THAILAND

PME REGIONAL CONFERENCE 2023

PROCEEDINGS

of the Regional Conference of the International Group for the Psychology of Mathematics Education 2023

MATHEMATICAL THINKING SKILLS IN THE DIGITAL ERA: REGIONAL EPISODE

2 - 4 DECEMBER 2023

EDITORS:

Maitree Inprasitha,
Anake Sudjamnong, Ariya Suriyon, Jensamut Saengpun,
Pimlak Moonpo, Rachada Chaovasetthakul,
Suwarnnee Plianram, and Wipaporn Suttiamporn

Plenary Lectures, Research Reports,
Oral Communications, Poster Presentations

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Preface

We are pleasure to welcome you to the PME Regional Conference 2023. PME is one of the most important international conferences in mathematics education, and draws together academic community of educators, researchers, and mathematicians from all over the world. The community is continue working to enrich research and practices in many regions, PME has decided to support the organization of a members of regional conference in order to enhance the capacities of research communities from their regions to be a prominent community in PME. PME Regional conference 2023 was approved by the Annual General Meeting of PME members in the PME 44 conference 2021 held in Khon Kaen, Thailand, virtually hosted by Technion Israel, and has support Mae Kong sub-region specifically.

PME regional conference 2023 take place by December 2-4, 2023, is the first time such as conference is being held in Thailand with CLMV (Cambodia, Laos and Myanmar and Vietnam) countries, where mathematics education is underrepresented in the community. Thus, this conference will provide chances to facilitate the activities and networks associated in mathematics education in the region and PME community.

“Mathematical thinking skills in the digital era: regional episode” has been chosen as the theme of the conference, which is very timely for this era. The theme focuses on how can we enrich the strength of mathematics education research on mathematical thinking skills in the digital era facing with volatility, uncertainty, complexity and ambiguity among the region of CLMV. It will be enhanced mathematics education community in the region.

The three plenary speakers for the PME conference 2023 are Hideyo Emori (Japan), Einat Heyd- Metzuyanim (Israel) and Maitree Inprasitha (Thailand). Moreover, the conference has 9 research reports, 15 oral communications and 19 poster presentations included. The participants of the conference attend physically and present in virtual platform included 149 peoples from 14 countries.

To organisations of PME regional conference 2023 is a collaboration effort presenting jointly by Thailand Society of Mathematics Education (TSMEd), Institute for Research and Development in Teaching Profession for ASEAN (IRDTP), Khon Kaen University (KKU), Centre of Excellence in Mathematics (CEM), Center for Promotion of Mathematical Research of Thailand (CEPMART), the Mathematical Association of Thailand Under the Patronage of His Majesty the King, the Institute for the Promotion of Teaching Science and Technology (IPST), Center for Research in Mathematics Education (CRME), and the Educational Foundation for Development of Thinking Skills (EDTS). Moreover, all the members of the Local Organizing Committee are also supported by the International Program Committee.

I appreciate the support of all those involved in making this conference possible, as well as the PME regional conference 2023 participants for their valuable contributions to this conference and the PME communities.

Yongwimon Lenbury

PME Regional Conference Chair

SPONSORS

We are grateful for the generous support received for this conference from:

The International Group for the Psychology of Mathematics Education (PME)

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Center for Research in Mathematics Education (CRME), Khon Kaen University, Thailand

The Educational Foundation for Development of Thinking Skills (EDTS)

The Mathematics Certification Institute of Thailand (SUKEN THAILAND)

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The International Group for the Psychology of Mathematics Education (PME) and the PME Regional Conference 2023

History of IGPME

The International Group for the Psychology of Mathematics Education (PME) is an autonomous body, governed as provided for in the constitution. It is an official subgroup of the International Commission for Mathematical Instruction (ICMI) and came into existence at the Third International Congress on Mathematics Education (ICME 3) held in Karlsruhe, Germany in 1976.

THE GOALS OF PME

The major goals of the organization are:

1. to promote international contacts and exchange of scientific information in the field of mathematical education;
2. to promote and stimulate interdisciplinary research in the aforesaid area; and
3. to further a deeper and more correct understanding of the psychology and other aspects of teaching and learning mathematics and the implications thereof.

PME has between 700 and 800 members from about 60 countries all over the world. The main activity of PME is its annual conference of about 5 days, during which members have the opportunity to communicate personally with each other about their special and general interests. There are plenary lectures, research paper presentations, working groups, poster sessions and many other activities. Every year the conference is held in a different country.

Its former presidents have been:

Efraim Fischbein, Israel	1977-1980	Stephen Lerman, UK	1995-1998
Richard R. Skemp, UK	1980-1982	Gilah Leder, Australia	2001-1998
Gerard Vergnaud, France	1984-1982	Rina Hershkowitz, Israel	2004-2001
Kevin F. Collis, Australia	1986-1984	Chris Breen, South Africa	2007-2004
Pearla Nesher, Israel	1988-1986	Fou-Lai Lin, Taiwan	2010-2007
Nicolas Balacheff, France	1990-1988	João Filipe Matos, Portugal	2010-2013
Kathleen Hart, UK	1992-1990	Barbara Jaworski, UK	2013-2017
Carolyn Kieran, Canada	1995-1992	Peter Liljedahl, Canada	2017-2021

The current president is Wim Van Dooren, Belgium, elected from 2021.

All information concerning PME and its constitution can be found at the PME website: http://www.igpme.org/.

THAILAND PME 44: The Challenge Conference

The 44th Annual Meeting of the IGPME, held on July 21-22, 2020, is the first time the conference hosted by Thailand and CLMV countries (Cambodia, Laos, Myanmar, Vietnam) where mathematics education is underrepresented in the community. Thus, this conference provides chances to facilitate the activities and network associated in mathematics education in the region. The theme of the conference is *"Mathematics Education in the 4th Industrial Revolution: Thinking Skills for the Future"*, which is very timely for this era. The conference held fully online cause of COVID-19 pandemic.

The Conference jointly presented by Center for Research in Mathematics Education (CRME), Centre of Excellence in Mathematics (CEM), Thailand Society of Mathematics Education (TSMEd), Institute for Research and Development in Teaching Profession for ASEAN Khon Kaen University (IRDTP), The Educational Foundation for Development of Thinking Skills (EDTS) and the institute for the Promotion of Teaching Science and Technology (IPST).

PME REGIONAL CONFERENCE

PME Regional Conference are an initiative approved by PME members in their Annual General Meeting in PME 40 (Hungary, 2016). These events have the goal of supporting researchers from regions underrepresented within PME to organize a conference in their region with the objective of promoting both within- and extra-regional networking. In other words, PME Regional Conferences aim at:

1) supporting a regional research community whose goals are aligned with those of PME, and

2) attracting researchers from the region to actively participate in future PME conferences and supporting them in preparing high quality PME contributions.

The Annual General Meeting in PME 41 (Singapore, 2017), one year later, approved a proposal for organizing a PME Regional Conference for South America during 2018. This year the PME Regional Conference will be held in Thailand, 2-4 December 2023. The theme of the conference is *"Mathematical Thinking Skills in the Digital Era: Regional Episode".* This is the first time that PME Regional Conference take place in Mekong Sub-region.

The conference will be presented jointly by Thailand Society of Mathematics Education (TSMEd), Institute for Research and Development in Teaching Profession for ASEAN (IRDTP), Khon Kaen University (KKU), Centre of Excellence in Mathematics (CEM), Center for Promotion of Mathematical Research of Thailand (CEPMART), the Mathematical Association of Thailand Under the Patronage of His Majesty the King, the Institute for the Promotion of Teaching Science and Technology (IPST), Center for Research in

Mathematics Education (CRME), and the Educational Foundation for Development of Thinking Skills (EDTS).

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WEBSITE OF PME REGIONAL CONFERENCE 2023

All information about PME Regional Conference 2023 can be found at the website: https://www.tsmed.or.th/PME_Regional_Conference/index.php

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PLENARY LECTURES

LEARNING TO THINK MATHEMATICALLY IN THAI CLASSROOM USING THAILAND LESSON STUDY INCORPORATED WITH OPEN APPROACH (TLSOA)

Maitree Inprasitha

Khon Kaen University, Thailand

In this paper, I would like to share my experiences how to expand the current literature related to mathematical thinking development by proposing a descriptive framework of Thailand Lesson Study incorporated with Open Approach to develop students' learning to think mathematically. The pedagogical implications are using students' prior knowledge to adapt to their problem-solving methods. Following this line of reasoning, students are able to develop their abilities to solve the mathematical problems, find various strategies, connect different related issues to solve these problems, and finally interpret the learning outcomes by themselves. On the other hand, the practical implications of using TLSOA model is to propose a transformational process of Lesson Study in the Thai local context through a weekly Plan-Do-Check-Act cycle, rather than focusing intensively on the details of the quality of the lesson itself, as has been conducted in many other countries including Japan. In conclusion, I found that the TLSOA model has been successfully adapted into a more successful professional development practice to cultivate students' mathematical thinking.

INTRODUCTION

Mathematical thinking is crucial in teaching mathematics for several reasons such as foundation for problem solving, understanding concepts deeply, transferability of skills, critical thinking skills, preparation for advanced mathematics, promoting creativity, life skills, enhancing communication skills, building confidence, preventing mathematics anxiety, and meeting the demands of the 21st century. Mathematical thinking provides a solid foundation for problem-solving. It equips students with the skills to analyze problems, identify relevant information, and develop effective strategies for finding solutions hence forming foundation for problem solving (Inprasitha, 2022). Following this line of reasoning, I define mathematical thinking development as cultivating a mindset and skills that enable students to approach problems, analyze situations, and make reasoned decisions using mathematical principles. Therefore, mathematical thinking development in classroom is somehow important in the teaching of mathematics.

In addition, I found that mathematical thinking development is not only encourages students go beyond memorization and rote learning but also promotes a deep understanding of mathematical concepts, allowing students to grasp the underlying principles and connections between different ideas (Inprasitha & Loipha, 2007). Moreover, mathematical thinking is a transferable skill. If students have undergone an appropriate

2023. In Inprasitha, M., Sudjamnong, A., Suriyon, A., Saengpun, J., Moonpo, P., Chaovasetthakul, R., Plianram, S., & Suttiamporn, W. (Eds.). *Proceedings of the Regional Conference of the International Group for the Psychology of Mathematics Education 2023*, pp.1-8. Khon Kaen, Thailand: PME.

teaching and learning process using Thailand Lesson Study incorporated with Open Approach (TLSOA) model, they possessed the ability to think mathematically that can be applied not only to solve mathematical problems but also to address challenges in various disciplines and real-life situations (Inprasitha, 2022).

We cannot deny that cultivating mathematical thinking can foster students' critical thinking skills. This is because students learn to evaluate information, make logical connections, and construct well-reasoned arguments. These skills are valuable in both academic and professional contexts (Inprasitha, 2006). Consequently, mathematical thinking development provides the groundwork for more complex mathematical concepts and helps students navigate higher-level courses with confidence thus mathematical thinking is essential for success in advanced mathematics (Inprasitha, 2023). There is no doubt that mathematical thinking encourages creativity by challenging students to explore different approaches to problem-solving. It involves thinking outside the box, making connections between seemingly unrelated concepts, and developing innovative solutions (Inprasitha, 2023). In other words, we can consider mathematical thinking as a life skill that extends beyond the classroom. It equips students with the ability to analyze data, make informed decisions, and solve real-world problems, contributing to their success in various personal and professional endeavors.

In general, teachers who utilize the TLSOA model involves developing communication skills of students because students learn to express their mathematical ideas clearly and coherently, both in written and verbal form. In my opinion, this skill is valuable for collaborating with others and explaining mathematical reasoning to enhance their communication skills along the way of mathematical thinking development (Inprasitha, 2022). As students develop problem-solving skills and a deeper understanding of mathematical concepts, they become more comfortable tackling challenges, which positively impacts their overall attitude toward mathematics, ultimately mathematical thinking helps to build confidence in students.

On top of that, encouraging mathematical thinking among students can help reduce mathematics anxiety. When students approach mathematics with a mindset focused on understanding and problem-solving, they are less likely to feel overwhelmed or discouraged by the subject. In the 21st century, where analytical and quantitative skills are increasingly important, mathematical thinking is a key competency. Therefore, teachers have to be aware that mathematical thinking skill is essential in preparing students for the demands of a rapidly changing world and enhances their ability to adapt to new challenges.

In summary, mathematical thinking is fundamental to teaching mathematics as it lays the groundwork for problem-solving, deep understanding of concepts, critical thinking, creativity, and the development of skills that are valuable both within and beyond the realm of mathematics. It empowers students to become effective learners and critical thinkers in a variety of contexts.

LITERATURE REVIEW OF MATHEMATICAL THINKING DEVELOPMENT

John Dewey, a prominent educational philosopher, made significant contributions to the understanding of education and learning. In his work *"How We Think"*, published in 1933, Dewey discussed general principles of reflective thinking and problem-solving, which can be applied to the development of mathematical thinking. Even though Dewey did not specifically focus on mathematics in his work, his ideas have been widely influential in educational theory and practice, including the teaching and learning of mathematics. There are seven key aspects of Dewey's philosophy, namely reflective thinking, problem-solving as learning, experiential learning, active engagement, contextual learning, social interaction and collaboration, and continuity and integration of learning. Dewey (1933) related these seven key aspects to the development of mathematical thinking. Although Dewey's work does not explicitly focus on mathematics, his general philosophy of education aligns with many principles that are considered essential in the development of mathematical thinking. Teachers and curriculum designers often draw on Dewey's ideas to create learning experiences that foster deep understanding, critical thinking, and problem-solving skills in mathematics (Dewey, 1933).

George Pólya was a Hungarian mathematician whose work, particularly in problem-solving methodology, has had a significant impact on mathematics education. His book *"How to Solve it,"* originally published in 1945, outlines a problem-solving process and principles that are still influential in mathematics education. Although Pólya's work did not extend into the 1980s and 1990s, his ideas continued to shape mathematics education during that time and remain relevant until today. Pólya (1945) emphasized the following seven key aspects include problem-solving heuristics, metacognition in problem solving, problem-centered instruction, mathematical communication, mathematical modeling, problem-solving in the curriculum, and constructivist pedagogy and their impacts on mathematical thinking, Teachers continued to draw on Pólya's problem-solving principles to enhance mathematical thinking and problem-solving skills in students. His influence can be seen in the ongoing efforts to promote a more student-centered, problem-solving oriented approach to teaching mathematics.

Alan H. Schoenfeld is a renowned mathematics education researcher who has made significant contributions to the study of mathematical thinking and problem-solving. In the early 1980s, Schoenfeld conducted research that delved into the cognitive aspects of mathematical problem-solving. Schoenfeld (1982) explored the cognitive processes involved in mathematical problem-solving in his seminal work. He analyzed the strategies that students use to solve mathematical problems and identified various factors influencing problem-solving performance. The book emphasizes the importance of metacognition, strategy use, and the development of flexible problem-solving approaches. Schoenfeld's 1985 paper focuses on the concept of teaching-in-context. He argued that effective mathematics instruction should go beyond teaching isolated procedures and concepts, instead placing mathematics in meaningful, real-world contexts. Schoenfeld (1985) discusses the role of cognitive apprenticeship, where students learn problem-solving heuristics and strategies through guided practice in authentic mathematical situations.

Richard Lesh (1985) is associated with the development of Realistic Mathematics Education that emphasizes problem-centered instruction and the use of real-world contexts to make mathematics more meaningful for students. This approach aligns with the broader theme of considering metacognition in mathematical thinking and problem-solving. Furthermore, Frank K. Lester et al.'s (1989) work often focused on mathematical problem-solving, and the cognitive processes involved. In his 1989 publication "Thinking and Reasoning in Problem Solving," Lester et al. explored the driving forces for mathematical problem-solving, including metacognition. His work contributed to the understanding of how students think and reason when faced with mathematical problems.

Edward A. Silver published an article in "For the Learning of mathematics", titled "The mathematical problem posing," in 1999, focuses on the concept of problem posing in mathematics education. Silver (1994) explores the idea that posing mathematical problems is an essential aspect of the learning process. Problem posing involves students not only solving problems but also creating and formulating their own mathematical problems. Silver (1994) argues that problem posing helps students develop a deeper understanding of mathematical concepts, promotes creativity, and enhances problem-solving skills.

The above literature review showed that Dewey (1933) and Pólya (1945)'s contributed to create an innovative approach in order to develop students' mathematical thinking while they are learning mathematics. However, Schoenfeld (1982, 1985), Lesh (1982, 1985), and Lester (1989) contributed to the exploration of metacognition in mathematical problem-solving. On the other hand, Silver (1994) contributed to the ways of problem posing to promote problem-solving skills. Most of the past researchers concentrated on the issues of how to develop students' metacognition using problem-solving approach through an open-ended problem which was found to be an effective approach. However, there is an obvious research gap that I should focus on such as how to develop mathematical thinking and how students learn to think mathematically in the classroom.

INTRODUCING TLSOA MODEL IN THAI CLASSROOM

I transformed the Japanese ideas of Lesson Study and Open Approach to create and sustain the TLSOA model in order to successfully create the Thai local contexts of how students learn to think mathematically in the classroom. I have been actively involved in promoting Lesson Study as a professional development that involves teachers collaboratively planning, observing, and analyzing lessons to improve their teaching practices. Moreover, I have been instrumental in adapting and implementing Lesson Study in various educational contexts, emphasizing its potential for enhancing both teachers' professional development and students' mathematical thinking. Unlike Japanese Lesson Study model, I incorporate Open Approach that emphasizes a "unique collaboration" in every step of the Lesson Study cycle; this is because traditional Thai classroom culture denies students opportunities to explore various forms of mathematical free thinking (Inprasitha, 2022).

The uniqueness of TLSOA model is teachers must change their teaching approach from emphasizing the rote learning of mathematics content, formulas or theories to an innovative method that encourages students to express their mathematical thinking (Inprasitha, 2022). In 2004, I proposed the TLSOA model consists of a three-steps Lesson Study process, namely collaboratively design lesson (Plan), collaboratively observe lesson (Do), and collaboratively reflect on teaching practice (See) to change the paradigm of teaching practices using the Japanese Lesson Study as a reference. Besides, I incorporated Open Approach in the Do step and the weekly Plan-Do-Check-Act (PDCA) procedure in the See step. The rationale of incorporating Open Approach in the Lesson Study process is to generate students' independent learning and mathematical thinking development. Likewise, the weekly PDCA procedure uses the "Kaizen" concept as a cycle of instructional improvement in which teachers work together to formulate goals for student learning in long-term practices so that they learn to think mathematically in the classroom. This weekly PDCA is found to be more important than the numbers of steps in the Lesson Study model. The unit of analysis in Japan context is the "lesson" but the unit of analysis in Thailand context is the "classroom".

Before the use of TLSOA model, students in Thailand were rarely given opportunities to pose their mathematics problems. I acknowledge the importance of student generated problem posing and formulated that along with problem-solving, there are two components of instructional activity, as emphasized by Silver (1994) in the contemporary constructivist theories of teaching and learning. As discussed above, Silver (1994) defined problem-posing as both the generation of a new problem and the re-formulation of given problems to be included in the first phase of Open Approach. This was followed by adopting the "What-if-not?" process from Brown and Walter (1983) in the following phases of Open Approach. The urgent issue of Thai education can be transformed from the product-oriented approach to that of Open Approach, which emphasizes both the product and the process (Inprasitha, 2006).

The details of the TLSOA model are illustrated in Figure 1 as follows:

Step 1: Collaboratively design a plan (PLAN)

The Lesson Study group members are required to meet, discuss, and design a lesson plan by taking into account students' thinking skills development in their learning. Therefore, they determine open-ended mathematical problem situations and organize the learning activities by engaging students in open-ended problems in terms of the Open Approach which is formulated to have multiple answers.

Step 2: Collaboratively observe the lesson (DO)

One of the Lesson Study group members implements the lesson that has been planned in Step 1. The other Lesson Study group members observe the learning activities targeting to identify students' mathematical thinking processes. Inprasitha (2006) emphasizes the importance of teachers understanding students' ideas so that teachers know the methods to support students' progress in their learning processes. The four phases of the Open Approach consist of (i) posing open-ended problems; (ii) students' self-learning; (iii) whole-class discussion and comparison, and (iv) summarizing by connecting students' mathematical ideas is incorporated in the DO step.

Step 3: Collaboratively discuss and reflect on the lesson (SEE)

Those observers in Step 2 are required to reflect on what they have observed in a weekly cycle. Moreover, two more cycles of reflection to make sure the Lesson Study group members have sufficient reflection chances, such as Open Class Activity at the district level and National Open Class at the national level for the respective semester and yearly basis. Figure 1 above illustrates the process of the TLSOA model in a weekly cycle.

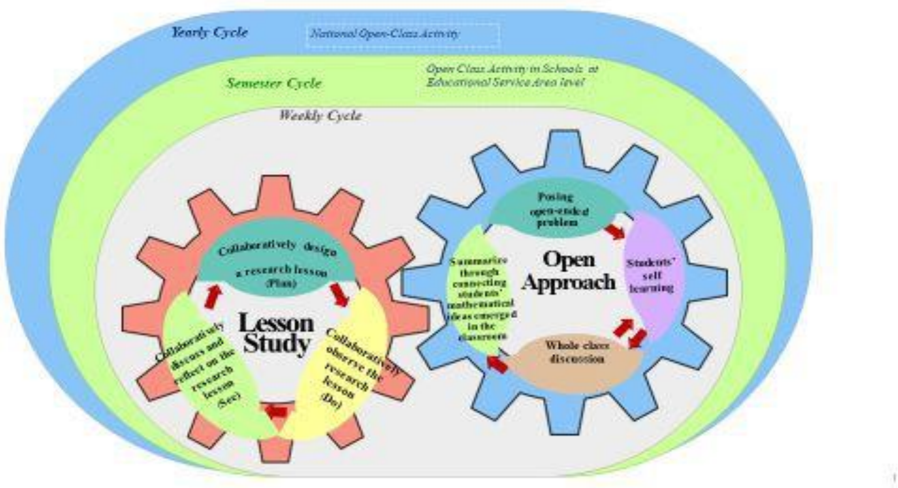

Figure 1: TLSOA model

Inprasitha (2004) has associated mathematical thinking development and promotion of the Open Approach in TLSOA model. The Open Approach encourages students to engage in problem-solving, exploration, and discussions, fostering a deep and flexible understanding of mathematical concepts. This approach aligns with the broader trend in mathematics education that emphasizes student-centered learning, critical thinking, and the application of mathematical knowledge in real-world contexts.

"Bansho" refers to a teaching method adopted from Japanese education means "blackboard writing" is used in TLSOA model. In the context of Lesson Study, Bansho involves the collaboration creation of a detailed, high-quality blackboard display that serves as a visual representation of the lesson content. Bansho is used during the "Do" step for writing and drawing key points, examples, and explanations related to the lesson content. The board becomes a dynamic visual aid that evolves as the lesson progresses. After the lesson, the teachers engage in PDCA cycle of the "See" step. The Lesson Study group members analyze the Bansho board, discuss student responses, and evaluate the effectiveness of the instructional strategies employed. This discussion aims to refine teaching practices and enhance student learning in subsequent lessons.

CONCLUSION

The TLSOA model plays a crucial role in cultivating students' mathematical thinking development. This model combines both Lesson Study and Open Approach to provide teachers with effective strategies for improving mathematical instruction. By incorporating Lesson Study, teachers can collaboratively plan and refine lessons and through the Open Approach, they can create an environment that encourages students to actively participate in mathematical thinking and problem-solving. Moreover, my contributions to the integration of these approaches in TLSOA model have had a positive impact on mathematics education globally, providing teachers with practical tools and methodologies for fostering a deeper and more meaningful understanding of mathematics among their students.

In addition, the TLSOA model provides a structured framework for teaching mathematics. It outlines the sequence of problem situations, teaching methods, and learning activities. This structure helps to guide both teachers and students through the mathematical curriculum, ensuring a systematic and organized approach to learning. The Bansho method in the TLSOA model emphasizes the importance of visual representation and collaboration reflection. The visual nature of the Bansho board allow teachers to make their thought processes explicit, fostering a shared understanding of effective teaching practices.

In conclusion, the TLSOA model is aligned with specific learning objectives. It ensures that the teaching methods and activities chosen are purposeful and contribute directly to the development of mathematical thinking skills. This alignment helps the teachers focus the educational process on the desired outcomes. This helps students develop the ability to think critically about mathematical concepts and apply their knowledge to real-world situations.

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THE DEVELOPMENT OF FAILURE TO LEARN MATHEMATICS: COMPLEXITY, COMMOGNITION, AND IDENTITY

Einat Heyd-Metzuyanim

Technion – Israel Institute of Technology and University of Pittsburgh

In this lecture, I offer a new theoretical and methodological perspective for studying the development of failure to learn mathematics (FtLM). I start with a short review of the reasons for FtLM identified in the literature, divided into three main categories: cognitive, affective, and social. I then claim that FtLM cannot be understood by only one of these lenses. Therefore, a complex, holistic lens is needed. I introduce complexity theory to push forward our understanding of FtLM, including complex dynamic systems, emergence, feedback loops, and attractor states. I then zoom in on one particular theory – that of commognition, which I claim is a theory that can be thought of as a sub-type of complexity theories. I demonstrate the theory on two cases of students who developed identities of failure to learn mathematics.

INTRODUCTION

The notion of *failure to learn mathematics* is not a very common one. Researchers and educators often talk about "difficulties," "disabilities," "marginalization," or even "disengagement" from mathematics. They do not talk about “failure”. Perhaps it is because the word “failure” has negative connotations. Perhaps it is because, as educators, we are usually focused on the other side – on success (or simply on learning) – and not on how this learning fails to happen. Nevertheless, I do not think anyone would contest the claim that some students do not learn mathematics as expected by their teachers, curriculum, or test standards.

It is not a coincidence that I chose not to use more common words such as "difficulties" or disabilities. It is because, as I will herein claim, failure to learn mathematics is not an attribute of an individual student (as mostly associated with the word “disability" or "difficulty"); rather it is an attribute of a *system*. More specifically, a *complex dynamic system*. The student and her actions have a significant role in this system. They are not merely passive recipients of the forces of a social system, which is why terms such as “marginalization” or “lack of access” are not entirely appropriate either. Thus, I remain with the term *failure to learn mathematics* (or FtLM), whose precise definition will need to await the outline of my theory.

My goal in this talk is to provide a new theoretical lens on the phenomenon of failure to learn mathematics. This theoretical lens rests primarily on my former work on how middle school students' identities of failure develop, intertwined with their actual mathematical discourse. The empirical data I will present is not very new. Most of it was collected during my doctoral studies and was published in several papers. In this

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talk, my main innovation is to connect the former theorizing of these cases to broader theories of complex dynamic systems.

My plan for the talk is thus as follows: I will start by succinctly reviewing the vast literature on failure to learn mathematics. The point of this short review will be to raise the need for a holistic, systems-based view of FtLM. I will next move to review, again, very succinctly, the theory of complex dynamic systems, focusing on several central concepts that may be helpful for understanding FtLM. Next, I will focus on commognition and attempt to show how this theory can be considered a type of complexity theory. This will lead me to demonstrate some commognitive and complexity-related findings in cases of FtLM. Finally, I will examine the implications of viewing FtLM as a complex dynamic system theoretically and practically.

THE THREE STRANDS OF RESEARCH ON FTLM

The individual cognitive view: Locating FtLM in students' cognitive capacities

By far, the most significant amount of research on students' failure to learn mathematics has been done under the assumption that the "blame" for the failure can be located in the individual student's brain, specifically in his/her cognitive capacities. Despite being somewhat lagging after the literature on failure to learn to read and write (captured by terms such as dyslexia and dysgraphia), the literature on failure to learn mathematics has seen an ever-growing number of diagnostic terms such as dyscalculia, acalculia, and Mathematical Learning Disability (MLD) (e.g., Butterworth, 2005; Geary, 2004).

Anchored in developmental psychology and neuroscience, the central metaphor of learning underlying this literature strand is learning as a biological development. Studies within this strand often background the teaching and learning processes happening at school. When they do relate to them, this is often through the metaphor of learning as acquiring knowledge. Thus, the underlying assumption of much of the individual-cognitive strand is that mathematical performance can be divided to that which naturally "develops" in the child (much like children develop their ability to walk) and that which is "acquired" through instruction. Instruction and the various ways instruction may meet students with various cognitive or neurological profiles are rarely dealt with in these studies.

Empirically, the vast literature in the individual-cognitive strand shows that low achievements in mathematics are highly persistent. Thus, in many studies (e.g. Zhang et al., 2020), students located on the lowest end of the achievement spectrum in one year were found to populate this low end of the spectrum several years up the line. In fact, it seems that at no point are children "equal" in their propensity to succeed or fail in mathematics. Right from the start of schooling, gaps between high and low-achieving students exist, which keep getting wider as schooling progresses (Dowker, 2019).

Even with the enormous amount of literature on MLDs, most of it is highly focused on diagnosis, prediction, and intervention. Much less has been done in terms of understanding *how* persistent difficulties in mathematics develop over time.

The Individual-affective strand – locating the blame on individuals' affect: emotions, motivation, and beliefs

The second prominent strand of research that focuses on FtLM, and could shed light on its development is the strand focusing on affective elements. Factors used in this individual-affective strand to explain FtLM involve low self-efficacy, lack of motivation, unhelpful beliefs (about mathematics and self), negative attitudes towards mathematics, and above all – mathematics anxiety (Ashcraft, 2012; Dowker, 2019).

This vast literature has revealed many important insights, among which are the reciprocal links between mathematics anxiety and achievement (Ramirez et al., 2018); the manifestations of mathematics anxiety, which often resemble those of other "cognitive" mathematics difficulties (Ashcraft, 2012); and importance of gender to self-perceptions and attitudes towards mathematics (Dowker, 2019). Generally, the literature on affect and its relation to FtLM takes into account social and even cultural factors as impacting students' negative feelings towards mathematics. Nevertheless, I still consider this strand to be primarily focused on the individual, since the mechanisms that it assumes to be working to produce failure in mathematics (such as emotions, anxiety, and beliefs) are located, first and foremost, in the individual's mind.

The social strand – Locating the fault outside the individual

Most of the literature explaining mathematical failure focuses on the individual and locates the problem in the individual student and their cognition or affect. Nevertheless, in the past several decades, researchers have started locating the problem of failure in mathematics in social processes and social systems, such as lack of access to the domain (Nasir & Hand, 2008) or lack of appropriate opportunities to learn (Hiebert & Grouws, 2007). The main metaphor underlying much of the literature of the social strand is of learning as participating in a collective activity or community (Lave & Wenger, 1991).

A significant force in this social strand has been studies of the relation between FtLM and racism or other forms of oppression on the experiences of Afro-American youth and the ways by which they are marginalized from quality mathematical experiences in the United States (e.g., Martin (2000); Nasir & Hand, (2008)). Studies on race and mathematical learning detail the intricate ways by which power, positioning, and identity lead students to disengage from mathematics. These studies have shown that FtLM can occur due to students being born to a certain race, ethnic group, or socio-economic situation, not due to their individual cognitive (or emotional) capacities.

The above short and admittedly shallow review of literature on FtLM is not meant to be an extensive review. It aims to show that the literature on this phenomenon has generally placed "the blame" for FtLM on different factors (cognitive, affective, and social). However, studies have rarely examined how all these factors function together. In addition, despite the extensive research, we still need answers to many of our questions. How and why do certain students develop identities of failure over time? How do specific teaching environments perpetuate failure, at least for some students? Viewing FtLM as a phenomenon of a complex dynamic system may offer some answers.

A COMPLEX DYNAMIC SYSTEMS VIEW ON FTLM

Complexity theories, or theories dealing with complex dynamic systems, have been flourishing in multiple scientific domains including mathematics, computer science, biology, neuroscience, and more (Mitchell, 2009). Familiar examples of complex dynamic systems (CDS) include flocks of birds, ant colonies, economies, and the brain. A CDS is “a collection of interacting components … that gives rise to complex behavior” (Hilpert & Merchand, 2018) where the behavior of the system as a whole is not only bigger than the sum of its parts but also cannot be explained by the actions of each part separately. In that way, *emergence* is one of the main properties of complex dynamic systems. It is the mechanism by which "dynamic micro-interactions of system components give rise to novel macro-system behavior” (ibid, p. 191). Moreover, a CDS is characterized by non-linear relationships. Thus, it is impossible to decompose a CDS into simple cause-effect relationships (Koopmans, 2020).

Complex dynamic systems have certain characteristic mechanisms. One such mechanism is *feedback loops*, where the elements of the system give rise to a particular state of the system, which in turn impacts the elements of the system and so on (Koopmans, 2020). Another important mechanism of a CDS is *attractor states*, whereby the system fixes on a particular state. Perturbations to the system, although changing elements locally, still lead to the system falling back again and again to the attractor state (Hilpert & Marchand, 2018). Attractor states are used, for example, to explain how micro-scale economic interactions emerge into a stable state of the overall economy.

The first step to characterize the phenomena of FtLM as a phenomenon of a complex dynamic system would be to pinpoint the elements of this system (Kaplan & Garner, 2020). For that, previous works that looked at learning and teaching mathematics in systemic ways may be helpful. Thus, the well-known teaching triad, which includes the student, the teacher, and the mathematics (Kilpatrick et al., 2001), offers a good starting point. Additional elements identified as relevant for learning success include the student's family members, social group norms, culture, and political discourses (Crick, 2012).

Although FtLM has not, to the best of my knowledge, been treated or conceptualized with tools of CDS theory, other tangential concepts have. These include, for example, learning engagement (Crick, 2012), role identity (Kaplan & Garner, 2017), and learning a second language (Larsen-Freeman, 1997). In mathematics education, the major work in complex systems was authored during the last two decades by Davis, Sumara, and Simmt (Davis & Simmt, 2003; Davis & Sumara, 2010). However, their work concentrated mainly on the work of teachers and the web of complex interactions involved in learning to teach mathematics and less on the learning of mathematics.

Commognition and its alignment with complexity theory

Commognition, the amalgam of communication and cognition, was put forward by Anna Sfard (2008) as an alternative to the mainstream cognitivist frameworks of mathematics learning. Relying on three primary inspirations: Vygotzky’s socio-cultural theory of learning, Lave & Wenger's (1991) idea of "learning as participating in a

community," and Wittgenstein's (1953) late philosophy, Sfard offered in her book a radical new look at what mathematics learning is all about. In what follows, I introduce the central tenets and conceptual toolset of commognition while attempting to show that much of it is aligned with complexity theories.
The central claim of Sfard (2008) is that mathematics is a discourse, and learning mathematics can be thought of as the process of an individual becoming a participant in that discourse. The way that Sfard conceptualizes the mathematics discourse is very similar to complex dynamic systems. For example, she states that “mathematical discourse … can be seen as a multi-level structure, any layer of which may give rise to, and become the object of, yet another discursive stratum” (Sfard, 2008, p. 129). Moreover, Sfard conceptualizes mathematics as an *autopoietic* system in which the objects emerge “from inside”, that is, the system gives rise to the objects of talk (the mathematical objects) along with the talk (narratives) about these objects. Similar to CDSs that are hierarchical and contain nested forms (sometimes referred to as fractal), Sfard conceptualizes mathematical discourses as nested (or subsumed) within each other. For example, the discourse on natural numbers is subsumed under the discourse on rational numbers, which is subsumed under the discourse on real numbers, and so forth.

Another tenet of commognition that is shared with complexity theories is the rejection of simple cause-effect relationships in the study of learning. Thus, for example, Sfard (2008) claims that thinking and communicating cannot be separated, and thinking cannot be considered as a "cause" of communicating. Rather, it is better to conceptualize thinking as an intra-personal type of interpersonal communication. In that way, the socio-cultural origins of thinking become more evident. In terms of CDS, this conceptualization brings forth the idea that individual thinking is a nested form of broader, collective forms of interaction. In addition, Sfard claimed that terms that refer to mental concepts (such as the "concept of number") and that carry the metaphor of learning as "acquisition" of a particular good ("knowledge") are unhelpful for understanding learning-teaching interactions at the microscale. Similar to complexity theories, one of Sfard’s main critiques against these “individualist” concepts was that they do not capture the complexity of learning and the fact that learning happens in a wide, rich, and ever-changing social sphere.

A central concept in commognition is that of *routine*, or a “set of meta-rules defining a discursive pattern that repeats itself in certain types of situations” (Sfard, 2008, p. 301). These repetitive patterns can be characterized according to their task (goal, what the routine aims to achieve) and procedure (the series of steps to achieve the goal) (Lavie, Steiner & Sfard, 2019). Complexity theorists have described such patterns as attractor states. For example, Cameron and Larsen-Freeman (2007) conceptualize the everyday classroom discourse routine of IRF (Initiation, Response, Feedback) as "The discourse system will tend to return to the IRF attractor because it is a pattern that works; it is a preferred behaviour of the system.” (Cameron & Larsen-Freeman, 2007, p. 11). Routines are conceptualized in commognition as elements of the broader system (the discourse), together with keywords, visual mediators, and narratives that dynamically shape each other.

Two major concepts of commognition that are helpful for understanding FtLM are *ritual* and *explorative* participation. Sfard and Lavie (2005), who studied the development of young children’s numerical discourse, noticed that these children had peculiar ways of talking about numbers. Their discourse was not *objectified*. That is, they did not talk about numbers as objects (as grown-ups or more experts of the arithmetic discourse do) but rather as *processes* (such as descriptors of the act of counting). Sfard and Lavie reasoned that this made sense from within a "learning as participating in a discourse" lens: How else could novice participants start communicating in a discourse whose objects are not yet objects? After all, numbers are not material objects to which one can point and say, "here, this is two floating around in the room." They are discursive objects whose existence *emerges* from the discourse that produces them. Novice participants enter a new discourse as ritual participants. They imitate the experts, relying mainly on the procedures that they observe these experts enact, and attempt to reproduce those results (or narratives) that would gain the approval of the experts. Only repeated experience in the practices and routines of the discourse can lead novices to base their narratives on their own authority and to participate *exploratively*. Such explorative participation is aimed at producing new narratives in the discourse (rather than gaining the approval of experts).

Importantly, the process of de-ritualization happens repeatedly whenever a new discourse is introduced. In school, these discourses include the discourse on natural numbers, then the discourse on rational numbers, on algebraic expressions, etc. The process of learning mathematics is thus a never-ending advancement from ritual to exploration in one discourse, only to move on to ritual participation in the higher level (or subsuming) discourse. However, as some studies have shown, this "natural" ritual-to-explorative progression does not always occur as expected, as seen in the ritual participation of students identified as having difficulties with mathematics. This phenomenon was termed by Heyd-Metzuyanim and Graven (2016) as “ritual gone wrong”.

Much of my work in commognition has focused on the affective and social, or identity related, aspects of learning, namely *subjectifying* – communicating about the participants of the discourse (Heyd-Metzuyanim & Sfard, 2012). The idea is that mathematizing and subjectifying mostly happen concomitantly with mathematizing in the actual, real-time activity of learning. Subjectifying narratives, such as “I don’t understand this” can gradually become reified into identity narratives such as “I’m not good in math”. These 1st Person identity narratives interact and are shaped also by 3rd person and 2nd person stories that significant others tell about the person, such as "She has difficulties with math" or "You are quick in calculations" (Sfard & Prusak, 2005).

A similarity between commognition and complexity theory can be found here in that, in analyzing mathematizing and subjectifying activity as it co-occurs in interaction, the researcher moves from searching for affective *cause-effect* relationships in the classroom (such as students’ emotions or beliefs causing engagement or disengagement) to the *co-shaping* of different types of discursive activities (e.g., mathematizing and subjectifying). Similarly, complexity theorists often talk about how complex dynamic systems *shape* each other rather than how elements in the system directly influence each other (Turner & Baker, 2019).

Within this theorizing, it can now be defined more precisely what is meant by failure to learn mathematics. It is *a consistent impasse of an individual within a certain mathematical community to develop an identity of inclusion and expertise and de-ritualize one's mathematical discourse*.

Commognition shares with complexity theories the rejection of simple cause-effect relationships in the phenomenon of learning; the idea that various forms and states of the system at a macro-level (such as mathematical discourses) emerge from interactions between the elements of the system (such as routines); and the idea that various activities and elements in the CDS (such as mathematizing and identifying) shape and interact with each other (rather than directly *impact* each other). In the two case studies presented below, I aim to exemplify some of the insights that can be gained with such a complexity/commognitive lens on the development of FtLM.

FTLM AS A COMPLEX DYNAMIC SYSTEM – TWO CASE STUDIES

The case of Idit – tracking the development of FtLM over time

When I first met her, Idit studied in 7th grade (12-13 years old). She enrolled in a mathematics learning project that I led in an out-of-school learning center, where I taught small groups of students as part of my Ph.D. study. My goal in that study was to examine the ways by which emotional and social factors intertwined with mathematical learning (Heyd-Metzuyanim, 2011). Idit stood out as an interesting case because she often subjectified and identified herself during learning, mostly negatively. Her main, repeated narrative was "I'm not good with fractions". Other than that, Idit identified herself as quite a good mathematics student, even as an "exceling" one in her former, elementary school years (in Israel, elementary school continues up to 6th grade).

The reason Idit's case became even more influential was not her participation in the original study. In that study, she was placed in the intermediate group and generally succeeded, achieving good enough grades to be tracked to the highest track in the following year and even managed to stay in that highest track until 9th grade. The reason Idit's story became so interesting is that two years after she graduated my 5-months course, her parents approached me with a desperate plea: "Idit is failing in all her mathematics tests. She knows the material just fine, but gets blackouts on tests. Please help!"

Well, discourse studies advance very slowly, I must confess. Thus, during that time, I was just finalizing my analyses of Idit's participation in my course and the characteristics of her discourse in the interviews of the two past years. I thus knew that Idit was not "just fine" when she had left my course. I knew there were major indications of ritual participation in 7th grade, which I was not able to overcome in my teaching of her over 5 months. Yet, I was still surprised to find the extent to which Idit had developed an identity of failure in mathematics over the two years. She was reporting having severe “mathematics anxiety” in tests and her mathematical interview revealed ritual participation in almost all the algebra tasks that were posed to her.

Let us now go back to Idit at 7th grade, and demonstrate two excerpts of what characterized her discourse at that stage most clearly: ritual performance on tasks dealing with fractions, along relatively explorative participation in discourse on whole (or natural) numbers.

Following is an illustrative example of Idit’s response when she was faced with a problem dealing with fractions. This problem was posed to her during a mathematical interview conducted with her at the beginning of the 5-month course. In this task, Idit was asked to compare 1/5 and 1/7 (i.e., place a sign of >, < or =) :

1. **Idit**: So Um.. (looks at the interviewer, smiles) here I'm completely unsure, I told you I'm not good with fractions. (Looks back at the sheet), so I think… that this is bigger (points to 1/5).. [I’m] not sure.
2. **Interv**: Why do you think so?
3. **Idit**: ‘cause, if I’m not mistaken, in elementary school we were told that the smaller the number, so it’s bigger. That’s what we were told (shrugs shoulders, slightly smiles).

The first issue to notice in the excerpt above is that Idit is participating in two, mutually interacting discourses. One is the mathematical discourse (in which she is *mathematizing*), the other is the identifying discourse (in which she subjectifies and identifies herself). Thus, statements such as “this is bigger” [1] and “the smaller the number, so it’s bigger” [3] are mathematizing narratives, while “I’m completely unsure” and “I’m not good with fractions” are identifying narratives.

Idit’s solution of the 1/5[]1/7 problem is also a clear example of *ritual participation* and enables exemplifying those characteristics of ritual routines as put forward by Lavie, Stein & Sfard (2019). First, we see in it *lack of agency*, seen by the hesitations and the “I’m not sure” hedge [1]. Next, the talk is *syntactically mediated* in that the digits 1, 5, and 7, are treated as separate signifiers of “the smaller” and “the bigger” numbers. There is no *objectification* of the fractions 1/5 and 1/7. That is, the pairs of numerals are not treated as signaling a single object. The performance is *procedure oriented*, where the procedure can be described as “decide which number is bigger or smaller, then switch your answer to the opposite”. There is a lack of *substantiality* in this procedure (Why switch? Why compare the numbers under the dividing line?) and it relies on external authority (“That’s what we were told” [3]).

In contrast to Idit’s engagement with tasks that included fractions, her discourse on natural numbers had much more explorative characteristics. Here is an example of how she solved a word problem with natural numbers:

1. Idit: (*Reads the question*) Um.. Four shirts cost 200 shekels, six pairs of pants cost 300 shekels. Rina bought two shirts and two pairs of pants. How much did she pay? So, she bought two shirts which is 100 shekels (*looks down*)
2. Interv: Umhmm
3. Edit: And two pants which is… also 100 shekels

4. Interv: OK
5. Idit: (*Writes*) so that gives 200 shekels

The first issue to notice in this excerpt is that it lacks any subjectifying talk. All of Idit's communications are purely mathematical. Secondly, the characteristics of performance on this task are almost opposite of her performance in the task with fractions. There, Idit relied on external authority, and here, she expresses confidence, does not ask for reassurance, and generally relies on her own authority for her mathematizing ("*so* that gives…"[5]). In addition, Idit talks about the numbers as objects ("two shirts which *is* 100" [1], "so that *gives* 200" [5]). There is no human action in her reporting of the calculation procedure. The numbers just "behave" in that way in the world. Finally, her procedures are all bonded. Although much of her sub-routines are performed internally (such as dividing 200:2, in accordance with two shirts being half of 4 shirts), their outputs flow directly to the next subroutine (such as adding 100 and 100 to get 200). Notably, Idit's performance is so fluent, that we do not have full access to the procedures she performed internally. In any case, Idit had no problem manipulating the whole numbers in the problem, dividing them, as well as adding them up in a multistep procedure.

The stark difference between Idit's discourse on fractions and her discourse on whole numbers exemplifies that rituals and explorations are not an attribute of a *student* (one cannot say "Idit was a ritual student"). Neither are they an attribute only of a certain mathematical subject (it is not that students are always ritual with fractions and explorative with whole numbers). Rather, ritual/explorative characteristics are an attribute of a *system* made of the student *and* the mathematics. In many cases, the teacher (or expert) also forms a part of that system.

Idit's case, however, did not just provide insights into the various emergent properties of her participation in different mathematical discourses; it also provided a telling case of the development of an identity of failure. A series of interviews I conducted with her parents, her school teacher and with Idit herself (in 7^{th} and 9^{th} grade) revealed that, most probably, Idit's system of learning had evolved into a "ritual cycle of failure" (Heyd-Metzuyanim, 2015).

In this cycle, various feedback loops were operating between different elements. These included *Idit's mathematizing routines*, which progressively became more and more ritual as the tasks she was asked to engage with (in the discourse of rational numbers, then in that of algebra and functions) grew wider apart from the routines that she had mastery of. In interaction with Idit's mathematizing routines, Idit's identity narratives became progressively more reified and general. From the local narrative of being "not good with fractions" yet "good in math," Idit's identity narratives became "math anxious" and "knows the material but fails in tests" (with no ostensible differentiation with what particular topics were failed in those tests). These identity narratives emerged in response to the mathematical activity and then fed back to the mathematizing. Thus, for example, Idit would disengage from tasks that included fractions with the claim that "she is not good with them."

Another crucial element in the system of Idit's FtLM were 3rd person identity narratives authored around her. Among the most significant authors of these narratives were her parents, who authored an identity of Idit as "knowing the material", coupled by the expectation that Idit be placed (as they had been in their youth) in the highest "5 units" track of mathematics high-school. These 3rd and 2nd Person identity narratives probably functioned in Idit's system to encourage Idit to seek even more shortcuts and opportunities for ritual (rather than explorative) participation, so that she produces the results expected of her. Such "shortcut-seeking" can often be the response of a system of learning (including teacher and student) to growing disparities between the student's current ability to participate in a particular discourse and the expectation of the curriculum. After all, imitating procedures (without bonding their sub-steps, objectifying the symbols involved, and using flexible procedures) is often quicker and easier to produce than explorative routines.

To the elements enlisted in Idit's system of FtLM, one should add the narratives about what successful mathematics learning looks like in Idit's mathematics classroom and school. These were of the "traditional" type, valuing the execution of correct and accurate procedures. "Understanding", "engaging," and "making sense" were not part of the valued actions in that culture. Thus, Idit, when evaluating that she "knew the material," was relying on the narrative that "knowing" in mathematics means producing correct answers in homework or test-preparation tasks.

We thus see that Idit's FtLM, as assessed in 9th grade, was far from being a result of one component of the system. It was an *emergent* property of a complex system that included Idit's mathematizing routines, her identity narratives, her parents' stories about her, the norms of her school mathematics classroom, and probably other factors that were less explored in this study (such as narratives about gender and mathematics in Idit's culture). Each one of these, in and of itself, could not explain Idit's system of learning evolving into an attractor state of failure in 9th grade. Neither could a simple adding up of these components. For example, Idit identified herself negatively in relation to mathematics (locally, concerning fractions) when she was in 7th grade. However, at that time, she was still relatively high-achieving. Only later on, in 9th grade, did Idit's identity narratives (as authored by various actors) form together with her mathematizing an emergent pattern of failure.

The overall view of Idit's progression of participation, from locally ritual in 7th grade to the wide and general failure in 9th grade, hints that Idit's ritual participation was formed by feedback loops that perpetuated and strengthened Idit's routines of participation, to eventually form an attractor state of FtLM.

The case of Dana – capturing the co-construction of a ritual system

As reviewed above, FtLM is often "blamed" on one of two elements: the student or the teacher. The case of Dana, the lowest achieving student in my PhD study, showed me, in minute detail, how failure, and more specifically, extreme ritual participation, can emerge from the interaction of these two elements in the system: the student *and* the teacher.

Dana was placed into the low-achieving group of 4 students in my course after having had a history of low achievements in mathematics. Her mother reported that this started very early in her schooling career. Both her school teacher and myself (after having worked with her for five months) concluded that she was a "lost cause." Simply, nothing that I had done seemed to help Dana advance. Her participation was so extremely ritual and often so remote from the expectations of skills supposed to be mastered by 7^{th} grade (or even much lower grades such as 3^{rd} or 4^{th}), that it led me to identify her as "clueless" in mathematics (Heyd-Metzuyanim, 2013). To my dismay, however, when I was ready to write up her case in my dissertation and to show that Dana was indeed, ‘un-teachable’, I discovered two alarming findings. (a) In many cases, careful analysis could reveal some sense in Dana’s mathematical performances (albeit highly non-canonical, but they had some systematicity). (b) I, as her instructor, had a large part in forming together with Dana ritual routines of participation. In other words, the ritual routines that Dana engaged in were not her individual achievement. We co-constructed them together. The following excerpt, taken from Heyd-Metzuyanim (2013), is an example of such an occurrence. The context for this episode is a lesson where I handed the group a worksheet on integer sums, which had a set of simplified tasks taken from the curriculum of their school mathematics. Dana came in late, so I gave her a short explanation on how to add integers using the "procedure of arrows", namely, deciding how to add the numbers on the number line (by moving left or right) according to the sign of the integer. Dana nodded all along my explanation, went to her seat, and started working, only to jump up after a couple of minutes and approach me:

77. Dana: Oh, oh, listen (*walks up to the teacher)* what do I do in this situation, that I have this plus this, like, here (*points to (+4) + (-1)=)* I write the biggest?

78. Teacher: First of all, this is excellent (*marks a V on a former exercise that Dana had solved correctly*). What are you asking? About plus four and minus three?

79. Dana: Like, what do I do, the closest to zero, for instance, plus three, minus three?

80. Teacher: You start, look. You do.. uh with the arrows. Plus four is (going) right, right? (*points with thumb to the right*), so one, two- this is zero (*points to the zero on the number line*) so one, two, three, four. I arrived at four. Now, what do I have to add? Minus one, which is one arrow (points to the left), one arrow to the left. OK? I arrived at the three.

81. Dana: (*impatiently*) So, no, this, I don't have problem with this.

Just minus or plus?

82. Teacher: Well, what is it here? (*points to the +3 on the number line*) (It's) a plus

83. Dana: Oh, >plus, plus< (*goes back to her chair*)

The most ostensible characteristics of ritual participation in this excerpt probably come from Dana's push towards a ritual routine, evident in her focus on procedures ("What do I do?" [77, 79] and in her syntactic mediation “Just minus or plus?” [81]). This made it evident that Dana was showing no interest in the objects at hand and their various realizations (number line, arrows, etc.). She was only concerned with which signifier to attach to the number (+ or -) and was seeking a short, easy-to-memorize rule for that (as seen from her suggestion, "I write the biggest?" [77]).

Yet, a more careful look at this interaction also shows that the teacher (myself) also had an important part in this ritual participation. The first indication could be found in [78], where I make a point to first mark Dana's previous exercise as "excellent," even though I have no idea at that moment how she arrived at the correct answer. By this, I indicated that what is important in this activity is getting the right answers. Second, I joined Dana in the talk about *doing* (“you start.. you do" [80]). Thus, my explanation focused on the procedure and the actions, not the meaning of the signifiers and the visual mediators. In addition, I never stopped to ask Dana what *she* sees in this task, what it means for *her*, and how *she* views the different signs. That way, I could have no idea whether my explanation ("Plus four is (going) right" [80]) made any sense to Dana, and whether it connected to how she interpreted the number line. Thus, I gave very little *agency* to Dana. The authority for what is right and what is wrong remained fully with me.

The system of Dana’s FtLM was much more complex than the teaching-learning interactions that she had with me, exemplified in the short interaction above. In Dana’s case, a major element in the system that attracted towards failure was the huge disparity between Dana’s mathematical skills and the expectations of her environment (mainly, the school curriculum). There was simply no perceivable way that, in 7th grade, Dana could be engaged in tasks that fit the level of her “intact” discourse. (Namely, the discourse where routines were relatively explorative, mathematical signifiers were treated as objects, and she could rely on her own agency to author mathematical narratives). Doing that would have probably demanded returning to tasks associated with 2nd or 3rd grade. Not only would such “catching up” with the 7th grade curriculum be improbable in the sense of how much time Dana and I (or any other teacher) could allocate for mathematical activity, Dana was completely unwilling to engage in any activities that she did not perceive as directly beneficial for helping her participate in classroom activities and "getting good grades." This unwillingness, in turn, was shaped, as in Idit’s case, by the norms in Dana’s culture of what it means to do mathematics successfully.

Nevertheless, the excerpt above, despite being only a small part of the mechanism of Dana's FtLM, is highly significant mainly because of the *unawareness* of all parties to the fact that they were participating in a feedback loop that preserved an attractor state of failure. As Sfard (2023) points out, in mathematical interactions (and in any teaching-learning interactions, at that), the devil may be in the details. As my tone of voice (inaccessible through the transcript) and other tacit utterances (such as the “well, what is it here?” [82]) could have hinted, I was sure at the time that I was directing Dana towards sense-making. According to my own story of the situation at that moment, I was affording her explorative opportunities to learn. Yet I was not.

Dana’s case thus illustrates that FtLM systems may function above and beyond the interests and aspirations of the individual actors taking part in them, particularly the teacher and the student. It is this evasive characteristic of the mechanisms of FtLM, which often manifests itself only in the micro-details of interaction, that makes them so important to unearth and expose. Only such awareness may give hope that actors within the system of FtLM will act to reverse or halt it.

SOME CONCLUDING COMMENTS

Viewing FtLM as a complex dynamic system changes our thinking of mathematical "difficulties" or under-achievement. It makes it clear that there is no "simple solution" to this phenomenon, no one element that can be changed to fix it. In the case studies above, I concentrated mainly on demonstrating the unique affordances of a commognitive lens, as one type of complexity theory, to understand the development of FtLM. This uniqueness stems from two major characteristics of the commognitive framework: first, it offers alternative concepts for “cutting” (as in cookie cutters) the phenomena of teaching and learning. Instead of working along old divisions such as emotion vs. cognition, social vs. individual, or thinking vs. behavior, commognitive concepts look at all human actions through the lens of communication and discourse and divide them according to categories such as "mathematizing," "subjectifying," and "identifying." By that, phenomena that may have seemed to belong to different ontological worlds (such as attitudes towards mathematics vs. the development of mathematical concepts) can now be treated with the same set of conceptual and methodological tools. Second, the commognitive framework is productive for studying micro-details of interactions, and by that, can often shed light on “the devil in the details”, as Sfard (2023) calls the micro-actions that form the substance of more general, wider patterns of teaching and learning.

My argument is, however, not that commognition is *the* ultimate framework for studying FtLM as a complex dynamic system. My claim is that commognition can be helpful for studying particular aspects of this system that often go unnoticed. These are the *micro-processes* (rather than the states) of the complex dynamic system. Moreover, by cutting across the traditional levels that have usually been used in the study of FtLM (and learning more generally), commognitive terms turn our attention to the self-similarity of processes in the learning systems. It is through such “cutting-across” concepts, such as "discourse," "narratives," and "routines," that are applicable both at the individual and the collective level, that commognition enables us to see more clearly how individuals’ learning is nested within broader systems.

However, as said above, commognition is just one type of complexity theory. Recognizing it as such affords the potential of connecting the commognitive theory of the development of FtLM, with other concepts and methodologies that have yet to be used in connection with commognition. In particular, commognition is effective at unearthing the micro-details of interaction. However, along with the micro-scale come the limitations of extensive workload on relatively small data sets. Hence, commognition is less helpful for studying long-term processes that occur over long periods or for studying representative samples of a population. One benefit of nesting commognition within the broader array of complexity theories is that we may find ways to connect it with methods that are more appropriate for larger datasets.

OK, so what does a complexity view of FtLM mean, practically?

I believe the most important implication of viewing FtLM as a complex dynamic system is practical. Teachers, students, parents and policymakers should start seeing FtLM as a phenomenon of a collective rather than placing the blame on only one of the elements (be it the student, the teacher, the curriculum or any other element that is easy to take the blame). It is my firm belief that understanding the mechanisms of FtLM as a system, or at least drawing closer to understanding them, will be the best way to overcome them.

Second, I believe the view of FtLM as a complex dynamic system opens an array of empirical, methodological, and theoretical possibilities. It is good and well to understand that we are facing an issue of complexity; it is less clear how to study it. Thankfully, the field of complexity studies is ever-growing and has reached increasing impact in "neighbouring" fields such as organizational and developmental psychology. Many of these studies have introduced new, primarily quantitative, methods that overcome the drawbacks of traditional linear statistical methods. Unfortunately, to my knowledge, less work has been done in connecting complexity theories with qualitative methods. Connecting commognition with complexity can offer a step forward in that regard.

Above all, the concepts of complexity science provide a rich source for new conceptual tools and research questions. For example, recognizing FtLM as a CDS leads us to question whether mechanisms discovered in other domains also work in mathematics learning systems. For example, attractor states often set into place after a period where the system was relatively dynamic and volatile. Then, a "tipping point" sets them into a relatively stable state. Is that the case also in the development of FtLM? Are there certain "tipping points" that set the system into a state of failure? We need many more studies to figure that out, but the new conceptual tools offered by complexity science seem promising and exciting for that journey.

However, before I lead you to believe that solving the problem of FtLM is right around the corner – there is one last concept to be borrowed from complexity theories: the concept of *Wicked Problems*. Turner and Baker (2019) explain that there are problems in the world of three types: simple, complex, or “wicked." Simple problems have an identifiable problem and solution. Complex problems have an agreed-upon problem but differing potential solutions. Wicked problems, however, have no agreed-upon definition of a problem or a solution. Well, I believe that in this talk, I have shown that FtLM is certainly not a simple problem. It remains to be seen whether it is complex or "wicked." In any case, future studies that attack this problem from a complex dynamic systems perspective are bound to provide exciting insights.

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INTROSPECTIVE THINKING AND CONTEMPLATIVE THINKING IN MATHEMATICAL COMMUNICATION

– THE POWER OF SEEING THE INVISIBLE TO OVERCOME A FRAGMENTED WORLD –

Hideyo Emori

Otani University, Kyoto, Japan

We identified that reflective thinking consists of "introspective thinking" and "contemplative thinking [(1)]" (Emori, 2010). This paper explains the difference of the directions of the actions of cognition to be seen in "introspective thinking" and "contemplative thinking." To achieve this purpose, we have developed a research methodology based on two types of knowledge theories in Buddhism: nirākārajñānavāda (Sanskrit: the theory of knowledge without an image) and sākārajñānavāda (Sanskrit: the theory of knowledge with images). The scientific significance of this paper is to provide some basic knowledge about the development of mathematical communicative competence as the power to see an invisible thing.

INTRODUCTION

There are various conflicts and divisions in the world, not limited to wars and armed conflicts, but also including social conflicts, economic disparities, racial discrimination, and other issues. There are many conceivable reasons for conflict and division, one of which is the difference in worldviews of how we perceive this world as a potential cause of conflict and division. We naively believe that we perceive the world through sight, hearing, and other sense organs. Therefore, we believe that the world perceived through these sense organs, which are possessed by everyone, is the same world equally for everyone. However, our perception of the world is also influenced by our upbringing, our learning history, and the environment in which we live. Our perception of the world depends on how we individually see and feel things. And these differences in the way we see and feel things influence our values, which in turn form two divided worldviews, A, and non-A.

In contrast to the educational goal of developing mathematical communication skills based on the view of mathematics as a language, which has been supported worldwide since the 1980s (Cockcroft, 1982), we believe that a new educational goal of "developing the ability to see the invisible" is necessary to overcome a fragmented world. We assume that the world we can perceive is the only correct world. It is necessary for the children who will play important roles in the future come need to have the ability to understand that the world as I see it is not visible to others, and that the world as others see it is not visible to me.

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To understand that there is a world that we cannot see with the knowledge and experience we have acquired, we must strive to see the invisible beyond our own perspective. By setting the educational goal of "developing the ability to see the invisible," we believe that the enrichment of the worldview of each one of us, the powerless, will give us the strength to overcome a divided society (cf. Havel, 1978).

RESEARCH QUESTION

In Berlo's (1960) SMCR model of the communication process, the process of sending a message from a sender to a receiver through a certain channel was described in terms of four elements: sender (S), message (M), channel (C), and receiver (R). Berlo's communication theory is characterized by the fact that the message externalized by the sender to convey his or her intention to the receiver is considered a physical stimulus that contains no meaning. If we accept the notion that messages are physical stimulus objects, the aspect requiring clarification in communication research can be summarized in the question of what type of cognitive process is engaged in the receiver's interpretation of the message. For example, in mathematical problem-solving situations, it is known that how the receiver interprets the diagram presented by the sender influences the receiver's thinking (Emori, 1996, 2006, 2007, 2009, 2010). In particular, Emori's (2010) case analysis and discussion showed that "reflective thinking (Dewey, 1933) [(2)]" occurring between the receiver and the figure (the object of thought) includes two components: "introspective thinking," in which the receiver actively engages with the figure, and "contemplative thinking," in which the figure brings about some awareness in the receiver, representing a passive form of thinking from the receiver's perspective.

Emori (2010) divided reflective thinking into two phases, introspective thinking and contemplative thinking, to explain the process of hypothesis formation and discovery that develops in the chain of introspective thinking and contemplative thinking about the message presented by the sender with using Peirce's logic (Peirce, 1961) [(3)]. Introspective thinking is thinking about the expressive activity of representing the object of thought. The reason for giving this thought the name of "introspective thinking" is that some improvement is made to some of the pre-expressive activities as a result of this thought. And furthermore, this introspective thinking will lead to second and third introspective thinking, and the activity of representing the object of thought, which is done as a second and a third expressive activity, will be elevated to an activity of creating productive expressions that bring innovative ideas. On the other hand, contemplative thinking is characterized by the fact that this thinking does not bring about an improvement of the expression. Contemplative thinking is the thinking that gives selective perception as a new interpretation to the exemplified expression by contemplating the expression that has been elaborated through previous trial and error.

In this paper, we focus on the cognitive activity of seeing the invisible in order to deepen our discussion on introspective thinking and contemplative thinking. It then solves the research question, "How can the difference in the orientation of the cognitive action seen in introspective thinking and contemplative thinking be explained as a cognitive process that occurs between the object and the knowledge possessed by the learner?” (Figure 1)

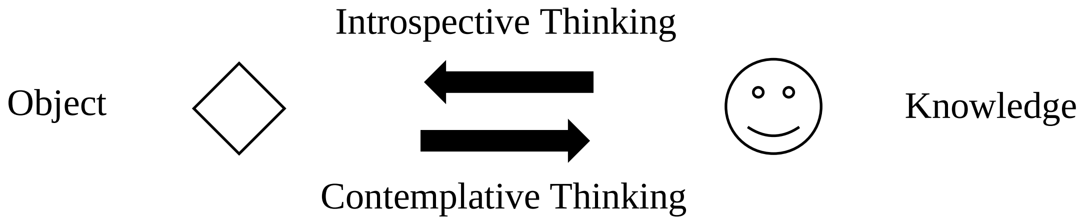

Figure 1: Introspective thinking and Contemplative thinking

RESEARCH METHODS

To solve the research problem, this paper constructs an eclectic research methodology based on two types of theories of knowledge in Buddhism between “*nirākārajñānavāda* (Sanskrit: the theory of knowledge without an image; Kajiyama, 1977, p. 123)” and “*sākārajñānavāda* (Sanskrit: the theory of knowledge with images; Kajiyama, 1977, p. 123) (4).” *Nirākārajñānavāda* is the doctrine that the perception of the outer world does not arise from images impressed on the mind (Monier-Williams, 1998, p. 540). *Sākārajñānavāda* is the doctrine that ideas consist of forms or images which exist independently of the external world (Monier-Williams, 1998, p. 1197).

In this section, we do this by discussing how new perceptions gained through seeing the invisible are used to solve problems in mathematics. To construct an epistemology based on Buddhist knowledge theories, we can utilize concrete examples. For instance, consider the following conditions: “Condition 1: Two diagonals are equal in length. Condition 2: The two diagonals intersect perpendicularly. Condition 3: The two diagonals intersect at their midpoints.” Now, let us contemplate how we would respond if asked, “What type of rectangle satisfies these three conditions? (5) (Table 1)” We can see that there are two types of reactions regarding the question as shown below.

Question: Which quadrilateral satisfies the following three conditions?

Condition 1: Two diagonals are equal in length.

Condition 2: The two diagonals intersect perpendicularly.

Condition 3: The two diagonals intersect at their midpoints.

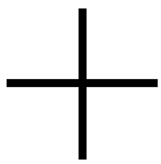

Table 1: Diagonal conditions and quadrangles

The first reaction is to imagine a figure connecting the four end points of the two segments attached to the question. By imagining the rectangle indicated by the dotted line in Figure 2, the learner believes that the rectangle being asked about in the question is a rhombus.

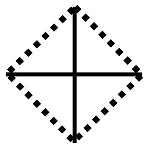

Figure 2: Image of rectangle connecting the four endpoints

The first reaction is one form of seeing the invisible. We can connect the ends of two diagonals and imagine that there is a rectangle there. And we can also infer, as a primitive inference, that the quadrilateral shown in Figure 2 is a “rhombus.” The quadrilateral that we see by imagining the invisible quadrilateral may be more natural to guess as a “rhombus” than as a “square” in the initial stage. In the first reaction, by observing the imaginary rectangle that was the object of the introspective thinking, the probable inference that it is most likely a “rhombus” is brought about by the observation that the four sides appear to be equal in length, and that the four corners also appear to be right angles, leading to a new perception that it is a “square.” This cognitive process of redefining the figure in front of us as a square, which was previously thought to be a rhombus, is what Emori (2010) identifies as introspective thinking.

The introspective thinking that occurs in this scene allows us to imagine an invisible rectangle and observe its shape by being keenly aware of its form. If we abandon our preoccupation with the invisible, we can draw the rectangle freehand. However, in any case, the rectangle in front of our eyes, which we perceive visually, may contain noise in terms of the length of its sides, the size of its angles, or other factors. In other words, the introspective thinking we are engaged in here is a reaction to remove noise from the figure in front of us and to recognize that the figure we are looking at is a genuine “square” as a mathematical concept, and this act is the very act of seeing the invisible. Our visual perception is not based solely on pure visual perception, such as the fact that the four sides are equal in length or that the four corners are right-angled. Even if the image in front of our eyes is slightly distorted, we recognize that the image we are seeing now is a square through our imagination.

We can determine the difference in the lengths of the sides and the distortion of the angles by observing the object, but we find that the lengths of the sides are approximately equal, or the side-to-side relationship is approximately perpendicular through this observation. In the theory of knowledge without an image, which is one of the positions of the knowledge theory in Buddhism, this process of recognition is explained by using the concept of “*Jñātatā*” (Sanskrit: the being known or understood. Monier-Williams, 1998, p. 425) (6). “Kumārila (circa 650-700) said, ‘The action of recognizing an object is caused by the nature of *Jñātatā*, which the object possesses’ (Yoshimizu, 2015) *.” Since a similar concept is not found in Western thought, in this paper, the term “knownness” is introduced as an English translation of the Sanskrit word “*Jñātatā*,” which means that which is known or understood.

As illustrated in Figure 3, the “representations of quadrilateral” described in the theory of knowledge without an image are nothing more than structured knowledge that transcends language. In the representations resulting from these cognitive processes, the characteristics that are verbalized through observation of the object remain as the memory of having observed the object. We then remember this representation as knowledge based on mathematical experience. By attaching various experiences as memories, we can create representations that contain structured knowledge (cf. Piaget, 1945).

The experience of verbally articulating an awareness derived from observing an object will return as a mathematical experience stored as knowledge the next time one encounters the object. The function of transforming this experience into knowledge related to the subject is vested in the subject. These functions of the object are fostered by the learning experience as a way of looking at the object. The nature of "knownness" in the theory of knowledge without an image is something that is imparted to an object by the learning experience of how to see the object, which has been fostered by the learning process of understanding the object for the first time. In other words, the question "What is the rectangle I am imagining?" that arises from the cognitive act of imagining a rectangle connecting the four endpoints makes us aware of a clear representation, as if the experience and knowledge we have accumulated in our previous studies were assigned to the diagram in front of us (Figure 3).

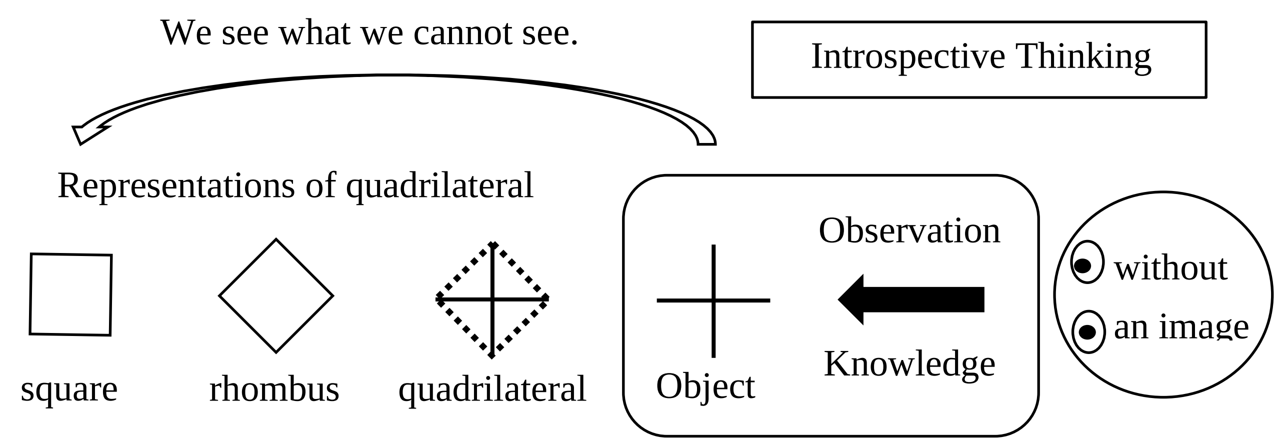

Figure 3: Cognitive process of seeing the invisible through introspective thinking

The second reaction is a view that selectively recognizes diagonals discarded in the first reaction (Figure 4). The view with the second reaction shows a cognitive process of applying the three conditions presented to compensate for visual perception by not excluding the diagonals from the visual picture. The view that four isosceles right triangles can be formed by two diagonals that satisfy the three conditions leads to the view that each of these four isosceles right triangles is congruent, and this view allows us to see that the quadrilateral connecting the four end points is a square.

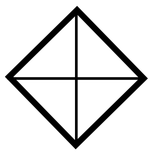

Figure 4: The view of classification of quadrilaterals based on diagonal conditions

In the second reaction, the knowledge that "a quadrilateral meeting the three conditions is a square" facilitates the cognitive process of perceiving the invisible. The theory of knowledge with images, which forms another school in the theories of knowledge in Buddhism, holds that knowledge necessarily has forms, and that we cannot recognize anything other than knowledge. Mokşākaragupta, at the end of his theory of perception in the first chapter of The *Tarkabhāşā* (Sanskrit: *Language of Logic*) [(7)], states, "Knowledge must be considered as endowed with the image of its object (*sākāra*). If knowledge is not admitted as having an image, it is not possible to establish objects separately from one another, since such knowledge without the imprint [left by each object] would remain the same on cognizing all objects (Kajiyama, 1966, pp. 61-62)." The fact that the images being recognized do not belong to the object of cognitive action but are part of knowledge is based on the fact that recognition is, in essence, the self-recognition of knowledge (Kajiyama, 1983, p. 45) [*]. Thus, the fact that we are looking at an object means that knowledge is aware of itself, which is nothing other than awareness of knowledge (Kajiyama, 1983, p. 47) [*].

We see the knowledge recalled by the object as an "image." The theory of knowledge with images holds that the objects we perceive are not external objects themselves, but images that we hold as knowledge. When we recall images about squares, we incidentally recall the mathematical experience of acquiring knowledge about squares. Recalling the mathematical experience implies an instant retracing of the learning process of the geometric concept of the square. Recalling learning experiences collectively as images, rather than recalling individual verbalized knowledge, means that "the images recalled are endowed with a directness, concentration, and high degree of demand that cannot be reached forever by concepts and explanations (Grassi, 1970/2016, p. 29) [*]."

Regarding the epistemology based on the theory of knowledge with images, Hattori states, "The recognition of an object means that there are two opportunities in our knowledge at the same time: the object illuminated by knowledge and the knowledge that illuminates that object. Knowledge always has these two opportunities within itself. If this is the case, then the object of knowledge is the object inside knowledge, i.e., the object that has already been incorporated into and become part of knowledge (Hattori, 1997, p. 116) [*]." Emori (2010) described the cognitive process in which a stimulus in front of our eyes stimulates the observer's knowledge and recall of related knowledge as shown in Figure 5.

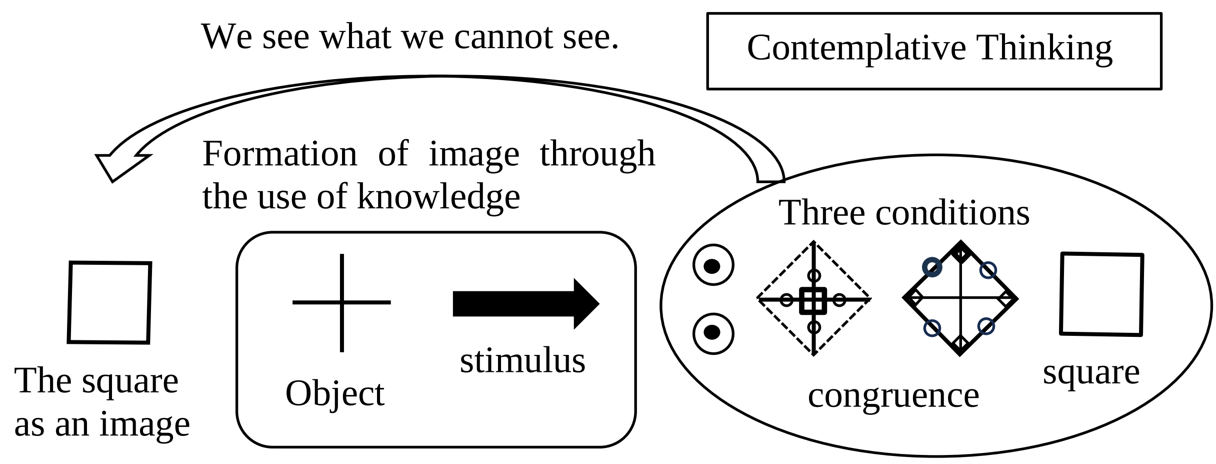

Figure 5: Cognitive process of seeing the invisible through contemplative thinking

CASE ANALYSIS

In this paper, we adopt Berlo's (1960) SMCR model, which considers the one-way process in which a message (M) is sent from a sender (S) to a receiver (R) through a certain channel (C) as the smallest unit of communication process (Figure 6).

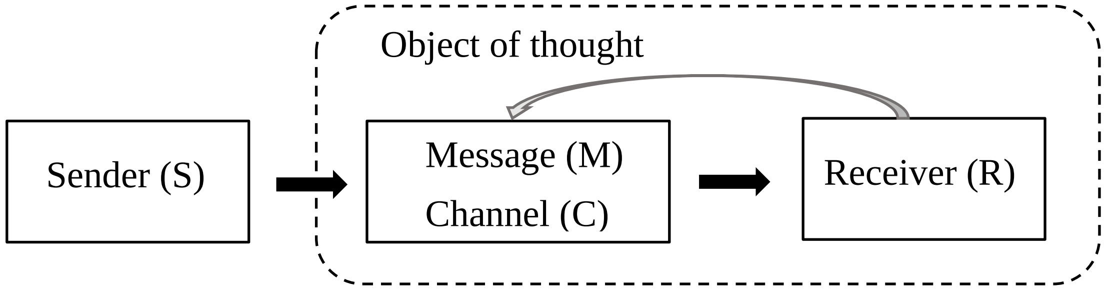

Figure 6: Basic unit of communication process

A distinctive feature of the SMCR model is that it defines a message as a physical stimulus without meaning. Therefore, in this model, the most critical issue in communication process research is the elucidation of the cognitive process of how the receiver interprets the message sent by the sender to understand the sender's intentions. Therefore, in this paper, we will focus on the cognitive process enclosed by the dotted line in Figure 6, where the receiver considers the message, problem statement and diagram, as the object of thought. The case study presented in this paper examines the processes of problem-solving of five ninth graders: Student A, Student B, Student C, Student D, and Student E. They were given the following problem: "There is a square ABCD with sides of length 2. Draw two semicircles inside the square with sides AB and BC as diameters. When the intersection of these two semicircles is point E, find the area of the ginkgo leaf shape that is the sum of the arrowhead shape AECD and the rugby ball shape EB (Table 2) [(8)]." This study is discussed in detail in Emori's article (Emori, 2022, 327(20)-306(41)).

Problem: There is a square ABCD with sides of length 2. Draw two semicircles inside the square with sides AB and BC as diameters. When the intersection of these two semicircles is point E, find the area of the ginkgo leaf shape that is the sum of the arrowhead shape AECD and the rugby ball shape EB.

Table 2: Problem statement

In this study, only the problem statement was presented at the beginning, and the students solved the problem, starting with a notebook representation of the ginkgo leaf-shaped diagram they sought from the given problem. At this stage, the freehand drawing shows that the two semicircles do not intersect at the center of the square. It was observed that none of the students paid attention to the location of the intersection of the semicircles (Figure 7). Students who have drawn up diagrams in this initial stage begin to feel uncomfortable with the freehand diagrams they have drawn themselves. The intuition that "something is wrong" will lead to what Dewey calls "reflective thinking," in which the intersection of the semicircles is selectively perceived.

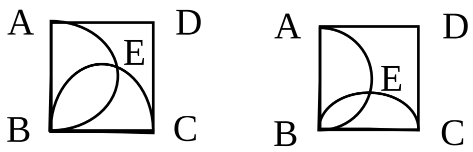

Figure 7: Freehand diagrams

Student A's solution using calculations

The question of where the two semicircles intersected paved the way for students who began to think about finding the area using formulas they knew. Student A said, "Subtract two semicircles from the area of a square, and then add the overlapping part twice to compensate for the subtraction and to add the overlapping part. The area of the square is 2 × 2 = 4, and the area of the two half circles is 1 × 1 × π × 1/2 × 2 = π. The area of the rugby ball shape is two black areas. The area of the black area (Figure 8) can be obtained by removing the triangle from the quarter circle, so 1 × 1 × π × 1/4 – 1 × 1 × 1/2 = π/4 – 1/2. The area of the rugby ball shape is equivalent to two of these, so (π/4 – 1/2) × 2 = π/2 – 1. Therefore, the area to be found is 4 – π + 2(π/2 – 1) = 4 – π + π – 2 = 2 (Table 3)."

The area of the square is subtracted by two half circles, and the overlapping area is subtracted twice to compensate for the subtraction and to add the overlapping area. The area of the square is 2 × 2 = 4, and the area of the two half circles is 1 × 1 × π × 1/2 × 2 = π. The area of the rugby ball shape is 2 times the area of the black area. The area of this black area is 1 × 1 × π × 1/4 – 1 × 1 × 1/2 = π/4 – 1/2, since the triangle can be removed from the quarter circle. The area of the rugby ball shape is the equivalent to two of these, so (π/4 – 1/2) × 2 = π/2 – 1. Therefore, the area to be determined is 4 – π + 2(π/2 – 1) = 4 – π + π – 2 = 2.

Table 3: Student A's solution

After the solution, when Student A was asked how he thought about the fact that "the area of the black part is one-fourth of the area of the original square," Student A stated that he did not have to explain it in particular because he intuitively thought that was correct. Student A emphasized that he knew empirically that this shape could be formed by the difference between a quarter circle and a right-angled isosceles triangle.

Although no explanation was added to Student A's solution, we can say that Student A had the shape recognition (image) to divide the square into four parts. Student A's intuitive understanding that the two semicircles intersect at the center of the square was essential knowledge for solving this problem. Student A can say that the knowledge that "two semicircles intersect at the center of a square" led him to recall the image of a square divided into four.

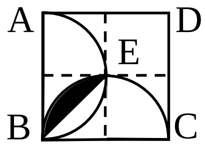

Figure 8: Black area shown by Student A (dotted lines are drawn by the author)

Student B's view of the diagram

Only a few students, like Student A, were able to find the area by calculation. Therefore, the author gave the students a question, "Is there any way to find the area by looking at the figure?" and encouraged them to find the area by observing the figure. This question made the students realize that the first incorrect diagram they drew was not a target for observation or thought, and they began to redraw it individually. In redrawing activities, student B draws a semicircle on side AB and then draws a semicircle with a diameter not on the adjacent side BC but on the opposite side CD, realizing that the two semicircles are tangent in a square. Student B said in a post-interview, "Suddenly, a diagram of two semicircles touching each other flashed into my mind, and I decided to draw it. When I was trying to draw a semicircle, I thought that the radius of a circle is equal everywhere, and I came up with such a figure," he said, adding that "a particular image" came to his mind before drawing Figure 9.

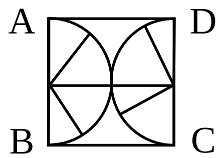

Figure 9: Figure drawn based on an image that came to Student B's mind

Student B then adds two more semicircles to draw up Figure 10. By adding two semicircles that are not in the problem statement, Student B discovers that the four semicircles intersect at the center of the square and that their intersection coincides with the intersection of the two tangent lines.

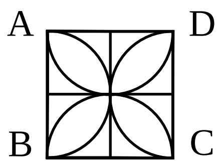

Figure 10: Four half circles and two tangent lines

Student B, through the contemplative thinking brought about by his selective perception of Figure 10, understands the mathematical structure presented by the problem statement as shown in Figure 11. Student B can be said to have recalled the image “the figure of two semicircles intersecting on the point of the intersection of two tangent lines” due to his knowledge that the radii of the circles are equal everywhere.

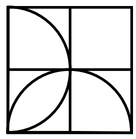

Figure 11: Mathematical structure brought about by selective perception

In Figure 9, many radii are drawn on the two semicircles, and a selective perception is at work in which one of the radii is extended to a neighboring radius that is dual-paired, resulting in a segment equal to the length of one side of the square. In order for such a realization to come to mind as an image, there must have been a prior background experience of having seen such a diagram. We cannot form mental images of things for which we have no knowledge or experience. Here it is shown that images have the power to instantly constitute mathematical knowledge, the power of images.

We do not recall all the knowledge contained in the object in the form of language, but rather, we make it known to ourselves in the form of images (as seen in mathematical experience, cf. Davis & Hersh, 1981), referred to as empirical intuition (Emori & Morimoto, 2019). This is what is meant by “knowledge itself perceiving its own knowledge.” In this scenario, communication takes place not through language, but rather as a dialogue with oneself, achieving a high degree of communication efficiency. It manifests itself as an image that condenses various forms of knowledge (Emori, 2006, pp. 53-54)

Student C’s thinking

What we have learned from observing the students’ drawings is that they recall the knowledge required for drawing as individualized knowledge for each student. For example, when observing Student C’s drawing, it becomes clear that Student C created a different image compared to Student B. Student C was attempting to depict a semicircle as a set of points equidistant from the center of the circle, focusing on the property of the circle’s radius. The addition of a vertical line extending straight up from the top of the semicircle inspired Student C (Figure 12).

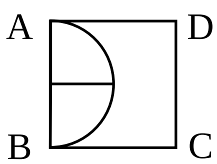

Figure 12: Vertical line extending from the top of the semicircle

Through introspective thinking, Student C strives to improve the diagram by drawing a semicircle as accurately as possible within the square. This process eventually leads Student C into a phase of contemplative thinking, prompted by the addition of a vertical line to the semicircle. In an instant, this inspiration conjures the image depicted in Figure 13 within Student C's mind. Prior to embarking on the task of sketching the second semicircle and vertical lines onto the diagram, integrated knowledge forms a mental image.

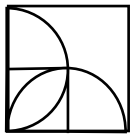

Figure 13: The mental image recalled by Student C

Student C then begins to draw a diagram to represent the image formed in his mind, and the diagram (Figure 14) that he draws brings another realization: "Each vertical line is a tangent line to the other semicircle, if you look at it differently."

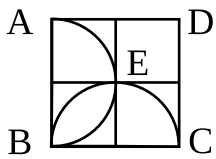

Figure 14: Figure drawn based on an image

The reaction that emerged in Student C is brought about by the knowledge that "the intersection of the two semicircles lies on the perpendicular bisector that passes through the center of each semicircle, the midpoints of sides AB and BC." The recall of such relevant knowledge instantly brings to the mind an image that depicts the state of affairs. We do not draw the two perpendicular bisectors into the diagram to recall and verify this relevant knowledge, but rather we draw them into the diagram as concrete lines in order to make the images that come to our minds the object of our thoughts. In the theory of knowledge without an image, after drawing a diagram, we are supposed to capture it as a visual picture, but from the standpoint of the theory of knowledge with images, we can obtain an accurate image of the situation, along with the recall of relevant knowledge, before drawing a diagram outside of it. Why should we go to the trouble of drawing it out as a diagram when we have obtained an exact image in our brains? This is because we are aware that the images obtained at this time alone will not help us solve the problem. The resulting figure does not need to be altered any further, and Student C uses it to solve the problem of finding the area. Student C, while looking at Figure 14, which he had scribbled out himself, notices that there are similar shapes, selective perception X_1 brought about by contemplative thinking (Figure 15, left). This selective perception then leads to the other selective perception X_2 (Figure 15, right).

Figure 15: Selective perception X_1 (left) and selective perception X_2 (right)

Student C thus discovers that the area he seeks is half of a square (Figure 16).

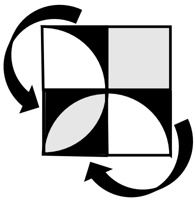

Figure 16: Equivalent deformation aggregated into two squares (image X)

Student C, who acquired the view of Figure 16, wrote on his answer sheet, "I divided this square into four parts. The square in the upper right corner is colored on all sides, so 1 × 1 = 1. Focusing on the remaining three squares, the black areas of the upper left and lower right square and the black area of the lower left square are equal, respectively, because the areas of the quarter circles are the same. So, the central part of the square in the lower left corner and these two areas together make a square, so the answer is 1 + 1, and which is 2. The area required is half the area of the square (Table 4)."

I divided this square into four parts. The square in the upper right corner is colored on all sides, so 1 × 1 = 1. Focusing on the remaining three squares, the black areas of the upper left and lower right squares and the black area of the lower left square are equal, respectively, because the areas of the quarter circles are the same. So, the central part of the square in the lower left corner and these two areas together make a square, so the answer is 1 + 1, and which is 2. The area required is half the area of the square.

Table 4: Student C's solution

The latter part of Student C's thought is an example of contemplative thinking in the sense that it is a thought found by contemplating the left side of Figure 17, which he drew. When Student C says, "The area I want is half the area of the square," the visual image he is recalling is the image of two squares divided into four parts (Figure 17, right). Student C can be said to have recalled the image of a "quarter square" by virtue of his knowledge that "the intersection of the two semicircles lies on the perpendicular bisector through the center of each semicircle."

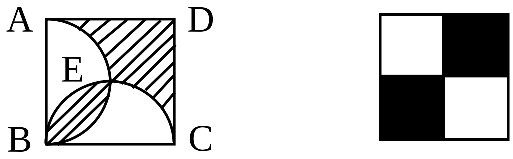

Figure 17: Student C’s figure (left) and image (right)

Mathematical activity of student D

Student D came up with the idea of rotating the figure in the process of thinking about the location of the intersection. Student D, who felt that his drawing was not beautiful, repeatedly rotated the figure and noticed that the intersection changed position as he rotated the figure (Figure 18: contemplative thinking).

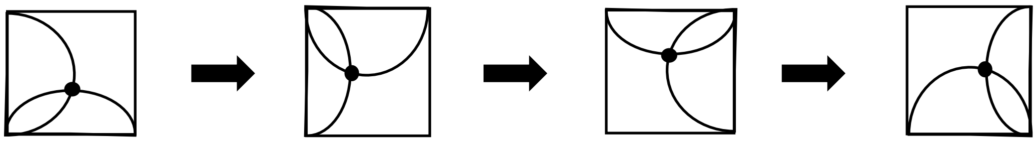

Figure 18: Figure rotation by Student D

Student D recalls the knowledge of “symmetry” by repeating the operation of rotation as mathematical activity. From the symmetry of the square and the symmetry of the circle, he noticed that the intersection of the two semicircles always comes to the same position, rather than shifting its position as it is rotated. With this notice, Student D verbalizes that the intersection point comes in the center of the square and that the center point coincides with the intersection of the two diagonals of the square. We can say that student D recalled the image “the intersection of the two semicircles comes in the same position as the intersection of the two diagonals of a square” based on his knowledge of “the symmetry of squares and circles” (Figure 19).

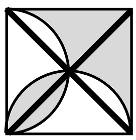

Figure 19: Student D’s image with two overlapping intersections

Student E’s Approach

By contemplating the diagram, Student E realizes that the intersection of the two semicircles is at the center of the square, just as Student D did, contemplative thinking. Student E’s realization that the intersection is at the center of the square brings the related knowledge that the two diagonals of the square intersect at the center of the square. This related knowledge results in the same realization as Student D’s, where the point of intersection of the two diagonals of the square coincides with the intersection of the two semicircles (Figure 20).

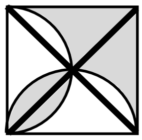

Figure 20: Same image as Student D recalled by Student E

When Student E draws this image externally, he initiates another equivalent deformation that differs from Student C's. The image of a rugby ball-shaped figure bisected by a diagonal line encompasses the selective perception of a crescent-shaped figure inset in the rounded-off portion from the upper right isosceles right triangle ACD (Figure 21).

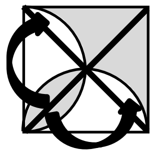

Figure 21: Selective perception on equivalent deformation

Furthermore, the selective perception obtained by student E results in a different interpretation than that of student C: the required area is perceived as being equal to half of the area of the square (Figure 22).

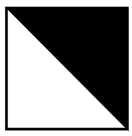

Figure 22: Image of half of the area of the square

Student E arrived at the same finding that the intersection recalled by Student C coincides with the intersection of the two diagonals of the square, but he said, "If we divide the shape EB into two parts by the line EB, we can fit each of the two exactly into the two curved parts of the shape AECD, respectively. Then the ginkgo leaf-like shape AECD transforms into triangle ACD." He solves the problem in a differing way from Student C (Table 5).

If we divide the shape EB into two parts by the line EB, we can fit each of the two exactly into the two curved parts of the shape AECD, respectively. Then the ginkgo leaf-like shape AECD becomes triangle ACD, so the equation: $2 \times 2 \div 2 = 2$, and the sum of the areas is 2.

Table 5: Student E's solution

Student E uses the knowledge that "the intersection of the two semicircles coincides with the intersection of the two diagonals of a square" through recall of knowledge related to the object, such as squares, semicircles, and intersection points. The diagram drawn based on this knowledge leads to the discovery that by moving the same shape, the area of the first given ginkgo leaf area is the same as the area of a right-angled isosceles triangle divided by the diagonals of the square. Here, contemplative thinking is brought about by the contemplation of a self-drawn diagram. Student E's knowledge that "the two diagonals of a square intersect at the center of the square" led him to recall the same image as Student D: "The intersection of the two semicircles overlaps the intersection of the two diagonals of a square."

In the five student cases employed in this paper, there were students who were looking at the same object, but who obtained different or the same image from it. The cases of Student C and Student D, who recalled different images for each, show that different knowledge recalls different images. On the other hand, the cases of Student D and Student E, who recalled similar images, show that similar images may be recalled even with different knowledge.

DISCUSSION

From introspective thinking to contemplative thinking

Given only the problem statement, the students first attempted to represent the problem situation in a diagram. However, freehand drawings often have various troubles and have not been a sufficient source of information to solve the problem. Introspective thinking encourages efforts to draw diagrams accurately. Then, the realization that "it is the location of the intersection of the two semicircles that matters" brought about by the improved diagram recalls knowledge related to the location of the intersection. At this stage, students A and B observe the figure, students C and E draw lines on the figure, and student D rotates the figure. Students arrive at contemplative thinking as various realizations by manipulating the figures as a physical stimulant represented outside of themselves.

Such case analysis indicates that the process of "initial primitive diagram→ introspective thinking: manipulation of representations, adding lines and rotating the diagram → improved diagram → contemplative thinking: recalling images → problem solving," as shown in Emori (2010), was also confirmed in the cases analyzed in this paper. Figure 23 shows a model of two types of "reflective thinking" using metaphor of incidence and reflection of the light.

Figure 24 shows that there are two types, X and Y, in the processes of "knowledge → image → solution". The fact that students may see the same figure but recall different images was found in the cognitive processes of Student C and Student E in this case study. Thus, a wide diversity is derived from manipulation of representations in introspective thinking and from the relationship between images and knowledge recalled in contemplative thinking.

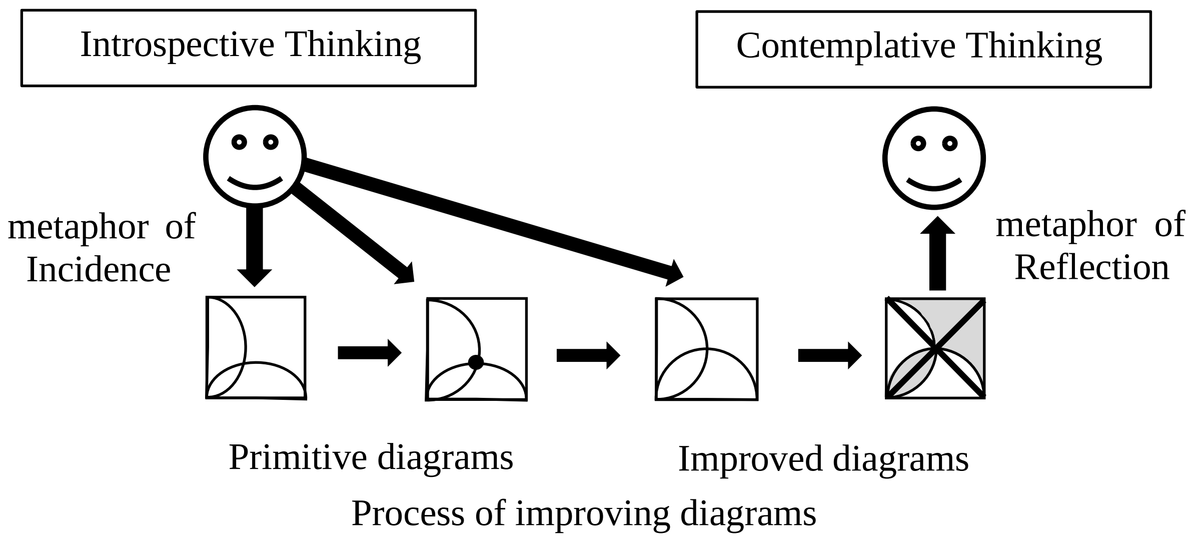

Figure 23: Model of two types of reflective thinking using metaphor of the light

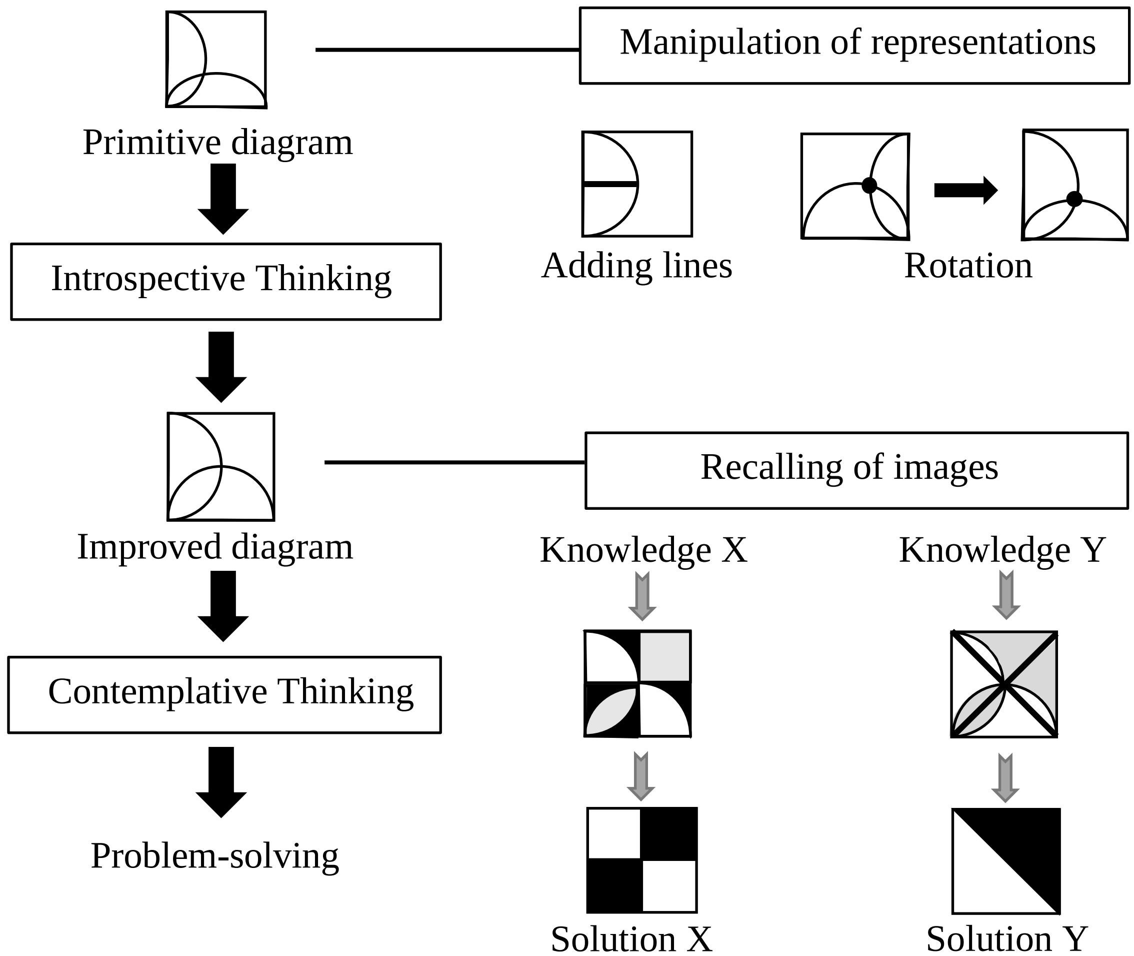

Figure 24: Representation and image in introspective and contemplative thinking

Seeing the invisible

If we organize the perception of the figure as an object of thought from the perspective of what is visible, what is invisible, and what becomes visible, we can say that: (1) what is visible = the figure perceived as a collection of lines, (2) what is invisible = the relationship between the two semicircles and the tangent lines of them; the relationship between the intersection of semicircles and the intersection of diagonals of a square, and (3) what becomes visible = the shape of two squares connected (half of the whole); the right-angled isosceles triangle (half of the whole). The diagram that became visible through the use of knowledge was, in the case of Student C, in the form of two squares divided into four parts, and in the case of Student E, in the form of a right-angled isosceles triangle. The instance of tangents and diagonals becoming visible in the figure means, as Hattori (1997) points out that "the recognition of an object means that there are two opportunities in our knowledge at the same time: the object illuminated by our knowledge and the knowledge that illuminates the object (p. 116) [*]." The relationship between object and knowledge proposed by Dignāga [(9)], who takes the position of contemplative thinking, can be organized as (1) knowledge of how to see the object and (2) knowledge related to the object. In the context of mathematics education, these two categories can be rephrased as (1) knowledge of finding mathematical structures and (2) knowledge of verbalizing mathematical structures.

A consideration of introspective thinking based on the theory of knowledge without an image

Selective perception is the perception of the parts of an object illuminated by knowledge, and the recognition of the mathematical structure produced by the combination of the parts of the perceived object. Hence, in order to understand the mathematical structure required to solve this problem, we must possess the knowledge that "the intersection of the semicircles coincides with the intersection of the diagonals." The primordial diagram, first drawn by each student, makes the meaning of the problem known as the object of selective perception of the lines and shapes to be focused on as a result of the cognitive action of how the diagram is viewed.

The fact that this external figure contains within it mathematical knowledge that is not directly reveal is what the theory of knowledge without an image claims to mean what it says "the nature of knownness arises in the object." The nature of knownness suggests that the message, or the object, possesses the function of representing a mathematical meaning that remains invisible when we attempt to observe it.

A consideration of contemplative thinking based on the theory of knowledge with images

In the examples presented in this paper, there was a conscious process of recognition by the students, who tried to find mathematical structures in the diagrams, by expressing a certain awareness as a diagram, by contemplating their own diagrams, not by looking at the diagram itself, but by comprehensively adding their own knowledge to it. These cognitive processes can be organized as follows.

“The initial representation acquired when observing the first drawing of the primitive figure produces an initial image of the figure as a more genuine mathematical object, which leads to the operation of improving the first drawing in order to make it the object of thought. Then, by using mathematical knowledge to contemplate the improved diagram, the image as a mathematically authentic visual picture is produced in the mind.”

In the cognitive process that continues, “what do you see first (selective perception) → what hypothesis do you make based on what you see (primitive inference) → how do you change your selective perception when it doesn’t work → improved inference,” the perception and cognition when the invisible becomes visible will never be lost again. Looking back on that state of matter, it seems as if the message was saying, “Perceive, recognize in that way,” and the mode of perception, once acquired, has spread to the present and the future. We have described this state of affairs as “contemplative thinking brought about by a message telling us something.” When we study such contemplative thinking within the framework of object and knowledge, we are forced to interpret that the message speaks to us in this way because the message acts as if it already possesses some information.

In this paper, we refer to the property of a message (i.e., object) that behaves in such a way that it encourages the receiver to interpret it in a certain way as the “knownness of the message.” As previous studies have indicated, messages do not contain information or meaning within them. However, the message in front of the eyes has the power to encourage the receiver to perceive it as if it were being presented there. It is power as a stimulant, as defined by Berlo (1960). In the process of recognizing external stimuli, we use our experiential intuition to instantly recall the knowledge we possess as relevant knowledge, but this recall is done as the recall of an image. The concept of “knownness (Sanskrit: *jñātatā* = the being known or understood)” adopted in this paper is a research methodology that transcends the principle of “subjective interpretation” that we have maintained in our research to date [10].

CONCLUSION

This paper tries to explain the research question, “How can the difference in the orientation of cognitive action in seen introspective thinking and contemplative thinking be explained as a cognitive process that occurs between the object and the knowledge possessed by the learner?” by using the theory of knowledge without an image and the theory of knowledge with images in Buddhism.

Conclusions of the study

Introspective thinking based on the theory of knowledge without an image

The direction of the cognitive action on the object, which we have called “introspective thinking,” from the receiver to the object, can be explained by the theory of knowledge without an image in the discussion of this paper.

Introspective thinking, in which object recognition takes place by looking at the object, is done in such a way as to search for the problem as a representation of the object. In the theory of knowledge without an image, which holds that it is possible to perceive external objects directly, the external object of thought itself is depicted in the brain as a visual image. We then approach the solution to the problem, manipulating its external figure. When we manipulate the figure, we take the visual image as a representation and manipulate that representation in our mind.

Contemplative thinking based on the theory of knowledge with images

The direction of the cognitive action of the object, which we have called "contemplative thinking," from the object to the receiver, can be explained by the theory of knowledge with images in the discussion of this paper.

The object, refined by introspective thinking, gives rise to a view of the object called selective perception, and the contemplation by that selective perception gives rise to contemplative thinking. In contemplative thinking, external stimulants function as if they make known the information needed to solve the problem. The receiver is stimulated by the object that has been refined as a representation and visualizes the newly recalled relevant knowledge as an image. Recall of images leads to economical communication that reduces the difficulty of recalling all of the recalled relevant knowledge as verbalized knowledge.

What makes mathematical communication mathematical?

When we use the term mathematical communication, we have always faced the question: What makes mathematical communication mathematical? In this paper, we answer this question by saying that the power of seeing the invisible is what makes mathematical communication mathematical. Because mathematics is invisible.

Future issues

In this paper, we have been discussing the research problem under the assumption that experience is also included in knowledge. As Dewey (1916) stated, "The nature of experience can be understood only by noting that it includes an active and a passive element peculiarly combined (p. 139)." If the essence of experience is to manipulate the object and to perceive something from the object, then the two-phase relationship between object and knowledge in introspective thinking and contemplative thinking can be reintegrated as reflective thinking from the perspective of experience. A discussion of introspective thinking and contemplative thinking, based on the perspective of experience, is the theme for future research.

Acknowledgments

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Footnotes

(1) In this paper, the English term "reverberative thought" used in Emori (2010) is renamed "contemplative thinking."

(2) "Reflective thinking: the kind of thinking that consists in turning a subjective over in the mind and giving it serious and consecutive consideration (Dewey, 1933, p. 3)." When Dewey used the term reflection, it may have been foreseen that reflective thinking consists of two kinds of thinking: introspective thinking and contemplative thinking, much like there are two different actions, incidence and reflection, when light is reflected off of an object.

(3) Pierce (1961, pp. 427-428) states that much the same thing about the idea of "knownness of the message," which I adopt in this paper, as follows. "I note that a Percept cannot be dismissed at will, even from memory. Much less can a person prevent himself from perceiving that which, as we say, stares him in the face. […] Suffice it to say that the perceiver is aware of being compelled to perceive what he perceives. Now existence means precisely the exercise of compulsion [4-541]."

(4) The knowledge theory in Buddhism, which developed as a branch of Buddhist epistemology, consists of two theories of knowledge; the theory of knowledge without an image (Sanskrit: *nirākārajñānavāda*) and the theory of knowledge with images (Sanskrit: *sākārajñānavāda*) (Kajiyama, 1965, 1977; Monier-Williams, 1998). The theory of knowledge without an image (3rd century B.C. to 1st century B.C.) nullifies the difference between perception and thought, giving the object of perception and the object of thought the same existence (Kajiyama, 1983, pp. vii-viii) *. On the other hand, the theory of knowledge with images (4th to 5th century A.D.) holds that what we directly perceive are the images given to us in this knowledge, but the existence of the external world that gave them to us is inferred (Kajiyama, 1983, p. 39) *.

(5) In Japan, fifth-grade students learn to classify quadrilaterals into squares, rectangles, parallelograms, rhombuses, and trapezoids using the three diagonal conditions.

(6) "*Jñāta*" is a Buddhist term meaning "what is to be known" (Nakamura, et al., 2002, p. 946) *. Ueki (2019) provides an explanation for the Sanskrit suffix "*-tā*" in *jñātatā* (knownness): "In Sanskrit, all nouns, adjectives, and adverbs become abstract nouns simply by adding '*-tā* (-ness)' to the end of the word. The abundance of abstract nouns is an indication of the (Indian) national character, which emphasizes the universality behind things rather than what is right in front of it (pp. 255-256) *."

(7) The *Tarkabhāṣā* (Sanskrit: Language of Logic) was written by Mokṣākaragupta at some time between 1050 and 1202 A.D. (Kajiyama, 1966, p. 1).

(8) This problem was provided by Prof. Akira Morimoto of Fukushima University.

(9) Dignāga (circa 480-540) was an establisher of Buddhist logic (Nakamura, et al., 2002, p. 579). He established the theory of knowledge with images on the basis of the self-awareness theory of knowledge, in which knowledge recognizes images within the self (see above, p. 1024).

(10) Schutz (1970, pp. 265-293) states that to understand human bodily activity as human action is nothing other than to attach the subjective meaning of the actor to the action and gives the principle of "subjective interpretation (see above, pp. 274-275)" as a basic principle of sociology (Emori, 2006, pp. 77-80). This paper, which introduces the concept of knownness, does not deny the principle of "subjective interpretation" in that it considers knowledge at work in the interpretation of the subject.

* For quotations, the author of this paper has translated the Japanese texts into English. Therefore, the author is responsible for any mistranslations or misinterpretations.

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THAILAND

RESEARCH REPORTS

MATHEMATICAL WELLBEING OF MATHEMATICS TEACHERS IN THAILAND: AN INITIAL EXPLORATION

Araya Piyakun[1], Wee Tiong Seah[2]

[1]Faculty of Education, Mahasarakham University, Thailand
[2]Faculty of Education, The University of Melbourne, Australia

110 mathematics teachers from across Thailand were surveyed for their perception of their own mathematical wellbeing (MWB), as well as the extent to which personally-held values were able to be fulfilled. Data analysis revealed that the mathematics teachers' MWB was generally positive, with no statistically significant difference amongst teachers from different school levels. The teachers' MWB relates to the embracing and fulfillment of nine ultimate values, all of which teachers nearly often got to fulfil, namely: accomplishment, autonomy, cognitions, compliance, engagement, meaning, perseverance, positive emotions, and relationships. While eight of these are known in the literature, compliance emerged as a value that is unique to teacher MWB.

INTRODUCTION

Mathematical wellbeing (MWB) refers to a person's wellbeing relating to the discipline of mathematics. It is a subject-specific component of general wellbeing. MWB was first conceptualised by Clarkson, Bishop and Seah (2010), and more recently investigated in greater detail by Julia Hill (e.g., Hill, Kern, et al, 2022).

While students' MWB has been the subject of research studies (e.g., Hill, Bowmar, et al, 2022; Hill & Seah, 2023), teachers' MWB is also important and should not be assumed. Our belief is that a mathematics teacher with a positive MWB would not only be teaching optimally, they will also be influencing their students affectively and conatively in constructive ways. Yet, to our knowledge no prior research has been conducted to assess or to understand the MWB of mathematics teachers.

In this context, we embarked on an exploratory study of the MWB of primary and secondary school mathematics teachers in Thailand, guided by the following research problem: What does teacher MWB look like for mathematics teachers in Thailand?

MATHEMATICAL WELLBEING

Adapting Huppert and So's (2013) definition of wellbeing, mathematical wellbeing refers to a person's subjective experiences of feeling good and functioning well with regards to the discipline of mathematics. Thus, a mathematics teacher's MWB reflects the extent to which they are feeling well and teaching mathematics well.

We subscribe to Tiberius' (2018) value fulfillment theory of wellbeing, which when applied to mathematics education posits that mathematical wellbeing is a function of the extent to which personally held values related to mathematics learning and teaching are fulfilled and actualised. In other words, a teacher with a positive MWB would be one who finds that they get to fulfil or actualise what they regard as important in mathematics teaching

2023. In Inprasitha, M., Sudjamnong, A., Suriyon, A., Saengpun, J., Moonpo, P., Chaovasetthakul, R., Plianram, S., & Suttiamporn, W. (Eds.). *Proceedings of the Regional Conference of the International Group for the Psychology of Mathematics Education 2023*, pp.49-56. Khon Kaen, Thailand: PME.

or learning. If a teacher values *practice*, say, but if their school has a zero-homework policy, then *practice* will likely not be fulfilled, leading to disappointment, uncertainty or discontent. Accordingly, MWB is affected.

To this end, Hill, Kern, et al (2022) identified a set of seven ultimate values (UVs) the fulfillment of which is required for positive student MWB. These UVs are, namely, *accomplishment, cognitions, engagement, meaning, perseverance, positive emotions,* and *relationships*. Each of these seven UVs would be served by different instrumental values (IVs), which can differ from culture to culture. For example, the UV *accomplishment* might be attained through the IV of *performance* (in assessments), yet other teacher colleagues might actualise the same UV via different IVs, such as (student) *understanding*, or (students' in-class) *participation*. Indeed, we were curious if the same set of seven UVs apply to teacher MWB.

The nurturing and sustaining of MWB represents a strength-based approach to optimising students' affect and motivation. For example, a positive student MWB reduces the chance of these students experiencing mathematical anxiety and disengagement. In so doing, we are being proactive towards the prevention of mental issues affecting large numbers of mathematics students around the world. Similarly, nurturing mathematics teachers' MWB would also be a proactive approach against the potential for teacher burnout, negative attitude, or even mathematics anxiety.

MATHEMATICS TEACHING IN THAILAND

Nearly all (mathematics) teachers in Thailand received their accreditation through completing a four-year Bachelor of Education degree. Most of these programs are specialized, with a specific focus on disciplines such as Mathematics. However, it is also worth noting that certain programs may focus on producing teachers for primary school teaching only. Pre-service teachers are also relatively more exposed to mathematics courses compared to their peers in other countries (Tatto, Rodriguez, & Lu, 2015). Quality assurance and degree accreditation are overseen by the Ministry of Higher Education, Science, Research and Innovation, and the Teachers' Council of Thailand.

The relative low pay for teachers has contributed to a shortage of mathematics teachers, especially in rural schools. The authorities' response has been to allow mathematics graduates to become teachers without completing teacher training. Thailand was placed 57th out of 78 systems taking part in PISA 2018 Mathematical Literacy, with its performance considered to be statistically significantly below the OECD average.

Given these realities above, what is the state of teacher MWB in Thailand? Could it be harnessed by mathematics teachers to support their professional practice, or might it be affected by the issues confronting mathematics education?

In this context, we posed three research questions for the current exploratory study:
RQ1: How do the MWB of primary mathematics teachers in Thailand compare with the MWB of secondary mathematics teachers there?
RQ2: What are the values associated with mathematics teacher MWB in Thailand?
RQ3: To what extent are these teacher values fulfilled when there is positive MWB?

METHODOLOGY

Given the exploratory nature of this study, and the intention to map teacher values and teacher opinions of their sense of wellbeing in relation to mathematics teaching, the questionnaire method was deemed to be the most practical. An online questionnaire was constructed in Thai and distributed widely via a flyer posted on (mathematics) teacher discussion fora across several social media platforms in May – Jun 2023.

Once the questionnaire items were identified, the questionnaire was piloted with four teachers in Thailand. As the teachers responded to the questionnaire, they were encouraged to check that the vocabulary used could be understood by typical primary school students, that the questions and instructions were clear and explicit, and to record the time taken to complete the survey. This pilot testing stage led to several changes made to the questionnaire, thus increasing its construct validity. The questionnaire items are listed below, having been translated into English.

Item 1: Level of teaching (primary / secondary)

Item 2: When teaching maths, how often do you feel good AND function/teach well? (never / sometimes / half of the time / often / always)

Item 3: During those times when you were feeling good and functioning well in your maths teaching, what might be the reasons?

Items 4/6/8: What is the first/second/third important aspect of maths teaching to your professional practice?

Items 5/7/9: When teaching maths, how often do you get to express or show the first/second/third important aspect? (never / sometimes / half of the time / often / always)

We used a convenience sample technique to recruit participants, since the intention had been to explore the MWB of mathematics teachers in general across the kingdom.

As the questionnaire was conducted online using Google Form, data from within the submitted questionnaires were collated automatically by the software. Raw data were exported in the form of a spreadsheet.

The responses gathered from items 2, 5, 7, and 9 were transformed into numerical scores (Never = 1, Sometimes = 2, Half of the time = 3, Often = 4, and always = 5). The data collected from items 4, 6, and 8 were read by us individually to distil the UVs. This was guided by the seven MWB UVs evident in the literature (see above), while we were open to potential new UV categories. Inter-rater agreement was 97.8% (308 agreements out of 315 responses). The UV frequencies were then tallied. Frequency, mean, standard deviation, and Kruskal-Wallis test were used to interpret the data.

RESULTS

A total of 110 valid responses were received. The self-reported MWB of these Thai teacher participants, categorised according to their teaching levels (i.e., primary, primary and lower secondary, secondary), is shown in Table 1. The overall mean score of 3.94 indicates that amongst this group of mathematics teachers practising in different parts of Thailand, on the whole, their teacher MWB is pretty much positive. They are nearly often feeling good and functioning well when teaching mathematics. In particular, primary mathematics teachers reported the most positive MWB, followed by their secondary counterparts. Those teachers who were teaching across primary and lower secondary school years reported the lowest MWB, even though it is still positive.

Teaching levels	Tertiary major				Total	Mathematical Wellbeing	
	Mathematics		others				
	Teach only Maths	Teach other subjects	Teach only Maths	Teach other subjects		$\bar{x}$	S.D.
Primary	18	26	0	3	47	4.11	0.81
Primary and Lower secondary	12	5	1	1	19	3.68	0.89
Secondary	35	5	2	2	44	3.86	0.82
Total	65	36	3	6	110	3.94	0.83

Table 1: Mathematics teachers' MWB.

However, as shown in Table 2, there is no significant difference in the MWB mean scores among these three groups of teachers ($\chi 2 = 4.179$, $p = .124$, $df = 2$), with a mean rank of 61.56 for primary school teachers, 46.45 for primary/lower secondary school teachers, and 52.93 of secondary teachers. Thus, the difference in MWB between the primary and primary/lower secondary, and between primary/lower secondary and secondary are not statistically significant.

A total of 315 different values were encoded from the teachers' responses. Even though 330 responses were expected (i.e., 3 responses from each of the 110 respondents), 15 of these were either non-responses or meaningless terms (e.g., weather, location). As shown in Table 3, 304 of these values were categorised into the seven UVs that made up the

Teaching levels	N	Mean Rank
Primary	47	61.56
Primary / Lower secondary	19	46.45
Secondary	44	52.93
χ^2	4.176	
df	2	
p-value	.124	

Table 2: A comparison of MWB among three groups of teachers.

student MWB framework. Of the remaining 11, nine were categorised into the UV *autonomy*, which incidentally is also the additional UV identified for students' wellbeing in science education, in addition to the set of seven UVs associated with MWB (Hill et al, 2023). The last two values nominated by two teachers are similar, reflecting the importance for practice to be aligned with the mathematics curriculum in order for them to experience positive MWB. We propose to name this valuing *compliance*, the ninth UV underlying mathematics teacher MWB in Thailand.

UVs associated with teacher MWB (examples of contributing IVs)	Frequency
Accomplishment (Goal attainment, Leadership, Learning, Mastery, Outcomes, Success)	7
Autonomy (Agency, Autonomy)	9
Cognitions (Active learning, Analytical thinking, Analyzing problems, Assessment, CK, Cognition, Comprehension, Content, Differentiation, Explanation, Knowledge, Mathematical reasoning, PCK, Problem-solving, Recall, Remediation, Review, Skills, Teaching methods, Understanding)	139
Compliance (Alignment with curriculum)	2
Engagement (Activities, Attention, Engagement, Experimental learning, Hands-on, Interactions, Motivation, Readiness, Responsiveness, Teacher noticing)	44
Meaning (Application, Connections, Informal language, NoM, Prior knowledge)	27
Perseverance (Practice)	5
Positive emotions (Emotions, Enjoyment, Fun, Happiness, Interest, Learning atmosphere, Learning environment, Love for maths, Passion for teaching, Passion in maths, Positive atmosphere, Positive attitude, Praise, Safety, Self-esteem)	43
Relationships (Communication, Family support, Learning agreements, Relationships, Student cooperation, Students, Teachers, Teamwork)	39

Table 3: The values governing the teachers' MWB.

Amongst the nine UVs, *cognitions* are overwhelmingly the most often nominated. 139 of the 315 (or 44.1%) values the fulfillment of which contribute to teacher MWB are related to knowledge and skills, such as content knowledge and mathematical reasoning respectively. This is followed by a distant second most often nominated UV, *engagement*, and then, *positive emotions*, *relationships*, *meaning*, *autonomy*, *accomplishment*, *perseverance*, and *compliance*.

UV	Frequency	Fulfillment
cognitions	139	4.00
engagement	44	3.68
positive emotions	43	3.84
relationships	39	3.85
meaning	27	3.78
autonomy	9	3.89
accomplishment	7	4.46
perseverance	5	4.25
compliance	2	5.00

Table 4: Teacher fulfillment of MWB UVs.

How often do the mathematics teachers get to fulfil or express these UVs underlying their MWB? Table 4 provides the mean scores for the fulfillment of the UVs, with a maximum of 5 meaning ‘always’, 3 for ‘half of the time’, and 1 for ‘never’. Thus, the two teachers who nominated *compliance* were able to experience fulfillment of this valuing all of the time, which would contribute towards their teacher MWB. Three other UVs – including the most popular, *cognitions*, as well as *accomplishment* and *perseverance* – get to be fulfilled often. The other five UVs are fulfilled nearly often enough, and well more than half of the time, with the lowest mean score being 3.68.

DISCUSSION

This exploratory study, probably the first of its kind researching teacher MWB, has found that with the perspective that MWB represents the extent to which personally-held values are fulfilled in a teacher’s mathematics teaching, then these values can be categorised into nine groups of UVs. Seven of these are exactly the same as the ones which define student MWB, namely, *accomplishment, cognitions, engagement, meaning, perseverance, positive emotions,* and *relationships*. Then there is also the UV *autonomy* which has been the value that had been identified in the construct of students’ science wellbeing (Hill et al, 2023). In addition, a new UV, *compliance,* has been identified. This is perhaps not surprising, since it is often expected of classroom teachers to ‘follow the curriculum’ when lessons are being designed, delivered, and students’ learning assessed. So, for many teachers, being able to not only value *compliance* but actually getting to abide by the relevant policies and expectations, is a source of their teacher MWB. The uniformity of teacher education across the kingdom as discussed above might be a contributing reason. Not surprisingly perhaps, student MWB is not dependant on this valuing (of *compliance*).

Amongst the nine UVs underlying teacher MWB, *cognitions* have been the most nominated. After all, it is a teacher’s job and responsibility to facilitate student learning of mathematical knowledge and skills, thereby developing and strengthening their cognitions.

At the same time, the teachers' valuing of *cognitions* might also refer to their own professional knowledge and skills, as illustrated by such IVs as *pedagogical content knowledge (PCK)* (see Table 3). Thus, being able to develop students' mathematical knowledge and skills through a teacher's lessons, and/or being able to utilise their own content or pedagogical content knowledge and skills in their practice, would contribute to a sense of positive MWB for the teacher.

Each of the nine UVs was fulfilled at least nearly often enough by the teachers, which supports the finding that teacher MWB is generally positive in Thailand. Given that the nine UVs were nominated a different number of times, ranging from 2 to 139, we are unable to draw conclusions from the different degrees of fulfillment. If the current model of student MWB also extends to Thai students, then our finding that the nine UVs associated with teacher MWB include the seven UVs of student MWB implies that student MWB possibly correlates with their teacher's. On the other hand, while *cognitions* might be the most emphasised for mathematics teachers (in Thailand) and students in Australia, *accomplishment* was most emphasised for students in China (Hill & Seah, 2023). What are the implications here for mathematics classrooms with migrant students (from China, say)? Would this constitute a form of values misalignment in the lessons? These are indeed implications for future research.

CONCLUSION

110 mathematics teachers from across Thailand responded to an online questionnaire surveying teacher perception of their own MWB, and the extent to which personally-held values were able to be fulfilled. In view of the Research Questions posed, data analysis revealed that the mathematics teachers' MWB was generally positive, with no statistically significant difference amongst primary school teachers, primary/lower secondary school teachers, and secondary school teachers. The teachers' MWB was found to relate to the embracing and fulfillment of nine ultimate values. That the teachers reported that they nearly often got to fulfil these ultimate values reinforced their overall positive teacher MWB. The nine ultimate values are: *accomplishment, autonomy*, *cognitions, compliance, engagement, meaning, perseverance, positive emotions,* and *relationships.* Seven of these values are already associated with student MWB and science wellbeing, and *autonomy* has been the eighth value for science wellbeing (Hill et al, 2023). *Compliance* is found to be unique to teacher MWB.

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ETHNOMATHEMATICS, FUNDS OF KNOWLEDGE, WORK-CONTEXT MATHEMATICS: NEED FOR A DIALOGUE?

Arindam Bose

Tata Institute of Social Sciences, Mumbai, India

It is now well acknowledged that school is not the only site of mathematics learning. Coming from a developing world context, large conglomerations of immigrants with diverse hand skills and strong social networks contain rich funds of knowledge with mathematical ideas embedded in various work-contexts. This paper calls for a need for examining intersection of work-context mathematics with funds of knowledge and ethnomathematics which holds diverse opportunities and affordance for engaging with different forms of mathematics. Invoking critical engagement, this paper calls for a dialogue between ethnomathematics and its epistemological underpinnings with funds of knowledge to locate embedded mathematics and productive force of knowledge creation for empowering the latter with voice and acceptance.

ETHNOMATHEMATICS FOR LOCATING MATHEMATICS

Mathematics as a school subject calls for precision and rigour manifested in symbols and proofs, and is seen as a tool for moving towards abstraction and generalisation. In its endeavour towards abstraction and generalisation, school mathematics gets detached from routine everyday settings and requirement, even though the origin of mathematics lies in human ideas and endeavours which however may appear obscure and counter-intuitive. This picture of school mathematics as an esoteric collection of rules and formulae, unconnected with reality leads it to becoming a subject which causes the greatest fear and sense of failure among the learners (National Council for Educational Research and Training [NCERT], 2006). Even though knowledge of mathematics is valorised in many societies and continues to be an integral part of the school curriculum for all children, it is ironical that many who study mathematics formally in schools often find it difficult and frightening and also opt out midway, but those who are outside of schools or have dropped out, continue successful use of mathematics in different everyday settings and in work-contexts with or without being aware of it. It is therefore of value to education researchers to explore diverse nature of mathematical expressions embedded in work-contexts and community practices in out-of-school settings which children are able to successfully engage with but not similar mathematical forms in formal school settings. This paper argues that such tensions can be suitably explored from an ethnomathematical lens but that would require revisiting and broadening the domain of subject ethnomathematics.

Previous studies indicate that school students and non-school going children from low-income settlements both in rural and urban conglomerations, living in disadvantaged conditions often experience difficulties in learning mathematics in schools and face failure and often drop out (Bose, 2015; Bose & Kantha, 2014). Informal knowledge gained in out-of-school contexts remains poorly built on in the classrooms, which may

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be the reason for knowledge gaps that are detrimental to their progress in formal mathematics learning (Saxe, et al., 1987). However, children who drop out of formal studies as well as children from economically poor families often get into income generating practices, where they acquire and successfully use their informal knowledge of mathematics. My past study has shown that economically active surroundings offer diverse contexts to children to acquire, learn and use mathematical knowledge (Bose, 2015). This study with middle graders living in one such neighbourhood analysed the opportunities and affordances available in diverse work-contexts and everyday settings for mathematics learning, such as, how handling of diverse goods or limited access to them, optimising resources and decision-making processes create different possibilities of dealing with mathematical knowledge. This paper argues in particular, that these diverse contexts which are resource-rich, linguistically diverse and come with strong social networks, can be sites for furthering ethnomathematical exploration. These resource rich everyday contexts call for critical engagement with ethnomathematical research for tracing implications for pedagogical practices. Ethnomathematics whose role is to unpack social role of formal mathematics (D'Ambrosio, 2001) can help in understanding the "many ways of knowing and doing mathematics that relates to the values, ideas, notions, procedures, and practices in a diversity of contextualized environments" (Rosa & Shirley, 2016, p.1). So far, there has been a silence in ethnomathematical studies to learn from the productive force of knowledge creation in work-contexts. Explorations of the above kind can help broaden the purview of subject ethnomathematics and its epistemological underpinnings, and also widen the perspective of mathematics knowledge generation besides valuing such practices.

WHY ETHNOMATHEMATICS LENS?

It is certainly of interest to mathematics educators to explore whether students of a certain grade know a particular mathematical concept from their out-of-school experiences, ways in which their prior mathematical knowledge help them in gaining new knowledge in the classroom by building on their out-of-school knowledge and whether their school mathematics learning illuminates their everyday mathematical knowledge. Also of interest are the questions about the knowledge organisation in everyday mathematics, for example, its forms of representations, different and flexible ways to arrive at solutions, and how they differ from the school mathematics in terms of abstractions and procedures. It is felt that the prevalence of everyday mathematics in the culture is a potential resource that can make school mathematics learning more effective. There are not many studies on how this can in fact be realised. Criticisms of efforts to use the insights from out-of-school mathematics in the school contexts are usually based on a clear separation between out-of-school and school knowledge with different goals. In fact, ethnomathematical perspective which locates mathematical activities embedded in cultural contexts can consider school mathematics as one cultural practice of mathematics among others (de Abreu, 2008; Bose, 2015). On the other hand, the developing world contexts are abounded with examples where hand skills are valorised as community practices and connectedness of social networks with communities give rise to diverse and rich knowledge and experience that can be drawn for the purposes of school learning. However, there are not many studies from an Ethnomathematical lens on such community work practices located in low-income settlements in large metropolis under disadvantaged conditions but possessing strong knowledge connects. Many of these work practices have mathematical elements embedded in them. Ethnomathematical lens can help unpack mathematical knowledge production and practices in such cultural groups and reflect on how members of distinct communities seen as units of cultural groups, negotiate their own mathematical practices in the work-contexts. Contrastingly, locating these social roles of mathematical practices can enrich ethnomathematical knowledge base that currently exists. Furthermore, it is also appropriate that subject ethnomathematics is broadened to extend power and value to such knowledge production and practice.

THE STUDY

The research study from which data for this paper is drawn, explored the nature and extent of middle grade school students' everyday mathematical knowledge who lived in an urban low-income settlement that has embedded in it a thriving micro-enterprise economy. Children living in this settlement either have exposure to the diverse work-contexts prevalent in their neighbourhood or participate in and contribute to the production and income generation right from an early age. In the course of exploration of the potentially rich opportunities available to the middle graders to gather everyday mathematical knowledge, the work-contexts were characterised from a mathematics learning perspective. The objective of the study was to unpack and document the connections between students' mathematical knowledge, work practices and identity formation, and inquire into the implications of these connections for school learning. In this paper, I draw instances of mathematical practice embedded in the work-contexts as part of community practice from an ethnomathematical lens.

This old and established settlement is a large, densely populated, multilingual, multicultural, multireligious neighbourhood located in central Mumbai with residents who are economically poor. The settlement has a high economic output and generates huge employment opportunities and attracts skilled and unskilled workers from all parts of India who come to Mumbai in search of livelihood. Having an estimated population of around one million, this settlement is spread over a 2 square kilometre area beside locations that fetch some of the highest property values (real estate) in the world (Campana, 2013). It was observed that almost every child in the settlement was involved in house-hold based economic activities as well as in micro enterprise in the neighbourhood in a variety of ways. Common house-hold occupations included embroidery, zari (needle work with sequins), garment stitching, making plastic bags, leather goods (bags, wallets, purses, files, folders, belts, briefcases, waist and hand pouches, shoes), textile printing (dyeing), recycling work, pottery making, cellular phone repairing, food delivery and so on. The goods produced in this locality were not only sold in Mumbai but also exported overseas. An ethnographic qualitative research design was adopted for data collection as a non-participant observer to explore students' everyday mathematical knowledge and the work-contexts that they had exposure to. The entire study followed a blend of ethnography, case study and action research methodology in three broad overlapping phases.

MATHEMATICAL ELEMENTS IN WORK-CONTEXTS

Diverse modes of work-contexts often embed within them mathematical elements in various forms. Community valorises hand skills and extends such knowledge to its children. Examples of mathematical knowledge required in major work-contexts are:

Embodied measurement and art skills

Rizwi (all names are pseudonyms), 13 years old sixth grader knew about the different raw materials used in his textile printing (called "dyeing") work, namely, stoppers (blocks used in printing the design), dyeing frames of varying sizes (typical frame-sizes are 16"×12", 28"×12" costing between Rs 2000-3000), dyes producing glossy or matt finish of different colours, thinner, adhesive, coating material, their prices and the units in which they are sold. He knows which colours are to be mixed to obtain a particular colour or shade and the proportion in which they are mixed. He chooses suitable dye-frame and "stopper" of a particular size by looking at the logo design to be printed. Rizwi makes the design estimation in inches and accordingly takes a call about the "stopper" to be used. He has developed accurate embodied skills for measurement in inch and foot and does not require to use measuring tapes, although he does not know the conversion between old British units and modern Standard International units (only SI units are part of the school curriculum). Rizwi also has to scale up or scale down the design and pattern in varying proportion for which he uses his embodied skills learnt from his father who is an expert in dyeing work.

Estimation and approximation

Ishaan's (12 yr old, Grade 6) involvement in his father's ready-made garment business gives him an opportunity to handle different kinds and sizes of garments with varying price-tags. He learnt garment size and age connection: that frock size 22 is for 4-year olds while size 30 for 10-12 year old girls and 32 for girls who are 13-14 years old. He knows costs incurred in making of different sizes of garments and therefore estimates the profit margins. He reported that he calculates ("hisaab karta hoon") and prepares the bills by adding quickly ("fatafat jodkar usko bill banana rahta hai"). This practice creates opportunities for him to handle numbers in different ways. In the following interview excerpt he explains connection between frock-sizes and age and how he learnt about it. T and S are researcher and student respectively.

115	T:	to yeh char saal ka baaees hua to tees wala size kitne saal wale bachche ka?	so, for a four year olds it's twenty-two, then size thirty is for which age-group of children?
116	S:	Dus-barah saal wala/	for ten-twelve years old/
117	T:	Dus barah saal wale? Aur battees?	for ten-twelve years old? and thirty-two?
118	S:	Battees terah-chaudah ko aayega/	thirty-two will fit thirteen-fourteen years old/

119	T:	to yeh sab tumhe kahan se pata chala? Kaun kisne sikhaya?	so where did you get to learn all this? who taught this?
120	S:	abbu ne bataya/ bahrhal na mere ghar mein kuchh bhi aata hai na to woh number sab pata kar lete hain kitne number hai, woh shirt ko dekh lete hain to bata dete hain/	Dad told me/ meanwhile when anything comes to my house then he finds out the numbers, what all numbers there are, he looks at shirts and tells us [the numbers]/

Table 1: Work-context interview excerpt on garment selling

Ishaan’s knowledge of the profit margins of different garment sizes helps in making decision about discounts to offer during bargaining with customers. Frocks are sold for about Rs 70-160 with a profit margin of Rs 5 to Rs 40 depending upon the sizes. He claimed that bigger sizes like 30, 32 incur loss because more amount of cloth is wasted in comparison to other smaller sizes and their making charges are more too.

Optimisation skills

Aman (12 yr, Grade 6) engaged in garment recycling work is required to make quick decisions about the rate at which he can close a deal for selling the collected cloth leftover pieces called *chindi*. He reported that once he earned Rs 640 on a single day's collection of 95 kg of *chindi*. This was like a benchmark against which he could make a decision of when to stop collection for the day and to assess the success of a particular day’s collection and earning. His decision of *chindi* selling price depended on different variables such as colour of the cloth-pieces, their texture, sizes, material quality and so on. He knows that other groups of boys like him are at the job and he quickly optimises a deal working with the variables and different price they fetch. All the calculations are done mentally and quickly without using calculator, pen or paper.

Decision making techniques

Sultan’s (13 yr, Grade 6) mobile phone repairing work requires knowledge of spares and tools market, connection with shops selling mobile phone spares, parts, repairing tools, electrical appliances like soldering machine, and other instruments. He has knowledge of different mobile phone parts and their functions; costs of both original spare parts and locally made low-cost substitute parts and about different products of different brands and their prices. His work involves optimisation skills for quoting a price by guessing a customer's paying capacity. This helps in deciding whether to use an original part or a low-cost substitute, keeping in the mind the expected profit. Such optimisation requires quick mental computation and making estimations such as how much of a particular part to buy and stock and at what price, time required to carry out repairs, repair charges and quick decision skills. Sultan also has to keep in mind what other shopkeepers in the vicinity are charging for the repair work.

FUNDS OF KNOWLEDGE

The "funds of knowledge" perspective brings to mathematics education research insights that are related to, but different from the perspectives embedded in studies of "culture and mathematics". In contrast to restrictive and sometimes reified notions of "culture", "funds of knowledge" emphasise the hybridity of cultures and the notion of "practice" as "what people do and what they say about what they do" (Gonzalez, 2005, p. 40). Funds of knowledge (FoK) are acknowledged to be broad and diverse. They are embedded in networks of relationships that are often thick and multi-stranded, in the sense that one may be related to the same person in multiple ways, and that one may interact with the same person for different kinds of knowledge. In other words, FoK points to the diversity of contexts and settings from which knowledge is acquired. FoK are also connected and reciprocal. When they are not readily available within households, then they are drawn from outside the household from the networks in the community. The concept thus emphasises social inter-dependence. FoK perspective illuminates how the connectedness of social networks gives rise to diverse and rich knowledge and experience. From this perspective, children in households are active participants, not passive by-standers. Thus, Funds of knowledge can be seen as a resource pool that emerges from people's life experiences and is available to the members of the group (including children from an early age) which could be households, communities or neighbourhoods.

The above data shows that children from the low-income settlement have wide access to the community's FoK and consequently the FoK that children acquire is also diverse. Access to FoK ensures knowledge about different work procedures, work requirements, raw materials, profit margins, wage calculations depending upon the kinds of work, duration of work, optimisation tasks and so on. FoK about work-contexts are also shared between friends who often help each other in arranging jobs and in supplementing earning. Indications of such a knowledge base emerged during the discussion with students and adults about the work-contexts and the details that they shared. Often these funds of knowledge are not seen as tools for teaching or for giving instructions but as aids for working. This study shows that students' access to such knowledge base creates opportunities for furthering their knowledge including mathematical knowledge. One such example is the familiarity that all study participants had about the inch tape or about the work-context of tailoring or about different binary fractions. They gathered such mathematical knowledge not because they were taught in their communities but by sharing of the rich funds of knowledge available within the community which are valued and seen as important to learn.

ETHNOMATHEMATICS FOR BUILDING ON FUNDS OF KNOWLEDGE

What can ethnomathematics offer in working with heterogeneous cultural groups (with varied linguistic, religious, ethnic, caste diversity) but with access to rich funds of knowledge? What can be ways to explore connections between mathematics, culture, community and social justice using reciprocal relations while going beyond the non-essentialised understandings emerging from ethnomathematical studies? For example, there is a need to understand how fairness is seldom taken into consideration in the world of work, which is governed far more by possibilities and bargains. For poor children in the settlement, fairness is not easy to grasp. Wages are often not correlated with hours of work, entitlements are not equally or fairly distributed in society, rewards and punishments are socially manipulated to favour a few over the majority (Bose & Kantha, 2014). Going forward, Ethnomathematics while unpacking community's FoK can examine and inform critical mathematics education for revisiting pedagogy of social justice within school mathematics learning. Experiences and knowledge drawn from such contexts are intimately familiar and include elements of mathematical knowledge, with aspects of it even embodied in students as part of the community's FoK. Ethnomathematical tools can help unpack conceptual, cognitive, historical and epistemological dimensions of work-contexts emerging from the available FoK. This approach of drawing from FoK calls for a dialogue between subject ethnomathematics and mathematics embedded in work-contexts to include the domain of work-context communities and practices as well-defined cultural groups and new sites for ethnomathematical learning.

NEW DIALOGUE FOR REDEFINING PEDAGOGY?

With education becoming politicised throughout the world, mathematics education is becoming even more political. Socio-cultural studies in mathematics and science education argue that cultural resources and funds of knowledge of people from non-dominant and underprivileged backgrounds are often not leveraged in school teaching and learning practices. Neither is learners' knowledge from everyday life experience valorised (de Abreu, 2008) and built upon in the classrooms nor is their identity acknowledged. Moreover, studies from developing world contexts show that there exists a strong influence of low socio-economic conditions on mathematics learning which draws from dominant class's socio-cultural-political dimension. Access to such school education that is seen as meaningful and relevant by the underprivileged communities and connected to their life settings has remained elusive. What is offered in schools at present is a structured educational package detached from most students' everyday life experiences, yet accepted as "legitimate" knowledge since it acts as the "gate-keeper" (Skovsmose, 2005) to different kinds of opportunities and future social well-being. The legitimacy and necessity of the "formal" school mathematics renders all other forms of mathematical knowledge not only insignificant but also ineffective. This "package" of formal school mathematics either repels or attracts people depending largely on their socio-economic status. In this backdrop, it is widely accepted that hierarchical social structure (for example, caste and class division in the Indian society) has bearings on academic achievements including mathematics learning. Addressing this pedagogical challenge requires bringing knowledge production from such "other" sites of learning to the classrooms.

Hand-skills and cultural practices as part of FoK are valorised in many low-income communities since these are sources of knowledge production and practice but have remained elusive in ethnomathematical explorations. This study highlights the need for dialoguing the epistemological underpinnings of subject ethnomathematics to give these practices newer voice, power and acceptance in the dominant discourse. Such broadened ethnomathematical subject domain is necessary for raising sensitivity towards work-context knowledge production, its preservation and empowerment, and also for creating hope for redefining mathematics pedagogy transformation.

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UNCOVERING THE COMPLEXITIES OF A TEACHER'S PEDAGOGICAL REASONING AND NOTICING DURING LESSON STUDY

Ban Heng Choy

National Institute of Education, Nanyang Technological University, Singapore

Lesson study has been promoted as a form of job-embedded professional development that can potentially improve teaching practices. Despite its promise, learning from lesson study is not straightforward. In this paper, I aim to revisit the notions of pedagogical reasoning and teacher noticing to model how a teacher might learn from the processes of lesson study. Specifically, I will use this dual lens to analyse how a mathematics teacher, Anita, learned from lesson study as she taught an introductory lesson on gradient and uncover some of the complexities underpinning her learning.

LESSON STUDY: A WAY FOR TEACHERS TO LEARN?

Lesson study is a site-based and collaborative professional learning approach that can potentially enhance teachers' adaptive teaching competence—an essential component of teaching expertise in diverse classroom environments. Despite the promise of lesson study to improve teaching practices (Schipper et al., 2017), learning from lesson study is challenging (Lee & Choy, 2017). As highlighted by Bryk (2009), gathering "teachers together to talk about practice" does not necessarily lead to improvement in teaching (p. 599). Instead, what is needed is for teachers to deliberate on the three key aspects of their teacher knowledge: understanding of mathematics, curriculum materials, and knowledge of how students learn for teacher learning to take place (Sherin, 2002). Hence, it is not what but how teachers engage with these professional learning activities that matters (Choy & Dindyal, 2019). However, the mechanisms which drive teacher learning during lesson study is not well-understood and more research is needed (Vermunt et al., 2019). I aim to add to ongoing conversations about these mechanisms by examining the complexities of a teacher's learning through the lenses of pedagogical reasoning (Shulman, 1987) and productive teacher noticing (Choy, 2016). The paper is framed by the following question: How does a secondary mathematics teacher learn from her own teaching via her participation in lesson study?

THREE KEY THEORETICAL IDEAS

I begin by revisiting Shulman's (1987) ideas of pedagogical reasoning and Choy's (2016) notion of productive noticing as ways to examine teacher learning. Following that, I will argue how teacher learning can be associated with one's new comprehension (Shulman, 1987), which leads to an expansion of one's resources, orientations, and goals related to the teaching of mathematics (Schoenfeld, 2011).

Seeing teaching as "an act of reason", which "continues with a process of reasoning" to culminate in a series of pedagogical actions (Shulman, 1987, p. 13), learning from one's

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teaching involves taking what one understands about content and "making it ready for effective instruction" (Shulman, 1987, p. 14), through a cycle of comprehension, transformation, instruction, evaluation, and reflection leading to new comprehension. Briefly, *comprehension* involves understanding the content and its purpose before the teacher *transforms* that knowledge into pedagogically powerful forms to be presented to students during *instruction*. Shulman (1987) used the term *evaluation* to mean the assessment of student learning and teaching processes, which provides the evidence to precipitate a teacher's *reflection* in ways that lead to *new comprehension* of content, student learning, and teaching approaches. It is pertinent to note that *new comprehension does not necessarily follow through from reflection* (Shulman, 1987). This explains that some teachers learn from their teaching experiences, while others do not.

Since teachers' actions can be modelled as a function of their clusters of resources, orientations, and goals (Schoenfeld, 2011), it is reasonable to assume that new comprehension can lead to an expansion of one's set of resources, orientations, and goals. In this case, one's resources refer to teacher knowledge about content, student, and teaching (Sherin, 2002; Shulman, 1987). By orientations, Schoenfeld (2011) refers to the set of beliefs, preferences, and values which one holds in relation to his or her goals of teaching and learning. An expansion of a teacher's current cluster of resources, orientations, and goals, which in turn becomes the base from which the teacher make sense of instruction, can then fuel changes in one's teaching practices. But how does a teacher gain new comprehension of content, student learning, and teaching actions?

To this end, I argue that new comprehension of content, student learning, and teaching actions occurs when a teacher has a shift of attention, gaining awareness of new possibilities in teaching and learning (Mason, 2002; Sherin, 2002), or simply when a teacher *notice* critical aspects of teaching and learning (Choy, 2016). However, not all noticing is productive. Choy (2016) suggests noticing is productive when teachers focused their attention on the key mathematical points of the content taught, students' confusion when learning mathematics, and the teaching actions directed at addressing these points of confusion. Moreover, as Choy (2016) has highlighted, teachers are more likely to make instructional decisions that promote student thinking when they attend and make sense of how their teaching actions are targeted at students' confusion when learning the content.

In the context of Lesson Study, as Lee and Choy (2017) have suggested, teachers have opportunities to hone their noticing expertise when examining the curriculum materials, or what is known as *kyouzai kenkyuu* (Choy & Lee, 2020). The process of detailed lesson planning also requires teachers to scrutinize aspects of mathematical content, student thinking, and teaching approaches as they transform their knowledge into tasks, instructional materials, and lesson plans. When teachers are more attuned to noticing relevant and critical aspects of what and how they are teaching, they are more likely to focus on how students think during the actual research lesson (Choy, 2016). This focused attention and sense-making can potentially lead to new comprehension after a more nuanced reflection during the post-lesson discussion.

Putting these ideas together, teachers' learning can be modelled through the lenses of pedagogical reasoning and productive noticing as shown in Figure 1. First, teachers work through *kyouzai kenkyuu* and lesson planning with some prior resources, orientations, and goals (ROG) to comprehend the key ideas for the lesson. They then transform their initial comprehended ideas into a form suitable for teaching their students. The iterative and cyclical processes of transformation, actual instruction, and assessment feed forward to the reflection of the teacher (to different extents for different teachers). This process leads to some new comprehension, which may lead to a new expanded set of ROGs and the cycle repeats. What this model affords us is the opportunity to capture the complexity of the dialogic processes involved when teachers learn from their practice.

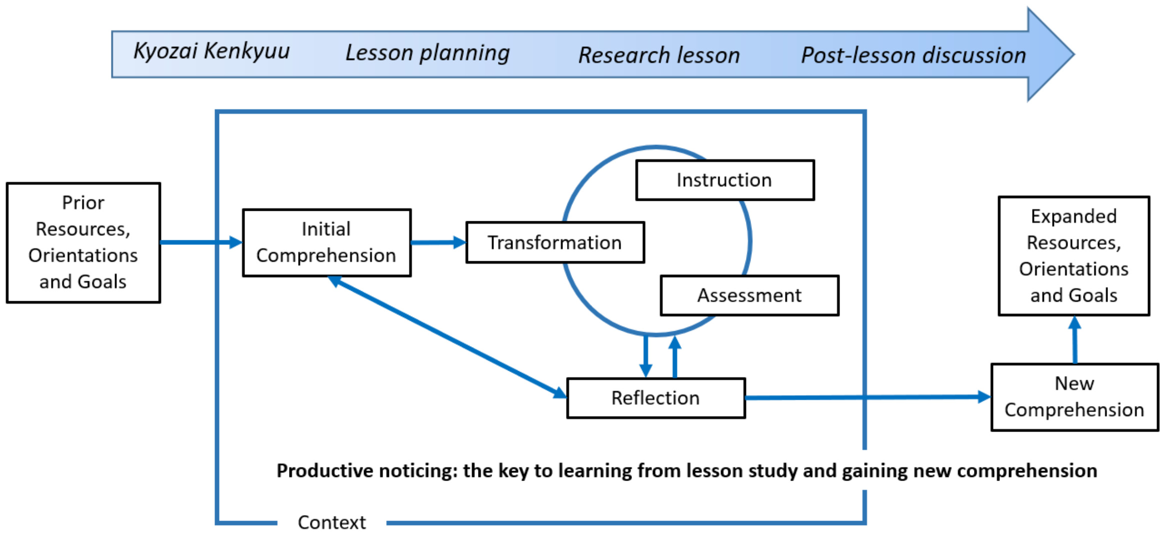

Figure 1: Learning from Lesson Study.

METHODS

The vignettes described in this paper centred about the pedagogical reasoning and noticing of Anita, a mathematics teacher with 12 years of experience. In the vignettes, she planned a lesson on gradient with five other teachers from Springside Secondary School—Eddie, Winston, Teresa, Don, and Kent—and taught the research lesson for Secondary One (aged 13) students. Data were generated mainly from the voice recordings of the lesson study discussions and video recordings of the research lesson. Photographs of lesson artifacts such as lesson plans, discussion notes, and when available, samples of students' work were also included. Segments of recordings related to the key tasks of lesson study and notable episodes involving mathematically or pedagogically significant moments were marked for further analysis. Following Choy (2016), I examined what the teachers noticed in terms of the mathematical points, students' confusion, and teaching approaches. The reviewed segments were then transcribed before they were coded using a "thematic approach" (Bryman, 2016, p. 578) to highlight aspects of what and how Anita learned from the lesson study sessions. Corresponding teacher and student artifacts, when available, were also analysed as triangulations of my analysis.

VIGNETTES FROM THE LESSON STUDY SESSIONS

Here, we present three vignettes from the Lesson Study sessions and discuss them in relation to Anita's noticing, pedagogical reasoning, and teaching actions. The first vignette was developed from data collected during the first session, which examined the teaching and learning of gradient. The second vignette focused on the research lesson, and the last vignette described conversations during the post-lesson discussion.

Rise over Run: "Not a problem for my students!"

In the first session, the teachers discussed the key mathematical points related to the topic of gradient and the potential confusion that students might have. Specifically, for Anita, she identified the notion of gradient as "rise over run" as a key mathematical point for the lesson and highlighted that her students might have difficulties when the coordinate axes were introduced. Here, Anita was very specific about students' errors and was able to provide possible examples of them. For instance, Anita highlighted that students might determine the "run" wrongly by taking reference to the origin (See Figure 2).

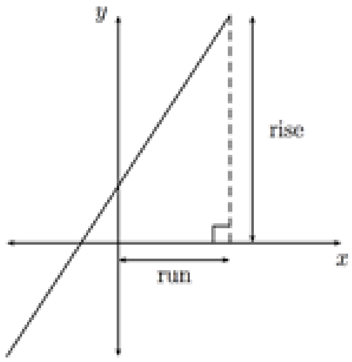

Figure 2: A possible student's error as identified by Anita.

Even though Anita had a relatively strong grasp of the content and possible students' errors, her comprehension did not necessarily transform into the design of her instructional materials and teaching approach. For instance, Anita used three types of questions in her worksheet: four questions on gradients of lines set in unmarked gridlines; six questions that involved the use of the standard Cartesian coordinates; and six which were situated in a non-homogeneous coordinate system. In each of the three types of questions, what counts as distance is different, and this may pose problems for students trying to find the "rise" and "run" especially when they are learning about coordinate systems for the first time. When Eddie asked whether her students could see the "rise" and "run", Anita highlighted that her students should be able to:

1. Eddie: So, can they see? Can they see the rise over run? (He laughs.) So… we want to make sure that…
2. Anita: Can see one… I don't think they cannot see… It's [obvious.] Can see…
3. Eddie: So, the students know that for the "rise", they know how to count the squares?
4. Author: Is this the first time they encounter this?

5. Anita: Yeah. This is the first time. They are Secondary One [students].
6. Eddie: So, do they know this one? (Points to the formula "rise over run".)
7. Anita: Sorry, I have been doing this for a long time already. At the beginning, the teacher will tell them that gradient is the measure of steepness or slope, so they will know that gradient is rise over run… then tell them that to measure the slope, it's rise over run. Just start with [calculating gradient.]
8. Author: So, you define for them?
9. Anita: Yes. Just tell them. It's just as simple as that. Gradient is just "rise over run". Some students, the better ones, can even tell you if they read the textbooks.

… …

14. Eddie: So, the rise is referring to? Let's say I am a student. What is rise?
15. Anita: Eddie, this one they will have no problems… I can tell you they know. (Eddie laughs.) They know that rise is the straight one [perpendicular]. This is not a problem to lower secondary students.

Anita seemed confident that students could understand gradient simply by telling them (Turns 7 and 9), and she expected them to see "rise" and "run" the same way she did. Yet, she was also the one who highlighted the possible confusion by students as shown in Figure 2. Eddie tried to get Anita see the possibility that this might not be the case by adopting the perspective of a student (Turn 14). However, Anita's insistence that her students would not have any problem seemed to suggest that she could have overlooked students' possible confusion about the effect of scale on distances.

"Is it start from here?"

Contrary to Anita's expectation, students had trouble "counting the run" and an unanticipated confusion about the "rise" arose during the lesson. For example, when a student S4, asked Anita about the determination of "rise", she reacted by "telling" without listening to find out what Student S4 was thinking about. Specifically, she told Student S4 that the "rise is the height", and she "should find the perpendicular and count how many units are there". Moreover, Anita told the students that they were supposed to "count by the boxes". It was not clear whether Student S4 understood the key idea, that what matters is the ratio of the "vertical change" to the "horizontal change" at this point. Instead, Anita's emphasis on "counting" might cause Student S4 to associate the "counting of boxes" with the measurement of "rise" and "run". This could be seen when student S4 encountered difficulties with Question 1b (See Figure 3). Student S4, like several other students, mistook the run as "negative three" because they took reference from the origin:

42. S4: Is it start from here or here [Pointing to (0,0) and (-3,0)]?
43. Anita: You must start at the base, right? Which is the right-angle triangle, right?
44. S4: But I thought we must start from zero? Then zero, should be minus three?
45. Anita: So, if today we don't have all these things (referring to the grid), then there's no more zero. So, we look at the line. Remember just now there's no zero at

all, we just try to form a right-angle triangle? Ok. Don’t bother about the zero first, just try to get the right-angle triangle…

46. S4: But...
47. Anita: Never mind. I will show you again at the board.

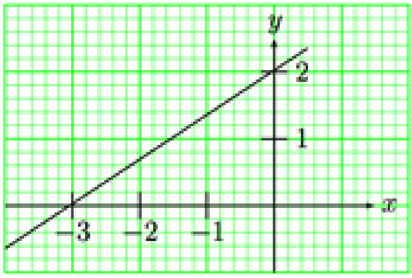

Figure 3: Question 1(b).

“You didn’t tell them why…”

Anita continued to see students’ difficulty with finding “rise” and “run” as non-critical during the post-lesson review. Instead of analysing students’ responses in relation to the content and difficulty, she attributed the issues highlighted by other teachers to factors such as time constraints. For example, during the review of her lesson, Anita expressed surprise at students’ mistake of seeing “gradient as the height of the mountain” even though this is a well-documented error (Zaslavsky et al., 2002):

> Like [Question] 1b, I expected some problems, but I did not manage to go around the class. […] But from the response of the class, I think most of them could get concept of the height and the run. I linked back to the starting, because at the start, I did not foresee them telling me that gradient is the height of the mountain. It was something that I did not expect… I thought it was very straightforward.

While she repeatedly acknowledged that she was surprised by students’ responses, Anita did not go beyond her initial surprise to analyse what went on during the lesson. Moreover, she thought that “most of them could get the concept of *height* and the run”, whereas other teachers had observed several misconceptions demonstrated by students during the lesson. For example, Eddie described how Anita’s students had problems counting the “rise” and “run” and highlighted that students might be confused about the definitions and conventions involved in understanding gradient. This corroborated with Kent’s observations:

> Once we include the axes, they were confused. I liked the second page, where the numbers are the same, but the directions are different. I asked one of the students sitting at the corner, who got it wrong. She did not put in the negative sign. I asked her whether there was any difference. It seemed that she was a bit confused about the convention from left to right, pointing left, pointing right, highest point etc. There are a lot of conventions…. They are also unable to identify the convenient points, as well as the drawing the triangles and reading of the scales.

Kent not only highlighted the possibility that students might be confused by the different conventions, but he also raised a new point about the calculation of the “rise” and “run” when the scales of the axes were different. This observation (which can be counted as a *new comprehension*) is important because in non-homogeneous (different scale on different axes) graphs, distance is not preserved. The confusion between geometric and analytic distance has been known to cause cognitive difficulties for understanding gradients (Zaslavsky et al., 2002). However, Anita persisted in viewing these “mistakes” as “expected” and she thought that students would learn from their mistakes without any intervention and by repeating her explanation. The turning point came about when another teacher Don referred to another observation:

> There’s one thing I realised. Every triangle you drew is the big one, and so the students also drew the big one. And you don’t always tell them why they have to start here or there. They also don’t get a hang of why they have to start here or there. You just drew the triangle and said, the rise is 5...

It was at this point that Anita noticed that she had only used the biggest possible triangle to find gradient in all her examples; and the reasoning behind her choice of triangle was not made explicit. This revelation prompted Anita to consider the possibility that she may have overlooked this in her teaching.

DISCUSSION

The three vignettes suggest a much more nuanced and complex portrait of Anita’s learning from her participation in Lesson Study. When planning the lesson, it is evident that Anita has a strong grasp of the key concepts in the topic of gradient, and she has a clear understanding of the potential confusion that students might have when coordinate axes are introduced. Yet, her comprehension did not transform into a more adaptive teaching approach. Instead, she “trivialized” the anticipated student’s error and did not think about other possible teaching approaches beyond just telling them (see turns 7 and 9). During the lesson, Anita attended to her students’ confusion, but she thought that most students “could get concept of the height [rise] and the run”. This despite the case that her colleagues observed otherwise. Even at the post-lesson discussion, while the comments offered by Eddie, Kent, and Don provided some affordances for enriching Anita’s noticing, she did not really take them up and chose to stick with her current approach of telling students—until Don’s comments provided some motivation for her to consider another possibility.

When viewed through the lens of teacher noticing (Choy, 2016), what Anita attended to, and how she interpreted these observations to make instructional decisions during lesson planning, teaching, and post-lesson discussion could have led to a more discursive approach to teaching. However, her noticing was deemed to be unproductive because her decision to “just tell them” did not align with what she noticed about student learning. Looking at her learning through the lens of pedagogical reasoning, aspects of her transformation, instruction, and reflection would need to be looked into even though it was clear that she had a relatively strong comprehension of the topic and was able to assess her students’ learning. What Anita did not do was to consider multiple viewpoints during her reflection to gain new comprehension that would lead to an expansion of her cluster of resources, orientations, and goals. Instead, she *looped back* into her usual approach of telling instead of using the comments from her colleagues to feed-forward her teaching. Schoenfeld (2011) describes "highly accomplished teachers" as those "who engage in noticing as a matter of practice, and use their observations to shape their instruction", or those who "are largely engaged in diagnostic teaching" (p. 235). The use of these two lenses can provide a more targeted locus of action for both Anita and others to engender a more adaptive teaching approach. This assertion places both a teacher's noticing and pedagogical reasoning at the center of our efforts to improve mathematics teaching.

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AN EYE-TRACKING STUDY ON PROBLEM-SOLVING STRATEGIES OF PLANE VECTORS AMONG SENIOR HIGH SCHOOL STUDENTS

Haode ZUO

Yangzhou University

To explore the differences in eye movement characteristics of high school students with different mathematics academic levels and gender in-plane vector problem-solving, this study selected some high school graduates in Jiangsu Province as research samples and collected and analysed their eye movement data in the process of plane vector problem-solving. In this study, we found differences in eye movements of high school students with different academic levels in solving plane vector problems. The male participants are more inclined to read pictures when solving plane vector problems, while female participants are more inclined to read text materials when solving plane vector problems.

INTRODUCTION

Plane vector, one of the main concepts of secondary school mathematics, combines the properties of both geometry and algebra. Many areas of high school mathematics, including trigonometry, linear equations, analytical geometry of the plane, and solid geometry, are explained based on plane vectors. The geometry and algebra duality of plane vector makes it difficult for students to solve problems related to plane vectors (Dorier, 1997; Goos, 2004). Existing research(Rasmussen & Marrongelle, 2006) has shown that students struggle with geometric or algebraic representations of vectors and their intersections. The research (Yilmaz & Argun, 2018) shows that high school students need help constructing the plane vector concept, an abstract mathematical concept. At the same time, these students have different understandings of geometric visualisations of plane vectors. The close connection between geometrical thinking and visual perception of plane vectors makes eye tracking a suitable and pertinent method (Strohmaier, MacKay, Obersteiner, & Reiss, 2020) for research on problem-solving strategies of plane vectors.

Although eye tracking is a nonintrusive method to obtain information about visual attention and cognitive processing while students solve problems, particularly where visual strategies are involved, such as plane vectors. The existing research has primarily focused on the space vector to solve the physics problem (Klein, Viiri, Mozaffari, Dengel, & Kuhn, 2018). A study using eye tracking (ET) techniques in mathematics education focusing on plane vectors needs to be addressed.

THEORETICAL FRAMEWORK

The research is eye-tracking analysis research on solving plane vector geometry problems for general high school students. The key research question of this study was what visual strategy, identified by eye-tracking measures, can promote better plain vector problem-solving. However, recent meta-analyses(Strohmaier et al., 2020) converge to suggest that the analytic framework of using an eye-tracking method for mathematical

2023. In Inprasitha, M., Sudjamnong, A., Suriyon, A., Saengpun, J., Moonpo, P., Chaovasetthakul, R., Plianram, S., & Suttiamporn, W. (Eds.). *Proceedings of the Regional Conference of the International Group for the Psychology of Mathematics Education 2023*, pp.73-80. Khon Kaen, Thailand: PME.

problem-solving is relatively fixed: it starts from the two dimensions of eye movement visual analysis and eye movement data analysis; the eye movement heat map, eye movement focus map and combination time when solving problems in plane vectors are examined; furthermore, objective indicators such as saccades, blinks, the ratio of eye movement and gaze time during the problem-solving process are analysed.

Also, there has yet to be an initial domain-specific interpretation framework for eye-tracking data in mathematics(Schindler & Lilienthal, 2019). Therefore, we explored an alternative approach that could answer our research problem: applying eye-tracking measures to discriminate strategies of plane vectors between participants of high and low mathematics achievements identified by the high-stakes college entrance examination. We chose this approach because participants with higher college entrance examination scores are believed to have a better visual strategy that promotes better-solving vector problems.

In particular, the following issues will be addressed:

1. What are the similarities and differences in the eye movement characteristics of students with different academic levels in solving plane vector problems?

In addition, the affective variable of plane vectors problem-solving is also examined, investigating relations between eye movements and gender:

2. What are the similarities and differences in the eye movement characteristics of students of different genders in solving plane vector problems?

METHODS

This experiment selected 24 graduating students who had been admitted by a university in Jiangsu Province, including 12 males and 12 females, aged between 18 and 19. All participants had normal vision or corrected vision. After high school in Jiangsu Province, they normally entered the university through the college entrance examination. The college entrance examination is a comprehensive reflection of the knowledge learned by senior high school students. It is an authoritative test material prepared according to senior high school curriculum standards and is the basis for selecting talents in colleges and universities.

To some extent, the math test paper of the college entrance examination in Jiangsu Province can reflect the academic level of the participants, so it is of particular significance to use the math test paper of the college entrance examination for sample screening in this experiment. They could skillfully use computers, keyboards and mice, have normal hearing, and understand Chinese instructions. They have not participated in this experiment's pre-experiment or similar eye movement experiment, nor have they participated in other plane vector geometry problem-solving experiments.

Among the 24 samples selected in this experiment, only 23 qualified data were obtained. During the comparison of academic level dimensions, the sample with the college entrance examination score greater than 125.95 was selected as the high academic level group (N=8), the sample with the college entrance examination score less than 125.95 was chosen as the low academic level group(N=8). The average score of the participants in the high academic level group was 131.4 points, the average score of the low academic level group was 120.5 points, and the standard deviation of the two groups was 4.6 and 2.593, respectively. The sample variance of the two groups is not homogeneous (f=5.27 p=0.034) through the Levene variance homogeneity test. Still, its p-value is smaller than 0.001, and the effect size is 0.8, which has a significant inter-group difference.

When comparing the gender variables, ten males and ten females were selected. There is little difference in academic level between gender groups (p=0.723).

This experiment uses the high-speed 1250 eye movement tester produced by SMI as the main instrument. This instrument mainly comprises a screen and an eye movement data collection collector. In the experiment, the participants need to put their heads on the device stably and look at the screen for proofreading and experiment. The instrument can collect multiple indicators such as eye movement trajectory, eye movement hot spots, eye movement focus, fixation count, fixation times, saccades, blink count, and problem-solving time. The eye movement data of the experiment is collected by *Iviewer* software, exported to a txt file, and then analysed by the *BeGaze* software.

The finalised instrument for our main study is composed of 4 plane vector problems, which are selected from selected by a high school mathematics teacher from a textbook or related exercises and verified by our pilot study.

RESULTS

Academic level variance

According to the result of the t-test, there are two insignificant ($\alpha = 0.05$) eye movement indexes of the two groups, the average number of saccades per minute ($p = 0.164$), and the time spent on each problem (all $ps > 0.05$), whereas significant ($\alpha = 0.05$) eye movement indexes of the two groups include the average count of fixations per minute($p = 0.015$)and the average count of blink per minute($p = 0.041$). Table 1~4 indicates the descriptive statistics of the indexes mentioned above.

	Group	N	Mean	SD	95% CI
The average count of fixations per minute	High	8	185.7534	16.03289	172.3496-199.1573
	Low	8	133.0891	46.52541	94.1929-171.9853

Table 1: Descriptive statistics of the average per minute fixation of high and low academic level grouping samples

Of the 16 participants in the two groups selected, the average number of fixations per minute in the high academic level group was 185.75, which was 39.6% higher than the 133.09 in the low academic level group, and its standard deviation was small. The samples in the high academic level group showed the characteristics of high fixation per minute more intensively, while the standard deviation of the low academic level group was significant, with some samples deviating from the centre.

	Group	N	Mean	SD	95% CI
Average saccade per minute	High	8	199.7923	39.25978	166.9703-232.6143
	Low	8	252.3673	93.24821	174.4098-330.3247

Table 2: Descriptive statistics of the average number of saccades of high and low academic-level grouping samples

	Group	N	Mean value	SD	95% CI
Blink every minute on average	High	8	41.0072	18.42459	25.6039-56.4106
	Low	8	83.6535	47.34654	44.0708-123.2362

Table 3: Descriptive statistics of average blink per minute in high and low academic level groups

Time	Group	N	Mean value	SD	95% CI
1st question	High	8	36.6300	16.66052	22.7105-50.5585
	Low	8	34.1425	10.26917	25.5573-42.7277
2nd question	High	8	39.9288	18.89501	24.1321-55.7254
	Low	8	40.9725	27.48830	17.9917-63.9533
3rd question	High	8	91.0713	26.92641	68.5602-113.5823
	Low	8	111.1863	68.48779	53.9290-168.4435
4th question	High	8	127.4863	66.60212	71.8055-183.1670
	Low	8	114.4338	45.64434	76.2741-152.5934
Total time	High	8	295.1163	88.24470	221.3418-368.8907
	Low	8	300.7350	86.24057	228.6361-372.8339

Table 4: Descriptive statistical table of time spent on grouping problem-solving at high and low academic levels

The analysis (figure 1~4) of the focus map and hot spot map indicates that: in the high academic level group, the contrast between light and dark is strong, the red area of the eye movement hot spot is concentrated, the green area is less, and the purple area and white area are larger; In the group of students with learning difficulties, the contrast between light and shade is not apparent, and the red regions of eye movement hot spots are scattered, while the green areas are more.

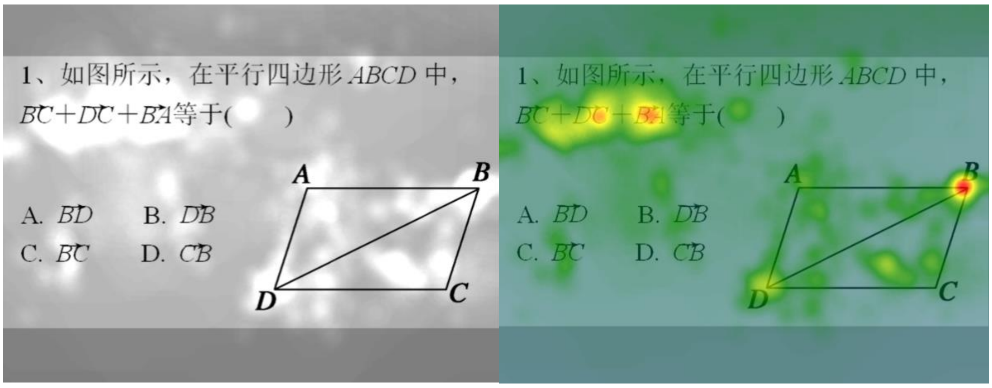

Figure 1: Focus and hot spots of the first question in the low academic level group

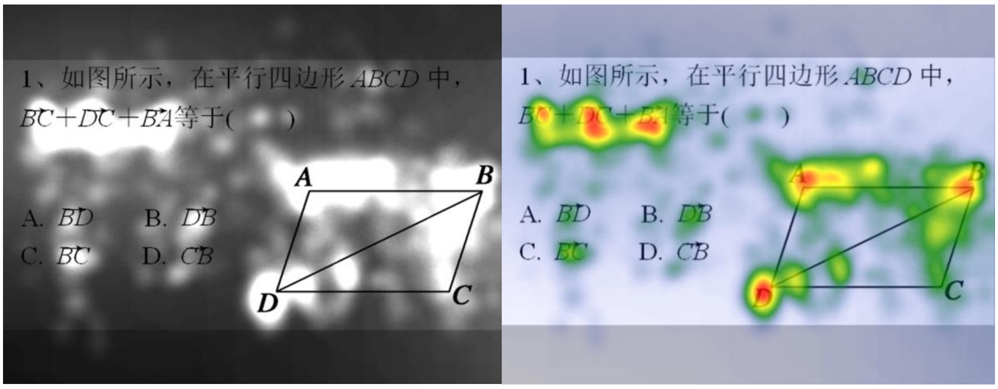

Figure 2: Focus and hot spots of the first question in the high academic level group

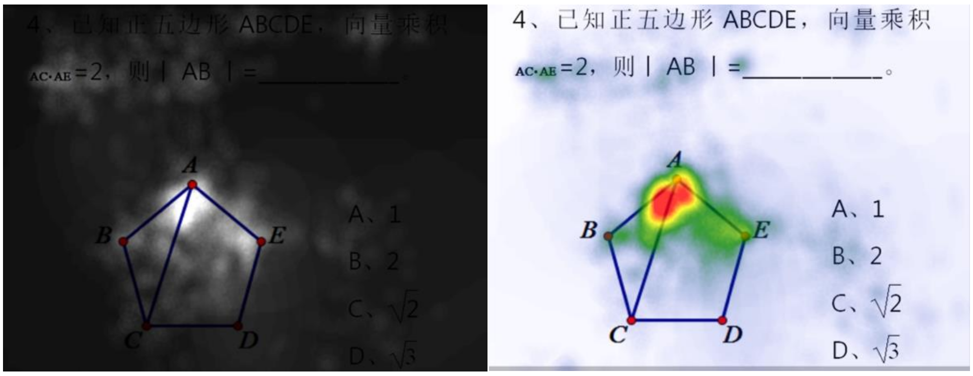

Figure 3: Focus and hot spots of the fourth question in the high academic level group

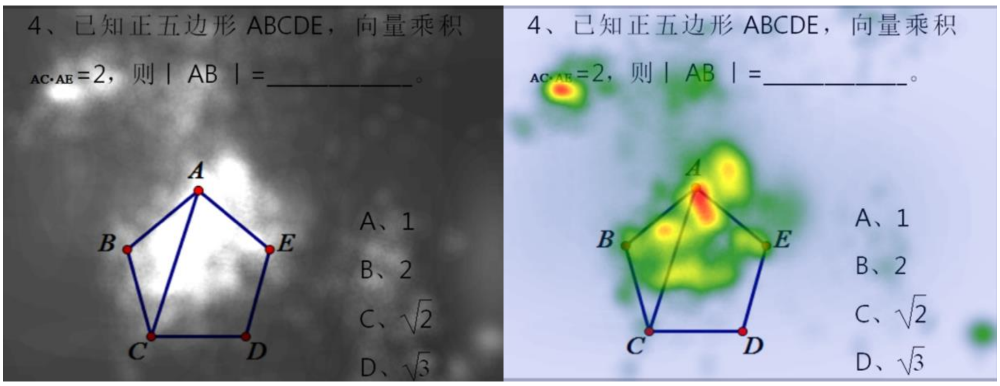

Figure 4: Focus and hot spots of the fourth question in the low academic level group

Gender variance

According to the result of the t-test, the four eye movement indexes (the average number of saccades per minute, the time spent on each problem, the average count of fixations per minute and the average count of blinks per minute) is not significantly related to gender. It is speculated that the difference in eye movement between male and female students in plane vector problem solving is the difference in perception and attention. The secondary data obtained from *BeGaze* software analysis, such as eye movement partition histogram and eye movement partition basic data map, are selected for qualitative analysis of male and female participants.

The eye movement partition histogram and the eye movement partition basic data map are images that reflect the time data of the participants' eyes staying in the divided area of interest during the eye movement process. The eye movement partition histogram can compare the degree of attention of the participants to different areas of interest and the proportion of the degree of attention to other areas of interest during different time periods of problem-solving; The basic data map of eye movement zones is the total data reflecting the degree of attention of participants in different groups to the divided areas of interest during eye movement.

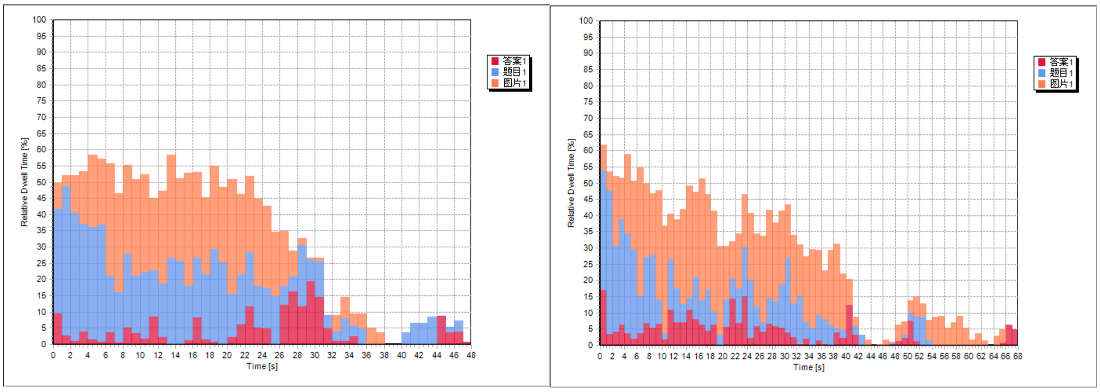

Figure 5: Histogram of fixation time for comparison between female and male of the first item (Left female and right male)

The comparison histogram of the first question shows that the proportion of female participants reading the text part (the blue part in the histogram) is significantly lower than that of male participants. The proportion of male and female participants reading the text increases in the first stage of reading the question, then gradually decreases. The proportion of reading the answer area (the red part in the histogram) is continually deficient.

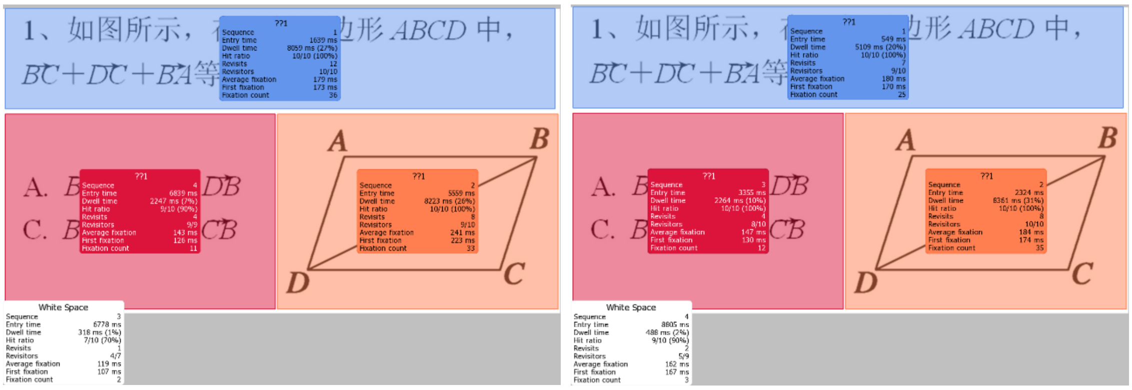

Figure 6: Comparison chart of male and female reading areas of the first item

It can be seen from the comparison chart of male and female areas in the first question that text reading accounts for 20% of the total reading time of male participants, picture reading accounts for 31% of the full reading time of male participants, and answer reading accounts for 10% of the total reading time of male participants; Text reading accounted for 27% of the full reading time of female participants, picture reading accounted for 26% of the total reading time of female participants, and answer reading accounted for 7% of the total reading time of female participants. The effective reading time of male and female participants accounted for 61% and 60% of the total reading time, respectively.

Limited to space, we only present the comparison of secondary data of the first item here. In the process of answering the four questions, male participants paid attention to the answers for 10%, 5%, 6% and 5%, respectively, while female participants paid attention to the solutions for 7%, 5%, 4% and 3%, respectively. There is no significant difference between male and female participants in their attention to the answers, and the proportion of their attention time to the total time is low. The attention of male participants to pictures was 31%, 41%, 29% and 38%, respectively, which was higher than that of female participants by 26%, 35%, 22% and 34%, and 19.2%, 17.1%, 31.8% and 11.8% respectively. In terms of attention to pictures, male participants were higher than female participants. The male participants' attention to words was 20%, 18%, 23% and 15%, respectively, while the female participants' attention was 27%, 18%, 24% and 12%, respectively. The male participants' attention was - 25.9%, 0%, - 4.2% and 25% higher than the female participants. There was a significant difference between the simplest first and the most difficult fourth questions, while the difference between the second and third questions was insignificant. Regarding effective reading time, the male was 61%, 64%, 58% and 58%, while the female was 60%, 58%, 50% and 49%. The male performed better in attention, and the overall attention of both men and women showed a slight downward trend. At the same time, the male sample was 1.7%, 10.3%, 16% and 18.4% higher than the female sample.

CONCLUSION AND DISCUSSION

In this study, we found that the average count of fixations per minute and the average count of blinks per minute of high academic participants were significantly different from that of low academic participants. The number of fixations per minute in the high academic level group is nearly 40% higher than in the low academic level group. The fixation is an eye movement indicator indicating that the subjects conduct information acquisition and processing. The higher number of fixations per minute indicates that the high academic level group has a higher ability in reading and processing topic information than the low academic level group. In terms of the average saccade per minute, the high academic level group is lower than the low academic level group, and the saccade index can reflect the reading habits of the participants, which means the high academic level group is more concentrated on getting information from the reading the materials. Moreover, the study also found that male participants are more inclined to read pictures when solving plane vector problems. In contrast, female participants are more prone to read text materials when solving plane vector problems.

Despite the potential generalizability concerns with small samples, this study produced results that supply previous eye-tracking studies on the mathematical domain of geometry, shape and form.

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FOSTERING SUCCESS: THE ROLE OF IMMIGRANT PARENTS IN MATH EDUCATION

Maliheh Manzouri

The University of British Colombia

Understanding math education for immigrant students is a multifaceted endeavor, marked by distinct challenges and prospects. In this study, I aimed to explore the impact and significance of immigrant parents in shaping their children's math education. I conducted a qualitative approach and selected seven Iranian immigrant students between the ages of 14 and 17 to participate in fourteen interviews. The results of the Thematic Analysis showed that reducing parental involvement in the mathematics education of Iranian students after immigration impairs their math performance and leads to academic setbacks. Not just Iranian parents, but all immigrant parents should be aware that changes in their behavior after immigration can significantly impact their children's education, especially in mathematics.

INTRODUCTION

This Research in mathematics education recognizes that mathematics, despite its reputation for objectivity, is susceptible to influence from various factors such as teaching approach, curriculum content, or academic background of students (Hannula, 2015; McDonough & Clarke, 2002). Similarly, the process of mathematical learning among immigrant students (Bishop et al., 1997) is subject to the sway of multiple factors, encompassing educational, personal, social, cultural, historical, and political dimensions (Leung, 2021; Sayers et al., 2022; Urick et al., 2022; Wan & Lee, 2022).

Parents exert a substantial influence on their children's academic success, including their performance in mathematics. Research underscores their pivotal role, encompassing emotional support, progress monitoring, and various direct and indirect roles such as motivator, resource provider, and math learning counsellor.

The process of immigration may introduce unique challenges and dynamics that can potentially modify the nature and extent of parental involvement. In this study, I investigate how immigrant students infer the role of their parents in their education after immigrating and how these changes affect their mathematics learning.

LITERATURE REVIEW

Parental involvement is fundamental to children's academic outcomes, with numerous studies underscoring the significance of varied forms of involvement, such as providing support, monitoring progress, and nurturing positive relationships (Antony-Newman, 2019; Bates et al., 2018, 2019; Ixa Plata-Potter & de Guzman, 2012). In the domain of mathematics, parents play specific roles, including serving as motivators, monitors, resource providers, and advisors (Cai et al., 1999b). Liu & Leighton (2021) found that students perform better in math and have improved attitudes when parents are actively involved. Diverse roles taken up by parents, especially indirect ones like motivation and monitoring, have shown stronger predictors of math success than more direct roles like

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content advising (Whitaker, 2019). Additionally, parental academic attainment influences their children's education. Parents with higher education not only have elevated expectations but also engage in cognitive stimulation, fostering an environment conducive to their children's academic success (Davis-Kean et al., 2021; Ludeke et al., 2021). However, the narrative changes with immigrant parents. Immigration brings about unique challenges, including unfamiliar parenting dynamics and limited social support, potentially impacting their psychological welfare (Bernhard, 2013; Cartwright & Chacon, 2021). Adolescents in immigrant families face compounded issues influenced by the intricacies of both immigration and adolescence (Choi et al., 2022; Ixa Plata-Potter & de Guzman, 2012).

While positive involvement of immigrant parents correlates with better educational outcomes for children (Liu & Leighton, 2021; Ludeke et al., 2021), there is a crucial gap in the literature comparing parental involvement before and after immigration, especially regarding mathematics education. Interactions between immigrant parents and educational institutions play a pivotal role. Teachers are frontline facilitators in guiding immigrant parents through unfamiliar educational terrains (Celedón-Pattichis et al., 2022; Lee & Kim, 2021). However, hindrances such as language barriers often make this task daunting (Artigue, 2010; Celedón-Pattichis et al., 2022; Chapman, 2003).

One salient challenge for immigrant parents is navigating often unfamiliar mathematics curricula and teaching methodologies (Antony-Newman, 2019; Cai et al., 1999a). The transition is not limited to the educational domain. Cultural shifts often affect parental perceptions, inducing stressors like language barriers, cultural gaps, and economic challenges, which collectively hamper their involvement in their children's education (Artigue, 2010; Leung, 2021; Ludeke et al., 2021).

Religion offers another layer of complexity. The transmission of faith becomes a balancing act for immigrant parents, aiming to preserve their beliefs while assimilating into the new culture (Horwitz, 2021; Smith & Adamczyk, 2021). It may introduce additional stressors or time commitments related to religious practices (Dein, 2020), potentially impacting the time and focus available for math studies. Additionally, conflicts or cultural differences related to religious beliefs can create emotional or psychological challenges for immigrant children, potentially affecting their overall well-being and, consequently, their academic performance in mathematics (Noth & Lampe, 2020).

Dynamics within immigrant families also play a role in children's education, including their math education. There is a higher risk of parental separation among immigrant couples, which can have significant implications for the well-being and educational outcomes of their children (Cartwright & Chacon, 2021; González-Ferrer et al., 2016; Stadelmann et al., 2010).

In conclusion, while parental involvement is universally acknowledged as vital for children's academic outcomes, the intricacies and challenges faced by immigrant parents add layers of complexity that require deeper understanding and nuanced interventions. Furthermore, it is imperative to recognize that the diverse cultural backgrounds, varying religious beliefs, and unique ethnicities among immigrant populations introduce additional dimensions that demand special investigation. In this study, I focused on the role of Iranian immigrant parents in the mathematics learning of their children.

METHOD

In exploring the mathematics experiences of Iranian immigrant high school students in Canada, I employed a qualitative research approach. Such methodological approaches aim to understand phenomena by exploring individuals' experiences, emotions, and behaviors, often using methods such as interviews, observations, and document analysis (Adedoyin, 2020). Qualitative research provides a powerful method to delve deeply into educational and significant social topics by interpreting the lived experiences of individuals intertwined with these issues (Bikner-Ahsbahs et al., 2015). Given the exploratory nature of my study, I employed semi-structured interviews. This format allowed for probing questions to clarify any uncertainties during interactions. Face-to-face interviews enriched the data, capturing both verbal responses and non-verbal signals. Care was taken to navigate sensitive topics while encouraging participants to elaborate on their responses. In addition to these interviews, I drew on my experiences as an Iranian immigrant mother who supports two kids in the Canadian education system and my communication with my students' parents as their math tutor. The focus group of the study consisted of Iranian immigrant high school students aged 14-18, attending Grades 8 to 12, who had transitioned to Vancouver, BC, Canada, between 2013 and 2018. This timeline was chosen because those who migrated earlier generally began their education in Iran and might not possess comprehensive insights relevant to the research questions. Notably, in Iran, high school begins in Grade 7. To ensure that students had experienced both Iranian and Canadian high school settings, a minimum grade requirement of Grade 8 was established.

Taking cues from Francis (2010) and Johanson and Brooks (2010), an initial sample size of seven Iranian immigrant students was targeted for the research. However, to enhance the depth of the data, an eighth participant was included. After analyzing this additional input, I found that the insights mirrored previous discussions. This overlap echoed Seidman's (2015) observation of data saturation, leading me to exclude the eighth interview.

To recruit participants, I initiated engagements with Iranian migrant families at community events in Vancouver. These families had relocated to Vancouver within the past 1-5 years and had adolescents aged 14-18 who were enrolled in schools across Greater Vancouver. For data analysis, I utilized Thematic Analysis (TA) to scrutinize the students' dialogues, identifying recurring patterns and motifs. Data from the preliminary research were not integrated into this analysis. The immersion process began with multiple readings of transcripts and careful listening to audio recordings. Subsequently, common themes were distilled, and data segments were labeled with concise descriptors (Bikner-Ahsbahs et al., 2015).

RESULTS AND DISCUSSION

Although initially apprehensive about their ability to answer questions, these three boys and four girls provided comprehensive responses. They come from affluent families, with most having attended private schools in Iran known for their intensive curricula and better facilities. Discussions with these children (referred to as 'MP') revealed their awareness of the disconnection between school and parents. They expressed a lack of obligation to invest time in completing their homework assignments. While in Iran, they faced pressure from their parents to complete their homework, and most did so without fully comprehending

the underlying reasons for this academic demand. They simply followed their parents' request and finished their assignments without truly understanding the consequences of procrastination and neglecting their studies. However, in Canada, where their parents pressure subsides, these MP students often feel a sense of freedom and tend to disengage from their studies, assuming that there will be no negative repercussions.
As a tutor within the Iranian community, I observed that beyond the challenges of adjusting to a new educational system and forging connections with teachers, Iranian parents are deeply concerned about their children's struggles and the new challenges they face in Canada, particularly within the school environment. These challenges encompass adapting to a new language of instruction, navigating a different education system, acclimating to a new environment, adjusting to a new curriculum, building new friendships, and interacting with new teachers, all of which add significant stress to their children's lives. When discussing their children's education in Canada with other Iranian immigrant parents, they often express empathy and reduce the pressure they exert on their children to excel academically compared to the expectations they held in Iran. Despite these lowered expectations, these parents still desire their children to achieve high grades. They acknowledge the difficulty of the new environment, the lack of awareness about the Canadian education system, and the language of instruction as major factors contributing to their children's lower academic performance. Nonetheless, it became evident during our discussions that if these parents were aware of their children's homework assignments, they would encourage them to complete the tasks promptly. However, most of the time, the children either concealed their homework or claimed they would complete it at school and they do not have anything to study.

Through my conversations with MP, it became apparent that these students recognized the change in their parents' expectations since arriving in Canada. They noted that their parents now understood the challenges of studying in a foreign language and were more lenient when it came to lower grades. While their parents encouraged them to study diligently, receiving a lower grade did not elicit strong negative reactions. The students believed that their parents recognized the difficulties of studying in a new country and were more understanding.

I believe that reducing parental pressure and involvement in their education sends a message to Iranian immigrant students (MP) that their education is no longer a top priority. This aligns with Chiu and Xihua's (2008) assertion that students' beliefs about the value of education influence their engagement in learning and studying. Furthermore, research suggests (Ludeke et al., 2021; Xerxa et al., 2020) that parental engagement in their children's learning correlates with higher academic achievement. A decrease in parental involvement in school matters and extracurricular activities diminishes students' intrinsic motivation for academics (Liu & Leighton, 2021). Additionally, the reduction in educational resources (Dronkers & Levels, 2007; Kiernan & Mensah, 2011), such as tutoring or access to books, has left students disheartened and demotivated to learn. The combination of reduced parental pressure and supervision limited educational resources, and a new education system has caused my participants to prioritize their math education less than they did in Iran.

These MP students had developed a habit of diligently doing their math homework after school during their more than seven years of education in Iran. They were closely supervised by their parents at home and their teachers at school to ensure the completion of their assignments. However, this habit underwent a significant transformation upon their arrival in Canada. They gained newfound freedom in deciding whether or not to complete their homework, with many choosing not to do it. After several years of studying in Canada and facing challenges in mathematics, some students, like Liam and Ali, recognized the importance of homework in their academic success. They realized that homework played a crucial role in understanding complex topics and improving their math performance. However, despite making promises to themselves, like Sam and Liam, they struggled to consistently follow through and complete their homework. Reverting to their previous habits of completing homework appeared to be a challenging endeavor.

According to Lally and Gardner (2013), habit formation involves four stages: decision-making, taking action, repeating the new action, and converting it into a new behavior. Throughout these stages, individuals require continuous encouragement and motivation. Some of the Iranian students (MP) acknowledged the need to cultivate good work habits and took the initial step of deciding to develop them (the first stage). They began to complete some of their homework assignments (the second stage). However, they continued to face challenges in consistently dedicating time to complete all their assignments, indicating that they were in the third stage of re-establishing their former work habits. The process of repeating these actions and solidifying them into new habits requires time and patience. For students like Liam, Ali, and Sam, who are currently in Grade 11 and nearing the end of their high school journey, there is limited time to fully reintegrate their old homework habits. Although they have come to recognize the importance of good work habits after several years of education in Canadian schools, it may be too late for them to realize their full academic potential. They could have achieved better academic outcomes if they had grasped this understanding earlier.

CONCLUSION

In conclusion, discussions with the students referred to as 'MP' have shed light on their awareness of the disconnection between their schooling experiences in Iran and Canada. These students displayed a reduced sense of obligation to complete their homework assignments and study for mathematics tests in Canada, as they no longer faced the same academic pressures they did in Iran. This change in perspective reflects a shift in parental expectations and involvement, with parents in Canada often exhibiting greater understanding and empathy toward their children's challenges in a new educational environment.

While parents have tempered their academic expectations, their desire for their children to excel academically remains intact. This disconnect between parental expectations and reduced involvement in their children's education sends a message that education is no longer a top priority. This finding aligns with prior research indicating that students' beliefs about the value of education significantly influence their engagement in learning.

Furthermore, studies suggest that parental engagement in their children's learning correlates with higher academic achievement. A reduction in parental involvement, coupled with limited educational resources and the challenges posed by a new education system, has led many of these students to deprioritize their math education compared to their experiences in Iran.

The transformation in study habits among these students serves as a poignant example of the impact of changing circumstances on educational practices. In Iran, they had established a routine of diligent homework completion and study, closely monitored by parents and teachers. In Canada, they encountered newfound freedom and faced difficulties in maintaining consistent homework habits.

The process of habit formation, as outlined by Lally and Gardner (2013), involves multiple stages, and these students find themselves in the midst of this journey. Recognizing the need for better work habits and taking initial steps toward improvement, they now grapple with the challenge of consistently dedicating time to homework. For some, time is running out as they approach the conclusion of their high school years.

In hindsight, these students acknowledge the crucial role of homework and study in their mathematics academic success, but the challenge lies in reestablishing their former habits. It is a process that demands time, patience, and ongoing motivation.

In essence, the experiences of these Iranian immigrant students in Canada underscore the importance of maintaining a supportive and engaging educational environment. Parental involvement, understanding, and a consistent emphasis on the value of education can significantly influence students' academic motivation and achievement. By recognizing these dynamics and offering appropriate support, we can better equip immigrant students to navigate the complexities of their educational journeys and help them reach their full potential.

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EXPLORING TEACHERS' BELIEFS ABOUT TEACHING IN THE CONTEXT OF BLENDED LEARNING CLASSROOM

Monita Chanroath[1], Maitree Inprasitha[2], Narumon Changsri[3]

[1,2,3]Department of Mathematics Education, Faculty of Education, Khon Kaen University, Thailand

[2,3]Institute for Research and Development in Teaching Profession for ASEAN, Khon Kaen University, Thailand

[2,3]Center for Research in Mathematics Education, Khon Kaen University, Thailand

The COVID-19 pandemic in 2019 transformed education, compelling a shift from traditional methods to online platforms. The Blended Learning Classroom Model (BLC model) by Inprasitha (2021) acknowledges that learning extends beyond the classroom. This study explores teachers' beliefs on blended learning. Data were collected from fifty-four teachers who were comprising teacher educators, school principal, prospective teachers and each with a minimum of two years of experience in lesson study incorporated with the Open Approach (TLSOA). They faced the challenge of incorporating Blended Learning during the pandemic. The results highlight the benefits of blended learning: fostering independent thinking and collaboration. It proves effective for continuous and adaptable student learning.

INTRODUCTION

The world faces pressing issues like climate change and environmental degradation due to rapid economic development in some nations, leading to poverty and inequality (Hepzibah, 2021). The United Nations' Sustainable Development Goals (SDGs) offer a solution, particularly SDG Goal 4: Quality education, which promotes lifelong learning for sustainable development (UN, 2015). Education for Sustainable Development (ESD) is a key element of SDGs, focusing on creating an inclusive educational future that values diversity, equality, and peace (UNESCO, 2021).

Mathematics classes, as Inprasitha (2015) highlights, play a crucial role in developing skills and cognitive processes through problem-solving. Instruction emphasizing the learning process is essential (Isoda & Katagiri, 2012). In Thailand, education often prioritizes knowledge transfer through lectures, but teachers should support students' independent problem-solving and cognitive development (Inprasitha, 2016; Changsri, 2012). Teachers must also understand their students and encourage individual learning processes, allowing students to tackle math problems creatively (Isoda & Nakamura, 2010).

Inprasitha (2021) introduced the Blended Learning Classroom Model (BLC) to emphasize that learning goes beyond physical classrooms. BLC focuses on, the benefit of (integration) between self-learning and interaction learning teachers and students meet, offering flexibility in student locations and interactions. These new conditions for blended learning go beyond physical classroom. It does not a matter of "onsite" vs

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"online" but a matter of when students learn by/for themselves (i.e., self-learning) according to their peace, time, interests and share their ideas when meeting other students and teachers.

Shifting from content-oriented to problem-solving classrooms through lesson study and an open approach changes teachers' beliefs about teaching and learning (Inprasitha, 2003). Changsri (2006, 2012) found that implementing lesson study and an open approach altered teachers' beliefs. However, many teachers still doubt students' ability to learn outside the classroom. This study aims to explore teachers' beliefs regarding teaching and learning in blended learning classrooms through qualitative analysis, emphasizing protocol analysis and analytic description.

BLENDED LEARNING CLASSROOM MODEL: BLC MODEL

According to Inprasitha (2021), blended learning blends self-paced learning with live and non-live or on-demand interactions and is Type 1 in time allocation. Students can learn without teacher or peer meetings.

In contrast, interaction learning in type 2 requires in-person or virtual encounters with teachers and peers. Inprasitha (2021) calls this the Blended Learning Classroom Model (BLC Model).

The main classroom models are explained below:

- Non-live classroom, ODC (On-Demand Classroom): Students can study at their own pace in this format. Well designing tasks and problem situations are set by lesson study team.
- Live classrooms, including VLC and FFC: These classrooms emphasise collaborative learning through teacher-scheduled live sessions. Reflecting on learning outcomes in FFC classes promotes deep learning and self-awareness. It's about "learning how to learn."

METHODOLOGY

The questionnaire was distributed to fifty-four teachers who were lesson study team comprising teacher educators, in-service teachers, researchers, and prospective teachers working with twenty-nine different schools. They have experiences to do lesson study incorporated with Open Approach (TLSOA) at least two years. During Covid-19 situation they challenge to use BLC into the process of TLSOA. The questionnaire consisted of 15 questionnaires and background related questions (gender, age, grade of teaching and subject of teaching, length of time using Thailand lesson study incorporated with open approaches). The study utilizes a qualitative research design, employing interviews, surveys, and classroom observations to gather data. A diverse sample of teachers from various grade levels and subject areas is selected to capture a comprehensive understanding of teachers' beliefs across different contexts. Data were analyzed using Inprasitha's conceptual framework of BLC Model, the most crucial condition for BLC is creating "context" and condition in terms of 'interaction' in order to set 'students' ideas' for future discussion in the following phases. This condition is the most difference from various types of 'online classroom'.

RESULTS

Traditional Beliefs Prior to Engaging; Results from Open-ended questions

An analysis of the teacher's responses to the open-ended questions posed during the interview regarding their instructional approach prior to engaging in the blended classroom setting indicated that a significant proportion of them employed a pedagogical strategy centered around delivering lectures or elucidating principles and equations, followed by providing illustrative examples, before assigning practice exercises to the students. The following findings are some examples of opinions as expressed by the teachers:

"Since teachers and students are unable to meet, the teacher's teaching will be tight, explained, guidance, ways to think for students to follow the steps and let the students do as the teacher teaches." — from a female teacher, age between 26-34 years old and had taught mathematics for 8 years. Se's played the roles of an instructor and observer and had taken part in lesson planning and reflection sessions.

"The teacher studies the lesson plan most of the activities according to the plan, the teacher explains the examples in the video as specified in the textbook for the students to understand clearly and then lets the students do the exercises my them self." — from a female teacher, age between 25-35 years old and had taught mathematics for 5 years. She's played the roles of an instructor and observer and had taken part in lesson planning and reflection sessions.

"My students don't want to come to school or meet up with me or their friend because they said that they cannot do the on-hand exercise that I gave them on the Monday." — from a male teacher, age between 40 - 45 years old and had taught mathematics for 19 years. He's played the roles of an instructor and observer and had taken part in lesson planning and reflection sessions.

Results shows that even though they have experienced using TLSOA in the physical classroom but when going beyond physical classroom, teachers' belief that they cannot menage students to learn by themselves and also interaction between students and teachers. The teaching practices observed in the context of blended learning classroom using lesson study and the Open Approach demonstrate the teachers' enduring conviction that their role is to deliver lectures, provide explanations of examples, laws, or formulas as outlined in the textbooks, with the expectation that these methods will facilitate student comprehension of subject matter and aid in the memorization of said laws or formulas.

Beliefs related to Blended Learning Classroom; Results from 5-likert scale and open-ended questionnaire.

When this group of teachers participated in the blended learning classroom, they were assigned to operate according to the ODC (On-demand Classroom) model. The ODC is a classroom that emphasizes students' self-learning, allowing them to study on their own schedule without fixed appointments with teachers. The key requirement for self-learning is that students generate their own ideas for discussion. The self-learning component involves teachers preparing problem situations, including teaching and learning materials, and possibly coordinating with parents to facilitate implementation. For example, this can take place at students' homes or other locations. Students learn independently, organizing their time according to their needs and pace, gathering evidence, and preparing for communication. The interaction-based learning aspect includes two modes: VLC (Visual/Virtual Live Classroom) and FFC (Face-to-Face Classroom). VLC classrooms emphasize "Learning Together" and collaborative learning, with scheduled appointments with the teacher. On the other hand, FFC classrooms focus on raising awareness through deep learning by reflecting on learning outcomes, promoting the development of effective learning strategies. These learning activities are significantly different from conventional teaching methods, such as lecturing, explaining, or demonstrating examples.

The teachers who took part in the project had various roles, such as helping to plan lessons, teaching the lessons for ideas collection/records, being an observer, and taking part in the reflection both online and onsite session. Also, as part of their weekly practice of doing each step of Lesson Study, they had many chances to talk about ideas elaborate clearly who they are comprising teacher educators, a school principal, prospective teachers, and Lesson study team expert from Khon Kaen University's Faculty of Education. Such a working environment forced the teachers to learn how to organize learning tasks and better understand how students learn. This, in turn, would help them get a broader view of how to teach math.

The following table shows the levels of opinions concerning teacher beliefs about teaching in the context of blended learning classroom as expressed by the teachers after they had participated in the project:

Teachers' beliefs about teaching in the context of blended learning classroom	Mean	S.D.
In On-demand Learning Classroom (ODC)		
1. Teachers explain and lecture the contents to the students.	1.29	0.46
2. Teachers should teach the content according to the curriculum.	1.38	0.49
3. Teachers are the one who's create videos for conveying content/knowledge to students.	1.24	0.43
4. Teachers design challenging task based on student interests.	4.71	0.42
5. Students have the opportunity to decide and choose their own way of solving problems.	4.47	0.5
6. Teachers should present the problem situation so that the students can enter the problem and create their own problem.	4.82	0.39

In Virtual Learning Classroom (VLC) or Face-to-face Learning Classroom (FFC)		
7. Students learn through teacher lectures.	1.16	0.37
8. Teachers evaluate through student answering questions.	1.11	0.32
9. Students apply previously learned to solve problem.	4.47	0.5
10. Students dare to share/present their ideas or opinions to class.	4.71	0.46
11. From observing the class, the teacher learns the way students think and the flaws in teaching through observation, learning and reflection.	4.71	0.5
12. Students appreciated others ideas.	4.82	0.39

Table 1: Teachers' beliefs about teaching in the context of blended learning classroom

It was found that the items that teachers had the most agreement about the beliefs about teaching in the context of blended learning classroom was:

- In On-demand Learning Classroom (ODC) teacher should present the problem situation so that the students can enter the problem and create their own problem. This was consistent with the results of the analysis of the teachers' interview about exploring teachers' beliefs about teaching in the context of a blended learning classroom, where teachers found out that their students enjoy the problem situation that the teacher has offered. Also, the students can enter the problem and create, solve the problem by themselves.
- In Virtual Learning Classroom (VLC) or Face-to-face Learning Classroom (FFC) Students appreciated others' ideas. This was consistent with the results of the analysis of the teachers' interview about exploring teachers' beliefs about teaching in the context of a blended learning classroom, where teachers found out that their students able to learn from each other ideas and they appreciate each other idea during the summarization and grouping the ideas that emerged in the classroom. Students able to express their ideas and help them to express their reasons and shared with their friends.

In addition to the above quantitative data, there are qualitative data from the semi structure interview of teachers' belief about teaching in the context of blended learning classroom prior to engaging in as the following:

"…As for on-demand classroom students themselves, they will learn by listening to stories telling or simply reading a few lines on the Internet. That's where it's a very important part. It wasn't because the teacher told them what to do, but it's what they will be done by their own way. Who should they ask? Which line should they write? Where should they write? Of course, the children they will have their experience. They would write down everything they heard. That's a very good part, how good it is, it's very good that we can build on that. Students will be able to discuss long texts that they come across

with their friends and will come to their own conclusion. This is where the class changes are enormous. Which in normal class the teacher will be the one who's told them to do or write or ask if they see it or not? but when it's come to on-demand classroom, they'll have more time to discuss there. Which has changed a lot in the classroom." — from a male teacher, age between 30- 39 years old and had taught mathematics for 10 years. He's played the roles of an instructor and observer and had taken part in lesson planning and reflection sessions.

"…My students don't have enough equipment like other teachers' students, so I decided to use face to face classroom method. Everyday in the evening they will meet me and their friends at school for 10 minutes. I will show them the video or documents that I have prepared form them throughout a big tv in the classroom. After they done with it, they will bring their work back home and solve the problem by them self. The next day we will meet again for 30 minutes so that they can share their ideas with their friends. What I have noticed is all my students very concentrate when their friends are talking, and they check their work at the same time. As a teacher I am very happy to see this situation." — from a female teacher, age between 26-34 years old and had taught mathematics for 8 years. Se's played the roles of an instructor and observer and had taken part in lesson planning and reflection sessions.

"…for me Virtual live classroom or Face to face classroom are the same. It's depended on how or where teacher and students agree to meet each other. My students have no problem with meet me online. We are decided to meet in zoom, so that everyone has space to share ideas to their friends. Every time after they are done with their on-demand work they will bring their work to zoom to share their ideas and learn from their friends work as exchange idea. I saw how my students want to talk and want to share with their friends respectfully. It's better than I used to do before." — from a male teacher, age between 31- 39 years old and had taught mathematics for 10 years. He's played the roles of an instructor and observer and had taken part in lesson planning and reflection sessions.

From the questionnaire distributed to the target group, it was found that the item that teachers had the most agreement about the beliefs about teaching in Blended learning classroom was in BLC Model helps students develop independent learning skills and become lifelong learners. This was consistent with the results of the analysis of the teachers' interview about exploring teachers' beliefs about teaching in the context of a blended learning classroom, where teachers found out that blended learning helps their students to improve their skills in various areas such as cooperative skills, self-learning skills, critical thinking, problem solving skills and leadership skills. Since blended listening classes are new and challenging for some students, the students said that they need more time to practice at the beginning.

RESEARCH FINDING:

Prior to engage in BLC model, teachers' beliefs that ODC, the first phase of BLC whereas the students expected to learn by themselves seems to be difficult. However, after engage going in BLC model, teachers have their perceived beliefs that well designed problem situation is the most crucial condition for students' self-learning, not the interaction between teachers and students as they used to beliefs.

In VLC or FFC, teachers have perceived beliefs that their students able to learn through solving problem by themselves, sharing ideas in VLC and reflecting their own and other ideas on FFC, students appreciated other students' ideas. This is consistent with OECD's (2018) that one among other things students should understand other's perspective.

CONCLUSION

The results of this study have substantiated the notion that there exists a relationship between teaching and teachers' views, albeit one that is independent and not of a cause-effect nature. Teaching practices and the beliefs held by teachers are inherently influenced by cultural environments. The integration of the enculturating blended learning model as a novel teaching style, in conjunction with lesson study, offers teachers an opportunity to critically examine their deeply ingrained views and begin to develop fresh perspectives towards classroom practices. There is a strong belief, consistent with Inprasitha's findings in 2003, that teachers' beliefs in students' ability to learn through hands-on experiences, whether it involves trial and error or comparing their ideas with peers. This interactive learning process allows students to acquire knowledge in a meaningful manner, potentially leading to the generation of new knowledge through data obtained from participatory observation. Supported by Philipp (2007) the most lasting impact arises from professional development programs that help teachers align their evolving beliefs with corresponding changes in their teaching practices.

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MATHEMATICS TEACHER EDUCATORS SUPPORTING COLLABORATIVE ACTION RESEARCH: TWO CASES FROM NEPAL

Ruchi S. Kumar

Tata Institute of Social Sciences, India; ruchi.kumar31@gmail.com

The paper presents journeys of two mathematics teacher educators (MTE) as fellows from Nepal participating in an international project to scale design principles of using blended courses to provide continuous professional development and developing professional learning communities to support engagement and reflection. The fellows conducted collaborative action research with 5 other teachers and mentored them in implementing the pedagogy or the resource identified as suitable for teaching the particular mathematics topic after literature review and deliberation with teachers. The different phases of the fellowship provided the opportunities for the fellows to reflect and build on their knowledge of mathematics teaching as well as about supporting mathematics teacher' professional development. The findings show implications for designing for scale by considering the variety of contexts in which the design principles have an impact while also considering the diversity of the audiences including the marginalised and underserved population.

INTRODUCTION

There is an urgent need to address the gaps in the professional development of teacher educators particularly in response to the requirements of the crisis like pandemic. The use of distance learning technologies, action research, and professional learning communities remain largely untapped opportunities for Teacher Professional Development (TPD), especially in South Asia. Additionally, the crisis in the education sector due to the Covid-19 pandemic, which led to school closure for extended periods of time has further exacerbated the inequities in the South Asian countries, highlighting the need to enhance the capacity of teachers and thus teacher educators' ICT preparedness. Considering that Mathematics Teacher Educators (MTE) and Mathematics Teachers (MT) have common goals of improving teaching and addressing students learning, how MTE and MT can collaborate to contribute towards this goal is a promising research endeavour. Several frameworks discussing the knowledge and skills of the MTE consider the knowledge and skills of the MT as the foundation for talking about the knowledge and skills of the MTE (Beswick & Goos, 2014; Zaslavsky& Leikin, 2004).

There is scant literature about the processes of the development of the MTEs. While Zazkis and Mamolo (2011) argue for the role of knowing advanced mathematics for working with teachers Masingila et al. (2018) discuss how novice MTE learn through the process of being mentored by experienced MTEs and reflection on their learning experiences. However, much remains to be researched about the kinds of opportunities and skills help in addressing teachers' learning. In this paper, I argue that the context

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also plays a big role in developing MTE knowledge and skills. In countries like Nepal which have a diversity of under-resourced contexts, it is important for MTEs to develop their knowledge of these contexts to address the issues that teachers face in implementing new approaches in the classroom. The paper builds on the researchers' experience of co-leading the project – Multimodal approach to Teacher Professional Development (MATPD)1. Important aspect of the project involves exploring scaling a model of Teacher educators' development through provision of Fellowships and engaging in collaborative action research and mentoring, Paper discusses engagements of MTE in professional development activities that provided opportunities for development of knowledge and skills required as MTE. Often MTE in South Asian context is referred to faculty working in university engaged in teaching math education courses, However, the paper offers a perspective that practicing mathematics teachers can serve as MTE and develop the capacities and identity of an MTE.

THE TPD LANDSCAPE IN NEPAL

Nepal has majority of people following Hinduism (81.3%) along with other religious and ethnic groups and declares itself as a secular state where people are free to practice any religion. Nepal has faced considerable political instability and devastating earthquakes in the recent past and recently transitioned to Federalism in 2015. There is hope that this transition will lead to inclusion of voices of the marginalised. However, there are concerns that these inclusions are only tokenism without sharing of any power with the marginalised.

The Landscape Mapping Study (2022) in MATPD project was done with the purpose of understanding policy space, the gaps between policy and practice and identifying the gaps in supporting teacher educators and teacher professional development. Key stakeholders at various positions in governmental and NGOs were interviewed. The study highlighted that in Nepal, Maldives and Afghanistan, there is lack of opportunities for teacher educators to work with teachers and develop a contextualised knowledge of teachers work and challenges There are few avenues for supporting the development of communities of educators and spaces to share and build on the knowledge of teaching learning process and of teacher professional development. Also, there is need for supporting the development of research culture among teacher educators and teachers through opportunities to engage in field-based research, identifying challenges and finding contextualised solutions. Further, need for online/blended curriculum for teacher educators and teachers to improve their practice and incorporate the skills of reflection and collaboration in their practice. It was observed that the use of top-down/ centralised approaches for teacher professional development indicate need for the system to explore collaborative modes of professional development including mentoring and coaching. Both teachers and teacher educators needed to develop the knowledge about how constructive and student-centred pedagogies can be contextualised in each country's contexts to support meaningful learning of students in school.

Some of the country specific challenges identified through the study indicate the need to focus on the use of ICT in remote areas of Nepal and developing inclusive pedagogy in classrooms while there is a supporting policy.

THE INTERVENTION – MULTIMODAL APPROACH TO TEACHER PROFESSIONAL DEVELOPMENT (MATPD) PROJECT

MATPD is a South-South collaborative endeavour funded by International Development Research Centre under the Global Partnership for Education Knowledge and Innovation Exchange (GPE-KIX) to address the poor quality of teacher professional development through distance teaching and learning. The Project is implemented in three countries – Nepal, Maldives and Afghanistan. The Consortium involved in the collaboration includes Villa College, Maldives, Swedish committee of Afghanistan, Kabul as collaborating partner and Tata Institute of Social Sciences (TISS) India as the knowledge partner.

An important focus of the programme is to develop an understanding of and engage the fellows in these concepts to enable them to design action research by working with teachers using distance teaching-learning modes through South Asian Teacher Educators (SATE) Fellowship. Fellows were expected to design, implement and report the action research under the guidance of academic mentors and field mentors assigned to them. The programme was designed to equip Fellows to include reflection, adaptable distance teaching and learning tools, and mentoring in their practice for working with teachers. We hoped that Fellows may develop self-agency and problem-solving skills to find solutions to educational problems and understand how action research can be used to address the educational issues at various levels.

The SATE fellowship had three phases as represented in Figure 1. The first phase involved all the fellows in undergoing continuous professional development courses in blended mode. The courses were on the themes of Meaningful use of ICT, Mentoring and Action research. Some of the sessions were conducted in the face-to-face mode for the fellows while some were done through webinar. The course platform also provided avenues for discussing insights, experiences in the field and innovative solutions through discussion forums and prompts. The second phase overlaps the first phase as the action research course paved the way for developing an action research proposal while being mentored from academic mentor from India and a field mentor from their respective country. This mentoring continued in the third phase of implementation of the collaborative action research with teachers which involved designing TPD on the chosen topic and supporting teachers in creating lesson plans and implementing the same in their classroom through the mentoring support of the fellow. The Fellow thus experienced both the role of the mentee and being a mentor 15 fellows were recruited through open call, interview and statement of purpose in each country- Nepal, Maldives and Afghanistan. 3 Fellows in Nepal, 4 fellows in Afghanistan and 1 fellow in Maldives opted to do action research on Mathematics Education. In this paper, only the Nepal context is discussed as the fellows from Maldives and Afghanistan have not yet finished their action research at the time of writing this paper.

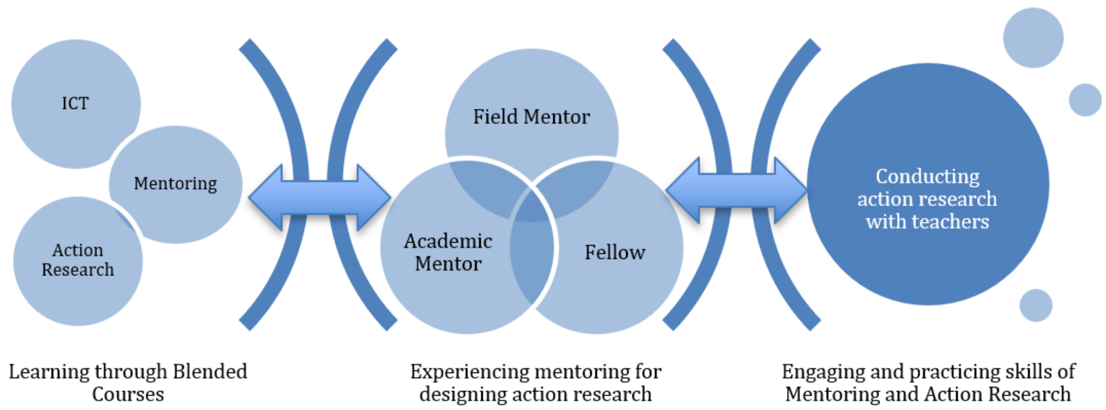

Figure 1: The three phases of SATE fellowship and the activities engaged in them

THEORY OF CHANGE

The design of the project includes scaling the design principles and the insights from earlier large-scale action research project implemented in India in under-resourced and under-served contexts. One of the key principles involved the use of practice-based tasks in the online courses designed to develop the knowledge and skills for teacher educators in under-resourced contexts focusing on action research, mentoring and constructive use of Information and Communications Technology (ICT). The courses were initially designed for implementation in action research projects within India and were contextualised to adapt to the country contexts of the fellows by having both synchronous interactions through webinars/ online meetings and non-synchronous interactions through the discussion forum within the course. The practice-based tasks in each of the course involved the learners in implementing an idea/part of the course with the students/teachers from their own country contexts to bridge the gap between theory and practice and also involved peer evaluation and discussion tasks to provide opportunities to learn from each other. These practice-based tasks thus provided opportunities for reflection on their practice as teachers and as teacher educators to construct contextualised knowledge and contribute to the knowledge of the community.

Another key design principle was to support the development of Professional Learning Communities (PLC) through forming mobile based chat groups (on Telegram app) at different levels (cross country, within country, theme-wise with teachers). These groups not only consisted of fellows but also the consortium members, course facilitators, academic and field mentors. The role of the academic mentors from India from TISS and its collaboration network was to provide guidance as an expert from the research field (having experience of conducting research and publishing papers) and providing technical inputs about the action research with the respect to the themes selected by the fellows (e.g., Maths education, Science Education). Field mentors were located within the implementing country within a university or a Non-Governmental organisation engaged in education. Having had experience of working in the country and knowledge of challenges as well as the field, the Field mentors were expected to provide contextualised inputs to support fellows designing, implementing and problem solving in doing action research collaboratively with a group of at least 5 teachers across the schools. Since the fellows were expected to work with at least 5 teachers, most fellows had to select the teachers across at least two schools. This was another design criteria to explore how the professional learning communities can be established across the schools for same subject teachers and to understand what kind of modes are used by fellows to work with teachers and what are the success and challenges.

METHODOLOGY

The main research questions that this research attempts to answer are

1. What are the barriers and levers for enabling the ownership and agency of the engaged teacher educators and teachers in engaging in collaborative action research and finding solutions for under-resourced contexts?

2. Whether practice based learning and professional learning communities as a design principle contributes towards the capacity development of the teacher educators for developing ownership and agency to design meaningful TPD for teachers? If so, how?

The study adopts a largely qualitative approach based on qualitative data in form of interviews and artifacts. The current paper discusses research which is part of the larger research project for analysing the impact of the innovative TPD interventions on the participation and engagement of the fellows in the programme. The fellows that have participated in the research project were granted the fellowship post an open call and going through interview and analysis of their basic proposal. In this paper I report the experiences of Fellow A and Fellow B who engaged in the action research on the theme of mathematics education. The data was collected by taking a semi-structured baseline and endline interview of both the fellows. Their participation in the online course, face to face and online webinars as well as reflections on meetings with the academic and field mentors were used for triangulation of the findings from the analysis of the baseline and endline interviews. The action research reports of the both the fellows were also used for triangulation. The analysis of the baseline and endline interviews were done to identify the main learnings and insights gained by the fellows as a result of engagement in the collaborative action research as well as to identify the challenges faced by the fellows in the engagement. Thematic analysis followed the approach suggested by Clarke, Braun and Hayfield (2015) to identify patterns within and across the data to identify and understand participants' lived experiences by analysing their perspectives, behaviour and practices and identifying the personal and social meanings that participants have derived from the experience. The findings from the analysis of the baseline-endline interviews of the both the fellows is presented in the next section.

ENGAGEMENT IN ACTION RESEARCH IN NEPAL

In Nepal, three fellows had opted for the theme of Mathematics education. All provided informed consent to participate in the research. The research topic selected by Fellow A was to focus on the teaching of fractions at primary level using manipulatives to explain concept of unit and fraction addition. She is a teacher in her thirties with experience of more than 5 years as a primary teacher. Fellow B focused on teaching of the concept of Calculus using GeoGebra at senior secondary level specifically area under the curve using Riemann sum. He is an independent consultant in his thirties helping secondary teachers in integrating ICT through workshops. Both had only known about action research through others and it was the first experience for them to engage in action research.

There were several experiences that both the fellows faced and it paved for their learning about the type of challenges that teachers face and their role as teacher educators in supporting teachers through those challenges. Both the fellows reported that teachers taught mathematics through traditional teacher-centric method where they start by showing how to solve the problems from the textbook rather than focusing on developing students' conceptual understanding. However, both were able to engage teachers in changing their practice to adopt more active learning pedagogies by trying out activities in the first cycle of the action research and giving students more opportunities to express their ideas in the second cycle of the action research. Both Fellows worked with 5 teachers from different schools which was challenging as teachers were pressed for time due to busy school calendar and other responsibilities. They had to devise several ways to engage with teachers as groups and as individuals through online and offline meetings. They shared that they learnt through the process of collaboration with teachers, especially through sharing of teaching strategies and active engagement with students.

There were differences in the Fellows' journey due to the grades and mathematics topics on which they focused the action research. To design and implement action research with teachers, fellows themselves had to explore the concept and the resources for teaching the concept deeply making them move beyond the textbook and focusing on students' difficulties. Fellow A was able to identify the key concept of the unitising of the fraction (Lamon, 1996) and was able to use it in concrete materials for cutting and sharing and focusing on the size of the whole. Later she was able to use this to explain to teachers why addition of fractions work by taking Lowest Common Multiple (LCM) of the denominators. The plan was made collaboratively with the teachers, who were keen to connect fraction with daily life. Though the school textbook did not have conceptual discussion about the role of unit in determining the fractions, the fellow was able to engage the teachers in conceptual discussion and teachers were able to implement the teaching of fraction using materials in daily life like vegetables and clay. While earlier the teachers completed the lesson in a hurry, while conducting action research teachers were able to understand students' difficulties for learning fractions and found the manipulative like fraction chart and fraction pizza as useful. Fellow A felt that she was able to enhance her own knowledge about pedagogical content knowledge of fractions by reading research and designing activities and worksheets with other teachers.

Fellow B focused on developing the skills of using GeoGebra for teaching calculus. He shared that although some schools in Nepal have computer labs and audio-visual rooms with projectors it is very difficult to schedule classes as they are mostly used to teach computer applications and rarely for integrating ICT with any subject matter. The teachers loved the animation of how area under the curve can be divided into a number of rectangles and how these numbers can be increased or decreased. While using the example of $f(x)= x^2$ teachers earlier used to teach to find the area under the curve by using the formula. However, when they showed the animation in first cycle and allowed the students to explore GeoGebra in second cycle the lesson became very interactive with students' questions as well as responses. Students were able to generalize that the more the number of partitions, the better the estimation of the area under the curve is. They also discussed why the Riemann Upper sum will be more than the Lower sum considering the area of rectangles. Teachers felt that GeoGebra helped in visualization and the same topic was very difficult to teach on the black/whiteboard while drawing the partitions, while the animations made it so much easier for students to study the pattern, Although, they had the knowledge of this approach for teaching calculus before the intervention, the difficulty of drawing the figure and static nature of the figure dissuaded the teachers from using the approach. The dynamicity and interactivity of the GeoGebra software made the teachers so comfortable that they asked for help to explore other topics like probability using GeoGebra. Teachers felt that even if the access to computer lab or projector is not available, engagement with this resource for teaching calculus helped them in imagining an alternative pedagogy for teaching mathematics.

WHAT DOES IT MEAN TO SCALE?

The learning trajectory of two fellows, though brief, reflects the diversity in the settings, the types of resources available including material/logistical, human and affective resources (Brodie, 2020). This diversity in under-resourced contexts can either causes despair or encourages adoption, adaptation and innovation in design and use of resources. This has implications for understanding scale of interventions which cannot be just considered on the criteria of the large numbers which are scaled but we need to consider the diversity of the contexts where the intervention was scaled and for a diversity of audiences including the marginalised ones. Here the scaling is not about the fidelity of the form of the intervention but about the design principles which were contextualised according to the situations in the country contexts and adaptations made. For example, the fellows in Nepal felt that they needed more inputs for designing the tools of research and maintaining the collaboration with the teachers and therefore the enrichment sessions were designed to address them. The process of identifying what kind of inputs are required to support and sustain the fellows was an ongoing process that continued throughout the project. Although, no subject specific course was included to address the fellows and teachers understanding of content and pedagogical content knowledge, the system of academic mentor as a technical expert supported both fellows with readings and discussed pertinent ideas, pedagogy and resources for addressing understanding of teachers and students for the selected topic. The field mentor help in contextualisation of the proposal, negotiating sticky situations of convincing management and teachers and use of local language of discussion provided a bridge between what was discussed in the courses and meetings with Academic mentors. We hope that fellows too would have learnt about mentoring through the process of interaction with academic and field mentors, as that is yet another aspect that need to be researched.

Additional information

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CHARACTERISING A TEACHER'S SPECIALISED CONTENT KNOWLEDGE IN TEACHING DIVISION THROUGH SORT-SEQUENCE-ACT METHOD

Su Ngin, Chia

Nanyang Technological University

In this paper, I introduce a novel sort-sequence-act (SSA) method to characterise a teacher, Emma's specialised content knowledge (SCK) for the teaching of division. The method consists of three tasks. For the first and second tasks, Emma sorted and sequenced a set of division expressions through a semi-structured interview. In the third task, Emma demonstrated her teaching of a division question. Through the three tasks, I uncover several characteristics of her SCK. In addition, I also revealed insights of SSA method.

INTRODUCTION

The Sort-Sequence-Act (SSA) method is adapted from the traditional card sorting task. This novel method consists of three tasks and is designed in the context of the teaching of division at the primary level. As teachers face difficulties in the teaching of division topics, it would be important to understand this issue better. In the first sorting task, participants will be presented with a set of printed cards. The prints on the cards include different representations of division expressions. Participants would sort the representations into groups according to similarities that are meaningful to them. For sequencing task, participants would sequence the division expressions according to their preference. In the last task, participants will be asked to demonstrate the teaching of a division expression.

I argued that SSA method is able to capture a more complete and dynamic view of a teacher's knowledge in the teaching of division (Goldin, 2000) as opposed to more traditional method such as pen and paper test and closed-ended interviews, which can only reflect a more static view of a teacher's knowledge (Lamb & Booker, 2004). Through SSA method, teachers' knowledge, more specifically, subject matter content and pedagogical knowledge can be analysed (Shulman, 1986, 1987). The uncovering of the different types of knowledge in the teaching of division may allow us to understand teachers' difficulties in teaching division more meaningfully. In this paper, I will explain in detail what SSA method is and the strengths of this method. I will also provide examples of the characteristics of specialised content knowledge (Ball et al., 2008) in the teaching of division demonstrated through the SSA method.

LITERATURE REVIEW

Card Sorting method is a powerful method to capture participants' deep conceptual thinking and thought processes. As participants sort and sequence the cards, they use mathematical processes such as making connections, discerning mathematical structures and properties to make sense of the expressions printed on the cards (Goldin, 2000).

2023. In Inprasitha, M., Sudjamnong, A., Suriyon, A., Saengpun, J., Moonpo, P., Chaovasetthakul, R., Plianram, S., & Suttiamporn, W. (Eds.). *Proceedings of the Regional Conference of the International Group for the Psychology of Mathematics Education 2023*, pp.105-112. Khon Kaen, Thailand: PME.

In mathematics education, researchers have used card sorting and sequencing methods to examine teachers' knowledge. For instance, Eli et al. (2011) used sorting tasks to develop teachers' geometrical teaching and Lloyd and Wilson (1998) used them to examine a teacher's conception of function. For sequencing tasks, Galant (2013) had highlighted teachers' weaknesses in teaching multiplication and Livy et al. (2017) instructed pre-service teachers to sequence students' work samples to reveal their beliefs on teaching approaches.

Division is an important but difficult topic to teach. Students need to understand division well as it is a foundational topic for other numeracy topic such as fractions and decimal. However, students often face difficulties as they do not have important pre-requisite knowledge such as place value concept (Holland, 1942; Martin, 2009). In addition, they may also not understand the key ideas of division such as relationship between multiplication and division and the partitive and quotative concept (Graeber, 1993). Ramsingh (2020) studied how teachers teach division and concluded that teachers lack knowledge in teaching division in the study. Hence, it may be useful to characterise teachers' knowledge in the teaching of division and better understand how their existing knowledge or lack of knowledge influence the teaching and learning of division in the classroom more deeply. More studies on division need to be conducted to mediate the difficulties in the teaching and learning of division. Furthermore, although, there are existing studies on sorting and sequencing tasks, there are limited studies that have used sorting and sequencing task in the teaching of division. Hence, it is worthwhile to design a card sort and sequencing method on division as the methods have potential to capture teachers' deep understanding of the teaching of division.

Shulman (1986) categorised teachers' knowledge into subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Subject matter knowledge refers to knowledge of the teacher of the subject discipline. However, to teach mathematics well, teachers also need to have pedagogical content knowledge. Teachers also need to transform their knowledge to their learners in a way that the learners can understand (Shulman, 1987). Although card sort and sequencing method is useful to elicit teachers' knowledge and thought processes, the teachers' responses would reveal more SMK than PCK. To capture a more complete view of the teachers' knowledge, I added Task 3 which is a demonstration of teachers' teaching of a division problem. Through the analysis of a teachers' demonstration of teaching, we will be able to understand teachers' PCK in teaching division as well.

Ball et al. (2008)'s six domains of Mathematical Knowledge for Teaching (MKT) was extended based on Shulman's notion of subject matter content knowledge and pedagogical content knowledge. For subject matter knowledge, it is further categorised into common content knowledge (CCK), specialised content knowledge (SCK) and horizon content knowledge (HCK). For pedagogical content knowledge, it is further categorised into knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum (KCC). The sort-sequence-act method is designed to characterise the six domains of the Mathematical Knowledge for Teaching in teaching division. In this paper, I will only discuss the characterisation of a teacher, Emma's SCK in the division using the SSA method. SCK refers to mathematical knowledge and skills that is required and specific to mathematics teaching. In the 'method' section, I will explain how a SSA method can be designed to capture teachers' knowledge in division. I will elaborate the examples of the characteristics of a teacher's SCK in the teaching of division in the 'finding' section.

METHOD

This paper demonstrates how Emma's SCK can be characterised using the sort-sequence-act (SSA) method. Emma is a Singapore mathematics teacher with five years of experience teaching Mathematics. SSA method consists of three tasks. The first and second tasks are sorting and sequencing tasks, and the third task is demonstration of teaching. The interview was an hour long.

The table below shows the division expressions that was printed on the sorting cards.

No renaming With remainder	No renaming No remainder	With renaming With remainder	With renaming No remainder
7 ÷ 3	20 ÷ 2	17÷ 4	16÷ 4
*21 ÷ 2	8 tens ÷ 4	21 ÷ 4	20 ÷ 4
43 ÷ 2	9 hundreds ÷ 3	43÷ 3	42÷ 3
54 ÷ 5	6÷ 3	54÷ 4	54÷ 3
*205 ÷ 2	42÷ 2	205 ÷ 4	204 ÷ 4
483 ÷ 2	*204 ÷ 2	482 ÷ 3	483 ÷ 3
	482 ÷ 2		

Table 1: Division expressions printed on Sorting Cards

Some key considerations that I made when selecting the division expressions are the renaming and reminder concepts and students' difficulties such as zero in the place holder. In addition, the choice of numbers in the expressions are adjusted among the categories by applying the theory of discernment and variation (Kullberg, 2017). For example, in the four expressions '43 ÷ 2', '42 ÷ 2', '43 ÷ 3' and '42 ÷ 3', using '43 ÷ 2' and '42 ÷ 2', participants may be able to discern the concept of remainder or renaming.

Tasks 1 and 2 are conducted as part of a semi-structured interview. Some examples of key questions for Sorting and Sequencing Task are:

- How would you categorise the 20 division expressions?
- Why do you categorise them this way?
- What headings would you give to the different categories?
- How would you sequence the categories in your teaching?
- Why did you sequence it this way?
- How would you sequence it differently?

Task 3 is a demonstration of teaching of a division expression from Table 1. Materials such as based ten blocks, number discs, writing materials and whiteboards are prepared for the participants. Some examples of key questions for Task 3 are:

- Select any expression from the sorting cards and demonstrate how you would teach it.
- Why did you select these expressions and the corresponding concrete or pictorial representations?

The interviews and demonstration of teaching are recorded using iPads. Next, I typed out the transcripts manually and capture screenshots of key teaching actions in the video recording. Using the transcript, I identify evidence of the characteristics of Emma's different domains of Mathematical Knowledge for Teaching in teaching division. In the following section, I will elaborate on her SCK that I uncover using the SSA method.

FINDINGS

From the analysis of Emma's responses in the three tasks, I characterized Emma's subject matter knowledge in the three following ways. She used the relationship between division and multiplication in teaching division (C1). She also used place value concept (C2) and partitive and quotative division model in teaching division (C3) shown in Table 2 below.

	C1: Use relationship between division and multiplication	C2: Use place value concept	C3: Use partitive and quotative model
Task 1: Sorting	Sorted the expressions according to divisors	She sorted the equations based on placed values.	She used partitive/quotative model to explain 42÷2.
Task 2: Sequencing	Introduce expressions that is linked to multiplication facts first.		
Task 3: Demonstration of teaching		Use place values chart and read and write the values of the digits of the dividends.	She used partitive/quotative model to explain 21÷4.

Table 2: Summary of Emma's SCK across the 3 tasks

Figure 1 below shows the results of Emma's sorting in task 1. She first sorted according to the number of divisors, followed by the number of digits in the dividends.

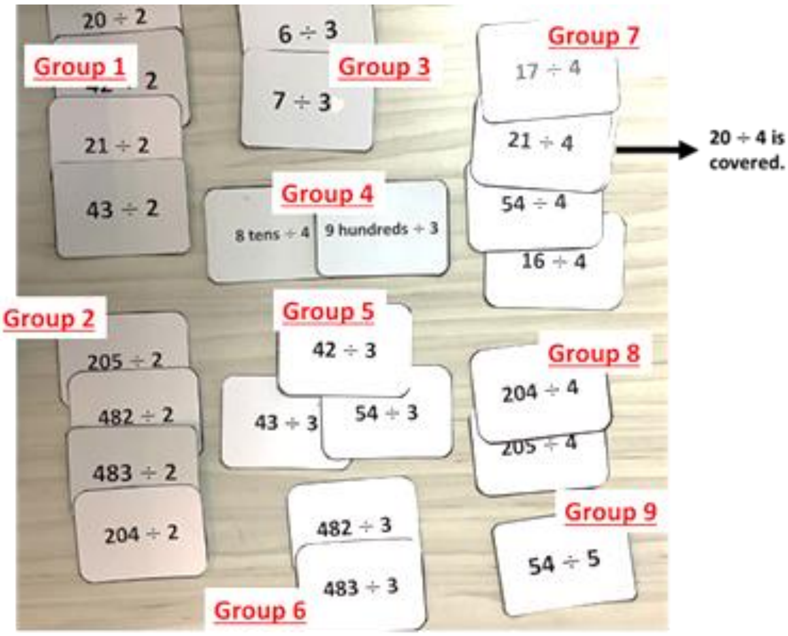

Figure 1: Emma's sorting of the division cards

Using the relationship between division and multiplication

During the justification of her sorting cards as shown in the transcript below, Emma explained that the relationship between division and multiplication was one of her considerations in her sorting. More specifically, she explained that she sorted according to the divisors (2,3,4 and 5) which is related to times table.

1 E: For example, the 1st consideration will be the divisors.

2 E: So I was thinking that you know division is quite related to multiplication. So they need to know their times table well.

3 E: 2 times 2 times 3 times 5

4 E: So they 1st group, I actually separated them according to the divisor.

5 E: So, at first, there will only 5 main groups. Divide by 2, 3, then 4, 5.

Next, during her sequencing task, she explained that she would introduce the expressions which have '0' remainder in group 3 and group 7 first because division is related to multiplication, and she would use multiplication facts in teaching division.

Using place value concept to teach division

In the 1st and 2nd tasks, Emma had demonstrated the use of place value concept to teach division. As part of task 1, she was asked to further sort the expression. She regrouped group 7 into group 7a and group 7b as shown in Figure 2. In group 7a, when the 3 expressions are represented using a division algorithm, the quotient would begin in the ones place and in group 7b, the quotient would begin in the tens place. The use of 'place value' in her justification reflects her knowledge of place value concepts.

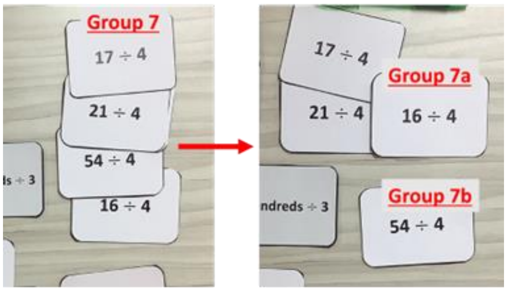

Figure 2: Result of Emma's further sorting

Her knowledge in places value concept is also demonstrated during her demonstration of teacher. During her explanation of '42 ÷ 2'. She drew the place value chart and wrote 'tens' and 'ones' as shown in Figure 2 below.

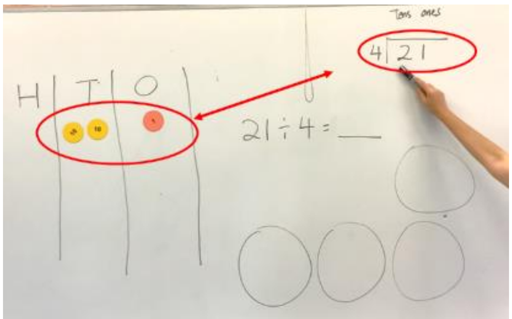

Figure 3: Evidence of using place value concept

Using partitive/ quotative model to teach division

During the sorting cards task, Emma was asked to explain how she would teach 42÷2. She explained that she will get the students to think about groupings and ask them how many 2s can get into 4 when dividing 4 tens by 2 as shown in the transcript below.

1 E: Normally, I will try to get them to think about the groupings.

2 E: So example 2 right [She pointed to the '2' divisor.]?

3 E: I will ask them like how many 2s can get into 4?

4 E: So after that right, at the tens place, they will say that 2 out of 2 will get into 4.

She continued to explain that she would use pictorial explanation in her teaching. She would draw 2 circles to represent 2 groups.

1 E: I will give them the tens, the ones, so I will show the pictorial.

2 E: I will draw 2 circles/ And then they will say, there are 20 in each group.

3 E: How many tens can you place in each group? We write 2 here but we will say that it's 20 in each group.

From her narration above, it seemed that she would introduce the expression using partitive model of division. On the other hand, her explanation of 'how many 2s can get in to 4' suggests that she is using the quotative model of division instead.

The contradiction in her explanation of division was also consistent in her demonstration of teaching of 21÷ 4 when she represented 4 groups by drawing the 4 circles as shown in Figure 4 below. She asked, 'How many can be placed into the 20?'

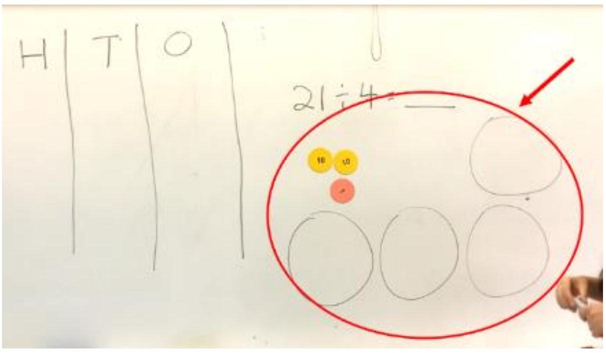

Figure 4: Evidence of using partitive model of division

From the evidence across Task 1 and 3, Emma did not show the difference between the two models clearly.

DISCUSSION

This paper has introduced a novel SSA method to characterise teachers' knowledge in teaching division topic. As the SSA method is still a work in progress, the selection of expressions and key interview questions can be further refined to capture a wider view of teachers' SCK. In addition, although this paper only shows examples of the characteristic of a teacher's SCK, future papers can focus on the use of SSA method to uncover the other domains of MKT and other Mathematics topics.

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STUDENTS' MATHEMATICAL ATTITUDES IN CLASSROOM USING LESSON STUDY AND OPEN APPROACH

Supanida Silaba[1], Narumon Changsri[2,3], Maitree Inprasitha[2,3]

[1]Mathematics Education Program, Faculty of Education, Khon Kaen University, Thailand

[2]Institute for Research and Development in Teaching Profession for ASEAN, Khon Kaen University, Thailand

[3]Center for Research in Mathematics Education, Khon Kaen University, Thailand

One of the challenges of the 21st century is developing positive attitudes (OECD, 2019). Mathematics is a suitable subject for cultivating these attitudes in students. In Thailand, Inprasitha (2004) introduced an innovation called Lesson Study (LS) incorporated Open Approach (OA) to improve mathematics teaching practices. Multiple studies found that this innovation had significantly contributed to more positive attitudes toward mathematics. The research school implemented this innovation for a year; the LS team was formed to design lesson plans and anticipate possible mathematical attitudes that will emerge in the classroom. It has been proven that using OA encourages students to attempt to solve problems by fostering their mathematical attitudes. After the lesson, the team members reflected on their teaching practice and how to handle unexpected ideas that occurred, or students' ideas related to mathematical attitudes.

INTRODUCTION

The competencies needed for a successful life and a well-functioning society are defined as 21st century skills (OECD, 2005). In the 21st century, the curriculum encompasses learning, literacy, and life skills. Students should be able to effectively use language and technology to communicate their ideas and thoughts, make rational decisions autonomously, and interact positively with their community (Mangao, Ahmad, & Isoda, 2017). To achieve the challenges of the 21st century, students need to be empowered and feel inspired to help shape a world for themselves, others, and the planet. The OECD Learning Compass 2030 was proposed to identify transformative competencies as the knowledge, skills, attitudes, and values students need to transform society and shape the future for better lives (OECD, 2019). The nature of mathematics has been seen from different points of view; research shows that many conceptions influence how teachers and mathematicians approach the teaching and development of mathematics)Brown, 1985; Bush, 1982; Cooney, 1985; Good, Grouws, & Ebmeier, 1983; Kesler, 1985; McGalliard, 1983; Owens, 1987; Thompson, 1984cited in Dossey, 1992). The teacher's conception of mathematics is strongly influenced by how mathematics is taught in the classroom (Cooney, 1985). Mathematics is now widely accepted as a human activity in the modern era. (Davis & Hersh 1980; Hersh, 1986 cited in Dossey, 1992).

2023. In Inprasitha, M., Sudjamnong, A., Suriyon, A., Saengpun, J., Moonpo, P., Chaovasetthakul, R., Plianram, S., & Suttiamporn, W. (Eds.). *Proceedings of the Regional Conference of the International Group for the Psychology of Mathematics Education 2023*, pp.113-120. Khon Kaen, Thailand: PME.

Normally, Thai mathematics teachers used the conventional method by describing new knowledge, *explaining* how to solve problems, and then assigning homework with a "closed" answer. Therefore, students were rarely given the chance to pose their mathematical problems and became passive learners who accepted and absorbed everything that the teacher said, unable to initiate their own learning (Inprasitha, 2004; 2015; 2022). The teaching innovation is considered an improvement if it effectively supports students' progress towards learning goals as compared to typical forms of instruction in a region or country (Maass, Cobb, Krainer, & Potari, 2019).

In Thai context, Inprasitha (2004; 2009; 2011; 2022) introduced Lesson Study (LS) incorporated Open Approach (OA) as an innovation to improve teacher teaching practice. The LS process consists of three steps: (i) Collaboratively design a research lesson (Plan); (ii) Collaboratively observe the research lesson (Do); (iii) Collaboratively discuss and reflect on the research lesson (See) (Inprasitha, 2004). The four stages of OA were incorporated in the Do step: (i) Posing open-ended problem; (ii) Students' self-learning; (iii) Whole class discussion, and (iv) Summarize through connecting students' mathematical ideas emerged in the classroom (Inprasitha, 2022).

Attitudes should be promoted in the mathematics classroom. According to Katagiri (2004), mathematics is an appropriate subject for fostering a desire and attitude aimed at learning things independently. Hannula (2002) has studied a case study student who had a negative attitude toward mathematics dramatically changed after engaging in problem-solving situations many times. Moreover, many studies have shown that the classrooms using LS incorporated OA, which generated mathematical activity utilized open-ended problem situations, have contribute significantly positive attitudes toward mathematics (Rakakaew, 2008). Because of this, researchers need to investigate how classrooms using LS incorporated OA can enhance students' mathematical attitudes.

MATHEMATICAL ATTITUDES

In this study, mathematical attitudes—a condition of "attempting to do" or "working to do" something—are the essential element of mathematical thinking. Additionally, according to Isoda & Katagiri (2012), mathematical attitudes serve as the driving force for mathematical methods and ideas. The four mathematical attitudes that we utilized were as follows:

Objectifying is attempting to grasp one's own problems or objectives and substance clearly on one's own. For example, attempting to pose questions, to be aware of problems, to realize mathematical problems from situations.

Reasonableness is attempting to be logically reasonable, for instance, attempting to be consistent with objectives, forming a perspective, and thinking using the information at hand, prior knowledge, and assumptions.

Clarity is attempting to represent matters clearly and simply. For example, attempting to record and explain problems and results clearly and simply, to classify and arrange objects when representing them.

Sophistication is to seek better ways and ideas. Students attempt to shift thinking away from objects and to operations to assess one other's thinking both objectively and subjectively for refining. They also economize thought and effort.

METHODOLOGY

Context of Study

The research school in northern Thailand has been using LS and OA to teach mathematics to thirteen third graders since 2022. Three phases of LS were implemented as follows: 1) The six members of LS team, three in-service teachers and three graduate students in the Mathematics Education Program, Faculty of Education, Khon Kaen University who have more than five years of experience in using LS and OA, collaboratively planned research lesson plans for three times a week. 2) Members of the LS team, except for one teacher who conducted the lesson plan, collaboratively observed the lessons for 2-4 hours per week. 3) All members of the LS team collaboratively reflected on the activities. This cycle of LS is repeated weekly.

Research Procedure and Instruments

A qualitative research design was employed to observe the teaching practices and to see how these innovations could affect the mathematical attitudes of the students. The research procedure consisted of three phases along the side of the three steps of the LS process.

During the Plan step, the LS team members meticulously designed seven research lesson plans on division. They incorporated different strategies based on curricular expectations and their understanding of how students learn. They referred to the Thai version of a Japanese mathematical textbook for guidance. Additionally, they anticipated mathematical attitudes that students may emerge during research plans. During the Do phase, the LS team, excluding the teaching teacher, observes the implementation of the research lesson plans they designed in the first phase. They take field notes to record the students' mathematical attitudes during the hour-long lessons. Post lessons, the LS team gathered to reflect on the teaching they had observed. This was the final phase of the procedure.

A total of seven research lesson plans were recorded in both audio and video formats. The researchers had to go through each lesson repeatedly to ensure that no phenomena were missed during the second phase. The research tools used included seven research lesson plans, field notes, and student worksheets. Additionally, audio and video recordings were used to capture any overlooked data from the three research tools. To sum up, the research procedure began with collaborative lesson planning, followed by collaborative classroom teaching and observation, and ended with a collaborative reflection on the research lesson.

Data Collection and Data Analysis

The data collected from classroom observations, student worksheets, and field notes were qualitatively analyzed. The audio and video recordings of class observations were converted into the protocol, field notes were taken during the lesson observations, and student worksheets were systematically triangulated based on the cycle of the LS to identify the characteristics of students' mathematical attitudes. This study used the data to analyze four types of mathematical attitudes by Isoda & Katagiri (2012).

RESULTS AND DISSCUSSION

Results from Step 1 of LS: Collaboratively Plan

The LS team members initiated the research by designing seven lesson plans that would account for the mathematical attitudes of the students. During the process, the team had to consider the overall development of the topic content, which was division, as part of the third-grade mathematical syllabus. They also referred to the content domain of the Thai version of the Japanese textbook and anticipated possible solutions through the task, focusing on students' behaviors and the flow of the lessons. The team aimed to promote the comprehensive development of students' mathematical attitudes using OA method. Details of the task used in the third lesson plan showed below.

Task: Distribute 15 cubic blocks equally among 3 children. How many blocks will each child get?

Condition: Demonstrate "how to" find the answer without using blocks.

Besides, the LS team needed to understand as many students' ideas as possible to sophisticate them through students' discussions with their peers and teachers' assistance.

Mathematical Attitude	Students' behaviors
Objectifying	The students raise the question such as "how to solve the problem without using cubic blocks". The students know what they were given from the task and anticipate ways to solve the problem.
Reasonableness	The procedures they had already learned from earlier lessons, such as division, multiplication, and one-by-one distribution, are used by the students to predict potential answers. The students take five from fifteen three times.
Clarity	Students' express equations of division or multiplication. The terminology can be used by students to solve problems.
Sophistication	Division issues can be resolved with multiplication. Students clarify how the equations for division and multiplication relate to one another.

Table 1: Planned mathematical attitudes.

The LS members designed the lesson plan to develop mathematical attitudes. They projected attitudes based on a variety of potential solutions.

Results from Step 2 of LS: Collaboratively Do

One of the members of the LS team designed a lesson plan, which was then conducted the lesson plan while the rest of the team observed and provided support. The lesson focused on identifying empirical evidence of the students' mathematical attitudes while solving division-related problems using the four stages of OA. The 13 students were identified as S1 to S13.

(i) Posing an open-ended problem

Before beginning teaching activities, the teacher asked the pupils what they had learned from the previous lesson, which dealt with dividing the cubes among the kids equally. The teacher then posed an open-ended problem by placing fifteen cubes. The pupils then informed the instructor that they needed to be given a specific number of children to distribute. When pictures of three children were placed, students suddenly expected each child to get five cubes. After the teacher stated that they had to discover the solution without using cubes, the students anticipated utilizing the division approach.

Mathematical Attitudes	Students' behaviours from field notes
Objectifying	Students thought about dividing fifteen cubes to three children without using cubes.
Reasonableness	The answer and previous experience of utilizing division were expected

Table 2: Results of field notes

(ii) Students' self-learning

Worksheets were distributed for students to discuss the open-ended problem situation and record their ideas. As a result, researchers found that students' self-learning can assist the students to express four types of mathematical attitudes through their participation in mathematical activities.

Mathematical Attitudes	Students' behaviours from field notes
Objectifying	Students assert to solve problem without using cubes
Reasonableness	The answer is expected before using the division method.
Clarity	Write division equation as $15 \div 3$. Write multiple equation as $5 \times 3 = 15$
Sophistication	Write timetable of 5 as $5 \times 1 = 5$, $5 \times 2 = 10$, $5 \times 3 = 15$ Take away number from 15 in groups of 2, 3, 4, 5 until there are none left

Table 3: Results of field notes

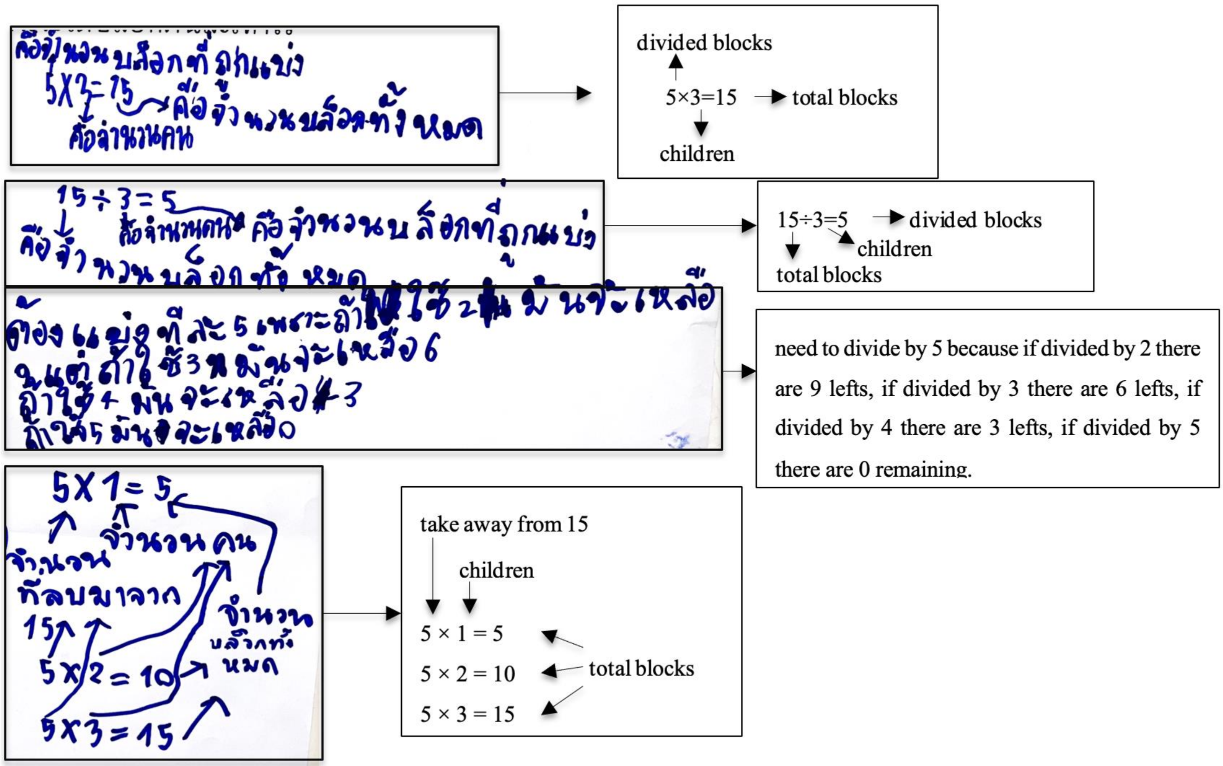


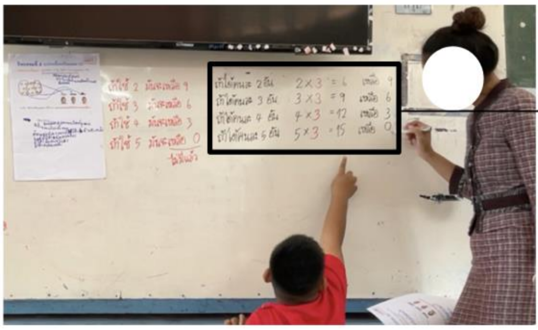
Figure 1: Results of student worksheets

(iii) Whole-class discussion and comparison

After the students finished their worksheets, the teacher selected different problem-solving methods and then posted them on the blackboard. Students were enthused to learn collaboratively and were required to explain verbally their solutions, and the teacher wrote down all the additional information on the student worksheets. The teacher turned their role as a facilitator to contrast and connect each students' idea to fostering students' discussion and comparison among their peers. At this stage, students expressed three types of mathematical attitudes: reasonableness, clarity, and sophistication.

(iv) Summarize through connecting students' mathematical ideas that emerged in the classroom

The teacher connected the students' ideas from (iii) and sophisticated them through negotiations with others or with the teacher's advice. Finally, the teacher summarized today's lesson by eliciting all the students' ideas based on the posted worksheets, emphasizing the mathematical concepts related to the relationship between multiplication and division.



If the number of blocks for each child is 2, 2 × 3 = 6, there is 9 lefts.
If the number of blocks for each child is 3, 3 × 3 = 9, there is 6 lefts.
If the number of blocks for each child is 4, 4 × 3 = 12, there is 3 lefts.
If the number of blocks for each child is 5, 5 × 3 = 15, there is 0 left.

Figure 2: Students' ideas

The result of using OA at the second step of the LS process revealed that students demonstrated four types of mathematical attitudes in attempting to solve problems by themselves. Besides, the results also indicated that students need well-planned research lesson plans to simultaneously cultivate their mathematical attitudes as a driving force of mathematical thinking in problem-solving.

Results from Step 3 of LS: Collaboratively See

After the research lesson, a reflection session was conducted to reflect on the research lessons and the collected data from student worksheets and field notes on students' mathematical attitudes. The LS team identified some not anticipated ideas in Step 3 of the LS. For instance (Fig.2), students found that the sum of remainder and blocks would equal 15. Meaning that if each child had 2 blocks, only 6 blocks would be used, and 9 blocks would remain; thus, the sum of 6 and 9 would be 15. The LS team considered this idea as a mathematical attitude, which is sophisticated. The researcher found that the final step of LS was crucial in identifying empirical evidence of students' mathematical attitudes while they participated in mathematical activities. Moreover, the results of Step 3 can be used to enhance teacher professionalism.

CONCLUSION

This study significantly contributed facts about the importance of LS incorporated OA, which could enhance students' mathematical attitudes through LS team in collaboratively designing research lesson plans and anticipating students' mathematical attitudes, collaboratively observing the implementation of research lessons, and collaboratively reflecting on the research lessons. The results confirm that the Thailand LS incorporated OA model can increase teachers' ability to create a teaching situation based on students' knowledge and improve their teaching.

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ORAL COMMUNICATIONS

UPPER ELEMENTARY SCHOOL STUDENTS' DESIRABLE ATTRIBUTES OF COLLABORATIVELY WORKING AS A TEAM

Anucha Koyata[1], Narumon Changsri[1,2], Dhanachat Anuniwat[3],

Kanita Pamuta[1], Maitree Inprasitha[1,2]

[1]Mathematics Education Program, Faculty of Education, Khon Kaen University, Thailand
[2]Center for Research in Mathematics Education, Khon Kaen University, Thailand
[3]University of Fukui, Japan

Acquiring the skill of effective collaboration and fostering an appreciation for individuals' diverse perspectives and backgrounds has become increasingly crucial (OECD, 2019). In the classroom using Lesson Study (LS) incorporated with Open Approach (OA) improves students' learning behavior and attitude toward learning (Inprasitha, 2022). So, it is imperative to instill in students the desirable attributes of collaboratively working as a team. This study investigates students' desirable attributes of collaboratively working as a team, classified by gender and grade level. The sample purposively selected includes 959 Grades 4-6 students from 22 schools that have started using LS incorporated with OA, which the LS comprises of collaboratively (i) plan, (ii) do, and (iii) see. In LS (ii) the OA as a teaching approach, (i) posing open-ended problems, (ii) students' self-learning, (iii) whole-class discussion, and (iv) comparison and summarizing by connecting students' mathematical ideas, were incorporated in this step (ibid). Mixed research methods were employed. Questionnaires with a 5-point Likert scale with an index of consistency between 0.60-0.80 and recorded videos were utilized as a research instrument. Data analysis used Percentage, Mean, Standard Deviation, and T-test for independent and one-way ANOVA. The results reveal that students have a high level of the desirable attributes of collaboratively working as a team $\bar{x}$=4.23 and s.d.=0.59. Classified by gender showed that females ($\bar{x}$=4.32, s.d.=0.56) have a higher level of this aspect than males ($\bar{x}$=4.11, s.d.=0.61) significantly distinct at 0.05 level. Although the gender is significantly different, both are at a high level. When examining the same attributes categorized by grade level, there was a statistically significant difference at 0.05 level. A pairwise analysis, employing Scheffé's method, revealed that grade 6 ($\bar{x}$=4.44, s.d.=0.45) students vary from grade 4 ($\bar{x}$=3.96, s.d.=0.61) students at a statistical significance of 0.05 level, and grade 5 ($\bar{x}$=4.39, s.d.=0.54) students are higher than grade 4. Even though the grade level is significantly distinct, it is still high. From the qualitative data showed that during OA (ii), students actively exchange and deliberate upon each other's ideas. They gauge the acceptance of these ideas, and typically, there is a harmonious convergence of viewpoints leading to the collective development of group ideas. Subsequently, they assign roles and jointly present their ideas during the OA (iii). This process enables students to actualize their individual thoughts and fosters a deep appreciation for the perspectives and contributions of their peers.

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TEACHER’S MATHEMATICS LESSON PLAN CONSTRUCTION, SALAVAN PROVINCE, LAOS

Bachieng INTHAVONGSA, Senhak BOUNMY, Bounneua INTHALATH, Inthava SENGCHAN, Souliphone SIVIXAY

Salavan Teacher Training College, Laos

Lesson plan construction is generally used as a guide and model for delivering classroom activities. Many researchers have explored how teachers plan their lessons. The content and activities involved in the lesson plan. This shows that teaching-learning planning plays an important role in teaching and creating an effective learning environment (Clark and Dunn, 1991; Reiser and Dick, 1996; Shauelson, 1983 cited in Koszalka et al., 1999). Preparing lesson plans required to set instructional goals, available resource consideration, and suitable activity design (Yo-An Lee and Takahashi, 2011).

In this study was to how do Primary school teacher’s comprehension on lesson plan construction? The research applied a qualitative method. The sampling comprised 4 teachers. 2 from demonstration primary school, Salavan teacher training college, and 2 from Nakokpho primary school, Salavan District, Salavan Province, academic year 2021-2022. Researchers analyzed data derived from video recorder, and teaching plan recorder form during teaching observation. Data analysis was reliant on the conceptual framework of the teaching plan which included 4 steps: (1) mathematical content comprehension and teaching objective setting, (2) Mathematical content comprehension of students, (3) Possibility of mathematical problem-solving in learning-teaching activity and (4) Understanding mathematical content (Watanabe et al., 2008).

In lesson planning, teacher implemented clear objectives, consumed time designing activities, outlined lesson for individual class, and constructed problem situations. Teacher created learning-teaching materials based on lessons and activity implementation. This facilitates students in problem-solving. Teacher stimulated students to solve problems by their own strategies.

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EXPLORING ERRORS OR MISCONCEPTIONS TO DEVELOP STATISTICAL THINKING

Daya Weerasinghe

Federation University, Australia

Enabling programs provide pathways for those who aim to achieve basic qualifications to enrol in an undergraduate program in a higher education institute. This presentation discusses about errors or misconceptions students make in Statistics while preparing for university studies. Students can figure out their errors by themselves because errors are usually due to carelessness. They cannot do the same for misconceptions. Misconceptions are committed because students think they are correct.

The data used are gathered from answers provided by students who submitted their Statistics assignment in the unit of 'Mathematics and introductory Statistics' in an enabling program delivered by a multi-campus university in Australia. Some of the submitted assignments with errors or misconceptions were purposefully selected and analysed in this study. It was found that some students were excellent statisticians, but they found it difficult to express their ideas using relevant statistical terms. These students find it easier to provide numerical, tabular, or graphical responses than to respond to worded questions. Reviewing literature on errors, misconceptions and statistical thinking, answers to the following research questions are of interest.

What are the errors and misconceptions in learning Statistics? How can teachers support tertiary students to develop statistical thinking?

Ruiz (1984) proposed basic orientations toward language and its role in society as: language-as-problem, language-as-right, and language-as-resource. This framework provides a deeper understanding of the complexity of language use in the multilingual classrooms. Content analysis techniques were used in this study. Even though students have the required knowledge and know how to do a given task, understanding a given problem and finding out what exactly is to be done can be difficult for them due to incorrect interpretation of the problem. The study found three areas where teachers see students' errors and misconceptions in Statistics. These are difficulties in understanding a question; interpreting results; and problems due to multilingualism. Subsequently, teachers may develop strategies which can be used to encourage students to reflect on their understanding. Misconceptions and errors must not be seen as obstacles but must be regarded as an opportunity to reflect and learn.

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ELEMENTARY SCHOOL STUDENTS' MOTIVATIONS IN LEARNING MATHEMATICS

Elaine Yu Ling Cai, Gregory Arief D. Liem

National Institute of Education, Nanyang Technological University

Despite mathematics being a core subject in elementary school, there has been little study on elementary school students' motivation in the subject, specifically behavioural regulations as reasons underlying achievement goals. The present study examines the tole of task-, self- and other-based goals (Elliot, Murayama, & Pekrun, 2011) in mediating the links between autonomous and controlled motivation (Deci & Ryan, 2000) and student engagement towards mathematics learning among elementary school students in Singapore.

491 students from Grades 4 to 6 were administered questionnaires assessing their motivations, achievement goals and level of student engagement. Path analysis and Mplus syntax for multiple mediation were used to analyse the data.

The study found that achievement goals significantly linked motivation to student engagement. Students who endorsed autonomous motivation and found mathematics interesting and meaningful would think more elaborately. Self-based goals were the only achievement goal which significantly mediated motivation to elaboration. It also strengthened the benefits of autonomous motivation for elaboration and channelled the adaptive role of controlled motivation on elaboration.

This study has shown that even in an Asian society, elementary school students learn best when they possess autonomous motivation and task-based goals. Mathematics educators could therefore modify the six dimensions of effective motivation, collectively known as TARGET which comprises tasks, authority, recognition, grouping, evaluation and time (Ames, 1992) to better nurture autonomous learning and task-based goals in students.

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STUDENTS' MATHEMATICAL THINKING ABOUT GENERALIZATION: THE CASE OF DECIMAL NUMBERS

Kanokwan Rattanajamnong, Sudatip Hancherngchai

Mathematics Education, Faculty of Education, Phuket Rajabhat University

This study aimed to analyze students' mathematical thinking focusing on generalization. The Target group was 23 pupils from elementary schools in Phuket Province that have been using TLSOA since 2020. As a method, participant observation and teaching experiments were employed in accordance with the collaborative lesson plan, do, and see of the Lesson Study. Research tools were lesson plans, an IC recorder, a camera, video, and field notes. Data were collected in the first semester, of the 2023 academic year. Data were analyzed by protocol and descriptive analysis following Isoda & Katagiri's ideas (2012). The result revealed that when using TLSOA in the classroom, students' mathematical thinking consisted of three things: 1) students took how to learn from previous lessons to solve the multiplication on decimal with whole number problems, 2) students compared the methods and selected the simple and proper methods to solve the multiplication on decimal with whole number problems, 3) students extended their ideas from previous lessons to solve the different problem situations, and 4) students summarized the algorithms of the multiplication on decimal numbers with whole numbers in vertical form.

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FACTORS THAT INFLUENCE THAI MATHEMATICS TEACHERS' PERSPECTIVES ABOUT CLASSROOM NORMS

Nitchada Kamlue[1], Sakon Tangkawsakul[2], Thadapanitsadee Sukonwiriyakul[3], Laura R. Van Zoest[1]

[1]Western Michigan University, [2]Kasetsart University, [3]Chulalongkorn University Demonstration Secondary School

Our earlier work found that Thai mathematics teachers' perspectives about teaching ideal classes seemed to support more productive sociomathematical norms than their perspectives on teaching realistic classes (Kamlue et al., 2022). For example, teachers tended to use *purposeful questions* (NCTM, 2014) and listen to students' mathematical thinking in their ideal classrooms, but not in their realistic classrooms. Knowing the reasons for these differences would identify where to focus professional development to support the development of productive sociomathematical norms. We used task-based individual interviews to investigate the factors that influenced the differences between the teachers' perspectives when teaching ideal and realistic classes that were identified in the earlier study of six Thai mathematics teachers from three settings (government, demonstration, and private schools). These differences were: students' characteristics (active learners), teachers' roles (teachers' responses to students' contribution), and teaching approaches (discussion). Our analysis indicated that the teachers perceived that ideal students were more active than their actual students because of two factors. Four said it was because of the students' backgrounds, while the other two said the differences were because of the teachers' actions. Regarding teachers' responses to students' contributions, all six teachers said that the differences were because of the teachers' actions. The most common reason given was that teachers' goals for instruction dictated how they would teach the given task. Finally, three teachers claimed that discussion was limited in reality because of the lack of students' confidence to express their ideas in classes. The others focused on the need for better teacher preparation for teaching students in this era under several circumstances, such as time limits and dense content. Our results identify promising foci for professional development.

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AN EXPLORING OF STUDENTS' MATHEMATICAL LITERACY THROUGH MATHEMATICAL LITERACY TESTING

Pantawat Sundara, Manita Kingket, Narumon Changsri

Mathematics Education Program
Center for Research in Mathematics Education (CRME), Khon Kaen University.

Mathematical literacy is a student's ability to solve problems and think logically according to the principles of mathematics (OECD, 2023). Mathematical literacy is measured and assessed by the Program for International Student Assessment (PISA) mainly through multiple-choice or fill-in tests. This research has designed a tool by adding space for students to express their ideas and explain how to solve problems in order to reflect students' Mathematical literacy.

The aim of this study was to explore students' mathematical literacy through Mathematical Literacy Testing. The target group consisted of two classrooms comprising sixty-four students who were fifteen years old and in grade 9 at an extra-large secondary school, using Open Approach as a teaching approach. The research method was qualitative research. Researchers had been collecting data by using 1) Mathematical Literacy Testing of PISA, 5 situations, 9 sub-questions, 60 minutes for the test, 2) field note recording of students' behavior and 3) interview form after testing. Data were analyzed according to the framework of Mathematical Literacy of PISA (2022).

The results of the study found that students' mathematical literacy occurred at 8 levels as follows: Level 1c had 1.56 %, students drew illustrations in real world to solve problem, and they could follow the conditions set by the situation. Level 1b had 6.25 %, students could distinguish which information are necessary or not for solve problems. Students used integers to solve problems and had short explanation of the solution. Level 1a had 6.25 %, students used various methods and explained how to solve problems step by step. Level 2 had 20.31 %, students created equations in algebraic form, a three-dimensional geometric model, and found a relationship between two quantities to be used in solving problems. Level 3 had 17.19 %, students could use strategies for solving one problem as a tool for solving another problem. Level 4 had 18.75 %, students criticized incomplete conditions and instructions in problem situations. There were arguments and reasons for the different solutions. Level 5 had 20.31 %, students could create complex models to solve problems systematically, creatively. Level 6 had 9.38 %, students solved the problems using a various of creative methods.

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MATHEMATICAL BEAUTY OF PATTERN IN LEARNING SEQUENCES AND SERIES IN MATHEMATICS CLASSROOM USING OPEN APPROACH

Passra Putklang, Chanathip Nakharangsu, Narumon Changsri

Mathematics Education Program,
Center for Research in Mathematics Education (CRME), Khon Kaen University.

The experience of mathematical beauty has a positive influence on students' motivations and attitudes towards mathematics and its study (Brinkmann, 2009). If they recognize the mathematical beauty of the number pattern, it presents the opportunity for them to appreciate the beauty of mathematics which lies beyond calculations (Isoda & Katagiri, 2012).

The aim of this study was to study students' perception of mathematical beauty in patterns through demonstrating the concepts of sequence and series. The target group was 29 students in grade 12 that enrolled the advanced science-mathematics program. The tools used in the study were 1) lesson plans with open-ended problem situations, 2) students' worksheets, 3) post lesson reports and 4) class observation forms. This research was qualitative research. Four steps of Open Approach were employed as the teaching approach (Inprasitha, 2022). Data were analyzed using the framework of using patterns under the number sequence (SEAMEO RECSAM, 2017).

The results of the study found that students perceived mathematical beauty in number patterns in learning sequence and series as follows: 1) They know the beautifulness of patterns in cases of arranging the objects depending on number sequence by writing a sequence to show the relationship between problem situations and sequences. 2) Arrange objects according to number sequence to find simple patterns. They can see the number patterns while solving problems 3) Arrange expressions such as addition and multiple to find simple patterns and then they can write in general terms. 4) They express the representation of patterns using variables. 5) They enjoy the arrangement depending on the number sequence in problem situations by observing how they think and discuss together in their group. Obviously, seeing mathematical beauty in number patterns is the tool that forces the mathematical thinking process, and ability to be solving problems by themselves.

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DEVELOPING MATHEMATICS TEACHER EDUCATORS' PEDAGOGICAL CONTEN KNOWLEDGE THROUGH LESSON STUDY IN LAO PDR

Phailath Sythong, Maitree Inprasitha, Narumon Changsri

Khon Kaen University, Thailand

The purpose of this study is to investigate the development of mathematics teacher educators' pedagogical content knowledge (PCK) on the practice of lesson study (LS) at primary schools in Lao PDR. This research classifies features of PCK as utilizing the conversation of mathematics teacher educators (MTEs) through the three steps weekly cycle of Thailand LS incorporated Open Approach (TLSOA) model (Inprasitha, 2022). The PCK components that will be described in this study consist of content knowledge (CK) and pedagogical knowledge (PK). 1) CK includes (1) mastery of mathematical concepts, (2) anticipate students' ideas and students' misconception 2) PK includes: (1) create and post mathematical problem situation, (2) design and use media according to students' ideas, (3) anticipate students' difficulties. This descriptive qualitative research was conducted in the classroom of 4th grade at primary school of Savannakhet Teacher Training College (STTC) in Lao PDR during semester 2 of academic year 2022, the study involved 3 MTEs in STTC, and angle was content unit chosen to conduct LS. Data sources included field notes, lesson plan, videotapes and reflection journal writing. Data were analyzed using analytical description.

The findings revealed that most components of PCK of MTEs were developed through the cycle of TLSOA as follows: in the step 1, LS group were able to create real-world problem situations about how to draw, measure and find an angle for students to be engaged and LS group could anticipate students' misconception and students' difficulties that will arise from the ideas of students used to solve problems, in addition, LS group were able to design the media according to students' ideas. In the step 2, the MTEs were able to pose problem situations to engage students to solve the problems, able to prioritize students' ideas and able to ask meaningful questions to students for discussion and compare ideas. However, they have difficulties in creating discussion questions to make students aware of the tools obtained by oneself. In the step 3, the LS group could explain students' ideas in each problem situation, aware of the students' misconception about the mathematical content, and they could reflect on the point to be improved for further development. By conducting LS practice, LS group were able to deeper understand the mathematics curriculum and textbook of primary school, and they have learned more about students' learning and environment of primary school classroom. They discovered the benefits of collaboration for the success of their lessons.

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LEARNING HOW TO CONDUCT DISCUSSION AND SUMMARIZATION OF STUDENT TEACHERS FOR MATHEMATICAL PROBLEM-SOLVING CLASSROOM

Saastra Laah-on, Pimpaka Intaros, Chanika Senawongsa

Mathematics Department, Science Faculty, Lampang Rajabhat University, Thailand

A thinking classroom is not only teaching how to think, but students need to originate both individual and collective thinking through activity and discussion (Liljedahl, 2016). This research was aimed to investigate how to conduct discussion and summarization of forty-eight third year mathematics student teachers who had enrolled Mathematics Learning Management in Junior High School, Bachelor Degree of Education in 2023 academic year. This course engages the student teachers to promote a mathematical problem-solving classroom using the Thailand Lesson Study incorporated Open Approach (TLSOA) as a teaching approach and a way to improve the teaching approach (Inprasitha, 2022). Qualitative methodology was employed to collect and analyze data of an analytical thinking about the student teachers' micro teaching classroom which designed from Japanese textbook 'Gateway to the Future for Junior High School' (Kerinkan, 2013). All researchers were co-lecturers for this course, therefore, participatory research design was brought to collaboratively plan, observe and reflect the classroom (Inprasitha, 2011; 2022). Research results related to student teachers' decision making of 1) anticipated ideas are adequate for discussion and connected to summarization, 2) how to discuss and connect emerged ideas to summarization, and 3) how to improve teaching to support the students for creating those anticipated ideas. At the final part of this course, the student teachers are aware of opportunities to improve their teaching at the stage of discussion and summarization by their team as peers and lecturers to conduct the mathematical problem-solving classroom.

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SECONDARY STUDENTS' MATHEMATICAL THINKING IN SOLVING MATHEMATICS PROBLEMS ABOUT BASIC SETS: THE CASE OF AN ATTACHED SCHOOL GRADE 6

Sommay SHINGPHACHANH

Khangkhay Teacher Training College, Lao PDR

It is widely accepted that mathematical thinking is a key to higher-order learning goals in mathematics education. Typically, it involves problem-solving, modelling to solve a real-world problem, and abstract perspective (Drijvers, Kodde-Buitenhuis & Doorman, 2019). Mathematical thinking pays much "attention to the process rather than content" even though they are both important in school mathematics curricula (Goos & Kaya, 2020). To reveal student mathematical thinking, eliciting from "clinical-task based interviews, verbal problem-solving protocols or carefully designed questionnaires, tasks or tests" is commonly used especially to see students' problem posing abilities, reasoning and proof (Goos & Kaya, 2020). This study focuses on one of attached schools/demonstration schools or laboratory schools of a Teacher Training College (TTC) in Laos. The study has 3 objectives, 1) to describe the situation of secondary student learning in the attached school of the TTC; 2) to find out secondary students' mathematical thinking about basic sets and 3) to demonstrate student learning achievement on learning basic sets. The researcher employed video records of some lessons, classroom observations and a test with 7 different math problems to 99 secondary students in grade 6 in the attached school in March 2023 a week after the lesson about the basic sets was taught. In this paper, however, the researcher only illustrated the second research objective regarding different secondary students' mathematical thinking about the basic sets. Preliminary findings revealed that secondary students expressed different mathematical thinking regarding writing set builder notation and drawing diagrams. For another 6 math problems, however, students solved them in the form given by the teacher and had mathematical thinking similarly. It seems they are not able to figure out other solutions for fear of making mistakes. The study suggested that encouraging students to have mathematical thinking differently to promote higher order thinking and expressing their idea of multiple solutions is recommended.

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AN EXPLORATION OF INTERACTIVE LEARNING IN VIRTUAL CLASSROOMS - USING THE PERCENTAGE OF ACCOMMODATION ROOMS AS AN EXAMPLE

Ting-Sheng Weng, Ting-Hsuan Lin
National Chiayi University

The virtual classroom, a model in the e-learning field, brings diversified choices to current teaching methods and attracts learners' learning interest. This study uses APP-BUD to develop a metaverse virtual classroom. The transformable virtual classroom scenario enables students' mobile learning without being limited by space or distance, meaning that they can experience the class situation on campus or in the classroom, which helps to improve students' adaptability before entering school. The questionnaire survey results regarding the experimental teaching of percentage interval estimations and assumptions of accommodation rooms found that the metaverse virtual classroom aroused most students' curiosity and interest; however, it also caused burden on some students' learning, thus, students needed time to adapt.

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SOCIOMATHEMATICAL NORMS IN MATHEMATICS CLASSROOM USING LESSON STUDY AND OPEN APPROACH

Weeraphat Sarapat[1], Narumon Changsri[2]

[1]Mathematics Education Program, Faculty of Education, Khon Kaen University
[2]Center for Research in Mathematics Education, Khon Kaen University

Sociomathematical norms refers to the normative aspects of mathematical discussion specific to students' mathematical activity (Yackel & Cobb, 1996). Every mathematics classroom has its own unique norm that is characteristic of its microculture (Güven & Dede, 2017). Inprasitha (2022) introduced Lesson Study incorporated with Open Approach (TLSOA) to improve teaching practice. The TLSOA has enabled teachers and students to foster classroom norms. The study was aimed to investigate sociomathematical norms in a three-year practice school using TLSOA in Khon Kaen, Thailand. Data collected by using field notes, questionnaires, interviews, and videos from 30 third-grade students during the 2022-2023 school academic year. The data obtained from student interviews, teacher-filled questionnaires, and classroom protocols were systematically triangulated through Lopez & Allal (2007) framework. The results revealed that the classroom using TLSOA enclosed two perspectives of sociomathematical norms. The first sociomathematical norm regarding problem solving was where students had expectations on how to solve the problems in several ways, differentiate the given situations from what is to be found, and confirm the results. The second sociomathematical norm regarding participation in joint problem-solving activity was where students explained one's problem-solving procedure and expressed opinions about other students' ideas.

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AN ANALYSIS OF SIXTH GRADE STUDENT'S MATHEMATICAL PROFICIENCY ASSESSMENT

Worapat Tohon[1], Narumon Changsri[1,2]

[1]Mathematics Education Program, Faculty of Education, Khon Kaen University
[2]Center for Research in Mathematics Education, Khon Kaen University

In our information-driven society, the world sees increased demands for people who have the skills and abilities that are cultivated through the study of mathematics. This is likely due to a growing recognition that such values are necessary for maintaining a prosperous and affluent society (Shimizu, 2015). Mathematical proficiency described in Adding It Up is strategic competence that ability to formulate, represent, and solve mathematical problems (NRC, 2001). Conducting a proficiency test using a test called SUKEN, which measures the test examinee's own mathematical abilities and in order to go back and evaluate what they have learned from school (MCIT, 2017).

This study aimed to analyze proficiency from the mathematical proficiency test (SUKEN test). The target group was 88 examinees from 5th the mathematical proficiency test (SUKEN test) in level 6. They study in Grade 6 to Grade 7 or aged 11-13 years old. This research used quantitative and qualitative research methods. The research tool was analyzed by analysis of answer sheets of examinees in level 6. SUKEN test in level 6 had 30 questions that translated from English to Thai by experts and examinees write their answers on answer sheet by themselves. Data analysis follows the mathematical proficiency framework from The Mathematics Certification Institute of Japan (MCIJ, 2015).

The results found that examinees have mathematical proficiency from 2 factors as follows :1) arithmetic abilities useful for handling matters in everyday life that divided into details as follows: examinees had (1) the ability to measure liquids, etc. in containers (35.23%). (2) The ability to calculate actual size and width using a map (57.95%). (3) The ability to display the relationship between two things using ratios and graphs (47.02%). (4) The ability to organize simple materials, and combine them on a chart (46.59%). And second factor found arithmetic useful for everyday life situations that divided into details as follows: (1) The ability to count coins and paper money, and to accurately calculate/transfer money amounts (52.50%). (2) The ability to display multiple items, compare amounts, etc. using pie charts and bar graphs (60.98%). It can be seen that the test examinees' ability for measure thing as the least of all proficiency and the ability to display multiple items, compare amounts, etc. using pie charts and bar graphs had the most performance.

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DEVELOPMENT AND PROSPECTS FOR MATHEMATICS ASSESSMENT USING A COGNITIVE DIAGNOSTIC MODEL IN THE REPUBLIC OF ZAMBIA

Yoshitaka ABE

Hiroshima University

This study explores the learning trajectory (Clements&Sarah,2013) in Zambia, links it to existing assessment criteria, and provides educational improvement. Zambia reported that 92% of students lacked basic math skills (SACMEQ III ,2011). However, Uchida (2012) and Baba et. al. (2019) indicates that existing assessment criteria do not adequately capture learning trajectories in Zambia since there are gaps in math proficiency, even among low-level students. However, even these studies in Zambia have not adequately studied learning trajectories because of limitations, such as a focus on natural numbers only (Baba et. al., 2021) and time constraints due to interview methods have hindered research across grades and domains.

To develop a diagnostic assessment using a cognitive diagnostic model (Tatsuoka, 2009), extending its scope to include decimals to overcome these limitations. This paper-based test enables broader surveys and resolves the previous time constraints. Cognitive diagnostic models apply to the knowledge and skills needed in the domain, called Attribute”. The concepts, procedures, and construct knowledge were selected as Attribute” and determined the 46 test items using researches of McCloskey et. al. (1991) and Baturo (1997). To assess the validity of the items, a preliminary survey of approximately 150 Zambian Grade 6 and 7 students was conducted using Rasch modelling. The results showed that of the 21/46 items were inappropriate (Holster & Lake, 2014). The results also indicate the consideration of previous studies and Zambia's unique responses.

Currently, three studies are being conducted to recreate properly items for “Attribute" in Zambia: first, an error analysis based on pupils' responses; second, an analysis of the knowledge and skills used in the textbook; and third, a reselection of the questions based on previous research. The learning trajectory in Zambia clarifies the actual situation of Zambian children by mapping them to both the new criteria and existing levels. Moreover, a user-friendly interface needs to develop for the cognitive diagnostic model.

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THAILAND
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POSTER PRESENTATIONS

THE ANXIETY OF HEARING-IMPAIRED COLLEGE STUDENTS ASPIRING TO BECOME MATHEMATICS TEACHERS

Akira MORIMOTO[1], Mao OBARA[2], Hideyo EMORI[3]

[1]Fukushima University [2]Fukushima University Graduate School [3]Otani University

In Japan, opportunities are available for hearing-impaired college students to become mathematics teachers at special needs education schools for the hearing impaired. However, these college students often face various anxieties when aspiring to become mathematics teachers. There are researches that points out mathematics teachers need to acquire expertise in both deaf and hard of hearing education and mathematics education(e.g., Pagliaro, 1998). While hearing-impaired college students aspiring to become mathematics teachers may have a passion for mathematics and mathematics learning, they may be unsure if their own understanding of mathematics is sufficient to teach it to children who face difficulties in language acquisition. They may also wonder if they can effectively share information and communicate with these children.

The survey was conducted, targeting hearing-impaired college students aspiring to become mathematics teachers. It was carried out through questionnaires and observations of interaction among students toward designing children's mathematical activities. The collected responses and records were used as data for analyzing the anxiety of students regarding mathematics teaching and learning. Reflections based on data analysis shows that although they have anxieties regarding understanding mathematics and interacting with children, in order to encourage hearing-impaired children to construct mathematical knowledge, it is important for children to negotiate with each other and with teachers until they are satisfied. It was thought that they had a positive view of the necessity of enhanced mathematics communication.

From the analysis results, it was found that among hearing-impaired college students aspiring to become mathematics teachers, there are students who have not experienced in negotiate through what they didn't understand during junior high school and high school mathematics classes. For them, the experience of becoming active learners is important. It was implicated that to overcome the anxiety, they need to accumulate and emphasize the experience of mathematics communication among themselves before becoming teachers.

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THE DEVELOPMENT OF GRADE 5 INSTRUCTIONAL MATERIALS ABOUT CIRCLE

Aphichaya Sumalai, Nisakorn Boonsena

Khon Kean University

The purpose of this research was to study the development of instructional materials. The target group were a lesson study team of 5 teachers and 12 fifth grade students selected by a specific method from students who have experience through open approach and were able to participate in research data collection during the coronavirus 2019 outbreak, in semester 2, academic year 2021. Teaching was conducted according to an Open Approach. Data collection was carried out through 6 lesson plans for Unit 10, Circles, Grade 5. Data analyzed from the protocol of classroom video, work sheet and field notes Based on the framework of LSOA (Inprasitha, 2014) and Flow of lesson (Inprasitha, 2017).

The results found that 1. Planning phase, lesson study team designed lesson plans and developed instructional materials. They started from reading textbook, anticipated student ideas, created instructional materials based on students' previous experiences or knowledge, taking into students' representations of the real world in order to connect them to students' semi-concrete, and students' mathematical world. 2. Doing phase, lesson study team collaborated to observe the ideas that emerged from the students' instructional materials. According to the concept of flow of lesson (Inprasitha, 2017), students' ideas were found as following; 1) real world material, students explained how rolling a circle compared to rolling a ball or turning the wheels of a car. Students learned diameter and circumference through a box of cookies. 2) semi concrete material, students used instructional materials to solve problems and were tangible instead of real objects, such as circles, strips of paper, cookie boxes, pictures, stickers. Through semi concrete material, there was a transfer from the real world to the mathematics world. 3) mathematical world material, students found relationship between diameter and circumference of a circle. 3. See phase, lesson study team reflects on students' ideas and instructional materials which using in classroom. They noticed the points for developing lesson plan and materials as follows:

1) representation of the real world. Instructional materials should be created link to classroom activities and students' previous experiences or knowledge.
2) semi concrete aids and representation of the mathematical world. Instructional materials should be created appropriately, such as being easily accessible, using appropriate colors and sizes for whole class. Therefore, students can be clearly seen.

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PROMOTING STUDENTS' CONCEPTUAL UNDERSTANDING OF DECIMAL DIVISION: A CASE OF GRADE 6 STUDENTS IN THAILAND

Chanchai Duangmala, Weerasuk Kanauan, Pariya Pariput

Ubon Ratchathani Rajabhat University, Thailand

This research used a qualitative approach to explain the development of learning activities by using the concept of flow of lesson within an open approach based on Inprasitha (2011) to promote students' conceptual understanding of decimal division. The target group is 5 students of grade 6 students, semester 1, academic year 2023, in a public school, Thailand. Data were collected by participant observation, semi-structured interviewing, video analyzing, and document analyzing. Data were analyzed by content analysis according to conceptual understanding: comprehension of mathematical concepts, operations, and relations based on Kilpatrick et al. (2001). The findings revealed that the way to promote students' conceptual understanding of decimal division: 1) Promoting students' understanding of concepts: Design learning activities where students solve problems together and create tasks from situations related to students' daily lives. Then focus on converting real-world situations into simulations with tape diagrams, number lines, and proportional number lines. 2) Promoting students' understanding of operations: Design activities by allowing students to present their own methods and compare the methods of mutual friends. Give students an opportunity to explain the steps by making connections from proportional number lines that represent problem situations and have them write an explanation of the steps to calculate the answer. 3) Promoting students' understanding of relations: Design activities through problem-solving by the students and discussion with the whole class. The discussion point is seeing the connections between the methods that students have already learned and then using them to solve new problem situations and extending the methods they have learned to use to solve the next problem situation.

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AN EXPLORATION OF JAPANESE MATHEMATICS TEACHERS' PRACTICE RESEARCH REPORT'S STRUCTURE

Dhanachat Anuniwat, Masato Kosaka, Yoshiko Hanbara

University of Fukui, Japan

Japanese Lesson Study encompasses various forms, structures, and goals. These diversities can be broadly classified into four modes: Mode 1, checklists and evaluations; Mode 2, planning and verification; Mode 3, dialogue and evidence; and Mode 4, multi-spiral and inquiry (Kimura, 2019). Differences in the modes can steer the teachers' writing style of their Practice Research Report (PRR) in dissimilar ways. The writings are a result from conducting Lesson Study, which document the study or research on teaching practices conducted either independently or by an association of teachers aiming to improve teaching practice (Miyakawa & Winsløw, 2019) and describe classroom events and interactions among teachers and students from a first-person perspective (Ishii, 2017). In Mode 4, the inquiry process is perceived as similarly and mutually beneficial for both children and teachers. This perspective creates a multi-spiral approach to learning, with teachers narratively recording it—a practice distinct from other modes—to enhance their longitudinal professional development (Kimura, 2019). There is research focusing on other modes, however, there is limited information on the research related to Mode 4. This study aimed to explore the structure of PRR written by mathematics teachers. A qualitative research method was employed. Data were collected from 10 practice research records written under Mode 4 during 2018-2022.

The result reveals that the writing structure comprises of two parts; the first part, the practice of research lessons, is comprised of 1) designing the learning that elaborates on the intention of designing a unit of the research lesson, 2) learning stories that narrate the learning student and how the teachers weave each lesson together through the whole unit which also includes the students' discourse, and 3) reflecting on the content, learning design, and the teachers' and students' learning process throughout the unit. The second part is theory which weaves and links the practice of research lessons and the school research theme.

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STUDENTS' MATHEMATICAL REPRESENTATIONS ON ADDITION

Janthira Sumrit, Nuengruthai Nhulung, Wipaporn Suttiamporn

Faculty of Education, Phuket Rajabhat University

Mathematical representations are visible or tangible creations. Examples of these creations include diagrams, number lines, graphs, arrangements of concrete objects or manipulatives, physical models, written words, mathematical expressions, formulas, and equations, or depictions on the screen of a computer or calculator – that encode, stand for, or embody mathematical ideas or relationships (Goldin, 2020). This study aimed to survey the first students' mathematical representations on addition. Qualitative methods, participant observation, and teaching experiments were used in accordance with the collaboration lesson plan, do, and see of the Lesson Study processes. The target group consisted of 21 first-grade children from a school in Phuket Province that employed the Thailand Lesson Study incorporated with Open Approach. Research tools include the ten lesson plans, observation form, the camera, lesson notes, and reflection notes. Data were collected in the first semester of the 2023 academic year. Data were analyzed through content and descriptive analysis following Owens & Clements's ideas (1998). The results revealed that the first-grade students' mathematical representations included 1) students drawing pictures such as flowers, hearts, etc. to represent their ideas, 2) drawing arrows to show the direction of addition both adding and together, 3) students writing mathematical expressions to show the addition situations, and 4) students described the addition situations both adding and together.

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THE 7TH GRADE STUDENTS' GEOMETRICAL REASONING THROUGH OPEN-ENDED PROBLEM-SOLVING

Jidapa Limdun, Paweena Pakdee, Sudatip Hancherngchai

Mathematics Education, Faculty of Education, Phuket Rajabhat University

As geometry evolves to encompass the understanding of diverse visual phenomena, it is important to be clear about what is meant by the geometrical reasoning necessary to solve mathematical problems involving visual phenomena, and how such reasoning develops (Jones, 1998). This study aimed to explore the 7th-grade students' geometrical reasoning. Participant observation was used as methodology. The target group was 40 people from Secondary school in Krabi province. which school used Thailand Lesson Study Incorporated with Open Approach. Research tools such as lesson plans, observation notes, cameras, IC-recorder, and field notes. Data were collected in the first semester, of the 2023 academic year. Descriptive and content analysis were analyzed date following Duval's idea (1995). The results revealed that 1) students categorized the 3D shapes into various groups and identified criteria of physical characteristics of 3D shapes 2) students reasonably grouped the 3D shapes by 2D shapes 3) students argued and discussed the characteristic 2D and 3D shapes, and 4) students identified the components of prism and pyramids.

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STUDENTS' MATHEMATICAL IDEAS IN MATHEMATICAL ACTIVITIES BY USING PATTERN BLOCKS

Kaona Simlee[1], Narumon Changsri[2], Auijit Pattanajak[3]

[1]Mathematics Education Program, Faculty of Education, Khon Kaen University
[2,3]Center for Research in Mathematics Education, Khon Kaen University, Thailand

Pattern blocks in 6 different wooden geometric shapes are teaching material that helps to develop students' mathematical thinking abilities (Takahashi & Yanase, 1997). The objective of this study was to analyse students' mathematical ideas in solving problem by using pattern blocks in classroom using Thailand Lesson Study and Open Approach (TLSOA). TLSOA plays major role to create mathematical activity to enhance students' mathematical ideas (Inprasitha, 2022). Research participants were lesson study team and 16 third-grade students in the 2022 academic year. This school participated in TLSOA for two years. The obtained data from field notes, transcript of video recording, and student worksheets was systematically triangulated through the framework of Mangao, Ahmad, & Isoda (2017). The findings showed that process of TLSOA promoted students to elicit their mathematical ideas. **In designing research lesson**, lesson study team created mathematical activity using problem situations from Thai version of Japanese mathematics textbook to anticipate students' mathematical ideas and their difficulties in solving-problems. **While research lesson was implemented**, two mathematical ideas, mathematical idea of unit and mathematical idea of comparison, occurred in this phase. **After the research lesson**, the team reflected on students' mathematical ideas and identified those that could be useful for students in solving problem for the next lessons. Inconclusion, the overall findings of this research contribute to that TLSOA process have developed students' mathematical ideas through their participation in mathematical activity.

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ABILITY TO TASK DESIGN IN MATH DAILY LIFE OF STUDENT TEACHERS BY USING COMMUNITY-BASED LEARNING

Katanyuta Bangtho, Kittisak Jai-on

Faculty of Education, Nakhon Si Thammarat Rajabhat University

Mathematical task design continues to be an important area of research in mathematics education (Johnson, Coles, and Clarke, 2017). The mathematical tasks with which students engage can shape students' mathematical learning opportunities and their experiences with mathematics as a whole (Watson & Mason, 2007). This article concern tasks design that involve the use of tools in the mathematics classroom and consequently how, under such design, tools can mediate the representation ability of student teacher's designing math daily life. The authors focus particularly on daily life and community-based learning can be used to transform an artifact into a pedagogical instrument. Math is very useful in everyday life and help us do many things that are important in our everyday lives. The research objective is to study the abilities to task design in math daily life of student teachers using community-based learning. The employed research instruments consisted of 1) Lesson Plan 2) assessment form for ability task design and 3) observing learning management. Conceptual framework for data analysis is Inprasitha (2016, 2019).

Almost every student studies math during school. However, the majority of students question the importance of math. Most students are unaware that math is not just a part of many other disciplines, it is part of everyday life. Therefore, the authors would like to make student teachers aware of the importance of mathematics in everyday life and use it in designing mathematics problems so that students can solve problems in a meaningful. The results of the research found that student teachers were able to use mathematics in the community to task design at a good level, that is, they had knowledge and understanding of the context and conditions used as elements in designing mathematics problems that were meaningful to students.

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AN EXPLORING STUDENT'S HIGHER-ORDER THINKING SKILLS IN MATHEMATICS THROUGH CREATING CIRCLE ACTIVITIES

Luksana Tonchan[1], Khem Khenkhok[2], Piyasak Pakkhothanang[2], Jumpol Rachvijit[2], Narumon Changsri[1,3]

[1]Mathematics Education Program, Faculty of Education, Khon Kaen University, Thailand
[2]Art Education Program, Faculty of Education, Khon Kaen University, Thailand
[3]Center for Research in Mathematics Education, Khon Kaen University, Thailand

Higher-order thinking skill is the ability to handle data or contents that have been obtained, which has to go through many steps and relies on thinking strategies that are more complicated, insightful, and creative in processing several bodies of knowledge (Zohar & Schwartzer, 2005). This study aimed to explore students' higher-order thinking skills through creating circle activities. The target group was 29 higher education students comprising faculty members, school principals, educational supervisors, in-service teachers, and graduate students in non-degree teacher professional development programs. The tools for collecting data were three open-ended problem situations regarding creating circles, a video observing the problem-solving of the target group, a worksheet, and a reflection. Teaching is conducted using the Open Approach, and data analysis follows the CCRLS framework for mathematics and the aims of mathematics learning (Mangao et al., 2017).

The results found that most participants had a positive attitude toward the creating circle activity in the classroom. They developed ideas by creating circles to use in designing logos by using creativity and imagination to connect various concepts. Moreover, they accepted and appreciated different ideas. And they also saw the beauty of using circles in friends' logo designs. They could use the ideas from creating circles to design a variety of logos, that connected to their own experiences.

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ANTICIPATION AS A TOOL FOR TEACHING PRACTICE TO DEVELOP STUDENTS' THINKING

Nisakorn Boonsena, Aphichaya Sumalai

Khon Kaen University

OECD (2019) launched the conceptual learning framework for the future of education and skills 2030 that mentioned the anticipation-action-reflection (AAR) cycle and anticipation played a role as not only asking questions but also projecting the consequences and potential impact of doing. Moreover, educators and researchers across the globe reported the important of anticipation phase as a process which encourage students to have their authentic problem, intentions, and curiosity to solve the problem as well as teachers to understand students' learning/thinking, typical misunderstanding, and carefully designing problem situation (Inprasitha, 2022; Lewis, 2016). Therefore, the aim of this study was to investigate teacher' s teaching practice while using designed materials. The researchers utilized the context and condition of the problem situation in Thailand Lesson Study incorporated with the Open Approach (TLSOA) (Inprasitha, 2022) as the conceptual framework. Data were collected through classroom video recordings and transcriptions, and classroom observation using students' worksheets. Participants were 12 fifth grade students, and a lesson study team consisted of 1 in-service teacher and 4 pre-service teachers in mathematics education program, faculty of education of a public university.

The research results revealed that teachers in TLSOA classroom changed their procedures in anticipation process from anticipate students' learning/ thinking by teachers' experiences to anticipate students' learning/ thinking through teachers' problem solving, carefully design problem situation with the context from textbook such as students' previous-future learning, and context from students' real world, and finally create the condition as instruction with short and clear keyword. Teachers also created the materials following the flow of the lesson. Furthermore, there were noticed that designed problem situations and materials support teachers to not talkative and not using a set of questions in the classroom. Teacher's teaching practices were gradually following the steps of open approach.

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PROBLEM POSING ON CLEAN ENERGY IN DAILY LIFE CONNECTS TO STUDENTS' MATHEMATICAL IDEAS

Noppasorn Boonsena, Nisakorn Boonsena

Mathematics Education, Khon Kaen University, Thailand.

Earth's environmental emergencies and human well-being such as SDG 7: universal access to clean energy and SDG 13: addresses climate change was nowadays addressed as the important issues (United Nations, 2015). Students as young generation have to engage in these kinds of situations for preparing their future through higher-order thinking in mathematics.

This study aimed to explore students' mathematical ideas regarding problem posing on clean energy in daily life. The target group were 146 ninth-grade students. The data were collected from lesson plan, students' worksheets, transcription of classroom recording, and classroom observation. The data were analyzed from a framework of Isoda & Olfos (2021) and Thailand Lesson Study incorporated Open Approach (TLSOA) of Inprasitha (2003, 2022).

The result described based on TLSOA revealed that in Step 1, LS team cooperated to select and revised problem situations from the 15th the national open class. Problem situations set the context related to global warming. LS team created conditions in order to lead students to solve it. Step 2, in problem posing phase, the teacher posed a problem situation to explore and select clean energy and gives reasons. In problem solving phase, first problem situation found 45% solar energy, 32% hydro energy, and 23% wind energy. In second problem situation found that 69% students used the concept of Quotative division, and 27% students used the concept of inverse operation of multiplication. In third problem situation found that 25% students used the concept of multiplication to find the area. Furthermore, Step 3, LS team reflected on students' ideas regarding designed problem situation. Students engaged in the problem situations and continued do problem solving. Moreover, students were aware of global warming, and how to use of renewable energy.

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STUDENTS' MATHEMATICAL THINKING ON SUBTRACTION

Nuengruthai Nhulung, Janthira Sumrit, Thamonwan Sermsakulkiet, Wipaporn Suttiamporn

Faculty of Education, Phuket Rajabhat University

Mathematical Thinking is the ultimate goal of education. Because mathematical thinking supports sciences, technology, life, and development in the economy (Stacey, 2007). Moreover, mathematical thinking as well as an attitude that presented an endeavor to do and work progressively not only products but also processes (Isoda & Katagiri, 2012) This study aimed to survey the first students' mathematical thinking on subtraction. Qualitative methods, participant observation, and teaching experiments were used. The target group was the first-grade students 21 people of school in Phuket Province, which school used Thailand Lesson Study incorporated with Open Approach. Research tools include the 5 lesson plans, observation form, camera, and lesson notes. Data were collected in the first semester of the 2023 academic year. Data were analyzed through content and descriptive analysis following Stacey's ideas (2007). The results revealed that the first-grade students' mathematical thinking included 1) students recognizing their problem and describing the meaning of subtraction 2) students arguing and reasoning their ideas with friends 3) students using their previous knowledge as the tools to learn the next lesson 4) students summarized the meaning of subtraction clearly and simply, and 5) students anticipating the results or ideas from problem situations that teachers posed.

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THE STUDY OF ON-THE-FYE ASSESSMENT IN MATHEMATICS CLASSROOM

Patchawan Aphiromchawan, Kiat Sangaroon, Nisakorn Boonsena

Khon Kaen University, Thailand

The learning evidence that can be collected through interactions on-the-fly depends on how these assessment conversations are set up, on what type of questions are used and how the teacher interprets and acts upon this evidence (Harison et al, 2018). Lesson Study and Open Approach (LSOA) are teaching approach that focuses on students in problem-solving, so that students can think for themselves (Inprasitha, M., 2018). This study aims to study teachers' approach on-the-fly assessment in mathematics classroom.

Data were collected from a sample group of lesson study team of four graduate students and two teachers from schools using LSOA during June 2023 to July 2023. The lesson study team recorded field notes to observe students' ideas in five groups, teacher's questions, and supplementary materials used by teachers or students during collaborative discussions in the classroom through analyzing data from the instructional plan construction form, field notes, protocols, and work sheets. This study used the students' learning assessment framework of Inprasitha (2018) to analyze the data and developed it by incorporating of teacher questions and supplementary materials for on-the-fly assessment.

The results showed that teachers and the lesson study team collaborate in step of lesson design (Plan). The assessment process comprises four steps. 1) Assessment Framework is based on predictions of student concepts. 2) Developing assessment tools. 3) Linking the assessment to previous knowledge. 4) Designating assessment positions. In the step of collaborative observation (Do) found that students had a variety of problem-solving approaches, both incorrect and correct ones. Teachers conduct on-the-fly assessments. It is found that there are both anticipated and unanticipated questions, as well as supplementary materials prepared for each concept. In step of Collaborative Discussion and Reflecting on Lesson Results (See), teachers also use questions that go beyond predictions, depending on students' responses at the time, and the teacher's objectives in asking questions that align with the position to be assessed.

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PRE-SERVICE TEACHERS' BELIEFS ABOUT MATHEMATICAL PROBLEM SOLVING

Piyathida Srihamongkhon[1], Ratthapong Surakhai[1], Narumon Changsri[1,2]

[1] Mathematics Education Program, Faculty of Education, KKU
[2] Center for Research in Mathematics Education, KKU

Problem solving is not merely a process that concludes when an answer is found; it is a scientific process that evolves from understanding the problem to evaluating the solution. This process is influenced by several factors, with belief being one of the most significant (Ozturk & Guven, 2016).

This study aimed to explore the beliefs of pre-service teachers regarding mathematical problem solving. The target group was twenty graduate students majoring in Mathematics Education during their first semester of study (five months). The target group had previously earned a bachelor's degree unrelated to the field of Education. During the first semester, they gained experience in mathematical problem solving through classroom observations, reflected on the results of observing classes using the lesson study and Open Approach (LSOA) (Inprasitha, 2022), and solved open-ended problems by themselves. Moreover, they participated in the Higher-order Thinking course. Data was collected both before and after entering the program using open-ended questionnaires concerning beliefs about mathematical problem solving and interviewing, with subsequent data analysis performed through content analysis.

The research results revealed that prior to entering the program, there were two primary beliefs about solving mathematical problems: one group believed that mathematical problem solving involved finding answers using mathematical rules and formulas, while the other group believed it was about finding only one correct answer. However, after gaining experience in Mathematics Education Program, they began to view mathematical problem solving as a process that involves various methods, steps, or approaches, depending on the context, prior knowledge, or previous experiences. The emphasis shifted from focusing solely on the end result to valuing the thought process. Additionally, they developed the belief that solving mathematical problems increased their ability to tackle problems independently and promoted diverse ways of thinking.

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SEMIOTICS ACTIVITY IN ZONE OF PROXIMAL DEVELOPMENT IN CLASSROOM USING OPEN APPROACH

Tanakorn Sanghaow[1], Narumon Changsri[2]

[1]Mathematics Education Program, Faculty of Education, Khon Kaen University
[2]Center for Research in Mathematics Education, Khon Kaen University, Thailand

Many studies have found that semiotic activity is a helper for solving mathematical problems. The zone of proximal development (ZPD) was developed by Vygotsky (1978) to account for children's learning potential. This study aimed to analyze semiotic activity in the zone of proximal development in the classroom using an open approach. The teaching practice set under lesson study and open approach was as follows: 1) A lesson study team was formed to design a research lesson at least once a week. 2) Team members collaboratively observed the research lesson with four steps of OA, which comprised posing an open-ended problem, students' self-learning, whole class discussion and summarizing through connecting students' mathematical ideas that emerged in the classroom. 3) All members collaboratively discuss and reflect on the research lesson (Inprasitha, 2022). A qualitative research method focused on two teachers and 18 third-grade students during the 2020 academic year. Data were gathered from field notes, classroom videos, and six lesson plans within the division unit. The obtained data, for example, protocols while planning lesson plans, teacher-filled field notes, and student worksheets, was systematically triangulated through the framework of Radford (2004). The finding showed that the lesson study team was formed to anticipate the actual development of children while planning the lesson. As the lesson plan was conducted, students were able to enhance their potential development level by engaging in semiotic activity, which helped them to solve problems and share their ideas with their peers. Moreover, this phase demonstrated that the ZPD exists between the actual and potential development levels.

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CHARACTERISING PRE-SERVICE SECONDARY TEACHERS' DISCOURSE WHEN DEFINING

Verónica Martín-Molina, Aurora Fernández-León, José María Gavilán-Izquierdo, Rocío Toscano

Universidad de Sevilla

An essential part of learning mathematics is learning definitions, but many pre-service mathematics teachers (PMTs) have problems with mathematical definitions (Molitoris Miller, 2018). In this work, we employed the commognitive framework (Sfard, 2021), in which mathematics is considered a particular type of discourse and learning is conceptualised as a change in that discourse. A source of learning is the resolution of commognitive conflicts, which occur when the participants in the discourse produce incommensurable narratives, i.e., narratives that are not "endorsable according to the same truth-defining rules" (Sfard, 2021, p. 9). In our research, we examined how the presence of non-prototypical figures influenced the way that PMTs employed to construct and select definitions for solids. In particular, our research questions were: (1) how can the presence of non-prototypical geometric solids influence the appearance of commognitive conflicts? (2) were the PMTs aware of those conflicts?

The participants of the study were 35 PMTs organised into 9 groups of 3-4 students (M1,…, M9). Each group had a worksheet with five questions about defining three different geometric solids. We collected their written answers and recorded their discussions while answering the questions. We inferred, through the lens of the commognitive framework, the appearance of several commognitive conflicts due to the presence of non-prototypical solids (e.g., in M7 and M8). In M8, the conflict appeared when the PMTs found in the worksheet narratives that were incommensurable with narratives of their own discourse, while in M7 the conflict appeared due to different narratives concerning what a definition is. The PMTs of M7 were aware of the conflict (but not of the reason for it), while the ones in M8 were not.

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THE 1ST-GRADE STUDENTS' CONCEPTUAL UNDERSTANDING ON NUMBER

Waleerat Rakdecha, Budsabawan Nammon, Julaluk Jai-on

Faculty of Education, Phuket Rajabhat University

The objectives of learning mathematics were to develop and deeply understand mathematical concepts and these relations the students could create, compare, and represent variously (NCTM, 2000). Conceptual understanding is one of the mathematical competencies that support students in organizing their knowledge by themselves and using these ideas to describe new mathematical ideas and reach integrational and relational mathematical ideas (Kilpatrick et al., 2001; Inprasitha, 2015). This study aimed to explore the first-grade students' conceptual understanding of the numbers of elementary school in Phuket province that school was in the project school innovated by the Thailand Lesson Study incorporated with Open Approach. Participant observation was used as methodology. The target group was the first-grade students 27 people. Data were collected in the first semester of the 2023 academic year. Research tools such as lesson plans, cameras, and field notes. Data were analyzed by content analysis and descriptive analysis following Kilpatrick et al.'s ideas (2001). The results revealed that 1) students could explain the relationship between concepts and mathematical situations. 2) students use various representations to communicate their mathematical ideas 3) students describe part-whole relationships, and 4) students express mathematical expression from problem situations.

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SUPERVISION OF MATHEMATICS STUDENT TEACHERS THROUGH LESSON STUDY PROCESSES

Weerasuk Kanauan[1], Tidarat Rujirawinitchai[1], Chanika Senawongsa[2]
[1]Ubon Ratchathani Rajabhat University, Thailand
[2]Lampang Rajabhat University, Thailand

The purpose of this research was to explain the approach for supervision of department of mathematics, faculty of Education, Ubon Ratchathani Rajabhat University and Lampang Rajabhat University through lesson study process based on Inprasitha (2022). Research was conducted on five schools' projects. The target group consists of 12 mathematics student teachers, 10 mentor teachers, 3 lecturers and 5 school directors. Data were collected by participant observation, semi-structured interviews, focus group discussions, and document analysis. Qualitative data were analysed by content analysis according to components of communities of practice based on Wenger, McDermott & Snyder (2002) and using methodological triangulation. The findings revealed that the approach to supervision stresses group engagement and includes the following components: 1) Community: Participants in the process include mathematics student teachers, mentors, school directors, and lecturers. 2) Domain: Establish supervisory goals in three stages: Phase 1 focuses on improving student teachers' abilities to respond to student ideas. Phase 2 focuses on improving student teachers' abilities to manage student ideas. Phase 3: Student teachers' ability to summarize student ideas to mathematical concepts. 3) Practice: It was found that there were three periods of supervision: before teaching, during student teaching, and after teaching. There are three phases of practice, consisting of: 3.1) Collaboratively plan that understanding the goals of the lesson, ways of managing learning activity, anticipating students' response to problem situations, or learning materials, the anticipation of the student's ideas, and defining issues for observing the class. 3.2) Collaboratively observe practices: Student teachers act as learning organizers. The observers consisted of mentor teachers, school directors, and lecturers. 3.3) Collaborative reflection: reflecting is a main tool for supervision according to three phases, consisting of the first phase of reflecting on the student teachers' ability to understand student ideas. The second phase focused on the student teachers' ability to manage the students' ideas. The third phase reflected on student teachers' ability to summarize and connect ideas to mathematical concepts.

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