of interest, namely, doing, successfully doing (pp.94-95).

To the above question about what arouses curiosity, Young answered no less forcefully: "Few of the subjects of instruction are intrinsically interesting to the child: they must be 'made interesting', and here is the teacher's main opportunity and obligation" (p. 96). The center of gravity is thus shifted to the question of what a mathematics teacher should do to arouse curiosity (note that this very question is an achievement—it had not been thought about for many centuries).

Some techniques have been mentioned above: "getting ahead" and direct questioning by the teacher or textbook as well as reliance on the concrete experience of the child, from which generalization grows. We can say that the feasibility of what was being studied was put forward as a necessary condition for interest and for the emergence of curiosity (this is the condition for doing, successful doing). What else do we see in history?

Curiosity is aroused by a riddle, i.e., a report about some unexpected and even contradictory phenomenon or some unexpected task. Examples of such riddles can be found in ancient manuscripts and even in the textbook of Magnitsky quoted above. Most likely they were given for entertainment, though they were related to what was being studied. An example of a deeper question can be found in a book translated into many languages, *Interesting Algebra* by Yakov Perelman (1955). The book tells of the will of Benjamin Franklin, who left a thousand pounds to the city of Boston to be lent to young artisans at 5% interest per annum. Franklin then disposed of the millions to be accumulated 100 and 200 years afterward. It is natural to ask the question, how did he calculate how much money would accumulate? Did he multiply 200 times 1.05? There were, after all, no calculators.

Examples of this kind can be found in history, in natural science, and in other fields. Of course, the effect is enhanced if the corresponding situation occurs in the classroom, and the teachers emphasize its mystery, using their pedagogical skills and a certain artistry. Almost half a century ago, the Leningrad teacher Aron Maizelis set a certain task before the class, and when children started shouting their questions and comments from their seats, he asked them not to make noise but to write the questions about additional details and context of the task on special sheets of paper, while he would write the answer. The questions were then read out, after which Maizelis would read out his answer that whatever was asked did not matter—the answer was still the same (Karp, 2007). The children's curiosity during this scene knew no bounds.

Pedagogical skills, including an understanding of how students think, played a major role here in drawing students to the task and in engaging students in the process of discussing the problem posed. But here we must mention another principle of arousing curiosity—gradualness and systematicity, which helps students to experience a problem as their own so that there will be a personal need to solve it.

If we turn to an old geometry textbook, e.g., Robinson (1867), we will see that the few problems included in the book are given rather chaotically; each of them is given separately, in isolation. If we look at newer problem books, such as Hart (1934), we see that the problems are organized in a very slender system, in which the movement from a simple problem to a more complex one and from one type to another is clearly visible (Karp, 2023). A student who has solved a problem is thus inevitably encouraged to think about how to do the next one, which is similar but still not solvable by the same approach. In this way, without special declarations, curiosity is awakened.

History, of course, does not and cannot provide answers to all of today's questions, and although we can learn from history some techniques for awakening curiosity, it does not replace today's reflections. But the experience accumulated by humanity is important. Russian psychologist Bozhovich (2012) once divided the motives for learning into social and cognitive. In fact, cognitive and social sides are deeply connected. Cognitive interest, including the awakening of curiosity, became more talked about when society and schools began to pay more attention to the individual

lives of children, their feelings, and their development. At the same time, the social and related to the social, including children's environment, their teachers, books, and classmates have bearing on cognitive interests, including the awakening of curiosity. How this can be done is worth exploring both in history and in the present.

CURIOSITY FROM EMOTIONAL AND COGNITIVE PERSPECTIVES: CHARACTERISTICS AND CONCEPTUAL OVERLAPS

Stanislaw Schukajlow, Pia Gödecke, & Janina Krawitz

Introduction

Emotions as affective constructs are crucial in individuals' lives, careers, learning, and overall well-being. As a result, they also significantly influence the school environment and mathematical learning. Affect is an essential prerequisite for cognitive outcomes, such as knowledge acquisition and strategy use. Additionally, affective constructs function as learning outcomes or goals in mathematics education and mediate the effects of instruction on other learning outcomes, including mathematical performance (Schukajlow et al., 2023). Recently, researchers have observed a growing interest in affect, suggesting a possible affective turn in mathematics education research (Schukajlow et al., 2023). However, with some exceptions such as mathematical anxiety, only few research analyzed specific affective constructs. A recent meta-analysis identified only nine studies on curiosity in mathematics out of 57 studies that analyzed emotions in mathematics between 2020 and 2024 (Schoenherr et al., 2025). In this contribution, we analyze curiosity as an emotional and cognitive construct by (1) addressing its theoretical background within the control-value theory of emotions and the information-gap theory, (2) exploring its conceptual overlap with interest, and (3) examining key characteristics of curiosity. At the end, we offer a summary and future directions for research.

Curiosity: theoretical underpinnings

Rather than being a simple feeling, curiosity is a complex, dynamic process that involves affective, cognitive, motivational, expressive, and physiological components. From the perspective of Darwin’s theory of evolution, curiosity is not exclusive to humans but also appears in animals and has evolved over a long period. In the control-value theory of human emotions, curiosity is classified as an epistemic emotion—one that drives the pursuit and acquisition of new knowledge. Curiosity arises in situations in which humans feel the gap between desired and current knowledge (Pekrun, 2024). In control-value theory, the desire to gain new knowledge is reflected in a high value of the activity (e.g., high value of solving an equation) and the ability to close the knowledge gap in a high control over actions (e.g., high confidence regarding the ability to learn how equations can be solved). Further, the awareness of the knowledge gap is a crucial condition for the emergence of curiosity. The relation between the value, control, and awareness of knowledge gap appraisals is complex and non-linear.

For example, if the knowledge gap is perceived as too high, this can hinder the rise of curiosity (Pekrun, 2024). Like other epistemic emotions (e.g., surprise), curiosity is expected to play a crucial role in mathematical thinking by fostering deeper engagement and cognitive exploration. That is why curiosity is one of the activating epistemic emotions.

Cognitive-oriented theoretical frameworks such as the information-gap theory by Loewenstein (1994), emphasize the knowledge-related component of curiosity. The information gap refers to the discrepancy between available knowledge and the knowledge one wishes to acquire. The information-gap theory emphasizes the role of expectations regarding the discrepancy between perceived knowledge and the knowledge one expects to perceive (Loewenstein, 1994). If an individual experiences an incongruity due to violated expectations, they identify the knowledge gap, and curiosity arises. In the information-gap theory, curiosity is proposed to be stimulated by both internal and external factors. Furthermore, the desire to feel competent is suggested as one of the factors that might lead to curiosity. Perspectives from control-value theory and information-gap theories come together in a literature review, in which Grossnickle (2016) identifies four dimensions of curiosity: the need for knowledge or information, emotions or arousal, exploratory behavior, and collative variables (e.g., novelty or complexity).

Conceptual overlaps of curiosity and interest

Overlapping definitions are typical for affective constructs. Research repeatedly emphasized the jingle-jangle fallacies in research in social sciences and affect is no exception. The "jingle" fallacy occurs when different labels are used to describe the same affective construct, whereas the "jangle" fallacy arises when the same label is applied to different affective constructs (Schukajlow et al., 2023). The jingle-jangle fallacies, however, must be differentiated from conceptual overlap, which occurs when theories use similar constructs or mechanisms of action as their foundations. Interest is the construct that conceptually overlaps with curiosity and has been extensively discussed in past research. For a long time, interest and curiosity were not distinguished from each other (Donnellan et al., 2022).

Interest is an affective construct characterized by a strong and lasting relationship between a person and the object of interest (e.g., mathematics). It develops from unstable situational interest to stable individual interest across different situations that catch and hold interest, such as working on mathematical problems (Hidi & Renninger, 2006). Persons with a high individual interest in mathematics engage repeatedly with the object of their interest and seek situations in which they can interact with mathematics. Interest consists of two dimensions: an affective and a cognitive dimension (Frenzel et al., 2012; Krapp, 2005). The affective dimension is associated with positive emotions that a person gains from interacting with the object of interest (e.g., enjoyment of solving problems). Interest theorists view curiosity as one of the components of the affective dimension of interest (Frenzel et al., 2012). In this

conceptualization, the cognitive dimension comprises the perceived value of the object of interest (e.g., subjective importance of problem solving), the willingness to acquire new knowledge (e.g., desire to learn how to solve the problem), and the feeling of competence. The comparison of the concepts of curiosity and interest indicates a conceptual overlap on the theoretical level regarding the affective/emotional and cognitive dimensions, as both constructs are associated with positive emotions and acquiring new knowledge. Examining key characteristics of affect can help clarify the unique role of curiosity.

Key characteristics of curiosity: valence, temporal stability, situational specificity, and its objects

Valence, temporal stability, situational specificity, and object were proposed as the key characteristics of emotions (Schukajlow et al., 2023). Valence reflects an individual's perception of a construct's desirability or pleasantness, ranging from positive to negative or neutral. While curiosity is typically linked to positive valence, some researchers argue that it carries a negative valence (Pekrun, 2024). Temporal stability differentiates between more enduring traits and transient states that fluctuate over time. Whereas early discussions of curiosity focused primarily on curiosity as a stable trait, more recent research primarily views it as a state that arises in response to a specific task or activity (Loewenstein, 1994; Pekrun, 2024). Situational specificity describes situational characteristics of affect, which are related to tasks, learning environments, and socio-cultural contexts. For example, due to social-cultural norms, curiosity may vary across mathematics classes, schools, and cultures. At the classroom level, teachers should create opportunities for students to generate and explore their hypotheses and assumptions, allowing them to actively channel their curiosity into meaningful learning experiences. If a teacher strictly follows a pre-defined lesson plan and directly answers students' questions, it could establish social norms that hinder students' curiosity. The object of curiosity may range from mathematics and various mathematical activities at a broad, overarching level, through a specific topic or competence at an intermediate level, to a particular task, mathematical activity, or strategy at the most specific level.

Summary and Future directions

In this contribution, we offered a brief overview of the theoretical underpinning of curiosity. From a theoretical perspective, curiosity is mostly based on two main concepts: emotions and knowledge. The leading theories include these two concepts. Control-value theory underlines the emotional aspect and emphasizes curiosity as a specific emotion, whereas information-gap theory highlights curiosity as an important factor in the acquisition of knowledge. Interest overlaps conceptually with curiosity.

Valence, temporal stability, situational specificity, and object help to characterize curiosity as an affective construct. Based on the characteristics of curiosity we suggest future directions of research:

- Position the research within one of the theoretical conceptualizations such as curiosity as an epistemic emotion (Schubert et al., 2023) or the information-gap theory.
- Differentiate between curiosity as a temporary stable trait vs. a temporary unstable state to capture its dynamic nature.
- Address different objects of curiosity, such as mathematics, specific topics, tasks, or strategies (Schukajlow et al., 2023).

TRIGGERS, PITFALLS, AND TASK DESIGN

Rina Zazkis

On curiosity triggers

There are numerous events that can trigger or inspire mathematical curiosity. For example, it can be a puzzle or a card trick, where we are eager to uncover why it "works." It can be a problem posed by a student or a colleague, or a surprising result of a personal investigation. In this note I focus on curiosity that is triggered by personal mistakes, or personal incorrect assumptions. I describe how such mistake occurred, what mathematical investigation followed, and how it resulted in tasks for student engagement.

In technology we trust

Consider a parallelogram. Connect each vertex to the midpoint of an adjacent side, without loss of generality, clockwise. The intersections of these connecting segments create an "inner" parallelogram, as shown in Figure 1(a). The ratio of the areas of the two parallelograms – Area (outer) : Area (inner) – has a constant value of 5:1, invariant under the shape of the parallelograms. This can be proved, for example, by decomposing areas, as demonstrated in Figure 1(b) and 1(c).

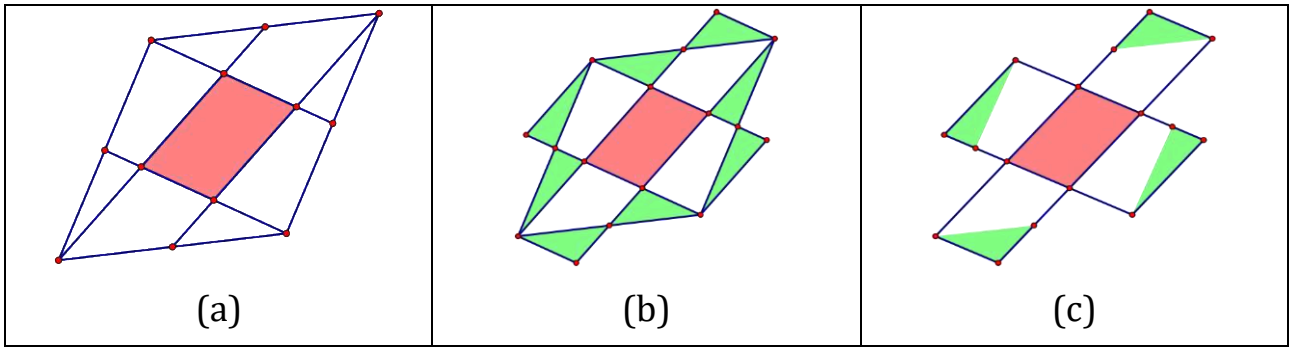

Figure 1: Diagram in support of a proof for parallelogram areas ratio of 5:1.

Consider a quadrilateral with no special constraints and its inner quadrilateral constructed in a similar way. Geometer' Sketchpad (GSP, Dynamic Geometry software of my choice) appears to suggest that the ratio is 5:1. Dragging one of the vertices of the quadrilateral changes the areas of the inner and outer quadrilateral, but the ratio

appears constant (see Figure 2). Availability of a proof for the special case of a parallelogram supports this assumption.

However, the same construction with a different software (Cinderella, rather than the GSP) calculated the ratio in a similar construction as 5.0024, and gentle dragging of any of the vertices of the quadrilateral also altered the ratio in the thousandth digits (e.g., 5.0021 or 5.0053). I initially dismissed this result as "oversensitivity of Cindrella", treating the rounding of the result to two decimal places as a constant value. Then, I was led to realize that even when using the GSP the ratio changes, for example, to 5.09:1 or 5.30:1 (allowing for only 2 decimal places) when one of the quadrilateral sides is very short relatively to the other sides (Figure 3). Curious indeed.

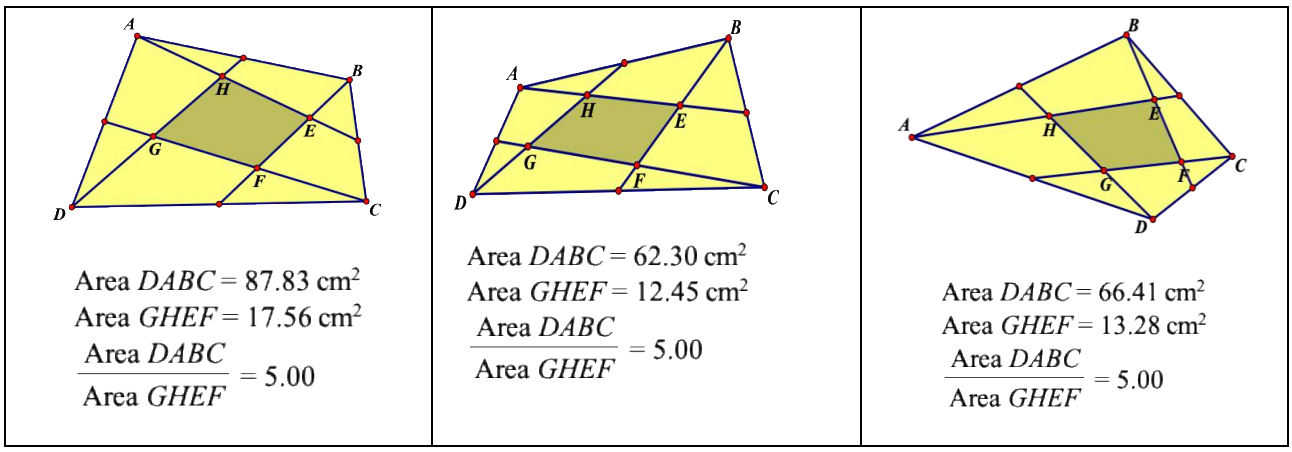

Figure 2: Is the ratio constant?

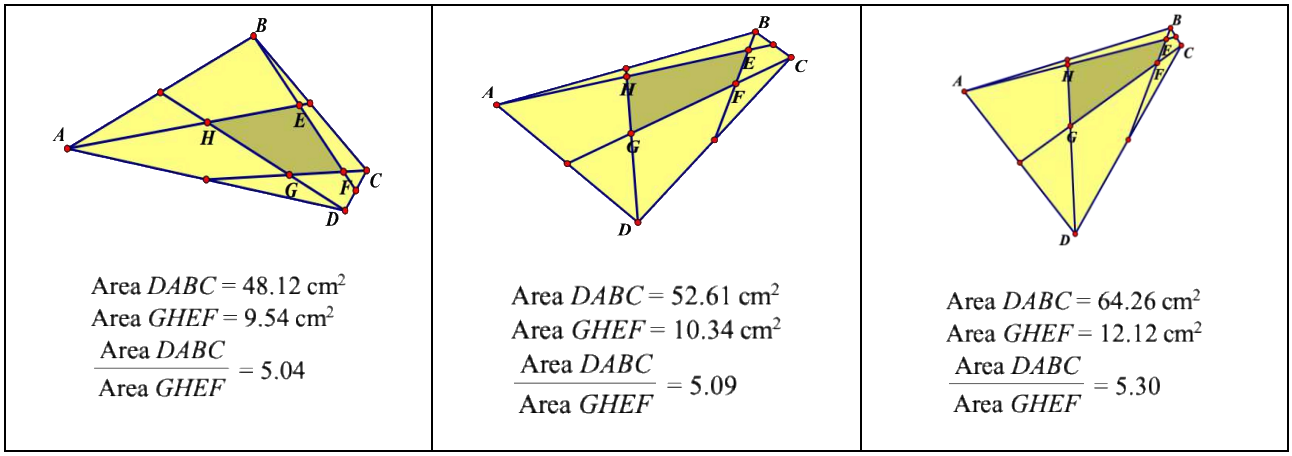

Figure 3: Variation in ratio

Problem solving

Following intense investigation, John Mason and I proved that this ratio fluctuates between 5:1 and 6:1 (Mason & Zazkis, 2018). It is exactly 5:1 when the inner parallelogram is a trapezoid (a parallelogram can be seen as a special case of a trapezoid), and it is 6:1 only when the quadrilateral degenerates to a triangle, that is, point C coincides with B or D, respectively (Figure 4).

The latter case (ratio 6:1) is easy to prove by considering medians in a triangle. The proof of the former is labor intensive, even with the help of CAS. The proof and its generalization are found in Mason and Zazkis (2018). The generalization attends to the case in which the sides of the quadrilateral are partitioned in ratio $: (1 - \mu)$.

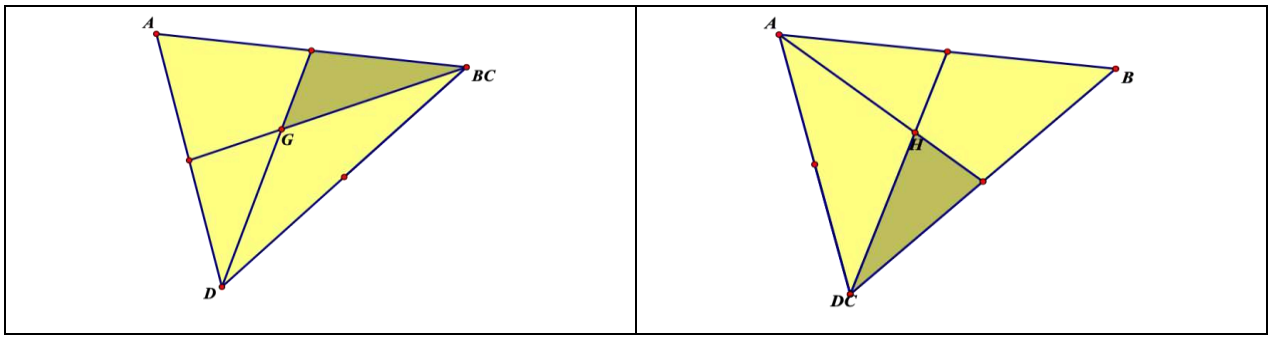

Figure 4: Degenerative case of matching point C with B or D, respectively.

Theoretically speaking

The curiosity to find a proof and to investigate further can be seen as an intellectual need for certainty. The need for certainty, according to Harel (2013), is the need to prove, to remove doubts. According to Harel, one's certainty is achieved when one determines — by whatever means he or she deems appropriate — that an assertion is true. The presumed certainty suggested by Dynamic Geometry can be misleading, as clearly demonstrated above. However, even when certainty is achieved, the use of CAS in a proof, which helps with tedious calculations, may appear unsatisfying. Intellectual curiosity may keep seeking synthetic proof in geometry, that is, without relying on algebraic computation with CAS.

Further, while GSP interfered with the need for certainty, it was instrumental in satisfying the intellectual need for causality, which is, according to Harel (ibid), the need to determine a cause of a phenomenon. Gradual shortening of one of the sides of the parallelogram, as seen in Figure 3, results in graduate increasing of the ratio, where the degenerative case (as in Figure 4) provides the upper bound.

Teacher educator's looking back

When a problem is solved, Polya suggests to "look back" (Polya, 1957/1984). This is the fourth step in the classical 4-step approach to problem solving (understand, plan, carry out the plan, and look back). Looking back attends to checking solutions for errors, considering alternative solutions or solution approaches. Looking back may also involve varying the givens and generalizing, which provide a strong connection to problem posing (Cai & Brook, 2006). This is looking back from a perspective of a problem solver.

However, there is also looking back from a perspective of a mathematics teacher and teacher educator. That is, as my curiosity was triggered, how can I occasion curiosity in my students? One possible step is to present the task: Given a quadrilateral and the construction of the inner quadrilateral described above, what is the ratio of the areas of

these two quadrilaterals? The invitation to examine special cases may follow, as proofs are more accessible when the given quadrilateral is a rectangle. However, this assigned task may or may not trigger curiosity.

A more explicit trigger can demonstrate the problematics of presumed rounding as a prompt for a scripting task. That is, a task can present beginning of a conversation between students, that teachers-scriptwriters are invited to develop further, in a form of a script for an imagined dialogue.

Annie:	I explored the construction (e.g., Figure 3) and I believe the ratio is 5.
Benny:	Somehow, when I move one of the vertices close to another, I do not get exactly 5. This is very strange.
Annie:	I get 5 all the time, look!
Benny:	Prove it!
Annnie:	You always want a proof, but I am totally convinced, this 5 does not change and it is proof-enough for me.
Teacher:	…

Presenting a disagreement between students is a type of a prompt for a scriptwriting activity discussed in prior research (e.g., Marmur et al., 2022; Zazkis & Zazkis, 2014). This is a mathematical task, presented as an assignment with a pedagogical flavor, that would direct teachers to examine by themselves the situation in order to join a conversation with students. The scriptwriter can have teacher-character join the conversation at any point, or imagine further exchange between the student-characters before letting the teacher-character intervene. The conversation can focus on examining special cases, or on examining the affordances and pitfalls when relying on technology.

MATHEMATICAL CURIOSITY EXHIBITED IN PROBLEM POSING

Jinfa Cai and Victoria Robison

It is well-documented from both national and international surveys that many students do not view mathematics positively. These negative affective responses to school mathematics frustrate efforts to improve students' learning, and research is needed to investigate potential avenues for addressing such challenges. One approach is to study mathematical curiosity, which could help students overcome the hurdles of negative dispositions and disengagement (Peterson & Cohen, 2019). In this paper, we explore this idea from the perspective of problem posing.

Problem posing is a fundamental creative mathematical process that has long been promoted in curriculum frameworks and supported by empirical research (Silver, 1997). More specifically, problem posing refers to several related types of activity that entail or support teachers and students formulating (or reformulating) and expressing

a problem or task based on a particular context (the *problem situation*; Cai & Hwang, 2020; Silver, 1994). Research has clearly shown that both students and teachers are capable of posing mathematical problems based on given situations (Cai et al., 2015). In what follows, we will explore the potential of problem posing with respect to evidence of mathematical curiosity exhibited in students' problem posing.

Attributes of Mathematical Curiosity

Peterson and Cohen (2019) conceptualized curiosity as a multidimensional construct with three attributes: (1) knowledge gaps, (2) heightened arousal, and (3) question-asking. Of these, the third attribute is the most directly and obviously related to problem posing. Curiosity often manifests through "question-asking or exploratory behaviors," and "students who exhibit curiosity may ask questions, form and test hypotheses" (Peterson & Cohen, 2019, p. 809). That is, mathematically curious students are likely to exhibit that curiosity by posing their own problems—problems that reflect the mathematical phenomena about which they are curious. Problem posing is also connected to Peterson and Cohen's other two attributes, knowledge gaps and heightened arousal. Moreover, research on problem posing has described various attributes of the problems that students (and teachers) pose, including both semantic and syntactic features and whether problems are solvable (e.g., Silver & Cai, 1996). Such research has generally found that students can pose many kinds of problems, including complex and unusual problems as well as unexpected responses.

What Is Problem Posing?

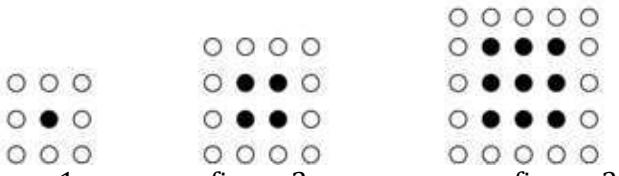

figure 1 figure 2 figure 3

Problem-solving task	Problem-posing task
Look at the figure above. A. Draw the 4th figure B. How many black dots are there in the 6th figure? Explain how you found your answer. C. How many while dots are there in the 6th figure? Explain how you found your answer. D. Figure 1 has 8 white dots. If a figure has 44 while dots, which figure is it? Explain how you found your answer.	Teacher Ma drew the figures above in a pattern, as shown above. For his student's homework, he wanted to make up three problems based on the above situation: an easy problem, a moderate problem, and a difficult problem. These problems can be solved using the information in the situation. Help Teacher Ma make up three problems and write these problems in the space below.
	The easy problem:
	The moderately difficult problem:
	The difficult problem:

Figure 5: Sample problem-solving and problem-posing tasks.

Figure 5 shows a pair of tasks (Cai & Hwang, 2002). The left-hand side is a typical problem-solving task, and the right-hand side is a typical problem-posing task. The problem-solving task requires students to answer several given questions. In contrast, the problem-posing task asks students to pose three mathematical problems: one easy problem, one moderately difficult problem, and one difficult problem.

A problem-posing task has two parts: the situation and the prompt (Cai, 2022). The design of both the situation and prompt is one of the important focuses on problem-posing research. In discussing the mathematical curiosity exhibited in problem posing, our focus will be on problem-posing tasks' situations and prompts.

Mathematical Curiosity Exhibited in Problem Posing

Mathematical curiosity can be exhibited in problem posing because of its low-floor effect, which helps students use whatever knowledge and experience they possess to pose problems. This comes from their desire and capability to know. This contrasts with some problem-solving tasks, where students either get it or they don't. One of the major findings from problem-posing research is that both students and teachers are capable of posing interesting and innovative mathematical problems.

Students' responses to mathematical problem-posing tasks are typically analyzed using both qualitative analytic and quantitative holistic methods. In what follows, we discuss how problem-posing tasks are analyzed qualitatively, including both expected and unexpected posed problems, and quantitatively. We also discuss the challenges of assessing mathematical curiosity exhibited in mathematical problem posing.

Mathematical Curiosity Exhibited in Qualitative Analysis: Using qualitative analytic methods, students' responses can be classified into mathematical problems, nonmathematical problems, and statements. Their curiosity can be exhibited the posed problems in various ways. Below are some examples of different problem-posing tasks and students' responses.

> Task 1: The Driving Task
>
> Jerome, Elliot, and Arturo took turns driving home from a trip. Arturo drove 80 miles more than Elliot. Elliot drove twice as many miles as Jerome. Jerome drove 50 miles.

In this situation, Jerome drove 50 miles. What about Elliot or Arturo? How many miles did Elliot or Arturo each drive? What about the total they drove altogether? Cai et al. (2023) found that some students added conditions and then posed a mathematical problem, such as "If they each had to drive there and back how many miles would they drive and how many total miles would there be?"

> Task 2: The Doorbell Task
>
> Sally is having a party. The first time the doorbell rings, 1 guest enters. The second time the doorbell rings, 3 guests enter. The third time the doorbell rings, 5 guests enter. The fourth time the doorbell rings, 7 guests enter. This pattern continues in the same way. On

the next ring a group enters that has 2 more guests than the group that entered on the previous ring.

On this task, implemented in Cai and Hwang (2002), students were able to go beyond the situations and were curious to know the amount of pizza needed to feed those in the party, such as "If Sally bought boxes of pizza with 6 slices each, how many boxes would she need to feed everyone in the house after the 6^{th} ring? (including Sally)."

These sample student-posed problems show aspects of what students want to know based on their respective situations; such desire demonstrates their curiosity to fill their knowledge gaps and motivation to establish new relationships. Thus, the above sample of problem-posing tasks and student-posed problems provide a window into what students want to know when they engage in problem-posing tasks.

Mathematical Curiosity Exhibited in Unexpected Responses: Researchers typically qualitatively analyze students' "desirable responses," such as students' posed solvable mathematical problems. These problems can demonstrate students' curiosity as in the above examples. However, some percentages of students' responses are unexpected or even undesirable. For example, in Silver and Cai (1996), almost one quarter of the students' responses were statements or nonmathematical problems. It is not entirely clear why students provide unexpected responses, such as statements or nonmathematical problems, even though the directions clearly ask students to pose mathematical problems. However, a recent study found that such unexpected responses may also demonstrate students' mathematical curiosity and innovative ideas (Ran et al., in press).

Mathematical Curiosity Exhibited in Quantitative Analysis: Students' responses to problem-posing tasks can also be analyzed quantitatively. In such quantitative analysis, a numerical score is assigned to a problem-posing response according to the nature of the response; for example, nonroutine posed problems are often scored higher than routine ones. One such quantitative analysis rubric is from Liu et al. (2020), who used the abovementioned Doorbell task to ask students to pose an easy problem, a medium difficult problem, and a difficult problem.

Challenges to Assessing Mathematical Curiosity in Problem Posing: The above examples show how mathematical curiosity can be exhibited not only in different posed mathematical problems and unexpected responses but also how they have been considered in quantitative scoring. However, it remains a challenge to systematically assess mathematical curiosity through problem posing. Future studies are needed to develop mathematical problem-posing tasks as assessment tools in general and problem-posing assessment tools for assessing mathematical curiosity specifically. We suggest three directions to overcome these challenges.

Conclusion and Looking to the Future

Investigating the mathematical curiosity exhibited in and through problem posing is theoretically justified by the conceptualization of mathematical curiosity by Peterson

and Cohen (2019) and is consistent with current views about the roles problem posing can play in students' learning (Cai, 2022; Cai et al., 2015). However, as we have indicated, actually assessing students' mathematical curiosity in and through problem posing involves a number of challenges. We call on other researchers to join the effort to conduct research in this area. We hope this chapter serves as a springboard for researchers to use problem posing as a lever to assess and nurture students' mathematical curiosity.

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STUDENTS' NARRATIVES ON MATHEMATICAL CURIOSITY AND ITS RELATION TO CPS

Patricio Felmer & Boris Koichu

Introduction

This note is written in connection with an initiative started in a meeting in Haifa, Israel, with the aim to move forward studies in mathematics curiosity (Leikin, 2023). In this context, a recent exploratory study of undergraduate students' narratives on mathematical curiosity and its relation to problem solving was carried out by the authors (Felmer & Koichu, in progress). In this chapter, we introduced an interview methodology, in which explicit questions on curiosity as a desirable human trait were asked in proximity to students' specific collaborative problem solving (CPS) experiences so that students can recall their state emotions. With this methodology, we addressed two research questions: What do participants in an undergraduate mathematics course featuring CPS mean by curiosity and mathematical curiosity? and How do the students associate curiosity and mathematical problem solving when reflecting on their CPS experiences in the course?

The data for this research came from undergraduate students at one of the leading engineering schools in Chile. The participants were in their second semester in the university, taking a Calculus course. Each week, the students participated in two 90-minute CPS sessions, where they solved problems in of four. The interviews took place one or two days after the CPS session and with the same groups that worked together during the CPS session. The interview protocol consisted of six questions, but in this note we only consider the following two:

1. Do you think that mathematical curiosity helps to solve problems?
2. Do you think CPS can help develop mathematical curiosity?

In CPS sessions, students are organized into random groups and work collaboratively on mathematics problems given by the facilitators. Once a group solves a given

problem, their members are required to explain to the facilitator how the problem was solved and why the solution is correct, only then the group is given the next problem. Thus, at the end of the CPS session, most groups will be in the process of solving one problem (their last problem). After the CPS session, students do not have the chance of having the *correct solutions* by facilitators, either in written form or in lectures. The interaction strategy of facilitators with students is through guiding questions. In summary, the main characteristics of CPS sessions are: Random groups, Collaboration, Problem (Receiving the problem, working on the solution and reaching a solution), Problems-delivering strategy, No official solutions and Interaction strategy.

In this note we are aiming to explore the relation that students make between the nature of the CPS session and curiosity. Precisely, our research question: What characteristics of CPS are associated by students with mathematical curiosity?

The data analysis

The audio-recorded interviews for 14 students were transcribed and then translated to English. Then the translated transcripts of the two questions were organized by students. The analysis started by identifying, for each of the 14 students, all single sentences having a complete meaning, which we called them *segments.* The next step was to select 43 segments that contain a relation of curiosity or mathematics curiosity with the characteristics of the CPS session. In Table 1, we display the number of segments associated with each characteristic of CPS, where we omit Random groups and Interaction strategy rows because no's segment associated them with curiosity and Problem (reaching a solution) which had only two segments.

Table 1. in parentheses the number of segments presented later as examples.

	Question 1	Question 2
Collaboration	4 (1)	9 (2)
Problem (receiving the problem)	2 (0)	5 (2)
Problem (working on the solution)	12 (2)	2 (0)
Problem (reaching a solution)	2 (0)	0
Problems delivery strategy	1 (1)	3 (1)
No official solutions	0	3 (2)

Students' narratives

In this section, we discuss the contents of the segments with the experiences narrated by the students.

Collaboration. Many segments show that students' mathematical curiosity with collaboration, as shown in Table 1. For example, Amelia says that the mathematical curiosity of the group members allows advancing on the solution to the problem:

…because if we all experience this mathematical curiosity, I can come up with ideas because I'm focused on seeing this problem [...] and suddenly, I reach a point where I can't go on, but Harold can also feel that same curiosity […] and collaboratively sort of form this synergy… (1, Amelia).

In a different direction, some students mention that they experience curiosity experiences on the way their peers think. For example:

…there are people who do it very fast, they see it "ah but look, if you do the graph", and they don't even graph it, it's already in their mind, so it causes me a lot of intrigue to notice the... the different types of mathematical understanding that exist in people (2, Matthew).

The number of students (13) making these connections with collaboration is a sign of the role that collaboration may have in students' curiosity when they solve problems.

Problem (receiving the problem). One source of curiosity that students identify when question 2 was asked is the problem itself. One student says, "… sometimes the curiosity of... the natural curiosity of - what is this problem like?" (2, Noel). Others realize that problems in CPS sessions are not *test problems* "…there are the test type exercises […] but the... the CPS exercises are different, and that opens up more... like types of problems that you can solve, and that can generate curiosity in some people" (2, Christopher).

Problem (working on the solution). Students clearly think that curiosity helps them to solve problems. In the process of starting to work on the problem:

… where the educational curiosity lies... when we started to do it eh... we started right away with drawing […] Then it's like...I just solve it, but I think it's like imagining how to solve it or... as well as looking for the relationships, that's where it is. (1, Preston).

In finding different approaches to the problem "It kind of leads you to look for... different points of view about the same problem" (1, Damian). When perseverance is essential for continuing:

> I remember that mathematical curiosity helped me to... to follow the problem [] I said "no, I want to know... I want to know... I want to know... I want to know how this thing happens" and I said "you know what? I'm going to continue, I'm going to do it again, I'm going to look at it from another angle, I'm going to draw... (1, Noel)

Problems delivery strategy. When a group solves a problem, it receives a new problem and curiosity about the problem is renewed: "Yes, another moment that makes you curious would be when you get a... a problem that is kind of easy and you want the next one […] you are kind of curious to see how... how it will be…" (2, Garret). A very interesting situation is reported by a student that did not complete the solution of the last problem "I leave, but I keep thinking about the exercise […] and the exercise comes to my mind and... and the desire, the curiosity to do it unconsciously tells me - well, what if I were to divide this one […]?" (1, Isabella).

No official solutions. The following students' comments may be related to fact that no official solution will be delivered to students after CPS session. One student says:

> Yes, because I think that every time you solve a problem, new questions arise, like I don't know, when they tell you […] "this function is continuous, invertible, derivable"

> but what would happen if it were not continuous, if it were not...? then you have new doubts that you want to... reflect on and then... it is like a domino effect… (2, Peter)

and another student feels that even when the problem was solved in the CPS session, doubts remain "… but one is also left with many doubts, one is left with many things to... to answer, that are generated in this... in this process of solving problems" (2, Perry)

Discussion

The analysis of students' narratives allows approaching their views on curiosity and mathematical curiosity and its relationship with the main features of the CPS session, where they participate regularly along the semester. Next, we analyze these findings.

The fact that Random groups are not connected with curiosity may mean that simply they do not see relation or because of the questions asked, students deal with the time after these random groups were organized. More interesting is that students do not connect curiosity with the interaction strategy. This may be associated with the fact that this interaction occurs when students need guidance, so when they get it, they have a way to continue searching for the solution.

Among all features of CPS sessions, the higher number of segments are connected with Collaboration (13) and Problem (working on the solution) (14). Collaboration is connected with question 2, which may mean that students believe that collaboration helps to development curiosity. On the other hand, that Problem (working on the solution) is mostly related to question 1, may mean that students feel that curiosity is important to solve problems, especially when they are fighting with the problem.

Even though, problem delivery strategy is not associated with many segments, the way students express the connection is obvious. The fact that a new problem comes after the one solved provokes curiosity. One may ask what would happen if a list of three problems is delivered at the beginning of the CPS session. On the other hand, the fact that the last problem is usually left unsolved is also identified as a source of curiosity.

After a CPS session, no official solutions will be delivered to students. This feature left students with certain doubts that possibly will later become in questions and even in new problems. This feature may also move students to ask new questions once the problem is solved. In this way, we see an interesting connection with Problem Posing, which is set up once a problem in solved (Cai & Leikin, 2020; Knuth & Peterson, 2003;)

Finally, we would like to observe that students' experiences with mathematical problem solving in relation to curiosity that has been studied in this note are in a collaborative setting. It would be interesting to study if problem solving sessions are implemented with all features of a CPS session, except for collaboration. Would narratives go in a similar way?

EXPLORING CURIOSITY FROM STUDENTS' METACOGNITIVE BEHAVIORS TAKEN DURING PROBLEM SOLVING

Hui-Yu Hsu

Background and Aim of the study

Curiosity refers to a positive emotional-motivational system associated with the recognition, pursuit, and self-regulation of novelty and challenge (Kashdan et al., 2004). Literature has recognized curiosity as a significant motivational factor influencing learning processes and educational outcomes. Piaget (1969) posited innate curiosity as a fundamental drive in cognitive development that can lead learners to actively construct knowledge by interacting with their environment. Learners thrive when encountering manageable levels of novelty and complexity that embed uncertainty or information gaps. Renninger et al. (2015) identified curiosity as a central component in developing student interest, emphasizing that cultivating curiosity enhances engagement, persistence, and deeper mathematics learning. Pekrun (2019) similarly found a positive correlation between curiosity and mathematical achievement, noting its mediating role between intrinsic motivation and academic outcomes. Peterson and Cohen (2019) specifically elaborated the concept of epistemic curiosity within mathematics education, defining it as intrinsic motivation driven by the desire to acquire knowledge and resolve uncertainty. They proposed three distinctive attributes of mathematical curiosity: recognition of an information gap with a belief in its resolvability; heightened arousal characterized by urgency and emotional reactions such as excitement or agitation; and behaviors triggered by curiosity (e.g., question-asking, exploration).

While previous research primarily investigated how curiosity influences students' cognitive behaviors and learning outcomes, relatively few studies have examined the specific metacognitive actions students undertake during mathematical problem solving. Understanding these metacognitive behaviors help identify particular actions and cognitive patterns that potentially foster curiosity. This understanding is particularly crucial since curiosity involves actively exploring challenging yet manageable problems while maintaining a sense of control. Therefore, deeper knowledge of metacognitive behaviors can illuminate how curiosity operates within individual students' mathematical problem-solving experiences and how it interacts effectively with the learning environment. Consequently, this study aimed to investigate the metacognitive actions of 4th grade and 8th grade students while solving high cognitive-demand problems.

Methodology

A total of 137 Taiwanese 4th grade students from six classes across three elementary schools and 131 Taiwanese 8th grade students from six classes across two middle schools participated in this study. Both samples included students with varying levels of mathematics achievement. Participants were asked to solve assigned high cognitive-demand mathematical problems. After completing each problem, students had to

respond to two items to illustrate their metacognitive behaviors during the problem-solving process. Specifically, the study aimed to identify students' problem-solving approaches and their self-perceived strategies when encountering difficulties. Table 2 presents the two items designed to capture students' metacognitive behaviors.

Table 2: Metacognitive items

Item 1: When solving this problem, ( ) I understood immediately and completed it quickly. ( ) I did not know how to solve it at first, but after thinking, I found the solution. ( ) I attempted using my ideas but encountered difficulties and did not finish. ( ) I did not know how to solve it initially, and still did not find a solution after thinking. ( ) I immediately recognized that I could not solve it and did not give it a try.
Item 2: When thinking about solving this problem, I considered (multiple choices allowed): ( ) Some strategies based on the problem conditions provided and then saw what could be done next. ( ) If I have encountered similar problems previously that could help solve the problem. ( ) If I have learned concepts or properties that could be applied to this problem.

Findings

The analysis results of students' performance on two high cognitive-demand geometry problems showed similar levels of difficulty for both grade levels. The average scores were 0.61 for 4th grade and 0.67 for 8th grade. Scoring criteria were as follows: 2 points for fully correct answers; 1 point for partially correct answers, and 0 points for incorrect or incomplete answers. The comparable performance between grades established an appropriate baseline for analyzing differences in metacognitive behaviors during problem-solving.

Table 3 highlighted differences between grades in attempting challenging mathematics problems. Initially, 8th graders (28.2%) were slightly more likely to directly complete problems compared to 4th graders (22.6%), suggesting greater immediate familiarity or confidence. However, a higher proportion of 4th graders (60.6%) completed problems through sustained cognitive effort compared to eighth graders (40.5%), indicating younger students' greater persistence. Notably, more 8th graders (10.7%) struggled and failed compared to 4th graders (1.5%), reflecting the older students' engagement with more complex strategies and greater challenge perception. Furthermore, the inability to solve problems even after considerable thought was higher among 8th graders (15.3%) than 4th graders (8.0%), possibly indicating older students' greater awareness of problem complexity. The percentage immediately giving up was slightly higher among

fourth graders (7.3%) than eighth graders (5.3%), suggesting younger students may more quickly perceive tasks as overly challenging.

Table 3: Analysis result for item 1

When solving the problem, I...	4th grade students	8th grade students
Direct completion (I understood immediately and completed it quickly)	22.6%	28.2%
Effortful completion (I did not know how to solve it at first, but after thinking, I found the solution)	60.6%	40.5%
Struggled and failed (I attempted using my ideas but encountered difficulties and did not finish)	1.5%	10.7%
Unable to solve (I did not know how to solve it initially, and still did not find a solution after thinking)	8.0%	15.3%
Giving up (I immediately recognized that I could not solve it and did not give it a try)	7.3%	5.3%

Table 4: Analysis of the item 2

During problem-solving process, I considered to		4th grade students	8th grade students
Effortful completion	Randomly trying	30.1%	47.2%
	Recalling similar problem-solving experiences	60.2%	43.4%
	Applying prior learned mathematics knowledge	63.9%	54.7%
Struggled and failed	Randomly trying	50.0%	50.0%
	Recalling similar problem-solving experiences	0%	71.4%
	Applying prior learned mathematics knowledge	50%	35.7%
Unable to solve	Randomly trying	0%	40.0%
	Recalling similar problem-solving experiences	54.5%	50.0%
	Applying prior learned mathematics knowledge	90.9%	45.0%

Table 4 showed the strategies employed by 4th grade and 8th grade students. For students who successfully completed the problems through effortful thinking (Effortful completion), 8th graders (47.2%) were more likely to use random trial-and-error approaches compared to 4th graders (30.1%). Conversely, 4th graders were more inclined to recall similar problem-solving experiences (60.2%) and apply previously learned mathematical knowledge (63.9%), compared to 8th graders (43.4% and 54.7%). For students who initially tried but eventually failed (Struggled and failed), both grades equally relied on random attempts (50.0%). However, while none of the 4th graders explicitly recalled similar experiences (0%), a substantial majority of 8th graders (71.4%) did so, suggesting older students were more likely to rely on prior experience even when unsuccessful. Interestingly, 4th graders more frequently applied learned

mathematical knowledge (50%) than 8th graders (35.7%) in this scenario. In cases where students were unable to solve the problem at all (Unable to solve), 8th graders commonly resorted to random trial approaches (40.0%), whereas none of the 4th graders did so (0%). Furthermore, both grades similarly relied on recalling similar experiences (54.5% for 4th graders; 50.0% for 8th graders). However, a significant difference appeared in the application of learned mathematical knowledge, where almost all 4th graders (90.9%) attempted this strategy, compared to fewer 8th graders (45.0%).

Discussions

The study indicated that 4th grade typically demonstrated higher persistence and effortful problem-solving approaches, though slightly fewer felt immediately confident. 8th grade showed a higher likelihood of immediate recognition or direct completion, but they also encountered more significant challenges, reflected by higher failure and "unable to solve" percentages. In addition, 4th grade students demonstrate greater reliance on formal knowledge and structured approaches, whereas 8th grade students show a higher tendency towards random exploration and recalling prior problem-solving experiences. These results highlight developmental differences in metacognitive approaches and provide insights into the intertwined relationship between curiosity and mathematical problem solving.

CONCLUDING NOTES

As mentioned earlier in the introduction part of this document, we see this Research Forum as a promoter for evoking curiosity within the PME community about the construct of mathematical curiosity and for advancing research in this area. We believe that the collection of short study descriptions included in this document exemplifies the viability of curiosity-focused research in mathematics education.

The Research Forum participants employed a variety of research methodologies: historical analysis (by Karp), review study (by Schukajlow et al.), reflective account of personal experience (by Zazkis), task analysis (by Cai and by Zazkis), empirical qualitative study (by Felmer & Koichu), and empirical quantitative study (by Hsu). The studies utilized different research tools, including mathematical tasks such as problem-solving (Zazkis and Hsu), problem-posing (Cai & Robinson), and mathematical investigations (Zazkis). Karp's study focused on textbook analysis, Felmer and Koichu conducted individual interviews on students' conceptions of mathematical curiosity, while Hsu employed Likert scale questionnaires.

Several works included analysis of curiosity-linked *task design*. Karp's examination of mathematics educational policy and mathematical textbooks (across several centuries) led to characterization of educational *curiosity-directed instructional approaches* (i.e., aimed at the development of mathematical curiosity) and *curiosity-evoking tasks* (i.e. tasks that serve a trigger for curios exploration). In contrast to Karp, Zazkis's work

focuses on *curiosity-driven tasks design* (i.e., process that embeds curiosity) evoked by the feeling of uncertainty during mathematical investigations and problem solving. Cai and Robinson connect problem-posing with question asking, which is typical for a curious state, and argue that curiosity is exhibited in problem-posing activities. They analyze several problem-posing tasks from the perspective of their power to evoke curiosity and examine the quality of posed problems as a function of curiosity.

The two empirical studies included in this paper involved participants of different ages, and we believe that similar investigations can be performed with teachers and students across various age groups and ability levels.

The research on mathematical curiosity has many open questions that we invite PME participants to discuss. *Some of the issues and questions we will address are*:

Historical perspectives on mathematical curiosity

- What role does curiosity play in mathematical research and how has it evolved throughout history?

Theoretical perspectives on curiosity in mathematics education

- What philosophical and psychological theories have been adopted when developing theoretical bases of curiosity in mathematics education?
- What cognitive, affective, and social factors affect curious mathematical thought?
- What psychological and neuropsychological mechanisms drive curiosity-based mathematical processing?

The nature and nurture of curiosity-driven mathematics instruction

- What is the nature of the mathematical tasks and instructional strategies that stimulate students' curiosity?
- What are the challenges and barriers when implementing curiosity-driven instruction in mathematics classrooms?
- Are there specific types of mathematical problems/tasks/activities (e.g., modelling tasks) that more effectively stimulate curiosity-driven explorations?

The role of technology in encouraging curiosity-driven instructional processes

- What kinds of technological tools (e.g., AI, educational platforms, games) be used to promote curiosity-driven mathematical explorations? How can they be used effectively?

Tools for measuring curiosity

- What are reliable measures and methods for measuring students' curiosity levels during mathematical instruction?

Impact of curiosity on students' learning and performance

- How does encouraging curiosity influence students' mathematical performance, creativity, and persistence in problem-solving?
- How does curiosity impact student engagement and motivation in mathematics?
- What long-term effects does curiosity-driven instruction have on students' mathematical performance and attitudes toward mathematics?
- How does curiosity-driven instruction affect students with different levels of mathematical achievement?

Differentiation and inclusion in curiosity-driven instruction

- How does curiosity-driven instruction affect students with different needs and abilities?
- What modifications can make curiosity-driven instruction accessible to students with different learning needs?

Curiosity across different age groups

- How does curiosity function differently in early childhood mathematics education versus higher-level mathematical instruction?
- Can early interventions foster curiosity to have long-term impacts on students' attitudes toward mathematics?

Teachers' role in fostering curiosity

- What role does teachers' curiosity play in the effectiveness of curiosity-driven instruction?
- How do teachers' beliefs about curiosity and learning affect their instructional practices in mathematics?

Teacher education and professional development

- What professional development programs can help educators understand and effectively implement curiosity-driven teaching methods in the mathematics classroom?

Note: Expanded versions of the RF presentations will be included in the edited book by Leikin, Cai, and Karp (in press). *Mathematical Curiosity: Conceptualizations and Multiple Perspectives*.

REFERENCES

Aristotle (1983). *Sochineniya v chetyrekh tomakh* [Works in Four Volumes]. V. 4. Mysl'.

Bozhovich, L.I. (2012). O motivatsii ucheniya [On the motivation to study]. *Vestnik prakticheskoy psikhologii obrazovaniya. 9*(4), 65–67.

Cai, J. (2022). What research says about teaching mathematics through problem posing. *Éducation & Didactique, 16*, 31–50.

Cai, J., & Brook, M. (2006). Looking back in problem solving. *Mathematics Teaching Incorporating Micromath*, *196*, 42–45.

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in U.S. and Chinese students' mathematical problem solving and problem posing. *Journal of Mathematical Behavior, 21*, 401–421.

Cai, J., & Hwang, S. (2020). Learning to teach through mathematical problem posing: Theoretical considerations, methodology, and directions for future research. *International Journal of Educational Research, 102*, 101420.

Cai, J., Hwang, S., Jiang, C., & Silber, S. (2015). Problem-posing research in mathematics education: Some answered and unanswered questions. In *Mathematical problem posing* (pp. 3–34). Springer.

Cai, J., & Leikin, R. (2020). Affect in mathematical problem posing: Conceptualization, advances, and future directions for research. *Educational Studies in Mathematics, 105*, 287-301.

Cai, J., Ran, R., Hwang, S., Ma, Y., Han, J., & Muirhead, F. (2023). Impact of prompts on students' mathematical problem posing. *Journal of Mathematical Behavior, 72*, 1–15. https://doi.org/10.1016/j.jmathb.2023.101087

Colburn, W. (1823). *First lessons in arithmetic on the plan of Pestalozzi*. Cummings, Hillard and Co.

Diesterveg, A. (1956). Rukovodstvo k obrazovaniiu nemetskikh uchiteley. In *Izbrannye pedagogicheskie sochineniya* [Guide for the Instruction of German. In Selected Works in Pedagogy] (pp. 136–203). Uchpedgiz.

Donnellan, E., Aslan, S., Fastrich, G. M., & Murayama, K. (2022). How are curiosity and interest different? Naïve Bayes classification of people's beliefs. *Educational Psychology Review, 34*(1), 73–105.

Engel, S. (2011). Children's need to know: Curiosity in schools. *Harvard Educational Review, 81*(4), 625–645.

Frenzel, A. C., Pekrun, R., Dicke, A.-L., & Goetz, T. (2012). Beyond quantitative decline: Conceptual shifts in adolescents' development of interest in mathematics. *Developmental Psychology, 48*(4), 1069–1082.

Grossnickle, E. M. (2016). Disentangling curiosity: Dimensionality, definitions, and distinctions from interest in educational contexts. *Educational Psychology Review, 28*(1), 23–60.

Gruber, M. J., Valji, A., & Ranganath, C. (2019). Curiosity and learning: A neuroscientific perspective. In A. S. Benjamin, & B. H. Ross (Eds.), *Psychology of learning and motivation* (Vol. 71, pp. 397–417). Academic Press.

Gurevich, A. (2005). *Individ i sotsium na srednevekovom Zapade* [Individual and Society in the Western Middle Ages]. ROSSPEN.

Harel, G. (2013). Intellectual need. In K. Leatham (Ed.), *Vital directions for mathematics education research* (pp.119–151). Springer.

Hart, W.W. (1934). *Progressive first algebra*. D.C. Heath and Company.

Hidi, S., & Renninger, K. A. (2006). The four phase model of interest development. *Educational psychologist, 41*(2), 111–127.

Krapp, A. (2005). Basic needs and the development of interest and intrinsic motivational orientations. *Learning and Instruction, 15,* 381–395.

Karp, A. (2023). Problem sets in school textbooks: Examples from the United States. In R. Leikin (Ed.), *Mathematical Challenges for All* (pp. 459–479). Springer.

Karp, A. (Ed.). (2007). *Pamiati A.R.Maizelisa*. (A.R.Maizelis: in Memoriam).SMIO-PRESS.

Kashdan, T. B., Rose, P., & Fincham, F. D. (2004). Curiosity and Exploration: Facilitating Positive Subjective Experiences and Personal Growth Opportunities. *Journal of Personality Assessment, 82*(3), 291–305.

Knuth, E. J. (2002). Fostering mathematical curiosity. *The Mathematics Teacher, 95*(2), 126–130.

Knuth, E. & Peterson, B. (2003). Fostering mathematical curiosity: Highlighting the mathematics. *Mathematics Teacher, 96*(8), 574–579.

Leikin, R. (Ed.) (2023). *Mathematical curiosity as means for the developing of mathematical proficiency in a wide range of students*. Collection of position documents presented at the International Workshop of Israel Science Foundation. University of Haifa, Israel. https://curiosity-isf.edu.haifa.ac.il.

Leikin R., Cai J., & Karp, A. (in press). *Mathematical curiosity: Conceptualizations and Multiple Perspectives*. Springer

Liu, Q., Liu, J., Cai, J., & Zhang, Z. (2020). The relationship between domain and task specific self-efficacy and mathematical problem-posing: A large-scale study of eighth-grade students in China. *Educational Studies in Mathematics, 105*, 407–431.

Loewenstein, G. (1994). The psychology of curiosity. *Psychological Bulletin*, 116(1),75–98.

Magnitsky, L. F. (1703). *Arifmetika, sirech' nauka chislitel'naya…*[Arithmetic, or learning of numbering…]. Moscow.

Marmur, O., Moutinho, I., & Zazkis, R. (2022). On the density of $\mathbb{Q}$ in $\mathbb{R}$: Imaginary dialogues scripted by undergraduate students. *International Journal of Mathematical Education in Science and Technology*, 53(6), 1297–1325.

Mason, J., & Zazkis, R. (2019). Fooled by rounding. *Digital Experiences in Mathematics Education, 5*(3), 252–265.

Pekrun, R. (2006). The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review, 18,* 315–341. https://doi.org/10.1007/s10648-006-9029-9.

Pekrun, R. (2019). The Murky Distinction Between Curiosity and Interest: State of the Art and Future Prospects. *Educational Psychology Review*, *31*(4), 905-914. https://doi.org/10.1007/s10648-019-09512-1

Perelman, Ya. (1955). *Zanimatel'naya algebra* [Interesting algebra]. Gosudarstvennoe izdatel'stvo tekhniko-teoreticheskoy literatury.

Peterson, E. G., & Cohen, J. (2019). A case for domain-specific curiosity in mathematics. *Educational Psychology Review, 31*, 807–832.

Piaget, J. (1952). *The origins of intelligence in children.* (M. Cook, Trans.). W. W. Norton & Co.

Piaget, J. (1969). *Psychology of intelligence*. Littlefield, Adams.

Polya, G. (1957/1984). *How to solve it: A new aspect of mathematical method*. Princeton University Press, Princeton.

Posamentier, A. S. & Lehmann, I. (2014). *Mathematical Curiosities: A Treasure Trove of Unexpected Entertainments.* Prometheus

Ran, H., Cai, J., Muirhead, F., & Hwang, S. (in press). An analysis of unexpected responses in middle school students' mathematical problem posing from the perspective of problem-posing processes. *Educational Studies in Mathematics*.

Renninger, A., Hidi, S., & Nieswandt, M. (2015). *Interest in Mathematics and Science Learning*. Casemate Group.

Robinson, H. N. (1867). *Elements of geometry and plane and spherical trigonometry with numerous practical problems.*

Schubert, S., Pekrun, R., & Ufer, S. (2023). The role of epistemic emotions in undergraduate students' proof construction. *ZDM–Mathematics Education, 55,* 299–314.

Schukajlow, S., Rakoczy K., & Pekrun R. (2023). Emotions and motivation in mathematics education: Where we are today and where we need to go. *ZDM – Mathematics Education, 55*(2), 249–267.

Schoenherr, J., Schukajlow, S., & Pekrun, R. (2025). Emotions in mathematics learning: a systematic review and meta-analysis. *ZDM–Mathematics Education.*

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM–Mathematics Education, 97*(3), 75–80.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521–539.

Stefanova, N., Podkhodova, N. (Eds.) (2005*). Metodika i technologiya obucheniya matematike* [Methods and technology of teaching mathematics]. Drofa.

Young, J. W. A. (1925). *The teaching of mathematics in the elementary and the secondary school*. Longmans, Green and Co.

Zazkis, R. & Zazkis, D. (2014). Script writing in the mathematics classroom: Imaginary conversations on the structure of numbers. *Research in Mathematics Education, 16*(1), 54–70.

MAKING SURE THAT MATHEMATICS EDUCATION
RESEARCH REACHES THE CLASSROOM

WORKING GROUPS

MATHEMATICS IN THE SERVICE OF HUMANITY: DATA, DIGNITY, DEMOCRACY AND SOCIAL CHANGE

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A community in the field of mathematics education research has been commited to putting forth teaching and learning of mathematics that is necessary to deal with issues of social justice. In this working group, we focus on mathematics as a tool of advocacy for social change, with a focus on data, dignity and democracy.

Recent events, including the rise of far-right states, war, and occupation have highlighted the power of the global economy, emphasizing the urgent need for advocacy for social change. Integrating a social advocacy component into mathematics education "means connecting the mathematics learned to pressing real-world issues" (Kingan, 2008, p. 238), such as defending dignity, human rights, and the protection of nature. As a starting point, we draw on Nussbaum's (2013) definition of dignity as the unconditional respect for individuals' powers of self-definition and democratic participation—expressing opinions while engaging in mutual learning, respect, critical thinking, and dialogue (Hauge et al., 2021). The central purpose of this working group is to invite participants to explore and discuss how mathematics can be used as a tool to uphold human dignity and democracy—and how this approach can drive social change. Research in mathematics education research has focused extensivly on democracy and democratic participation, and to a lesser degree on human dignity. For example, explaining that mathematics and its learning and teaching are neither the causes of nor the solution to intolerance and inequity, Valero et al. (2012) argue that decentralisation of the school mathematics curriculum could be an alternative for a democracy that reclaims human dignity. Abtahi (2022) argues for imagining the world from the point of view of others— "people and things whose experiences, contexts and ways of living are very different from our own" (p. 157) as a bases for standing up for human dignity. In this working group, we will analyze various data sets to explore how mathematics can serve as a powerful tool for advocacy and social change. In particular, we propose the following objectives:

1. To explore diverse perspectives about the concepts encompassed by the term human dignity democracy and social change;
2. Explore how mathematics as a knowledge system can lead to more awarness of issues of dignity and democracy leading to possibility of social change?
3. To explore the possibilities and challenges for such incorporation.

The following question will guide our exploration through various activities and discussions: How can a focus on dignity and democracy, through the exploration of

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, pp. 183-184. PME.

data, make mathematics a tool for advocacy and social change? What challenges and opportunities arise in this process?

Activity A – Using data from various international UN and NGO-based organizations, we will examine issues where human dignity and democracy are either positively or negatively magnified. Participants will then reflect on these issues and discuss their implications for the teaching and learning of mathematics.

Activity B – We will explore how and to what extent mathematics can serve as a catalyst for advocacy and social change.

Activity C – By engaging with mathematics as a system of knowledge, we will analyze social, political, and economic challenges and opportunities.

We propose to organise the sessions as follows:

Day 1 90 mins	• Introduction to the guiding question and exploration of the key terms. Q & A (15 min.) • Exploration of current literature, on methodological issues in researching human dignity (15 min.) • Small group working on Activity A (30 min.) • Whole-group reflection on implications of activity A (20 min). • Gathering themes and questions for session 2 and recapitulation. Q & A (10 min.)
Day 2 90 mins	• A short summary of themes collected in the first session (10 min.) • Small group working on Activity B. (25 min.). • Whole group discussion. Relating the work to our stance towards social change (10 min.). • Small group working on Activity C. (25 min.). • Whole group discussion, Q & A, closing remarks (20 min.)

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References

Abtahi, Y. (2022). What if I was harmful? Reflecting on the ethical tensions associated with teaching the dominant mathematics. *Educational Studies in Mathematics, 110*(1), 149-165.

Kingan, S. (2005). Rethinking and connecting algebra to real-world issues. In Gutstein, E. & Peterson, R. (Eds>) *Rethinking mathematics: Teaching social justice by the numbers*, pp. 238-242. Rethinking Schools.

Nussbaum, M. C. (2013). *The therapy of desire: Theory and practice in Hellenistic ethics*. Princeton University Press.

Valero, P., García, G., Camelo, F., Mancera, G., & Romero, J. (2012). Mathematics education and the dignity of being. *Pythagoras*, *33*(2), 1-9.

WHAT DOES AN INTEGRATED STEM CURRICULUM APPROACH OFFER THE TEACHING AND LEARNING OF MATHEMATICS?

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After working/discussion groups into integrated STEM education at PME conferences in 2017, 2018, 2019 and 2024, questions remain about the benefits of this approach to mathematics teaching and learning. In their narrative review of the literature from 2017 to 2022, Goos et al. (2023, p. 1201) reported it was rare to find studies showing how integrated STEM education helps students develop understanding of mathematics. When such studies do exist, they often also report challenges in curriculum design and/or delivery. One example was provided by Tytler et al. (2024) who reported successful learning outcomes in their interdisciplinary mathematics and science research in primary grades classrooms. While they described "potential learning payoffs" they also reported "multiple challenges for teachers and students in negotiating the interactions of the two subjects" (p. 39). In a similar vein, Honey et al. (2014) argued the need for a "strategic approach to implementing integrated STEM curriculum that accounts for the potential trade-offs in cognition and learning" (p. 5). It would be useful to identify the 'payoffs' and 'trade-offs' to support integrated STEM curriculum design and implementation, particularly for mathematics teachers and leaders.

Some studies have reported greater affective than cognitive benefits for students as they develop an appreciation for the relevance and usefulness of mathematics (e.g., Anderson et al., 2019). Others have recommended a focus on providing rich opportunities for students to develop 21st century skills such as critical and creative thinking, collaboration and problem solving (e.g., Gravemeiger, 2024). Li and Schoenfield (2019) problematize current approaches to teaching mathematics and argue for a new approach of 'making sense' and 'sense making' in the curriculum that can be applied to all STEM subjects through project-based learning and design-based learning. They argue this needs to be accompanied by a change from "what should the teacher do" to "what mathematical experiences should students have for them to develop into powerful thinkers?" (p. 8).

This discussion group will provide an opportunity for further research to be shared and to foster new international collaborative research that supports mathematics teachers' efforts to design and implement integrated STEM curriculum. During our discussions, several questions need to be considered including what is the balance in the curriculum between disciplinary knowledge and cross-disciplinary knowledge, or between content and practices, or between the STEM subjects themselves. If the role of mathematics in

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, pp. 185-186. PME.

the integrated STEM curriculum is as a goal, as a tool, or as an integrative bridge (such as mathematical modelling) (Goos, 2024), what benefits are evident for the learner of mathematics and how does that approach benefit the learner more than a different approach? How can we better support mathematics teachers and leaders as they design and implement an integrated STEM curriculum in schools?

PLAN FOR DISCUSSION GROUP SESSIONS 1 AND 2

30 mins	**Session 1** – Brief introduction, Judy Anderson and Ban Heng Choy
45 mins	Participants share STEM education perspectives to develop shared understandings of research and to raise questions for research agendas
15 mins	Summary of common (existing and/or emerging) themes, topic areas, questions and making connections with scholars with similar interests
30 mins	**Session 2** – Summary of Session 1. Short presentations on research projects from representatives in the discussion group
45 mins	Discussing possible future research collaborations and publications
15 mins	Developing a final set of outcomes for the discussion group and strategies for continued communication and collaboration

REFERENCES

Anderson, J., Wilson, K., Tully, D., & Way, J. (2019). Can we build the wind powered car again?" Students' and teachers' responses to a new integrated STEM curriculum. *Journal of Research in STEM Education*, *5*(1), 20-39. https://doi.org/10.51355/jstem.2019.61

Goos, M. (2024). Reflecting on the contribution of mathematics to STEM education. In J. Anderson & K. Makar, *The contribution of mathematics to school STEM education: Current understandings* (pp. 337-349). Springer

Goos, M., Carreira, S., & Namukasa, I. K. (2023). Mathematics and interdisciplinary STEM education: Recent developments and future directions. *ZDM – Mathematics Education*, *55*: 1199-1217. https://doi.org/10.1007/s11858-023-01533-z

Gravemeijer, K. (2024). Mathematics and STEM, preparing students for their future. In Y. Li, Z. Zeng & N. Song (Eds.), *Disciplinary and interdisciplinary education in STEM: Changes and innovations* (13-32). Springer

Honey, M., Pearson, G., & Schweingruber, H. (2014). *STEM integration in K-12 education: Status, prospects and an agenda for research.* National Academies Press.

Li, Y., & Schoenfeld, A. H. (2019). Problematizing teaching and learning mathematics as a "given" in STEM education. *International Journal of STEM Education*, *6*:44. https://doi.org/10.1186/s40594-019-0197-9

Tytler, R., Mulligan, J., White, P. J., & Kirk, M. (2024). Promoting effective interactions between mathematics and science : Challenges of learning through interdisciplinarity. In Y. Li, Z. Zeng & N. Song (Eds.), *Disciplinary and interdisciplinary education in STEM: Changes and innovations* (33-62). Springer.

REPLICATION STUDIES IN MATHEMATICS EDUCATION

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The aim of this working group is to examine the role quantitative replication studies may have in mathematics education research. The workshop aims to clarify when a study is considered a replication study, the potential impact and value of replication studies for the field, the way in which they are handled by journals, and the challenges researchers face in their execution and publication.

Replication studies are a very important common practice in scientific research, recognized as a crucial method for validating empirical knowledge claims based on quantitative and experimental research. Methodological textbooks consistently highlight the necessity of ensuring research results are replicable (Newby, 2013). There has recently been a growing discussion about the role of replication studies in educational research. In 2018, the U.S. National Science Foundation (NSF) and the Institute of Educational Sciences (IES) stressed that advancing evidence-based decision making requires a more prominent role of reproducibility and replication research within the education community. Despite this emphasis, research indicates that fewer than 5% of studies in the field involve replication (Makel et al., 2012).

Replication studies in the field of mathematics education are rare (Cai et al., 2018). At PME42, a first working group was held on Replication Studies in Mathematics (Inglis et al., 2018). This workshop aimed to initiate the conversation surrounding replication studies in mathematics education, focusing on specific issues in replication. The current Working Group will focus on quantitative and experimental research within mathematics education, seeking a clear and detailed examination of what replication studies involve, their potential impact and value for the field, the way they are handled by journals, and the challenges researchers face both in execution and publication. The Working Group will consist of two 90-minute sessions. The First Session topic is Conceptualization, Rationale, Contribution, and Impact of Replication Studies, and Second Session topic is Challenges and Evaluation of Replication Studies.

STRUCTURE OF THE WORKING GROUP

Session 1: Conceptualization and Rationale, Potential Impact and Contribution of Replication Studies

In the first session, we will start with a welcome round and a brief exploration of the participants' interest in the topic as well as their experience with replication studies (15'). The organizers will then provide an introduction to the idea of replication and its relevance for mathematics education based on recent publications in the field (e.g., Cai

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, pp. 187-188. PME.

et al., 2018). Different dimensions and aspects of replication research will be presented as well as points of discussion and controversy (25').

Next, a small group discussion will be held (25'). Participants' own experiences and ideas can form the basis of the discussion but the organizers will provide concrete (anonymized) abstracts of empirical studies that may or may not be seen as replication studies that provide a valuable contribution to the field. The focus will be on *what* is replicated (e.g., a theoretical idea, a specific finding), what—if any—relevant *differences* are in the study (e.g., exact replication, another country, another mathematical topic), and what *reasons* there are to conduct a replication study.

The session will be concluded by a plenary summary and discussion (25').

Session 2: Evaluation of Replication Studies and Challenges in Execution

The second session is divided into two parts. The first focuses on the way in which replication studies that are submitted to journals in mathematics education are evaluated by reviewers and handled by journal editors. We will provide input for the discussion based on a survey that we conducted among editors and former editors of major mathematics education journals. These will first be briefly presented (15') after which a small group discussion will focus on participants' own experiences and ideas about the results of the survey (30'). A brief plenary summary (10') concludes this half.

The second part focuses on the execution and challenges of replication studies. In small groups, participants will discuss (25') and make a list of challenges experienced. These may relate to several aspects, including the difficulty of finding sufficient information about the study to be replicated as well as difficulties in reporting the replication and in getting studies published. Again, a plenary summary will conclude this part (10').

References

Cai, J., Morris, A. K., Hohensee, C., Hwang, S., Robison, V., & Hiebert, J. (2018). The role of replication studies in educational research. *Journal for research in mathematics education. Journal for Research in Mathematics Education, 49*(1), 2-8. https://doi.org/10.5951/jresematheduc.49.1.0002

Inglis, M., Schukajlow, S., Van Dooren, W. & Hannula, M. S. (2018). Replication in mathematics education. In E. Bergqvist, M. Österholm, C. Granberg & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 195–196). PME.

Makel, M. C., Plucker, J. A., & Hegarty, B. (2012). Replications in psychology research: How often do they really occur? *Perspectives on Psychological Science, 7*(6), 537-542. https://doi.org/10.1177/1745691612460688

Newby, P. (2013). *Research methods for education*. Hoboken: Taylor and Francis.

VALUES FOR MATHEMATICS EDUCATION: NEXT STEPS

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BACKGROUND

This working group aims to further research within the domain of *values as a part of non-cognition research in mathematics education.* Although mathematics education research has incorporated psychological and sociocultural perspectives, the dominance of cognitive approaches have prevailed since the 1950s, often sidelining critical pedagogical factors. A more comprehensive understanding requires acknowledging the tripartite model of the human mind: cognition, affect, and conation. Incorporating conative elements, such as values, into mathematics education offers a more holistic framework for addressing the complexities of teaching and learning (cf. Seah, 2019). DeBellis and Goldin (2006) describe values as "personal truths" shaping long-term priorities and structured value systems.

Recently, research in mathematics education has increasingly explored how values influence decision-making and behaviour, revealing their interplay with related affective variables. A dynamic interplay between teaching and learning highlights the potential of values to enhance ethical and effective mathematics education (Chiu & Seah, 2024). Values have also been instrumental in the development and promotion of mathematical wellbeing, an affective variable (Hill et al, 2021).

This WG builds on foundations from previous PME working sessions (namely, PME36 in 2012, and PME39 in 2015) that focused on values and related affective constructs (DeBellis & Goldin, 2006; Tiberius, 2018) and looks to extend existing concepts and applications in educational environments (e.g. Dede, Marschall & Clarkson, 2024).

AIMS OF WORKING GROUP

- To revisit existent theories and concepts on values and other related affective constructs by exploring research experiences and a variety of frameworks.
- To formulate researchable questions in the field and to explore and develop potential methodologies that support these questions.

OUTLINE OF SESSIONS

Session1

- Introductions, then participants share experiences in values research: what theoretical constructs guide the identification of teacher and student values? What specific expertise do we have as mathematics education researchers? How can we learn from other fields (psychology, cultural studies etc)? The WG team

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will begin by sharing their view on frameworks as we come from different backgrounds and ethnicities. Attendees will be invited to share their stories leading to a discussion of themes, commonalities, differences, etc.

- Brief presentation of existing ways of research and then moving to some suggestions for possible new approaches and conceptualisations (e.g. JEDI approach, applying research to practice, etc).
- Discussion in groups with a focus on the development of frameworks that support the interaction between theory and practice.
-

Session 2:

- Building on session 1, groups will discuss: i) What researchable questions can be created in the area of values? ii) What theoretical frameworks can support this research? iii) What methodologies are appropriate for these questions? iv) What data would be needed?
- Each group will share their ideas and discuss next steps for future collaborations, including a possible joint output for participants such as a special issue for the *Journal of Mathematics Teacher Education*.

References

Clarkson, P., Seah, W. T., & Pang, J. S. (Eds.), (2019). *Values and valuing in mathematics education: Scanning and scoping the territory*. Springer.

Chiu, M.-S., & Seah, W. T. (2024). Values and valuing pedagogies in affect-focused mathematics teaching. *Social Sciences and Humanities Open, 10*, 101050. https://doi.org/10.1016/j.ssaho.2024.101050

Dede, Y., Marschall, G., & Clarkson, P. (Eds.). (2024). *Values and valuing in mathematics education: Moving forward into practice*. Springer Nature. https://doi.org/10.1007/978-981-99-9454-0

Hill J. L., Kern M. L., Seah W.T., van Driel J. (2021). A model of student mathematical wellbeing: Australian grade eight students' conceptions. In Inprasitha, M., Changsri, N. & Boonsena, N. (Eds). (2021). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*. PME.

Seah, W. T. (2019). Values in mathematics education: Its conative nature, and how it can be developed. *Research in Mathematical Education*, *22*(2), Article 2. https://doi.org/10.7468/JKSMED.2019.22.2.99

Tiberius, V. (2018). *Well-Being as value fulfilment: How we can help each other to live well*. Oxford University Press.

(RE)IMAGINING MATHEMATICS TEACHER EDUCATION AND RESEARCH AS A FUTURE-MAKING PROJECT

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BACKGROUND AND RATIONALE

Ideas about the future play a key role in education, influencing all levels of activity from our in-the-moment decision-making as practitioners and researchers, to the planning of large-scale educational reform. UNESCO's report from the *International Commission on the Futures of Education* (ICFE, 2021) prompts us to reconsider ways in which education can shape sustainable futures for the common good. As humans we are constantly anticipating futures. An awareness of futures is present in our "everyday anticipatory behaviour and explicit foresight" (Ahvenharju et al., 2018, p. 1). Futures scholars go beyond these everyday anticipatory behaviours and have developed techniques that support "futures consciousness" (Ahvenharju et al., 2018), that is, a humans' capacity to anticipate desirable futures as a way of enabling decision-making. Futures research is thus concerned with studying and building "images of possible, probable and preferable futures and paths to such futures" (p. 1).

In this working group, we draw on some of these futures ideas and techniques along with Skovsmose's (2011) notion of "pedagogical imagination" that he explains is "crucial for exploring educational possibilities" (p. 23). We extend Skovsmose's notion and use the term 'methodological imagination' to incorporate the idea of exploring research possibilities. One aspect of methodological imagination is a rethinking of potential (research) questions beyond those that take the form "what is". Hopkins (2019) prompts us to consider asking "what if" questions that balance imagination and action by assuming nothing is unmovable, and that "kick past that inner (and outer) critic who says that nothing can really ever change" (pp. 139-140). Other aspects of methodological imagination include, but are certainly not limited to: expanding our notion of data, how we collect and analyse it; using imaginative and speculative methods both in teacher education contexts and as research methods (e.g., Helliwell & Ng, 2022); and developing activities and technologies to engage with as-yet unimagined futures.

Taking a futures-oriented approach to teacher education and research means moving beyond the assumption that the anticipated state of the world is both bleak and non-sustainable. In this working group, we build on a recent research forum at PME46 on innovating the mathematics curriculum (Amico et al., 2023) and ask how our pedagogies and research practices might evolve to study, build images of, and create pathways to preferable futures. We will draw on futures techniques (e.g., backcasting; scenario building; creative mapping) to support participants in imagining alternative

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futures both in the work they do with mathematics teachers but also in relation to their research practices.

AIMS OF THE WORKING GROUP

(1) To explore what it means to take a futures-oriented approach and to build images of preferable futures in relation to mathematics teacher education and to mathematics education research; (2) To imagine and develop potential futures-oriented research questions and methodologies and/or teacher education pedagogies.

OUTLINE OF THE SESSIONS

Session 1: (1) Introduction to ideas of "sustainable futures", "futures studies" and "futures consciousness/literacy" with existing examples from mathematics teacher education research (10 mins); (2) Futures technique 1 (utopic scenarios) to support the imagining and articulation of preferable futures in mathematics education (40 mins); (3) Small group discussions to share experiences (30 mins); (4) Whole group discussion and feedback (10 mins).

Session 2: (1) Introduction to ideas of futures-oriented research questions and methodologies (10 mins); (2) Futures technique 2 (creative mapping) to support action planning towards preferable futures (35 mins); (3) Sharing mappings and collaborating to develop them into potential research questions and methodologies and/or teacher education pedagogies (30 mins); (4) Whole group discussion and feedback (15 mins).

References

Ahvenharju, S., Minkkinen, M., & Lalot, F. (2018). The five dimensions of futures consciousness. *Futures*, *104*, 1–13. https://doi.org/10.1016/j.futures.2018.06.010

Amico, A., Andra, C., Chorney, S., Coles, A. T., Doria, L., Helliwell, T., Hunter, J., Makramalla, M., Pezzutto, M., Rubel, L., & Solares, A. (2023). How socio-ecological issues are urging changes in curriculum (and beyond). In M. Ayalon, B. Koichu, R. Leikin, L. Rubel & M. Tabach (Eds.), *Proc. 46th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 71–101). PME. https://www.igpme.org/wp-content/uploads/2024/06/PME46-2023-Haifa.zip

Helliwell, T., & Ng, O. L. (2022). Imagining possibilities: Innovating mathematics (teacher) education for sustainable futures. *Research in Mathematics Education*, *24*(2), 128–149. https://doi.org/10.1080/14794802.2022.2079553

Hopkins, R. (2019). *From what is to what if: Unleashing the power of imagination to create the future we want*. Chelsea Green Publishing.

ICFE (2021). *Reimagining our futures together: a new social contract for education*. UNESCO. https://unesdoc.unesco.org/ark:/48223/pf0000379707

Skovsmose, O. (2011). Critique, generativity and imagination. *For the Learning of Mathematics, 31*(3), 19–23. https://www.jstor.org/stable/41319603

FAILURE TO TEACH/LEARN MATHEMATICS: CONCEPTUAL, DIAGNOSTIC, AND TECHNOLOGICAL PERSPECTIVES

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Failure to Teach/Learn Mathematics (FTLM) is a persistent challenge spanning cognitive, affective, and social-institutional dimensions. This Working Group (WG) aims to advance the theorization of FTLM by integrating perspectives from commognition, complexity theory, and technology-based interventions. We will explore nested failure systems and how technology can support productive engagement in mathematics learning. The WG will engage PME participants through empirical data, theoretical conceptualizations, and collaborative discussions.

Failure to Teach/Learn Mathematics (FTLM) is a complex phenomenon that spans multiple dimensions—cognitive, affective, interactional and social-institutional. Traditional perspectives on mathematics learning difficulties often focus on the individual, cognitive domain (e.g. Geary, 2011), yet recent research highlights the necessity of integrating broader frameworks that consider cognitive, emotional and social dimensions as functioning within nested complex dynamic systems (Heyd-Metzuyanim, 2025). This WG aims to advance the theorization of FTLM by integrating perspectives from commognition (Sfard, 2008), complexity theory (Davis & Sumara, 2006), and technology-based interventions (Baccaglini-Frank, 2021).

A key focus will be on understanding FTLM as a multi-layered phenomenon, operating within nested systems of failure that include the individual, classroom practices, and broader institutional and socio-cultural structures. We will explore:

1. How different theoretical perspectives conceptualize FTLM and its development over time.
2. The role of diagnostic tools in identifying FTLM and the extent to which they capture its complexity.
3. How technology-mediated interventions can address FTLM and support shifts towards more productive engagement with mathematics.

FORMAT AND STRUCTURE

Session 1: Conceptualizing Failure to Learn Mathematics

Introduction (20-30 min): Overview of theoretical perspectives: individual, affective and social aspects of FTLM. Introducing complexity theory and commognition as means for conceptualizing FTLM as a complex dynamic system.

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Data Engagement (40 min): Participants will analyze video excerpts and transcripts from at least two research groups and discuss instances of FTLM from different theoretical perspectives.
Discussion (20-30 min): Comparison of interpretations across groups and reflections on how different theoretical lenses highlight distinct aspects of FTLM.

Session 2: Diagnostics and Interventions

Introduction (25 min): The role of technology in addressing FTLM at different levels of nested failure systems. Presentation of diagnostic tools and intervention strategies, linking them to empirical data.
Data Engagement (40 min): Participants will work in self-selected sub-groups to analyze diagnostic tools and intervention strategies applied to different cases. They will be invited to share personal research insights around diagnostical tools and interventions targeting FTLM. The WG leaders will facilitate discussions, ensuring that contributions are aligned with the overall goals.

Discussion (25 min): Discussion of findings and potential for methodological advancements. Consideration of implications of the various theoretical viewpoints presented for designing more effective interventions. The sessions will conclude with a synthesis of insights and suggestions for continued collaboration beyond PME.

EXPECTED OUTCOMES

This WG will refine theoretical perspectives on FTLM, identify limitations in current approaches, and propose directions for future research. Participants will gain insights into integrating technology and theory to better address FTLM. Discussions will foster collaborations among researchers interested in mathematical learning difficulties, marginalization of students and technology-assisted interventions.

ACKNOWLEDGMENTS

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References

Baccaglini-Frank, A. (2021). To tell a story, you need a protagonist: How dynamic interactive mediators can fulfill this role and foster explorative participation to mathematical discourse. *Educational Studies in Mathematics*, *106*(2), 291–312.

Davis, B., & Sumara, D. (2006). *Complexity and education: Inquiries into learning, teaching, and research*. Routledge.

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology*, *47*(6), 1539–1552.

Heyd-Metzuyanim, E. (2025). Failure to teach/learn mathematics: A complexity-discursive perspective. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-025-10404-1

Sfard, A. (2008). *Thinking as communicating*. Cambridge University Press.

GLOBAL PERSPECTIVES OF EQUITY IN MATHEMATICS EDUCATION: DEFINITIONS, CHALLENGES, PATHS FORWARD

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The pursuit of equitable mathematics education is a fundamental challenge as well as a moral and global imperative. For example, the United Nations (U.N.) sustainable development goals include the creation of quality education systems that encompass equitable lifelong learning opportunities and inclusivity for all (U.N., n.d.). Disparities in mathematical achievement have long persisted, often along lines of differing cultural perspectives and knowledge systems, race, ethnicity, language, socioeconomic status, disabilities, sensory- or neurodiversity, and other social identities. These disparities are not just a reflection of inherent ability, but rather a consequence of systemic inequities that impact students' access to rich, meaningful mathematical learning experiences.

We invite scholars to engage in a conversation about equity—to share theoretical groundings, definitions, and operationalizations, as well as the literature that they read and contribute to. The goal of this working group is to establish a long-term collaborative conversation about equity and equitable teaching practices in classrooms in the context of mathematics education research. Equity has been defined and operationalized in various ways in mathematics education. Esmonde (2009) defined equity as "a fair distribution of opportunities to learn" (p. 249); Hunter and Hunter (2023) have operationalized equity as "explicitly connect[ing] and build[ing] on the cultural, social, and linguistic contexts of non-dominant students, their family and community" (p. 1768); whereas Ernest et al. (2019) posited that "participatory equity" occurs when "students from marginalized groups in mathematics […] participate at rates proportional to their demographic representation in a classroom" (p. 155). However, studies on equity often do not explicitly define or operationalize the term. For example, in a literature review that sought to identify definitions and operationalizations of equity in mathematics instruction, only 21 out of 171 articles in mathematics education provided either a definition or operationalization of equity; these included notions of fairness, access, participation, demanding mathematics, and references to bridging, identity, agency, and power. Often, we found that the terms equity, equality, diversity, and inclusivity were used interchangeably.

At the same time, in some countries we have seen a recent shift in policy direction that posits a simplistic view of addressing equity issues, and proposes that all students learn the same. Voices advocating for such a shift are claiming that if we therefore teach all students mathematics in the same way using direct instruction, all students will learn. We contend instead that a critical component of addressing inequities lies in fostering

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the ability to grasp the underlying nature of mathematical concepts based on what one knows, rather than relying solely on rote memorization.

We are cognizant that concepts of equity and equitable teaching vary across different nations and cultures, reflecting diverse historical experiences, social structures, and systemic challenges related to varied histories of colonialism, slavery, indigenous oppression, caste systems, and economic disparities. Our group seeks to develop an understanding of equity specifically in the contexts of mathematics education research and classroom instruction. The working group will focus on: (a) Comparing definitions and operationalizations of equity in research and in mathematics classrooms, (b) Developing sensitivity around definitions of understanding equity and equitable classrooms within international contexts, and (c) Synthesizing issues and research approaches to understanding equity and equitable classrooms.

Day 1: Introductions/discussion of goals (30 min). Main discussion points and defining equity in different settings (60 min): (1) Discuss the contexts in which this work is conducted (nations, cultures, diverse historical experiences, social structures, and systemic challenges); (2) Share literature, frameworks, and operationalizations of equity and equitable teaching practices, research approaches; (3) Create a shared database of sources, definitions, and operationalizations. **Day 2:** Summarize discussion points from the first session (15 min). Discuss the similarities, differences, constraints, and affordances across countries, cultures, and educational contexts for defining equity and equitable teaching (30 min). Small group activity to plan and develop a mini research initiative. Groups will define future work related to equity in mathematics education, formulate a research question, outline a research plan and potential impact of the research on mathematics classrooms striving for equity, and wrap-up (45 min).

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References

Ernest, J. B., Reinholz, D. L., & Shah, N. (2019). Hidden competence: Women's mathematical participation in public and private classroom spaces. *Educational Studies in Mathematics*, *102*, 153-172.

Esmonde, I. (2009). Mathematics learning in groups: Analyzing equity in two cooperative activity structures. *The Journal of the Learning Sciences*, *18*(2), 247-284.

Hunter, J., & Hunter, R. (2023, July). Working towards equity for diverse learners: Drawing on values and ways of being as strengths in mathematics classrooms. In *Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)* (No. 10). Alfréd Rényi Institute of Mathematics; ERME.

United Nations. (n.d.). Sustainable Development Goals, from www.un.org/sustainabledevelopment/

EXTENDING EMBODIED LEARNING TO MATHEMATICS TEACHER PROFESSIONAL DEVELOPMENT THROUGH MULTIMODAL LEARNING ANALYTICS

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Building on the PME 2023 Research Forum on Embodied Learning Design, this Working Group explores the role of multimodal learning analytics (MMLA) in mathematics teacher professional development (PD). Drawing on Francisco Varela's enactive cognition framework, we examine how exposure to first-person experience, multipersonal eye-tracking, and multimodal data analysis can enhance teacher reflection, professional vision, and instructional decision-making. Participants will engage in embodied mathematical activities, analyze MMLA-enhanced classroom data, co-develop PD frameworks, and propose methodological approaches for researching emerging phenomena in these innovative settings.

TOPIC AND CONNECTION TO PME 2023 RESEARCH FORUM

This Working Group (WG) builds upon the Research Forum on Embodied Learning Design at PME 2023 (Palatnik et al., 2023). The RF focused on theoretical and design principles of embodied learning, emphasizing interaction, perception-in-action, and multimodal engagement (Varela et al., 1991) in mathematics education. However, it did not address how these principles could inform mathematics teacher professional development (PD).

This WG aims to fill that gap by integrating multimodal learning analytics (MMLA) into embodied teacher PD. Recent research has demonstrated that MMLA tools such as eye-tracking and subsequent multimodal analysis can enhance teacher reflection and professional vision (Goodwin, 2018). Participants in the WG will explore together how embodied cognition and multimodal analytics can shape teacher learning in collaborative and inquiry-based professional settings.

GOAL AND STRATEGY TO REACH THE GOAL

The goal of this WG is to develop a framework for integrating embodied learning and multimodal analytics into teacher PD. Our strategy includes:

- Exploring theoretical connections: Revisiting key RF themes such as perception-in-action, multimodal interaction, and professional vision and discussing their role in teacher learning.
- Analyzing multimodal data: Engaging participants in video-based teacher reflection enriched with eye-tracking and action/gesture-tracking data, allowing for a deeper understanding of instructional attention and decision-making.

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- Co-developing PD activities: Drawing on participants' expertise to co-design embodied PD tasks that incorporate collaborative problem-solving and multimodal analytics for reflective practice.
- Mapping future research directions: Identifying opportunities for collaborative studies, networking of theories and methodological innovations.

The Working Group is led by researchers bringing their internationally acclaimed and complementary expertise: Alik Palatnik (Coordinator), specializing in embodied learning and multimodal analytics, and Jason Cooper, an expert in teacher education and the connection between academic research and classroom discourse and practices.

LAYOUT AND TIME STRUCTURING (2 X 90-MINUTE SESSIONS)

Session 1: Embodied Cognition and Teacher Learning (90 min)

- Introduction to embodied learning and multimodal analytics in teacher PD (15 min).
- Participation in the paradigmatic mathematical embodied activity including interactive engagement with MMLA tools (multiple mobile eye-tracking, motion sensors, 30 min).
- Small group work on case study analysis: examining video eye-tracking enhanced data of teachers who participated in the similar activity and their reflections on it in the frame of PD (30 min).
- Whole group discussion (15 min).

Session 2: Designing PD with Multimodal Analytics (90 min)

- Review of key insights from Session 1 (10 min).
- Small group collaborative design in shared digital workspace: developing teacher PD modules with MMLA (40 min).
- Whole group discussion on future research and PME engagement (40 min).

This WG expands on PME 2023 RF, applying its principles to teacher learning. We plan to continue this WG at PME 2026 to refine the framework and consolidate a thematic Colloquium and a series of Short Oral presentations to advance empirical research on embodied teacher PD.

References

Goodwin, C. (2018). *Co-operative action*. Cambridge University Press.

Palatnik, A., Abrahamson, D., Baccaglini-Frank, A., Ng, O.-L., Shvarts, A., & Swidan, O. (2023). Theory and practice of designing embodied mathematics learning. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 159–189). PME.

Varela, F. J., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. MIT Press.

MATHEMATICS TEACHER NOTICING: NEW CONTEXTS AND CONCERNS IN PROGRAMS DESIGN

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This Working Group (WG) is a new initiative in PME that originates from the "Mathematics Teacher Noticing Seminar" held previously in Chile. During these discussions in the seminar, two important themes emerged that are part of the work proposal in the WG: (a) noticing in new contexts and (b) the double role of the researcher and teacher educator in researching program design related to noticing.

INTRODUCTION

This Mathematics Teacher Noticing Working Group is an initiative to incorporate more new contexts and perspectives in mathematics teacher noticing (e.g., König et al., 2024) and aims to establish continuity in future PME conferences. The Working Group seeks to enhance research on professional noticing through collaboration, fostering dialogue among participants from diverse communities. It will facilitate the discussion, and identification of research challenges while building a network for academic exchange on noticing.

The Working Group originates from the "Mathematics Teacher Noticing Seminar," a monthly hybrid event hosted by three Chilean universities since March 2024. In this seminar on different perspectives about teachers' noticing, Chilean and international researchers from other universities worldwide participated. During these discussions, two important themes emerged: (a) noticing in new contexts (e.g., teacher orchestration, modeling, argumentation, or equity-focused noticing); (b) the double role of the researcher and teacher educator in researching pre-service or in-service teachers' noticing development.

With regard to the first theme, research on mathematics teacher noticing has significantly advanced our understanding of teaching competencies, particularly regarding how teachers attend to, interpret, and respond to students' mathematical thinking (e.g., Jacobs et al., 2010). Various frameworks have conceptualized noticing as a multidimensional skill essential for effective teaching, influencing both pre-service and in-service teacher training (e.g., Mason, 2002; van Es & Sherin, 2002). Additionally, researchers have developed assessment tools to measure and characterize teachers' growth in various contexts. However, significant gaps remain despite these advancements, and new contexts are emerging in coordination with noticing.

Considering the second theme, researchers and teacher educators design courses or professional development programs to develop noticing (Ivars et al., 2019). In the design of these courses and programs, different questions arise: What are the principles

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in designing these courses and programs to support noticing? What is the role of the researcher and the teacher educator in the design?

This Working Group seeks to build on these themes, fostering discussion on how noticing unfolds in different teaching and learning environments.

THEMATIC STRUCTURING

In Table 1, the structure and organization of the sessions is presented.

20 min	• Introduction of the theme: "Noticing in new contexts"
45 min	• Working in small groups to discuss noticing in various contexts
20 min	• Whole group discussion of the relevant ideas from the small groups
5 min	• Summarizing the relevant ideas and introducing the new theme for the second day
20 min	• Introduction of the theme: "Design principles and role of the researcher and teacher educator"
40 min	• Working in small groups to discuss the theme
20 min	• Whole group discussion of the relevant ideas from the small groups
10 min	• Summering the relevant ideas, and future perspectives and ideas for the group

Table 1: Structure of the sessions.

References

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169–202.

König, J., Santagata, R., Scheiner, T., Adleff, A. K., Yang, X., & Kaiser, G. (2022). Teacher noticing: A systematic literature review of conceptualizations, research designs, and findings on learning to notice. *Educational Research Review*, 36, 100453.

Ivars, P., Fernández, C., & Llinares, S. (2019). Principles in the design of tasks to support preservice teachers' noticing enhancement. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 408-415). PME.

Mason, J. (2002). *Researching your own practice. The discipline of noticing*. Routledge Falmer

van Es, E. A., & Sherin, M. G. (2002). Learning to Notice: Scaffolding New Teachers' Interpretations of Classroom Interactions. *Journal of Technology and Teacher Education*, 10, 571–596.

STRENGTHENING MENTORSHIP IN MATHEMATICS EDUCATION PUBLISHING

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Equity, diversity and inclusion (EDI) in academic publishing remain critical challenges in mathematics education research. This PME Working Group focuses on strengthening mentorship structures to support scholars in navigating the academic publishing process. Participants will explore mentorship models, develop structured guidelines for advisory support, and propose actionable strategies for a more inclusive publishing landscape. Through collaborative discussions, we aim to advance EDI initiatives in mathematics education publishing and develop sustainable mentorship.

BACKGROUND, RATIONALE AND STRATEGY

Academic publishing in mathematics education is characterised by systemic inequities that disproportionately affect scholars from underrepresented regions, particularly those who publish in English-medium journals. Challenges such as linguistic barriers, limited institutional support, and the dominance of Global North perspectives contribute to the exclusion of diverse voices in the field (Darragh et al., 2024). Our previous work, including an ICME-15 Discussion Group (Brodie et al., 2024), examined these challenges and explored solutions. The discussions underscored the need for structured mentorship programs that help (emerging) scholars navigate the complexities of academic writing, peer review, and international publishing norms.

In response to these concerns, initiatives such as the advisory mentoring system within the Journal of Mathematics Teacher Education have sought to support scholars by providing guidance on manuscript preparation and publication processes (see Scheiner et al., 2024). However, these efforts remain fragmented and require further refinement and expansion. Building on discussions initiated at ICME-15, this Working Group offers PME participants to engage in a critical and action-oriented discussion on EDI-focused mentorship in academic publishing. Through collaborative exchange, this initiative seeks to enhance mentorship practices in mathematics education publishing; strengthen support structures for scholars from underrepresented regions; and develop recommendations for fostering inclusive academic publishing practices.

The primary goal of this Working Group is to advance EDI in mathematics education research publishing by strengthening mentorship structures. Specifically, we aim to:

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(a) Identify key challenges faced by scholars from diverse world regions in publishing mathematics education research;

(b) Exchange best practices in mentorship, exploring existing models of support for scholars navigating the academic publishing process;

(c) Develop structured guidelines for regional mentorship that address barriers related to language, institutional support, and navigating peer review processes; and

(d) Propose follow-up initiatives (e.g., regional advisory programs) to sustain and expand mentorship efforts in our research community.

PROPOSED STRUCTURE AND PARTICIPANT ENGAGEMENT

Session 1 (90 minutes) – Understanding Challenges and Sharing Insights

1. Welcome and Introductory Presentation (20 min) – Overview of the Working Group's goals and insights into the main challenges in academic publishing.
2. Breakout Group Discussions (40 min) – Participants discuss specific challenges related to EDI-focused mentorship in mathematics education research publishing.
3. Synthesis and Reflections (30 min) – Groups report back, identifying common themes and preliminary recommendations.

Session 2 (90 minutes) – Co-Constructing Solutions and Future Directions

1. Refining Recommendations (30 min) – Groups work collaboratively to finalise mentoring guidelines for supporting (emerging) scholars in publishing.
2. Action Planning (30 min) – Participants co-develop concrete follow-up initiatives (e.g., mentorship networks, journal initiatives, collaborative research projects).
3. Final Reflections and Closing (30 min) – Summary of key takeaways, commitments for follow-up, and opportunities for continued engagement beyond PME.

References

Brodie, K., Darragh, L., Halai, A., Planas, N., Potari, D., Santos Trigo, M., Scheiner, T., & Walkoe, J. (2024). *Promoting equity, diversity, and inclusion in publishing mathematics education research*. Discussion Group at the International Congress on Mathematical Education. Sydney, Australia.

Darragh, L., Brodie, K., Halai, A., Planas, N., Potari, D., Santos-Trigo, M., Scheiner, T., & Walkoe, J. (2024). Publishing mathematics education research in English: amplifying voices from the field. *Journal of Mathematics Teacher Education*, *27*(5), 857–878. https://doi.org/10.1007/s10857-024-09659-x

Scheiner, T., Brodie, K., Planas, N., Darragh, L., Halai, A., Potari, D., Santos-Trigo, M., & Walkoe, J. (2024). Addressing equity, diversity and inclusion in academic publishing: key initiatives from JMTE. *Journal of Mathematics Teacher Education*. https://doi.org/10.1007/s10857-024-09636-4

MAKING SURE THAT MATHEMATICS EDUCATION
RESEARCH REACHES THE CLASSROOM

PUBLICATION REPORTS

BETWEEN COMPUTATIONAL THINKING AND LITERACY IN MATHEMATICS EDUCATION

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In this essay, I critically engage with arguments regarding the trend of including programming and other computing contents into school mathematics education. I differentiate and compare two perspectives on this regard: computational thinking and computational literacy. Based on Niss' (1996) view on purposes of mathematics teaching, I delineate four aspects for consideration: political, contingent, disciplinary and empirical. My position is threefold. First, computational thinking is more accessible for implementation and assessment purposes. Second, computational literacy is a more radical view on the role of computing, beyond a tool for teaching traditional mathematics. Third, computational literacy has a more robust grounding, which may work as a necessary alternative to the state of the actual.

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THE INHERENT STRUCTURE OF TEACHERS' KNOWLEDGE FOR HIGH QUALITY INSTRUCTION IN MATHEMATICS

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[1]University of Tübingen,
[2]University of Education, Karlsruhe

Teachers need technology-related knowledge for high-quality instruction in mathematics, commonly conceptualized as Technological Pedagogical and Content Knowledge (TPACK). Despite TPACK's importance for effective teacher education, findings on its inner structure are mixed, likely due to the use of self-reports in prior research. This study applied test-based instruments to examine TPACK's relation to Pedagogical Content Knowledge (PCK) and Technological Knowledge (TK) and compared self-reported with test-based TPACK. Among 141 mathematics pre-service teachers, results show that PCK and TK are moderately related to TPCK to a similar extent. Further, self-reported TPACK diverges from test-based TPACK, moderated only by metacognitive accuracy but not gender or prior experience.

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DEVELOPING AND IMPLEMENTING TASK-FOCUSED GUIDANCE FOR LANGUAGE-RESPONSIVE MATHEMATICS TEACHING

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Mathematics teaching that is responsive to the linguistic demands of the mathematical content, as well as the linguistic resources that students bring, is widely recognised as supporting the learning of mathematics. In this paper we report on one aspect of the video analysis that occurred as part of a larger development project focused on language-responsive mathematics teaching. Videos of lessons where experienced teachers implemented tasks they had collaborated in designing guidance for were analysed using a standardised observation framework focused on classroom discourse and the quality of mathematical activity. The analysis shows the teachers explicitly including opportunities to elicit students' thinking and to enable students to engage in a range of rich discourse practices.

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DOES PROBLEM POSING AFFECT SELF-EFFICACY, TASK-VALUE, AND PERFORMANCE IN MATHEMATICAL MODELLING?

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Problem posing is a promising teaching method for enhancing motivation and mathematical modelling. This study examines the effects of problem posing on modelling performance, self-efficacy, and task values in solving modelling problems and analyze whether problem posing affects modelling performance via self-efficacy and task values. Ninth- and tenth-graders (N = 210) were assigned to either a problem-posing and problem-solving group or to one of two problem-solving groups. Problem posing positively enhanced students' self-efficacy and indirectly affected modelling performance via self-efficacy. However, problem posing had no total effect on modelling performance. The findings highlight the importance of motivational constructs in problem-posing instructions aimed at fostering modelling.

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MAKING SURE THAT MATHEMATICS EDUCATION
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ORAL COMMUNICATIONS

INVESTIGATING TEACHER RESOURCES IN KENYA'S NEW CURRICULUM

Sheila Oyeila Amuko

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In 2019, Kenya introduced a competency-based curriculum (CBC) aimed at addressing learner needs, characterized by its dynamic and flexible nature. This oral communication forms part of a broader investigation into the implementation of this competency-based mathematics curriculum in a sub-county Kenya.

New mathematics learning materials for the updated curriculum were developed, reviewed, and approved by a panel of mathematics educators at the Kenya Institute of Curriculum Design (KICD). This process was guided by the 2020 Kenyan constitution and aimed at fulfilling the goals of Kenya Vision 2030, which emphasizes the need to reform the Kenyan education system by focusing on three essential elements: competencies, character, and creativity. This study examines the KICD-endorsed learning materials and how teachers utilized them to enhance their instruction. Among these resources were mathematical lesson plans, textbooks, and links provided within the textbooks that direct students to various web pages, including YouTube.

This study examines learning resources in primary schools in Kajiado Central, Kenya, for learners aged 9 to 11. Research questions include: What resources do upper primary teachers use to teach mathematics? What challenges do teachers encounter resources?

I report preliminary results as the study is ongoing. In grade five classes, teachers' actions were recorded to collect qualitative data on their perception of the approved resources and time spent on teaching and learning materials. One lesson focused on data collection and analysis. The teacher used YouTube videos provided in the textbook to demonstrate data representation with tally charts and activities. These videos significantly enhanced students' problem-solving skills, as demonstrated by improved testing outcomes. Mathematics teachers strategically maximized time for all tasks and activities in alignment with the CBC curriculum, offering prompt feedback that promotes learners' mastery. Yet, it's crucial to acknowledge the need for increased training for teachers on digital devices. Even when simply presenting videos, better mastery of technology is essential for effective lesson content management and discipline, ultimately fostering a more supportive and patient teaching environment.

References

Ulandari, L., Amry, Z. & Saragih, S. (2019). Development of learning materials based on realistic mathematics education approach to improve students' mathematical problem solving ability and self-efficacy. *International Electronic Journal of Mathematics Education, 14*(2), 375-383. https://doi.org/10.29333/iejme/5721

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 211. PME.

IMPACT OF CUSTOMIZED GEOGEBRA APPLICATIONS ON LEARNING ANGLES: A QUASI-EXPERIMENTAL STUDY

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Despite growing interest in technology integration in mathematics education, many digital tools are used in their generic form, lacking adaptation to specific curricula and pedagogical needs. Traditional instruction often fails to engage students meaningfully, particularly in abstract topics like angles, leading to persistent learning difficulties. This study addresses these challenges by developing and implementing customized interactive applications using GeoGebra, specifically designed to align with the lower secondary mathematics curriculum and grounded in variation theory (Marton, 2015) to enhance conceptual understanding. A quasi-experimental design was employed, involving two intact Grade 7 classes from a low-achieving school. The control group received traditional instruction, while the treatment group used the developed GeoGebra-based applications on tablet PCs. Pre-test and post-test assessments indicated that while both groups improved, the treatment group demonstrated significantly higher gains. Qualitative data from focus groups further revealed that students found the interactive approach engaging, intuitive, and conducive to self-directed learning. The dynamic exploration and real-time feedback provided by GeoGebra-based applications enabled deeper conceptual understanding, as students could visualize, manipulate, and test geometric properties promptly. Teachers also reported increased student autonomy and collaborative problem-solving, reducing reliance on direct instruction. These findings support previous research (Reis & Ozdemir, 2010) that emphasizes the importance of pedagogically driven digital tools in enhancing mathematics learning. This study contributes to the growing body of evidence supporting the need for tailored, theory-based digital interventions in mathematics education. The results suggest that well-designed interactive applications can bridge learning gaps, making abstract mathematical concepts more accessible. These insights hold implications for educators and policymakers seeking to modernize mathematics instruction through evidence-based digital resources. Future research could explore the long-term retention effects of such interventions and extend the approach to other mathematical topics.

References

Marton, F. (2014). *Necessary Conditions of Learning*. Routledge. https://doi.org/10.4324/9781315816876

Reis, Z. A., & Ozdemir, S. (2010). Using GeoGebra as an information technology tool: Parabola teaching. *Procedia-Social and Behavioral Sciences*, *9*, 565-572.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 212. PME.

TASK DESIGN FOR PARTIAL MEANINGS OF THE NOTION OF LIMIT

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INTRODUCTION

The notion of limit poses epistemological, cognitive, and didactic challenges in teaching and learning. Traditional methods focus on algebraic procedures, often overlooking conceptual depth. While previous studies have explored students' conceptions of limits, there is a lack of clear criteria for task design that address its complexity. This study proposes a task design based on the Onto-Semiotic Approach (OSA) (Godino et al., 2007) to foster the understanding of six partial meanings of limit identified by Araya et al. (2021).

METHODOLOGY

This qualitative study follows a Design-Based Research (DBR) approach (Bakker & van Eerde, 2015), focusing on task evaluation. The implementation involved 11 teacher-training students in a Differential Calculus course, with tasks conducted online via Teams and Moodle. Data collection included student responses, group recordings, and surveys.

CONCLUSIONS

The implementation of the designed tasks effectively promoted the intended epistemic configurations, though challenges remained in students' use of graphical representations. Future research will refine the tasks for in-person implementation to foster collaborative reasoning and oral argumentation. Adjustments in time allocation and technology training will also be incorporated.

References

Araya, D., Pino-Fan, L., Medrano, I., & Castro, W. (2021). Epistemic Criteria for Designing Limit Tasks on a Real Variable Function. *Bolema, 35*(69), 179-205.

Bakker, A., & van Eerde, D. (2015). An Introduction to Design-Based Research with an Example from Statistics Education. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Approaches to Qualitative Research in Mathematics Education*, 429-466. Springer.

Elia, I., Gagatsis, A., Panaoura, A, Zachariades, T., & Zoulinaki, F. (2009). Geometria and Algebraic Approaches in the concept of "limit" and the impact of the "Didactic Contract". *International Journal of Science and Mathematics Education, 7*, 765-790.

Godino, J. D., Batanero, C., & Font, V. (2007). The Onto-Semiotic Approach to Research in Mathematics Education. *ZDM – The International Journal on Mathematics Education, 39*(1), 127-135.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 213. PME.

CO-CONSTRUCTING MATHEMATICS LESSONS TO EMBED INDIGENOUS KNOWLEDGES WITH TRADITIONAL OWNERS: AN AUSTRALIAN CONTEXT

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Internationally education policies and strategies have aimed to improve outcomes for Indigenous students. However, these policies often adopt a deficit perspective, leading to ongoing disparities between Indigenous and non-Indigenous students, especially in mathematics (Rigney et al., 2021). The mathematics curriculum is predominantly based on Western knowledge systems, impacting teaching practices and student learning. Additionally, Indigenous Knowledges and perspectives are often overlooked in the mathematics curriculum as they are not often understood and are frequently interpreted through a Western lens (Martin, 2003). Underpinned by Rigney's (1999) Indigenist research theory, this study aims to use strengths-based approaches to include Aboriginal and Torres Strait Islander knowledges and perspectives in mathematics classrooms; guided by the following research question:

From the perspectives of Indigenous people, what is considered as best practice for co-constructing mathematics learning experiences that embed Indigenous knowledges?

What do Indigenous knowledge holders consider to be the best practices for co-constructing mathematics learning experiences that incorporate Indigenous knowledges?

Data were collected through recorded yarning sessions (4 sessions x 3hrs) with four Indigenous knowledge holders and community members. Transcripts of the yarning sessions were analysed using the systematic approach of constant comparative method to identify themes. These themes informed the design of the evidence-based framework. In this presentation we share the findings of the analysis and provide a worked example of how the framework is used to co-construct mathematics learning experiences that include Indigenous knowledges.

References

Martin, K. (2003). Ways of knowing, being and doing: A theoretical framework and methods for Indigenous and Indigenist research. *Journal of Australian Studies, 76*, 203–214.

Rigney, L. I. (1999). Internationalization of an Indigenous anticolonial cultural critique of research methodologies: A guide to Indigenist research methodology and its principles. *Wicazo sa review*, *14*(2), 109-121.

Rigney, L., Garrett, R., Curry, M., & MacGill, B. (2021). Culturally responsive pedagogy and mathematics through creative and body-based learning: *Urban Aboriginal schooling. Education and Urban Society, 52*(8), 1159-1180

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 214. PME.

DOES WEBCAM EYE TRACKING WORK FOR MATHEMATICS EDUCATION? A CASE STUDY WITH AI-ASSISTED ENHANCED DATA PROCESSING

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Eye tracking (ET) has shown great potential in mathematics education research (Schindler et al., 2020). Since high-quality ET (hqET) systems are expensive, cost-efficient alternatives are needed for practical applications, e.g., in schools. Webcam-based ET (wcET) provides an affordable alternative, yet comes with a lower data quality. Our research aims to make wcET usable for practical applications in mathematics education. To achieve this, we intend to enhance wcET data through, e.g., automatic offset calibration (Asgahri et al., 2023) and semantic remapping (using knowledge about the presented tasks), so that relevant information can be extracted from the students' eye movement patterns *as with hqET data*. Here, we consider number line (NL) tasks and ask: To which degree can wcET data support math education in comparison to hqET data and how can we increase the degree?

Using our KI-ALF system (https://ki-alf.de), we collected gaze data from 136 fifth-grade students working on NL tasks using a Logitech BRIO webcam (wcET) and a Tobii Pro X3-120 eye tracker (hqET). We pre-processed the wcET data using AI for (1) correcting systematic wcET data inaccuracies using task information (automatic offset calibration); (2) grouping gaze points into areas of interest, e.g., specific numbers or landmarks, to improve data interpretation and reduce noise (semantic remapping). We evaluated the pre-processing pipeline using clustering algorithm to analyse strategy patterns in terms of cluster consistency compared the clusters to expert-labelled strategies based on hqET data. With our domain-specific post-processing, the consistency of strategy identification reached a level close to that of hqET data. Our findings show that wcET affords scalable and cost-effective solutions to identify students' NL strategies. Based on our development, KI-ALF wcET systems are currently in use in a comprehensive school in Germany to support students' mathematics learning utilizing eye-tracking-based identification of their strategies.

References

Asghari, P., Schindler, M., & Lilienthal, A. J. (2023). Eye tracking auto-correction using domain information. In *International conference on human-computer interaction* (pp. 373–391). Springer Nature.

Schindler, M., Schovenberg, V., & Schabmann, A. (2020). Enumeration processes of children with mathematical difficulties: An explorative eye-tracking study on subitizing, groupitizing, counting, and pattern recognition. *Learning Disabilities: A Contemporary Journal, 18*(2), 193–211.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 215. PME.

NUMBER TALKS IN GRADES 4–8: TEACHER PERSPECTIVES

Sandy Bakos and Richelle Marynowski

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Aiming to build number sense, Number Talks involves students using mental mathematics to solve numerical expressions and then explaining their solutions to the class (Humphreys & Parker, 2015). As part of a larger research project, 32 teachers in grades 4–8 in Alberta, Canada, volunteered to implement Number Talks for five months in their mathematics classes. This paper shares some of the initial interview data gathered from teacher participants.

Several teachers reported noticing significant improvements in their students' ability to communicate mathematically and articulate their thinking more clearly during Number Talks. One teacher described how his students needed more precise language to describe their interpretation of dot configurations to others in the first Number Talks he did. Students shifted from vague expressions such as 'the bottom one' or 'that one over there' to more specific vocabulary such as left, right, center, horizontal, vertical and diagonal. Additionally, students' use of terms like 'plussing' and 'minusing' were gradually replaced with more formal mathematical terminology during Number Talks.

The significance of teacher modelling and prompting during students' explanations of the processes used to find solutions in Number Talks was highlighted by multiple teachers. One described how she became more intentional about modelling correct mathematical language. Although her students did not always use the vocabulary correctly, their attempts to do so had increased, which she believed was beneficial. "When students are not vocalizing – when I am the only one vocalizing math in class, they're not incorporating the words into their own vocabulary." Another teacher noted that her students "were much more concise in their mathematical communication" during Number Talks in January compared to September

Teachers also shared how student mistakes became learning opportunities. The culture of Number Talks allowed everyone to share their thoughts while also showing that nobody is correct all the time. Even though her students initially struggled to express their ideas, a fifth-grade teacher noticed they became more willing to participate once they recognised there was a safety net. Number Talks allowed students to discuss their mistakes positively and fostered a safe space for participation. "I was amazed at how many of my students were willing to acknowledge their own mistakes in thinking and explain how they learned from what they did wrong."

References

Humphreys, C., & Parker, R. (2015). *Making number talks matter: Developing mathematical practices and deepening understanding, Grades 4–10*. Stenhouse Publishers.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 216. PME.

NUMBER TALKS IN GRADES 4–8: STUDENT PERSPECTIVES

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This paper presents one piece of a larger research project conducted in Alberta, Canada, where teachers implemented Number Talks in their grades 4–8 mathematics classes over five months. Number Talks are a brief routine wherein students mentally solve numerical expressions and explain their problem-solving processes, which the teacher records for all students to see (Humphreys & Parker, 2015). To gain a deeper understanding of student perceptions of Number Talks and whether this routine influenced their mathematical competence and confidence, semi-structured interviews were conducted. By adopting a theoretical perspective that values student voice, we position the student participants as agents and holders of knowledge, equipped with contextually grounded insights to share with researchers (Brasof & Levitan, 2022).

When interviewed, many students noted an increase in confidence and described how hearing diverse approaches and learning to communicate their strategies more clearly to others enhanced their understanding. An eighth-grade student remarked that learning different strategies from her classmates expanded her thinking and shifted her focus from simply calculating answers to considering multiple perspectives. This helped her understand her mistakes and boosted her confidence. The experience of expressing their thoughts in front of classmates seemed to empower many students. A fifth-grade student found that discussing mathematics with his peers helped him persevere through challenges and deepen his understanding of the material. Multiple students reported that sharing their ideas with the class not only boosted their confidence but also provided a supportive classroom community to assist them when mistakes occurred.

Furthermore, students noted that Number Talks expanded their thinking, helped them visualise mathematics and required them to think more critically and creatively. One sixth grader explained, “I feel like they kind of expand your brain a bit. They make you think more and, like, more productive thinking. It's like your brain expands almost – opens up to more ideas.” Overall, students described how Number Talks fostered a deeper, more interconnected understanding of mathematics in a community that promoted confidence, critical thinking, and collaboration.

References

Brasof, M., & Levitan, J. (Eds.). (2022). *Student voice research: Theory, methods, and innovations from the field*. Teachers College.

Humphreys, C., & Parker, R. (2015). *Making number talks matter: Developing mathematical practices and deepening understanding, Grades 4–10*. Stenhouse Publishers.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 217. PME.

INVESTIGATING DIFFERENCES IN ACHIEVEMENT IN AN EARLY GRADE MATHEMATICS PST COURSE

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INTRODUCTION

In South Africa learners in their last three years of schooling can choose to study core mathematics or Mathematical Literacy (ML), however entrance to science degrees or careers require mathematics. Students with an ML background can be accepted for some studies in the Humanities, including the foundation phase teaching degree which is the focus of this paper. The subject ML is very different from core mathematics in scope because of its focus on the use of contexts to develop self-management skills and civic responsibility. Previous research (Long et al., 2014) found that ML learners struggle with algebraic reasoning because of the lack of focus on algebra in the subject. In this paper we look at the results of a mathematics course for early grade pre-service teachers (PSTs) based on number sense and operations with a particular focus on the differences in results between those students who had studied ML and core mathematics in school respectively. The research question is: To what extent is there a difference in performance in an early grade mathematics course according to their mathematics background?

The instrument used was the final assessment in the course taken by 490 students. The data made up from the individual scores obtained on each item was subjected to a Rasch analysis. The results showed significant differences between the pass rates for two groups. The Rasch analysis allowed us to check for differential item functioning (DIF) which occurs when people from two groups, who have the same ability ranking, perform differentially on the item concerned.

The findings showed that overall, students with an ML background achieved lower scores overall than those with core Maths. On an item level, ML students struggled more on the items focusing on algebraic manipulation than the maths students. However, they displayed strengths in items which required intuitive rather than procedural responses. It is important to recognize the differences and to design support that is tailored for their needs.

References

Long, C., Bansilal, S., & Debba, R. (2014). An investigation of Mathematical Literacy assessment supported by an application of Rasch measurement. *Pythagoras, 35*(1), 29-45

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 218. PME.

ENGINEERING DESIGN IN MATHEMATICS: EXPLORING THE ROLE OF DIGITAL MAKING IN TEACHER EDUCATION

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The integration of digital making, CAD technologies, and 3D printing, into teacher education represents an advancement in mathematics instruction. Furthermore, the ability to design and build math manipulatives through Engineering Design (ED) offers a means to bridge abstract mathematical concepts with tangible experiences (Läufer & Ludwig, 2021). However, there is still limited research on how pre-service teachers engage with these technologies, within the ED cycle (Barbosa et al., 2024). This study aims to explore how pre-service elementary teachers interact with digital making tools to design and create math manipulatives, addressing two questions: 1) What knowledge and skills emerge throughout the ED cycle when pre-service teachers use these technologies? 2) How do they perceive the integration of Tinkercad and 3D printing as tools for designing and building manipulatives? The study highlights the potential of ED as a systematic, iterative process that fosters creativity and critical thinking (Barbosa et al., 2024). Digital making aligns with these principles by allowing learners to conceptualize, design, and refine instructional resources (Läufer & Ludwig, 2021). A qualitative exploratory study was conducted with thirteen pre-service elementary teachers. Data collection methods included classroom observations, artifacts, written reports, and visual records. Participants worked collaboratively to design manipulatives, documenting their decisions and revisions throughout the process. Findings indicate that the iterative nature of ED reinforced technical knowledge, as participants engaged in CAD modelling and 3D printing, refining their designs through iterations. Although they struggled with mathematical precision and unit conversions in Tinkercad, they developed a deeper understanding of spatial reasoning and problem-solving through hands-on trial and error (Läufer & Ludwig, 2021). Participants reflected on the role of manipulatives in mathematics instruction, proposing classroom applications and recognizing the potential of digital making for active learning.

References

Barbosa, A., Vale, I., & Alvarenga, D. (2024). The use of Tinkercad and 3D printing in interdisciplinary STEAM education: A focus on engineering design. *STEM Education, 4*(3), 222-246. https://doi.org/10.3934/steme.2024014

Läufer, T., & Ludwig, M. (2023). "Building" knowledge by creating manipulatives with the 3D printer: A course for mathematics student teachers. *Proceedings of the 9th International Conference on Higher Education Advances (HEAd'23)* (pp. 19-23). https://doi.org/10.4995/HEAd23.2023.16225

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 219. PME.

MATHEMATICAL PROCESSES IN HICH SCHOOL STEAM PROJECTS: A CASE STUDY

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INTRODUCTION

In recent years, STEAM projects have been incorporated into secondary education teaching practice, highlighting the need to work on problem solving. This approach is aligned with the mathematical process standards of the National Council of Teachers of Mathematics (2000), which include reasoning, representation, connection, communication and problem solving.

METHOD

In this context, from a qualitative approach, we propose to analyse the integration of mathematical competence in STEAM projects in four phases: 1) analysis of mathematical integration in the theoretical design of the project; 2) observation and analysis of classroom classes (Berciano et al., 2024); 3) analysis of students' written productions; 4) contrast of the results of all phases and proposals for improvement with the school's teaching team. This study analyses phase 3 of a project called 'Sorginkeria eta Sukaldea', where a connection is established between witchcraft, chemical processes and cooking to address various curricular contents in history, chemistry, physics and mathematics. This project is carried out over 4 weeks with 32 students in the second year of secondary school in a public school in the Basque Country. In relation to mathematical content and processes, proportionality and number sense and problem solving are addressed, where students have to identify which of the five proposed food discount representations is most beneficial: '25% discount per pack', '50% off the second unit', '2x1', '70% off the second unit' and '3x2'.

RESULTS AND CONCLUSIONS

The results indicate that students analyse, on average, 3.2 representations (SD= 1.2). However, they perform correctly, on average, 2.4 (SD=1.3). About reasoning, only 22% of students are able to generate some kind of answer and only 13% give correct mathematical reasoning. In conclusion, although students show an ability to deal with various representations, accuracy and quality of reasoning are areas that require further development.

References

Berciano, A., Salgado, M., & Alsina, A. (2024). Selección y análisis de vídeos en la formación inicial del profesorado de educación matemática infantil: Rúbrica R4. *Uniciencia, 38*(1), 1-25. http://dx.doi.org/10.15359/ru.38-1.10

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston Virginia: NCTM.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 220. PME.

EXPERIENCE OF AN ACTION RESEARCH: TEACHING PROPORTIONS WITH VISUAL SUPPORT IN GRADE 7

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Proportional reasoning is an important pillar in learning mathematics, and in many aspects of everyday life. Students show different levels of success on different types of tasks. Knowing that proportional reasoning is affected by whether the quantities are discrete or continuous (Vanluydt, 2020), and since external visualization is an effective tool to support learning mathematics (Schoenherr et al., 2024), we aimed to develop a training program to foster proportional reasoning by using different visualizations.

This paper presents the results of an action research on intense use of visual representations among 7th grade students (N = 60) in three schools. Researchers and participating teacher worked together on five lesson plans to implement. During the experiment, all lessons were video-recorded, and teachers wrote reflections after each lesson. They reflected both on their classroom experiences and on their changing views and beliefs.

Before and after the experiment, students completed tests of arithmetic skills and proportional reasoning (Cronbach' alpha ranged from .81 to .85) and an adapted version of the MRBQ (Mathematics-Related Beliefs Questionnaire).

The results show that students' performance increased in tasks containing discrete quantities ($p < .05$, Cohen's d ranging from .23 to .39, indicating medium effects), and no significant difference was found when the tasks contained continuous quantities. The questionnaire indicated that students were less nervous in class ($p = .048$) and seemed to feel less challenged in learning mathematics ($p = .043$) after the action research. They preferred to visualize the tasks after the experiment ($p = .026$) and felt that their teachers thought they were better in mathematics ($p = .003$). In their reflective diaries, teachers gave positive feedback on their changing beliefs and found the method extendable to other domains, e.g., to teaching fractions.

Additional information

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References

Schoenherr, J., Strohmaier, A. R., & Schukajlow, S. (2024). Learning with visualizations helps: A meta-analysis of visualization interventions in mathematics education. *Educational Research Review*, 45, 100639.

Vanluydt, E., Degrande, T., Verschaffel, L., & Van Dooren, W. (2020). Early stages of proportional reasoning: A cross-sectional study with 5-to 9-year-olds. *European Journal of Psychology of Education*, *35*(3), 529-547.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 221. PME.

ETHNOMATHEMATICS AND CULTURE IN RURAL SCHOOLS

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This study explores the intersection of Ethnomathematics and rural culture within a Brazilian school, examining how local practices and knowledge can enrich mathematics education. The central question guiding this research is: *How can the teachings and practices of rural schools connect with and be enhanced by Ethnomathematics?* To answer this, a qualitative ethnographic study (Creswell, 2014) was conducted at a community-based rural school in southern Brazil from February to July 2023. Data was gathered through documentary analysis, participant observation, and semi-structured interviews with teachers and staff.

Content analysis of the collected data revealed key themes: the connection between theory and practice, interpersonal relationships within the school, interdisciplinarity, the value of local knowledge, and cultural context. Three prominent aspects emerged: (i) valuing local culture, rural youth, and traditional knowledge (Knijnik, 2006; D'Ambrosio, 2012); (ii) fostering positive school relationships as a foundation for peace education (D'Ambrosio, 2012); and (iii) employing interdisciplinary approaches and dialogue to promote transdisciplinarity (D'Ambrosio, 2016).

These findings suggest that the power of Ethnomathematics lies in its departure from traditional curricula that often prioritize urban perspectives and abstract academic knowledge. By grounding education in practical applications and the lived experiences of students, this rural school demonstrates the potential for a more relevant, culturally responsive, and knowledge-rich learning environment. This invites us to reconsider the valuable lessons that rural schools can offer in creating an ethnomathematical approach to education that truly values students and their communities.

Additional information

Grant 2021/11937-0, FAPESP

References

Creswell, J. W. (2014). *Investigação qualitativa e projeto de pesquisa – Escolhendo entre cinco abordagens*. Penso.

D'Ambrosio, U. (2016). *Educação para uma sociedade em trânsito*. Editora Livraria da Física.

Knijnik, G. (2006). *Educação matemática, culturas e conhecimento na luta pela terra*. EDUNISC.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 222. PME.

FIFTH GRADE STUDENTS' BELIEFS ABOUT ERRORS IN MATHEMATICS: A LARGE-SAMPLE STUDY

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Depending on the educational context, avoiding errors is sometimes necessary, while they can also be valuable for learning. However, both teachers and students are apprehensive about making and dealing with errors (Pan et al., 2020). As Steuer et al. (2013) highlight, classroom error culture is shaped by both cognitive and affective factors. How students feel about errors has far-reaching consequences for their behaviour and performance, yet no research in Hungary has explored their beliefs on this issue. Therefore, beyond assessing the reliability of a questionnaire combining internationally recognised measures with culturally specific items, this study aims to provide novel, practice-oriented insights into 5th-grade students' beliefs about errors.

The sample comprised 2247 fifth graders, representing socioeconomic background, gender and other variables on a nationally representative scale. Students completed an online questionnaire with 35 five-point Likert scale items, demonstrating strong internal consistency (Cronbach's alpha = .89). Exploratory factor analysis (KMO = .94) identified six factors: (1) Persistence in overcoming errors, (2) Errors as means of learning, (3) Concealment of errors, (4) Honest admission of errors, (5) Errors as demotivating factors, and (6) Resilience after making errors. Descriptive statistics indicated that while students recognise that errors can be important as means of learning (M = 3.92) and try to overcome difficulties once identified, they also show a tendency to conceal mistakes from classmates and teachers (M = 3.18).

These findings suggest that although 5th-grade students acknowledge the role of errors in learning mathematics, social and emotional factors in the classroom may discourage them from open engagement with errors. Addressing these affective barriers may foster a more supportive learning environment, enhancing both motivation and learning outcomes.

Additional information

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References

Pan, S. C., Sana, F., Samani, J., Cooke, J., & Kim, J. A. (2020). Learning from errors: Students' and instructors' practices, attitudes, and beliefs. *Memory, 28*(9), 1105-1122.

Steuer, G., Rosentritt-Brunn, G., & Dresel, M. (2013). Dealing with errors in mathematics classrooms: Structure and relevance of perceived error climate. *Contemporary Educational Psychology, 38*(3), 196-210.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 223. PME.

REDEFINING SUCCESS: UNVEILING THE HIDDEN CAPITAL OF NORWEGIAN FIRST-GENERATION COLLEGE STUDENTS

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Nordic countries are consistently praised for their exceptional education systems, including their emphasis on increasing access to higher education by decreasing economic disparity across social strata (Reisel, 2011). However, there is limited research on how Norwegian first-generation college students (FGCS), students whose parents or guardians did not receive a college degree or higher, navigate first-year mathematics courses, key gatekeepers for Science, Technology, Engineering, and Mathematics degrees. We employ Community Cultural Wealth (CCW) (Yosso, 2005) as an asset-based framework to highlight non-dominant forms of capital FGCS possess, including aspirational, linguistic, familial, social, navigational, and resistant (Yosso, 2005). We seek to answer: What forms of capital do Norwegian FGCS identify as beneficial to their success in first-year mathematics courses?

The qualitative study includes ten FGCS enrolled in first-year mathematics courses (e.g., Calculus 2, Mathematics 2) at a Norwegian university. Hour-long, in-person interviews followed a semi-structured protocol aligned with CCW (Yosso, 2005). We conducted a thematic analysis to determine themes within each form of capital that supported students in their first-year mathematics courses.

We found that students identified all six forms of capital and two additional forms of capital, vocational and age-earned, when discussing their success in first-year mathematics courses. For example, standards-based assessments in conjunction with the tutoring center on campus fostered a collaborative environment that allowed Norwegian FGCS to grow their social capital by engaging in peer-to-peer collaboration digitally and in-person. Additionally, students who previously held a vocational position spoke of utilizing their practical knowledge of topics to support their newfound theoretical knowledge of mathematics. We argue that by acknowledging and implementing instructional practices that support FGCS' capital, educators and institutions can develop more inclusive learning environments to help FGCS thrive.

References

Reisel, L. (2011). Two paths to inequality in educational outcomes: Family background and educational selection in the United States and Norway. *Sociology of Education, 84*(4), 261–280. https://doi.org/10.1177/0038040711417012

Yosso, T. J. (2005). Whose culture has capital? A critical race theory discussion of community cultural wealth. *Race Ethnicity and Education, 8*(1), 69–91.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 224. PME.

FOSTERING GEOMETRIC REASONING USING TECH-ENABLED CLIX MODULES: TRACING CRITICAL NEEDS

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Geometric reasoning proficiency plays a crucial role in nurturing students' problem-solving, reasoning, and spatial understanding skills. However, fundamental geometric concepts often remain poorly built even among the secondary graders leading to a significant gap in their ability to reason geometrically with effective communication (Usiskin, 2017). To address this gap, *Geometric Reasoning* (GR) module of the *Connected Learning Initiative* (CLIx) project aimed at improving and strengthening students' learning of formal Euclidean Geometry, and at a scale using technological affordances and targeting learners at the margins in India. This blended module with digital and classroom activities including students' workbook and teachers' handbook aimed at supporting ideal implementation by the teachers. The CLIx pedagogical pillars of providing safe space for learning from mistakes and collaboration with peers were essential components of a high-fidelity implementation, and teachers were supported to adopt these in their classrooms to provide an ideal learning environment for the students (Srinivas *et al.*, 2019). The module has discerned students' level-wise progression in geometry tasks and activities and support that can help them progress to the next higher level of thinking following van Hiele framework.

This short presentation will respond to the research question of tracing the critical needs for facilitating learning environments of ninth graders and recording changes in their mathematical communication and practice. Drawing students' conceptual gaps in understanding hierarchical class relationships between quadrilaterals and conflated concept of parallel with congruence, data shows discernible difference in teachers' actions between the Intervention and Non-Intervention classrooms. Besides learning gains and a positive effect on students' mathematical discourse, this Learning Research study of CLIx GR module showed increased confidence, interest and motivation, enhanced collaboration skills and teamwork, initiative, autonomy among the students to learn the subject and confidence in handling computers. About the teachers, the study showed that planned teacher support led to *Increased Engagement* in *Communities of Practice* (CoPs) with enhanced pedagogical content knowledge.

References

Srinivas, S., Rahaman, J., Kumar, R. & Bose, A. (2019). Analysing the role of language in students' conception of parallel lines. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 327-334). Pretoria: PME.

Usiskin, Z. (2018). Approaching Euclidean Geometry Through Transformations. *Mathematics Matters in Education: Essays in Honor of Roger E. Howe*, 233-244.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 225. PME.

COMPETENCE FOR THE TEACHING OF MATHEMATICAL MODELING, A CASE STUDY IN PRIMARY

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Teachers need knowledge and competencies for mathematical modeling (MM) (Borromeo-Ferri, 2018). This paper evaluates the effectiveness of a Teacher Professional Development (TPD) program for primary teachers aimed at strengthening such competencies, particularly those related to their conception and to the characterization of a mathematical task (MT) that promotes it.

MM is the "treatment of a real-world problem using mathematical means, whereby a modeling process is carried out" (Kaiser et al., 2023, p. 399). A MT of MM is open-ended, complex, realistic, meaningful, clear, and go through the phases of a MM cycle (Borromeo-Ferri, 2018; Maaß, 2010).

The TPD was designed based on the Model for competencies needed in teaching mathematical modeling by Borromeo-Ferri (2018), implemented in 16 sessions with eight primary school teachers from two schools in Chile. Data was obtained from the teachers' responses to a questionnaire composed of two items (What do you understand by mathematical modeling in primary? and What characteristics must a mathematical task have to be considered mathematical modeling?) applied at the beginning and at the end of the TPD. It was analyzed using the content analysis technique (Bardín, 2002).

Seven of the eight teachers modified their conception of MM from understanding it, for example, as an explanation of steps to solve a MT, to defining it as a skill that students develop when they solve real problems using mathematics. Regarding the characteristics of a MT of MM, at the beginning of the TPD they do not recognize its key features. At the end, they identify that these are open-ended and involve the MM process. In summary, the TPD program was effective in terms of strengthening teachers' conception of MM and its characterization in a MT.

References

Bardín, L. (2002). *Análisis de contenido*. Ediciones Akal.

Borromeo-Ferri, R. (2018). *Learning how to teach mathematical modeling in school and teacher education*. Springer. https://doi.org/10.1007/978-3-319-68072-9

Kaiser, G., Blum, W., Borromeo Ferri, R., & Greefrath, G. (2023). Mathematisches Modellieren. In Bruder, R., Büchter, A., Gasteiger, H., Schmidt-Thieme, B., Weigand, H.G. (Eds.), *Handbuch der Mathematikdidaktik (*pp. 399-428). Springer. https://doi.org/10.1007/978-3-662-66604-3_13

Maaß, K. (2010). Classification Scheme for Modelling Tasks. *Journal Für Mathematik-Didaktik, 31*(2), 285-311. https://doi.org/10.1007/s13138-010-0010-2

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 226. PME.

CONCEPTUALIZATION OF PERIODIC FUNCTION IN A SOUND CONTEXT

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The present research is focused on the process of conceptualisation of periodic functions from the conceptualisation of a particular case of these, the trigonometric functions.

Research reports that there is a predominance of algebraic work on these functions, which means that the property of periodicity remains invisible compared to others, appearing as a transfer of the sine function. As a result of the above, we have proposed to characterise the conceptualised mathematical work of trainee teachers in solving a task of modelling periodic functions.

Due to the research objective, we have proposed to establish a networking between the theory of Mathematical Working Space (Kuzniak et al., 2022) and Activity Theory (Vandebrouck, 2018), to propose the notion of conceptualized mathematical work.

The design of the proposal aimed to generate a technology-mediated modeling process for conceptualize periodic functions from sound phenomena. For this purpose, the incorporation of technological tools such as Geogebra and Audacity was considered. The sequence designed is composed of four didactic situation whose objectives promote the achievement of the proposed research objective.

The implementation is composed of one case of 13 trainee teachers in the fifth semester of a nine-semester programme at a Chilean university. The data were extracted from the written productions of the trainee teachers and from the recording of the screen of the work computer.

Among the results obtained, we conclude that the modelling tasks were a favourable environment for the development of the conceptualisation of periodic functions, based on the design of tasks centred on a sound context. Given that not all the trainee teachers conceptualised periodic functions as established in the a priori analysis, we can conclude that there are other factors such as the relationship between physical-musical knowledge or sound perception over theoretical knowledge that are involve in the conceptualisation process.

References

Kuzniak, A., Montoya Delgadillo, E., & Richard, P. (Eds.) (2022). *Mathematical Work in Educational Context – The MWS Theory Perspective*, Mathematics Education in the Digital Era, *18*, Springer.

Vandebrouck, F. (2018). AT in French didactic research. In *Invited lectures from the 13th International Congress on Mathematical Education* (pp. 679-698). Springer International Publishing.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 227. PME.

TEACHERS' PERCEPTIONS ON THE TEACHING AND LEARNING OF GEOMETRY IN THE PK-2 TRANSITION

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In order to establish continuity strategies in educational processes, learning trajectories represent a tool with the potential to enhance the transition experiences of children in early childhood and elementary education as they learn mathematical ideas. Clements and Sarama (2015) define mathematical learning trajectories as sequences that take into account a mathematical objective, levels of progression and a set of mathematical tasks. An understanding of how teachers comprehend these trajectories and how they implement them in the classroom is essential to facilitate an adequate transition.

In this paper, we analyse teachers' perceptions of geometry teaching and learning in transition from early childhood to primary education (ages 4-8). Semi-structured open-ended interviews are conducted, and responses are coded following the guidelines of grounded theory (Strauss & Corbin, 2015). Four early childhood education teachers and four primary education teachers from two Chilean schools participated.

The results indicate that teachers organise their ideas around three aspects. Firstly, they say that from preschool to grade 2, 2D shapes (squares, circles, triangles and rectangles) are the most important. Secondly, they talk about the use of manipulatives and the need to connect shapes with the environment. Finally, they express a positive self-concept of their geometric and didactic knowledge related to their own ability to teach these subjects between levels.

Teachers' views on how well they know about discipline and teaching, as well as their professional identity, can be understood. We believe it is important to recognise these aspects as a basis for creating teacher training proposals. We want to get teachers involved in understanding and designing geometry learning paths from early childhood to elementary education. This will encourage them to think about how important it is to make a suitable transition (Ballam, et al., 2017).

References

Ballam, N., Perry, B., & Garpelin, A. (Eds.). (2016). *Pedagogies of educational transitions: European and antipodean research*. Springer International Publishing. https://doi.org/10.1007/978-3-319-43118-5

Clements, D. H., & Sarama, J. (2015). *El aprendizaje y la enseñanza de las matemáticas a temprana edad: el enfoque de las trayectorias de aprendizaje*. Learning Tools.

Corbin, J. & Strauss, A. (2015). *Basics of Qualitative research*. Sage.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 228. PME.

PERCEPTION OF INCLUSIVE TEACHING PRACTICES: AN EXPLORATORY STUDY OF FIRST-YEAR ENGINEERING STUDENTS IN A LINEAR ALGEBRA COURSE

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In this exploratory research, we examine the perception of first-year engineering students regarding their perception of inclusive teaching practices implemented in a Linear Algebra course. Given the increasing focus on inclusive education in higher education contexts, it is essential to understand how students perceive inclusive teaching practices in challenging courses such as Linear Algebra to inform future pedagogical improvements. The study included 46 engineering students from a Chilean university. Data collection was conducted using an adapted version of the "Inclusive Teaching Practices Scale" (S-ITPS) instrument developed by Schwab et al. (2022), which assesses two fundamental dimensions: personalization and differentiation in teaching. Participants responded to 14 items using a four-point Likert scale.

Our results reveal a highly positive perception of inclusive practices and show a balance between the perception of personalization and differentiation practices. Students identified as strengths the diversity in content presentation techniques, clarity in communicating rules and instructions, variety of assessment methods, and consideration of individual academic performance. The only dimension with a relatively lower score, though still positive, was teacher collaboration with colleagues.

These findings suggest that implementing inclusive practices was effective in a traditionally challenging context such as university mathematics teaching. Our study provides evidence that it is possible to create inclusive mathematical learning environments at the university level when strategies that address both personalization and differentiation of teaching are implemented. This research provides a first approach to understanding how engineering students perceive inclusive practices in a traditionally challenging course such as Linear Algebra.

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References

Schwab, S., Sharma, U., & Hoffmann, L. (2022). How inclusive are the teaching practices of my German, Maths and English teachers? – psychometric properties of a newly developed scale to assess personalisation and differentiation in teaching practices. *International Journal of Inclusive Education*, *26*(1), 61-76.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 229. PME.

THE INFLUENCE OF LANGUAGE IN INTERPRETING MATHEMATICAL EQUATION: AN ERP STUDY

Yu Chen, Wei-Jung Lai, Zai-Fu Yao, Tsu-Jen Ding, Chen-Yu Yao and Hui-Yu Hsu

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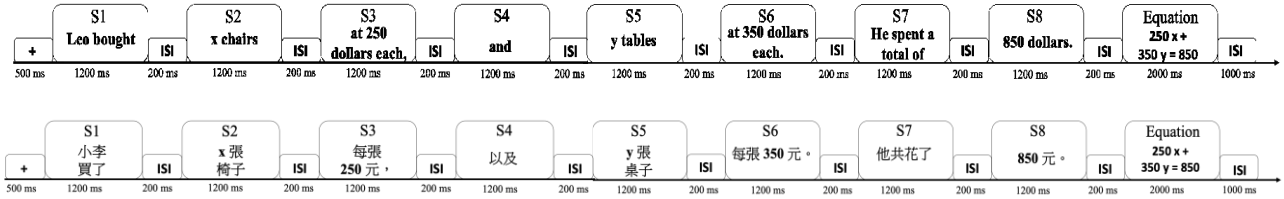
The mathematics education community emphasizes the value of linking mathematical concepts to real-world applications in teaching (Gainsburg, 2008). Our study focuses on the transformation of real-world contexts into linear equations with two variables and investigates how language influences this process. We designed a 168-trial instrument where students were asked to judge whether an equation was consistent with real-world contexts presented in Chinese and English, with Chinese serving as the participants' first language (L1) and English as their second language (L2). Stages for both languages were carefully aligned (e.g., coefficients, variables, relationships, total quantities) to minimize differences and reduce ambiguity between the two languages. The participants in this study were college students in Taiwan.



We employed an ERP methodology to analyze the participants' brain activities. Behavioral data showed no significant difference in accuracy but revealed a significant difference in reaction time between the two languages, with English (EN) requiring longer reaction times than Chinese (CH). Regarding ERP data, CH generally facilitated greater engagement, especially in language-related tasks, which may reflect the advantage of one's native language in cognitive processing. However, during the answer verification stage (Equation) in both CH and EN, the amplitudes were highly similar, suggesting that students had already transformed the contexts into equations while reading the language materials. The P3 component provides valuable insights into cognitive processes such as attention allocation, concentration, and the speed of mental processing. Analysis of P3 revealed significant differences in S3, S4, S6, and S7, while no significant difference was found in other stages. In S3, S6, and S7, the CH elicited significantly higher amplitudes than the EN, suggesting greater cognitive engagement or familiarity. However, in S4, the EN's brainwave response exceeded that of CH, indicating that language strongly affects how students process and transform contextual information into equations.

References

Gainsburg, J. (2008). Real-world connections in secondary mathematics teaching. *Journal of Mathematics Teacher Education*, *11*(3), 199-219. https://doi.org/10.1007/s10857-007-9070-8

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 230. PME.

LEARNING TO TEACH THROUGH PROBLEM SOLVING FOR LOW-PROGRESS LEARNERS: THE CASE OF MR FAIZAD

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Unlike the usual approach of instruction, Takahashi's (2021) teaching through problem solving (TTPS) approach positions problem solving as the "means" to introduce new concepts to students by giving them opportunities to solve problems. Although this approach puts problem solving at the center of a mathematics lesson, many teachers struggle to design and implement TTPS lessons, especially when working with low-progress students—those who struggle with mathematics in schools. This is the case for many Singapore teachers, who teach the strategies first and get their low-progress students to apply them to solve similar questions (Kaur et al., 2019). In this paper, we focus on Mr Faizad, a mathematics teacher who taught a class of 15 low-progress learners at Zane Hall Primary School (pseudonyms used) in Singapore. Mr Faizad started out with a "teach first, then apply" approach to problem solving but his practices changed significantly towards the end of the project. We are interested to examine the changes in his teaching as he learned to enact TTPS, framed by the following question: What were the changes in Mr Faizad's task design and implementation as he learned to enact TTPS lessons for his low-progress students?

We worked with Mr Faizad to develop and trial the TTPS tasks to be used. For each lesson, we used a three-camera approach (Kaur et al., 2019) to capture the teacher view, students view, and the whole class view. We collected his instructional materials and sampled students' work related to the lesson observed. Fieldnotes of the lessons were taken and we also conducted a post-lesson discussion with Mr Faizad to understand his thinking and instructional decisions. To create a snapshot of his lessons, we analysed the lessons in terms of their structures, the tasks used, his teaching actions, and students' responses to the tasks. In this presentation, we will share snapshots from two of his TTPS lessons, which took place almost a year apart and highlight how Mr Faizad had changed in his task design and implementation. Findings suggest he had noticed and harnessed the affordances of the problems he used in ways that supported his students to make sense of mathematics. Some implications for research and practice will be discussed during the presentation.

References

Kaur, B., Toh, T. L., Lee, N. H., Leong, Y. H., Cheng, L. P., Ng, K. E. D., Yeo, K. K., Yeo, B. W. J., Wong, L. F., Tong, C. L., Toh, W. Y. K., & Safii, L. (2019). *Twelve Questions on Mathematics Teaching*. National Institute of Education, Nanyang Technological University.

Takahashi, A. (2021). *Teaching mathematics through problem solving: A pedagogical approach from Japan*. Routledge.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 231. PME.

ANALYZING THE COGNITIVE DEMAND OF A PROBLEM DESIGNED TO IDENTIFY MATHEMATICAL GIFTED

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Addressing diversity in the classroom is a constant challenge for teachers, including the integration of mathematical gifted (MG) students, who often feel underserved in mathematics classes (Singer et al., 2016). This paper presents advances in research on identifying MG students using the cognitive demand (CD) model proposed by Smith and Stein (1998), which classifies mathematical tasks into four levels based on their complexity: memorization, procedures without connection, procedures with connection, and doing mathematics.

The research aims to design mathematical tasks that require high cognitive effort and make it possible to identify characteristics specific to MG students by evaluating their solutions (Jaime & Gutiérrez, 2017). In particular, it presents the design and analysis of a probability and statistics problem with six questions, considering a progressive increase in difficulty. A questionnaire was administered to 78 seventh-grade students (ages 12 and 13) from three types of institutions in the Chilean school system.

The qualitative analysis focused on identifying the various strategies used by students in each question, which allowed them to be categorized into different levels of CG. In the initial questions, 31% and 24% of the students correctly answered the items aimed at performing compound random experiments and determining the probabilities of compound events, respectively. In contrast, in the final questions, designed to deduce the multiplicative combinatorial principle and the product rule, and which required greater cognitive effort, only 13% and 10% managed to solve them correctly, demonstrating that the CD model is a valid tool for designing challenging problems and identifying MG students.

Additional information

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References

Jaime, A. & Gutiérrez, Á. (2017). Investigación sobre estudiantes con alta capacidad matemática. In J. M. Muñoz-Escolano, A. Arnal-Bailera, P. Beltrán-Pellicer, M. L. Callejo, y J. Carrillo (Eds.), *Investigación en Educación Matemática XXI* (pp. 71-89). SEIEM.

Singer, F. M., Sheffield, L. J., Freiman, V. & Brandl, M. (Eds.) (2016). *Research on and activities for mathematically gifted students*. Springer Open.

Smith, M. S. & Stein, M. K. (1998). Selecting and creating mathematical tasks: from research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344-350.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 232. PME.

DEVELOPMENT OF ARGUMENTATION IN PRESCHOOL MATHEMATICS

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We present the main findings of a study on the development of argumentation in kindergarten children, through the characterization of Argumentative Situations when constructing numbers. The study had two main objectives: (1) to characterize the argumentation of kindergarten children as they construct numbers up to 20, and (2) to describe the development of their argumentation throughout a school year. Argumentation is defined as a communicative and situated activity through which children provide reasons (for others or for themselves) to justify and convince (or become convinced) of a certain position or reflexively question it (Cornejo-Morales et al., 2021). The study introduces a theoretical-methodological construct: the Argumentative Situation (AS). This construct comprises five interconnected components: 1) argument (what is argued? and why?); 2) interaction (who argues?); 3) function of the argumentation (what is the argument for?); 4) character of the argumentation (how is it argued?); and 5) mathematics (what is argued about?).

Twenty-two kindergarten lessons were video recorded over a school year to capture students' number construction up to 20 and analyze argumentation and its development. This involved reconstructing central arguments, transcribing and characterizing Argumentative Situations (AS), and tracking their components over time. Thematic analysis was used to identify key themes and co-occurrences related to argumentation.

The main results relate to the character and function of argumentation. Regarding its character, a homogeneous presence of diagrammatic and narrative argumentation is observed throughout the year. Moreover, co-occurrences between the diagrammatic and narrative types were evident, having implications for the study of this skill. On the other hand, Verifying and Explaining are the functions that present the highest frequency, showing a homogeneous distribution throughout the school year. Moreover, the function of Refuting is added to the initial set of functions to fully account for children argumentation. These findings highlight the importance of studying mathematical argumentation in early ages, characterizing both arguments and argumentation at this level and demonstrating the relevance of argument functionality and character.

Additional information

Acknowledgements. FOVI240238, Milenio NCS2021_014, FONDECYT 11190135

References

Cornejo Morales, C., Goizueta, M., & Alsina, Á. (2021). La Situación Argumentativa: un modelo para analizar la argumentación en educación matemática infantil. *PNA: Revista de Investigación en Didáctica de la Matemática, 15*(3), 159-185.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 233. PME.

BLACK VOICES RISING AND REACHING THE CLASSROOM

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Mathematics has frequently been used as a gatekeeper. In addition, across Canada, education has a long and persistent history of excluding Black people, resulting in racial achievement gaps at almost all levels of education. These facts together can be determinant in decreasing access to professional opportunities while perpetuating the exclusion of students and reinforcing negative stereotypes about Black students and mathematics. Triggered by this reality, this study investigated the racialized mathematics learning experiences of Black undergraduate students in STEM programs in Ontario, Canada. We sought to unfold these experiences, understand how they shape students' academic endeavours, and reveal what resilience strategies students employ to navigate challenging learning environments.

Our literature review highlights the risk of shifting away from social-political factors toward blaming students for poor academic achievement instead. It also emphasizes how Black students shape their identities based on common but false academic stereotypes and how they must deal with detrimental microaggressions (Corrêa & Oloo, 2023). The study employed narrative inquiry; data collection involved dialogic conversations centred on semi-structured questions meant to have students share their lived learning experiences and the meanings they attached to these experiences. Data was analyzed from the viewpoint of the Community Cultural Wealth model (Yosso, 2005), which offers an asset-based perspective, understanding that racialized students have funds of knowledge and forms of capital that they draw upon to achieve academic success. Participants shared, for example, how they successfully deal with stereotypes and microaggressions in an environment that is not always welcoming or supportive.

Inspired by participants' narratives, we were encouraged to organize a Sharing Experiences Series, a research knowledge mobilization channel for sharing racialized learning experiences through stories. The series sought to amplify Black voices and share knowledge that has the potential to create opportunities to change learning environments. Since educators are catalysts for change, we invited them to the series to generate awareness and potentially contribute to developing equitable and conscientious teaching approaches that counteract racism, stereotypes, and microaggressions, providing students with transformative learning experiences.

References

Corrêa, P. D., & Oloo, J. A. (2023). Racialized mathematics learning experiences of Black undergraduate students. *International Educational Review, 1*(1), 41-55.

Yosso, T. J. (2005). Whose culture has capital? A critical race theory discussion of community cultural wealth. *Race, Ethnicity and Education, 8*(1), 69–91.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 234. PME.

CONSTRUCTING LEARNER IDENTITIES WHEN USING DIGITAL PLATFORMS FOR MATHEMATICS

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Digital and online platforms for the learning of mathematics are already ubiquitous in the post COVID-19 world (Engelbrecht et al., 2020) and likely to become even more so with advances in Artificial Intelligence. Whilst the question of whether student learning may be enhanced in this context is relevant, there are many other questions we might also consider. For example, how do such platforms contribute to students' developing positive relationships with the subject of mathematics?

This research project aimed to explore how students develop mathematics learner identities (MLIs) using online mathematics instructional platforms. Data were collected via a multiple embedded case study of MLI in online and in-class settings. Up to five students in each of four classrooms at four different schools were recruited as the individual cases. The classrooms included two secondary school Year 10 classes and two primary school classes (Years 5/6 and 7/8). Three schools were English medium and one was a Māori medium school. Data were analysed using the concept of "identity sites" and drawing on Butler (1988) to see identity as a performative act.

In this oral communication I share findings from analysis of the platform *Matific* as it was used in the English medium primary school classroom, and present the case of one 9-year old boy who named himself David Bowi. Analysis centred on identity construction the context of *Matific* and I sought answers to the questions: What is mathematics in this context? What roles are available to the mathematics learner? How does the individual take up those roles and access that mathematics? The platform *Matific* presented mathematics variously as an adventure; a competition; skills training; a currency for purchasing consumables; and a set of tasks assigned by the teacher. David Bowi took up the roles of 'high ranked' and 'accomplished'. He was an agentic learner who did all assigned tasks but was particularly motivated to earn points for spending on his avatar. He also described a mathematics world full of interesting characters and friends. This world contrasted with his experience in the regular classroom where access to learning and friendship during mathematics was more restricted. Further analysis of his case and others will be presented at the conference.

References

Butler, J. (1988). Performative acts and gender constitution: an essay in phenomenology and feminist theory. *Theatre Journal, 40*(4), 519–531. http://www.jstor.org/stable/3207893

Engelbrecht, J., Borba, M., Llinares, S., & Kaiser, G. (2020). Will 2020 be remembered as the year in which education was changed? *ZDM - Mathematics Education, 52*, 821–824.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 235. PME.

PAULO FREIRE'S PROBLEM-POSING EDUCATION

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We address a discussion about Paulo Freire's Problem-Posing Education perspective (Freire, 1987). This approach involves ensuring the development of mathematical concepts and ideas from a social-cultural-political view. Its objective is to enhance the development of mathematical concepts and processes in multiple and socially relevant contexts that involve collaborating in students' mathematics understanding, as well as broader extensions in citizenship promoting. Problem-Posing Education is a critical method for doing so. This study discusses how socially relevant problem-posing (PP) tasks have broaden students' mathematical and social dimensions. A qualitative approach is used, with a description of the participants' development in the tasks. How many different couples could be formed with three boys and four girls available? An entire class of students gave the answer of 12 couples, except one who answered 21 couples. It brings different voices and cultures in mathematics classroom. This problem-posing situation has been thought in socially relevant contexts (Crespo, 2015). It has been developed with pre-service teachers. It has been possible reflects on the concept of couples in today's society. It considers a couple as not just a boy-girl, but a group of two people. We have reached understandings such as: $C_{7,2} = 21$ (girl-boy, boy-boy, and girl-girl); $C_{4,2} = 6$ (girl-girl); $C_{3,2} = 3$ (boy-boy); $C_{7,2} - C_{4,2} - C_{3,2} = 21 – 6 – 3 = 12$ (Combinatorial Analysis – Combination); $C_{7,2}$ - C4,2 = 15 (only girl-boy or boy-boy); $C_{7,2} - C_{3,2} = 18$ (only girl-boy or girl-girl); $C_{4,2} + C_{3,2} = 9$ (boy-boy or girl-girl), and so on. All these syntheses are full of social and mathematical dimensions. It is concluded Mathematics has a social dimension and is a tool that helps to critically understand the social world, which brings new meanings to mathematical knowledge. Freirean Problematization allows a deeper mathematical understanding of the PP activity, along with its different social contexts and the different posed problems around the activity. Problematization generating new problems, and so on.

Additional information

Financial support. This project has financial support of CNPQ, Brazil.

References

Crespo, S. (2015). Chapter 24: A collection of problem-posing experiences for prospective mathematics teachers that make a difference. In: Singer, F. M., Ellerton, N. F., Cai, J. (Eds.). (2015). *Mathematical problem posing*. New York: Springer, p. 493-511.

Freire, P. (1987). *Pedagogia do Oprimido*. 17. ed., Rio de Janeiro, Paz e Terra, 1987.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 236. PME.

SUPPORTING PRE-SERVICE MATHEMATICS TEACHERS' NOTICING OF STUDENTS' ALGEBRAIC IDEAS: VIDEO BASED PROFESSIONAL DEVELOPMENT

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Algebra is a critical subject in school mathematics. It is the base for most advanced mathematics and science courses and careers. The ways teachers notice (attend and interpret) (van Es & Sherin, 2021) student algebraic thinking holds consequences for student learning (Levin & Walkoe, 2022). For instance, when teachers look beyond attending to formal algebraic concepts and consider the pre-instructional algebraic resources (seeds) children bring to the classroom, opportunities for learning are and better connected to children's lived experiences (Levin & Walkoe, 2022). This study explores pre-service mathematics teachers' (PMTs) noticing of students' pre-formal and formal algebraic ideas in a video club-based professional development (PD) that was designed using the perspective of "Seeds of Algebraic Thinking," (Walkoe & Levin, 2020) and aims to show the potential impact of the design on PMTs' noticing student algebraic thinking. This study was conducted as part of a mathematics methods course offered to six PMTs in their third year at a public university in the US. An intervention involving six Video Club (VC) sessions was administered as a part of this course. In the sessions, PMTs annotated short video clips of children's algebraic reasoning in interview and classroom discussions, identifying "important" moments of student algebraic thinking. Next, PMTs discussed the algebraic ideas they tagged in the video clips, led by a facilitator. The analysis of VC sessions revealed that PMTs could notice and reason about children's small-grained (pre-conceptual) algebraic ideas. They could attend to several key moments of students' ways of algebraic thinking, and they made connections between these pre-formal and formal algebraic ideas. This study suggests that the PD has the potential to support PMTs' noticing students' nascent/developing algebraic thinking in the classroom.

Additional information

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References

Levin, M., & Walkoe, J. (2022). "Seeds of algebraic thinking: A knowledge in pieces perspective on the development of algebraic thinking". *ZDM*, *54*, 1303–1314.

Walkoe, J., & Levin, M. (2020). Seeds of algebraic thinking. *For the Learning of Mathematics, 40*(2), 27–31.

van Es, E. A., & Sherin, M. G. (2021). Expanding on prior conceptualizations of teacher noticing. *ZDM–Mathematics Education*, *53*, 17-27.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 237. PME.

INFLUENCE OF ANGULAR DISPARITIES IN GEOMETRIC MENTAL ROTATION AND MAP-BASED PERSPECTIVE-TAKING

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Angular disparities play a critical role in performance on mental rotation (object-based rotation) and spatial perspective-taking (self-based rotation). Previous psychological research suggests that larger angular disparities elicit higher cognitive load, as indicated by increased response time and decreased accuracy (Kozhevnikov et al., 2006). We sought to investigate how domain-specific content—geometry and cartography—may influence performance in rotation tasks. Thus, we developed a geometric mental rotation test consisting of 400 trials and a map-based perspective-taking task comprising 320 trials. Accuracy (Acc) and reaction time for correct responses (Rtc) served as key indicators of rotation-based cognitive processes. Both tests were implemented using E-Prime software, and 76 high school or undergraduate students participated in this study.

Analysis of reaction time for correct responses (Rtc) for both geometric mental rotation and map-based perspective-taking tests supported findings from existing psychology literature: larger angular disparities led to longer reaction times. In particular, 180-degree rotations—whether they involved object-based rotation of geometric diagrams or self-based rotation in map reading—yielded the longest evaluation times. These results indicate that larger angular transformations require more processing time, regardless of whether the rotation is based on the self or on geometric objects. However, the accuracy (Acc) data presented a different pattern. When performing geometric mental rotation, participants achieved their highest accuracy when no mental rotation was required (0°), and there were no significant differences among rotations at 45°, 90°, 135°, or 180°. By contrast, in the map-reading spatial perspective-taking test, accuracy at the 90-degree rotation (i.e., rotating one's perspective clockwise) was significantly lower than at other rotation angles. Overall, these findings demonstrate that both object-based and self-based rotations at larger angular disparities require greater processing time, but additional factors influence accuracy. Particularly low accuracy at 90 degrees clockwise in the map-based task may be attributable to participants' right-handedness, as performing rotations in a non-dominant direction poses greater strategic difficulty. Meanwhile, the absence of accuracy differences across non-zero angles in the geometric mental rotation test may stem from the complexity of creating mental images and rotating the images accordingly. These discrepancies in accuracy underscore the importance of considering subject-specific contexts in studies of spatial ability and highlight the need for further research to elucidate how domain factors shape self-based and object-based rotations.

References

Kozhevnikov, M., Motes, M. A., Rasch, B., & Blajenkova, O. (2006). Perspective-taking vs. mental rotation transformations and how they predict spatial navigation performance. *Applied Cognitive Psychology*, *20*(3), 397-417.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 238. PME.

THEORETICAL FRAMEWORKS IN MATHEMATICS EDUCATION THESES: A BIBLIOMETRIC ANALYSIS

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Completing a thesis is a requirement in most undergraduate programs. In Teacher Education, this allows for various objectives, including the development of didactic proposals that draw on theoretical frameworks to study mathematical phenomena in the classroom. Research highlights the importance of drawing from theory to engage in research about pedagogical practices (Lara et al., 2007). Analyzing theses provides insight into research trends and the role of theoretical frameworks in mathematics education.

This study employs a bibliometric analysis to address the following research question: What are the predominant types of research conducted in mathematics education graduation theses, and how are theoretical frameworks applied?

To explore this question, the study aims to:

- Classify the types of research conducted in mathematics teachers' theses.
- Identify the most frequently cited theoretical frameworks.
- Examine the mathematical concepts addressed and their relation to the chosen frameworks.

This analysis focuses on graduation theses from the Secondary Mathematics Teacher Education program of a major private university in Santiago, Chile and considers production since 2022. A bibliometric approach was used to examine publication trends, citation networks, and keyword distributions, inspired by the work published by Zapata et al. (2021).

Preliminary findings indicate that most theses consist of didactic proposals, followed by textbook analyses and classroom activity studies. Constructivist frameworks, such as the Theory of Didactic Situations and Levels of Cognitive Demand, are the most commonly used. Greater variation is observed in the selection of mathematical objects and concepts, which include topics in algebra, arithmetic, geometry, statistics, and financial mathematics. Further details will be presented in the oral communication.

References

Lara, M., Forster, C. & Gorichon, S. (2007). Transferencia de la investigación educacional a la formación inicial de profesores. *Calidad en la Educación*, (27), 178-204.

Zapata, R., Rojas, J., Molina, D., Lagos, I., Sanhueza, C., Núñez, M., Sanzana, G, & Cigarroa, I. (2021). Características de las tesis de pregrado de Educación Básica, Parvularia y Diferencial de una Escuela de Educación: Un análisis bibliométrico. *Revista de estudios y experiencias en educación*, 20(44), 73-88

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 239. PME.

EXPLORING PRIMARY MATHEMATICS STUDENT TEACHERS' PRIOR KNOWLEDGE IN PREPARATION FOR A MODELLING UNIT

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This presentation reports on strengths, weaknesses, errors and misconceptions in primary mathematics student teachers' prior mathematical knowledge, in the topic fractions, before embarking on a mathematical modelling unit developed in the project Studies in Teaching Approaches for Mathematical Modelling at Primary Level (STAMM&prim). The project, which follows from CoSTAMM (Durandt, Blum & Lindl, 2022), has been launched early in 2025 at a public university in Johannesburg and aims to link essential elements of an effective teaching design with contextual factors in the development of primary mathematics student teachers' modelling competence. One contextual factor is students' level of prior knowledge.

To solve a new mathematical problem prior conceptual knowledge should support the production of new procedures and the application of old procedures in new situations. Enhancing topic-specific prior knowledge of students impacts their outcomes in mathematics (Alreshidi, 2023). This result is particularly promising for situations where topic-specific teaching interventions are planned, like in STAMM&prim.

Data were collected from 80 participants through a diagnostic test, based on guidelines from Larson and Silbey (2015). The test consists of 16 items grouped in four sections (Concept and formulas, Operations, Applications and modelling, Teaching fractions). Acceptable internal consistencies are reported for sections and the overall test. Results reveal students are rather weak in fractions. A further qualitative analysis of their mathematical work provided more insights related to errors and misconceptions, particularly in the *Application and modelling* section. This information will be used as a covariate for the evaluation of the modelling that follows later in 2025.

References

Alreshidi, N. A. K. (2023). Enhancing topic-specific prior knowledge of students impacts their outcomes in mathematics. *Frontiers in Education, 8*, 1050468. https://doi.org/10.3389/feduc.2023.1050468

Durandt, R., Blum, W., & Lindl, A. (2022). Fostering mathematical modelling competency of first year South African engineering students: What influence does the teaching design have? *Educational Studies in Mathematics*, *109*, 361–381. https://doi.org/10.1007/s10649-021-10068-7

Larson, K. D., & Silbey, R. (2015) *Mathematical Practices, Mathematics for Teachers: Activities, Models, and Real-life Examples*. Cengage: Boston

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 240. PME.

REASONING WITH MATH RESEARCHER VIDEOS

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Many students find reasoning in mathematics lessons quite complex, so teachers need to offer help. Interactive educational videos, such as math researcher videos, may make this support as engaging and iterative as possible.

THEORY

Argumentation consists of four steps (Bezold & Ladel, 2014). Toulmin (2003) structures argumentation as a well-founded transfer from reliable knowledge (*data*) to a statement (*conclusion*). This may require a guarantor (*warrant*) supported by backing.

LEARNING ENVIRONMENT AND MATH RESEARCHER VIDEOS

According to the operative principle, a substantial digital learning environment has been developed using the substantial task number grids (Wittmann, 2021). The learning environment has been enriched with various educational videos particularly math researcher videos which support students in discovering, describing, and reasoning.

METHODS

The argumentation structure of students in two transcripts (two students work together on the task), is elaborated using the Toulmin scheme (Toulmin, 2003) to clarify the roles of each participant, especially of the math researcher video.

ANALYSIS

While Wiebke and Leon use the math researcher video to complete a missing number grid and begin their reasoning process**,** Meike and Luke implicitly justify their discoveries during the exploration phase, filling in the blocks of the Toulmin scheme. They then utilize the math researcher video to articulate their reasoning.

CONCLUSION

Math researcher videos can help students to focus on their discoveries and to transfer them into reasoning.

References

Bezold, A., & Ladel, S. (2014). Reasoning in primary mathematics–an ICT-supported environment. *Bildung und Erziehung*, *67*(4), 409-418.

Toulmin, S. E. (2003). *The uses of argument.* Cambridge University.

Wittmann, E. C. (2021). *Connecting mathematics and mathematics education: Collected papers on mathematics education as a design science*. Springer Nature.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 241. PME.

MOVEMENT-BASED LEARNING OF PROBABILITY: CONCEPTUAL GAINS AND CHALLENGES

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INTRODUCTION

Probability is a complex field encompassing fundamental concepts such as sample space (Ω), certainty, and probability that pose challenges (Batanero & Sánchez, 2005). Based in embodied cognition (Varela et al., 1991) and embodied design (Abrahamson, 2012), this study explores how collaborative, movement-based activities with middle school students link physical experiences to abstract concepts.

RESEARCH GOALS AND METHODOLOGY

A qualitative, design-based study involving 10th graders examined movement-based probability tasks in a reconfigured classroom. Hand movements represented events in a sample space, leg movements expanded it, and creative combinations defined probabilities. Data collection included video recordings and observations, analyzing verbal and physical expressions, standardized comparisons, and multimodal interactions.

FINDINGS AND CONCLUSION

Movement-based activities promoted probabilistic reasoning, aiding students in internalizing key concepts. Initially, they found it difficult to define discrete events, but guided engagement improved their movement patterns and understanding. While Abrahamson's approach utilized structured tools, this study revealed challenges in transitioning from movement to formal expression, proposing movement notation as a potential bridge. Spatial awareness enhanced probability understanding, although complex tasks led to processing difficulty, avoidance, and disruption. This study emphasizes the role of embodied learning in connecting intuition with formal reasoning, highlighting the need for balanced instructional design. Future research should refine strategies and tackle classroom management challenges.

References

Batanero, C., & Sánchez, E. (2005). High school students' conceptions and misconceptions about probability. In G. A. Jones (Ed.), *Exploring probability in school* (pp. 241–266). Springer.

Varela, F. J., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. MIT Press.

Abrahamson, D. (2012). Embodied design: Constructing epistemic forms. *Educational Studies in Mathematics, 80*(1), 27–47.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 242. PME.

SPATIAL SENSE: CLASSROOM TASKS IN SECONDARY SCHOOL AND PISA TESTS

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The importance of spatial sense is demonstrated by its identification as a special dimension of intelligence, spatial intelligence, which defines the ability to perceive the spatial world and accurately represent it mentally. This report describes the characteristics of the PISA assessment tasks (TEP) in students aged 15-16 who are in the first year of Upper Secondary Education, in the space and form content, and the classroom assessment tasks (TEA) in geometry.

Four categories are established with their corresponding subcategories. The task contexts category has four subcategories: personal, work, social and scientific. The degree of cognitive complexity is measured in: reproduction, connection and reflection. The typology of the task statements has three subcategories: in text form, schematic and arithmetic. The response format has: openly constructed response, closed constructed response and multiple choice selected response questions (OECD, 2017). The analysis is carried out based on the different resolutions of the 19 tasks, thirteen TEP and six TEA, that are part of this research. For each task, a table is made with the evidence of the subcategories corresponding to the context, the typology of the statement, the cognitive complexity and the response format of each task.

A similarity between the tasks arises in the face of cognitive complexity, all TEA and TEP require a performance of connection or reflection in their resolution, although reflection in TEP concludes in a measure as a response and in TEA it implies explaining and justifying. Another similarity occurs in the typology of tasks, all present their data in a schematic form and in the text they present relevant data. However, they differ by the presence of irrelevant data in the TEP.

Another difference between the tasks is given by: the context, most of the TEPs present a work context, while all the TEAs a scientific context; the response format, all TEAs require an open constructed response, which implies the individual construction of the response, whereas TEPs provide the three response formats presented, that is, open constructed response, closed constructed response, and simple multiple choice selected response questions (OECD, 2017) with the exception that open constructed responses require a measure.

Reference

OECD (2017). *PISA 2015 assessment and analytical framework: science, reading, mathematic, financial literacy and collaborative problem solving.* PISA, OECD Publishing.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 243. PME.

PROBLEMS OR EXERCISES? UNDERSTANDING PRE-SERVICE TEACHERS' PERSPECTIVES

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Pre-service teachers (PTs) frequently encounter difficulties in posing problems for elementary education, opting instead for exercises that restrict their students' opportunities to develop problem-solving abilities (Mallart et al., 2018). This study aims to identify PTs' preconceived notions regarding problems in contrast to exercises.

Kantowski (1977) characterizes problems as tasks requiring inquiry to reach a solution, whereas exercises provide a straightforward and clear path. Leavy & Hourigan (2022) established the F-PoSE framework, which comprises eight indicators — context engagement, clarity, curriculum alignment, cognitive demand, solution steps, multiple solutions, strategy variety, and success opportunities — to assist educators in recognizing mathematical problems aligned with primary education goals.

We designed and applied a broad questionnaire aimed at exploring the perspectives of 174 PTs. We analysed PTs' written responses to: "What do you think differentiates a mathematical problem from an exercise in primary school?" using the F-PoSE.

Broadly speaking, data analysis revealed that PTs tended to blend elements of both problems and exercises, focusing on data extraction, operation identification, and solution development within realistic contexts. The most frequently mentioned F-PoSE elements were context engagement and cognitive demand (Leavy & Hourigan, 2022). Further research should focus on providing PTs with opportunities to identify problems aligning with the F-PoSE, with emphasis on the less frequently observed elements in their responses, such as multiple solutions, strategy variety, and success opportunities.

Additional information

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References

Kantowski, M. G. (1977). Processes involved in mathematical problem solving. *Journal for Research in Mathematics Education, 8*(3), 163-180.

Leavy, A., & Hourigan, M. (2022). The framework for posing elementary mathematics problems (F-PosE): Supporting teachers to evaluate and select problems for use in elementary mathematics. *Educational Studies in Mathematics, 111*(1), 147-176. https://doi.org/10.1007/s10649-022-10155-3

Mallart, A., Font, V., & Diez, J. (2018). Case Study on Mathematics Pre-service Teachers' Difficulties in Problem Posing. *EURASIA Journal of Mathematics, Science and Technology Education, 14*(4). https://doi.org/10.29333/ejmste/83682

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 244. PME.

VALUING OF AND CONFIDENCE IN MATHEMATICS: IMPACT ON PERCEIVED INSTRUCTIONAL SUPPORT

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INTRODUCTION

Students' confidence in mathematics reflects how well they believe they can do mathematics, which also affects their perceptions of instructional support available. Values refer to socially mediated qualities deemed important by individuals and communities, and we are curious how students' valuing of mathematics might affect how their confidence in mathematics is related to their perceptions of instructional support. Thus, in this study, we posed the Research Question: How do German secondary students' valuing of mathematics play a role in the relationship between student confidence in mathematics and student-perceived instructional support?

METHODOLOGY

The data for this study were collected from 146 Grade 8 secondary school students in the southern part of Germany using a questionnaire. The questionnaire consisted of three validated inventories: The 9-item Inventory of Mathematics Confidence, the 3-item Inventory of Mathematics Instructional Support, and an item asking for students' Value in Mathematics.

RESULTS

A moderation analysis was conducted to examine the interaction effect between Mathematics Confidence and Value in Mathematics on Mathematics Instructional Support. The overall model was significant, F (3, 138) = 7.45, $p < .001$, $R^2 = .14$, indicating that the predictors explained 13.94% of the variance in Mathematics Instructional Support. The interaction between mathematics confidence and value in mathematics was significant, $b = .23$, $t = 2.30$, $p = .02$, suggesting that students' confidence in mathematics had a stronger impact on their perceived instructional support when they placed higher value in mathematics as a core subject in school.

INTERPRETATION

These findings highlight the importance of students' value in mathematics in enhancing the relationship between their confidence and perception of instructional support. When students value mathematics more, their confidence in mathematics more strongly influences their perception of instructional support. This highlights the importance of fostering students' valuing of mathematics.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 245. PME.

HARNESSING THE EPISTEMIC POTENTIAL OF MULTIPLE LANGUAGES IN TRANSITIONAL MULTILINGUAL MATHEMATICS CLASSROOMS IN SOUTH AFRICA

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The transitional model is the most common approach for including indigenous languages in formal education in most African countries and involves using an indigenous language for instruction over a certain number of years for formal education before transitioning to a 'global' language (Trudell, 2023). In South Africa, this transition takes place in Grade 4 (on average, age 10). However, this shift raises questions as to whether students have gained access to both communicative and academic proficiency in the home language and English and can, therefore, be taught mathematics in English from Grade 4 onwards (Essien & Sapire, 2022).

This paper proposes a framework for rethinking teaching in transitional classrooms in a way that draws on and exploits the epistemic potential of the multiplicity of languages in classrooms to benefit students who have experienced language transition. Drawing on data from a research and development project that combines teacher professional development and classroom observations of language transition classrooms, we bring together both the translanguaging framework and the language responsive mathematics teaching framework. The resulting merger framework reifies and recontextualises the 'jobs' of teachers in transitional multilingual mathematics classrooms in such a way that accounts for 12 horizontal, vertical and diagonalised bi-directional movements between languages and their different forms for epistemic access.

Preliminary findings from the use of this merger framework for teacher development indicate that it is a useful tool for showing how teachers (can) draw on and exploit the epistemic potential of multiple languages in multilingual mathematics classrooms to benefit students. This is particularly important as South Africa moves towards building a strong and efficient mother-tongue based bi/multilingual education.

References

Essien, A. & Sapire, I. (2022). Language policy implementation in early grade mathematics in South Africa: A 2010-2020 overview. In, Venkat, H. & Roberts, N. (Eds.) *Early Grade Mathematics in South Africa*. Chapter: 5. Cape Town: Oxford University Press. pp.81-96.

Trudell, B. (2023). *Early-exit language transitioning programming: The rationale, the benefits and the limitations*. Paper commissioned for International Mother Language Day 2023. © UNESCO 2023.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 246. PME.

SENSES ON VALIDATION PRODUCED BY A GROUP OF ARGENTINEAN PROSPECTIVE TEACHERS

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The study of validation in educational settings using mathematical modelling (MM) and its role in sense-making is of interest to researchers, though various theoretical approaches are recognized (e.g., Leung et al., 2021). This applies particularly to future mathematics teachers (FMT) while they are involved with MM activities.

Building on that research interest, we conducted a study with the aim of exploring and analysing the senses of validation produced by 17 FMT from an Argentine public university, who participated in a MM scenario within a geometry course seeking to inform about differences between senses produced by FMT in their educational trajectory prior to their immersion in the scenario and those produced during and after that immersion. We conducted qualitative research following an interpretative paradigm and a design research approach. Data sources include narratives on pre-immersion and in-situ validation, class videos, written work, and a post-immersion interview, all corresponding to a purposefully selected group of 4 FMTs. For the scenario design we draw on Bassanezi's advances in MM. For analysing sense on validation we draw on Ballachef and Lakatos' work. Regarding sense-making, we build on several contributions, including those of Levrini et al. (2015).

Based on our analysis, we observed that initially, FMTs associate validation with demonstration using the axiomatic deductive method and Euclidean modes. After engaging with the MM scenario, they distinguish validation from demonstration, seeing it as a complex, reflective process involving model testing, self-regulation, and social consensus. The observed change shows that constructing senses about validation requires experience and contextualization. Early discussions on validation in teacher education would build understanding and promote recognition of diverse methods beyond formal demonstrations and single-example verification.

References

Leung, G.A. Stillman, G. Kaiser y K. L. Wong (Eds.) (2021). *Mathematical Modelling Education in East and West, International Perspectives on the Teaching and Learning of Mathematical Modelling.* Springer.

Levrini O., Fantini P., Tasquier G., Pecori B. & Levin M. (2015). Defining and Operationalizing Appropriation for Science Learning. *Journal of the Learning Sciences* (24) 1, 93-136. https://doi.org/10.1080/10508406.2014.928215

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 247. PME.

SCALABLE AI-ASSISTED IDENTIFICATION OF MATH SUPPORT NEEDS FOR FIRST GRADERS

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Early identification of mathematical difficulties of young learners is critical for their individual development, yet traditional assessments such as standardized tests or diagnostic interviews face challenges, e.g., in scalability and practicality. To address these challenges, we developed an AI-assisted, pedagogical domain-informed digital tool, DIDUNAS, to evaluate first graders' individual skill profiles and identify their need for support. As our research question we ask, *How accurately can AI identify first graders' need for support in mathematics?*

Our study involved 290 students from 8 European schools (in Germany & Cyprus). The participants included multilingual students, students with disabilities, and diverse socioeconomic backgrounds. We used the ZAREKI-K standardized test (von Aster et al., 2009) as the ground truth of the students' mathematical skill profiles. The test results indicated whether a student needed support or not. In our study, 71 students were identified as being in need of support in mathematics.

For developing the digital tool, we first let 55 students engage with 12 digitally presented mathematical activities (e.g., number line, quantity comparison, etc.) designed by pedagogical experts, and take individual standardized ZAREKI-K tests. We assessed both results with machine learning techniques to have an insight about which tasks contributed most to accurate identification of support needs based on students' errors in each task. The analysis allowed us to refine the assessment to just 10 tasks. The AI-assisted DIDUNAS app with the refined task set was validated with 235 students, and it was able to identify students in need of support with 85% accuracy with a much less time consumption compared to one-by-one diagnostic interviews.

Our results demonstrate that AI can enhance scalability for accurately identifying first graders' need for support in mathematics. We use the results from the digital app DIDUNAS, which runs on typical tablet computers and PCs, to support teachers with overviews over their students' skill profiles and needs for support. Future work will explore real-time adaptive feedback to improve user experience, longitudinal studies to assess long-term learning outcomes and applications across additional grade levels.

References

von Aster, M., Bzufka, M. W., Horn, R. R., & Zulauf, M. W. (2009). *ZAREKI-K: Neuropsychologische Testbatterie für Zahlenverarbeitung und Rechnen bei Kindern-Kindergartenversion.* Pearson.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 248. PME.

CONTRIBUTIONS OF THE DIGITAL EDUCATIONAL RESOURCE ALJABAR KINGDOM TO THE DEVELOPMENT OF ALGEBRAIC THINKING IN EARLY YEARS

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The contribution of Digital Educational Resources (DER) to the development of algebraic thinking has been studied by several authors (Castro-Filho, Castro & Freire, 2021). However, the same is not found in the initial years of schooling (Haspekian et al., 2023). This study investigates the use of a DER called Aljabar Kingdom (http://bit.ly/4h6u1bY) to the development of algebraic thinking in eight 3rd and 4th grade elementary school students (ages 7 to 10) in Fortaleza, Ceará, Brazil. Individual sessions using the DER were conducted involving manipulating known and unknown values on a virtual scale to explore concepts of unknown, equivalence and inequality. Data consisted of video recordings during the RED use, field diaries and clinical interviews.

Results indicate the use of strategies relates to the algebraical thinking. Strategies included: interval analysis, in which values were adjusted iteratively; half-way search, which restricts the search for possible values by half; observation of recorded weights, in which values already found were used to adjust hypotheses; interleaving of weights, in which values were tested in an interleaved manner; and sequence of weights, in which values were tested systematically to establish relationships.

From these strategies, key concepts of algebraic thinking emerged, such as understanding equivalence, with visual exploration of the relationship between mathematical expressions, and generalization of patterns and symbolic representation, evidenced in the students' transition to formal algebraic notations. These findings indicate that technology can contribute to the teaching of algebra, even in the early years. Future research will investigate the use of RED in formal teaching situations with a full class of students.

References

Castro Filho, J.A., Castro, J.B., & Freire, R.S. (2021). Contributions of Digital Technologies to the Development of Algebraic Thinking at School. In: Spinillo, et al. (eds), *Mathematical Reasoning of Children and Adults*. Springer, Cham.

Haspekian, M., Kieran, C., Drijvers, P., Bråting, K., & Tabach, M. (2023*).* Algebra Education and Digital Resources: A Long-Distance Relationship? In: Pepin, et al. (eds), *Handbook of Digital Resources in Mathematics Education*. Springer International Handbooks of Education. Springer, Cham.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 249. PME.

ANALYSIS OF YOUCUBED.ORG ACTIVITIES FOCUSING ON FOSTERING CRITICAL AND CREATIVE THINKING IN MATHEMATICS

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Currently, there is a wealth of research and documentation advocating for the development of critical and creative thinking in mathematics (CCTM) in basic education (Vicent-Lancrin et al., 2019). However, one of the difficulties reported by teachers is the lack of training and teaching materials specifics. Aiming to contribute to filling this gap, we cite the Analysis Matrix of Didactic Materials for Stimulating CCTM, composed of 16 rubrics, grouped into 3 dimensions: Idea Generation, Idea Evaluation, and Classroom Climate (Fonseca, Gontijo & Carvalho, 2023). We used this Matrix to analyze items from the youcubed.org website, which offers resources aimed at "an open and creative mathematical mindset". One analyzed item was: "When coins are placed in groups of 2, one coin is left over. When they are placed in groups of three, five, and six, one coin is also left over. But when they are placed in groups of seven, none are left over. What could be the possible number of coins?" A summarized evaluation of the item is presented below, based on the three dimensions of the Matrix: (a) Idea Generation - admits mathematical investigation, allowing for the construction and testing of hypotheses, as well as the undertaking of multiple solution strategies (and multiple solutions); (b) Idea Evaluation - provides opportunities for the analysis of generated ideas, stimulating reflection on the answers generated; and (c) Classroom Climate - favors aspects cited in the literature as a good classroom climate, stimulating student engagement and active and collective work, which can also contribute to minimizing math anxiety. It is expected to communicate the analysis process in detail during the event, based on other items from the site. It is also expected that the dissemination of the instrument will contribute to teachers being able to select/create/adapt items from several sources. Such an initiative aims to foster the approximation of research focused on CCTM with basic education classrooms, as proposed by the theme of this conference.

References

Vincent-Lancrin, S., González-Sancho, C., Bouckaert, M., De Luca, F., Fernández-Barrerra, M., Jacotin, G., Urgel, J., & Vidal, Q. (2019). *Fostering Students' Creativity and Critical Thinking: what it means in school*. OCDE Publishing.

Fonseca, M. G., Gontijo, C. H. & Carvalho, A. T. (2023). Pensamento crítico e criativo em matemática: rubricas avaliativas hapman, *O. Zetetiké 31*(0), 1-16.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 250. PME.

EMERGING STUDENT AGENCY: LEARNING MATHEMATICS IN PROGRAMMING ENVIRONMENT

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In this study, we explore how integrating CT into mathematics education through a programming environment supports students' agency. While programming is increasingly promoted as a way to enhance both mathematical understanding and agency (Shaw et al., 2023), limited research explores how learners interact with their environment through authentic tasks that support the development of agency. Framed by enactivism (Varela et al.,2016) and stigmergy (Parunak, 2005), which conceptualizes agency as a co-emergent process shaped by reciprocal interactions and co-actions between learners and their environments, we examine how student's agency emerges in computer-based mathematics investigation projects.

This qualitative study involved six participants from a sequence of three courses in the Mathematics Integrated with Computer Applications (MICA) program at an Ontario university where students develop Exploratory Objects (EOs) – original programs created to explore mathematical concepts or real-world problems. Data was collected through semi-structured, stimulated recall interviews using participants' EOs, course reflections, and field notes. Thematic Analysis was employed to identify patterns of significance.

Findings highlight three themes—Learning Strategies and Student Agency, Pedagogical Approaches and Strategies, and Sensing and Perception—that revealed how student agency in computer-based mathematics investigation projects emerges dynamically through interactions with the learning environment and other agents. Findings show agency emerged as participants responded to perceived instructional and evolving environmental affordances which conditioned adaptive learning strategies and action. The findings highlight the importance of integrating computational thinking into math education through project-oriented approaches.

References

Parunak, H. V. D. (2005). A survey of environments and mechanisms for human-human stigmergy. In D. Weyns, H. V. D. Parunak, & F. Michel (Eds.), *Environments for multi-agent systems II* (pp. 163-186). Berlin, Germany: Springer Berlin Heidelberg.

Shaw, A., R. Lawler, B., Crombie, W., McKlin, T., & Richards, T. (2023). Computational Thinking and The Algebra Project: From Voice to Agency. *Prometeica (Mar Del Plata)*, *27*, 565–574. https://doi.org/10.34024/prometeica.2023.27.15348.

Varela, F. J., Thompson, E., & Rosch, E. (2016). *The Embodied Mind, revised edition: Cognitive Science and Human Experience*. MIT press.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 251. PME.

FEATURES OF TEACHING THAT PROMOTE ARGUMENTATION IN MATHEMATICS CLASSROOMS

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Promoting argumentation in classrooms is an essential curriculum goal in mathematics. It supports understanding and productive discussions (Conner et al., 2014), and creates equitable learning opportunities (Bieda, 2010). However, how teaching supports argumentation is an open question (Solar et al., 2021). This study addressed this issue. Argumentation is a verbal, social, and rational activity of trying to establish claims by putting forward propositions that support the claims, playing functions such as data, warrants, backings, qualifiers, and rebuttals (Toulmin, 2003). Teaching is the interaction between teachers and students over content to facilitate learning and can be modelled as a system of interacting features or strategies (Hiebert & Grouws, 2007). This study addressed the following question: What features of teaching facilitate the development and support of claims? Analysis using video methodology for studying reasoning (Powell et al., 2003) and Toulmin's argumentation scheme of interactions between researchers and students as they tried to justify their solutions to challenging tasks in a longitudinal study revealed several features that support argumentation, including fostering "sense-making," "generalization," and "empirical investigation of warrants." These and other features will be discussed with a focus on how they support argumentation, their significance, and implications for practice and future research.

References

Bieda, K. N. (2010). Enacting proof-related tasks in middle school mathematics: Challenges and opportunities. *Journal for Research in Mathematics Education, 41*(4), 351–382.

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics, 86*, 401–429

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371– 404). Information Age.

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *Journal of Mathematical Behavior, 22*, 405–435.

Solar, H., Ortiz, A., Deulofeu, J., & Ulloa, R. (2021). Teacher support for argumentation and the incorporation of contingencies in mathematics classrooms. *International Journal of Mathematical Education in Science and Technology, 52*(7), 977–1005.

Toulmin, S. E. (2003). *The uses of argument* (updated ed.). Cambridge University Press. Original work published 1958.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 252. PME.

CO-DESIGN WITH FAMILIES FOR TEACHER LEARNING

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Authentic collaborations with nondominant families are essential for teachers to develop culturally sustaining pedagogy (CSP) and practice (Hong et al., 2025). Involving families in designing mathematics learning opportunities can restore their epistemic authority (Booker & Goldman, 2016) and challenge teachers' deficit perspectives. Despite exemplary cases of successful CSP enactment (Hong et al., 2025) and CSP professional development (PD), many US schools lack this kind of PD opportunity. This study explores the evolving sensemaking of a first-grade Latina teacher during two years of PD focused on responsive mathematics teaching. In the second year, a bilingual math leadership team was formed with families to co-design supports for children's mathematics learning. Within these two settings, we explore the opportunities for developing CSP by asking: How does a Latina teacher frame families, language, power, and identity? Andrea's case is unique as she is one of two Latina teachers at a school serving almost 90% Latine students and is also a parent in the community. Although she expresses a need to support parents, she does not center her Latina identity in her teaching. Data sources include interviews, focus groups, field notes, and lesson videos. For this paper, we conducted thematic analyses of two one-hour semi-structured interviews at the end of each year using Marshall's (2023) framework for CSP. We also coded her contributions to a focus group with two other teachers. Findings show that at the end of her first year, Andrea's framing of mathematical authority includes that she learns from students, does not consider students' multiple identities but focuses solely on their mathematical success, and positions non-English forms of expressions as barriers for families. By the end of her second year, she frames her classroom as a space of shared authority with and between students. While she still does not fully embrace bilingualism, she moves beyond framing it as a barrier. In the family team, she frames caregivers as valuable supports for children's learning. This case study shows the promise of co-design spaces centered on shared epistemic authority as opportunities for teachers' development of CSP.

References

Booker, A., & Goldman, S. (2016). Participatory design research as a practice for systemic repair: Doing hand-in-hand math research with families. *Cognition and Instruction*, *34*(3), 222-235.

Hong, S., Baloch, M. H., Conklin, K. H., & Warren, H. W. (2025). Teacher-family solidarity as culturally sustaining pedagogy and practice. *Urban Education*, *60*(1), 90-122.

Marshall, S. (2023). But what does it look like in maths? A framework for culturally sustaining pedagogy in mathematics. *International Journal of Multicultural Education, 25*(1), 1-29.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 253. PME.

UNDERSTANDING PEDAGOGICAL PRACTICE: TEACHER'S REFLECTIONS ON THEIR OWN CLASSROOM VIDEOS

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Starting in 2025, Chilean teachers will be able to download and review to recordings of their own lessons. This recording, which is part of the national teacher evaluation system, provides a space for reflection, offering opportunities for teacher learning and professional growth, as videos serve a powerful tool which teachers' interpretation of their own practice can play a critical role in their learning (Hollingsworth & Clarke, 2017). Given this, the guiding question of this study is: How do teachers interpret their pedagogical practice when watching videos of their own classes?

The sample for this descriptive study was based on responses to a question from a semi-structured interview with eleven primary school math teachers as part of a larger research project. The interview was focused on observing a selected segment from their own lessons to illustrate a distinctive moment, followed by the question: "Could you explain what was happening there?". For data analysis, inductive thematic analysis (Huberman, 2014) was used, resulting in six themes, two of which appeared consistently across most cases: Critical evaluation of their practice and considerations about students.

These results highlight the evaluative approach teachers take when interpreting their actions, being highly critical of their practice and focusing on self-perceived actions as errors. Additionally, the considerations about students suggest that teachers try to understand students' thinking when observing segments of their classes. Given these findings, it is crucial explore how teachers interpret the recordings, considering the potential of review and use the instrument national teacher evaluation system provide. Therefore, it is essential to provide teachers with tools to make productive use of their recordings lessons.

Additional information

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References

Hollingsworth, H., & Clarke, D. (2017). Video as a tool for focusing teacher self-reflection: Supporting and provoking teacher learning. *Journal of Mathematics Teacher Education, 20(5)*, 457-475.

Huberman, A. (2014). *Qualitative data analysis a methods sourcebook*. Sage

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 254. PME.

USING DRAWINGS TO IDENTIFY THE EMOTIONS OF PRIMARY SCHOOL STUDENTS IN MATHEMATICS

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Drawing has proven to be an effective tool for exploring students' emotions and conceptions, as it allows them to express themselves spontaneously (Stiles et al., 2017). However, most studies focus on adolescents or contexts outside Mexico, leaving a gap in understanding how primary school children (ages 6-12) express their emotion with mathematics through drawings. Since childhood is crucial for shaping attitudes toward the discipline, this research aims to examine how primary school students in Mexico portray their emotions through drawings.

The study is based on the Theory of the Cognitive Structure of Emotions (Ortony et al., 1988), which suggests emotions arise from the appraisal of events, agents, and objects concerning individual goals. Using a qualitative methodology, the research focuses on students' subjective experiences. A total of 285 students from a public primary school in southern Mexico participated. For data collection, each student was given a blank sheet of paper and asked to create two drawings: one representing a positive emotion and another a negative emotion related to their mathematics classes. They were also asked to write the emotion's name and describe the situation that triggered it, to prevent overinterpretation during analysis.

A total of 558 drawings were collected, of which 56 were excluded due to unclear emotional interpretations, leaving 502 for analysis. The most frequently expressed emotions were joy and distress, triggered by situations such as being called to the board, succeeding or struggling with basic operations, receiving grades, and understanding or not understanding topics.

As a whole, the results suggest that emotional triggers were directly related to academic goals, indicating that emotions in mathematics lessons significantly affect students' performance and well-being. They also highlight the potential of drawing as a way for children to communicate their emotions.

References

Stiles, D., Adkisson, J., Sebben, D., & Tamashiro, R. (2008). Pictures of hearts and daggers: strong emotions are expressed in young adolescents' drawings of their attitudes towards mathematics. *World Cultures eJournal*, 16(2), 1-14.

Ortony, A., Clore, G., & Collins, A. (1988). *The cognitive structure of emotions*. Cambridge University Press.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 255. PME.

TEACHING MATHEMATICS TO STUDENTS WITH ADHD: A SCOPING REVIEW

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Attention Deficit Hyperactivity Disorder (ADHD) is a neurodevelopmental condition that affects executive functions, primarily attention regulation, hyperactivity, and impulsivity (Robles Bermejo, 2023). Students with ADHD tend to exhibit lower academic performance from childhood through adolescence (Tarver et al., 2014). In the field of mathematics, executive functions influence learning in multiple ways. For instance, problem-solving requires following sequential steps, sustaining attention, planning, applying procedures that rely on working memory, and carefully reading statements to extract relevant information (González-Calleros et al., 2018). Mathematics teachers must work with students with ADHD and require practical tools to provide an adequate learning experience for this group. In light of this, the present study aims to identify teaching strategies that foster mathematics learning among students with ADHD. To this end, we conducted a scoping review using the WOS, SCOPUS, SCIELO, and ERIC databases, employing the keywords *ADHD* and *mathematics*. An initial screening based on titles and abstracts identified 589 studies, of which 76 were selected for full-text review. The in-depth analysis of these studies is still in progress. Preliminary findings have allowed us to categorize teaching strategies into two main groups: (1) those aimed at improving executive functions, and (2) those focused on adapting mathematical tasks and activities to better suit students with ADHD. We consider that the strategies with the greatest applicability for mathematics teachers belong to the second group, as they include, among others, gamification, the use of videos, project-based and interdisciplinary learning, high levels of scaffolding in instruction, and environmental mediation. Additionally, we identified areas requiring further research to better understand the opportunities available for mathematics teachers working with students with ADHD.

References

González-Calleros, C. B., Guerrero-García, J., & Navarro-Rangel, Y. (2018). Matemáticas para Niños con TDAH a través de Juegos Serios: Una Revisión del Estado del Arte. In M. E. Prieto-Méndez, S. J. Pech-Campos & A. Francesa-Alfaro (Eds.), *Tecnologías y Aprendizaje: Investigación y Práctica* (pp. 207-214). CIATA.

Robles Bermejo, F. (2024). Attention deficit hyperactivity disorder: Neuropsychological profile and study of its impact on executive functions and academic performance. *Anales de pediatría* (English Edition), *100*(2), 87-96.

Tarver, J., Daley, D., & Sayal, K. (2014). Attention-deficit hyperactivity disorder (ADHD): an updated review of the essential facts. *Child: care, health and development*, *40*(6), 762-774.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 256. PME.

ON THE ART OF FORMULATING GOOD STATISTICAL PROBLEMS FOR STUDENTS AT EARLY AGES

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Posing appropriate statistical problems is essential in the training of prospective primary teachers (PPTs). However, the process of creating an understandable and interesting statistical problem that can be analyzed with data collected by schoolchildren, is a complex iterative procedure (Leavy & Frischmeier, 2021). We assume that it is possible to facilitate this creation process using well-oriented feedback based on research evidence about what makes a good statistical problem (Bargagliotti et al., 2020). The research question: How do statistical problems posed by PPTs evolve based on feedback from a rubric and tailored investigative questions?

The sample comprised 36 PPTs (working in groups of 5 or 6), that were 3rd year students in a 4-year undergraduate degree in primary teacher education. The intervention included two phases: First, PPTs formulated a statistical problem to be developed in primary education, and in the second phase, they refined their proposal based on the feedback provided by their teacher, which was based on a rubric and included some tailored investigative questions. The rubric considered two components (context and statistical adjustment) and three performance ratings for each of them. Additionally, the expert posed two or three investigative questions to each group, to highlight the limitations of their proposal.

In the first phase, 3 out of 7 groups posed problems with improvable contexts. In terms of statistical adjustment, all of the groups formulated problems that required refinement because they were deterministic, or data search was not feasible or did not include survey questions. After receiving feedback, 5 out of 7 teams successfully proposed good statistical problems in both context and statistical adjustment.

The proposed feedback has enabled the students to refine their proposals by introducing substantive improvements, being a useful tool to make sure that mathematics education research reaches the classrooms.

Additional information

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References

Leavy, A., & Frischemeier, D. (2022). Developing the statistical problem posing and problem refining skills of prospective teachers. *Statistical Education Research Journal, 21* (1).

Bargagliotti, A., Franklin, C., Arnold, P., Gould, R., Johnson, S., Perez, L., & Spangler, D. (2020). *Pre K–12 Guidelines for Assessment and Instruction in Statistics Education (GAISE) report II*. American Statistical Association and NCTM.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 257. PME.

IMPLEMENTATION FIDELITY AND ITS IMPACT ON THE SUCCESS OF INTERVENTION PROGRAMMES: THE CASE OF TWO MENTAL STARTERS PROJECTS

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This study reports on the impact of implementation fidelity on two ongoing intervention projects – Base-Ten Thinking, and Mental Starters Assessment Project – that sought to foster number sense through mental mathematics for Grades 2 and 3 students in South Africa. These projects integrate Freudenthal's theory of number structuring with mental mathematics strategies – the Jump and Bridging Through Ten strategies. In our context, implementation fidelity is the extent to which the intervention is executed as intended (Carroll et al., 2007) and is measured through adherence to the correct use of mathematical tasks and the structural integrity of tasks provided in our instructional materials.

A total of 12,423 learners participated in the study across 120 schools, with 1416 in the control group and 11007 in the experimental group. Using a design research approach, the study employed a combination of quantitative pre- and post-tests and qualitative analysis of video-recorded lessons to assess the intervention's success. Schools were categorised based on their fidelity of implementation ranging from high, moderate, low, to no fidelity. Schools that demonstrated high fidelity exhibited structured adherence to the 3-step instructional model (warm-up – whole class – individual/group activities), resulting in statistically significant improvements. Our findings revealed that contrary to findings by Essien, Venkat, et al. (2015) (where teachers with moderate implementation fidelity tended to be more successful in teaching mathematical concepts due to teacher agency), higher-fidelity schools in our study tended to demonstrate stronger gains in student performance, particularly in jump strategy tasks, compared to schools with moderate fidelity. These findings highlight the importance of attention to, and deliberate sequencing and coherence in the development of instructional tasks in intervention programmes.

References

Carroll, C., Patterson, M., Wood, S., Booth, A., Rick, J., & Balain, S. (2007). A conceptual framework for implementation fidelity. *Implementation science*, *2*, 1-9.

Essien, A. A., Venkat, H., Takane, T., & Tshesane, H. (2015). *An evaluation of the use and efficacy of the GPLMS FP multilingual mathematics materials: Issues and prospects—Final report*. Johannesburg: Tshikululu.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 258. PME.

IMPACT OF METACOGNITIVE STRATEGIES ON LOW ACHIEVING SECONDARY SCHOOL STUDENTS' LEARNING EXPERIENCE OF GEOMETRY

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Recent Grade 9 national examination reports have highlighted persistent struggles among learners in coordinate geometry and geometric shapes. Prior research (Bessoondyal, 2005) attributes low achievement in mathematics to predominantly outcome-based and teacher-centered instruction, which often fails to engage students in meaningful learning. While previous studies have explored the use of dynamic software like GeoGebra in mathematics education, limited research has investigated its integration with metacognitive strategies to support low-achieving students in geometry. This study seeks to bridge this gap by examining how metacognitive strategies, combined with GeoGebra, can enhance students' conceptual understanding and engagement in geometry lessons. A quasi-experimental design is adopted, involving a treatment group, exposed to metacognitive strategies and dynamic software, and a control group. The study is guided by Kuzle's (2013) framework, which emphasizes metacognitive strategies in problem-solving through seven key episodes: reading, understanding, analysing, exploring, planning, verifying, and transitioning. Data collection methods include pre- and post-tests, interviews, observations, and think-aloud protocols to assess students' metacognitive awareness. Thematic analysis will be used to identify emerging patterns and insights. A pilot study with 20 low-achieving Grade 8 students revealed key challenges, such as language barriers in understanding metacognitive prompts and the need for culturally relevant instructional scaffolding. While immediate conceptual improvements were limited, students demonstrated increased engagement in class discussions and collaborative problem-solving. These findings suggest that refining instructional prompts and providing structured guidance may foster deeper metacognitive reflection and learning gains. The findings from this study will provide valuable insights into the integration of metacognitive strategies and dynamic software in mathematics education. By fostering self-regulation and engagement, this approach has the potential to empower low-achieving students, enhance their problem-solving skills, and bridge the gap between research and classroom practices. The study contributes to the growing body of research on metacognition in education and offers practical recommendations.

References

Bessoondyal, H. (2005). Challenges in teaching mathematics: An outcome-based approach. *Journal of Mathematics Education*, *10*(2), 45-60.

Kuzle, A. (2013). Metacognitive strategies in mathematics education: A framework for problem-solving. *Journal of Mathematics Education*, *6*(2), 123-135.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 259. PME.

FINNISH TEACHER LANGUAGE IN RESEARCH TEXTS

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Based on the international Lexicon project, the appearance of Finnish mathematics teachers' terminology in mathematics education research texts was examined. Results show differences between how often research articles, mathematics education master theses, and educational sciences master theses.

TEACHER PROFESSIONAL LEXICON IN RESEARCH LANGUAGE

Building from the international Lexicon project I examined the use of the Finnish mathematics teachers' professional terminology in mathematics education research texts. I chose 31 searchable terms out of 47 most important terms in the Finnish Lexicon (Hannula et al., 2021), dividing them into 4 categories based on the focus the terms: student, structures for teaching, pedagogy, and mathematics (Table 1). These terms were searched for in three text corpora: articles from the Finnish Mathematics and Science Education Research Association publications (23 articles, 87,680 words), master theses from mathematics teacher education program in the University of Helsinki (48 theses, 525,544 words) and master theses from elementary teacher education program in the University of Helsinki (7 theses, 130,516 words).

Focus on (n terms)	Articles	Math ed. theses	Elem. ed. theses
Student (6)	21 (7%)	91 (7%)	10 (2%)
Structures (10)	81 (26%)	509 (37%)	214 (42%)
Pedagogy (11)	204 (64%)	694 (50%)	261* (51%)
Mathematics (4)	11 (3%)	91 (7%)	26 (5%)

Table 1: Occurrences of selected terms from Finnish Lexicon in three corpora.

* An outlier term had 407 occurrences. This was capped to 100 occurrences.

The differences between the three corpora were statistically significant: X^2 (6, N =2213) = 44.8, $p < 0.00001$. These differences suggest that the three author groups focus with different intensity on different aspects of mathematics education.

References

Hannula, M. S., Sahlström, F., Kiviharju, J. (2021). The Finnish mathematics teachers' lexicon: A focus on organisation and relationships. In C. Mesiti, M. Artigue, H. Hollingsworth, Y. Cao, & D. Clarke, (Eds.).*Teachers talking about their classrooms : Learning from the professional lexicons of mathematics teachers around the World* (pp. 147-158). Routledge.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 260. PME.

A STUDY ON STUDENT AUTONOMY THROUGH USING DIGITAL TEXTBOOKS IN MATHEMATICS EDUCATION

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This study explores how digital textbooks in elementary mathematics education support student autonomy, with particular attention to students' and teachers' perspectives on their use. Grounded in Self-Determination Theory (SDT; Ryan & Deci, 2017), the research investigates how digital textbooks can fulfill the basic psychological need for autonomy, thereby fostering intrinsic motivation and self-regulated learning.

With the implementation of GIGA School initiative in Japan, digital tools such as GeoGebra, spreadsheets, and digital drills have become increasingly common in mathematics classrooms. Despite this, empirical research focusing specifically on digital textbooks and their role in promoting students' autonomy remains limited, especially regarding their emotional and motivational effects.

A questionnaire survey was conducted in May 2024 with 126 fifth- and sixth-grade students and four mathematics teachers at one elementary school. The findings indicate that while about 20% of students preferred using both digital and paper textbooks, digital materials were particularly valued for features that support independent learning—such as visual clarity, interactive elements, and flexible content navigation. These aspects were perceived to promote a greater sense of control and self-directed engagement in learning. Students described digital textbooks as convenient (52 responses), enjoyable (28), and motivating (20). However, relatively few students engaged in autonomous creative activities such as writing equations or drawing, suggesting current digital platforms may not yet fully enable expressive or open-ended student participation. Reported barriers included battery concerns, eye strain, and technical difficulties.

Although teachers currently rely more on traditional paper textbooks, many expressed increasing interest in approaches that promote student autonomy through a combination of digital and print formats. A majority (64.1%) supported a hybrid approach that takes advantage of the unique affordances of both.

In conclusion, digital textbooks have the potential to enhance student autonomy when implemented with pedagogical intention. To better support independent and self-regulated learning, digital tools should be complemented by materials and strategies that encourage active participation and creative expression. A hybrid learning model aligned with the goal of fostering autonomy may provide a more balanced and effective approach to mathematics instruction.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 261. PME.

LEARNING OF MATHEMATICS TEACHERS IN TRAINING ON DIVERSITY: EXPERIENCES OF DESIGN WITH AN INCLUSIVE APPROACH

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Reflection is a fundamental process within communities of practice (CoP) of mathematics teachers to negotiate meanings and develop practical knowledge. In this sense, international education policies consider the need to promote spaces for reflection to recognize the challenges that arise in the classroom, especially those related to the diversity of students, a responsibility that falls on teachers who need to diversify content, methods, resources, assessment strategies, among other aspects, to ensure that all students achieve the learning goals. In light of this, some research reports that a way to provide this initial training is to involve future teachers in the design and adaptation of activities for students with specific learning characteristics, as this allows them to approach and become sensitized to the experience of collaborating in the learning of all people, assuming the existence of neurodiversity and neurodivergence, and recognizing that different social and cultural contexts, as well as interests, are at the center of meaningful learning. The research still in progress, to be communicated in the framework of the PME, relates two aspects: the model of Reflection and Action of teachers RyA (Parada, 2018) and a flexible and adaptable curricular proposal to promote attention to diversity in mathematics class PADCMA (Jácome et al., 2024), which seeks to answer What meanings negotiate teachers in training who participate in a CoP that proposes a curricular proposal to address diversity in mathematics class? In two years, 24 designs were created in the CoP, 8 of which were made by teachers-in-training who systematized their experiences in research proposals, bachelor theses, papers and articles. Based on the RyA model, the negotiation of meanings of Reflective Thinking was identified. For example, teacher P_1 states: "...History as a didactic resource not only allows epistemological orientation of the teaching, but also of the student's learning and mathematical activity" (Rey, 2022, p. 78).

References

Jácome, I., Parada, S. & Fiallo, J. (2024). Curricular proposal to address diversity in mathematics class: A design on sequences and patterns. *EURASIA Journal of Mathematics, Science and Technology Education*, *20*(6), em2458.

Parada, S. (2011). *Reflexión y acción en comunidades de práctica: Un modelo de desarrollo profesional.* [Tesis de doctorado, Centro de Investigación y de Estudios Avanzados del IPN] Distrito Federal, México.

Rey, J. (2022). *Estudio del razonamiento proporcional en educación primaria: un acercamiento histórico-epistemológico para favorecer la inclusión* [Tesis de pregrado]. Universidad Industrial de Santander.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 262. PME.

RESEARCH ON INTERPRETATIVE KNOWLEDGE: TRENDS AND FUTURE DIRECTIONS

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We present findings from a review on Interpretative Knowledge (IK), showing trends and suggesting future directions for research. Since Lee Shulman's introduction of teacher knowledge, research in this area has expanded, and several structured reviews on teacher knowledge have been conducted (e.g., Hoover et al., 2016). However, none of these reviews include reports on research into teachers' IK—the specialised mathematical knowledge required to interpret students' productions (e.g., Mellone et al., 2020).

We commenced the investigation by searching the ERIC and Web of Science databases, using the terms "interpretative knowledge" and "mathematics" (as well as their equivalents in Spanish and Portuguese). We focused on peer-reviewed empirical research papers on IK published between 2010 and 2024 – when 2010 was when IK started to be conceptualized. Subsequently, we conducted a manual search by scanning proceedings from CERME and PME for the same years.

A total of 23 papers were identified and systematically coded by three researchers. The first article was published in 2013, with the number of publications increasing from five (2013–2016) to six (2017–2020) and ten in the period 2021 to 2024. The majority was published in English (13), with the remaining in Portuguese (8). Nineteen of the articles reported on studies where participants were given a task involving the interpretation of students' productions, while two did not. The studies mainly focused on the process of teaching and learning IK (13), while five examined IK as a construct (i.e., its definition), and two investigated the long-term effects of students learning IK as part of an intervention. Most studies reported working with medium-sized samples of pre-service or in-service teachers (two studies with over 70 participants, four with 30–70 participants, eight with 10–29 participants, and one with fewer than 10 participants). Six studies did not report sample size. Future secondary teachers where the most frequent studied group (7). We will present a deeper analysis and outline future directions informed by the reviewed papers, further elaborating on the results of the different clusters. One crucial point is that all studies were qualitative.

References

Hoover, M., Mosvold, R., Ball, D. L., & Lai, Y. (2016). Making progress on mathematical knowledge for teaching. *The Mathematics Enthusiast, 13*(1), 3–34.

Mellone, M., Ribeiro, M., Jakobsen, A., Carotenuto, G., Romano, P., & Pacelli, T. (2020) Mathematics teachers' interpretative knowledge of students' errors and non-standard reasoning. *Research in Mathematics Education, 22*(2), 154–167.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 263. PME.

DIAGNOSTIC TESTS TO ASSESS MATHEMATICAL KNOWLEDGE

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Diagnostic tests are typically short quizzes or tests used in mathematics education as a type of formative assessment, helping teachers identify students' current knowledge and potential knowledge gaps. Over the years, advances in assessment have prioritized increased precision, which has resulted in simple tests in terms of right-wrong answers and with a high reliability, rather than tests that focus on mathematical processes (Burkhardt & Schoenfeld, 2018). Mathematical knowledge is usually described as mathematical competencies (e.g., Lithner et al., 2010; Niss & Højgaard, 2019). A strong emphasis on reliability in assessments risks favouring competencies that are easier to measure, while more complex competencies receive less attention (Burkhardt & Schoenfeld, 2018). This raises important questions about how the design of diagnostic tests influences the type of information they provide about students' mathematical knowledge. More research is needed to explore this relationship and to develop assessments that better capture a broad range of mathematical competencies.

In this ongoing study, we used an established framework (Lithner et al., 2010) to analyse a mathematics diagnostic test used for all beginning student at an upper-secondary school, together with student solutions, to capture which competencies were activated when engaged with the tasks. As is common for diagnostic tests, this test only consisted of right-wrong answer tasks. As opposed to the arrangements for this study, teachers do not have access to students' full solutions, but only their final answer. Preliminary results show that the tasks mainly allow students to show the connection and procedure competencies, and particularly with a focus on the "do and use" aspect. In the next step, we will modify the design of some of tasks to explore which aspects may promote activation of other competences, thereby striving to obtain a more comprehensive picture of students' mathematical knowledge. The results will be discussed and exemplified further in the presentation.

References

Burkhardt, H., & Schoenfeld, A. (2018). Assessment in the service of learning: Challenges and opportunities. *ZDM: Mathematics Education*, *50*(4), 571-585.

Lithner, J., Bergqvist, E., Bergqvist, T., Boesen, J., Palm, T., & Palmberg, B. (2010). Mathematical competencies: A research framework. In C. Bergsten, E. Jablonka & T. Wedege (Eds.), *Mathematics and mathematics education: Cultural and social dimensions. Proceedings of MADIF 7*, (pp 157-167). SMDF.

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics*, *102*(1), 9–28.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 264. PME.

TASK-BASED INTERVIEW AS BOUNDARY OBJECT IN MATHEMATICS EDUCATION RESEARCH

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As mathematics education researchers (MERs) continue to employ a wide range of theoretical perspectives, a pressing concern involves the relationship between theories and research methods. Drawing on Wenger's (1998) Community of Practice theory, we investigate how and why MERs may use a common research method in conjunction with different theoretical perspectives. We ask: In what ways are MERs' sensitivities to theoretical assumptions evidenced in their design, use, and analysis of task-based interviews in empirical studies? We report a case of the use of task-based interviews in two recent studies (Rodriguez & Jones, 2024; Uscanga et al., 2024) employing different theoretical perspectives on functions and graphs. Results illuminate how the research method of the task-based interview can serve as a boundary object between different *Communities of Theoretical Practice* (CoTPs), consisting of MERs employing a shared theory (or theories) in the practice of research.

By serving as a boundary object, a research method can provide space for elements of different CoTPs to overlap with each other, while still adhering to principles laid out by the theory (or theories) guiding the communities. When a research method is used among different CoTPs, the design, use, and analysis adapt in relationship to the theoretical perspective in which the method is nested. To illustrate, Rodriguez and Jones (2024) interviewed students from different universities, treating them as a single group for analysis, to develop a set of graphical forms. In contrast, Uscanga et al. (2024) interviewed students from the same university, because an assumption from their theoretical perspective was that students' graphing techniques were linked to the institution (i.e., the university) in which the techniques emerged. Hence, the underlying logic of the method changes in response to the theory. As theories proliferate, research methods can be a place for different CoTPs to dialogue around the practice of research.

References

Rodriguez, J.-M. G., & Jones, S. R. (2024). How students understand graphical patterns: Fine-grained, intuitive knowledge used in graphical thinking. *Journal for Research in Mathematics Education*, *55*(2), 96–118. https://doi.org/10.5951/jresematheduc-2022-0167

Uscanga, R., Melhuish, K., & Cook, J. P. (2024). Students' techniques for approaching defining properties of functions. *Educational Studies in Mathematics*, *117*, 457–484. https://doi.org/10.1007/s10649-024-10344-2

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge University Press.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 265. PME.

ENHANCING PRE-SERVICE TEACHERS' SPATIAL REASONING IN GEOMETRY THROUGH HANDS-ON ACTIVITIES

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Spatial reasoning is a crucial cognitive skill in learning geometry, as it enables individuals to visualize, manipulate, and understand spatial relationships between objects (Clements & Sarama, 2011). Research has shown that developing special reasoning skills enhances students' abilities to engage in mathematical problem solving and geometrical thinking (Uttal et al., 2013). For pre-service teachers, strengthening these skills is particularly important, as their spatial reasoning abilities influence how they teach geometry in classrooms and foster students' understanding of spatial concepts (Ng & Sinclair, 2015).

This study investigates the relationship between one-dimensional (1D) to three-dimensional (3D) shapes by having pre-service teachers construct a quadrilateral using squares created from popsicle sticks. The study involved a target group of 30 second-year pre-service teachers. The research was designed according to a three-phase framework: Co-Plan, where pre-service teachers collaboratively devise a method to construct a quadrilateral using the provided materials; Co-Do, in which they engage in problem-solving through the construction process; and Co-See, wherein each group presents their completed quadrilateral and reflects on their learning experiences. Data were collected through classroom observations, participant reflections, and documentation of their constructed work.

The findings indicate that the integration of practical activities with geometric concepts fosters effective learning and retention of spatial reasoning skills. The use of tangible materials such as popsicle sticks creates an engaging educational environment that enhances students' abilities to visualize and manipulate geometric shapes, thereby bridging the gap between mathematical theory and real-world applications. Moreover, this hands-on approach significantly contributes to improving pre-service teachers' understanding of geometry and spatial reasoning.

References

Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: the case of geometry. *Journal of Mathematics Teacher Education*, *14*(2), pp.133-148

Ng & Sinclair. (2015). Young children reasoning about symmetry in a dynamic geometry environment. *ZDM: the international journal on mathematics education*, *47*(3), pp. 421-434.

Uttal et al., (2013). The Malleability of Spatial Skills: A Meta-Analysis of Training Studies. *Psychological Bulletin*. *139*(2), pp. 1-51.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 266. PME.

ENHANCING PRESERVICE TEACHERS' MATH IDENTITIES AND LEADERSHIP SKILLS BY FOSTERING COLLABORATIVE CULTURE

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This study explores how pre-service teachers (PSTs) develop mathematical identities and leadership skills through collaborative activities in a mathematics methods course. Activities include math mindset modules, unit planning, and professional development (PD) webinars. Using complexity science (Davis & Simmt, 2003), which views learning environments as dynamic social systems (Reid, 2014), the study examines how collaboration fosters PSTs' leadership and develops their identities as math educators. The central question asks: How does working in small groups for unit planning and PD webinars contribute to PSTs' math identities and leadership skills?

A mixed-methods approach (Creswell & Creswell, 2018) was used. The online questionnaire, which consisted of 11 questions, including six Likert-scale and five open-ended questions, was employed to study PSTs' mathematical and pedagogical knowledge, leadership skills, collaborative experiences, and development of their math identities. Of 150 invited PSTs from a university in Ontario, Canada, 24 participated. Quantitative data was analyzed using descriptive statistics, while qualitative responses were thematically examined.

Findings indicate that 71% of PSTs felt confident in taking on leadership roles in mathematics, with one stating, "My confidence levels have grown over my time in this course. I can use this to take on math leadership roles at my future schools." Regarding math identities, 71% of PSTs strongly agreed that working in small groups and sharing ideas enhanced their confidence in understanding math concepts and pedagogy. One PST noted, "The course helped deepen my understanding and to listen to new perspectives and ideas." These findings suggest that structured, collaborative small groups helped PSTs build confidence, strengthen their math identities, and recognize their potential as future math leaders.

References

Creswell, J. W., & Creswell, J. D. (2018). *Research design: Qualitative, quantitative, and mixed methods approaches (5th ed.)*. SAGE Publications.

Davis, B. & Simmt, E. (2003). Understanding Learning Systems: Mathematics Education and Complexity Science. *Journal for Research in Mathematics Education, 34*(2),137-167.

Reid, D. A. (2014). The coherence of enactivism and mathematics education research: A case study. *Avant (Toruń)*, *2*, 137–172. https://doi.org/10.26913/50202014.0109.0007

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 267. PME.

BELIEFS AND MATHEMATICAL KNOWLEDGE FOR TEACHING IN COMMUNITY COLLEGE ALGEBRA

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This study examined whether U.S. community college instructors' mathematical knowledge for teaching college algebra (MKT-CCA) and their reported frequency of engaging in specific tasks of teaching predict their beliefs about teaching and learning mathematics, as well as their awareness of their students' mathematical dispositions. Building on the acknowledged mutually informing relationship between teacher' knowledge, beliefs, and teaching practice, we hypothesized that increasing instructors' knowledge for teaching mathematics and their engagement in specific tasks of teaching can lead to changes in their beliefs about teaching and learning of mathematics. To test our hypothesis, we analyzed data collected from 341 community college algebra instructors. The dataset consisted of responses to 19 multiple-choice MKT-CCA items (Mesa et al., 2024), 16 beliefs and awareness items (Clark et al., 2014), and questions about their teaching experiences. After confirming the factor structure of the constructs, we conducted structural equation modelling (SEM) to examine how instructors' beliefs and awareness factors were predicted by their MKT-CCA and reported frequency of two tasks of teaching – understanding student work and choosing problems. Results showed that MKT-CCA positively predicted beliefs about student-centred instruction and negatively predicted beliefs about teaching with incremental mastery, memorization and procedures, and awareness of students' mathematical dispositions. The reported frequency of engaging in understanding student work predicted higher awareness of students' mathematical dispositions.

Additional information

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References

Clark, L. M., DePiper, J. N., Frank, T. J., Nishio, M., Campbell, P. F., Smith, T. M., ... & Choi, Y. (2014). Teacher characteristics associated with mathematics teachers' beliefs and awareness of their students' mathematical dispositions. *Journal for Research in Mathematics Education, 4*5(2), 246-284.

Mesa, V., Duranczyk, I., Wingert, K., & VMQI-AI@CC 2.0 Team. (2024). *Measuring mathematical knowledge for teaching college algebra at community colleges.* Research in Undergraduate Mathematics Education Conference, Omaha, NE, United States.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 268. PME.

PREDICTING SPATIAL ABILITY PERFORMANCE FROM GAZE AND BRAIN ACTIVITY

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An effective learning approach depends on a child's cognitive level and individual characteristics. Real-time assessment of current proficiency and post-learning performance could enable personalized learning strategies. Previous studies have demonstrated that performance can be predicted using gaze and neural activity during learning (Brucker et al., 2015; Chettaoui et al., 2022). However, few studies in mathematics education have integrated both gaze and brain activity. We hypothesized that combining these data could enhance predictive accuracy. This study investigated whether spatial ability performance, both pre- (reflecting initial proficiency) and post-learning, could be predicted using gaze and brain activity during ICT-based learning.

A total of 25 sixth-grade students participated; however, due to incomplete data from eight participants, analyses were conducted on 17 students. The experiment comprised three phases: (1) Pre-learning Task: Participants completed 30 spatial reasoning problems, assessing whether an unfolded figure would reconstruct into an identical cube. (2) ICT Learning: They manipulated virtual 3D cubes to observe their assembly and disassembly. (3) Post-learning Task: They solved another 30 spatial reasoning problems identical to the pre-learning phase. Eye-tracking data were recorded using the Tobii Pro Nano, while neural activity in the prefrontal cortices was measured via the Spectratech OEG-16. Data were randomly partitioned into 12 training and 5 test sets, with predictive modeling conducted using Partial Least Squares Regression.

Results indicated a significant positive correlation between predicted and actual pre-learning performance ($r = .926$, $p = .024$), whereas no significance was observed for post-learning ($r = .753$, $p = .142$). While gaze and neural activity effectively predicted initial ability, they were less reliable in forecasting post-learning performance. The relatively high post-learning correlation (i.e., $r = .753$) suggests that the lack of statistical significance may be attributable to the limited sample size.

References

Brucker, B., Ehlis, A.-C., Häußinger, F. B., Fallgatter, A. J., & Gerjets, P. (2015). Watching corresponding gestures facilitates learning with animations by activating human mirror-neurons: An fNIRS study. *Learning and Instruction, 36,* 27–37.

Chettaoui, N., Atia, A., & Bouhlel, M.S. (2022). Student Performance Prediction with Eye-Gaze Data in Embodied Educational Context. *Education and Information Technologies, 28*, 833-855.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 269. PME.

CONCEPTUALIZING AND MEASURING VALIDATING COMPETENCE WHEN MODELING WITH FUNCTIONS

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Modeling is considered a key activity in mathematics and refers to describing the real world with mathematical concepts. Modeling processes have been systematized using modeling cycles, with validation as a sub-process (Cevikbas et al., 2022). Validation can be understood as a constant activity to evaluate each step of the modeling process, considering prior steps (Czocher, 2018). Working on complete modeling tasks is assumed to promote validating competence (Cevikbas et al., 2022). However, to our knowledge, there are no test instruments to measure validating competence when modeling data with functions. Thus, we address the following questions in this context:

- How can validating competence be conceptualized and measured?
- Can validating competence be fostered with an intervention consisting of three sessions, each working on one complete modeling task?

We draw on a sample of $N = 616$ secondary students (27 classes) who modeled real-world data with functions in self-regulated and structured learning environments.

We conceptualized validating competence as the ability to evaluate and modify a model or result during a modeling process, distinguishing whether it is based on the situation model or data. The final test consists of 4 open-ended and 16 multiple-choice items, focusing on validating when modelling data with linear or exponential functions.

Using a multi-matrix design, students answered half of the items before and the other half after the intervention. Thus, we make use of item-response theory to scale pretest and posttest jointly and successfully applied a background model to draw 20 plausible values (PV) for validating competence (mean EAP/PV reliability T1 & T2: .97). A t-test for multiple imputed data reveals a significant increase in validating competence from pre to posttest ($t(47.77) = 2.70$, $p < .01$) with a small effect (Cohen's $d = 0.11$).

In sum, we give insights into the development of a test to assess validating competence when modeling with functions and its application to evaluate an intervention. As the intervention promotes validating competence to a very limited extent, it seems students need to be explicitly instructed on how to validate when modeling with functions.

References

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2021). A systematic literature review of the current discussion on mathematical modelling competencies. *Educational Studies in Mathematics, 109*(2), 205-236. https://doi.org/10.1007/s10649-021-10104-6

Czocher, J. A. (2018). How does validating activity contribute to the modeling process? *Educational Studies in Mathematics, 99*(2), 137-159. https://doi.org/10.1007/s10649-018-9833-4

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 270. PME.

FLEXIBLE ABILITY GROUPING AS A SOURCE OF INEQUALITY IN MATHEMATICS EDUCATION

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Ability grouping has been a common practice for teaching mathematics in many countries and known to affect achievement of low and middle groups (Boaler, 2020). Finnish school education sets a special context where ability grouping was officially removed after launching public basic education in 1985. The ethos has been to bring support and individualisation to the classroom of mixed abilities, and thus, avoid differentiation by grouping students. Finnish mathematics education is to provide uniform and equal learning opportunities for all students despite their background and learning history. However, it has become more common to use flexible grouping (Lou et al., 1996) in Finnish mathematics classes. The idea of flexible grouping is to allow students to change groups in accordance with individual progress, i.e. development of mathematical skills and knowledge shown in tests. We investigate the realisation of flexible grouping and its impact on treating students equally.

We present a case of a Finnish lower secondary school based on various data sets. The case study is based on interviews with students (N = 53) and responsible teachers (N = 4) as well as results of a diagnostic test and 12 course tests designed by the respective teachers during three academic years from 2018 to 2021. The analysis focused on investigating differences of student groups, analysis of the tests, changes of grouping and understanding the reasons for experienced practice by students and their teachers over three academic years.

Our data shows that flexible grouping treats students unequally and reinforces differences between the groups. On the one hand, grouping based on mathematical ability seems to have an impact on learning objectives, and thus, expectations on mathematical progress. Analysis of the tests reveals that not all ability groups are expected to achieve the same learning outcomes despite the common curriculum. On the other hand, design of the mathematics tests seems to reinforce the difference of low-performance and high-performance groups and inflexibility of grouping. The tests included both common and mandatory items for all students and optional items of varied difficulty level. However, students stated in the interviews that they chose mainly optional items 'according' to their assumed ability group. Social aspects were not considered in ability grouping even if students expressed them to be essential for their overall learning process. Interestingly, both teachers and students claim that flexible grouping makes it possible to address students' individual needs.

References

Boaler, J. (2020). Ability grouping in mathematics classrooms. In S. Lerman (ed.), *Encyclopedia of Mathematics Education* (pp. 1-5). doi 10.1007/978-94-007-4978-8

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 271. PME.

UNDERSTANDING STUDENTS' READING COMPREHENSION OF REAL-WORLD PROBLEMS UTILIZING ERP TECHNIQUES

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Mathematics education aims to establish a strong connection between real-world contexts and abstract mathematical concepts. This study examines reading comprehension in converting real-world problems into mathematical linear equations with two variables in both Chinese and English. It focuses on cognitive processes involved in reading comprehension, including decoding, meaning construction, and integrating prior knowledge (Kintsch, 1998). We designed a new reading comprehension test, ensuring consistency by using identical problems and contexts in both languages. Each problem was structured into stages, incorporating key information (e.g., coefficients, variables, key relationships, and total quantities) and non-key information providing contextual background. Students were asked to determine whether the equation in the final stage aligned with the real-world context. The test included various real-world problems, with each test item offering both correct and incorrect options, totaling 168 trials. Forty Taiwanese university students participated in the study, and their English proficiency was assessed using the Common European Framework of Reference for Languages (CEFR).

The test measured response accuracy (Acc) and reaction time for correct responses (RTc) as indicators of cognitive complexity. Brainwave activity during problem-solving was recorded and analyzed using event-related potential (ERP) techniques. Behavioral data analysis showed no significant difference in Acc between the Chinese and English versions. However, RTc was significantly faster in Chinese than in English. Brain activity analysis revealed higher amplitudes in Chinese across all components, suggesting that language familiarity influences cognitive processing in reading comprehension. Despite this, the amplitude patterns in both languages were very similar in terms of reading comprehension processes. Decoding and meaning construction were analyzed through the P300 component, revealing that students allocated greater neural resources to keywords, with significant differences between key and non-key information stages. Integration and monitoring, a metacognitive process associated with higher-level reading comprehension, was linked to P600, which peaked at Stage 7 when students evaluated equation-context alignment. Expectation and error detection, analyzed through N400, showed a complementary pattern to P600. While P600 amplitudes increased in later stages, N400 peaked at Stage 2 and then steadily declined, indicating reduced cognitive load over time. These findings offer key insights into bilingual mathematical literacy, underscoring the importance of keywords in reading comprehension within real-world context and deepening our understanding of cognitive processes at a neurophysiological level.

References

Kintsch, W. (1998). The representation of knowledge in minds and machines. *International Journal of Psychology*, *33*(6), 411-420.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 272. PME.

EXPLORING THE IMPACT OF REFLECTIVE TEACHING ON MATHEMATICAL PEDAGOGICAL KNOWLEDGE: A CASE STUDY IN RATIO AND PROPORTION INSTRUCTION

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The PISA 2021 Mathematics Framework (OECD, 2018a) defines mathematical literacy as an individual's ability to engage in mathematical reasoning and apply, interpret, and utilize mathematical concepts to solve real-world problems. In addition to mathematical literacy, PISA 2022 incorporates eight key 21st-century skills: Critical Thinking, Creativity, Research & Inquiry, Self-Direction, Initiative and Persistence, Information Use, Systems Thinking, Communication, and Reflection. Dewey (1993) defines reflection as a purposeful mode of thinking that involves reviewing past experiences and improving learning behaviors. Moreover, self-reflection refers to the ability to observe and evaluate one's cognitive, emotional, and behavioral processes.

This study adopts a case study approach to investigate the implementation of ratio and proportion instruction in a sixth-grade elementary school classroom. It examines a veteran teacher's reflections before and after a teaching intervention. The participant is an experienced teacher with 20 years of teaching experience who had not previously engaged in reflection-based teaching. Qualitative data were collected through interviews and students' classroom notes. The study utilized the Mathematical Pedagogical Content Knowledge (MPCK) Perception Scale for Elementary School Teachers (Chen, 2014) to assess whether the teacher's mathematical pedagogical knowledge improved after implementing reflective teaching practices.

Following the instructional intervention, data were analyzed and coded, leading to the following findings: (a) Before the teaching intervention, the teacher's reflection involved reviewing cases from nearby schools as part of lesson preparation; (b) After the intervention, the teacher demonstrated an increased ability to guide students in employing diverse problem-solving strategies and multiple representations; (c) The teacher's knowledge of mathematical instructional strategies and mathematical representations improved.

References

Chen, Y.-T. (2014). The development of the Mathematical Pedagogical Content Knowledge (MPCK) perception scale for elementary school teachers. *Journal of Educational Measurement and Evaluation, 61*(1), 51–78.

Dewey, J. (1993). *How we think: A restatement of the relation of reflective thinking to the educative process*. D.C. Heath & Co.

Organization for Economic Co-operation and Development. (2018a). *PISA 2021 mathematics framework.* OECD Publishing.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 273. PME.

RESEARCH-BASED TEACHER EDUCATION IN MATHEMATICS FOR FUTURE PRIMARY TEACHERS

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For decades scholars have suggested that mathematics teaching should involve mathematical discussions and reasoning to improve students' conceptual understanding (Kazemi & Stipek, 2001). Rather than disseminating research findings through lectures and various forms of assessments, we have chosen to embody the principles we teach within the teacher education program for future primary teachers (FPT)

In short, we treat the mathematical content in the FPT course by presenting mistakes that research has exposed as common among students, such as believing that 0.2×0.8 equals 1.6 since 'multiplication makes bigger' (Van Dooren et al., 2015). Several of the mistakes we use are made by the FPTs themselves, hence realistic to the FPT-group. The FPTs are invited to engage in a discussion regarding the reasons for the mistake and to explore pedagogical strategies that can enhance conceptual understanding of the mathematics. This discussion is overseen and guided by us, the instructors, utilizing the ideas from Stein et al. (2008). In whole-class discussion we function as role-models leading the discussion by questions that press the FPTs to think conceptually and use mathematical reasoning (Kazemi & Stipek, 2001). Thus, we operationalise research outcomes both for content and methods.

Our changed teaching has resulted in higher quality of the solutions in the mathematics exam, measured by more students passing the exam and a greater proportion with the higher grade. Before the change between 65–70 % passed the exam and 20–25% with the higher grade. In the autumn of 2024 80 % of the FPT passed the exam and 46 % with the higher grade.

References

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal 102*(1), 59–80.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 3133–40.

Van Dooren, W., Lehtinen, E., & Verschaffel, L. (2015). Unraveling the gap between natural and rational numbers. *Learning and Instruction, 37*, 1–4.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 274. PME.

MAPPING OF STANDARDIZED INSTRUMENTS TO ASSESS NUMBER SENSE IN CHILDREN

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Number sense refers to the general understanding of numbers and concepts, their properties and relations, their uses and meanings in a variety of situations (Spinillo et al., 2021; Whitacre et al., 2020). Its relevance has been recognized since this construct is a powerful predictor of children's later mathematical achievement. Due to this, number sense has been assessed through standardized instruments in studies that aimed to determine performance levels in children of different ages and school grades. Given the large number of these instruments, it seems important to examine how these studies are characterized. Thus, a Scoping Review of the literature was carried out on papers published from 1954 (date of the first occurrence of the term "number sense") to 2024. Articles were searched in the databases Periódico CAPES, Scielo, Web of Science, PsycInfo, and PePSIC, using the keywords in Portuguese, Spanish, and English: "number sense" AND children AND number sense instrument OR number sense test. A total of 358 papers were found, of which 74 were duplicates. Titles and abstracts were read to apply the inclusion criteria: full articles reviewed by specialists, addressing the creation/validation/adaptation/use of instruments to assess number sense in children up to 12 years old. After applying these criteria, 40 articles were identified, of which only 15 focus on the creation/validation and/or adaptation, presenting psychometric indices of internal consistency, with limited emphasis on content validation of the items. The instruments adopted are primarily based on the tests created by Jordan and Yang, ranging from 8 to 165 items, and are generally administered individually, using pencil and paper. It was also found that standardized tests assess a great diversity of components of number sense and that few instruments are administered online. Most of the tests are developed for use with children from Kindergarten to Elementary School II. The Asian continent leads in the development of instruments, followed by the United States. The results indicate a lack of tests on Spanish and Portuguese speaking children.

References

Spinillo, A.G., Correa, J., & Cruz, M. S. Silva (2021). Number sense in a developmental perspective: comparing the mastery of its different components in children. In A. G. Spinillo, S. L. Lautert, R. E. S. R. Borba. (Eds.). *Mathematical Reasoning of Children and Adults* (pp.41-68). Springer Nature.

Whitacre, I., Henning, B., & Atabas, S. (2020). Disentangling the Research Literature on Number Sense: Three Constructs, One Name. *Review of Educational Research, 90*, (1), 95–134.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 275. PME.

STUDENTS' PERCEPTIONS OF GENERATIVE ARTIFICIAL INTELLIGENCE FOR PROVING MATHEMATICAL STATEMENTS

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This study explores students' perceptions of proving mathematical statements using generative artificial intelligence (genAI). Yoon et al. (2024) proposed a framework, explaining how undergraduate students construct proofs using genAI. Yoon et al. (2024) highlighted that students' factors—such as conceptions of proof and ChatGPT, ethical considerations—can be connected to the mathematical proving process using genAI. Building on this, the present study focuses on analyzing undergraduate students' perceptions of engaging with genAI in proving mathematical statements.

The study was conducted in a *Mathematics and Educational Technology* course offered during the Fall 2024 semester at a university in Korea, with 33 students enrolled. Within this course, ChatGPT was introduced as an emerging technology in genAI and applied in the context of mathematical proving. We administered pre- and post-surveys using an identical set of 22 items based on three factors identified by Yoon et al. (2024), measured on a 5-point Likert scale and analyzed through paired t-tests. Students' written reflections on proving using genAI were collected and analyzed using open coding, focusing on their perceptions of proving using genAI.

The quantitative analysis revealed a significant difference in students' perceptions of ChatGPT conceptions between the pre- and post-surveys. However, no significant changes were observed in their perceptions of proof conceptions or ethical considerations. In the qualitative analysis focusing on ChatGPT conceptions, students perceived that using ChatGPT made the proving process easier, and therefore, they considered it more effective to first draft an outline and then use ChatGPT to complete the proof. Additionally, while students found ChatGPT's responses to be sometimes lacking in justification or inconsistent, leading to concerns about its capability, they generally regarded its responses as logically sound and perceived it as a useful tool for subsequent proving tasks. These findings offer implications for designing instructional strategies that shape students' perceptions of genAI for mathematical proving.

References

Yoon, H., Hwang, J., Lee, K., Roh, K. H., & Kwon, O. N. (2024). Students' use of generative artificial intelligence for proving mathematical statements. *ZDM–Mathematics Education, 56*(7), 1531-1551.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 276. PME.

GAME-BASED TEACHING FOR FIFTH-GRADE MATHEMATICS CURRICULUM CASE STUDY OF FACTORS AND MULTIPLES

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The integration of diverse teaching methods in the classroom can effectively capture students' attention and enhance their learning experiences. This study investigates the application of game-based pedagogy to facilitate students' understanding of factors and multiples, recognize relationships among numbers, and identify patterns in the factors or multiples of two numbers. According to Noda & Nakao (2019), the incorporation of games in mathematics instruction can significantly improve students' comprehension of factors and multiples. The integration of number-based games with game-oriented teaching strategies fosters increased student engagement in the learning process. Similarly, Krath & Korflesch (2021) emphasized the positive impact of gamified instruction on students' ability to master factors and multiples in mathematics education.

This study was conducted in a fifth-grade classroom with a small cohort of 10 students. The instructional approach focused on the implementation of game-based learning centered around the theme of "factors and multiples." Specifically, students participated in the "Math War Chess" game, a resource developed by Taiwan's Ministry of Education, designed to enhance students' understanding of these mathematical concepts. The primary aim of this study was to evaluate the effectiveness of this method and to gather students' perceptions of game-based learning. A case study methodology was employed, utilizing worksheets and assessments related to factors and multiples to evaluate learning outcomes.

The main findings of this study are as follows: (a) Student participation in class discussions and presentations significantly increased, reflecting a higher level of engagement. (b) Students exhibited greater proficiency in identifying multiples than factors, suggesting a disparity in the cognitive processing required for these two concepts.

References

Krath, J., Schürmann, L., & von Korflesch, H. F. O. (2021). Revealing the theoretical basis of gamification: A systematic review and analysis of theory in research on gamification, serious games, and game-based learning. *Computers in Human Behavior, 125*, 106963.

Noda, S., Shirotsuki, K., & Nakao, M. (2019). The effectiveness of intervention with board games: A systematic review. *BioPsychoSocial Medicine, 13*, 22.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 277. PME.

UNVEILING THE COMPONENTS OF MATHEMATICAL THINKING EXPERIENCE: A FACTOR-ANALYTIC FRAMEWORK FOR MATHEMATICAL CREATIVITY ENHANCEMENT

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Mathematical creativity is a key competency in 21st-century education, requiring both deep knowledge and rich experience, with mathematical thinking experience playing a crucial role in fostering creativity (Guo & Shi, 2012; Leikin & Sriraman, 2022). However, the specific components of mathematical thinking experience and their role in promoting creativity remain unclear. This study addresses two questions: (1) What is the framework for the components of mathematical thinking experience? (2) How does this framework contribute to the development of mathematical creativity?

Drawing on Krutetskii's (1968/1983) component analysis of mathematical ability, the study integrates cognitive characteristics from Hadamard's four-stage model of the creative process and surveys 69 high school teachers about the cognitive traits of students who excel at mathematical thinking. Based on these insights, eight potential components were hypothesized: observation, comparison, mathematical expression, induction, analogy, proof, association, and classification. A 22-item measurement tool was developed (Cronbach's $\alpha = 0.87$) and administered to 138 high school students.

Factor analysis ($KMO = 0.83$, $p < 0.001$) revealed four components, explaining 88.5% of the variance: Inductive generalization (observation, comparison, classification, induction), Analogy reasoning (analogy, association), Mathematical expression, and Proof. A series of teaching cases were designed based on this framework, and cross-case analysis showed that the framework-based teaching significantly enhanced students' creativity, especially in flexibility and originality. For example, inductive generalization helped students view familiar problems from new perspectives, while expansive associations fostered insightful solutions. These findings provide valuable insights for designing creativity-oriented mathematical tasks and teaching strategies.

References

Guo, Y., & Shi, N. (2012). Basic mathematical activity experience: Proposal, understanding, and practice. *Journal of the Chinese Society of Education*, (4), 42 – 45.

Krutetskii, V. A. (1968/1983). *The psychology of mathematical abilities in schoolchildren* (L. Bóshù, Trans.). Shanghai Education Press.

Leikin, R., & Sriraman, B. (2022). Empirical research on creativity in mathematics (education): from the wastelands of psychology to the current state of the art. *ZDM, 54*(1), 1 – 17. https://doi.org/10.1007/s11858-022-01340-y

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 278. PME.

THE ASSOCIATION OF GENERIC AND SUBJECT-SPECIFIC MINDSETS WITH MATHEMATICS PERFORMANCE

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Research indicates that mindsets can be both domain-general and domain-specific. While domain-specific mindsets predict specific outcomes, global mindsets more strongly predict overall outcomes. In mathematics education, domain-specific interventions have shown more consistently positive results than general interventions (Bui et al., 2023). However, some studies suggest that generic mindsets might be more predictive of learning outcomes (Cutumisu & Lou, 2020). These findings underscore mindsets' complexity and varying influences in different contexts. This study investigates the roles of generic and subject-specific mindsets in students' math performance. The growth mindset scale in the PISA 2022 student questionnaire includes four questions, two of which pertain to subject-specific mindsets ("Some people are just not good at mathematics / in Chinese, no matter how hard they study.") while the other two relate to general ability mindsets ("Your intelligence/creativity is something about you that you cannot change very much."). This study conducted a latent class analysis (LCA) on Taiwan students' responses to these four questions. The results indicated that students' mindsets can be categorized into four types: 30.34% belong to the "fixed mindset for subject learning" category, believing that while intelligence and creativity may change, subject-specific abilities (mathematics and Chinese) cannot. 25.13% have a "fixed mindset for all," meaning they believe neither subject-specific nor general abilities can change. 29.07% exhibit a "growth mindset for all," holding that both subject-specific and general abilities can change. Finally, 15.46% have a "growth mindset for subject learning," believing that intelligence and creativity cannot change, but subject abilities can with hard work. This study further employed hierarchical linear modeling (HLM). The results showed that after controlling for gender and socioeconomic status, the mindsets still made a unique contribution. The additional explanatory power for inter-school and intra-school variation is 1%. If students with a fixed mindset shift to a growth mindset, their PISA math performance could increase by 18 to 22 points, representing an improvement equivalent to approximately one academic year's difference.

References

Bui, P., Pongsakdi, N., McMullen, J., Lehtinen, E., & Hannula-Sormunen, M.M. (2023). A systematic review of mindset interventions in mathematics classrooms: What works and what does not? *Educational Research Review, 40,* 100554.

Cutumisu, M., & Lou, N.M. (2020). The roles of generic and domain-specific mindsets in learning graphic design principles. *Interactive Learning Environments, 31*, 1781-1795.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 279. PME.

MATHEMATICAL MODELING AND ETHICAL MATHEMATICAL AWARENESS: THE INFLUENCE OF AI

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Recent research highlights the importance of integrating ethical considerations into mathematics education. Some authors, for instance, examines ethical dilemmas arising from the official objectives of mathematics education at different levels, showing how the discipline can both promote social well-being and perpetuate inequalities.

Ethical mathematical awareness refers to recognizing the social, political, and cultural implications of mathematics and its teaching. Some studies emphasize that mathematics is not neutral but embedded in contexts that both influence and are influenced by it. An educational approach that embraces this awareness fosters more reflective and responsible teaching.

The incorporation of artificial intelligence further expands ethical concerns. The concept of humans-with-ChatGPT highlights the collaboration between humans and AI in knowledge construction. ChatGPT can act as an epistemic agent, reorganizing human thought and enhancing mathematics education. However, its use requires a critical approach to prevent dependency and ensure students' reflection and autonomy (Lopes & Borba, 2024).

An empirical study with engineering students investigated ChatGPT's role in mathematical modeling applied to social and environmental issues. Results indicated that this interaction fosters reflections on ethical dilemmas, contributing to ethical mathematical awareness. However, findings also suggest that teacher mediation is crucial, as it fosters critical discussions and guides knowledge construction more consciously and responsibly.

Integrating ethics into mathematics education is essential for shaping critical citizens aware of the discipline's social impact. AI, such as ChatGPT, expands learning opportunities but also raises ethical challenges, requiring a balance between innovation and critical reflection. Mathematics education must evolve to address ethics and technology, with future research focusing on pedagogical strategies and teacher mediation to strengthen this awareness.

References

Lopes, A. P. C., & Borba, M. C. (2024). Seres-humanos-com-ChatGPT em Modelagem Matemática. *Seminário Internacional De Pesquisa Em Educação Matemática*, 1-15. https://www.sbembrasil.org.br/eventos/index.php/sipem/article/view/260

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 280. PME.

INTERACTIONS CREATED DURING SHAPING BY MATHEMATICS TEACHERS WHEN TEACHING THE PYTHAGOREAN THEOREM

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The literature has increasingly emphasized the need for studies that focus on the mathematics teachers' noticing in "action" (in-the-moment) that is, the noticing that occurs while teaching mathematics in the classroom (e.g., Kersting et al., 2021). According to van Es and Sherin (2021), noticing involves three dimensions: attending, interpreting, and shaping. This study focuses on the shaping dimension, which "involves the *creation* of interactions for the purpose of gaining access to additional information" (van Es & Sherin, 2021, p. 24). In this context, and as part of a doctoral dissertation, we aim to answer the following research question: What interactions do mathematics teachers construct within the shaping dimension when teaching the Pythagorean Theorem?

This research is based on a case study of three Chilean mathematics teachers: Misty (9 years of experience) and Max (22 years of experience), both with master's degrees, and May (8 years of experience). Data collection was conducted through non-participant observation of these teachers while they taught lessons on the Pythagorean Theorem to eighth-grade students (ages 12–14) in Chilean educational institutions. Classroom recordings were analyzed to identify moments of attending, interpreting, and shaping. Once moments of shaping were identified, they were coded and classified according to the types of interactions constructed.

Preliminary results reveal different interactions during teachers' shaping processes, such as requesting additional information, reformulating students' thinking, or connecting it to their previous strategies. In our presentation, we will provide evidence of the different identified interactions.

These findings suggest that the interactions constructed depend on multiple factors, including mathematics teachers specialized knowledge, which will be the focus of our future research.

References

Kersting, N. B., Smith, J. E., & Vezino, B. (2021). Using authentic video clips of classroom instruction to capture teachers' moment-to-moment perceiving as knowledge-filtered noticing. *ZDM: The International Journal on Mathematics Education*, *53*(1), 109–118. https://doi.org/10.1007/s11858-020-01201-6

van Es, E. A., & Sherin, M. G. (2021). Expanding on prior conceptualizations of teacher noticing. *ZDM – Mathematics Education*, *53*(1), 17-27. https://doi.org/10.1007/s11858-020-01211-4

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 281. PME.

EMERGENCE OF ALGEBRAIC REASONING IN PROBABILITY PROBLEM SOLVING IN PRIMARY EDUCATION

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Various authors advocate for the importance of incorporating the study of probability from primary education, leveraging students' intuitions and progressing toward the description, quantification, and modelling of uncertainty (Pratt & Kazak, 2018). Numerous studies highlight children's ability to successfully solve probability comparison tasks in the context of urn and, to a lesser extent, their ability to construct the sample space associated with an experiment. While previous studies explore the link between probabilistic and proportional reasoning (Hernández-Solís et al., 2021), the connection with the degree of algebraic reasoning involved has not been explored. In this study, we analyze the responses of 48 fifth-grade primary school students (10–11 years old) to a task in which they had to determine how many balls to transfer from one urn to another to achieve the same probability of success. We examine the level of generalization achieved, the semiotic representations used, and the emergence of algebraic reasoning in their solutions. The results show that 50% of the students correctly solved the problem. Additionally, 27% developed strategies reflecting pseudo-structural reasoning (Burgos et al., 2024), which led them to identify the underlying structure of the situation. However, only a minority of students demonstrated proto-algebraic reasoning by considering the equivalence relationship between probabilities. Furthermore, a positive relationship was observed between the accuracy of responses and the emergence of pseudo- or proto-algebraic traits.

Additional information

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References

Burgos, M., Tizón-Escamilla, N., & Godino, J. D. (2024). Expanded model for elementary algebraic reasoning levels. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(7), em2475.

Hernández-Solís, L. A., Batanero, C., Gea, M. M. y Álvarez-Arroyo, R. (2021). Comparing probabilities in urns: A study with primary school students. *Uniciencia, 35*(2), 1-19.

Pratt, D. & Kazak, S. (2018). Research on uncertainty. In D. Ben-Zvi, K. Makar & J. Garfield (Eds.), *International handbook of research in statistics education* (pp. 193-227). Springer.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 282. PME.

TEACHER-STUDENT INTERACTIONS, OPPORTUNITIES AND CHALLENGES

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Teachers frequently use questions to facilitate learning, and the kind of questions used affects, among other things, the students' cognitive opportunities (Boaler & Brodie, 2004). Further, the feedback teachers verbally communicate can hinder or help the students' reasoning and thereby their learning possibilities (Hattie & Timperley, 2007). Thus, studying how teachers interact with students when they work with mathematics is vital.

As part of a bigger project in which four upper secondary teachers implemented the teaching approach Thinking classroom (Liljedahl, 2016), this study focuses on teacher-student interactions while the students work with mathematical tasks in groups.

The data analysed consists of 44 video recordings of students working in groups, ranging from 31 to 55 minutes, where every teacher-student interaction was transcribed. A theoretical framework that combines Boaler and Brodies (2004) categorisation of teacher questions with Hattie and Timperley's (2007) model of feedback was constructed to analyse how the teacher-student interaction affected the students' continued work with the task. Whether the interaction was initiated by the teacher or the students was also incorporated into the framework.

In this oral presentation, the preliminary results will be presented and discussed. One hypothesis is that who initiated the interaction will have an impact on both the questions the teacher asks and what feedback they give to the students. Another hypothesis is that certain kinds of questions and feedback will hinder the students' possibility to think freely about the mathematics at hand.

References

Boaler, J., & Brodie, K. (2004). The importance, nature and impact of teacher questions. In D. E. McDougall & J. A. Ross (Eds.), *Proceedings of the twenty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 774–782). OISE/UT.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research*, *77*(1), 81–112. https://doi.org/10.3102/003465430298487

Liljedahl, P. (2016). Building thinking classrooms: Conditions for problem-solving. In P. Felmer, E. Pehkonen, & J. Kilpatrick (Eds.), *Posing and Solving Mathematical Problems* (pp. 361–386). Springer International Publishing. https://doi.org/10.1007/978-3-319-28023-3_21

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 283. PME.

AI ROLE IN TEACHER TRAINING FOR GEOMETRY INQUIRY

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Increasing student AI use challenges educators. Teacher training must move beyond reluctance to integrate AI meaningfully into pedagogy and learning. This study explores how prospective mathematics teachers (PSTs) interact with generative AI during inquiry-based geometry, illuminating pedagogical opportunities and potential pitfalls. Geometry, combining abstract reasoning with visual and deductive thinking, provides a powerful context for examining how AI interaction influences PST engagement, autonomy, and critical thinking during inquiry. Our research question: What characterizes PSTs-AI interaction during geometry inquiry task?

To address this question, 30 PSTs enrolled in two parallel Euclidean geometry courses completed an inquiry-based task on an isosceles triangle with medians to its legs. Participants identified and proved geometric properties and used ChatGPT during the process. Data sources included task submissions, submitted AI transcripts and related reflective writing.

Analysis revealed a spectrum of PST engagement with AI, ranging from skeptical to balanced and to intensely reliant. Skeptical PSTs challenged AI outputs, often correcting its errors or 'teaching' it, and demonstrated independence by ignoring AI suggestions and proving different properties in their submissions. Balanced users used AI iteratively to refine and complement their own thinking, leveraging AI as a cognitive partner to scaffold their inquiry process. Intensely reliant users leaned uncritically on AI, often copying responses and missing errors. Findings demonstrate that *balanced engagement*, characterized by critical and selective use, can support inquiry, metacognitive development, and reflective thinking. However, uncritical reliance undermines deep learning and can lead to misconceptions. These results *underscore the imperative* for teacher training programs to prepare PSTs not just to use AI, but to model and facilitate productive, balanced AI engagement in the classroom.

This research contributes to growing scholarship on AI integration in education (Holmes et al., 2023; Yeh, 2024), and underscores the importance of scaffolding and reflection in AI-supported learning environments. By training teachers to engage critically and thoughtfully with AI, we empower them to cultivate similar habits in their students and support meaningful, technology-integrated inquiry.

References

Holmes, W., Smith, J., & Taylor, R. (2023). AI-powered learning: Enhancing inquiry and engagement through pedagogical agents. *Educational Technology Advances*.

Yeh, H. C. (2024). The synergy of generative AI and inquiry-based learning: transforming the landscape of English teaching and learning. *Interactive Learning Environments*, 1-15.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 284. PME.

PROBLEM SOLVING – A COMMOGNITIVE PERSPECTIVE

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Problem solving (PS) has long been recognized as an important mathematical activity yet teaching and learning of non-routine PS remain a challenge. Advances in Commogniton (Lavie et al., 2019) may shed new light on this challenge. Learners who can participate exploratively in the discourse of *routine* PS (i.e. drilling recently taught procedures) will find that the familiar metalevel rules of the discourse are not appropriate for non-routine PS. They may mis-interpret the task situation as one calling for a familiar solution strategy, and thus their search for a relevant precedent routine may be guided by surface features that served them well in drilling (e.g., try a procedure that the teacher demonstrated recently). In contrast, non-routine problems are likely to be *unprecedented*, i.e., the learner has no previously encountered strategy for solving the problem and hence must construct one from routines encountered less recently. If this fails, it may be necessary to search for an appropriate *heuristic* (Polya, 1945) routine that learners may have encountered, such as working backwards or finding a simpler analogous problem.

Challenges of initial steps in unprecedented PS are demonstrated in task-based think-aloud interviews that the first author held with six pre-service teachers (PSTs) working individually on a geometry task that was unprecedented for them. While there are indications of explorative participation motivated by an intrinsic goal of solving the task, solvers relied on guidance from the interviewing teacher when they were unable to find and apply a precedent routine (e.g., "can you point me towards a certain direction please I need to find the answer to this problem"). Often, their search for precedent routines was not appropriate for unprecedented PS (e.g., "what I normally do [in similar situations], since we are trying to prove that some given angle equals another angle, I start by looking at similarity"). Bonding procedures - a hallmark of exploration – was difficult ("I don't see how these things relate to one another").

In our talk we will present analysis of data that demonstrates the inadequacy of an accomplished precedented-PS discourse for solving unprecedented problems and suggest teaching moves – theoretically and empirically derived – that may guide learners in their initial steps in unprecedented PS. Capitalizing on learners' explorative, intrinsically motivated, discourse of precedented PS, we challenge the commognitive assumption that metalevel learning must begin with ritualized participation.

References

Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies in Mathematics*, *101*(2), 153–176. https://doi.org/ghsqgx

Polya, G. (1945). *How to solve it*. Princeton University Press.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 285. PME.

THE TRANSFER OF PROBLEM-SOLVING REASONING THROUGH POSING ANALOGOUS PROBLEMS AND THE FEELING OF BEING STUCK

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Problem posing is key to the development of mathematical competence. Lack of understanding of problem structures is an inhibiting factor in the development of transfer skills that can be improved by problem posing. On the one hand, comparing the problem to be solved with an analogous one, or generating an analogous one, helps to recover structurally analogous problems without superficial similarities (Minervino et al., 2017). On the other hand, problem posing has a positive effect on mathematical attitudes, with benefits in solving skills (Rosli et al., 2014). Mathematical anxiety interferes when performing mathematical tasks that require working memory.

The aim of this study was to prove that posing and solving analogous problems facilitates the transfer of mathematical reasoning reducing the feeling of being stuck. The sample consisted of 96 Psychology students from the University of Barcelona. The intervention consisted of four moments. First, a Bayesian problem was posed to be solved and then explained. Second, a different task of equal duration was given to three randomly created subgroups: A) posing a problem with an analogous resolution to the one explained; B) creating a problem analogous to an unrelated one; C) explaining the resolution of the given problem in their own words. Third, two Bayesian problems were posed followed by questionnaires. Fourth, a final questionnaire was proposed. $\chi 2$ comparison tests and correlation (Pearson) were carried out.

We found differences between the three subgroups in transfer of reasoning ($\chi 2(8)=16.3, p=0.038$).Developed confidence in problem solving was positively related to making transfer of reasoning ($r=0.458, p<0.001$), and negatively related to the feeling of being stuck ($r=-0.609, p<0.001$). Based on these results, posing analogous problems improves the reasoning transfer in a better way than the reflection of an example or the posing of a non-analogous problem; it reduces the feeling of being stuck. Nonetheless, the sample was small and the students had little time to consolidate strategies.

Additional information

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References

Minervino, R., Olguín, V., & Trench, M. (2017). Promoting interdomain analogical transfer: When creating a problem helps to solve a problem. *Memory & Cognition*, *45*, 221-232.

Rosli, R., Capraro, M. M., & Capraro, R. M. (2014). The Effects of Problem Posing on Student Mathematical Learning: A Meta-Analysis. *International Education Studies 7* (13), 227–241. http://dx.doi.org/10.5539/ies.v7n13p227

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 286. PME.

IMPROVING TEACHER LEADERS' NOTICING OF MATHEMATICAL MODELLING

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Key to modeling is the space between a real-world problem and the mathematical world, where modeling comprises a set of cycles to follow and implement (Blum & Borromeo-Ferri, 2009). Students face challenges when performing modeling tasks, and it is critical to recognize those challenges and address them. Therefore, we focus our efforts on noticing (van Es & Sherin, 2002) how teachers *identify* and *interpret* modelling orchestration in the classroom and how students respond to orchestration during the phases of the modelling cycle. Professional Development Programs (PDP) aimed to teacher leaders that focus on teacher noticing can support teachers in learning how to promote modelling in the mathematics classroom**.** This study addresses the research question: How does the noticing of teacher leaders participating in a mathematical modelling PDP improves over time? To this end, we built an instrument composed by 4 classroom videos, 4 questions on identifying and 4 on interpreting modelling, and an analysis rubric (Martinez et al., 2024). The rubric contains 4 levels of identification and interpretation to classify teachers' responses when watching the videos. We will show results from two of four instrument applications to a group of nine teacher leaders as they began their participation in a PDP aimed to develop skills to support other teachers in promoting modelling with their students. Preliminary findings of the first application indicate that the teacher leaders show different noticing levels. On the one hand, their levels range from *incipient* to *deeply focused* when identifying and interpreting modelling orchestration, and on the other hand, their levels range from *general* to *focused* when interpreting the actions taken by the teacher and by the students in the modelling task portrayed in the video. The preliminary findings of the first application are promising given the role of the teacher leaders in supporting other teachers noticing of modelling in the mathematics classroom.

References

Blum, W. & Borromeo-Ferri, R. (2009). Mathematical modelling: can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45-58.

Martínez, M., Arriagada, V., & Solar, H. (2024). Bulding an instrument to characterize teacher noticing about argument and mathematical modeling competencies. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.). *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). PME.

van Es, E. A., & Sherin, M. G. (2002). Learning to Notice: Scaffolding New Teachers' Interpretations of Classroom Interactions. *Journal of Technology and Teacher Education, 10,* 571–596.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 287. PME.

EMPOWERING RURAL SOUTH AFRICAN MATHEMATICS FOUNDATION PHASE CLASSROOMS THROUGH VIDEO-STIMULATED RECALL INTERVIEWS WITH TEACHERS

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Self-reflection is an integral part of effective teaching. In this paper, I argue that the focus on teachers' learning through self-reflection is crucial in fostering pedagogical unlearning and relearning. Nel (2015, p. 14) emphasises that "the lessons learnt by teachers from their exposure to reflective practice in their teaching should result in their engagement in new practices". In the current study, Video-Stimulated Recall Interview (VSRI) technique, defined as the teacher's introspection process in which video recorded classroom practices are replayed to the teachers to stimulate recall of their pedagogic moves during the lessons were employed, allowing participating teachers to recall their pedagogical thoughts and actions. In relation to this, Mbhiza (2019) averse that VSRIs technique aids as a continuous professional development (CPD) instrument that can meaningfully augment teaching and learning outcomes. The usage of this research tool is fundamental in rural areas, especially within the South African context where professional development initiatives are limited due to the remoteness of those contexts and isolation from various educational facilities. This research question is:

> - What is the effect of using VSRIs on the pedagogical unlearning and relearning of foundation phase teachers in relation to teaching number and geometric patterns within rural classrooms?

The study utilized a multiple case study design, presenting diverse realities through the perspectives of 33 teachers across three different school sites, thus facilitating a comprehensive analysis of their teaching practices. In this article I only focus on the reflections of 5 Grade 2 teachers on their teaching of number and geometric patterns. The findings reveal that the use of VSRI between teachers and researchers is a powerful tool for enhancing reflective practices in the teaching of number and geometric patterns, particularly in rural classrooms. In the presentation, further findings will be discussed in detail.

References

Nel, B. P. (2015). *An evaluation of a mathematics professional teacher development programme* [Doctoral dissertation]. University of South Africa.

Mbhiza, H. W. (2019). Using video-stimulated recall interviews: teachers' reflections on the teaching of algebraic functions in rural classrooms. *The Independent Journal of Teaching and Learning, 14*(2), 92-107.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 288. PME.

COGNITIVE DEMAND FRAMEWORKS IN MATHEMATICS SECONDARY TEACHERS' THESES: A CASE STUDY

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Tasks are not created equal. Different tasks will elicit different types of thinking and reasoning in students (Smith & Stein, 1998). A previous work about the feedback that Mathematics Teacher Educators (MTEs) provide to Preservice Teachers' (PSTs) Mathematics lesson plans found that cognitive demand plays a significant role in the process of planning a lesson, yet this role is often underestimated or even unknown to PSTs and even some MTEs (Medel, 2022). Further experience has shown that focusing on the cognitive demand of mathematical tasks proves useful to develop a deeper understanding of varied Mathematics Education phenomena. For instance, PSTs engage with these frameworks writing their theses. This opens an opportunity to explore how PSTs incorporate cognitive demand into their academic work.

The current study focuses on the question: In which ways do secondary Mathematics PSTs use theoretical frameworks about cognitive demand in their graduation theses?

To address this question, the study aims to:

- Identify the main frameworks about the cognitive demand of mathematical tasks found in the literature and their salient features.
- Analyze the use of cognitive demand frameworks in the graduation theses of secondary Mathematics PSTs.
- Develop a characterization of the different ways cognitive demand frameworks are used in these theses.

The term 'ways of use' is intentionally left open-ended, as its working definition will be shaped by an in-depth exploration of each case. This study follows an instrumental case study methodology (Stake, 2005), where each case contributes to a broader understanding of how cognitive demand is incorporated into PSTs' research.

Preliminary results suggest that PSTs are increasingly integrating cognitive demand into their theses, mainly through teaching proposals and textbook task analysis. Further details will be presented in the oral communication.

References

Medel, L. (2022). *Feedback that matters: mathematics teacher educators' feedback of student teachers' lesson plans.* [Doctoral dissertation, Michigan State University]. MSU Digital Repository. https://d.lib.msu.edu/etd/50844?q=Life

Smith, M., Stein, M. (1998). Reflections on practice. Selecting and creating mathematical tasks. *Mathematics Teaching in the Middle School*, *3*(5), 344-350.

Stake, R. E. (2005). Qualitative Case Studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *The Sage handbook of qualitative research* (3rd ed., pp. 443-466). Sage Publications Ltd.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 289. PME.

STUDY OF THE MATHEMATICAL WORKING SPACE OF A FORMATIVE TRIAD IN THE CONTEXT OF INTERNSHIPS

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INTRODUCTION

Research has shown the importance of strengthening the formative triad (preservice teachers who take internships, teachers supervising internships, and teachers of the school system) in initial teacher education, as it is a scenario where theory converges with practice (Crecci & Fiorentini, 2018).

The objective of this research is to characterize the individual and collective mathematical work of the triad, when planning, implementing and analyzing a class on the conceptualization of the number π, for seventh grade (12 years old approximately).

THEORETICAL FRAMEWORK AND METHOD

The analyses are framed in the theory of Mathematical Working Spaces (MWS) (Kuzniak et al., 2022), which allows us to look at epistemological and cognitive aspects in the work of individuals who solve a task and in the organization that is carried out for the teaching of mathematical objects.

Methodologically, we conducted design research (Gravemeijer & Cobb, 2006), since we created a collaborative scenario where we placed the triad, in order to characterize their mathematical work.

RESULTS AND CONCLUSIONS

The results allow us to conclude about the depth that the teachers' reflections reach thanks to the collaborative work, in relation to the understanding of objects such as radians, diameter and perimeter of the circumference, and number π, as well as to characterize the dynamics of the MWS built by the triad, and how the commitment to carry out a common project allowed decreasing the hierarchical roles.

References

Crecci, V. & Fiorentini, D. (2018). Reverberations of learning by Math teachers in a borderland community between university-school. *Educar EM Revista, 34*(70), 273–292.

Gravemeijer, K. & Cobb, P. (2006). Design research from a learning design perspective. In J. Van den Akker, K. Gravemeijer, S. McKenney & N. Nieveen (Eds), *Educational design research*. (pp. 17-51). Routledge.

Kuzniak, A., Montoya-Delgadillo, E. & Richard, Ph. (2022). Mathematical Work in educational context. The perspective of the theory of Mathematical Working Spaces. Springer Nature.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 290. PME.

LINKING EDUCATIONAL RESEARCH AND CLASSROOM TEACHING: A CHARACTERISTIC EXAMPLE FROM MATHEMATICS.

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According to Heid, Middleton, Larson, Gutstein, Fey, King, Strutchens & Tunis (2006) the challenge of linking research and practice in Mathematics Education is bidirectional. The classroom practice needs to be better informed by research and researchers should be better attuned to the practitioners' needs. The study reported here attempts to address this challenge by transforming theory into practical ideas on the topic of integrating sustainability into mathematics learning. Furthermore, it was implemented by a community of researchers and teachers working together and learning from each other. The research took place in a typical Greek primary school having as participants all of its students (110). A course, with the aim to elicit sustainable development skills (SDK) during playful mathematics problem solving, was designed for the 5th and 6th Year classes of the school (10-12 years old).

A characteristic activity example from the course, is the one titled "How can you combine your clothes to create outfits for your trip?" which aimed to reinforce the mathematical concept of "combinations". The pupils were told that the suitcase for their trip had only enough space for 3 trousers, 6 tops and 2 pairs of shoes. They needed to discuss how many different outfits they could create from them. Each group was given appropriate visual aids for their investigations.

The pupils worked together effectively, constructing the desired mathematical knowledge and on the same time exhibiting SDK skills such as "critical thinking". Most importantly, the teachers of the community, agreed that the teaching ideas were easily replicable in every classroom whereas the researchers, based on the teachers' feedback, developed more stimulating variations of the above activity.

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References

Heid, M., Middleton, J., Larson, M., Gutstein, E., Fey, J., King, K., Strutchens, M. & Tunis, H. (2006). The Challenge of Linking Research and Practice. *Journal for Research in Mathematics Education, 37* (2).

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 291. PME.

LEARNING ALGEBRAIC STRUCTURES AND TEACHING EQUATIONS: CONNECTIONS AND DISCONNECTIONS

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The disconnect between advanced and secondary mathematics has been widely documented (Wasserman, 2018). The objective of this research was to analyse the connections that prospective mathematics teachers (PMTs) could establish in specific domains, based on specially designed instructional sequences.

The study considered various types of connections, ranging from those strictly related to mathematical content to pedagogical ones, with a focus on desirable teaching models for PMTs. The design process followed a methodological model for teaching advanced mathematics to PMTs: (i) building from teaching practice, (ii) learning advanced mathematics, (iii) stepping down to practice (Wasserman et al., 2019). This paper presents one of the three sequences designed and implemented with 25 PMTs. The first task involved analyzing a secondary student's solution to an equation (i). The second task aimed to develop knowledge of algebraic structures (ii). The final task prompted PMTs to synthesize their reflections from the previous phases through an imagined dialogue between a teacher and secondary students (iii).

The results highlight the importance of establishing connections between algebraic structures and equation solving, consistent with findings by Murray and Badinger (2018). However, PMTs' responses revealed difficulties in using their knowledge of algebraic structures to justify the steps required for solving equations. Additionally, PMTs struggled to integrate this advanced mathematical knowledge into secondary mathematics instruction. Both findings provide a foundation for an ongoing, broader project dedicated to generating pedagogical knowledge for teacher training.

References

Murray, E. & Baldinger, E. (2018). Impact of Abstract Algebra on Teachers' Understanding of and Approaches to Instruction in Solving Equations. In N. Wasserman (Ed.), *Connecting Abstract Algebra to Secondary Mathematics, for Secondary Mathematics Teachers* (pp. 403–429). Springer.

Wasserman, N (2018). Exploring advanced mathematics courses and content for secondary mathematics teachers. In N. Wasserman (Ed.), *Connecting Abstract Algebra to Secondary Mathematics for Secondary Mathematics Teachers* (pp. 1–15). Springer.

Wasserman, N., Weber, K., Fukawa, T & McGuffey, W. (2019). Designing advanced mathematics courses to influence secondary teaching: fostering mathematics teachers' "attention to scope". *Journal of Mathematics Teacher Education*, *22*, 379–406.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 292. PME.

CHARACTERIZING PROFESSIONAL LEARNING AMONG CHILEAN MATHEMATICS TEACHER EDUCATORS

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PURPOSE AND THEORETICAL PERSPECTIVE

Scholars and policymakers increasingly recognize the professional learning needs of teacher educators related to their dual roles as teacher of teachers and researchers (Cochran-Smith et al., 2020; Ping et al., 2018). However, few studies examine the impact of formal and systematic professional learning experiences on their professional development. This study explores the professional learning of mathematics teacher educators who participated in RedFID (https://www.redfid.cl/), an innovative professional learning support system that promotes professional learning through online courses and workshops.

METHOD

We used a quasi-experimental, pre-post design with a sample of 28 mathematics teacher educators from 14 Chilean universities. Participants completed a 20-hour online course entitled *Inquiry into Teaching Practice* and attended six workshops focused on developing teaching and research skills. Quantitative data were collected through a self-reported instrument that comprised five subscales to assess research practices and professional learning. Data analysis included descriptive statistics, paired t-tests, and cluster analysis (Hair et al., 1999).

RESULTS AND DISCUSSION

Findings show that after one year, mathematics teacher educators perceived improvements in their teaching and research skills, with post-test scores increasing across all related scales. Cluster analyses revealed distinct patterns: one group of mathematics teacher educators valued formal academic knowledge in research, linked to their academic background. In contrast, another group showed greater professional learning needs due to limited support during their professional induction.

References

Cochran-Smith, M., Grudnoff, L., Orland-Barak, L., & Smith, K. (2020). Educating teacher educators: International perspectives. *The New Educator*, *16*(1), 5–24.

Hair, J., Anderson, R., Tatheam, R., & Black, W. (1999). *Análisis multivariante* (5a. ed.). Prentice Hall.

Ping, C., Schellings, G., & Beijaard, D. (2018). Teacher educators' professional learning: A literature review. *Teaching and Teacher Education*, *75*, 93–104.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 293. PME.

LEVELS OF NOTICING IN RELATION TO SPECIALIZED KNOWLEDGE OF FUNCTIONS IN PRESERVICE TEACHERS

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The noticing competence involves teachers identifying noteworthy classroom events, distinguishing those that present learning opportunities for students (attending dimension), and making sense of what is observed (interpreting dimension). For each dimension, van Es (2011) proposes a series of categories, which allow for classification into four progressive levels: baseline, mixed, extended, and focused. This research is part of a project aimed at examining how the level of noticing competence changes in relation to specialized knowledge among preservice secondary mathematics teachers (PTs). The literature in this field suggests that to attend to and interpret significant classroom events (e.g., students' mathematical thinking), teachers rely on their mathematical and didactic knowledge. Therefore, it is essential to explore noticing in the context of the interactions between these types of knowledge. To this end, the study employs the Mathematics Teacher's Specialized Knowledge (MTSK) model (Carrillo et al., 2018). This paper analyzes written narratives by 35 PTs enrolled in a bachelor's degree program in mathematics instruction at a Chilean university. These PTs participated in a workshop where they analyzed various classroom videos. The paper reports data from one video on the introducing functions, focusing on three moments that the PTs identified as noteworthy in their narratives. Content analysis, the noticing levels categories, and the subdomains of the MTSK model (which address both mathematical and didactic content knowledge) were used to analyze each narrative. Findings suggest that most PTs exhibit mixed and focused attention levels of noticing, while their interpretations tend to be mixed. In the current phase of the project, the goal is to determine which knowledge subdomains are most prevalent at each level of participant noticing. Lower levels of noticing are expected to correspond with a focus on only one type of knowledge (either didactic content or mathematical knowledge) or show limited evidence of both. In contrast, higher levels of noticing are anticipated to demonstrate a greater integration of MTSK subdomains.

Additional information

Acknowledgements. Research supported by the Chilean Government Doctoral Scholarship ANID [folio: 21210589] and FONDECYT Regular Nº1230434.

References

Carrillo, J., Climent, N., Montes, M., Contreras, L. C., Flores, E., Escudero, D., Vasco, D., Rojas, N., Flores, P., Aguilar, A., Ribeiro, M., & Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialized knowledge (MTSK) model. *Research in Mathematics Education*, *20*(3), 236–253. https://doi.org/10.1080/14794802.2018.1479981

van Es, E. (2011). A framework for learning to notice student thinking. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Studies in mathematical thinking and learning. Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 164–181). Routledge.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 294. PME.

CHARACTERIZATION OF FOURTH GRADERS' GENERALIZATION PROCESS THROUGH A TASK INVOLVING A LINEAR FUNCTION

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This work is part of a research project that aims to explore the algebraic thinking of elementary school students (www.pensamientoalgebraico.es). In this work we characterize the generalization process of fourth grade students when working with a generalization task involving a linear function. Rivera and Becker (2007) argue that abduction and induction are key forms of reasoning in the process of generalization. First, students generate conjectures based on nearby cases (abduction), then test them with more distant cases (induction), and finally extend the validated conjectures to general cases, demonstrating an understanding of the underlying regularity and structure between variables, and generalizing. The purpose of our study is to characterize the stages of the generalization process that fourth grade students exhibit when solving a linear function problem. A qualitative and descriptive approach was used, analyzing the responses of 25 (9-10-year-old) students of a low socioeconomic and cultural school in southern Spain. The task, contextualized in an amusement park, consisted of calculating the total cost given a fertilizer of 1 euro and a price of 2 euros per attraction ($f(n)=2n+1$).

The analysis of the fourth graders' work revealed three distinct phases in the generalization process: abduction, induction, and generalization. Most students were able to produce correct outputs for near (abduction), but only a portion of them successfully validated these patterns with far cases (induction). A smaller number of students progressed to the generalization phase, where they attempted to express general rules using symbolic notation often requiring teacher support. The results show a progression from concrete reasoning to more abstract representations, underscoring the critical role of teacher guidance and contextualized tasks in fostering algebraic generalization.

Additional information

Acknowledgements. Project reference PID2020-113601GB-I00, funded by the Spanish State Research Agency (AEI) MCIN/AEI/10.13039/501100011033.

References

Rivera, F. D., & Becker, J. R. (2007). Abduction–induction (generalization) processes of elementary majors on figural patterns in algebra. *The Journal of Mathematical Behavior, 26*(2), 140-155. https://doi.org/10.1016/j.jmathb.2007.05.001

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 295. PME.

DESIGN FACTORS THAT AFFECT PERFORMANCE ON TASKS OF NUMBER LINE ESTIMATION

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Young children learn to count from one to ten, but grasping the relative magnitudes of these numbers develops over time. A widely used tool to assess this understanding is the number line estimation task, which has been shown to possess key predictive properties in numerical learning research (Siegler et al., 2012). However, subtle design variations in these tasks can activate different numerical representations, leading to significant differences in children's performance. This study investigates how specific design features of number line estimation tasks influence performance. We examined two variations of a 0-to-10 number line estimation task. The *Ship Task*, is based on the classic number line paradigm, where participants are asked to locate a target number (displayed at the top center of the screen) on a horizontal line with 0 on the left and 10 on the right. The *Pantry Task*, embeds the number line within a pantry context, where participants locate a numbered food can on a shelf. Unlike the Ship Task, this task presents the target number directly on the can, which moves with the participant's marker, offering a more discrete and contextually grounded experience. This study employed a mixed-methods design, combining qualitative and quantitative approaches. The qualitative component involved structured interviews with 20 children to explore their task interpretations. The quantitative component analyzed data from 333 kindergarten-level children. Informed consent was obtained from all parents.

Qualitative findings revealed three key insights (1) The position of the target numeral significantly influences task interpretation. (2) The contextual framing of the task (e.g., pantry vs. ship) affects children's spatial reasoning and approach, and (3) Clear game instructions enhance participants' ability to interpret and engage with the task. For quantitative analysis, we calculated Percent Absolute Error (PAE = 10*|Estimation - Target Number|) for each child and task, averaging across all items. The Ship Task yielded a mean PAE of 32% (SD = 12%), while the Pantry Task showed a significantly lower mean PAE of 14% (SD = 10%). A paired t-test confirmed that the difference in PAE between the two tasks was statistically significant, $t(332) = 23.0$, $p < .0001$.

These findings show that even minor design modifications in number line estimation tasks can substantially impact children's performance. They highlight the importance of task context, target presentation, and instructional clarity in shaping numerical reasoning. This study underscores the need for continued refinement in the design and application of estimation tasks to capture and support numerical understanding.

References

Siegler, R., Duncan, G., Davis-Kean, P., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early Predictors of High School Mathematics Achievement. *Psychological Science*, *23*(7), 691–697.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 296. PME.

INTERPLAY BETWEEN CONCEPTUAL AND PROCEDURAL MATHEMATICAL KNOWLEDGE IN CHILDREN

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BACKGROUND

Research highlights the bidirectional relation between procedural and conceptual mathematical knowledge, where conceptual understanding supports procedural mastery, and procedural fluency reinforces conceptual insights (Dlugosch et al., 2022; Rittle-Johnson et al., 2015). This study builds on this perspective by investigating the cognitive abilities underlying both types of knowledge, offering deeper insights into the mechanisms that drive mathematical learning.

METHOD

127 Brazilian students from 3rd and 4th grades of primary school, with a mean age of 9.3 years (SD=0.7), were assessed on three cognitive abilities—number transcoding (NT), number line estimation (NLE), and working memory (WM)—as well as on both conceptual (quantitative reasoning) and procedural (arithmetic computations) mathematical knowledge.

RESULTS

The results show that all cognitive abilities predict conceptual knowledge, with NT as the strongest predictor (R^2=0.466, $F(3,121)$=35.208, $p<.001$). For procedural knowledge, only NT and WM were significant (R^2=0.402, $F(3,121)$=27.103, $p<.001$). Separate regression analyses identified a bidirectional relation between conceptual and procedural knowledge, while mediation analyses further confirmed that NT mediates this connection in both directions.

CONCLUSIONS

The findings highlight the bidirectional relation between conceptual and procedural mathematical knowledge, with NT playing a central role. Classroom strategies that integrate both types of knowledge and strengthen number transcoding can enhance students' overall mathematical proficiency.

References

Rittle-Johnson, B., Schneider, M. & Star, J.R. (2015). Not a One-Way Street: Bidirectional Relations Between Procedural and Conceptual Knowledge of Mathematics. *Educ Psychol Rev* 27, 587–597. https://doi.org/10.1007/s10648-015-9302-x

Dlugosch, A. C., Simacek, T., & Schneider, M. (2022) Predictive relations between conceptual and procedural knowledge in mathematics: A meta-analysis. In *Proceedings of the WOG 2022*. https://ppw.kuleuven.be/onderwijskunde/en/wog2022/predrelations

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 297. PME.

SPECIALIZED KNOWLEDGE ABOUT THE REMAINDER OF DIVISION OF DECIMAL NUMBERS OF A FUTURE ELEMENTARY SCHOOL TEACHER

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It is proposed to characterize the specialized knowledge about the remainder in divisions with decimal numbers of a future elementary school teacher. Through a qualitative case study, using the MTSK model (Carrillo et al., 2018), the responses to a questionnaire validated by experts were analyzed.

Although their training did not explicitly address the interpretation of the remainder in this type of division, the future teacher evidenced knowledge of definitions (KoT) based on their experience with natural numbers and knowledge about relevant mathematical connections (KSM), such as the link to place value. Furthermore, they evidenced knowledge of how to adapt meaningful didactics (KMT) for the remainder of decimal division. The future teacher evidenced knowledge of pictorial and symbolic representation registers (KoT) of the remainder, showing signs of knowing learning progressions (KFLM) related to the topic. They claimed to know the Theory of Didactic Situations and the Theory of Semiotic Representation Registers (KFLM), recognizing common errors in students (KFLM) over the rest and knowing teaching strategies consistent with such difficulties (KMT). Lee (2021) states that the interpretation of the remainder varies according to the context; in this line, the future teacher evidenced knowing and selecting appropriate tasks (KMT) on the remainder, which are aligned with Zorrilla et al. (2021), who highlight using realistic problems for the understanding of the remainder (KFLM).

This study opens up the possibility of future research on the teaching of remainder in decimal divisions, suggesting the need to strengthen teacher training to address the difficulties associated with this concept.

References

Carrillo, J., Climent, N., Montes, M., Contreras, L., Flores-Medrano, E., Escudero-Ávila, D., ... & Ribeiro, M. (2018). The mathematics teacher's specialized knowledge (MTSK) model. *Research in Mathematics Education, 20*(3), 236-253. https://doi.org/10.1080/14794802.2018.1479981

Lee, H. (2021). Exploring the issues and improvements of the quotient and the reminder of the decimal division. *Education of Primary School Mathematics, 24*(2), 103–114. https://doi.org/10.7468/JKSMEC.2021.24.2.103

Zorrilla, C., Ivars, P., y Fernández, C. (2021). Problemas realistas de división con resto. *Revista Mexicana de Investigación Educativa. 26*(91), 1313-1339.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 298. PME.

A FRAMEWORK FOR DESIGN DECISIONS FOR DEVELOPING AN AUGMENTED REALITY NUMBER LINE RESOURCE

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Augmented Reality (AR) is an increasingly popular and rapidly evolving, often freely available technology that enables you to see 3D digital objects anchored to your view of the physical world. We believe the increasing development and adoption of this technology will apply to learners in school and that AR manipulatives may even become a staple of the mathematics classroom – as common as a ruler or calculator. Given the ubiquity of number lines, especially in primary mathematics classrooms, we thus seek to develop a prototype AR number line resource. Although there is mounting evidence on the cognitive and affective affordances of AR in mathematics education (Bulut & Ferri, 2023), little research has aimed at integrating the benefits of AR and number lines for learning arithmetic, especially where negative quantities are included.

Our immediate aim was therefore to systematically evaluate design characteristics of an AR number line in terms of these benefits, to allow for its most effective layout and implementation. We developed a conceptual framework to compare various layouts (e.g. horizontal vs. vertical, on the floor vs. the wall) and forms (e.g. a line, a ladder or a helix). We created a detailed table organising and comparing different forms against an extensive set of characteristics informed by the related literature. These included orientation (horizontal/vertical), embodiment, degree of augmentation and cost (minimal if using GeoGebra). These categories afford some degree of subjective rating and were thus scored (ordinally) with reference to relevant literature and frameworks.

Results so far indicate that out of 10 types of AR number line considered, only three distinctly emerge as highly effective: a flight of stairs (20% of total scores); a helix/spiral staircase (17%); and a horizontal line on the floor (13%). These scores collectively outweighed the others and thus provide a steer for future design priorities. We caveat this as the output of the first iteration of a Design-Based Research process so far using researcher inputs towards developing an optimal AR number line; in the presentation, further iterations will be detailed that triangulate inputs from multiple stakeholders (including teachers and students), with weightings for each characteristic, aligned with design priorities such as ease of implementation. This semi-quantitative approach provides a systematic process that lends clarity to complex, research-informed multifactorial AR toolkit design that may well be applied more broadly.

References

Bulut, M., & Borromeo Ferri, R. (2023). A systematic literature review on augmented reality in mathematics education. *European Journal of Science and Mathematics Education, 11*(3), 556-572. https://doi.org/10.30935/scimath/13124

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 299. PME.

MODES OF THINKING WHILE IMAGINING TEACHING

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This paper presents results from a case study conducted during the initial phase of an ongoing project investigating the difficulties prospective mathematics teachers (PMTs) face in understanding school-level linear algebra concepts, with the aim of designing their training. The research question addressed in this phase was: What difficulties related to the concept of solution of a system of linear equations can be identified when writing an imaginary lesson (IL) for lower secondary education?

The theoretical framework is based on the perspective of Sierpinska (2000), who identifies three modes of thinking in linear algebra: synthetic-geometric (SG), analytic-arithmetic (AA), and analytic-structural (AS). In the SG mode, objects are presented to students through a geometric representation, such as a shape or a set of points. In the AA mode, mathematical objects are conceived through numerical relationships. In the AS mode, the focus is instead on the properties of the objects or their characterization through axioms. Each of these modes of thinking leads to different interpretations of the object, as each provides a distinct perspective on the algebraic entity in question.

This case study involved four PMTs. In this paper, we report only the findings related to one of the participants. Data collection was carried out through the writing of an IL to teach systems of linear equations with two variables to 14- to 15-year-old students. The starting point was an imaginary student's response: “Because there is no numerical equality”. The PMT had to elaborate on what the student was responding to and how the lesson progressed.

The results indicate that the IL was influenced by the PMT's understanding of the solution concept, which was exclusively framed within the SG mode of thinking and limited to the case of parallel lines as the only graphical configuration of a system with no solution. This is consistent with Oktaç (2018); however, the findings show how these conceptions come into play not only in the context in which the PMT learns linear algebra but also in the context of teaching—that is, when the PMT assumes the role of a teacher, an aspect that this project aims to address.

References

Sierpinska, A. (2000). On some aspects of students' thinking in linear algebra. In J. Dorier (Ed.), *On the Teaching of Linear Algebra* (pp. 209–246). Kluwer Academic Publishers.

Oktaç, A. (2018). Conceptions about system of linear equations and solution. In Stewart, S., Andrews, C., Berman, A., Zandieh, M. (Eds.), *Challenges and Strategies in Teaching Linear Algebra, ICME–13 Monographs* (pp. 71–101). Springer.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 300. PME.

ANALYSIS OF THE PRESENCE OF PISA COGNITIVE PROCESSES IN CHILEAN TEACHER EDUCATION STANDARDS

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The PISA framework evaluates student performance through fundamental cognitive processes: formulate, employ, and interpret and evaluate, which enable the transformation and analysis of mathematical information. In Chile, the standards for initial teacher training (CPEIP, 2022) represent a significant advance by delineating what future teachers must master in both disciplinary and didactic knowledge. Although the PISA test is not administered to teachers, it is essential that they have these skills and to know how to develop them in their students. The objective of this study is to analyze the presence of the cognitive processes defined in the PISA framework (formulate, employ, interpret, and evaluate) in the disciplinary mathematics standards for initial teacher training in Chile.

Based on the PISA Mathematics framework (OECD, 2019), a matrix was developed to identify the presence of each cognitive process. Using this tool, we analyzed the 85 descriptors of the disciplinary mathematics standards for initial teacher training (CPEIP, 2022). To ensure reliability, two evaluators independently applied the matrix, resulting in a Cohen's kappa of 0.847. Discrepant cases were discussed until consensus was reached on the coding. Finally, a count was performed by type of descriptor (disciplinary knowledge or disciplinary didactics) and the corresponding percentages were calculated. The results are shown in Table 1.

	Cognitive process		
Type of descriptor	Formulate	Employ	Interpret and Evaluate
Disciplinary knowledge	19 (37%)	24 (46%)	40 (77%)
Disciplinary didactic	2 (6%)	2 (6%)	23 (70%)

Table 1: Analysis results.

These results show that, in the standards for teacher education in Chile, the three cognitive processes evaluated by PISA appear unbalanced. In particular, it is striking that little emphasis is placed in the standards on learning to teach how to formulate and employ mathematical concepts, facts and procedures.

References

Centro de Perfeccionamiento, Experimentación e Investigaciones Pedagógicas (CPEIP) (2022). *Estándares pedagógicos y disciplinares para carreras de pedagogía en educación general básica*. Ministerio de Educación.

OECD (2019). *PISA 2021 mathematics framework*. OECD Publishing.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 301. PME.

TRANSLATING EQUATIONS AND INEQUALITIES: FROM VISUALLY SUPPORTED PROBLEMS TO ALGEBRAIC SYMBOLISM

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Within the context of a design research teaching experiment focused on students' development of algebraic thinking, we attend to the structures that 3rd and 4th grade students show when translating equation and inequation problems stated with visual support into algebraic symbolism. Referring to and generalising structure are our focuses as components of algebraic thinking (Kieran, 2022) which have proven to have an impact in promoting students' success in mathematics as well as into their preparation for the later study of formal algebra (Blanton et al., 2018).

We qualitatively analysed the written responses and oral discussions carried out by 51 students when solving an equation problem and an inequality problem. Subsequently, we conducted semi-structured interviews to 12 of these students when solving other similar problems. From the answers obtained, we analysed the structures perceived by the group. The results inform of the elements that students included in their expressions: relationship of equality and inequality, indeterminate quantities, the use of operations, as well as the composition, decomposition, and re-composition of numbers. They perceived and expressed multiple structures and equivalences between them, both for equations and inequalities, and reasoned about the equivalence of structures. Differences between the case of equations and inequations revel difficulties to be addressed in teaching. The role of visual aids is highlighted as a way to contribute to facilitate structural reasoning even without specific instruction. It was a key element in recognising the equivalence of different symbolic representations of a problem.

Additional information

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References

Blanton, M. L., Brizuela, B. M., Stephens, A., Knuth, E., Isler, I., Gardiner, A. M., Stroud, R., Fonger, N. L., & Stylianou, D. (2018). Implementing a framework for early algebra. In C. Kieran (Ed.), *Teaching and learning algebraic thinking with 5-to-12-year-olds, ICME-13* (pp. 27-49). Springer. https://doi.org/10.1007/978-3-319-68351-5_2

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: Background, overarching dimensions, and new directions. *ZDM Mathematics Education, 54*(6), 1131-1150. https://doi.org/10.1007/s11858- 022-01435-6

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 302. PME.

MATHEMATICAL LANGUAGE DIVERSITY IN PROBLEM-SOLVING INSTRUCTION: A COMPARISON BETWEEN READING AND MATHEMATICS BACKGROUND TEACHERS

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Mathematical language helps children build more sophisticated mathematical understandings. Word-problem solving necessitates language comprehension, with an emphasis on mathematics-specific language (Fuchs et al., 2015; Powell et al., 2019). Therefore, it is essential to provide students with high-quality mathematics-language instruction.

This study aims to investigate how teachers with two distinct specializations utilize mathematical language in various stages of problem-solving. The participants were two teachers and their students. One teacher(A) was trained to teach Chinese reading comprehension, the other(B) one specializes in mathematics education.

Classroom observations and teacher interviews were used to collect data. Video recordings were used, and subsequently, six decimal division lessons from each teacher were transcribed. Video conversations were coded by three educators (two elementary teachers and one professor of mathematics education) to determine the use of mathematical language and its presence in each stages of the problem-solving process, proposed by Mayer (1992), which are problem translation (PT), problem integration (PI), solution planning and monitoring (SPM), and solution execution (SE).

The findings of this study showed as follows. The frequency of mathematical language used by Teacher A are SE (54%), and then SPM (21%), PT (14%) and PI (11%). The frequency of mathematical language used by Teacher B are SE (87%), and then PT (7%), SPM (4%), and PI (3%). Teacher A, who was trained in reading comprehension instruction, pays more attention to SPM, PI and PT than Teacher B did. Teacher A spend more time discussing on problem representation with students, including the clarification of mathematical concepts, the meaning of problems, the integration of sentences, than Teacher B. However, Teacher B, who specializes in mathematics education, spent more time and effort on solution execution than teacher A. More details will be presented at the conference.

References

Powell, S. R., Stevens, E. A., & Berry, K. A. (2019). Effects of a word-problem intervention on word problem language features for third-grade students with mathematics difficulty. *Learning Disabilities: A Multidisciplinary Journal, 24*(2), 1–14.

Fuchs, L. S., Fuchs, D., Compton, D. L., Hamlett, C. L., & Wang, A. Y. (2015). Is word-problem solving a form of text comprehension? *Scientific Studies of Reading, 19*(3), 204–223.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 303. PME.

WHAT DOES THE EQUAL SIGN MEAN? FROM TEACHER TRAINING TO TEACHING PRACTICE

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Understanding the equal sign is crucial for developing algebraic thinking. However, pre-service teachers (PSTs) face difficulties anticipating responses, interpreting student work, and making decisions when analyzing tasks related to equal signs. This paper presents findings from a case study conducted during the initial phase of an ongoing project aimed at generating pedagogical knowledge to improve teacher training. The research question addressed in this phase was: What meanings of the equal sign can be identified in the classroom practice of a PST?

The theoretical framework categorizes meanings attributed to the equal sign in school mathematics (Molina et al., 2009). The authors distinguish between *operational meanings*, such as the equal sign used as an operator (3+4=7) or a separator (3+4=7+2=9), and *relational meanings*, such as the expression of a numerical equivalence (3+4=5+2) or symbolic equivalence (x+3x=6x-2x). Research has shown that a relational understanding of the equal sign promotes better performance in solving algebraic equations.

This case study involved a fourth-year PST (23 years old). Data was collected by analyzing a narrative in which she described her class implementation, centered on discussing equal sign meanings. The data comprise the lesson plan, the 45-minute lesson audio recording, and the PST's reflection. The analysis identified segments that revealed key aspects of teaching and learning related to the equal sign.

The results indicate that the PST prioritized correctly writing equations without explicitly addressing the meanings of the equal sign. This approach implicitly validated the equal sign as a separator while neglecting its interpretation as a conditional equivalence. These findings show how these difficulties emerge not only in analyzing student responses outside the classroom (e.g., Pinto et al., 2024) but also in teaching practice. The study highlights the need for training programs that promote reflection on the role of the equal sign and strategies to strengthen its relational interpretation.

References

Molina, M., Castro, E., & Castro, E. (2009). Elementary students' understanding of the equal sign in number sentences. *Electronic Journal of Research in Educational Psychology, 7*(1), 341-368.

Pinto, E., Piñeiro, J., Cortés, C., & Martínez, M. (2024). The Decision-Making of Prospective Elementary School Teachers in Engaging with Children's Algebraic Thinking. *Pensamiento Educativo, 61*(2), 01-11.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 304. PME.

BRIDGING THE GAP: STUDENTS' STRUGGLES WITH ALGEBRAIC AND GRAPHICAL REPRESENTATIONS OF FUNCTIONS

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INTRODUCTION

Understanding functions is a fundamental aspect of mathematics education, with algebraic and graphical representations serving as key tools for conceptual development. Prior research (Knuth, 2000, Van dyke & White, 2004) has documented students' difficulties in transferring knowledge between these representations. Despite the informational equivalence of algebraic equations and graphs, students exhibit a marked preference for symbolic manipulation over graphical reasoning. This study builds upon constructivist learning theories and metacognitive perspectives to analyze students' struggles with function representations.

STUDY OVERVIEW AND KEY FINDINGS

This presentation explores how constructivist approaches can enhance students' understanding of functions. While constructivist theory emphasizes active knowledge-building through multiple representations, many students struggle to transition between algebraic and graphical forms. A study of 300 Grade 12 STEM students in Nepal found 67% relied solely on algebraic methods, while only 33% attempted graphical approaches, even when more efficient. Additionally, 48% were unaware of graphical solutions**,** and only 6.6% successfully connected equations to graphs. These challenges stem from an overemphasis on procedural algebraic skills, limiting graphical literacy and metacognitive awareness.

IMPLICATIONS

This study underscores the need for curricular adjustments that balance algebraic and graphical methods in teaching functions. To address these challenges, educators should integrate visual tools like dynamic graphing software to enhance conceptual understanding. Additionally, metacognitive training should be encouraged, helping students reflect on multiple problem-solving approaches. Finally, balanced assessment strategies should be implemented to ensure that algebraic and graphical reasoning are equally emphasized in evaluations. These steps will foster a more flexible and well-rounded mathematical understanding.

References

Knuth, E. J. (2000a). Student understanding of the Cartesian connection: An exploratory study. *Journal for Research in Mathematics Education, 31*(4), 500-507.

Van Dyke, F., & White, A. (2004). Examining students' reluctance to use graphs. *The Mathematics Teacher, 98*(2), 110-117.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 305. PME.

A FIFTH-GRADE STUDENT'S CONSTRUCTION AND UNDERSTANDINGS OF FUNCTION TABLES AND GRAPHS

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This study explores how a fifth-grade student constructed and understood function tables and graphs. We built on Strachota et al. (2024) by expanding their trajectory of students' *understandings* of function tables and graphs. Through a case study, we describe a new set of levels that include how students *construct* these representations along with their understandings. We address the following question: *How does a fifth-grade student construct and understand function tables and graphs?*

We selected Mary from a group of 26 fifth-grade students who had participated in four instructional sessions focused on generalization tasks with linear functions and various representations: verbal, symbolic, tabular, and graphical. Following these sessions, we interviewed Mary due to the breadth of her responses, which provided insight into varying levels of both construction and understandings. To analyze Mary's interview, we developed a parallel trajectory to Strachota et al.'s (2024), focusing on construction. This trajectory includes stages ranging from pre-interiorization to representation as a mathematical object. In our presentation, we will describe these levels in detail.

Findings revealed a mismatch between Mary's levels of construction and understandings. When working with graphs, Mary reached more sophisticated levels in understanding than in construction. Additionally, Mary provided evidence of more sophisticated levels of construction of function tables than in understanding.

Additional information

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References

Strachota, S., Lopez, M. A., Brizuela, B. M., Pérez-Martos, M. C., Murphy Gardiner, A., & Blanton, M. (2024). Young Students' Understandings of Function Graphs. In Lindgren, R., Asino, T. I., Kyza, E. A., Looi, C. K., Keifert, D. T., & Suárez, E. (Eds.), *Proceedings of the 18th International Conference of the Learning Sciences - ICLS 2024* (pp. 330-337). International Society of the Learning Sciences.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 306. PME.

PRE-SERVICE TEACHERS' PERCEIVED ON COMPUTATIONAL THINKING THROUGH MATHEMATICS TEXTBOOKS

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Computational thinking is a basic skill essential for citizens in the digital era (Wing, 2006). In developing students' computational thinking skills, it was important to emphasize computational thinking in teacher education program so that pre-service teachers could develop these skill for future classroom integration (Yadav et al., 2014).

This study aims to explore pre-service teachers' perceived on computational thinking through mathematics textbooks. The target group consists of 29 third-year pre-service teachers enrolled in a computational thinking in school mathematics course for 1 semester. They have had experience solving problems in computational problem situations, both in programming-based and unplugged activity. And They have used mathematics textbooks emphasizing problem solving approach to design lesson plans and micro teaching. Towards the end of the semester, they were assigned analyse mathematics textbook related to computational thinking. Data were collected from pre-service teacher presentation and reflection relate to their analyse textbook. Data were analyzed using the Computational Thinking framework (Hoyles and Noss, 2015).

The finding revealed that 39.29% of target group focused on decomposition such as problem situations related to positioning and separated large shapes to simplify calculations. Additionally, 17.86% of target group focused on abstraction such as problem situations that involved finding relevant quantities for calculations and reducing details in visuals. Furthermore, 25% of target group focused on algorithms such as particularly in calculation algorithms. And 25% of target group focused on pattern recognition such as identified patterns in problem situations related to changes in two quantities, divisibility properties, and proportional increases or decreases. The result show that pre-service teachers had perspectives on computational thinking through problem situations in mathematics textbooks. And it could be used as a tool to develop computational thinking skills in the classroom.

References

Hoyles, C., & Noss, R. (2015). Revisiting programming to enhance mathematics learning. In *Paper presented at math + coding symposium*. University of Western Ontario.

Wing, J. (2006). Computational thinking. Commun. *ACM, 49*, 33–35.

Yadav, A., Mayfield, C., Zhou, N., Hambrusch, S., & Korb, J. T. (2014). Computational thinking in elementary and secondary teacher education. *ACM Transactions on Computing Education (TOCE),* 14(1), 1–16. https://doi.org/10.1145/2576872

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 307. PME.

SEMIOTIC REGISTERS USED BY FIRST-YEAR ENGINEERING STUDENTS DURING PROBLEM SOLVING

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Students across STEM programs often to the transition to university, in part because of difficulties in their mathematics courses. Previous research has focused on the issues that students have with mathematical language, symbolic notation, and formal logic, suggesting that representational fluency is a major hurdle for students. Despite research showing that students struggle with transitioning between representations of mathematical objects, we know little about how they coordinate between modes of representation while reasoning in mathematics.

This project takes the perspective that learning in a discipline requires students to develop fluency in multiple modes of communication (Airey & Linder, 2017). As a result, we investigate how first-year engineering students, and their mathematics instructors use semiotic representations during mathematical activity. Specifically, the present study examines student use of representations in individual problem-solving. Its primary goal is to identify the semiotic representations that students favour and to provide an initial analysis of how they coordinate between them.

To achieve this, we conducted think-aloud interviews with 15 first-semester engineering students. Each participant solved two out of six possible mathematics problems. The data were coded based on student problem-solving activity (Schoenfeld, 1985) and categorized according to the registers of representations they used throughout their problem-solving process (Duval, 2006). Data for four of the problems is discussed. The data show that student use of semiotic representations largely depends on the problems' requirements and that students make consistent choices in the registers used for solving problems. However, regardless of context, more registers are called upon during the analysis and exploration phases of problem-solving but shift toward symbolic register use during the implementation phase.

References

Airey, John, and Cedric Linder. "Social Semiotics in University Physics Education." In *Multiple Representations in Physics Education*, edited by David F. Treagust, Reinders Duit, and Hans E. Fischer, 95–122. Cham: Springer International Publishing, 2017.

Duval, R. (2006). A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics. *Educational Studies in Mathematics* 61(1–2), 103–31.

Schoenfeld, A. (1985). *Mathematical problem solving.* Academic Press.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 308. PME.

GENDER DIFFERENCES IN MATHEMATICS LEARNING IN PRE-KINDERGARTEN CLASSROOMS IN CHILE

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FRAMEWORK AND OBJECTIVE

Recent national (SIMCE) and international (TIMSS, PISA) assessments reveal a growing gender gap in favor of boys, particularly as students' progress. While research on early childhood education is limited, a study by Pérez-Mejías et al. (2021) projected gender gaps in mathematics from kindergarten to grade 12 among Chilean students. This work aims to answer the question: Is there a difference in early mathematical learning between boys and girls? Does a gender gap favor boys in mathematics during early childhood education?

METHODOLOGY AND RESULTS

The study evaluated the mathematical abilities of 338 Pre-Kindergarten students (N=185 females) using the *Research-based Early Mathematics Assessment* (REMA). Multivariate analyses revealed significant sex differences: boys outperformed girls in numerical tasks, while no differences were found in geometry. Additionally, boys and girls employed different strategies.

DISCUSSION

Limited understanding exists regarding when math learning gaps emerge. This study shows that gender differences in mathematics can be observed as early as Pre-Kindergarten. One explanation may be the inadvertent reinforcement of gender stereotypes by adults in educational settings. Previous research indicates that cultural values and beliefs can significantly impact children's engagement with subjects based on these stereotypes (Nosek & Smyth, 2011).

References

Nosek , B.A., & Smyth , F.L. (2011). Implicit social cognitions predict sex differences in math engagement and achievement. *American Educational Research Journal, 48*, 1125–1156. doi: 10.3102/0002831211410683

Perez Mejias , P., McAllister , DE, Diaz, KG, & Ravest , J. (2021). A longitudinal study of the gender gap in mathematics achievement: Evidence from Chile. *Educational Studies in Mathematics, 107*(3), 583-605.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 309. PME.

OBSERVING MATHEMATICS AND BODY MOVEMENTS BEYOND WORDS: REFLECTIONS FROM AN OBSERVER

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Maturana (1988) reminds us,

> we observers never listen in a vacuum, we always apply some particular criterion of acceptability to whatever we hear (see, touch, smell....or think), accepting or rejecting it according to whether or not it satisfies such criteria in our listening (p. 3).

Based on what he said, when a movement involves the whole body, there is a criterion to apply to recognize if the body movement implies something mathematically. Furthermore, the criterion can also be shown through observation, where our conceptions and views as observers are confronted with what we consider to be mathematics in the movement. What is accepted as 'something' mathematically shares a common criterion (conscious or not) between observers under a situation or an action observed. In this oral communication, from an enactivist stance, I reflect on the position of the observer to observe "something mathematical" when body movement is taking place, extending Maturana's concept about distinction and languaging, which is defined as the "coordinations of coordinations of consensual behaviours" (Maturana & Verden-Zöller, 2008, p. 29), bringing an example from a toddler and their jumping in a street, which can be categorised as a mathematical movement, showing how the distinction in the actions act to generate coordination between the observer and the observed, noting how the operation of acceptance emerge in mathematical term through that observation. Finally, I discuss how the observer plays a crucial role in determining, beyond the words, what mathematics is brought forth by the participant, especially when it is recognised that mathematical concept is grounded in the physical and sensorimotor experience of the bodies (Abrahamson & Lindgren, 2014).

Additional information

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References

Abrahamson, D., & Lindgren, R. (2014). Embodiment and embodied design. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (2nd Edition) (pp. 358-376). Cambridge, UK: Cambridge University Press.

Maturana, H. R. (1988). Reality: The search for objectivity or the quest for a compelling argument. *The Irish journal of psychology*, *9*(1), 25-82.

Maturana H. R., & Verden-Zöller, G. (2008). *The Origins of Humanness in the Biology of Love*. Imprint Academic

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 310. PME.

PREPARATION FOR FINAL EXAMS WITH ADAPTIVE DIGITAL ENVIRONMENTS

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THEORETICAL BACKGROUND

In many countries, like Germany, students graduate from highschool with a final exam. Preparation for these exams happens during regular school lessons but also at home. For this purpose, students often use digital tools. Research suggests that the success of learning with digital environments depends on how the digital tools are designed regarding adaptivity, feedback and scaffolding (Reinhold et al., 2024). This applies especially for learning at home, which requires a large amount of self-regulation and motivation. Current research provides just little data about the learning procedures of highschool students in upper grades. This study aims to explain the impact of digital learning environments on the learning outcome and motivation of those students.

QUESTIONS AND METHODS

We investigate the effects of using digital adaptive tools regarding students' performance when preparing for their University entrance degree, intrinsic motivation, and perceived support. In an experimental pre-posttest design, we tested 30 students before and after the preparation phase for their final highschool exams in mathematics. Thereby they were free to use digital tools or not. We tested their mathematical competence and asked them about their intrinsic motivation and perceived support during different stages of learning. For analysing the data we used linear mixed models.

RESULTS AND DISCUSSION

On average, all students' performance increased from pretest to posttest, but there was no significant effect of the use of adaptive digital use. However, there was a significant difference concerning intrinsic motivation and perceived support, with higher increases for those students who used digital tools in their learning process. That means that the use of digital environments seems to be promising regarding the intrinsic motivation and perceived support. Further data ascertainment is planned to investigate the impact of the differences of the used digital environments on the learning procedure.

References

Reinhold, F., Leuders, T., Loibl, K., Nückles, M., Beege, M., & Boelmann, J. M. (2024). Learning mechanisms explaining learning with digital tools in educational settings: A cognitive process framework. *Educational Psychology Review, 36(1), 14*. doi:10.1007/s10648-024-09845-6

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 311. PME.

STUDENTS SUCCESS IN THE DEVELOPMENT OF INDIRECT MEASUREMENT STRATEGIES IN TEACHER EDUCATION

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The prevailing approach in the context of teaching and learning about measurement in real classrooms is typically oriented towards number-oriented tasks. Students frequently encounter challenges when confronted with contextualized tasks (Smith et al., 2011). To tackle this situation in primary teacher education, our study explores an indirect measurement task sequence as a strategy for getting prospective teachers to use their knowledge of measurement. Indirect measurements enrich measurement processes by determining the value of a quantity from the results of direct measurements of other quantities related to the one being measured

The study involved 68 first-year students on the primary education programme of the Universitat Autònoma de Barcelona. We used a learning activity to measure the diameter of a tree pit in the courtyard of the Faculty of Education. The placement of a tree at the center of the tree pit precluded direct measurement. The proposed teaching sequence comprised five distinct phases: individual proposal, group proposal, field measurement, sharing, and individual evaluation (Rojas & Albarracín, 2024). In our analysis we observed four main measurement strategies, three of which were successful, i.e. provided an adequate measurement of the diameter of the tree pit (diameter displacement, use of the perimeter/diameter formula, or a combination of both), and one unsuccessful (direct measurement of the inaccessible part). The results indicate that 75% of students plan unsuccessful strategies in their individual proposals. However, when working collaboratively in groups, 13 out of 17 groups successfully devised a viable strategy. After the field measurement, 15 groups manage to come up with a successful measurement procedure. In the sharing phase, all groups reach a successful strategy. Finally, in the evaluation test, 80.9% of students generate a successful proposal. These results show that the work sequence designed is effective; however, further attention is necessary to address individual learning needs.

The use of indirect measurement proved to be an enriching approach to develop key concepts such as measurable quantities, functional relationships and non-conventional strategies. This approach prepares future teachers to teach measurement concepts in a more meaningful and field-based way, not just with a numerical approach.

References

Smith, J. P., van den Heuvel-Panhuizen, M., & Teppo, A. R. (2011). Learning, teaching, and using measurement: introduction to the issue. *ZDM, 43*, 617-620.

Rojas, F. & Albarracín, L. (2024). Uso de actividades de medidas indirectas en la formación de maestros. *Números, Revista de Didáctica de las Matemáticas 113,* 23-42.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 312. PME.

DEVELOPING A PRIMARY STUDENT QUESTIONNAIRE FOR TEACHER EVALUATION

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Teaching quality in primary education has been measured mostly through extensive classroom observations. Less time-consuming are student ratings of teaching quality, which can also provide reliable and valid measures. However, some concerns have been raised about younger students' ratings. Although students have been found to be able to discriminate between different teaching quality constructs (Wagner et al., 2013), for younger students evaluating their teachers is unfamiliar and may be affected by their limit understanding. The focus of our study is to capture primary students' assessment of teaching quality in relation to two principles of conceptual focus and enhanced communication (Prediger et al., 2022). These principles, among others, guide the QuaMath project dealing with developing quality in mathematics education. Based on existing items, we developed a total of ten items for both scales. An example of conceptual focus is the statement 'In math class I can find patterns in problems', while 'In math class we talk about our ideas with other children' serves as an example of enhanced communication. To support the understanding of the statements, each item was paired with a corresponding picture. Furthermore, a cognitive lab study was conducted with three eight-year-old children in second and third grade to determine their understanding of the different items and whether specific phrases or words caused any misunderstandings. The students' answers were then categorized as "completely correct" (1) and "not correct or not completely correct" (0). Five items were well understood by all students, scoring 1 for both concise main sentences and subordinate clauses. The other five items received lower scores, ranging on average from 0 to 0.67. A detailed analysis of the students' responses revealed that specific words such as "similar", "idea", and "pattern" were the main sources of confusion. These findings highlight the need for further refinement of the questionnaire. Identifying and addressing problematic words and phrases is crucial to developing items that are easily understood by a wide range of primary school students, ensuring that the feedback collected is both meaningful and useful for improving teaching practice.

References

Prediger, S., Götze, D., Holzäpfel, L., Rösken-Winter, B., & Selter, C. (2022). Five principles for high-quality mathematics teaching: Combining normative, epistemological, empirical, and pragmatic perspectives for specifying the content of professional development. *Frontiers in Education, 7*(969212), 1-15.

Wagner, W., Göllner, R., Helmke, A., Trautwein, U., & Lüdtke, O. (2013). Construct validity of student perceptions of instructional quality is high, but not perfect: Dimensionality and generalizability of domain-independent assessment. *Learning and Instruction, 28,* 1-11.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 313. PME.

THE RELATION OF MATH ANXIETY WITH EMOTIONAL SKILLS OF PRIMARY STUDENTS IN MEXICO

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Math anxiety, a tension that interferes with number manipulation and problem-solving, affects learning across academic domains. Although most research has focused on adults, recent findings show that math anxiety begins in the early primary years. However, in Mexico, it remains under-explored, limiting opportunities for effective early interventions. Grounded mainly on the Control-Value Theory of Pekrun (2006), alongside other relevant frameworks, this study focused on three emotional skills identified in the author´s review of literature as relevant to managing math anxiety: self-efficacy, intrinsic motivation, and emotional intelligence.

The study examined the relationship between these emotional skills and math anxiety in 3rd- and 4th-grade students in a school in Mexico City. It was hypothesized that lower development of these skills is associated with higher levels of math anxiety. A mixed-method exploratory design was used with 103 students (ages 9–11); 94 remained after excluding those with general anxiety. Emotional intelligence, self-efficacy, intrinsic motivation, perceptions and beliefs, and math anxiety were assessed with instruments selected for their theoretical grounding, validity, and prior use with similar age groups. Questionnaires with emoji-supported Likert scales were completed by hand. Semi-structured interviews provided qualitative data that was thematically coded. Quantitative analysis was conducted in *Jamovi* to assess internal consistency and correlations among variables.

Results showed that 67% of students experienced nervousness or fear during math tasks, particularly when working under time pressure or faced with unfamiliar content. In addition, 14% showed high levels of math anxiety as measured by the corresponding scale. Despite these experiences, many students perceived themselves as competent in math. At the same time, some expressed a disconnection from its practical relevance, for example, a 3rd-grade student remarked, “Word problems are boring; they are not my problems.” Math anxiety showed significant negative correlations with self-efficacy and beliefs ($p < .001$), and motivation and emotional intelligence ($p < .05$).

The findings confirm a significant association between emotional skills and math anxiety. Students with higher self-efficacy, motivation, and emotional intelligence reported lower anxiety and more favorable attitudes toward math. These results highlight the importance of socioemotional development in early math education.

References

Pekrun, R. (2006). The control-value theory of achievement emotions: assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review*, *18*(4), 315–341. https://doi.org/10.1007/s10648-006-9029-9

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 314. PME.

CONCEPTIONS AND KNOWLEDGE OF PRE-SERVICE TEACHERS ON MATHEMATICS LEARNING ASSESSMENTS

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Mathematics learning assessment is essential for the development of students' skills. Beyond merely tracking progress, assessments should adopt a modern approach to teaching, promoting reasoning and the conceptual understanding of mathematics through the implementing of diverse evaluations practices (Suurtamm, 2018). However, current assessment practices often remain focused on one-time tests, overlooking the continuous appreciation of learning. Pre-service teachers struggle to interpret and apply formative assessment policies and still favour traditional methods such as tests, including question-and-answer formats. This situation complicates the implementation of assessment practices that support learning and negatively impacts student outcomes. Therefore, it is crucial to reinforce teacher training to promote more diverse and effective assessment approaches.

This study examines the conceptions and knowledge of pre-service middle-school teachers in Ontario's Francophone educational system regarding the assessment of learning in mathematics, at the beginning of their training. The research framework is based on the concepts of assessment for, of, and as learning (Black & Wiliam, 2018), as well as the principle that mathematics assessment should be continuous, embedded in teaching, and centered on meaningful tasks (Suurtamm, 2018). A descriptive approach was applied for analyzing responses from a questionnaire administered to thirty-three pre-service teacher participants. Data analysis was conducted through the study of relative frequencies and correlations. Results show that 57.6% of participants believe that assessments help identify students' learning difficulties in mathematics. while 51.5% perceive assessments as a formative tool enabling teachers to adjust and refine their teaching practices. A strong correlation exists between "assessment helps adjust mathematics teaching" and "assessment helps track student progress" ($r = 0.70$). The results show that assessment is perceived as a formative tool, although some traditional conceptions persist. Further results will be discussed in the presentation.

References

Black, P., & Wiliam, D. (2018). Classroom assessment and pedagogy. *Assessment in Education: Principles, Policy & Practice*, *25*(6), 551-575.

Suurtamm, C. (2018). Enhancing Mathematics Teaching and Learning Through Sound Assessment Practices. In A. Kajander, J. Holm, & E. J. Chernoff (Eds.), *Teaching and Learning Secondary School Mathematics : Canadian Perspectives in an International Context* (pp. 473-482). Springer International Publishing.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 315. PME.

INSTRUMENTS AND THEORIES ON STATISTICAL ANXIETY IN UNIVERSITY STUDENTS: A SYSTEMATIC REVIEW

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Statistical anxiety has been extensively studied due to its impact on students' academic performance. However, its conceptualization has evolved, leading to different theoretical perspectives and measurement instruments. This study conducts a systematic literature review to analyze how statistical anxiety has been defined and assessed with university students.

Following PRISMA 2020 guidelines, a systematic review was conducted using Web of Science (147), Scopus (173), and ERIC (126) with the search equation "(statistics anxiety) AND (student)." After removing duplicates, 297 manuscripts were screened based on theoretical conceptualization, measurement instruments, and empirical studies in higher education, resulting in 65 articles for in-depth analysis.

Findings confirm that statistical anxiety is multidimensional, shaped by three key research teams. Cruise et al. (1985) identified six factors, including worth of statistics, test/class anxiety, data interpretation anxiety, computational self-concept, fear of seeking help, and fear of instructors, leading to the Statistical Anxiety Rating Scale (STARS). Zeidner (1991) expanded this with social anxiety and cognitive intrusions. Onwuegbuzie and Wilson (2003) introduced a holistic model incorporating situational, dispositional, and environmental factors. Most assessment instruments stem from STARS but differ in structure and applicability.

Future research should focus on developing an integrative model of statistical anxiety, exploring individual and contextual influences, and designing evidence-based pedagogical interventions to mitigate its negative effects and improve statistical learning in higher education.

References

Cruise, R., Cash, R., & Bolton, D. (1985). Development and validation of an instrument to measure statistical anxiety. *Educational and Psychological Measurement, 45*(4), 789-795.

Onwuegbuzie, A., & Wilson, V. (2003). Statistics anxiety: Nature, etiology, antecedents, effects, and treatments–a comprehensive review of the literature. *Teaching in Higher Education, 8*(2), 195-209. https://doi.org/10.1080/1356251032000052447

Zeidner, M. (1991). Statistics and mathematics anxiety in social science students: Some interesting parallels. *British Journal of Educational Psychology, 61*(3), 319-328. https://doi.org/10.1111/j.2044-8279.1991.tb00989.x

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 316. PME.

ALGEBRAIC IDENTITIES AND THEIR REPRESENTATION TO SOLVE WORD PROBLEMS

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INTRODUCTION

Research in mathematical education highlights the importance of registers of representation for understanding mathematics. However, in basic education, often focus on memorization rather than conceptual understanding and problem-solving strategies, which persists in higher education. This research proposes a learning design aimed at assessing and improving students' knowledge and skills in solving problems involving algebraic identities through the coordination of semiotic representations, since they are fundamental in algebra and its applications. This document presents the results of an exploratory questionnaire administered to college students.

TEORETHICAL FRAMEWORK AND METHODOLOGY

The study is based on the theory of registers of semiotic representation proposed by Duval (2006) and on the perspective on problem solving by Puig and Cerdán (1988). An exploratory questionnaire was administered to 28 first-semester students at a public university in Mexico, which consisted of three sections: algebraic representation, geometric representation, and the coordination of both to solve a word problem.

RESULTS AND CONCLUSIONS

Only 21.4% of the students identified the square of a binomial as a special product, and 10% correctly performed polynomial multiplication. In the second section, a higher number of students provided a correct geometric representation compared to an algebraic one. Only one student successfully coordinated both representations to solve the word problems. Most of the students were unable to complete the tasks correctly. The results highlight the need for additional didactic interventions to teach special products from the perspective of the theory of registers of semiotic representation and problem solving, particularly by promoting the coordination of algebraic and geometric representations, which aids in the resolution of word problems.

References

Duval R. (2006) Semiosis et pensée humaine: Recherche epistemologique sur les représentatios et leurs rôles dans la cognition et l'enseignement des mathématique. *Recherches en Didactique des Mathémtatiques*, *26*(2), 219-241.

Puig, L., & Cerdán, L. (1988) *Problemas aritméticos elementales: Modelos de resolución en la escuela secundaria*. Graó

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 317. PME.

STEAM IN ELEMENTARY SCHOOL: EXPLORING "MOBFOG" IN MATHEMATICS EDUCATION

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STEM, an acronym for "science, technology, engineering, and mathematics", represents an educational approach aimed at fostering innovation (English, 2015). The STEM framework has expanded with the inclusion of the arts, forming STEAM.

During a visit to a public school in a town in the state of São Paulo, Brazil, we observed fourth-grade elementary students engaged in activities related to the Brazilian Rocket Exhibition (MOBFOG). It is an experimental competition, as it involves building and launching rockets obliquely from a launch base, as far as possible. Notably, at the day, the winning team launched their rocket to a distance of 44 meters. At one point, a student asked us: "We know the distance because we measured it, but what about the height the rockets reached?" This question sparked curiosity, prompting an investigation that culminated in the development of this research. The research sought to answer the following question: What aspects of STEAM pedagogy emerge when elementary students engage in activities related to MOBFOG?

Methodologically, we engaged four pairs of fourth-grade students in teaching experiments consisted of three parts: (1) identifying through drawings students' prior knowledge related to MOBFOG; (2) engaging students in an investigative scenario using GeoGebra; and (3) prompting students to create poems about their learning using ChatGPT. For data collection, all sessions were video recorded, and each pair of students used a laptop to explore the tasks (GeoGebra and ChatGPT). Data analysis employed triangulation and a video analysis framework.

As results, by revisiting projectile motion formulas and applying a fixed 45° launch angle, the GeoGebra App was design to offer ways to students to conjecture that the maximum height of the rocket is one-fourth of the maximum distance, modelling its path with the parabolic function $f(x) = -x^2/D_{máx} + x$. Within the humans-with-media framework, students engaged in "thinking-with-GeoGebra," constructing and refining mathematical understanding through testing and simulations. Moreover, students-with-ChatGPT created poem from defining key-words of their learning with GeoGebra, but the prompts were not deeply developed by them.

References

English, L. (2015). STEM: Challenges and opportunities for mathematics education. In *Proceedings of the 39th Meeting of the International Group for the Psychology of Mathematics Education, PME 39 (Volume 1)* (pp. 4-18). IGPME-The International Group for the Psychology of Mathematics Education.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 318. PME.

STUDENTS' READING BEHAVIORS IN CONNECTION TO READING COMPREHENSION OF MATHEMATICS TEXTS

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Mathematics texts are multisemiotic in nature, and experienced readers have learned to pay attention to all semiotic resources involved (Fang, 2023). Students, on the other hand, tend to prioritize symbolic expressions over written language, which might lead to the neglect of important content and impaired comprehension (Dee-Lucas & Larkin, 1991; Österholm, 2008). In this ongoing study, we investigate whether an excessive focus on symbols, as well as other reading behaviors concerning the relations between symbols and written language, can impact students' reading comprehension. Compared to previous research, we add eye-tracking data as methodological approach.

The study includes various mathematical texts that combine written language and symbols. The texts present concepts or procedures and cover various topics (e.g., symmetry and divisibility). A total of 75 participants at different educational levels, from lower secondary to tertiary, encounter texts with mathematical content that is new for them. Using eye-tracking techniques, their eye movements are recorded during reading. Afterwards, reading comprehension is tested through specific questions. To distinguish between comprehension levels, we ask for both content reproduction and applying the content in new situations, corresponding to textbase and situation model questions (see Kintsch, 1998). We use exploratory analyses to examine what reading behaviors distinguish the most successful readers from the least successful. This includes the proportion of time spent on symbols, the number of visits to certain text areas, and other reading behaviors that address relations between symbols and written language.

Connecting the reading process to reading comprehension provides insight into which reading skills are beneficial to develop to comprehend mathematical texts. Further research will be needed to determine how students should be taught to efficiently acquire these skills.

References

Dee-Lucas, D., & Larkin, J. H. (1991). Equations in scientific proofs: Effects on comprehension. *American Educational Research Journal, 28*(3), 661-682.

Fang, Z. (2023). *Demystifying academic reading: A disciplinary literacy approach to reading across content areas*. Taylor & Francis.

Kintsch, W. (1998). *Comprehension: A paradigm for cognition*. Cambridge University Press.

Österholm, M. (2008). Do students need to learn how to use their mathematics textbooks? The case of reading comprehension. *Nordic Studies in Mathematics Education, 13*, 53-73.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 319. PME.

VALIDATING A PERFORMANCE-BASED ASSESSMENT OF PRE-SERVICE TEACHERS' DIGITAL COMPETENCE

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Using digital tools in mathematics teaching—such as computer algebra systems, dynamic geometry, or spreadsheet software—requires a high level of digital competence from mathematics teachers (e.g., Weigand et al., 2024). In particular, teachers must be able to use the tools efficiently to solve mathematical problems (Technical-Mathematical Competence, TMC) and master the digital teaching demands (Technical-Pedagogical-Mathematical Competence, TPMC). However, self-reports, prone to bias, are currently the common method for measuring these competences, so alternative approaches should be considered. Seifert and Lindmeier (2024) have therefore developed a performance-based assessment: in addition to questionnaires on affective-motivational aspects of digital teaching, this assessment contains items on typical instructional demands of TMC (e.g., tool operation, knowledge of mathematical potential) and TPMC (e.g., development of digital tasks, digital feedback for students, knowledge of pedagogic potential), which are processed directly in the respective tool.

To validate the instrument and the underlying theoretical model of digital competence, we conducted a pre-post study with $N = 148$ pre-service mathematics teachers from five German universities. A confirmatory factor analysis, testing the hypothesized two-dimensional structure of TMC and TPMC, showed a moderate to good fit ($\chi^2 = 27.793$, $df = 7$, $CFI = .93$, $SRMR = .05$, $RMSEA = .10$). In addition, we evaluated the assessment's sensitivity to interventions using Wilcoxon's Signed-Rank Tests (TMC: large effect, $W(N = 256) = 1142$, $p < .001$; TPMC: small effect, $W(N = 256) = 2752.5$, $p = .004$). Finally, using linear regressions, we found significant contributions of affective-motivational aspects to TMC and TPMC ($.08 < \beta < .23$).

The results support the findings of Seifert and Lindmeier (2024) regarding empirical structure and sensitivity. In addition, they reveal the relations between affective-motivational and cognitive aspects. In conclusion, the instrument seems suitable for assessing TMC and TPMC and provides insights into the current state of digital competences of future teachers.

References

Seifert, H., & Lindmeier, A. (2024). Developing a Performance-Based Assessment to Measure Pre-Service Secondary Teachers' Digital Competence to Use Digital Mathematics Tools. *Journal Für Mathematik-Didaktik*, Article 25.

Weigand, H.-G., Trgalova, J., & Tabach, M. (2024). Mathematics Teaching, Learning, and Assessment in the Digital Age. *ZDM – Mathematics Education*, *56*(4), 525–541.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 320. PME.

THE DEVELOPMENT OF FACILITATORS' NOTICING OF TEACHERS' RESPONSES TO STUDENT MULTIPLICATIVE THINKING

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Professional development (PD) for mathematics teachers has received increasing attention in recent years, in tandem with facilitator PD programs (FPD) to enhance facilitators' expertise. Just as teachers improve their skills through PD, facilitators' expertise for leading PD (Karsenty et al., 2023), including their noticing of teacher learning during PD courses, is essential and subject to development. Defined by van Es and Sherin (2002), noticing consists of identifying the significance of a situation, establishing connections between interactions and broader teaching principles, and utilizing contextual knowledge to reason about interactions. Teacher noticing (van Es & Sherin, 2002) is also transferable to facilitators and PD settings (e.g., Jacobs et al., 2017), as facilitators need to be able to notice productive teacher learning during PD sessions, by perceiving noteworthy events in PD, interpreting these events, and making decisions accordingly. Through a situated examination, this study examines (N = 113) facilitators' expertise for noticing in a fictional PD situation concerning multiplicative thinking at the beginning and after one year of a content-specific FPD program. Findings highlight the knowledge elements with which facilitators started the FPD and that about half of the facilitators could only superficially interpret the situation and focused on irrelevant aspects. After a year of FPD, significant improvements were seen in facilitators' perception of and decision-making about the fictional situation, thereby underlining FPD as an opportunity to improve facilitators' noticing expertise.

References

Karsenty, R., Pöhler, B., Schwarts, G., Prediger, S. & Arcavi, A. (2023). Processes of decision-making by mathematics PD facilitators: the role of resources, orientations, goals, and identities. *Journal of Mathematics Teacher Education*, *26*(1), 27–51. https://doi.org/10.1007/s10857-021-09518-z

Jacobs, J., Seago, N., & Koellner, K. (2017). Preparing facilitators to use and adapt mathematics professional development materials productively. *International Journal of STEM Education, 4*, 30. https://doi.org/10.1186/s40594-017-0089-9

van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*(4), 571–596.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 321. PME.

FEEDBACK AS A SPECIALIZED PRACTICE: ONE FOUNDATION OF INTERPRETIVE KNOWLEDGE

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Providing feedback to students is a difficult practice for mathematics teachers. Specially constructive feedback, that is, feedback which help students move to develop knowledge at upper levels. The type of feedback teacher proposes is associated with the content of their Interpretative Knowledge – IK (Ribeiro, Mellone, & Jakobsen, 2013). IK corresponds to specialised mathematical knowledge, required to give meaning and interpret student's productions – even non-standard or based on errors (Jakobsen, Mellone, Ribeiro, & Tortora, 2016). Depending on the level of IK, the teacher performs a certain interpretative practice and proposes a type of feedback.

In previous work, we have focused on IK in different topics (e.g., powers of ten (Jakobsen et al., 2016) and fraction (Ribeiro et al., 2013)). Here, we moved towards the work on geometry and included a focus on feedback, in the direction of observing changes in interpretative practices. Considering the specificities of teacher's knowledge, and feedback to improve practice and teacher education proposals, it is essential to focus on topics in which teachers and students reveal difficulties. We present and discuss the type and content of feedback provided by 12 lower and upper secondary mathematics teachers enrolled in a professional development context (40 hours), when asked to give meaning and interpret student's productions regarding a mathematical task in the context of the composition of isometric geometric transformations. Such interpretation requires "listening to students' thinking", going beyond the immediatism of "what can be seen" and that was one of the critical features addressed by teachers when reflecting on the professional development they have experienced.

Additional information

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References

Jakobsen, A., Mellone, M., Ribeiro, M., & Tortora, R. (2016). Discussing secondary prospective teachers' interpretative knowledge: A case study. In C. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proc. 40th Conf. of the Intern. Group for the Psychology of Mathematics Education* (Vol. 3, p. 35–42). PME.

Ribeiro M., Mellone, M., & Jakobsen, A. (2013). Prospective teachers' knowledge in/for giving sense to students' productions. In A. M. Lindmeier & A. Heinze (Eds.). *Proc. 37th Conf. of the Intern. Group for the Psychology of Mathematics Education* (Vol. 4, p. 89-96). PME.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 322. PME.

AUTOMATIC FEEDBACK AND AFFINE FUNCTION LEARNING: GRAPHIC-ALGEBRAIC CONVERSION STRATEGIES IN GEOGEBRA

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This research presents a proposal for teaching affine functions using GeoGebra with gamification elements and automatic feedback, with the aim of analyzing how students perform the conversion between graphic and algebraic registers during interactive activities. Based on Duval's Theory of Registers of Semiotic Representation (2012), the study highlights that mathematical understanding requires the mobilization of different representations and their transformations, such as treatments and conversions. The conversion from graphic to algebraic register, for example, demands a global interpretation of the visual variables of the line, such as its inclination and position in relation to the origin (Duval, 2011), an approach that is often little used in traditional teaching.

In the research, four second-year high school students participated in activities developed in GeoGebra Classroom, in which they had to identify the coefficients of the affine function from graphical representations. Automatic feedback helped students with suggestions based on the types of errors made. Data were collected through observation, screen recording using the Loom platform, questionnaires and analyzed according to the methodology of Bogdan and Biklen (1994).

The results showed that 71% of the errors were related to the slope, requiring more complex treatments. Successful global interpretation strategies included visual analysis of the vertical (Δy) and horizontal (Δx) variation to identify the slope, as suggested in the feedback. Students reported that gamified activities with automatic feedback helped them to correct their errors and learn autonomously and in a fun way. The research concluded that automatic feedback and the use of GeoGebra with gamification enabled more effective learning of the affine function, providing a better understanding of the mathematical concepts covered.

References

Duval, R. (2011). Gráficos e equações: a articulação de dois registros. Tradução de M. T. Moretti. *Revemat: Revista Eletrônica de Educação Matemática*, *6*(2), 96-112.

Duval, R. (2012). Registros de representação semiótica e funcionamento cognitivo do pensamento. Tradução de M. T. Moretti. *Revemat: Revista Eletrônica de Educação Matemática*, *7*(2), 266-297.

Bogdan, R. C., & Biklen, S. K. (1994). *Investigação Qualitativa em Educação.* Porto Editora.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 323. PME.

DEVELOPING MATHEMATICAL THINKING THROUGH HARNESSING LINGUISTIC DIVERSITY

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As more than half the world's citizens are multilingual, learners of mathematics are frequently engaging in mathematical activities through languages that are not their first (Barwell *et al*., 2016). This means that they are often mediating between learning the content of the mathematics and of their additional language, potentially placing these learners at a linguistic disadvantage. There is growing evidence to suggest that enabling learners to draw on their full linguistic repertoire to support this process improves their educational outcomes (Cummins, 2021), but there is currently limited guidance for teachers on how to harness this linguistic diversity within the mathematics classroom context. This paper presents the preliminary findings of an ongoing action research project, which seeks to address this gap by exploring the following questions:

- How can multilingual learners be supported to draw upon their full linguistic repertoire during mathematical activities?
- How might teachers be supported to develop their pedagogical approaches to maximise crosslinguistic transfer during maths lessons?

This project, which adopted a social constructivist perspective, utilised interpretivist methodology, including the analysis of recorded maths lessons, semi-structured interviews with teachers and focus group discussions with learners, from one secondary and three primary schools in Jersey, UK. The findings highlight the benefits of drawing on learners' full linguistic repertoires flexibly, through strategies including translanguaging, dynamic use of multilingual glossaries and multiple representations to support cross-linguistic transfer (Cummins, 2021). Furthermore, through analysing the responses of these learners, six stages of mathematical thinking were identified and developed into a framework for supporting learners. This framework underpins the next phase of the study, which includes teachers and learners in an all-through English-French/Spanish/German bilingual school in England and a range of primary/ secondary schools in the Kalgidhar Trust in India. This phase involves developing training materials, initially in English, Hindi and Spanish. These aim to support the professional development of teachers of multilingual learners of mathematics globally.

References

Barwell, R., Clarkson, P., Halai, A., Kazima, M, Moschkovich, J., Planas, N., Setati-Phakeng, M., Valero, P. and Ubillús, M. V. (2016). *Mathematics Education and Language Diversity: The 21st ICMI Study Educational Studies in Mathematics,* Springer.

Cummins, J. (2021). *Rethinking the Education of Multilingual Learners*. Multilingual Matters.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 324. PME.

RESEARCH ON PROOF-RELATED ACTIVITIES AND THEIR RELATIONSHIP: A SYSTEMATIC LITERATURE REVIEW

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Proof is foundational in mathematics, with proving as a key activity for generating evidence. Thus, a major goal in mathematics education is to help learners grasp the concept of proof and successfully engage in proof-related activities. Frameworks (e.g., Mejía-Ramos & Inglis, 2009) distinguish activities like constructing, reading, and presenting proofs, each including multiple more specific activities, such as comprehending, validating, and evaluating proofs. Though distinct in goals, givens, and products, these activities are interconnected. For instance, comprehending a proof often requires (mentally) constructing (sub-)proofs, and constructing a proof typically entails validating the partial proof(s) already created.

While some publications explicitly relate these activities (e.g., Pfeiffer, 2009) or examine how training in one impacts another, the relationship remains mostly unclear. However, such knowledge is vital for research and education, for example, to evaluate the possibility of teaching proof through what might be easier activities for students. To address this gap, we conducted a systematic literature review guided by the questions: Which proof-related activities are examined if multiple proof-related activities are investigated? How are they and their relationship conceptualized, and is empirical evidence provided?

A systematic literature search for publications in 2000-2023 in public databases using terms such as "argument*" or "proof*" in combination with activities (e.g., "comprehension" or "validation") yielded 15371 publications. Based on a manual screening of titles and abstracts, the number was narrowed to 199. Subsequently, the full texts of these publications were coded. The interrater reliability was $\kappa > .72$.

Initial results indicate that connections between activities are rarely discussed. There is neither broad consensus on their relationship nor adequate evidence.

The review's results underline that to advance research on proof-related activities, the underlying cognitive processes must be examined more thoroughly, and empirical evidence must be gathered regarding their relationship. These results could be highly valuable for creating learning opportunities related to proof.

References

Mejía-Ramos, J. P., & Inglis, M. (2009). What are the argumentative activities associated with proof? *Research in Mathematics Education, 11*(1), 77-78.

Pfeiffer, K. (2009). The role of proof validation in students' mathematical learning. In M. Joubert (Ed.), *Proceedings of the British Society for Research into Learning Mathematics* (Vol. 29(3), pp. 79-84). British Society for Research into Learning Mathematics.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 325. PME.

WOMEN-IDENTIFYING GRADUATE STUDENTS' VALUES RELATED TO TEACHING AND LEARNING MATHEMATICS

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Women-identifying mathematics graduate students' values related to teaching and learning mathematics are critical in shaping their professional identities and pedagogical practices as future faculty members (Austin & McDaniels, 2006). Values tension may produce role conflict, decreased job satisfaction, and an increased likelihood of leaving the profession (Gardner & Holley, 2011). Given the historical alignment in the United States of the mathematics discipline with dominant masculine values (Levya, 2017), more work is needed to illuminate women-identifying graduate students' values related to teaching and learning and discuss how to support these values in graduate education spaces. Thus, this study explores three women-identifying math graduate students' values related to mathematics teaching and learning and areas in graduate education in which they encountered and negotiated these values. From this work emerged four values related to mathematics teaching and learning across the participants: collaboration, student-instructor perceptions, humanizing mathematics, and compassion and care for students' well-being. The intertwining of these four values suggest a more feminine view of mathematics teaching and learning, challenging dominant masculine values such as individualism, emotionless, and innate ability (Mendick, 2006). We further utilized a narrative approach to make sense of where and how the participants navigated these values within graduate school. We found that interactions with faculty and advisors, prior teaching and mentoring experiences, experiences as undergraduate and graduate students, and cohort experiences were critical sites for negotiating these values. These results suggest implications for departments to support a more diverse set of mathematics education values.

References

Austin, A. E., & McDaniels, M. (2006). Preparing the professoriate of the future: Graduate student socialization for faculty roles. In *Higher Education* (pp. 397-456). Springer. https://doi.org/10.1007/1-4020-4512-3_8

Gardner, S. K., & Holley, K. A. (2011). "Those invisible barriers are real": The progression of first-generation students through doctoral education. *Equity & Excellence in Education*, *44*(1), 77-92. https://doi.org/10.1080/10665684.2011.529791

Leyva, L. (2017). Unpacking the male superiority myth and masculinization of mathematics at the intersections: A review of research on gender in mathematics education. *Journal for Research in Mathematics Education, 48*(4), 397–433. https://doi.org/10.5951/jresematheduc.48.4.0397

Mendick, H. (2006). *Masculinities In Mathematics*. McGraw-Hill Education (UK).

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 326. PME.

INVESTIGATING THE PERCEPTUAL SPAN IN MATHEMATICS

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In reading, the eyes process written information from only a limited area of the visual field at a time. The perceptual span refers to the region from which useful information can be obtained (Rayner, 1998). The size and asymmetry of the perceptual span are affected by various factors, including the text's difficulty and the reader's language abilities. Therefore, studies on the perceptual span have substantially contributed to our understanding of cognitive processes during reading (Frei & Bosse, 2018).

To our knowledge, the perceptual span has not yet been studied for the visual perception of mathematics. Knowledge about the perceptual span could contribute to a deeper understanding of the commonalities and differences between reading text and reading mathematics, as well as open up a range of new approaches to investigate the cognitive processes underlying visual perception of mathematical tasks. Moreover, it could contribute to the ongoing question regarding which information can be processed outside the currently fixated gaze position– specifically, in which instances the eye–mind assumption may not apply (Schindler & Lilienthal, 2019).

In a pilot study with 30 undergraduate students, we employed a *boundary technique* in a parity judgement task. Students were shown a simple addition problem and asked whether its result was even or odd. Through eye tracking, the visual stimulus was changed once the gaze approached the target area (the final digit needed to solve the task). These changes occurred during saccadic eye movements, when visual perception is momentarily inhibited, so participants were typically not consciously aware of the change. We systematically varied both the boundary distance and the nature of the stimulus change. Our results show that the boundary technique can be successfully implemented for mathematical tasks and that some forms of preview may occur. In this talk, we also discuss methodological possibilities and limitations of the paradigm.

References

Frey, A., & Bosse, M.-L. (2018). Perceptual span, visual span, and visual attention span: Three potential ways to quantify limits on visual processing during reading. *Visual Cognition, 26*(6), 412-429.

Rayner, K. (1998). Eye Movements in Reading and Information Processing: 20 Years of Research. *Psychological Bulletin, 124*(3), 372–422.

Schindler, M., & Lilienthal, A. J. (2019). Domain-specific interpretation of eye tracking data: towards a refined use of the eye-mind hypothesis for the field of geometry. *Educational Studies in Mathematics, 101*, 123-139.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 327. PME.

A SOCIOECOLOGICAL MATHEMATICS TASK

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There are calls to shift from individualistic perspectives on teaching and learning to relationality, environmental consciousness, and attachment to nature within socioecological systems (Barwell et al., 2022). This challenges educators to create embodied mathematics activities that turn learners' attention to nature (i.e., socioecological tasks). We created a mathematics task (Parrish, 2011) that emerged from a composting activity exploring the interaction between the soil and objects such as cardboard, regular paper, plastic, and fabric. Learners were asked to share their predictions and when preparing to bury the objects, we asked learners how we should proceed so we could find the objects after a week. Children suggested putting the objects in rows and columns by object type. This situation brought us to create the mathematics task: How many objects are in total? How do you know?

DATA COLLECTION AND ANALYSIS

We worked with two different classes of nine-year-old learners at the beginning of the school year; each class had 14 learners. Data constitute teaching videos. To identify what strategies learners used and how they responded to the composting activity, we used qualitative content analysis to identify and create a priori codes.

RESULTS AND DISCUSSION

Learners used a variety of strategies, informed by their knowledge of addition: skip counting, doubling, and decomposing numbers by their place value. This work provides evidence of engagement when learners work on activities involving embodied interactions with nature (Barwell et al., 2022). The mathematics content of the task was not decided ahead of time; it emerged from our interactions with the learners, which answers calls to focus on relationships first. We argue that planning and teaching socioecological mathematics tasks require educators to be attentive to learners' conversations and knowledgeable of their backgrounds to build from these when creating mathematics situations.

References

Barwell, R., Boylan, M., & Coles, A. (2022). Mathematics education and the living world: A dialogic response to a global crisis. *The Journal of Mathematical Behavior, 68*, 101013. https://doi.org/10.1016/j.jmathb.2022.101013

Parrish, S. (2011). Number talks build numerical reasoning. *Teaching Children Mathematics, 18*(3), 198–206. https://doi.org/10.5951/teacchilmath.18.3.0198

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 328. PME.

MULTIPLE CHOICE VERSUS OPEN-ENDED QUESTIONS IN RETRIEVAL PRACTICE

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Solidifying a piece of information in your memory can be supported by retrieval practice. To ensure long-term retention, the strategic use of retrieval practice was shown to be efficient. Numerous studies were conducted in this area and proved the efficacy of retrieval practice, however, they mainly focused on word or text memorization (Roediger & Karpicke, 2006). The positive effects of retrieval practice have also been shown in the field of mathematics (Szabó et al, 2023). Aiming to reduce the workload of teachers, we measured the efficiency of open-ended and multiple-choice tests.

In this research, we investigated first-year pre-service mathematics teachers' performance in the Introduction to Algebra course during a 13-week period. Our sample comprises 78 students divided into four groups. After each seminar, students had to take a short quiz on the material learnt on that day. Two seminar groups had to take the test immediately after the lesson, while the other two groups could take it later that day. We divided the students according to test types into two groups. Half of the students got open-ended tests, the other half got multiple-choice tests. Each student took the same type of test throughout the 13-week period. Students' prior mathematics knowledge was assessed before the start of the course. During the semester, we measured their content-related progress twice, on the 6th and on the 13th week. At the end of the semester, a post-test was also administered.

The pre-tests were analysed by ANOVA, and we found that the groups were similar. Their average scores in percentage were the following: 62, 63, 67, 70. During the first test, the group, who took open-ended tests at home, scored significantly lower than the rest ($p=0.044$), while their pre-tests were similar ($p=0.262$). The results of the second test were indifferent among students, taking open-ended and multiple-choice tests. Similar results emerged in the post-test. In the presentation, results will be discussed in detail. Our findings suggest that multiple-choice tests are just as efficient in the short-, and medium-term as open-ended tests. Thus, tests could be digitalised, and the workload of teachers could be diminished.

References

Roediger, H. L. III, & Karpicke, J. D. (2006). Test-enhanced learning: Taking memory tests improves long-term retention. *Psychological Science, 17*(3), 249–255. https://doi.org/10.1111/j.1467-9280.2006.01693.x

Szabó, C., Bereczky-Zámbó, C., Muzsnay, A., Szeibert, J., & Bernáth, L. (2023). Investigación de la eficacia depráctica de recuperación en matemáticas universitarias. *Revista De Educación*, 401(1). https://doi.org/10.4438/1988-592X-RE-2023-401-584

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 329. PME.

PROBLEM-POSING IS SOLVED - OR HOW WE CREATED A FRAMEWORK FOR STUDENTS' COMPLEX DEVELOPMENT

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Well-adapted problem-posing (PP) in mathematics lessons may promote student mathematical attitudes and enhance academic knowledge (Cai & Hwang, 2023). While the positive effects of PP on students' mathematical learning have been explored, most educators have limited experience in implementing PP with their students (Papadopoulos et al., 2022). Therefore, several studies suggest that research on supporting secondary school teachers should now be at the forefront of research in this area. This is why, we aimed to create a tool to support teachers in implementing PP activities in the classroom. We designed a framework of a learning model based on several works in the literature (i.e. Rosli et al., 2015). The goal of the framework was to reflect, among other aspects, students' key skills in PP, such as creative originality, language proficiency, and mathematical content knowledge. We validated it in a classroom environment. The participants were 16 tenth-grade students from a high-ranking secondary school. To this end, an 8-lesson action plan comprising 20 PP activities, based on the created framework, was developed and implemented. The results showed that the teacher and students, who were inexperienced in PP, understood the framework and were engaged in PP activities. The problems were analysed, and the PP was successful in the sense that the students were able to pose proper-quality problems. The framework was also able to provide feedback to the teacher and his/her students on the areas where they were successful in their PP and on which aspects could be improved. Our research can be a basis for future research on how to support teachers and make the PP method more popular.

References

Cai, J., & Hwang, S. (2023). Making mathematics challenging through problem posing in the classroom. *Research in Mathematics Education*, 115-145. https://doi.org/10.1007/978-3-031-18868-8_7

Papadopoulos, I. & Patsiala, N. & Rott, B. & Baumanns, L. (2021). Multiple Approaches to Problem Posing: Theoretical Considerations Regarding its Definition, Conceptualisation, and Implementation. *Center for Educational Policy Studies Journal*. https://doi.org/10.26529/cepsj.878.

Rosli, R., Capraro, M. M., Goldsby, D., Y Gonzalez, E. G., Onwuegbuzie, A. J., & Capraro, R. M. (2015). Middle-grade Preservice teachers' mathematical problem solving and problem posing. *Mathematical Problem Posing*, 333-354. https://doi.org/10.1007/978-1-4614-6258-3_16

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 330. PME.

MATH ANXIETY OF PRIMARY SCHOOL TEACHER TRAINEES

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Math anxiety poses a major challenge at all levels of the education system (Bernáth & Krisztián, 2017), including teacher education. In addition to possibly negatively affecting performance, it also impacts on attitudes and expectations related to the subject of mathematics. The aim of our research was to get a picture about the mathematical anxiety attitude and expectations of first-year students, tracking possible changes over their first semester at the university.

The sample comprised 118 students participating in elementary teacher training. The first data set was collected just before the first semester, and the second data collection was right after the first semester. The questionnaire consisted of three parts, and 44 Likert-scale items: 22 items about mathematical anxiety in different contexts and situations (Anxiety, Bernáth & Krisztián, 2017), 9 items concerned students' attitude towards learning mathematics (Attitude, Carey et al., 2017), and 13 items addressed their feelings about their future mathematics teaching challenges (Expectations, newly developed). All subscales had appropriate reliability in both datasets (Cronbach's alpha ranged from .84 to .94.) Comparison between the two datasets yielded significant difference on the first part of the questionnaire, where a positive change occurred after one semester of math learning at the university ($t(117) = 2.93$, $p = .004$), while on the other two parts no significant changes were observed ($p = .53$ and $p = .24$, respectively).

Our main finding is that one semester of learning mathematics in a tertiary education context could significantly reduce students' anxiety in mathematical situations. One of the most remarkable changes (out of the 22 items) occurred on the item about calculating several multiplications by written algorithm. The fact that students' attitudes and expectations for their future math teaching remained on the same level after one semester suggests that these psychological constructs are more difficult to foster and are more resistant to change. Since mathematics has a reputation as being a tough subject to learn and to teach, our results help in re-shaping the course topics and the teaching methods used in primary teacher training.

Additional information

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References

Bernáth, L. & Krisztián, Á. (2017). A matematikai szorongás és a MAS-UK kérdőív. [Math anxiety and the MAS-UK questionnaire]. In A. Bóna, K. Lénárd, & M. Pohárnok (Eds.), *Bontakozó jelentés: Tanulmányok a 60 éves Péley Bernadette köszöntésére* (pp. 61–69). Oriold és Társai Kiadó.

Carey, E., Hill, F., Devine, A. & Szűcs, D. (2017). The modified abbreviated math anxiety scale: A valid and reliable instrument for use with children. *Frontiers in Psychology, 8*, 11.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 331. PME.

DEVELOPING A TEACHING REFLECTION TOOL

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Teacher reflection on the teaching and learning of mathematics is essential for driving instructional change. While teacher noticing is key to reflecting, this skill does not necessarily develop through teaching experience alone. Professional learning (PL) opportunities can play a critical role in supporting teachers' use of reflection to purposefully cultivate their understanding of ways to attend, interpret and act on significant instructional moments (van Es et al., 2017).

This oral communication focuses on the steps used to design, develop and pilot a reflection framework used with coaching support to enhance teachers' ability to notice key aspects of high-quality mathematics discourse in the classroom. The study was conducted in the context of Project AIM (All Included in Mathematics), a professional development program focused on high-quality discourse (Sztajn et al., 2021) that provides K-2 teachers with 30 hours of PL over one year and opportunities to engage in coaching cycles using the reflection framework in the following year.

The framework organizes reflection around intentional, systematic examination of two key aspects of instruction (powerful mathematics and mathematics discourse) and two groups of instructional participants (teachers and students). Five iterative design steps used to develop the framework will be discussed: a) identifying critical aspects of mathematics instruction through a systematic literature review, b) refining teacher and student actions, c) enhancing usability and relevance of the tool by incorporating stakeholders' feedback, d) embedding the tool within structured professional learning support, and e) piloting the framework in coaching cycles. The presentation will include findings from the current pilot study in which one coach worked with six elementary school teachers across six coaching cycles. Preliminary analysis of coaching meetings and participant interviews suggests that the framework provided a focused yet flexible support for teachers to reflect on their practice.

Additional information

This work was supported by NSF awards DRL-2200370 and DRL-22003071.

References

Sztajn, P., Heck, D., & Malzahn, K. (2021). *Activating math talk: 11 purposeful techniques for your elementary students*. Corwin.

van Es, E.A., Hand, V., & Mercado, J. (2017). Making visible the relationship between teachers' noticing for equity and equitable teaching practice. In Schack, E.O., Fisher, M.H, & Wilhelm, J.A. (Eds.) *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks*. (pp. 251-270). Springer International Publishing.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 332. PME.

FIFTH GRADERS' STRATEGY USE IN SUBTRACTION

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Mental calculation strategies are characterized by splitting tasks into easier subtasks and can help reinforce and deepen essential basic concepts (like understanding place values) and provide a foundation for algebra. Despite their importance in the transfer to secondary school, teachers in primary school often prioritizes standard algorithmic calculation (Kuzu & Nührenbörger, 2021). To foster mental calculation strategies, we`ve developed within the divomath project a digital learning environment emphasizing the conceptual understanding of mental calculation strategies (stepwise; hundreds, tens, units; adding up; auxiliary tasks) for first-year secondary students in Germany. For this study, we aim to identify the subtraction strategies fifth graders use before and after our intervention. In a first trial of the learning environment, pre- and post-assessment tasks of 22 pupils were analyzed. In one task, $191 - 46$ had to be solved in different ways – two times symbolically and two times on a number line. The data were analyzed using a qualitative content analysis, whereby the categories were mainly determined deductively based on established mental calculation strategies (e.g., Selter, 2001). Figure 1 presents Toms' representation of the strategies "simplifying" and "auxiliary task" on a number line before the intervention. He uses the "simplifying" strategy incorrectly by adding the units of 191 to the six units of 46. The "auxiliary task" is used correctly, but represented incorrectly, as he also jumps back the added five units. An initial analysis of the frequencies of strategies used for both symbolic and iconic representations shows that before the intervention, more strategies were used, but often incorrectly (see Figure 1). After the intervention, students applied strategies more accurately and confidently. This shows that the intervention led to a more precise and safer use of the chosen strategies.

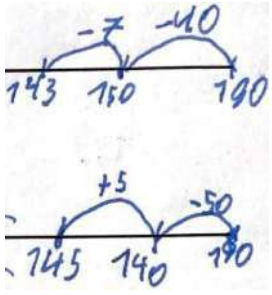

Figure 1: Toms' representation of $191 - 46$ on a number line

References

Kuzu, T. E., & Nührenbörger, M. (2021). The conceptual understanding of mental calculation strategies: Meaning-making in the case of the 'auxiliary task'. In J. Novotná & H. Moraová (Eds.), *Proceedings: Broadening experiences in elementary school mathematics. International Symposium Elementary Mathematics Teaching* (pp. 270–280).

Selter, C. (2001). Addition and subtraction of three-digit numbers: German elementary children's success, methods and strategies. *Educational Studies in Mathematics, 47*, 145–173.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 333. PME.

ESSENTIAL ILLUSTRATIONS IN REALISTIC WORD PROBLEMS: RESULTS OF A DESIGN EXPERIMENT

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Word problems are one of the most complex development tools for fostering mathematical problem solving (Verschaffel & De Corte, 1997). Research has confirmed that visual representations and drawings presented with word problems help pupils in the problem-solving process (Csíkos et al., 2012). In one type of mathematical word problems, students need to get the information they need for problem solving from a figure called essential illustration (Berends & Van Lieshout, 2009).

The focus of our research was to help students solve realistic word problems containing essential drawings. Our hypotheses were as follows. (1) The rate of realistic answers increases in the experimental group. (2) Students in the experimental group perform better on tasks containing an essential illustration.

86 students took part in our development experiment in the 3rd grade of a public school in the suburbs of Budapest. In our research we followed the PPC (pretest-posttest-control) design, so the control group (N=83) took the same tests as the experimental group. Although designed for 15-year-old students, a released PISA-task with essential diagram was included in the post-test. In the development experiment, over 12 lessons, students were asked to solve pairs of mathematical word problems, one of them containing an essential figure. Teachers implementing the training program took part in a one-hour training. Our results show that students in the experimental group outperformed their control group peers concerning the rate of realistic answers. Also, an independent samples t-test revealed a significant difference in the PISA dice test scores between the experimental group (M=0.67; SD=0.47) and the control group (M=0.34; SD=0.48), $t(167) = 4.627$, $p<0.001$. The effect size, as measured by Cohen's d, was $d = 0.712$, indicating a medium effect.

Additional information

Acknowledgment. This work was supported by the Research Programme for Public Education Development of the Hungarian Academy of Sciences.

References

Berends, I. E. & Van Lieshout, E. C. (2009). The effect of illustrations in arithmetic problem-solving: Effects of increased cognitive load. *Learning and Instruction, 19*(4), 345–353.

Csíkos, C., Szitányi, J., & Kelemen, R. (2012). The effects of using drawings in developing young children's mathematical word problem solving: A design experiment with third-grade Hungarian students. *Educational Studies in Mathematics*, *81*, 47-65.

Verschaffel, L. & De Corte, E. (1997). Word problems: A vehicle for promoting authentic mathematical understanding and problem solving in the primary school? In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 69–97). Psychology Press.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 334. PME.

CIVIC STATISTICS IN CHILEAN STANDARDS FOR MATHEMATICS AND SCIENCE TEACHER EDUCATION

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Statistics plays a fundamental role across disciplines, supporting data-driven decision-making. However, different analyses of the same dataset may yield biased conclusions, highlighting the need for robust statistical education to foster critical statistical literacy as a pillar of active citizenship. In Chile, although statistics and decision-making are included in the mathematics, natural sciences, and history curricula (Ubilla et al., 2024), classroom implementation remains limited. This raises the question: How do Chile's Initial Teacher Education standards incorporate statistics teaching and its societal relevance?

To explore this, we adopt Civic Statistics framework, which encompasses the knowledge and skills required to critically interpret, evaluate, and react to statistical information on social issues. This model comprises 11 interrelated facets grouped into three dimensions: (1) Engagement and Action, (2) Statistical, Methodological, and Contextual Knowledge, and (3) Enabling Processes as ICT (Gal et al., 2023).

Through a content analysis of Chile's Standards for Mathematics and Science Pedagogy Programs (CPEIP, 2021), we preliminarily found that science teacher standards consistently promote connections between scientific inquiry and societal issues. In contrast, mathematics teacher standards emphasize inquiry only in probability and statistics, alongside the use of technology for problem-solving.

By characterizing these standards through the lens of Civic Statistics, this study seeks to identify opportunities within teacher training programs to enhance the development of critical data analysis skills for informed decision-making across disciplines.

References

Centro de Perfeccionamiento, Experimentación e Investigaciones Pedagógicas (CPEIP) (2021). *Estándares Pedagógicos y Disciplinarios para Carreras de Pedagogía en Matemáticas y Ciencias*. Ministerio de Educación de Chile.

Gal, I., Nicholson, J., & Ridgway, J. (2023). A conceptual framework for Civic Statistics and its educational applications. In *Statistics for Empowerment and Social Engagement: Teaching Civic Statistics to Develop Informed Citizens* (pp. 37-66). Cham: Springer International Publishing.

Ubilla, F.M., Giaconi, V., & Ruz, F. (2024). Civic Statistics and interdisciplinarity in the Chilean context. In: EM Jones (Ed.), *Fostering Learning of Statistics and Data ScienceProceedings of the Satellite conference of the International Association for Statistical Education (IASE),* July 2023, Toronto, Canada.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 335. PME.

GALLERY WALK - AN ACTIVE LEARNING STRATEGY TO SHARE INSIGHTFUL AND ORIGINAL SOLUTIONS

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An effective teaching of mathematics engages students collectively in solving and discussing tasks that promote mathematical reasoning. This means that the nature of the context and the tasks used in classroom have a crucial role for the students' performance. Whereby challenge and multiple-solutions tasks must be contemplated, allowing students to use their personal different thinking processes, choosing the most convenient strategy to achieve the solution, giving them the chance to think outside the box. So, we propose a GW (Fosnot & Jacob, 2010) that emerges as an active instructional strategy that allows students to solve multiple-solution tasks collaboratively, displayed in posters around the classroom, having the opportunity to give and receive written/oral feedback about their work, discussing ideas collectively, contributing for their learning.

In this presentation, we report part of a qualitative and interpretative study, with an exploratory approach, carried out with 21 preservice teachers (PST) (children aged 6–12), during a learning experience in the context of a didactical course. The aim of the study was to characterize the contribution of a GW to highlight the strategies used by the PST when solving multiple-solutions tasks, in particular to identify the use of visual strategies with at least one insightful solution (Leikin & Guberman, 2023). Data was collected through observations and students' written productions (tasks' solutions, posters' constructions, participants comments) and photo records, and analysed in a holistic, descriptive and interpretative way. Preliminary results allow us to identify: a positive reaction to the GW experience; that the GW allowed PST to share, in a "non-threatening" way, the problem-solving solutions, using their own skills and increasing their repertoire of problem solving strategies; that the multiple solution tasks provoked PST creative potential, fluency, flexibility and originality, leading to the emergence of visual solutions, which they highly valued.

References

Fosnot, C., & Jacob, B. (2010). *Young mathematicians at work: Constructing Algebra***.** Portsmouth, NH: Heinemann.

Leikin, R., & Guberman, R. (2023). Creativity and challenge; task complexity as a function of insight and multiplicity of solutions. In R. Leikin (Ed.), *Mathematical Challenges for All* (pp.325-342). Springer.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 336. PME.

TEACHERS' PRACTICES THAT PROMOTE GENERALIZING ACTIVITY THROUGH CLASSROOM DISCUSSION

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During mathematical classroom discussions, students have opportunities to share their mathematical thinking with peers and engage in peers' ideas by conjecturing, justifying, clarifying, and elaborating, which contribute to students' generalizating activity (SGA) (Goos, 2004). However, how the teachers' practices in facilitating classroom discussions that support SGA remains underexplored. To respond to this issue, we conducted a study with the following research question: *How do teachers use the 5 practices framework to support their goals for promoting SGA?*

The 5 practices (5Ps) framework (Smith et al., 2020) comprises five practices for orchestrating mathematical classroom discussions: anticipating, monitoring, selecting, sequencing, and connecting. In contrast, to characterize SGA, we use the relating-forming-extending generalization framework (RFE) (Ellis et al., 2022), which characterizes students' mental processes when they engage in SGA. Our data corresponds to a videorecorded 3-lesson mini unit implemented by an in-service teacher (IST), Felix, with the aim of exploring the identity $(a+b)^2=a^2+2ab+b^2$. The data analysis was constituted by, first, deductively coding each lesson using the 5Ps to characterize Felix's teaching practices and, second, using RFE to infer Felix's goals for promoting SGA.

The preliminary findings account for an emergent framework that encompasses teacher's practices and goals that promote SGA *within* each of the 5Ps. Specifically, initial findings suggest that, while progressing in the classroom discussion, SGA evolves in terms of RFE. For example, while practices like selecting may prompt initial SGA such as *establishing a way of operating* or *searching for regularity or similarity,* the practices of sequencing and connecting to learning goals may prompt higher SGA such as *identifying a regularity.* Further analysis on Felix's and other ISTs' practices may allow us to expand the identification of links between the 5Ps and the RFE frameworks.

References

Ellis, A. B., Lockwood, E., Tillema, E., & Moore, K. (2022). Generalization across multiple mathematical domains: relating, forming, and extending. *Cognition and Instruction*, 40(3), 351-384, DOI:10.1080/07370008.2021.2000989

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258–291. https://doi.org/10.2307/30034810

Smith, M., Steele, M. D., & Sherin, M. G. (2020). *The Five Practices in Practice [High School]: Successfully Orchestrating Mathematics Discussions in Your High School Classroom.* Corwin.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 337. PME.

PROSPECTIVE MATEMATICS TEACHERS' DISPOSITIONS TOWARD STUDENT RACIAL AND ETHNIC DIVERSITY

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The preparation of teachers to teach mathematics in racially and ethnically diverse school contexts is a prominent issue in mathematics education. In addition to acquiring knowledge and skills to teach high-quality mathematics, teachers in these contexts also must develop a specific form of "sociopolitical consciousness" or *dispositions* to recognize and understand the different forms of inequity that shape the students' mathematical experiences. The development of such dispositions is critical for teachers to act upon existing barriers to mathematics learning that students from racial and ethnic minority backgrounds consistently encounter in school. However, research has documented prospective mathematics teachers' reluctance to engage with social justice issues, particularly those pertaining to racial and ethnic discrimination (Shah & Coles, 2020). Typically, future teacher complete their teacher education programs without developing the capacity to decipher the racial grammar that structures institutions and social practices (Guinier, 2004), including mathematics teaching and learning. In this oral communication, we will share preliminary findings of an ongoing study focused on the preparation of teachers to teach mathematics in rural, immigrant, and indigenous school settings in a Chilean region marked by historical racial and ethnic conflicts. We examine the extent to which local teacher education programs offer opportunities for prospective teachers to develop dispositions to recognize and act upon racial and ethnic discrimination that shapes students' mathematical experiences in this region. To this end, we conducted in-depth interviews with 12 prospective teachers in their final year of preparation, along with two focus groups. The results to be discussed in detail underscore the pivotal role of the practicum, which emerges as a signifying and socializing space where prospective teachers recreate, reinforce, or reconstruct understandings about how to "deal with" racial and ethnic diversity while teaching mathematics. The processes through which these understandings are shaped are influenced by various factors, including the prospective teachers' backgrounds, identities, and relationships with cooperating teachers in the practice schools.

Additional information

Acknowledgement. This research is supported by ANID under the grants Fondecyt #11240575 and Millenium program NSC2024_028.

References

Guinier, L. (2004). From racial liberalism to racial literacy: Brown v. Board of Education and the interest-divergence dilemma. *Journal of American History*, *91*(1), 92-118.

Shah, N., & Coles, J. A. (2020). Preparing teachers to notice race in classrooms: Contextualizing the competencies of preservice teachers with antiracist inclinations. *Journal of Teacher Education*, *71*(5), 584–599.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 338. PME.

ADDITION AND SUBTRACTION PROFICIENCY INVOLVING NEGATIVE INTEGERS IN MALAWI AND ZAMBIA

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Mathematics proficiency in Malawi and Zambia remains low, especially regarding addition and subtraction involving negative integers. While Zambian students' incorrect responses are often based on recognizable algorithms rather than random guessing (Sudo et al., 2024), little was known about the underlying calculation processes in Malawi. This study aimed to (1) examine the error patterns and underlying algorithms used by Malawian students when performing addition and subtraction with negative integers, and (2) compare the level of knowledge acquisition between Malawi and Zambia using cognitive diagnostic models (CDM).

A 24-item test was administered to Grades 8 and 9 students: 615 in Malawi and 971 in Zambia. Twelve arithmetic expressions were tested under two conditions (a>b and a<b). Data analysis involved descriptive statistics, correspondence analysis, and the Additive Cognitive Diagnosis Model (A-CDM) in R.

Zambian students performed significantly better than Malawian students. Both groups showed systematic incorrect responses, suggesting the use of internalized but incorrect algorithms. Unique to Malawi were errors where subtractions (e.g., -4-2) were mistakenly computed as additions (e.g., 4+2). A-CDM results showed that only 19.1% of Malawian and 21.6% of Zambian students had mastered all three attributes: understanding negative numbers, addition, and subtraction involving negatives.

Students in both countries apply structured, albeit incorrect, algorithms when working with negative integers. Malawian students demonstrated more varied incorrect strategies. A clear implication is the need for instruction that strengthens the conceptual understanding of negative numbers and connects it with correct operational procedures. Teaching approaches that address these misconceptions could substantially enhance computational skills in both countries.

References

Sudo, S., Watanabe, K., & Chileya, G. (2024). Addition and Subtraction Proficiency Involving Negative Integers in Zambia. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.), *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 137-144). PME.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 339. PME.

INFLUENCE OF DIAGRAM CHARACTERISTICS ON STUDENTS' PERFORMANCE OF MENTAL ROTATION

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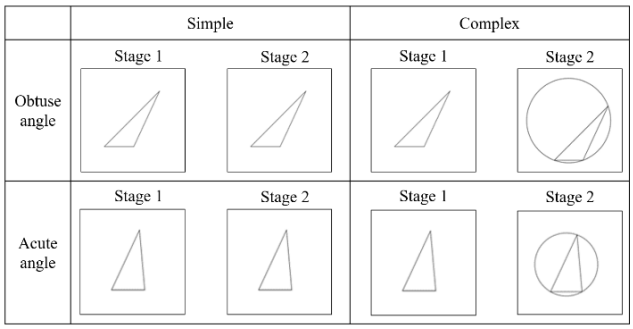

Understanding mental rotation is essential for the development of spatial reasoning skills in mathematics (Shepard & Metzler, 1971). In this study, we investigated the effects of two geometric diagram characteristics on mental rotation performance, examining both behavioral outcomes and neural activity through event-related potential (ERP) analysis. Specifically, we focused on two key factors: triangle type (obtuse vs. acute), and diagram complexity (complex vs. simple). We designed a test consisting of 400 items, in which participants were required to determine whether presented diagrams at various rotation angles were identical to an original reference image. This test was implemented using E-Prime software, with response accuracy (Acc) and reaction time to the correct response (RTc) serving as indicators of cognitive processing efficiency. Concurrently, brainwave activity was recorded during problem-solving and ERP techniques were used to analyze the corresponding neural processes. A total of 36 university students participated in the study.

Behavioral data revealed no significant interaction between the two factors (traingle type and diagram complexity). However, further analyses showed that participants achived higher accuracy and shorter reaction times when performing mental rotations on simple diagrams compared to complex diagrams, as well as on obtuse triangles compared to acute triangles. These findings suggest that complex diagrams and acute triangles may impose higher cognitive demands. Regarding neural activity, we focused on Rotation-Related Negativity (RRN), an ERP component indicative of the cognitive load associated with mental rotation. Consistent with the behavioral results, complex diagram and acute triangles elicited larger RRN amplitudes than simple diagrams and obtuse triangles, respectively, underscoring the greater neural resources required for mentally manipulating these stimuli. In summary, both behaviora and neural analyses confirmed that diagram characteristics—specifically complexity and triangle type—exert a significant influence on mental rotation perforamcne. This research underscores the importance of carefully considering diagram design in tasks that involve spatial reasoning.

References

Shepard, R. N., & Metzler, J. (1971). Mental rotation of three-dimensional objects. *Science*, *171*(3972), 701-703.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 340. PME.

"THE OLD WOMAN": COMMUNITY KNOWLEDGE IN MATHEMATICS CLASSROOMS

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We draw here on a focus group interview with five newly migrated teachers from Nigeria and Ghana living in Canada. We sought to identify storylines from their lived experience, and positioning of the teachers, their cultural knowledge, and mathematics, using African Indigenous Knowledges (AIK) as a theoretical lens. We interrogated how AIK could help us re(imagine) different positionings of mathematics teachers.

While we found some of the mathematics teaching experiences of these immigrant teachers similar to what others have reported in the literature, others were relatively unique, informed by AIK. In discussion about community decision making, the teachers described how it is common to say one will consult with the "old woman." In Ghanaian folklore, the old woman is a revered and a complex figure who symbolises wisdom, and spirituality. The old woman's responsibility is to provide wise counsel and spiritual guidance to kings and elders who consult her.

One immigrant teacher asked, "who is the old woman in the mathematics classroom?" A lively discussion followed, envisioning classroom discourse through this AIK concept/tradition. One teacher asked "if the teacher becomes the old woman (...) what happens if she makes mistakes (…). Some teachers if they don't know they will consult colleagues and others may not." Others noted that teachers normally *initiate* communication, which is unlike the old woman whom people come to: "the old woman could be anyone outside the mathematics classroom including parents" or "students and peers". The conversation pointed to the importance of who is consulting whom, and which questions are being consulted on.

AIK emphasizes the importance of community and collective knowledge construction (Dei, 2020). Hunter & Civil (2021) described sophisticated collaborations in and out of mathematics classroom such as peers working collectively and parents' shared responsibility for children's success in mathematics. The concept of the old woman embodies community knowledge construction.

References

Hunter, R., & Civil, M. (2021). Collaboration in mathematics: Taking a sociocultural perspective. *AIEM - Avances de Investigación en Educación Matemática*, *19*, 7–20.

Dei, G. J. S. (2020). Elders' cultural knowledges and African indigeneity. In: Abidogun, J., & Falola, T. (Eds.) *The Palgrave handbook of African education and Indigenous knowledge*, (pp. 279–301*)*. Palgrave Macmillan, Cham.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 341. PME.

EXPLORING TEACHER PREPAREDNESS FOR IMPLEMENTING INQUIRY-BASED METHODS IN MATHEMATICS CLASSROOMS

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Inquiry-based learning (IBL) promotes student agency by shifting the teacher's role from a knowledge transmitter to a learning facilitator, highlighting its pedagogical benefits in undergraduate mathematics (Laursen & Rasmussen, 2019). This approach aligns with reform-oriented standards like the Common Core State Standards (CCSS), which emphasize reasoning, sense-making, and perseverance in problem-solving.

A key concern is that teacher education programs often lack authentic, inquiry-oriented mathematical experiences for preservice teachers. Consequently, many revert to the traditional, didactic methods they experienced as students, leading to a disconnect between modern standards and classroom practices.

Grounded in Vygotsky's social constructivist theory—which views learning as a social process shaped by dialogue and scaffolding—this study emphasizes the value of collaborative environments that promote questioning, investigation, and active construction of mathematical understanding. IBL supports these practices, fostering modeling, analysis, problem-solving, and justification, all essential for conceptual depth (Vidic, 2023).

To explore how IBL is modeled in college mathematics courses for preservice teachers, two courses were observed: one on mathematical proofs and another in second-level calculus. Though both had content suitable for inquiry—like proving statements or integrating functions—student exploration was often overlooked. Instructors prioritized content coverage, guiding students through problems instead of encouraging discovery.

These observations reveal a critical gap: without consistent modeling of IBL throughout the mathematics curriculum, preservice teachers are unlikely to adopt it in their own classrooms. They risk continuing lecture-based instruction that limits student voice and deeper learning. For IBL to take root in K–12 education, it must be integrated across all teacher preparation content areas—not just methods courses.

References

Laursen, S. L., & Rasmussen, C. (2019). I on the prize: Inquiry approaches in undergraduate mathematics. *Int. Journal of Research in Undergraduate Mathematics Education, 5,* 129–146.

Vidic, D. A. (2023). Comparison of interdisciplinary connections between mathematics and other subjects through student-centered approaches. *Journal of Research in Mathematics Education, 12*, 29–55.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 342. PME.

DEVELOPING THE MATHEMATICS REGISTER THROUGH COLLABORATIVE PROBLEM-SOLVING

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In mathematics classrooms, developing the mathematics register is crucial for students' mathematical learning. According to Halliday (1978), the mathematics register comprises terminology, vocabulary, symbols, and structures expressed meaningfully. A well-developed register supports precise reasoning and communication, which can deepen students' mathematical understanding. Research has examined students' transition from everyday language to mathematical discourse. However, more needs to be known about how students develop the mathematics register (Sigley & Wilkinson, 2015). This study shows that collaboration can influence this process based on an analysis of the mathematical activity of two pairs of students who worked collaboratively on a combinatorial task in an afterschool research project that investigated how students reason and build mathematical ideas while working on challenging tasks. The combinatorial task asked the students to determine all possible outfits given a set number of shirts and pants. Analysis of the students' mathematical activity using Powell et al. (2003) video methodology for studying reasoning revealed three ways in which collaboration supported the development of students' mathematics register. First, classroom discourse enhanced students' co-construction of the register through dialogue, helping them transition from using everyday language (e.g., referring to outfits by color) to using formal mathematical terms (e.g. combinations). Second, teacher scaffolding through questioning and revoicing (e.g., "Show me an outfit on your paper") guided the students toward understanding combinatorial concepts as they went from describing outfits as "clothes" to using combinatorial terminology. Third, multiple representations also helped bridge informal and formal language. The students used symbolic notations such as lines to visually represent the concept of combination and organize their thinking. The results reveal that developing a mathematics register is a complex process involving discourse, scaffolding, and multimodal representations. Details of these and other findings will be shared in the presentation.

References

Halliday, M. A. K. (1978). *Language as social semiotic*. London: Edward Arnold.

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *The Journal of Mathematical Behavior*, *22*(4), 405-435.

Sigley, R. & Wilkinson, L.C. (2015). Ariel's cycles of problem solving: An adolescent acquires the mathematics register, *The Journal of Mathematical Behavior*, *40*, 75-87.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 343. PME.

CURRICULAR NOTICING: RELATIONSHIPS BETWEEN IDENTIFYING AND INTERPRETING SKILLS

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Curricular noticing is conceptualized as the way teachers make sense of the complexity of content and pedagogical opportunities in curricular materials, involving: curricular attending (viewing information contained in curricular materials to inform teaching), curricular interpreting (connecting what they have attended to their own mathematical and pedagogical content knowledge for teaching), and curricular responding (making curricular decisions based on the interpretation) (Dietiker et al., 2018). As other studies have shown that students' hypothetical learning trajectories (HLTs) are potential tools to support pre-service teachers' noticing of students' thinking (e.g., Ivars et al., 2020), we hypothesize that HLTs could focus pre-service teachers' attention on recognizing opportunities and limitations of the curricular materials (e.g., textbooks). Our study explores how 171 pre-service primary school teachers (PTs), using a students' HLT, identify and interpret characteristics of a sequence of activities from a textbook about the part-whole meaning of the fraction concept (particularly in three aspects: mathematical elements, types of activity, and modes of representation). A qualitative analysis was performed aimed at looking for similar patterns of curricular attending and interpreting. Five groups of PTs were identified, showing features of the relationships among both skills. While some PTs showed that identifying important aspects supported their interpretation, others PTs interpreted less characteristics than what they had identified, and others were able to interpret without having identified any characteristic. Although the HLT seems to have supported some PTs' curricular noticing, others PTs had difficulties when identifying and/or interpreting. These difficulties highlight the need to develop the curricular noticing skill in teacher training programs.

Additional information

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References

Dietiker, L., Males, L. M., Amador, J. M., & Earnest, D. (2018). Curricular noticing: A framework to describe teachers' interactions with curriculum materials. *Journal for Research in Mathematics Education*, *49*(5), 521-532.

Ivars, P., Fernández, C., & Llinares, S. (2020). A learning trajectory as a scaffold for pre–service teachers' noticing of students' mathematical understanding. *International Journal of Science and Mathematics Education*, *18*(3), 529-548.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 344. PME.

MAKING SURE THAT MATHEMATICS EDUCATION
RESEARCH REACHES THE CLASSROOM

POSTER PRESENTATIONS

CONSIDERING MATHEMATICS EDUCATION IN RURAL CONTEXTS

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OVERVIEW

Mathematics education in rural schools presents a complex and often overlooked landscape, requiring targeted research and policy interventions (Banilower et al., 2018). With many schools internationally classified as rural, there is considerable variability in demographics, resources, and student outcomes. Rural students tend to have lower graduation rates in many contexts, and they often face persistent inequities, particularly among historically marginalized populations. Similar disparities are found in many world geographies (Echazarra & Radinger, 2019). Limited resources, geographic isolation, and technological challenges further complicate the delivery of high-quality mathematics education. As a result, researchers emphasize the need for innovative, context-specific strategies to address these challenges and support mathematics educators and students in rural settings.

Opportunity gaps in rural mathematics education further highlight the need for targeted research and teacher support (Banilower et al., 2018). Rural students are significantly less likely than their suburban peers to complete foundational mathematics courses by the end of primary education, limiting their mathematics progression in secondary education. Rural mathematics teachers often report feeling underprepared in key content areas and have fewer professional development opportunities. Extant research has shed light on these disparities, but more research is needed to understand the specific factors that enable or constrain ambitious mathematics teaching (Lampert et al., 2010). Mathematics educators can work toward equitable access to high-quality instruction by prioritizing research and professional development in rural contexts. The purpose of the poster is to share progress from two projects, both of which research and support mathematics teachers in rural locations. The poster will describe the opportunities and challenges in rural contexts, specifically emphasizing mathematics-related aspects.

References

Banilower, E. R., Smith, P. S., Malzahn, K. A., Plumley, C. L., Gordon, E. M., & Hayes, M. L. (2018). *Report of the 2018 NSSME+*. Horizon Research, Inc.

Echazarra, A., & Radinger, T. (2019). Learning in rural schools: Insights from PISA, TALIS and the literature. *OECD Education Working Papers*, (196), 0_1-77.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. *Instructional explanations in the disciplines*, 129-141.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 347. PME.

RESEARCH INSIGHTS ON PROSPECTIVE TEACHER TRAINING ON MODELLING

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Teaching mathematical modelling requires specific training for teachers being effective to implement it in the classroom (Borromeo Ferri & Blum, 2010). Prospective teachers, in particular, need opportunities to both develop their own modelling skills and learn how to develop this mathematical competency in their future students (Wickstrom & Arnold, 2023). In recent years there have been advances in relation to the tools and strategies to fostering and evaluating modelling, obtained by researchers in a variety of contexts. Nevertheless, most of these investigations put no specific emphasis on pre-service teachers, persisting a gap in this line (Hidayat, 2024).

This study aims to identify, analyze and categorize the advances in the recent research related to the development of modelling in future teachers, its evaluation and measurement. A systematic literature review in WoS and Scopus databases was conducted, to date of December 31, 2024, following the guidelines corresponding to PRISMA 2020.

Researches in the last decade present a diversity of evaluation strategies and instruments. The main strategy to train future teachers in the development of this skill consists in the design of tasks and their subsequent implementation. As for assessment, most studies focus on small samples and qualitative evaluations.

Additional information

Acknowledgements. Project PID2022-139007NB-I00, by MCIN/AEI/10.13039/ 501100011033/FEDER, EU.

References

Borromeo Ferri, R., & Blum, W. (2010). Mathematical modelling in teacher education – Experiences from a modelling seminar. In *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (pp. 2046–2055) Lyon, France. www.inrp.fr/editions/cerme6

Hidayat, R. (2024). Validity Of Stem-Based Modelling Instrument For Pre-Service Teachers Of Mathematics Education. *Journal of Institutional Research South East Asia*, (Vol. 1, Special Issue, pp. 154-178). Scopus.

Wickstrom, M. H., & Arnold, E. G. (2023). Investigating secondary pre-service teachers as teachers and learners of mathematical modeling. *Teaching Mathematics And Its Applications*, (Vol. 42 (1), pp. 86-107). https://doi.org/10.1093/teamat/hrab031

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 348. PME.

DIAGNOSING MATHEMATICAL SKILLS: CHALLENGES IN NUMBER LINE ESTIMATION TASKS

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Michael Gaidoschik[3], Michael Kleine[4] and Eva Lasnitzer[5]

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The Erasmus+ project DiToM develops diagnostic tests for assessing mathematical key skills. Trials show high failure rates in Number Line Estimation Tasks, especially with rational numbers. Italian and German results are analysed using a semiotic lens.

The DiToM project (Diagnostic Tool in Mathematics) is an Erasmus+ funded project that addresses the need for national and international diagnostic instruments to assess mathematical key skills (Brings et al., accepted) in arithmetic and algebra. Tests for five levels (Grades 0, 2, 4, 6 and 8) are developed, each of them addressed to students attending the end of the respective grade or the beginning of the next one. The trials show that in all seven involved countries, Number Line Estimation Tasks (NLETs) (e.g. Gaidoschik, 2024) have high failure rates (e.g., between 69.3% and 83.4% when positive rational numbers are involved in the Grade 6 test and 94.9% when positive and negative rational numbers are involved in the Grade 8 test). First results from Italy and Germany confirm what is known from literature: students' difficulties with NLETs are particularly evident when aspects of measurement and decimal system understanding are required (Gaidoschik, 2024), but they also show how basic properties of natural numbers – e.g., each natural number has an successor – seem to be implicitly extended by students to rational numbers preventing them from adopting a geometrical perspective which allows considering numbers as segments rather then only symbolic representations. Adapting the SIK (Semiotic Interpretative Knowledge) framework (Asenova et al., 2023), based on the semiotic functions of choice of distinctive features, treatments and conversions, according to Duval's semiotic perspective, we analyse students' errors in the NLETs through different grades.

References

Asenova, M., Del Zozzo, A., & Santi, G. (2023). From Interpretative Knowledge to Semiotic Interpretative Knowledge in prospective teachers' feedback to students' solutions. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, M. Tabach (Eds.), *Proceedings of the 46th of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 51-58).

Brings, L., Kleine, M., Coronado Alvarez S., Pilet, J., Rukavina, S., Tatsis, C. Fesakis, G., Moutsios-Rentzos, A., Asenova, M., & Drumčić Danilović, D. (accepted). Development of screening instruments for mathematics education: Theoretical Framework and item development. *Proceedings of CERME 14*.

Gaidoschik, M. (2024). Number line estimation tasks: Ein fachdidaktischer Blick auf einen in den kognitiven Neurowissenschaften prominenten Aufgabentyp. *Journal für Mathematik Didaktik, 45*(1). https://doi.org/10.1007/s13138-023-00229-x

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 349. PME.

NUMBER TALKS TO IMPROVE NUMERACY CONFIDENCE AND COMPETENCE WITH STUDENTS IN GRADES 4–8

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This poster presents a collaborative research project involving three school divisions in Alberta, Canada, and researchers at the University of Lethbridge. The project's focus was to examine whether the integration of Number Talks (Humphreys & Parker, 2015) in middle school mathematics classrooms would enhance student competence and confidence in numeracy

By targeting grades 4–8, it was hoped that Number Talks would assist students in developing the numeracy skills necessary for success in high school mathematics. Research clearly shows that possessing strong numeracy skills positively correlates with success in higher-level mathematics; however, much of this research has been conducted at the preschool and primary levels (e.g., Fuson et al., 2015). By concentrating on numeracy development in middle school grades, we aim to enhance student competency and confidence while expanding upon prior research regarding Number Talks. The main research question was: Does the implementation of Number Talks in grades four to eight mathematics classrooms increase student confidence and improve competence in numeracy? We were also interested in teachers' and students' perceptions of the impact of Number Talks on student confidence and competence numeracy.

In response to the main research question, we found that the implementation of Number Talks indeed increased student confidence and competence in numeracy. Teachers also perceived that Number Talks enhanced both student competence and confidence in numeracy. In particular, it was noted that students sharing their thoughts had been beneficial for all students. As one teacher expressed: "I have a higher percentage of kids ready to defend their answers, take risks, and work through a problem even if they're not quite sure how they got there." Students described feeling that the use of Number Talks had helped them become better at math and more confident in their thinking.

References

Fuson, K., Sarama, J., & Clements, D. (2015). Making early math education work for all children. *Phi Delta Kappan*, *97*(3), 63–68.

Humphreys, C., & Parker, R. (2015). *Making number talks matter: Developing mathematical practices and deepening understanding, Grades 3–10*. Stenhouse Publishers.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 350. PME.

COLLABORATIVE SENSE-MAKING FOR SOCIAL JUSTICE

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Mathematics teachers who privilege critical perspectives in their pedagogical decision-making empower themselves and their students to take action for a more just society (Planas & Civil, 2009). This inquiry brings together two conceptually similar PhD studies, each with distinct methodologies and contexts, that aim to investigate how teachers can individually and collectively challenge the dominant discourse of mathematics through their agency and decision-making. Critical perspectives in mathematics education, despite epistemic plurality, are unified by shared concerns, such as critiquing power structures and recognizing patterns of injustice. The researchers, both teachers, have had markedly different life experiences, yet share similar concerns with respect to knowledge diversity and human rights. In this collaborative inquiry we ask: How are individual and collective decisions made among two critical teacher-researchers similar or different from each other? What insights emerge from connecting our perspectives through co-constructed sense- making?

One PhD study uses narrative inquiry to explore critical teachers' lived experiences through conversational interviews, revealing how they critiqued the status quo and changed their actions, offering insights into their decision-making. The other PhD study examines the culturally responsive collaborative decision-making of Inuit and non-Inuit teachers, grounded in Inuit perspectives, to design mathematics pedagogy in the Canadian Arctic. While one study uncovers personal critique and action, the other emphasizes collaboration's role in transforming pedagogy. With its relational approach, "making sense as research methodology" (Nolan & Tupper, 2013) helps us understand teachers' decision-making at both individual and collective levels. The researchers will visualize their co-constructed meaning making from ongoing dialogue over two years. They will share tensions they experienced while persisting in keeping hope alive as they challenge the status quo to promote justice-oriented teaching together.

References

Nolan, K., & Tupper, J. (2013). Field" trips with Bourdieu: Making sense as research methodology in teacher education. *International Journal of Humanities and Social Science, 6*(3), 11-19.

Planas, N., & Civil, M. (2009). Working with mathematics teachers and immigrant students: an empowerment perspective. *Journal of Mathematics Teacher Education, 12*(6), 391–409. https://doi.org/10.1007/s10857-009-9116-1

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 351. PME.

STATIC AND DYNAMIC TEXT – DIFFERENT OPPORTUNITIES TO EXPLORE LOGICAL EXPANSIONS

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All texts are characterized by cohesive relations (see e.g., Halliday & Matthiessen, 2014). In mathematical texts a facet of cohesion is networks of information between the various semiotic resources. A network is anchored by a central aspect introducing a theme relevant throughout an entire section of text. This anchoring expression is followed by different types of expansions of a concept, creating a cohesive network throughout the text. In this study, we focus on *logical* expansions as they express mathematical connections, and interpreting those are central to understanding mathematics. A logical expansion is expressed when a previously introduced central aspect is justified, by a reason, condition, or comparison in another instance. The theoretical understanding of expansions has been developed from van Leeuwen's (2005) work. When dynamic functions are introduced, new ways of representation are also introduced, which means that new ways to express logical expansions in mathematics may appear. On this basis, our aim is to explore how meaning is offered to students via logical expansions in static and dynamic texts.

The texts analysed in this study are the theory parts of five items developed in a larger research project, specifically designed to introduce new mathematics to Swedish ninth-grade students. For each item, there are two versions of the theory part: one static version like a PDF and one dynamic and interactive version. The analysis comprised all central aspects of a particular mathematical concept and the logical expansions between these aspects were identified. In the dynamic version, we detected all expansions that could be offered dynamically depending on the readers actions.

The result reveals that in the static text, logical expansions are less common, but on the other hand, directly accessible to the reader. In the dynamic/interactive media it is inherent that often, logical expansions are only made explicit when some actions are taken. When the visual appearance change, several different logical expansions can occur. However, the reader *must* choose to make the action and thereafter grasp how the instances are related. This means that logical expansions are offered implicitly in dynamic media, placing demands on the reader. At a fist glance it may seem that dynamic features are solely beneficial due to all the possibilities for expressing logical expansions, but it is important to remember that active interaction is required to access these expansions.

References

Halliday, M A K., & Matthiessen, C. M. (2014). *An introduction to functional grammar* (4 ed.). Arnold. https://doi.org/10.4324/9780203431269

van Leeuwen, T. (2005). *Introducing social semiotics*. Routledge.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 352. PME.

TIMELINE ANALYSIS OF STUDENT READING – EXPLORING EYE TRACKING DATA

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Mathematics text is multisemiotic and reading such texts is typically not linear due to the interrelation between the resources natural language, symbols and images (O'Halloran, 2015). The current research explores timeline analysis of student reading as a method to investigate how students connect different resources in the mathematical text, and thus which information is gained. The focus is laid on reading central aspects of mathematical concepts, expressed in different semiotic resources, and how the meaning is expanded throughout the text (e.g. see, van Leeuwen 2005).

Using eye tracking, we can observe in real time how students read. In contrast to text containing only natural language, it is reasonable to assume that mathematical multimodal texts are read in a nonlinear manner. By applying a structured timeline analysis that distinguishes central aspects in the text, it is possible to scrutinize how the readers' choices contribute to making different messages available. To enable the analysis, the texts were first coded regarding how the information about central aspects was expanded throughout the text. Based on eye tracking data and the coding of central aspects in the text, the reading paths were thoroughly transferred to a timeline scheme. To test the aptness of this method we analyse the reading of two students with different reading approaches. The timeline scheme is designed to enable a detailed analysis of the reading and thereby to understand the kind of information that is made available depending on the chosen reading path.

The analysis of case A highlights that the reader skips instances of central aspects that introduce basic facts and rather focus on instances with expanded meaning directly. The analysis of case B reveals a more deliberate reading following the information path of the central aspects, first the introduction and thereafter the expansions of the central aspects, ultimately leading to a coherent message that conveys the core of the text. A conclusion is that the analysis model is an apt tool to describe and understand crucial choices made by readers, as well as the consequences of these choices.

References

O'Halloran, K. L. (2015). Mathematics as multimodal semiosis. In E. Davis & P. J. Davis (Eds.), *Mathematics, Substance, and Surmise: Views on the Meaning and Ontology of Mathematics.* (pp. 287–303). Springer.

van Leeuwen, T. (2005). *Introducing social semiotics.* Routledge. https://doi.org/10.4324/9780203647028

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 353. PME.

PRE-SERVICE TEACHERS' NON-ROUTINE PROBLEM-SOLVING PROCESSES: AN EYE-TRACKING STUDY

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Eye-tracking technology provides an in-depth analysis of mathematical problem-solving processes and enhances our understanding of the cognitive mechanisms involved (Paskovske & Kliziene, 2024). The nature of the problem plays a critical role in interpreting these processes. Unlike routine problems that follow standard procedures, non-routine problems require higher-order thinking such as organizing data, recognizing patterns, and logical reasoning (Souviney, 1989). Understanding pre-service teachers' non-routine problem-solving processes is critical for improving their problem-solving skills and cognitive processes. In this context, this study aims to examine the eye movements of pre-service teachers from different programs in non-routine problem-solving through various metrics obtained with the eye-tracking tool. Pre-service teachers from various teaching programs will participate in the study, utilizing the Tobii X2-60 eye-tracking device. The expected outcomes of the study are anticipated to reveal significant differences in eye movement patterns between mathematics and elementary pre-service teachers. It is expected that mathematics teacher candidates will exhibit longer fixations and more structured scanpaths on key data regions, indicating more analytical problem-solving strategies. In contrast, elementary teacher candidates may demonstrate more scattered fixations and frequent regressions, reflecting a more intuitive approach. Additionally, differences in total time spent and cognitive effort are also expected. These findings will provide insight into how professional background influences visual attention and problem-solving strategies in non-routine mathematics tasks. This research will be conducted in April 2025, and the findings will be presented at the related conference.

References

Souviney, R. J. (1989). *Learning to teach Mathematics*. Merrill Publishing Company.

Paskovske, A., & Kliziene, I. (2024). Eye tracking technology on children's mathematical education: systematic review. *Frontiers in Education, 9*, Article 1386487.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 354. PME.

STRATEGIES FOR STUDENT-FACULTY PARTNERSHIPS IN CRITICAL TRANSFORMATIONS IN MATHEMATICS

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To challenge ingrained cultural practices of mathematics departments, Callahan et al. (2020) promote involving students as partners in undergraduate STEM department reform. However, limited research exists on the use of students as partners in mathematics department change efforts (e.g., Reinholz et al., 2021; Bolick et al., in press). We expand current research by examining the ways that faculty and students in a networked improvement community (NIC) fostered a collaborative environment underscored by respect, reciprocity, and responsibility (Cook-Sather et al., 2014). Our research question is: *What structures and practices were leveraged by the NIC to support students as partners in critical change efforts?*

As a part of a larger study (NSF ECR# 2201486), we discuss a NIC from Alpha University, a medium-sized, doctorate-granting, racially diverse institution on the west coast of the United States. The NIC ran for four semesters and in the final semester, consisted of five students or former students and three returning faculty members. A longitudinal analysis identified key structures and practices as being influential in forming student-faculty partnerships. Effective structures included a neutral meeting space, relationship building tasks (e.g., reading norms, icebreakers), anonymous spaces for sharing of ideas (e.g., Post-It notes, Google Forms), and shared leadership through rotating facilitation and small group settings. Faculty practices involved thoughtful responsiveness, intentional stepping back, and maintaining distinct student relationships across contexts. Our findings highlight the importance of intentionally designing structures that mitigate power differentials to promote respect, reciprocity, and shared responsibility among faculty and student partners.

References

Bolick, M.A., Pai, L., Funk, R., & Voigt, M. (in press). It's hard to ignore the data when the data is in the room: Examining the role of students as partners in critically-oriented reform of tertiary mathematics. *International Journal for Students as Partners*.

Callahan, K. M., Williams, K., & Reese, S. (2020). *Student leaders as agents of change*. Transforming Institutions: Accelerating Systemic Change in Higher Education.

Cook-Sather, A., Bovill, C., & Felten, P. (2014). *Engaging students as partners in learning and teaching: A guide for faculty*. John Wiley & Sons, Inc.

Reinholz, D. L., White, I., & Andrews, T. (2021). Change theory in STEM higher education: A systematic review. *International Journal of STEM Education, 8*(1), 37. https://doi.org/10.1186/s40594-021-00291-2

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 355. PME.

INVESTIGATING PROSPECTIVE TEACHERS' SKILLS THROUGH THE OPEN CLASS OF TLSOA

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Lesson Study (LS) has gained global recognition for its effectiveness in teacher professional development, particularly following The Teaching Gap (Stigler & Hiebert, 1999). In Thailand, Inprasitha (2015; 2022) adapted LS into the Transformative Lesson Study incorporated with Open Approach (TLSOA), integrating it into teacher education at Mathematics Education Program, Khon Kaen University. TLSOA emphasizes developing core teaching competencies: designing problem situations, understanding students' thinking, anticipating student responses, observing student thinking, and reflecting through the Open Class (Inprasitha, 2024). This study investigated the development of TLSOA-related competencies among 103 prospective mathematics teachers during a one-year teaching practicum. Data were collected at three time points — prior to the practicum, after the first semester, and after the second semester — using self-assessment questionnaires on a 10-point scale and open-ended reflection prompts. Quantitative analysis indicated progressive improvement across all five competencies. Qualitative analysis further revealed an increasingly sophisticated understanding of the competencies, particularly regarding observation and reflection. Specifically, 33% of participants reported significant growth in their observation skills, while 32% identified reflection as an area for further development. Moreover, 33% expressed a need to continue improving their observation skills during their initial three years of beginning teachers. These findings suggest that the structured implementation of TLSOA during the internship period meaningfully supports the cultivation of key competencies necessary for effective classroom teaching and contributes to the development of skills related to structuring the flow of lessons or task sequences.

References

Inprasitha, M. (2015). "Prospective teacher education in mathematics through Lesson Study", in Inprasitha, M., Isoda, M., Wang-Iverson, P. and Yeap, B.H. (Eds), Lesson Study: Challenges in Mathematics Education, World Scientific, Singapore, pp. 185-196.

Inprasitha, M. (2022). Lesson study and open approach development in Thailand: A longitudinal study. International Journal for Lesson & Learning Studies, 11(5), 1–15. https://doi.org/10.1108/IJLLS-04-2021-0029

Inprasitha, M. (2024). Basic Skills Set. Paper presented in A Capacity-Building Workshop for Expert Assessors on Evaluating Higher-Order Thinking Skills in Open Class: Emphasis on Students' Ideas on April 2024.

Stigler, J. and Hiebert, J. (1999), The Teaching Gap, Free Press, New York, NY.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 356. PME.

CREATING AND JUSTIFYING ALGEBRAIC EQUALITIES AND INEQUALITIES WITH PRIMARY EDUCATION STUDENTS

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Research on early algebra highlights its role in enriching mathematical activity, facilitating the transition from arithmetic to algebra and promoting relational thinking (Carraher and Schliemann, 2018). Although the equal sign is often considered only as operational in primary school (Unteregee, 2022) and students show limited understanding of the concepts "equal," "more" or "less" (Warren, 2008), children may come to adopt a relational view of equalities (Nührenbörger & Schwarzkopf, 2015).

In our research, we conducted an experience with fifth-grade primary education students (ages 10-11), having their first encounter with early algebra, in which they were asked to recognize when an algebraic equality is true and to generate valid algebraic identities or inequalities from algebraic expressions. The results of our work show that students did not have difficulty successfully solving the first task, as well as justifying their assessment (always true or not) of the identity validity. However, they found the second task more complex. Moreover, although we anticipated greater difficulty with order relations than with equality relations, as deciding what the relationship is (equal to, less than, greater than) involves comparing the values of the expressions for different values of the literal symbol, this was not the case.

Additional information

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References

Carraher, D., & Schliemann, A. (2018). Cultivating early algebraic thinking. In C. Kieran (Ed.), *Teaching and learning algebraic thinking with 5- to 12-year-olds: The global evolution of an emerging field of research and practice* (pp. 107-138). Springer.

Nührenbörger, M., & Schwarzkopf, R. (2015). Processes of mathematical reasoning of equations in primary mathematics lessons. In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 316–323). ERME

Unteregge, S. (2022). Primary school children's justifications of equalities. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education* (pp. 638-645). ERME.

Warren, E. (2006). Comparative mathematical language in the elementary school: A longitudinal study. *Educational Studies in Mathematics, 62*(2), 169–189.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 357. PME.

UNDERSTANDING COVARIATION FROM AN APPROACH WITH MULTIPLE REPRESENTATIONS PROVIDED BY DIGITAL TECHNOLOGIES AND THAT CONSIDERS ARITHMETIC AND ALGEBRA BY 6TH GRADE STUDENTS

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Covariation involves understanding situations in which two variables vary together. For example, to analyze the best purchase to make between two boxes of laundry detergent, it is necessary to compare the price per pound of each of the products. The proportional calculation with the sense of covariation can be observed in multiplicative situations with quaternary relations, called proportion - simple, double and multiple (Omited, 2016). In these situations, it is possible to perform two types of analysis: a horizontal one, corresponding to the functional operator that relates two details of different nature; and a vertical one, associated with the scalar operator that operates with quantities of the same magnitude (Omited, 2016; Vergnaud, 1988).

This study aims to verify the understanding of covariation by 6th grade elementary school students, through strategies developed in diagnostic assessments after intervention with 12 students. Due to the educational level of the research subjects, an approach was chosen that considered arithmetic, but also some elements of algebra. This methodological option was also chosen because it allows students to focus on the search for generalizations, establishing relationships, regularities and patterns, rather than algebraic symbolism. In addition, multiple representations were explored through digital technologies (Omited, 2016).

The analyses regarding knowledge of covariation were carried out considering: (1) performance in situations of proportions; (2) performance in understanding quantities; (3) performance in tabular and graphical representation; and (4) general performance, with all aspects analyzed in the intervention. The results show that the students, although they were unable to understand a set of data initially (pre-test), after the intervention developed schemes that enabled the understanding of invariance and covariation. The ability to transpose one representation to another (tables into graphs, verbal language into diagrams, iconic representation into symbols, for example) brought greater meaning to the understanding of the concepts studied.

References

Omited. (2016).

Vergnaud, G. (1988). Multiplicative structures. In: Hiebert, H.; Behr, M. *Research Agenda in Mathematics Education*: *number concepts and operations in the Middle Grades*. Hillsdale, NJ: Lawrence Erlbaum, pp. 141-161.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 358. PME.

DEVELOPING LOGICAL THINKING IN EARLY MATHEMATICS THROUGH COLOURING ACTIVITY

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Logical thinking is crucial for developing algorithms, which are fundamental to computational and mathematical thinking. Introducing quantifiers in the colouring activities is a practical application of logical thinking. Students are tasked with proposing instructions using these logic types, reinforcing their understanding of how logical statements can be formulated and interpreted (Araya & Isoda, 2023). This study aimed to 1) explore the effects of colouring activities on students' interpretations of quantifiers and 2) cultivate students' abilities to create pseudocode instructions. The target group comprised 40 fourth-grade students. The research design was structured around the cycles of the TLSOA model; collaboratively plan the flow of the lesson, the fish tank colouring task was designed based on a problem situation including context and conditions; collaboratively do with four steps of Open Approach, and collaboratively see (Inprasitha, 2022). Data were collected from the students' worksheets, classroom observation, and reflections. Data analysis employed descriptive analytics and content analysis.

The results revealed a spectrum of interpretations regarding the instruction, "In each fish tank paint the tails of at least two fish." Specifically, a divergence emerged between students who adhered strictly to the minimum requirement of two fish and those who exceeded it. By facilitating small group discussions, students actively explored diverse interpretations. Following this, students were tasked with designing their own instructions related to the fish tank worksheet. The results indicated that 45% of the students incorporated the quantifiers "least" and "at least," while 30% employed "the most." Furthermore, 25% of the students utilized "all," alongside other quantifiers. Notably, 30% of the students' instruction designs were influenced by aesthetic considerations related to colouring. This study demonstrated that engaging fourth-grade students in colouring activities with quantifiers enhances their interpretation skills and fosters the ability to create pseudocode instructions, reinforcing foundational concepts in logical thinking and collaborative learning.

References

Araya, R. & Isoda, M. (2023). Unplugged Computational Thinking with Colouring Book. In M. Isoda, N. Nakao, J.R. Siena & C. Nonthachai (Eds.). *Journal of Southeast Asian Education, Vol. 1, Special Issues, pp. 72-91.*

Inprasitha, M. (2022). Lesson Study and Open Approach development in Thailand: a longitudinal study. *International Journal for Lesson and Learning Studies*, Vol. 11 No.5, pp.1-15. https://doi.org/10.1108/IJLLS-04-2021-0029

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 359. PME.

USE OF PRIME FACTOR COLOR CARD GAME TO PROMOTE STUDENTS' ACADEMIC, SOCIAL, AND EMOTIONAL LEARNING IN MATHEMATIC

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In recent years, significant achievements and contributions have been made by scholars and institutions on how to enhance students' academic performance and positive emotional responses in the classroom (CASEL, 2022; Hannula, 2019; Hill & Seah, 2022). These abilities required to promote mathematics learning in the classroom refer to students demonstrating their best function and potential in mathematics classrooms through healthy social relationships and positive coping with stress. By integrating the "prime factor color card" with ASE (academic, social, and emotional) learning core competencies into mathematics teaching, this research examines the impact on students' performance in multiple aspects. This research aims to explore whether there is a difference in students' academic performance in learning factors, as well as the performance of positive behaviour in mathematics learning and their social and interpersonal relationships, self-awareness, and management learning. Two classes of fifth-grade elementary students participated as experimental and control groups for a teaching experiment lasting six periods. Data on students' learning performance were collected and analyzed mixed methods. The results indicated that students in the experimental group showed significant improvements in the factor concept tests, especially in finding factors and in solving application problems. In terms of positive behaviour in mathematics learning, students in the experimental group showed significant progress in the subtests of self-control and management and the test on interpersonal relationships and interactions. In summary, this card games enhanced students' understanding of factors and increased multiple measures on students' outcomes in mathematics learning.

References

Collaborative for Academic, Social, and Emotional Learning (2022, CASEL). n.d. “Fundamentals of CASEL.” https://caCASEL.org/fundamentals-of-CASEL/.Accessed March 2, 2022.

Hannula, M. S. (2019). Young learners’ mathematics-related affect: A commentary on concepts, methods, and developmental trends. *Educational Studies in Mathematics, 100*(3), 309–316.

Hill, J. L., & Seah, W. T. (2022). Student values and wellbeing in mathematics education: perspectives of Chinese primary students. *ZDM—Mathematics Education*, *55*, 385-398. https://doi.org/ 10. 1007/ s11858- 022- 01418-7

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 360. PME.

CHARACTERISTICS OF ARGUMENTS IN PRESCHOOL MATHEMATICS

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This presentation aims to describe the arguments made by kindergarten children throughout a school year while arguing in mathematics in the context of a study on the development of argumentation in kindergarten children (Cornejo-Morales et al., 2021).

Argumentation is defined as a communicative and situated activity through which children provide reasons (for others or for themselves) to justify and convince (or become convinced) of a certain position or reflexively question it. In the early ages, arguments can be considered as a product of the cognitive and emotional activity of children, as well as their imitative behavior. These arguments correspond to the oral, gestural, pictorial, and written productions that children use to justify or question, as well as to the subsequent linguistic reconstructions through which the argumentative aspect of such productions is synthetically expressed by an observer. Methodologically, an argument has two key elements: the position adopted and the reasons that support that position.

Twenty-two preschool lessons were video-recorded over a school year to capture students' number construction up to 20, allowing for analysis of arguments and their development. This involved reconstructing central arguments and characterizing them.

We identified 100 arguments constructed over the course of the year. These arguments comprised 120 positions and 115 reasons, indicating that some arguments involved multiple positions or reasons. In the case of multiple positions or reasons (up to three of each), refutations were also analyzed. Results show that the number of arguments, positions, and reasons increased throughout the school year. Positions were typically implicit or brief, averaging four words, with "Yes", "No" and numbers being the most common. Reasons averaged eight words and, while initially consisting of numerical sequences, later in the year they demonstrated causal reasoning, employing words like "because", "first", and "after." While 49 arguments had positions and reasons formulated by one student, 51 arguments involved multiple students, highlighting the presence of collective argumentation within the preschool mathematics classroom.

Additional information

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References

Cornejo Morales, C., Goizueta, M., & Alsina, Á. (2021). La Situación Argumentativa: un modelo para analizar la argumentación en educación matemática infantil. *PNA: Revista de Investigación en Didáctica de la Matemática, 2021, vol. 15, núm. 3, 159-185*.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 361. PME.

PRE-SERVICE TEACHERS' EVALUATION OF STUDENTS' INCORRECT REASONING IN FRACTION COMPARISON TASKS

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Teachers' ability to identify misconceptions in students' mathematical thinking is important (e.g., Brodie, 2013). By accurately identifying misconceptions, teachers can adjust their instruction to address them effectively. Fraction magnitude is a common area of students' misconceptions (e.g., Gómez & Dartnell, 2019), so teachers should be able to identify and correct these.

This pilot study examined how 20 German pre-service mathematics teachers (PMTs) evaluated three fictional students' responses in two fraction comparison tasks (Vignette 1: 3/5 vs. 4/9; Vignette 2: 4/7 vs. 1/3). The responses in both vignettes reflected three forms of incorrect reasoning: (1) natural number bias (i.e., the larger the numerator and denominator, the larger the fraction), (2) reverse bias (i.e., overgeneralizing that smaller denominators lead to larger fraction magnitudes), and (3) gap thinking (i.e., the smaller the difference between numerator and denominator, the larger the fraction). PMTs completed a paper-pencil test, indicating agreement with any response and, if not, explaining the reason of disagreement and their own reasoning.

Most PMTs accurately identified all three responses as incorrect (Vignette 1: 70%, Vignette 2: 80%). PMTs explanations fell into four types: (1) disagreement without elaboration (V1: 21%, V2: 13%), (2) providing the correct solution without addressing the flawed reasoning (V1: 43%, V2: 44%), (3) explicitly explaining the flaws in reasoning (V1: 14%, V2: 31%), and (4) using a counterexample (V1: 21%, V2: 13%). The most common explanation type given was (2).

The results show that while most PMTs identified incorrect reasoning, there were qualitative differences in their explanations. Many focused on providing the correct solution rather than addressing the underlying misconceptions or providing counterexamples that can show the incorrectness of the reasoning.

References

Brodie, K. (2013). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics*, *85*(2), 221-239. https://doi.org/10.1007/s10649-013-9507-1

Gómez, D., & Dartnell, P. (2019). Middle schoolers' biases and strategies in a fraction comparison task. *International Journal of Science and Mathematics Education*, *17*(6), 1233-1250. https://doi.org/10.1007/s10763-018-9913-z

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 362. PME.

HIGH COGNITIVE DEMAND DIDACTIC EXPERIENCE FOR TEACHING INFERENTIAL STATISTICS

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The International Baccalaureate Diploma Programme requires students to conduct a Simple Experimental Study in Psychology, applying descriptive and inferential statistics to evaluate hypotheses. However, prior results indicate persistent difficulties in understanding sample representativeness, discussing biases, and interpreting hypotheses, consistent with findings in other contexts (Batanero et al., 1994). To address these challenges, a high cognitive demand task was designed and implemented to enhance conceptual understanding and improve student performance.

According to Smith and Stein's (1998) Cognitive Demand Theoretical Model, effective mathematical tasks should engage students in meaningful problem-solving, providing better opportunities for thinking and reasoning. High cognitive demand tasks foster deeper learning by strengthening connections between procedures and concepts, unlike low-demand tasks that emphasize memorization (Ni et al., 2017).

A pre-experimental design was applied to a convenience sample of 60 Psychology students from 10th and 11th grade. The high-demand task involved four stages: an initial discussion on key statistical concepts, a comprehension check, an experimental task where teams designed studies and analyzed data from a simulated population, and a final reflection comparing sample findings with full population data, focusing on Type I and II errors. Institutional approval was obtained prior to implementation.

While the long-term impact will be assessed in the coming months, preliminary results suggest that high cognitive demand tasks effectively promote discussions on key statistical concepts, fostering engagement with issues such as sampling limitations, statistical biases, and hypothesis testing errors. This approach appears to support a deeper understanding of inferential statistics, reinforcing its potential as an effective educational strategy.

References

Batanero, C. et al. (1994). Errors and difficulties in understanding elementary statistical concepts. *International Journal of Mathematics Education in Science and Technology*, 25(4), 527-547.

Ni, Y. et al (2017). Improving cognitive and affective learning outcomes of students through mathematics instructional tasks of high cognitive demand. *The Journal of Educational Research*, 1–16.

Smith, M., & Stein, M. (1998). Reflections on Practice. Selecting and Creating Mathematical Tasks. *Mathematics Teaching in the Middle School*, 3(5), 344–350.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 363. PME.

OBSTACLES TO LEARNING AND TEACHING WITH VIDEOS

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Since extensive research already exists on the quality criteria educational videos must meet, this paper presents the first results on students and teachers' obstacles and problems.

THEORY

As defined by Wolf (2018), educational videos are self-produced videos that explain various topics, including how-tos, explanations of how things work, and abstract concepts. Previous research has explored the promotion of process-related skills in substantial tasks (Wittmann, 2001). These so-called substantial task formats play a crucial role not only in enhancing content-related skills in arithmetics but also in fostering discovery-based learning and the development of process-related skills.

METHODS

The study is structured cyclically within the framework of research on subject didactic development. The initial cycles were analyzed using qualitative content analysis, which involved the inductive extraction of categories from the data (Mayring, 2015).

LEARNING ENVIRONMENT AND MATH RESEARCHER VIDEOS

A learning environment was created for the substantial task format number grid, which contains various research tasks following its operational nature (Wittmann, 2001). To support the students, interactive math researcher videos have been integrated using a QR code to help them develop and formulate reasoning.

FIRST RESULTS

The evaluation of the studies resulted in the following main categories: obstacles to teaching and to learning with educational videos. These are divided into several subcategories, including technical obstacles, representation and argumentation obstacles, and the learning and teaching process. These are confirmed and, if necessary, expanded based on further analyses.

References

Mayring, P. (2015). Qualitative content analysis: Theoretical background and procedures. Approaches to qualitative research in mathematics education: Examples of methodology and methods, pp. 365-380.

Wittmann, E. (2001). Designing, Researching and Implementing mathematical learning environments, In: M. van den Heuvel-Panhuizen (ed.), *Proceedings of PME25*, Utrecht, The Netherlands, vol. 1 pp. 189-192.

Wolf, K. D. (2015). Video-Tutorials und Erklärvideos als Gegenstand, Methode und Ziel der Medien- und Filmbildung. *Filmbildung im Wandel*, *2*, pp. 121-131.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 364. PME.

VALID, RELIABLE AND EASY TO USE: TOWARDS THE DEVELOPMENT OF A TEXTBOOK SELECTION CHECKLIST

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Mathematics textbooks are regarded a relevant factor for teaching quality. In many countries, teachers or local authorities are responsible for selecting textbooks. Polikoff et al. (2020) analyzed such textbook selection processes in California school districts and concluded that teachers rarely consulted objective evaluation sources, often failed to identify high-quality textbooks, and that limited resources hinder textbook selection, as textbooks are extensive and teachers are not trained for textbook assessment. Despite the development of research on mathematics textbooks to date, there is still a lack of guidance for textbook assessment in school practice. To address this gap, our research goal is to develop a checklist for elementary school teachers that is valid, reliable, and easy to use. For this, we apply the design-based research methodology and address the obstacles identified by Polikoff et al. (2020). To ensure validity, we use established assessments from textbook research, which we simplify to enable untrained teachers to use them. Despite the simplification, the assessment should lead to the same quality ranking of the textbooks as the original scientific evaluation. To reduce the time required, we aim for a high inter-rater reliability of the assessment indicators, so that not all teachers of a selection committee have to analyze all pages of all textbooks.

As a first result, we developed a prototype assessment scheme for textbook units on addition and subtraction strategies. As a basis, we used the assessment framework of the research study by Sievert et al. (2019). This framework was simplified by removing indicators with low influence, re-operationalizing indicators for low-inference ratings and reducing the number of textbook units to be analyzed. The revised scheme was applied to the same textbooks analyzed by Sievert et al. (2019). Two independent raters achieved high interrater agreement ($\kappa_w = .95$). The ranking of the textbooks remained unchanged, confirming the validity of the simplified scheme. This first step toward a practical checklist for textbook selection in elementary schools shows that a research framework can be simplified while maintaining reliability and validity. We will address further curriculum topics and test the final checklist with in-service teachers.

References

Polikoff, M. S., Campbell, S. E., Rabovsky, S., Koedel, C., Le, Q., T., Hardaway, T., & Gasparian, H. (2020). The formalized processes districts use to evaluate mathematics textbooks, *Journal of Curriculum Studies, 52*, 451-477.

Sievert, H., van den Ham, A.-K., Niedermeyer, I., & Heinze, A. (2019). Effects of mathematics textbooks on the development of primary school children's adaptive expertise in arithmetic. *Learning and Individual Differences, 74,* 101716.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 365. PME.

THE MECHANISMS OF ASSIMILATION AND ACCOMMODATION IN THE EQUILIBRATION OF THE DEFINITE INTEGRAL SCHEMA

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This exploratory study examined the role of the mechanisms of assimilation and accommodation in the equilibration of the definite integral schema (Piaget, 1975). Drawing on Piagetian theory within the APOS framework (Action–Process–Object–Schema), we developed a genetic decomposition of the concept of the definite integral, consisting of the key mental structures necessary for its construction (Arnon et al., 2014). We then implemented a teaching sequence following the ACE cycle (Activities, Classroom discussion, and Exercises) and evaluated the results through a questionnaire comprising six tasks.

Data analysis revealed that students initially employ assimilation to interpret the definite integral using their existing schemas from algebra, geometry, and area. Given the conceptual complexity of the definite integral, this initial assimilation is limited, prompting students to recognize the inadequacy of their existing schemas and compelling them to modify their cognitive structures through accommodation. This shift allows them to conceive of the integral beyond mere geometric area, viewing it as a functional operator, a limit of a summation process, or a tool for modeling accumulation phenomena.

The data also indicated that these mechanisms interact to form a dynamic cycle of equilibration, enabling the progressive development of a more comprehensive definite integral schema. In our view, this is evidence that the designed activities—based on the genetic decomposition—and the tasks in the questionnaire effectively fostered cognitive disequilibrium.

The findings offer evidence underscoring the importance of designing instructional sequences grounded in a robust genetic decomposition, one that provides opportunities for both assimilation and accommodation.

Additional information

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References

Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Roa Fuentes, S., Trigueros, M. y Weller, K. (2014). *APOS Theory. A framework for research and curriculum development in mathematics education*. New York: Springer. https://doi.org/10.1007/978-1-4614-7966-6.

Piaget, J. (1975). *L'équilibration des structures cognitives: Problème central du développement*. Paris: PUF.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 366. PME.

DEVELOPING NOTICING DISCOURSE PRACTICES FOR EQUITY WITH COLLEGE MATH INSTRUCTIONAL TEAMS

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Teacher noticing- attending to noteworthy aspects in instructional interactions, reasoning about them, and making decisions- is at the core of responsive mathematics teaching (Santagata et al., 2021). Taking a situative perspective, learning to notice involves engaging in discourse-rich spaces where educators collectively make sense of teaching and learning (van Es et al., 2025). Recent scholarship has explored noticing for equity- noticing to enhance opportunities for historically marginalized learners (van Es et al., 2022)- though examples of how teachers develop this expertise are scarce. This study contributes to understanding how participants' discourse during professional development (PD) can promote noticing for equity. Instructional teams (3 college instructors, 8 graduate and 3 undergraduate teaching assistants) participated in a PD that emphasized noticing student thinking and participation for equity. Participants engaged in whole and small group discussions around video clips. In this study, we ask: What are features of the discursive activity that enable participants learning to notice for equity? Small group discussion segments were identified, transcribed, and double-coded by two members of the team. Qualitative analysis was informed by literature on teachers' noticing discourse practices (van Es et al., 2025) to characterize productive features of discursive activity in the PD. Emergent findings reveal that participants negotiated noteworthy phenomena by making assertions about key aspects of equity, such as students' feelings and classroom setup to promote student participation. In some instances, participants engaged in problematizing objects of teaching by offering contrasting viewpoints about students' mathematical authority in the interactions. This discursive move allowed them to expand their noticing in generative ways with implications for equitable instruction. Our findings contribute to the limited literature on learning to notice at the college level and shed light on noticing discourse that supports teacher noticing for equity.

References

Santagata, R., König, J., Scheiner, T., Nguyen, H., Adleff, A. K., Yang, X., & Kaiser, G. (2021). Mathematics teacher learning to notice: A systematic review of studies of video-based programs. *ZDM–Mathematics Education*, *53*(1), 119-134.

van Es, E. A., Hand, V., Agarwal, P., & Sandoval, C. (2022). Multidimensional noticing for equity: Theorizing mathematics teachers' systems of noticing to disrupt inequities. *Journal for Research in Mathematics Education*, *53*(2), 114-132.

van Es, E. A., Luna, M., & Tekkumru-Kisa, M. (2025). An exploration of noticing discourse as a mechanism of teacher learning. In *Teacher Professional Vision: Empirical Perspectives* (pp. 13-30). Routledge.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 367. PME.

TRACKING SELF-EFFICACY DEVELOPMENT: A LONGITUDINAL ANALYSIS OF STUDENTS IN CHILE

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Self-efficacy, understood as students' perception and judgment of their own effectiveness and response capability, plays a crucial role in their academic trajectory, directly influencing their motivation and future performance (Bandura, 1986). The development of self-efficacy is nourished by various sources, such as family, peer influences, and adolescent transitions, making it particularly relevant to understand how this belief evolves over time. The longitudinal component is especially important, as there is evidence demonstrating that mathematics performance and self-efficacy have a reciprocal relationship (Hannula et al., 2014). Identifying the factors that shape self-efficacy beliefs allows us to understand students' academic and personal life trajectories.

This study proposes to analyze the self-efficacy trajectories of students in Chile, differentiating between general academic self-efficacy and mathematics-specific self-efficacy, with a gender-comparative approach. The data for this study comprises approximately 142,000 students (50.9% female), sourced from a national test for students. Data were collected when students were in 4th, 8th, and 10th grade, corresponding to years 2016, 2018, and 2022, respectively. Students' self-efficacy was measured at two levels: general and mathematics-specific. General self-efficacy was assessed through multiple Likert-scale items during the national test, while mathematics self-efficacy was measured using a single item each period.

Results indicate that for 4th and 6th grade, general self-efficacy levels are higher in females, but this reverses in 10th grade. In mathematics self-efficacy, males show higher levels from 4th grade onward, with this gap widening over time. This reveals a concerning pattern: while self-efficacy decreases in all students throughout their academic path, the decline is more pronounced in women, particularly in mathematics. This disparity is troubling as it may perpetuate women's historical underrepresentation in STEM disciplines. The reasons behind these results will be discussed in the session.

References

Bandura, A. (1986). *Social foundations of thought and action: a social cognitive theory*. Prentice-Hall.

Hannula, M., Bofah, E., Laura Tuohilampi, L., and Metsämuuronen, J. (2014). A longitudinal analysis of the relationship between mathematics-related affect and achievement in Finland. In Oesterle, P. Liljedahl, C. Nicol, and D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36*. Vancouver, BC: PME.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 368. PME.

USING QUANTITATIVE STRATEGIES TO UNDERSTAND THE "SIZE" OF CLIMATE CHANGE: A LEARNING EXPERIENCE WITH PRESERVICE ELEMENTARY TEACHERS IN CHILE

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Climate change education is increasingly recognized as essential for preparing future generations to address global environmental challenges. Integrating climate change topics into mathematics lessons can enhance students' understanding of environmental issues through quantitative analysis. Therefore, preservice teachers (PSTs) need opportunities to explore these topics before they start teaching. This poster presents research on a learning unit designed to integrate mathematics and climate change education for PSTs in Santiago, Chile. It focuses on how PSTs used quantitative strategies to understand the magnitude of climate change impacts such as greenhouse gas emissions, temperature increases, and sea level rise. This initiative responds to the Ministerio del Medio Ambiente (MMA) and Ministerio de Educación (MINEDUC) efforts to develop climate-related curricular materials (Kong & Garrido, 2024).

The research question guiding this study was: *How do PSTs use quantitative strategies to comprehend the "size" of quantities related to climate change impacts?*

The unit was implemented twice, in fall 2023 and fall 2024, with 25 PSTs enrolled in an elementary education program. It included group labs and mathematical modeling projects that required using mathematical concepts to analyze climate change scenarios. For example, PSTs examined the decline of sea ice extent at the poles over the years. To visualize this decline, PSTs used mathematics to demonstrate the rapid decrease in sea ice extent. Data were collected from PSTs' work on group labs and modeling projects to assess how they used quantitative strategies to understand and communicate the magnitude of climate change impacts.

Analysis of PSTs' responses revealed struggles with dimension analysis and interpreting units of measure for intensive quantities. However, using ratios helped them develop a sense of the size of quantities measuring climate change impacts. This underscores the importance of enhancing PSTs' quantitative reasoning skills to effectively teach climate change in their future classrooms and suggests addressing their difficulties with dimension analysis and unit interpretation.

References

Kong, F., & Garrido, M. (2024, July 11). *Bienvenido el cambio climático al currículum escolar*. CIPER Chile. Retrieved from https://www.ciperchile.cl/2024/07/11/bienvenido-el-cambio-climatico-al-curriculum-escolar/

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 369. PME.

SECONDARY-TERTIARY TRANSITION: FOSTERING AUTONOMY AND MATHEMATICAL COMPETENCIES

Chiara Grisendi

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RATIONALE, THEORETICAL FRAMEWORK, AND COLLECTING DATA

In Mathematics Education Research, the secondary-tertiary transition is a significant topic examined from various perspectives, spanning epistemological and cognitive viewpoints to sociocultural dimensions (Gueudet et al., 2016). The resource presented here explores cognitive and metacognitive perspectives. On the one hand, from a metacognitive perspective, according to Gueudet and colleagues (2016), during this transition, "students experience an increase in autonomy" (p. 19). However, if they are not accustomed to autonomy, this shift may cause anxiety. To develop autonomy, students must feel responsible for their learning (Zan, 2007). Various assessment approaches can highlight "the importance of a variety of personal features–self-concept, self-attribution, self-efficacy, and assumptions about the nature of learning" (Black & William, 1998, p. 25), which enable students to work on their sense of learning responsibility. On the other hand, from a cognitive perspective, this research project aims to design and test educational resources to reinforce basic mathematics knowledge and promote mathematical competencies (in the sense of Niss and Højgaard, 2019) essential for university-level mathematical studies.

Another key aspect of the project is integrating this resource into daily classroom practices. Four upper secondary teachers have collaborated on designing the resource to achieve this. Indeed, the project consists of three phases organised in a cyclical structure. First, the design of the resources involves critically comparing existing materials, including the syllabus for university entrance tests. Second, the created activities will be tested in schools, and the collected data is currently undergoing qualitative analysis. Finally, the activities will be revised and refined.

References

Black, P., & Wiliam, D. (1998). Assessment and Classroom Learning. *Assessment in Education: Principles, Policy & Practice, 5*(1), 7–74. https://doi.org/10.1080/0969595980050102

Gueudet, G., Bosch, M., DiSessa, A. A., Kwon, N., & Verschafel, L. (2016). *Transitions in mathematics education*. Springer Nature. https://doi.org/10.1007/978-3-319-31622-2

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics 102*(1)*,* 9–28. https://doi.org/10.1007/s10649-019-09903-9

Zan R. (2007). *Difficoltà in matematica. Osservare, interpretare, intervenire.* Springer.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 370. PME.

ENHANCING TEACHER LEARNING IN COMPUTATIONAL THINKING THROUGH CULTURALLY RESPONSIVE PROFESSIONAL DEVELOPMENT

Lynn Liao Hodge, Bethany Parker, Amy Maples, Chulin Chen and Shande King

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The analysis reported in this presentation examines a professional development initiative in a rural region of the United States known as East Tennessee that integrated community resources and culturally responsive pedagogy to deepen teachers' understandings of computational thinking (CT). We focus on understanding the affordances and limitations of the activities and instructional design of the professional development in supporting teacher learning and fostering engaged identities in relation to CT.

The analysis is part of Computer Science (CS) for East Tennessee, a three-year project funded by the National Science Foundation (#2219418), aimed at increasing elementary students' access to CT through its integration with literacy. Building on a prior NSF planning grant, the project leverages community practices such as making and storytelling to create meaningful connections between teachers' (and students') experiences and key CT concepts.

Since its launch in Fall 2022, the project has undergone two cycles of design-based research. This presentation focuses on the second cycle, implemented in 2024, in which a cohort of elementary teachers from multiple schools across East Tennessee participated in six professional development sessions spanning Summer and Fall. Our analysis frames the professional development context through the lens of access, emphasizing how learning environments are designed and structured to distribute opportunities for engagement (Engle & Conant, 2002). Using this framework, we examine discourse from professional sessions, including large and small group discussions, teacher artifacts, and interviews. Preliminary findings indicate that teachers drew on their own and their students' prior knowledge, leveraging cultural connections to engage with CT. Additionally, the data highlights the significance of linking community aspects to computational thinking in shaping teachers' professional identities. The presentation will discuss implications for teacher learning and the role of culturally responsive approaches in expanding access to computational thinking in elementary and teacher education.

References

Engle, R. A., & Conant, T. (2002). Guiding principles for fostering productive disciplinary engagement: Explaining an emergent argument in a Community of Learners classroom. *Cognition and Instruction, 20*(4), 399-483.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 371. PME.

USING MATH IN BIOLOGY: EXAMINING WHAT STUDENTS NOTICE

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Using math is a critical skill for undergraduate biology. However, many students struggle with applying math in biological contexts, and it remains unclear why some struggle with sensemaking (i.e., drawing conceptual meaning) when doing mathematical analyses in biology. Here, we investigate student noticing to ask: 1) What do students notice when doing math in undergraduate biology lab courses? 2) How does what students notice influence their mathematical and biological sensemaking?

We apply the framework of student noticing, which has been used in the context of K-12 math (Hohensee 2016; Lobato 2006, 2012). Under this framework, students notice and mentally isolate different features during instruction, which forms the foundation of their sensemaking. Here, we focus what students notice when doing math in biology lab courses. We conducted observations and interviews with >60 students, where we asked students about what they noticed, the factors that impacted their noticing, and how they made sense of their data. After transcription, two coders independently read through a subset of transcripts and independently generated codebooks. Following discussion, we generated a consensus codebook. We then collaboratively coded the interviews, resolving disagreements through discussion.

We identify broad variation in what students notice, which varied from more perceptual noticing not directly relevant to the underlying math or biology, to conceptual noticing of features more salient to the math and/or biology. These factors, in turn, likely impact students' sensemaking. For example, our preliminary results indicate that students who notice more perceptual features were less likely to exhibit blended sensemaking that merges mathematical and biological principles, while those who noticed deeper, more salient features were more likely to do so.

References

Hohensee, C. (2016). Student noticing in classroom settings: A process underlying influences on prior ways of reasoning. *The Journal of Mathematical Behavior, 42*, 69–91.

Lobato, J. (2006). Alternative Perspectives on the Transfer of Learning: History, Issues, and Challenges for Future Research. *Journal of the Learning Sciences, 15*(4), 431–449.

Lobato, J., Rhodehamel, B., & Hohensee, C. (2012). "Noticing" as an Alternative Transfer of Learning Process. *The Journal of the Learning Sciences, 21*, 433–482.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 372. PME.

USING VIDEO DESIGN TO INNOVATE PROBLEM SITUATIONS THAT FOSTER CHALLENGE AND ENGAGEMENT IN STUDENTS

Kamonchanok Japa, Narumon Changsri, Maitree Inprasitha, Khem Khenkhok and Ua-Jit Pattanajak

Khon Kean University, Thailand

Designing productive problem situations is vital for fostering meaningful Mathematical Thinking (MT), with attention to both context and condition (see Inprasitha, 2023). This study aimed to design video problem situations scenarios to foster 4th grade students' engagement. In a context where open-ended mathematical tasks remain underdeveloped. The problem situations used in this study adapted from Somsaman et al. (2024), involved coloring fish in a tank. This study employed a qualitative research design through three cycles of the Lesson Study (LS) and Open Approach (OA). Data were collected from the LS team's planning, teaching, and reflective sessions arising from planning discussions and classroom observations, as well as student worksheets and field notes. Content analysis was used, focusing on improving context and conditions to increase student engagement in MT to understand how problematic situations developed.

The findings revealed that in the first cycle, students were able to connect the context of the problem-situation video with their everyday experiences. They showed excitement when encountering familiar elements, which helped capture their attention. However, the three given conditions were still too complex for their level of understanding. In the second cycle, the number of conditions was reduced to two, and the segment showing the fish tank was changed to black and white to minimize visual distractions. Students expressed curiosity about what they saw, and showed a desire to take action or further explore the situation on their own. These reactions persisted in the third cycle. Although the number of conditions remained the same, the language was simplified and the visuals were enlarged to support better comprehension, while the context remained unchanged. These progressive improvements to the problem situation led to cognitive enhancement, this reflects the potential of well-designed video to foster meaningful and in-depth mathematical thinking.

References

Inprasitha, M. (2023). Blended learning classroom model: a new extended teaching approach for new normal. *International Journal for Lesson & Learning Studies, 12*(4), 288-300. https://doi.org/10.1108/IJLLS-01-2023-0011

Somsaman, K., Isoda, M., & Araya, R. (Eds.). (2024). *Guidebook for unplugged computational thinking*. SEAMEO STEM-ED.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 373. PME.

SHIFTING SECONDARY MATH TEACHERS' PERCEPTIONS OF RESOURCES, INSTRUCTION, AND ASSESSMENT WITH AI

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The rise of artificial intelligence, or AI, has posed both an interesting challenge and unique teaching opportunity for K-12 educators. Focusing on secondary mathematics teachers, we aim to understand how the mathematics department of an upper school shifted their perspective of AI usage in their classrooms through refined incorporation of AI into lesson planning, instruction and reinstruction, and assessment of student understanding. Using a situated learning perspective (Lave & Wenger, 1991), we as researchers sought to understand how secondary math teachers refined ways to use AI to assist in their instruction and assessment of student learning over time. The research questions are:

- How have teachers shifted their perception of the use of AI in their preparation or assessment of secondary mathematics lessons?
- In what ways have secondary math teachers found AI useful in the growth of student understanding?

The sample comprises 8 secondary mathematics teachers of various age and experience, although all licensed and equipped with extensive mathematics teacher training. The intervention takes place over one semester of teaching, with teachers taking an initial survey concerning both their perceptions and use of AI in secondary math classrooms before the incorporation of AI. A similar survey is taken at the end of the semester to see change in the teachers' perspectives and measure shifts in their use of AI in math classes.

Survey results show a slightly more positive favor towards the use of AI in mathematics classrooms at the secondary level, citing that the mistakes AI makes have been decreased by appropriately incorporating AI as a resource maker and teaching tool instead of relying on it for full instruction. Further, results show teachers more creatively using AI as a way to give students autonomy in their own learning and understanding of their own learning.

References

Canonigo, A. M. (2024). Levering AI to enhance students' conceptual understanding and confidence in mathematics. *Journal of Computer Assisted Learning, 40*(6), 3215–3229. https://doi-org.utk.idm.oclc.org/10.1111/jcal.13065

Lave, J. and Wenger, E. (1991) *Situated learning: legitimate peripheral participation*. Cambridge University Press.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 374. PME.

PROMOTING STUDENT WELL-BEING IN THE CLASSROOM THROUGH ROBOT PROGRAMMING ACTIVITIES

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Promoting student well-being in the classroom is a critical mission that focuses on developing a balance between cognitive, emotional, and social dimensions (Australian Catholic University & Erebus International, 2008). This research aimed to investigate the impact of robot programming activities within environmental problem-solving contexts on the well-being of 18 ninth-grade students. The research was designed by combining Transformative Lesson Study incorporated with Open Approach (TLSOA) framework (Inprasitha, 2022), emphasizing collaborative Plan, collaborative Do and collaboratively See within the context of local environmental challenges. Specifically, the students focused on addressing issues pertinent to Kaeng Lawa, a significant water basin proximate to their community. Data were collected through 180 minutes of classroom observation, students' worksheets, and reflection from the lesson study team. The data were analyzed using interpretive analysis with triangulation. The findings revealed that robot programming activities in environmental problem-solving contexts effectively promoted student well-being across 3 dimensions 1) Cognitive Dimension: Students demonstrated significant development in computational thinking skills, including decomposition, pattern recognition, abstraction, and algorithm design for controlling robots, linked to mathematical reasoning. They developed abilities to apply logical sequences for robot control, all of which strengthened both their computational thinking and mathematical thinking. 2) Emotional Dimension: Students exhibited positive emotions, happiness, and enthusiasm during the learning process. Their confidence and perseverance improved as they tackled complex challenges and resolved errors in their programming tasks. 3) Social Dimension: Collaborative group work fostered positive relationships and acceptance of diverse perspectives and problem-solving approaches. Students developed a sense of connectedness to their community and a heightened responsibility toward local environmental issues. Additionally, the activities enhanced teamwork, communication skills, and environmental awareness, all of which are essential components of overall well-being.

References

Inprasitha, M. (2022). Lesson study and open approach development in Thailand: a longitudinal study. *International Journal for Lesson and Learning Studies, 11*(5),1-15.

Australian Catholic University & Erebus International. (2008). *Scoping study into approaches to student wellbeing: Literature review.* Report to the Department of Education, Employment and Workplace Relations: Canberra.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 375. PME.

MATHEMATICS TEACHERS' POSITIONING IN MATHEMATICS TEACHER PD PROGRAMS ON EMERGING TECHNOLOGIES

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This study examines how mathematics teachers' positioning evolves through interactions with instructors and participating teachers in mathematics teacher PD programs designed to incorporate emerging technologies into mathematics classrooms. Positioning theory provides a framework for understanding how participants construct and transform their roles through interactions in a learning context (Herbel-Eisenmann et al., 2015). In mathematics teacher PD programs, instructors are often positioned as absolute experts, while participants are assumed to be novices. When instructors are positioned as pioneers of emerging technologies rather than as absolute experts, participants may adopt more active and engaged roles, moving beyond passive novices.

Since July 2023, the Center for Research in Mathematics Education at Seoul National University in Korea has conducted four PD programs focusing on integrating generative artificial intelligence as an emerging technology into mathematics classrooms. The instructors in these programs were not positioned as authoritative figures but as pioneers, guiding participants toward practical implementation. The instructors' discourse and participants' chat records from the synchronous PD programs were collected. We conducted discourse analysis to examine how the instructors' discourse influenced the participants' positioning and to analyze the transitional processes of their positioning throughout the PD program.

Three distinct forms of positioning among participating mathematics teachers were identified. First, in the early stages of the PD program, teachers were positioned as passive knowledge recipients, focusing on absorbing the expertise and practical cases provided by the instructor. Second, during the PD program, some teachers were positioned as semi-experts, sharing their own experiences and practical knowledge with both the instructor and fellow participants. Third, by the end of the PD program, teachers were positioned as potential practitioners, demonstrating their intention to apply the instructional strategies introduced in the program to their own classroom settings. These findings highlight the significance of the instructor's and participants' positioning dynamics and their evolution within the mathematics teacher PD programs.

References

Herbel-Eisenmann, B. A., Wagner, D., Johnson, K. R., Suh, H., & Figueras, H. (2015). Positioning in mathematics education: Revelations on an imported theory. *Educational Studies in Mathematics, 89*(2), 185-204.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 376. PME.

MATHEMATICAL COMPETENCE OF PRIMARY SCHOOL TEACHERS IN INITIAL TRAINING WHEN SOLVING MATHEMATICAL PROBLEMS IN A COOPERATIVE LEARNING CONTEXT

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INTRODUCTION

Mathematical problem solving is considered a central component of mathematics learning and is part of the mathematical process standards of the NCTM (2000). On the other hand, cooperative learning is considered a solid and effective method for teaching mathematics at any school stage (Herrada & Baños, 2018). This study aims to analyse the degree of achievement and correction in mathematical problem solving by primary school teachers in initial training, in a didactic proposal based on the principles of cooperative learning and focused on solving mathematical problems with basic content from all areas of school mathematics.

METHOD

In this context, a mixed approach is adopted to analyse students' written productions, to determine: 1. degree of problems completed; 2. degree of correctness of the solutions given; 3. mathematical contents and processes used (NCTM, 2000). The sample consists of the work of 51 students of the Degree in Primary Education at the University of the Basque Country/Euskal Herriko Unibertsitatea (UPV/EHU).

RESULTS AND CONCLUSIONS

Preliminary results indicate that: 1) 80% of the problems are solved; 2) 59% of the solutions given are correct; 3) some kind of pictorial representation appears in almost all problems, but the presence of mathematical notation is scarce. In conclusion, although students show an ability to deal with mathematical problems, problem solving needs to be improved. Also, the use of appropriate mathematical notation is an issue that needs further development.

References

Herrada Valverde, R. I., y Baños Navarro, R. (2018). Experiencias de Aprendizaje Cooperativo en Matemáticas. *Espiral. Cuadernos Del Profesorado, 11*(23), 99-108. https://doi.org/10.25115/ecp.v11i23.2131

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 377. PME.

DIAMAT: A VALIDATED ADAPTIVE MATHEMATICS ASSESSMENT TOOL FOR CHILEAN MIDDLE SCHOOLS

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Accurate and timely assessment of student learning is crucial for instructional decision-making in mathematics education (Hattie, 2008). DIAMAT is an adaptive mathematics assessment tool designed to offer individualized and group-level reports, supporting teachers in identifying students' needs and designing targeted instructional strategies. Based on the principles of Curriculum-Based Measurement (Deno, 2003) and Computer Adaptive Testing (Martin & Lazendic, 2017) principles, DIAMAT aligns with the Chilean national curriculum for middle school students from Grades 5 to 8.

The development of DIAMAT involved two stages. In the pre-assembly stage, experts reviewed and calibrated an item bank to ensure alignment with curriculum objectives. The post-assembly stage included adaptive test implementation with 4,875 students from 26 schools. This process gathered evidence to support the tool's validity and reliability. Validity evidence was collected from test content, response processes, internal structure, and relationships with other variables (AERA, APA & NCME, 2014). Expert reviews confirmed content validity, while student feedback helped refine clarity of the items. Test structure evidence demonstrated that the test effectively measured mathematical ability at each curriculum strand. Item Response Theory (IRT) analyses were used to ensure item discriminations quality and to locate item difficulty level. Moderate correlations with school mathematics grades further supported its validity. Test-retest correlations showed consistent results, indicating that DIAMAT provides reliable results over time. DIAMAT's adaptive algorithm enhances precision and reduces testing time. DIAMAT empowers educators with timely, curriculum-aligned data to support instructional decision-making, making it a valuable tool for improving educational outcomes and equity in mathematics learning.

References

American Educational Research Association, American Psychological Association, & National Council on Measurement in Education. (2014). *Standards for educational and psychological testing*. American Educational Research Association.

Deno, S. L. (2003). Developments in curriculum-based measurement. *The Journal of Special Education, 37*(3), 184–192. https://doi.org/10.1177/00224669030370030801

Hattie, J. (2008). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. Routledge.

Martin, A. J., & Lazendic, G. (2018). Computer-adaptive testing: Implications for students' achievement, motivation, engagement, and subjective test experience. *Journal of Educational Psychology, 110*(1), 27–45. https://doi.org/10.1037/edu0000205

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 378. PME.

EXPLORING THE IMPACT OF TEACHER ENCOURAGEMENT ON STUDENTS' MATHEMATICAL THINKING AND LITERACY

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Teachers' encouragement of mathematical thinking is crucial as it fosters deep understanding, enhances problem-solving skills, and builds student confidence. This study aims to explore the level of encouragement teachers provide for mathematical thinking in the context of cognitive activation in mathematics, as perceived by students, and the relationship between this encouragement and students' mathematical literacy. This research examines nine items from the "Cognitive Activation in Mathematics: Encourage Mathematical Thinking" scale in the PISA 2022 student questionnaire. These nine items are categorized into three groups: "Application of Mathematics in Everyday Life," "Mathematical Thinking and Logic," and "Connections Between Mathematical Topics." The study conducts latent class analysis using data from 15-year-old students in Taiwan participating in PISA 2022. The results indicate that students perceive the encouragement of mathematical thinking by Taiwanese teachers as falling into four classes: the first class represents moderate encouragement of mathematical thinking, accounting for 16.27%; the second class indicates low encouragement, with 29.56%; in other words, students in this class may perceive their teachers' instruction as leaning toward traditional lecturing. The third class represents high encouragement of mathematical thinking, comprising 42.87%; additionally, 11.31% of students reported that their teachers particularly emphasize the connections between mathematical topics. From a gender perspective, there are differences in the encouragement classes of mathematical thinking among genders: in the high encouragement class, the ratio of boys is significantly higher than that of girls, whereas, in the other three classes, the ratio of female students is higher than that of male students. When examining the relationship between different encouragement classes and students' mathematical literacy, the results indicate that students in the high mathematical connections class have the highest literacy, followed by the moderate encouragement class. The highest and lowest encouragement classes for mathematical thinking show relatively low mathematical literacy, and the differences in mathematics between these two classes do not reach statistical significance. Promoting mathematical thinking is currently an important orientation in mathematics education. This study suggests that mathematics teachers should first encourage mathematical thinking moderately in the classroom. They could start by strengthening the connections between mathematical topics and everyday life, gradually enhancing students' mathematical thinking.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 379. PME.

MATHEMATICAL CONCEPTS IN CRYPTOGRAPHY TASKS IN TEXTBOOKS

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The role of textbooks in the teaching and learning processes of Mathematics is widely recognized in the literature, as they are essential resources for both planning and developing lessons (Amaral, Mazzi, Andrade & Perovano, 2022). Textbooks also present tasks, which direct proposals to be solved by students, and which can be analyzed in terms of their mathematical concepts and other characteristics (Litoldo, 2021). In particular, we are developing a broader investigation on the presence of Cryptography in Brazilian textbooks and, in this poster, we aim to analyze the tasks of two textbooks in terms of the mathematical concepts presented, as well as the formulation and encryption techniques. A total of 36 tasks were analyzed. Of these, some mathematical concept is explicitly addressed in 20, such as prime numbers, modulo operation, permutations and matrices. As for the formulation and encryption techniques, they were largely related to the mathematical concepts presented, and included, for example, Caesar cipher, RSA and prime factorization. The mathematics present in the tasks worked, at various times, as a tool to apply the encryption process (Buchmann, 2004), without there being, at times, a broader and deeper discussion about the security of messages, and how this encompasses implications for the world today (Litoldo & Guimarães, 2023).

References

Amaral, R. B., Mazzi, L. C., Andrade, L. V., & Perovano, A. P. (2022). *Livro didático de matemática: Compreensões e reflexões no âmbito da educação matemática* [Mathematics Textbook: Understandings and Reflections in the Field of Mathematics Education]. Mercado de Letras.

Buchmann, J. (2004). *Introduction to cryptography* (Vol. 335). New York: Springer.

Litoldo, B. F. (2021). *A contextualização e os níveis de demanda cognitiva de tarefas de Geometria presentes em Livros Didáticos de Matemática sob a perspectiva do Opportunity-to-Learn* [The Contextualization and Cognitive Demand Levels of Geometry Tasks in Mathematics Textbooks from the Opportunity-to-Learn Perspective]. (Doctoral Thesis). Instituto de Física "Gleb Wataghin", Universidade Estadual de Campinas, Campinas, SP, Brasil.

Litoldo, B. F., & Guimarães, D. R. (2023). Posibles diálogos entre el poder, las matemáticas y la criptografía: Una reflexión crítica sobre sus interrelaciones. *Prometeica - Revista de Filosofía y Ciencias*, (27), 306-315. https://doi.org/10.34024/prometeica.2023.27.15302

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 380. PME.

LOGISTICS AND TRANSPORTATION ENGINEERING STUDENTS' CONCEPTIONS OF SOLUTION SET OF SYSTEMS OF LINEAR EQUATIONS

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The research focuses on the learning of the concept of solution set of Systems of Linear Equations (SLE) by university students. Previous studies have revealed the difficulties that students of diverse educational levels face in learning SLE and its solution set, related to the use of algorithmic procedures without understanding its properties, the use of different representations and their ability to interpret and apply SLE to solve contextual problems.

Based on APOS theory, this qualitative research focuses on how individuals construct their knowledge through the reflection of the properties of mathematical objects during the resolution of problems. This construction goes through levels or mental structures of Action, Process and Object, which are coherently grouped in a Schema (Arnon et al., 2014). The study inquiries into what are the conceptions about SLE's solution set that university students construct while solving problematic situations.

METHODOLOGY

Methodology is based on the qualitative paradigm and the strategy of the studies based on the APOE theory, which consists of three stages or phases: 1) theoretical analysis, to establish a genetic decomposition (GD), which describes a path towards the construction of a mathematical concept and serves as a tool to evaluate learning; 2) the design and implementation of an instructional treatment, which is based on the constructions and mental mechanisms described in the GD, in a group of 31 students of the logistics and transportation engineering career; and 3) the data collection and analysis stage, in which information is obtained to verify the mental constructs described in the GD, achieved by the students and how well they learned the concepts in question (Arnon et al. , 2014).

In the poster to be showcased at the 48th Conference of the International Group for the Psychology of Mathematics Education, the research trajectory and findings of the study on students' conceptions of solution set of SLE will be presented.

References

Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Roa Fuentes, S., Trigueros, M., & Weller, K. (2014). *APOS Theory: A Framework for Research and Curriculum Development in Mathematics Education* (1a ed.). Springer.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 381. PME.

MOORING PHASE: EMBODIED PROOF IN STEM'S GEOMETRY

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This study's purpose is to examine how concreteness fading (Fyfe & Nathan, 2019), compared to traditional instruction, influences *conceptual understanding* of geometric proof. According to an embodied cognition (Barsalou, 2008) theoretical framework, engaging students' body-based sensorimotor systems, such as manipulating concrete representations, enhances *conceptual understanding* of abstract concepts, like mathematical theorems. The research question is: How do learners utilize concrete and symbolic representations to *conceptually understand* proof in STEM's geometry?

METHOD

Participants: non-math majors (N=4); U.S. university; volunteered, $40 compensation. Materials: a) concrete representations of construction paper shapes (invert and adjoin two identical triangles along common side, overlay symmetrical parallelogram, fold parallelogram along diagonal to demonstrate original triangle's median); b) symbolic representations of formulas (identify coordinates of given triangle's vertices, use coordinates in midpoint formula to locate intersection of corresponding median). Design: random assignment; concreteness fading condition (n=2, concrete then symbolic representations); traditional condition (n=2, symbolic representations). Tasks: watch videos of constructions/justifications for proof using concrete or symbolic representations and mimic with same materials. Procedures: pre-test (equilateral), training (isosceles), post-test (equilateral), novel test (scalene). Measures: coded by historical-cultural mathematical proof (Harel & Sowder, 2007).

RESULTS AND EDUCATIONAL IMPLICATIONS

3 of 4 participants improved from empirical proof (pre-test) to superior deductive proof (post-test); 3 of 4 achieved conceptual understanding (novel test). Both concreteness fading participants demonstrated Mooring Phase: drew a physical structure attached (moored) to given triangle emulating their embodied training – reinforcing importance of teaching geometry through embodied concrete representations before symbolic.

References

Barsalou, L. W. (2008). Grounded cognition. *Annual Review of Psychology, 59*, 617-645.

Fyfe, E. R., & Nathan, M. J. (2019). Making "concreteness fading" more concrete as a theory of instruction for promoting transfer. *Educational Review, 71*(4), 403-422.

Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 805-842). Charlotte: Information Age Publishing.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 382. PME.

THE CURRICULAR NOTICING OF PROSPECTIVE MATHEMATICS TEACHERS WHEN MODIFYING A HYPOTHETICAL LEARNING TRAJECTORY

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Our study characterizes prospective mathematics teachers' curricular noticing when modifying a hypothetical learning trajectory (HLT) (Simon, 2020) while planning a lesson on quadratic functions within design research. Curricular noticing is the process of making sense of the content and pedagogical opportunities of curricular resources (Amador et al., 2017).

In the preparation phase, two groups of prospective mathematics teachers modified the HLT when planning a lesson. In the retrospective analysis and redesign phase, the prospective teachers reflected on implementing the learning activities and changed the HLT when planning a lesson on quadratic functions. We analyzed written documents and semi-structured interviews about the changes in the HLT by the prospective mathematics teachers in terms of the curricular noticing skills (attending, interpreting, and deciding).

The results show that the aspects attended by the prospective mathematics teachers focused on the following: (i) characterization of artifacts, (ii) incorporation/elimination of information and formulation of specific questions, and (iii) anticipation of students' difficulties, mistakes, and actions. They decided to leave the learning objective of the HLT unchanged, but they modified the learning activities related to focuses of attention (i) and (ii). Furthermore, they suggested including aspects of the focus of attention (iii) to the hypothetical learning process. The prospective mathematics teachers demonstrated their interpretative skills by justifying the changes made to the HLT.

Our findings suggest that using HLTs as curricular resources supports the development of curricular noticing, enabling prospective teachers to attend to relevant aspects, interpret mathematical opportunities, and make informed pedagogical decisions. This contributes to understanding how prospective teachers develop curricular noticing skills in lesson planning.

References

Amador, J.M., Males, L.M., Earnest, D. & Dietiker, L. (2017). Curricular noticing: Theory on and practice of teachers' curricular use. In E., Schack, M., Fisher y J. Wilhelm (eds.), *Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks. Research in Mathematics Education* (pp. 427-443). Springer, Cham.

Simon, M. (2020). Hypothetical Learning Trajectories in Mathematics Education. In: Lerman, S. (Ed.), *Encyclopedia of Mathematics Education* (pp. 354-357). Cham: Springer International Publishing.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 383. PME.

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EXPLORING TEACHER STATISTICAL KNOWLEDGE USING AI: AN ANALYSIS WITH CHATGPT

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Statistics and Data Science are essential across various fields, including education (cf. Ben-Zvi & Aridor-Berger, 2016). Investigating the knowledge of teachers who teach statistics is key to strengthening statistical reasoning processes. The STSK framework (Vidal-Szabó & Estrella, 2020), an extension of the MTSK framework, distinguishes between Statistical Knowledge (SK) and Pedagogical Content Knowledge (PCK). However, identifying this knowledge in authentic teaching contexts remains a methodological challenge. This study explores the potential of artificial intelligence (AI), specifically ChatGPT, as a systematic method to detect patterns of knowledge in complex qualitative datasets, thus contributing to research on teacher education and discourse analysis in mathematics education.

Two cycles of Lesson Study were carried out with four Chilean teachers (grades K–3) focused on the teaching of statistics and informal inference. A transcribed session was analyzed using ChatGPT, which had been previously trained on theoretical descriptions of each domain and subdomain of the STSK. A custom prompt guided the model to classify fragments of teacher discourse according to STSK categories.

The model generated 103 classifications: 23 corresponding to the SK domain, 60 to the PCK domain and 20 as 'Unclassified'. ChatGPT assigned the latter label to discourse segments that did not align with the descriptions provided by the researchers. This result demonstrates ChatGPT's ability to identify traces of teaching knowledge without generating subjective classifications, thereby strengthening the data analysis that had been carried out manually, prior to the use of the software. The use of ChatGPT also revealed that primary school teachers who teach statistics mainly exhibited knowledge linked to the PCK domain, highlighting the need to strengthen their disciplinary (SK) knowledge. These findings are consistent with the manual analysis conducted by the research team, highlighting the potential of AI-based tools as methodological resources in mathematics education research.

References

Ben-Zvi, D., & Aridor-Berger, K. (2016). Children's wonder how to wander between data and context. In D. Ben-Zvi & K. Makar (Eds.), *The teaching and learning of statistics* (pp. 25–36). Springer. https://doi.org/10.1007/978-3-319-23470-0_3

Vidal-Szabó, P., & Estrella, S. (2020). Extension of the MTSK model to the statistical domain. In Y. Morales-López and Á. Ruíz (Eds.), *Educación Matemática en las Américas 2019* (pp. 1036–1042). Inter-American Committee on Mathematics Education. https://conferencia.ciaem-redumate.org/index.php/xvciaem/xv/paper/viewFile/692/327

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 385. PME.

SUPPORTING THE MATHEMATICS TEACHER EDUCATORS' PRACTICAL KNOWLEDGE: SOME PIECES OF EVIDENCE

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PURPOSE AND THEORETICAL PERSPECTIVE

Exploring how teacher educators and mathematics teacher educators (MTEs) use their knowledge (Chapman, 2021) contributes to advancing teacher and mathematics teacher education. We are an interdisciplinary group of teacher educators who worked alongside a group of MTEs in researching their practices for the first time. We utilized practical knowledge (PK), defined as "the experiential, moral, emotional, embodied knowledge" (Clandinin et al., 2009, p. 141), to illustrate how we supported the MTEs in conducting and publishing their research on mathematics teacher education.

DATA COLLECTION AND ANALYSIS

We recorded our conversations and wrote narratives about our motivations for working with the MTEs. We also analyzed videos of our editorial meetings and meetings with the MTEs. We used conversations (Guilfoyle et al., 2004) as an analytical tool to identify our collective PK when supporting the MTEs in conducting their studies.

PRELIMINARY RESULTS AND DISCUSSION

We encouraged the MTEs to identify moments in their practice that revealed challenges or puzzlement. We discussed and analyzed the MTEs' moments through open conversations among us and with them. We fostered MTEs' new perspectives in their practice by offering artistic activities and readings. We encouraged the MTEs to use mathematics education literature. Our collective use of PK might inform MTEs in conducting similar endeavours. This study answers calls (Chapman, 2021) to document how MTEs use their knowledge in practice.

References

Chapman, O. (2021). Mathematics teacher educator knowledge for teaching teachers. In M. Goos & K. Beswick (Eds.), *The learning and development of mathematics teacher educators* (pp. 403–416). Springer. https://doi.org/10.1007/978-3-030-62408-8

Clandinin, D. J., Downey, C. A., & Huber, J. (2009). Attending to changing landscapes: Shaping the interwoven identities of teachers and teacher educators. *Asia-Pacific Journal of Teacher Education, 37* (2), 141–154. https://doi.org/10.1080/13598660902806316

Guilfoyle, K., Hamilton, M. L., Pinnegar, S., & Placier, P. (2004). The epistemological dimensions and dynamics of professional dialogue in self-study. In J. J. Loughran, M. L. Hamilton, V. K. LaBoskey, & T. Russell (Eds.), *International handbook of self-study of teaching and teacher education practices* (pp. 1109–1167). Springer.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 386. PME.

FOSTERING TEACHER QUESTIONING THROUGH VIDEO ANNOTATION

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INTRODUCTION

Teacher noticing (Jacobs et al., 2011) is a framework to understand how teachers make decisions in the classroom, including where teachers place their attention, how they understand what they attend to, and what they act upon. With this study, we sought to examine the impact of a six-session video club on teachers' practice. Preservice teachers (PSTs) participated in six video clubs that introduced them to the tool, encouraged them to *tag* moments of student thinking they found interesting. Four teacher candidates participated in the study. As an outcome, we examined the 645 questions they asked over the course of six sessions of an after-school teaching program for middle schoolers.

After the initial three video club sessions, questions focused on student thinking made up a consistent one-third of all questions asked by PSTs throughout the after-school program. These questions about student thinking increased in proportion to total questions asked, which increased over the course of the program. Teachers' questions were mostly disjoint (i.e., did not sustain focus on an element of student thinking) until midway through the program, after which point we saw several examples of PSTs using a series of questions to sustain focus on particular student strategies, ideas, or other methods of solving problems.

Our results suggest that a small intervention is enough for PSTs to start attending to and asking about student thinking, since they do this in a small volume after just three sessions of the video club. However, further sessions of the video club and experience with the after-school program seemed to help them grow confident in asking a greater volume of questions, including those focused on student thinking. Instances where PSTs sustained focus on student thinking through a series of related questions were rare, but did appear towards the end of the program, suggesting that this is a skill requiring more practice and time.

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References

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. Journal for Research in Mathematics Education, 41(2), 169–202.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 387. PME.

THE INFLUENCE OF GRADE 12 LEARNERS' CREATIVE THINKING SKILLS ON THEIR PROBLEM-SOLVING ABILITIES IN SOLVING QUADRATIC FUNCTIONS

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Over the years, studies on critical thinking and problem-solving have reported learners' difficulties with quadratic functions (Bannister, 2014; Abdul Rahman, & Mohamad, 2021). For example, these authors report that quadratic equations pose several obstacles for learners, including difficulty with mathematical processes, especially factoring, and difficulty applying meaning to the equations. This paper responds to the question: How do grade 12 learners' creative thinking skills influence their problem-solving abilities when solving problems involving quadratic functions? Creative thinking skills involve use of (i) fluency, (ii) flexibility, (iii) novelty and (iv) elaboration. Creative thinking entails problem-solving, which includes initiatives to generate, implement, and lead to new ideas. This study draws from the Creative Problem-Solving (CPS) model, and constructivist learning theories which emphasize active learning and engagement. In this study learners were provided with opportunities to explore, experiment, and collaborate, and take ownership of different ways of solving quadratic equations. A purposive sample of 20 grade 12 learners in groups of 4 participated in a qualitative study in which structured worksheets covering problem solving of quadratic equations. This was followed by focus group interviews and discussions in which learners explained their thought processes that led to solutions given in the test. Data were collected through classroom observations, focus group interviews together with structured worksheets.

Findings revealed the computational tools used by learners when solving quadratic functions and provided insights into their thought processes. The problem-solving approaches used by learners were dominated by fallacious and pseudo propositions, marked by incorrect computations. This problem-solving approach challenged the learners such that they were curious, motivated, and persistent in their pursuit of mathematical knowledge and solved complex quadratic function problems.

References

Bannister, V. R. P. (2014). Flexible conceptions of perspectives and representations: an examination of pre-service mathematics teachers' knowledge. *International Journal of Education in Mathematics, Science and Technology*, *2*(3).

Abdul Rahman, T. F., & Mohamad Foad, M. S. (2021). Quadratic functions in additional mathematics and mathematics: an analysis on students' errors. *Academic Journal of Business and Social Sciences (AJoBSS), 5*(1), 1-16.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 388. PME.

IDENTIFYING CHARACTERISTICS OF ANSWER PATTERNS FOR JAPANESE SECONDARY SCHOOLERS IN PISA 2018

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INTRODUCTION

While PISA and TIMSS have influenced mathematics education in Japan, most comparative analyses rely heavily on average scores or correct response rates. This study applied IRT, focusing on latent ability to analyse item difficulty patterns among 13 countries/regions in PISA 2018. The study aimed to uncover distinct national characteristics in mathematical literacy, specifically among Japanese students.

METHODS

Using R (ver. 4.2.1), data from 64 PISA 2018 maths items were analysed through the Rasch model. The difficulty values were equated using the mean-sigma method to allow cross-national comparison. Countries included in the comparison were Japan, Australia, Canada, Finland, France, Germany, Ireland, Italy, South Korea, New Zealand, the USA, Netherlands, and Hong Kong. Difficulty patterns were analysed in four PISA domains: 'space and shape', 'uncertainty and data', 'change and relationships', and 'quantity'.

RESULTS AND DISCUSSION

Japan ranked second highest in mean mathematical literacy scores, following Hong Kong. The difficulty patterns of Japan closely resembled those of South Korea. These findings may reflect curriculum similarities and shifts following the 2017 curriculum revision. Notably, the standard deviation of difficulty in the domain 'change and relationships' was low for Japan, while it was high in 'quantity', suggesting less variation in the former and more in the latter.

CONCLUSION

Japan's high achievement and distinctive difficulty patterns in PISA 2018 reflect unique aspects of its educational context. Notably, the consistency in 'change and relationships' and the variability in 'quantity' highlight areas of strength and diversity. The resemblance with South Korean results suggests curricular alignment.

Additional information

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AN EXAMINATION OF IMAGE CONTEXTUALITY AND TYPES OF TASKS IN CHILEAN PRIMARY EDUCATION TEXTBOOKS

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Textbook images are vital for math understanding; quality visuals ensure effective material selection. This work offers an in-depth exploration of one aspect from a broader study that examines images in Chilean textbooks using Kim's (2012) framework. We focus on contextuality—how images relate to real-life contexts—and its relationship with task types, as classified by van Zanten and van den Heuvel-Panhuizen (2018): simple tasks, grey-area tasks, and non-routine problems. In Chile, publicly tendered textbooks (PT)—distributed free to 90% of schools—are developed under government guidelines, emphasizing real-life contextualization. In recent years, an alternative adapted mathematics textbook, based on the Japanese teaching approach (authored textbook, AT) has also been used, offering a contrasting framework. The difference between these texts was also included as a study variable.

The study's objectives are: 1) Analyze the relationship between textbook type and the level of image contextuality; 2) Analyze the relationship between textbook type and the tasks associated with images; 3) Explore the relationship between contextuality and the types of tasks in each textbook.

We analyzed three textbooks used in recent years: two PT textbooks and one AT textbook. Each unit of analysis comprised an image paired with its corresponding task. We evaluated image contextuality using Kim's criteria and classified tasks per van Zanten and van den Heuvel-Panhuizen's categories, examining 996 units in total. Data indicate that PT textbooks exhibit 46.9% of images with high contextuality, compared to 35.7% in AT. Moreover, AT shows a higher percentage of images with low contextuality (38.1% vs. 30.8% in PT), suggesting that the mandatory guidelines in PT promote greater contextual integration. In PT, 72.8% of tasks are simple, with non-routine problems rarely recorded (1.0%), whereas AT has fewer simple tasks (58.8%) and more grey-area tasks (36.6%) and non-routine problems (4.6%), evidencing greater task diversity. The results suggest that a high level of contextuality, predominant in PT, is associated with simple tasks, while in AT, where contextuality is lower, there is a higher inclusion of grey-area tasks and non-routine problems, implying that the need to contextualize images could limit the complexity of the problems presented.

References

Kim, R. Y. (2012). The quality of non-textual elements in mathematics textbooks: an exploratory comparison between South Korea and the United States. *ZDM, 44*, 175-187.

Van Zanten, M. y van den Heuvel-Panhuizen, M. (2018). Opportunity to learn problem solving in Dutch primary school mathematics textbooks. *ZDM*, *50*(5), 827-838.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 390. PME.

MATHEMATICS TEACHERS' SELF-EFFICACY BELIEFS ON REFLECTION SKILLS: A CASE STUDY

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Mathematics teachers' self-efficacy beliefs (MTSEB) significantly influence instructional quality and student learning processes (Lee & Smith, 2022). However, most studies have focused on one-off training contexts, which often overlook long-term professional development. Addressing this gap, the study aim to examines MTSEB regarding their reflection skills using a case study methodology. Three cases were developed within a Mathematics Education program that integrates a Multi-level Professional Learning Community (PLC) with the Transformative Lesson Study incorporated with Open Approach (TLSOA) model (Inprasitha, 2022). Participants gained experience through supervised classroom observations, participation in Lesson Study (LS) teams during the National Open Class, micro-teaching, and structured reflection, all conducted within the Multi-level PLC framework. Data were collected using self-assessments on a scale from 0 to 5, supplemented by semi-structured interviews and reflection after each Open Class session.

The results revealed improvements in all three teachers, who had 4, 5, and 6 years of experience in PLC, respectively. The MTSEB scores increased from 1 to 3.5 in both cases 1 and 2, and from 1 to 4 in case 3. This indicates progress in their skills for managing and reflecting on mathematical ideas, as well as in their ability to anticipate students' ideas and assess student learning outcomes. Reflection enabled each teacher to recognize the importance of preparation, participate in LS team, and develop their learning at each level of the multi-level PLC. Additionally, the teachers demonstrated greater confidence and clearer skills in analyzing students' mathematical ideas. However, there are still areas for further development, such as the use of language (keywords) and the addition of conditions to create more challenging problem situations.

This study highlights that increased MTSEB leads to greater confidence, which in turn enhances their ability to improve mathematics classrooms focused on mathematical thinking. The research demonstrates that as teachers develop stronger self-efficacy, they implement more effective instructional strategies that prioritize students' mathematical ideas and problem-solving processes.

References

Inprasitha, M. (2022). Lesson Study and Open Approach Development in Thailand: A Longitudinal Study. *International Journal for Lesson and Learning Studies, 11*(5), 1-15.

Lee, C., & Smith, J. (2020). Self-Efficacy Beliefs in Mathematics Pedagogy. *Teaching and Teacher Education, 95*, 103-124.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 391. PME.

CONSTRUCTION OF CATEGORIES FOR THE TASKS DESIGN THAT INTEGRATE GEOMETRY AND ART

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Interdisciplinarity addresses complex real-world problems by integrating multiple disciplines to create comprehensive solutions. Integrating geometry and art contextualizes mathematics, linking geometric concepts to skills like creativity, reasoning, and visualization (Diego-Matecón et al., 2019). However, designing integrative tasks for mathematics education and teacher training remains challenging (Diego-Matecón et al., 2019). This study seeks to identify categories for task design from an interdisciplinary perspective, activating a *Complete Mathematical Working Spaces* (Kuzniak et al., 2022) by integrating geometry and art through transitions between 2D and 3D figures and linear perspective.

The theoretical framework is grounded in the theory of Mathematical Working Spaces, which characterizes epistemological and cognitive elements organized into two planes: the cognitive plane, which includes visualization, construction, and proof; the epistemological plane, which encompasses representamen, artifacts, and referential (Kuzniak et al., 2022).

This study adopts a deductive approach, using documentary analysis to interpret historical documents, scientific articles, and curricular resources, including teacher training programs and textbooks. Qualitative content analysis identifies relevant categories for task design and their implementation in teacher training.

The study aims to identify categories for designing tasks that integrate geometry and art. This work is part of an ongoing doctoral thesis, contributing to the advancement of mathematics teacher training.

Additional information

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References

Diego-Mantecón, J. M., Blanco, T. F., Búa Ares, J. B., & González Sequeiros, P. (2019). Is the relationship between art and mathematics addressed thoroughly in Spanish secondary school textbooks? *Journal of Mathematics and the Arts*, *13*(1–2), 25–47. https://doi.org/10.1080/17513472.2018.1552068

Kuzniak, A., Montoya-Delgadillo, E., & Richard, P. R. (2022). *Mathematical Work in Educational Context*. Springer Cham. https://doi.org/10.1007/978-3-030-90850-8

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 392. PME.

PERCEPTIONS OF FUTURE MATHEMATICS TEACHERS AFTER EXPERIENCING SELF-LEARNING MODULES ON AI

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This poster shows prospective mathematics teachers' perceptions of AI in a teaching practice course,. It explores the factors that influence their intention to learn and use this technology in their practices. Understanding these perceptions is crucial, as they can significantly shape the future integration of AI tools in mathematics classrooms, potentially transforming pedagogical approaches and student learning experiences.

THEORETICAL FOUNDATION

Sanusi et al. (2024) used the model of planned behavior (TPB) (Ajzen, 1991) to explore the factors that influence prospective teachers' intention to learn AI. This poster shows attitude toward behavior, subjective norm, and perceived behavioral control based on the TPB of prospective mathematics teachers at a Chilean university.

METHODOLOGY

A test (Sanusi et al., 2024) was applied that measured prospective teachers' intention to learn about AI after experiencing with self-learning modules for 6 weeks about AI in their initial teacher education. The questionnaire with 8 sections was based on TPB and other constructs, such as basic AI knowledge, AI anxiety, personal AI relevance, self-efficacy to learn AI, AI for social good, and self-transcendence goals.

RESULTS

The tendency of the responses of the 22 Future Mathematics Teachers was framed in favor of teaching AI to their students, having a good perception of it in education. Highlighting their majority of perceptions, strongly agree with the personal relevance of AI (0.48), self-transcendent goals (0.34); and Behavioral intention to learn AI (0.39) respectively with respect to the total responses per section. In no section was there any tendency to disagree that dominated the perceptions. It is posited that AI is a potential working tool for teachers, so they should be literate in its effective use in the classroom.

Additional information

Acknowledgments to ANID/Exploración 13240075; ANID/PIA/Fondos Basales para Centros de Excelencia FB0003; VINCI 039.493/2024; ANID Becas/Doctorado Nacional 21241378.

References

Ajzen, A. (1991). The theory of planned behavior. Organizational Behavior and Human Decision Processes, 50(2), 179-211. https://doi.org/10.1016/0749-5978(91)90020-T

Sanusi, I., Ayanwale, M., y Tolorunleke, A. (2024). Investigating pre-service teachers' artificial intelligence perception from the perspective of planned behavior theory. *Computers and Education: Artificial Intelligence, 6*(1), 100202.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 393. PME.

PROSPECTIVE MATHEMATICS TEACHERS' PERFORMANCE ON COMPLEX NUMBERS TASKS: REVEALING THE REAL NUMBER BIAS

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Recent studies have identified a cognitive phenomenon vaguely discussed in the literature, the real number bias (Kontorovich, 2018), in which learners extrapolate concepts and properties valid in the real domain to the complex domain where they are not valid anymore.

To study the manifestation of this bias and the performance of prospective mathematics teachers on complex number tasks, a questionnaire consisting of 14 tasks (T) was administered to 85 future teachers. This questionnaire included four groups of tasks: i) definition and representation (T1, T5); ii) real-complex context (T2, T3, T4); iii) operations (T6, T7, T8, T9, T10, T13); and iv) properties and order (T11, T12, T14). The questionnaire was analyzed through a confrontation of a priori and a posteriori perspectives in terms of the correctness of each task.

The results indicate that 65 out of the 85 participants answered more than half of the tasks in the questionnaire, although most scored 14 or less out of 29, showing incorrect responses. In fact, in eight tasks, the real number bias is observed. These tasks include T5, where $\sqrt{-1}$ is presented on the real axis; T2, where $\sqrt[4]{1}$ is considered a function of real variable; T3, where $f(x) = x^2 - 4$ $(x \in \mathbb{C})$ is represented by a parabola in the complex plane; T4, where $z = \pm 5$ is indicated as a solution of $|z| = 5$ $(z \in \mathbb{C})$; T9, where $z = (3,2)$ and $w = (1,2)$ are multiplied through a dot product; T12, when $-1 - i$ is considered a negative number; T13, where the function $f(x) = ix$ is represented as a line passing through the origin of the complex plane with a slope of i; and T14, where the inequalities $i < 2i < 1 + 2i < 2 + 2i$ are incorrectly established as true. Additionally, in T2, there is an inappropriate extension in the reverse sense when $\sqrt{-4}$ is presented in the real numbers and is considered from the perspective of complex numbers. Based on these results, it is hypothesized that teaching centered on a nested view of real and complex numbers may be influencing the manifestation of the real number bias, which would necessitate further studies.

Additional information

Acknowledgements. ANID fellowship Doctorado Nacional/ 2019-21191199, ANID projects Basal/FB0003 and Support 2024/AFB240004

References

Kontorovich, I. (2018). Undergraduates' images of the root concept in R and C. *The Journal of Mathematical Behavior*, *49*, 184-193.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 394. PME.

FOSTERING ENGAGEMENT AND CLASSROOM CLIMATE THROUGH COLLABORATIVE PROBLEM-SOLVING: INSIGHTS FROM THE ARPA PEDAGOGICAL INNOVATION

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Collaborative Problem Solving is a cross-cutting competence embedded in 21st-century skills (Saadati & Felmer, 2021). The ability to collaborate is a central competence to prepare students for the social and work realities they will face in 21st century (Liebech-Lien & Sjølie, 2021). In this sense, intrapersonal and interpersonal skills in learning have become essential (Lee, 2022). This study aimed to evaluate the impact of the ARPA mathematics initiative (Spanish acronym for Activating Problem-solving in Classrooms) on engagement in mathematics, classroom climate, and sense of school community.

A pre-post evaluation was conducted with a sample size of 196 students from upper elementary and secondary education in four public schools in Santiago, Chile. Data collection included measurements taken before and after ARPA's implementation to evaluate changes in the selected constructs. The analysis was conducted using mean difference tests, comparing the results between students who participated in ARPA activities in the mathematics class and those who did not.

Analyses showed that students who participated in problem-solving activities in math improved their perception of the classroom climate compared to their peers who did not participate. Additionally, students who got involved in ARPA mathematics activities maintained their level of engagement in math, while those who did not participate showed a decrease in this indicator. No significant differences were found regarding the sense of community. These findings suggest that the ARPA initiative contributes positively to the learning environment and students' engagement with mathematics. This also contributes to the broader understanding of the socio-emotional and interpersonal factors of collaborative mathematics learning (Lee, 2022).

References

Lee, Y.-J. (2022). Promoting social and emotional learning competencies in science, technology, engineering, and mathematics project-based mathematics classrooms. *School Science and Mathematics*, *122*(8), 429-434. https://doi.org/10.1111/ssm.12557

Liebech-Lien, B., & Sjølie, E. (2021). Teachers' conceptions and uses of student collaboration in the classroom. *Educational Research*, *63*(2), 212-228. https://doi.org/10.1080/00131881.2020.1839354

Saadati, F., & Felmer, P. (2021). Assessing impact of a Teacher professional development program on student problem-solving performance. *ZDM – Mathematics Education*, *53*(4), 799-816. https://doi.org/10.1007/s11858-020-01214-1

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 395. PME.

SUBLIMINAL MESSAGING IN MATH SONGS CREATED BY PRESERVICE MATHEMATICS TEACHERS

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Subliminal messages (SM), those designed to pass below the normal limits of conscious perception, are now studied for their potential in learning. Zdravkovska et al. (2021) argue that priming techniques have the potential to enhance memory and motivation. Their findings suggest that the use of subliminal stimuli can improve cognitive processes, thereby supporting the achievement of educational objectives.

In this PP we present initial results of an ongoing research that aims to investigate how preservice mathematics teachers explore SM in the process of creating math songs with artificial intelligence (AI). The study conceives the production of math songs as an aesthetic mathematical experience (Gadanidis et al., 2016). Methodologically, we conducted 16 hours of teaching experiments with two pairs of preservice mathematics teachers of a Brazilian public university (4 sessions of 2 hours with each pair).

The first two sessions focused on exploring figurate numbers through the use of manipulatives. Participants examined sequences and series of natural, odd, and even numbers linked to triangular, square, and rectangular representations, extending their investigation to pyramidal and cubic figurate numbers. They conjectured results for Σr, Σr^2, and Σr^3, emphasizing the surprising aspects of pattern identification, generalization, and geometric representation. In the final two sessions, participants engaged in creating math songs about AI tools by generating lyrics with ChatGPT, incorporating conceptual mathematical inputs derived from earlier sessions. Using Suno.AI, they transformed these lyrics into songs and developed a "road map" to integrate subliminal sounds aligned with the lyrical content. Through Logic Pro X, they embedded subliminal auditory messages, including whispers of number sequences and keywords, Morse code, instrument sounds, natural setting noises, and participant voices. The approach highlighted how AI-driven creativity and subliminal messaging can innovate mathematics education by enhancing engagement, reinforcing learning, and creatively conveying mathematical concepts through subtle auditory elements.

References

Gadanidis, G., Borba, M. de C., Hughes, J., & Lacerda, H. D. (2016). Designing aesthetic experiences for young mathematicians: a model for Mathematics Education reform. *Revista Internacional de Pesquisa Em Educação Matemática, 6*(2), 225-244.

Zdravkovska, S. D., Haque, S., & Szydlak, W. (2021). Subliminal Messages-Motivating Learners Using Visual Cues: An Experimental Study. *Balkan Social Science Review, 18*, 367-385.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 396. PME.

ENHANCING TEACHERS' ANTICIPATION SKILLS THROUGH TLSOA AND THE 4D FRAME

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Most teacher training programs in Thailand have traditionally focused more on enhancing teachers' knowledge rather than developing practical teaching skills. However, this approach has proven insufficient. Teachers need not only to acquire new ideas and knowledge but also to enhance their teaching skills. A key challenge for teacher educators is identifying the most effective relationship between content development and teacher training programs to improve classroom teaching practices. This study explores how the 4D frame (Park, 2023) can be integrated into Transformative Lesson Study incorporated with Open Approach (TLSOA) (Inprasitha, 2022) to support the development of teachers' anticipation skills—an essential competency for effective classroom teaching. The research employs a qualitative design, using case studies from three teacher training programs involving students, pre-service teachers, and in-service teachers. Data were collected through anticipation skills surveys, classroom observations, and teacher reflections.

The findings reveal three key challenges: (1) The 4D frame, used as a problem situation related to geometry and dimensions in teacher training, incorporates content development within each cycle, requiring improvements keywords in problem conditions, teaching materials, and flow of lesson within the Open Approach. (2) Engaging in problem-solving by themselves helps teachers develop anticipation skills, progressing from general anticipation to a more focused understanding of students' ideas. (3) Teachers apply their anticipation of students' ideas as a lens to observe students' thinking intentionally, allowing them to reflect and refine the problem situation in subsequent cycles. These findings offer valuable insights into improving teacher education by demonstrating how TLSOA can bridge the gap between content development and teacher training, emphasizing the importance of developing teachers' skills alongside their knowledge.

References

Inprasitha, M. (2022). Lesson study and open approach development in Thailand: A longitudinal study. **International Journal for Lesson and Learning Studies**, 11(5), 1–15. https://doi.org/10.1108/IJLLS-04-2021-0029

Park, H. (2023, October 31). **4D frame**. Presentation at the UNITWIN Cooperation Programme for Quality Teacher Education Workshop, 4D Math and Science Creativity Researcher Institute, South Korea.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 397. PME.

BUILDING A MPP TASK FOR TEACHING AND LEARNING LINEAR TRANSFORMATION

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International research indicates that the difficulties in Linear Algebra Teaching and Learning are closely related to the topic abstract nature, as well as to historical, epistemological, linguistic, and representational aspects. These challenges have led to the development of diverse theoretical perspectives on how students engage with and learn Linear Algebra concepts. While several researchers have investigated Mathematical Problem Posing (MPP), studies focusing on the use of MPP within the advanced mathematics teaching and learning are still at an early stage. In this context, our guiding research question is: How could a MPP task be designed to support the Linear Transformation teaching and learning? This study has adopted a qualitative approach, analyzing the research participants' responses and two workshop feedback. The MPP task used in the workshops was developed based on a comprehensive literature review and consisted of an unstructured problem situation within an advanced mathematical context, accompanied by an open-ended prompt (Baumanns & Rott, 2021; Cai & Hwang, 2023). Workshop 1 was conducted in October 2024 with six in-service mathematics teachers, with a two-hour workload, and Workshop 2 took place in November 2024 with eight licentiate degree in Mathematics students, also with a two-hour workload, Brazil. Results include: (1) difficulties in understanding the notion of "problem posing," (2) challenges related to the content of Linear Transformation, (3) research participant feedback enabled categorization of the posed problems, and (4) collaborative discussions around problem-reformulation and posing process. These insights led to improvements to the MPP task, including a narrative-context development – inviting participants to create an assessment activity based on three given vectors – and a revised, closed prompt (Cai et al., 2024). Such results have contributed to identifying of response mathematical-complexity subcategories and levels.

References

Baumanns, L., & Rott, B. (2021). Rethinking problem-posing situations: A review. *Investigations in Mathematics Learning*, 13 (2), 59-76.

Cai, J., & Hwang, S. (2023). Making mathematics challenging through problem posing in the22, classroom. In *Mathematical challenges for all* (pp. 115-145). Cham: Springer International Publishing.

Cai, J., Koichu, B., Rott, B., & Jiang, C. (2024). Advances in research on mathematical problem posing: Focus on task variables. *The Journal of Mathematical Behavior*, 101186.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 398. PME.

AFFINE FUNCTION IN GEOGEBRA: IMPACT OF AUTOMATIC FEEDBACK ON ACTIVITY PROPOSALS

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With technological advances and the growing search for new teaching practices that provide more attractive classes, this study aims to present how activities with automatic feedback built using GeoGebra helped students to identify their mistakes and overcome obstacles, reducing their mistakes throughout the tasks.

Ten activities were built with automatic feedback for the learning of affine functions based on gamification techniques in order to promote greater engagement of the participants. Four second-year high school students from a public school in the state of Rio de Janeiro, Brazil, participated in the research. The tasks were carried out individually, in the school's laboratory and with a duration of two hours.

As a theoretical contribution, the Theory of Registers of Semiotic Representation, by Duval (2012), was used, which associates the mathematical object with its various representations for a better understanding of the student. Among the main representation registers worked, algebraic, natural language, tabular and graphic stand out.

The results showed that the automatic feedback contributed to the reduction of students' errors, who after receiving suggestions, were able to understand their difficulties and get it right in the following attempts. Among the main results, the conversion from the graphic register to the algebraic register stands out, in which the feedbacks reduced by up to 44% the number of errors of the students at the end of the activity in relation to the initial state.

The gamified activities provided playful and fun learning, capable of motivating the students, who claimed to enjoy learning this way. The use of GeoGebra enabled the participants to perform the conversion between the representation registers in a dynamic way, contributing to the mobilization of mathematical knowledge.

References

Duval, R. (2012). Registros de representação semiótica e funcionamento cognitivo do pensamento. Tradução de M. T. Moretti. *Revemat: Revista Eletrônica de Educação Matemática*, *7*(2), 266-297.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 399. PME.

REASONING IN TEXTBOOKS POST-CURRICULUM REFORM

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This study investigates how Brazilian high school mathematics textbooks (TB) address different types of reasoning in chapters on Spatial Geometry. We compare five collections adopted in two government evaluation cycles: the last before (2018) and the first after (2021) the curricular changes. It is important to contextualize that Brazil has nearly 86% of high school students enrolled in public schools, and for these, the government runs a program that distributes TB free of charge.

The analysis draws on the framework proposed by Sevinc et al. (2022), which integrates earlier studies (e.g., Stylianides, 2009; Bieda et al., 2014) for examining forms of reasoning. In terms of methodology, the TB were analyzed following the approach of the Theorem System (Amaral et al., 2022), coding for the presence of reasoning and conducting a qualitative analysis.

Findings reveal a predominance of one-step deductive reasoning and inductive generalizations based on specific cases suggesting that students are offered limited opportunities to engage with diverse forms of reasoning, reinforcing a view of mathematics as a finalized structure. The study offers perspectives for TB authors, teachers, and researchers by highlighting how TB tends to prioritize conventional forms of mathematical presentation over the reasoning processes involved in the development of mathematical ideas.

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References

Amaral, R. B., Mazzi, L. C., Andrade, L. V., & Perovano, A. P. (2022). *Livro didático de matemática: compreensões e reflexões no âmbito da educação matemática*. Mercado de Letras.

Bieda, N. K., Ji, X., Drwencke, J., & Picard, A. (2014). Reasoning-and-proving opportunities in elementary mathematics textbooks. *International Journal of Educational Research, 64*, 71–80. https://doi.org/10.1016/j.ijer.2013.06.005

Sevinc, S., Kohanová, I., Isiksal-Bostan, M., et al. (2022). Developing an integrated framework for analyzing ways of reasoning in mathematics. *In Proceedings of ICERI2022* (pp. 2082–2089). https://doi.org/10.21125/iceri.2022.0529

Stylianides, G. J. (2009). Reasoning-and-proving in school mathematics textbooks. *Mathematical Thinking and Learning, 11*(4), 258–288. https://doi.org/10.1080/10986060903253954

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 400. PME.

STUDENTS' MATHEMATICAL HABITS OF MIND IN MATHEMATICS CLASSROOM: PROBABILITY

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Although artificial intelligence (AI) enables quick access to vast amounts of data, there is still limited information on how to effectively manage this data to provide meaningful insights for decision-making. Utilizing statistical and probability skills facilitates problem-solving in various contexts. When students encounter situations where they can use measurements and numbers to determine probabilities, it is crucial to begin with their intuitive thinking or mathematical habits of mind. Mathematical habits of mind are developed and nurtured alongside the acquisition of content knowledge (GAN et al., 2021).

This study aims to explore students' mathematical habits of mind in the mathematics classroom: Probability in the context of TLSOA (Thailand Lesson Study incorporated with Open Approach) (Inprasitha M., 2022). The target group consisted of 12 eighth-grade students who had continuously studied under the TLSOA framework for four semesters. Qualitative research methods were employed to analyze the data. Findings revealed that consistent practice in a problem-solving environment fostered sustainable mathematical habits of mind. Students applied these habits both individually and collaboratively, engaging in discussions to refine reasoning and make informed, evidence-based decisions. The open approach enabled students to solve problems independently using diverse methods, demonstrating adaptability in mathematical thinking. This study provides evidence that systematic and continuous opportunities for analytical thinking cultivate strong mathematical habits. It highlights the importance of the learning context, open teaching methods, and sustained practice as critical factors for intellectual growth. A structured yet flexible approach enhances problem-solving abilities and supports long-term skill development by fostering adaptability, collaboration, and critical thinking, equipping students to address complex challenges effectively and achieve enduring cognitive and academic growth.

References

GAN, T. H., ISODA, M., & TED, K. H. (2021). *Mathematics Challenges for Classroom Practices at the LOWER SECONDARY LEVEL Based on SEAMEO Basic Education Standards: Common Core Regional Learning Standards in Mathematics. SEAMEO RECSAM.*

Inprasitha, M. (2022). Lesson Study and Open Approach development in Thailand: a longitudinal study. *International Journal for Lesson and Learning Studies*, Vol. 11 No.5, pp.1-15.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 401. PME.

LEARNING FROM AN UNSUCCESSFUL INTERDISCIPLINARY COLLABORATION IN MATHEMATICS EDUCATION

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THEORETICAL BACKGROUND

Disciplines are social phenomena evolving and increasing in specialization. Practitioners of disciplines use practices, i.e., ways of being, interacting, and operating (Suazo-Flores et al., 2024) that are distinct from other disciplines and might hamper interdisciplinary interactions (Roth, 2020). Mathematics education researchers (MERs) are called to join interdisciplinary collaborations (Bakker et al., 2021). Yet, little research is conducted to identify what hampers interdisciplinary collaborations. We explored: What practices hampered an MER's interdisciplinary collaboration?

METHODS AND ANALYSIS

Transcripts of interviews constitute the data, collected as part of a larger research program (Suazo-Flores et al., 2024). For this study, we used an existing codebook to identify and analyze practices from a MER's transcript. We selected this data because the MER reported it as an unsuccessful interdisciplinary collaboration. The data were coded independently and compared. Disagreements were discussed and resolved.

FINDINGS AND DISCUSSION

Out of the 49 practices identified in the data, 27 are operating practices, 15 are being practices, and seven are interacting practices. Our findings and the literature allow us to argue that interdisciplinary collaborations may be unsuccessful when group members focus on operating practices (i.e., doing research tasks) and neglect being and interacting practices (i.e., developing language). We will contribute examples of operating practices and suggestions for future MERs' interdisciplinary collaborations.

References

Bakker, A., Cai, J., & Zenger, L. (2021). Future themes of mathematics education research: An international survey before and during the pandemic. *Educational Studies in Mathematics, 107*(1), 1–24. https://doi.org/10.1007/s10649-021-10049-w

Roth, W. M. (2020). Interdisciplinary approaches in mathematics education. *Encyclopedia of Mathematics Education*, 415–419.

Suazo-Flores, E., Walker, W.S., Kastberg, S.E. et al. (2024). Mathematics education researchers' practices in interdisciplinary collaborations: Embracing different ways of knowing. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-024-00489-x

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 402. PME.

SCHOOL-UNIVERSITY PARTNERSHIP AND TECHNOLOGIES USE WITH TEACHERS

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The thematic project, its theoretical assumptions and developments

The thematic project "Teaching and learning Mathematics with calculators: possibilities for teacher practice", approved in process nº 3221/2021 – CCP – FUNDUNESP, develops teacher training activities in different cities in the state of São Paulo, Brazil, and aims to investigate possibilities for the use of calculators in teaching Mathematics in Basic Education. To this end, we established a partnership between the university and the school based on collaboration and experimentation with technologies. Foerste (2004, p. 3) highlights that university-school partnerships occur when professionals from the university and basic school build "new possibilities for integrated work, favored by cooperative practices". For us, this practice goes beyond cooperation, because when talking to teachers we envision working together, in order to understand their context and demands (D'Ambrosio & Lopes, 2015). Regarding experimentation with technologies, Borba & Villarreal (2005) point out that their use should be related to the exploration of their potential, that is, enabling students and teachers to test and conjecture hypotheses, producing knowledge through the feedback provided by the artifact. In this way, researches are developed in order to articulate theory and practice, through planning, applications and evaluations carried out collaboratively. Examples of topics already addressed in activities with teachers and students, within this perspective: cases of dengue fever and concepts of Statistics, use of pesticides and linear regression, among others. We noticed that by establishing these partnerships with teachers, there is a movement that demystifies the view that the academic environment is the holder of knowledge and the school the holder of practice, furthermore, when aligning the calculator in the actions, we notice the production of mathematical knowledge with and by the students.

References

BORBA, M. C., & VILLARREAL, M. E. (2005). *Humans-with-media and the reorganization of mathematical thinking: Information and communication technologies, modeling, experimentation and visualization*. Springer. (Mathematics Education Library, 39).

D'AMBROSIO, B. S., & LOPES, C. E. (2015). Creative insubordination: An invitation to the reinvention of the mathematics educator. *Bolema, 29*(51), 1-17. http://dx.doi.org/10.1590/1980-4415v29n51a01.

FOERSTE, E. (2004). Partnership in teacher education. *Revista Iberoamericana de Educación, 34*(3), 1-12. https://doi.org/10.35362/rie36133547.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 403. PME.

FIGURATIVE PATTERNS – A BRIDGE BETWEEN CREATIVE PROBLEM SOLVING AND ALGEBRAIC THINKING

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Algebraic thinking is a part of mathematical reasoning that extends beyond the mere manipulation of symbols. It involves the ability to identify patterns, generalize relationships and solve problems in an abstract, generalized way. One of the most effective strategies to develop algebraic thinking is the exploration of patterns, in particular visual ones. Visualization plays an essential role, mainly with figurative patterns, which facilitate natural transitions from arithmetic to algebra. In this context the use of multiple solutions tasks is essential to stimulate dimensions of creativity (fluency, flexibility, originality) (Leikin & Guberman, 2023). The use of visualization supports the emergence of alternative approaches, based on the manipulation of figures and identification of spatial relationships, which highlight different aspects of the same problem (Vale et al., 2018). In this presentation, we report a qualitative study, with an exploratory approach, carried out with 21 preservice teachers (PST), in the context of a unit course, where we used a didactical sequence for algebraic thinking, with focus on visualization, structured into three phases that allows students to explore patterns, formulate conjectures and generalize. The aim of the study was to characterize the PST performance in solving challenging, multiple solution tasks, involving the generalization of figurative patterns, and identify dimensions of creativity that emerge from their (re)solutions. Data were collected and analysed in a descriptive and interpretative way, and include observations and written productions of the proposed tasks. Results show that the didactical sequence contributes to identify the dimensions of creativity, that emerged in different ways in the participants' productions. Fluency was identified through the diversity of solutions, with multiple approaches to generalize patterns (visual, verbal, algebraic); flexibility was highlighted by the shift between different representations and processes used; and originality was also identified through the uncommon solutions.

References

Leikin, R., & Guberman, R. (2023). Creativity and challenge; task complexity as a function of insight and multiplicity of solutions. In R. Leikin (Ed.), *Mathematical Challenges for All* (pp.325-342). Springer.

Vale, I., & Barbosa, A. (2018). Mathematical problems: the advantages of visual strategies. *Journal of the European Teacher Education Network (JETEN)*, 13, 23-33.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 404. PME.

FRACTION SCHEMES FOR SOLVING FRACTION DIVISION PROBLEMS IN TWO PRESERVICE TEACHERS

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Since learning fractions is critical for students in K-8, preservice elementary teachers (PSTs) must be prepared to teach fractions to children. However, researchers have found that many PSTs may not solve essential fraction problems such as finding a whole bar given 3/5 of it (Lovin et al., 2018). Therefore, understanding how PSTs learn their fraction knowledge is a significant goal for K-8 teacher education programs. This research contributes to that understanding and is guided by the following research question: *What schemes and operations do PSTs construct as they solve fraction division problems?*

I regard learning as the *accommodations* people make of their *schemes*, which are cognitive structures constituted by three parts: a situation, an activity, and a result. When a person experiences an incongruence between the result of their activity and their expectation(s), i.e., a *perturbation,* they may reorganize one or more parts of their scheme (von Glasersfeld, 1995). I explore three main hierarchical fraction schemes, the *partitive fraction scheme* (PFS)*, reversible partitive fraction scheme* (RPFS)*,* and *iterative fraction scheme* (IFS) (cf. Steffe & Olive, 2010), and their role in the PSTs' solution of *measurement fraction division* problems (MFDPs) and *sharing fraction division* problems (SFDPs). The participants were two PSTs, Annie and Kim, who engaged in solving fraction division problems in a 10-episode teaching experiment.

The findings associated with Kim's work demonstrated that an RPFS and an IFS are needed to solve SFDPs. In contrast, Annie, who had not constructed an RPFS at the beginning of the experiment, used her PFS to solve SFDPs as MFDPs while facing perturbations. Toward the end of the teaching experiment, Annie showed evidence of having constructed an RPFS and an initial scheme to solve SFDPs. The two PSTs' work contributes to our understanding of how the construction of fraction schemes may impact the learning of other mathematical ideas, e.g., fraction division. Moreover, Annie's advancement in her construction of an RPFS to solve SFDPs may constitute a resource to support PSTs' learning of fraction knowledge.

References

Lovin, L. H., Stevens, A. L., Siegfried, J., Wilkins, J. L. M., & Norton, A. (2018). Pre-K-8 prospective teachers' understanding of fractions: An extension of fractions schemes and operations research. *Journal of Mathematics Teacher Education*, 21(3), 207–235. https://doi.org/10.1007/s10857-016-9357-8

Steffe, L. P., & Olive, J. (2010). *Children's Fractional Knowledge*. New York: Springer.

von Glasersfeld, E. (1995). *Radical Constructivism: A Way of Knowing and Learning. London:* The Falmer Press.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 405. PME.

MATHEMATICAL ANXIETY OF TECHNICAL-VOCATIONAL HIGH SCHOOL STUDENTS

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Mathematical Anxiety (MA) is defined as a feeling of tension that interferes with Mathematical Performance (MP), hindering learning while positive emotions enhance it (McLeod, 1994). Chilean 15-year-old students exhibit one of the highest levels of MA among OECD countries (OECD, 2023). MA negatively impacts MP, reducing PISA scores and increases with schooling, affecting 11% of students at basic levels and tripling in higher grades (Devine et al., 2018). Perception of being "good at math" influences their Careers Trajectories (CT) (Street et al 2024). In Chile, 944 TVHS educate over 156,355 students. Given the impact of MA on CT, understanding MA in this group is crucial since research indicates that MA predicts STEM enrolment, influencing CT.

The objective was to determine MA experienced by students (N=110) exposed to a mathematical outreach activity (at University of Santiago). MA was measured using the PISA MAQ (following ethical guidelines). Results indicate that 92% of students worry about poor grades, 82% fear difficulties in math classes and 71% feel lost when solving math problems. These findings highlight the need for interventions to help students develop confidence and problem-solving skills.

Some questions remain open: does the university environment influence MA perceptions? Would teaching problem-solving strategies reduce MA levels? Literature emphases the importance of addressing emotional aspects in mathematical education. Early interventions are necessary to mitigate MA effects. Promoting a growth mindset, viewing mistakes as learning opportunities can help reduce MA. Teachers require tools to support students emotionally, particularly in TVHS, where mathematics is key to future academic and professional success. Addressing MA is vital to improving educational outcomes and encouraging STEM participation at university.

References

Devine, A., Hill, F., Carey, E., & Szu, D. (2018). Cognitive and emotional math problems largely dissociate: Prevalence of developmental dyscalculia and mathematics anxiety. Journal of Educational Psychology, 110(3), 431-444.

Mcleod, D. B. (1994). Research on affect and mathematics learning in the JRME: 1970 to the present. Journal for Research in Mathematics Education, 25(6), 637-647.

OCDE. (2023). PISA 2022 Results (Volume I) The State of Learning and Equity in Education.

Street, K. E. S., Malmberg, L. E., & Schukajlow, S. (2024). Students' mathematics self-efficacy: A scoping review. ZDM Mathematics Education, 56, 265-280.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 406. PME.

DECISION MAKING UNDER RISK AND UNCERTAINTY IN SECONDARY SCHOOL STUDENTS

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The teaching of probability and statistics has gained increasing relevance due to the need to train citizens capable of making data-driven decisions (Batanero, 2020). Research on decision-making in contexts of uncertainty, as well as in risky situations, highlights the importance of strengthening both the conceptual understanding of probability and the heuristic processes aimed at evaluating real-life situations. These contexts range from games of chance to contemporary problems of the 21st century. However, within the educational field, research on how these contexts influence decision-making processes remains limited. This article presents the design and content validation by experts (Escobar-Pérez and Cuervo-Martínez, 2008) of a forced-choice questionnaire designed to explore how secondary school students make decisions under conditions of risk or uncertainty. The development of the questionnaire is based on previous research, which sought to provide an epistemological basis for the study of decision-making processes under conditions of uncertainty. The theoretical framework employs the conceptual distinction between risk and uncertainty, as well as the characterization of heuristic reasoning (Mousavi & Gigerenzer, 2014), paying attention to the importance of contexts and models (Gal, 2019).

The methodology involved six stages. The stage-by-stage organization facilitated the identification and justification of the contexts and reference problems used to construct the questionnaire items. The questionnaire specifically examines students' opinions or preferences regarding contextualized situations that require decision-making, rather than their knowledge of mathematical procedures or algorithms. The results of the analysis indicate a high level of validity and agreement for 80% of the items, allowing for adjustment of the instrument. This adjusted version could be subjected to additional empirical validation processes or contribute to future research.

References

Batanero C. (2020) Probability Teaching and Learning. In Lerman S. (Eds.), Encyclopedia of Mathematics Education (pp. 682-686). Springer.

Escobar-Pérez, J., & Cuervo-Martínez, Á. (2008). Validez de contenido y juicio de expertos: una aproximación a su utilización. Avances en medición, 6(1), 27-36.

Gal, I. (2019). Understanding statistical literacy: About knowledge of contexts and models. In J. M. Contreras, M. M. Gea, M. M. López-Martín y E. Molina-Portillo (Eds.), Actas del Tercer Congreso International Virtual de Educación Estadística.

Mousavi, S., & Gigerenzer, G. (2014). Risk, uncertainty, and heuristics. *Journal of Business Research, 67*(8), 1671-1678.

Research funded by FONDECYT Initiation

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 407. PME.

STUDENTS' PERSPECTIVES IN COLLEGE ALGEBRA LEARNING ENVIRONMENTS

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In this poster, we share our progress in exploring community college students' views regarding their mathematical learning environments in college algebra courses. Our goal is to document their perspectives about their sense of belonging and about how their experiences and perspectives, as well as those of their peers, are considered in their courses. We will share our design process and data collection protocols, which have been developed to provide a thorough understanding of how students perceive these environments. The study involved 80 students from eight community colleges across the Northwest, Southwest, Central, and Midwest regions of the U.S. We used the grounded theory approach (Charmaz, 2006), and conducted a series of four interviews with each student over the course of a semester. These interviews are designed to address students' personal and mathematical histories, their perception of environments in college algebra by examining classroom instruction videos, a Q-sort exercise to identify how students describe belonging, agency, and rigor in mathematics (Rieber, 2020), and reflections on the research process itself. By the time of the conference, we will have completed at least two of the four rounds of interviews with all students. We will present initial themes from students' personal and mathematical histories, preliminary analysis of their commentary on classroom instruction videos, and about environments for learning college algebra. We anticipate that this research will contribute valuable insights for improving instructional practices and informing policies in community college mathematics education. Additionally, our study will shed light on how students' personal and mathematical histories influence their perceptions of belonging and sense of agency in college algebra learning environments.

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References

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis*. Sage.

Rieber, L. P. (2020). Q methodology in learning, design, and technology: An introduction. *Educational Technology Research and Development*, *68*(5), 2529-2549.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 408. PME.

SECOND GRADERS' ADAPTIVE USE OF STRATEGIES: EFFECTS OF TEXTBOOK AND TEACHERS' INSTRUCTION

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The adaptive use of strategies, i.e. using an advantageous strategy to solve a given addition or subtraction problem, is an important goal in elementary arithmetic education, which is challenging for many students. Previous studies indicate that the textbook quality for strategy learning varies significantly between textbooks and has an impact on students' skills in adaptive strategy use (e.g., Sievert et al., 2019). However, these studies have not yet examined the role of the teacher. To better understand the influence of textbook and teacher on student learning, we address the following research question: Which effect does the quality of teachers' instruction regarding the adaptive use of strategies have on students' adaptive strategy use in grade 2 (a) as direct effect and (b) as moderator of the effect of textbook quality?

The sample comprises 627 students from 41 grade 2 classes in Germany using one of four common textbooks. Students' adaptive strategy use was assessed with an 8-items test on two-digit addition and subtraction ($\alpha = .84$). Adapting the four dimensions of strategy competence from Lemaire and Siegler (1995), we assessed the quality of learning opportunities in textbooks (interrater agreement: $\kappa_w > .88$) and the quality of teachers' strategy instruction (by teacher questionnaires). Students' prior arithmetic knowledge and quality of classroom management were included in the analysis as control variables. For the prediction of students' adaptive strategy use, a multilevel analysis revealed no significant direct effect of textbook quality ($\beta = -.12$, $p = .095$) or interaction effect *textbook quality* × *teachers' instructional quality* ($\beta = -.07$, $p = .444$), but a direct effect of teachers' instructional quality ($\beta = .14$, $p = .046$). The findings emphasize the pivotal role of teachers in fostering students' adaptive strategy use. The absence of textbook effects may stem from limited variance in textbook quality. Future research should examine in more detail how teachers deal with the strengths and weaknesses of textbooks in order to achieve high quality strategy teaching.

References

Lemaire, P., & Siegler, R. S. (1995). Four Aspects of Strategic Change. Contributions to Children's Learning of Multiplication. *Journal of Experimental Psychology General, 124*, 83–96.

Sievert, H., van den Ham, A.-K., Niedermeyer, I. & Heinze, A. (2019). Effects of mathematics textbooks on the development of primary school children's adaptive expertise in arithmetic. *Learning and Individual Differences, 74*, 101716.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 409. PME.

ENHANCING STUDENTS' PROOF SKILLS WITH AN ONLINE SELF-REGULATED LEARNING TOOL BY INCOPRATING THE MATHEMATICAL ARGUMENTATION PROCESS

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Students often struggle with learning proof, especially solving proof exercises. Encouraging students to explain their steps can help identify and correct misconceptions. Mathematical proof is a formalized mathematical argument. Lin (2018) suggested a five-step conjecturing process to improve students' argumentation skills. While self-regulated learning is known to enhance math outcomes, its effect on higher-order thinking skills is less studied. This research expands the conjecturing process by developing an online tool for self-regulated learning. During the self-regulation phase, students engage with GeoGebra to generate examples and propose conjectures. In the social regulation phase, students present their conjectures in online group discussion forums. This study addresses two questions: 1) Is the Online Self-Regulated Learning Tool more effective than physical group discussions? 2) What are the discussion patterns in online group discussion forums?

Forty ninth-grade students from a Taiwanese junior high school were selected to participate in the field experiment, with 19 in the experimental group and 21 in the control group. The curriculum on Properties of Triangles and Reasoning and Proof was covered in five class sessions. ANCOVA results showed the experimental group's mean was 74.36, and the control groups was 71.05 [$F=51.85$, $p < .05$, $\eta^2=0.15$]. The findings suggest that the intervention had a substantial impact on the students' proficiency in proof skills. This study coded the content of group discussions in the discussion forum and applied Epistemic Network Analysis to examine the interactions among group members during the argumentation process. The results indicate that in groups with greater progress, the connections between "answer-acceptance" and "help-prompt" were particularly strong. The findings indicate that students can utilize self-regulated tools and refine their claims through online discussions, thereby enhancing their proof skills.

References

Lin, P. J. (2018). Improving knowledge for teaching mathematical argumentation in Primary Classrooms. *Journal of Mathematics Education*. 11(1), 17-30.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 410. PME.

SUB-SAHARAN AFRICAN IMMIGRANT FAMILIES' VIEWS OF GIFTEDNESS IN MATHEMATICS

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We have been interviewing newly migrated students from Sub-Saharan Africa to listen to their experiences of learning mathematics in Canada, and interviewing their parents to understand how they support their children in this. Our analysis has focused on identifying storylines at play, and the positioning of the students, their cultural knowledge and mathematics. This analysis of the interactions is done through the lens of Afrocentricity (African Indigenous Knowledges). One goal of this work is to inform Canadian mathematics teachers on how their new African students may be experiencing their math classes, and how to address these students well.

The students and parents talked about many ideas that are familiar themes in mathematics education research. Interpreting these ideas through Afrocentric lenses has been interesting. However, one concept that comes up in the interviews is not familiar to our field of research: their references to God. We argue that an understanding of Sub-Saharan African students' experience of mathematics learning cannot be understood well without acknowledging the importance of their worldviews.

One parent said of herself, "I would say God gifted me in that mathematics, so I never struggled with math." In a separate interview, her daughter complained about things people have said to her about her math, "you're so smart or like you really know a lot of stuff. […] one person said to me like you were born with the gift of math. And to be honest, I was not born with the gift of math. I really did do a lot of practice."

We are *not* interested in saying whether ability in mathematics is God-given or not. We *are* interested in the impact of people saying it comes from God. What happens when this is said to a student from Sub-Saharan Africa? Or when someone refers to their giftedness (when at home giftedness is positioned as coming from God). What does it mean to the student? And does it matter who says it? Our field has considerable literature on giftedness (described nicely by Leikin, 2021) and active efforts to argue against referring to success in math as giftedness (e.g., Boaler, 2022).

References

Boaler, J. (2022). *Mathematical mindsets: Unleashing students' potential through creative mathematics, inspiring messages and innovative teaching*. John Wiley & Sons.

Leikin, R. (2021). When practice needs more research: the nature and nurture of mathematical giftedness. *ZDM–Mathematics Education, 53*(7), 1579–1589.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 411. PME.

IMPROVING MATHEMATICAL THINKING BY PLAYING BOARD GAMES IN MATHEMATICS LESSONS – INVESTIGATING TOWARDS IMPLEMENTATION

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There can be several motives for implementing board games in mathematics education, among them neuroscience considerations implying that board games can be used to improve students' mathematical thinking (Newman, Hansen & Gutierrez, 2016) and empirical research from several aspects suggesting the same (Sunay & Erdogan, 2024).

In our previous investigations, we examined the possible benefits of implementing non-digital, commercially available board games in mathematics classroom work by replacing one lesson per week with playing board games in grade 10 (i.e., students aged 16-17 years). We obtained promising results regarding students' logical and geometrical skills without any negative change in their subject knowledge, suggesting that it is worth using adequate board games to improve students' thinking (Zámbó et al., 2024). Besides, it was also reinforced that there is a need for further investigation on how board games can be effectively integrated into mathematics classroom work.

In our ongoing work, we focus on how to establish an optimal environment for improvement of mathematical thinking using board games – especially, but not exclusively, on the choice of game(s) – and the construction of a framework to facilitate the process. To do so, we have used the quantitative results obtained in our previous experiments; insight into pedagogical observations and perceived outcomes by participating teachers via half-structured deep interviews; the framework of Sunay & Erdogan (2024); analysis of the games' properties (e.g., cognitive demands, strategic complexity, mathematical content and supposed connections between game mechanisms and activated cognitive processes) and last, but not least, student characteristics (e.g., player types, previous experience). In our presentation, we will present the provisional framework and the plans for the next experiment's design.

References

Newman, Sh., Hansen, M., & Gutierrez, A. (2016). An fMRI Study of the Impact of Block Building and Board Games on Spatial Ability. *Frontiers in Psychology*. *7.* http://dx.doi.org/10.3389/fpsyg.2016.01278

Sunay, M. A. & Erdogan, A. (2024.) Abstract strategy board game for mathematical reasoning: Opening the black box.*Thinking Skills and Creativity.* (52) *101507.* https://doi.org/10.1016/j.tsc.2024.101507.

Zámbó, Cs., Muzsnay, A., &Szeibert, J. (2024). Improving mathematical thinking by playing board games in mathematics lessons. In Tanya, Evans; Ofer, Marmur; Jodie, Hunter; Generosa, Leach; Jyoti, Jhagroo (Eds.), *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). p. 247

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 412. PME.

TEACHERS' EXPERIENCES OF MIGRANT STUDENTS' CULTURAL-HISTORICAL MATHEMATICAL REPERTOIRES

Sona Zendedel, Karli Bergquist, Richard Barwell, Christine Suurtamm

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Despite widespread international migration, there is relatively little research on the experiences of migrant students or their teachers in mathematics classrooms (Lüssenhop & Kaiser, 2024). In Canada, provincial mathematics curricula include statements acknowledging the value of diversity among students. We used the concept of *cultural-historical repertoires* to examine how teachers engage with migrant students' experiences in the mathematics classroom and reflected on adapting their own practices in response. We define cultural-historical repertoires as ways of engaging in activities that flow from individual and collective backgrounds, experiences, and interests. The formation of these repertoires is based on participation in cultural practices (Gutiérrez & Rogoff, 2003). We first collected and analyzed the experiences of migrant students in Canada. Participants were asked to write a letter to someone to explain what mathematics classes are like in Canada. Based on 57 letters and 10 interviews, narratives highlighting the challenges and unfamiliar situations they faced in mathematics classes were shared with teachers via an online survey. We collected 31 teacher survey responses and transcripts from two focus groups. The stages of the research design and extracts from the data will be displayed on the poster.

Our analysis showed that participants valued multiple ways of knowing, understanding that mathematics may be expressed in different languages and conceptualized in diverse contexts (through multiple models, algorithms, and symbols). They spoke positively about creating space for migrant students to demonstrate "their way" of doing things. However, the focus on supports such as differentiation for language proficiency, scaffolding for learning in English, and recognizing that students may find mathematics in Canada "harder or easier" suggests a perspective on mathematics as universal; rather, it is the language and organization of the curriculum that differ across contexts. These findings indicate that migrant students and their mathematics teachers bring multiple, not necessarily complementary, perspectives to mathematics learning and teaching. Ongoing work aims to understand the significance of these perspectives for supporting migrant mathematics learners and their teachers.

References

Gutiérrez, K. D., & Rogoff, B. (2003). Cultural ways of learning: Individual traits or repertoires of practice. *Educational Researcher, 32*(5), 19–25.

Lüssenhop, M., & Kaiser, G. (2024). Mathematics teaching for migrant students in German schools—How do teachers respond to their students' diverse needs? *Asian Journal for Mathematics Eduation, 3*(1), 60–80.

2025. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 413. PME.

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2025
SANTIAGO, CHILE

MAKING SURE THAT MATHEMATICS EDUCATION
RESEARCH REACHES THE CLASSROOM

Universidad de Chile

Universidad de O'Higgins